3-D RECONSTRUCTION OF VELOCITY PROFILES IN RECTANGULAR MICROCHANNELS

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ABSTRACT

Particle Image Velocimetry (PIV) is a non-intrusive technique widely used nowadays to experimentally obtain the velocity field of a flow. During the last decade, it has been adapted to microscale and μ-PIV is now a useful tool for providing information about microflows. One of the main differences between μ-PIV and PIV is related to the depth of field which is negligible at macroscale but which could become non-negligible compared with microchannels dimensions and could lead to a poor experimental accuracy. In this paper, we propose a way to obtain an accurate 3-D velocity profile from a series of 2-D μ-PIV data, when the depth of field is not negligible compared with the microchannel depth. The method is analysed and validated from virtual experiments for benchmark flows. The improvement is very noticeable and the method also allows to accurately determine the position of the walls and the real depth of field.

1. INTRODUCTION

In the last twenty years, the methodology and correlation algorithms for macro-scale PIV have been developed [1, 2]. This was extended to the microscale by Santiago [3, 4] who used μ-PIV technique to study a Hele-Shaw flow cell around a 30 μm elliptical cylinder, with a bulk velocity of 50 μm/s. In that paper it was outlined how the spatial resolution and the accuracy of the measurement are limited by the diffraction of the optics, the noise due to out-of-focus particles, the Brownian motion and interactions between fluid and particles.

μ-PIV differs from classic PIV on three fundamental aspects [5]: (i) the size of the particles is small in regards with the light wavelength; (ii) the Brownian motion of the particles is more important and needs to be taken into account when the flow is slow and the particles diameter is sub-micrometric; (iii) the whole volume of the test section is illuminated, instead of a slice in the conventional technique. The depth of field is defined by the focusing characteristics of the optics and the size of the particles [6], which can be non-negligible compared to the depth of the channel. It is one of the main issues in μ-PIV which could imply a poor accuracy along the microchannel depth. Some solutions are proposed in the literature. Most of the authors use channels with a depth largely bigger than the width but consequently limit their study to 2-D flow and are not able to scan the velocity profile along the depth. To limit noise due to a great number of particles inside the illuminated slice, some authors reduce the particles concentration but this solution needs to increase the number of images and in this case new algorithms should be developed to process the data [7].

This paper details a method for processing experimental data when the depth of field is not small compared with the depth of the microchannel and when the evolution of the velocity along the depth is needed.

The method is firstly analyzed from virtual clean experiments, in order to separate issues due to data acquisition and data post-processing.

2. MICRO-PIV OPTICS: SPECIFICITIES AND LIMITATIONS

A μ-PIV system is essentially an adaptation of a macro-PIV system through a microscope. Volume illumination rather than a light sheet is employed, with the measurement depth dictated through the objective of the microscope. Fluorescent particles are used to avoid saturation of the detector array by flare from the micro-device surfaces — a consequence of volume illumination.

The emitted light illuminates a so-called “cone of illumination” volume; each particle in this cone emits light but essentially particles inside the in-focus slice emit enough light to be taken into account in the correlation function during post-processing (Figure 1). In practice, the depth of measurement is slightly different from the depth of field and it can be calculated in different ways. One expression is given by Inoué and Spring [8] and has been adapted by Meinhart et al. [6], in the following form:

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\[ \Delta y = \frac{3n_i \lambda_0}{NA^2 \tan \theta} + \frac{2.16d_p}{\tan \theta} + d_p. \]  

The first term of the right-hand-side of this equation characterises the effect of diffraction in the optical device. It is calculated from the characteristics of the optics: \( n_i \) is the refractive index of the immersion fluid between the lens and the illuminated section, \( \lambda_0 \) is the wavelength of the light, \( NA \) is the numerical aperture of the lens given by \( \sin \theta = NA/n_i \), \( \theta \) is the half-angle of the light cone. The second term is due to geometrical effects and take into account the high value of the particles diameter \( d_p \) compared to the \( e/M \) ratio, where \( e \) is the smallest distance that can be resolved by the detector (for a CCD camera it is the distance between two photosites) and \( M \) is the lens magnification.

In our experiments, an Olympus air-immersion lens \((n_i = 1)\) with a magnification of 20 and a numerical aperture of 0.6 and particles with a diameter of 1 \( \mu \)m (Molecular Probes) were used to study the flow through a long rectangular microchannel with a \( 44 \times 300 \mu \)m \(^2\) cross-section. According to equation (1), the depth of measurement was 26 \( \mu \)m. This value is important compared with the 44 \( \mu \)m microchannel depth. The flow was screened in the \( y \)-direction – as well as in the \((x,z)\) plane – by means of a micrometric lens board.

\[ \Delta y = \frac{3n_i \lambda_0}{NA^2 \tan \theta} + \frac{2.16d_p}{\tan \theta} + d_p. \]

**Figure 1**: Definition of the focal depth from [9]

For screening along the depth, it should be noted that when the lens of the microscope moves along distance \( \delta_{obj} \), the focal plane moves along a distance \( \delta \) equal to:

\[ \delta = \frac{n_f}{n_i} \delta_{obj}. \]

where \( n_f \) is the refractive index of the studied fluid, \( n_i \) is the refractive index of the immersion fluid of the lens and \( \delta_{obj} \) is the displacement of the microscope lens controlled by the micrometric board of the microscope (Figure 2).

If the lens is an oil-immersion lens and if the flow is a flow of water, a 1 \( \mu \)m lens displacement provokes a \( n_{water}/n_{oil} = 1.33/1.515 = 0.88\mu \)m displacement of the focal plane. If the lens is an air-immersion lens, the displacement of the focal plane in water is \( n_{water}/n_{air} = 1.33/1.0 = 1.33\mu \)m.

**Figure 2**: Light path and relationship between lens and focal plane displacements

### 3. METHOD FOR IMPROVING MEASUREMENT ACCURACY

#### 3.1 Procedure

As the flow is fully-developed in the \( z \)-direction (Fig 4.), the study of the velocity profile can be limited to the \((x,y)\) plane. The accuracy on the \( x \)-axis is only dependant on the field of view (which is fixed by the objective magnitude) and the seeding density (which will define the size of the interrogation area). Then, only the precision along the \( y \)-axis needs to be improved. Raw experimental data provide information about the velocity averaged in the \( y \)-direction on a layer whose thickness is the depth of measurement \( \Delta y \). This thickness is not small compared with the channel height.

The method consists in calculating a more local velocity averaged in a sub-layer of thickness \( \delta \) screening the depth of the channel with a screening step \( \delta < \Delta y \), and post-processing the raw experimental data as follows: considering the conservation of the mass flow rate, each experimental flow rate in a layer \( \Delta y \) in depth is the sum of the flow rate in the sub-layers \((\delta, \delta \delta, \delta \Delta y)^t \) in depth. Thus, for the \( i \)-layer, by subtracting the processed velocities of the previous layers from the raw experimental velocity in this layer, the velocity in the \( i \)-sub-layer, \( \delta \) in depth, can be determined. Step by step, the velocity profile is build along the channel height with layers \( \delta \) in depth. The screening step \( \delta \) is obtained using the micrometric board for translating the lens of the microscope with a step \( \delta_{obj} \) in the \( y \)-direction. Relationship between \( \delta \) and \( \delta_{obj} \) was given in equation 2.

Adding to the improvement of the accuracy, the method also permits to solve two problems: the determination of the position of the wall –characterised by the parameter...
Δy/Δy—and the real value of Δy. The method can be applied along the two scanning directions A and B (Figure 3). The processed velocity profiles obtained for each direction should be the same. Comparing them for different values of Δy and Δy/Δy, it is then possible to find the optimum value of these two parameters which minimise the deviation between the two velocity profiles.

3.2 Parameters

The main parameters required for the calculation appear in the following figure (Figure 3). Direction A and direction B indicate the two different scanning directions in which the calculation can be processed. 1A, 2A (1B, 2B) are the numbers representing the experimental layer in the scanning direction A (resp. B).

![Figure 3: Schematic representation of the method](image)

For a given experiment, the screening step δ the depth H of the channel and the number of layers n, (Figure 3) are supposed to be known. The other parameters can then be deduced, although an accurate evaluation of the depth of the first layer in direction A Δy/Δy, and Δy is not easy. A method for obtaining their most probable value is proposed further (section 3.4). Once these two parameters are fixed, the others, Δy/Δy and Δy/Δy, the depths of the last layers in direction A and B (residual depth?), and Δy/Δy, the depth of the first layer in direction B can be determined from the following equations:

\[ \Delta y_A + \Delta y_B + (n - 1)\delta = H \]  
\[ \Delta y_A = \Delta y_A - \delta' \]  
\[ \Delta y_A + \Delta y_B = \Delta y_B + \Delta y_B \]

where δ<δ verifies the equation Δy = kδ + δ' in which k is an integer.

The main hypothesis used is that the velocity measured during the experiment for the i-layer, \( \bar{V}_i \), is the mean velocity in the illuminated slice, Δy in depth. This can be written as follows:

\[ \bar{V}_i = \frac{1}{\Delta y_{\delta - \Delta y + i\delta}} \int_{\Delta y_{\delta - \Delta y + i\delta}} Vdy . \]  

(6)

In the same way, the post-processed velocity calculated for the i-sub-layer, \( \bar{V}_i' \), is the mean velocity in the grey region (light-grey when processing in direction A and charcoal-grey when processing in direction B) of the i-layer, whose thickness is δ (Figure 3). It is defined by:

\[ \bar{V}_i = \frac{1}{\delta} \int_{\Delta y_{\delta - \Delta y + i\delta}} Vdy . \]  

(7)

The flow rate conservation involves a relation between \( \bar{V}_i' \) and \( \bar{V}_i' \) as:

\[ \frac{1}{\Delta y_{\delta - \Delta y + i\delta}} \int_{\Delta y_{\delta - \Delta y + i\delta}} Vdy = \frac{1}{\Delta y_{\delta - \Delta y + i\delta}} \int_{\Delta y_{\delta - \Delta y + i\delta}} Vdy + \frac{1}{\Delta y_{\delta - \Delta y + i\delta}} \int_{\Delta y_{\delta - \Delta y + i\delta}} Vdy , \]  

(8)

which gives

\[ \bar{V} = \frac{1}{\Delta y_{\delta - \Delta y + i\delta}} \int_{\Delta y_{\delta - \Delta y + i\delta}} Vdy + \bar{V}_i' . \]  

(9)

where \( \frac{1}{\Delta y_{\delta - \Delta y + i\delta}} \int_{\Delta y_{\delta - \Delta y + i\delta}} Vdy \) only depends on the position of the studied layer in the channel. Four cases are encountered:

- For the first layer, it is assumed that Δy < δ. The measured velocity obtained by μ-PIV analysis is the mean velocity in the sub-layer of thickness Δy:

\[ \bar{V}_1 = \bar{V}_1 . \]  

(10)

- For layers which are in part in the channel, (e.g. layer 2 in Figure 3),

\[ \bar{V}_2 = \left[ (i-1)\delta + \Delta y \right] V_2 - \left[ (i-2)\delta + \Delta y \right] V_{i-1} . \]  

(11)

- For layers totally inside the channel, i.e. when the illuminated plane is entirely in the channel,

\[ \bar{V}_r = \Delta y \bar{V}_r - \sum_{j=1}^{k} \delta \bar{V}_{i} - \delta' \bar{V}_r , \]  

(12)

where \( \bar{V}_r \) is the mean velocity of the thin sub-layer, δ’ in depth. The velocity \( \bar{V}_d \) is obtained by linear interpolation between the mean velocities in the two consecutive sub-layers, δ in depth, which surround the thin sub-layer, δ’ in depth.

- For the n-layers, a specific processing is also required because the thickness of the sub-layer in which the velocity
must be calculated is only $\Delta y_2$. The velocity is expressed as follows:

$$
\Delta y_2 \bar{V} = ((k-1)\delta + \delta + \Delta y_2) \bar{V} - \delta \sum_{i=1}^{k-1} \bar{V}^i - \delta \bar{V}^0.
$$

(13)

4. VALIDATION OF THE METHOD ON A VIRTUAL EXPERIMENT

The method is first tested on virtual clean experiments, in order to separate possible image acquisition issues from data processing issues. Two benchmark flows were tested. The first one is a laminar pressure driven flow in a rectangular channel (Figure 5a). The analytical velocity is calculated from the Navier-Stokes equations and can be expressed as follows [10, 11] using the notations mentioned in Figure 4:

$$
V(x, y) = \frac{v(x, y)}{V} = \sum_{n=1, odd}^{\infty} \sum_{m=1, odd}^{\infty} v_{n,m} \sin(n\pi x) \sin \left( \frac{m\pi y}{\beta} \right),
$$

(14)

where

$$
v_{n,m} = \frac{\pi^2}{4mn(\beta^2 n^2 + m^2)} \sum_{i=1, odd}^{\infty} \sum_{j=1, odd}^{\infty} \frac{1}{i^2 j^2 (\beta^2 i^2 + j^2)}
$$

and $v$ is the local fluid velocity, $\beta = H/W$ is the aspect ratio of the channel, $\bar{V}$ the mean velocity in the cross section.

![Figure 4: Schematic of channel cross-section and notations](image)

To extend the analysis to more complex configurations, the second studied flow has an asymmetric profile (Figure 5b) built with the following expression:

$$
V_d(x, y) = \frac{V(x, y)}{HW} \left( y + \frac{H}{2} \right) \left( x + \frac{W}{2} \right).
$$

(15)

At a $(x,z)$ location, we assume that the raw measured velocity (i.e. the velocity which would be directly obtained from a real $\mu$-PIV experiment) corresponds to the mean velocity inside a layer, $\Delta y$ in depth. After data processing, we obtain the mean value of the velocity inside a sub-layer, $\delta$ in depth. Values of the parameters used for this simulation are summarized in Table 1, they correspond to the real experimental data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y$</td>
<td>26 $\mu$m</td>
</tr>
<tr>
<td>$\Delta y_1$</td>
<td>1.5 $\mu$m</td>
</tr>
<tr>
<td>$\delta$</td>
<td>6 $\mu$m</td>
</tr>
<tr>
<td>$H$</td>
<td>44 $\mu$m</td>
</tr>
</tbody>
</table>

Table 1: Values of the parameters used for virtual testing

![Figure 5: Tested velocity profiles](image)

3. EXPERIMENTAL

The processed velocity profile is plotted in Figure 6 for the symmetric case and in Figure 7 for the asymmetric case and compared to the raw velocity. The accuracy is widely improved: the processed velocity is in very good agreement with the real velocity for a screening step $\delta = 6\mu$m. The good results obtained with the asymmetric profile prove that the method can be extended to more complex velocity profiles.

Note that the data processing has been completed in both A and B directions as outlined in Figure 3.
Let us consider a specific location \((x,z)\). For each processing direction A and B and for each value of \(\Delta y\), a discrete velocity profile along the height of the channel is obtained using the method described in the previous section and considering first that \(\Delta y\) is accurately known. Then, a function \(F\), sum of the square differences of the processed velocities is calculated:

\[
F = \sum (V_{ym} - \bar{V}_{ym})^2
\]  

(16)

\(F\) is function of \(\Delta y\) and its value is minimum for the real experimental value of \(\Delta y\).

As an example, the results presented above were obtained from a virtual experiment for which \(\Delta y = 1.5 \mu m\). The algorithm was applied for \(\Delta y\) between 0 and 2 \mu m. Figure 8 shows clearly the minimum of the function \(F\) is found for \(\Delta y = 1.5 \mu m\).

3.4 Determination of parameters \(\Delta y\) and \(\Delta y\)

In the previous section, we have supposed that the depth of the first layer \(\Delta y\) and the depth of measurement \(\Delta y\) were known. But in practice, it is really difficult to detect exactly the position of the microchannel wall, which means that \(\Delta y\) is not well known. Moreover, as it has been shown by [6], equation 1 does not accurately determine the value of the depth of measurement.

If \(H\) and \(\delta\) are known, \(n_c\) is deduced as \(n_c = \text{integer}(H/\delta)\) and the following geometric relation \(\Delta y_1^A + \Delta y_2^B = H - n_c \delta\) leads to \(0 \leq \Delta y \leq H - n_c \delta\).

For example in our experimental case, \(H = 44 \mu m\) and \(\delta = 6 \mu m\), consequently \(n_c = 7\) and \(0 \leq \Delta y \leq 2 \mu m\). Our goal is to find the real value of \(\Delta y\) in this range.

Once the real value of \(\Delta y\) is found, the same approach is used to find the real value of the depth of measurement \(\Delta y\), in the vicinity of its supposed value provided by equation 1. The function \(F\) is calculated for trial values of \(\Delta y\) and a minimum is found (Figure 9) which corresponds closely to the real value of our virtual experiment.

![Figure 8: Minimum value of \(F\) as a function of \(\Delta y\)](image)

![Figure 9: Minimum value of \(F\) as a function of \(\Delta y\)](image)
To complete the study, the function $F$ is directly plotted as a function of the two independent parameters $\Delta y_1$ and $\Delta y$. The minima for $\Delta y_1$ and $\Delta y$ are found (Figure 10), which allows to conclude that even if during an experiment two parameters remain imprecisely known it is possible to find their optimum values using the method described above.

![Figure 10: $F$ variation in function of $\Delta y_1$ and $\Delta y$](image)

5. CONCLUSION AND PERSPECTIVES

A method for improving the accuracy of $\mu$-PIV measurements has been developed. It has been tested with different virtual experiments built on analytical expressions of velocities in a rectangular channel 44 $\mu$m in depth and 300 $\mu$m in width. The main advantages of this method are:

- accuracy is improved: in the above example the spatial resolution along the depth is improved from 26 $\mu$m to 6 $\mu$m leading to an accurate representation of the real velocity;
- the position of the walls of the channel can be accurately determined, which is very difficult during the experiments;
- the real depth of measurement $\Delta y$, which is another key parameter for using the experimental data, can also be precisely calculated;
- real 3-D velocity cartographies can be obtained from a series of 2-D $\mu$-PIV measurements.

The method is directly applicable for smooth raw velocity profiles along the y-direction. However, if the raw profile has a high noise content, or if one datum is abnormal, local errors from the raw profile could be spread to a series the post-processed velocities, because one processed velocity depends on several raw velocity data. In that case, the raw data require a specific smoothing before processing.

Our current work consists in determining the best way to perform such a smoothing. In parallel, we are studying the effects of the particles concentration on the measurements accuracy. With the same objective, some authors recently tried to improve the $\mu$-PIV accuracy by controlling the seeding injection in a selected part of the microchannel [12, 13]. This selective seeding technique could very likely be coupled with our method to provide more accurate results.

Our first tests dealt with microchannels 44 $\mu$m in depth and with a 26 $\mu$m depth of measurement. With a more higher-performance optical equipment, the same technique could be used to reach high precision measurements in much smaller microchannels.

6. REFERENCES

7. NOMENCLATURE

\[ d_p \] Particle diameter (m)

\[ e \] Smallest distance that can be resolved by the detector

\[ H \] Depth of the channel (m)

\[ M \] Lens magnitude

\[ NA \] Numerical aperture of the lens given by
\[ \sin \theta = NA / n_i \]

\[ n_e \] Number of layers

\[ n_i \] Refractive index of the immersion fluid between the lens and the illuminated section

\[ n_f \] Refractive index of the studied fluid

\[ v_i^t \] Mean velocity of the thin sub-layer, \( \delta \) in depth (m/s)

\[ v_i^p \] Post-processed velocity for the \( i \)-sub-layer (m/s)

\[ v_i^r \] Raw velocity measured during the experiment for the \( i \)-layer (m/s)

\[ \delta \] Screening step (m)

\[ \Delta y \] Depth of measurement (m)

\[ \Delta y_1^A (\Delta y_1^B) \] Depth of the first layer in direction A (resp. B) (m)

\[ \Delta y_2^A (\Delta y_2^B) \] Depth of the last layers in direction A (resp. B) (m)

\[ \lambda_0 \] Wavelength of the light (m)

\[ \theta \] Half-angle of the light cone