Phase engineering with all-dielectric metasurfaces for focused-optical-vortex (FOV) beams with high cross-polarization efficiency

HAMMAD AHMED,1 ARBAB ABDUR RAHIM,1,* HUSNUL MAAB,1 MUHAMMAD MAHMOOD ALI,2 NASIR MAHMOOD,3 AND SADIA NAUREEN4

1Faculty of Electrical Engineering, Ghulam Ishaq Khan Institute of Engineering Sciences and Technology, Topi 23640, District Swabi, Khyber Pakhtunkhwa, Pakistan
2Optical Fibre Sensors Research Centre, Department of Electronics and Computer Engineering, University of Limerick, Limerick, V94 T9PX, Ireland
3Department of Electrical Engineering, The University of Lahore, J-KM Defense Road, 54590, Lahore, Pakistan
4Department of Electronic Engineering, The Islamia University of Bahawalpur, 63100, Bahawalpur, Pakistan
*arbab@giki.edu.pk

Abstract: Metasurfaces, the two-dimensional (2D) metamaterials, facilitate the implementation of abrupt phase discontinuities using an array of ultrathin and subwavelength features. These metasurfaces are considered as one of the propitious candidates for realization and development of miniaturized, surface-confined, and flat optical devices. This is because of their unprecedented capabilities to engineer the wavefronts of electromagnetic waves in reflection or transmission mode. The transmission-type metasurfaces are indispensable as the majority of optical devices operate in transmission mode. Along with other innovative applications, previous research has shown that Optical-Vortex (OV) generators based on transmission-type plasmonic metasurfaces overcome the limitations imposed by conventional OV generators. However, significant ohmic losses and the strong dispersion hampered the performance and their integration with state-of-the-art technologies. Therefore, a high contrast all-dielectric metasurface provides a compact and versatile platform to realize the OV generation. The design of this type of metasurfaces relies on the concept of Pancharatnam-Berry (PB) phase aiming to achieve a complete $2\pi$ phase control of a spin-inverted transmitted wave. Here, in this paper, we present an ultrathin, highly efficient, all-dielectric metasurface comprising nano-structured silicon on a quartz substrate. With the help of a parameter-sweep optimization, a nanoscale spatial resolution is achieved with a cross-polarized transmission efficiency as high as 95.6% at an operational wavelength of 1.55 $\mu$m. Significantly high cross-polarized transmission efficiency has been achieved due to the excitation of electric quadrupole resonances with a very high magnitude. The highly efficient control over the phase has enabled a riveting optical phenomenon. Specifically, the phase profiles of two distinct optical devices, a lens and Spiral-Phase-Plate (SPP), can be merged together, thus producing a highly Focused-Optical-Vortex (FOV) with a maximum focusing efficiency of 75.3%.

© 2020 Optical Society of America under the terms of the OSA Open Access Publishing Agreement

1. Introduction

Tailoring the wavefronts of electromagnetic waves has always been an enticing topic in the field of optics and electrodynamics. The electromagnetic waves can be characterized with the help of the phase, polarization, and amplitude. The conventional optical systems deal with three-dimensional (3D) bulky optical elements such as prisms, lenses, beam splitters, and waveplates, etc. These
are used to manipulate the attributes of incident light waves based on the concepts of geometrical optics (reflection, diffraction, and refraction). However, such type of bulky optical components makes the overall system larger in size and present meagre performance at the subwavelength scale due to the diffraction limit. Therefore, their integration with rapidly growing photonic integrated circuits (PICs) is very difficult. With recent developments in the field of nano-optics and nano-photonics, 3D optical metamaterials have opened a new esplanade for designing the optical systems with distinct functionalities. These artificially engineered devices possess the ability to control the attributes of an incident electromagnetic wave through their tailored permittivity and permeability. With the inspiration of such control, it has led to a variety of practical applications. Some of them are invisibility cloaking [1], zero and negative index material [2,3], and sub-diffraction imaging [4]. However, along with the novel and fascinating phenomena provided by 3D metamaterials, strong dispersion [5], difficulty in fabrication at optical frequencies [6,7] and on-chip application limitations [8], provide significant challenges towards their integration for useful devices. To circumvent the limitations imposed by the 3D bulky optical metamaterials, the advent of two-dimensional (2D) metasurfaces has provided a planar substitute for the realization of miniaturized optical devices and systems. With the help of sub-wavelength meta-atoms, metasurfaces can manipulate the phase, amplitude, and polarisation of incident waves at the interface between two mediums. In recent years, metasurfaces have opened up the boulevard for many enthralling applications such as flat-lens [9–11], meta-axicon [12,13], beam shaping [14–16], holography [17], self-accelerating beams [18,19], optical vortex [8] and non-diffracting Bessel beams [20,21].

The Optical-Vortex (OV) generation is one of the important application of metasurfaces, which has aroused remarkable curiosity, since its discovery in 1992 [22]. The OV has a number of applications in the field of particle manipulation [23] and optical communication [24]. It has been mentioned elsewhere that a number of various approaches have been adopted to generate OVs. In an approach, the tunnig of optical retardation of Q-plates has been demonstrated either with the temperature/electric field to realize the efficient generation of OVs [25,26]. The concept of a spiral zone plate and forked grating have been combined through exclusive OR (XOR) logic to generate highly focused OVs for orbital angular momentum superposition applications [27]. Besides, the spiral phase has been imprinted on the Gaussian beam, using the computer-generated hologram, to produce OV for trapping of micron-scale dielectric particles [28,29]. Furthermore, the spiral phase plate based on the blazed fractal zone plate has been employed to produce the focused OV, for optical trapping of high indexed particles [30]. For quantum computing, micromanipulation, and optical communication applications, optically reconfigurable and electrically tunable liquid crystal fork gratings have been demonstrated to produce reconfigurable and switchable OVs [31].

It is worth mentioned here that the integration conventional of OV generators with PICs is not possible owing to its bulky nature and hard to downsize. To overcome these restrictions, recently, transmission-type metasurfaces comprised of plasmonic meta-atoms have been demonstrated for the OV generation [32–34]. The temperature instability and significant Ohmic losses, caused by inevitable electron-phonon and electron-electron scattering, in plasmonic metasurfaces made it unsuitable choice. It is due to the fact that because of these losses it reduced the overall efficiency of optical wavelengths in transmission mode [35]. Although, this issue was partially tackled by reflection-type metasurfaces [36]. However, such metasurfaces are still incompatible with the current complementary-metal-oxide-semiconductor (CMOS) technology. Thus, restricting the miniaturization of practical optical devices operating in the transmission mode. Hence, there is a dire need of metasurfaces, which manifest better performance in the transmission mode than plasmonic metasurfaces.

In this regard, high-index dielectric materials possessing the transparent window ($n_{img} \approx 0$; where $n_{img}$ represents the imaginary part of the refractive index also known as the extinction coefficient [37]) for the wavelengths of interest, can be considered as the best choice. An
additional feature of these all-dielectric metasurfaces is the exhibition of high transmission efficiency due to the excitation of electric and magnetic resonances at optical wavelengths [38]. Moreover, due to the low loss characteristics, these metasurfaces can be considered better even in reflection mode as well. Recently, all-dielectric OV generating reflection-type metasurfaces have been presented with the efficiency significantly better than the plasmonic OV generators [36,39]. The realization of such highly efficient all-dielectric metasurfaces is completely dependent on the complex refractive index \((n_{\text{real}} + jn_{\text{img}})\) of the concerned dielectric material. For a particular spectrum of interest, the complex refractive index should have a sufficiently larger real part \((n_{\text{real}} > 2)\) with a low extinction coefficient \((n_{\text{img}} \approx 0)\) [40]. The sufficiently large real part of the refractive index plays a vital role in achieving complete \((2\pi)\) phase distribution due to the strong confinement of an incident wave, while a low extinction coefficient ensures a high transmission efficiency.

Keeping in view the aforementioned discussion, in this paper, a highly efficient transmission-type metasurface for Focused-Optical-Vortex (FOV) generation is proposed. In the proposed device, silicon material (Palik) is used which has a negligible loss at telecommunication wavelengths. The main advantages of silicon over other reported materials are its simple fabrication, cost-effectiveness, and CMOS compatibility [41]. The concept of phase and polarization engineering is utilized to realize the FOV generation. The manipulation of phase and polarization is accomplished through the rotation of a silicon nanobar. The rotation angle of a silicon nanobar is swept to attain the complete \((2\pi)\) phase modulation of the spin-inverted transmitted wave through the Pancharatnam-Berry (PB) phase (also known as geometric phase) mechanism [42]. PB phase provides an effective way of controlling the spin of incident circularly-polarized (CP) wave by adjusting the orientation angle of nanobar instead of changing its geometric parameters [37,43]. Furthermore, due to the excitation of quadrupole electric resonances and low absorption, the silicon nanobar act as a lossless half-wave plate at 1.55 \(\mu\)m (infrared domain). In comparison with recently reported work [8,33,44–47], the proposed silicon-based FOV generator seems promising in transmission mode due to the excitation of quadrupole electric resonances. Moreover, the proposed FOV generator operates on 1.55 \(\mu\)m which is an important optical communication wavelength. Due to higher data transmission capacity, the OV generated by this device is very useful in fiber optics for data transmission [24], encoding, and decoding of data [48], OAM multiplexing and de-multiplexing of data [24]. It will also establish the platform for future practical applications of the structured beams in the relevant research fields such as particle trapping [23], and microscopy [49].

2. Theory

Depending upon the geometry, a high refractive indexed meta-atom can provide a peremptory effect on the localized phase, amplitude, and polarization of an incident wave. The schematic diagram of the proposed meta-atom is shown in Fig. 1(a). A silicon nanobar is patterned on a quartz substrate in order to obtain the desired phase with maximum possible transmission efficiency. Silicon nanobar is numerically optimized in a way to act as a miniaturized half-wave plate, which reverses the spin of an incident wave and manipulates the phase of a spin-inverted wave as a function of the orientation angle \(\theta\). The PB phase mechanism is performed to acquire the complete \((2\pi)\) phase coverage. The definition of PB phase states that instead of varying geometric parameters, silicon nanobars are rotated from 0° to 180° to achieve 2\(\pi\) phase coverage upon CP incidence. By virtue of Jones calculus (detailed derivation is provided in Appendix A), the Jones matrix \(J\) for silicon nanobar oriented at an angle \(\theta\) can be written as [50,51]:

\[
J(\theta) = R(\theta)^\ast J_T R(\theta)
\]
\[
\begin{bmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
t_a & 0 \\
0 & t_b
\end{bmatrix}
\begin{bmatrix}
\cos(\theta) & \sin(\theta) \\
-\sin(\theta) & \cos(\theta)
\end{bmatrix}
\]
\[
\begin{bmatrix}
\cos(\theta) & \sin(\theta) \\
-\sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
t_a & 0 \\
0 & t_b
\end{bmatrix}
\begin{bmatrix}
t_a\cos^2(\theta) + t_b\sin^2(\theta) & (t_a - t_b)\cos(\theta)\sin(\theta) \\
(t_a - t_b)\cos(\theta)\sin(\theta) & t_b\sin^2(\theta) + t_a\cos^2(\theta)
\end{bmatrix}
\] (2)
\[
\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}, \text{[see Eq. (26) in Appendix A]}.
\]
\[
E_t = \frac{t_a + t_b}{2} E_r + \frac{t_a - t_b}{2} e^{-j\theta} E_l, \quad (4)
\]

where \( R(\theta) \) is the rotation matrix, \( J_T \) is Jones matrix for transmission with rotation effects omitted. The total transmitted electric field \( E_t \) from silicon nanobar for the right-handed-circular-polarized (RCP) incident wave can be extracted by multiplying Eq. (3) with Jones’ vector of RCP wave which is given by \( \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix} \), [see Eq. (26) in Appendix A]. Therefore, the \( E_t \) can be expressed as [38]:

![Fig. 1. Meta-atom optimization. (a) Schematic of silicon nanobar on quartz substrate; length (\( L = 445 \) nm), width (\( W = 190 \) nm), height (\( H = 920 \) nm), and periodicity (\( P = 620 \) nm) are geometric parameters obtained through the parameter-sweep optimization at 1.55 \( \mu \)m. (b) The comparison between theoretically predicted (red line) and numerically plotted (black asteriks) phase profiles. (c) and (d) Show the simulated transmission (T) (purple line), co-polarized (red line), and cross-polarized (green line) efficiencies versus wavelength and versus rotation angle (\( \theta \)) of silicon nanobar at 1.55 \( \mu \)m, respectively. The optimized silicon nanobar is investigated for a broad wavelength range. The analysis shows a very high average cross-polarized efficiency of 87.5% with the maximum value of 95.6% at 1.55 \( \mu \)m. Moreover, the amplitude of cross-polarized and transmission efficiencies remained uniform as silicon nanobar is rotated from \([0^\circ - 180^\circ]\).]
here, \( t_a \) and \( t_b \) are the complex transmission coefficients when the polarization of an incident wave is parallel to the slow axis and fast axis of silicon nanobar, respectively. For RCP incidence, the transmitted electric field \( (E_t) \) comprised of two terms; co-polarized term with exactly the same phase and handedness as of the incident electric field and the cross-polarized term which possesses an extra phase of \( 2\theta \) with opposite handedness (i.e. left-handed-circular-polarized (LCP)). An angle \( \theta \) denotes the rotation angle of silicon nanobar between the \( x \)-axis and the slow axis (as shown in Fig. 6 under Appendix B). Therefore, the phase of an outgoing LCP wave can be engineered from 0 to \( 2\pi \) as nanobar is rotated from 0° to 180°.

3. Materials and methods

To verify the linear dependence of the geometric phase with the rotation angle, the numerical simulation of silicon nanobar is performed using the commercially available Finite-Difference-Time-Domain (FDTD) solver from Lumerical Inc. [52]. The size of a mesh cell is kept at 0.035 \( \mu \)m which is smaller than \( \lambda / 10 \) (where \( \lambda = 1.55 \mu \)m). The convergence is ensured by changing the mesh cell size to 0.025 \( \mu \)m and it is made sure that the results are identical [53,54]. Moreover, the Perfectly-Matched-Layer (PML) boundaries are considered in \( z \)-direction while the periodic boundary conditions are applied along \( x \)- and \( y \)-direction. Figure 1(b), shows the comparison between the theoretically predicted phase using Eq. (1) and the numerically simulated phase, where the observation monitor was placed in the far-field at a distance of 5 \( \mu \)m away from the top surface of a silicon nanobar. The simulated results show a good agreement with theoretical predictions. Therefore, it is concluded that the phase of the cross-polarized component can be controlled by controlling the rotation angle of a silicon nanobar.

Furthermore, it is necessitated that the variation of the rotation angle should interrupt only the phase of the cross-polarized component to maintain the high transmission efficiency. This could be accomplished by carefully optimizing the geometric parameters of silicon nanobars. Therefore, the parameter-sweep optimization [17,55,56] is performed at 1.55 \( \mu \)m for cross-polarized and co-polarized efficiencies (shown in Fig. 7 under Appendix B). The length and width of a silicon nanobar are swept from 150 nm to 450 nm with an optimal periodicity \( (P = 620 \text{ nm}) \) and height \( (H = 920 \text{ nm}) \). In Fig. 7(a), the region of interest is represented by white dashed lines which indicates the maximum cross-polarized efficiency, which is the ratio of the power of the cross-polarized component of exiting wave to the total power of incident wave [57]. It is clearly evident from Fig. 7 that the choice of length \( (L) \) and width \( (W) \)–denoted by a white asterisk–is made entirely on the basis of maximum cross-polarized (Fig. 7(a)) and minimum co-polarized (Fig. 7(b)) efficiencies. The optimization reveals that the maximum cross-polarized efficiency of 95.6\% is achieved at \( L = 445 \text{ nm} \) and \( W = 190 \text{ nm} \). As illustrated in Fig. 1(c), the broadband analysis of an optimized nanobar shows an average cross-polarized efficiency of 87.5\% in the range of 1.5 \( \mu \)m – 1.7 \( \mu \)m. In Fig. 1(d), the high transmission (T) and cross-polarized efficiency remained uniform as the silicon nanobar is rotated from 0° to 180°. The high transmission efficiency is achieved due to the presence of resonance modes inside the silicon nanobar. For better understanding, the vector field orientations and cross-sectional intensities of electric and magnetic fields for a 920 nm thick nanobar are plotted in Figs. 2(a) – 2(d) at 1.55 \( \mu \)m. It can be seen that the two electric resonance modes with different quadrupole mode intensities (as depicted in Figs. 2(b) and 2(c)) are highly confined into the silicon nanobar for \( x \)- and \( y \)-polarized incident waves. Comparing with recently reported work [35,57], it is evident that an optimized silicon nanobar produces large magnitude resonance modes because of its negligible absorption \( (n_{\text{img}}) \) at telecommunication wavelengths. Furthermore, complete \( 2\pi \) phase control is achieved using the PB phase mechanism due to the strong confinement of an incident wave inside a silicon nanobar [40]. This can be observed in Fig. 8 under Appendix C that most of the incident wave is concentrated inside the nanobar and has the negligible coupling with neighboring nanobars.
Thus, each nanobar can act as an independent nano waveguide which imparts different phase with various rotation angle and shape the transmitted wave to a desired form [58,59].

![Fig. 2](image_url) Excitation of resonance modes in a silicon nanobar. (a) and (c) Show the cross-section intensities and vector field profiles of electric fields under $x$- and $y$-polarized wave incidence, respectively. (b) and (d) Show the cross-section intensities and vector field profiles of magnetic fields under $x$ and $y$ polarized wave incidence, respectively. These results are plotted at 1.55 $\mu$m where maximum cross-polarized efficiency is achieved.

4. Results and discussion

An interesting phenomenon of FOV is realized here to prove the concept of phase merger via polarisation and phase manipulation. For illustration purposes, a picturesque view of the FOV phenomenon is presented in Fig. 3(a). Here, the metasurface is encoded with the combined phase profile of an SPP ($\phi_{SPP}$) and a lens ($\phi_{lens}$). Therefore, the expression for phase profile can be written as follow

$$\phi_T(x, y) = \phi_{SPP}(x, y) + \phi_{lens}(x, y), \quad (5)$$

where

$$\phi_{SPP}(x, y) = l\tan^{-1}(\frac{x}{y}), \quad (6)$$

and

$$\phi_{lens}(x, y) = -k\sqrt{x^2 + y^2 + f^2} - f. \quad (7)$$
Here, \( k = \frac{2\pi}{\lambda} \) is the wavenumber, \( f \) is the focal length, \( l \) is the topological charge defining the degree of helicity at the focal point while \( x \) and \( y \) are the transverse coordinates representing the location of each silicon nanobar. Equation (5) provides the required spatial distribution of phase which is then realized by adjusting the rotation angle of well-optimized silicon nanobars placed at each coordinate point. The rotation angles are acquired from Fig. 1(b). Hence, four metasurfaces are designed with topological charges \( l = \pm 2 \) and \( l = \pm 4 \), and the focal length \( f = 9 \) \( \mu \)m. The size of all devices is kept \((18.6 \times 18.6) \mu m^2\) in order to achieve the numerical aperture (NA) of 0.70. The calculation for NA is demonstrated through Fig. 3(b). Here, \( S \) is the total size of a metasurface along y-dimension, and \( f \) is the focal length. By considering the \( \Delta OCA \) as a right triangle, the hypotenuse \( h = 12.9 \) \( \mu \)m is obtained using the Pythagoras theorem. The \( NA = 0.72 \) is then calculated using the formula \( NA = \sin \phi \). Figures 4(a), 4(f) and Figs. 5(a),

Fig. 3. (a) Schematic of a transmission-type metasurface capable of generating the FOV under the RCP incidence. The donut-shaped ring is observed at the focal plane for the cross-polarized component (LCP in this case) with the desired topological charge. (b) Demonstration of numerical aperture calculation for the design of the FOV generator.
5(d) depict the helical distribution of silicon nanobars for topological charges \((l = 2\) and \(4)\) and \((l = -2\) and \(-4))\), respectively.

Fig. 4. Numerical simulation results. (a) and (f) Helical distribution of silicon nanobar for \(l = 2\) and 4, respectively. (b) and (g) Electric field intensity profile in \(xz\)-plane for (b) \(l = 2\) and (g) \(l = 4\). The white dashed line shows the focal plane. The donut-shaped vortex rings are imaged at the focal plane (\(z = 8.6\) µm) along with their phase pattern for (c) and (d) \(l = 2\) and (h) and (i) \(l = 4\) in \(xy\)-plane. The black dashed circles in (c) and (h) indicates the annular opening of donut-shaped vortex rings with diameters 1.2 µm and 2.1 µm for \(l = 2\) and \(l = 4\), respectively. (e) and (j) Show the corresponding horizontal cuts of donut-shaped rings having the FWHM at 0.9 µm and 1.1 µm, respectively.

Full-wave numerical simulations of all designed metasurfaces are performed through the FDTD solver. Open boundary conditions are applied in all three \((x, y,\) and \(z))\) dimensions. Simulation results of designed metasurfaces are shown in Figs. 4 and 5 with RCP illumination. Figures 4(b) and 4(g) show the cross-sectional intensity profiles in \(xz\)-plane for \(l = 2\) and 4, respectively. The focusing of FOV is represented by the white dashed line at \(z = 8.5\) µm. The difference between analytically calculated and simulated focal length is due to the discrete phase distribution of silicon nanobars. The donut-shaped vortex ring as demonstrated in Figs. 4(c) and 4(h) (for \(l = 2\) and \(4))\) are observed at the focal plane with minimum intensity at the center. In Figs. 4(d) and 4(i), the corresponding phase patterns consist of 2 and 4 spirals represent the topological charges. From the electric field intensity profile shown in Figs. 4(c) and 4(h), it can be seen that
5. Conclusion

In this article, an ultra-thin, highly efficient all-dielectric metasurface is proposed that demonstrate FOV generation. The all-dielectric metasurface comprises nano-structured silicon on a glass substrate. The periodic arrangement of silicon-based rectangular nanobars has enabled the proposed metasurface to manipulate the phase of spin-inverted transmitted waves with a high cross-polarized efficiency of 95.6%. This high efficiency is achieved due to the existence of electric quadrupole resonances of high magnitude. Based on this concept, FOV with distinct topological charges ($l = \pm 2$ and $\pm 4$) for $f = 9$ $\mu$m is generated. The simulated results revealed the focusing of OV with a slight deviation from analytical calculation at $f = 8.6$ $\mu$m, owing to discrete phase distribution. The designed metasurfaces focus with the maximum efficiency...
of 75.3%. The creative approach of generating FOV at various values of topological charges using low-loss dielectric would find potential applications in providing CMOS compatible and cost-effective on-chip realization in the field of integrated optics.

Appendix A: Derivation of the Jones matrix

The time-harmonic electromagnetic wave propagation in $z$-direction can be expressed as:

$$E(z, t) = E_o e^{i(kz - \omega t)},$$

(8)

where $E_o$ is given by

$$E_o = E_x \hat{x} + E_y \hat{y}$$

(9)

Let us assume $E_y = -j E_x$, then by taking the real part of Eq. (9), we get

$$\text{Re}[E(z, t)] = \text{Re}\left\{E_x e^{i(kz - \omega t)} \hat{x} + Re\left\{e^{i\pi/2} E_x e^{i(kz - \omega t)}\right\} \hat{y}\right\}$$

(10)

$$E_x \cos(kz - \omega t) \hat{x} - E_x \cos\left(kz - \omega t + \frac{\pi}{2}\right) \hat{y}$$

(11)

Equation (12) represents the electric field expression for the RCP wave which is incident on silicon nanobar. The amplitude of E-field is constant and rotating in a circular pattern in $xy$-plane. In order to introduce the Jones vector form of the RCP wave, we combine Eqs. (8) and (9):

$$E(z, t) = (E_x \hat{x} + E_y \hat{y}) e^{i(kz - \omega t)}$$

(13)

For convenience, we compute the effective strength of E-field ($E_{\text{eff}}$). This field is neither $E_x$ nor $E_y$, but it is the effective strength of the linearly polarized wave that would render the same intensity that Eq. (13) would give. We can write Eq. (13) in terms of $E_{\text{eff}}$ as:

$$E(z, t) = E_{\text{eff}} (U \hat{x} + V e^{i\phi} \hat{y}) e^{i(kz - \omega t)}$$

(14)

Using $E_x$ and $E_y$, the parameters $E_{\text{eff}}$, $U$, $V$, and $\Psi$ can be calculated. The general form of the Jones vector can be presented as [62]:

$$\text{Jones Vector} = \begin{bmatrix} U \\ V e^{i\phi} \end{bmatrix}$$

(15)

For RCP wave, Eq. (15) can be expressed as

$$\text{Jones Vector} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix}$$

(16)

Using Eq. (16), the total E-field equation transmitted from silicon nanobar can be calculated. The E-field components are transformed from $\hat{x}$, $\hat{y}$ to $\hat{a}_1$, $\hat{a}_2$ [63] respectively, in order to find the general solution for silicon nanobar positioned at an arbitrary angle. An incident E-field with new basis before transmission can be expressed as:

$$E_{\text{in}}(z, t) = E_1 \hat{a}_1 + E_2 \hat{a}_2$$

(17)

Where

$$E_1 = E_x \cos(\theta) + E_y \sin(\theta)$$

(18)

and

$$E_2 = -E_y \sin(\theta) + E_x \cos(\theta)$$

(19)

Now we are in a position to introduce the effect of silicon nanobar on total transmitted E-field ($E_t$) by a parameter '$\Upsilon$', where '$\Upsilon$' manifests the effect of silicon nanobar on incident E-field.
This parameter only affects the $E_2$ component of incident E-field due to a newly created basis. Consequently, the E-field expression after nanobar will become:

$$
E_t(z, t) = E_1 \hat{a}_1 + \Upsilon E_2 \hat{a}_2
$$

(20)

It would be nice to rewrite Eq. (20) in the form of original $x - y$ coordinates. $E_t$ will become:

$$
E_t(z, t) = (E_x \cos(\theta) + E_y \sin(\theta))(\cos(\theta)\hat{x} + \sin(\theta)\hat{y}) + \Upsilon (-E_y \sin(\theta) + E_x \cos(\theta))(-\sin(\theta)\hat{x} + \cos(\theta)\hat{y})
$$

(21)

$$
E_t(z, t) = \begin{bmatrix}
\cos^2(\theta) + \Upsilon \sin^2(\theta) & \sin(\theta) \cos(\theta) - \Upsilon \sin(\theta) \cos(\theta) \\
\sin(\theta) \cos(\theta) - \Upsilon \sin(\theta) \cos(\theta) & \Upsilon \cos^2(\theta) + \sin^2(\theta)
\end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}
$$

(22)

Here, the matrix is known as the Jones matrix, used for operating the Jones vector. The parameter $\Upsilon = -1$ for the half-wave plate with the rotation angle $\theta$ between the $x$-axis and slow axis of silicon nanobar as illustrated in Fig. 6. The Jones matrix $J(\theta)$ for the half-wave plate from Eq. (23) can be deduced to [64]:

$$
J_m(\theta) = \begin{bmatrix}
\cos 2(\theta) & \sin 2(\theta) \\
\sin 2(\theta) & \cos 2(\theta)
\end{bmatrix}
$$

(24)

Similarly, using Eq. (18) and (19), we can express rotation matrix through the coordinate rotation.

$$
R(\theta) = \begin{bmatrix}
\cos(\theta) & \sin(\theta) \\
-\sin(\theta) & \cos(\theta)
\end{bmatrix}
$$

(25)

Hence, we can define Jones matrix $J(\theta)$ of silicon nanobar with arbitrary $\theta$ as [50,51]:

$$
J(\theta) = R(\theta)^{\top}J_{T}R(\theta)
$$

(26)

**Appendix B: Unit Cell Optimization**

Fig. 6. Top view of meta-atom showing the slow and fast axis of silicon nanobar. Here, $\theta$ is the orientation angle between $x$-axis and slow axis.
Fig. 7. (a) Simulated cross-polarized and (b) Co-polarized efficiency maps at 1.55 µm. Each point on maps shows the corresponding efficiency of silicon nanobar as a function of length \( L \) and width \( W \). The white dashed line shows the region where maximum value of cross-polarized and minimum value of co-polarized component occur. The white asterisk indicates the selection of \( L = 445 \) nm and \( W = 190 \) nm.

Appendix C: Simulated Energy Density Inside the Periodic Array of Unit Cells.

Fig. 8. The top views and side views of simulated energy densities in silicon nanobar. The array of nanobar is rotated by 45°. The x-polarized plane wave is incident from substrate side. The boundaries of nanobars are indicated by solid black lines.

Acknowledgments
This work was supported by the Microwave and Photonics Research Group (MPRG), GIK Institute, Swabi, Pakistan. All authors are grateful to anonymous reviewers and associate editor for their valuable comments which have greatly improved the quality of the manuscript.

Disclosures
The authors declare no conflicts of interest.

References


