A Blended Learning Teaching Approach
Aimed at Improving Post-Primary Students’
Conceptual Understanding of Mathematics –
A Focus on Algebra Supported by Digital Pencasts

Submitted by Jason Gallagher

Supervised by:
Patrick Johnson
&
Joe English

For the award of Masters of Science by Research.
Submitted to the University of Limerick, February 2016.
Abstract

This research project investigated whether students’ conceptual understanding of algebra was improved with a taught intervention supported by digital pencasts. The motivation for this project was inspired by observing students’ difficulties when learning algebra in the author’s own classroom. Digital pencasts were used as a support tool throughout this investigation. Digital pencasting is the process of viewing web-based hand written notes with accompanying audio.

The teaching intervention focused on the elements of Niemi’s (1996) framework for measuring conceptual understanding within the two separate groups; an adult education group and a transition year group. A statistically significant improvement was observed in the transition years’ conceptual understanding of algebra whereas there was no statistically significant improvement observable in the adult education group.

The completion of this project unveiled the dearth of research currently available on digital pencasting. This project contributes to the literature already available on digital pencasting and may also be of benefit for practitioners by providing teachers with a new technology tool which students can use as an additional resource. A further contribution is the design of a teaching intervention which complements the aims of Project Maths that has the potential to enhance students’ conceptual understanding of algebra.
Declaration

This thesis is presented in fulfillment of the requirements for the degree of Masters by research in Mathematics Education. It is entirely my own work and has not been submitted to any other University or higher education institution. When reviewing the work of other people it has been fully acknowledged and referenced.

Signed: __________________________________________________

(Jason Gallagher)

Date: _________________________________
I would like to thank the following:

My supervisors Patrick Johnson and Joseph English for their guidance and support. Also to Miriam Liston who was my supervisor for the first year of my project.

The students and adults who took part in the research project.

The teachers of Rosses Community School for their honest feedback on the Livescribe Smartpen.

My friends and family for their support and patience.

Thanks to the NCE-MSTL / EPI-STEM for the financial support.
# Table of Contents

Abstract ........................................................................................................................................... ii  
Declaration ...................................................................................................................................... iii  
Acknowledgement ......................................................................................................................... iv  
Table of Contents .......................................................................................................................... v  
List of Appendices ........................................................................................................................... viii  
List of Figures ................................................................................................................................. ix  
List of Tables ...................................................................................................................................... x  

## Chapter 1: Introduction  
1.1 Introduction ............................................................................................................................ 1  
1.2 Background ............................................................................................................................. 1  
1.3 Rationale .................................................................................................................................. 2  
1.4 Research Questions .................................................................................................................. 3  
1.5 Scope & Significance of Research ............................................................................................ 4  
1.6 Overview of Thesis ................................................................................................................... 4  

## Chapter 2: Literature Review  
2.1 Introduction ............................................................................................................................. 6  
2.2 Issues facing Mathematics Education ...................................................................................... 6  
  2.2.1 Issues facing Mathematics Education Internationally ......................................................... 6  
  2.2.2 Issues facing Mathematics Education in Ireland ............................................................... 11  
2.3 Blended Learning ..................................................................................................................... 14  
2.4 Technology in Mathematics Education ................................................................................... 15  
2.5 Digital Pencasting ................................................................................................................... 18  
2.6 Attitudes & Anxieties towards Mathematics .......................................................................... 22  
2.7 Students’ Difficulties with Algebra ......................................................................................... 23  
2.8 Frameworks for Measuring Conceptual Understanding ......................................................... 25  
2.9 Teaching Methodologies ......................................................................................................... 29
Chapter 3: Methodology

3.1 Introduction ......................................................................................................... 35
3.2 Rationale for Undertaking the Research Project ................................................. 35
3.3 Research Aims / Questions .................................................................................... 36
3.4 Research Design .................................................................................................... 37
3.5 Action Research .................................................................................................... 39
3.6 Sampling ................................................................................................................. 40
3.7 Intervention Design & Execution ......................................................................... 41
   3.7.1 Pre and Post-Tests ............................................................................................. 41
   3.7.2 Lesson Plans & Handouts ............................................................................... 42
   3.7.3 Digital Pencasts .............................................................................................. 46
   3.7.4 Focus Groups ................................................................................................... 47
3.8 Validity & Reliability .............................................................................................. 48
3.9 Ethical Guidelines .................................................................................................. 49
3.10 Chronology of Project ......................................................................................... 51
3.11 Conclusion ............................................................................................................ 53

Chapter 4: Research Findings

4.1 Introduction ............................................................................................................ 55
4.2 Analysis of Data ...................................................................................................... 55
4.3 Research Question 1 ............................................................................................... 57
4.4 Research Question 2 ............................................................................................... 58
   4.4.1 Explanation Task – Transition Year Students .................................................. 59
   4.4.2 Representational Task – Transition Year Students ......................................... 60
   4.4.3 Problem Solving Task – Transition Year Students ........................................ 61
   4.4.4 Justification Task – Transition Year Students ............................................... 63
   4.4.5 Explanation Task – Adult Education Group .................................................... 64
   4.4.6 Representational Task – Adult Education Group ............................................. 65
   4.4.7 Problem Solving Task – Adult Education Group ............................................ 67
   4.4.8 Justification Task – Adult Education Group .................................................... 68
List of Appendices

Bibliography .................................................................................................................................113
Appendix A: Lesson Plans & Scheme of Work...........................................................................126
Appendix B: Pre & Post-Tests, Grading Rubric and Handouts.............................................162
Appendix C: Data Collection – Focus Group Transcripts....................................................194
Appendix D: Information Sheets & Consent Forms.............................................................202
List of Figures

2.1 Usiskin’s (2012) Framework for Conceptual Understanding in Mathematics .................................................................27
2.2 Niemi’s (1996) Framework for Conceptual Understanding in Mathematics .........................................................28
2.3 Hiebert et al’s (2003) Framework for Conceptual Understanding in Mathematics .................................................................29
2.4 Polya’s (1945) Problem Solving Heuristic .........................................................30
3.1 Problem Solving and Justification for Digital Pencasts .........................................................47
5.1 Comparison of Mean scores for Transition Year Students – Explanation Task .................................................................79
5.2 Comparison of Mean scores for Adult Education Group – Explanation Task .................................80
5.3 Comparison of Pre & Post-Test Scores for Transition Year Students – Representational Task .........................................................82
5.4 Comparison of Pre & Post-Test Scores for Adult Education Group – Representational Task .........................................................83
5.5 Comparison of Pre & Post-Test Scores for Transition Year Students – Problem Solving Task .........................................................85
5.6 Comparison of Pre & Post-Test Scores for Adult Education Group – Problem Solving Task .................................................................87
5.7 Comparison of Mean scores for Transition Year Students – Justification Task Part 1 .........................................................89
5.8 Comparison of Mean scores for Transition Year Students – Justification Task Part 2 .........................................................90
5.9 Comparison of Mean scores for Adult Education Group – Justification Task .................................................................91
## List of Tables

4.1 Reliability Statistics ........................................................................................................56
4.2 Test for Normality...........................................................................................................56
4.3 Transition Year Students – Comparing Pre & Post Test Scores ..........................58
4.4 Pre & Post Test Scores for Representational Task (Part 1) ...............................59
4.5 Pre & Post Test Scores for Representational Task (Part 2) ...............................60
4.6 Pre & Post-Test Scores for Problem Solving Task (Part 1) .............................61
4.7 Pre & Post-Test Scores for Problem Solving Task (Part 2) .............................62
4.8 Pre & Post-Test Scores for Problem Solving Task (Part 3) .............................62
4.9 Mean & Standard Deviation of Justification Task ......................................................63
4.10 Transition Year Students – Comparing Pre & Post-Test Scores ......................64
4.11 Adult Education Group – Comparing Pre & Post-Test Scores .........................64
4.12 Pre & Post-Test Scores for Representational Task (Part 1) ..............................65
4.13 Pre & Post-Test Scores for Representational Task (Part 2) ..............................66
4.14 Pre & Post-Test Scores for Problem Solving Task (Part 1) ..............................67
4.15 Pre & Post-Test Scores for Problem Solving Task (Part 2) ..............................67
4.16 Pre & Post-Test Scores for Problem Solving Task (Part 3) ..............................68
4.17 Mean & Standard Deviation of Justification Task ..................................................69
4.18 Adult Education Group – Comparing Pre & Post-Test Scores .........................70
Chapter 1 - Introduction

1.1 Introduction

This chapter describes the background around why this research is being undertaken, the purpose of the research as well as outlining its scope and significance. The author also provides an overview of the thesis, chapter by chapter.

1.2 Background

Due to the perceived importance of mathematics, mathematics education is viewed as an area of interest both in Ireland and around the world. Project Maths is a new mathematics curriculum which was introduced in 2008 by the National Council of Curriculum and Assessment (NCCA). In Ireland, many articles have been written about the introduction of Project Maths, and in particular whether its introduction has led to a ‘dumbing down’ of the mathematics curriculum in Ireland. In an article in the Irish Times by Stack, Stack & Hurley (2012) they state how Project Maths is simply not challenging enough for students. Keane (2012) backs up these claims in the Irish Examiner by insisting the curriculum is being dumbed down. He adds that this will affect students when the move on to third level education since there is a decreased emphasis on calculus. Project Maths was developed in order to increase students’ conceptual understanding, to provide for an enhanced student learning experience and greater levels of achievement for all. Conceptual understanding is one of the key focuses of this research study. Another area that is written about frequently by many media outlets, such as the Irish Times and the Irish Independent, is the poor performance of Irish students in mathematics in comparison to students in other countries. However since the NCCA introduced Project Maths in 2008, the number of students taking up higher level mathematics has increased. In 2011, only 16% of Leaving Certificate students were taking higher level mathematics, whereas in 2014 27% took the higher level paper. Contrary to what the statistics suggest, some educators believe that the rise in the uptake of students sitting the
higher level paper is due to the 25 bonus points which are awarded to students who sit the paper. The Irish Independent (2014) agree with these views by insisting that the “striking change in higher level uptake came after the introduction of bonus points”. In light of all these conflicting arguments around mathematics education in Ireland the author decided to conduct research into whether a blended learning approach to instruction would increase students’ conceptual understanding of algebra. This rationale behind this decision will now be presented.

1.3 Rationale

The author is a post-primary school teacher who has experience teaching both Junior and Leaving Certificate mathematics. The author’s interest in algebra was sparked by personal experiences in the classroom specifically when teaching algebra. The author constantly received negative feedback about algebra and quickly realised that algebra was not a popular topic among students. Some seemed to grasp the topic relatively quickly, however a majority of students seemed to struggle from the beginning. While carrying out some research, it was clear that this was not a phenomenon limited to the author’s classroom. According to the Chief Examiner’s Report in Ireland (2005, p. 49), it was stated that “average candidates experience difficulty with all but the most basic of algebraic manipulations and can cope only with basic routines in solving equations”. This was also evident in the Chief Examiners Report in 1999 and 2003, where they also concluded that attention must be focused on improving students’ understanding of algebra. It is expected that if students struggle with these areas in the Junior Cycle, they tend to have added difficulty with the more in-depth areas at Senior Cycle.

Additionally the author has a keen interest in technology and tries to incorporate it into lessons frequently. Graham (2003, p. 334) explains that integrating a “combination of face-to-face and online instruction with the aim of complementing each other” is known as blended learning. This approach will be used in this research study. Garret (2014, p. 1) comments how essential technology is in mathematics education arguing “technology has the potential to
empower students, to aid them in understanding, reasoning and communication”. The technology tool, which will be used in this research project, is the Livescribe Smartpen that creates digital pencasts. It is a novel technology tool that has been developed over the past decade. Digital pencasting is the “process of viewing the web-based handwritten notes with accompanying audio” (Baig, 2008, p. 14). The Livescribe Smartpen “combines all four modes of communication - reading, writing, speaking and listening - in the simple, low cost, and convenient format of pen and paper” (Livescribe Smartpen, 2007). The digital pencasts have a number of key benefits such as the twenty-four seven accessibility of the pencasts. This enables students to revisit challenging concepts whenever they wish.

With algebra and the digital pencasts in mind, the author set about enhancing students’ conceptual understanding of algebra. This research project decided to examine different approaches to enhance conceptual understanding and implement one that would complement the aims of Project Maths.

### 1.4 Research Questions

The author designed, developed and implemented an intervention based on educational literature, which aimed to improve students’ conceptual understanding of a specific topic within the Algebra strand. With the use of the Livescribe Smartpens, digital pencasts were used as an additional resource for students to use. The author established three specific research questions:

1. What frameworks exist for measuring students’ conceptual understanding of mathematics and how closely do these frameworks align with the aims of Project Maths?

2. Does a blended learning teaching intervention, supported by digital pencasts, enhance students’ conceptual understanding of algebra?

3. What are students’ opinions with regards to using the digital pencasts as a supportive tool to enhance their understanding of algebra?
These are the study’s primary research questions. Now that these questions have been outlined, the author will explain the significance of these questions within the Irish context.

1.5 Scope and Significance of the Research

The findings of this research have significance as the answers to the research questions above, focus on enhancing students’ conceptual understanding of mathematics. The Expert Group on Future Skills Needs reported in 2009 that

“Raising our national mathematical achievement can improve Ireland’s competitiveness and each individual’s ability to contribute and participate in an increasingly globalized and technological society”

(EGFSN, 2009, p. 3)

This research project should also add to research available on digital pencasting since there currently is a lack of literature available on this area. This investigation also has the potential to benefits for practitioners. As stated in section 1.2, algebra is an area that a large percentage of students struggle with. The digital pencasts developed will provide teachers with a new resource which students can use as a support tool. Teachers may also see benefits in a teaching intervention which is used to enhance conceptual understanding of algebra, especially one which compliments the aims of Project Maths.

1.6 Overview of Thesis

Chapter 2: This chapter reviews the general issues facing both mathematics education internationally and nationally. The literature reviewed also examines many key areas discussed in this research such as Blended Learning, Technology in Mathematics Education, Digital Pencasting, Students’ Difficulties with Algebra, Frameworks for Measuring Conceptual Understanding, Teaching Methodologies and Adults Learning Mathematics.
Chapter 3: This chapter outlines the methodology used in this research project. It explains action research methodology as well as outlining the research design and the instruments used to collect the data. The importance of validity and reliability are discussed and the ethical considerations involved in this research project are explained.

Chapter 4: This chapter presents the main findings from the data. The data is presented under key themes that emerge from both the ‘Literature Review’ and the focus group discussions.

Chapter 5: This chapter analyses the main findings from the data. The data is analysed under the same key themes from the previous chapter.

Chapter 6: In this chapter a summary of the research is presented and conclusions are stated. The author makes recommendations based on these conclusions and the research undertaken. The contribution made by the research is also outlined, and the limitations of the research project are stated.
Chapter 2 - Literature Review

2.1 Introduction

This chapter presents the existing literature related to teaching interventions supported by digital pencasts to enhance students’ conceptual understanding of algebra. In particular the following areas will be investigated and reviewed:

- Issues Facing Mathematics Education
- Blended Learning
- Technology in Mathematics Education
- Digital Pencasting
- Students’ Difficulties with Algebra
- Frameworks for Measuring Conceptual Understanding
- Teaching Methodologies
- Adults Learning Mathematics

2.2 Issues Facing Mathematics Education

2.2.1 Issues Facing Mathematics Education Internationally

This section will focus on international issues in mathematics education. First of all, the author will explore how the design of the mathematics curriculum in many countries worldwide is continuously developing and changing. Lynch, Pyke & Jansen (2003) claim that changing the curriculum increases the workload of teachers since they must implement new strategies in the classroom. Anderson, White & Wong (2012, p. 238) insist that the changes in curriculum exist to “provide students with experiences in school mathematics which will enable them to be prepared for the 21st century”. Gordan (2010) continues by adding that more disciplines recommend that there should be a strong emphasis on problem solving, critical thinking and reasoning skills, technology and
probability and statistics, while he adds that mathematics departments within schools should cooperate and share information more. Along with this, he adds that conceptual understanding is more important than skill development.

Taking Australia as the first example of continual curriculum change Anderson et al. (2012) describe the content and restructuring of the Australian curriculum model structure which now includes three content strands: number and algebra, measurement and geometry; and probability and statistics. As well as the three strands which have been illuminated, the curriculum also employs four proficiencies; understanding, fluency, solving real life problems and reasoning. The understanding proficiency which Anderson et al. (2012) talk about is focused on students developing an understanding between the ‘why’ and the ‘how’ of mathematics.

In Ireland, the curriculum has also changed substantially since the introduction of Project Maths in 2008. Around the time of the introduction of Project Maths, Sarah Lubienski’s (2011) article ‘An Outsiders Analysis of Project Maths’ compared the Irish mathematics education system to that of the United States. She has written about the large number of similar goals between Project Maths and a mathematics curriculum reform in the United States a number of years ago. The connections include the “call for more real world examples, increased emphasis on statistics and probability, multiple representation of algebra and geometric reasoning” (Lubienski, 2011, p. 30). Some teachers who provided feedback in the report agree that there have been difficulties with Project Maths, but conversely they insist that students are developing a better grasp of the subject. Lubienski (2011) also states that, in Ireland, teachers were disappointed at the lack of materials and curricular assistance available.

From an American point of view, Lynch et al. (2003) explains some of the difficulties teachers experience when delivering the curriculum. Many curricula currently focus on teaching students real life applications of mathematics and educators are encouraged to use new instructional technology. Changing the curriculum in such a significant way, puts a substantial amount of pressure on the teacher. However Lynch et al. (2003) insist the implementation of these ideas are
extremely time consuming considering educators already have a sizable curriculum to cover. Showing students real life applications will take up more class time. A further difficulty which Lynch et al. (2003) allude to is the ineffective nature of curriculum materials which Irish teachers also highlighted in Lubienski’s research project. As a result of low availability of quality teaching resources, teachers internationally resort to making their own resources which is not only time consuming but more worryingly it encourages educators to revert to traditional approaches in lessons to ensure their students meet the required standards in time for assessment (Lynch et al., 2003).

Another issue reported by Lloyd (2009), is the over reliance on textbooks by mathematics teachers in America and also internationally. She adds that textbooks “can influence teachers’ beliefs, knowledge and classroom practices” (p. 763). She continues by revealing that the findings suggest newly qualified teachers gain guidance from textbooks about how and what to teach but textbooks are only a guideline. Educators must use the syllabus to ensure they are teaching the required content. A final issue to be discussed regarding the curriculum involves the lack of calculus in most mathematics curricula. Gordon (2010) comments that mathematicians were concerned about the state of calculus education 20 years ago. He insists this decline is continuing nowadays since more time is being spent on algebraic skills and less time on calculus.

A further issue facing mathematics education internationally is the continued professional development of mathematics teachers. Guskey (2000, p. 16) defines teacher professional development as “those processes and activities designed to enhance the professional knowledge, skills and attitudes of educators so that they might in turn, improve the learning of students”. He stresses that professional development must be an intentional and purposeful process where one must clearly outline their goals. According to Roesken (2011) together teaching and learning are an important aspect of professional development. She adds that teachers’ professional development takes place every day inside and outside the classroom. She states this takes place through reflecting and talking about practice, preparing for the next day and many other areas. Ball (2000) concurs that professional development takes place every day. Ball (2000) explains that if
she teaches the same lesson to two different groups, she ends up with a different lesson, such is the diversity of teaching.

Kuzle and Biehler (2015) insist professional development in-service course are one key support of mathematics curriculum reform. Roesken (2011) alludes that in many countries, in-service training programs are viewed as professional development events. In-services are given to teachers as a method of compensating for shortage in knowledge and improving their teaching. Unfortunately he explains that it is up to the teacher to decide whether a teaching strategy is suitable or not. Zehetmeier and Krainer (2011) state that the teaching strategy must be designed in a way to ensure it has a sustainable impact. They believe that many innovative ideas are either never used or stopped relatively quickly by educators because the methods are not appropriate for their class or they are too time consuming to create or implement.

Rogers, Abell, Lannin, Wang, Musikul, Barker & Dingman (2007) conducted a research project in Taiwan with 72 teachers and 23 professional development facilitators about effective professional development. Both the teachers and the facilitators indicated some difficulties with professional development events. The biggest issue teachers provided was when they are given activities to implement in their classroom but they have to be modified or adapted. The teachers explained they do not have the time to sit down and modify activities during the school year. They believe that these events need to be set up in a way to target teachers’ needs by asking teachers what do they want or need. They insist it is very beneficial to receive the necessary resources for different mathematical topics but if they do not have the materials or if they have to be modified, what good are they? Another issue these teachers identified was the lack of hands-on experience at the in-services to enable the teachers to transfer what they had learned, back to the classroom. One stated how they sent the materials back to get a refund because they forgot how to use the materials once they returned from the in-service. Overall, Rogers et al. (2007, p. 523) explain that it is “essential for the professional development facilitators to provide teachers with simple ready-made materials, equipment and resources to assist them with implementing the activities in their own classroom”. Finally Roesken (2011)
explains the effects of professional development are measured in terms of improving teachers’ content knowledge. This leads on to the next section, the lack of teacher knowledge in mathematics education internationally.

The final issue to be discussed from an international perspective is the lack of teacher knowledge in mathematics education internationally. Tchoshanov (2011) conducted a research project in Texas on how teachers’ content knowledge is associated with student achievement. His results concur with a number of other studies that were conducted in America which state that teachers lack the essential knowledge for teaching mathematics. He believes one of the reasons for the lack of knowledge is because the knowledge of a large number of teachers is limited to mathematical procedures rather than teachers having an overall conceptual understanding of mathematics.

According to Zazkis & Zazkis (2011) there are three mathematical components that are essential for teachers of mathematics; mathematical knowledge, ability to break down material and attention to a student’s difficulties. The one area which they believe many teachers lack is the ability to break down mathematical content into basic steps in order to enhance students’ understanding. They add that some teachers are unable to make information accessible to others due to their own high level of mathematical content knowledge. Marshall & Sorto (2012), who conducted their research project on teachers’ mathematical knowledge in Guatemala, reject these claims by insisting teachers with a deep understanding of mathematics are better equipped to deal with students’ mistakes and misconceptions in the classroom. They add that teachers with a lack of content knowledge struggle to provide effective explanations when students are having difficulty.

Hicks, Taylor & Bruton (2013) insist that while teachers have certain levels of mathematical knowledge, they still must gain knowledge through professional development programs. They add that a large number of countries offer professional development courses designed to develop teachers’ mathematical knowledge. Hicks et al. (2013) state that by providing the opportunity for newly qualified teachers to develop their ability to break down content material for
students this will impact greatly on their students’ understanding. Marshall & Sorto (2012) state that there is one key issue which is vital to consider when organising professional development programs. What form of teacher knowledge do the professional development programs focus on? One must understand the underlying dynamics that makes one teacher more effective than another.

2.2.2 Issues Facing Mathematics Education in Ireland

Mathematics education in Ireland faces many of the problems illuminated in the international context section 2.2.1 but there are also issues in mathematics education which are unique to Ireland. These include; transitions in mathematics in the Irish education system and Project Maths which will both be examined in detail. The first national issue to be discussed is the transition from primary to post-primary mathematics. This transition has been described as “a critical educational step for many children” (Irish National Teachers’ Organisation (INTO), 2008, p. 7). In a report by the Project Maths Implementation Support Group (2010), it was stated that at primary school level, teaching and learning of mathematics must be improved. This is because a significant number of students are entering secondary level mathematics without the required skills. They suggested that there should be a support service set up in order to ensure a successful transition from primary to post-primary school which in turn should positively impact on mathematical achievement. The INTO’s (2008) report postulates that up to 40% of pupils experience a decline in academic progress when they commence their post-primary education. They insist the key root of this issue is the lack of communication between primary and post-primary schools.

The ‘Literacy & Numeracy for Learning and Life’ report released by the Department of Education and Skills (2011, p. 78), supports these claims, insisting there is a weakness in the transferring of student information between primary and post-primary schools. They argue that these discontinuities could cause a student’s learning to regress. The curriculum states that consistency in the approach between primary and the Junior Cycle curriculum should help to
ease pupils’ progress from one level to the next (Primary School Curriculum, 1999). The Literacy & Numeracy report (2011) states that there is a mistaken belief that students’ numeracy skills should be developed by the end of primary school. It is essential to build on the learning which they acquired at primary school so all students can master the essentials of mathematics. The Literacy & Numeracy Report (2011) recommend that rather than assuming that numeracy learning concludes in primary school, it is important to recognise that students arrive into post-primary school with different levels of achievement. The focus groups in the INTO report suggested that “policies on transition, which involve parents and pupils, primary and post-primary teachers are likely to improve the process of communication, which, in turn, should enhance the experience of transition for pupils” (2008, p. 32). Continuity in teaching strategies and methodologies between primary and post-primary is central to the smooth transition.

A final worrying trend in Irish mathematics education is that when asked how familiar they are with mathematics in the primary curriculum, only 24% of post-primary teachers were very familiar and 50% just somewhat familiar (Primary School Curriculum, 1999). The remaining 26% were not at all familiar. Therefore as a result of the literature examined it can be held that the transition from primary to post-primary is a major issue facing mathematics education in Ireland as the curriculum does not follow on naturally from primary to post-primary level. The introduction of Project Maths in 2008 aimed to ratify this issue.

On a national level, mathematics education has changed dramatically over the past decade with the introduction of Project Maths in 2008. The main aims of this new curriculum are “to provide an enhanced student learning experience and greater levels of achievement for all” (Project Maths Development Team, 2013). Project Maths was developed due to “increasing concern about student performance in mathematics and calls for an urgent review of the syllabus” (NCCA, 2008, p. 6). The Project Maths Development Team stated greater emphasis will be placed on student understanding of mathematical concepts and
applications which will enable students to relate mathematics to everyday experiences. This was reinforced by the NCCA (2008), who insisted that new approaches to teaching and learning were needed if students were to gain a deeper understanding of mathematics. A further aim is to encourage investigation, meaning that students take on the role of a self-directed learner in developing various mathematical skills (Cosgrove, 2012). An additional objective of Project Maths is the importance of deepening students’ conceptual understanding of each topic. The importance of real life examples in mathematics was also highlighted by the NCCA. The previous curriculum did not place an emphasis on highlighting the links to real life examples and classrooms were teacher-centred, as opposed to the new student-centred classrooms. By relating topics to real life, the new curriculum has created an active classroom with the hope of increasing the quality of the teaching and learning experience of students (Project Maths Development Team, 2013). This in turn, it is hoped, will build a more positive attitude among students towards mathematics.

The NCCA (2008, p. 1) have noted the need for a “greater emphasis on developing the students’ essential numeracy skills and on the use of contexts and modern applications of mathematics that are relevant to students’ present and future lives”. The structure of the exams has also been reformed to support these changes to teaching and learning. These variations to the syllabus means that teachers must transform their teaching style. This has led to access to a larger number of resources both on the Project Maths website and elsewhere. Students have also been affected by this major transformation in the curriculum. They have to alter their approach in learning mathematics since classrooms are encouraged to be more student-centred and active due to Project Maths. The collection of changes due to Project Maths has presented a major challenge. It is a national issue in mathematics education as educators have had to alter teaching styles and students have been forced to adapt a novel method of learning.

In 2013, the National Foundation for Educational Research (NFER) issued a report on the impact of Project Maths on student achievement, learning and motivation. The report states that there “does not appear to have been a
substantial shift in what teachers are asking students” and that “traditional approaches to mathematics teaching and learning continue to be widespread” (2013, p. 71). Despite the above concerns, the NFER report also remarks that there is emerging evidence of positive impacts on students’ experiences and attitudes towards mathematics, that links are being made between different mathematical topics and that considerable progress is being made. One new method of using differentiated teaching strategies is the use of a blended learning approach, which will be discussed in the next section 2.3.

2.3 Blended Learning

Blended learning is recognised as one of the major trends in higher education today. Driscoll (2002, p. 54) defines blended learning as “intermixing of any instructional forms to achieve an educational goal”. Garrison and Kanuka (2004) explain that to blend, simply means integrating classroom teaching with online experiences. They explain their definition is clear and simplistic but the implementation of blended learning is complex and rather challenging since “virtually limitless designs are possible depending on how much or how little online instruction is inherent in blended learning” (p. 96). Graham (2003, p. 334) views blended learning as the “combination of face-to-face and online instruction with the aim of complementing each other”.

Garrison & Kanuka (2004) believes blended learning is beneficial since integrating classroom teaching with online experiences provides better learning outcomes. They add that blended teaching can facilitate independent and collaborative learning experiences for students. Kasraie & Alahmed (2014) conducted a study in the USA and Canada investigating the reasons why higher education use a blended learning approach. Interestingly, they suggest students being digitally literate enhance the chances of extending their lessons and conversations beyond the classroom. Students were able to explore information in their own time and at their own pace, to be followed by classroom discussion or debate. These advantages are also applicable to second level assuming students can remotely access the content. Graham (2003) adds that blending
technology with face-to-face instruction can stimulate learning and provide more collaborative learning experiences. Kasraie & Alahmed (2014) claim that combining technology with class debates helps students gain more understanding of the subject matter. Okaz (2015) continues that combining teaching methods enhances students learning experience and allows teachers to use a variety of teaching strategies. Finally he concludes that blended learning empowers students with the ability to become self-directed learners. It was evident that the key areas of blended learning align with the primary reasons for the introduction of Project Maths.

Despite the large number of benefits of a blended learning approach, many educators have reasons not to use this method. Vonderwell (2003) explains that one of the key reasons is that some students fail to connect with teachers due to the usage of technology in remote settings. Vonderwell (2003) insists this leads to a loss of a sense of classroom community and it may remove immediate feedback. Okaz (2015) adds that some students may struggle to use technology or may not have access to technology material at home due to different social economic backgrounds. He continues that teachers may also need training in technology in order to use it in a classroom setting. Prensky (2001) explains that those born within the last twenty years can be referred to as “digital natives” and as a result are more adept at using technology. Contrastingly, those older than twenty can be referred to as “digital immigrants” (Prensky 2001) and are less at ease with technology. Teachers fall into this latter category. Okaz (2015) explains that another reason for not using a blended learning approach may be due to students’ unwillingness to collaborate with others. Some of the above views on technology in education will be discussed in more detail in the next section (2.4).

2.4 Technology in Mathematics Education

In the developing world that we live in, technology is progressing constantly and is playing a key role in general education and in mathematics education. According to Kocak & Gulcu (2013, p. 294) “using developing technology in teaching-learning process is inevitable”. They also explain how most teachers
use technology actively in their daily life. However, a study in Baltimore by Holden & Rada (2008) conversely states eight out of ten teachers rely on computers for administrative reasons, but only half of them are incorporating computers into their daily lessons. In their opinion (2008, p. 115), the reason for the low numbers is because teachers need both training and time to “acquire technological skills and develop new teaching strategies for integrating technology into the classroom”. In support of this, Kocak and Gulcu (2013) note that teachers must be given professional support in order to use the technology more efficiently. This includes giving information on material creation along with the technical difficulties that may arise when using the technology.

Ertmer, Ottenbreit-Leftwich, Sadik, Sendurur & Sendurur (2012) assert that there are two types of barriers that impact on teachers’ use of technology in the classroom. The first types of barriers are external to the teacher. For example, resources and training. However the second type are internal to the teacher. These include teachers’ confidence, beliefs and values on technology. The researchers remark that, “simply increasing computer access was not sufficient to change teachers’ technology practices especially if this increased access was not accompanied by a corresponding shift in teachers’ pedagogical beliefs” (p. 423). Lei & Zhao (2007) provide more insight into the problem of technology in education by insisting that the quantity of technology use is not the critical issue. It is when “the quality of technology use is not ensured, more time on computers may cause more harm than benefit” (p. 284). They reiterate that teachers need to be aware that not all uses of technology are beneficial. They state the key issue concerning technology use in education is how it is used and for what purpose, since the major goal of integrating technology into schools is to increase student learning. Chuang (2013, p. 81) echoes that it is important to “understand the constraints and abilities of various technologies, along with the pedagogical and content knowledge necessary for further adaptations if successful instructional practices are to take place”.

Along with ensuring that the quality is ensured and the barriers that teachers experience are overcome, there is also the TPACK framework, which Yigit (2014) insists is extremely beneficial when applying technology in a classroom
setting. He states that TPACK, which is the combination of Technological, Pedagogical And Content Knowledge, is crucial for successfully incorporating technology into teachers’ lessons. Yigit (2014) illuminates that if teachers use any type of dynamic geometry software to teach a geometrical concept, they must understand the following; how to use the representations, any challenges with the software, the pedagogical techniques to best illustrate the concept and finally how the software could be helpful for the students. This framework is an extension of Lei & Zhao’s views.

Technology is a practical tool in mathematics education. Harvey (1991, p. 4) suggests “that computer software, when designed properly and used with appropriate materials, can provide an opportunity for engaging students in significant mathematical inquiry”. GeoGebra is an award winning software, which aids students understanding, and makes lessons more interesting for students (Reis & Ozdemir, 2010). GeoGebra was developed in 2001 by a Masters student who carried out a thesis on creating a technology tool. The corresponding website explains that it is a “free and multi-platform dynamic mathematics software for all levels of education that joins geometry, algebra, tables, graphing, statistics and calculus in one easy-to-use package” (GeoGebra, 2001). The software has received many awards including the National Technology Leadership Award in 2010 in Washington, USA. In an article from Reis & Ozdemir (2010), they imply that integrating GeoGebra into a lesson improves academic achievement and students’ attention is increased due to the visual impact of the lessons. The Project Maths Development Team promotes the use of GeoGebra in Irish classrooms as it covers a wide variety of topics and is a cost free package. Along with information on how to download the free software, there are also GeoGebra packs and tutorials available on the Project Maths website. The GeoGebra packs and tutorials were created by the Project Maths Development Team to aid teachers and students to navigate the software. The creators of the software have recently developed GeoGebraTube, which means that people can upload and share their GeoGebra creations. A study in Florida by Escuder & Furner (2011), completed on a group of in-service teachers showed that teachers gained experience in using GeoGebra and enhanced their own knowledge. This led to the enhancement of students’ grades. Teachers’ use of
GeoGebra increased from 27.8% to 64.3% and the results found a relationship between the use of GeoGebra and student achievement.

Kocak and Gulcu (2013) stated that technology makes lessons more enjoyable and interesting along with increasing motivation and aiding understanding. Teachers from six schools in Turkey who took part in this research, also explained that it is important to attract students’ attention during lessons. The one issue they identified was that controlling students’ behaviour became challenging. The results in the study from Holden & Rada (2008) reported a strong positive relationship between technology being beneficial to students’ learning, confidence, motivation and achievement. Additionally they stated that teachers are beginning to acknowledge that technology makes their lessons more interesting.

As already discussed, GeoGebra makes connections between various topics such as geometry, algebra, calculus and statistics. Students are able to make graphical representations of algebraic equations. Escuder & Furner (2011) explain that GeoGebra follows the TPACK framework, which Yigit (2014) believes is the basis of teaching with technology. Garret (2014, p. 1) comments on how essential technology is in mathematics education arguing “technology has the potential to empower students, to aid them in understanding, reasoning and communication”. He insists that technology provides students with the opportunity to explore and discover, while the teacher uses careful questioning to aid their understanding. Khouyibaba (2010, p. 638) asserts that technology allows the teacher “to capture the attention of the students and enables them to better understand and master mathematical concepts”. In the next section a more detailed look at the Livescribe Smartpen will be provided as this was the technology tool utilized by the author during this project.

2.5 Digital Pencasting

A novel technology tool that has been developed over the past decade is digital pencasting. Digital pencasting has evolved from screencasts which are a “digital
movie in which the setting is partly or wholly a computer screen, and in which audio narration describes the on-screen action” (Kopel, 2010, p. 297). The screencasts show the narrator via a webcam and one can listen to their voice narration at the same time. Kopel (2010) postulated that screencasts are a low costing, high effective tool since it is a reusable technology tool that can be watched on a computer or mobile device with video playback capabilities. Digital pencasting is the “process of viewing the web-based handwritten notes with accompanying audio” (Baig, 2008, p. 14). Powers, Bright & Bugaj (2010) state that one of the key qualities of digital pencasting is the twenty-four seven accessibility of the pencasts that enables students to revisit challenging concepts whenever they wish. The advantageous part is that students can independently improve their “insufficient level of previous knowledge” (Calm, Ripoll, Olive, Masia, Sancho–Vinuesa, Pares, Pozo, 2012, p. 1).

Mathematics support websites such as Kahn Academy provide an extensive range of screencasts to support post-primary school and third-level students. Loch, Jordan, Lowe & Mestel (2014) state the delivery of good revision material will enable students to develop their knowledge by revisiting topics they may have forgotten. In the designing process of the screencasts Loch et al. (2014, p. 258) insist that it is important to “present a carefully designed step-by-step solution and to explain the thinking process when selecting a solution method”. They also add how it is essential to explain why each step within the process is important. This helps the student to grasp an understanding of the process. They continue by suggesting educators should create short pencasts as opposed to long pencasts since mathematical problems are generally demanding and difficult to process. They further add that students should be encouraged to try additional problems once they have watched one of the screencasts.

In a study by Loch, Jordan, Lowe, Mestel & Wilkins (2012) in which 140 participants gave their opinion on screencasts, 94% of respondents found the screencasts useful. Encouragingly 94% also believed that their understanding of mathematical concepts can be improved by watching screencasts. In a separate study by Loch, Dunn & Mc Donald (2015) where they also investigated participants (555) views on screencasts, 90% of respondents found the
screencasts very helpful. One of the key benefits that the respondents mentioned was the ability to pause, rewind and fast-forward the screencasts. One participant explained that this was essential because you can dictate the pace yourself and you have the option of repeating a step continuously until you understand it. Meanwhile another participant insisted they were helpful since they are not as content-heavy compared to lectures. Another benefit Loch et al. (2014) state is that students’ confidence towards mathematics was increased by watching the screencasts. The screencasts made content more accessible. Other positives were that the screencasts are much easier to understand compared to text-based material due to the audio explanation reinforcing the techniques required to complete a problem and the advantage of them being focused.

Loch et al. (2012) asked respondents whether they prefer handwritten screencasts or typed screencasts. They majority of students insisted they preferred the handwritten screencasts because they were more engaging, user friendly and personal. The participants who opted for the typed screencasts said they were faster to read and easier to follow. One of the negative aspects of the screencasts which Loch et al. (2014) reported was their passive nature. They explained that students could not verify their understanding immediately as opposed to if they were in a classroom setting.

Digital pencasts are the creation of material developed by smartpens such as the Livescribe Smartpen. The Livescribe Smartpen “combines all four modes of communication - reading, writing, speaking and listening - in the simple, low cost, and convenient format of pen and paper” (Livescribe, 2007). Unfortunately due to the Livescribe Smartpen’s recent release in 2008, not much research has been conducted on this product to date. According to Powers, Bright & Bugaj (2010, p. 144), “92.6% of students (n=108) who reported using the Livescribe digital pencasts believe that they enhanced learning”. These third level students also suggested using the Livescribe Smartpen in modules other than mathematics. In the authors research project, the digital pencasts were intended as an additional study tool. Calm et al. (2012) explain that the digital pencasts provided by the Livescribe Smartpen have many benefits such as providing students with support, easy access for both the creator of the digital pencasts and
the viewer and one can alternate from one point to another of the digital pencast. For mathematics, Calm et al. (2012, p. 3) comment that “the videos enable you to follow the real sequence in which mathematical calculations are performed” and “the explanations given through the built-in sound support help students to more easily understand the development and intention of the solving strategy”. Calm et al. (2012) added that they received extremely positive feedback in a survey that the students completed and there was an upward trend on students’ grades.

A deeper understanding of the Livescribe Smartpen can be obtained by comparing it to podcasts. A podcast is “any digital media file distributed over the internet for playback on portable media players or computers” (Lonn & Teasley, 2009, p. 88). Lonn & Teasley (2009) maintain that students only benefit from audio podcasts if they take notes or listened to them frequently. Lonn & Teasley (2009) state that podcasts allow students to review lectures they attended. However, Kay (2012) and Kay & Kletskin (2012) researched video podcasts, which are audio-visual files similar to the digital pencasts created by the Livescribe Smartpen. From their results, both reports provided the following benefits of video podcasts; they enhance test scores, learning was improved and students developed a deeper understanding of concepts. The only disadvantages which Kay (2012) stressed was students missed being able to ask questions or gain immediate clarification on issues they may have. Kay & Kletskin (2012, p. 622) state video podcasts can “help to review old material, visualization, and helping to solve and understand problems better”. Kay & Kletskin (2012) also conclude the quality of the video podcasts give students the ability to control the pace of teaching and to follow clear, step by step explanations. Finally it was found that the video podcasts are easier to follow than a written example. 90% of the students who used the video podcasts found them useful or very useful.

To conclude, an in depth discussion and critique of the strengths and weaknesses of the Livescribe Smartpen has been completed. The use of the Livescribe Smartpen is central to this research project and the mathematical topic that will be focused on, and supported by use of the Livescribe Smartpen, is the topic algebra.
2.6 Attitudes and Anxieties towards Mathematics

First of all, students’ attitudes and anxieties towards mathematics will be explored. According to Belbase (2013), a major concern of mathematics educators is the relationship between students’ attitudes and their overall achievement in mathematics. Yaratan & Kasapoglu (2012, p. 162) insist that attitude is another “psychological construct that affects students’ performance in mathematics”. Critically for them, they believe attitude is one of the key determinants of performance in mathematics. Supporting the above claim, Zan & Di Martino (2007) believe that attitude is either positive or negative towards mathematics. They continue by remarking that attitude plays a key role in students’ learning of mathematics. A positive attitude towards mathematics leads to high achievement, however a negative attitude leads to low achievement in the subject. Belbase (2013, p. 233) concluded that attitudes “could change dramatically in a relatively short time and a negative attitude towards mathematics could be a successful defense strategy of a positive self-conception”.

Mathematics anxiety is closely linked to attitude. Mathematical anxiety is an “anxious state in response to mathematics-related situations that are perceived as threatening to self-esteem” (Belbase, 2013, p. 232). Hoffman (2010) supports Belbase’s claims by agreeing that a strong negative correlation exists between mathematical anxiety and achievement. He believes that anxiety generally occurs when one is faced with an unfamiliar problem or a problem which is considered overly complex. He insists it is essential that educators intercept students with high mathematical anxiety early to ensure students do not become overwhelmed by mathematical problems.

Attitudes and anxieties have significant implications in the teaching and learning of mathematics with knock-on effects on both performance and achievement. Zan & Di Martino (2007) explain that once a teacher diagnoses a negative attitude, they can begin to plan an intervention, which will aim to alter the negative factors identified by the student. Yaratan & Kasapoglu (2012) focus on the students learning of mathematics by stating that attitudes and mathematics
anxiety affects students’ mathematical experiences. They suggest that students should concentrate on developing an understanding of the mathematical concepts as opposed to learning the procedures. They believe this will lower the anxiety and help to build a positive attitude. Developing an understanding in mathematics is one of the key components of this research project. This leads on to the next area to be discussed which is students’ difficulties with algebra.

2.7 Students’ Difficulties with Algebra

Before discussing the difficulties associated with algebra it is important to understand what exactly algebra is. Prendergast & Sterritt (2011) hold that there are seven themes of algebra. It can be described as a school subject, generalised arithmetic, a tool, a language, a culture, a way of thinking and an activity. The word algebra derives from the Arabic word ‘al-jabr’ and it translates roughly to the word “balancing”. Algebra, as we know it today, arose out of the need to solve for unknown quantities in equations. In the 17th century, symbols were first introduced for the unknowns. The French mathematician and philosopher René Descartes was the first to denote the unknown quantities with letters at the end of the alphabet \((x, y, z)\) and the known quantities with letters from the beginning of the alphabet \((a, b, c)\). These letters and values are still used in the primary curriculum and the post primary curriculum in the 21st century.

Prendergast & Sterritt (2011) observe an important application of algebra which is its effectiveness in solving many types of problems, which establishes its relevance to everyday life. Algebra has many benefits such as enhancing “students’ power of communication, facilitate simple modeling and problem solving, and hence illustrate the power of mathematics as a valuable subject of study” (Bednarz, Kieran & Lee, 1996, p. 163). Similarly, Skouras (2014, p. 14) asserts “an important conceptual leap for students is to move from the concrete grounded world of arithmetic problems, to the more abstract world of algebra problems with variables”. Prendergast & O’Donoghue (2010, p. 251) comment that despite the importance of algebra, post-primary students are oblivious as to where it is used in everyday life. Hence the introduction by the NCCA of Project Maths, which aims to show the importance of mathematics in everyday life.
Usiskin’s (1995) article on ‘Why is algebra important to learn?’ gives many real life applications of the topic. He insists without a knowledge in algebra, you will be unable to perform many jobs, you will make unwise decisions in life (financial) and you may not be able to participate fully in our technological society. Some of the applications of algebra include; converting from degrees celsius to degree fahrenheit, calculating how many miles per gallon your car is getting, analysing various aspects of your health (i.e. weight loss and diets), predicting things such as population growth and the growth of diseases and tumors and calculating the win or loss percentage of a sports team.

Artigue & Assude (2000) explain that many students see algebra as a domain where mathematics becomes a non-understandable world. In a study by Samo (2009, p. 57), he commented “pupils experienced great difficulty in framing problems in algebraic terms, and in accepting that letters can be used in a general sense and not just stand for one specific unknown number”. As stated previously, the Chief Examiner’s Reports in 1999, 2003 and 2005 explained that students experience difficulty with a significant number of algebraic concepts. They concluded that attention must be focused on improving students’ understanding of algebra, hence why developing understanding is one of the key reasons for the introduction of Project Maths. According to Linchevski & Herscovics (1994) there are numerous reasons for students’ difficulties with algebra. These include the pace at which it is covered and the formal approach used to explain the topic. They continue to explain that students “fail to construct meaning for the new symbolism and are reduced to performing meaningless operations on symbols they don’t understand” (1994, p. 60).

Prendergast & Sterritt (2011) explain that students are graded on their mathematical skills and manipulations and not the understanding of the concepts. They continue by saying “students need a sound understanding of algebraic concepts and the ability to use knowledge in new and often unexpected ways” (Prendergast & Sterritt, 2011, p.29). The introduction of Project Maths has changed this approach since one of its key components is its focus on developing students’ understanding (NCCA, 2011).
Coupled with the importance of students’ understanding of algebra, it is also essential that teachers have the required understanding of algebra. In a study in the U.S. by Kulm & Huang (2012), they assessed 115 teachers’ knowledge of algebra. Overall the teachers had a relatively limited knowledge of the topic. They investigated the three areas of knowledge in algebra teaching; school mathematics, teaching mathematics and advanced mathematics and found that the participants knew less than 40% of the required content in each area. These conclusion were backed up in a study in Australia by Wilkie (2014) who concluded that most teachers’ had an inadequate level of mathematical understanding. Along with a lack of understanding, De Castro (2004) explains how it is important for teachers to encourage students to justify their answers and constantly question their teacher in order to develop their understanding. In order for this to take place, a high level of teacher content knowledge is required. Again, teachers’ continued professional development is extremely important.

Prendergast & O’Donoghue (2010) list numerous ways to increase students’ interest in the topic of algebra such as showing mathematics can be fun and establishing relevant links between students and algebra. In an article by Mc Convey (2006), he discusses how he and the Junior Certificate Maths Support Service developed an active learning methodology for teaching algebra. Alge-tiles are aimed at addressing students’ difficulties with algebra by “providing teachers with the skills and resources to introduce algebra in a concrete way to deepen understanding and increase confidence” (Mc Convey, 2006, p. 2). Following on from this, Artigue & Assude (2000, p. 10) believe that there is potential in computer technology to “overcome the identified learning difficulties and to develop more effective teaching strategies”. Teaching for conceptual understanding will now be explored with reference to algebra.

2.8 Frameworks for Measuring Conceptual Understanding

Before the author discusses different frameworks for measuring conceptual understanding, it is important to define what conceptual understanding is. According to Case & Marshall (2004), students process information in two different ways. Surface-level processing is when the students try to memorise as
much as they can. Contrastingly, deep-level processing involves gaining an understanding. This is also known as conceptual understanding. Interestingly Skemp (1976) defines surface-level processing as instrumental understanding meanwhile he calls deep-level processing relational understanding. He admits that one of the key benefits of instrumental understanding is that it is easier to understand in particular with topics like multiplying, where one only needs to remember certain ‘rules’. He continues by insisting students get a huge feeling of success from getting a page of answers correct. Relational understanding on the other hand also has its advantages. Relational understanding deals with understanding why a particular method worked. This enables students to adapt methods to new problems compared to instrumental knowledge where one memorises a particular method. Panasuk (2010, p. 237) states that conceptual understanding is when one can “grasp the full meaning of knowledge”. Similarly Hiebert, Morris & Glass (2003, p. 203) defines conceptual understanding as “comprehension of mathematical concepts, operations and relations”. They continue to state more emphasis is placed on procedural understanding at the expense of conceptual understanding. However due to the introduction of Project Maths and changes in curricula in other countries around the world more emphasis is being placed on conceptual understanding nowadays. Next, a number of different frameworks for measuring conceptual understanding will be presented and critiqued.

In an article by Usiskin (2012), he alludes to ‘what it means to understand some mathematics’ from a learner’s point of view. He proposes that there are at least five dimensions of understanding as outlined in Figure 2.1. However students only delve into four of these dimensions since the fifth one is outside of the scope of students according to Usiskin. They are called dimensions of understanding since each aspect can be conquered individually. Nonetheless, Usiskin (2012) believes the dimensions taught together are better than those being taught alone. The skill-algorithm is the first dimension outlined in his framework. Critically Usiskin (2012) insists that procedural understanding goes beyond applying a skill, which has been learned. He argues that procedural understanding is not lower-level understanding. For example, there are at least seven different ways of multiplying fractions. He continues by stating “we
exhibit a higher form of this same type of understanding when we know many ways of getting the right answer” (2012, p. 6). Usiskin (2012) ponders that one can change a higher order activity to a lower order one if they continuously work on a concept. The Property-proof dimension is the second dimension discussed by Usiskin (2012). This dimension requires understanding why the chosen method worked. The third dimension is the use-application dimension. This dimension entails knowing when to carry out the necessary steps. The concept that Usiskin (2012) talks about is multiplication, where he informs us that students do not know where they can use multiplication. It is added that teachers have only begun to realise that uses can be taught, hence the emphasis in Project Maths on stressing the different uses of mathematics in real life. Unfortunately, he claims that people believe applications involve a higher order level of thinking rather than skill. He insists this is untrue. Consequently less time is spent teaching the applications, hence the poor results in this area. The fourth and final dimension from a students’ standpoint is the representation dimension. Usiskin (2012, p. 9) explains, “from psychology we obtain the notion that a person does not really understand mathematics unless he or she can represent the concept in some way”. The fifth dimension of understanding, which is not important from a students’ perspective, is the history-cultural dimension. This dimension requires an understanding of the work of the inventor and how they developed a concept. One must also understand the significance of the developed concept.

Figure 2.1 Usiskin’s (2012) Framework for Conceptual Understanding in Mathematics
The second framework to be presented is Niemi’s (1996) framework of measuring conceptual understanding in mathematics, as outlined in Figure 2.2. He explains the first task one must be able to complete is a representational task. This involves fluency in identifying, generating and using representations. There are a range of technology tools, such as GeoGebra, which has been previously discussed (section 2.3) which could aid in creating these representations but additionally students could just represent the concept via sketches or drawings.

The second task for understanding is a problem solving task. One must use their skills to solve the problems at hand. The third task, justification, is a follow-on from both the representational task and the problem solving task. To justify your answer, you must show and explain why your solution is correct. The final task in measuring conceptual understanding is the explanation task. Niemi (1996) explains that this task requires one to explicitly explain a concept and its principles.

Figure 2.2. Niemi’s (1996) Framework for Conceptual Understanding in Mathematics

Similar to Usiskin (2012) and Niemi (1996), Hiebert et al. (2003) proposes five strands, that when integrated together, ensure one becomes mathematically proficient. These strands include conceptual understanding (comprehension of mathematical operations, concepts and relations), procedural fluency (skill in
carrying out procedures accurately and appropriately), strategic competence (ability to formulate, represent and solve problems), adaptive reasoning (logical thought, reflection, explanation and justification) and productive disposition (ability to see mathematics as useful and worthwhile). They believe that by increasing a teachers’ mathematical proficiency the teachers can, in turn, increase a students’ proficiency. Hiebert et al. (2003) continue by insisting these five strands listed here, as presented in Figure 2.3, must be included in teacher education courses in order to improve teacher and student proficiency in mathematics.

![Figure 2.3. Hiebert et al.’s (2003) Framework for Mathematical Proficiency](image)

**2.9 Teaching Methodologies**

As stated in the curriculum design section 2.2.1, mathematics teaching is changing rapidly in the past decade and will continue to change in the future. Lynch, Lyons, Boland, Close & Sheerin (2003) explain that mathematics teaching in Ireland was previously very teacher-centred since a ‘chalk and talk’ approach was used by many teachers. However this has changed both in Ireland and also internationally as the author has previously stated in section 2.2.2. Classrooms are much more student-centred meanwhile the teacher is more of a facilitator within the classroom. Guo (2011) explains that constructivism is one of the modern teaching theories which many teachers and curricula follow.
Constructivism which was founded by John Piaget involves learners actively constructing their own knowledge from their experiences. Constructivism compels teachers to change their teaching style from the traditional ‘chalk and talk’ to a new method that involves helping students through a process of learning. Watt (1997) explains that although traditional approaches to teaching must take place on some occasions, it is important for teachers to develop a balance between both methods.

From a student perspective, O’Shea & Leavy (2013) explain that constructivist theory orders students to engage actively in lessons, collaborate with fellow learners and to problem solve independently and in groups. These are also some of the key components of Project Maths. They add that it is important for students to test ideas, formulate solutions and to discuss their answers collaboratively. They explain that Polya’s (1945) four-stage problem solving heuristic is essential. They insist this method will enable students to build on their knowledge and design more effective problem solving strategies in the future. The first step of Polya’s problem solving heuristic is that one must understand the problem they are given. Without a solid understanding of the problem at hand, learners cannot begin the question. The second step involves devising a plan to solve the problem. He suggests that ideally one should think of similar problems they have solved previously. The execution of the plan is the third step of the method. The fourth and final step of Polya’s heuristic is where one must reflect on their answer and justify their solution.

![Polya’s (1945) Problem Solving Heuristic](image)

**Figure 2.4. Polya’s (1945) Problem Solving Heuristic**
The study by O’Shea & Leavy (2013) investigated 5 mathematics teachers view on the constructivist theory and highlighted a number of concerns that were expressed by the teachers. One teacher pointed out that if you are teaching a class of varying levels of ability, the constructivist approach would not be of benefit to the weaker students since they have a lack of basic mathematical knowledge. Another teacher supported these claims whilst adding that she would only adopt the constructivist approach if she believes that her students have significant content knowledge. As O’Shea & Leavy (2013, p. 293) state, “constructivism is a theory of learning and not teaching”. They add that this is why traditional teaching approaches are still widespread considering teachers have to teach a large amount of content within a specific timeframe. Another aspect of teaching that is more demanding on teachers as a result of the constructivist approach is classroom management. Due to student interactions and discussions, students are generally louder and more disruptive as a result. Watt (1997) believes teachers must be aware of students’ prior conceptual knowledge in order to plan effective strategies to reconstruct their learning.

Another teaching methodology theory used by teachers is problem-based learning. Neufeld & Barrows (1974) explain that problem based learning began in the 1960’s in McMaster University Medical School in Canada. It was primarily developed for medical students but it soon became embedded into other disciplines. Savery (2006, p. 12) defines problem based learning as “an instructional learner-centered approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem”. He also adds that the success of the approach is restricted by appropriate selection of the problems and also on a teacher who guides the student through the learning process. Barrows & Tamblyn (1980) on the other hand have a more simplistic definition. They define it as “the learning that results from the process of working toward an understanding or resolution of the problem” (p. 18). Furthermore Savery & Duffy (2001, p. 14) state that problem based learning is “consistent with the principles of instruction arising from constructivism”. Similar to constructivism, they explain that learners are actively engaged in tasks and the focus is on constructing their own knowledge while they collaborating with their peers.
From a mathematics point of view, Roh (2003) asserted that problem based learning encourages problem solving skills, creative thinking, critical thinking and maths communication skills. However MacMath, Wallace & Chi (2009) explain that one difficulty is that students lack the necessary conceptual understanding to make connections. They added that students previously learned procedures without having a conceptual understanding in the subject. This is why they believe problem based learning is essential. They express that the key components of problem based learning are collaborative learning, student-centred approach, teacher as facilitator and the use of real life problems. These are all essential components of Project Maths. MacMath, Wallace & Chi (2009) believe that one of the key difficulties of this approach is the requirement of teachers to move from direct instructors to a facilitator in the classroom.

2.10 Adults Learning Mathematics

When adult education was introduced in the 1920’s many teachers of adults experienced numerous problems. They taught adults using pedagogical methods, which is the art and science of helping children learn. Knowles (1980) explained that dropout rates of adults were high since they thought teachers’ methods were insufficient. It was around this time that Knowles (1980, p. 43) defined andragogy as “the art and science of helping adults learn”. After years of educationalists questioning whether andragogy was a theory or just principles of good practice, Knowles (1989, p. 112) explained that it is “a model of assumptions about learning or a conceptual framework that serves as a basis for an emergent theory”. He added that both pedagogy and andragogy are appropriate for both children and adults depending on the situation at hand.

Knowles (1980) proposed a number of assumptions about adult learners. First of all, he insisted that adults should feel at ease in their learning environment. He added that they should feel accepted, respected and supported in a friendly and informal atmosphere. He continued by stating they should not feel like they are being talked down to, and the classroom should be set up in an adult friendly manner. Finally he explained that the teacher should take up a role of facilitator since adults are self-directed learners. This was also highlighted by Merriam
(2001) who specified that self-directed learning was another model that helped define adult learners in comparison to children.

As Schloglmann (2006) explains, education used to be a process that finished at the beginning of adulthood. However that situation has changed. There are numerous reasons for the emergence of lifelong learning. Both Schloglmann (2006) and Coben (2006) state that adults are no longer considered experts in their jobs and also developing technology have contributed to the need for adults to further educate themselves. Coben (2006) also believes that adult teaching and learning is finally receiving attention and therefore more adults are participating. She adds that it is only an emerging practice rather than a well-established one.

The emergence of adult education has seen a rise in the number of adults returning to learn mathematics. Schloglmann (2006) explains that this is because mathematics plays a pivotal role in the development of new technologies and it is also central to economic progress and development. He adds that another reason why the trend of adults returning to learning mathematics is because people learn it for personal development and fulfillment. He adds that some adults participate in adult education to raise their status or to increase their chances of promotion since they are highly motivated to learn. Essentially they see it as an opportunity. However Schloglmann (2006) insists some adults are forced into entering further education in order to prepare for a new job or because of company restructuring. It is added that for those who enter further education involuntary, learning can be difficult. Knowles (1980) claims that many adults lack motivation, they have anxiety towards mathematics due to past experiences and they fear that memory loss may occur during learning due to their age in comparison to post-primary level students.

For an educator teaching these adults, it is essential to include a process of managing this fear and to change their negative views. Coben (2006) suggests when teaching adults it is important to treat them as adults, not as students. It is important to be aware that their mathematical skills may be unpracticed for some time. It is also helpful to consider and acknowledge there may be a technology
gap, be creative and choose activities that engage adults, similar to how you would engage teenage students.

2.11 Conclusion

It has been established from this Literature Review that there are numerous issues in mathematics education both nationally and internationally. These issues range from students attitudes and anxieties towards mathematics to the design of the mathematics curriculum. The design of the mathematics curriculum nationally in Ireland was researched in detail due to the introduction of Project Maths. It was highlighted that technology is playing a key role in education with developments such as the Livescribe Smartpen. It has been determined that the key to the successful integration of a technology tool in the classroom is ensuring the technology tool is effective and useful for both the teacher and the students.

The research which was explored suggested that the digital pencasts increased confidence and students grades improved as a result. The Chief Examiners Reports in 1999, 2003 and 2005 stated that algebra is an area of great difficulty for students and so algebra is the topic chosen for this research project. In terms of student understanding, focus has changed from procedural understanding to conceptual understanding. Three frameworks for measuring conceptual understanding were researched and explored. One of these frameworks will be used in this research project, the one which aligns closest with the aims of Project Maths.
Chapter 3 - Methodology

3.1 Introduction

This research project is an examination by the author to determine if students’ conceptual understanding of algebra is altered as a result of a blended learning teaching intervention supported by digital pencasts. The purpose of the current chapter is to describe the methodology and varying research methods used in this research project from its initial stages to its conclusion. The methodology section will be presented under the following headings:

- Rationale for Undertaking the Research Project
- Research Aims / Questions
- Research Design
- Action Research
- Sampling
- Intervention Design & Execution
- Validity & Reliability
- Ethical Guidelines
- Chronology of Project

3.2 Rationale for Undertaking the Research Project

The author’s research was inspired by students’ difficulty with algebra in the author’s classroom. The author received negative feedback from the majority of students’ regarding algebra. As discussed in the Literature Review, anxiety and a negative attitude towards a topic often leads to lower achievement.

After carrying out research on students’ difficulties with algebra, the author realised these views were shared by other mathematics educationalists such as Harvey (1991). As outlined in the Literature Review, the Chief Examiners Reports in 1999, 2003 and 2005, also emphasises these views. It is expected that
if students struggle with algebra in Junior Cycle, they tend to have added difficulty with the more in-depth areas of algebra in the Senior Cycle. The Chief Examiner concluded that some of the reasons for student difficulty with algebra is due to the pace and formal approach of lessons. The introduction of Project Maths aims to reducing students’ difficulty with the topic. The author decided to create a teaching intervention that was structured around the elements of Niemi’s (1996) framework aimed at improving students’ conceptual understanding of algebra. Niemi’s (1996) framework was used as opposed to Usiskin’s (2012) or Hiebert et al.’s (2003) since it aligns closest with Project Maths. This will be discussed in more detail in sections 4.3 and 5.2. Students were supported throughout the intervention with digital pencasts created by the author.

3.3 Research Aims / Questions

The aim of this research is to determine if conceptual understanding is altered as a result of a teaching intervention supported by digital pencasts. Niemi’s (1996) framework was utilized by the author to improve students’ conceptual understanding of algebra. Firstly the literature related to both mathematics education and more specifically the area of algebra was explored. The author also reviewed literature on conceptual understanding in mathematics, which examined three frameworks to potentially use when conducting the research. The three of these frameworks have been discussed extensively in the previous chapter.

Once the research aims were devised the author established three specific research questions.

1. What frameworks exist for measuring students’ conceptual understanding of mathematics and how closely do these frameworks align with the aims of Project Maths?

2. Does a blended learning teaching intervention supported by digital pencasts enhance students’ conceptual understanding of algebra?
3. What are students’ opinions with regards to using the digital pencasts as a supportive tool to enhance their understanding of algebra?

3.4 Research Design

A fundamental element of any research study is the research design. Cohen & Manion (2000, p. 75) state that “research design is governed by the notion of fitness of purpose”. Before the author delves into the research design, it is important to be able to answer questions such as ‘what is the purpose of the research?’ and ‘how do you plan to carry out the research?’ A number of different approaches can be used; quantitative, qualitative or mixed methods approach (Cohen & Manion, 2000).

Quantitative studies generally involve either surveys or experiments whose outcomes can be measured numerically. It is used to generalize results from a large population. Cohen & Manion (2000) insist that quantitative research methods are structured in comparison to qualitative research methods because they uncover patterns and relationships in research. Kuhn (1961) states that quantitative data has the advantage that “numbers register the departure from theory with an authority and finesse that no qualitative technique can duplicate” (p. 180).

Qualitative studies on the other hand focus on people’s values and opinions and are context specific. Observations are made and analysed and general themes emerge. Qualitative data are limited as these interpretations can only lead to an hypothesis on the general population rather than specific conclusions. On the other hand one of the key reasons why this type of research was chosen in this research project is because “qualitative research methods are known to be appropriate and effective when little or nothing is known about the situation, as they do not require a predictive statement and therefore seek the answers to open questions” (Hartley & Muhit, 2003, pg.108). They continued by adding that formulating open-ended questions and facilitating a discussion of this nature takes a lot of planning and practice to ensure participants are comfortable throughout the duration of the interview process. Qualitative research is data
verifiable by observations made by the researcher or the experience of its participants rather than pure logic and takes the form of the spoken or written word rather than numbers (Punch, 2005; Roberts-Holmes, 2012). Examples of qualitative research methods include interviews, observations and the review of documents. “Qualitative research can also be used for providing possible explanations for quantitative survey results, which would be otherwise unexplicable” (Hartley & Muhit 2003 pg.110). In the context of the current research project, qualitative findings were gathered from the focus group part of the project.

Cohen & Manion (2000, p. 112) explain that “exclusive reliance on one method may bias or distort the researcher’s picture of the particular slice of reality they are investigating”. Hence they define triangulation as the use of two or more methods of data collection in a study. This method has become more popular in recent times. Researchers believe it strengthens their study and it has numerous advantages. One advantage of the triangulation approach is that the researcher will gain greater assurance if the outcomes of the quantitative inquiry are replicated by the qualitative research (Cohen & Manion, 2000). A second advantage of this approach is enhancing the validity and reliability of data by providing triangulation (Cohen & Manion, 2000).

For this research project, a triangulation approach was employed since both qualitative and quantitative methods were used. This research was conducted with both transition year students and with an adult education group. The author worked with the transition year students for the duration of the intervention. He currently teaches the adult education group who are completing the Leaving Certificate mathematics course. At the beginning of the three week intervention, a pre-test was given to the participants to assess their conceptual understanding of a particular topic in algebra. The topic that was chosen was simultaneous equations because it was the next topic to be covered. In the following 6 lessons, the author covered the topic of simultaneous equations with the participants. Participants were given access to digital pencasts as a supporting resource with the intention of developing their understanding of simultaneous equations. These digital pencasts were uploaded onto a website which the author created. At the
end of the 6 lessons a post-test similar to the pre-test was administered to the participants. Cohen & Manion (2000) explain it is essential to ensure both the pre and post-tests incorporate the same content and include the same level of difficulty. Both tests were designed by the author, in conjunction with the framework set out by Niemi (1996), and focused on assessing conceptual understanding. Along with the pre and post-tests, the lesson plans for the intervention, the handout which students received at the end of each lesson and the digital pencasts, were all designed following Niemi’s (1996) framework.

Subsequent to carrying out the data analysis on both tests, two focus groups were conducted with a random selection of the participants from both the transition year students and the entire adult education group. One advantage of a focus group is that “they are very focused on a particular issue and yield insights that might not otherwise have been available in a straightforward interview” (Cohen & Manion, 2000, p. 288). Cohen and Manion (2000) continue to state that in a focus group, participants interact with each other rather than with the interviewer. In this way participants voice their true opinions.

3.5 Action Research

There are many definitions of action research. Ebbutt (1985) suggests that action research is a form of inquiry in which one tries to improve and reform practice. Similarly Kemmis and McTaggart (1992, p. 22) claim “action research is an approach to improving education by changing it and learning from the consequences of changes”. Cohen & Manion (1994, p. 186) define action research as “a small scale intervention in the functioning of the real world and a close examination of the effects of such an intervention”. In practice, the process of action research begins with an idea of which an improvement is desirable. In the case of this research an improvement in students’ conceptual understanding of algebra is sought. Action research is known as “teacher research” in literature because of the researcher’s role as the teacher in the process. This is evident in this investigation since a taught intervention was carried out in the process of this research project.
Action research has many benefits but it also has its limitations. One limitation is that action research produces results which are not generalisable. The findings of action research are only relevant to the specific classroom being investigated. Action Research has also come under criticism due to the validity and reliability of the results as findings are often unique to the specific research. Baskerville (1996) states that if the research was carried out again with a different group results would not be the same.

One of the key reasons why action research was the most suitable research inquiry method for this project is because it inspires teachers to solve internal classroom issues themselves instead of using an external method. This is backed up by Elliot (1991, p. 49) who states that “the fundamental aim of action research is to improve practice rather than to produce knowledge”. It must be noted that an independent inquiry is being carried out. Cohen & Manion (2000) explain that action research is generally used when a problem has been identified, one tries to add something different to current practice and in the end an alternative method is provided. Therefore action research fitted appropriately with the author’s goals for the investigation.

3.6 Sampling

Sampling is important in both qualitative and quantitative research as “we cannot study everyone, everywhere doing everything” (Punch, 2006, p.187). Sampling means that not everyone in a research population needs to be included in the research due to issues surrounding resources and time. Cohen & Manion (2000, p. 102) claim that this is called a non-probability sample because it “derives from the researchers targeting a particular group, in the full knowledge that it might not represent the wider population”. Convenience sampling was used in this study because it is easy to carry out the intervention and it takes less time to carry out in comparison to probability sampling techniques. As stated in the research design (section 3.4), the research was conducted with both transition year students and with an adult education group in the author’s own school. The transition year students whom the author teaches are a mixed-ability group who completed either higher level or ordinary level mathematics for their junior
certificate. The author worked closely with this group of 24 students (12 male and 12 female) out of a total of 80 transition year pupils in the school for the duration of the intervention. The adult education group, who are completing the leaving certificate mathematics course, are a group which the author currently teaches at night. There are only six participants in this group, one male and five female. This raises validity issues due to generalising findings for a wider population since this is an action research study and hence the small numbers. The use of convenience sampling should be viewed in the context of this action-research study which by definition restricts teacher-researchers to a study of their own classroom with the intention of improving practice. It would be hoped that if action-research becomes a widespread practice then insights gained can collectively help to generalise results more widely. Whilst convenience sampling should be treated with caution, its ease of use, low cost and potential to improve practice justifies its use in this study.

3.7 Intervention Design & Execution

As discussed in chapter 2, Niemi (1996) stated that one doesn’t have a conceptual understanding of mathematics unless they can identify, generate and use representations, complete an explanation task, and problem solve and justify their solution. The majority of the intervention design was structured around the elements of Niemi’s (1996) framework from the creation of the pre and post-tests, to the designing of the intervention and the digital pencasts. The only element of the research project that was not specifically structured around Niemi’s framework were the focus group questions. Instead the focus group questions were created by the author with the aim of discovering participants’ opinions on digital pencasts.

3.7.1 Research Instruments: Pre and Post-Tests

The pre and post-tests (see Appendix B) were designed by the author and they focused extensively on the key tasks outlined by Niemi’s (1996) framework. Both tests were structured the same. Firstly, participants had to answer an explanation task about simultaneous equations. In this explanation task
participants had to explain in detail what is a linear equation, what are simultaneous equations and what is the meaning of your answer when you solve simultaneous equations. They also had to answer what methods can be used to solve a pair of simultaneous equations and how are simultaneous equations used in real life. For the representation task, participants were given a pair of algebraic simultaneous equations and six graphical representations of simultaneous equations. Their task was to identify which graph corresponded with the pair of simultaneous equations given. Participants had to complete three of these examples to lessen the likelihood that they had just guessed the correct answers. Finally participants had to complete three problem solving questions. The first two problem solving questions required participants to create a pair of simultaneous equations based on a worded problem. Then they had to represent the pair of simultaneous equations graphically. Finally they had to justify their answer by completing a number of additional questions based on the problem. The final problem solving question again required participants to create a pair of simultaneous equations based on the information given but in this instance they were not required to sketch the equations. They then had to use their procedural knowledge to find the solution and following this they had to justify their answer by completing additional questions. The problem solving questions for the pre and post-tests were based on Usiskin’s (1995) article titled “Why is Algebra Important to Learn?” The author scrutinized the examples provided in Usiskin’s (1995) paper and developed his own questions based on suitable real-life scenarios. The real life scenarios created for the pre and post-tests and the handouts were based on simultaneous equations. Therefore the author could only select appropriate examples provided by Usiskin (1995). Along with this the author ensured the real life examples would be of interest to both the transition year students and the members of the adult education group.

3.7.2 Lesson Plans and Handouts

Similar to the creation of the pre and post-tests, the lesson plans and the handouts (see Appendix A) focused extensively on the key tasks outlined by Niemi’s (1996) framework. A blended learning teaching approach was also used throughout the six lesson intervention as it combines the use of “face-to-face and online instruction with the aim of complementing each other” (Graham, 2003, p.
Along with Niemi’s framework and a blended learning approach, the lesson plans also followed a teaching method called constructivism which was discussed in the Literature Review, section 2.7. This method encourages students to engage actively and to collaborate with fellow learners during lessons. Hence active learning and pair work were used extensively during the six lesson intervention. The lessons were taught to the transition year students and the adult education group separately. Due to time constraints the six lessons were taught to the adult education group over two class periods of two hours.

Lesson One
Lesson one focused mainly on the explanation task of Niemi’s (1996) framework. The lesson began by allowing participants to discuss in pairs. Discussion questions included what is a linear equation, what are simultaneous equations, what are you trying to find when you solve simultaneous equations, what methods can be used to solve simultaneous equations and how are they used in real life. The author used oral questioning to challenge all learners. When participants were finished discussing the above questions, a whole class discussion on their answers commenced. The author clarified any uncertainty that prevailed. The next part of lesson one required participants to graph a pair of simultaneous equations. Graphing a pair of simultaneous equations is part of the representational task of Niemi’s framework. First of all, a pair of simultaneous equations were written on the whiteboard. Participants had to graph the pair of simultaneous equations on the graph paper provided and locate the point of intersection. Once participants completed the example, two participants were selected to go through the solution on the board using GeoGebra. At the end of the lesson, oral questioning was used to recap on the lesson. Higher cognitive questions such as ‘what are you trying to find when you solve a pair of simultaneous equations?’ were used. Once the recap was complete, participants were given a handout to complete for homework. The handout consisted of questions similar to those asked during class. Question 1 and 2 required participants to graph two pairs of simultaneous equations and locate the point of intersection. Meanwhile question 3 focused on the explanation task.
Lesson Two
Lesson two focused on solving a pair of simultaneous equations graphically. At the beginning of lesson two participants were asked, ‘what other methods can be used to solve a pair of simultaneous equations?’ Participants were provided with the same pair of simultaneous equations as the first example from lesson one. They were asked to solve using the algebraic method. Once they completed this task, they were asked what do they notice. Following the attainment of the answer participants were probed to check their solution was correct by using substitution. Again if participants had any difficulty throughout the lesson they were encouraged to seek help from the person beside them. Feedback was given to all participants throughout while they were being observed by the author. To conclude the lesson, a pair of simultaneous equations were solved using the algebraic method and the answer was checked using substitution. Once the recap was complete, participants were given a handout to complete for homework. Both questions from handout two required students to solve a pair of simultaneous equations using both the algebraic and graphical method. As a check strategy, students were encouraged to validate their answer using substitution.

Lesson Three
Lesson three focused on identifying representations. Once the homework from lesson two was corrected, participants were given a sheet with six graphs on it. Each graph had a pair of simultaneous equations represented. Above the six graphs a pair of simultaneous equations were given. The task for students was to identify which of the six graphs represented the pair of simultaneous equations. Participants discussed in pairs ‘what method could be used to assess which diagram represents the pair of simultaneous equations given?’ Once this discussion concluded, a whole class discussion took place. Either the participants or the author identified three methods of solving this problem. Method 1 (Algebraic Method) required solving the pair of simultaneous equations algebraically. From this they were able to eliminate four graphs. Next they had to pick points on each line from the remaining graphs and check are they a solution of the pair of simultaneous equations given. Method 2 (Finding Point of Intersection from graph) required participants to write down the point of
intersection from each diagram and check is it a solution of the simultaneous equations. From this they were able to eliminate four graphs. Next they had to pick points on each line from the remaining graphs and check are they a solution of the pair of simultaneous equations given. Method 3 (Graphical Method) required participants to find two points on each line and graph their solution on graph paper. From this, they were able to match their graph with one of the six graphs given. Once all three methods were explored, a recap on the lesson was given. Participants were given a handout to complete for homework similar to the example they completed in class, were they had to identify which of the six graphs represented the pair of simultaneous equations given using all three methods.

Lesson Four
Lesson four focused on generating and using representations and also justification. To begin participants were given a real life example about two cars. In pairs, their task was to make a pair of simultaneous equations based on the information given and graph the solution on graph paper in order to find the point of intersection between the two cars. Once participants completed the task a video of the example was shown on the whiteboard via GeoGebra. If participants made a mistake they were given time to assess where the mistake was made. The author went through the example in detail with the help of participants. Once this was complete, additional questions were given to participants to assess their understanding of the problem. These questions included; if both cars were travelling for 2 hours which car travelled the longest distance, how much further did it travel compared to the other car and which car would win the race if the race lasted 3 hours. To recap on the lesson, the position and the speed of the cars were changed and another similar example was completed orally with the help of the participants. A handout was provided for homework which included two real life examples. Participants were required to create and graph a pair of simultaneous equations and justify their solution by completing additional questions.
Lesson Five

Lesson five concentrated on problem solving and justification. Participants were given a real life example to complete. In pairs they had to make a pair of simultaneous equations using the information they were given. Then they had to solve the pair of simultaneous equations algebraically to gain the solution. To conclude the example participants answer additional questions to assess their understanding of the problem. They discussed their answers with their partners and then a whole class discussion took place. A second example was given to students to be completed in the same manner. Participants were given a handout to complete for homework. The handout consisted of similar problem solving questions to those examined during class.

Lesson Six

The final lesson recapped on all previous lessons. To begin participants had to complete an explanation task about simultaneous equations. Next they had to complete the identifying representation task similar to those given in lesson 3. Then participants were given two problem solving questions with accompanying justification questions. One question was a representational problem similar to those given in lesson four and the other question was a problem solving and justification problem similar to those in lesson five where they must solve using the algebraic method. Again using constructivism, participants completed these problems in pairs and all participants engaged actively by discussing solutions actively.

3.7.3 Digital Pencasts

Before making the digital pencasts for this investigation the author researched work on the creation of screencasts and digital pencasts. Since not a lot of research has been conducted on digital pencasts the author had to rely on the research based on screencasts. Loch et al. (2012) explained that short, focused screencasts would be more effective that one long screencasts since they are easier to understand and remember. They also suggested removing unnecessary content as this would distract learners’ attention away from the main instructional messages. Loch et al. (2014) further state that in class problems are generally very content intensive, therefore shorter screencasts would be less overwhelming for students. This is one of the reasons why four short digital
Pencasts were created by the author instead of one long digital pencast. A screenshot of one of the four short digital pencasts (problem solving & justification) is shown here in Figure 3.1.

![Problem Solving & Justification Digital Pencast](image)

**Figure 3.1. Problem Solving & Justification Digital Pencasts**

Like the tests and lesson plans, the digital pencasts were structured around the elements of Niemi’s framework. A digital pencast was created for each of the four tasks: representation, problem solving, justification and explanation. The design of the digital pencasts required careful consideration so the content was not overwhelming for the students. Along with this, Loch et al. (2014) state that it is essential to explain the step-by-step process when selecting a solution method so that students understand why each step is important. This is why numerous solution methods were provided by the author when solving the representational task as students will be able to decide which solution method they prefer and want to use.

Similar to what Loch et al. (2012) suggested, several recordings were made before a satisfactory outcome was achieved as the creation of the digital pencasts took longer than expected. This was due to the author not being happy with the quality of the digital pencasts or an accidental slip of the tongue. The author was also aware of the different viewpoints that participants in Loch et al.’s (2014)
study had. Some participants suggested shortening the screencasts by skipping some of the straightforward details, however another participant complained that the screencasts assumed a level of knowledge which they didn’t have. Since the digital pencasts include the option of pausing, rewinding and fast forwarding the author included all necessary information to ensure participants had all of the information they needed. Another concern raised by these participants was the claim that the audio quality was distorted at times. The author addressed the audio issue by purchasing a pair of Livescribe earphones. More detail on the timeline of events is provided in the Chronology of Project, section 3.10. It must be noted both the transition year students and the adult education group used the digital pencasts as a supportive tool. They accessed the digital pencasts in their own time if they had any difficulties with any particular concept. They were shown in detail how to access the digital pencasts via the authors website prior to the six lesson intervention. All participants in the focus groups were asked whether they used the digital pencasts or not.

3.7.4 Focus Groups
To conclude the intervention two focus groups were held. One focus group was held with a random selection of eight transition year students and another with the entire adult education group due to the small size of this class group. These were held to determine whether participants believed that their conceptual understanding of algebra had improved and if they found the digital pencasts useful in improving their conceptual understanding of algebra. It must be noted that students and adults conceptual understanding was likely to improve over the six lesson intervention even if traditional methods were used. Therefore it is difficult to say that one’s understanding improved due to the digital pencasts specifically. Hence this is why participants were asked their opinions on the usefulness of the digital pencasts.

It must be acknowledged that the author and the creator of the digital pencasts conducted both of the focus groups. This may influence the respondents when they are asked about the effectiveness of the technology tool. The analysis of this qualitative research was carried out by using a method called constant comparative analysis (Taylor & Bogdan, 1984). This method is used when
interviews, questionnaires or focus groups are carried out and when participants are given the opportunity to express their opinion freely. Taylor & Bogdan (1984, p. 126) explain that constant comparative analysis is when the researcher “codes and analyses data in order to develop concepts; by continually comparing specific incidents in the data, the researcher refines these concepts, identifies their properties, explores their relationships to one another and integrates them into a coherent explanatory model”.

In the case of this research project, the constant comparative method was used since two focus groups were carried out with a small number of participants. Open-ended questions were asked and the participants stated their views. The focus groups were carried out in an unbiased manner since the transition year participants were selected at random by selecting names out of a hat and the entire adult education group took part. All participants were asked the same questions (see Appendix C). Once the focus groups were carried out, the data was organized by grouping all the data from each question together so it was clear to see respondents’ consistencies and differences. Once this was complete, data was organized into categories utilizing the constant comparative method. These categories were selected by looking for emerging themes which were clear from both focus groups. Both the author and an expert examined the data to determine the emerging themes. The two sets of themes were compared to ensure both parties came up with the same themes. After selecting the emerging themes, all of the data related to these themes were inputted into NVivo (version 10). This is a software program designed to analyse quantative data. Once this was complete the process of interpreting the data began.

3.8 Validity and Reliability

A researcher should be familiar with reliability and validity when conducting research and analysing findings. The importance of reliability and validity when conducting research is that every effort to minimize errors is taken to enhance the trustworthiness of findings (O’Hara et al., 2011). Cohen & Manion (2000) explain that there are two types of validity; internal and external. They state that internal validity “seeks to demonstrate that the explanation of a particular event,
issue or set of data which a piece of research provides can actually be sustained by the data” (Cohen & Manion, 2000, p. 107). On the other hand external validity refers to generalizing the results of a study. This research project used a triangulation approach which incorporates both quantitative and qualitative data. Within qualitative research, reliability relates to the concept of good quality research and the dependability of the data collection method used. For the qualitative researcher, validity is dependent on accurately representing the voices and experience of the research participants (Roberts-Holmes, 2011).

The quantitative and qualitative data was collected by providing students with a pre and post-test. To ensure reliability when devising the pre and post-test it is essential to guarantee the two tests differ in terms of form and wording, although they must test the same content (Cohen & Manion, 2000). The data was analysed using SPSS (Statistical Software for the Social Sciences) data analysis programme and no claims were made unless a significance value of p < 0.05 was achieved. According to Cohen & Manion (2000) research is only reliable if “it were carried out on a similar group of respondents in a similar context, then similar results would be found”. In terms of the pre and post-test scores, given the limited sample of students in the adult education group it is difficult to tell whether this would be the case. As stated in the Action Research section 3.5, this is one of its limitations.

After the intervention and tests were complete, focus groups were conducted with a random selection of students from the transition year group and another with the entire adult education group. A focus group is a form of group interview, where the participants interact with each other rather than with the interviewer and they were provided with the opportunity to express their true voice (Cohen & Manion, 2000). To ensure qualitative reliability and validity as part of this research, participants were all asked identical questions and an accurate record was kept of their responses.
3.9 Ethical Guidelines

Within social research ethical issues must always be considered because research involves collecting data from people about themselves or other individuals (Cohen & Mannion, 2000). Ethical issues can arise in both qualitative and quantitative approaches to research but are more common and acute in qualitative research. The following ethical issues will be discussed as part of this section; consent, confidentiality and bias.

With regards to consent, the interviews were conducted on the basis of informed consent. Informed consent refers to the “research participants voluntarily agreeing to participate in a research project” (Robert-Holmes, 2011, p.48). Each participant of this research project and their parents / guardians, received a participant information form and consent form (see Appendix D). The participant information form outlined details of the research project, conveyed that participation was voluntary, and explained the potential risks and benefits of participation. It was also stated that this research project would not in any way have a negative effect on the participating students, particularly in terms of performance within the classroom. Ethical clearance was granted from the principal to carry out this research project (see Appendix D).

Confidentiality limits access to, or places restrictions on, certain types of information and there is a duty on all researchers not to divulge confidential information to others. This includes the storage of data deemed to be confidential. The Data Protection (Amendment) Act (2003) legislates that it is the responsibility of the researcher to store the data securely. All data on written paper will be stored in a secure locked filing cabinet in the author’s school when not in use and shredded when finished. Data will also be stored electronically on a password-protected laptop using a data-encrypted hard-drive. All electronic data will be deleted securely by the researcher from the laptop five years from the date of first collecting it as per the ethical guidelines of the University of Limerick Research Ethics Committee.
The confidentiality of participants was included as part of the consent form they received, detailing that their information would be used but that the anonymity and confidentiality of each participant was preserved. Also in the findings and discussion chapters, the names of the students and the school name were not used. In addition, focus group transcripts were only accessible to the author and his supervisors. Within social research there are always dangers of bias by researchers who have strong views about the topic they are researching. Bias includes the distortion of judgment or an unfair influence (Bell, 2008). As a result the author’s focus group questions were open-ended and were carefully worded in a way that would not influence the students in their responses.

The author maintained the highest level of professional competence in the work carried out in this research project. Furthermore, the rights, dignity and worth of all participants were respected throughout.

3.10 Chronology of Project

November 2013:
Developed the general outline of the project.

January 2014:
Before the commencement of the research, a pilot group of 20 third year students were provided with a sample of digital pencasts in order to gain feedback on their opinion of the technology tool. One of the main negative attributes discussed by the students was in relation to the difficulty opening the pencasts as part of the email attachment sent by the author. To rectify this, the author created a website where the digital pencasts could be accessed. On the homepage of the website, step-by-step instructions were provided detailing how to download and navigate through the digital pencasts. A further issue identified by the pilot group was a malfunction in the digital pencasts audio. Again the author easily rectified this minor issue by contacting the Livescribe support team who advised the use of compatible Livescribe earphones to eliminate unwanted noise. This pilot group experiment was a worthwhile exercise as it ensured that students
participating in the intervention were faced with as little technical issues as possible.

May 2014:

In May 2014, the author made a presentation on the Livescribe Smartpen to 29 teachers. Subsequent to the presentation, the teachers gave feedback about the Livescribe Smartpen by completing a short questionnaire. The questionnaire asked teachers; ‘what subject did you teach’ and ‘do they believe the Livescribe Smartpen would be beneficial in your subject as an additional study tool’. 27 out of the 29 (93%) believe the Livescribe Smartpen would be beneficial in their subject. The subjects range from mathematics and Irish to Art and Music. The two teachers who disagreed were Physical Education teachers. Some of the reasons why it would be beneficial are given;

- Extremely useful for sending notes to students so they can access them at home, especially if they are struggling with homework. The pencasts would explain the questions step-by-step.
- Excellent tool to help students with special educational needs take notes. Therefore time saving for both student and teacher.
- Beneficial to the theory area of Construction Studies.
- Would be extremely helpful for revision in the area of accounting and book keeping.
- Beneficial for diagrams and notes in Geography
- Would be great to show how to illustrate art works for art history while recording notes to go along with it. Would be very useful for both practical and art history lessons.
- Excellent to show sample exam answers in English and History.
- Useful in Irish when helping students with pronunciation, grammar points and comprehension.
October / November 2014
The pre and post-tests were created by the author and based on the key elements of Niemi’s (1996) framework. The pilot tests were adjusted based on feedback and the final tests were created.

February 2015:
The pre-test was given to both the transition year students and the adult education group. Once the tests were completed, the author began the six lesson intervention. Upon the conclusion of the intervention, all participants were given a post-test to complete.

March 2015:
Subsequent to the conclusion of the intervention and the pre and post-tests, a focus group was held separately with both a random selection of eight transition year students and the entire adult education group. Transcripts of the focus groups were typed up and inputted into NVIVO.

March 2015:
Upon the conclusion of the research, the analysis of the results began.

June 2015:
Final write up of thesis commences.

3.11 Conclusion

In this chapter, the reader was introduced to the rationale for undertaking this research project. The research aims and questions, the intervention design and execution and the research methodology that was used. Issues regarding reliability and validity, sampling and ethics were also discussed. The findings from this research will be presented in the next chapter, Chapter 4.
Chapter 4 - Research Findings

4.1 Introduction

In this chapter the author presents the main findings from the data collected during the study. The author adopted a triangulation research design as described in Chapter 3. Hence the data collected is a mixture of both qualitative and quantitative data. The author is primarily concerned with examining if an intervention to improve students’ conceptual understanding of algebra, supported by digital pencasts, was successful or not. It should be noted that all 30 participating students made use of the pencasts throughout the intervention.

This chapter includes a description of the evaluation procedures undertaken and the presentation of the findings that emerged from this research. A discussion of the results will take place in Chapter 5 with the conclusions of the research being presented in the final chapter.

4.2 Presentation of Findings

The research questions, which the author examined in this action research project were:

1. What frameworks exist for measuring students’ conceptual understanding of mathematics and how closely do these frameworks align with the aims of Project Maths?

2. Does a blended learning teaching intervention supported by digital pencasts enhance students’ conceptual understanding of algebra?

3. What are students’ opinions with regards to using the digital pencasts as a supportive tool to enhance their understanding of algebra?
The results of the pre and post-tests were analysed using SPSS. SPSS is one of the most popular statistical software packages which is generally used to analyse highly complex data. The author separated the tests into four sections based on the framework provided by Niemi; explanation task, representational task, problem solving and finally justification. The results for research question two will be presented under these headings. Both the explanation task and the justification task resulted in qualitative data but these tasks were also graded using a Likert-type score system depending on the correctness of their answers; 1 = incorrect, 2 = partially correct, 3 = mostly correct, 4 = fully correct. A grading rubric for these questions has been provided in Appendix B. The results of the explanation task and justification task from the pre-tests will be compared to their results in the post-tests for each participant to check for a change in their conceptual understanding of these tasks. The representational task along with the problem solving tasks were graded either 0 for incorrect or 1 for correct. Chi-Squared & McNemar’s test was used in the analysis of the results to compare both the transition year students and the adult education groups pre and post-test scores in both the representational and the problem solving tasks. The results from the pre-test were compared with the students’ post-test results to check for a change in their conceptual understanding.

The transcripts for each of the two focus groups were inputted into NVIVO and analysed. Three key themes emerged. The three key themes were Difficulty with Algebra, Difficulty with Technology and Positivity towards Digital Pencasts. The results related to research question three will be presented under these themes later in this chapter.

A test of reliability or internal consistency of the data obtained using each of the scales in both the pre and post-test was undertaken. The author wished to assess the reliability of the pre-test in comparison to the post-test. The value of Crobach’s alpha was calculated in each case and is outlined in Table 4.1. The Crobach’s alpha value indicated very good reliability where $p > 0.8$ is acceptable. Therefore all the data produced by the scales can be considered to be of a reliable quality. 20 questions in the pre and post-test used the scale explained above.
Table 4.1: Reliability Statistics

<table>
<thead>
<tr>
<th>Cronbach's Alpha</th>
<th>Cronbach's Alpha Based on Standardized Items</th>
<th>Number of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>.880</td>
<td>.906</td>
<td>20</td>
</tr>
</tbody>
</table>

A test for normality was also conducted and the results below show they data is normally distributed since a p-value of 0.316 is greater than 0.05. These results are shown in Table 4.2. The Shapiro-Wilk test was used for the normality test since only 20 items were tested. The Kolmogorov test is only needed when a large number of elements are tested.

Table 4.2: Test for Normality

<table>
<thead>
<tr>
<th></th>
<th>Kolmogorov-Smirnov*</th>
<th>Shapiro-Wilk</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Statistic</td>
<td>df</td>
</tr>
<tr>
<td>Data</td>
<td>.137</td>
<td>20</td>
</tr>
</tbody>
</table>

4.3 Research Question 1:

*What frameworks exist for measuring students’ conceptual understanding of mathematics and how closely do these frameworks align with the aims of Project Maths?*

The three frameworks for measuring students’ conceptual understanding of mathematics which were closely looked at in this intervention were Usiskin’s (2012) framework, Niemi’s (1996) framework and Hiebert et al.’s (2003) framework. These have been examined in detail in the Literature Review, Chapter 2. Firstly recapping on Usiskin’s (2012) framework, he suggested there
are five dimensions of understanding. The five dimensions are skill, algorithm, property proof, use application, representation and history-culture dimension. The final dimension that Usiskin explains is unnecessary for secondary level students. Niemi (1996) on the other hand proposes four tasks in order to gain conceptual understanding in mathematics. The four tasks are explanation task, representational task, problem solving task and finally a justification task. Lastly Hiebert et al. (2003) suggested five strands in order to become mathematically proficient. These five strands are; conceptual understanding (comprehension of mathematical operations, concepts and relations), procedural fluency (skill in carrying out procedures accurately and appropriately), strategic competence (ability to formulate, represent and solve problems), adaptive reasoning (logical thought, reflection, explanation and justification) and productive disposition (ability to see mathematics as useful and worthwhile).

Out of the three frameworks, Niemi’s (1996) framework was chosen for this investigation since the four tasks outlined complement the aims of Project Maths. The primary reason this framework was selected over the other two is because Niemi’s (1996) tasks focus on explanation, representing, problem solving and justification which are all objectives of Project Maths. Niemi’s (1996) framework was used in the creation of the pre and post-tests, the lesson plans and structure of the intervention, the creation of the handouts during the intervention and finally the creation of the digital pencasts. In Chapter 5 (Discussion of Findings) the three frameworks in question will be discussed with the aim of measuring how closely these frameworks align with the aims of Project Maths. In particular, Niemi’s (1996) framework will be discussed in detail since this is the framework that was chosen for this intervention.

4.4 Research Question 2:

Does a teaching intervention supported by digital pencasts enhance students’ conceptual understanding of algebra?

The results for Research Question 2 will be presented under the headings Explanation Task, Representational Task, Problem Solving Task and
Justification. These four headings were chosen since they are the key components of Niemi’s (1996) framework for measuring conceptual understanding in mathematics, as explained in Chapter 2. The results will be presented separately for the transition year students and the adult education group. Note again that there were 24 transition year students in the sample, 12 male and 12 female. There were six adults in the sample, one male and five female. First of all, the author will look at each group separately then a comparison between the two groups will follow in the discussion chapter. The pre and post-tests can be found in Appendix B.

4.4.1 Explanation Task – Transition Year Students

The explanation task was split into five questions (See Appendix B). The overall average score (out of 20) for the transition year group rose from 5.59 in the pre-test to 9.63 in the post-test.

<table>
<thead>
<tr>
<th>Pre - Post</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1a</td>
<td>1.13 – 1.17 = – .042</td>
<td>.550</td>
<td>.714</td>
</tr>
<tr>
<td>Q1b</td>
<td>1.21 – 1.92 = – .708</td>
<td>.624</td>
<td>.000</td>
</tr>
<tr>
<td>Q1c</td>
<td>1.08 – 2.00 = – .917</td>
<td>.929</td>
<td>.000</td>
</tr>
<tr>
<td>Q1d</td>
<td>1.13 – 2.08 = – .958</td>
<td>1.398</td>
<td>.003</td>
</tr>
<tr>
<td>Q1e</td>
<td>1.04 – 2.46 = – 1.417</td>
<td>1.412</td>
<td>.000</td>
</tr>
</tbody>
</table>

From Table 4.3 we can see that for Q1a (What is a Linear Equation?), the p-value from the paired t-test is 0.714 which is greater than 0.05. From this we can conclude that there is not a statistically significant difference in the mean scores on this question for the transition year students. However comparing the other four pairs of questions in the explanation task, it is clear since all of the p-values are less than 0.05 with the mean differences all negative, that there is a statistically significant increase in the mean scores.
4.4.2 Representational Task – Transition Year Students

A Chi-Squared test followed by McNemar’s test was used to compare the transition year students’ scores in the representational task from their pre-test to their post-test. The Chi-square test measures how likely an observed distribution is due to chance. McNemar’s test is used to analyse pre-test / post-test study designs. It measures if there are differences between the two related groups. It is similar to a paired-samples t-test, however for a dichotomous dependent variable in comparison to a continuous one. It was graded using score of 0 for an incorrect answer or 1 for a correct answer. This representational task included Q2a, Q2b and Q2c in which students were given six graphs with the aim of identifying which represented the pair of simultaneous equations given. Two other questions, which will be graded identically, are Q3ai and Q4ai. In these questions the transition year students were required to construct a graph to represent the simultaneous equations given. The results from these tests can be seen in Tables 4.4 and 4.5 below.

<table>
<thead>
<tr>
<th>Table 4.4: Pre and Post-Test Scores for Representational Task (Part 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Q2a Incorrect</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Q2a Correct</td>
</tr>
<tr>
<td>Q2b Incorrect</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Q2b Correct</td>
</tr>
<tr>
<td>Q2c Incorrect</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Q2c Correct</td>
</tr>
</tbody>
</table>

Comparing the transition year students’ pre and post-test scores for Q2a. Encouragingly the number of students who answered the question correctly rose from 17 to 22. Since such a large number succeeded to find the correct answer in
both tests, the p-value of 0.180 is greater than 0.05. Consequently there was no statistically significant change in the percentage of participants who got Q2a correct in the pre-test (70.8%) when compared to the percentage who got question Q2a correct in the post-test (91.7%).

Focusing on the transition year students’ pre and post scores for Q2b, none of the participants answered this question correctly in the pre-test. However, this number increased significantly to 17 in the post-test. Since the p-value of 0.000 is less than 0.05, it can be suggested that there was a statistically significant change in the percentage of participants who got Q2b correct in the pre-test (0%) when compared to the percentage who got question Q2b correct in the post-test (70.8%).

Similarly for Q2c there was a significant change since the p-value of 0.000 is less than 0.05. 4 students answered correctly in the pre-test compared to 22 in the post-test. Again this infers that there was a statistically significant change in the percentage of participants who got Q2c correct in the pre-test (8.3%) when compared to the percentage who got Q2c correct in the post-test (91.7%).

Table 4.5: Pre and Post-Test Scores for Representational Task (Part 2)

<table>
<thead>
<tr>
<th></th>
<th>Pre Test</th>
<th>Post Test</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q3ai</td>
<td>Incorrect</td>
<td>22 (91.7%)</td>
<td>3 (12.5%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>2 (8.3%)</td>
<td>21 (87.5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td>Q4ai</td>
<td>Incorrect</td>
<td>22 (91.7%)</td>
<td>9 (37.5%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>2 (8.3%)</td>
<td>15 (62.5%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.001</td>
</tr>
</tbody>
</table>

Matching the transition year students pre and post scores for Q3ai, 22 failed to answer correctly in the pre-test but only 3 students failed to do so in the post-test. This illustrates that the number of students who secured the correct answer rose from 2 to 21. As a result of this increase, the p-value of 0.000 is less than 0.05 and shows that there was a statistically significant change in the percentage of participants who got Q3ai correct.
The final question in the representational task is Q4ai. From Table 4.5 we can observe that the number of students who answered incorrectly plummeted from 22 to 9 while the number who answered correctly then increased from 2 to 15. Similar to Q3ai the p-value of 0.001 is smaller than 0.05 therefore there was a statistically significant change in the percentage of participants who got Q4ai correct in the pre-test (8.3%) when compared to the percentage of participants who got Q4ai correct in the post-test (62.5%).

4.4.3 Problem Solving Task – Transition Year Students

A Chi-Squared test followed by McNemar’s test was also used to compare the transition year students’ scores for the problem solving task. This problem solving task involved questions 3bi, 3ci, 4aii, 4bi, 4ci, 5ai, 5bi and 5ci. A sample of these questions can be viewed in Appendix C. Each of these questions were graded 0 for incorrect or 1 for a correct answer.

Table 4.6: Pre and Post-Test Scores for Problem Solving Task (Part 1)

<table>
<thead>
<tr>
<th></th>
<th>Pre Test</th>
<th>Post Test</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q3bi</td>
<td>Incorrect</td>
<td>12 (50%)</td>
<td>4 (16.7%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>12 (50%)</td>
<td>20 (83.3%)</td>
</tr>
<tr>
<td>Q3ci</td>
<td>Incorrect</td>
<td>15 (62.5%)</td>
<td>3 (12.5%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>9 (37.5%)</td>
<td>21 (87.5%)</td>
</tr>
</tbody>
</table>

Comparing the transition year students’ pre and post scores for Q3bi, the number of students who answered correctly increased from 12 to 20. This resulted in a p-value of 0.039 when comparing between pre and post-test results. This is less than 0.05 and implies that there was a statistically significant change in the percentage of participants who got Q3bi correct in the pre-test (50%) when compared to the percentage who got question Q3bi correct in the post-test (83.3%). Similarly for Q3ci, a significant change was evident due to the number of students who answered correctly rising from 9 to 21. Along with a p-value of 0.002, which is less than 0.05, this suggests a statistically significant change in
the number who answered correctly in the post-test compared to the pre-test.

Table 4.7: Pre and Post-Test Scores for Problem Solving Task (Part 2)

<table>
<thead>
<tr>
<th></th>
<th>Pre Test</th>
<th>Post Test</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q4aii</td>
<td>Incorrect</td>
<td>22 (91.7%)</td>
<td>14 (58.3%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>2 (8.3%)</td>
<td>10 (41.7%)</td>
</tr>
<tr>
<td>Q4bi</td>
<td>Incorrect</td>
<td>22 (91.7%)</td>
<td>6 (25%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>2 (8.3%)</td>
<td>18 (75%)</td>
</tr>
<tr>
<td>Q4ci</td>
<td>Incorrect</td>
<td>18 (75%)</td>
<td>10 (58.3%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>6 (25%)</td>
<td>14 (41.7%)</td>
</tr>
</tbody>
</table>

For Q4aii and Q4bi, both the p-values, 0.021 and 0.000 respectively are less than 0.05. This suggests there was a significant change in the percentage of students who got Q4aii (8.3%) and Q4bi (8.3%) correct in the pre-test when compared to the percentage who got Q4aii (41.7%) and Q4bi (75%) correct in the post-test. Meanwhile in Q4ci, the percentage changed from just 25% to 41.7% which suggests there is no significant change since the p-value (0.375) is greater than 0.05.

Table 4.8: Pre and Post-Test Scores for Problem Solving Task (Part 3)

<table>
<thead>
<tr>
<th></th>
<th>Pre Test</th>
<th>Post Test</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q5ai</td>
<td>Incorrect</td>
<td>21 (87.5%)</td>
<td>12 (50%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>3 (12.5%)</td>
<td>12 (50%)</td>
</tr>
<tr>
<td>Q5bi</td>
<td>Incorrect</td>
<td>21 (87.5%)</td>
<td>22 (91.7%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>3 (12.5%)</td>
<td>2 (8.3%)</td>
</tr>
<tr>
<td>Q5ci</td>
<td>Incorrect</td>
<td>24 (100%)</td>
<td>24 (100%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
</tbody>
</table>

Matching the transition years pre and post-test scores for Q5ai it is evident that 3 students got the question correct in the pre-test whereas this number increased to
12 in the post-test. This lead to a p-value of 0.012 which is less than 0.05. This suggests that there was a statistically significant change in the percentage of participants who got Q5ai correct in the pre-test (12.5%) when compared to the percentage who got question Q5ai correct in the post-test (50%). In question 5bi, only 3 students answered correctly in the pre-test and only 2 answered correctly in the post-test. Similarly, in Q5ci, all 24 students failed to answer the question correctly in both the pre and post-tests. Both questions had p-value of 1 which are greater than 0.05. Hence, statistically there is no change in scores for both of these questions.

4.4.4 Justification – Transition Year Students

The justification is a follow on from the problem solving task. These justification questions checked that students understood the answers which they had given. A sample of these questions can be viewed in Appendix B. As stated in the Presentation of Findings (4.2) section, these questions were graded using a score system; 1 = incorrect, 2 = partially correct, 3 = mostly correct, 4 = fully correct. The mean score and standard deviation for each of these questions is given below. From Table 4.9 it can be computed that the overall average score (out of 20) for the transition year students rose from 7.71 in the pre-test to 12.51 in the post-test.

Table 4.9: Mean and Standard Deviation of Justification Task

<table>
<thead>
<tr>
<th>Question</th>
<th>Mean Pre</th>
<th>Mean Post</th>
<th>N</th>
<th>Std. Deviation Pre</th>
<th>Std. Deviation Post</th>
<th>Std. Error Pre</th>
<th>Std. Error Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q3aii</td>
<td>1.21</td>
<td>1.88</td>
<td>24</td>
<td>.415</td>
<td>1.035</td>
<td>.85</td>
<td>.211</td>
</tr>
<tr>
<td>Q3bii</td>
<td>1.83</td>
<td>3.29</td>
<td>24</td>
<td>1.090</td>
<td>1.268</td>
<td>.223</td>
<td>.259</td>
</tr>
<tr>
<td>Q3cii</td>
<td>2.21</td>
<td>3.21</td>
<td>24</td>
<td>1.414</td>
<td>1.103</td>
<td>.289</td>
<td>.225</td>
</tr>
<tr>
<td>Q4bii</td>
<td>1.08</td>
<td>2.00</td>
<td>24</td>
<td>.282</td>
<td>.659</td>
<td>.058</td>
<td>.135</td>
</tr>
</tbody>
</table>

Comparing each of the five questions it can be seen that the p-values are less than 0.05 and the mean differences are all negative (Table 4.10). Therefore we
can conclude that there is a statistically significant increase in the mean scores for all questions in the justification task.

**Table 4.10: Summary Statistics of Justification – Comparing Pre & Post Test Scores for Transition Year Students (Paired Samples Test)**

<table>
<thead>
<tr>
<th>No.</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre Q3aii – Post Q3aii</td>
<td>-.667</td>
<td>1.049</td>
<td>.005</td>
</tr>
<tr>
<td>Pre Q3bii – Post Q3bii</td>
<td>1.458</td>
<td>1.888</td>
<td>.001</td>
</tr>
<tr>
<td>Pre Q3cii – Post Q3cii</td>
<td>1.000</td>
<td>1.934</td>
<td>.019</td>
</tr>
<tr>
<td>Pre Q4bii – Post Q4bii</td>
<td>-.917</td>
<td>.654</td>
<td>.000</td>
</tr>
<tr>
<td>Pre Q4cii – Post Q4cii</td>
<td>-.750</td>
<td>1.539</td>
<td>.026</td>
</tr>
</tbody>
</table>

**4.4.5 Explanation Task – Adult Education Group**

The explanation task was split into the same five questions as outlined in Appendix B. As you can see from Table 4.11 the overall average score (out of 20) for the 6 adult students rose from 10.83 in the pre-test to 15.67 in the post-test.

**Table 4.11: Comparing Pre & Post-Test Scores for Adult Education Group.**

<table>
<thead>
<tr>
<th>Pre - Post</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1a</td>
<td>2.00 – 2.33 = -.333</td>
<td>.516</td>
<td>.175</td>
</tr>
<tr>
<td>Q1b</td>
<td>2.33 – 3.00 = -.667</td>
<td>.516</td>
<td>.025</td>
</tr>
<tr>
<td>Q1c</td>
<td>2.00 – 2.67 = -.667</td>
<td>.816</td>
<td>.102</td>
</tr>
<tr>
<td>Q1d</td>
<td>2.00 – 3.67 = 1.667</td>
<td>1.506</td>
<td>.042</td>
</tr>
<tr>
<td>Q1e</td>
<td>2.50 – 4.00 = 1.500</td>
<td>1.643</td>
<td>.076</td>
</tr>
</tbody>
</table>

Looking at Q1a the p-value is 0.175 which is greater than 0.05 thus we can conclude there is not a statistically significant difference in the mean scores on Q1a for the adult education group. Similarly the p-values for Q1c and Q1e are 0.102 and 0.076 respectively, which again are not statistically significant. Despite the fact that p-values are greater than 0.05, it is important to note that the
mean differences for each of these three questions negative. Looking at Q1b and Q1d we can see that the p-value for these questions are 0.025 and 0.042, respectively. Since both of these p-values are less than 0.05, with negative mean differences, we can conclude that there is a statistically significant increase in the mean scores of Q1b and Q1d in the adult education group.

4.4.6 Representational Task – Adult Education Group

A Chi-Squared test followed by McNemar’s test was used to compare the adult education groups’ scores from their pre-test to post-test. The following is the presentation of the results for the representational task.

| Table 4.12: Pre and Post-Test Scores for Representational Task (Part 1) |
|-------------------------------------------------|-----------------|-----------------|-----------------|
| Q2a                                             | Incorrect       | Pre Test | Post Test | P-Value |
| Correct                                         | 1 (16.7%)       | 0 (0%)    | 6 (100%)  | 1.000   |
|                                                | 5 (83.3%)       |           |           |         |
| Q2b                                             | Incorrect       | Pre Test | Post Test | P-Value |
| Correct                                         | 3 (50%)         | 0 (0%)    | 6 (100%)  | 0.375   |
|                                                | 3 (50%)         |           |           |         |
| Q2c                                             | Incorrect       | Pre Test | Post Test | P-Value |
| Correct                                         | 3 (50%)         | 1 (16.7%) |
|                                                | 3 (50%)         | 5 (83.3%) |

Comparing the adult education groups’ pre and post scores for Q2a we can see that 5 people got the question correct in the pre-test but everyone answered correctly in the post-test. Since such a large number acquired the correct answer in both tests, the p-value of 1 which is not less than 0.05. This suggests that there was no significant change in the percentage of participants who got Q2a correct in the pre-test (83.3%) when compared to the percentage who got question Q2a correct in the post-test (100%).

In Q2b, only 3 people answered correctly in the pre-test but all 6 students got it correct in the post-test. Since the p-value of 0.375 is not less than 0.05 this suggests that there is no statistically significant change in the percentage of
participants who got Q2b correct in the pre-test (50%) when compared to the percentage who got question Q2b correct in the post-test (100%).

For Q2c there was again no significant change as the p-value in this was 0.625. Hence there was no statistically significant change in the percentage of participants who got Q2c correct in the pre-test (50%) when compared to the percentage who got Q2c correct in the post-test (83.3%).

Table 4.13: Pre and Post-Test Scores for Representational Task (Part 2)

<table>
<thead>
<tr>
<th></th>
<th>Pre-Test</th>
<th>Post-Test</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q3ai</td>
<td>Incorrect</td>
<td>2 (33.3%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>4 (66.7%)</td>
<td>6 (100%)</td>
</tr>
<tr>
<td>Q4ai</td>
<td>Incorrect</td>
<td>4 (66.7%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>2 (33.3%)</td>
<td>6 (100%)</td>
</tr>
</tbody>
</table>

Matching the adult education groups pre and post scores for Q3ai, 2 failed to answer correctly in the pre-test however all students answered correctly in the post-test. However the p-value of 0.625 is greater than 0.05 and so there was no significant change in the percentage of participants who got Q3ai correct in the pre-test (66.7%) when compared to the percentage who got Q3ai correct in the post-test (100%).

The final question in the representational task is Q4ai. From the table we can see that the number of adults who answered correctly increased from 2 to 6. Hence since the p-value of 0.021 is smaller than 0.05 therefore there was a statistically significant change in the percentage of participants who for Q4ai correct in the pre-test (33.3%) when compared to the percentage of participants who for Q4ai correct in the post-test (100%).
4.4.7 Problem Solving Task – Adult Education Group

Table 4.14: Pre and Post-Test Scores for Problem Solving Task (Part 1)

<table>
<thead>
<tr>
<th></th>
<th>Pre Test</th>
<th>Post Test</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q3bi</td>
<td>Incorrect</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>6 (100%)</td>
<td>6 (100%)</td>
</tr>
<tr>
<td>Q3ci</td>
<td>Incorrect</td>
<td>0 (0%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>6 (100%)</td>
<td>6 (100%)</td>
</tr>
</tbody>
</table>

For Q3bi and Q3ci, all 6 adults answered correctly for both the pre and post-test. Therefore there is no statistically significant change in the scores for both questions.

Table 4.15: Pre and Post-Test Scores for Problem Solving Task (Part 2)

<table>
<thead>
<tr>
<th></th>
<th>Pre Test</th>
<th>Post Test</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q4aii</td>
<td>Incorrect</td>
<td>3 (50%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>3 (50%)</td>
<td>6 (100%)</td>
</tr>
<tr>
<td>Q4bi</td>
<td>Incorrect</td>
<td>1 (16.7%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>5 (83.3%)</td>
<td>6 (100%)</td>
</tr>
<tr>
<td>Q4ci</td>
<td>Incorrect</td>
<td>1 (16.7%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td></td>
<td>Correct</td>
<td>5 (83.3%)</td>
<td>6 (100%)</td>
</tr>
</tbody>
</table>

Looking at the adult education groups’ scores on Q4aii in the pre-test we see that 3 answered incorrectly. Although these results improved in the post-test due to all 6 answering the question correctly, a p-value of 0.375 suggests there was no statistically significant change in the percentage of participants who got Q4aii correct in the pre-test (50%) when compared to the percentage who got question Q4aii correct in the post-test (100%). In both Q4bi and Q4ci there was no statistically significant change in scores since 5 people answered correctly in the
pre-test in both of the questions, meanwhile everyone answered correctly in the post-test.

Table 4.16: Pre and Post-Test Scores for Problem Solving Task (Part 3)

<table>
<thead>
<tr>
<th></th>
<th>Incorrect</th>
<th>Correct</th>
<th>Pre Test</th>
<th>Post Test</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q5ai</td>
<td>2 (33.3%)</td>
<td>4 (66.7%)</td>
<td>0 (0%)</td>
<td>6 (100%)</td>
<td>0.625</td>
</tr>
<tr>
<td>Q5bi</td>
<td>3 (50%)</td>
<td>3 (50%)</td>
<td>3 (50%)</td>
<td>3 (50%)</td>
<td>1.000</td>
</tr>
<tr>
<td>Q5ci</td>
<td>3 (50%)</td>
<td>3 (50%)</td>
<td>5 (83.3%)</td>
<td>1 (16.7%)</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Looking at the adult education groups’ scores on Q5ai in the pre-test 4 adults answered the question correctly. These results improved in the post-test due to all 6 answering the question correctly. A p-value of 0.625 suggests there was no significant change in the percentage of participants who got Q5ai correct in the pre-test (66.7%) when compared to the percentage who got question Q5ai correct in the post-test (100%). In Q5bi there was no significant change since the same number of adults answered the question correct in both the pre and post-test while less adults answered correctly in Q5ci’s post-test in comparison to the pre-test.

4.4.8 Justification – Adult Education Group

As with the transition year group, the justification task is a follow on from the problem solving task. These justification questions checked that students understood the answer they gave. As stated in the Presentation of Findings (4.2) section, these questions were graded using a score system; 1 = incorrect, 2 = partially correct, 3 = mostly correct, 4 = fully correct. The mean score and standard deviation for each of these questions are given below. From Table 4.16 it can be computed that the overall average score (out of 20) for the adult education group rose from 13.67 in the pre-test to 17.17 in the post-test.
Table 4.17: Mean and Standard Deviation of Justification

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>N</th>
<th>Std. Deviation</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre</td>
<td>Post</td>
<td>Pre</td>
<td>Post</td>
</tr>
<tr>
<td>Q3aii</td>
<td>1.67</td>
<td>2.67</td>
<td>6</td>
<td>.816</td>
</tr>
<tr>
<td>Q3bii</td>
<td>3.33</td>
<td>4.00</td>
<td>6</td>
<td>.816</td>
</tr>
<tr>
<td>Q3cii</td>
<td>3.67</td>
<td>3.83</td>
<td>6</td>
<td>.516</td>
</tr>
<tr>
<td>Q4bii</td>
<td>2.00</td>
<td>3.17</td>
<td>6</td>
<td>.632</td>
</tr>
<tr>
<td>Q4cii</td>
<td>3.00</td>
<td>3.50</td>
<td>6</td>
<td>1.265</td>
</tr>
</tbody>
</table>

Table 4.18: Comparing Pre & Post Test Scores for Adult Education Group (Paired Samples Test)

<table>
<thead>
<tr>
<th>No.</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre Q3aii – Post Q3aii</td>
<td>- 1.000</td>
<td>.894</td>
<td>.041</td>
</tr>
<tr>
<td>Pre Q3bii – Post Q3bii</td>
<td>-.667</td>
<td>.816</td>
<td>.102</td>
</tr>
<tr>
<td>Pre Q3cii – Post Q3cii</td>
<td>-.167</td>
<td>.753</td>
<td>.611</td>
</tr>
<tr>
<td>Pre Q4bii – Post Q4bii</td>
<td>- 1.167</td>
<td>.753</td>
<td>.013</td>
</tr>
<tr>
<td>Pre Q4cii – Post Q4cii</td>
<td>-.500</td>
<td>1.225</td>
<td>.363</td>
</tr>
</tbody>
</table>

Looking at Q3aii and Q4bii, the p-values are 0.041 and 0.013 respectively, which are < 0.05 thus indicating there is a statistically significant difference in the mean scores on these questions for the adult education group in the justification task. Contrastingly the p-values associated with the remaining questions are 0.102, 0.611 and 0.363 respectively, thus there is not a statistically significant difference in the mean score for questions 3bii, Q3cii and Q4cii.

4.5 Research Question 3:

*What are students’ opinions with regards to using the digital pencasts as a supportive tool to enhance their understanding of algebra?*
The results for Research Question 3 will be presented under the following headings: Difficulty with Algebra, Difficulty with Technology and Positivity towards Digital Pencasts. These were the three key themes to emerge from the two focus groups with both the transition year students and the adult education group. These focus group interviews were recorded using the Livescribe Smartpen and then they were transcribed and inputted into NVIVO (version 10) to be analyzed. Responses are indicated by single quotation marks and presented in their original form. The findings from each of the three themes will be discussed in the context of the literature reviewed in chapter 2 in the next chapter. The focus group transcripts for both the transition year student and the adult education group can be found in Appendix C.

The first theme to emerge from the focus groups was ‘Difficulty with Algebra’. 14% of the transcript covered the topic of algebra and students’ difficulties with the topic. When students were asked about their feelings with algebra, one student responded that they ‘hate it’ and another explained ‘well algebra is my least favourite topic’. Subsequently, the transition year group were asked ‘In your opinion, what if any, are the benefits of algebra’. Student 6 claimed that ‘It probably makes you use your brain but I don’t know how it would help in everyday life’. Similar difficulties have also emerged regarding technology, which will be discussed in the next paragraph.

The second theme to emerge from these focus groups was ‘Difficulty with Technology’. 13% of the transcript focused on transition year students difficulties with technology. Firstly, they were asked whether they encountered any difficulties with the digital pencasts. One students explained ‘I couldn’t get the audio going in the beginning but I used the handout you gave us and then it worked’. Following on from this another student asked ‘how come the digital pencasts don’t work on an iPhone’. It was explained by the interviewer that iPhones do not support Adobe Flash Player and hence the digital pencasts will not play on such a device. However the interviewer added that ‘I am trying to find a way around that. I believe they will fix this issue soon since the majority of people have a smartphone’. Subsequent to talking about students issues with
the digital pencasts, the positive nature of the digital pencasts will be discussed in the next paragraph.

The third and final theme to emerge from these focus groups was ‘Positivity towards Digital Pencasts’. This emerged because 61.2% of the focus group interview with the transition year students was spent talking about the positives of the digital pencasts. Students were asked ‘Do you believe that a supportive tool like the digital pencasts has aided your understanding of algebra’. All students agreed that the digital pencasts aided their understanding while student 1 added that ‘you can go through problems at your own pace, compared to the fast pace we go through topics in class’. Along with this student 4 insisted that ‘at least when we go into 5th year, if there are any areas we forget we can revise using the pencasts’. Next the transition year students were asked if they found the digital pencasts a valuable tool when learning algebra. Students 1 responded ‘definitely, I couldn’t remember how to graph simultaneous equations. Even when you went through it in class I was still confused. At least I could go through them at my own pace at home’. Student 4 added ‘yeah I forgot everything also. They were great for revision’. When asked about the audio explanation student 1 responded ‘yes that’s the main part. Without the audio, the pencasts are just like a normal textbook’. Finally students were asked ‘In your opinion, do you believe the digital pencasts helped to improve your understanding of algebra’. Student 1 stated ‘Yes. I never realised that solving simultaneous equations using algebra and also using the graphical method, you got the same answer. Going through the representation pencast at home, it clicked with me that’s what it meant. So they definitely helped improve my understanding’. Then student 7 explained that the only drawback is the inability to asked questions.

Moving on to the adult education group, again the first theme to emerge from this focus groups was Difficulty with Algebra. This emerged because 20% of the transcript addressed the issue of algebra and the adults’ difficulties with the topic. Initially they were asked their feelings towards algebra. Adult 1 explained ‘I really struggle with the topic. I always have, its one area of mathematics I really did not enjoy at school. I find it very difficult and I do not see its
relevance’. Meanwhile adult 3 added ‘I am less weary of the subject due to this intervention using the digital pencasts but I am still not overly confident with the topic’. However adults 2 and 5 admit they enjoy algebra due to its challenging nature. When the adult education group were asked if there are any benefits of algebra they responded that it is used in many key areas of life such as engineering and computer programming.

The second theme to emerge from the focus group with the adult education group was Difficulty with Technology. 20% of the conversation was spent talking about using technology at home, the difficulties and the difficulties with the digital pencasts. Firstly they were asked if they have a computer at home and if they use it often. All responded that they have a computer at home while adult 1 added ‘I try to use it more often because I have 4 young daughters who use technology all the time. I have to keep up to date’. Next the adults were asked if they are comfortable using technology. Again adult 1 explained that ‘its constantly changing and I cannot keep up to date. I sometimes feel quite intimidated by technology and have a fear of doing something wrong’. Finally when asked whether they encountered any difficulties with the digital pencasts one adult said ‘I struggled to get the audio working in the beginning but when I followed the required steps which you explained, it worked for me then’. The other adults agreed.

The third and final theme to emerge from these focus groups was Positivity towards Digital Pencasts. This emerged because 53% of the focus group with the adult education group was spent talking about the positives about the digital pencasts. Initially the adults were asked ‘do you believe that a supportive tool like the digital pencasts has aided your understanding of algebra?’ Adult 4 responded ‘yes most definitely because we cover one topic per night here so I use it a lot for revision purposes’. Adult 6 added ‘they help because I like to see the steps worked. The book solutions can be overwhelming compared to the step by step solution of the digital pencasts’. Finally adult 5 explained ‘listening to the steps explained and being able to pause between steps to digest what going on is great I found’. Next the interviewer asked ‘did you find the digital pencasts a valuable tool when learning algebra?’ All of the adult education group agreed it
is a valuable tool while adult 6 added that ‘it is extremely beneficial for students if they are willing to use it’. Adult 6 also believes the digital pencasts are extremely beneficial since you can access them whenever you want. Adult 5 added that ‘it’s like having someone there beside you explaining the answers’. The ability to pause between steps was also seen as an extremely beneficial feature of the digital pencasts since it allows one to digest new information. When asked about the audio feature and the benefits of the audio all adults agreed that this is the most important part of the digital pencasts. Adult 6 explained that ‘without the audio explanation the solution is just like a book solution. Subsequently the interviewer asked ‘In your opinion, do you believe the digital pencasts helped to improve your understanding of algebra?’ Adult 6 stated ‘Yes, without a doubt. If you are stuck on something and you can listen to the directions again. In my day if you were stuck with your homework, you either asked your parents or else you couldn’t complete it. I have used them numerous times already to revise difficult parts of algebra’. Meanwhile adult 5 added ‘it’s like having the teacher beside you, which is great’. When asked if they would recommend the digital pencasts to others they all said yes and adult 2 concluded ‘I think it’s an excellent tool for students to have. I find it excellent when revising’.

4.6 Other themes to Emerge:

On examining the data other minor themes emerged, outside the three central research questions. These themes were largely concerned with the usefulness of the digital pencasts.

The final question in the focus group to both the transition year students and the adult education group was: In your opinion, would digital pencasts be useful in other subjects, and if so, what subjects do you think it would be most helpful in?

The feedback from this was extremely positive. Everyone agreed it would be beneficial in other areas of mathematics and also other subjects. The transition year students elaborated by adding:
Student 5:
'It would be great in history and English for planning essays.'

Student 6:

'Yeah because I always forget what we are meant to do.'

Student 4:

'It would probably be handy enough in French also I think.'

Student 3:

'Yes for orals it would be great.'

The adult education group had similar views to this:

Adult 2:

'I think it would be fantastic for business and accountancy. Anything you would need instruction for.'

Adult 6:

'Also for languages from a pronunciation point of view.'

Adult 5:

'And maybe for diagrams and key words in geography and science too.'

Adult 3:

'Yes it’s probably excellent for other topics too but I feel it is most beneficial in mathematics because it’s a subject most people struggle with.'

The findings from ‘Other Themes to Emerge’ will be discussed in the next chapter, Chapter 5: Discussion of Findings.

4.7 Summary of Findings:

At the outset of this chapter the three research questions were outlined. They were:

1. What frameworks exist for measuring students’ conceptual understanding of mathematics and how closely do these frameworks align with the aims of Project Maths?
2. Does a blended learning teaching intervention supported by digital pencasts enhance students’ conceptual understanding of algebra?

3. What are students’ opinions with regards to using the digital pencasts as a supportive tool to enhance their understanding of algebra?

After examining the data the key findings to emerge are:

1. There was a significant change (p < 0.05) in transition year students’ scores in a large number of the questions when comparing their pre and post-test scores.

2. There was no significant change (p < 0.05) in adult education group scores in the majority of their questions when comparing their pre and post-test scores.

3. No claim of significance can be made overall as to whether the use of an intervention based on Niemi’s framework enhanced students’ conceptual understanding of algebra.

4. The general theme that emerges from the qualitative data is the positivity that all respondents showed towards the digital pencasts. All respondents believe they are a beneficial revision tool and that they helped improve their understanding of algebra.

5. Both the transition year students and the adult education group agree that the digital pencasts would be beneficial both in other areas of mathematics and other subjects entirely.
Chapter 5 – Discussion of Findings

5.1 Introduction

The aim of this research project is to assess whether a blended learning teaching intervention supported by digital pencasts would improve students’ conceptual understanding of algebra. In this chapter the author discusses the main findings from the data collected for the purpose of the study. These findings will be discussed by examining a number of themes that have been identified from the analysis of the data and in the context of the literature reviewed in chapter 2.

5.2 Research Question 1:

*What frameworks exist for measuring students’ conceptual understanding of mathematics and how closely do these frameworks align with the aims of Project Maths?*

The three frameworks for measuring students’ conceptual understanding of mathematics which were analysed in the ‘Literature Review’ in Chapter 2 were Usiskin’s (2012) framework, Niemi’s (1996) framework and Hiebert et al.’s (2003) framework. Firstly, Usiskin’s (2012) four dimensions of understanding from a students’ standpoint are as follows; the skill-algorithm dimension, the property-proof dimension, the use-application dimension and finally the representation dimension. Niemi’s (1996) framework explains that one must be able to complete a representational task, a problem solving task, a justification task and finally an explanation task in order to gain a conceptual understanding of a particular mathematics topic. Lastly, Hiebert et al. (2003) proposed five strands in order to become mathematically proficient. These five strands are; conceptual understanding (comprehension of mathematical operations, concepts and relations), procedural fluency (skill in carrying out procedures accurately and appropriately), strategic competence (ability to formulate, represent and solve...
problems), adaptive reasoning (logical thought, reflection, explanation and justification) and productive disposition (ability to see mathematics as useful and worthwhile).

Out of the three frameworks, Niemi’s (1996) framework was the chosen framework for this investigation since the four tasks outlined compliment the aims of Project Maths. The primary reason that this framework was deemed the more suitable framework for the current research is because Niemi’s (1996) tasks focus on explanation, representing, problem solving and justification which are all objectives of Project Maths. It is fundamental to understand a concept in detail in order to complete the explanation task. Likewise, the completion of the justification task requires a problem solving task to be completed and the learner must understand the problem in order to justify their answer. As stated in the ‘Literature Review’ in Chapter 2, one of the key aims of Project Maths is to deepen students’ conceptual understanding of each topic. This was also highlighted by the NCCA (2008) who explained that new approaches towards teaching and learning were needed in order to enhance students understanding in mathematics. These novel approaches to teaching and learning included a greater emphasis being placed on real life applications in which mathematics would relate to everyday experiences. Again this objective of Project Maths is also a key component of Niemi’s framework, the problem solving task. Taking the perspective of the classroom environment these real life problems are provided with the aim of creating an active classroom. This in turn will increase the quality of the teaching and learning experience of students. Focusing on the current research, Niemi’s (1996) framework was used as a guide in the creation of the pre and post-tests, the lesson plans, structure of the intervention, the creation of the handouts, and also the creation of the digital pencasts.

Although it has been previously alluded to that Usiskin’s five dimensions of understanding are also key components of the Project Maths curriculum, in terms of the actual representation dimension, Usiskin (2012) only discusses the importance of being able to represent a concept. Niemi (1996) on the other hand insists one must be able to identify, generate and use representations. A further shortcoming in Usiskin’s (2012) framework is that there is no explanation task.
As outlined above, one cannot explain a concept in detail unless they have a deep understanding of it. In the five strands proposed by Hiebert et al. (2013) the explanation task is evident and is referred to as adaptive reasoning. The fifth strand, productive disposition, which is the ability to see mathematics as useful and worthwhile didn’t feature in the objectives of Project Maths. Therefore Niemi’s framework was the version the author selected for the purpose of this research project due to how closely it aligns to the aims of Project Maths.

5.3 Research Question 2:

*Does a teaching intervention supported by digital pencasts enhance students’ conceptual understanding of algebra?*

As stated in the ‘Research Findings’ chapter, Niemi’s framework was implemented in the evaluation of this investigation therefore the findings of Research Question 2 will be discussed under the following four headings: Explanation Task, Representational Task, Problem Solving Task and Justification.

5.3.1 Explanation Task

The explanation task was the first question on the pre and post-tests. As stated in Chapter 2, Niemi (1996) explains that this task requires one to explicitly explain a concept and its principles. The questions asked in the Explanation Task are listed below.

- What is a linear equation?
- What are simultaneous equations?
- What is the meaning of your answer when you solve simultaneous equations?
- What methods can be used to solve a pair of simultaneous equations?
- How are simultaneous equations used in real life?
Transition Year Students

![Graph showing comparison of mean scores for Transition Year Students](image)

**Figure 5.1: Comparison of Mean Scores for Transition Year Students**

Firstly concentrating on the transition year students’ scores, it was clear that there is an improvement in the mean score on each of the five questions (see Figure 5.1). Furthermore in four of the five cases, the p-value is less than 0.05 therefore there is a statistically significant increase in the mean scores. Based on the scale which these questions were marked on, the mean scores for the transition year students in the pre-test was approximately 1 (i.e. incorrect). This explanation task created by Niemi (1996) focuses on developing students’ ability to explain a concept. There was a statistically significant improvement on 4 out of the 5 areas in the post-test for the transition year students, the mean scores for these students in the post-test was approximately 2 (i.e. partially incorrect). Therefore while there is a statistically significant improvement in the post-test mean scores, there is still room for improvement in order for students to have a deeper conceptual understanding of simultaneous equations in terms of the explanation task. Perhaps the reason for a lack of conceptual understanding in this area is because students are used to answering questions in a procedural-like fashion. Additionally Linchevski & Herscovics (1994, p. 60) stated that students are “reduced to performing meaningless operations on symbols they don’t understand” as a result of a lack of understanding about a concept. Niemi (1996)
alludes to the fact that if students complete explanation tasks then their conceptual understanding about a topic will improve.

**Adult Education Group**

![Figure 5.2: Comparison of Mean Scores for Adult Education Group](image)

**Figure 5.2: Comparison of Mean Scores for Adult Education Group**

Similar to the transition year group, the adult education group results also displayed an improvement in mean scores in each of the five questions when comparing the pre and post scores (see Figure 5.2). However there is only a statistically significant increase in the mean scores of Q1b and Q1d since both p-values are less than 0.05. Based on the scale that these questions were scored against, the mean scores across all the questions for the adult education group in the pre-test were between 2.00 and 2.50 (i.e. partially correct) however the mean scores for the adults in the post-test ranged between 2.33 and 4.00. This explains that while there is only a statistically significant improvement in two out of the five questions, adults’ conceptual understanding has improved since they achieved either ‘mostly correct’ or ‘fully correct’ in the majority of questions in the post-test. Another reason for the lack of a statistically significant improvement in all areas of the explanation task may be due to the small group of adults in the group. A large database would be more likely to yield results which were statistically significant.
It is clear that the adult education group received a higher grade overall in the explanation task in comparison to the transition year students. Prendergast & Sterritt (2011) explains that post-primary students are graded on their mathematical skills and manipulations and not on their understanding of concepts. In the context of this task, it would be understandable then to conclude that students are not familiar with explaining a concept in this fashion, hence the poor results. Perhaps this is why the transition year students underperformed at this task. It may also be due to the fact that adults are more used to this type of task from previous work or life experiences in comparison to transition year students who are more familiar with just mastery of procedural skills. Essentially adults have better communication skills in comparison to the transition year students. Knuth and Peressini (2001) state that “in any social interaction involving spoken communication, each individual must both decipher what is said and generate his or her own meaning from it” (p. 325). They add that one has to develop these skills over a period of time. Also, a large number of adults are in education because they want to be there. Schloglmann (2006) explains that adults who have returned to learning mathematics may do so for personal development and fulfillment. He adds that some adults participate in adult education to raise their status or to increase their chances of promotion since they are highly motivated to learn. Essentially they see it as an opportunity. However some adults are forced into entering further education in order to prepare for a new job or because of company restructuring. In this particular case, all adults entered further education for personal development and fulfillment. A further reason why these adults outperformed the transition year students may be because they are also more used to encountering tasks in their everyday lives they may never have seen before.

5.3.2 Representational Task

Niemi (1996) stated that in order to complete a representational task, one must be able to identify, generate and use representations. For the first part of the representational task, participants were provided six graphs with the aim of identifying which graph represented the pair of simultaneous equations given.
Three of these tasks Q2a, Q2b, Q2c (see Appendix B for questions) were given to ensure participants didn’t merely guess the answer. The second part of the representational task required participants to construct a graph to represent a pair of simultaneous equations given. Both representational tasks were graded using a score of 0 for an incorrect answer or 1 for a correct answer.

Transition Year Students

Figure 5.3: Comparison of Pre & Post Scores: Transition Year Students

Figure 5.3 shows a comparison between transition year students’ pre and post-test scores in the representational task. It is clear from the diagram that there was only a small increase when comparing Q2a pre and post scores. This was backed up by the p-value of 0.180 which is greater than 0.05. Hence there is no statistically significant change in the proportion of participants who got Q2a correct. Contrasting Figure 5.3 shows there was a significant increase in transition year students’ scores for each of the other four representational task questions. P-values of 0.000 for Q2b and Q2c along with p-values of 0.000 and 0.001 for Q3ai and Q4ai highlight that there is a statistically significant change when comparing the pre-test scores to the post-test scores. Niemi (1996) highlights that graphing representations is only the first step, one must also be able to use representations. He also highlights the importance of fluency in
identifying, generating and using mathematical representations. Shulman (1986) reinforces this by insisting that there is no single most powerful form of representation. Niemi (1996) concludes that graphs convey so much information and hence their importance.

**Adult Education Group**

![Bar chart showing comparison of Pre & Post scores for Adult Education Group](chart.png)

**Figure 5.4: Comparison of Pre & Post Scores: Adult Education Group**

In the first part of the representational task for the adult education group, there was no statistically significant change in the proportion of participants who got Q2a, Q2b and Q2c correct in the post-test when compared to the pre-test. However, from a positive viewpoint all 6 participants got Q2a and Q2b correct in the post-test meanwhile five out of the six adults got Q2c correct. Therefore, despite the fact that there is no statistically significant change, there is evidence to suggest that a large proportion of the adults have displayed competence in the representational task activity. Similarly for Q3ai and Q4ai, all of the adults received a correct answer in the post-test. This is emphasized in Figure 5.4. The importance of this cannot be understated. Hiebert & Carpenter (1992) explain that the ability to draw on multiple representations is an important aspect of students’ mathematical understanding. Highlighting Niemi’s (1996) views, Usiskin (2012, p. 8) explains that “a person does not fully understand
mathematics unless he or she can represent the concept in some way”. He continues by insisting if students are brought to understand (from a representational sense) what they are doing then they will ultimately be better at the skill. As stated previously, a key reason for the lack of a statistically significant improvement may be due to the small group of adults in the group. A large database would be more likely to yield results which were statistically significant.

Similar to the explanation task, the adult education group performed better than the transition year students. However the transition year students’ scores improved significantly in the post-test. Perhaps they forgot how to represent a pair of simultaneous equations and the six lesson interventions plus the support of the digital pencasts reinforced the necessary steps. Since the adults already represented a pair of simultaneous equations a number of weeks prior to the pre-test, this is probably why the majority of them got all of the questions correct. Also reinforcing what was said in the previous section, the transition years are removed somewhat from formal exam work since they have not completed much course work in comparison to the adult education group who are completing the leaving cert syllabus. Overall both the transition year students and the adult education group performed extremely well in the representational task. Ball (2008) describes how selecting representations for a particular purpose and linking them to underlying ideas is unique to teaching and defined as ‘specialised content knowledge’. The ability to represent a mathematical concept has already been highlighted by Niemi (1996) and Usiskin (2012) as a key determinant in developing conceptual understanding in mathematics. The Irish National Teachers Organisation (INTO) (2014) also highlight the importance of representation skills. They state that it is essential one can “represent their thinking so that others can understand how they solved the problem” (p. 3).

Transition year students’ scores may have been better if they were exposed to the possibilities of relevant mathematical software. From a teachers’ perspective, one of the most effective ways to show students how to represent a concept is by using GeoGebra. As explained in the Literature Review, GeoGebra is an award winning software, which was developed by a Masters student in 2001. The
software that has received many major awards, aims to aid students understanding and make lessons more interesting for students’. Kocak & Gulcu (2013) insist that one key issue surrounding technology tools such as GeoGebra is that teachers must be provided with professional support in order to use the technology efficiently. GeoGebra allows teachers the opportunity to visually demonstrate to students how to identify, generate and use representations.

5.3.3 Problem Solving Task

The third task of the pre and post-tests which participants had to perform was a problem solving task. Each of the three problem solving tasks in these tests required students to solve a real life application. In each of the three questions they had to create a pair of simultaneous equations based on the information provided. Each of the problem solving questions were graded 0 for incorrect and 1 for a correct answer. Usiskin (2012) stated that many believe applications involve a higher order level of thinking compared to procedural questions. Usiskin (2012) believes this is untrue because we exhibit a higher form of procedural understanding when one knows multiple ways of completing the same question. He continues by explaining when one knows many ways of getting the correct answer then they must also have the ability to choose the most effective method for a particular problem. As stated in the Literature Review in chapter 2, real life applications are one of the key components of Project Maths.
Transition Year Students

In terms of the transition year students there was a substantial increase in the number of correctly answered questions in the problem solving tasks (Q3bi and Q3ci). P-values of 0.039 and 0.002 respectively, which are less than 0.05 suggests that there is a statistically significant change in the number who answered correctly in the pre-test compared to the post-test. The same results applied for Q4aii and Q4bi since p-values of 0.021 and 0.000 respectively are lower than 0.05. However for Q4ci since the p-value is greater than 0.05, the suggestion is that there is no significant change in students’ answers. This could be due to the fact that the students found the post-test Q4ci more difficult than the pre-test question. Mc Leod (1988) describes how inexperienced problem solvers express frustration if exposed to a non-routine mathematical problem. Perhaps this is one of the reasons why students struggled with the post-test Q4ci.

As explained in the ‘Literature Review’, the problem solving task was one of Niemi’s (1996) four tasks in measuring conceptual understanding in mathematics. It has already been explained how Niemi’s (1996) framework aligns closely with the aims of Project Maths. Problem solving using real life examples is one of the key components of Project Maths, as discussed in the Literature Review. By relating concepts to real life, classrooms have become more student-centred as opposed to the previously teacher-centred classrooms.
Students have become more active within the classroom nowadays with the hope of increasing the quality of teaching and learning and in turn improving students’ attitude towards mathematics. From the results presented above, it is clear that for transition year students their problem solving skills statistically improved for all but one of the questions. In the final problem solving question there was a statistically significant change in Q5ai (p-value 0.012). However Q5bi and Q5ci both had p-values of 1 from which we can conclude that statistically there is no change in scores. Both the pre and post-tests required students to use their answers from the first part of the question, to complete the other problem solving questions. See Appendix B for pre and post-tests. Students may have found the wording of this problem considerable more difficult in comparison to the pre-test example. Garnett (1998) suggests that the inability of students to understand the mathematical language of a question might result in students making various errors and confusion in the process of completing a problem. Polya’s (1981) problem solving heuristic details how to solve a problem solving question. The four stage ladder begins with one to understanding the problem. Next one must devise a plan to solve the problem. The third stage is to carry out the plan while finally looking back and examining the solution. Polya (1981, p. 104) added that “what the teacher says in the classroom is not unimportant but what the students think is a thousand times more important. The idea should be born in the students’ mind and the teacher should act only as midwife.”

Adult Education Group

![Figure 5.6: Comparison of Pre & Post Scores: Adult Education Group](image)
Since all of the adult education group gained the correct answer in both the pre and post-test for Q3bi and Q3ci, there is no change in scores. In Q4aii despite an increase from 3 correct answers to 6 correct answers, a p-value of 0.375 which is great than 0.05 suggests that there is no statistically significant change in the scores. Likewise for Q4bi and Q4bi there is no statistically significant change when comparing the pre-test scores to the post-test scores since the number of correct answers only increased from 5 to 6 in both questions. It must be noted that the adult education group gained the correct answer in all of the problem solving questions in question 3 and question 4 in the post-test. These adults are covering the leaving cert syllabus at the minute in comparison to the transition year students who only covered the junior cert course thus far. This could be a major factor as to why the adult education group performed much stronger in this area. The adults’ good scores in this area is very positive because as previously highlighted in the Literature Review, problem solving has a key role in the Project Maths syllabus. Polya (1981) insisted on giving students responsibility for their own learning by letting them discover problems by themselves as much as possible. Perhaps one of the reasons for such a positive adult score in comparison to the transition year students is due to their experience in discovering real life problems by themselves. Again for Q5ai, Q5bi and Q5ci a p-value of 1 suggests there is no statistically significant change. Again similar to the transition year students, this is because students may have found the wording of the post-test questions more difficult in comparison to the pre-test questions. Geary (2004) agrees with Garnett’s (1998) views by stating that students have difficulty breaking down worded problems and understanding what they are being asked.

O’Connell (2000) insists that problem solving in the classroom should not only be incorporated in algebra, it should be integrated in multiple topics. McGregor (2007) explains that developing students’ problem solving skills is critical to solving real-life problems. O’Connell (2000) states that problem solving allows students’ to persist at questions and to take risks. He adds that students need to be taught problem solving strategies along with ways to organise their thinking and how to begin a problem.
From a teacher perspective, problem selection is important. Oliver & Omari (1999) state that problems must be well-structured and include open ended questions. They also add that including problems based on real-life events of interests are recommended to catch the attention of students. They insist these types of problems improve motivation since they are relevant while also developing students conceptual understanding of a particular topic. In this study, the author spent a number of weeks creating the problem solving questions for the pre and post-tests and also for the handouts that students were given through the six lesson intervention. Lynch et al. (2003) explained how the implementation of real life examples and other new aspects of a new curriculum was extremely time consuming for educators, considering they already have a sizable curriculum to cover. Along with spending a large amount of time creating real life examples, going through the explanations of these real life examples in a classroom setting also take up more class time than teaching procedural skills. It is important for educators to take both of these ideas into account when creating and implementing real life examples.

5.3.4 Justification Task

The final task which participants had to perform was the justification task. As Niemi (1996) specified, to justify your answer, you must be able to show and explain why your solution is correct. Participants had to justify their solutions to each of the problem solving questions. Similar to the explanation task, the justification task was graded using a Likert-type score system depending on the correctness of the participants answers; 1 = incorrect, 2 = partially incorrect, 3 = mostly correct, 4 = fully correct. A grading rubric is provided in Appendix B.
Transition Year Students

Figure 5.7: Comparison of Mean Scores for Transition Year Students

Comparing pre and post-test scores, transition year students’ justifications improved significantly. In Q3aii, Q3bii and Q3cii (see Appendix for pre & post-test questions) p-values of 0.005, 0.001 and 0.019 which are all significantly less than 0.05 suggests that there is a statistically significant increase in students’ ability to justify their answers. Figure 5.7 highlights the increase in the mean scores for the transition year students. Furthermore the graph emphasizes that a mean score of approximately 3 stands for students gaining a ‘mostly correct’ answer. It is clear from Figure 5.7 that students had more difficulty with Q3aii in comparison to the other two justification questions. In the other two questions participants were given specific values, however for Q3aii participants were asked to give their opinion on which car / DVD they should rent.

Figure 5.8: Comparison of Mean Scores for Transition Year Students
Similarly in Q4bii and Q4cii, p-values of 0.000 and 0.026 respectively are both less than 0.05. Hence there is a statistically significant increase in students’ scores as highlighted in Figure 5.8. Unfortunately despite this increase, students’ mean scores of 2.00 and 2.13 in the post-tests reflect that they gained only a ‘partially correct’ answer. For these transition year students moving into the leaving certificate it is essential they are able to justify their answers. Despite the statistically significant increase in students’ mean score for the justification task, improvements still must be made. Glass & Maher (2004) explain how students are not used to justifying their solutions. They believe this is due to teachers only asking students to explain their reasoning when they answer incorrectly. The key reason why students must improve in this area is because justification is such an essential part of Project Maths. Not only is it a vital skill for students to possess within a mathematics setting it is also extremely important for one to be able to justify and communicate outside of the mathematics classroom. It is an essential skill that students must develop. Niemi (1996) adds that in each real life example, students must complete additional questions to justify the answer they have provided.

**Adult Education Group**

Similar to the transition year students’ results, the mean differences (pre-post) for the adult education group were all negative. However despite this fact, only Q3aii (p = 0.041) and Q4bii (p=0.013) resulted in p-value of less than 0.05 when comparing the mean scores. A larger database would more likely result in statistically significant results. Looking at Figure 5.9, it is clear that mean scores of between 3 and 4 for the post-test suggests that the adult group gained a ‘mostly correct’ or ‘fully correct’ answer in each of the five different justification tasks. As stated above, justification of answers is essential. In order to complete the task, one must understand why your solution is correct. Hence why it is one of the four tasks proposed by Niemi (1996) in developing a conceptual understanding in mathematics. De Neys (2010) adds that justification skills require students to solve a problem and then justify their answer based on evidence and reasoning. He also declared that students’ justifying their decisions with thorough reasoning is an essential contribution of developing students’ expertise in a topic.
To conclude, it is clear that the transition year students’ scores improve significantly in the majority of the questions in the post-test. Conversely, the adult education group scores did not improve significantly in the majority of the questions. This is probably down to the fact they received a correct answer in a large number of the pre and post-test questions, hence little improvement was made. Overall the adult education group outperformed the transition year students in each of the tasks. As stated previously this could be partially due to the fact the adult education group are completing the leaving cert course. On the other hand, the transition year students only recently completed the junior cert course and they are removed somewhat from formal exam work. Ball (1976) states that justification can be used to validate claims and provide insight into a result or solution. Cohen & Ball (2001) add that along with enhancing understanding and increasing proficiency at doing mathematics, justification is also a means of learning and doing mathematics.

### 5.4 Research Question 3:

*What are students’ opinions with regards to using the digital pencasts as a supportive tool to enhance their understanding of algebra?*
The discussion of findings for Research Question 3 will be presented under the following headings: Difficulty with Algebra, Difficulty with Technology and Positivity towards Digital Pencasts. As stated in the ‘Research Findings’ chapter (section 4.5), these are the three key themes to emerge from the focus groups conducted with both the transition year students and the adult education group. The findings from each of the three themes above will be discussed in the context of the literature reviewed in chapter 2. As previously stated in chapter 3, it must be noted both the transition year students and the adult education group used the digital pencasts as a supportive tool. They accessed the digital pencasts in their own time if they had any difficulties with any particular concept.

5.4.1 Difficulty with Algebra

As indicated in the Literature Review, on average, students experience great difficulty with the majority of algebraic concepts. These views were highlighted in the focus groups with both the transition year students and the adult education group. When questioned about their feelings towards algebra, the majority of the students taking part in the focus group said they ‘hate it’ and it is their ‘least favourite topic’. Most of the adult education group shared these views stating it is a very difficult topic. Conversely, one of the adults insisted they enjoy algebra because they find it extremely challenging. Echoing the students’ and adults’ opinions of algebra, Linchevski & Herscovics (1994) stated that many learners found the concept difficult since a formal approach is used to explain the topic and generally it is covered at a swift pace. They added that students “fail to construct meaning for the new symbolism and are reduced to performing meaningless operations on symbols they don’t understand” (1994, p. 60). In this context, adult 1 stated ‘I really struggle with the topic. I always have, its one area of maths I really did not enjoy at school. I find it very difficult and I do not see its relevance.’

Murphy & Horgan (2003) outlined that steps are being made to improve students’ views on algebra, and other areas of mathematics, and to develop their understanding. One of the key changes was the introduction of Project Maths which focused on conceptual understanding as opposed to procedural
understanding. Hopefully in the near future students’ opinions on algebra will change and conceptual understanding will improve. Despite these changes one of the transition year students explained ‘It’s probably good for difficult maths in the future but I can’t see how it will help us in secondary school’. Prendergast & O’Donoghue (2010) explained that it is important to show learners that mathematics can be fun by establishing links between the learners and algebra. The use of real life examples is essential in this process. Another method put forward by Artigue & Assude (2000) insisted that developing computer technology has the potential to play an important role in overcoming learners difficulties and negative views on algebra.

Moving on to the benefits of algebra, the transition year students struggled to see how algebra could be beneficial in their lives. They believe it may be important for third level mathematics courses in the future but unfortunately they failed to see its present relevance to everyday life. Student 6 commented, ‘it probably makes you use your brain but I don’t know how it would help in everyday life’. Similarly, the adult education group understood that engineers use algebra in real life but they fail to see how it is used in areas of their personal lives. This is confirmed by experts in the field such as Predergast & O’Donaghue (2010) who stated that students are oblivious as to where algebra is used in everyday life. Another aim of Project Maths is to show students the importance of mathematics in everyday life. Hence why real life examples were used in both the pre and post-test. Usiskin (1995) stated many applications of algebra including; converting from degrees Celsius to degree Fahrenheit, calculating how many miles per gallon your car is getting, analysing various aspects of your health (i.e. weight loss and diets), predicting things such as population growth and the growth of diseases and tumors and calculating the win or loss percentage of a sports team. The mathematical questions in the pre and post-tests were based on the above real life applications.

5.4.2 Difficulty with Technology

In the ‘Literature Review’, it was outlined how technology is being used daily by teachers. Nevertheless, Holden & Rada (2008) also insisted only half of the
teachers who use technology daily, incorporate it into their daily lessons. They are mainly used for administration purposes. The key issue concerning technology in education is how it is used and what its purpose is because the goal of integrating technology into the classroom is to increase students learning and understanding. For this intervention the technology that was used as a support for students, was the Livescribe Smartpen. The Livescribe Smartpen which incorporates reading, writing, speaking and listening all at once, creates digital pencasts. As Baig (2008) explains, digital pencasting is the process of viewing handwritten notes with accompanying audio. Both the transition year students and the adult education group had access to the digital pencasts. These digital pencasts were provided as a support tool while completing the intervention. They accessed these digital pencasts from a website which the author created. The website was created to make the process of sharing the digital pencasts easier since some of the adults did not have an email address. Thankfully all participants had wifi at home.

The first question both focus groups were asked regarding technology was ‘Do they have a computer at home and, if so, did they use it often?’ All of the transition year students have a computer but they mainly use their smartphones or iPads. On the other hand, the entire adult education group have computers at home and four participants use it for work purposes only. The other two respondents, Adult 1 and Adult 3 explained that they dislike technology, however Adult 3 insisted she tried to use it more because she has four young daughters so she must keep up to date with technology. When asked how comfortable they are with technology again Adults 1 and 3 maintained they struggle to keep up to date with technology since it is constantly changing. Adult 1 continued “I sometimes feel quite intimidated by technology and have a fear of doing something wrong”.

Holden & Rada (2008) stated that educators must be given professional support in order to use the technology more efficiently in the classroom. Perhaps the same training is appropriate for students. The key issue which both the transition year students and the adult education group had with the digital pencasts, regarded the audio. Most participants struggled to get the audio working in the
digital pencasts in the beginning since one of the requirements to play the digital pencasts is that you must have the latest version of adobe flash player (a free software package required when viewing content created on the adobe flash platform). This was explained in detail and all participants were shown the step by step guidelines in order to get the audio working. A transition year student asked ‘why do the digital pencasts not work on an iPhone’. It was explained that iPhones do not support adobe flash player. This student was concerned that the digital pencasts would never work on an iPhone since the majority of people use their smartphones more than their computers. One of the adults also raised the same concern since she mainly uses her iPad at home. She stated ‘the digital pencasts won’t work on iPad because Adobe flash player won’t work on those’.

In early 2015, Livescribe released new software to enable everyone to play the digital pencasts on their smartphone. Livescribe (2015) state “Watch your digital world expand and your notes become more useful when everything you write and draw on paper is instantly synced to Livescribe+. Everything you write appears on your tablet or smartphone”. Along with being able to play the digital pencasts on any smartphone, you are not required to download adobe flash player to play them on your computer any more either.

In the first year of this investigation the author used a pilot group to gain their views on the Livescribe Smartpen. In particular the author wished to see what difficulties and issues students would find with the digital pencasts. This pilot group of 18 students were a third year class which the author taught and the questioning took place in January 2014. The pilot group also highlighted the issue with getting the audio working. This has now been amended. However a different issue which these students brought to the authors attention was a scratching noise that could be heard in the background of the audio. This happened since the microphone was at the base of the Livescribe Smartpen. This issue was investigated by reading forums on the Livescribe website and it was noticed that many people across the world shared these views. Livescribe corrected this issue by releasing special earphones which insert into the Livescribe Smartpen and now the recording comes from the ear. Thanks to the pilot group, this issue was avoided. The positives of the digital pencasts will be discussed in the next section.
5.4.3 Positivity towards Digital Pencasts

The aim of this intervention was to improve students’ conceptual understanding of algebra. The digital pencasts were used as an additional support for students to achieve this aim. By enhancing conceptual understanding, students’ anxiety towards algebra decreases (Zan & Di Martino, 2007). Zan & Di Martino (2007) state that if students have high mathematical anxiety, this affects their achievement in the subject. It was also mentioned that mathematics anxiety is closely linked to attitude. Again by enhancing conceptual understanding, ones attitude becomes more positive. The Livescribe Smartpen was used as a supportive tool to lessen students’ anxiety towards mathematics and in turn, assist in improving conceptual understanding.

Both the transition year students and the adult education group agreed that the digital pencasts aided their conceptual understanding of algebra. The transition year students claimed they aided understanding because ‘you can go through problems at your own pace compared to the fast pace you go through topics in classes’ and ‘if there are any areas we forget, we can revise using the digital pencasts’. The adults also praised their usefulness by declaring the ‘book solutions are overwhelming compared to the step by step solution of the digital pencasts’. Participants’ views were also echoed by Loch et al. (2014, p. 258) who highlighted the importance of presenting “a carefully designed step-by-step solution and to explain the thinking process when selecting a solution method”. They also stressed the significance of helping students grasp an understanding of the process by explaining why each step is important. Participants in the current research project also found the ability to pause between steps to digest information as a great addition. In Loch, Dunn & Mc Donald’s (2015) study, respondents also highlighted the same views while adding that one can dictate the pace and repeat a step until one gains an understanding.

Calm et al. (2012) assert that the digital pencasts have many benefits such as providing students with support and enabling students to revisit challenging concepts whenever they wish. He added that students have the ability to alternate from one point to another in the digital pencasts and follow the real sequence in
which mathematical calculations are performed. The transition year students and the adult education group echoed these views. The transition year students thought the digital pencasts are ‘extremely beneficial because the notes are always there twenty four seven and it is an excellent revision tool’. Adult 6 commented how it will be extremely beneficial for her son when he moves into senior cycle next year. Meanwhile adult 3 mentioned that ‘you teach my daughter in first year, she uses the pencasts to revise areas she finds difficult. That is why I began to use them myself’. Another adult commented how it is like ‘having the teacher beside you explaining the answers’. Subsequently adult 4 added: ‘I really like how you can pause the pencasts between steps. For me it takes a while to digests new information, especially with algebra so this feature was extremely beneficial personally’.

While Calm et al. (2012) listed many benefits of the digital pencasts they insisted that the explanation through the built-in sound support was the main benefit of this device since it helped students understand the development and intention of the solving strategy. In the author’s study all participants shared these views and agreed that this was the most important part of the digital pencasts. Student 4 explained that you are not just ‘staring at words and letters, you can hear the explanation too’. While another student insisted ‘without the audio the pencasts are just like a textbook’. This was echoed by one of the adults who explained ‘without the audio, the pencasts are just like a book solution. It is extremely helpful because you have a detailed explanation in the background’. Another adult added that they forget steps easily, even looking back at their notes so the pencasts helped her remember the steps.

Due to all of the benefits provided by the participants, everyone claimed they would recommend the digital pencasts to other students, in particular those ‘who struggle with mathematics’ and those who wish ‘to revise certain topics and concepts’. The only negative that a transition year student reported is that you cannot ask questions. This view was also shared by respondents in Loch et al.’s (2012) study where they explained that they could not verify their understanding immediately unlike a classroom setting due to the passive nature of the digital pencasts. However the author explained that it is easy for students to email any
questions to the author and another digital pencast can be forwarded to answer their questions.

5.5 Other themes to Emerge:

With the support of the digital pencasts this study aimed to improve students’ conceptual understanding of algebra by mean of a pedagogically sound intervention. There is no reason to suggest digital pencasts cannot be used in other areas of mathematics and indeed other subjects entirely. Calm et al. (2012, p 1) explained that by using digital pencasts students can independently improve their ‘insufficient level of previous knowledge’. They add that the digital pencasts enable students to revisit challenging concepts whenever they wish. Meanwhile Loch et al. (2015) reported that 90% of 555 respondents found screencasts very helpful. One of the key reason is because they were not as content-heavy compared to lectures. Likewise, at the end of the focus groups with both the transition year students and the adult education group, it was highlighted that the digital pencasts were beneficial from more than just an algebra point of view. All participants believed they would be beneficial in other areas of mathematics but also in other subjects entirely. The transition year students believed they would be valuable in History and English when planning essays and also in French and other language subjects to help pronunciation. The adults added that the pencasts would be an excellent addition for Business and Accounts because similar to mathematics, following steps and instructions are extremely important in these subjects. One participant also added they would be helpful in Geography and Science when drawing diagrams. On the Livescribe website they have created digital pencasts-based diagrams of the bodystems of the body and also various geography diagrams.

As previously stated (3.10), the author presented the Livescribe Smartpen to 29 teachers (May 2014) from the school which he teaches in. Teachers provided feedback on the Livescribe Smartpen by answering a short questionnaire which asked questions such as ‘what subject did you teach’ and ‘do they believe the Livescribe Smartpen would be beneficial in your subject as an additional study tool’. Significantly 27 out of the 29 (93%) believe the Livescribe Smartpen
would be beneficial in their subject. These subjects range from Construction Studies and Art to English and Geography. The only two teachers who disagreed were Physical Education teachers. Two of the benefits given by teachers were how the Livescribe Smartpen would be of ‘great benefit for sending notes to students so they can access them at home, especially if they are struggling with homework’ and ‘the pencasts would help explain sample exam answers to students’.

From the feedback from both of the focus groups and also the 29 mathematics teachers, it is clear that the Livescribe Smartpen would be beneficial in areas other than algebra and indeed for teachers of varied subjects.

5.6 Summary of Findings:

At the outset of this chapter the research questions were outlined. They were

1. What frameworks exist for measuring students’ conceptual understanding of mathematics and how closely do these frameworks align with the aims of Project Maths?
2. Does a blended learning teaching intervention supported by digital pencasts enhance students’ conceptual understanding of algebra?
3. What are students’ opinions with regards to using the digital pencasts as a supportive tool to enhance their understanding of algebra?

After examining the data the key findings to emerge are stated below.

The first key finding is that out of the three frameworks researched, Niemi’s (1996) is the framework that aligns closest with the aims of Project Maths. The primary reason that this framework was deemed the more suitable framework for the current research is because Niemi’s (1996) tasks focus on explanation, representing, problem solving and justification which are all objectives of Project Maths.

The second key finding proposes that there was a significant change (p < 0.05) in transition year students’ scores in a large number of the questions when
comparing their pre and post-test scores. Out of the 23 different questions based on each of Niemi’s tasks for measuring conceptual understanding, the transition year students’ scores recorded statistically significant scores \((p < 0.05)\) on 18 of these questions. Contrastingly there was no significant change \((p < 0.05)\) in adult education group scores in the majority of their questions when comparing their pre and post-test scores. Out of the 23 different questions based on each of Niemi’s tasks for measuring conceptual understanding, the adult education groups’ scores statistically significant changes \((p < 0.05)\) on only four of these questions. It is important to note that the adult education group got more pre-test questions correct when compared to transition year students and hence ‘less room’ for improvement. Furthermore despite the no significant improvement, the adults received either a ‘mostly correct’ or ‘fully correct’ answer on the majority of the questions relating to either the explanation or justification tasks. Furthermore the adults received a correct answer on all but two of the representational and problem solving tasks.

Despite the introduction of Project Maths, transition year students still struggle to see to how algebra is used in real life. This was evident in the focus group with the transition year students when one student explained ‘it probably makes you use your brain but I don’t know how it would help in everyday life’. Another student also stated ‘it’s probably good for difficult mathematics in the future but I can’t see how it will help us in secondary school’.

The general theme that emerged from the focus group data is the positivity that all respondents showed towards the digital pencasts. All respondents believe they are a beneficial revision tool and that they helped improve their understanding of algebra. Furthermore all respondents explained that the digital pencasts would be beneficial in other areas of mathematics and in other curriculum subjects. Likewise the pilot teachers’ who teach a wide variety of subjects believe they would be useful in their subject area.
Chapter 6 - Conclusions and Recommendations

6.1 Introduction

The aim of this research study was to improve students’ conceptual understanding of algebra via a blended learning teaching approach supported by digital pencasts. The research was focused around a taught intervention while students could use digital pencasts as a supportive tool. The research started with the author identifying a problem. It was recognized that algebra was an area which students struggle greatly with. The next step was to undertake research of areas that were key to this project including; Technology in Mathematics Education, Digital Pencasting, Students’ Difficulties with Algebra, Frameworks for Measuring Conceptual Understanding, Teaching Methodologies and Adults Learning Mathematics. This research was presented in chapter 2.

The author employed an action research methodology. Prior to the beginning of the intervention, both the transition year students (n = 24) and the adult education group (n = 6) were given a pre-test to assess their base knowledge. Next the six-lesson intervention took place and students were granted the use of specifically designed digital pencasts as an additional support. Once this was complete the transition year students and the adult education group were given a post-test to complete. Subsequent to the completion of the post-test, a random selection of eight students from the transition year group and the entire adult education group took part in separate focus groups. It is important to note that the pre and post-test, the intervention and the digital pencasts were all structured around the elements of Niemi’s (1996) framework.

In chapter 4, Research Findings, the main findings from this study were presented and the analysis of these results was offered in chapter 5. In chapter 5 the author discussed the links between the research questions posed in the introduction chapter and the theoretical frameworks discussed in chapter 2. The
results were also discussed in relation to the context of this study (in terms of Irish Education system) and the specific issues with algebra and key findings that emerged from the analysis of the data.

6.2 Conclusions

1. *What frameworks exist for measuring students’ conceptual understanding of mathematics and how closely do these frameworks align with the aims of Project Maths?*

As stated previously, three frameworks were researched for measuring students’ conceptual understanding of algebra. They were Usiskin’s (2012) framework, Niemi’s (1996) framework and Hiebert et al.’s (2003) framework. Out of the three, Niemi’s (1996) framework was chosen since the explanation task, representational task, problem solving task and justification task are all objectives of Project Maths. One objective of Project Maths is to deepen students understanding of the subject (NCCA, 2008). Students must understand a concept in detail in order to complete an explanation task where they must list everything they know about a concept (Niemi, 1996). Since the introduction of Project Maths in 2008, exam question have asked students to complete all three of the representational tasks of identifying, generating and using representations. Another key objective of Project Maths is to show students where mathematics is used in real life, hence the introduction of real life examples and problem solving (NCCA, 2008). Along with problem solving, to ensure students understand the problem at hand, Niemi (1996) explains that they are asked additional questions to justify their solution. Both of these which are tasks in Niemi’s (1996) framework. To conclude it is clear that Niemi’s (1996) framework aligns closely with the aims of Project Maths.

2. *Does a blended learning teaching intervention supported by digital pencasts enhance students’ conceptual understanding of algebra?*

The enhancement of students’ conceptual understanding was measured using pre and post-tests. A comparison of these scores was used to determine if statistically
significant improvements were made. As stated in the summary of findings in section 5.6 there was a significant change (p < 0.05) in transition year students’ scores in a large number of the questions when comparing their pre and post-test scores. Out of the 23 different questions based on each Niemi’s (1996) tasks for measuring conceptual understanding, the transition year students’ scores returned statistically significant results (p < 0.05) for eighteen of these questions. Conversely there was no significant change (p < 0.05) in adult education group scores in the majority of their questions when comparing their pre and post-test scores. Out of the 23 different questions based on each Niemi’s (1996) tasks for measuring conceptual understanding, the adult education group’s scores statistically changed significantly on only 4 of these questions. However despite this result, the adults received either a ‘mostly correct’ or ‘fully correct’ answer on the majority of the explanation task or justification task questions. Furthermore the adults received a correct answer on all but two of the representational and problem solving tasks.

In the end, no claim of significance can be made as to whether the use of an intervention based on Niemi’s (1996) framework enhanced students’ conceptual understanding of algebra. Although there are positive indicators that this framework could be used to measure a students’ conceptual understanding of a mathematical topic.

3. What are students’ opinions with regards to using the digital pencasts as a supportive tool to enhance their understanding of algebra?

The three key themes to emerge from the focus groups with both the transition year students and adult education group were Difficulty with Algebra, Difficulty with Technology and Positivity towards Digital Pencasts. Students’ opinions regarding use of the digital pencasts as a supportive tool to enhance their understanding of algebra were presented in chapter 4 (Research Findings) and discussed in relation to the literature review in chapter 5 (Discussion of Findings).
Prior to delving into both the transition year students and the adult education groups opinions on the digital pencasts as a supportive tool it is important to note their views on algebra. Echoing the literature reviewed in chapter 2, both the transition year students and the majority (five of the six) of the adult education group disliked algebra because they found it difficult and challenging. Along with finding the topic difficult they also failed to see its relevance in real life. In particular they were unaware of its usefulness in areas such as population growth, analysing various aspects of our health and analysing diseases and tumors. It is hoped that the introduction of Project Maths in 2008 will change this due to one of the key component of Project Maths being the emphasis on and the use of real life examples.

Technology is growing and changing constantly particularly in education. In the Literature Review in chapter 2 it was noted that many people struggle with technology. These views were highlighted in this study. In particular some of the adults felt ‘intimidated by technology and have a fear of doing something wrong’. Changes in technology were also evident with the Livescribe Smartpen. As mentioned in the section 3.10, the author had to purchase Livescribe earphones to remove a scratching noise and give all participants a step by step guide handout on how to play the digital pencasts. Once the research was complete in early 2015, Livescribe released new software to enable the playing of digital pencasts on smartphones. Unfortunately this information was only released subsequent to the completion of the research with the adult education group and the transition year students.

The Livescribe Smartpen is a new technology tool (2007) which creates digital pencasts. Both the transition year students and the adult education group gave very positive reviews on the digital pencasts which flowed from the intervention in this research project. They found the technology tool extremely beneficial for revision purposes. Two transition year students claimed they aided understanding because ‘you can go through problems at your own pace compared to the fast pace you go through topics in class’ and ‘if there are any areas we forget, we can revise using the digital pencasts’. The adults also praised their usefulness by declaring the ‘book solutions are overwhelming compared to the step by step
solution of the digital pencasts’. In Loch, Dunn & Mc Donald’s (2015) study, respondents also highlighted the same views while adding that one can dictate the pace and repeat a step until one gains an understanding.

It is clear from the two focus groups (transition year students and the adult education group) that in the opinion of students the digital pencasts would be beneficial in other areas of mathematics and also other subjects. Both the transition year students and the adult education group believe the Livescribe Smartpen would be useful in subjects such as history and English for planning essays, language subjects for pronunciation, science and geography for diagrams and in business and accounting for step by step procedures. The 29 teachers in Rosses Community School who gave their view on the Livescribe Smartpen also highlighted these views. 27 out of the 29 (93%) believe the Livescribe Smartpen would be beneficial in their subject. The two teachers who disagreed were physical education teachers. As stated in section 5.4, teachers cited numerous reasons for their positivity towards digital pencasts including the digital pencasts would be of ‘great benefit for sending notes to students so they can access them at home, especially if they are struggling with homework’. Other reasons included how they would be an ‘excellent tool to help students with special educational needs take notes’, for revision in the area of accounting and booking keeping, for diagrams and notes in geography and to show how to illustrate art works for art history while recording notes to go along with it.

It is clear from both of the focus groups and the Livescribe Smartpen presentation offered to the teachers by the author that the digital pencasts would be of benefit to the author and fellow mathematics teachers in areas other than algebra and indeed for teachers of other subjects.

6.3 Thesis Contribution

This study has made a number of contributions that add to the literature available. The study has also critiqued existing literature on blended learning, technology in mathematics education, digital pencasting, students’ difficulties
with algebra, frameworks for measuring conceptual understanding, teaching methodologies and adults learning mathematics.

A blended learning approach was used in this study. As previously stated, Garrison & Kanuka (2004) believe blended learning is beneficial since integrating classroom teaching with online experiences provides better learning outcomes. They also add that blended learning can facilitate independent and collaborative learning experiences for students. This approach has comparisons to the Project Maths curriculum in Ireland, mainly the student-centred approach and the use of a variety of teaching strategies to enhance the student learning experience. Due to the similarities between both the blended learning approach and some of the key elements of Project Maths there is the possibility of employing a blended learning approach within the Irish post-primary classroom.

Another contribution this study makes is that is provides and trials a theoretical framework originally developed by Niemi (1996) for measuring students’ conceptual understanding of mathematics. Moreover, it was highlighted that this framework aligns closely with the aims of Project Maths. This framework was used to create a teaching intervention which complements the aims of Project Maths to enhance students’ conceptual understanding of algebra. The teaching intervention may be of benefit to other practitioners to use.

The third contribution this study makes is the strong case for digital pencasts as a beneficial support tool for students to enhance their conceptual understanding of algebra. All participants in this study believed the Livescribe Smartpen to be a very helpful and supportive learning tool. Importantly, the evidence presented also indicated that students believed that the Livescribe Smartpen has benefits to areas in mathematics apart from algebra and indeed to other subject areas of the curriculum. Along with this, teachers (of most subjects) reacted positively towards the digital pencasts and believe it would be an excellent tool to use in their classroom. This also highlights the benefits of blended learner which incorporates a mixture teaching approaches.
6.4 Research Limitations

The first limitation is that an intervention of only six lesson may not have been long enough. Unfortunately due to the circumstances of the author inheriting the transition year group for the intervention it was not possible to teach them for longer. Another limitation in this research study was that the author conducted the two focus groups. This may have led to bias since the transition year students and the adult education group may have told the author what he wanted to hear. A further limitation regarding the sample size of the adult education group was small (n = 6) and it would be unwise to generalise in the circumstances. A larger group would yield more productive outcomes in future research studies. However this was not possible during this study.

Niemi’s (1996) framework was chosen out of a selection of three different frameworks for this research study. These were the only three frameworks which the author sourced when researching the measurement of conceptual understanding. Niemi’s (1996) framework stated that one can develop a conceptual understanding in mathematics if they can represent, complete an explanation task, problem solve and justify their solution. This is why Niemi’s framework aligned closely with the aims of Project Maths. However perhaps there are other frameworks of conceptual understanding that may have suited this research project better.

Whilst the digital pencasts were viewed by participants as an excellent tool when revising algebra, the digital pencasts had their limitations since students were unable to ask questions when using the pencasts in their own independent study time. This meant that students could not verify their understanding immediately, which would not be the case if they were in a classroom setting. In the author’s opinion, none of the above limitations undermine the conclusions and recommendations made in this chapter.
6.5 Recommendations

A number of recommendations are now offered based on the findings and conclusions from this research:

1. Perhaps when structuring future research, it would be beneficial to divide the participants into two equal groups with one group completing the pre-test first and the post-test second. The other group can complete the post-test first and the pre-test second. This will remove bias about the standards of the two tests.

2. A second recommendation would be to select an adult group of participants which is larger than six. In this study, statistical significance was difficult to achieve with the data from the adult group due to the small number involved. A sample size similar in size to the transition year group (n = 24) would have added additional credibility to the research study outcomes.

3. A third recommendation would be to have a control group being taught the same material and completing the same pre and post-tests without blended learning methods. These results could then be used to measure the effectiveness a blended learning approach to teaching algebra against a more traditional approach.

6.6 Future Research

This research study leads to several questions which merit further investigation. Firstly, if this study was completed in a number of schools the results would be more generalizable. Perhaps conducting the research in three or four different schools. Also the results may be more generalizable if the study was conducted with students other than transition year students (2nd & 5th years).
Another option for further research would be to incorporate a blended learning approach in other areas of mathematics. Algebra was chosen for this study due to students difficulties with the topic but it would be interesting to see how a blended learning approach would work in other areas such as probability and statistics.

When the research was complete, the author realised that perhaps when structuring future research, it would be beneficial to divide the participants into two equal groups with one group completing the pre-test first and the post-test second. This would remove any question of bias in the pre and post-tests. The other group can complete the post-test first and the pre-test second.

Regarding the digital pencasts, this research study leads on to several questions which merit further investigation:

- Do the actual digital pencasts themselves improve students’ conceptual understanding of algebra? This was not possible in the current study because the research centred around the teaching intervention and the digital pencasts acted merely as a support tool.
- Are digital pencasts more effective for students as a revision tool on previously studied topics or would they be more useful for topics that students have not yet been taught?
- Are digital pencasts more effective as a tool to refresh what was covered in class, or for last minute study in preparation for a mathematics test?
- Do digital pencasts build student confidence, particularly students who struggle greatly with a particular topic?
- What is the effectiveness of the digital pencasts over a wide range of mathematical levels?
- What is the effectiveness of the digital pencasts over a wide range of mathematical topics and to other subjects on the curriculum?
- What design features lead to effective mathematical digital pencasts?
6.7 Final Comments

The conclusions from the findings of this research, the contribution of this research and the recommendations for future research have all been outlined in this chapter. This study was initiated after the author first identified a specific issue regarding algebra within his own mathematics classroom. This study succeeded in addressing this issue for the author while it also provided insights and support for other mathematics teachers who are faced with similar difficulties in the area of algebra. In particular, this study noted the effective use of digital pencasts as an important factor in improving students’ conceptual understanding of algebra via a blended learning approach. The results of this research may not have provided definitive answers regarding the usefulness of digital pencasts in conjunction with Niemi’s (1996) framework, however the results still suggest a positive impact.
Bibliography


Barrows, H. S., & Kelson, A. C. (1995) Problem-Based Learning in Secondary Education and the Problem-Based Learning Institute, Springfield, IL.


Department of Education and Skills (2011) Literacy and Numeracy for Learning and Life: The National Strategy to Improve Literacy and Numeracy among Children and Young People. Dublin: DES.


Irish National Teachers’ Organisation (INTO), (2014). We Need to Talk About Maths Problem Solving. Available at: https://www.into.ie/ROI/Publications/InTouch/FullLengthArticles/FullLengthArticles2014/WeNeedToTalk_ProblemSolving.pdf [Accessed July 2015]


Literacy and Numeracy Report (2011) The National Strategy to Improve Literacy and Numeracy Among Children and Young People


Appendix A

Lesson Plans and Scheme of Work
Scheme of Work

- Subject: Mathematics
- Topic: Algebra – Simultaneous Equations
- Class: Transition Year Students & Adult Education Group
- No. of Students: 24 Transition Year Students and 6 Adult Education
- Level of Students: Common
- No. of Lessons: 6

Previous Knowledge and Experience:
Students will have extensive prior knowledge in simultaneous equations from junior cycle. Students will be able to graph a pair of simultaneous equations and find the point of intersection. Students will be able to solve a pair of simultaneous equations algebraically. Students will be able to solve a pair of simultaneous equations given a real life example.

Selection and Structuring of Subject Matter
I choose to teach simultaneous equations because algebra is an area which students have difficulty with. Simultaneous equations is also the next part of the course to be covered. In lesson one, an explanation about simultaneous equations will be given to students because they must understand what they are trying to find when they solve a pair of simultaneous equations. Next students must recap on how to graph a pair of simultaneous equations. They will use this knowledge when they are identifying graphical represents in lesson 3. Once students are able to graph and identify representations, they will learn how to graph a problem solving question. Afterwards they will justify their answer by answering additional questions. This will take place during lesson 4. In the next lesson, students will use self guided discovery to note the algebraic solution is the same as the graphical solution. Then in lesson 5, students will solve problem solving questions using algebra and justify their answers by answering additional questions. The penultimate lesson will have a mixture of problem solving using
algebra and using graphical representations. Again, students will answer additional questions to justify their answer.

**Aim:**
At the end of the six lessons, I hope that students will be able to explain what you are trying to find when you solve a pair of simultaneous equations. I also hope that students will be able to identify representations. Finally, students will be able to solve simultaneous equations both graphically and algebraically, while justifying their solution by answering additional questions.

**Objectives:**
Students will be able to explain what you are trying to find when you solve a pair of simultaneous equations. Students will also be able to explain any other relevant information about simultaneous equations. Students will be able to graph a pair of simultaneous equations. Students will be able to identify representations. Students will be able to solve problem solving questions using both the algebraic method and the graphical method. Students will be able to justify their answers by answering additional questions.

**Organisation of Learning Experiences:**
Group work will be used throughout most lessons.
Oral questioning will be used to stretch all learners.
Handouts will be given to students for homework, at the end of each lesson. This will be one mode of assessment.
Geogebra will be used (lessons 1, 2, 3, 6) to graphically represent the simultaneous equations.
Digital Pencasts will be given to students as an additional support.

**Resources:**
GeoGebra
Digital Pencasts
Handouts for homework
Content

Lesson 1 – Introduction to Simultaneous Equations & Graphing

- Students will discuss what you are trying to find when you solve a pair of simultaneous equations.
- Explain any other important information about simultaneous equations including where they are used in real life.
- A pair of simultaneous equations will be written on the whiteboard. In pairs, students will locate two points on each line. Then two students will graph the two lines on the whiteboard via GeoGebra. Students will find the point of intersection.
- Students will be given one more similar question to graph, with the aim of locating the point of intersection.

Lesson 2 – Solving Simultaneous Equations using Algebra

- Students will solve the pair of simultaneous equations from the previous lesson, using the algebraic method. They will use self-guided discovery to note the solutions are the same, using both methods.
- Students must check their answer using substitution.
- Students will also complete the same task for the other two questions, which they were given in the previous lesson.

Lesson 3 – Identifying graphical representations

- Six diagrams will be drawn on the board. Students will be given a pair of simultaneous equations and they must discuss which method they could use to assess which diagram represents the pair of equations. They will complete this task in pairs.
- Then students will solve the problem using all of the methods which they discussed.
Lesson 4 – Graphing Real Life Examples & Justify Solution

• Students will be given a real life problem. In groups, they must make a pair of simultaneous equations. Then they will graph the equations on graph paper and hence find the point of intersection.

• Then the video of the real life problem will be shown on the whiteboard via GeoGebra.

• Again in their groups, students will have to answer additional questions based on their representation. Then the class will have an open discussion about these solutions.

Lesson 5 – Problem Solving and justification using Algebra method

• Students will be given real life problems to solve using algebra. They must make a pair of simultaneous equations from the information they were given. Then solve to calculate the value of the two unknowns.

• Once they have gained their solution, they must answer additional questions about their solution. This will take place in pairs.

• A handout with similar questions will be given to students.

Lesson 6 - Recap

• The final lesson will recap on all previous lessons. Students will be given two problem solving questions with accompanying justification questions. One question will include a representational problem and the other question must be answered using the algebraic method. Students can discuss and compare answers with their peers.

Assessment:

Students will be assessed throughout each lesson using the following:

• Oral questioning
• Self guided discovery
• Feedback sandwich
• Peer tutoring
• Homework
• Test
**Self Evaluation:**

Oral open ended questioning will be used throughout to stretch all learners. GeoGebra will be used to graphically represent the simultaneous equations. Positive reinforcement will be used while the investigator circulates around the classroom assessing students understanding. Assessment will be carried out using the assessment techniques described above.
Lesson Plan 1

Day:  Monday (Transition Years) / Wednesday (Adults)
No. of Pupils:  24 (Transition Years) / 6 (Adults)

Date:  23rd February 2015 (Transition years) / 25th February 2015 (Adults)
Time:  11.30 – 12.10 (TY’s) / 6.00 – 6.40 (Adults)

Class Group:  Transition years / Adults (Separate)
Length of Lesson:  40 minutes

Subject:  Mathematics
Topic:  Simultaneous equations

Lesson Number:  1

Previous Knowledge and Experience:
Students will have extensive prior knowledge of simultaneous equations from junior cycle. Students should be able to solve a pair of simultaneous equations graphically and algebraically.

How has the Previous Lesson informed my Planning for this Lesson?
Simultaneous equations is the next topic to be covered.

Aim:
At the end of this lesson, I hope that students will be able to explain what you are trying to find when you solve a pair of simultaneous equations. I also hope that students will be able to graph a pair of simultaneous equations.

Objectives:
Students will be able to explain what you are trying to find when you solve a pair of simultaneous equations. Students will also be able to explain any other relevant information about simultaneous equations. Students will be able to solve a pair of simultaneous equations graphically.

**Resources:**
GeoGebra – graphing simultaneous equations. Handout 1
Digital Pencasts – Additional resource for students

**Content**

<table>
<thead>
<tr>
<th>Time</th>
<th>Introduction</th>
<th>Pupil Activity</th>
<th>Assessment of the Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 mins</td>
<td><strong>Part 1:</strong> Students will be asked to discuss in groups what you are trying to find when you solve a pair of simultaneous equations. Then a whole class discussion will be held to discuss their answers. An explanation of any additional information will be given. Students will also be asked where simultaneous equations are used in real life.</td>
<td>• Pupils will discuss in groups and then as a class group.</td>
<td>• Questionning What is a linear equation? What are simultaneous equations? What is the meaning of the answer when you solve simultaneous equations? What methods can be used to solve a pair of simultaneous equations? • Peer tutoring</td>
</tr>
<tr>
<td>5 mins</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 mins</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Development**

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
<th>Pupil Activity</th>
<th>Assessment of the Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 mins</td>
<td>A pair of simultaneous equations</td>
<td>Students must find two points on</td>
<td>Questionning</td>
</tr>
</tbody>
</table>

134
| 5 mins | will be written on the whiteboard. Students must find two points on each line and graph their solution on graph paper.  
• Go through the solution in detail with the help of students. GeoGebra will be used to graph the lines. If GeoGebra fails to work, a graph will be drawn on the board.  
• Students will be given one more similar question to complete. | each line and graph their solution on graph paper. If students are having difficulty their peers can help.  
• Two students will come up to the board and graph their solutions via GeoGebra.  
• Students will complete another similar example. The teacher will observe and help any student who is struggling. | How many point do you need to draw a line? Why do you only need two points to graph a line?  
• Peer tutoring  
• Feedback sandwich – teacher will give feedback while observing. |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5 mins</td>
<td>Conclusion</td>
<td>5 mins</td>
<td>Conclusion</td>
</tr>
</tbody>
</table>
| | • Oral questioning will be used to recap on what students have learned during the lesson.  
  - what are you trying to find when you solve a pair of simultaneous equations?  
  - How are simultaneous equations used in real life?  
  - How many points do you need to find to graph a line? | - See Higher Cognitive Questions | - Oral Questioning - See Higher Cognitive Questions |
2 mins  |  • Students will be given their homework and a short explanation will be given of what they must do.  |  • Students will note their homework in their diaries.  |

**Assessment:**
- Oral open ended questionning
- Feedback sandwich – a positive, one thing they should improve on, followed by another positive.
- Peer tutoring
- Homework – Handout

<table>
<thead>
<tr>
<th>Higher Cognitive Questions</th>
<th>Prompts</th>
<th>Sample Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>What are you trying to find when you solve a pair of simultaneous equations?</td>
<td></td>
<td>Looking for the solution that satisfies both equations at the same time.</td>
</tr>
<tr>
<td>How are simultaneous equations used in real life? Explain.</td>
<td></td>
<td>Simultaneous equations are used in sales when searching for the equilibrium supply and demand price. Example if the supplier overcharges for a product the demand decreases. However if the supplier doesn’t provide enough product then they are missing out on potential sales. Hence they are looking for the equilibrium point.</td>
</tr>
<tr>
<td>Apart from the graphical method, do you know any other method of solving simultaneous equations?</td>
<td></td>
<td>Algebraic method</td>
</tr>
</tbody>
</table>
Worked solutions:

Simultaneous Equation – Graphing 1

Graph the following pair of Simultaneous Equations

l : \( x + y = 3 \)

k : \( 2x + y = 4 \)

STEP 1

Find two points on line l & k:

l : when \( x = 0 \)

\[
0 + y = 3 \\
y = 3 \quad (0,3)
\]

l : when \( y = 0 \)

\[
x + 0 = 3 \\
x = 3 \quad (3,0)
\]

k : when \( x = 0 \)

\[
2(0) + y = 4 \\
y = 4 \quad (0,4)
\]

k : when \( y = 0 \)

\[
2x + 0 = 4 \\
x = 2 \quad (2,0)
\]

STEP 2

Draw both lines on coordinate axis

Students may have difficulty locating points on each line. Tell them to let \( x = 0 \) to find the \( y \) coordinate and let \( y = 0 \) to find the \( x \) coordinate.

STEP 3

Read of the point of intersection

Point of Intersection = (1,2)

Students may have difficulty plotting points on coordinate axis. To practice, a link to a plotting points game will be given to students.
Worked solutions:

Simultaneous Equation – Graphing 2
Graph the following pair of Simultaneous Equations
l : \( x + 2y = 4 \)
k : \( x - y = 1 \)

STEP 1
Find two points on line l & k:

l : when \( x = 0 \)
\[ 0 + 2y = 4 \]
\[ y = 2 \quad (0,2) \]

l : when \( y = 0 \)
\[ x + 0 = 4 \]
\[ x = 4 \quad (4,0) \]

k : when \( x = 0 \)
\[ 0 - y = 1 \]
\[ y = -1 \quad (0,-1) \]

k : when \( y = 0 \)
\[ x - 0 = 1 \]
\[ x = 1 \quad (1,0) \]

STEP 2
Draw both lines on coordinate axis

STEP 3
Read of the point of intersection
Point of Intersection = (2,1)

Students may have difficulty locating points on each line. Tell them to let \( x = 0 \) to find the y coordinate and let \( y = 0 \) to find the x coordinate.

Students may have difficulty plotting points on coordinate axis. To practice, a link to a plotting points game will be given to students.
Worked solutions:

Simultaneous Equation – Graphing 3 (Handout)

Graph the following pair of Simultaneous Equations

l: \( x + y = 5 \)
k: \( 2x - y = 1 \)

STEP 1
Find two points on line l & k:

l: when \( x = 0 \)
\[
0 + y = 5 \\
y = 5 \quad (0,5)
\]
l: when \( y = 0 \)
\[
x + 0 = 5 \\
x = 5 \quad (5,0)
\]
k: when \( x = 0 \)
\[
2(0) - y = 1 \\
y = -1 \quad (0,-1)
\]
k: when \( y = 0 \)
\[
2x + 0 = 1 \\
x = \frac{1}{2} \quad (\frac{1}{2},0)
\]

STEP 2
Draw both lines on coordinate axis

STEP 3
Read of the point of intersection
Point of Intersection = (2,3)

Students may have difficulty locating points on each line. Tell them to let \( x = 0 \) to find the y coordinate and let \( y = 0 \) to find the x coordinate.
**Worked solutions:**

Simultaneous Equation – Graphing 4 (Handout)

Graph the following pair of Simultaneous Equations

1: \[3x - 2y = 2\]

\[k : 2x + 3y = 10\]

**STEP 1**

Find two points on line 1 & k:

\[l: \text{when } x = 0\]

\[3(0) - 2y = 2\]

\[y = -1\]

\[(0, -1)\]

\[l: \text{when } y = 0\]

\[3x - 2(0) = 2\]

\[x = 2/3\]

\[(2/3, 0)\]

\[k: \text{when } x = 0\]

\[2(0) + 3y = 10\]

\[y = 10/3\]

\[(0, 10/3)\]

\[k: \text{when } y = 0\]

\[2x + 3(0) = 10\]

\[x = 5\]

\[(5, 0)\]

**STEP 2**

Draw both lines on coordinate axis

**STEP 3**

Read of the point of intersection

Point of Intersection = (2,2)
Lesson Plan 2

**Day:** Tuesday (Transition Years) / Wednesday (Adults)  
**No. of Pupils:** 24 (Transition Years) / 6 (Adults)  
**Date:** 24th February 2015 (Transition years) / 25th February 2015 (Adults)  
**Time:** 9.10 – 9.50 (TY’s) / 6.40 – 7.20 (Adults)  
**Class Group:** Transition years / Adults (Separate)  
**Length of Lesson:** 40 minutes  
**Subject:** Mathematics  
**Topic:** Simultaneous equations  
**Lesson Number:** 2

**Previous Knowledge and Experience:**
Students are able to graph a pair of simultaneous equations and locate the point of intersection from yesterday’s lesson.

**How has the Previous Lesson informed my Planning for this Lesson?**
In the previous lesson students graphed a pair of simultaneous equations to find the point of intersection. All students completed this task without any problems.

**Aim:**
By the end of this lesson, students will be able to solve a pair of simultaneous equations algebraically.

**Objectives:**
Students will be able to solve a pair of simultaneous equations graphically or using the algebra method. Students will be able to check their answer using substitution.
**Resources:**
GeoGebra – graphing simultaneous equations.
Digital Pencasts – Additional resource for students

### Content

<table>
<thead>
<tr>
<th>Time</th>
<th>Teacher Activity</th>
<th>Pupil Activity</th>
<th>Assessment of the Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 mins</td>
<td>• Homework will be corrected by showing the graphical solutions to questions 1 and 2 on the board via GeoGebra.</td>
<td>• Students will look at the diagrams presented on the board via GeoGebra and assess did they gain the same answer. If they got the answer incorrect they will assess where they went wrong and take down the correct solution.</td>
<td>• Observe if students had difficulty with their homework. If so then spend a number of minutes recapping on the areas they found difficult.</td>
</tr>
<tr>
<td>5 mins</td>
<td>• Question 3 of students homework will be produced on the board via the projector. Students will be asked to read their answers out loud. A short discussion will take place discussing their answers.</td>
<td>• Selected students will read their answers to the rest of the class. All students will get involved in a whole class discussion.</td>
<td>• Oral questioning and classroom discussion.</td>
</tr>
<tr>
<td>2 mins</td>
<td>• A pair of simultaneous equations (same as yesterday’s 1st example) will be written on the whiteboard. Students will be asked what other</td>
<td>• Students will explain that the other method is solving using the algebra method.</td>
<td>• Oral questioning</td>
</tr>
</tbody>
</table>

142
<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 mins</td>
<td>Students will solve the simultaneous equations in pair and I will observe their progress.</td>
</tr>
<tr>
<td>4 mins</td>
<td>Go through the solution in detail with the help of students. GeoGebra will be used to show the solution using the graphical method.</td>
</tr>
<tr>
<td>5 mins</td>
<td>Students will be asked to check their answer using substitution. I will go through the solution on the board when students have completed the task.</td>
</tr>
<tr>
<td>8 mins</td>
<td>The other three simultaneous equations questions from yesterday’s lesson will be written on the board.</td>
</tr>
</tbody>
</table>

**Conclusion**

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 mins</td>
<td>Students will give me directions and explain to me what step I must complete next.</td>
</tr>
</tbody>
</table>

**Pair work & Feedback sandwich – teacher will give feedback while observing.**

**Oral Questioning**

‘Compare your answer to yesterday’s solution?’

‘What answer did you get when you sub the point into both equations?’

**Feedback sandwich – teacher will give feedback while observing.**

**To recap on the lesson, I will pick one of the three examples and solve using the algebra method and the graphical method (using GeoGebra). I will also check the answer using**

**Oral Questioning – ‘what step is next and why?’**
<table>
<thead>
<tr>
<th>2 mins</th>
<th>substitution.</th>
<th>Students will note their homework in their diaries.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Students will be given their homework and a short explanation will be given of what they must do.</td>
<td></td>
</tr>
</tbody>
</table>

**Assessment:**
- Oral open ended questioning
- Feedback sandwich – a positive, one thing they should improve on, followed by another positive.
- Peer tutoring
- Homework – Handout
**Worked solutions:**  
**Simultaneous Equation – Algebra 1**  
**Solve the following pair of Simultaneous Equations using Algebra method:**  
**l:** \( x + y = 3 \)  
**k:** \( 2x + y = 4 \)

You need the number in front of \( y \) (or \( x \)) in both equations to be the same and the signs opposite.

\[
\begin{align*}
-1 \cdot (x + y &= 3) \\
2x + y &= 4
\end{align*}
\]

\[
\begin{align*}
-x - y &= -3 \\
2x + y &= 4 \quad \text{(ADD)}
\end{align*}
\]

\( x = 1 \)  
when you find the value for \( x \),

sub it back into either \( l \) or \( k \) to find the \( y \) value.  
Sub back into \( l \):  
\( 1 + y = 3 \)  
\( y = 2 \)  
**Answer = (1,2)**

**Graphing method**  
**Point of Intersection = (1,2)**

**Substitution**  
**Sub the solution back into both \( l \) and \( k \)**  
**l:** \( x + y = 3 \)  
\( (1) + (2) = 3 \)  
\( \text{Yes} \)

**k:** \( 2x + y = 4 \)  
\( 2(1) + (2) = 4 \)  
\( \text{Yes} \)

Students may have difficulty starting the question. Start by ensuring the number in front of \( y \) (or \( x \)) in both equations is the same and the signs are.

Students may only sub into one equation. It will be explained that they MUST sub the solution into both \( l \) and \( k \).
**Worked solutions:**
Simultaneous Equation – Algebra 2

Solve the following pair of Simultaneous Equations using Algebra method

\[ l: x + 2y = 4 \]
\[ k: x - y = 1 \]

You need the number in front of \( x \) (or \( y \)) in both equations to be the same and the signs opposite.

\[
\begin{align*}
x + 2y &= 4 \\
-1(x - y) &= -1 \\
3y &= -1 \\
\text{So } y &= 1 \\
\text{when you find the value for } y,
\end{align*}
\]

sub it back into either \( l \) or \( k \) to find the \( x \) value.

Sub back into \( k \):
\[
x - 1 = 1 \\
x = 2 \quad \text{Answer = (2,1)}
\]

**Graphing method**
Point of Intersection = (2,1)

**Substitution**
Sub the solution back into both \( l \) and \( k \)
\[
\begin{align*}
l: x + 2y &= 4 \\
(2) + 2(1) &= 4 \quad \text{Yes} \\
k: x - y &= 1 \\
(2) - (1) &= 1 \quad \text{Yes}
\end{align*}
\]

Students may have difficulty starting the question. Start by ensuring the number in front of \( y \) (or \( x \)) in both equations is the same and the signs are opposite.

Students may only sub into one equation. It will be explained that they MUST sub the solution into both \( l \) and \( k \).
**Worked solutions:**
Simultaneous Equation – Algebra 3

**Solve the following pair of Simultaneous Equations using Algebra method**

l: \( x + y = 5 \)
k: \( 2x - y = 1 \)

You need the number in front of \( y \) (or \( x \)) in both equations to be the same and the signs opposite.

\[
\begin{align*}
x + y &= 5 \\
2x - y &= 1 \quad \text{(ADD)} \\
3x &= 6 \\
x &= 2
\end{align*}
\]

when you find the value for \( x \),

\[
\begin{align*}
x &= 2 \\
y &= 3 & \text{Answer} = (2,3)
\end{align*}
\]

**Graphing method**

Point of Intersection = (2,3)

**Substitution**

Sub the solution back into both l and k

l: \( x + y = 5 \)

\[
(2) + (3) = 5 \quad \text{Yes}
\]

k: \( 2x - y = 1 \)

\[
2(2) - (3) = 1 \quad \text{Yes}
\]

Students may have difficulty starting the question. Start by ensuring the number in front of \( y \) (or \( x \)) in both equations is the same and the signs are opposite.

Students may only sub into one equation. It will be explained that they MUST sub the solution into both l and k.
**Worked solutions:**

Simultaneous Equation – Algebra 4

Solve the following pair of Simultaneous Equations using Algebra method

l : \(3x - 2y = 2\)

k : \(2x + 3y = 10\)

You need the number in front of \(y\) (or \(x\)) in both equations to be the same and the signs opposite.

\[3 \times (3x - 2y = 2)\]
\[2 \times (2x + 3y = 10)\]

\[9x - 6y = 6\]
\[4x + 6y = 20\]  (ADD)
\[13x = 26\]
\[x = 2\]

When you find the value for \(x\),
sub it back into either l or k to find the \(y\) value.

Sub back into l:
\[3(2) - 2y = 2\]

**Graphing method**

Point of Intersection = (2,2)

**Substitution**

Sub the solution back into both l and k

l: \(3x - 2y = 2\)
\[3(2) - 2(2) = 2\]  Yes

k: \(2x + 3y = 10\)
Solution satisfies both equations
\[2(2) + 3(2) = 10\]  Yes
Lesson Plan 3

Day: Friday (Transition Years) / Wednesday (Adults)  
Date: 27th February 2015 (Transition years) / 25th February 2015 (Adults)  
Class Group: Transition years / Adults (Separate)  
Subject: Mathematics  
Lesson Number: 3

No. of Pupils: 24 (Transition Years) / 6 (Adults)  
Time: 11.30 – 12.10 (TY’s) / 7.20 – 8.00 (Adults)  
Length of Lesson: 40 minutes  
Topic: Simultaneous equations

Previous Knowledge and Experience:
Students are able to graph a pair of simultaneous equations and locate the point of intersection from lesson 1. Students are also able to solve a pair of simultaneous equations using algebra and check their answer using substitution.

How has the Previous Lesson informed my Planning for this Lesson?
In the previous lesson students solved a pair of simultaneous equations using algebra and check their answer using substitution. All students completed this task without any problems.

Aim:
By the end of this lesson, students will be able to identify a pair of simultaneous equations from a list of 6 diagrams.

Objectives:
Given 6 graphs, students will be able to identify which one of six graph corresponds to a pair of simultaneous equations. They will be able to complete this task using three different methods.
**Resources:**

GeoGebra – graphing simultaneous equations.  
Handout 3  
Digital Pencasts – Additional resource for students

## Content

<table>
<thead>
<tr>
<th></th>
<th><strong>Teacher Activity</strong></th>
<th><strong>Pupil Activity</strong></th>
<th><strong>Assessment of the Learning</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time</strong></td>
<td><strong>Introduction</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 6 mins | **Part 1:**  
- Correct homework. Solutions will be shown on the board via the projector. | • Pupils will ask any questions they may have about the homework. | • Check students homework. Give individual feedback OR take up handouts and correct. |
| 5 mins | **Part 2:**  
- To introduce identifying representations, a pair of 6 graphs with be drawn on the board. A pair of simultaneous equations will be written beside the diagrams. | • Students will have to discuss in pairs what methods they could use to identify which graph represents the pair of simultaneous equations.  
• After, a whole class discussion will take place to discuss their answers. | • Oral questionning  
- ‘What method could be used to assess which diagram represents the pair of simultaneous equations given?’ |
|  | **Development** | | |
| 2 mins | • The next part of the lesson will depend on what methods students believe would identify which graph represents the pair of simultaneous equations given. | • A whole class discussion will take place to discuss their answers. Expected answers:  
- Method 1: Algebra Method  
- Method 2: Finding Point of Intersection from graph  
- Method 3: Draw graph of pair | Self guided discovery |
| | | • Oral questionning to guide | |
| 7 mins | • Method 1 (Algebra method) will be used to identify which graph represents the pair of simultaneous equations given. If students are struggling with step 2 then I will guide them in the right direction. |
| 7 mins | • Method 2 (Finding Point of Intersection from graph) will be used to identify which graph represents the pair of simultaneous equations given. If students are struggling with step 2 then I will guide them in the right direction. |
| 7 mins | • Method 3 (Draw graph of pair of simultaneous equations) will be used to identify which graph represents the pair of simultaneous equations given. If students are struggling with step 2 then I will guide them in the right direction. |

| 7 mins | • STEP 1: Students will solve the pair of simultaneous equations to find the point of intersection. From this they will be able to eliminate 4 graphs. STEP 2: Pick points on each line from the remaining graphs and check are they a solution of the pair of simultaneous equations given. |
| 7 mins | • STEP 1: Students will write down the point of intersection from each diagram and check is it a solution of the simultaneous equations. From this they will be able to eliminate 4 graphs. STEP 2: Pick points on each line from the remaining graphs and check are they a solution of the pair of simultaneous equations given. |
| 7 mins | • Students must find two points on each line and graph their solution on graph paper. From this, students will be able to match their graph with one of the 6 |

| students in the right direction. | • Pair work |
| students in the right direction. | • Oral questioning to guide students in the right direction. |
| students in the right direction. | • Pair work |
| students in the right direction. | • Oral questioning to guide students in the right direction. |
| students in the right direction. | • Pair work |
I will guide them in the right direction.

<table>
<thead>
<tr>
<th>Conclusion</th>
<th>graphs given.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4 mins</strong></td>
<td></td>
</tr>
<tr>
<td>• To recap on the lesson, a short explanation of each of the three methods will be given.</td>
<td></td>
</tr>
<tr>
<td>• Students will be given their homework and a short explanation will be given of what they must do.</td>
<td></td>
</tr>
<tr>
<td><strong>2 mins</strong></td>
<td></td>
</tr>
<tr>
<td>• Students will give directions and explain what step must be complete next.</td>
<td></td>
</tr>
<tr>
<td>• Students will note their homework in their diaries.</td>
<td></td>
</tr>
<tr>
<td>• Oral Questioning – ‘what step is next and why?’</td>
<td></td>
</tr>
</tbody>
</table>

**Assessment:**
- Oral open ended questioning
- Feedback sandwich – a positive, one thing they should improve on, followed by another postive.
- Self guided discovery
- Peer tutoring
- Homework – Handout

<table>
<thead>
<tr>
<th>Higher Cognitive Questions</th>
<th>Prompts</th>
<th>Sample Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>What method could be used to assess which diagram represents the pair of simultaneous equations given?</td>
<td>Think of the tasks we completed in both lesson 1 and lesson 2 – solving simultaneous equations graphically and algebraically while checking your answer via substitution.</td>
<td>Method 1: Algebra Method Method 2: Finding Point of Intersection from graph Method 3: Draw graph of pair of simultaneous equations</td>
</tr>
</tbody>
</table>
**Worked Solutions:** (In class example)
Which of the graphs below represent:

\[ l: x - y = 2 \]
\[ k: x - 6y = -3 \]

(Circle the correct answer)

**Method 1: Algebra Method**

**STEP 1** - Solve algebraically to find the point of intersection

You need the number in front of \( y \) (or \( x \)) in both equations to be the same and the signs opposite.

\[
\begin{align*}
x - y &= 2 \\
-1(x - 6y &= -3)
\end{align*}
\]

Sub back into \( l \) and \( k \) to find the y value.

\[
\begin{align*}
x - y &= 2 \\
-x + 6y &= 3 \quad \text{(ADD)}
\end{align*}
\]

\[5y = 5 \]
\[ y = 1 \]

From diagrams, can eliminate (a), (c), (d), (f)

Remaining (b), (e)

**STEP 2** - Find a point on each line from graphs (b) and (e), check are they a solution of the simultaneous equations \( l \) and \( k \).

(b) Blue: (-3,0) \[
l: -3 - 0 = 2 \quad X \]
\[
k: -3 - 6(0) = -3 \quad \text{YES}
\]

(e) Blue: (0,3) \[
l: (0) - (3) = 2 \quad X \]
\[
k: (0) - 6(3) = -3 \quad X
\]

Red: (2,0) \[
l: 2 - 0 = 2 \quad \text{YES}
\]
\[
k: (2) - 6(-3) = -3 \quad X
\]

Conclusion: Graph (b) represents the simultaneous equations.
**Method 2: Finding Point of Intersection from graph**

STEP 1: Write down the point of intersection from each of the six graphs. Find them visually from the diagrams. Check if it is a solution of the simultaneous equations \( l \) and \( k \) by substituting each of the points back into the simultaneous equations and see if the point satisfies both equations.

<table>
<thead>
<tr>
<th>Point</th>
<th>( l )</th>
<th>( k )</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (-2,2)</td>
<td>((-2) - (2) = 2)</td>
<td>((-2) - 6(2) = -3)</td>
<td>X</td>
</tr>
<tr>
<td>(b) (3,1)</td>
<td>((3) - (1) = 2)</td>
<td>YES</td>
<td></td>
</tr>
<tr>
<td>(c) (0,0)</td>
<td>((0) - (0) = 2)</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>(d) (No point of intersection)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(e) (3,1)</td>
<td>((3) - (1) = 2)</td>
<td>YES</td>
<td></td>
</tr>
<tr>
<td>(f) (2,1)</td>
<td>((2) - (1) = 2)</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

**It is possible that more than one graph will have the same point of intersection.**

STEP 2 – Find a point on each line from graphs (b) and (e), check are they a solution of the simultaneous equations \( l \) and \( k \).

<table>
<thead>
<tr>
<th>Graph</th>
<th>Point</th>
<th>( l )</th>
<th>( k )</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue:</td>
<td>(-3,0)</td>
<td>((-3) - 0 = 2)</td>
<td>((-3) - 6(0) = -3)</td>
<td>X</td>
</tr>
<tr>
<td>Red:</td>
<td>(2,0)</td>
<td>(2 - 0 = 2)</td>
<td>YES</td>
<td></td>
</tr>
<tr>
<td>Blue:</td>
<td>(0,3)</td>
<td>((0) - (3) = 2)</td>
<td>((0) - 6(3) = -3)</td>
<td>X</td>
</tr>
<tr>
<td>Red:</td>
<td>(0,-3)</td>
<td>((0) - (-3) = 2)</td>
<td>((0) - 6(-3) = -3)</td>
<td>X</td>
</tr>
</tbody>
</table>

**Method 3: Draw graph of pair of simultaneous equations**

STEP 1 - Find two points on line \( l \) & \( k \):

\[ l: \begin{align*} (0) - y &= 2 \\ y &= -2 \end{align*} \]

\[ k: \begin{align*} (0) - 6y &= -3 \\ y &= 0.5 \end{align*} \]

**Remaining (b), (e)**

<table>
<thead>
<tr>
<th>Graph</th>
<th>Point</th>
<th>( l )</th>
<th>( k )</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue:</td>
<td>(0,3)</td>
<td>((0) - (3) = 2)</td>
<td>((0) - 6(3) = -3)</td>
<td>X</td>
</tr>
<tr>
<td>Red:</td>
<td>(0,-3)</td>
<td>((0) - (-3) = 2)</td>
<td>((0) - 6(-3) = -3)</td>
<td>X</td>
</tr>
</tbody>
</table>

**Conclusion: Graph (b) represents the simultaneous equations.**
**Worked Solutions: (Homework example)**

Which of the graphs below represent:

(Circle the correct answer)

\[ l: 4x + y = -2 \]
\[ k: x - y = -3 \]

**Method 1: Algebra Method**

**STEP 1 - Solve algebraically to find the point of intersection**

You need the number in front of \( y \) (or \( x \)) in both equations to be the same and the signs opposite.

\[ 4x + y = -2 \]
\[ x - y = -3 \] (ADD)

\[ 5x = -5 \]
\[ x = -1 \]

When you find the value for \( x \), sub it back into either \( l \) or \( k \) to find the \( y \) value.

\[ -1 - y = -3 \]
\[ y = 2 \]

Answer = \((-1, 2)\)

From diagrams, can eliminate (b), (c), (e), (f)
Remaining (a), (d)

**STEP 2 – Find a point on each line from graphs (a) and (d), check are they a solution of the simultaneous equations \( l \) and \( k \).**

(a) Blue: \((-4, 0)\)
\[ l: 4(-4) + 0 = -2 \]
\[ K: (-4) - 0 = -3 \]

(d) Blue: \((-3, 0)\)
\[ l: 4(-3) + 0 = -2 \]
\[ K: (-3) - 0 = -3 \]

Red: \((1, 0)\)
\[ l: 4(1) + 0 = -2 \]
\[ K: (1) - 0 = -3 \]

Red: \((0, -2)\)
\[ l: 4(0) + (-2) = -2 \]

Conclusion: Graph (d) represents the simultaneous equations.
Method 2: Finding Point of Intersection from graph

STEP 1: Write down the point of intersection from each of the six graphs. Find them visually from the diagrams. Check is it a solution of the simultaneous equations l and k by substituting each of the points back into the simultaneous equations and see if the point satisfies both equations.

(a) (-1,2)
- l: 4(-1) + (2) = -2  YES
- k: (-1) - (2) = -3  YES

(b) (No point of intersection)

(c) (2,0)
- l: 4(2) + (0) = -2  X
- k: (2) - (0) = -3  X

(d) (-1,2)
- l: 4(-1) + (2) = -2  YES
- k: (-1) - (2) = -3  YES

(e) (-2,2)
- l: 4(-2) + (2) = -2  X
- k: (-2) - (2) = -3  X

(f) (0,1)
- l: 4(0) + (1) = -2  X
- k: (0) - (1) = -3  X

**It is possible that more than one graph will have the same point of intersection.**

**Eliminate (b), (c), (e), (f)**
**Remaining (a), (d)**

Conclusion: Graph (d) represents the simultaneous equations.

Method 3: Draw graph of pair of simultaneous equations

STEP 1 - Find two points on line l & k:

l: when x = 0
- 4(0) + y = -2
- y = -2  (0, -2)

k: when y = 0
- x - (0) = -3
- x = -3  (-3, 0)

STEP 2 - Draw both lines on coordinate axis

Conclusion: Graph (d) represents the simultaneous equations.
Lesson Plan 4

Day:  Monday (Transition Years) / Wednesday (Adults)  
Date:  2\textsuperscript{nd} March 2015 (Transition years) / 4\textsuperscript{th} March 2015 (Adults)  
Class Group:  Transition years / Adults (Separate)  
Subject:  Mathematics  
Lesson Number:  4  
No. of Pupils:  24 (Transition Years) / 6 (Adults)  
Time:  11.30 – 12.10 (TY’s) / 6.00 – 6.40 (Adults)  
Length of Lesson:  40 minutes  
Topic:  Simultaneous equations

Previous Knowledge and Experience:
Students are able to graph a pair of simultaneous equations and locate the point of intersection from lesson 1. Students are able to solve a pair of simultaneous equations using algebra and check their answer using substitution. Students are able to identify which one of six graphs corresponds to a pair of simultaneous equations. They will be able to complete this task using three different methods.

How has the Previous Lesson informed my Planning for this Lesson?
In the previous lesson students had to identify which one of six graphs represented the pair of simultaneous equations given.

Aim:
By the end of this lesson, students will be able to generate and use a graphical representation of simultaneous equations.

Objectives:
Given a real life simultaneous equations example, students will be able generate a graph and use the graph to answer additional questions.
Resources:
GeoGebra – graphing simultaneous equations.
Digital Pencasts – Additional resource for students
Handout 4
Video – Car example [http://www.geogebratube.org/student/m93693](http://www.geogebratube.org/student/m93693)

<table>
<thead>
<tr>
<th>Time</th>
<th>Teacher Activity</th>
<th>Pupil Activity</th>
<th>Assessment of the Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 mins</td>
<td><strong>Introduction</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Part 1:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Correction of homework:</td>
<td>• Students will actively participate in</td>
<td>• Oral questionning</td>
</tr>
<tr>
<td></td>
<td>One student will go through solution using method 1</td>
<td>the correction of homework.</td>
<td>‘Which method do you prefer and why?’</td>
</tr>
<tr>
<td></td>
<td>One student will go through solution using method 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>One student will go through solution using method 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Part 2:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• A real life example will be written on</td>
<td>• Students will read the real life example. In pairs they</td>
<td>• Self guided discovery.</td>
</tr>
<tr>
<td></td>
<td>the board about a Car Race. It will be explained</td>
<td>will try to make the two simultaneous equations using</td>
<td>• Oral questionning will be used to guide any students</td>
</tr>
<tr>
<td></td>
<td>that this can be solved using</td>
<td>the information provided.</td>
<td>who are struggling.</td>
</tr>
<tr>
<td></td>
<td>simultaneous equations.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 mins</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Development</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 mins</td>
<td>• Once students have made their two</td>
<td>• Students will try to graph their</td>
<td>•</td>
</tr>
<tr>
<td></td>
<td>simultaneous equations, they must graph their</td>
<td>simultaneous equations to find the point of</td>
<td>•</td>
</tr>
<tr>
<td></td>
<td>simultaneous equations.</td>
<td>intersection between the two cars.</td>
<td>•</td>
</tr>
<tr>
<td></td>
<td>Km on one axis Vs hrs on the other.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

158
<table>
<thead>
<tr>
<th>3 mins</th>
<th>• The video of the cars racing will be shown on the board via GeoGebra.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 mins</td>
<td>• With the help of the students, I will go through the example in detail. Explaining how to make the pair of simultaneous equations to begin and then show students how to use a graph to find the solution.</td>
</tr>
<tr>
<td>6 mins</td>
<td>• Additional questions will be given to students to answer which will assess their understanding of the problem. - ‘If both cars were travelling for 2 hours, which car travelled the longest distance? How much further did it travel compared to the other car?’ - ‘Which car would win the race if the race lasted 3 hours?’</td>
</tr>
<tr>
<td></td>
<td>• Students will use the diagram from the video to check is their answer correct. They will be given two minutes to assess where they went wrong if they made a mistake.</td>
</tr>
<tr>
<td></td>
<td>• Students will actively participate in the explanation.</td>
</tr>
<tr>
<td></td>
<td>• In pairs, students will answer the additional questions which have been asked.</td>
</tr>
<tr>
<td></td>
<td>• Students will use the diagram from the video to check is their answer correct. They will be given two minutes to assess where they went wrong if they made a mistake.</td>
</tr>
<tr>
<td></td>
<td>• Students will actively participate in the explanation.</td>
</tr>
<tr>
<td></td>
<td>• In pairs, students will answer the additional questions which have been asked.</td>
</tr>
<tr>
<td><strong>Conclusion</strong></td>
<td></td>
</tr>
<tr>
<td>4 mins</td>
<td>• To recap on the lesson, I will change the starting position of the two cars and also the speed they will travel at. Then with the help of the students I will go through with solution on the board.</td>
</tr>
<tr>
<td>2 mins</td>
<td>• Students will be given their homework and a short explanation will be given of what they must do.</td>
</tr>
<tr>
<td></td>
<td>• Students will give me directions and explain to me what step I must complete next.</td>
</tr>
<tr>
<td></td>
<td>• Students will note their homework in their diaries.</td>
</tr>
<tr>
<td></td>
<td>• Self assessment</td>
</tr>
<tr>
<td></td>
<td>• Pair work Oral Questionning will be used to guide any students who are struggling.</td>
</tr>
</tbody>
</table>
Assessment:
- Oral open ended questioning
- Feedback sandwich – a positive, one thing they should improve on, followed by another positive.
- Peer tutoring
- Homework – Handout

<table>
<thead>
<tr>
<th>Higher Cognitive Questions</th>
<th>Prompts</th>
<th>Sample Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>If both cars were travelling for 2 hours, which car travelled the longest distance? How much further did it travel compared to the other car?</td>
<td>Use your graph for help</td>
<td>See solutions below</td>
</tr>
<tr>
<td>Which car would win the race if the race lasted 3 hours?</td>
<td>Use your graph for help</td>
<td>See solutions below</td>
</tr>
</tbody>
</table>
Worked Solutions:
Blue Car starts at 35 km and travels at 25km/hr. Red Car starts at 5km and travels at 35km/hr. Draw a graph to represent the distance covered by both the Blue and Red Cars over a 6 hour period.

Blue: \[ y = 25x + 35 \]
where \( x \) = number of hours
Red: \[ y = 35x + 5 \]
y = total distance

<table>
<thead>
<tr>
<th>Number of hrs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Distance</td>
<td>60</td>
<td>85</td>
<td>110</td>
<td>135</td>
<td>160</td>
<td>185</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of hrs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Distance</td>
<td>40</td>
<td>75</td>
<td>110</td>
<td>145</td>
<td>180</td>
<td>215</td>
</tr>
</tbody>
</table>

Justification

a) If both cars were travelling for 2 hours, which car travelled the longest distance?
Clearly from the graph the blue car travels further after 2 hours.

How much further did it travel compared to the other car?
It travels 10 km further than the red car.

b) Which car would win the race if the race lasted 3 hours?
They both travelled the same distance after 3 hours. If the race lasted longer than 3 hours then the red car would win.
Worked Solutions:

Daniel wants to rent an apartment in Dungloe. Apartment 1 costs an initial €180 plus an additional €30 per night. Apartment 2 costs an initial €150 plus an additional €35 per night.

Draw a graph to represent the cost of Apartment 1 and 2 over a 10 day period.

Apartment 1: \( y = 30x + 180 \)
where \( x \) = number of nights
Apartment 2: \( y = 35x + 150 \)
\( y \) = total cost

Apartment 1

<table>
<thead>
<tr>
<th>No. of nights</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (180)</td>
<td>210</td>
<td>240</td>
<td>270</td>
<td>300</td>
<td>330</td>
<td>360</td>
<td>390</td>
<td>420</td>
<td>450</td>
<td>480</td>
</tr>
</tbody>
</table>

Apartment 2

<table>
<thead>
<tr>
<th>No. of nights</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (150)</td>
<td>185</td>
<td>220</td>
<td>255</td>
<td>290</td>
<td>325</td>
<td>360</td>
<td>395</td>
<td>430</td>
<td>465</td>
<td>500</td>
</tr>
</tbody>
</table>

Justification Questions

Which apartment would you recommend Daniel renting and why?

If Daniel was staying less than 6 nights apartment 2 would be the cheaper option, however if he is staying longer than 6 nights apartment 1 would be the cheaper option. If he is staying exactly 6 nights, both apartments are the same price.

If Daniel was only renting the apartment for 8 nights which apartment should he rent and why?

He should rent apartment 1 because it is €10 cheaper.

If Daniel was only renting for 6 days which apartment should he rent and why?

If he is staying exactly 6 nights, both apartments are the same price.
Appendix B

Pre & Post-Test, Grading Rubric and Handouts
Algebra Pre-Test

Explanation Task

You have been approached by a popular children’s T.V. show and asked to give a talk on T.V. The purpose of your talk will be to help students learn about simultaneous equations.

Below are some questions you should try to answer. For each question you should draw as many pictures as you can to show what you mean.

1) What is a linear equation?
2) What are simultaneous equations?
3) What is the meaning of your answer when you solve simultaneous equations?
4) What methods can be used to solve a pair of simultaneous equations?
5) How are simultaneous equations used in real life?
Representational Knowledge - Identify

Question 1

Which of the graphs below represent:  

2x + 4y = 8  
3x – 2y = 4

(Circle the correct answer)
Question 2

Which of the graphs below represent: $x + 3y = 6$

$3x - y = -2$

(Circle the correct answer)
Question 3

Which of the graphs below represent:

3x + y = 7  
2x – 2y = -6

(Circle the correct answer)
Problem Solving 1

Peter wants to rent a car for a maximum period of three weeks (21 days). A BMW 3-Series costs an initial €160, plus an additional €40 per day. An Audi A6 costs an initial €240, plus an additional €35 per day.

Represent both rental plans as linear equations and then graph them on the graph paper provided.

Justification

Answer the following questions, based on your graph above.

a) In terms of cost, which car would you recommend Peter renting and why?

b) If Peter was only renting the car for 10 days which one should he rent and why?

c) If Peter was only renting the car for 16 days which one should he rent and why?
Problem Solving 2

Vodafone are offering two plans for the new iPhone 6.

Plan A costs €30 a month plus an additional 10cent per text.
Plan B costs €50 a month plus an additional 5cent per text.
Both plans include free calls to all networks.

Represent both plans as linear equations and graph them on the graph paper provided.

Justification

Answer the following questions, based on your graph above.

a) How many texts must be sent in order for both plans to be equivalent?

b) Which plan would you choose and why?

c) If you sent 450 texts the previous month, which plan would you choose and why?
Problem Solving 3

20,000 tickets were sold for The Script in the 3Arena, Dublin. Adult tickets cost €35, children (under 18) tickets cost €22, and a total of €648,000 was collected.

a) How many tickets of each kind were sold for The Script concert?

b) A Beyoncé concert was also held in the 3Arena and the same number of adults and children attended. However adult tickets cost €38 and children tickets cost €20. Which concert produced more money?

c) If the Beyoncé concert was changed to an over 18’s event, how much would the 3Arena need to charge to collect the same money?
Algebra Post-Test

Explanation Task

You have been approached by a popular children’s T.V. show and asked to give a talk on T.V. The purpose of your talk will be to help students learn about simultaneous equations.

Explain in detail, everything you know about simultaneous equations. You should draw as many pictures as you can to show what you mean.
Representational Knowledge - Identify

Question 1

Which of the graphs below represent:

2x + y = 5
x + y = 4

(Circle the correct answer)
Question 2

Which of the graphs below represent:

2x + 3y = 12
x – y = 1

(Circle the correct answer)
Question 3

Which of the graphs below represent:

3x + 2y = 12
x - 2y = -4

(Circle the correct answer)
Problem Solving 1

The weekly rentals for the newly released ‘Hunger Games – Catching Fire’ DVD in HMV decreased each week by 24 rentals after its release in December. During the first week of its release the DVD was rented 360 times. At the same time, the ‘Horrible Bosses 2’ DVD was released and during the opening week it was only rented 24 times. After the opening week the rental of the ‘Horrible Bosses 2’ DVD increased by 18 per week.

Represent both weekly rentals as linear equations and graph the first 10 weeks of rentals.

Justification

Answer the following questions, based on your graph above.

d) Which film had more rentals in week 6?

[Blank]

e) Which film had more rentals in week 9?

[Blank]

f) What can you conclude about the rentals?

[Blank]
Problem Solving 2

You are offered two jobs selling digital boxes. Sky offers you €480 plus an additional €7.50 per sale. UPC offers a salary of €500 per week plus €5 per sale.

Assuming you will sell at most, 10 digital boxes, represent both job offers as linear equations and graph them on the graph paper provided.

Justification

Answer the following questions, based on your graph above.

   d) How many digital boxes would you have to sell in a week in order to make both offers equivalent?

   e) Which job offer would you choose and why?

   f) If an average of 10 digital boxes are sold each week by each sale representative, which offer would you choose and why?
Problem Solving 3

A fruit grower uses two types of fertilizer in an orange grove, brand A and brand B. Each bag of brand A contains 8 pounds of nitrogen and 4 pounds of phosphoric acid. Each bag of brand B contains 7 pounds of nitrogen and 6 pounds of phosphoric acid. Tests indicate that the grove needs 720 pounds of nitrogen and 500 pounds of phosphoric acid.

d) How many bags of each brand should be used to provide the required amounts of nitrogen and phosphoric acid?

Brand A was removed by the company and brand C was introduced in its place. You can assume brand A and brand C use the same number of bags. Use your answer from part a) to solve the following:

e) How many pounds of nitrogen would be present in brand C, if the Orange Grove needed 802 pounds of nitrogen?

f) How many pounds of phosphoric acid would be present in brand C, if the Orange Grove needed 541 pounds of phosphoric acid?
## Grading Rubric for Explanation Task & Justification

### Questions

<table>
<thead>
<tr>
<th>Question</th>
<th>1 (Incorrect)</th>
<th>2 (Partially Correct)</th>
<th>3 (Mostly Correct)</th>
<th>4 (Fully Correct)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1) What is a linear equation?</td>
<td>Incorrect or no explanation given</td>
<td>Gives an example of a linear equation but is unable to explain what exactly it is.</td>
<td>Good explanation but leave out the fact that the variables must be of degree 1.</td>
<td>Correct detailed explanation given (i.e. in secondary school we traditionally only see linear equations involving two variables, were all variables are of degree 1)</td>
</tr>
<tr>
<td>2) What are Simultaneous Equations?</td>
<td>Incorrect or no explanation given</td>
<td>Gives an example of a pair of simultaneous equations but is unable to explain what exactly they are.</td>
<td>Good explanation but leave out the fact that the variables must be of degree 1.</td>
<td>Correct detailed explanation stating that a pair of simultaneous equations generally involve 2 variables in secondary school. They must be solved at the same time so that the solution satisfies both equations at the same time.</td>
</tr>
<tr>
<td>3) What is the meaning of your answer when you solve Simultaneous Equations?</td>
<td>Incorrect or no explanation given</td>
<td>Poor explanation (i.e. it is the point of intersection between the two lines).</td>
<td>Good explanation but more detail needed (i.e. the solution satisfies the</td>
<td>Correct explanation given (i.e. the solution satisfies both equations are the same)</td>
</tr>
<tr>
<td>Question</td>
<td>Incorrect or no explanation given</td>
<td>Gives one example of one method to solve a pair of simultaneous equations but fails to explain how this method works.</td>
<td>Gives two methods to solve a pair of simultaneous equations but fails to explain how these methods work. OR Fails to explain these methods in detail.</td>
<td>Both methods given (algebraically &amp; graphically) with a detailed explanation of how to complete these methods.</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>4) What methods can be used to solve a pair of Simultaneous Equations?</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5) How are Simultaneous Equations used in real life?</td>
<td>Incorrect or no example given</td>
<td>Real life example given without explanation</td>
<td>Real life example given with partial explanation</td>
<td>Real life example given with detailed explanation</td>
</tr>
<tr>
<td><strong>Justification Questions</strong></td>
<td>Incorrect or no answer given</td>
<td>Correct answer given (e.g. correct plan) with no explanation</td>
<td>Correct answer given (e.g. correct plan) with partial explanation</td>
<td>Correct answer given with detailed explanation</td>
</tr>
</tbody>
</table>
Handout 1: Simultaneous Equations

Question 1

Find the point of intersection between lines l and K (Show all your work)

l: $x + y = 5$

k: $2x - y = 1$
**Question 2**

Find the point of intersection between lines $l$ and $k$  (Show all your work)

$l: 3x - 2y = 2$

$k: 2x + 3y = 10$
**Question 3**

Explain in your own words what are a pair of Simultaneous Equations and what do you find when you solve them? Also give a brief explanation of how they are used in real life?
Handout 2: Simultaneous Equations

Question 1

a) Solve the following pairs of simultaneous equations using algebra.

l: \( x + 2y = 12 \)

k: \( 3x - 5y = 3 \)

b) Check your answer using substitution
c) Check your answer using the graphical method.  (Show all your work)

\[ l: x + 2y = 12 \]
\[ k: 3x - 5y = 3 \]
Question 2

a) Solve the following pairs of simultaneous equations using algebra.

l: \( 3x + y = 5 \)

k: \( 5x - 4y = -3 \)

b) Check your answer using substitution

c) Check your answer using the graphical method. (Show all your work)

l: \( 3x + y = 5 \)
k: \( 5x - 4y = -3 \)
Handout 3: Simultaneous Equations

Question 1

Which of the graphs below represent:

\[ x - y = 2 \]
\[ x - 6y = -3 \]

(Circle the correct answer)

(a) [Graph A]

(b) [Graph B]

(c) [Graph C]

(d) [Graph D]

(e) [Graph E]

(f) [Graph F]
Which of the graphs below represent:

4x + y = -2
x - y = -3

(Circle the correct answer)
Handout 4: Simultaneous Equations

Question 1

Blue Car starts at 35 km and travels at 25km/hr. Red Car starts at 5km and travels at 35km/hr.

Draw a graph to represent the distance covered by both the Blue and Red Cars over a 6 hour period.

Justification

a) If both cars were travelling for 2 hours, which car travelled the longest distance?
   How much further did it travel compared to the other car?

b) Which car would win the race if the race lasted 3 hours?
**Question 2**

Daniel wants to rent an apartment in Dungloe. Apartment 1 costs an initial €180 plus an additional €30 per night. Apartment 2 costs an initial €150 plus an additional €35 per night.

Draw a graph to represent the cost of Apartment 1 and 2 over a 10 day period.

- **a)** Which apartment would you recommend Daniel renting and why?

- **b)** If Daniel was only renting the apartment for 8 days which apartment should he rent and why?

- **c)** If Daniel was only renting for 6 days which apartment should he rent and why?
Handout 5: Simultaneous Equations

Question 1

Jasmine wants to use milk and orange juice to increase the amount of calcium and vitamin A in her daily diet. An ounce of milk contains 37 milligrams of calcium and 57 micrograms of vitamin A. An ounce of orange juice contains 5 milligrams of calcium and 65 micrograms of vitamin A.

a) How many ounces of milk and orange juice should Jasmine drink each day to provide exactly 500 milligrams of calcium and 1200 micrograms of vitamin A.

b) From your answer, what can you conclude?

c)
Question 2

50,000 tickets were sold for the Ireland v’s France in the Aviva Stadium, Dublin. Adult tickets cost €65, children (under 18) tickets cost €30, and a total of €3,075,000 was collected.

a) How many tickets of each kind were sold for rugby match?

b) The Ireland v’s England match was also held in the Aviva Stadium and the same number of adults and children attended. However adult tickets cost €68 and children tickets cost €25. Which match produced more money?

c) If the Ireland v’s England match was changed to an over 18’s event, how much would the Aviva need to charge to collect the same money?
Handout 6: Simultaneous Equations

Question 1

At a price of €1.88 per pound, the supply for eggs (dozen) in Dublin is 16,000 pounds and the demand is 10,600 pounds. When the price drops to €1.46 per pound, the supply decreases to 10,000 pounds and the demand increases to 12,700 pounds. Assume that the price-supply and price-demand equations are linear.

Draw a graph to represent the above information on the graph paper provided.

a) What is the difference in supply / demand if the price of eggs (per dozen) is €2.09 per pound?

b) What is the difference in supply / demand if the price of eggs (per dozen) is €1.32 per pound?

c) From your graph, what can you conclude?
Question 2

A company markets ‘Exercise DVDs’ that sell for €19.95, including shipping and handling. The monthly fixed costs (advertising, rent, etc.) are €24,000 and the variable costs (materials, shipping, etc) are €7.45 per DVD.

a) How many DVDs must be sold each month for the company to break even?

b) If 1800 DVD’s were sold, how much profit or loss was made?

c) From the above calculations, what can you conclude?
Appendix C

Data Collection – Focus Group Questions & Transcripts
Focus Group Transition Years

P1: Niamh       P5: Sean
P2: Caoimhe     P6: Declan
P3: Bridget     P7: Chelsea
P4: Cara        P8: Conor

I: Q1 - What are your feelings towards Algebra (S.E.’s)?
P6: ‘I hate it.’
P2: ‘It depends if I can do it or not.’
P8: ‘Yeah, some parts are easier than others.’
P4: ‘To be honest, I don’t mind it compared to geometry and trigonometry. I really hate those.’
P1: ‘Well algebra is my least favorite topic.’

I: Q2 - In your opinion, are there any benefits to learning Algebra (S.E.’s)?
P6: ‘It probably makes you use your brain but I don’t know how it would help in everyday life.’
P1: ‘It’s probably good for difficult maths in the future but I can’t see how it will help us in secondary school.’
P6: ‘Or ever’ (laugh)
I: ‘Anyone else have anything to add?’
P3: ‘No, I just agree with the rest.’

I: Q3 - Do you believe that a supportive tool like the digital pencasts has aided your understanding of algebra?
P1: ‘Yes definitely, you can go through problems at your own pace, compared to the fast pace we go through topics in class.’
P5: ‘I didn’t use the pencasts much but for our leaving leaving cert it would be great.’
P4: ‘At least when we go into 5th year, if there are any areas we forget we can revise using the pencasts.’
P2: ‘Yes I agree because I completely forgot everything about simultaneous equations. The pencasts helped me revise.’

I: Q4 - Do you have a computer at home?
‘Yes’ (everyone)

And do you use it often?
P6: ‘To be honest I use my phone more. I rarely use the computer anymore.’
P5: ‘Same here.’
I: Q5 - Are you comfortable using technology?
‘Yes’ (everyone)

I: Q6 - Do you enjoy using technology in school or at home or do you avoid using it when you can?
P5: ‘Yes because you can find out anything you want on the internet.’
P7: ‘And you can shop online now too.’

I: Q7 - Looking at the digital pencasts, do you think they were valuable to you when learning algebra?
P2: ‘Definitely, I couldn’t remember how to graph simultaneous equations. Even when you went through it in class I was still confused. At least I could go through them at my own pace at home.’
P4: ‘Yeah, I forgot everything aswell. They were great for revision.’

I: Q8 - Can you see any benefits from these digital pencasts?
P6: ‘Yes, you wouldn’t have to write as much.’ (laugh)
P4: ‘If you forget what something meant when you go home at least you have the notes and they’re always there.’
P3: ‘Yeah, with the voice explanation too, which is great.’

I: Q9 - In your opinion, do you believe the digital pencasts helped to improved your understanding of algebra?
P1: ‘Yes. I never realised that solving simultaneous equations using algebra and also using the graphical method, you got the same answer. Going through the representation pencast at home, it clicked with me that’s what it meant. So they definitely helped improve my understanding.’
P5: ‘I was the same. There was a number of things I always found hard, however listening to the pencasts helped.’
P7: ‘The only draw back is not being able to ask questions.’
I: ‘Yes that is true but if you have any questions you can email me. Anyone else?’
P8: ‘Nothing really apart from they are really handy.’

I: Q10 - Have you encountered any difficulties with the digital pencasts?
P3: ‘I couldn’t get the audio going in the beginning but I used the handout you gave us an then it worked.’
P2: ‘Same here, my dad got it working for me.’
P6: ‘How come it doesn’t work on an iPhone?’
I: ‘You need Adobe Flash Player but iPhones do not support that.’
P6: ‘So the pencasts will never work on your phone?’
I: ‘I am trying to find a way around that. I believe they will fix this issue soon since that majority of people have a smartphone.’

I: Q11 - Did you listen to the audio?
P1: ‘Yes that’s the main part. Without the audio, the pencasts are just like a normal textbook.’
I: ‘So everyone used the audio?’
‘Yes’

I: Q12 - If yes, what are the benefits of having the accompanying audio?
P4: ‘They help you remember because you are not just staring at a load of words and letters, you can hear the explanations too.’
P5: ‘Yeah, otherwise I would still be lost. When I look at the solutions in the book, I get so confused.’

I: Q13 - Would you recommend the digital pencasts to other students?
P2: ‘Yes, I like it, its quite cool.’
P7: ‘It’s a fancy wee gadget.’
P3: ‘And it’s very useful for school sir.’

I: Q14 – Finally, in your opinion, would digital pencasts be useful in other subjects, and if so, what subjects do you think it would be most helpful in?
P5: ‘History and English for planning essays.’
P6: ‘Yeah because I always forget what we are meant to do.’
P8: ‘That because you don’t listen.’ (laugh)
P4: ‘It would probably be handy enough in french aswell I think.’
P3: ‘Yes for orals it would be great.’
I: ‘Any concluding comments?’
P6: ‘How do you charge the pen?’
I: ‘You plug it into the computer via a USB lead.’
P6: ‘Cool, that’s handy.’
Focus Group Adults

Themes:
1) Positivity towards digital pencasts
2) Difficulties with algebra
3) Difficulties with technology

I: Q1 - What are your feelings towards Algebra (S.E.‘s)?

P1: ‘I really struggle with the topic. I always have, its one area of maths I really did not enjoy at school. I find it very difficult and I do not see its relevance.’
P4: ‘Yes, I agree. Prior to the adult classes I disliked algebra and couldn’t see how it could be used in real life. Now I am beginning to see its relevance, however I still dislike it.’
P2: ‘I think algebra is extremely challenging, which is why I like the topic. I enjoy progressing through the steps to get my answer.’
P5: ‘I enjoy Algebra because you know when you get the correct answer, compared to the topics in paper 2.’
P6: ‘I like algebra because I love numbers. I work with numbers every day in my job so I didn’t find it overly challenging.’
I: ‘Anyone else?’
P3: ‘I am less weary of the subject due to this intervention using the digital pencasts but I am still not overly confident with the topic.’

I: Q2 - Ok so, in your opinion, are there any benefits to learning Algebra (S.E.‘s)?

P2: ‘It helps to solve real life problems. Used in a lot of key areas in life’
I: Such as?
P2: ‘Well engineers use it a lot. As do computer programmers.’
P4: ‘Before this course I thought it was irrelevant but now I see its importance.’
P1: ‘Maybe for some careers, but not useful for me as a housewife.’ (laughs)
P3: ‘Personally, I don’t use it in real life. But I see how it is essential for engineering or similar jobs.’
P6: ‘Yes it’s used everywhere.’
I: ‘Where do you see it used?’
P6: ‘Well like Hugh says, engineers’ use it everyday. Also like you already said, it is used in air traffic control to ensure two planes don’t collide.’

I: Q3 - Do you believe that a supportive tool like the digital pencasts has aided your understanding of algebra?
P4: ‘Yes most definitely because we cover one topic per night here so I use it a lot for revision purposes.’
P1: ‘I would see how it is useful but I dislike using technology.’
I: Ok but did you find the pencasts easy enough to navigate?
P1: ‘Oh yes, very easy especially when you showed us how to assess them in class before we left.’
I: ‘That’s good any other opinions?’
P6: ‘They help because I like to see the steps worked. The book solutions can be overwhelming compared to the step by step solution of the digital pencasts.’
P5: ‘Yes listening to the steps explained and being able to pause between steps to digest what going on is great I found.’

I: Q4 - Do you have a computer at home? Do you use it often?
P6: ‘I do, a PC. I would use it for work. Not for personal point of view.’
P1: ‘No I am not a huge fan of technology apart from my nokia phone.’ (laughs)
P4: ‘I mostly use technology for social media, checking emails and planning for work.’
P2: ‘I use technology a lot both in my job but also at home to stream documentaries, etc…’
P3: ‘I try to use it more often because I have 4 young daughters who use technology all the time. I have to keep up to date.’

I: Q5 - Are you comfortable using technology?
P3: ‘I try to keep up with technology like I said but its difficult when you are older. It seems to be ever changing. The younger generation seem so at ease with technology.’
P1: ‘Yes that is my issue. Its constantly changing and I cannot keep up to date. I sometimes feel quite intimidated by technology and have a fear of doing something wrong.’
P2: ‘I actually use it for work everyday so I am comfortable using it.’
P6: ‘Yeah, same here’

I: Q6 - Do you enjoy using technology in work or at home or do you avoid using it when you can?
P6: ‘Well it is a great tool because you can find out anything you want on a computer. I would say I use it daily.’
P1: ‘Personally, I dislike it so I don’t use it.’
P4: ‘I enjoy using it, use it mainly as a pass-time in the evenings.’

I: Q7 - Looking at the digital pencasts, do you think they were valuable to you when learning algebra (S.E’s)?
P5: ‘Fantastic tool for young teachers and students who are able to use it.’
I: ‘And for you personally?’
P5: They definitely aided my learning of algebra.’
P6: ‘Yes it is extremely beneficial for students if they are willing to use it. My son is in transition year and I know it would be a huge benefit to him when he goes on to complete his leaving cert. Not only for maths but for all subjects if other teachers used it.’
P3: ‘You teach my daughter in 1st year, she uses the pencasts to revise areas she finds difficult. That is why I began to use them myself. Definitely very useful.’
P1: ‘As I said before I dislike algebra but I did find the pencasts useful. I just am not a fan of technology so I am not sure if I would use them again.’

I: Q8 - Can you see any benefits from these digital pencasts?
P6: ‘Yes, you can access them whenever you want. If you are at home and you are struggling with a particular question, you can open up the pencasts and it will take you through the question, step by step.’
P5: ‘I second that. When you are on your own, its like having someone there beside you explaining the answers.’
P1: ‘I can certainly see the benefits but technology is an issue for an older person.’
P4: ‘I really like how you can pause the pencasts between steps. For me it takes me a while to digest new information, especially with Algebra so this feature was extremely beneficial personally.’

I: Q9 - In your opinion, do you believe the digital pencasts helped to improved your understanding of algebra?
P6: ‘Yes without a doubt. If you’re stuck on something and you can listen to the directions again. In my day if you were stuck with your homework, you either asked your parents or else you couldn’t complete it. I have used them numerous times already to revise difficult parts of algebra.’
P2: ‘Yes. It helps you grasp questions you don’t understand.’
I: ‘Anyone else have anything to add?’
P5: ‘Just reinforcing what I said in the previous question, its like having the teacher beside you, which is great.’

I: Q10 - Have you encountered any difficulties with the digital pencasts?
P1: ‘I struggled to get the audio working in the beginning but when I followed the required steps which you explained, it worked for me then.’
P5: ‘Yes, I had the same difficulties in the beginning.’
P6: ‘The digital pencasts won’t work on iPad because Adobe flash player won’t work on those.’
I: ‘Yes, I understand numerous people have had difficulty with this. Students cannot get the pencasts working on their iPhones either due to the same issue. Did you get the pencasts working on another device?’
P6: ‘Yes they worked on my other laptop.’
I: Q11 - Did you listen to the audio?
P6: ‘Yes the audio is the most important part of the technology, without it the solution is just like a book solution.’
P2: ‘Exactly, that’s why it is so useful.’

I: Q12 – Right, so this is the key benefit of having the accompanying audio?
P6: ‘Yes, it is extremely helpful because you have a detailed explanation in the background.’
P5: ‘And I find it helpful because I struggle to use my notes to help me. I forget steps easily.’

I: Q13 – So would you recommend the digital pencasts to other students?
P3: ‘Yes it would be helpful for parents who wish to help their son / daughter with their homework.’
P6: ‘Same here, my son really struggles with maths. I will make sure he uses the pencasts for his leaving cert.’
P1: ‘Yes from the positive reviews of others it’s great if you can use technology.’
I: ‘Any further comments?’
P2: ‘I think it’s an excellent tool for students to have. I find it excellent when revising.’

I: Q14 - In your opinion, would digital pencasts be useful in other subjects and if so, what subjects do you think it would be most helpful in?
P2: ‘I think it would be fantastic for business and accountancy. Anything you would need instruction for.’
P6: ‘Also for languages from a pronunciation point of view.’
P5: ‘And maybe for diagrams and key words in geography and science too.’
P3: ‘Yes its probably excellent for other topics too but I feel it’s most beneficial in maths because it’s a subject most people struggle with.’
I: ‘Any concluding comments?’
P4: ‘Nope that’s me anyways.’
P1: ‘Me Neither.’
I: ‘That’s us then. Thanks folks.’
Appendix D

Information Sheets & Consent Forms
Principal Information Sheet

Title of Project
Assessing Students’ Conceptual Understanding of Algebra – A Teaching Intervention supported by Digital Pencasts.

The Research Project
The research project involves research into determining whether students believe that their conceptual understanding of the topic has improved as a result of the taught intervention supported by the digital pencasts.

Participation Information
Adults education students may participate in the following research project should they wish to by:

- Completing exercises in class and allowing the results to be analysed for the final report.
- Completing a focus group discussing their opinions on the Livescribe Smartpen.

Students may participate in the following research project should they wish to by:

- Completing exercises in class and allowing the results to be analysed for the final report.
- Completing a focus group discussing their opinions on the Livescribe Smartpen.

There are no risks involved in this research project. All information gathered will remain confidential and will be used only for the purpose of this research project. No information about the participants or the school will be identified in the final report. All information will be stored safely with access only available to the investigators. The focus groups will be recorded using the Livescribe Smartpen. The participants are under no obligation to participate in this research project. Should you/they have any questions or do not understand something just ask the investigator to clarify the issue.
Contact Details:
Jason Gallagher, Dr Patrick Johnson,
Glenties, Dept. of Mathematics and Statistics,
Co. Donegal, University of Limerick,
13186272@studentmail.ul.ie 061 202208.

If you have concerns about this research project and wish to contact someone independent, you may contact:

Prof. Thomas Waldmann (Chairman),
Faculty of Science & Engineering Research Ethics Committee,
University of Limerick,
Tel: 061 – 202802.
Principal Consent Form

I give consent for you to approach pupils and adult education students to participate in the research project titled “Assessing Students’ Conceptual Understanding of Algebra – A Teaching Intervention supported by Digital Pencasts”.

I have read the Project Information Sheet explaining the purpose of the research project and understand that:

• The role of the school is voluntary.
• I may decide to withdraw the school’s participation at any time without penalty.
• Teachers and pupils will be invited to participate and that permission will be sought from them and their parent/guardians.
• All information obtained will be treated in strictest confidence.
• The teachers’ and students’ names will not be used and they will not be identifiable in any written reports about the research project.
• The school will not be identifiable in any written reports about the research project.
• Participants may withdraw from the research project at any time without penalty.
• A report of the findings will be made available to the school if desired.
• I may seek further information on the project from Dr Patrick Johnson on 061 202208 if necessary.

Principal: _______________________

Signature: _______________________

Date: ___ / ___ / _____
Student/Participant Information Sheet

Title of the research project:
Assessing Students’ Conceptual Understanding of Algebra – A Teaching Intervention supported by Digital Pencasts.

The research project:
The research project involves research into determining whether students believe that their conceptual understanding of the topic has improved as a result of the taught intervention supported by the digital pencasts.

What will I have to do as a Participant
You will complete a pre-intervention test on simultaneous equations to determine your base conceptual understanding of the topic. The investigator will teach you for 6 lessons. The digital pencasts will be shared with you via the investigator’s website. You will use the digital pencasts as an additional support. At the end of the 6 lessons, you will complete a post-test on simultaneous equations. If you agree to partake in this research project, you allow for your results to be analysed as part of the research. A focus group will be conducted with a random selection of students after tests are complete. This focus group will be asked a number of questions and their responses will be recorded for use. The questions will be answered anonymously and the information will be used as a means to determine whether students believe the taught intervention supported by the digital pencasts has enhanced students’ conceptual understanding of algebra. The focus groups will be recorded using the Livescribe Smartpen.

What are the risks
There will be no risks involved in the research project and you are under no obligation to participate in this research project.

Confidentiality
All information will be kept confidential. All information gathered will be stored in a secure location in the University of Limerick’s Department of Mathematics and Statistics and shall be available to the researcher and his supervisor only. No information about you, the subject, will be identified in the final report. Students will remain anonymous throughout the research project.

**Contact Details:**
Jason Gallagher, Dr Patrick Johnson,
Glenties, Dept. of Mathematics and Statistics,
Co. Donegal, University of Limerick,
13186272@studentmail.ul.ie 061 202208.

**What if I have more questions or do not understand something?**
If you have any concerns and wish to contact someone independent, you may contact the Chairman of the research ethics committee. Contact details given below.

Prof. Thomas Waldmann (Chairman),
Faculty of Science & Engineering Research Ethics Committee,
University of Limerick,
Limerick.
061-202802
Student/Participant Consent Form

Title of Research: Assessing Students’ Conceptual Understanding of Algebra – A Teaching Intervention supported by Digital Pencasts.

Please read the following questions and tick the appropriate yes or no box. Please sign the bottom of the page if you consent to participate in this study.

<table>
<thead>
<tr>
<th>Question</th>
<th>Participant</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have read and understand the participant information sheet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I understand what the project is about and what the results will be used for</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I am fully aware of all procedures and of any risks and benefits associated with the research project</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I know that my participation is voluntary and that I can withdraw from the project at any stage without giving reason</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I am aware that the results will be kept confidential</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Participant (Block Letters):
_______________________

Signature: ______________________

Date: ___ / ___ / ______

Witness (Block Letters):
_______________________

Signature: ______________________

Date: ___ / ___ / ______

Parent/Guardian (Block Letters):
_______________________

Signature: ______________________

Date: ___ / ___ / ______

Investigator (Block Letters):
_______________________

Signature: ______________________

Date: ___ / ___ / ______
Parent Information Sheet

Title of research project:

Assessing Students’ Conceptual Understanding of Algebra – A Teaching Intervention supported by Digital Pencasts.

The research project:

The research project involves research into determining whether students believe that their conceptual understanding of the topic has improved as a result of the taught intervention supported by the digital pencasts. From the research project we hope to implement the digital pencasts into the classroom so students can use them as a supportive tool to develop their understanding.

Procedures:

You will complete a pre intervention test on simultaneous equations to determine your base conceptual understanding of the topic. The investigator will teach you for 6 lessons. The digital pencasts will be shared with you via the investigators website. You will use the digital pencasts as an additional support. At the end of the 6 lessons, you will complete a post-test on simultaneous equations. If you agree to part take in this research project you allow for your results to be analysed as part of the research. A focus group will be conducted with a random selection of students after tests are complete. The focus groups will be recorded using the Livescribe Smartpen. This focus group will be asked a number of questions and their responses will be recorded for use. The questions will be answered anonymously and the information will be used as a means to determine whether students believe the taught intervention supported by the digital pencasts has enhanced students’ conceptual understanding of algebra.

Benefits:

I would hope that as a result of taking part in this research project that pupils’ will be able to access the Livescribe Smarptens, digital pencasts. The main benefit of these digital pencasts
will be in the deepening students’ conceptual understanding of the topic of Algebra. This then is of obvious benefit in all areas of mathematics as most areas are heavily reliant on Algebra.

**Risk:**

There will be no risks to parent or child involved in the research project but you are still under no obligation to participate in this study. By agreeing to partake in this research project you allow your child’s results to be used as part of the research.

**Voluntary Participation:**

Taking part in this research project is purely voluntary. There is no money or any other reward for taking part in this research project.

**Permission:**

Permission to do this research project has been sought from the University of Limerick Research Ethics Committee.

**Confidentiality:**

All information will be kept confidential and used only for the purpose of this research project. All information gathered will be stored in a secure location at all times and shall only be available to the researchers involved. No information about the subjects will be identified in the final report.

**Contact Details:**

Jason Gallagher, Glenties, Co. Donegal.
13186272@studentmail.ul.ie

Dr Patrick Johnson, Dept. of Mathematics and Statistics, University of Limerick,
061 202208.

**Further Information:**

If you have any concerns and wish to contact someone independent, you may contact the chairperson of the Faculty of Science & Engineering Research Ethics Committee. Contact details given below:

Prof. Thomas Waldmann, Faculty of Science & Engineering Research Ethics Committee, University of Limerick, Limerick.
061-202802
Parent Consent Form

Title Of Research: Assessing Students’ Conceptual Understanding of Algebra – A Teaching Intervention supported by Digital Pencasts.

Please read the following questions and tick the appropriate Yes or No box.

Please sign the bottom of the page if you consent to participate in this research project.

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have read and understood the information sheet.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I know my son/daughter’s participation is voluntary and he/she can withdraw from the project at any time.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I know and understand the relevant risks and benefits attached to participating in this research project.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I understand that my son/daughter’s anonymity will be ensured throughout this research project.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I agree to my son/daughter participating in this research project.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Signature</th>
<th>Block Capitals</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investigator</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>