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Cepstrum of Bispectrum - A New Approach to Blind System Reconstruction

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Abstract

In this paper, an improved approach to blind deconvolution of LTI systems incorporating phase unwrapping is presented. The method can recover a noise-free estimate of the logarithm of the system transfer function which enables reconstruction of the system. The algorithm is fast due to simple computation and accurate as it includes phase unwrapping. The proposed method is compared via simulation with other methods, selected as representative of both bispectrum and bicepstrum based techniques. In general, it performs as well as or much better than the other methods considered. The proposed method is also shown to perform well under low signal-to-noise ratios.

Key words: Higher Order Statistics, Bispectrum, Skewness, Bicoherence, Cepstrum, Power Cepstrum, Bicepstrum, Phase Unwrapping, Blind Deconvolution

1 Introduction

Identification of an unknown linear time-invariant (LTI) system when the only available information is the output of the unknown system is an important problem which arises in many signal processing applications. Compared to second-order statistics, higher order statistics can provide more information, as higher order spectra carry the system phase information when the input random process is non-Gaussian and white.

There are many established blind deconvolution techniques that have been developed using higher order statistics, in particular, third order statistics. However, bispectrum based system reconstruction typically requires a large

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amount of computation [1]-[4], although in recent years as computing power and memory continue to increase dramatically, this has become less of a consideration. To reduce the amount of computation required, many researchers [5] - [9] have utilized one or two slices of the bispectral plane to estimate the system from the output data. These approaches may exhibit high variance depending on the selection of slices and errors may accumulate due to the use of recursive computation.

For blind system reconstruction, the cepstra of higher order statistics have also been of interest as they are computationally simpler and can uniquely characterise system phase information. The power cepstrum, bicepstrum and bispectrum of bicoherence are used in system reconstruction techniques such as the Bispectrum Signal Reconstruction (BSR) approach and the Bicepstrum Iterative Reconstruction Algorithm (BIRA) presented in [10]-[14].

A technique for computing the minimum and maximum phase components of a system directly from the bicepstrum by using third-order moments or cumulants and without phase unwrapping was introduced in [15]. The main problem in this approach is the need to estimate the exact number of phase component parameters needed as the minimum and maximum phase components are theoretically of infinite extent [16]. Moreover, a large number of points are required for least-squares estimation if there are many poles and zeros.

In [17]-[19], it is established that the n th order spectrum of the system output signal is factorizable when the LTI system is driven by non-Gaussian white noise, and the system transfer function can be computed from $n + 1$ lines projected onto the n th order cepstral plane. The projected lines are the origin crossing axes; e.g. in the case of the bispectrum, the system transfer function can be estimated from 3 projected lines in the bicepstrum plane: (i) $m = 0, -\infty < n < \infty$; (ii) $n = 0, -\infty < m < \infty$ or (iii) $m = n$. The method proposed in this paper effectively recovers the system information which lies along one of these axes, but as it is arrived at directly by taking the 1-D inverse Fourier transform of the complex logarithm of the bispectrum, and as it also provides further system information off these axes, it has been named the Cepstrum of the Bispectrum. Furthermore, our approach incorporates phase unwrapping [3], [19] and recovers the phase from the overdetermined bispectral equations [1] which provides greater accuracy.

Our approach has been successfully used in recovery of biomedical signals [20]. In the present work, we present the details of the method more fully, in particular providing a detailed account of the phase unwrapping approach used. We also present the results of simulations which allow comparison of our approach with other popular HOS-based techniques in a more general system-recovery problem.

We also note that some slice-based approaches to system recovery [7], [9] have achieved related results. In [7], projection onto 1-D slices of the bispectral plane was used to recover an estimate of the complex cepstrum of a complex LTI system, whereas the phase was recovered recursively from a single slice for real LTI systems. In [9], the complex logarithm followed by the 1-D inverse Z transform (in practice, the 1-D inverse discrete Fourier transform) was applied to oblique slices of the bispectrum. Our approach may be viewed as a special case for real, discrete-time LTI systems of these more general approaches.

The organization of the paper is as follows. The problem definition and preliminaries are discussed in Section 2. The cepstrum of bispectrum and its properties are established in Section 3 and its use in blind system reconstruction is then developed. Results of computer simulation and comparison with other established methods are presented in Section 4. Finally, concluding remarks are drawn in Section 5.

2 Problem Definition - Preliminaries

Consider a signal $x(n)$ observed at the output of an LTI moving average system as follows:

$$x(n) = e(n) \otimes h(n) + w(n) \quad (1)$$

where \otimes denotes the convolution operation, $e(n)$ is a zero mean, stationary, non-Gaussian i.i.d random signal whose second- and third-order moments are assumed non-zero and finite, $h(n)$ is the system impulse response and $w(n)$ is a zero mean, stationary Gaussian noise process uncorrelated with $e(n)$. It is assumed that only the system output signal $x(n)$ is available and the system $h(n)$ is to be estimated from this.

Let $X(k)$, $E(k)$, $H(k)$ and $W(k)$ be the frequency domain representations of $x(n)$, $e(n)$, $h(n)$ and $w(n)$ respectively; $k = 0, 1, 2, \dots, N - 1$ are the discrete frequency indices; and N is the number of Fourier frequency points. The bispectrum of the output signal $B_x(k, l)$ can be defined as:

$$B_x(k, l) = \gamma_e H(k)H(l)H^*(k + l) \quad (2)$$

where $*$ denotes the conjugate term; k, l are the frequency indices and γ_e is the skewness of the input random process. The output bispectrum estimated by (2) does not have any contribution from the system's noise signal since it is Gaussian. The cepstrum, $c_x(m)$, power cepstrum, $p_x(m)$, and the bicepstrum, $b_x(m, n)$, can be written as [21], [22]:

$$\begin{aligned} c_x(m) &= F_1^{-1}[\log\{X(k)\}] \\ &= c_e(m) + c_h(m) + c_w(m) \end{aligned} \quad (3)$$

$$\begin{aligned} p_x(m) &= F_1^{-1}[\log\{|X(k)|^2\}] \\ &= p_e(m) + p_h(m) + p_w(m) \end{aligned} \quad (4)$$

$$\begin{aligned} b_x(m, n) &= F_2^{-1}[\log\{B_x(k, l)\}] \\ &= F_2^{-1}[\log\{\gamma_e\}] + F_1^{-1}[\log\{H(k)\}]\delta(n) \\ &\quad + F_1^{-1}[\log\{H(l)\}]\delta(m) + F_1^{-1}[\log\{H^*(k+l)\}]\delta(m-n) \end{aligned} \quad (5)$$

where $F_1^{-1}[\circ]$ and $F_2^{-1}[\circ]$ denote the 1-D and 2-D inverse Fourier transform respectively; $\delta(\circ)$ denotes the Kronecker delta function; m, n are the cepstral indices; c_e, c_h, p_e and p_h are the cepstrum and power cepstrum of $e(n)$ and $h(n)$ respectively; and c_w and p_w are error terms due to the additive white noise [22]. From (5), it is noted that the bicepstrum exists only for $m = 0, n = 0, m = n$ and $m = n = 0$.

3 Cepstrum of Bispectrum

The cepstrum of bispectrum was introduced in [23]. The cepstrum of bispectrum can be found by applying a 1-D inverse Fourier transform operation to the logarithm of the bispectrum (a 2-D frequency domain signal).

$$c_{B_x}(k, m) = F_1^{-1}[\log\{B_x(k, l)\}] \quad (6)$$

where k, l are the frequency indices and m is the cepstral index. The choice between k, l for the cepstrum of bispectrum computation is arbitrary. The cepstrum of the bispectrum is a representation of frequency vs. time vs. its log-amplitude. Using (2) in (6) gives [23]:

$$\begin{aligned} c_{B_x}(k, m) &= F_1^{-1}[\log\{\gamma_e\}]_l + F_1^{-1}[\log\{H(k)\}]_l + F_1^{-1}[\log\{H(l)\}]_l \\ &\quad + F_1^{-1}[\log\{H^*(k+l)\}]_l \\ &= \log\{\gamma_e\}\delta(m) + \log\{H(k)\}\delta(m) + F_1^{-1}[\log\{H(l)H^*(k+l)\}]_l \end{aligned} \quad (7)$$

where $F_1^{-1}[\circ]_l$ denotes the 1-D inverse Fourier transform to be applied on the frequency axis l ($= 0, 1, 2, \dots, N-1$). The last term of (7) is the correlation sequence of two (normal and conjugated) Fourier frequency terms of the system. Note that the conjugated frequency term is always ahead of the other term by a constant value k (between 0 and $N-1$). Due to the periodic nature of the Fourier frequency component, (7) can be expressed as:

$$\begin{aligned}
c_{B_x}(k, m) &= \log\{\gamma_e\}\delta(m) + \log\{H(k)\}\delta(m) + F_1^{-1}[\log\{H(l)\}]_l \\
&\quad + e^{-j\frac{2\pi km}{N}} F_1^{-1}[\log\{H^*(l)\}]_l \\
&= \log\{\gamma_e\}\delta(m) + \log\{H(k)\}\delta(m) + c_h(m) + e^{-j\frac{2\pi km}{N}} c_h(-m)
\end{aligned} \tag{8}$$

From (8), it can be concluded that the cepstrum of bispectrum is a linear combination of the complex cepstrum of the system modified by a linear phase term and the logarithm of the Fourier system response. As, theoretically, there is no contribution from Gaussian noise in the bispectrum, (8) could be used after suitable manipulation (for example see [7]) to provide a noise-free estimate of the cepstrum of the system impulse response, which is not the case when the cepstrum is taken directly. At $m = 0$, (8) can be simplified to:

$$\begin{aligned}
c_{B_x}(k, 0) &= \log[\gamma_e] + p_h(0) + \log\{H(k)\} \\
&= \log[\gamma_e] + \log[|\alpha|^2] + \log\{H(k)\}
\end{aligned} \tag{9}$$

Thus, at $m = 0$, $p_h(m)$ is a constant amplification factor (α) for the system [16], [24] and the system transfer function is directly related to the cepstrum of bispectrum because γ_e and $|\alpha|^2$ are constant with respect to the index k .

3.1 System Estimation

A cepstrum of bispectrum value is both a frequency and time domain parameter that can be computed by applying a 1-D inverse Fourier transform to the logarithm of the bispectrum. Letting $H(0) = 1$, (9) can be rewritten as [24]:

$$c_{B_x}(0, 0) = \log[\gamma_e] + \log[|\alpha|^2] \tag{10}$$

The above equation describes the scale indetermination due to the fact that the true value of $H(0)$ cannot be determined. By subtracting (10) from (9) and considering only the axis $m = 0$, the logarithm of the system transfer function can be expressed as:

$$\begin{aligned}
c_{B_x}(k, 0) - c_{B_x}(0, 0) &= \log[H(k)] \\
&\text{for } k = 1, 2, \dots, N/2 - 1
\end{aligned} \tag{11}$$

where N is the length of the Fourier transform used in the cepstrum of bispectrum estimation. The frequency domain system transfer function can then be found by applying the exponential operator to (11).

$$H(k) = \exp[c_{B_x}(k, 0) - c_{B_x}(0, 0)] \quad (12)$$

for $k = 1, 2, \dots, N/2 - 1$

It is noted that the scale factor of the recovered system transfer function in (12) is still indeterminate as the true value of $H(0)$ is not known.

3.2 Phase Unwrapping

In system estimation, we are generally working with an estimate, $\hat{B}_x(k, l)$, of the bispectrum as expressed by (2). Equation (6) may be rewritten in terms of the magnitude and phase of the estimated bispectrum as:

$$c_{\hat{B}_x}(k, m) = \frac{1}{N} \sum_{l=0}^{N-1} \log[|\hat{B}_x(k, l)| \exp^{j\hat{\psi}_x(k, l)}] \exp^{j2\pi \frac{ml}{N}} \quad (13)$$

where $\hat{\psi}_x(k, l)$ is the phase of the estimated bispectrum. At $m = 0$ this becomes:

$$c_{\hat{B}_x}(k, 0) = \frac{1}{N} \sum_{l=0}^{N-1} \log|\hat{B}_x(k, l)| + j \frac{1}{N} \sum_{l=0}^{N-1} \hat{\psi}_x(k, l) \quad (14)$$

and the recovered system phase is determined by the right-hand term in (14). Equation (2) which relates the bispectrum to the system transfer function can be expressed in terms of its magnitude and phase [3]. Considering only the phase we have:

$$\psi_x(k, l) = \phi(k) + \phi(l) - \phi(k + l) \quad (15)$$

where $\psi_x(k, l)$ and $\phi(k)$ refer to the true bispectral and system phase respectively and it is furthermore assumed that the system phase is the principal argument, $-\pi < \phi(k) \leq \pi$. Substituting (15) in (14) and using the periodicity and symmetry of the system Fourier coefficients for discrete, real systems, it can be shown that $\frac{1}{N} \sum_{l=0}^{N-1} \psi_x(k, l) = \phi(k)$. Thus, the cepstrum of bispectrum could, in theory, produce a perfect phase estimate but this depends on whether $\hat{\psi}_x(k, l)$ as obtained from the data is a good estimate of the true bispectral phase, $\psi_x(k, l)$, as calculated by (15). Unfortunately, when estimating the bispectrum, only the principal argument of the bispectrum phase is available [3], [25] whereas in (15), assuming the principal argument of the true system phase, the resulting true bispectral phase $\psi_x(k, l) \in (-3\pi, 3\pi)$ [3].

To quantify the error which results from using the principal argument bispectral phase, we express it in terms of the true bispectral phase as $\hat{\psi}_x(k, l) = \psi_x(k, l) - 2\pi n(k, l)$ where $n(k, l) \in [-1, 0, 1]$ from (15) [3]. Thus

$$\begin{aligned} \frac{1}{N} \sum_{l=0}^{N-1} \hat{\psi}_x(k, l) &= \frac{1}{N} \sum_{l=0}^{N-1} \left(\psi_x(k, l) - 2\pi n(k, l) \right) \\ \hat{\phi}(k) &= \phi(k) - \frac{2\pi}{N} \lambda(k) \end{aligned} \quad (16)$$

where $\lambda(k) = \sum_{l=0}^{N-1} n(k, l)$, i.e., the phase recovered from the cepstrum of bispectrum is the true system phase perturbed by a phase error term which is an integer multiple of $2\pi/N$ where N is the size of the FFT.

The proposed phase unwrapping technique is based on the approach introduced by [3] which aims to determine the elements $n(k, l)$. Multiplying the vector of unique (for a discrete-time, real system) system phase estimates, $\hat{\phi} = [\hat{\phi}(1), \hat{\phi}(2), \dots, \hat{\phi}(N/2 - 1)]^T$, obtainable from (16), by the matrix \mathbf{A}_ϕ , we obtain:

$$\mathbf{A}_\phi \hat{\phi} = \psi_x - \mathbf{A}_\phi \frac{2\pi}{N} \lambda \quad (17)$$

where ψ_x is the $\frac{N^2}{16} \times 1$ vector of the principal domain terms of the true bispectral phase [3] and λ is the $(\frac{N}{2} - 1) \times 1$ vector of phase error terms and

$$\mathbf{A}_\phi = \begin{bmatrix} 2 & -1 & 0 & 0 & \dots & 0 \\ 1 & 1 & -1 & 0 & \dots & 0 \\ 1 & 0 & 1 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 2 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 2 & \dots & -1 \end{bmatrix}$$

and we note that $\hat{\phi}(0)$ is directly obtainable from $\hat{\psi}_x(0, 0)$. Subtracting the principal domain elements of the principal argument bispectral phase $\hat{\psi}_x$ from (17) and dividing by 2π gives

$$\frac{1}{2\pi} \left\{ \mathbf{A}_\phi \hat{\phi} - \hat{\psi}_x \right\} = \mathbf{n} - \mathbf{A}_\phi \frac{1}{N} \boldsymbol{\lambda} \quad (18)$$

where \mathbf{n} is the vector of phase differences between the true bispectral phase and the principal argument bispectral phase and the right-hand term is an error term. The resulting vector clearly does not always have integer values, so the elements are rounded to the nearest integer value. Once an estimate of the elements $n(k, l)$ is found, they can be added to the principal argument bispectral phase and a least squares estimate of the true system phase value is then found as in (19) [3]. Although finding the least squares estimate is computationally more intensive it produces a better result and, as mentioned in the introduction, ever increasing computing power makes this objection less significant.

$$\phi = \left(\mathbf{A}_\phi^T \mathbf{A}_\phi \right)^{-1} \mathbf{A}_\phi^T \psi_x \quad (19)$$

3.3 Time Domain System Impulse Response

The estimated frequency domain system impulse response is computed by taking the system Fourier magnitude from (12) and the system Fourier phase term from (19) as

$$\hat{H}(k) = |\hat{H}(k)| \exp(j\phi(k)) \quad (20)$$

where $|\hat{H}(k)|$ is the magnitude of estimated value of $H(k)$. For the time domain system impulse response, we use a 1-D inverse Fourier transform on the estimated $H(k)$

$$\hat{h}(n) = F_1^{-1}[\hat{H}(k)] \quad (21)$$

4 Simulation Result

4.1 Description of Simulated Signals

In this section, we demonstrate the performance of the proposed algorithm on minimum phase, non-minimum phase and maximum phase LTI systems. To simulate each test signal (data length is 4096), we use a 5th order MA model where the system input signal is zero mean, non-Gaussian white noise (Poisson random process) and the system noise is zero mean, white Gaussian noise (uncorrelated with the input random process). The time domain value of the systems and the position of their zeros are noted in Table 1.

We perform 50 Monte Carlo runs for different inputs and noise realizations considering three fixed signal to noise ratios (infinity, 20dB and 10dB) for each type of system. To compute the signal to noise ratio (SNR) we use

$$SNR = 10[\log(\frac{E[y(n)^2]}{E[\{c.w(n)\}^2])}] \quad (22)$$

where $y(n) = e(n) \otimes h(n)$ is the noise free system output signal, c is a constant used here to set the level of the Gaussian white noise $w(n)$ and $E[\circ]$ denotes the statistical expectation. In the simulations the size of the FFT is $N = 128$ points which seems to deliver a good compromise between frequency resolution and variance due to noise.

4.2 Results and Analysis

Since the system noise and the input sequences to the system differ each time, the algorithm may reconstruct the system with a different amplification scaling factor, as the algorithm is blind to the true value of $H(0)$. Therefore, the comparison of true vs. estimated time domain system impulse response is arranged in normalized form. We present the average estimated values and the standard deviations for each of the systems and each SNR in Table 2, Table 3 and Table 4. For each value of SNR in the three different systems, we plotted the true normalized value versus the average estimated value and the 95% confidence intervals in Fig. 2, Fig. 3 and Fig. 4.

From the figures and tables, it may be observed that the shape of the true and average estimated systems are quite similar. Hence, it can be concluded that the proposed algorithm can reconstruct minimum, non-minimum and maximum phase systems for different non-Gaussian white noise input sequences and for different levels of additive Gaussian white noise.

We note that the standard deviation in the estimated values is generally quite small, but increases with decreasing SNR as expected. This occurs even though the bispectrum is theoretically blind to additive Gaussian noise because estimators are finite and thus have variance. The variance of the bispectral estimator is also proportional to the energy present at a particular bifrequency [16] and this may explain why there is more variance in some estimates than others. However, we also observed that this standard deviation can be decreased by increasing the data length used in the estimation.

4.3 Different System Reconstruction Techniques - A Comparison

We compare three different system reconstruction algorithms with the new cepstrum of bispectrum based system reconstruction algorithm. The algorithms are: (i) Log-Bispectra by [3] (R-G), (ii) Bicepstrum by [26] (P-N) and (iii) HOS factorization by [19] (T-E). Among these algorithms R-G and P-N are well established and are used in the Matlab Higher Order Spectral toolbox. The T-E and P-N methods are chosen because they are quite closely related to our approach, whereas the R-G is bispectrum based. The R-G algorithm uses least-squares estimation over the whole principal domain for both magnitude and phase estimation. This approach should be more accurate, because phase unwrapping is used and the set of overdetermined equations is used.

The P-N algorithm is based on the partial differential of the bicepstrum but, in practice, the computation deals with the third order cumulant and a 2-D Fourier transform operation. This technique does not need any phase unwrapping. In the P-N method the minimum and maximum phase components are computed and used to recover the system information. Here the main problem is to compute the exact number of phase components needed - as the number of these components are theoretically infinite [16]. For the purpose of blind deconvolution it is not practical to use the P-N method initially. Moreover, if the system has a large number of minimum and maximum phase components, the P-N method is also computationally intensive.

The T-E method is based on the n th-order spectrum factorization principle. Clearly, use of the cepstrum of bispectrum approach assumes the existence of a 3rd-order spectrum factorization, as demonstrated in [19]. The T-E approach is also not very computationally intensive if the bicepstral projection method proposed in [19] is used. The main differences between the T-E algorithm and the proposed algorithm are: (a) the algorithm used and the bias term that needs to be deducted from the frequency domain impulse response are different; (b) the T-E method does not use a phase unwrapping step whereas the proposed method includes a phase unwrapping technique by which a more accurate system can be estimated.

The comparison between the proposed cepstrum of bispectrum algorithm and the three methods described above is carried out for three different LTI systems (minimum, non-minimum and maximum phase). In these comparisons, the average SNR is 10dB and 50 Monte Carlo runs are considered with data length of 2048. Table 5 shows the true values of the coefficients of each system and the position of the zeros.

Table 6 shows the normalized true versus average estimated system impulse response with the associated standard deviations. From Table 6, we note that

the standard deviation for the P-N algorithm is always very low. This algorithm uses minimum and maximum phase components to reconstruct the system and these components are free from any skewness function - as the bispectrum separates the skewness parameter from the minimum and maximum phase components. Fig. 5 to Fig. 7 have been plotted from Table 6 on a system by system basis. These figures illustrate the normalized actual vs. estimated impulse response. From the graphical comparison, it is observed that both the P-N and the proposed algorithm show high performance as the average estimated system is very close to the actual system. On the other hand, the T-E algorithm shows the worst performance. The main reason is that phase unwrapping is not included in the T-E algorithm.

In conclusion, the P-N and the proposed algorithm have both reconstructed the minimum phase system perfectly and the proposed algorithm also has low variance (see the data for the minimum phase system in Table 6). The same performance is shown in the case of the maximum phase system (see Fig. 7 and Table 6). But in the case of the non-minimum phase system, the P-N algorithm shows better performance. Nevertheless, it can be stated that the proposed algorithm has highly acceptable performance.

4.3.1 *Special Case: When the signal to noise ratio is extremely low*

In signal processing it is normally assumed that the sampled signal contains some system noise and the power of any signal of interest is greater than the power of noise. However, it is not impossible that a signal (e.g. a biomedical signal) contains a noise level that is higher than the level of the signal of interest. Therefore, a system has been chosen where the signal to noise ratio is extremely low, that is, the noise level is higher than the expected signal level. A 5th order non-minimum phase MA system with time domain system impulse response $[0.7999, -1.5381, 2.1401, 0.3827, -0.9115, 0.5532]$ is chosen for this experiment. 50 Monte Carlo runs are considered here for each reconstruction method and the length of each data sequence is 2048. The chosen SNR is -5.8192. Fig. 8 illustrates the normalized actual vs. the average estimated system impulse response obtained by the four algorithms.

It is observed that the proposed algorithm and the P-N algorithm are successful in reconstructing the system impulse response. It is important to note that the variance for this case is high due to the noise level. Thus, the proposed cepstrum of bispectrum based algorithm can recover the system at a very low SNR level.

5 Conclusion

In this paper, we have presented a blind system reconstruction approach. The cepstrum of bispectrum is a hybrid time-frequency function. It allows the straightforward extraction of the logarithm of the frequency domain system representation and also presents the possibility of obtaining a noise-free estimate of the cepstrum. The method incorporates phase unwrapping and the use of least-squares phase estimation to improve accuracy. The proposed algorithm is computationally simple and its performance has been tested with simulated signals. The simulation results are promising. We found the variance of the results is generally low even in the presence of additive noise. We also found that the approach worked well in very low signal to noise ratio conditions. We have provided plots and tabulated results comparing the performance of the proposed algorithm with three other established methods. We found that our approach performed as well as the best other method examined [26] and it does not have some of the computational drawbacks of that method.

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Table 1
Description For Minimum, Nonminimum, And Maximum Phase System

Phase System	Minimum Phase System	Nonminimum Phase System	Maximum Phase System
Time domain system impulse response	1.6563	0.4679	-0.9010
	-0.2807	0.3511	-0.0595
	-0.4487	1.6384	0.3480
	0.4890	-0.3855	-0.2973
	-0.2274	0.9573	0.7241
	-0.0606	-0.0304	2.6789
Position of Zeros	-0.8030,	-0.5976±j1.7530	-1.0458
	0.6321,	0.2063±j0.7398	1.0082±j0.802
	-0.1842,	0.0321	0.4522±j1.2278
	0.2623±j0.5680		

Table 2
Minimum Phase System - Real Vs. Average Estimated System (50 Runs) Comparison With Respective Standard Deviation For Different Input Signal And Different Signal To Noise Ratio.

		SNR = inf.	SNR = 20.0547	SNR = 10.6891
	Actual	Mean ± std	Mean ± std	Mean ± std
h(1)	1	1 ± 0	1 ± 0	1 ± 0
h(2)	-0.1695	-0.1901 ± 0.0005	-0.1895 ± 0.0024	-0.1887 ± 0.0072
h(3)	-0.2709	-0.3236 ± 0.001	-0.3234 ± 0.0030	-0.3238 ± 0.0062
h(4)	0.2952	0.3004 ± 0.0016	0.3003 ± 0.0024	0.3013 ± 0.0067
h(5)	-0.1373	-0.1298 ± 0.0008	-0.1289 ± 0.0026	-0.1289 ± 0.0066
h(6)	-0.0366	-0.0576 ± 0.0005	-0.0567 ± 0.0022	-0.0568 ± 0.0048

Table 3
Nonminimum Phase System - Real Vs. Average Estimated System (50 Runs) Comparison With Respective Standard Deviation For Different Input Signal And Different Signal To Noise Ratio.

		SNR = inf.	SNR = 20.5025	SNR = 10.3658
	Actual	Mean ± std	Mean ± std	Mean ± std
h(1)	0.2856	0.2411 ± 0.0025	0.2409 ± 0.0028	0.2428 ± 0.0081
h(2)	0.2143	0.2194 ± 0.0024	0.2194 ± 0.0025	0.2195 ± 0.0077
h(3)	1	1 ± 0	1 ± 0	1 ± 0
h(4)	-0.2353	-0.2693 ± 0.0039	-0.2698 ± 0.0032	-0.2687 ± 0.0077
h(5)	0.5843	0.5555 ± 0.0028	0.5551 ± 0.0027	0.5552 ± 0.0085
h(6)	-0.0186	-0.0249 ± 0.0032	-0.0249 ± 0.0033	-0.0242 ± 0.0071

Table 4

Maximum Phase System - Real Vs. Average Estimated System (50 Runs) Comparison With Respective Standard Deviation For Different Input Signal And Different Signal To Noise Ratio.

		SNR = inf.	SNR = 20.1139	SNR = 10.2799
	Actual	Mean ± std	Mean ± std	Mean ± std
h(1)	-0.3363	-0.3422 ± 0.0034	-0.3415 ± 0.0040	-0.3408 ± 0.0076
h(2)	-0.0222	-0.0182 ± 0.0004	-0.0183 ± 0.0021	-0.0177 ± 0.0057
h(3)	0.1299	0.1226 ± 0.0009	0.1229 ± 0.0021	0.1243 ± 0.0053
h(4)	-0.111	-0.1073 ± 0.0006	-0.107 ± 0.0024	-0.1066 ± 0.0049
h(5)	0.2703	0.2633 ± 0.0006	0.2634 ± 0.0018	0.2639 ± 0.0053
h(6)	1	1 ± 0	1 ± 0	1 ± 0

Table 5

Systems Used For Test Signal Simulation - System Sequence And Its Position Of Zeros

System	Minimum Phase System	Nonminimum phase System	Maximum Phase System
System Sequence	1.2534	0.7999	-0.0067
	-0.4645	-1.5381	-0.5484
	-0.9833	2.1401	-1.2027
	0.7425	0.3827	1.1479
	0.1877	-0.9115	-0.1886
	-0.1722	0.5532	2.1373
Position of zeros	-0.8579	-0.7277	-80.4923
	-0.5389	0.9212 ±	-3.1395
	0.5155	i1.3871	1.2483
	0.6259 ±	0.4040 ±	-0.1953 ±
	i0.4298	i0.4237	i0.9922

Table 6
 Comparison Of Different Algorithms - Real And Average Estimated System (50
 Runs) With Respective Standard Deviation. The Average Signal To Noise Ratio.
 (SNR) is 15 db

		Actual	RG Mean \pm std	P-N Mean \pm std	T-E Mean \pm std	Proposed Mean \pm std
M I N I M U M P H A S E	h(1)	1	1 \pm 0	1 \pm 0	1 \pm 0	1 \pm 0
	h(2)	-0.3706	-0.397 \pm 0.0197	-0.4248 \pm 0.0364	-0.5734 \pm 0.0222	-0.389 \pm 0.021
	h(3)	-0.7845	-0.7602 \pm 0.0165	-0.7471 \pm 0.0272	-0.6175 \pm 0.0112	-0.8341 \pm 0.0175
	h(4)	0.5924	0.6014 \pm 0.0126	0.5912 \pm 0.0129	\pm 0 \pm 0.012	0.5914 \pm 0.0125
	h(5)	0.1498	0.1035 \pm 0.0106	0.1249 \pm 0.0172	0.0764 \pm 0.0095	0.1903 \pm 0.0111
	h(6)	-0.1374	-0.1235 \pm 0.0106	-0.1351 \pm 0.0068	-0.1146 \pm 0.0093	-0.1729 \pm 0.0119
N O N M I N I M U M P H A S E	h(1)	0.3738	0.3829 \pm 0.0125	0.3661 \pm 0.008	0.571 \pm 0.02	0.233 \pm 0.0141
	h(2)	-0.7187	-0.7225 \pm 0.0142	-0.7256 \pm 0.0094	-0.9173 \pm 0.0361	-0.7528 \pm 0.017
	h(3)	1	1 \pm 0	1 \pm 0	0.6693 \pm 0.0514	1 \pm 0
	h(4)	0.1788	0.1699 \pm 0.014	0.1671 \pm 0.0092	1 \pm 0	0.2388 \pm 0.0138
	h(5)	-0.4259	-0.4332 \pm 0.0137	-0.4311 \pm 0.0107	-0.8979 \pm 0.0361	-0.4734 \pm 0.0151
	h(6)	0.2585	0.2668 \pm 0.0119	0.2532 \pm 0.0088	0.1528 \pm 0.0166	0.2348 \pm 0.0112
M A X I M U M P H A S E	h(1)	-0.1015	-0.1178 \pm 0.0126	-0.099 \pm 0.0068	-0.1576 \pm 0.0255	-0.0894 \pm 0.0132
	h(2)	-0.0583	-0.0564 \pm 0.0094	-0.063 \pm 0.0055	0.3507 \pm 0.0285	-0.0815 \pm 0.009
	h(3)	-0.2843	-0.3092 \pm 0.0107	-0.2884 \pm 0.0079	-0.0985 \pm 0.0206	-0.3068 \pm 0.0114
	h(4)	0.4028	0.3963 \pm 0.011	0.3974 \pm 0.0066	0.8639 \pm 0.0286	0.3522 \pm 0.0108
	h(5)	-0.5146	-0.5035 \pm 0.0106	-0.5167 \pm 0.0085	-0.9064 \pm 0.0283	-0.5286 \pm 0.011
	h(6)	1	1 \pm 0	1 \pm 0	1 \pm 0	1 \pm 0

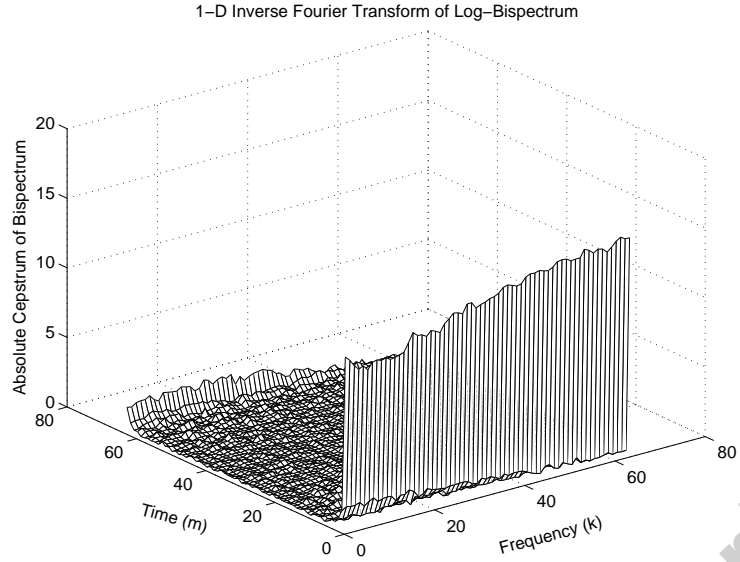


Fig. 1. An example of typical cepstrum of bispectrum.

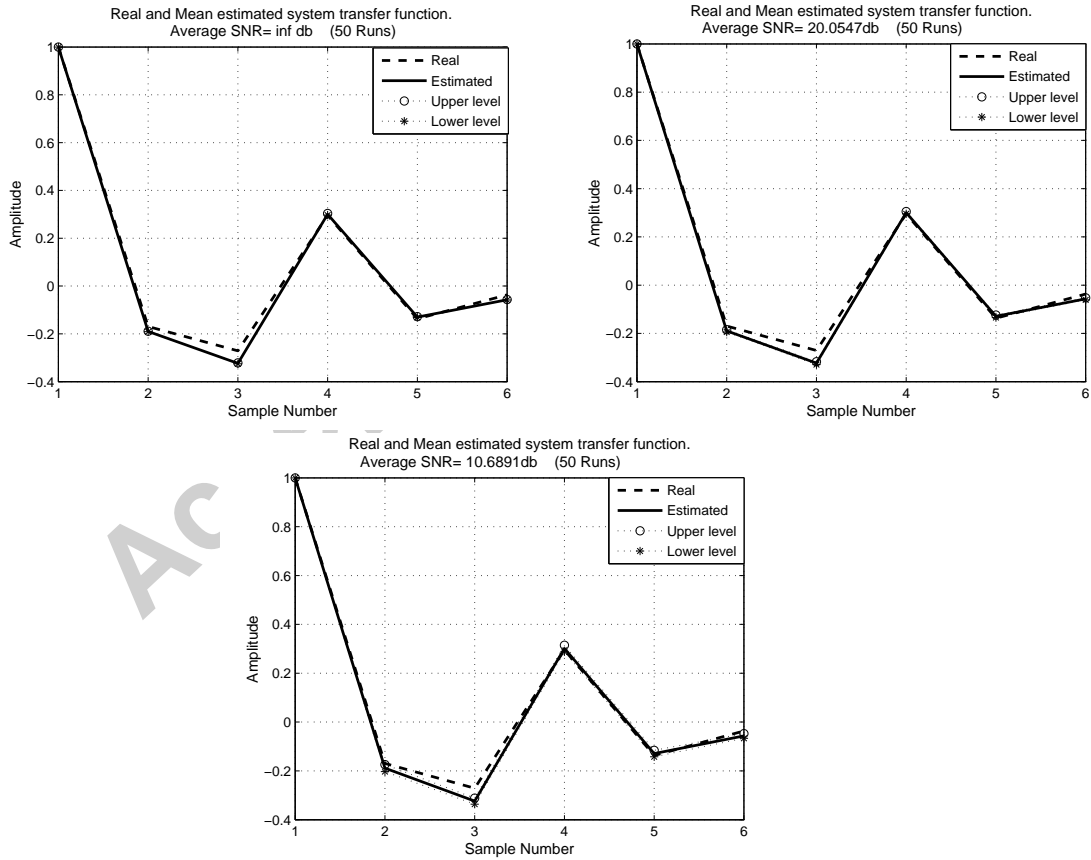


Fig. 2. Real and estimated system (minimum phase system) transfer function using cepstrum of bispectrum based algorithm (a) SNR=Infinity, (b) SNR=20db and (c) SNR=10db

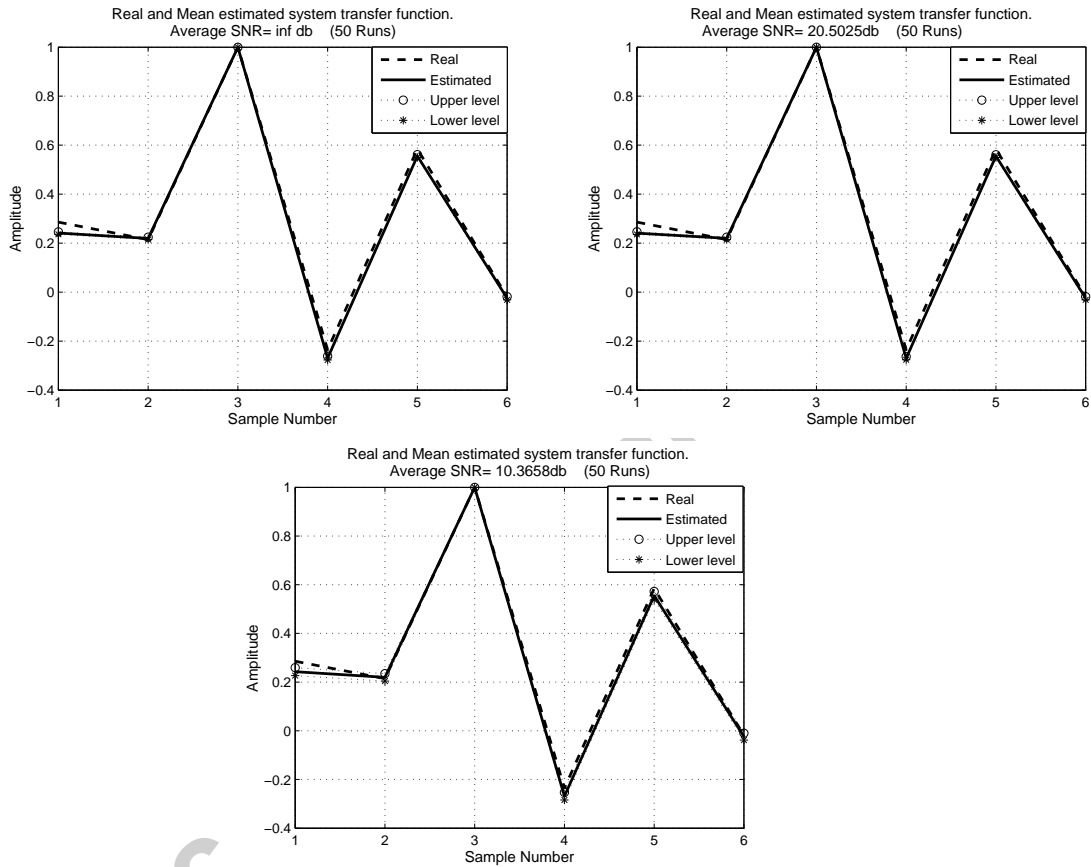


Fig. 3. Actual and estimated system (nonminimum phase system) transfer function using cepstrum of bispectrum based algorithm (a) SNR=Infinity, (b) SNR=20db and (c) SNR=10db

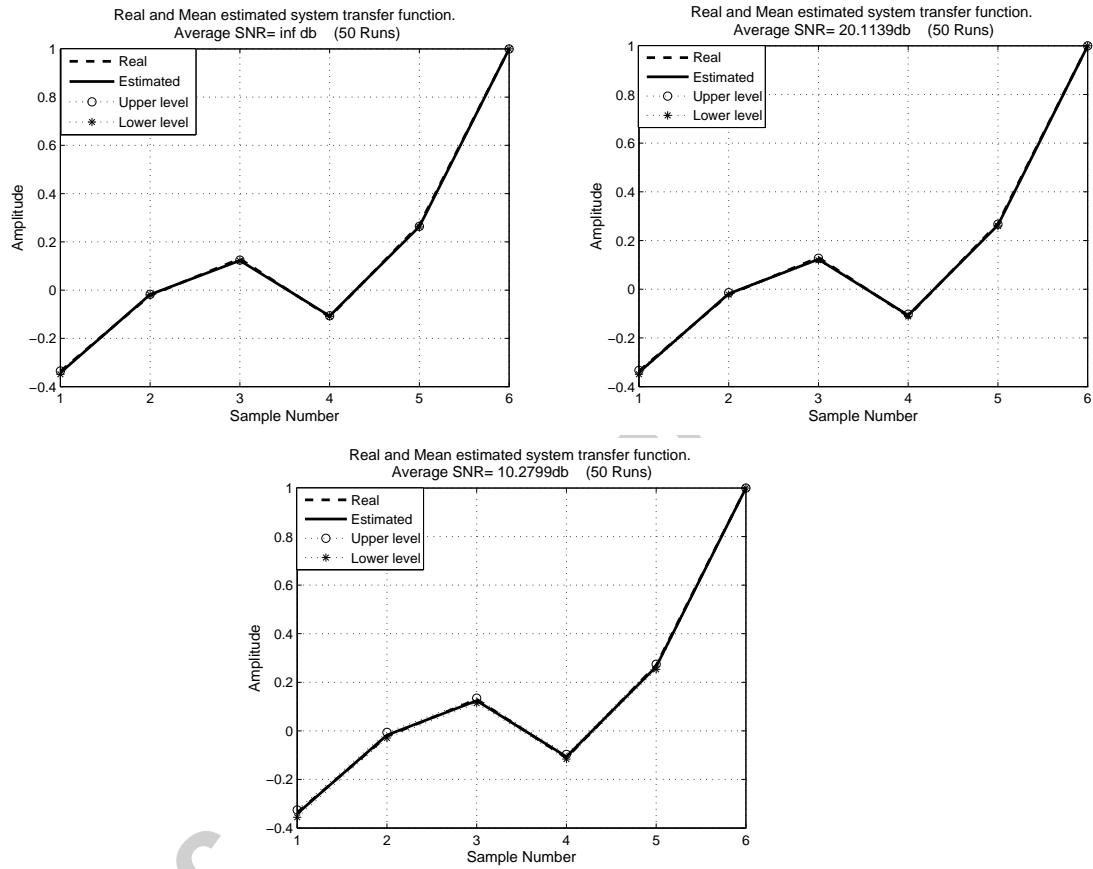


Fig. 4. Real and estimated system (maximum phase system) transfer function using cepstrum of bispectrum based algorithm (a) SNR=Infinity, (b) SNR=20db and (c) SNR=10db

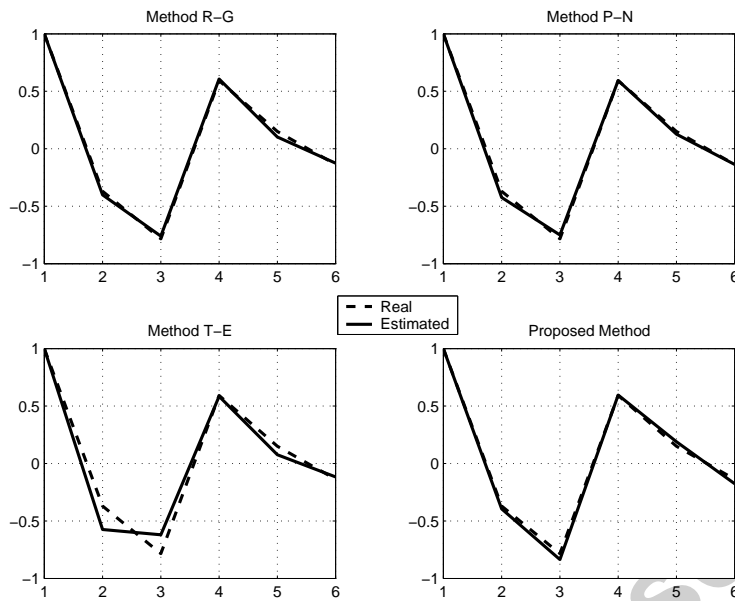


Fig. 5. Actual vs. estimated minimum phase system. Reconstructed by the methods of R-G, P-N, T-E and Proposed in Section III. (average SNR=10.3577 db)

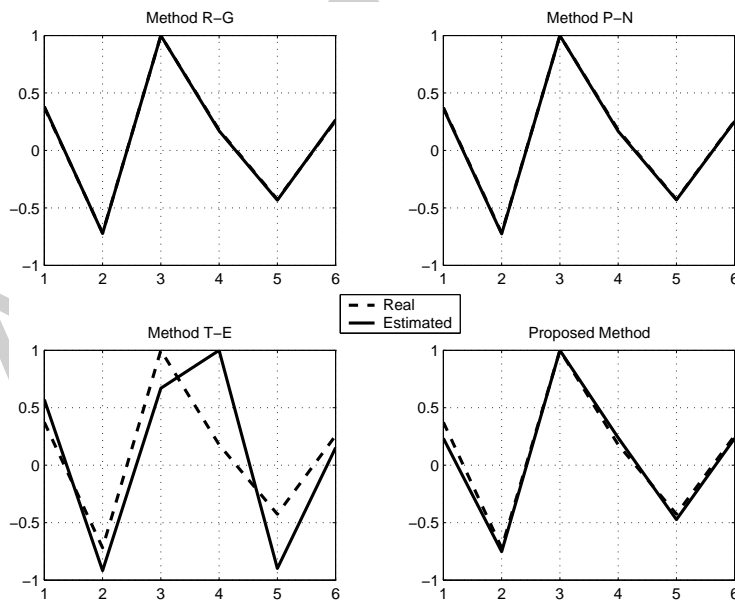


Fig. 6. Actual vs. estimated nonminimum phase system. Reconstructed by the methods of R-G, P-N, T-E and Proposed in Section III. (average SNR=10.1421 db)

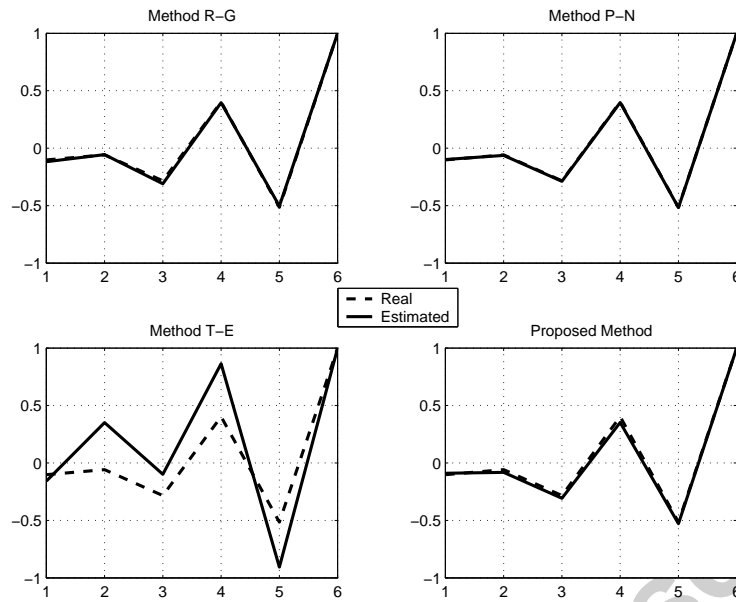


Fig. 7. Actual vs. estimated maximum phase system. Reconstructed by methods of R-G, P-N, T-E and Proposed in Section III. (average SNR=10.2947 db)

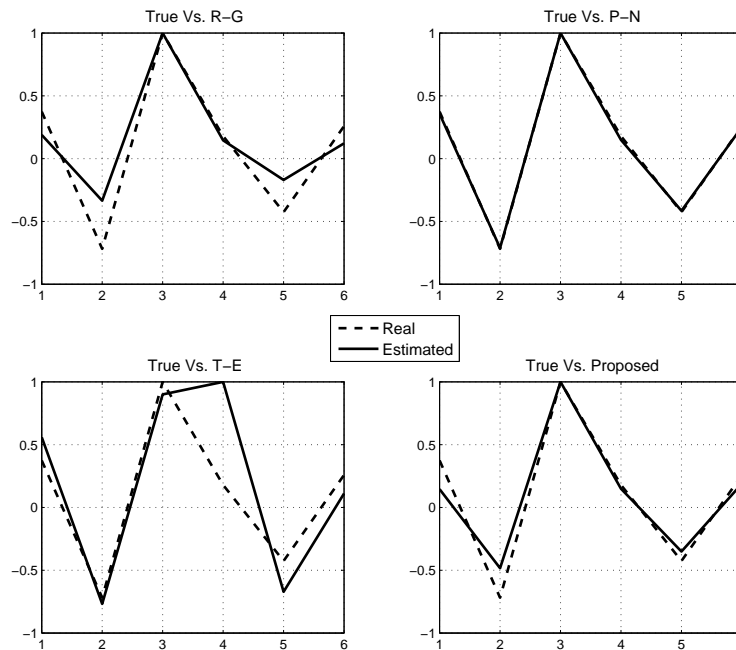


Fig. 8. System reconstruction when the test signal is extremely contaminated by the noise. (average SNR=-3.7634 db)