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A Simple Model to Estimate Yield Stress and Hardness Variation in Railheads

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Abstract

This technical note presents a macroscopic model capable to estimate the variation of hardness and yield stress at different railhead distances (depths) from the running surface. Published data, including results reported in past work by the authors, have been utilised to calibrate and test the validity of the model. From this preliminary investigation it was found that the model can predict accurately the measured hardness and yield stress values, as well as represent the variation profile exhibited in the examined railhead material. This model, subject to further validation, has the potential to be used in practical applications.

Keywords

Rail, steel, hardness, yield stress, isotropic hardening, macroscopic model.

Background and objectives

Wear and ratcheting modelling and simulation in steel railhead requires accurate knowledge of the material mechanical properties under cyclic loading. In particular, the description of isotropic hardening (yield stress change) is an important element in material modelling, since railhead material exhibits a non-uniform variation of the yield stress at different distances from the railhead running surface (depth levels) due to heat treatments. To this end, tensile tests on coupons obtained from various depths are used to determine the yield stress values, which are then fed into analytical and computational models predicting damage accumulation (e.g. wear, fatigue, ratcheting) in rails. Moreover, hardness measurement across the railhead profile is commonly employed as an alternative and less cumbersome method to identify indirectly the material varying mechanical properties, since yield stress can be estimated from hardness values.

However, discrete values can only offer a nonhomogeneous description of the varying railhead material mechanical properties (yield stress / hardness) across its depth. This technical note presents a model that represents the head-hardened rail steel yield stress and hardness as a function of distance from running head surface (depth). Such model can be used as a prediction tool, when sufficient data are available for its calibration. Moreover, employing a continuous function of this kind can be embedded in existing isotropic hardening models for steel alloys, which in turn offers a unified mathematical representation of the material plastic properties.

Research method

Experimental data

The proposed model was developed and calibrated with the use of yield stress and hardness data from different railhead depths, sourced from previously published research[1-5]. The research studies, rail material, yield stress/hardness measurement details and the corresponding figures presenting the experimental data are listed in Table 1.

Table 1 Experimental data type, sources and railhead material.

Research Study	**Railhead Material**	**Measurement Type**	
		Yield Stress	**Hardness**
Athukorala et al[1]	Virgin rail samples from Australian AS1085.1 heavy haul track	Yes (Tensile Tests)	Yes (HV100; 30 measurements/specimen for four depths; ASTM E384 and E122)
		Fig. 1a	*Fig. 1b*
Bandula-Heva & Dhanasekar[2-4]	Virgin rail samples from Australian AS1085.1 heavy haul track	Yes (Tensile Tests)	N/A
		Fig. 2	
Ahlstrom & Karlsoon[5]	Virgin rail samples from common rail steel quality UIC grade 900A	N/A	Yes (HV20; 81 measurements throughout the rail cross-section at various depths)
			Fig. 3

Model description

The model was developed on the basis of macroscopic observations from yield stress and hardness experimental data (measurements). These experimental data are presented, for brevity, combined with the output (prediction) of the model in the results section of this technical note (namely, Fig. 1, 2 and 3).

The model represents the variation of yield stress σ_y as a function of depth d (distance from the railhead running surface) and it is given by the following expression:

$$\sigma_y\left(d\right)=k_\infty+k_v e^{-\mu d^{2.5}} \tag{1}$$

Where k_∞ is the saturation value of the yield stress and k_v and μ the parameters controlling respectively the magnitude and evolution pace of the right-hand side (variable) term of the model ($k_v e^{-\mu d^{2.5}}$). Moreover, the 2.5 exponent in the variable term was selected on the basis of obtaining a good fit of data, avoiding the introduction of another parameter in the equation.

Similarly, on the basis of a linear relationship between yield stress and hardness (an assumption confirmed by the experimental data), hardness H can be described by the following expression:

$$H\left(d\right)=h_\infty+h_v e^{-\mu d^{2.5}} \tag{2}$$

Where h_∞ is the saturation value of the hardness and h_v the parameter controlling the magnitude of the variable term of the model. It is noted that parameter μ is the same in both Eq. 1 and 2.

The model was developed through observing to the macroscopic characteristics exhibited by the material at different depth levels. In particular, as illustrated in the sequel, it is the model's nonlinear nature that enables fitting of the yield stress and hardness experimental data. The model calibration is presented in detail in the results section.

One may notice that the $\sigma_y(d)$ function follows largely the formulation of the well know microstructure-dependent Hall-Petch equation:

$$\sigma_y(D) = \sigma_i + k_y D^{-0.5} \tag{3}$$

Where D is the average grain size and the σ_i , friction stress, and k_y , stress intensity coefficient, independent parameters.

However, the proposed model does not intend to relate microstructure characteristics with macroscopic mechanical properties. Further investigation of a possible relation requires insight on the underlying mechanisms in microstructure level, which escapes the aims of the technical note.

Results

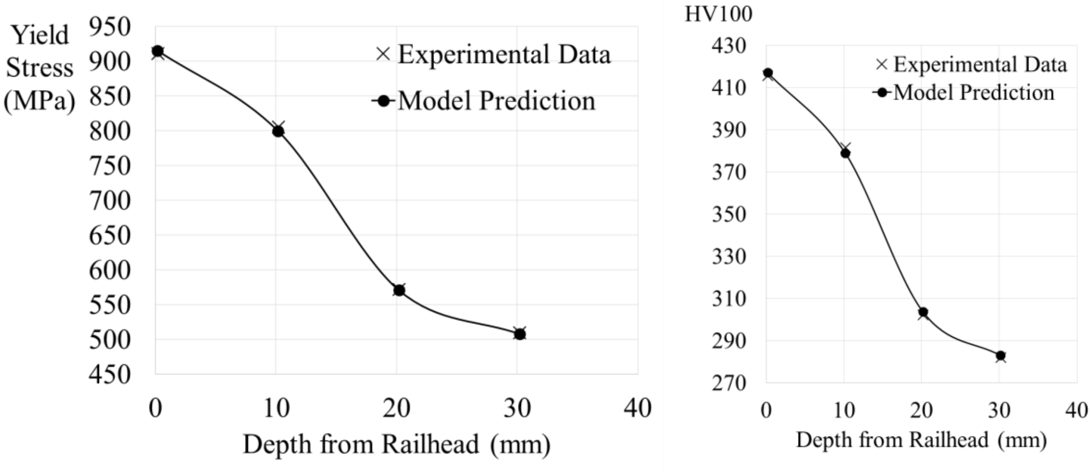
The model has been applied both for yield stress and hardness estimation in the cases (experimental data) presented in Table 1. For the calibration of the $\sigma_y(d)$ model parameters (eq. 1) the following conditions had to be met:

- $d = 0: \quad \sigma_y(0) \cong k_\infty + k_v$ (4)
- $d = \max(d_{measured}) = d_{\max}: \quad \sigma_y(d_{\max}) \cong k_\infty$ (5)

While, the parameter μ was adjusted to best fit the data points, namely to minimise the error at each point [error = (experiment-calculation)/experiment]. Similarly, the aforementioned conditions (Eq. 4 and 5) can be met with relative flexibility (thus the approximate equal notation), which can be beneficial in achieving an overall (average) error reduction goal. The $H(d)$ model was calibrated in the same fashion. The obtained $\sigma_y(d)$ and $H(d)$ model parameters are summarised in Table 2.

The model output (predictions) are presented in conjunction with the experimental data in Fig. 1, 2 and 3. In all cases examined a very good agreement between experimental and predicted data is evident. This is also confirmed by the % average error, which calculated to 0.4%, 3.8% and 0.7% for the data Fig. 1, 2 and 3 data respectively. The noticed differences between measured and calculated data in Fig. 2 (yield stress) and Fig. 3

(hardness) may be attributed to experimental error (such as the outlier hardness data points shown in Fig. 3).



(a) (b)

Fig. 1 Evolution of (a) yield stress and (b) hardness (HV100) at different railhead depths: Experimental data[1] and model prediction.

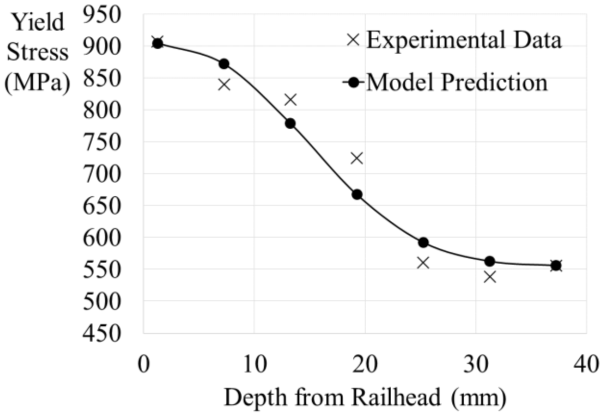


Fig. 2 Evolution of yield stress at different railhead depths: Experimental data[2-4] and model prediction.

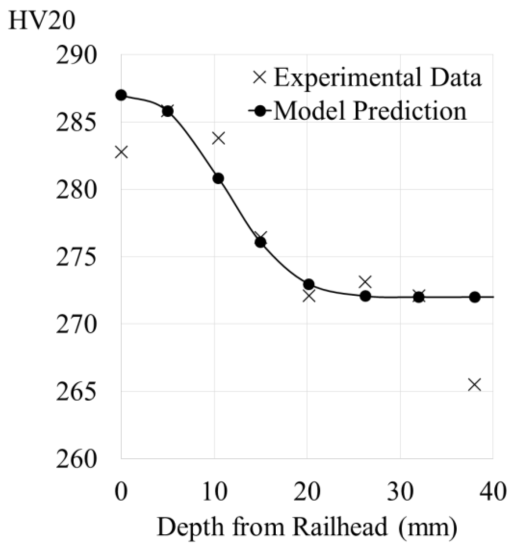


Fig. 3 Evolution of hardness (HV20) at different railhead depths: Experimental data[5] and model prediction.

Table 2 Model parameters for all cases examined.

Research Study	Measurement Type				
		Yield Stress		Hardness	
	Parameters				
	μ	k_∞ (MPa)	k_v (MPa)	h_∞ (HV)	h_v (HV)
Athukorala et al[1]	0.0010	505	410	282	135
		Fig. 1(a)		*Fig .1(b)*	
Bandula-Heva & Dhanasekar[2-4]	0.0007	555	350	N/A	
		Fig. 2			
Ahlstrom & Karlsoon[5]	0.0015	N/A		275	15
				Fig. 3	

Discussion and Conclusions

The model predicts with high accuracy (96.2% to 99.6%) the yield and hardness variation in the railhead depth in all cases examined. This is attributed to the model's exponential nature and the three parameter formulation, which offers flexibility in capturing the experimental data points. Overall, this simple macroscopic model offers, through a generalised representation of the plastic properties' variation, the capability to fit experimental data from different material.

Further validation of the model is, however, necessary and a comprehensive experimental campaign is currently being planned for this purpose. This effort can be complemented by future published data, especially where these data have been collected with the implementation of robust (standardised) methodologies and these results are reported in detail. Moreover, this preliminary investigation can act as a guide for an independent evaluation of the model by other researchers working in the field.

The model may be further examined from a physical point of view, investigating a possible connection of the macroscopically obtained parameters with those microstructural parameters contributing to the variation of yield stress / hardness at different depths (e.g. grain size, interlamellar spacing, etc). This would require a comprehensive analysis of the microstructure, in relation to the macroscopic mechanical properties.

The proposed macroscopic model, subject to further validation, can be useful for head-hardened rail steel elastoplastic analysis and simulation, in cases where isotropic hardening is necessary to describe a varying initial (pre-cycling) yield surface, either at uniaxial or multiaxial stress/strain conditions. So far, the model has been implemented successfully within a unified cyclic plasticity model used to simulate ratcheting in head-hardened rail steel[6].

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