

Abstract

In constant bit-rate timing transfer, the reference clocks which encode and reconstruct the service clock at origin and destination may be jittered. We present new, straightforward approaches to finding and visualising jitter spectra in timing transfer for jittered destination and reference clocks and confirm our results by simulation.

Keywords: synchronization, non-uniform sampling, jitter, SRTS, sigma-delta modulator

Modeling the effect of non-ideal reference clocks on the jitter generated in timing transfer.

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I Introduction

To transfer timing information for a constant bit rate service from source to destination involves encoding the timing information at the source using a local network reference and reconstructing the source clock at the destination using the encoded information and a network reference available at the destination. An example of such an approach is used in the transfer of timing information for CBR transmission over ATM Adaptation Layer 1 (AAL1) by the Synchronous Residual Time Stamp (SRTS) method [1]. It has been shown that such timing transfer techniques are similar to rate adaptation by bit stuffing [2] and pointer adjustment [3] and the jitter generated is similar to waiting-time jitter [1], [3], [4]. Waiting-time jitter is deterministic jitter and can accumulate significantly from repeater to repeater [5], whereas jitter accumulation in synchronous networks is prevented by limiting the number of allowed pointer adjustments [3]. The rate adaptation process is illustrated by the timing diagram in Fig. 1 where the service clock period, T , is synchronized to the reference clock, of period T_{nx} . As the exact number of reference clock cycles in T is usually not an integer, the result is a quantized representation of the service clock with inter-pulse intervals $S_n T_{nx}$, where S_n takes on one of two possible values: $\lfloor M \rfloor$ and $\lfloor M \rfloor + 1$, such that the long term average period is M [6]. The resulting jitter on the quantized clock, $d_n T_{nx}$ is also the jitter present on the recovered timing signal at the destination and is given by:

$$d_n T_{nx} = (d_0 - nM)T_{nx} - \lfloor d_0 - nM \rfloor T_{nx} \quad (1)$$

The spectrum of this jitter can be expressed as:

$$J_{SRTS}(f) = F \sum_{k=-\infty}^{\infty} \alpha_k \sum_{n=-\infty}^{\infty} \delta(f - k\rho F - nF) \quad (2)$$

where $\rho = \langle M \rangle = M - \lfloor M \rfloor$, $M = T/T_{nx}$, $F = 1/T$ and the coefficients α_k are

$$\alpha_k = \begin{cases} T_{nx}/2, & k = 0 \\ T_{nx} e^{-j2\pi k d_0 / (j2\pi k)}, & k \neq 0 \end{cases}. \text{ In previous analyses it has been assumed that the clocks are}$$

ideal, whereas in real systems the assumption of ideal clocks does not hold. In [2], [7],

consideration was given to the effect of jitter on the service clock. It may also be the case that the reference clocks are neither ideal nor identical at origin and destination. Within SONET/SDH networks, for example, the primary cause of pointer adjustments while the network remains synchronized is noise on the reference timing signals [8].

II The Effect of a Non-Ideal Reference Clock at the Destination

In most approaches to SRTS reconstruction, the service clock is regenerated from the sequence S_n using the reference clock at the destination. In this section, jitter on the reference clock at the destination is modeled as a time-varying delay $\phi(t)T_{nx}$ between the significant edges of the two reference clocks, where $0 \leq |\phi(t)| < 1$. As shown in the timing diagram in Fig.

1, the resulting jitter on the service clock at the destination will be $w_n T_{nx} = (d_n + \phi(t_n))T_{nx}$, where $\phi(t)T_{nx}$ is a sample of the jitter on the reference clock at $t_n = nT + d_n T_{nx}$. Thus the jitter on the reference clock is sampled non-uniformly at time instants determined by the value of the SRTS jitter. The non-uniformly sampled jitter function can be written as

$$\phi_s(t) = \sum_{n=-\infty}^{\infty} \phi(t_n)\delta(t-t_n) = \phi(t) \sum_{n=-\infty}^{\infty} \delta(t-t_n) \text{ and in general its spectrum is given by [9]:}$$

$$\phi_s(f) = \Phi(f) \otimes F\left\{ \sum_{n=-\infty}^{\infty} \delta(t-t_n) \right\} \quad (3)$$

where \otimes represents convolution and $F\{ \}$ represents the Fourier Transform. A number of

approaches to determining the spectrum of $\sum_{n=-\infty}^{\infty} \delta(t-t_n)$ have been described [10]-[15]. A

general approach based on the properties of the delta function is given in [10], [11]. Specific solutions are presented in [12] for the case of t_n samples of sinusoidal jitter and in [13] for

those cases where the t_n are samples of a random variable. More specific solutions are given in [14] for sampling by a quasi-periodic sampling function and in [15] for those cases where the t_n are generated (for example in high speed waveform digitizers) such that they are not

uniformly spaced but structured to have an overall periodicity. In the case under consideration

here, the t_n are determined by samples of a nonlinear deterministic function which is quasi-periodic. When the value of ρ is rational and can be expressed as p/q , p and q co-prime, then the values of d_n and S_n will follow a fixed pattern of values over a total length of q cycles, such that, there will be $(q-p)$ S_n of length $\lfloor M \rfloor T_{nx}$ and p S_n of length $(\lfloor M \rfloor + 1)T_{nx}$, that is $qT = (q\lfloor M \rfloor + p)T_{nx}$. However, when the value of ρ is irrational, then the values of d_n and S_n do not follow a repeating cycle. The approach of [14], [15] for rational ρ and assuming the reference clock jitter is co-sinusoidal of amplitude A_j and frequency f_j , predicts that the spectrum will be given by:

$$\phi_s(f) = F \sum_{k=-\infty}^{\infty} A_p(k) \frac{A_j}{2} \delta\left(f - \frac{kF}{q} - f_j\right) + A_n(k) \frac{A_j}{2} \delta\left(f - \frac{kF}{q} + f_j\right) \quad (4)$$

where $A_p(k) = \frac{1}{q} \sum_{m=0}^{q-1} e^{j2\pi f_j d_m} e^{-\frac{j2\pi mk}{q}}$, $A_n(k) = \frac{1}{q} \sum_{m=0}^{q-1} e^{-j2\pi f_j d_m} e^{-\frac{j2\pi mk}{q}}$. While (4) provides a means

to calculate the spectrum, it does not provide a means to visualise it easily or provide a closed form which does not involve a summation of exponentials. To overcome this problem, we show that the spectrum of the non-uniformly sampled jitter can be determined by using the nonlinear time transformation originally developed by Papoulis [16]. As shown in [4], the jitter values $d_n T_{nx}$ are obtained by sampling $d(t)$, the sawtooth function, which is periodic with period T/M , at the times $t = nT$, i.e. $d(t)$ is undersampled, and so any other curve with a lower or higher bandwidth can be interpolated through the jitter samples [17]. Interpolating by

a lowpass filtered version of $d(t)$, for example, $\theta(\tau) = \sum_{k=-\infty}^{\infty} d_k T_{nx} \frac{\sin 2\pi f_2(\tau - kT)}{2\pi f_2(\tau - kT)}$, bandlimited

to f_2 and with its sample values equal to $d_k T_{nx}$ (which corresponds to an ideal lowpass filter of amplitude $1/F$ and bandwidth $f_2 = F/2$), following the example of [16], we form a nonlinear transformation of the time index using $\theta(\tau)$. Assuming that the function so created

$t = \tau + \theta(\tau)$ has a single-valued inverse, $\gamma(t) = \tau$ then it is possible to define a new function $\psi(\tau) \equiv \phi(\tau + \theta(\tau)) = \phi(t)$ which may be sampled at the equidistant points $\tau = nT$ to give

$$\psi(nT) \equiv \phi(nT + \theta(nT)) = \phi(t_n) \quad (5)$$

Thus the uniform samples in (5) are equal to the non-uniform samples of the reference clock jitter. The spectrum of the non-uniformly sampled reference clock jitter may be found using $\Psi(f) \approx \Phi(f) + \Theta(f) \otimes [j2\pi f\Phi(f)]$ [16] and so in the sampled case:

$$F\{\phi(t_n)\} = (\Phi(f) + \Theta(f) \otimes [j2\pi f\Phi(f)]) \otimes \left(F \sum_{n=-\infty}^{\infty} \delta(f - nF) \right) \quad (6)$$

where $\Theta(f)$ is the spectrum of the lowpass filtered waiting-time jitter. In (6) we predict that the spectrum of the non-uniformly sampled reference clock jitter will be the spectrum of the reference clock jitter convolved with a lowpass filtered waiting-time jitter spectrum. To compare the new approach with existing approaches, MATLAB was used to produce clocks jittered by waiting time jitter for different values of ρ and these were used to non-uniformly sample a reference clock jitter represented by 20 Hz co-sinusoidal jitter of amplitude 18 ms. The jitter samples were windowed with a Hamming window and a 2048 point FFT applied. The resultant spectrum of the non-uniformly sampled jitter for the particular case of $T_{nx} = 1$ ms, $T = 13.322$ ms, and $\rho = 0.322$ is plotted in Fig. 2(a). It shows that the effect of the non-uniform sampling on the overall spectrum is very small. The spectrum is plotted again in Fig. 2(c) at a scale which reveals the detail of the non-uniform sampling jitter and also permits comparison with the spectra predicted by (4) and (6) which are plotted in Fig. 2(b) and Fig. 2(d) respectively. Fig. 2(d) is plotted using MATLAB from (6) and (2), using the first 101 (including zero) k coefficients and with n running to ± 20 , then applying an ideal lowpass filter of amplitude $1/F$ and bandwidth $f_2 = F/2$ and re-sampling. Both approaches predict the spectrum correctly, although (4) seems to over-estimate some coefficients. For values of

$\rho = p/q$ which are irrational, results from (4) can become less accurate as the number of samples of d_n required, which is set by q , becomes very large compared with the FFT size.

III The Effect of Jitter on the Reference Clock at the Origin

The equivalence of pulse stuffing and the first-order sigma-delta modulator was demonstrated in [18] and the equivalence of pulse stuffing and an open-loop timing transfer method, such as the SRTS method, was shown in [4]. It is shown in [6] that the jitter equation for the SRTS method can be written as

$$d_n = d_{n-1} - \langle M_{n-1} \rangle + 1 - u(d_{n-1} - \langle M_{n-1} \rangle) \quad (7)$$

Defining the quantizing function as $q(a) = u(a) - 1 = \begin{cases} -1, & a < 0 \\ 0, & a \geq 0 \end{cases}$ and writing $\rho_n = -\langle M_n \rangle$, then

equation (7) can be rewritten as $q(\rho_{n-1} - e_{n-1}) = e_n - e_{n-1} + \rho_{n-1}$ which is the equation for a sigma-delta modulator [19] as represented in Fig. 3. Following the method of [19], it is possible to show that the error sequence, e_n , for the sigma-delta modulator representing the creation of SRTS jitter is given by

$$e_n = -\langle e_0 - \sum_{k=0}^{n-1} \rho_k \rangle \quad (8)$$

Now $\rho = \langle T/T_{nx} \rangle = \langle f_{nx}/F \rangle$ and if the reference clock is affected by timing jitter, $x_t(t)$, such that $T_{nx}(n) = T_{nx} + x_t((n+1)T_{nx}) - x_t(nT_{nx})$ then (8) becomes nonlinear. However, it can be shown that low levels of timing jitter can be approximately represented as phase jitter,

$$x_p(nT) = f_{\Delta} \int_{-\infty}^t \dot{x}_t(\tau) d\tau \Big|_{t=nT} \quad [7], \text{ where there is a time-varying frequency } f(t) = f_{nx} + f_{\Delta} \dot{x}_t(t)$$

and $\dot{x}_t(t)$ is the derivative of the timing jitter function. Then $\rho_k = \langle M_k \rangle = \langle (f_{nx} + f_{\Delta} \dot{x}_t(kT))/F \rangle$

and (8) evaluates to

$$e_n = -\langle e_0 - \langle \frac{nf_{nx}}{F} + \frac{f_{\Delta}}{F} \sum_{k=0}^{n-1} x_k \rangle \rangle \quad (9)$$

where $x_k = \dot{x}_i(kT)$. When the jitter on the reference clock is sinusoidal, $x_i(t) = -A_j \sin(2\pi f_j t)$, then (9) becomes [19]

$$e_n = -\langle e_0 - \langle \frac{nf_{nx}}{F} + \frac{f_\Delta}{2F} + \gamma \sin(2\pi f_j nT + \pi f_j T) \rangle \rangle \quad (10)$$

where $\gamma = f_\Delta / 2F \sin(\pi f_j T)$ and $f_\Delta = f_{nx} 2\pi f_j A_j$. Equation (10) is of the same form as the equation for the jitter generated by the timing transfer process when there is timing jitter on the service clock [7]. Following the analysis of [7], the spectrum of the jitter in (10) is

$$E(f) = F \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \alpha_k \sum_{m=-\infty}^{\infty} J_m(2\pi k \gamma) \delta\left(f - k\rho F - nF - m\frac{f_j}{F}\right) \quad (11)$$

To verify this prediction, a comparison was made between the spectrum in (11) and the spectrum plotted using an FFT, based on a simulation of jitter affecting the origin reference clock. Note that the prediction is only valid if the phase jitter accurately approximates the timing jitter affecting the reference clock. As shown in [7], this approximation is valid if $f_j A_j \ll \frac{1}{2\pi}$, $f_j T_{nx} \ll \frac{1}{2\pi}$, as is the case with the results presented here where the timing jitter affecting the reference clock is sinusoidal with $f_j = 20\text{Hz}$, $A_j = 0.09T_{nx}$. In Fig. 4, plots (a) and (c) show the predicted spectrum and plots (b) and (d) show the FFT of the simulated jitter (produced as in Section II) for two different values of ρ . The predicted spectra are plotted using MATLAB from (11) using the first 21 k coefficients and with n and m running to ± 5 . As can be seen, there is very good agreement between the predicted and simulated spectra.

IV References

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Figures

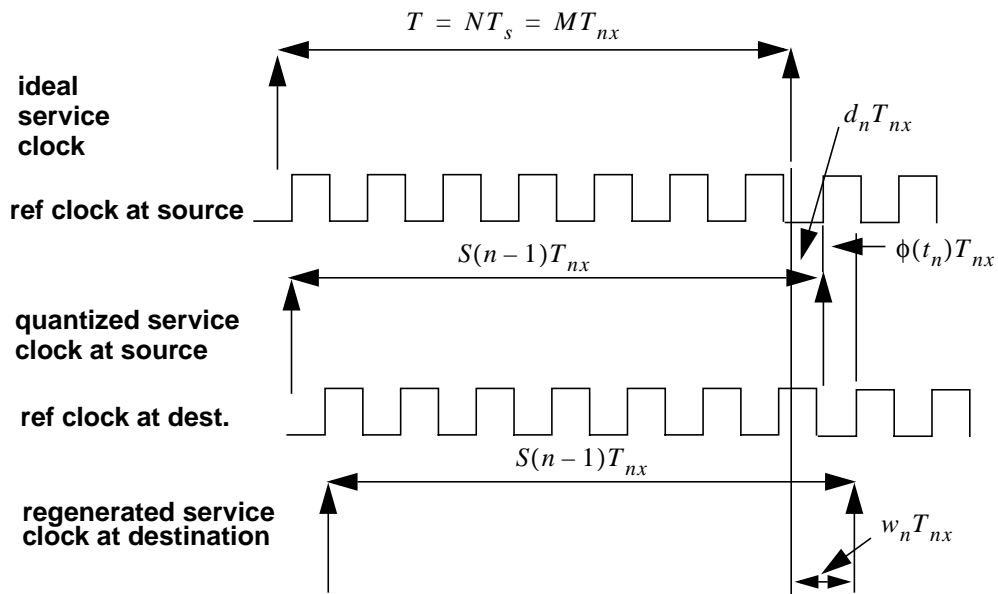


Fig. 1: Timing diagram for synchronization process with jitter on the destination reference clock.

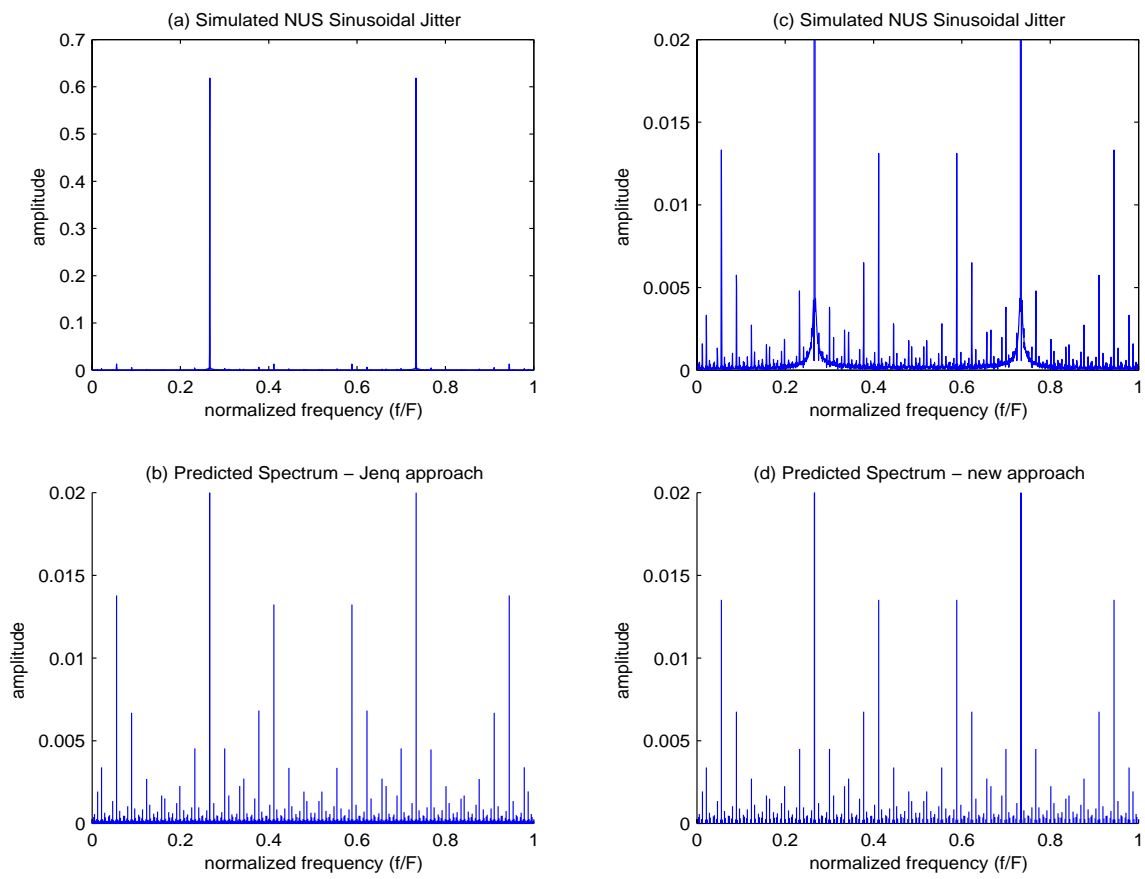


Fig. 2: Jitter on the destination reference clock: (a) Spectrum of simulated non-uniformly sampled (NUS) jitter; (b) Predicted spectrum of NUS jitter using Jenq approach [15]; (c) Spectrum of simulated NUS jitter (scale-change); (d) Predicted spectrum of NUS jitter using new approach.

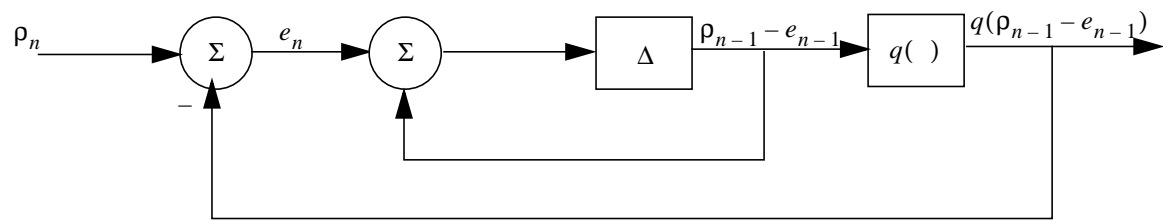


Fig. 3: Rate-adaptation synchronization process modeled as a sigma-delta modulator.

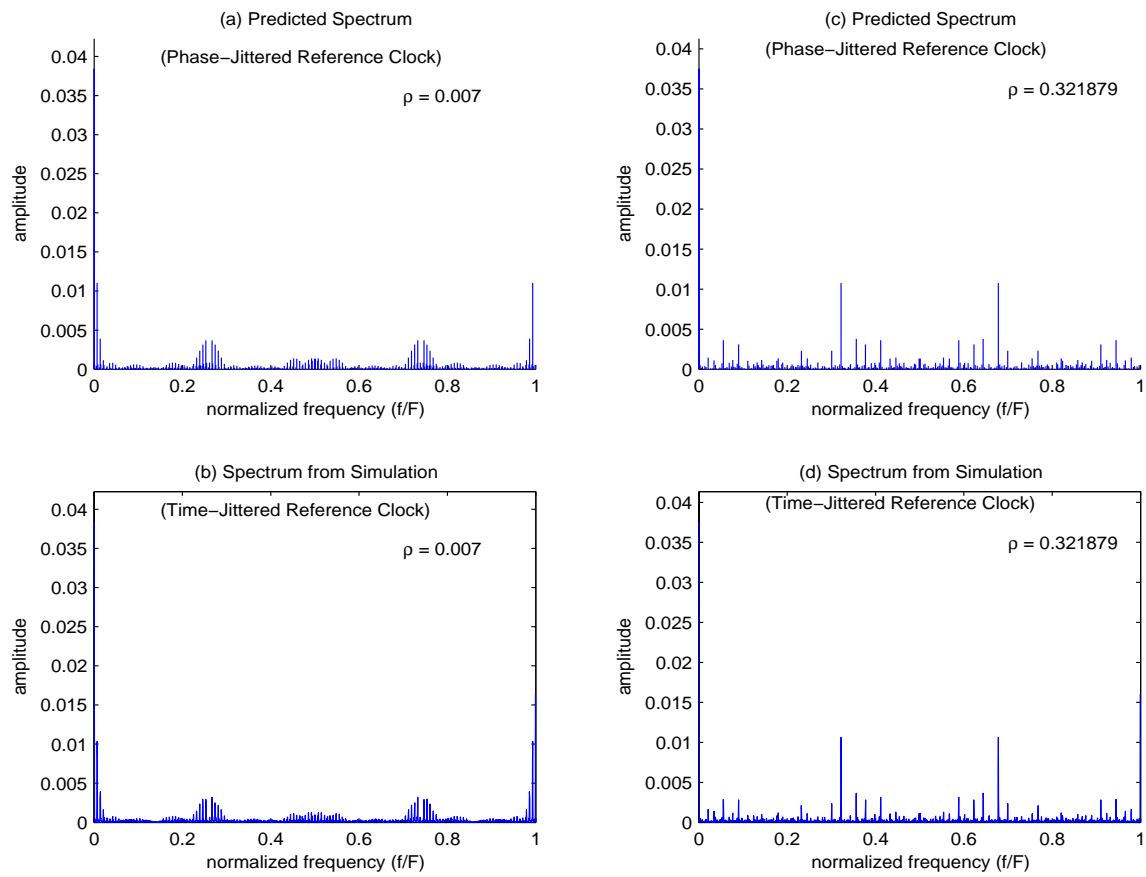


Fig. 4: Jitter on the source reference clock: (a) Predicted spectrum using phase jitter approximation ($\rho = 0.007$) (b) Simulated spectrum using timing jitter ($\rho = 0.007$); (c) Predicted spectrum using phase jitter approximation ($\rho = 0.321879$) (d) Simulated spectrum using timing jitter ($\rho = 0.321879$).