A questionnaire survey was conducted as part of a PhD study investigating post-primary students’ images of mathematics in Ireland. A definition of ‘image of mathematics’ was adopted from Lim (1999) and Wilson (2011). Students’ images of mathematics were proposed to include attitudes, beliefs, motivation, self-concept, emotions and past experiences regarding mathematics. A questionnaire was created with both quantitative and qualitative aspects. This paper focuses on the qualitative facet by reviewing students’ responses to the open-ended questions according to the five categories of image of mathematics found by Lim (1999). The qualitative data provides an in-depth insight into Irish students’ images of mathematics. Findings from the qualitative data afford an innovative insight into post-primary mathematics education from the student perspective, thus offering a means for mathematics educators to respond to students’ needs and encourage Irish post-primary students’ engagement with mathematics.

INTRODUCTION

Affective, cognitive and conative issues have come to the fore of mathematics education research in recent years. They have been examined in Ireland and abroad with studies by researchers such as McLeod (1992), Kelly and Oldham (1992), Ernest (1996), Lim (1999), Liston (2008), Eaton & O’Reilly (2009) and Wilson (2011) establishing a strong theoretical basis for examining the relationship between them and mathematics. Studies that have examined affective and cognitive issues (and, in some cases, conative issues) defined the combined issues as “mathematical identity” (Hill, 2008; Eaton & O’Reilly, 2009), “self-efficacy” (Tait-McCutcheon, 2008), “disposition” (Wilson, 2011) or “image of mathematics” (Kelly & Oldham, 1992; Ernest, 1996; Lim, 1999). This aspect of mathematics education had not previously been examined extensively in Ireland in relation to post-primary school students. Thus we identified an obvious gap in mathematics education research in the Irish context. As part of a PhD research study (Lane, 2013), we aimed to address these affective, cognitive and conative issues in Irish post-primary mathematics education by examining the image of mathematics held by students.

The general public’s view of mathematics, whether in Ireland or elsewhere, often depends on individuals’ experience of mathematics, particularly during school years. This is supported by Lim (1999) who, in her study of the public image of mathematics in the UK, found that most people did not distinguish between their image of mathematics and their image of learning mathematics. Therefore the process of teaching and learning mathematics plays a vital role in establishing a person’s image of mathematics. Mathematics education researchers have come to realise the significance of students’ attitudes, beliefs, emotions, motivation and self-concept regarding mathematics, with consequential effects on mathematical performance and achievement. As Lane’s (2013) study began, a phased introduction of a new mathematics curriculum for Irish post-primary schools, Project Maths, was under way and thus there was a
particular focus on post-primary mathematics education in Ireland. The current interest in innovation and change in mathematics education in Ireland and the relevance of affective, cognitive and conative issues to the mathematics education community both in Ireland and abroad provided strong motivation for the study.

We focused on 5th-year students in the Leaving Certificate cycle of post-primary school. It was hypothesized that these students would have formed a stronger image of mathematics than students who were in their first years of post-primary education. Furthermore, we chose to examine the images of mathematics of students of ordinary level mathematics specifically. It was expected that ordinary level students would have a more diverse range of images than students of the higher or foundation levels.

This paper focuses on the qualitative data findings from a questionnaire survey constructed to examine students’ images of mathematics. While most of the questionnaire consisted of quantitative fixed-response items from eight Likert scales (examining attitudes, beliefs, motivation, self-concept and emotions about mathematics), the qualitative questions provide important findings relating to students’ images of mathematics.

THEORETICAL FRAMEWORK

From the literature, it was clear that no universal definition exists for ‘image of mathematics’. However, for those researchers who define ‘image of mathematics’ (either explicitly or implicitly), there appears to be a general consensus that it should include attitudes, beliefs, self-concept, emotions and past experiences regarding mathematics (e.g., Kelly & Oldham, 1992; Lim, 1999). For our definition of ‘image of mathematics’, we draw on the theories of Lim (1999) and Wilson (2011). Lim (1999, p.73) conceptualizes the term ‘image of mathematics’ as:

“A mental representation or view of mathematics, presumably constructed as a result of social experiences, mediated through school, parents, peers or mass media. This term is also understood broadly to include all visual and verbal representations, metaphorical images and associations, beliefs, attitudes, and feelings related to mathematics and mathematics learning experiences”.

Lim divides the elements included in her definition of ‘image of mathematics’ into two categories: the affective domain (attitudes, emotions and feelings regarding mathematics) and the cognitive domain (knowledge and beliefs regarding mathematics). Wilson’s (2011) theory of ‘disposition’ overlaps to a certain degree with Lim’s definition of ‘image of mathematics’. In his definition he proposes four components of disposition as follows:

- Beliefs / values / identities
- Affect / emotions
- Behavioural intent / motivation
- Needs

The first two components coincide with Lim’s definition of ‘image of mathematics’, while the fourth component ‘needs’ is similar to the factors of influence (school, parents, peers and
media) included in Lim’s theory. The third component refers to the conative domain proposed by Ruffell, Mason and Allen (1998). Ruffell et al. suggests that a conative element works together with the affective and cognitive domains. We adapt the definitions of Lim (1999) and Wilson (2011) for our study, with ‘image of mathematics’ conceptualized as follows:

A mental representation or view of mathematics, presumably constructed as a result of past experiences, mediated through school, parents, peers or society. This term is also understood broadly to include three domains:

- The affective domain dealing with attitudes, emotions, and self-concept regarding mathematics and mathematics learning experiences.
- The cognitive domain dealing with beliefs regarding mathematics and mathematics learning experiences.
- The conative domain dealing with motivation regarding mathematics and mathematics learning experiences.

Self-concept is not included in the definitions of Lim or Wilson, but following our literature review which indicated the increasing significance of self-concept regarding mathematics (e.g., Gourgey, 1982; Tait-McCutcheon, 2008), it was decided that self-concept should be included in the affective domain of our definition for ‘image of mathematics’.

METHODOLOGY

The design for our research study was chiefly explorative. Ethical permission was sought and granted by the Ethics Committee in University College Cork prior to carrying out the investigative research. The main research question for the study was:

- What is the image of mathematics held by 5th-year post-primary students in Ireland?

This question was then refined through the literature review and the theoretical framework and broken down into nine sub-questions. These nine questions related to students’ (a) attitudes, (b) emotions, (c) self-concept, (d) beliefs, (e) motivation, (f) past experiences, (g) influences in mathematics, (h) causal attributions for success/failure in mathematics, and (i) prior achievement in mathematics.

A mixed-methodology was employed. The main method used to examine students’ images of mathematics was a questionnaire survey. The questionnaire contained both quantitative fixed-response items and qualitative open-ended questions. The quantitative aspect incorporated eight pre-established Likert scales to examine students’ attitudes, beliefs, emotions, self-concept and motivation regarding mathematics – see Lane, 2013. The Likert scales addressed the first five of the research sub-questions as well as the main research question. The qualitative element consisted of open-ended questions composed by the researcher and aimed to address the sixth, seventh and eighth sub-questions aforementioned as well as clarifying students’ overall image of mathematics. Both the quantitative and qualitative questions were used in examining the final (ninth) research sub-question.
This paper focuses on the qualitative element of enquiry. There were five open-ended questions in Section A of the questionnaire. The research purpose of each of the open-ended questions as well as research studies that support the relevance of these questions are shown in Table 1. Four of the five open-ended questions contained two parts with the first part consisting of a ‘tick the box’ option. The purpose of this tick the box option was, in part, to clarify questions which might otherwise be misunderstood by students, and in addition, to provide a means of analysing parts of the questions more quickly and efficiently using SPSS.

<table>
<thead>
<tr>
<th>Open-ended Question</th>
<th>Research Purpose</th>
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<tbody>
<tr>
<td>Q1: What is your earliest memory of mathematics?</td>
<td>To examine students’ past experiences of mathematics (Lim, 1999).</td>
</tr>
<tr>
<td>Q2: Have your past experiences of mathematics caused you to be interested/ disinterested in mathematics? (a) Yes/No (b) Please Explain.</td>
<td>To examine students’ past experiences of mathematics and the influence of past experiences on current engagement in mathematics, (Lim, 1999; Eaton &amp; O’Reilly, 2009).</td>
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<tr>
<td>Q3: Who influences you most in mathematics? (a) Mathematics Teacher/ Parents/ Peers/ Media/ Other (b) Why?</td>
<td>To examine influences on students’ images of mathematics and students’ engagement with mathematics (Lim, 1999; Lyons, Lynch, Close, Sheerin &amp; Boland, 2003; Hill, 2008).</td>
</tr>
<tr>
<td>Q4: Which of the following contributed most to the grade you received for mathematics in the Junior Certificate? (a) Mathematics Ability/ Effort/ Luck/ Level of Difficulty of Exam/ Other (b) Please Explain.</td>
<td>To examine students’ causal attributions for success and/or failure in mathematics (Weiner, 1974).</td>
</tr>
<tr>
<td>Q5: Do you use mathematics outside of school and school work? (a) Yes/No (b) If so please give examples. If not, please give reasons.</td>
<td>To examine students’ image of mathematics in terms of the utility of mathematics in everyday life (particularly relevant given the new Project Maths curriculum) (Lim, 1999; Hill, 2008).</td>
</tr>
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Table 1: Qualitative questionnaire items and research purpose of the questions

In the analysis of the qualitative data, the constant comparative method of analysis was used (Conrad, Neumann, Haworth & Scott, 1993). The responses to the open-ended questions were sorted into categories and then integrated into broader categories. These categories were then coded and analysed using SPSS. In addition, individual responses were also reviewed qualitatively, bearing in mind previous research findings.

This paper focuses on findings from the open-ended questions that inform the main research question regarding students’ images of mathematics. The data was analysed qualitatively, bearing in mind the five images of mathematics found by Lim (1999), namely – the absolutist/dualistic image, the utilitarian image, the symbolic image, the problem-solving image, and the enigmatic image. Answers by students to each of the open-ended questions were reviewed by the researcher and were found to contribute to creating an impression of students’ overall images of mathematics in terms of the five images found by Lim. Not all students’ responses fitted into one of the categories by Lim. Some of the students’ answers were only a direct response to the question asked, providing only a one or two word answer.
However, a considerable number of students gave more detailed comments and these were used to examine the existence of the five categories of image. These five images are not strictly separate, and a student’s image of mathematics may fall into one, several or even none of these categories. The images of mathematics categorized by Lim do however provide a guideline for assessing the images of mathematics found in the study, by providing a theoretical basis for categorizing responses and assessing common or widespread images among students in our sample.

**Sampling Methodology**

As mentioned above, no previous research has investigated the image of mathematics held by post-primary students. Therefore, we followed the sampling methodology used by Murray, McCrory, Thornton, Williams, Quail, Swords, Doyle and Harris (2011), in the ESRI’s study of nine-year olds’ attitudes to mathematics in Ireland. The sampling objective of our research survey was to select a representative random sample of 400 5th-year ordinary level mathematics students in post-primary school in Ireland. It is widely accepted that non-response is a major issue in surveys and particularly in postal surveys (see Wiersma, 1991). In order to achieve a sample as close to the required 400 as possible, we decided to choose a sample with 3 times as many students as was required (that is, 1200 students).

Similar to Murray et al. (2011), post-primary schools were stratified according to type of school (secondary, vocational, or community and comprehensive), co-educational status (single-sex girls’, single-sex boys’ or co-educational), and school ethos (fee-paying or non-fee-paying). A random sample of 60 schools was initially selected on a stratified systematic basis. As in the ESRI study, a maximum threshold of 20 students was set in any one school. Schools were provided with instructions for selecting a random sample of 20 students. An information sheet advised schools with fewer than 20 ordinary level mathematics students that all students should be asked to participate. For schools with more than 20 students, a random digit table with instructions was provided. Students and their parents were requested to sign an informed consent form prior to participation.

One of the limitations of our study was generalizability of findings. Ideally, a much larger sample size should be used as in the ESRI study, but given time and resource constraints this was not possible. A total of 356 students completed the questionnaires and while slightly below our sampling aim, this was sufficient to provide a preliminary examination of Irish post-primary students’ images of mathematics.

**QUALITATIVE DATA**

In this section we present findings from the qualitative open-ended questions. Students’ responses from the five open-ended questioned are considered in relation to the main research question: What is the image of mathematics held by 5th-year post-primary students in Ireland? The students’ answers and comments were reviewed for evidence of the five categories of image of mathematics found by Lim (1999). Not all of the comments made by students could be said to reveal their image of mathematics, but the qualitative analysis did reveal the existence of each of Lim’s image categories, with some categories found to be more prevalent.
than others. Due to the large amount of data collected in this study, not all of the students’ answers can be displayed in this paper. Rather we examine each of the five image categories in turn, provide quotes from the students’ answers which fall into each category and discuss whether students’ images are positive or negative.

Students’ Images of Mathematics

In order to analyse students’ images of mathematics from the qualitative data, we refer to the five images of mathematics found by Lim (1999). Within each of the image categories, students’ comments also reveal their image of mathematics with respect to attitudes, beliefs, motivation, self-concept and emotions.

The absolutist or dualistic image of mathematics is that of a set of ‘absolute truths’ with only one right answer. The absolutist image has been fostered by the style of teaching that has been experienced by many students, whereby the emphasis is placed on the product rather than the process of mathematics (e.g., Hill, 2008; Tait-McCutcheon, 2008; Eaton & O’Reilly, 2009). On the one hand the absolutist view may result in a positive image of mathematics as there is a solution to each mathematical problem which, if found, cannot be disputed. On the other hand, the absolutist view can produce a negative image of mathematics, that of a subject lacking creativity and it can be frustrating when the one true solution cannot be reached.

Examples of the absolutist image of mathematics held by students in our study can be seen from the following comments:

- “Some experiences make me interested – like when you work out a new sum … on your own it gives you a little kick and makes you interested to learn.”
- “If I get a hard question right I feel good to accomplish it …”
- “I knew all the answers when I looked at the paper.”
- “Getting frustrated if I couldn’t do it.”
- “When I got it wrong I didn’t want to do anything else.”
- “… used to get very stressed.”
- “Stressful and frustrating.”

The responses listed above illustrate the positive and negative effects of an absolutist image of mathematics. The importance of finding the right answer results in feelings of pleasure and achievement when the one true solution is obtained. On the other hand, when students were unable to find the answer they felt frustrated and stressed resulting in a negative image of mathematics. One student commented that he/she did not want to continue when they were unable to find the correct answer suggesting that the absolutist image of mathematics can lead to a lack of motivation in students. In the qualitative section of the questionnaire, students were asked in the first question about their past experiences or memories of mathematics – see Table 1. Although the majority of students (86%) simply stated that their earliest memories were of primary school or learning a particular type of mathematics at school, the remainder of the students (14%) gave detailed examples of their earliest memories, 24 students referring to negative experiences and 25 students recalling negative experiences. Of the students who
referred to negative memories of mathematics, 15 spoke of finding mathematics difficult or struggling, leading to negative emotions or feelings about mathematics, stating that they got “frustrated” and “upset”. The most explicit negative memories involved experiences with teachers. One student wrote that the teacher was “writing sums and answers on the board and we couldn’t learn anything”. A second student recalled a “teacher shouting at me when I got something wrong”. A third student remembered that the teacher was “angry at me when I couldn’t do a sum on the board and the class were laughing at me”. These descriptions by students are indicative of the culture of rote-learning which has been commonplace in our schools as found by De Corte, Greer & Verschaffel, (1996). Emphasis was placed on finding the right answer and negative memories held by students in our sample related to an inability to find correct solutions. As found in Ruffell et al. (1998), specific negative memories or experiences of mathematics can override more general, positive experiences of mathematics and in our study, the students who recalled negative memories of mathematics were found to have the most negative image of mathematics compared to all other students.

The utilitarian image views mathematics in terms of its usage and relevance. Usually, for those who like mathematics and have a positive image, mathematics is seen as a useful tool, while for those who dislike mathematics and have a negative image, mathematics is seen as irrelevant and not to be applied to everyday life. Some examples of the utilitarian image of mathematics can be seen the following comments by students:

- “Everyone uses mathematics in every situation if you think about it.”
- “The subject itself (mathematics) is a useful tool.”
- “You need maths for most things in life.”
- “I see maths everywhere. I will always need maths no matter what I choose to do in life.”
- “It’s a useful tool i.e. for budgeting and minding your money – it makes you think wisely.”
- “I am interested in the maths that would help me in the future not irrelevant maths.”
- “Maths is something you have to do there is no option, you will need it for the rest of your life.”
- “I love physics and see the usefulness of maths but think a lot of the school subject is unnecessary.”
- “Not everyone wants to become a scientist or engineer.”
- “I don’t see why we would need some of the maths course in the future.”
- “Nothing I do that makes me use it.”
- “Maths from post-primary school is not needed in daily life.”
- “I don’t know enough to be able to successfully use it elsewhere.”
- “I don’t know where it can be used.”
“Nothing involves maths.”

When asked in Question 5(a) of the questionnaire whether they used mathematics outside of school and school work, 65.2% of students acknowledged that they did so. However, a considerable number of students – 118 (33.1%) – responded that they did not, suggesting that a third of the students sampled are unaware of the utility of mathematics in everyday life. Students’ responses to Question 5(a) correlated significantly at the 0.01 level with students’ overall image of mathematics. Responses regarding the utilitarian image of mathematics in our study were varied, similar to Hill (2008). While some students were found to have a positive image of mathematics as being useful and relevant in everyday life and necessary for the future, other students’ responses in relation to the utilitarian image of mathematics highlight a lack of awareness regarding the utility of mathematics. Students stated that they were unable to use mathematics as they didn’t know enough to use it while others claimed that mathematics is not needed in day-to-day life. This is similar to the findings of Picker and Berry (2000) who, in their research on students’ images of mathematicians, found a distinct lack of awareness of the utility of mathematicians. Referring to post-primary mathematics education specifically, a number of students in our study stated that they enjoy studying mathematics which is relevant but that much of the mathematics they learn is unnecessary. There appears to be a lack of awareness for some students regarding the application of school mathematics to real world situations. While the introduction of Project Maths aims to address this issue, one student, referring to the new Project Maths course, stated that not everyone wants to become a scientist or engineer. This suggests that while the importance of mathematics in these careers is highlighted to students, the value of mathematics in other areas is not commonly known to students. One student commented that “the problem is that teachers don’t explain what jobs use different topics.” There is clearly a need for a more widespread and detailed explanation to students of the relevance of mathematics in a variety of careers.

The symbolic image of mathematics is that it is a collection of numbers and symbols, or rules and procedures to be followed and memorized. This image has similarities with the absolutist image, with the two categories of image often going hand in hand. Students’ comments that are indicative of the symbolic image of mathematics include the following:

- “Reciting multiples off by heart.”
- “…found learning theorems pointless.”
- “Every year you begin maths it’s just a step up of the same question just with different rules.”
- “Teacher encourages me to learn formulas.”
- “Teacher made us repeat questions over and over until we knew them perfect.”
- “Learnt formulas and equations.”
- “… questions were different from previous years and confused me.”
- “Practised exam papers again and again.”
➢ “Learnt off all the theorems so knew one that came up.”
➢ “Studied the formulas very thoroughly.”
➢ “… don’t work well with numbers.”

There is an emphasis on memorising rules and formulae in mathematics teaching in post-primary schools (e.g., De Corte et al., 1996). This well-established ethos of teaching mathematics by focusing on learning off rules, formula and theorems has already been witnessed in previous studies such as Eaton and O’Reilly (2009) and Liston (2008). There is also a connection between this symbolic image of mathematics and exam preparation. Indeed one student stated that because the exam was different from previous years he/she was confused, thus blaming an ‘unpredictable’ exam for a low mathematics grade in the Junior Certificate. Rather than memorising the mathematics rules, students need to understand what these rules mean and to understand where the formulae and theorems come from.

Further evidence of the symbolic image and rote-learning can be seen from students’ earliest memories of mathematics (Question 1). A total of 51 students (14.3%) recalled learning or memorising multiplication tables. This was the third most frequent response category for this question and suggests that rote learning played a significant role in students’ learning of mathematics. Furthermore, in students’ causal attributions for success or failure in mathematics (Question 4), over half the sample of students in our study, 51.1%, selected ‘effort’ as contributing to the grade they received for mathematics in the Junior Certificate. In cases where students described the type of effort made, repetition and rote-learning were frequently cited.

The problem-solving image of mathematics sees mathematics as a set of problems to be solved. Students’ responses to the qualitative questions that illustrate this problem-solving image include:
➢ “Interested because I do my best to solve maths problems.”
➢ “It’s like figuring out puzzles.”
➢ “I like to be challenged.”
➢ “I like working with figures and problem-solving.”
➢ “I love figuring out problems for myself.”
➢ “Playing maths games in primary school.”
➢ “…encourage me to figure out things on my own.”
➢ “Use maths to apply logic – to sort out problems.”
➢ “Puzzle-solving and order to things.”

For the majority of students, the problem-solving image of mathematics was a positive one and solving mathematical problems was an enjoyable and rewarding experience. Students referred to mathematics as a challenge and appeared to enjoy figuring out solutions to problems on their own. This was also found by Hill (2008). Some students in our study
mentioned problem solving when discussing the utility of mathematics, for example, in Sudoku, brain-training, computer games, applying logic and creating order. The problem-solving image has particular relevance in Project Maths, which focuses on problem solving and the application of mathematics to real-world problems.

Finally, the enigmatic image of mathematics is of something seen as mysterious but yet something to be explored and whose beauty is to be appreciated. Lim (1999) found that a small minority of the sample, particularly those who liked mathematics and were directly involved with mathematics, viewed the subject as an enigma, something foreign and at times incomprehensible but elegant at the same time. Evidence of the enigmatic image of mathematics can be seen in the following statements made by students:

- “It makes me think outside the box.”
- “As you advance, the more you want to know where the maths comes from.”
- “A strange interest as the years have progressed.”
- “It feels like you need to be excellent at it and if not there’s no place for you in the mathematics world.”
- “The teacher guides us through maths.”
- “Maths seems like an elitist subject.”
- “Einstein struggled in maths too but discovered amazing things and I find that inspiring.”
- “…not got a flare for maths.”
- “I don’t have a natural mathematical ability.”
- “I’m naturally good at maths.”
- “I find some things easier than the other students and they come to me naturally.”

Each of the comments made by students listed above could fit into an enigmatic image of mathematics. One student wrote that the teacher “guides” them through mathematics, which is similar to the metaphor of mathematics as a journey that was found by Lim (1999). There was also the impression of mathematics as being strange or making one “think outside the box”, which ties in with the image of mathematics as something foreign. Two of the responses given by students referred to the elitist nature of mathematics, implying that mathematics is only for the very smart and only accessible to a few with a very high mathematical ability. This was a common image of mathematics found by Ernest (1996). Finally, a number of students mentioned having a natural mathematical ability or lack thereof. The notion of the mathematical mind or having a natural ability in mathematics is also seen in Picker and Berry (2000) who found that mathematics may be seen more as a magical power than as a subject that anyone can learn. This can create a negative image of mathematics as an unattainable skill, and impossible for those who do not possess a ‘natural mathematical ability’.
CONCLUSIONS

The findings discussed in the previous section provide innovative information regarding the image of mathematics held by 5th-year, post-primary students in Ireland. Each of the five images of mathematics in Lim (1999) was found to exist among students in our study. The problem-solving image of mathematics was the most positively expressed image of mathematics, with a positive effect on enjoyment of mathematics. The qualitative data analysis highlights the need for more detailed and widespread explanation regarding the relevance of mathematics in daily life and in a variety of future careers. It is also necessary to emphasise understanding the mathematical process rather than memorising rules. Findings from this qualitative data are essential in giving students a voice. It is crucial to listen to students’ views in order to gain a better understanding of students’ needs and therefore to inform teacher education and the way we approach mathematics education in our schools.

REFERENCES


