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On the Utilisation of Nonlinear Plasticity Models in Military Aircraft Fatigue Estimation: A Preliminary Comparison

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Abstract

Strain-life methodologies are commonly employed for fatigue estimation in military aircraft structures. These methodologies rely on models describing the elastoplastic response of the material under cycling. Despite the numerous advanced plasticity models proposed and utilised in various engineering problems over the past decades, the Masing model remains a popular choice in fatigue analysis software, mainly due to its simplicity. However, in the case of military aircraft load spectra including scattered overloads the Masing choice fails to represent adequately transient cyclic phenomena, such as mean stress relaxation and ratcheting. In this
study, four well-known constitutive plasticity models have been selected as potential substitutes of the Masing model within a defence organisation in-house developed fatigue analysis software. The models assessed were the well known Multicomponent Armstrong-Frederick Model (MAF) and three of its derivatives: MAF with threshold (MAFT), Ohno-Wang (OW) and MAF with Multiplier (MAFM). The models were calibrated with the use of existing experimental data, obtained from aircraft aluminium alloy tests. Optimisation of the parameters was performed through a genetic algorithm-based commercial software. The models were incorporated in the fatigue analysis software and their performance was evaluated statistically and compared against each other and with the Masing model for a series of different flight load spectra for a military aircraft. In this preliminary investigation, all four models have achieved a drastic improvement in fatigue analysis, with the MAFT model indicating a slightly better performance.

**Keywords:** Cyclic plasticity, kinematic hardening, isotropic hardening, ratcheting, fatigue life; aircraft fatigue
1. Introduction

Fatigue damage can be influenced by transient phenomena exhibited in the cyclic plastic response of the material, especially under low cycle fatigue conditions. Aircraft structural fatigue assessment via strain-life methodologies has commonly been performed with the use of the Masing cyclic plasticity model [1]. The Masing model in conjunction with the Ramberg-Osgood curve fitting equation, define the hysteresis loops by using the stabilised stress-strain curve of the material. This modelling approach may be an acceptable solution for stabilised loading conditions, however it does not take into consideration transient cyclic plastic phenomena, such as strain ratcheting and mean stress relaxation. Therefore, in the case of more complex loadings conditions, where a benign load spectrum is interspersed with severe overloads leading to nonzero mean stress/strains, this could lead to a gross error in the prediction of fatigue life [2].

Improvements in fatigue life prediction can be achieved by adopting nonlinear kinematic hardening models, such as the Multicomponent Armstrong-Frederick (MAF) model [3], as opposed to approaches relying on the Masing model [4]. Since its inception, the MAF model has been a basis for numerous improvements modifications, including the following:

- MAF model with Threshold term (MAFT) [5] (addition of a fourth back stress containing a threshold term $\tilde{a}$);
- Ohno-Wang (OW) model [6] (superposition of a large number of back stress terms, each containing a slight nonlinearity introduced through a multiplier);
• Multicomponent Armstrong-Frederick model with Multiplier (MAFM) [7] (addition of a back stress multiplier $X^*$).

Both the MAFT and MAFM model differ from the basic MAF model through an added fourth back stress term, aimed at improving hysteresis loop shape and strain ratcheting simulation accuracy. In the MAFT model the fourth term evolves in a dual way (non-linearly below a threshold value and linearly above that value) [5], while in the MAFM model its evolution is controlled via a multiplier term, imposing a continuous nonlinear response [7, 8]. The OW model differs from the MAF model in the modification of the dynamic recovery [6]. However, the OW model typically requires a larger number of back stresses terms (as opposed to MAFT and MAFM), all of which containing the modification of the dynamic recovery term.

In order to represent the transient cyclic plasticity behaviour exhibited in aircraft aluminium alloys during fatigue cycling, a plasticity model should be able to capture adequately the strain ratcheting and mean strain relaxation phenomena. The aforementioned nonlinear hardening models are considered to have the capacity to account for the strain hardening and strain ratcheting phenomena. This paper reports interim results from the application of the MAF, MAFT, O-W and MAFM model in strain-life fatigue predictions, extending ongoing research by the authors in this field [9-12].

2. Formulation of Plasticity Models

The formulation of the models is presented in its uniaxial form both for simplicity but also due to the fact that their implementation, both for the material cyclic elastoplastic
simulations for the fatigue analysis (presented in the sequel), is performed in their uniaxial form.

The classical von Mises yield function \( f \) was selected in this study, described by Eq. 1.

\[
f = (\sigma - X(\epsilon^p))^2 - \left[ \sigma_{y0} - R(\epsilon^p) \right]^2 = 0
\]

(1)

Where \( \sigma \) is the stress, \( X \) the kinematic hardening back stress as a function of plastic strain \( \epsilon^p \), \( \sigma_{y0} \) is the initial yield stress and \( R \) the isotropic hardening evolution rule, also as a function of plastic strain \( \epsilon^p \).

The kinematic hardening rule (back stress \( X \)) is presented in its incremental form, as following:

\[
dX = \sum dX_i
\]

(2)

Where \( dX_i \) the increment of the constituent back stresses \( X_i \) (with \( i = 1, 2, 3, \ldots \)) for the case of the MAF model (summation of multiple back stresses).

In order to improve the model’s ability to simulate effectively plastic shakedown and cyclic hardening the following terms were included in the full model respectively:

- A nonlinear isotropic hardening rule given by Eq. 3 [13]:

\[
R = R_e \left[ 1 - e^{-b\epsilon^p} \right]
\]

(3)

Where \( R_e \) and \( b \) are model parameters determined from experimental data.

- A Prager linear kinematic hardening term (back stress), given by Eq. 4 [14]:

\[
dX_i = c_i d\epsilon^p
\]

(4)

Where \( c_i \) a model parameter.
The models’ kinematic hardening formulation (back stresses) are summarised in Table 1, with the reader referred to papers [3, 5-7] for further details. Moreover, the set of parameters \((c_i, a_i, \bar{a}, m, c_i^*, a_i^*)\) corresponding to each model is presented in Table 2 (labelled under kinematic and isotropic hardening parameters).

**Table 1** MAF, MAFT, OW and MAFM incremental uniaxial formulation, including the Prager linear kinematic hardening term.

<table>
<thead>
<tr>
<th>Model</th>
<th>Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAF</td>
<td>(dX_i = c_i d\varepsilon^p + \sum_{j=2}^{3} (a_{ij} d\varepsilon^p - c_i X_i [d\varepsilon^p]))</td>
</tr>
<tr>
<td>MAFT</td>
<td>(dX_i = c_i d\varepsilon^p + \sum_{j=2}^{4} (a_{ij} d\varepsilon^p - c_i X_i [d\varepsilon^p]) + \left[ a_{4j} d\varepsilon^p - c_4 X_4 \left(1 - \frac{\bar{a}}{X_4}\right)\right][d\varepsilon^p])</td>
</tr>
<tr>
<td>OW</td>
<td>(dX_i = c_i d\varepsilon^p + \sum_{j=2}^{9} (a_{ij} d\varepsilon^p - c_i X_i [d\varepsilon^p])\left(\frac{X_i}{a_i}\right)^m)</td>
</tr>
<tr>
<td>MAFM</td>
<td>(dX_i = c_i d\varepsilon^p + \sum_{j=2}^{4} (a_{ij} d\varepsilon^p - c_i X_i [d\varepsilon^p]) + \left[ c_4 + c_4^* (a_4^* - X_4^*)\right]\left(a_4 d\varepsilon^p - X_4 [d\varepsilon^p]\right))</td>
</tr>
</tbody>
</table>

with \(dX_i^* = c_4^* \left( a_4^* d\varepsilon^p - X_4^* [d\varepsilon^p]\right)\) (dimensionless back stress \(X_4^*\)).
3. Optimisation of Plasticity Models’ Parameters

The performance of the models is highly dependent on their calibration, which entails identifying their parameters through fitting experimental (mechanical testing) data. For this reason, the parameters were determined through an optimisation process performed with the modeFrontier [15] commercial software, in conjunction with the basic methodologies applicable for each model described in [3, 5-7] (for the initial parameter set determination). This process, by the authors in previous works [10, 11], is capable of improving the simulation accuracy for symmetric strain-controlled and asymmetric stress/strain-controlled simulations.

In brief, a genetic algorithm is used in the parameter optimisation process which involves the following key stages:

- Parameter initialisation (initial population of 25);
- Model simulation;
- Objective evaluation.

In each iteration the chosen parameters are given a “fitness rating” based on the output of the objective functions (hysteresis loop shape and strain ratcheting). A new set of parameters is derived with the use of these ratings. This set is subsequently trialled in the next iteration, based on the best output obtained from the previous iteration. The full optimisation process is presented in detail in [11].

Each model’s parameters, obtained from this iterative optimisation process, are presented in Table 2.
Table 2 AA7075-T6 MAF, MAFT, OW and MAFM kinematic hardening and isotropic hardening models’ parameters.

<table>
<thead>
<tr>
<th></th>
<th>MAF</th>
<th>MAFT</th>
<th>OW</th>
<th>MAFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\text{Ref}}$ (MPa)</td>
<td>451</td>
<td>345</td>
<td>435</td>
<td>361</td>
</tr>
<tr>
<td>$E$ (GPa)</td>
<td>69</td>
<td>69</td>
<td>69</td>
<td>69</td>
</tr>
</tbody>
</table>

**Kinematic Hardening Parameters**

<table>
<thead>
<tr>
<th></th>
<th>MAF</th>
<th>MAFT</th>
<th>OW</th>
<th>MAFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$ (MPa)</td>
<td>5900</td>
<td>563</td>
<td>3000</td>
<td>5783</td>
</tr>
<tr>
<td>$(a_2, c_2)$ (MPa, -)</td>
<td>(7, 55502)</td>
<td>(28, 30239)</td>
<td>(62, 1000)</td>
<td>(19, 100000)</td>
</tr>
<tr>
<td>$(a_3, c_3)$ (MPa, -)</td>
<td>(100, 435)</td>
<td>(3, 29418)</td>
<td>(3, 339)</td>
<td>(75, 989)</td>
</tr>
<tr>
<td>$(a_4, c_4)$ (MPa, -)</td>
<td>-</td>
<td>(14, 3229)</td>
<td>(9, 219)</td>
<td>(76, 53675)</td>
</tr>
<tr>
<td>$(a_5, c_5)$ (MPa, -)</td>
<td>-</td>
<td>-</td>
<td>(12, 163)</td>
<td>-</td>
</tr>
<tr>
<td>$(a_6, c_6)$ (MPa, -)</td>
<td>-</td>
<td>-</td>
<td>(3, 133)</td>
<td>-</td>
</tr>
<tr>
<td>$(a_7, c_7)$ (MPa, -)</td>
<td>-</td>
<td>-</td>
<td>(11, 115)</td>
<td>-</td>
</tr>
<tr>
<td>$(a_8, c_8)$ (MPa, -)</td>
<td>-</td>
<td>-</td>
<td>(10, 12)</td>
<td>-</td>
</tr>
<tr>
<td>$(a_9, c_9)$ (MPa, -)</td>
<td>-</td>
<td>-</td>
<td>(41, 6)</td>
<td>-</td>
</tr>
<tr>
<td>$\bar{a}$ (MPa)</td>
<td>-</td>
<td>118</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$a_i$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>44233</td>
</tr>
<tr>
<td>$c_i$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>18924</td>
</tr>
<tr>
<td>$m$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Isotropic Hardening Parameters**

<table>
<thead>
<tr>
<th></th>
<th>MAF</th>
<th>MAFT</th>
<th>OW</th>
<th>MAFM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_0$ (MPa)</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>$b$</td>
<td>6.8</td>
<td>6.8</td>
<td>6.8</td>
<td>6.8</td>
</tr>
</tbody>
</table>

4. Results

The results presented in this section were obtained from the numerical implementation of the models in Matlab and in the Defence Science and Technology (DST) Group CGAP fatigue analysis software [16].
4.1 Uniaxial Cyclic Elastoplastic Simulations

Simulations were conducted for symmetric strain-controlled and asymmetric stress-controlled uniaxial test cases and compared to experimental data, as a verification of the selected optimal parameters. The comparison between the AA7075-T6 symmetric strain-controlled experimental data at the ±1.5% strain range with each of the model simulations for the first cycle and the stabilised cycles (150th cycle) demonstrate a relatively small amount of cyclic hardening (Figure 1). Moreover, the results demonstrate that the parameters selected for each model capture accurately the shape of both the first and the subsequently stabilised cycles.

Figure 1 AA7075-T6 experimental symmetric strain-controlled hysteresis loop corresponding to ±1.5% strain range against models’ (MAF, MAFT, OW, MAFM) simulated data for: (a) First cycle; (b) Stabilised cycle (150th cycle).

The effectiveness of the models in simulating the ratcheting strain accumulation (maximum strain obtained at peak of each loading cycle) for varying stress amplitudes was also analysed using experimental data. In particular, the ratcheting strain outputs for
the following combinations of imposed minimum and maximum (min, max) stresses were used:

1. (-450 MPa, 550 MPa);
2. (-460 MPa, 540 MPa);
3. (-440 MPa, 520 MPa);
4. (-430 MPa, 520 MPa).

The simulation results are presented in Figure 2, where Figure 2(a) compares the accuracy of each model against larger stress loading cases (pairs 1 and 2), while Figure 2(b) shows the outputs for the lower stress cases (pairs 3 and 4).

Figure 2 AA7075-T6 experimental ratcheting curves against models’ (MAF, MAFT, OW, MAFM) simulated data corresponding to asymmetric stress-controlled loadings at minimum and maximum stresses of: (a) (-450MPa, 550MPa) (curve 1) and (-460MPa, 540MPa) (curve 2); (b) (-440MPa, 520MPa) (curve 3) and (-430MPa, 520MPa) (curve 4).
All models were able to predict the plastic shakedown phenomenon relatively well. However, under higher peak stresses the prediction accuracy tends to underestimate the experimentally observed limit (curves 1 and 2 in Figure 2), while for low peak stresses it overestimates the experimentally observed limit (curves 3 and 4 in Figure 2). It is of note that only the MAFT model was successful in simulating the correct strain at which the plastic shakedown phenomena occurs for both of the examined lower load cases [curves 1 and 2 in Figure 2(a)].

4.2 Strain-life Fatigue Simulations

Experimental fatigue data of AA7075-T6 notched coupons have been used in the CGAP software fatigue analysis, from 21 different load spectra obtained from the P-3C service life assessment program [17,18]. The CGAP simulations are compared to these experimental data. Although, in general the geometric mean of the experimental fatigue life under each spectrum is used to compare predictive accuracy, a statistical approach was adopted in addition. In particular, the comparison considered the probability distribution of experimentally determined fatigue lives for each spectrum, as a way to assess the experimental variability. The distributions of the experimental fatigue lives were generated with the bootstrap [19] non-parametric statistical method. Once distributions were generated, the bootstrap method was utilised to generate a characteristic distribution across selected fatigue critical locations. A similar distribution can be constructed using the predicted fatigue lives across the 21 locations. The Mean Integrated Squared Error (MISE) was used as the metric to compare the two distributions, where lower MIS corresponds to higher accuracy in the simulation results.
MISE calculations were repeated in order to form a distribution of MISE values for each model. The MISE distributions allowed the assessment of the accuracy of the models relatively to each other, including the known variability in experimental test results.

In Figure 3(a) the MISE distribution for each of the plasticity models is compared with the Masing model MISE distribution, while in Figure 3(b) the geometric mean fatigue life data are presented for each model under comparison. From the results illustrated in Figure 3(a), it stems that the MAFT and MAFM models, offer improved simulation accuracy when compared to the Masing model. It is particularly evident from the MAFT MISE distribution that the MAFT model achieves overall more accurate simulations when compared to all the other models. It is also of note that the majority of the simulation results are located in the lower (green shaded) portion of Figure 3(b), which indicates that the predictions are conservative.
Figure 3 (a) Mean Integrated Squared Error (MISE) distributions formed by each of the models in comparison (MAF, MAFT, OW, MAFM and Masing); (b) Geometric mean of simulated fatigue lives (predicted flight hours) against the corresponding experimental values (experimental flights hours) for each model (MAF, MAFT, OW, MAFM and Masing).
5. Discussion

From the preliminary comparison results, it was observed that the MAFT model is capable of achieving both the highest fatigue life estimation accuracy and the most accurate ratcheting simulation. However, one needs to consider that the optimisation process used in the parameters’ determination (for all models) was focused on identifying those set of parameters capable to simulate the cyclic phenomena observed in the uniaxial experiments and not the fatigue life. Thus, the process may be inherently biased towards the uniaxial data and further research efforts are currently in progress, including a comprehensive sensitivity analysis for each of the models’ parameters and the fatigue data influence on the overall performance of the models.

The parameter selection for this study was based on achieving improved ratcheting simulation rather than simulating accurately the mean stress relaxation phenomenon. This choice was made on the basis of achieving an acceptable balance between these two cases but also due to a limitation on the number of experimental data that were used (mainly associated with the scarcity of representative available data for AA 7075-T6). Further work planned is aimed to determine the difference in fatigue life predictions when the models’ parameters are optimised for mean stress relaxation. This work includes the collection of more extensive test data, which are essential for the validation of the models and a complete comparison study of the different cyclic plasticity models.
6. Conclusion

The fatigue estimation performance of four established cyclic plastic models (MAF, MAFT, OW and MAFM) incorporated in the CGAP strain-life fatigue analysis software is currently under investigation within the DSTG. The assessment of the cyclic plasticity models was conducted using MISE as a metric, based on the statistical distribution of the experimental and the numerical results. The assessment showed that the MAFT and MAFM nonlinear kinematic hardening models, coupled with a Prager linear back stress and a nonlinear isotropic hardening rule, improved fatigue life predictions when compared to the Masing model.

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