Fault Adaptation by Reconfiguring Flight Controls using Control Allocation

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<td>BESA</td>
<td>Bisecting Edge Search Algorithm</td>
</tr>
<tr>
<td>c.g.</td>
<td>centre of gravity</td>
</tr>
<tr>
<td>CA</td>
<td>Control Allocation</td>
</tr>
<tr>
<td>CGI</td>
<td>Cascade Generalised Inverse</td>
</tr>
<tr>
<td>DA</td>
<td>Direct Allocation</td>
</tr>
<tr>
<td>DASMAT</td>
<td>Delft University Aircraft Simulation Modelling and Analysis Tool</td>
</tr>
<tr>
<td>EMMAE</td>
<td>Extended Multiple Model Adaptive Estimation</td>
</tr>
<tr>
<td>EoM</td>
<td>Equation of Motion</td>
</tr>
<tr>
<td>FAA</td>
<td>Federal Aviation Authority</td>
</tr>
<tr>
<td>FRL</td>
<td>Fuselage Reference Line</td>
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<td>FTLAB747</td>
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</tr>
<tr>
<td>GA</td>
<td>Genetic Algorithms</td>
</tr>
<tr>
<td>ICAO</td>
<td>International Civil Aviation Organisation</td>
</tr>
<tr>
<td>KKT</td>
<td>Karush – Kuhn – Tucker</td>
</tr>
<tr>
<td>LICQ</td>
<td>Linear Independence Constraint Qualification</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Programming</td>
</tr>
<tr>
<td>m.a.c.</td>
<td>mean aerodynamic chord</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>QP</td>
<td>Quadratic Programming</td>
</tr>
<tr>
<td>RPI</td>
<td>Redistributed Pseudo Inverse</td>
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<tr>
<td>w.d.p.</td>
<td>wing design plane</td>
</tr>
<tr>
<td>ZOH</td>
<td>Zero Order Hold</td>
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<tr>
<td>MOIC</td>
<td>Mixed Optimisation Scheme with Intercept Correction</td>
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Cascade Generalised Inverse (CGI)

\( B^{inv} \) Pseudo inverse of control effectiveness matrix B

\( \hat{B} \) Partition matrix corresponding to saturated input

\( \bar{B}^{inv} \) Pseudo inverse of control effectiveness matrix corresponding to unsaturated input

\( v \) Desired virtual demand

\( \bar{v} \) Remaining virtual demand

Direct Allocation (DA)

\( v \) Virtual demand

\( v_{max} \) Maximum demand in the direction of \( v \)

\( u_f \) Feasible input in the direction of \( v \) producing maximum magnitude of \( v_{max} \)

\( B \) Control effectiveness matrix

\( S_f \) Scaling factor

Daisy Chain

\( v \) Virtual demand

\( B1 \) First of the two partition of control effectiveness matrix in 2-D problem
R1 \hspace{1cm} \text{Right inverse of } B1

B2 \hspace{1cm} \text{Second of the two partition of control effectiveness matrix in 2-D problem}

R2 \hspace{1cm} \text{Right inverse of } B2

x^B, y^B, z^B \hspace{1cm} \text{Body axis (FRL)}

x^s, y^s, z^s \hspace{1cm} \text{Relative stability axis}

x^w, y^w, z^w \hspace{1cm} \text{Wind axis}

x^e, y^e, z^e \hspace{1cm} \text{Earth fixed frame}

\alpha \hspace{1cm} \text{Angle of attack with respect to relative wind (rad)}

\beta \hspace{1cm} \text{Side slip angle with respect to relative wind (rad)}

V^T \hspace{1cm} \text{True airspeed with respect to relative wind (m/s)}

T^{B \to s} \hspace{1cm} \text{Rotation matrix from body fixed axis to stability axis}

T^{s \to w} \hspace{1cm} \text{Rotation matrix from stability axis to wind axis}

T^{s \to w} \hspace{1cm} \text{Rotation matrix from body axis to wind axis}

I \hspace{1cm} \text{Inertia matrix (kg/m}^2\text{)}

V \hspace{1cm} \text{Velocity vector (m/s)}

u \hspace{1cm} \text{Longitudinal velocity (m/s)}

v \hspace{1cm} \text{Lateral velocity (m/s)}

w \hspace{1cm} \text{Normal velocity (m/s)}
\( \mathbf{v}^w \) Velocity vector in wind axis (m/s)

\( \mathbf{\omega} \) Angular velocity vector (rad/s)

\( p \) Roll rate (rad/s)

\( q \) Pitch rate (rad/s)

\( r \) Yaw rate (rad/s)

\( \Phi \) Euler angles (rad)

\( \phi \) Roll angle (rad)

\( \theta \) Pitch angle (rad)

\( \psi \) Yaw angle (rad)

\( \mathbf{p}^e \) Aircraft position vector (m)

\( h^e \) Altitude (m)

\( x^e \) Position north (m)

\( y^e \) Position east (m)

\( \mathbf{u}_{\text{aero}} \) Vector of control surfaces inputs

\( \mathbf{u}_{\text{prop}} \) Input vector to propulsion system

\( \mathbf{F}_{\text{aero}} \) Vector of aerodynamic forces in body axes

\( \mathbf{M}_{\text{aero}} \) Vector of aerodynamic moments in body fixed frame

\( \mathbf{F}_{\text{prop}} \) Vector of propulsion forces in body fixed frame
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{prop}^B$</td>
<td>Vector of propulsion moments in body fixed frame</td>
</tr>
<tr>
<td>$F_{grav}^B$</td>
<td>Vector of gravity forces in body fixed frame</td>
</tr>
<tr>
<td>$F_{tot}^B$</td>
<td>Vector of total forces in body fixed frame</td>
</tr>
<tr>
<td>$M_{tot}^B$</td>
<td>Vector of total moments in body fixed frame</td>
</tr>
<tr>
<td>$\mathbf{x}$</td>
<td>Aircraft state variables vector in body fixed frame</td>
</tr>
<tr>
<td>$M$</td>
<td>Pitching moment (Nm)</td>
</tr>
<tr>
<td>$L$</td>
<td>Rolling moment (Nm)</td>
</tr>
<tr>
<td>$N$</td>
<td>Yawing moment (Nm)</td>
</tr>
<tr>
<td>$C_m$</td>
<td>Dimensionless pitching moment</td>
</tr>
<tr>
<td>$C_l$</td>
<td>Dimensionless rolling moment</td>
</tr>
<tr>
<td>$C_n$</td>
<td>Dimensionless yawing moment</td>
</tr>
<tr>
<td>$\bar{\bar{q}}$</td>
<td>Dynamic pressure (N/m$^2$)</td>
</tr>
<tr>
<td>$S$</td>
<td>Wing planform area (m$^2$)</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>Mean aerodynamic chord (m)</td>
</tr>
<tr>
<td>$b$</td>
<td>Wing span (m)</td>
</tr>
<tr>
<td>$\alpha_{wdp}$</td>
<td>Angle of attack with respect to w. d. p</td>
</tr>
<tr>
<td>$\alpha_{FRL}$</td>
<td>Angle of attack with respect to FRL</td>
</tr>
<tr>
<td>$C_{mbasic}$</td>
<td>Basic pitching coefficient for the rigid aircraft at the zero</td>
</tr>
</tbody>
</table>

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stabilizer angle

\[(\Delta C_{m0.25})_{\alpha_{w.d.p}=0}\]  Change in pitching moment coefficient at zero \(\alpha_{w.d.p}\).

\[C_L(c.g.-0.25)\]  change in pitching moment due to variation in the c.g. with respect to the 25% m.a.e.

\[\Delta \left(\frac{dC_{m0.25}}{d\alpha}\right)\alpha_{w.d.p}\]  change in basic pitch coefficient due to wing angle of attack, \(\alpha_{w.d.p}\).

\[\frac{dC_{m0.25}}{d\dot{\alpha}}\]  change in basic pitch coefficient due to rate of change of angle of attack, \(\dot{\alpha}\).

\[\frac{dC_{m0.25}}{dq}\]  change in basic pitch coefficient due to pitch rate, \(q\).

\[\frac{dC_{m0.25}}{dnz}\]  change in basic pitch coefficient due to normal load factor \(n_z\).

\(n_z\)  Load factor, normal acceleration (g)

\(K_\alpha\)  Elevators and stabilizer effectiveness factor

\[\frac{dC_{m0.25}}{d\delta_{th}}\]  Pitching moment stability derivatives for stabilizer \(\delta_{th f-r.1}^{th}\).

\[\frac{dC_{m0.25}}{d\delta_{ei}}\]  Stability derivatives for inboard elevator \(\delta_{ei}\).

\[\frac{dC_{m0.25}}{d\delta_{eo}}\]  Stability derivatives for outboard elevator \(\delta_{eo}\).

\(\Delta C_{m0.25}\)  Change in pitching moment due to spoilers

\(\Delta C_{m0.25}\)  Change in pitching moment due to inboard ailerons

\(\Delta C_{m0.25}\)  Change in pitching moment due to outboard aileron
ΔC_{\text{m0.25landing gears}} \quad \text{Change in pitching moment due to landing gears}

ΔC_{\text{m0.25ground effects}} \quad \text{Change in pitching moment due to ground effect}

ΔC_{\text{m0.25sideslip}} \quad \text{Change in pitching moment due to side slip}

ΔC_{\text{m0.25rudder}} \quad \text{Change in pitching moment due to rudders}

[ΔC_{\text{m0.25flap failure}}] \quad \text{Change in pitching moment due to flap failure}

dC_l \over d\beta \quad \text{Rolling moment stability derivatives for side slip } \beta

dC_l \over dp \ 2V_T \quad \text{Rolling moment stability derivatives for stability axis roll rate}

dC_l \over dr \ 2V_T \quad \text{Rolling moment stability derivatives for stability axis yaw rate}

ΔC_{C_{\text{inboard ailerons}}} \quad \text{Change in rolling moment due to inboard ailerons}

ΔC_{C_{\text{outboard ailerons}}} \quad \text{Change in rolling moment due to outboard ailerons}

K_{\delta_{\text{ai}}} \quad \text{Inboard ailerons effectiveness factor}

K_{\delta_{\text{ao}}} \quad \text{Outboard ailerons effectiveness factor}

(ΔC_{C_{\text{ai}}}_20 \quad \text{Change in rolling moment with change in } \alpha_{w.d.p} \text{ with inboard aileron deflected at 20 degrees upward or downward}

(ΔC_{C_{\text{ai}}}_20 \quad \text{Change in rolling moment with outboard aileron deflected at 25 degrees upward or other aileron 15 degrees downward}

\frac{ΔC_{C_{\text{ai}}}}{M} \quad \frac{\text{Rolling moment effects due to change in Mach number by inboard ailerons}}{M=0}}
\[ \frac{(\Delta C_{l_{ao}})_{M}}{(\Delta C_{l_{ao}})_{M=0}} \]
Rolling moment effects due to change in Mach number by outboard ailerons

\[ \frac{R_E}{R_R} \frac{L_{l_{ai}}}{} \]
Rolling moment effects due to aeroelastic effects by inboard ailerons

\[ \frac{R_E}{R_R} \frac{L_{l_{ao}}}{} \]
Rolling moment effects due to aeroelastic effects by outboard ailerons

\[ V_e \]
Equivalent airspeed (m/s)

\[ F_{i GE} \]
Ground effects

\[ \frac{d C_{n_{nl}}}{d \beta} \]
Yawing moment stability derivative for side slip \( \beta \)

\[ \frac{d C_{n_{\beta \beta b}}}{d \beta \ 2V_T} \]
Yawing moment stability derivatives for side slip rate \( \dot{\beta} \)

\[ \frac{d C_{n_{\beta p s b}}}{dp \ 2V_T} \]
Yawing moment stability derivatives for stability axis roll rate \( p \)

\[ \frac{d C_{n_{r s b}}}{dr \ 2V_T} \]
Yawing moment stability derivatives for stability axis yaw rate \( r_s \)

\[ \Delta C_{n_{spoilers}} \]
Yawing moment due to spoilers

\[ \Delta C_{n_{inboard ailerons}} \]
Yawing moment due inboard ailerons

\[ \Delta C_{n_{outboard ailerons}} \]
Yawing moment due to outboard ailerons

\[ \Delta C_{n_{rudders}} \]
Yawing moment due to rudders

\[ \Delta C_{n_{flap failure}} \]
Yawing moment due to flap failure

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\[ \Delta C_{\text{leading edge flap failure}} \] Yawing moment due to leading edge flap failure

\[ F^e_{\text{grav}} \] Force of gravity vector in earth fixed frame (N)

\[ F^B_{\text{grav}} \] Force of gravity vector in body fixed frame (N)

\[ F^B_{\text{prop}} \] Engine thrust vector in body fixed frame (N)

\[ M^B_{\text{prop}} \] Engines moment vector in body fixed frame (Nm)

\[ T_{n1}, T_{n2}, T_{n3} \text{ and } T_{n4} \] Thrust by engine 1, 2, 3 and 4 respectively (N)

\[ l_p \] Rolling moment by engine thrust (Nm)

\[ m_p \] Pitching moment by engine thrust (Nm)

\[ n_p \] Yawing moment by engine thrust (Nm)

\[ Z_{E_1}, Y_{E_1}, Y_{E_o} \text{ and } Z_{E_o} \] Effective engine inboard and outboard moment arms respectively (m)

\[ m \] Mass of aircraft (Kg)

\[ B_c \] Control effectiveness matrix

\[ u(t) \] Control vector

\[ v(t) \] Virtual control demand

\[ u_{\text{min}} \] Control vector minimum position limits (deg)

\[ u_{\text{max}} \] Control vector maximum position limits (deg)

\[ \dot{e}_{\text{min}} \] Control vector with minimum rate limits (deg/s)
\( \dot{\mathbf{e}}_{\text{max}} \) Control vector with maximum rate limits (deg/s)

\( T \) Sampling time (s)

\( \mathbb{I} \) Set of inequality constraints

\( \mathbb{E} \) Set of equality constraints

\( \mathbf{d} \) Feasible descent direction vector

\( \nabla \) Gradient

\( L(\mathbf{u}, \lambda) \) Lagrangian

\( \lambda \) Lagrange multiplier

\( \mathbb{W} \) Working set

\( \mathbb{A} \) Active set

\( \delta_{\text{aor}} \) Right outboard aileron (deg)

\( \delta_{\text{air}} \) Right inboard aileron (deg)

\( \delta_{\text{aol}} \) Left outboard aileron (deg)

\( \delta_{\text{ail}} \) Left inboard aileron (deg)

\( \delta_{\text{eor}} \) right outboard elevator (deg)

\( \delta_{\text{eir}} \) right inboard elevator (deg)

\( \delta_{\text{eol}} \) left outboard elevator (deg)

\( \delta_{\text{eil}} \) left inboard elevator (deg)
\( \delta_{th} \) stabilizer (deg)

\( \delta_{ur} \) upper rudder (deg)

\( \delta_{dr} \) down rudder (deg)

\( \dot{p} \) roll angular acceleration in body x-axis (rad/s²)

\( \dot{q} \) pitch angular acceleration in body x-axis (rad/s²)

\( \dot{r} \) yaw angular acceleration in body x-axis (rad/s²)
1 Introduction

1.1 Background

Japan Airlines Flight 123 crashed into the ridge of Mount Takamagahara in Ueno, Gunma Prefecture, 100 kilometres from Tokyo, on Monday August 12, 1985 (National 2006). The accident situation is shown in Figure 1.1. The vertical stabilizer had separated from the aircraft during flight, which cut off the four available hydraulic systems. The only control available was the engine thrust. The pilot had tried to stabilize the aircraft by controlling the engine thrust but all in vain. Due to the loss of pressure and oxygen in the cockpit, the decision-making capability of the pilots was highly affected. At the height of 13500 feet the pilot reported that the aircraft was uncontrollable. All 15 crew members and 505 out of 509 passengers died, resulting in a total of 520 deaths. It was one of the deadliest single-aircraft accidents. This accident inspired the concept of an automated system for the reconfiguration of an aircraft flight control in the event of a disaster. With the loss of oxygen the decision-making ability of pilots is significantly affected. In a case such as JAL 123 stabilizing the aircraft from unstable phugoid using engine control alone, the pilot actions could be augmented with an automated reconfigurable flight control.

Figure 1.1: JAL 123 air crash scenario
In classical aircraft control theory, for the three degrees of freedom (i.e. roll, pitch and yaw accelerations) there are three control surfaces each assigned to control an angular acceleration. Ailerons are primarily used to control rolling acceleration; elevators are for pitching accelerations and the rudder is used for yaw acceleration. There is some redundancy built in this classical design in terms of coupling of lateral and directional dynamics (roll and yaw axes are coupled); this coupling will give a partial control to either roll or yaw accelerations. In longitudinal dynamics there is no redundancy in the classical design. In the case of failure in either of lateral or directional axis there might be a chance to recover aircraft control strictly in the lateral and directional sense only. On the other hand, if there is a failure in the longitudinal axis, the classical aircraft design does not support recovery of the aircraft in the longitudinal axis until there is redundancy provided in new designs. A major focus of redundant system design is to facilitate the exploitation of the redundant systems and safely recover the aircraft.

This notion of redundancy has directed the new aircraft designs to have more control surfaces than control variables. Now the next question arises: How to allocate a demand to the control surfaces optimally and feasibly, fulfilling certain performance guideline defined by the designer and the requirements for handling qualities specified by some regulatory authority? This question can be formulated mathematically to form a constrained optimisation problem. To solve this problem it is required that the solution should converge to an optimal and a feasible point in a computationally efficient way. This means that the solution to the optimisation problem should be found within the sampling time of the aircraft (discretised control loop). Mathematically, control allocation can be defined by the underdetermined system of equations (i.e. number of equation are less than control variables). If this underdetermined system of equations is linear and well conditioned, then the solution could be unique. If these underdetermined systems of equations are nonlinear, then the solution could get stuck in some local minimum. In this case there are some convergence issues that need to be addressed.

To the author's knowledge, the most notable contributors in this area of control allocation in aeronautics are Doman, D., Oppenheimer, M., Bodson, M., Durham, W.C. and Härkegård, O. David Doman and his colleagues at Air Force Research Laboratory (AFRL). Their interests have ranged from static linear control allocation to dynamic control allocation and nonlinear control
allocation. The idea of piecewise linear approximation in control allocation proposed by Bolender and Doman (2005) could potentially solve a very important problem of non-monotonic behaviour of the control vs moment curve in control allocation. Michael Oppenheimer proposed the idea of compensating the interaction of control allocation and actuator dynamics. This approach has been further pursued by the author by using evolutionary computing technique (i.e. genetic algorithms) to tune the compensator when the actuator dynamics are unknown. Wayne C. Durham, to the author’s knowledge, is the pioneer of direct allocation in control allocation design (Durham 1993). Härkegård (2003) utilised the idea of control sufficiency and control redundancy branches proposed by Buffington (1999) and posed the problem elegantly in terms of quadratic programming and solved those programs using the active set method. The author has utilised this technique and exploited the modular design approach to design a multivariable proportional integral control law and control allocation for fighter aircraft using the active set method Härkegård (2003).

This research work expands upon the work previously reported by these authors. Some of these ideas are further developed and implemented (in simulation) for the first time on civil nonlinear aircraft model in this research programme. One of the benefits of this multi-branch design is that it is well suited to the application of system identification. In the event of a failure the control law is not changed but reconfiguration is done by redistributing the demand to the control surfaces using control allocation. This gives a metric of performance for control allocation with the same control law in closed loop.

The author has also given a means of quantification of open loop analysis of the uncertainty of the static control effectiveness matrix when the flight conditions are changed. It has been presented in chapter 5 (section 5.5) that the most notable uncertainty comes from lateral and directional dynamics, but there is negligible change in longitudinal dynamics by changing the flight condition in this specific transport B747-200. The reasons for using this aircraft are that it has many redundant control systems (which make it suitable for control allocation design) and the data for this aircraft is available in public domain.
System identification is central to estimating control derivatives which are then used in control allocation. The most worthy contribution is again by Doman and Ngo (2001) in which multi-branch linear programming control allocation is used with the desired vector selected in the null space of the control effectiveness matrix $\mathbf{B}_c$. The author has utilised this approach in multi-branch quadratic programming with excitation of the desired vector in the null space of $\mathbf{B}_c$ and produced satisfactory results. A fast update of $\mathbf{B}_c$ has been implemented by the author to avoid using the slow `linmod` command from Matlab.

Exploiting flight mechanics to get redundancies for some control axis is an interesting idea. The author has proposed integrating the control allocation and fuel management system by controlling the centre of gravity of the aircraft by the movement of fuel. This idea has been implemented in the analysis of a damaged aircraft based on the air crash of, El Al Flight 1862, a Boeing 747 cargo plane of the Israeli airline El Al, crashed into the Groeneveen and Klein-Kruitberg flats in the Bijlmermeer (colloquially "Bijlmer") neighbourhood (part of Amsterdam Zuidoost) of Amsterdam, Netherlands. It has been noted that the selection of the desired vector has an important effect on the performance of control allocation and has removed chattering of the control surfaces in the longitudinal axis, which is an important requirement for the flight worthiness and handling qualities requirements of an aircraft. Another important issue is the design requirement of selecting some control surfaces prior to others. The author has illustrated this control surfaces prioritisation and its effect on the solution in chapter 5 (section 5.9).

**1.2 Motivation**

In 1959 the world’s air carriers averaged 100,000 jet-flying hours per hull loss. Today, they average nearly 800,000 flying hours per hull loss (Boeing 2000). Notwithstanding this progress in airline safety, there is concern regarding forecast hull losses. The current accident rate, which is a little under 2 per million departures, is likely to become unacceptable due to the increase in commercial air traffic. On average over the next 20 years, passenger travel will grow at 5.0 percent and cargo at 5.8 percent (Boeing 2008).
Several agencies, including the Federal Aviation Authority (FAA), National Aeronautics and Space Administration (NASA) and the International Civil Aviation Organisation (ICAO) are pushing for a significant reduction in airplane accident rate (Bajpai 2001). For example, the NASA Aviation Safety program goal is to “develop and demonstrate technologies that contribute to a reduction in the aviation fatal accident rate by 90% by 2018” (NASA 2008).

A survey of airplane accidents from Kebabjian (2008), in which the probable cause was determined, is given in Table 1.1.
### Table 1.1: Probable cause of 1,843 airplane accidents: 1950–2006

<table>
<thead>
<tr>
<th>Probable cause</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pilot error</td>
<td>53%</td>
</tr>
<tr>
<td>Mechanical failure</td>
<td>21%</td>
</tr>
<tr>
<td>Weather</td>
<td>11%</td>
</tr>
<tr>
<td>Sabotage</td>
<td>7%</td>
</tr>
<tr>
<td>Other human error (e.g. air traffic controller)</td>
<td>7%</td>
</tr>
<tr>
<td>Other cause</td>
<td>1%</td>
</tr>
</tbody>
</table>

Note: Excludes military, private and charter aircraft.

Many of the accidents can be directly linked to failures of the control system or to malfunctions (or damage) that rendered the controls inadequate. In the past 25 years, many aircraft types (e.g. L-1011, DC-10, B-52, and C-5A) have experienced major flight control system failures (Burcham et al. 1997). Two examples of fatal accidents, in which control system failure or inadequacy under abnormal conditions were seen to occur, are given below:

1. A MD-83 of Alaska Airlines crashed off the coast of California on 31 January 2000 killing all 88 people onboard (ntsb undated)


It is evident that the problems are not restricted to any particular manufacturer, carrier or region of the world. The accidents also indicate a need for designing control systems that actively address failures. The motivation to work in this area is based on reducing fatal accidents. The main focus of this research is on flight safety.
Reconfigurable control systems are control systems that are characterised by the ability to perform in the presence of drastic changes in the system dynamics due to, for example, sudden component failures or changes in the operating conditions by reconfiguring some aspect of the control system. A reconfigurable control system’s task is twofold: firstly, it needs to guarantee safe performance (stability); and secondly, it needs to recover optimum control performance under impairments. In an emergency situation, the first objective of the flight control system is to maintain the aircraft in a stable, flyable state and then try to recover as much of the capability of the un-failed system as possible.

1.3 Control Allocation for Aircraft: Graphical Illustration

Control allocation is merely a mapping (i.e. linear or nonlinear) from total virtual demands in terms of body angular accelerations to the control position setting subject to rate and position constraints. An illustration of control allocation is given in Figure 1.3.

![Control allocation for aircraft: graphical illustration](image-url)
In the flight control system of FTLAB747, there is no actuator redundancy utilisation and similar control surfaces are considered to be the same (e.g. the four elevators are considered to be one surface). In the design of a control allocation scheme, the redundancy of all actuators should be exploited. The model was thus modified to mimic the actual aircraft control redundancies.

### 1.4 Objectives

The objectives of this research work are as follows:

1. To design and implement a control law and control allocation for the nonlinear civil aircraft model;
2. To use control redundancies to compensate for a failure without changing the baseline control law;
3. To update control derivatives for control allocation;
4. To identify control derivatives by exploiting control redundancies;
5. To change centre of gravity by moving fuel to have an additional pitch attitude control of the aircraft (integrating fuel management system and flight control allocation blocks);
6. To study the effects of the desired control vector and control surfaces prioritisation in control allocation;
7. To design and tune a compensator to mitigate the effects of control allocation and control surface dynamics using genetic algorithms.
1.5 Thesis Structure

The structure of the thesis is illustrated in the block diagram shown in Figure 1.4.

Chapter 2 introduces the literature in the area of control allocation and classifies control allocation into sixteen categories. Each category is then explained in the subsequent sections. In addition to this, any problem formulation that is the most appropriate for a specific category is also described. The novelty of the work presented in this thesis is based on research questions resulting from the detailed analysis and conclusions drawn from the review of the previous research available in the public domain.

Chapter 3 deals with the nonlinear dynamic model of the B747-200. The mathematical framework is introduced, including description of coordinate frames, which are the basis for any transformation. Also, aircraft variables (such as state variables, total forces and moments) and
aircraft properties (such as mass, inertia, and sign conventions) are described to explain the characteristics of an aircraft. A nonlinear dynamic model of the aircraft is introduced, with some explanations of nonlinear stability and control derivatives to show the complexity of the model.

Chapter 4 deals with control allocation. The general control allocation problem is defined as a constrained optimisation problem. Control deficiency and control sufficiency branches are defined and formulated. The active set method is introduced through a set of examples, providing insight into the geometry of the problem. The application of the active set method to solve the control allocation problem for the healthy aircraft is presented at the end of the chapter.

Chapter 5 is focused on fault-tolerant control allocation. The chapter starts with a short introduction to the basic terminology and concepts of fault-tolerant control systems, including fault categorisation. The effects of the static control effectiveness matrix in control allocation are investigated. The weakness of the linear static control effectiveness matrix for control allocation is identified, providing a framework for dynamic update of the control effectiveness matrix. The open loop analysis of control allocation with static and dynamic updated control effectiveness matrix is performed. A test to check whether the aircraft can be retrimmed in the case of a failure is described and closed loop analysis of a damaged aircraft is given. The use of centre of gravity as a redundant pitch control is described. The effects of weighting on control allocation are discussed. Finally, a methodology to identify control stability and control derivatives of the aircraft is introduced at the end of the chapter.

Chapter 6 describes the method of using genetic algorithms for the design and tuning of a compensator to mitigate the effects of control allocation and actuator dynamics interaction. The interaction of first order and second order actuator dynamics and control allocation is analysed. The structure of compensators to compensate for these interactions is described, including the genetic algorithm to tune the compensator parameters.

Chapter 7 reviews the thesis and summarises the objectives and contributions. Finally, suggestions for further work are given at the end of the chapter.
2 Literature Review

2.1 Introduction

The literature search is a very significant step in the research process. According to Reed (1998) the basic stages in a typical research project are:

i) To identify the topic of interest,
ii) To perform a literature review,
iii) To generate related questions,
iv) To state the unsolved problem or hypothesis,
v) To find or develop a solution,
vi) To document the results.

Control allocation is useful for the control of over-actuated systems, and deals with distributing the total control demand among the individual actuators. Using control allocation, the actuator selection task is separated from the regulation task in the control design. To introduce the ideas behind control allocation, consider the following system (Härkegård 2003)

\[ \dot{x} = u_1 + u_2 \]  

(2.1)

where

\( x \) is a scalar state variable, and \( u_1 \) and \( u_2 \) are control inputs. Variable \( x \) can be thought of as the velocity of a unit mass object affected by a net force \( v = u_1 + u_2 \), generated by two actuators. Assume that the net force \( v = 1 \) is to be produced to accelerate the object. There are several ways to achieve this. One way can be to utilize the first actuator individually and select \( u_1 = 1 \) and \( u_2 = 0 \) or to gang the actuators together and use \( u_1 = u_2 = 0.5 \). It is even possible to select \( u_1 = -11 \) and \( u_2 = 12 \) although this might not be very practical. Which combination to pick is essentially the problem of control allocation.
2.2 Applications

2.2.1 Aerospace applications
Most of the research in the area of control allocation is focused on aerospace applications. As mentioned in Chapter 1 (section 1.1), the most notable contributors in this area of control allocation in aeronautics are Doman, D., Oppenheimer, M., Bodson, M., Durham, W.C. and Härkegård, O. The detailed references are given in the sections 2.3 and 2.4.

2.2.2 Marine applications
For surface marine vessels, most notable contributors in the area of control allocation are Fossen, T.I. and Sorensen A.J. from Norwegian University of Science and Technology. The detailed references are provided in sections 2.3 and 2.4. Ship control has been an area of research for almost a century. There are wide varieties of underwater marine vessels like remotely operated vehicle (ROV) and semi-submersible rigs. An efficient algorithm for control allocation of thrusters-propelled ROVs, able to work in real time and able to compensate for faulty thrusters has been developed by Omerdic from Mobile & Marine Robotics Research Centre, UL (Omerdic 2004). The commercial control system of marine vessels usually consists of a control law and control allocation to distribute the total demand given by the control law to available actuators.

2.2.3 Automotive applications
The most notable contributors in the area of control allocation in automotive applications are Tondel, P. and Johansen, T.A. Since the 1980s, a variety of active chassis vehicle control approaches have been explored. In particular, vehicle dynamics stability control and electronic stability program systems have become a very vital and thorough research area. The main objectives of vehicle dynamics control include enhancements in the following:

- vehicle safety,
- steerability,
- manoeuvrability,
- passenger comfort,
• reduced driver workload, especially in poor driving situations.

Many different vehicle chassis control sub-systems have been developed based on advanced electronic technology, such as traction control, active steering control, direct yaw-moment control, anti-lock braking, active roll control, and so on. There are some kind of actuation redundancies in these sub-systems, which are needed to exploit the performance of the vehicle (e.g. fault tolerance, reliability etc.). The main task of control allocation is to achieve a control objective by having optimum settings of these sub-systems.

2.3 Control Allocation Issues Framework

2.3.1 Classification

Almost 80 papers that address the topic of control allocation have been reviewed in this section. In this literature review control allocation is classified into a framework with sixteen categories. Table 2.1 summarises the reviewed papers against these categories. The categories are as follows:

A. Control allocation with monotonic nonlinearities,

B. Control allocation with automatic update of control effectiveness matrix,

C. Control allocation with non-monotonic nonlinearities,

D. Control allocation with linear objective function,

E. Control allocation with multi-objective optimisation,

F. Control allocation with more than three virtual demands allocation,

G. Control allocation with constrained optimisation (i.e. rate and position constraints),

H. Control allocation with actuator dynamics,

I. Control allocation in case of fault and failure in the system,
J. Is control allocation flight worthy (e.g. smoothly varying actuator commands),
K. Control allocation as a non-convex problem,
L. To retrofit control allocation in the existing system,
M. Real time control allocation and pilot interaction,
N. Control allocation to attain maximum attainable control power,
O. Control allocation with system identification,
P. Control allocation and feedback control law interactions.
Table 2.1: Cross-reference table showing relations between papers and categories covered by them

<table>
<thead>
<tr>
<th>No</th>
<th>Paper's Reference</th>
<th>Classification of Control Allocation Defined in Section 2.3.1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
</tr>
<tr>
<td>1</td>
<td>(Wang et al. 2007)</td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td>(Ning et al. 2006)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(Liao et al. 2007)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(Johansen 2004a)</td>
<td>X</td>
</tr>
<tr>
<td>5</td>
<td>(Alwi and Edwards 2006)</td>
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2.3.2 A: Control allocation with monotonic nonlinearities

The literature, in general, views the control distribution problem as formulated under the assumption that the control effectors are linear functions of virtual control demand (i.e. desired angular and linear accelerations), and any non-conformity to this assumption is treated by the robustness of the controller. Failure to conform to this assumption would be the result of an effector failure and the healthy effectors moving in the highly nonlinear region of the effector to virtual demand curve, and then the linear assumption is not sufficient to compensate for the failure. If the virtual control demands are assumed to be nonlinear functions of effectors, this will be referred to as nonlinear control allocation.

An optimizing nonlinear control allocation algorithm was derived using a control Lyapunov design approach, this algorithm is in the form of a dynamic update law which together with the stable model reference control law, guarantees the stability of the closed loop nonlinear system (Liao et al. 2007; Johansen 2004a). The sliding mode nonlinear control allocation is used to determine the nonlinear (Alwi and Edwards 2006; Alwi and Edwards 2007) gain required to
maintain sliding (Alwi and Edwards 2006). In this methodology, the faults and even certain total effector failures can be compensated for directly, without reconfiguring the controller. A reconfigurable sliding mode control method was developed for aircraft, which automatically adjusts after aircraft damage and prevents actuators from deflection and rate saturation (Shtessel et al. 2002). This technique does not require online damage estimation; hence it improves robustness to noise and disturbances. A dynamic control allocation based on $H_{\infty}$ design was used to close the loop around the existing control allocation to improve the performance of the allocation module (Hodel and Callahan 2002). In (Fossen and Johansen 2006) non-convex optimisation is solved by using the control Lyapunov method proposed by Johansen (2004a) and by using quadratic programming and locally approximating the non-convex problem with a convex quadratic problem. The control allocation problem, defined as a nonlinear optimisation problem to which an explicit piecewise linear approximate solution function is computed offline, is proposed by Johansen et al. (2005). The boundary of attainable moments set for a class of multiple nonlinear effectors and for clipping unattainable moments was proposed in (Doman and Sparks 2002). A control allocation algorithm with a stability proof using a Lyapunov function was presented in (Tjoennas and Johansen 2006). The nonlinear allocation problem was posed as a piecewise linear program and solved exactly in (Bolender and Doman 2005; Bolender and Doman 2004b). Nonlinear control allocation is compared to other techniques, on the basis of performance metric. This was illustrated in (Poonamallee et al. 2004). An optimal nonlinear allocation method that ensures bounded internal dynamics as well as input constraints was proposed in (Benosman et al. 2007). A nonlinear control allocation was solved in each sample as a convex quadratic programming approximating the nonlinear program. In this method singularity avoidance was stated as an additional objective (Johansen et al. 2004b). Online identification of control derivatives to update the control effectiveness matrix to compensate for the nonlinear relationships between effectors and moments was presented in (Doman and Ngo 2001). A force allocation algorithm was proposed in (Lindegaard and Fossen 2003), which poses a non-convex optimisation problem. A technique for improving the performance of linear allocation by modifying the input to the allocation module and using an additional intercept term, means utilizing an affine relationship which has shown an improvement in performance. (Durham 1994a).
2.3.3 B: Control allocation with automatic update of control effectiveness matrix

Identifying control derivatives to update the control effectiveness matrix online is an important issue, but it has been addressed by few researchers. In (Page and Steinberg 1999) this parameter update law was used to configure the control effectiveness matrix. Online computation of control derivatives (i.e. slope) and intercept were done in the mixed optimisation scheme with intercept correction (MOIC) (Durham 1994a). The MOIC methodology was used in a comparative study with other methods of control allocation. The Online system identification to estimate the current control effectiveness matrix to update the control allocation module was shown in (Doman and Ngo 2001). The system is excited in the null space and decorrelates the effectors without degrading the system response. The adaptive law for estimation of the maximal tyre road friction parameter in the control allocation algorithm for the yaw stabilisation was presented in (Tjoennas and Johansen 2006).

2.3.4 C: Control allocation with non-monotonic nonlinearities

Non-monotonic nonlinearities are very important in control allocation research, as most of the studies were under the assumption that the control surfaces to moment curves are monotonically increasing or decreasing. The presence of non-monotonic nonlinearities could lead to a non-unique solution of the control allocation problem. The nonlinearities are normally encountered in the event of failure, but also when the actuators are operating near the bounds. This issue was addressed by quite a few researchers in the area of control allocation. The solution to the problem of monotonic control to the moment linear relationship was obtained by a parabolic curve fit to the data for the yawing moment generated by the right elevon deflection of the vehicle. As for the linear relationship, it was shown that the nonlinearity was introduced by sign changes of the slopes, and their estimation is inaccurate most of the time, as stated by (Bolender and Doman 2004a; Doman and Sparks 2002). In control allocation for ships, the non-convexity of the inequality constraint for the propeller and rudder coupling introduce discontinuous behaviour if an optimal solution is sought for all times. This problem was solved in (Lindegaard and Fossen 2003). The monotonic assumption introduces modelling uncertainty in the control allocation algorithm which will give incorrect results. In nonlinear allocation non-uniqueness in
the solution was accounted for by this non-monotonic behaviour (Bolender and Doman 2005). Two methods for solving this problem were introduced in (Doman and Oppenheimer 2002). The first method is to reduce the saturation limit before hitting this behaviour reversal, which will eventually have undesirable consequences in terms of loss of redundant control power. The second method is the clipping logic introduced by the author. In multi-parametric constrained control allocation for a ship, the monotonic linear relationship of an actuator to the generalised forces was assumed (Johansen et al. 2005). This non-monotonic nonlinearity was given in an example of F18 yawing moment coefficient which gives a V-shaped relationship with the right aileron with leading edge flaps at 0,-3, 15, 35 degrees (Bordignon 2004). In (Fossen and Johansen 2006) it was mentioned that the ship control forces due to propeller, rudder and fins can be given as a specific monotonic nonlinearity which satisfies the linear relationship.

2.3.5 D: Control allocation with linear objective function

The majority of the literature written has been under the assumption that the control surfaces are a linear function of commanded variables. This assumption is valid under the referenced operating conditions. However, as the system moves away from these conditions, the control effectiveness matrix needs to be updated. Otherwise, there will be a reduction in the overall robustness of the whole system due to the modelling error. This linear assumption worked well in model predictive control technology, where model requirements are not as stringent. If this assumption is valid in using reconfigurable control, then in the event of failure the remaining controls work in the nonlinear region, and the performance of a control allocation module is tremendously reduced. This degradation of performance should be dealt with by the robustness of the control law. By using the linear relationship the fast convergence to a unique solution is guaranteed under a properly formulated problem. Usually the sampling time of 0.01s (i.e. sampling frequency of 100Hz) is the upper limit for solving this optimisation. Otherwise, the system becomes an open loop (i.e. optimisation is not solved within this sampling time). As power is increasing with computer technology, this constraint can be lifted and more complex solutions can be implemented. There is another condition for the system to become open loop, when the actuators reach their saturation bounds.
2.3.6 E: Control allocation with multi-objective optimisation

The secondary objective is sought only when there is sufficient control authority available. In other words, if a solution set for a primary objective exists, and among the solutions in that set the one which is closest to the secondary objective is selected. In aircraft technology, the primary objective is to fulfil the virtual demand with the solution set consisting of the control effectors positions. In the secondary objective, a minimum norm solution is determined from the residual of preferred values and the set determined during primary optimisation. In aircraft the preferred values are those which give minimum drag, and hence minimum fuel consumption. Mixed optimisation is a kind of optimisation in which a single objective function with primary and secondary objectives are utilised together with prioritizing design variables in the function. $l_1$ norm was selected in mixed optimisation (Bolender and Doman 2005; Doman and Oppenheimer 2002; Petersen and Bodson 2005; Luo et al. 2007; Doman and Ngo 2001; Luo et al. 2004; Poonamallee et al. 2004). The $l_2$ norm was used in (Simmons and Hodel 2004; Laine and Andreasson 2007; Härkegård 2002; Plumlee et al. 2004b; Bolender and Doman 2004b; Petersen and Bodson 2006). Buffington (1999) has introduced two branches to a control allocation algorithm, namely control deficiency and control sufficiency branches. In the later branch the secondary objective is optimised. In marine applications the most important objective is to achieve minimum total power (Lindegaard and Fossen 2003). Another important secondary objective is singularity avoidance (Johansen et al. 2004b).

2.3.7 F: Control allocation with more than three virtual demands allocation

Previous research into aircraft control allocation has focused on the problem of choosing control positions to satisfy demands for specific angular accelerations generated by the control law (Härkegård 2002). Methods for satisfying the largest possible range of these demands have been developed and are termed optimal. One of the problems encountered with such optimal methods is the number and complexity of the computations required. Faster techniques for optimally allocating controls for three moments have been developed that could be used in real time operations (Ducard and Geering 2006a; Wang and Longoria 2006; Gundy-Burlet et al. 2003).
While traditional aircraft control design often disregards the force-generating effects of the control effectors, redundant effectors could be used to offer control over extra degrees of freedom for the aircraft motion. Some aircraft have attempted to use their redundant controls to control one or more of the translational degrees of freedom. Researchers using the X-22 and VAAC Harrier have investigated combinations of cockpit controls to allow pilot commands for this translational motion (Oppenheimer and Doman 2004; Johansen et al. 2004b). Direct force control in response to traditional pilot commands has been suggested for improving the instantaneous response to load factor commands (Durham 1994a), as well as for in-flight simulation (Lee et al. 2004). This research focuses on algorithms for solving redundant control allocation problems to meet four or more objectives with control effectors constrained by upper and lower limits. The initial focus is on extending an existing fast three-objective algorithm to handle a fourth objective. Improvements are made to allow allocation for an arbitrary number of objectives. As the intention is to eventually perform these computations online as part of a control system, the focus is on the computational performance of these methods (Härkegård 2004).

2.3.8 G: Control allocation with constrained optimisation (i.e. rate and position constraints)

Position and rate limitations greatly affect the flying and handling qualities of the aircraft. Therefore, unconstrained control allocation would be detrimental and lead to catastrophic results. Much of the research has been focused on constrained control allocation with effector position and rate limits among the constraints (Wang et al. 2007; Boskovic et al. 2002; Fossen and Johansen 2006; Ducard et al. 2006b; Härkegård 2004; Boskovic and Mehra 2002). In the early research of Durham (Durham 1993; Durham 1994b) the attainable moment set was generated by taking into account only the position limits. Rate limitations for generation of an attainable moment set were introduced in (Bodson and Pohlchuck 1998). One of the drawbacks is that for zero moment demand there are non-zero deflections because the current control settings are dependent on the path in the moment space. The problem was alleviated by continuously applying the unused rate capabilities and directing the solution towards the desired characteristics. This is performed in the null space of the control effectiveness matrix (Durham et
The nonlinear attainable moment set generation with position limits is described in (Doman and Sparks 2002).

2.3.9 H: Control allocation with actuator dynamics

With constrained control allocation the response of the actuators to deal with rigid body modes may not be fast enough. The method of overdriving the actuators to deal with this interaction when the above assumption is invalid (i.e. actuator not fast enough as compared to the rigid body modes) is given in (Oppenheimer and Doman 2004). Dynamic control allocation will minimize the error between the desired and actual moments for all the possible inputs to the linear effector model, which also makes the problem convex (Venkataraman et al. 2004). Dynamic control allocation in the form of Model Predictive Control (MPC) was used with actuator dynamics as an internal dynamic model for control allocation (Luo et al. 2005; Luo et al. 2004; Luo et al. 2007).

2.3.10 I: Control allocation in case of fault and failure in the system

Fault tolerant control is a vast area of research. Several methodologies for fault tolerant control are given in (Blanke et al. 2006). Generally, the rationale for using control allocation as fault tolerant control is to exploit the redundancies in the system to compensate for the faults/failures in the system without changing the closed dynamics of the system. Control allocation can be used for a reconfigurable control system under the assumption that there is a reasonably good fault detection and identification system (Burken et al. 2001). A method known as Extended Multiple Model Adaptive Estimation (EMMAE) is used for fault detection and identification (Ducard et al. 2006b) and (Ducard and Geering 2006a) for the unmanned aerial aircraft. This method has performed reasonably well to estimate the fault and to reconfigure the control allocation algorithm. Having one or more control surface jammed on the aircraft was considered in (Burken et al. 2001) for the comparison of two control designs (i.e. robust servomechanism and control allocation). In the case where controls are damaged, the control allocation would be asked to reconfigure controls to assist the pilot. The online sliding mode control allocation has shown that there is no need to reconfigure the controller for certain types of failures and damages and they are handled directly (Shtessel et al. 2002; Petersen and Bodson 2002). Preliminary studies were made to increase the control authority by employing “split actuators” (Poonamallee
et al. 2004). A novel approach of using null space injection for the identification of control derivatives in the case of failure was made in (Doman and Ngo 2001; Buffington et al. 1999). This methodology is applied in two phases with the desired values being selected randomly in the second phase. The minimum distance between those selected values and the solution set from the first phase is sought, which gives solutions in terms of control positions which are highly decorrelated. This generated decorrelated regressor matrix is the requirement of the least squares problem. The specific failure and damage cases that are investigated consist of single and multiple lost surfaces, actuator hardovers, and an oscillating stabilizer case. These are examined in nonlinear control law design with control allocation (Steinberg 2001). In ground vehicles the control allocation methodology, for the control strategy having conflicting objectives, worked well in the event of failures (Plumlee 2004a).

2.3.11 J: Is control allocation flight worthy (e.g. smoothly varying actuator commands)

Smoothly varying control signals are necessary for the flight worthiness requirement; otherwise, it will result in high structural loads, which consequently affect the handling qualities of the aircraft. The smoothing of the control signal was considered in discrete time control allocation (Buffington 1999). High level control design based on sliding mode control has shown control discontinuity by frequent switching of sliding surface and was made smooth in (Wang and Longoria 2006). For the control allocation algorithm to be flight worthy, the control setting should be smooth enough not to show chattering from one time step to the next. The results in (Petersen and Bodson 2006) have shown that the primal and dual method produce smoother results than the active set method. A model produced by cubic fitting would produce smooth desired values for model predictive control (MPC) control allocation (Luo et al. 2004). Smooth reference trajectories were produced for the control allocation algorithm in marine craft (Lindegaard and Fossen 2003). The optimal control surfaces assigned for alleviation of the structural loads without changing the aircraft behaviour was studied in (Gaulocher et al. 2007).
2.3.12 **K: Control allocation as a non-convex problem**

A challenging non-convex problem posed itself in terms of singularity avoidance for underwater vehicles with azimuth thrusters which were solved in (Fossen and Johansen 2006). The singular effectors must be avoided in control allocation as this will lead to loss of controllability, this is done by utilizing a non-convex term in the objective function (Johansen *et al.* 2004b).

2.3.13 **L: To retrofit control allocation in the existing system**

The only work for retrofitting in existing aircraft was done in (Gundy-Burlet *et al.* 2003). It was shown that a neural adaptive control allocation system has shown consistent handling qualities of aircraft, but required significant system upgrade.

2.3.14 **M: Real time control allocation and pilot interaction**

To assess the handling quality of an aircraft fitted with an on board control allocation module, the real time pilot in the loop flight simulation must be run. The real time piloted simulation for direct allocation using the facet search and BESA (Bisecting Edge Search Algorithm) technique was described in (Beck 2002). The idea behind using three moment control allocation was to give a pilot the classical three inceptors. One of the advantages of the control allocation is to have the pilot at ease. This also means that allocation to virtual demands is done automatically. If this is not done then the pilot would distribute the demand to the effectors simultaneously, e.g. in the harrier project the demand consisting of longitudinal force and three moments is allocated through stick, throttle and nozzle simultaneously by the pilot. This scenario can be automatically dealt with by the control allocation module (Beck 2002). The desired moment input in the form of a sinusoid will give low gain piloting behaviour and the saw tooth demand with abrupt changes accounts for high gain piloting associated with pilot induced oscillations (Beck 2002). The control allocation incorporates the time history of the pilot inputs by manoeuvre generator (Durham *et al.* 1997). In comparison to error minimisation and direct allocation most piloted simulations have given a more ’stable feel’ for error minimisation (Petersen and Bodson 2005). The control law takes the pilot’s input and performs control distribution to the effectors. The interrelation between pilot and effectors is complicated and cannot be straightforward (Durham 1995). The practicality of piloted simulation of the F15 with redundant controls using a bisection
method direct allocation is justified in (Durham 2001). The pilot should understand control allocation strategies on an abstract level (Langari et al. 2003). The inputs to the dynamic inversion control are either pilot inputs or outer loop guidance system commands (Langari et al. 2003). This law facilitates the analysis of the closed loop performance for control allocation. The linear program control allocation, despite its computational burden, was implemented on real time piloted simulations (Buffington et al. 1999). Unconstrained control allocation would lead to actuator rate limits which will result in integral windup, tracking degradation, loss of stability and pilot induced oscillations.

2.3.15 N: Control allocation to attain maximum attainable control power

Two and three moment problems were solved to attain maximum possible moment under the constraints of controls (Durham 1993; Durham 1994b; Durham 1999). The problem of attaining maximum possible moment with planar and coplanar control was investigated in (Petersen and Bodson 2002). The method for determining the nonlinear attainable moment set was given in (Bolender and Doman 2004a). This has formed a nonlinear optimisation problem: in this problem the feasibility of desired moment is checked first; if the moment is feasible then the constrained problem is solved.

2.3.16 O: Control allocation with system identification

Updating the control effectiveness matrix based on changes in operating conditions is vital for control allocation design. This is especially true in the event of faults when the slopes in the effectiveness matrix should be estimated again. If this is not the case, the controller must be robust enough to compensate for those uncertainties. Tire and road friction estimation is necessary to avoid actuator saturations on the ground vehicle control allocation design (Wang and Longoria 2006). The null space excitation of the control surfaces to get a well conditioned problem for identification of control derivatives without degrading the aircraft response is described in (Buffington et al. 1999; Doman and Ngo 2001). MPC control allocation would get information about the model parameters from the parameter estimation module (Luo et al. 2004). The parameter identification algorithm is used to identify the stability and control derivatives and
pseudo control effectiveness of the aircraft (Eberhardt and Ward 1999). Signals applied to control surfaces must be linearly independent in time for the identification of control derivatives for the control effectiveness matrix.

2.3.17 P: Control allocation and feedback control law interactions

Hitting rate limitation bounds in control allocation would lead to pilot induced oscillations and control law integrator windup which would cause catastrophic failure (Durham and Bordignon 1996). Discussions about the integrator windup and strategy to avoid it are discussed in (Steinberg and Page 2002; Fredriksson et al. 2004; Shtessel et al. 2002; Laine and Andreasson 2007; Steinberg 2001). Zero dynamics is very important in the design of a dynamic inversion control law. If there are internal non-minimum phase zeros, then the closed loop dynamic inversion controller would be unstable. The zero dynamic is dependent on nonlinear control allocation. A sufficient condition for global asymptotic stability of zero dynamics with some admissible allocation function is given in (Buffington et al. 1998). The high level controller interaction with a control distribution function would result in different solutions (Wang et al. 2007). The interaction of the control law and control allocation in closed loop under the occurrence of damage was under investigation in 2004 (Poonamallee et al. 2004).—The asymptotically optimal control allocation in interaction with an exponentially stable trajectory-tracking controller guarantees uniform boundedness and uniform global exponential convergence” (Johansen 2004a). The interaction of control, control allocation and actuator dynamics would lead to degraded performance of aircraft or closed loop stability (Luo et al. 2004). Control law and control allocation interaction is analyzed in (Page and Steinberg 1999). The alleviation of adverse interaction among different modules in modular control architecture is shown in nonlinear simulated results (Eberhardt and Ward 1999).

2.4 Framework of Problem Formulations

All of these issues have been more or less resolved by exploiting one of the following problem formulation schemes:
a. Redistributed pseudo inverse (RPI),

b. Cascade generalised iNverse (CGI),

c. Daisy chain control allocation,

d. Direct Allocation (DA),

e. Fixed point iteration,

f. Optimisation based algorithms (i.e. quadratic programming (QP) and linear programming (LP)),

g. Nonlinear allocation.

2.4.1 Redistributed Pseudo Inverse (RPI)

The algorithm for RPI is shown in Figure 2.1. Under the definition of a non-optimal set the issue N (i.e. control allocation to attain maximum attainable control power) is not solved. As can be seen from Table 2.2, only a few researchers have applied this technique.
Figure 2.1: Schematic for RPI algorithm

Table 2.2: RPI covering issues

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<td>P</td>
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<tr>
<td>(RPI)</td>
<td>35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>57</td>
<td>35,57</td>
</tr>
</tbody>
</table>

Note: The paper index is given in Table 2.1 on page 15

Even in (Poonamallee et al. 2004) the results of RPI are not published because a degraded metric performance was encountered using RPI. In (Eberhardt and Ward 1999) RPI was used as a primary allocation algorithm.
2.4.2 Cascade generalised inverse (CGI)

The schematic of CGI algorithm is shown in Figure 2.2. The implementation of CGI corresponds with various issues as is shown in Table 2.3. A discrete version of CGI is introduced in (Page and Steinberg 2000; Steinberg and Page 2002). This method is computationally more expensive than generalised inverse because it will take the maximum number of iterations equal to the number of controls. Another limitation of CGI may be that if the constraints are tight then it probably will not converge to a solution (Härkegård 2003).

![Figure 2.2: CGI in 2D problem](image)

Table 2.3: CGI covering issues

<table>
<thead>
<tr>
<th>Issues</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>(CGI)</td>
<td></td>
<td></td>
<td></td>
<td>14,21,2,8,77</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Issues</td>
<td>I</td>
<td>J</td>
<td>K</td>
<td>L</td>
<td>M</td>
<td>N</td>
<td>O</td>
<td>P</td>
</tr>
<tr>
<td>(CGI)</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td>75</td>
<td></td>
<td></td>
<td>21</td>
</tr>
</tbody>
</table>

Note: The paper index is given in Table 2.1 on page 15.
2.4.3 Daisy chain control allocation

The schematic of a daisy chain algorithm is shown in Figure 2.3. As the daisy chain is a non-cooperative scheme, it is at a disadvantage to the rate limited system (Durham 1993). It also introduces a phase delay in response to rate limited inputs (Beck 2002). In the design of a dynamic inversion control law the zero dynamics stability is of primary importance, it has been shown that the daisy chain has provided zero dynamics stability (Buffington and Enns 1996). The implementation of this algorithm corresponds to the various issues framework as shown in Table 2.4.

![Figure 2.3: Schematic for daisy chain algorithm in 2D problem](image)

**Table 2.4: Daisy chain covering issues**

<table>
<thead>
<tr>
<th>Issues</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daisy Chain</td>
<td></td>
<td></td>
<td></td>
<td>8,2,53,61,65,6,70,73,74,75,78</td>
<td>18,73</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Issues</td>
<td>I</td>
<td>J</td>
<td>K</td>
<td>L</td>
<td>M</td>
<td>N</td>
<td>O</td>
<td>P</td>
</tr>
<tr>
<td>Daisy Chain</td>
<td>77,76</td>
<td></td>
<td></td>
<td></td>
<td>77,73,75</td>
<td></td>
<td>65,73,70</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The paper index is given in Table 2.1 on page 15
2.4.4 Direct allocation (DA)

The schematic for DA is shown in Figure 2.4. This method gives a significant computational burden in terms of calculating the attainable moment set. The metric for closed loop performance is given in (Poonamallee et al. 2004) for different methodologies including DA. In addition, it turns out that DA is comparable in performance to the nonlinear control allocation scheme in the case of both fault-free and faulty scenarios. The method for attaining a nonlinear moment set is given in (Bolender and Doman 2005). For issue D many papers have been written; the pioneer of this method is Durham (Durham 1993; Durham 1994a). By the definition of DA it inherently deals with the position constraints. However, for rate limitation the moment rate allocation is given in (Durham and Bordignon 1996). It is evident that the design of the DA algorithm was to achieve the maximum moment in the direction of a desired value and thus has fulfilled the issue N automatically. The implementation of this algorithm corresponds to the various issues framework as shown in Table 2.5.

Figure 2.4: Schematic for direct allocation
### Table 2.5: Direct allocation covering issues

<table>
<thead>
<tr>
<th>Issues</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>DA</td>
<td>30,35</td>
<td>46</td>
<td>77,30,67</td>
<td></td>
<td>75,71</td>
<td>21,35,71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Issues</td>
<td>I</td>
<td>J</td>
<td>K</td>
<td>L</td>
<td>M</td>
<td>N</td>
<td>O</td>
<td>P</td>
</tr>
<tr>
<td>DA</td>
<td>42,35,77</td>
<td>77,35</td>
<td></td>
<td></td>
<td>77,42,5669</td>
<td>71,7375</td>
<td>42,7</td>
<td>21,5,46,73</td>
</tr>
</tbody>
</table>

Note: The paper index is given in Table 2.1 on page 15

#### 2.4.5 Fixed point iteration

The schematic for fixed point iteration is shown in Figure 2.5. Fixed point iteration was first introduced in (Burken et al. 2001), where it was stated that it is easy to incorporate rate and position constraints in the algorithm. This method is a kind of gradient search iterative technique where, at the end of each iteration, any variable exceeding the bounds is clipped (Zhang et al. 2007). The method has shown finite time convergence, but it is stated in (Petersen and Bodson 2006) that its global time convergence is slow. The comparison of the primal dual interior point method, the active set method and the fixed point method is also shown in (Petersen and Bodson 2006). Another comparison was done between RPI, QP, the fixed point method and mixed $l_1$ optimisation in (Bodson 2002).

![Diagram](image)

**Figure 2.5: Schematic for fixed point iteration**
2.4.6 Optimisation based algorithms

Optimisation based algorithms are classified into the following techniques:

2.4.6.1 Quadratic programming (QP)

An optimisation problem with a quadratic objective function and linear constraints is called quadratic programming. Fast implementation of control allocation using multi-parametric quadratic programming is given in (Fossen and Johansen 2006; Johansen et al. 2005). The problem formulation of dynamic control allocation is formulated as sequential quadratic programming (Härkegård 2004). The algorithm for solving QP using active set method is depicted in Figure 2.6.

![Figure 2.6: Quadratic programming using active set method](image-url)
A discrete version of quadratic programming is formulated in (Steinberg and Page 2002). Nonlinear programming approximated by local convex quadratic programming is solved efficiently in (Johansen et al. 2004b). Regular quadratic programming and sign preserving QP is compared in (Plumlee et al. 2004b; Simmons and Hodel 2004). Unlike linear programming, QP will distribute effort equally among the effectors (Plumlee 2004a). In practice an important feature of the active set method for quadratic programming is that the required matrix inverse does not need to be computed from scratch at each step, but can be updated efficiently as a constraint is either added or dropped from the working set.

2.4.6.2 Linear programming (LP)

Linear programs have a linear objective function and linear equality or inequality constraints or possibly both. The feasible set is a polytope, which is a convex, connected set with flat, polygonal faces. The contours of the objective function are planar. Figure 2.7 depicts a linear program in two-dimensional space, in which the contours of the objective function are indicated by dotted lines. The solution in this case is unique - a single vertex. A simple reorientation of the polytope or the objective gradient could however make the solution non-unique; the optimal value could take on the same value over an entire edge. In higher dimensions, the set of optimal points can be a single vertex, an edge or face, or even the entire feasible set. The problem has no solution if the feasible set is empty (the infeasible case) or if the objective function is unbounded below on the feasible region (the unbounded case) (Nocedal and Wright 1999).
The nonlinear program is transformed in to a linear program and solved by the iterative method in (Fossen and Johansen 2006). A linear program has been solved for checking whether the aircraft is still able to retrim or not (Burken et al. 2001; Steinberg and Page 2002).

Discrete time LP is solved in (Steinberg and Page 2002). In the design of MPC allocation scheme an internal optimisation based on linear programming is solved (Tjoennas and Johansen 2006; Luo et al. 2004). A linear programming problem is converted in to a piecewise linear program which effectively solves the non-monotonic behaviour of control surface to moment (Bolender and Doman 2005). The LP problem can be solved in a finite number of iterations which makes it amenable to real time implementation in the aircraft having a sampling rate of 50 – 100 Hz range (Luo et al. 2004; Beck 2002). The LP programming technique was used to train the neural network to emulate LP behaviour (Langari et al. 2003). In (Bodson 2002) it is shown that DA can be posed as LP, and solved using any LP solving techniques. Closed loop and open loop analysis of LP control allocation is given in (Page and Steinberg 2000).

2.4.7 Nonlinear allocation

The issue A effectively deals with nonlinear allocation techniques. The schematic of this methodology is shown in Figure 2.8. This problem is non-convex in nature, which makes it prone to falling into a local minimum. The convergence of a solution is one of the issues needing to be resolved.
2.5 Discussions

The detail study and analysis of the literature will help to find out how much the research objectives given in section 1.4 conforms to the gaps stated in Table 2.1. Now the thesis objectives are analyzed with the issues stated (i.e. A to P).

Most of the applications in control allocation are focused on military aircraft, very few publications are on the application of control allocation on civil aircraft (there are some civil aircraft publication like Alwi and Edwards 2007). So the first objective would be a novel approach to design and implement control law and control allocation for the nonlinear civil aircraft model (B747-200).

The issue of a fault tolerant system in terms of control allocation is addressed by few researchers (see column I Table 2.1). This shows an open area of research of using control allocation for fault tolerant system. The second objective (to use control redundancies to compensate for a failure without changing the baseline control law for the civil aircraft) conforms to the open question posed by analyzing the issue I (i.e. control allocation in the case of a fault and failure in
the system). The important condition for using the control allocation for damage adaptation is that there is a fault detection, isolation and identification system onboard.

The issue B (i.e. control allocation with automatic update of control effectiveness matrix) is another open area to be explored and has motivated the author to pursue the third objective. This issue becomes very important when the operating condition changes and the control effectiveness matrix is required to be updated. To gauge the effects of control allocation with the changes to the flight operating conditions in an open loop simulation testing is given in section 5.5. So the third objective also validates that the posed research question would be unique in the frame work of issue B.

All the control law and control allocation designs are highly dependent on the model used. The model must be good enough for these design processes and if there is a faulty system then the identification of stability derivatives (which are used for the design of the control law) and control derivative (which are used for the design of control allocation) become vital and need to be addressed. In this survey it was found that the most important of all is the issue O (i.e. control allocation with system identification) and must be the future research question.

To the author's knowledge nobody has undertaken the idea of using control allocation to control pitch of the aircraft by its centre of gravity as a redundant pitch attitude control in the aircraft systems (which is objective 5).

Objective 6 could be stated in the frame work of the issue J (i.e. is control allocation flight worthy?). It was noted that by carefully selecting the desired control vector the flying and handling qualities of the aircraft would be improved which eventually makes it more flight worthy. Checking and improving the flying and handling qualities with the control allocation in the loop remains an open research question. The second part of this objective was covered by very few researchers. This idea is studied in the case of the 2-D problem as explained in section 5.9. This objective (i.e. control prioritisation by using weighting matrix) in control allocation may improve the results if the positive definiteness of this weighting matrix is observed. This makes this research objective viable for the study.
The last objective 7 corresponds to issue H (i.e. control allocation with actuator dynamics). According to this survey of using an evolutionary technique (i.e. genetic algorithms) for the design of compensator to lessen the interaction between control allocation and actuator dynamics for the civil aircraft model (B747-200) is a novel approach in the area of control allocation.

2.6 Conclusions

In this chapter the control allocation is categorised. Control allocation is a constrained optimisation problem. This optimisation problem has been classified into 16 categories based on problem definition (i.e. objective function and constraints). When the objective function of the control allocation problem is defined as monotonic nonlinearities, non-monotonic nonlinearities, linear and multi-objectives, then they are categorised in A, C, D and E categories, respectively. The modelling uncertainties which are dealt within the objective function are categorised in B and O. The constraints in control allocation are handled in categories G and N and actuator dynamics effects are reviewed in category H. The objective to achieve three or more virtual demands is discussed in F. Changing the problem definition in control allocation problem in case of faults/failures is given in category I. Control allocation interaction with pilot and control law are cater in M and P respectively. The non-convex objective function in control allocation problem definition falls in to the category K. Formulating and solving control allocation problem to get flight worthy solutions are given in category J. The retrofitting of control allocation in to an existing flight control is given in category L. The non-optimal and optimal methodologies were used for solving this optimisation problem. The class of non-optimal methods (i.e. RPI, CGI and daisy chain) are fast to solve the control allocation problem but might give sub-optimal solution in some cases. The optimal methods require more computational power compared to non-optimal methodologies but the solution might be optimal in these methodologies. Among these optimal methods the quadratic programming (QP techniques) has proven to give smoothly varying solutions from one sample to the next. This methodology was opted throughout this thesis because of its good mathematical properties which will be discussed in Chapter 4. The outcome of this survey has provided a ground to defend that the proposed research objectives are
unique in some sense for this area of research. Also, it has given some guideline of seeing this research work novel.
3 Civil Aircraft Model (B747-200)

3.1 Introduction

The Boeing 747 is a four-fanjet intercontinental transport aircraft (see Figure 3.1). In low speed flight conditions high lift is obtained by wing triple-slotted trailing flaps and Krueger type leading edge flaps. The Krueger flaps outboard of the inboard nacelle are cambered and slotted while the inboard Krueger flaps are standard unslotted. The longitudinal control for the aircraft is provided by a movable stabilizer with four elevator segments. Lateral control is achieved by five spoiler panels, an inboard aileron amid the inboard and outboard flaps, and an outboard aileron, which only activates when the flaps are down. Speed-braking is provided by six spoiler panels on each wing that are set out symmetrically. Directional control is obtained with split rudders (i.e. upper and lower rudders) (Hanke and Nordwall 1970). The main performance features of the B747-200 are a range of 11,000 kilometres and a typical cruising speed of Mach 0.84 at an altitude of 35000 ft. The remainder of this chapter is based on the concepts presented in reputable textbooks (Stevens and Lewis 2003; Etkin and Reid 1994; Stengel 2004).

![Figure 3.1: Boeing 747 - 100 aircraft](image-url)
The following is an outline of the chapter. Coordinate frames and the transformations between them are introduced in section 3.2. In section 3.3 aircraft variables and properties are described. Section 3.4 includes a nonlinear dynamic model of the aircraft with some explanations of nonlinear stability and control derivatives to show the complexity of the model. A graphical illustration of the control allocation problem is presented in section 1.3. Finally, section 3.5 provides some concluding remarks.

### 3.2 Coordinate Frames

Different coordinate frames used to describe aircraft kinematics are depicted in Figure 3.2.

These frames are:

- Earth-fixed frame \( \{e\} \),
The aerodynamic forces and moments on an aircraft are produced by its relative motion with respect to air and depend on the orientation of the aircraft relative to the airflow. There are two relative orientation angles with respect to wind namely, angle of attack $\alpha$ and side slip angle $\beta$. The superscripts $e$, $B$, $s$ and $w$ denote axes of earth-fixed, body-fixed, relative stability and wind frames, respectively. Euler angles $\phi$, $\theta$ and $\psi$ are defined as roll angle, pitch angle and yaw angle respectively. The rotation matrix from body fixed axis to stability axis is given by

$$T^B_{\text{to} \ s} = \begin{bmatrix} \cos \alpha & 0 & \sin \alpha \\ 0 & 1 & 0 \\ -\sin \alpha & 0 & \cos \alpha \end{bmatrix}$$ (3.1)

The rotation matrix from stability axis to wind axis is given by

$$T^s_{\text{to} \ w} = \begin{bmatrix} \cos \beta & \sin \beta & 0 \\ -\sin \beta & \cos \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ (3.2)

And the rotation matrix from body axis to wind axis is given by

$$T^s_{\text{to} \ w} = \begin{bmatrix} \cos \beta & \cos \alpha \sin \beta & \sin \beta & \sin \alpha \cos \beta \\ -\cos \alpha \sin \beta & \cos \beta & 0 & -\sin \alpha \sin \beta \\ -\sin \alpha & 0 & 1 & \cos \alpha \end{bmatrix}$$ (3.3)

### 3.3 Aircraft Variables and Properties

#### 3.3.1 Mass properties

The inertia matrix of an aircraft is defined as
The zero entries show that the x-z plane is the plane of symmetry.

### 3.3.2 Aircraft linear velocity

The velocity vector $\mathbf{V}$ in body-fixed frame is given by

$$\mathbf{V} = [u \quad v \quad w]^T$$

Here $u$ is the longitudinal velocity, $v$ is the lateral velocity and $w$ is the normal velocity.

The velocity vector in the wind frame is given by

$$\mathbf{V}^w = [V^T \quad 0 \quad 0]^T$$

where $V^T$ is the true air speed defined as:

$$V^T = \sqrt{u^2 + v^2 + w^2}$$

$$\alpha = \tan^{-1} \frac{w}{v}$$

$$\beta = \sin^{-1} \frac{v}{V^T}$$

where $\alpha$ is the angle of attack and $\beta$ is the side slip angle.

### 3.3.3 Aircraft angular velocity

The angular velocity vector $\mathbf{\omega}$ in the body-fixed frame is given by

$$\mathbf{\omega} = [p \quad q \quad r]^T$$

where $p$ is the roll rate, $q$ is the pitch rate and $r$ is the yaw rate.
3.3.4 Aircraft orientation

The orientation of the aircraft is given by Euler angles, which relate the aircraft’s body-fixed frame with the earth-fixed frame

\[ \Phi = [\phi \ \theta \ \psi]^T \]  

(3.9)

where \(\phi\) is the roll angle, \(\theta\) is the pitch angle and \(\psi\) is the yaw angle.

3.3.5 Aircraft position

The aircraft position vector, giving the position of the aircraft with respect to the earth fixed frame, is given by

\[ p^e = [h^e \ x^e \ y^e]^T \]  

(3.10)

where \(h^e\) is the altitude, \(x^e\) is the position north and \(y^e\) is the position east.

3.3.6 Sign conventions

The sign convention for the angular velocities is shown in Figure 3.3. The stabilizer deflection with leading edge up is treated as positive. For the individual ailerons trailing edge down is defined positive under the standard convention.
3.4 Nonlinear Model of Aircraft

3.4.1 Introduction

The model used in this study is a derivative of the Simulink model FTLAB747 version 6.5 developed by (Esteban 2004). This software is an upgraded version of two previous programs (i.e. Delft University Aircraft Simulation Modelling and Analysis Tool, DASMAT and Flight Lab 747, FTLAB747) developed by (van der Linden 1998) in Delft University. In this section, the aerodynamic forces and moments are combined with the equations of motion to obtain a nonlinear aircraft model. This model is trimmed at certain flight conditions by a trimming algorithm and then at this trim point it is linearised to obtain a linear model for the purpose of control law design and control allocation. The schematic for the dynamic model of the aircraft is shown in Figure 3.4 and the whole section is structured according to this diagram. The symbols shown in Figure 3.4 are defined as follows:

\[ \mathbf{u}_{aero} = \text{Vector of control surfaces inputs,} \]
\( u_{\text{prop}} \) = Input vector to propulsion system,

\( F_{\text{aero}}^B \) = Vector of aerodynamic forces in body-fixed frame,

\( M_{\text{aero}}^B \) = Vector of aerodynamic moments in body-fixed frame,

\( F_{\text{prop}}^B \) = Vector of propulsion forces in body-fixed frame,

\( M_{\text{prop}}^B \) = Vector of propulsion moments in body-fixed frame,

\( F_{\text{grav}}^B \) = Vector of gravity forces in body-fixed frame,

\( F_{\text{tot}}^B \) = Vector of total forces in body-fixed frame,

\( M_{\text{tot}}^B \) = Vector of total moments in body-fixed frame,

\( x \) = Aircraft state variables vector in body-fixed frame.

\[
\dot{x} = f(x,F_{\text{aero}},M_{\text{aero}}) + \int dt
\]

Figure 3.4: Schematic of aircraft dynamic model
3.4.2 Aerodynamic model

The aerodynamic model is derived from (Hanke and Nordwall 1970) and used in lookup tables for the whole flight envelope. In this section a brief introduction to the aerodynamic coefficients and the point of uniqueness of the model for this research is highlighted. In the design of control allocation, three degrees of freedom as three objectives were utilised. Three angular accelerations are assigned to the control allocation module and it allocates this total demand to the control surfaces optimally and feasibly. So in this section only three non-dimensional moment equations are given and to study the non-dimensional forces governing this model, readers are referred to (Hanke and Nordwall 1970). The aerodynamic coefficients given below are in a stability axis frame.

3.4.2.1 Pitching moment coefficient $C_m$

A moment about the y-axis (or pitch axis) is called the pitching moment $M$. This moment may be written in dimensionless form

$$C_m = \frac{M}{\bar{q}S\bar{c}}$$

where $\bar{q}$, $S$, and $\bar{c}$ are the dynamic pressure, wing area, and mean aerodynamic chord respectively. The pitch moment about the c.g. is shown in Figure 3.5. The dimensionless pitching moment for the B747-200 is given as:

![Figure 3.5: Pitch moment of aircraft about c.g.](image)

48
\[ C_m = C_{mbasic} + (\Delta C_{m0.25})_{\alpha_{w.d.p}=0} + \Delta \left( \frac{dC_m}{da} \right)_{\alpha_{w.d.p}} + C_L (cg - 0.25) \]
\[ + \frac{dC_m}{d\alpha} \frac{\dot{\alpha}}{2V_T} + \frac{dC_m}{dq} \frac{\dot{q}}{2V_T} + \frac{dC_m}{dn_z} n_z \]
\[ + K_a \left[ \frac{dC_m}{d\delta_{th}} \delta_{th,f,r,i} + \frac{dC_m}{d\delta_{el}} \delta_{el} + \frac{dC_m}{d\delta_{eo}} \delta_{eo} \right] + \Delta C_{m0.25} \text{spoiler} \]
\[ + \Delta C_{m0.25} \text{ inboard ailerons} + \Delta C_{m0.25} \text{outboard ailerons} + \Delta C_{m0.25} \text{landing gears} \]
\[ + \Delta C_{m0.25} \text{ground effects} + \Delta C_{m0.25} \text{sid eslip} + \Delta C_{m0.25} \text{rudder} \]
\[ + \left[ \Delta C_{m0.25} \text{flap failure} \right] \]

The stability derivatives for the pitching moment coefficient are calculated with respect to the 25% of mean aerodynamic chord (m.a.c.) for this aircraft model. The mean aerodynamic chord is illustrated in Figure 3.6.

Figure 3.6: Centre of gravity (c.g.) at 25% of mean aerodynamic chord (m.a.c.)

The first term in the pitching aerodynamic coefficient, \( C_{mbasic} \), accounts for the basic pitching coefficient for the rigid aircraft at the zero stabilizer angle. This term \( (\Delta C_{m0.25})_{\alpha_{w.d.p}=0} \) is the change in pitching moment coefficient at zero \( \alpha_{w.d.p} \). The term \( C_L (cg - 0.25) \) measures the change in pitching moment due to variation in the c.g. with respect to the 25% m.a.c. This term is utilised as a redundant pitch attitude control of the aircraft. The non-dimensional lift coefficient is given by \( C_L \). The four terms \( \Delta \left( \frac{dC_m}{da} \right)_{\alpha_{w.d.p}}, \frac{dC_m}{d\alpha} \frac{\dot{\alpha}}{2V_T}, \frac{dC_m}{dq} \frac{\dot{q}}{2V_T} \), and \( \frac{dC_m}{dn_z} n_z \) measure the change in basic pitch coefficient due to wing angle of attack, \( \alpha \), rate of change of angle of attack, \( \dot{\alpha} \), pitch rate, \( q \), and normal load factor, \( n_z \), respectively. To make the terms non-dimensional, the relevant terms are multiplied by \( \alpha_{w.d.p}, \frac{\dot{\alpha}}{2V_T}, \frac{\dot{q}}{2V_T} \), and \( n_z \), respectively. The effectiveness factor of the elevators and stabilizer is given by \( K_a \). This term corrects the stability
terms for the stabilizer, \( \frac{dC_m}{d\delta_{th}} \delta_{th,f,r,t} \) and the inboard/outboard elevators \( \frac{dC_m}{d\delta_{ei}} \delta_{ei} \) and \( \frac{dC_m}{d\delta_{eo}} \delta_{eo} \). The effect of spoilers (or speed brake deflection) is added by \( \Delta C_m \delta_{spoiler} \). The ailerons influence on the basic pitching moment is measured by \( \Delta C_m \delta_{inboard\ ailerons} \) and \( \Delta C_m \delta_{outboard\ ailerons} \). Finally, the effects of landing gear, ground effects, side slip rudder and flap failure are respectively given by \( \Delta C_m \delta_{landing\ gears} \), \( \Delta C_m \delta_{ground\ effects} \), \( \Delta C_m \delta_{sidelip} \), \( \Delta C_m \delta_{rudders} \), \( \Delta C_m \delta_{flap\ failure} \), and \( \Delta C_m \delta_{leading\ edge\ flap\ failure} \).

### 3.4.2.2 Rolling moment coefficient \( C_l \)

A moment about the x-axis (or roll axis) is called a rolling moment \( L \). This moment may be written in dimensionless form

\[
C_l = \frac{L}{\bar{q}Sb}
\]  
(3.13)

where \( \bar{q}, S, \) and \( b \) are dynamic pressure, wing area and wing span respectively. The dimensionless rolling moment for B747-200 is given as:

\[
C_l = \frac{dC_l}{d\beta} \beta + \frac{dC_l}{dp} \frac{p_s b}{2V_l} + \frac{dC_l}{dr} \frac{r_s b}{2V_l} + \Delta C_l \delta_{spoilers} + \Delta C_l \delta_{inboard\ ailerons} + \Delta C_l \delta_{outboard\ ailerons} + \Delta C_l \delta_{rudders} + \Delta C_l \delta_{flap\ failure} + \Delta C_l \delta_{leading\ edge\ flap\ failure}
\]  
(3.14)

The stability derivatives affected by the states of sideslip \( \beta \) is \( \frac{dC_l}{d\beta} \), roll rate in \( p_s \) for the stability axis is \( \frac{dC_l}{dp} \frac{p_s b}{2V_l} \) and yaw rate \( r_s \) is \( \frac{dC_l}{dr} \frac{r_s b}{2V_l} \). The expression governing the control derivatives for \( \Delta C_l \delta_{inboard\ ailerons} \) and \( \Delta C_l \delta_{outboard\ ailerons} \) is governed by the following expressions:

\[
\Delta C_l \delta_{inboard\ ailerons} = \sum_{\text{left and right inboard ailerons}} K_{\delta_{ai}} \left( \frac{\Delta C_l \delta_{ai}}{20} \right) \left( \frac{\Delta C_l \delta_{ai}}{M=0} \right) \left( \frac{R_E}{R_R} \right)_{\delta_{ai}} F_{lGE}
\]  
(3.15)
\[ \Delta C_{l_{\text{outboard ailerons}}} = \sum_{\text{left and right outboard ailerons}} K_{\delta\alpha} \Delta C_{l_{\alpha\alpha}} \frac{(\Delta C_{l_{\alpha\alpha}})_{M=0}}{(\Delta C_{l_{\alpha\alpha}})_{M=0}} \left( \frac{R_E}{R_R} \right)_{C_{l_{\alpha\alpha}}} F_{l_{GE}} \]  

(3.16)

In Eq. (3.15) and Eq. (3.16) the first term \( K_{\delta_{ai}} \) and \( K_{\delta_{ao}} \) are the inboard and outboard ailerons effectiveness factors (see Figure 3.7). It can be seen that the effectiveness varies with the full movement of the ailerons, and for the outboard aileron the effectiveness is different for upward and downward deflections. The stability derivatives are calculated using the wing design plane angle of attack \( (\alpha_{wdp}) \) rather than the fuselage reference line angle of attack \( (\alpha_{FRL}) \). The expression for the wing design plane angle of attack is \( \alpha_{wdp} = \alpha_{FRL} + 2^\circ \) (Hanke and Nordwall 1970). The terms \( (\Delta C_{l_{ai}})_{20} \) and \( \Delta C_{l_{ao}} \), defined in Figure 3.8 and Figure 3.9, has shown that the rolling moment coefficient varies nonlinearly with \( \alpha_{wdp} \). During a manoeuvre, when the angle of attack changes, the control effectiveness matrix needs to be updated. Otherwise, the controller effort would be increased. The effects due to changes in Mach number are given by \( \left( \frac{\Delta C_{l_{ai}}}{\Delta C_{l_{ai}}}_{M=0} \right) \) and \( \left( \frac{\Delta C_{l_{ao}}}{\Delta C_{l_{ao}}}_{M=0} \right) \). The next terms \( \left( \frac{R_E}{R_R} \right)_{C_{l_{ai}}} \) and \( \left( \frac{R_E}{R_R} \right)_{C_{l_{ao}}} \), which are given in Figure 3.10, are the aeroelastic effects of the inboard and outboard ailerons respectively. They are very important to make the problem unique. The aeroelastic effects caused by outboard ailerons introduced in the model are functions of equivalent airspeed, \( V_e \) and show complete reversal after certain \( V_e \) so these outboard ailerons are locked with flaps during high speeds. This limitation of the outboard ailerons may be partially relaxed with advancements in structural design. The inboard aileron rolling moment coefficient is also affected by the aeroelastic effects in different flight conditions. The last term \( F_{l_{GE}} \) gives the effects of the ground, which is normally set equal to 1 as all studies have been conducted out of ground effect. The term \( \Delta C_{\delta_{rudders}} \) in Eq. (3.14) has also aeroelastic results affecting \( C_l \) as shown in Figure 3.11.

As can be seen from the above discussion, the aileron effectiveness is dependent on many factors which have made the problem definition distinctive for the purpose of control allocation (CA) design.
Figure 3.7: Inboard and outboard effectiveness factor

Figure 3.8: Rolling moment coefficient with inboard aileron up at 20 degrees
Figure 3.9: Rolling moment coefficient due to inboard aileron and inboard ailerons fully up and down

Figure 3.10: Aeroelastic effect on rolling moment coefficient by outboard aileron as a function of equivalent air speed $V_e$ and by inboard aileron as a function of Mach number and height
3.4.2.3 Yawing moment coefficient $C_n$

The moment about the z-axis (or yaw axis) is called the yawing moment $N$. This moment may be written in dimensionless form

$$C_n = \frac{N}{\bar{q}Sb}$$

(3.17)

where $\bar{q}, S$, and $b$ are dynamic pressure, wing area and wing span respectively. The dimensionless yawing moment for the B747-200 is given as

$$C_n = \frac{dC_{nl}}{d\beta} \beta + \frac{dC_n}{d\beta} \frac{\beta b}{2V_T} + \frac{dC_n}{dp} \frac{p_s b}{2V_T} + \frac{dC_n}{dr} \frac{r_s b}{2V_T} + \Delta C_{\text{spoilers}}$$

$$+ \Delta C_{n_{\text{inboard ailerons}}} + \Delta C_{n_{\text{outboard ailerons}}}
+ \Delta C_{n_{\text{rudders}}}
+ \left[ \Delta C_{n_{\text{flap failure}}} + \Delta C_{n_{\text{leading edge flap failure}}} \right]$$

(3.18)
Aerodynamic forces and moments are normally calculated in the stability axis and then rotated to the body axis frame to be used for the equation of motion (EoM).

\[
\mathbf{F}_{aero}^B = \bar{q}S \begin{bmatrix}
\cos \alpha & 0 & -\sin \alpha \\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{bmatrix} \begin{bmatrix}
C_x \\
C_y \\
C_z
\end{bmatrix}
\]

(3.19)

\[
\mathbf{M}_{aero}^B = \begin{bmatrix}
\cos \alpha & 0 & -\sin \alpha \\
0 & 1 & 0 \\
\sin \alpha & 0 & \cos \alpha
\end{bmatrix} \begin{bmatrix}
\bar{q}SbC_i \\
\bar{q}ScC_m \\
\bar{q}SbC_n
\end{bmatrix}
\]

(3.20)

Where \(\bar{q}, S, b, \) and \(\bar{c}\) are dynamic pressure, wing area, wing span and mean aerodynamic chord respectively and are used as multiplying factor to convert the non-dimensional forces and moments calculated in the stability axis to dimensional ones in the body fixed frame.

### 3.4.3 Gravity

The force of gravity is calculated in the earth fixed frame and then transformed to the body fixed frame to be used in the equation of motion.

\[
\mathbf{F}_{grav}^e = \begin{bmatrix}
0 \\
0 \\
mg
\end{bmatrix} \xrightarrow{transformation} \mathbf{F}_{grav}^B = mg \begin{bmatrix}
-\sin \theta \\
\sin \phi \cos \theta \\
\cos \phi \cos \theta
\end{bmatrix}
\]

(3.21)

### 3.4.4 Engine thrust

The thrust from the engine produces forces and moments in the body axis frame as given by the following expression:

\[
\mathbf{F}_{prop}^B = \begin{bmatrix}
T_{nx} \\
T_{ny} \\
T_{nz}
\end{bmatrix}
\]

(3.22)

where

\[
T_{nx} = \sum_{i=1}^{4} T_{ni}
\]

(3.23)
\[ T_{ny} = 0.0349(T_{n1} + T_{n2} - (T_{n3} + T_{n4})) \]

\[ T_{nz} = -0.0436 \sum_{i=1}^{4} T_{ni} \]

where \( i \) is the index for the four engines.

The vector of engine moments about the body fixed frame

\[ \mathbf{M}_{prop}^B = \begin{bmatrix} l_p \\ m_p \\ n_p \end{bmatrix} \] (3.24)

where \( l_p, m_p \) and \( n_p \) are rolling, pitching and yawing moments generated by the engine thrust and defined as follows:

\[ l_p = 0.0436n_p \]

\[ m_p = (T_{n1} + T_{n4})Z_{Eo} + (T_{n2} + T_{n3})Z_{Ei} \] (3.25)

\[ n_p = (T_{n1} + T_{n4})Y_{Eo} + (T_{n2} - T_{n3})Y_{Ei} \]

where \( Z_{Ei}, Y_{Ei}, Z_{Eo} \) and \( Y_{Eo} \), are the effective engine inboard and outboard moment arms respectively.

### 3.4.5 Total aircraft forces and moments equations

The aircraft forces and moments are combined in total forces and moments in the body axis frame and then given to the EoM.

\[ \mathbf{F}_{tot}^B = \mathbf{F}_{aero}^B + \mathbf{F}_{grav}^B + \mathbf{F}_{prop}^B \]

\[ \mathbf{M}_{tot}^B = \mathbf{M}_{aero}^B + \mathbf{M}_{prop}^B \] (3.26)

### 3.4.6 Aircraft equation of motion

The standard equation for rigid body motion in terms of velocity and angular velocity is as follows:
\[ F_{\text{tot}}^B = m(\dot{V} + \omega \times V) \]
\[ M_{\text{tot}}^B = I \dot{\omega} + \omega \times l \omega \]  

Eq. (3.27) is expanded:
\[ F_{\text{tot}}^B = m \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \]  

\[ M_{\text{tot}}^B = \begin{bmatrix} l_x & 0 & -l_{xz} \\ 0 & l_y & 0 \\ -l_{xz} & 0 & l_z \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} l_x p - l_{xz} r \\ l_y q \\ -l_{xz} p + l_z r \end{bmatrix} \]  

The kinematic equation gives a measure of pitch, roll and yaw rates:
\[ \dot{\phi} = p + \tan \theta \left( q \sin \phi + r \cos \phi \right) \]
\[ \dot{\theta} = q \cos \phi - r \sin \phi \]
\[ \dot{\psi} = \frac{\left( q \sin \phi + r \cos \phi \right)}{\cos \theta} \]  

\[ (3.30) \]

3.4.7 Aircraft control variables

The aircraft control variables mainly used in this work are the angular positions of redundant control surfaces (as shown in Figure 3.12), which are constrained by rate and position limits.
3.4.8 Aircraft control objectives

The control objectives for control allocation and control law designs are divided into two categories.

1. In open loop control allocation the control variables are non-dimensional moments (i.e. $C_L, C_M, C_N$).
2. For the closed loop control allocation with control law design the reference control variables are Euler angles and for control allocation it would be the angular accelerations (i.e. rolling, pitching and yawing accelerations).
3.5 Conclusions

The idea of this chapter is to present the notion that the problem formulation in terms of the model of the civil aircraft (i.e. B747-200) is unique enough for the control allocation design. In this aircraft there are two or more control surfaces to be selected for a control variable. The constrained control allocation problem would be defined by rate and position constraints on the control surfaces. These constraints and the actuator dynamics for each control surface are available for this model. All three angular accelerations (i.e. roll, pitch and yaw) are controllable to some extent by the engines, which gives some extra redundancy. The aerodynamic coefficients in the aerodynamic model of this civil transport have shown their limitations by the nonlinear relationship between control surfaces and the moments generated by them. These relationships are strongly dependent on the changes in the flight conditions. The introduction of aeroelastic effects has made this model more realistic. This model was used throughout this research work for the development and implementation of the control law and control allocation design.
4 Control Allocation

4.1 Introduction

In this chapter the control law and control allocation are designed and implemented on a nonlinear aircraft model of the B747-200. The active set method was utilised first by Ola Härkegård on high angle of attack military aircraft (Härkegård 2003). The application of the active set methods, to the author’s knowledge, is implemented on civil aircraft B747-200 for the first time in this research work (thesis). The online control allocation problem is solved at the sampling frequency of 50 Hz within the bandwidth of this aircraft.

Many questions dealing with "what is the 'best' approach", employ optimisation techniques. Such applications arise in various fields of science and engineering, which are often models of reality. The index of 'goodness' is measured by an objective function, which is iteratively minimised or maximised over constrained decision variables to get an optimal as well as a feasible solution.

Modern jet aircraft have many actuators required for flight path control (e.g. two or more engines, elevators, rudders, flaps). In essence aircraft are "over-actuated" as they possess control redundancy and the pilot commanded flight vector can be realised with more than one (often many) different combinations of settings of the actuators. With advanced control schemes this redundancy in the control of actuators can be taken advantage of to enhance aircraft safety in the event of an aircraft malfunction or damage. The research objective is to utilize the multiple redundancies within the control systems in the event of a system failure or other aircraft malfunction to control the aircraft by automatically switching control laws and control allocation techniques. This technique, which is based on online optimisation, has recently been explored for use in military air vehicles; however, little research has been undertaken for civil aircraft applications.

The idea of control allocation can be given by a simple example. The lateral and directional dynamics are coupled in the aircraft. On certain aircraft (e.g. B747), there are two rudders (i.e.
upper and lower) for directional control redundancy. In theory, it is possible to use this redundancy to control the aircraft following a certain type of failure. In the event of a failure affecting lateral control (e.g. an aileron jam) it is theoretically possible to still roll the aircraft by moving the two rudders in opposite directions, without yawing of the aircraft (i.e. moving the aircraft left or right).

The modular design approach is applied in this research work. The control law is designed separately to the allocation algorithm (see Figure 4.1). The benefits of this approach are as follows:

1. Position and rate constraints are treated separately in the CA block.
2. Actuator dynamics can be accounted for in this design, explicitly in model predictive control allocation design (Luo et al. 2007). An implicit approach is used here for relating the actuators suite bandwidths to control allocation.
3. In the event of failures the reconfiguration is typically done by the redistribution of control capability, but this is under the assumption that good fault identification and detection systems are in place.
4. A secondary objective optimisation can be done as explained in section 2.3.6.

Most of the issues treated in Chapter 2 explicitly or implicitly related to control allocation and can be considered to support the potential of CA in control design.

![Figure 4.1: Modular approach to control law and control allocation design](image)

Section 4.2 describes the basic definition of control allocation. The constrained optimisation and the properties of the solution are described in section 4.3. The general quadratic programming problem is defined and the properties of the solution to the quadratic program are explained and
demonstrated in section 4.4. The active set algorithm is explained and represented in section 4.5. In section 4.6 the multi-branch control allocation problem is defined and then solved by active set methods. An example of the application of the active set method algorithm is given in section 4.7. The simulation results of the control law and the control allocation design implemented on B747-200 are presented in section 4.8. Finally some concluding remarks are given in section 4.9.

### 4.2 Control Allocation Problem

The problem of control allocation is considered to be a constrained system of linear equations having fewer equations than the number of decision variables. Here the number of equations is governed by the virtual control demand (i.e. total demand) vector $\mathbf{v}(t) \in \mathbb{R}^k$. The number of decision variables is given by the control allocation output vector $\mathbf{u}(t) \in \mathbb{R}^m$. Here $m > k$ gives an underdetermined system. The nonlinear mapping is given by

$$\mathbf{v}(t) = h(\mathbf{u}(t)) \tag{4.1}$$

where $h: \mathbb{R}^m \rightarrow \mathbb{R}^k$.

In linear control allocation, $\mathbf{v}(t)$ is effectively mapped to $\mathbf{u}(t)$ by a coefficient matrix $\mathbf{B}_c$ of dimensions $m \times k$ and of rank $k$.

$$\mathbf{v}(t) = \mathbf{B}_c \mathbf{u}(t) \tag{4.2}$$

$\mathbf{B}_c$ is normally called the control effectiveness matrix, which consists of the information of the control derivatives for individual actuators in a certain flight condition. These control derivatives are normally slopes in rad/s$^2$ per degree deflection of the control surfaces.

The position and rate limitations are combined into one by exploitation of the fact that the control is part of a digital control system (Härkegård 2004). The position saturation band is given by:
\[ u_{\text{min}} \leq u \leq u_{\text{max}} \]  \hspace{1cm} (4.3)

And the rate limitation is expressed as:

\[ \dot{e}_{\text{min}} \leq \dot{u} \leq \dot{e}_{\text{max}} \]  \hspace{1cm} (4.4)

The rate of change for input in digital control can be approximated by:

\[ \dot{u}(t) = \frac{u(t) - u(t - T)}{T} \]  \hspace{1cm} (4.5)

where T is the sampling time. Eqs (4.3) to (4.5) are combined to get an integrated constraint accounting for both the position and rate limitations which gives

\[ \underline{u}(t) \leq u(t) \leq \overline{u}(t) \]  \hspace{1cm} (4.6)

here

\[ \underline{u} = \max\{u_{\text{min}}, u(t - T) + T\dot{e}_{\text{min}}\} \quad (4.7) \]

\[ \overline{u} = \min\{u_{\text{max}}, u(t - T) + T\dot{e}_{\text{max}}\} \]

The linear control allocation is defined by:

\[ \nu(t) = B_c u(t) \]

\[ \underline{u}(t) \leq u(t) \leq \overline{u}(t) \]  \hspace{1cm} (4.8)

To explain the control allocation concept consider the following example:

The general linearisation of Eq. (4.1) is given as
\[ h(\mathbf{u}) = h(u_1, u_2) \]
\[ \approx h(u_{10}, u_{20}) + \frac{\partial h}{\partial u_1}(u_{10} - u_{20}) \]
\[ + \frac{\partial h}{\partial u_2}(u_{10} - u_{20}) \]  
\[ (4.9) \]
\[ h(u_1, u_2) - h(u_{10}, u_{20}) \]
\[ \approx h(\mathbf{u}_0) + \left[ \frac{\partial h}{\partial u_1} \frac{\partial h}{\partial u_2} \right]_{(u_{10}, u_{20})} \begin{bmatrix} u_1 - u_{10} \\ u_2 - u_{20} \end{bmatrix} \]  
\[ \text{B}_c \]  
\[ (4.10) \]

Here \( u_{10} \) and \( u_{20} \) are the operating point of \( u_1 \) and \( u_2 \) around which the linearisation is performed.

**4.2.1 Example for 2D control allocation**

Consider Eq. (4.1) where \( h(\mathbf{u}(t)) \) is defined as
\[ h(\mathbf{u}(t)) = u_1^2 + 3u_2 \]  
\[ (4.11) \]

The function is shown in Figure 4.2. Now linearizing \( h(\mathbf{u}(t)) \) around \( \mathbf{u}_0 \) yields
\[ h(u_1, u_2) - h(u_{10}, u_{20}) \]
\[ \approx h(\mathbf{u}_0) + \left[ \frac{\partial h}{\partial u_1} \frac{\partial h}{\partial u_2} \right]_{(u_{10}, u_{20})} \begin{bmatrix} u_1 - u_{10} \\ u_2 - u_{20} \end{bmatrix} \]  
\[ \text{B}_c \]  
\[ (4.12) \]

where
\[ \text{B}_c = \left[ \frac{\partial h}{\partial u_1} \frac{\partial h}{\partial u_2} \right]_{[u_{10}=1][u_{20}=1]} \]
\[ (4.13) \]

Now
\[ \vec{v} = \text{B}_c \Delta \mathbf{u} \]
\[ (4.14) \]

where
\[ \tilde{v} = v - h(u_{10}, u_{20}) \]  

(4.15)

where \( h(u_0) = 4 \) and \( \mathbf{B} \mathbf{c} u_0 = 5 \)

\[ \tilde{v} = 2\Delta u_1 + 3\Delta u_2 \]

\[ -1 \leq \Delta u_1 \leq 1 \]

\[ -1 \leq \Delta u_2 \leq 1 \]  

(4.16)

\( \Delta \) is dropped to simplify the notation. Using the notation of Eq. (4.8) gives \( \mathbf{B} \mathbf{c} = \begin{bmatrix} 2 & 3 \end{bmatrix}, \mathbf{u} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \mathbf{\bar{u}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) and \( v = 3.5 \) which gives \( \tilde{v} = 3.5 \). The graphical solution to this problem is shown in Figure 4.3. The solutions to the problem with \( \tilde{v} = 2 \) have infinite number of solutions, unique solution with \( \tilde{v} = 5 \) and no solution with \( \tilde{v} = 7 \), in this case a minimum norm solution is sought. Any of the available vector norms could be used, but the most common ones in optimisation are \( l_1 \) and \( l_2 \) norms. The choice of \( l_2 \) norm will be discussed in section 4.4.

In the following discussion \( \sim \) is dropped from the variable \( v \).
Figure 4.2: Example 4.1.1 function plot

Figure 4.3: Virtual demand of $\tilde{v} = 2$ has an infinite number of solutions. Virtual demand of $\tilde{v} = 5$ has one unique solution and with a demand of $\tilde{v} = 7$ no solution is found
4.3 Constrained Optimisation

Consider a general problem formulation

$$\min_{x \in \mathbb{R}^n} f(u) \text{ subject to } \begin{cases} c_i(u) = 0, & i \in \mathbb{E} \\ c_i(u) \geq 0, & i \in \mathbb{I} \end{cases}$$ (4.17)

Where $\mathbb{E} \cap \mathbb{I} = \emptyset$. The function $f$ and the functions $c_i$ are all smooth, real-valued functions on a subset of $\mathbb{R}^n$, and $\mathbb{I}$ and $\mathbb{E}$ are two finite sets of indices. The function $f$ is called the objective function, while $c_i$, $i \in \mathbb{E}$ are the equality constraints and $c_i$, $i \in \mathbb{I}$ are the inequality constraints.

Now the notion of the active set is introduced.

4.3.1 Definition of the active set

The active set $\mathbb{A}(u)$ at any feasible $x$ consists of the equality constraint indices from $\mathbb{E}$ together with the indices for the inequality constraints $i$ for which $c_i(u) = 0$; that is,

$$\mathbb{A}(u) = \mathbb{E} \cup \{i \in \mathbb{I} : c_i(u) = 0\}$$ (4.18)

At a feasible point $x$, the inequality constraint $i \in \mathbb{I}$ is said to be active if $c_i(u) = 0$ and inactive if the strict inequality $c_i(u) > 0$ is satisfied.

In section 4.3.2 an example is given to explain the conditions for feasibility and optimality.

4.3.2 Example with one equality and two inequality constraints

The example is a two variable problem with single equality and two inequality constraints:

$$\min u_1 + u_2 \text{ subject to } \begin{cases} u_1 - u_2 - 1 = 0 \\ -u_1^2 - u_2^2 + 4 \geq 0 \\ -u_1^2 - u_2 \geq 0 \end{cases}$$ (4.19)

In terms of Eq. (4.17), $f(u) = u_1 + u_2$, $\mathbb{I} = \{2, 3\}$, and $\mathbb{E} = \{1\}$ and $c_1(u) = u_1 - u_2 - 1$, $c_2(u) = -u_1^2 - u_2^2 + 4 \geq 0$, $c_3(u) = -u_1^2 - u_2 \geq 0$. 

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Three cases are introduced in the next section. The first two cases explain how the initial point effects the reduction of minimisation of the objective function and the third case explains the properties of the solution.

4.3.2.1 Case 1
Consider the first case in which the point $u'$ is on the green line shown in Figure 4.4 so that the strict inequalities $c_2(u') > 0$ and $c_3(u') > 0$ hold. Any step in the direction of $d$ satisfying the following properties (Nocedal and Wright 1999) in Eq. (4.20) and Eq. (4.21) would decrease the objective function $f(u)$.

\[
\nabla c_1(u')^T d = 0 \quad (4.20)
\]

\[
\nabla f(u')^T d < 0 \quad (4.21)
\]

The selection of $d$ which satisfies the properties in Eq. (4.20) and Eq. (4.21) is given

\[
d = \frac{\hat{d}}{\|\hat{d}\|} \quad (4.22)
\]

where

\[
\hat{d} = \left(1 - \frac{\nabla c_1(u')^T \nabla c_1(u')}{{\|\nabla c_1(u')\|^2}}\right) \nabla f(u') \quad (4.23)
\]
4.3.2.2 Case 2

In this case $u'$ lies at the intersection of $c_1(u)$ and $c_3(u)$ (see Figure 4.5) such that $c_1(u) = 0$ and $c_3(u) = 0$ are valid and a strict inequality, $c_2(u) > 0$ holds for the second constraint. In order to decrease the objective function $f(u)$ the following properties must be satisfied.

$$\nabla c_1(u')^T d = 0$$

(4.24)
\[ \nabla c_3(u')^T d \geq 0 \quad (4.25) \]
\[ \nabla f(u')^T d < 0 \quad (4.26) \]

where the direction \( d \) is selected as
\[ d = -\nabla f(u') \quad (4.27) \]

which is satisfying the properties in Eqs. (4.24) to (4.26).

![Graph showing Case 2](image)

**Figure 4.5**: Case 2 showing the geometrical interpretation of direction \( d \) satisfying Eqs. (4.24) to (4.26)
4.3.2.3 Case 3

For the point $\mathbf{u}^*$ to be an optimal solution some conditions need to be satisfied. First the Lagrangian function is introduced for the general problem Eq. (4.17) as

$$ L(\mathbf{u}, \lambda) = f(\mathbf{u}) - \sum_{i \in \mathcal{E}} \lambda_i c_i(\mathbf{u}) - \sum_{k \in \mathcal{I}} \lambda_k c_k(\mathbf{u}) $$

(4.28)

The gradient of Lagrangian function is

$$ \nabla_{\mathbf{u}} L(\mathbf{u}, \lambda) = \nabla f(\mathbf{u}) - \sum_{i \in \mathcal{E}} \lambda_i \nabla c_i(\mathbf{u}) - \sum_{k \in \mathcal{I}} \lambda_k \nabla c_k(\mathbf{u}) $$

(4.29)

It should also be noted that if a feasible first order descent direction $\mathbf{d}$ does not exist at some point $\mathbf{u}^*$, then this point is the solution if it fulfils the following two conditions

$$ \nabla_{\mathbf{u}} L(\mathbf{u}^*, \lambda^*) = 0 \quad \text{with} \quad \lambda_k^* \geq 0 \quad \text{where} \quad k \in \mathcal{I} $$

(4.30)

$$ \lambda_k^* c_k(\mathbf{u}^*) = 0 \quad \text{where} \quad k \in \mathcal{I} $$

(4.31)

It is deduced from Eq. (4.31) for the equality constraint that $c_i(\mathbf{u}^*) = 0$ where $i \in \mathcal{E}$. The Eq. (4.31) is known as the complimentary condition; it implies that the Lagrange multiplier $\lambda_k$ can be strictly positive only when the corresponding constraint $c_k(\mathbf{u})$ is active. These conditions are necessary conditions for the point $(\mathbf{u}^*, \lambda^*)$.

Now geometrical interpretation of the solution for the example Eq. (4.19) is shown in Figure 4.6. It can be seen that Eq. (4.30) is satisfied when $\lambda_1$ and $\lambda_2$ are strictly positive. The complimentary condition Eq. (4.31) suggests that $c_1(\mathbf{u}^*) = 0$ and $c_2(\mathbf{u}^*) = 0$ and $\lambda_3 = 0$ which shows that $c_3(\mathbf{u}^*)$ is not active.
Figure 4.6: Case 3 geometrical interpretation of conditions of the solution at $u^*$

In the following first order necessary conditions are defined for solution point $(u^*, \lambda^*)$, but before defining these conditions linear independence constraint qualification (LICQ) is defined first.
4.3.3 Definition of LICQ

Given the point \( u \) and the active set \( A(u) \) (defined in Definition section 4.3.1), it is stated that the (LICQ) holds if the set of active constraint gradients \( \{ \nabla c_i(u), i \in A(u) \} \) is linearly independent (Nocedal and Wright 1999).

4.3.4 Theorem (first order necessary conditions)

Suppose that \( u^* \) is a local solution of Eq. (4.17), that the functions \( f \) and \( c_i \) in Eq. (4.17) are continuously differentiable and that the (LICQ) holds at \( u^* \). Then there is a Lagrange multiplier vector \( \lambda^* \) with components \( \lambda^*_i, i \in E \cup I \), such that the following conditions are satisfied at \( (u^*, \lambda^*) \)

\[
\nabla_{u^*} L(u^*, \lambda^*) = 0 \tag{4.32}
\]

\[
c_i(u^*) = 0 \quad \text{for all } i \in E \tag{4.33}
\]

\[
c_i(u^*) \geq 0 \quad \text{for all } i \in I \tag{4.34}
\]

\[
\lambda^*_i \geq 0 \quad \text{for all } i \in I \tag{4.35}
\]

\[
\lambda^*_i c_i(u^*) = 0 \quad \text{for all } i \in E \cup I \tag{4.36}
\]

The conditions defined in Eqs. (4.32) to (4.36) (Nocedal and Wright 1999) are often known as the Karush – Kuhn – Tucker conditions (KKT conditions). The condition Eq. (4.36) is a complimentary condition; and imply that either the constraint \( i \) is active or \( \lambda^*_i = 0 \), or possibly both. In particular, the Lagrange multipliers corresponding to inactive inequality constraints are zero, then it is possible to omit the terms for the indices \( i \notin A(u^*) \) for Eq. (4.32) and rewrite the condition as

\[
0 = \nabla_{u^*} L(u^*, \lambda^*) = \nabla f(u^*) - \sum_{i \in A(u^*)} \lambda_i \nabla c_i(u^*) \tag{4.37}
\]
4.3.5 Combinatorial difficulty of inequality – constrained problem

One of the main challenges in solving nonlinear programming problems lies in dealing with inequality constraints—in particular, in deciding which of these constraints are active at the solution and which are not. One approach, which is the essence of the active-set methods, starts by making a guess of the optimal active set $\mathbb{A}^*$, that is, the set of constraints that are satisfied as equalities at a solution. This guess is called the working set and is denoted by $\mathbb{W}$. Then the problem is solved in which the constraints in the working set are imposed as equalities and the constraints not in $\mathbb{W}$ are ignored. Then it is required to see if there is a choice of Lagrange multipliers such that the solution $\mathbf{u}^*$ obtained for this $\mathbb{W}$ satisfies the KKT conditions Eqs. (4.32) to (4.36). If so, $\mathbf{u}^*$ is selected as a local solution of Eq. (4.17). Otherwise, a different choice of $\mathbb{W}$ is considered and the process is repeated. This approach is based on the observation that, in general, it is much simpler to solve equality-constrained problems than to solve nonlinear programs (Nocedal and Wright 1999).

The number of choices for the working set $\mathbb{W}$ may be very large - up to $2^{||\mathbb{I}||}$, where $||\mathbb{I}||$ is the number of inequality constraints. This estimate is by observing that it is possible to make one of two choices for each $i \in \mathbb{I}$: to include it in $\mathbb{W}$ or leave it out. Since the number of possible working sets grows exponentially with the number of inequalities this phenomenon is referred to as the combinatorial difficulty of nonlinear programming.

This phenomenon is explained by the example given in section 4.7. There are two inequality constraints therefore the possible choice of the working set $\mathbb{W}$ is $2^2$ as it is known that the equality constraint is always in the working set. Now all the possible working set is considered for this problem.

1. First consider $\mathbb{W} = \{1\}$ which makes the problem as equality constrained problem (case 1, section 4.3.2.1). The first KKT condition Eq. (4.32) is not satisfied at this point. This means that a feasible descent direction exists.
2. Consider now $\mathbb{W} = \{1,2,3\}$ which is not possible in this case as three constraints do not share the common point of intersection. This can be seen in Figure 4.4, Figure 4.5 and Figure 4.6.
3. Consider now $\mathbb{W} = \{1, 3\}$ (case 2, section 4.3.2.2). As can be seen from Figure 4.5 $\lambda_3 < 0$ and KKT conditions are not satisfied.

4. Consider now $\mathbb{W} = \{1, 2\}$, (case 3, section 4.3.2.3) all KKT conditions are satisfied at this point so $\mathbf{u}^*$ is the solution point.

In the following it will be shown how to apply the first order necessary conditions for quadratic programs.

### 4.4 General Quadratic Problem Definition

The problem can be formulated mathematically as follows:

$$
\min_{\mathbf{u}} f(\mathbf{u}) = \|\mathbf{S}\mathbf{u} - \mathbf{q}\|
$$

$$
c_i(\mathbf{u}): \mathbf{B}_i\mathbf{u} = \mathbf{v} \quad \text{where } i = 1, 2, ..., k \text{ and } i \in \mathbb{E} \quad (4.38)
$$

$$
c_i(\mathbf{u}): \mathbf{C}\mathbf{u} \geq \mathbf{U} \quad \text{where } i = 1, 2, ..., 2m \text{ and } i \in \mathbb{I}
$$

where $k$ is the number of equality constraints and $m$ is the number of inequality constraints such that $\mathbf{C}_{2m \times 2} = \begin{bmatrix} \mathbf{I} & \mathbf{I} \end{bmatrix}$ and $\mathbf{U}_{m \times 1} = \begin{bmatrix} \mathbf{u} \\ \mathbf{u} \end{bmatrix}$. Eq. (4.38) defines the general constrained quadratic optimisation problem with $l_2$ norm. In many applications, the objective function is quadratic (such as to minimize the sum of squared errors in a linear system or a quadratic approximation of a nonlinear function). In this case it is first a case of minimizing the sum of the squared error and the constraint functions are linear (or affine). Such a formulation defines the quadratic programming problem.

The selection of $l_2$ norm is due to its good mathematical properties. It distributes the virtual demand among all the actuators available. The $l_2$ norm is a continuously differentiable function of the decision variable. The $l_2$ solution varies continuously with the problem parameters which is suitable for flight worthiness. If $\mathbf{S}$ is non-singular then Eq. (4.38) give a unique solution and
the objective function is then strictly convex which is not the case when defining the problem in $l_1$ norm (Härkegård 2003).

Thus, the goal of an optimisation algorithm is the solution of the quadratic program presented in Eq. (4.38) that is, search for a point, $\mathbf{u}^*$, such that the objective function $\| \mathbf{Su} - \mathbf{q} \|$ is minimised in addition to satisfying the set of constraints $\mathbf{Cu} \geq \mathbf{U}$ and $\mathbf{B}_c\mathbf{u} = \mathbf{v}$. Now an example is given to explain graphically the quadratic objective function, the constraints and the optimal and feasible solution.

### 4.4.1 Example for general quadratic program

Consider an example with $k = 1$ and $m = 2$ for the general problem defined in Eq. (4.38) and given the following formulation

\[
\min_{\mathbf{u}} f(\mathbf{u}) = \| \mathbf{Su} - \mathbf{q} \|
\]

\[
c_1(\mathbf{u}) : \mathbf{B}_c\mathbf{u} = \mathbf{v} \quad \mathbf{E} = \{1\}
\]

\[
c_i(\mathbf{u}) : \mathbf{C}\mathbf{u} \geq \mathbf{U} \quad \text{where } i = \{2,3,4,5\} \text{ and } i \in \mathbb{I}
\]

where $\mathbf{S} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\mathbf{v} = 2$ and $\mathbf{B}_c = \begin{bmatrix} 2 & 3 \end{bmatrix}$. Only two decision variables $\mathbf{u} = [u_1 \quad u_2]^T$ are considered to allow graphical display of various properties. In this problem $u_1$ and $u_2$ are constrained at $\mathbf{u} = [4 \quad 3]^T$ and $\mathbf{u} = [-4 \quad -3]^T$. It is should be noted that there is no rate limitation considered in this example. The objective function is represented itself as a family of circles (i.e. level curves) centred at (0,0). The linear constraints are represented by a straight line. These are shown in Figure 4.7. The black box shows the inequality constraints. It is clear from this figure that the minimum is achieved at $\mathbf{u}^*$, where the equality constraint is tangent to the circle of radius (0.55). In this problem there are no active inequality constraints.
4.4.2 Lagrange multipliers

In unconstrained optimisation, necessary and sufficient conditions for a minimum \( u^* \) are based on first and second order conditions for an unconstrained problem derived from Eq.(4.38).

\[
\begin{align*}
\text{gradient} \iff \nabla f(u^*) &= S^T(Su^* - q) = 0 \\
\text{hessian} \iff \nabla^2 f(u^*) &= S^T S \succeq 0
\end{align*}
\]
To generalize this concept of constrained optimisation, the notion of Lagrange multipliers is introduced. For \( u^* \) to be a minimum, no feasible descent direction must exist as explained in section 4.3.2.3.

4.4.2.1 Example for Lagrange multiplier in quadratic programming

Let us consider the problem formulation with an equality constraint only.

\[
\min_{u} f(u) = \|Su - q\| 
\]

\[
c_1(u) : B_c u = v \quad \mathcal{E} = \{1\} 
\]

where \( S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, q = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, v = 2 \) and \( B_c = [2 \ 3] \).

The feasible minimum point \( u^* \) must necessarily lie on that constraint. Let \( \delta \) represent an incremental feasible step in the direction \( d \) from the minimum point. Then the new position, \( u^* + \delta \), must also lie on the linear equality constraint. Thus,

\[
B_c(u^* + \delta) - v = 0 
\]

However, since \( u^* \) is a feasible point, it satisfies the equality constraint \( c_1(u) \). It therefore follows that

\[
B_c \delta = 0 
\]

Eq. (4.43) provides a means of identifying feasible directions. If in addition, the gradient \( \nabla f(u^*) \) has a negative slope along \( \delta \), that is

\[
\delta^T \nabla f(u^*) < 0 
\]

then the feasible directions along \( \delta \) will reduce \( f(u) \). However, since \( u^* \) is a local minimum this cannot occur, that is, no further feasible descent directions are possible at the minimum point. Thus, Eq. (4.43) and Eq. (4.44) cannot be satisfied simultaneously at the minimum point \( u^* \).
preceding statement will not be violated if $\nabla f(u^*)$ is a linear combination of the vector $\nabla c_1(u^*)$, that is,

$$\nabla f(u^*) = \lambda^* \nabla c_1(u^*)$$

(4.45)

Eq. (4.43) forms the necessary condition for a local minimiser. The coefficient $\lambda$ is referred to as the Lagrange multiplier. The superscript $^*$ indicates that the multiplier is associated with the minimum solution $u^*$.

These ideas are illustrated in Figure 4.8. Consider the point $u'$, which satisfies the equality constraint. At the point $u'$, which is not a local minimum point, $\nabla f(u') \neq \lambda^* \nabla c_1(u^*)$, since $\nabla f(u')$ and $\nabla c_1(u^*)$ are non-collinear. Thus, there exists an incremental feasible step, $\delta$, in the direction $d$ as shown in Figure 4.8 that will satisfy both Eq. (4.43) and Eq. (4.44). Thus, $d$ represents a feasible step in the descent direction. Taking this step will reduce the value of the objective function as is evident from Figure 4.8. On the other hand, at the minimum point, $u^*$, the gradient, $\nabla f(u^*) = \lambda^* \nabla c_1(u^*)$, satisfies Eq. (4.45) and hence no feasible descent direction exists.
Figure 4.8: The gradient of the objective function and constraints are collinear at the local minimum $u^*$. At any other non-stationary point, Eq. (4.45) is not satisfied

\[
L(u, \lambda) = f(u) - \sum_{i \in E} \lambda_i c_i(u) - \sum_{k \in W} \lambda_k c_k(u)
\]  

(4.46)

It can be noted that
\[ \nabla_u L(u, \lambda) = \nabla f(u) - \sum_{i \in E} \lambda_i \nabla c_i(u) - \sum_{k \in W} \lambda_k \nabla c_k(u) \]  

(4.47)

Eq. (4.47) can be conveniently written in terms of a problem statement Eq. (4.38) as

\[ \nabla_u L(u, \lambda) = S^T(Su - q) - [B_c^T \quad C_0^T] \lambda = 0 \]  

(4.48)

where \( C_0 \) contains the rows of \( C \) that correspond to constraints in the active working set \( W \) (Härkgård 2003a). The optimality of the solution is checked by the KKT conditions Eqs. (4.32) to (4.36).

It can also be shown (Fletcher 1987) that the Lagrange multiplier, \( \lambda_i \), of the \( i \)th constraint measures the rate of change in the objective function value, relative to changes in that constraint function. It is assumed that \( \lambda_i \) is a negative number for \( i \in A \) in the set of active inequality constraints. This implies that if the \( i \)th active inequality is relaxed, then the objective function decreases. Since at the minimum point \( u^* \) no further decrease in objective function is possible in the feasible region, the multiplier must have a non-negative value at the minimiser. Thus, at \( u = u^* \)

\[ \lambda_i \geq 0, \quad \forall i \in A \]  

(4.49)

This condition is very useful in evaluating the current set of active inequality constraints. Thus, Lagrange multipliers aid in the identification of constraints, which are not binding at a given feasible point.

### 4.5 Active Set Methods

Equality constraints force the minimum solution to lie on the intersection of the hyper-surfaces of those constraints, since \( u^* \) must satisfy \( B_c u = v \). However, inequality constraints do not necessarily require the solution to exist on the hypersurface of those constraints. The Lagrange method above involves active constraints (equality and active inequality constraints) only and searches along the intersection of these hyper-surfaces.
The primal active set method describes a method for identifying a correct set of active inequality constraints and temporarily disregards the remaining inequality constraints. The active constraints are treated as equality constraints. With this information, the KKT conditions Eqs. (4.32) to (4.36) are used to solve for the desired solution. Checks are made to ensure that the obtained solution is feasible with respect to the constraints not in the active set. If the solution is infeasible, then a new set of active constraints is formed based on certain criteria. An algorithmic description of the procedure is described below:

**4.5.1 Algorithm**

This algorithm is based on the algorithm described in (Nocedal and Wright 1999, Fletcher 1987). The algorithm is given in the following six steps:

1. An initial feasible point, \( u^1 \) is found (i.e. might be from a previous iterate) which satisfies the active constraints (i.e. inequality constraints which are active at \( u^1 \)) in the working set \( W^k \). Set \( k = 1 \).

2. Solve the following problem

   \[
   \min_\delta \| S(u^k + \delta) - q \| \\
   B_c \delta = 0 \\
   \delta_i = 0, \quad i \in W^k
   \]  

   If \( \delta \neq 0 \), go to (4), or else go to (3).

3. Compute Lagrange multipliers \( \lambda^k \) using Eq. (4.46) and solve for the minimum value of an active inequality constraint multiplier using,

   \[
   \min_{\lambda_i} \lambda^k_i 
   \]  

   Let \( \lambda_j \) represent the minimum value. If the minimum value is non-negative, then the solution \( u^k \) is a minimum and satisfies all constraints Eq. (4.48). Thus, set \( u^* = u^k \) and
terminate the program. On the other hand, if the minimum value is negative, the $j^{th}$ constraint is not binding and is removed from the current active set $\mathbb{W}^k$.

4. Take a step in the direction of $\delta$, by setting,

$$\mathbf{u}^{k+1} = \mathbf{u}^k + \alpha^k \mathbf{s}^k$$

(4.52)

where $\mathbf{s}^k$ represents the current search vector, $\delta$. $\alpha^k$ is chosen such that the new location $\mathbf{u}^{k+1}$ lies on the set of active constraints and this is calculated by,

$$\alpha^k = \min \left(1, \min_{i, i \notin \mathbb{W}^k} \frac{\mathbf{U}(i) - \mathbf{C}(i,:)^T \mathbf{u}^k}{\mathbf{C}(i,:)^T \mathbf{s}^k} \right)$$

(4.53)

where $(i,:)$ stands for $i^{th}$ row and all columns.

Let the minimum value in the bracket be satisfied by the $z^{th}$ constraint.

5. If $\alpha^k < 1$, add the $z^{th}$ constraint to the active set.

6. Set $k = k + 1$, and go to (2).

### 4.6 Multi-Branch Control Allocation Problem Definition

The control allocation problem is solved in two branches. In the first branch a set of feasible solutions is sought and then in the second branch, the closest value in the set of feasible solutions from the preferred values is determined (Buffington 1999).

#### 4.6.1 Control deficiency branch

The feasibility branch is formulated as follows:

$$\eta = \min_{\mathbf{u} \preceq \mathbf{u} \preceq \mathbf{P} \mathbf{v}} \| \mathbf{P} \mathbf{v} \mathbf{B}_c \mathbf{u} - \mathbf{v} \|^2_2$$

(4.54)

where $\mathbf{P}_v$ is the weighting matrix prioritizing virtual demands.
In this problem the $l_2$ norm is minimised over a set of actuator positions and rate limits. If $B_c u = v$ is satisfied, then the second branch is triggered or otherwise the minimum norm solution is the final outcome.

**4.6.2 Control sufficiency branch**

The redundancy branch is given by the following expression:

$$u = \min_{u \in \eta} \| P_u u - u_d \|_2^2$$

subject to

$$B_c u = v$$

$$u(t) \leq u(t) \leq \bar{u}(t)$$

where $P_u$ is the weighting matrix prioritizing individual actuators and $u_d$ is the vector of preferred values.

These two branches for solving the control allocation quadratic programming problem are built inside the active set method s algorithm. In the following section a simple example is given to provide a geometrical interpretation of the active set method.

**4.7 Application of Active Set Method – Simple Example**

**4.7.1 Control deficiency branch**

The control deficiency branch defined in Eq. (4.54) is rewritten as a problem of solving $u$ by minimizing $\| P_v B_c u - v \|$ subject to $Cu \geq U$.

$$\eta = \min_u \| P_v B_c u - v \|$$

subject to
\[ Cu \geq U \quad (4.57) \]

\[ C = \begin{bmatrix} 1_{m \times m} & \end{bmatrix} \]

\[ U = \begin{bmatrix} \frac{u}{-u} \end{bmatrix} \quad (4.58) \]

where \( B_c = [2 \ 3] \), \( P_v = 1 \), and \( v = 3.5 \), \( I_{m \times m} = \text{identity matrix} \), where \( m = 2 \) (i.e. number of control variable) \( u = [0 \ \ 1]^T \), \( \bar{u} = [1 \ \ 2]^T \). It should be noted that in this example the rate saturation is neglected.

For the active set method consider Eq. (4.38) as a general problem formulation, where

\[ S = P_v B_c \quad (4.59) \]

\[ q = P_v v \quad (4.60) \]

Substituting the above in Eq. (4.50) will give

\[ \min_{\delta} \left\| P_v B_c (u^k + \delta) - P_v v \right\| \]

\[ \delta_i = 0, \quad i \in A^k \quad (4.61) \]

Now the optimal perturbation \( \delta \) is found by minimizing the objective function \( f(\delta) \) subject to constraint \( c(\delta) \) by solving the following problem

\[ \min_{\delta} f(\delta) = \|\delta\| \]

\[ c_1(\delta) \iff S\delta = e \quad (4.62) \]

\[ \delta_i = 0, \quad i \in \mathbb{W}^k \]

where, \( e = q - Su^k \) is the residual vector. The problem in Eq. (4.62) has geometrical solution given by defining the gradient of the objective function \( f(\delta) \) as \( \nabla f(\delta^*) \) and the gradient of the
constraint \( c_1(\delta) \) as \( \nabla c_1(\delta^*) \). The geometrical interpretation of the solution \( \delta^* \) can be seen in Figure 4.9.

![Geometrical interpretation of solution to the problem Eq. (4.62)](image)

**Figure 4.9: Geometrical interpretation of solution to the problem Eq. (4.62)**

It can be seen from Figure 4.9 that the following first order condition is satisfied

\[
\nabla f(\delta^*) = \lambda^* \nabla c(\delta^*)
\]  

(4.63)

where \( \lambda^* \) is the lagrange multiplier and \( \lambda^* > 0 \)

Now \( \delta^* \) is added to the initial feasible point \( u^0 = [0.5 \quad 1.9]^T \), this initial point could be the solution from the previous iterate with working set \( \mathbb{W}^0 = \{ \} \) and gives \( u_\eta = u^1 = [0.0077 \quad 1.1615]^T \). At this point the feasibility with respect to the given constraints is checked.
If the solution is feasible, the next step is followed; otherwise, move to step 4 of the algorithm in section 4.5.1. (see Figure 4.10).

Figure 4.10: Solution to the first iteration shown in \( u \) coordinates

If the solution \( u_\eta = u^1 = [0.0077 \ 1.1615]^T \) satisfies the condition \( P_vB_c - P_vv = 0 \), then second branch (i.e. control sufficiency branch) is initialised. Otherwise the algorithm stops and the solution is \( u = u^1 = [0.0077 \ 1.1615]^T \). In this example \( P_vB_c - P_vv = 0 \) is satisfied and
the algorithm moves to the second branch in section 4.6.2. It should be noted that no inequality constraint is active in the first iteration.

## 4.7.2 Control sufficiency branch

Consider again Eq. (4.38) as general problem formulation for the control sufficiency branch where

\[ S = P_u \]  
(4.64)

\[ q = P_u u_d \]  
(4.65)

\[ C = \begin{bmatrix} I_{m \times m} \\ -I_{m \times m} \end{bmatrix} \]  
(4.66)

\[ U = \begin{bmatrix} u \\ -\bar{u} \end{bmatrix} \]

where \( P_u = I_{m \times m} \) and \( u_d = [0 \ 0]^T \). Substituting the above in Eq. (4.50) will give

\[
\min_{\delta} \| P_u (u^k + \delta) - P_u u_d \|
\]

\[ B_c \delta = 0 \]

\[ \delta_i = 0, \quad i \in \mathbb{W}^k \]

At this point no inequality constraints are active. Now the optimal perturbation \( \delta \) is found by minimizing objective function \( f(\delta) \) subject to constraint \( c(\delta) \) by solving the following problem

\[
 f(\delta) \triangleq \min_{\delta} \| \delta - e \|
\]

\[ c(\delta) \triangleq B_c \delta = 0 \]  
(4.68)

\[ \delta_i = 0, \quad i \in \mathbb{W}^k \]
where, $e = q - Su^k$ is the residual vector. The problem in Eq. (4.68) has a geometrical solution given by defining the gradient of objective function $f(\delta)$ as $\nabla f(\delta^*)$ and the gradient of the constraint $c_1(\delta)$ as $\nabla c_1(\delta^*)$. The geometrical interpretation of the solution $\delta^*$ can be seen in Figure 4.11.

![Figure 4.11: Geometrical interpretation of solution to the problem Eq. (4.68)](image)

In the second iteration the second branch Eq. (4.55) is solved without any active inequality constraints and gives $u_\eta + \delta = [0.5385 \quad 0.8077]^T$, which is an infeasible solution shown in...
Figure 4.12. Now step (4) of the algorithm is given in section 4.5.1 is activated to calculate $\alpha^k$ using Eq. (4.53) and then by using Eq. (4.52) results in $u^2 = [0.25, 1]$ and active set $W^1$ is updated with $u_2 = 1$. It can be seen in $u$ in Figure 4.13.

Figure 4.12: Infeasible solution to the second iteration shown in $u$ coordinates
Figure 4.13: Feasible solution after applying procedure in step 4 of the algorithm in section 4.5.1 at the end of second iteration shown in $u$ coordinates

In the third and final iteration again the following problem is solved

$$f(\delta) \equiv \min_{\delta} \|\delta - e\|$$

$$c(\delta) \equiv B_c\delta = 0$$

$$\delta_i = 0, \quad i \in \mathbb{W}^1$$

(4.69)
where \( e = q - Su^2 \) is the residual vector. This solution gives \( \delta = 0 \). Now the step 3 of the algorithm is used to calculate the Lagrange multipliers. The geometrical interpretation of the solution in terms of the Eq. (4.69) is explained in Figure 4.14. To show this inference the control sufficiency branch at the third iteration breaks down as

\[
\begin{align*}
&\quad u = \min_{u \in \eta} \| P_u u - u_d \|^2_2 \\
&\text{subject to} \\
&\quad c_1(u): B_c u = v \\
&\quad c_2(u): u_1 \geq 0 \\
&\quad c_3(u): u_2 \geq 1 \\
&\quad c_4(u): -u_1 \geq 1 \\
&\quad c_5(u): -u_2 \geq 2
\end{align*}
\]

(4.70)

At the solution the constraints \( c_1(u) \) and \( c_3(u) \) are active and satisfy Eq. (4.69). The geometrical deduction can be seen in

\[
\nabla_u L(u, \lambda) = S^T (Su - q) - [B^T_c \quad C^T_0] \lambda = 0
\]

(4.71)
Figure 4.14: Geometrical inference of the Eq. (4.69) at the solution $u^*$

So the solution at the second iteration is effectively the solution point which is confirmed by checking the first order necessary condition KKT at the solution point. The solution is shown in Figure 4.15 in $u$ coordinates.
In the following a control allocation algorithm is implemented on a nonlinear aircraft model and nonlinear simulation results are presented.

### 4.8 Closed Loop Analysis for B747-200 (Healthy Aircraft)

A nonlinear dynamic model of the Boeing 747-100/200 as discussed in chapter 3 is now considered for closed loop analysis of the healthy aircraft. The notations used in this section are given as follows:

- \( \delta_{aor} \) = right outboard aileron (deg)
- \( \delta_{air} \) = right inboard aileron (deg)
- \( \delta_{aol} \) = left outboard aileron (deg)
\(\delta_{ail}\) = left outboard aileron (deg)

\(\delta_{eor}\) = right outboard elevator (deg)

\(\delta_{eir}\) = right inboard elevator (deg)

\(\delta_{eol}\) = left outboard elevator (deg)

\(\delta_{eil}\) = left outboard elevator (deg)

\(\delta_{ih}\) = stabilizer (deg)

\(\delta_{ur}\) = upper rudder (deg)

\(\delta_{dr}\) = down rudder (deg)

\(p\) = roll rate about body x-axis (rad/s)

\(q\) = pitch rate about body y-axis (rad/s)

\(r\) = yaw rate about body z-axis (rad/s)

\(\dot{p}\) = roll angular acceleration in body x-axis (rad/s\(^2\))

\(\dot{q}\) = pitch angular acceleration in body x-axis (rad/s\(^2\))

\(\dot{r}\) = yaw angular acceleration in body x-axis (rad/s\(^2\))

\(V_T\) = true airspeed (m/s)

\(\alpha\) = angle of attack (rad)

\(\beta\) = side slip angle (rad)

\(\dot{V}_T\) = rate of change of true airspeed (m/ s\(^2\))

\(\dot{\alpha}\) = rate of change of angle of attack (rad/s)
\[ \dot{\beta} = \text{rate of change of side slip angle (rad/s)} \]

\[ \phi = \text{roll angle (rad)} \]

\[ \theta = \text{pitch angle (rad)} \]

\[ \psi = \text{yaw angle (rad)} \]

\[ \dot{\phi} = \text{rate of change of roll angle (rad/s)} \]

\[ \dot{\theta} = \text{pitch angle (rad/s)} \]

\[ \dot{\psi} = \text{yaw angle (rad/s)} \]

\[ v = \text{virtual control effort (rad/s}^2) \]

### 4.8.1 Trimming of B747-200

A trim point, also known as an equilibrium point, is a point in the parameter space of a dynamic system at which the system is in a steady state. In the case of aircraft steady state flight requires

\[ \dot{p}, \dot{q}, \dot{r}, \dot{V_T}, \dot{\alpha}, \dot{\beta} = 0 \]  \hspace{1cm} (4.72)

This also requires controls to be fixed. The aircraft model is trimmed at straight and level flight with a flight condition of \( V_T = 241 \) m/s and at 7000m height, with the flight path angle \( \gamma \) set to zero. For this flight condition the following optimisation problem Eq. (4.73) is solved using the \textit{fminsearch} command of Matlab to get the fixed aircraft control inputs. The solution of the trimming routine is trimmed control surfaces and the thrust of the engines. The control surfaces are all at zero except the stabilizer during steady state wing level flight.

\[ \min_{\text{Aircraft Controls}} p^2 + q^2 + r^2 + V_T^2 + \dot{\alpha}^2 + \dot{\beta}^2 \]  \hspace{1cm} (4.73)
4.8.2 Linearisation of B747-200

The aircraft is linearised around this searched equilibrium point by introducing the deviation $\Delta \mathbf{x} = \mathbf{x}_e - \bar{\mathbf{x}}$ and $\Delta \mathbf{u} = \mathbf{u}_e - \bar{\mathbf{u}}$. The linearisation will give a linear state space model of the aircraft with the system matrix $\bar{\mathbf{A}} \in \mathbb{R}^{n \times n}$, containing the stability derivatives, the control matrix $\bar{\mathbf{B}} \in \mathbb{R}^{n \times m}$, containing the control derivatives, output matrix $\bar{\mathbf{C}} \in \mathbb{R}^{p \times n}$ and feed through matrix $\bar{\mathbf{D}} \in \mathbb{R}^{p \times m}$ which is the null matrix in this case. The linear model is given as:

$$\Delta \dot{\mathbf{x}} = \bar{\mathbf{A}} \Delta \mathbf{x} + \bar{\mathbf{B}}_u \Delta \mathbf{u}$$

$$\Delta \dot{\mathbf{y}} = \bar{\mathbf{C}} \Delta \mathbf{x} + \bar{\mathbf{D}} \Delta \mathbf{u}$$

(4.74)

Where $\bar{\mathbf{x}} \in \mathbb{R}^n$ is the system state vector, $\bar{\mathbf{u}} \in \mathbb{R}^m$ is the control input vector to the system, and $\bar{\mathbf{y}} \in \mathbb{R}^p$ is the output vector of the system to be controlled. The state vector is

$$\bar{\mathbf{x}} = [p \quad q \quad r \quad V_T \quad \alpha \quad \beta \quad \phi \quad \theta \quad \psi]^T$$

and the input vector is

$$\bar{\mathbf{u}} = [\delta_{aor} \quad \delta_{aol} \quad \delta_{air} \quad \delta_{eor} \quad \delta_{eol} \quad \delta_{eir} \quad \delta_{eil} \quad \delta_{th} \quad \delta_{ur} \quad \delta_{dr}]^T.$$ The linear model is given by the following matrices.

$$\bar{\mathbf{A}} = \begin{bmatrix}
-0.82 & 0 & 0.32 & 0 & 0 & -3.48 & 0 & 0 & 0 \\
0 & -0.73 & 0 & -0.0009 & -1.2 & 0 & 0 & 0 & 0 \\
-0.03 & 0 & -0.15 & 0 & 0 & 1.29 & -0.0021 & 0 & 0 \\
0 & -0.08 & 0 & -0.0055 & 6 & 0 & 0 & -9.79 & 0 \\
0 & 1 & 0 & -0.0004 & -0.52 & 0 & 0 & 0 & 0 \\
0.02 & 0 & -1 & 0 & 0 & -0.10 & 0.041 & 0 & 0 \\
1 & 0 & 0.02 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$

$$\bar{\mathbf{B}}_u = 10^{-3} \begin{bmatrix}
-4.2 & 4.2 & -5.0 & 5.0 & 0 & 0 & 0 & 0 & 2.6 & 0.7 \\
-0.9 & -0.9 & -2.9 & -2.9 & -9.4 & -9.4 & -6.9 & -6.9 & -80.5 & -7.7 \\
-0.2 & 0.2 & -0.1 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.2 & -0.2 & -0.18 & -0.18 & -0.13 & -0.13 & -1.65 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 & 0.1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$
The dependency of $\dot{\alpha}$ and $\dot{\beta}$ on the control surfaces are neglected and give an approximated control matrix

$$
\mathbf{B}_u = 10^{-3}
$$

\[
\begin{bmatrix}
-4.2 & 4.2 & -5.0 & 5.0 & 0 & 0 & 0 & 0 & 0 & 2.6 & 0.7 \\
-0.9 & -0.9 & -2.9 & -2.9 & -9.4 & -9.4 & -6.9 & -6.9 & -80.5 & -7.7 & 5.8 \\
-0.2 & 0.2 & -0.1 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

And the approximate model is given as:

$$
\Delta \dot{x} = A \Delta x + \mathbf{B}_u \Delta u \\
\Delta y = C \Delta x + D \Delta u
$$

(4.75)

### 4.8.3 Control law design for B747-200

This control law is based on robust servomechanism design, which is a generalisation of proportional-plus-integral (PI) design. A PI controller is designed to stabilize the aircraft (Burken et al. 2001). This law is also treated as a baseline control law, whereas in the event of failure the redundant degrees of freedom are utilised to cancel the effect of the jammed surface using control allocation. The control matrix $\mathbf{B}_u$ is factored into

$$
\mathbf{B}_u = \mathbf{B}_v \mathbf{B}_c
$$

(4.76)

where the rank of $\mathbf{B}_u = k \leq m$, and $\mathbf{B}_u$ is $n \times m$, $\mathbf{B}_v$ is $n \times k$, and $\mathbf{B}_c$ is $k \times m$.

The system in Eq. (4.75) is given by:

$$
\dot{x} = Ax + \mathbf{B}_v \nu
$$

(4.77)
\[ v = B_c u \quad (4.78) \]
\[ y = Cx + Du \quad (4.79) \]

For the simplicity of notation \( \Delta \) is removed from the above expressions.

The controller dynamics are set to be
\[ \dot{x}_s = A_s x_s + B_s (r - y) \quad (4.80) \]
where \( x_s \in \mathbb{R}^p \) is the controller states, \( A_s \in \mathbb{R}^{p \times p} \) and \( B_s \in \mathbb{R}^{p \times p} \).

Consider the open loop system including the plant Eqs. (4.77) to (4.80) and gives Eq. (4.81) with \( r = 0 \).

\[
\begin{bmatrix}
\dot{x} \\
\dot{x}_s
\end{bmatrix} =
\begin{bmatrix}
A & 0 \\
-B_s C & A_s
\end{bmatrix}
\begin{bmatrix}
x \\
x_s
\end{bmatrix} +
\begin{bmatrix}
B_v \\
-B_s D
\end{bmatrix}
v
\quad (4.81)
\]

with \( v = [\dot{p} \quad \dot{q} \quad \dot{r}]^T \) and \( y = [\phi \quad \theta \quad \psi]^T \). The controllability of the augmented system Eq. (4.81) is checked by \( rank(C_g) = l \), where \( C_g = [B_g \quad A_g B_g \quad A_g^2 B \quad \cdots \quad A_g^{l-1} B_g] \), and \( n + p \).

The augmented system in Eq. (4.81) is controllable. Hence there exists a control law
\[ v = k x + k_s x_s \quad (4.82) \]
such that the closed loop system is stable.

The control law can be conveniently found by applying the LQR or Pole Placement techniques. Here the LQR approach is applied to Eq. (4.81). The weighting matrices for the LQR design are:

State weighting matrix, \( Q = diag([1 \quad 10 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1000 \quad 1]) \) and

Input weighting matrix, \( R = diag([1 \quad 5 \quad 1]) \)
In this special case $r$ is a constant command, therefore $A_s = [0_{3\times3}]$ and $B_s = [I_{3\times3}]$, according to their definitions. It can be seen from the controller dynamics given in Eq. (4.80) that $x_s = \int (r - y)dt = \int e dt$. This control law in Eq. (4.82) is simply a PI control law for the multi inputs and multi outputs (MIMO) system. The gain matrices $k$ and $k_s$ are:

$$k = \begin{bmatrix} -1.51 & 0 & -0.67 & 0 & 0 & 3.1 & -2.2 & 0 & 1.12 \\ 0 & -6.08 & 0 & 0.068 & 1.29 & 0 & 0 & -14.21 & 0 \\ -0.67 & 0 & -2.5 & 0 & 0 & -0.868 & -0.9208 & 0 & -3.86 \end{bmatrix}$$

$$k_s = \begin{bmatrix} 0.96 & 0 & -0.29 \\ 0 & 14.14 & 0 \\ 0.29 & 0 & 0.96 \end{bmatrix}$$

The control effectiveness matrix $B_c$ and virtual control matrix $B_v$ which are expressed in Eq. (4.76) are given as

$$B_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The three angular accelerations (i.e. roll, pitch and yaw accelerations) are used for the design of the control law. This selection conforms to the pilot expectation in terms of three classical controls of the aircraft (i.e. wheel, stick and pedals).

$$B_c = 10^{-3} \begin{bmatrix} -4.2 & 4.2 & -5.0 & 5.0 & 0 & 0 & 0 & 0 & 2.6 & 0.7 \\ -0.9 & -0.9 & -2.9 & -2.9 & -9.4 & -9.4 & -6.9 & -6.9 & -80.5 & -7.7 \\ -0.2 & 0.2 & -0.1 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In the design and implementation of the controller the aircraft dynamics are not decoupled into lateral/directional and longitudinal dynamics. The controller found has given a satisfactory performance, so the decoupling of the dynamics of the aircraft to design the controller is not taken up further. Moreover, the engine thrusts are not used in the design of control allocation as
it would add complexity which could be a topic of further research. The structural design limitation in terms of load factor for the B747 is considerably smaller than a highly manoeuvrable fighter aircraft. So it is better to control the position rather than the rotation rates (i.e. in roll, pitch, and yaw). In this way the aircraft will easily remain inside the design limits.

The control allocation is now designed by solving a multi-branch problem as given in section 4.5.1 using the active set method to distribute the online the virtual control (i.e. $\mathbf{v} = [\dot{p} \quad \dot{q} \quad \dot{r}]^T$) effort among the control surfaces optimally and feasibly.

### 4.8.4 Control allocation for B747-200

The control allocation module was designed first by formulating the problem and multi-branch problem and then solving these branches by the active set method as discussed before. The schematic for control allocation using the active set method is shown in Figure 4.16.
4.8.4.1 Control deficiency branch

A set of feasible solutions is calculated by using the active set method applied to the problem formulation in Eq. (4.54). It should be noted that $P_v^T P_v$ should be positive definite to ascertain unique solution. This condition is the second order condition mentioned in Eq. (4.40) for the hessian to be positive definite.

4.8.4.2 Control sufficiency branch

If the feasibility branch has generated a set of feasible solutions then the sufficiency branch Eq. (4.55) would trigger and the same second order condition having positivity of curvature is also applied to the priority matrix $P_u$. 

---

Figure 4.16: Control allocation by active set method
In the case of automatic updates of these priority matrices the resulting matrices must be projected towards the positive eigenvalues otherwise last best matrices should be selected.

The control allocation optimisation is solved online at the sampling time of 0.02 seconds. The closed loop results of the aircraft as follows. It can be seen from Figure 4.17 that the tracking performance of the aircraft is fine with the installed systems (i.e. control law and control allocation). The outboard right and left ailerons reaches there limits for this manoeuvre as shown in Figure 4.18 which further infer that during the time between 0 – 50 seconds the outboard right and left ailerons are in the active set. It should be noted that the lateral and directional dynamics are coupled. Symmetrical time responses of elevators have shown Figure 4.19 that the lateral/directional dynamics are decoupled from longitudinal dynamics. The time responses of stabilizer and rudders are given in Figure 4.20 and Figure 4.21, respectively.

![Figure 4.17: Time responses of reference trajectories and actual trajectories](image-url)
Figure 4.18: Time responses for ailerons with outboard right and left hitting the saturation limits.

Figure 4.19: Symmetrical time responses for elevators
Figure 4.20: Time response of stabilizer

Figure 4.21: Time response of rudders
4.9 Conclusions

This chapter reveals the application of the active set methods to solve the multi-branch quadratic program for civil aircraft. It is observed that the active set method has satisfied the rate and position constraints of the aircraft control surfaces. The multi-branch approach for control allocation has given a solution closer to the desired control surfaces positions, which could eventually satisfy some secondary performance criteria e.g. minimizing drag and, hence, reducing power consumption. This multi-branch approach is fruitful in estimating the control derivatives using system identification techniques. In general, this modular approach has shown that control allocation does not necessarily affect the closed loop properties of the system. If the control effectiveness matrix $B_c$ is known exactly, then the virtual demands are achieved and the tracking performance could be independent of the control allocation technique. But if the $B_c$ is not known exactly, then the closed loop performance could be the outcome of dynamic interaction between the control law and control allocation. The uncertainties in the estimate of control derivatives of $B_c$ are dealt with by the robustness of the control law. Tools to cope with uncertainty in $B_c$ will be presented in chapter 5, section 5.5
5 Fault-Tolerant Control Allocation

5.1 Introduction

A fault in a dynamical system is a divergence of the system structure or the system properties from the nominal condition (Blanke et al. 2006). Structural variations can be exemplified by a damaged actuator, a loss of an information link between system components or the failure of a sensor. All these structural anomalies will eventually induce variations in the set of interacting components of the plant and especially in the interface between the plant and controller. The variations in system properties in terms of the parametric uncertainty will be accounted for by wear or damage. The dynamical input output properties diverge from the nominal ones as a result of these faults and hence, change the performance of the closed-loop system which further results in the degradation or even loss of the system function. This section and its subsections are based on the concepts presented by Blanke et al. (2006) and discussed here in the framework of fault tolerant control allocation.

Section 5.1.1 gives the notion of faults and failures. Fault categorisation is given in section 5.1.2. Section 5.2 introduces the effects of the static control effectiveness matrix in control allocation. The validity of the linear static control effectiveness matrix for control allocation is shown to be null and void in this section. How this invalidity presents in the control to moment curve with lateral and directional coupling is explained in section 5.3. The need to automatically update $B_c$ is fully described in section 5.4. The open loop analysis of control allocation with static $B_c$ and dynamic update of $B_c$ is described in section 5.5. In the case of a failure, whether the aircraft can be retrimmed is checked in section 5.6. Closed loop analysis of damaged aircraft is given in section 5.7. In section 5.8 use of centre of gravity as a redundant pitch control is described. Section 5.9 deals with the effects of weighting on control allocation. Section 5.10 deals with a methodology to identify control stability and control derivatives of the aircraft. Finally conclusions are presented in section 5.11.
5.1.1 Faults and failure

Faults are those variations which change the properties of the component such that performance of the system diverges from the nominal one. For example, in Figure 5.1 a healthy system is given showing the feasible and infeasible regions for the control allocation problem. As soon as there is a fault (i.e. reduction in the range of operation) in the actuator $u_1$ the region of feasibility is reduced as shown in Figure 5.2.

However, the failure describes the incapability of the system or its components to perform their job. In Figure 5.3 complete loss of capability of $u_1$ will eventually result in the loss of redundancy in the whole system.

Control allocation as a fault tolerant system is given by an illustration shown in Figure 5.4. In the case of a healthy system, the solution lies on the intersection of the desired virtual demand line and the box constraint as depicted by $\star$. As the range of operation is decreased for $u_1$ the feasible region is reduced as well, then the solution will be in the degraded set of performance. So control allocation is reconfigured to find the solution on the intersection of the same line and the modified box constraint.
Figure 5.1: Healthy system feasible set

Figure 5.2: $u_1$ is faulty and has reduced its range of operation

Figure 5.3: $u_1$ is failed, there is no redundancy left in the system and only $u_2$ is operational

Figure 5.4: An abstract picture of control allocation as fault tolerant system
5.1.2 Fault categorisations

Faults are broken down in the following categories (Figure 5.5):

1. **Plant faults**: In this case the faults directly change the dynamical input/output properties of the system. In the linear case it changes the eigenvalues of the plant.

2. **Actuator faults**: The plant dynamics are not changed but the interaction between the plant and controller is disrupted or altered. In control allocation only faults and failure of actuators are dealt with. Control allocation would not change the plant dynamics as it affects the plant through control derivatives of the input matrix of the linearised plant.

3. **Sensor faults**: The plant properties are not influenced but sensor readings have substantial degradation in performance.

4. **Information link faults**: The plant characteristics are not affected explicitly. In this fault scenario communication links between different system components is degraded.

![Figure 5.5: Actuators, plant and sensors faults categories](image-url)
5.2 Static Control Effectiveness Matrix

The effects of the static control effectiveness matrix $B_c$ can be seen in the following example.

Defining a nonlinear function:

$$v(t) = h(u(t), a) = u_1 + 0.7a^2u_2$$

(5.1)

where $h: \mathbb{R}^m \mapsto \mathbb{R}^k$

Now linearizing $h(u(t), a)$ around $u_0$ and $a_o$ yields

$$h(u, a) \approx h(u_0, a_o) + \frac{\partial h}{\partial u}(u_0, a_o)(u - u_0)$$

(5.2)

where

$$B_c = \begin{bmatrix} \frac{\partial h}{\partial u_1}(u_{10}, a_o) & \frac{\partial h}{\partial u_2}(u_{20}, a_o) \end{bmatrix}_{[u_{10}=1]}$$

(5.3)

where $u_{10}$, $u_{20}$ and $a_o$ are the operating point of $u_1$ and $u_2$ around which the linearisation is performed. Now

$$\ddot{v} = B_c u$$

(5.4)

where

$$\ddot{v} = v - h(u_0, a_o)$$

(5.5)

where $\ddot{v}$ is the desired trajectory given in Figure 5.6, and the parameter $a$ is given as $-1 \leq a \leq 1$.

Find $u_1$ and $u_2$

$$\ddot{v} = u_1 + 1.4au_2$$

(5.6)

subject to
\[-1 \leq u_1 \leq 1\]
\[-1 \leq u_2 \leq 1\]

When the above control allocation problem formulation in Eq. (5.6) is solved at constant operating condition \(a_0 = 1\) it will give the following results as shown in Figure 5.6.

The static \(B_c\) has given a mismatch between the actual and the desired trajectory (the linear assumption in terms of static \(B_c\) is invalid). This can be seen in Figure 5.6. This problem is overcome by the continuous update of \(B_c\) with the different operating conditions which are specified by the scalar \(a\) and hence improve the control allocation results as shown in Figure 5.7. Control allocation for flight control with different flight conditions are given in the following sections. Before going into the details of control allocation open analysis, the insight into the nonlinear control surfaces to moment curve is necessary. This has been described in section 5.3.

![Figure 5.6: Desired trajectory compared with actual trajectory generated by static B](image-url)
5.3 Nonlinear Region of Control Surfaces to Moment Curve

Only the lateral and directional non-dimensional moment curves are given as a function of control surfaces angles at different angle of attack.

5.3.1 Lateral/directional non-dimensional derivatives as a function of aileron positions

The typical aerofoil is shown in Figure 5.8. The angle of attack with respect to the relative wind is illustrated. The position of the aileron with their maximum and minimum deflections are illustrated to show how the angle of attack changes the lift on the aerofoil and by deflecting the aileron from maximum to minimum positions will generate the rolling moment. The rolling of
the aircraft about the x-axis is shown in Figure 5.9. The yawing of the aircraft about the z-axis is also shown in Figure 5.9.

![Diagram of aerofoil with angle of attack](image1)

**Figure 5.8:** Typical aerofoil showing angle of attack with respect to relative wind (the maximum and minimum position of an aileron on the trailing edge).

![Diagram of aircraft with ailerons](image2)

**Figure 5.9:** Rolling of aircraft about x-axis by right inboard and outboard aileron

The non-dimensional rolling moment $C_l$ is shown as a function of aileron position and angle of attack in Figs. 5.10, 5.12, 5.14, 5.16. It can be seen from these figures how the angle of attack theoretically changes the slope of these curves which are essentially the coefficients of the control effectiveness matrix only in the lateral sense. This gives a measure of the dependency of $C_l$ on the flight condition for this specific nonlinear aircraft. In Figure 5.14 and Figure 5.16 it can be seen that for left and right inboard ailerons after a certain angle of attack there is a change in the sign of the slope. This non-monotonic nonlinearity may guide the operating range for the angle of attack in the commercial jets. The lateral and directional coupling can be seen by the non-dimensional yawing moment $C_n$ as a function of aileron position and angle of attack in Figs. 5.11, 5.13, 5.15, 5.17. In these figures it can be seen theoretically that the slope changes its
direction after some value of angle of attack. It can be seen from these figures how the angle of attack theoretically changes the slope of these curves which are essentially the coefficients of the control effectiveness matrix only in the directional sense.

**Figure 5.10**: $C_l$ as a function of left outboard ailerons and angle of attack

**Figure 5.11**: $C_n$ as a function of left outboard ailerons and angle of attack

**Figure 5.12**: $C_l$ as a function of right outboard aileron and angle of attack

**Figure 5.13**: $C_n$ as a function of right outboard aileron and angle of attack
5.3.2 Lateral/directional non-dimensional derivatives as a function of rudders positions

The side force generated by the aircraft by deflecting rudders would generate a yawing moment about the z-axis of the aircraft. The side forces generating yawing moments at a certain angle of attack by the incremental deflection of rudders from minimum to maximum positions would differ when the angle of attack is changed. The side force generated by deflecting rudders is shown in Figure 5.18.
The non-dimensional rolling moment $C_l$ is shown as a function of rudder position and angle of attack in Figs. 5.19 and 5.21. It can be seen from these figures how the angle of attack theoretically changes the slope of these curves which are essentially the coefficients of the control effectiveness matrix only in a lateral sense. In Figs. 5.19 and 5.21 after a certain angle of attack there is a change in the sign of the slope which introduce a non-monotonic nonlinearity theoretically. The non-dimensional rolling moment $C_n$ is shown as a function of rudder position and angle of attack in Figs. 5.20 and 5.22.
5.4 Automatic Update of Control Effectiveness Matrix

The control effectiveness matrix $B_c$ for one flight condition would not be valid for all conditions and needs to be recalculated for the whole flight envelope. The flight condition of $V_T = \SI{241}{m/s}$ and at $\SI{7000}{m}$ height with angle of attack $\alpha = 2 \text{ deg}$ is used for the control allocation. In control allocation only nonlinear non-dimensional moment equations are used to analyze the
performance of the open loop control allocation module. The example given in section 5.2 has shown that static allocation (by having a static control effectiveness matrix) would give an error between desired and actual trajectories.

### 5.4.1 Non-dimensional control derivatives (rolling moment)

The equation for change in the rolling moment coefficients for inboard/outboard ailerons (i.e. $\Delta C_{\delta_{ai}}$ and $\Delta C_{\delta_{ao}}$) are given in Eq. (3.15) and Eq. (3.16) In those equations the inboard/outboard aileron effectiveness factors (i.e. $K_{\delta_{ai}}$ and $K_{\delta_{ao}}$) shown in Figure 5.23 can be approximated by $K_{\delta_{ai}} = -0.0012\delta_{ai}^2 + 0.0745\delta_{ai}$ and $K_{\delta_{ao}} = -0.0019\delta_{ao}^2 + 0.0945\delta_{ao}$.

![Figure 5.23: Inboard and outboard ailerons effectiveness as function of inboard and outboard deflections](image)

\[
\Delta C_{l_{ai}} = \sum_{\text{left and right inboard ailerons}} (-0.0012\delta_{ai}^2 + 0.0745\delta_{ai}) \left(\frac{\Delta C_{l_{ai}}}{\Delta C_{l_{ai}}}\right)_{20} \left(\frac{\Delta C_{l_{ai}}}{\Delta C_{l_{ai}}}\right)_{M=0} \left(\frac{R_E}{R_R}\right) C_{l_{ai}} F_{lGE} \tag{5.7}
\]
\[ \Delta C_{ao} = \sum_{\text{left and right}} (-0.0019\delta_{ao}^2 + 0.0945\delta_{ao}) \Delta C_{lao} \frac{(\Delta C_{lao})_M}{(\Delta C_{lao})_M = 0} \left( \frac{R_E}{R_R} \right) C_{lao} F_{lGE} \]  
(5.8)

\[ \Delta C_{ao} = \sum_{\text{left and right}} (-0.0006\delta_{ao}^2 + 0.0554\delta_{ao}) \Delta C_{lao} \frac{(\Delta C_{lao})_M}{(\Delta C_{lao})_M = 0} \left( \frac{R_E}{R_R} \right) C_{lao} F_{lGE} \]  
(5.9)

The above equations are linearised at operating conditions \( V_e^T, \alpha_e \) and \( h_e \) and trimmed aileron deflections with subscript \( e \), which gives the following relationships in terms of control derivatives of the inboard and outboard ailerons.

\[
\frac{\partial \Delta C_{lai}}{\partial \delta_{ai}} = \sum_{\text{left and right}} \Delta C_{lai} \left. \frac{(\Delta C_{lai})_M}{(\Delta C_{lai})_M = 0} \left( \frac{R_E}{R_R} \right) C_{lai} \right|_{V_e^T, \alpha_e, h_e} \left( \delta_{ai} - \delta_{ai}^e \right) + (-0.0024\delta_{ai} + 0.0745) \delta_{ai} = \delta_{ai}^e
\]  
(5.10)

\[
\frac{\partial \Delta C_{lao}}{\partial \delta_{ao}} = \sum_{\text{left and right}} \Delta C_{lao} \left. \frac{(\Delta C_{lao})_M}{(\Delta C_{lao})_M = 0} \left( \frac{R_E}{R_R} \right) C_{lao} \right|_{V_e^T, \alpha_e, h_e} \left( \delta_{ao} - \delta_{ao}^e \right) + (-0.0038\delta_{ao} + 0.0945) \delta_{ao} = \delta_{ao}^e
\]  
(5.11)

\[
\frac{\partial \Delta C_{lao}}{\partial \delta_{ao}} = \sum_{\text{left and right}} \Delta C_{lao} \left. \frac{(\Delta C_{lao})_M}{(\Delta C_{lao})_M = 0} \left( \frac{R_E}{R_R} \right) C_{lao} \right|_{V_e^T, \alpha_e, h_e} \left( \delta_{ao} - \delta_{ao}^e \right) + (-0.0012\delta_{ao} + 0.0554) \delta_{ao} = \delta_{ao}^e
\]  
(5.12)

The terms marked blue would constitute the control derivatives for inboard and outboard ailerons.
The control derivatives by virtue of upper and lower rudders are now calculated, but it is first necessary to consider the following equations:

\[
\Delta C_{l_{ur}} = \left(\frac{-0.0003\delta_{ur}^2 + 0.0745\delta_{ur}}{K_{\delta_{ur}}^2} \right) \left(\Delta C_{l_{ur}}\right)_{25} \frac{\left(\Delta C_{l_{tr}}\right)_{M}}{M=0}
\]

(5.13)

\[
\Delta C_{l_{dr}} = \left(\frac{-0.0003\delta_{dr}^2 + 0.0745\delta_{dr}}{K_{\delta_{dr}}^2} \right) \left(\Delta C_{l_{dr}}\right)_{25} \frac{\left(\Delta C_{l_{tr}}\right)_{M}}{M=0}
\]

(5.14)

where \(K_{\delta_{ur}}\) and \(K_{\delta_{dr}}\) are upper and lower rudders effectiveness coefficients respectively (see Figure 5.24). These are approximated by a function \(K_{\delta_{ur}} = K_{\delta_{dr}} = \left(-0.0003\delta_{ur}/\delta_{dr} + 0.0745\delta_{ur}/\delta_{dr}\right)\).

The above equations are linearised at operating conditions \(V^T, \alpha_e\) and \(h_e\) and the trimmed rudder deflections are shown with subscript \(e\), which gives the following relationships in terms of control derivatives of the upper and lower rudders respectively.
\[
\frac{\partial \Delta C_{l_{ur}}}{\partial \delta_{ur}} = \Delta C_{l_{ur}} \bigg|_{(\delta_{ur}, \nu_e^T, \alpha, h_e)} + \left[ -0.0006 \delta_{ur} + 0.0745 |\delta_{ur}=\delta_{ure} \right]
\]
(5.15)

\[
\frac{\partial \Delta C_{l_{dr}}}{\partial \delta_{dr}} = \Delta C_{l_{dr}} \bigg|_{(\delta_{dr}, \nu_e^T, \alpha, h_e)} + \left[ -0.0006 \delta_{dr} + 0.0745 |\delta_{dr}=\delta_{dre} \right]
\]
(5.16)

The terms marked blue would constitute the control derivatives for the upper and lower rudders respectively.

### 5.4.2 Non-dimensional control derivatives (pitching moment)

The flight conditions used during this research work do not affect to a great extent the non-dimensional pitching moment coefficient of the aircraft. The changes in \(\Delta C_m\) with respect to the angular displacement of left inboard elevator \(\delta_{e_{il}}\) at angle of attacks in the range \(-5 \leq \alpha \leq 25\) can be seen in Figure 5.25. There is no significant change as compared to lateral dynamics which can be observed.
Figure 5.25: Change in pitching moment coefficient due to angular deflection of left inboard elevator at different angle of attacks

5.4.3 Non-dimensional control derivatives (yawing moment)

The non-dimensional yawing moment from ailerons is given by the following equations:

\[
\Delta C_{n_{ai}} = \sum_{\text{left and right inboard ailerons}} \left(-0.0012\delta_{ai}^2 + 0.0745\delta_{ai}\right) \left(\Delta C_{n_{ai}}\right)_{20} \frac{\left(\Delta C_{l_{ru}}\right)_{M}}{\left(\Delta C_{l_{ru}}\right)_{M=0}} F_{n_{GE}} \\
\]

(5.17)

\[
\Delta C_{n_{ao}} = \sum_{\text{left and right outboard ailerons upwards}} \left(-0.0019\delta_{ao}^2 + 0.0945\delta_{ao}\right) \Delta C_{n_{ao}} F_{n_{GE}} \\
\]

(5.18)

\[
\Delta C_{n_{ao}} = \sum_{\text{left and right outboard ailerons downwards}} \left(-0.0006\delta_{ao}^2 + 0.0554\delta_{ao}\right) \Delta C_{n_{ao}} F_{n_{GE}} \\
\]

(5.19)
where $\Delta C_{nai}$ and $\Delta C_{nao}$ are yawing moment coefficients for inboard and outboard ailerons respectively. The dimensionless yawing moment coefficient $(\Delta C_{nai})_{20}$ is generated by deflecting one inboard ailerons at 20 degrees upward or the other opposite 15 degrees downward as function of flaps and $\alpha_{wdp}$. The contributions to the yawing moment effects for the inboard ailerons due to Mach number is given by $\frac{(\Delta C_{lur})}{(\Delta C_{lur})_{\text{M}=0}}$. The ground effects are given by $F_{nGE}$.

The yawing moment coefficients due to upper and lower rudders are given by the following equations:

$$\Delta C_{nur} = (-0.0003\delta_{ur}^2 + 0.0745\delta_{ur})(\Delta C_{nur})_{25} \frac{(\Delta C_{nur})}{(\Delta C_{nur})_{\text{M}=0}}$$  \(5.20\)

$$\Delta C_{ndr} = (-0.0003\delta_{dr}^2 + 0.0745\delta_{dr})(\Delta C_{ndr})_{25} \frac{(\Delta C_{ndr})}{(\Delta C_{ndr})_{\text{M}=0}}$$  \(5.21\)

where $(\Delta C_{nur})_{25}$ and $(\Delta C_{ndr})_{25}$ are the yawing moments by fully deflecting upper and lower rudders and plotted as a function of $\alpha_{wdp}$. The aeroelastic effects by upper and lower rudders are given by $\frac{(\Delta C_{nur})}{(\Delta C_{nur})_{\text{M}=0}}$ and $\frac{(\Delta C_{ndr})}{(\Delta C_{ndr})_{\text{M}=0}}$.

The Eqs. (5.17) to (5.21) are linearised around the equilibrium point to give control derivatives in terms of the non-dimensional yawing moment.

### 5.5 Simulation Testing and Analysis for Open Non-Dimensional Aerodynamic Model

The open loop test is performed to see the performance of the control allocation algorithm at different flight conditions (see Figure 5.26) which cannot be seen in a closed loop analysis. The performance of control allocation in closed loop could be performed if dynamic inversion control law is utilised along with the control allocation algorithm (Doman and Ngo 2001).
It can be seen from Figure 5.27 that the mismatch in $C_l$ is more pronounced as the angle of attack moves towards the limits. The time trajectory of $\alpha$ is shown in Figure 5.28. As described in section 2.3.14, the desired moment input in the form of a sinusoid will give low gain piloting behaviour and the saw tooth demand with abrupt changes accounts for high gain piloting associated with the pilot induced oscillations. Hence the sinusoidal input is used for this test to emulate the commercial pilot. This analysis has shown at which point the control law will kick in to mitigate this mismatch in the case of static $B_c$ being employed. It should be noted that the model used is a commercial transport aircraft. For this aircraft the selection of angle of attack is sufficiently small (i.e. -5 to 10 deg) as compared to the fighter aircraft. Hence, for the simulation of the open loop control allocation, the range of angle of attack given is sufficient for this analysis.

Figure 5.26: Open loop simulation control allocation with nonlinear aerodynamic model
Figure 5.27: Non-dimensional desired rolling moment is tracked by static and updated $B_c$

Figure 5.28: Progression of angle of attack as the $C_l$ demand changes which is given in Figure 5.26
5.6 Retrimming of Aircraft after Damage

Before proceeding with the control allocation for damage adaptation, it is important to determine whether the aircraft can still be retrimmed with a particular aerosurface jammed at a given position. This approach is taken from (Burken et al. 2001). The post failure aircraft model as

\[
\dot{x} = Ax + B_{u_r}u_r + b_\delta \delta
\]  

(5.22)

where \( \delta \) is the jammed surface position, \( B_{u_r} \) is the post failure \( B_u \) matrix, \( u_r \) is the remaining control surfaces, and \( b_\delta \) is the control effectiveness vector corresponding to the jammed surface.

Let \( y_d \) represent the three body angular rates (i.e. roll, pitch and yaw) of the aircraft in the body fixed frame. Suppose that \( y_d = C_d x \), then

\[
y_d = C_d Ax + C_d B_{u_r}u_r + C_d b_\delta \delta
\]  

(5.22)

A necessary condition for retrimming the vehicle with the jammed surface is that the right hand side of the preceding equation can still be made to vanish at \( x = 0 \) with \( u_r \) in its allowable range. For the range of jammed position of aerosurface \( \delta \) for which retrimming is possible, the following linear programming (LP) problem is solved:

\[
\begin{align*}
\min_{u_r, \delta} & \quad \delta \\
\text{or} & \\
\max_{u_r, \delta} & \quad \delta
\end{align*}
\]  

(5.23)

subject to

\[
C_d B_{u_r}u_r + C_d b_\delta \delta = 0
\]  

(5.24)

\[
u_{r_{\min}} \leq u_r \leq u_{r_{\max}}, \quad \delta_{\min} \leq \delta \leq \delta_{\max}
\]  

(5.25)
The solution to the LP problem expressed in Eqs. (5.23) to (5.25) gives the minimum (most negative) or maximum jammed incremental position of $\delta$ that can be balanced at the trim condition by the remaining aerosurfaces $u_r$ within the saturation limits. The resulting range serves as a reasonable estimation within which the reconfigurable control can still possibly stabilize the system. In the present study, which concerned an inboard aileron jamming at full range (-20 to 20 degrees), it is required that the system can still be retrimmable and stabilizable.

### 5.7 Closed Loop Analysis for B747-200 (Damaged Aircraft)

In the event of complete failure the column corresponding to the failed surface is completely pulled out of the control effectiveness matrix (see Figure 5.29).

![Figure 5.29: The dynamics control effectiveness matrix in closed loop simulation](image)

However, if the control demand from the control law cannot be fulfilled, closed loop stability cannot be assured but the system does not necessarily become unstable. In this case closed loop stability is assured by constructing a control law, in terms of the virtual control signal, which stabilizes the system (sufficient condition). The control allocator merely distributes the total
control demand among the available effectors and, in principle, does not affect the closed loop behaviour. In this case the control demand is fulfilled by control law. And the nominal control law does not push the vehicle too hard for performance. The time response of trajectory tracking is given in Figure 5.30.

The time responses of control surfaces positions are given in Figure 5.31, Figure 5.32, Figure 5.33 and Figure 5.34. It can be seen from Figure 5.30 that lateral/directional dynamics is decoupled from longitudinal dynamics. When the left inboard aileron is jammed at $\delta_{ail} = -20$ deg the rolling angle tracking to second half of trajectory have less chattering than with the jamming at $\delta_{ail} = 20$ deg. In the first half of the trajectory all the lateral/directional controls are saturated at $\delta_{ail} = 20$ deg. If $\delta_{ail} = -20$ deg then its counterpart (i.e. $\delta_{air}$) would be allocated the same position angle of 20 deg to compensate for the jam. It should be noted that the base line control law is the same as described in section 4.8.3 when the damage is introduced. Figure 5.32 shows a symmetrical displacement of elevator for the tracking of $\theta$. In this analysis there is no switching taking place in real time, so there is no stability concern related to switching. As in control allocation the control effectiveness matrix $B_c$ (i.e. input matrix) is changed in the case of jamming of $\delta_{ail}$ and the column of $B_c$ corresponding to $\delta_{ail}$ is removed, and this jamming will be treated as an input disturbance. This input disturbance does not affect the closed-loop stability of the system.
Figure 5.30: Time response trajectory tracking with inboard aileron jammed from -20 degree to +20

Figure 5.31: Time responses of remaining ailerons with inboard left aileron jammed from -20 degrees to +20 degree
Figure 5.32: Time responses of elevators with left inboard aileron jammed from -20 degrees to +20 degrees

Figure 5.33: Time response of stabilizer with left inboard aileron jammed from -20 degrees to +20 degrees
5.8 Centre of Gravity as Redundant Pitch Control

The centre of gravity (c.g.) of an aircraft can theoretically be moved by moving fuel to-and-fro a trim tank installed in the tail of an aircraft; this cg movement is exploited as an additional pitch control redundancy. The c.g. movement is rate limited by 0.06% of mean aerodynamics chord (m.a.c.) per second. In the high speed region of the flight envelope, the full utilisation of the flight controls is not possible due to load factor limitations, so this redundancy can be used as a compensation system. The control system (i.e. control law and control allocation) is applied to the nonlinear model of the Boeing 747-200. The system was also tested on a nonlinear model with changes in longitudinal aerodynamics coefficients due to the wing damage, which is based on the damage that arose on the EL AL Flight 1862 accident near Schiphol airport close to Amsterdam. The reduced effectiveness of longitudinal control surfaces due to a loss of hydraulics in this crash is taken into account when the control allocation is performed.
5.8.1 Aerodynamic pitching model

The aerodynamic pitching model of the B747-200 is Eq. (5.26). The term $C_l(c_g - 0.25)$ measures the change in pitching moment due to variation in the cg with respect to the 25% of m.a.c. This term is utilised as the redundant pitch attitude control of the aircraft. The c.g. movement of a typical aerofoil is shown in Figure 5.35.

![Diagram of c.g. band of typical aerofoil](image)

Figure 5.35: c.g. band of typical aerofoil
\[ C_m = C_{mbasic} + (\Delta C_{m0.25})_{\alpha_{w.d.p}} + \Delta \left( \frac{dC_{m0.25}}{d\alpha} \right) \alpha_{w.d.p} + C_t(cg - 0.25) + \frac{dC_{m0.25}}{d\alpha} \frac{\dot{\alpha}}{2V_T} \]
\[ + \frac{dC_{m0.25}}{dn_x} n_x \]
\[ + K_d \left[ \frac{dC_{m0.25}}{d\delta_{th,f,r,l}} \delta_{th,f,r,l} + \frac{dC_{m0.25}}{d\delta_{ei}} \delta_{ei} + \frac{dC_{m0.25}}{d\delta_{eo}} \delta_{eo} \right] \tag{5.26} \]
\[ + \Delta C_{m0.25_{\text{spoiler}}} + \Delta C_{m0.25_{\text{inboard ailerons}}} + \Delta C_{m0.25_{\text{outboard ailerons}}} + \Delta C_{m0.25_{\text{landing gears}}} + \Delta C_{m0.25_{\text{ground effects}}} + \Delta C_{m0.25_{\text{sideslip}}} + \Delta C_{m0.25_{\text{rudder}}} + \Delta C_{m0.25_{\text{flap failure}}} \]

5.8.2 Fuel management system and control allocation (Flight control system)

The aircraft is equipped with several different fuel tanks and the fuel is transferred among these tanks during a flight. The transfer is regulated by different types of valves and this transfer has different routes to follow which gives structural redundancies in the system. An onboard program for the management and reconfiguration of the fuel system, which is linked to the flight control system, is considered here. Some aircraft are equipped with a fuel tank (trim tank) in the tail, especially those which are designed for the large intercontinental flights (e.g. A330 and A340). This tank helps to get a good aircraft trim angle along the flight (i.e. during cruise) by shifting the aircraft c.g. backwards through the transfer of fuel to the trim tanks (Jimenez et al. 2007). This will give 2% drag reduction in some aircraft. The control allocation with cg allocation through the fuel management system for the failure case is given in Figure 5.36. There is a limitation of this approach in terms of speed of movement of fuel but this could be another research question for future work. This approach was utilised to exploit any available redundancy in the case of failure.
5.8.3 Simulation testing and analysis

All the simulation testing is done on the nonlinear aircraft model. The results shown in this section are based on the longitudinal damage which occurred during the accident of EL AL 1862.

Control allocation of damaged aircraft based on EL AL Flight 1862 accident (Smaili 2000); the stabilizer rate of change is reduced by 50% due to the damage in hydraulic 3 and 4 and the total loss of the inboard left and outboard right elevators is considered in the control distribution scheme. The pitch attitude tracking using the control law and the control allocation on the damaged aircraft based on the accident of EL AL Flight 1862 is shown in Figure 5.37. The hydraulics 3 and 4 are responsible for left inboard and right outboard elevators respectively. During the EL AL Flight 1862 accident these hydraulics were lost which led to the loss of the corresponding surfaces. The distribution of the demand to the rest of elevator positions as a function of time are shown in Figure 5.38. The time response of the c.g. which is measured as the percentage of m.a.c. and the time response of the stabilizer positions in the event of the aforementioned damages are shown in Figure 5.39. The time responses of the non-dimensional
pitching moment from the c.g. position settings and the rest of system other than c.g. positions are shown in Figure 5.40. All the time responses are based on the careful selection of the desired position of the c.g. It is evident from the results that in the event of failure the longitudinal manoeuvre is highly affected by these desired positions. At the onset of a specific failure it is suggested that if the desired c.g. position is selected to be forward in the control allocation algorithm, the trajectory tracking performance may improve. It should be noted that the preferred values of control surfaces for the control allocation in the redundancy branch suggests a reduction of the secondary objective (i.e. drag). It can be observed in the selection of the desired c.g. position (i.e. forward or aft) the control allocation pushes the c.g. aft with different slopes. In the case of forward desired c.g. the slope is lower than the desired aft c.g. position shown in Figure 5.39. This shows that in the manoeuvring of the airplane the lower slope of c.g. gives more stability which is a well-known phenomenon.

Figure 5.37: Pitch attitude tracking with damage based on EL AL Flight 1862 accident.
Figure 5.38: Time responses of the inboard left and the outboard right elevators. The outboard right and the inboard left elevators effectiveness lost by EL AL Flight 1862 accident.

Figure 5.39: Time responses of the stabilizer positions with 50% rate reduction and the cg as % of m.a.c. based on EL AL Flight 1862 accident.
Figure 5.40: Time response of the aircraft non-dimensional pitching moment contribution due to the c.g. based on EL AL Flight 1862 accident

Figure 5.41: Time response of the aircraft non-dimensional pitching moment contribution due to anything other than c.g. based on EL AL Flight 1862 accident
5.9 Effect of Weighting on Control Allocation

The decision on prioritizing the control surfaces by using a weighting matrix is an important issue which may affect the solution. The selection of \( P_u \) to prioritize individual actuators may force the actuators not to saturate. In the knowledge of the author this area of research was explored in (Boskovic et al. 2002; Boskovic Mehra 2002; Kishore et al. 2006). The work from Boskovic is on the automatic update of the weighting matrix by using linear matrix inequalities with position and rate limitations. In this section the idea of how strong would be the effect of the weighting matrix in terms of the weighted pseudo inverse solution is explained. First consider a general problem formulation which is stated as minimizing \( P_u u \) over the range of \( u \) subject to \( B_c u = v \).

\[
\min_{u} P_u u \\
\text{subject to} \\
B_c u = v
\]

(5.27)

5.9.1 Example

Now consider an example where \( P_u = I \), \( B_c = [2 \ 1] \), \( v = 3 \), and \( u = [u_1 \ u_2]^T \). The control vector \( u \) is constrained between (-1 and 1) which is used later to check whether the weighted pseudo inverse solution is feasible or not. The weighted pseudo inverse solution given by

\[
u = P_u^{-1}B_c^T(B_cP_u^{-1}B_c^T)^{-1}v
\]

(5.28)

the solution to this example is \( u = [1.2 \ 0.6]^T \) and it can be seen that \( u_1 \) is violated. Now this 2D problem will be parameterised.

5.9.1.1 Parameterisation of Example

A tuning variable \( k \) is introduced in the priority matrix \( P_u \) at the diagonal element corresponding to the violated constraint.
\[ P_u = \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix} \]  
(5.29)

The other terms are parameterised as follows: \( u = [u_1 \ u_2]^T \), \( B_c = [b_{11} \ b_{12}] \).

\[ (B_c P_u^{-1} B_c^T)^{-1} = \frac{k}{b_{11}^2 + kb_{12}} \]  
(5.30)

\[ P_u^{-1} B_c^T = \begin{bmatrix} b_{11} \\ b_{12} \end{bmatrix} \]  
(5.31)

The solution is given as:

\[ u = \begin{bmatrix} b_{11} \\ \frac{b_{11}^2 + kb_{12}^2}{kb_{12}} v \\ \frac{b_{11}^2 + kb_{12}^2}{kb_{12}} v \end{bmatrix} \]  
(5.32)

There will be two equations and three unknowns as follows:

\[ u_1 = \frac{b_{11}}{b_{11}^2 + kb_{12}^2} v \]  
(5.33)

\[ u_2 = \frac{kb_{12}}{b_{11}^2 + kb_{12}^2} v \]  
(5.34)

Now the equation corresponding to the constraint violation is selected to calculate \( k \). The control variable is fixed at its constraint limit.

Substituting the corresponding values and setting \( u_1 \) at 1 in Eq. (5.33) yields in \( k = 2 \). The value of \( u_1 \) is selected at its limit because it has violated the constraint. Using Eq. (5.28) again with modified \( P_u = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \) will give \( u = [1 \ 1]^T \). This can be seen more clearly in the plot shown in Figure 5.42. It is noted that the dotted inequality constraint is not used to calculate the solution but used in post processing of the solution and it was made sure that the solution did not violate...
the constraint. It is clear from this example that in this specific problem the change in \( P_u \) has shifted the weighted pseudo inverse solution from infeasible to a feasible one.

\[
(2u_1 + 1u_2) - 3 = 0
\]

Figure 5.42: The effects of changing the \( P_u \) is illustrated by the changes in the level curves shapes. The solution is found at \((u_1 = 1\) and \(u_2 = 1\)) which is the required one by using the weighted least squares with modified \( P_u \).

**5.10 System Identification in Control Allocation**

In order for the control law to compensate for failures, damages and modelling errors, it is required to have accurate estimation of the control effectiveness matrix \( B_c \) during the flight. This requirement is fulfilled by on-line identification of \( B_c \). An efficient algorithm for this purpose has been published in (Doman and Ngo 2001) and successfully applied to the X-33 aircraft (in simulation). In this chapter the same methodology is expanded and applied for off-line
identification of the control effectiveness matrix $B_c$ and system matrix $A$ for the civil aircraft B747-100.

For successful identification of elements $B_c$ each control effector must be active at all times. In addition, each effector must move independently from each other, so that there is no correlation between movement of individual effectors, i.e. control deflections must be decorrelated. Decorrelated control deflections are a necessary requirement for system identification, since in this case a regressor matrix is well conditioned. One way to obtain decorrelated control deflections is to provide dithered effector commands that consist of an additive random signal, which is superimposed on the nominal effector command.

This section deals with the method of identifying the $B_c$ and $A$ in the control allocation algorithm without degrading the aircraft response. The vehicle having sufficient control authority during control allocation is excited in its null space by applying a dithering signal. In this way the vehicle response is not degraded. The null space excitation could also be applied for tuning the gains of the controllers of the system with redundant controls (i.e. more actuators than control variables).

An algorithm for the system identification in the null space is given in (Buffington et al. 1999) by decorrelating the control effectors commands for the valid estimation of control derivatives in real time. The static system identification algorithm which incorporates prior information of flight mechanics that enhanced the performance of the algorithm is presented in (Chandler et al. 1995).

For the identification of the control effectiveness matrix all control surfaces must move independently and decorrelated which necessitate a well conditioned regressor matrix for system identification. One way of fulfilling these conditions is to add the dither effector commands to nominal effector commands. This additive random signal in practice decorrelates control surfaces but on behalf of degraded vehicle response. As in general $B_c u = v$ when the dither signal is added to the nominal command then $B_c(u + udither) \neq v$. The remedy to this problem is to apply dither excitation in the null space of the control effectiveness matrix $B_c$, i.e.
\( \mathbf{B}_c \mathbf{u}_{\text{dither}} = \mathbf{0} \) so that \( \mathbf{B}_c (\mathbf{u} + \mathbf{u}_{\text{dither}}) = \mathbf{v} \). This is accomplished by randomly perturbing the vector \( \mathbf{u}_d \) in the control sufficiency branch. For this purpose the following linear programming problem is formulated:

Find \( \mathbf{u}_d \) by minimizing the objective function \( \mathbf{u}_d \mathbf{P}_u \mathbf{u}_d \) subject to the constraint \( \mathbf{B}_c \mathbf{u}_d = \mathbf{v} \)

\[
J = \min_{\mathbf{u}_d} \mathbf{u}_d \mathbf{P}_u \mathbf{u}_d
\]

subject to \( \mathbf{B}_c \mathbf{u}_d = \mathbf{v} \) \hspace{1cm} (5.35)

where

\[
\mathbf{P} = \mathbf{P}_u \mathbf{P}_r
\]

subject to \( \mathbf{P}_r = \text{diag}[10^{s_1} 10^{s_2} \ldots 10^{s_m}] \) \hspace{1cm} (5.22)

where \( \mathbf{s} = [s_1 \ s_2 \ \ldots \ s_m]^T \) is a vector of uniformly distributed random variables between -1 and 1. The solution to the weighted least squares problem described by Eq. (3.35) is

\[
\mathbf{u}_d = \mathbf{P}^{-1} \mathbf{B}_c^T (\mathbf{B}_c \mathbf{P}^{-1} \mathbf{B}_c^T)^{-1} \mathbf{v}
\]

(5.36)

In the control sufficiency branch \( \mathbf{u}_d \) is randomly changed and the constraint \( \mathbf{B}_c \mathbf{u}_d = \mathbf{v} \) is not violated. This method would ensure that the control surfaces are decorrelated and active without degrading the vehicle response.

### 5.10.1 Example

The control allocation problem is defined by \( \mathbf{B}_c = \begin{bmatrix} 2 & 1 \end{bmatrix}, \mathbf{v} = 1, \mathbf{u}_{\min} = \begin{bmatrix} -1 & -1 \end{bmatrix}^T \) and \( \mathbf{u}_{\max} = \begin{bmatrix} 1 & 1 \end{bmatrix}^T \). The other terms are \( \mathbf{u}_d \) from Eq. (5.36), \( \mathbf{P}_u = \mathbf{I} \) and \( \mathbf{P}_v = 1 \). The box constrained control space (i.e. \( \mathbf{u}_{\min} \leq \mathbf{u} \leq \mathbf{u}_{\max} \)) and solution line for \( \mathbf{B}_c \mathbf{u} = \mathbf{v} \) when \( v = 1 \) is shown in Figure 5.43.
The dotted line is the feasible set which will produce the same \( v \) with different combinations of controls \( u_1 \) and \( u_2 \). This is the null space of \( B_c \). The dither signal is used to calculate \( u_d \) by using Eq. (5.36).

Consider the following equation for identification:

\[
v(n) = au_1(n) + bu_2(n)
\]  
(5.37)

Putting \( n \) sampled measurements in Eq. (5.37) and setting \( z = 2 \), which is the number of control effectors, results in the following equation:

\[
\begin{bmatrix}
    v_1 \\
    v_2 \\
    \vdots \\
    v_n \\
\end{bmatrix} = \begin{bmatrix}
    u_{11} & \cdots & u_{1z_1} \\
    \vdots & \ddots & \vdots \\
    u_{n1} & \cdots & u_{nz_n}
\end{bmatrix} \begin{bmatrix}
    a \\
    b \\
\end{bmatrix}
\]  
(5.38)
where $\varphi$ denotes the $n \times 1$ vector of measured virtual demand. $H$ is a regressor matrix of measured control effector positions. The $z \times 1$ vector $\theta$, are control effectiveness matrix elements that need to be estimated. The least squares estimate for $\theta$ is given by

$$\theta = H^T (HH^T)^{-1} \varphi$$

(5.39)

The decorrelated control inputs in the null space of $B_c$ and the non-degraded virtual demand are shown in Figure 5.44. The method can be applied for the identification of the control effectiveness matrix $B_c$ in the case of a control effector failure which otherwise should be compensated by the robustness of the control law.

![Figure 5.44: Null space excitation with decorrelated control inputs and non-degraded actual virtual demand produced through control allocation](image)

**5.10.2 Identifying stability and control derivatives**

In the identification of the stability and the control derivatives only the lateral/directional equations of motion of the aircraft are presented to explain the working principal of the algorithm.
5.10.2.1 Derivatives for roll angular acceleration

The roll acceleration equation defined in the body axis frame:

\[ \dot{p} = \dot{p}_p p + \dot{p}_r r + \dot{p}_\beta \beta + \dot{p}_{air} \delta_{air} + \dot{p}_{ail} \delta_{ail} + \dot{p}_{aor} \delta_{aor} + \dot{p}_{aol} \delta_{aol} + \dot{p}_{ur} \delta_{ur} + \dot{p}_{dr} \delta_{dr} + \cdots (5.40) \]

where, the roll acceleration coefficient due to roll rate \( p \), yaw rate \( r \) and the side slip angle \( \beta \) are \( \dot{p}_p \), \( \dot{p}_r \) and \( \dot{p}_\beta \) respectively. The roll acceleration due to the right and left inboard aileron positions \( \delta_{air} \) and \( \delta_{ail} \) and the left and right outboard aileron positions \( \delta_{aor} \) and \( \delta_{aol} \) and the upper and lower rudder positions \( \delta_{ur} \) and \( \delta_{dr} \) are \( \dot{p}_{air} \), \( \dot{p}_{ail} \), \( \dot{p}_{aor} \), \( \dot{p}_{aol} \), \( \dot{p}_{ur} \) and \( \dot{p}_{dr} \) respectively.

5.10.2.2 Derivatives for yaw angular acceleration

The yaw acceleration equation defined in the body axis frame:

\[ \dot{r} = \dot{r}_p p + \dot{r}_r r + \dot{r}_\beta \beta + \dot{r}_{air} \delta_{air} + \dot{r}_{ail} \delta_{ail} + \dot{r}_{aor} \delta_{aor} + \dot{r}_{aol} \delta_{aol} + \dot{r}_{ur} \delta_{ur} + \dot{r}_{dr} \delta_{dr} + \cdots + \text{higher order terms} (5.41) \]

where, the yaw acceleration coefficient due to roll rate \( p \), yaw rate \( r \) and the side slip angle \( \beta \) are \( \dot{r}_p \), \( \dot{r}_r \) and \( \dot{r}_\beta \) respectively. The yaw acceleration due to the right and left inboard ailerons positions \( \delta_{air} \) and \( \delta_{ail} \) and the right and left outboard aileron positions \( \delta_{aor} \) and \( \delta_{aol} \) and the upper and lower rudder positions \( \delta_{ur} \) and \( \delta_{dr} \) are \( \dot{r}_{air} \), \( \dot{r}_{ail} \), \( \dot{r}_{aor} \), \( \dot{r}_{aol} \), \( \dot{r}_{ur} \) and \( \dot{r}_{dr} \) respectively.

5.10.3 Simulated identification results

In system identification the data is collected by applying dither signal in calculating the desired control in the null space of the control effectiveness matrix by applying Eq. (5.39). These desired position of control surfaces are used in a control redundancy check branch of the algorithm. The data collection in the identification is shown in Figure 5.45. The decorrelated control surfaces and unaffected control variables \( \psi \) and \( \phi \) are shown in Figure 5.46 and Figure 5.47 respectively. The method outlined in section 5.10 was followed to produce the following results presented in Table 5.1 and Table 5.2. It should be noted that any discrepancies in the results may be due to the
linear interpolation used in the lookup tables in the nonlinear dynamic model of the aircraft. Also the rudders and ailerons are rate and position constrained in the control allocation optimisation.

Table 5.1: Comparison of the results in lateral dynamics which are obtained by system identification are compared to the desired values

<table>
<thead>
<tr>
<th>Coefficient due to roll acceleration $\dot{p}$</th>
<th>Desired</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{p}_p$</td>
<td>-0.8242</td>
<td>-0.8020</td>
</tr>
<tr>
<td>$\dot{p}_r$</td>
<td>0.3211</td>
<td>0.4196</td>
</tr>
<tr>
<td>$\dot{p}_\beta$</td>
<td>-3.4843</td>
<td>-3.4550</td>
</tr>
<tr>
<td>$\dot{p}_{air}$</td>
<td>-0.0042</td>
<td>-0.0042</td>
</tr>
<tr>
<td>$\dot{p}_{ail}$</td>
<td>0.0042</td>
<td>0.0042</td>
</tr>
<tr>
<td>$\dot{p}_{aor}$</td>
<td>-0.005</td>
<td>-0.0045</td>
</tr>
<tr>
<td>$\dot{p}_{aol}$</td>
<td>0.005</td>
<td>0.0048</td>
</tr>
<tr>
<td>$\dot{p}_{ur}$</td>
<td>0.0026</td>
<td>0.0025</td>
</tr>
<tr>
<td>$\dot{p}_{dr}$</td>
<td>0.0007</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

Table 5.2: Comparison of the results in directional dynamics which are obtained by system identification are compared to the desired values

<table>
<thead>
<tr>
<th>Coefficient due to yaw acceleration $\dot{r}$</th>
<th>Desired</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{r}_p$</td>
<td>-0.0259</td>
<td>-0.0298</td>
</tr>
<tr>
<td>$\dot{r}_r$</td>
<td>-0.1539</td>
<td>-0.1656</td>
</tr>
<tr>
<td>$\dot{r}_\beta$</td>
<td>1.2884</td>
<td>1.2706</td>
</tr>
<tr>
<td>$\dot{r}_{air}$</td>
<td>-0.0002</td>
<td>-0.0002</td>
</tr>
<tr>
<td>$\dot{r}_{ail}$</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\dot{r}_{aor}$</td>
<td>-0.0001</td>
<td>-0.0003</td>
</tr>
<tr>
<td>$\dot{r}_{aol}$</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\dot{r}_{ur}$</td>
<td>0.0076</td>
<td>0.0076</td>
</tr>
<tr>
<td>$\dot{r}_{dr}$</td>
<td>-0.0058</td>
<td>-0.0058</td>
</tr>
</tbody>
</table>
Figure 5.45: Data collection for system identification

Figure 5.46: Non-degraded time response of yaw angle $\psi$.

Figure 5.47: Non-degraded time response of roll angle $\phi$. 
Figure 5.48: Time responses of decorrelated rudder positions with rate and position constraints

Figure 5.49: Time responses of decorrelated ailerons positions with rate and position constraints
5.11 Conclusions

This chapter describes the implementation of techniques and algorithms in fault tolerant control allocation for civil aircraft, including a fast update of control effectiveness matrix, fault accommodation in control allocation, motion of centre of gravity as a redundant pitch control, the effect of weighting on control allocation and system identification by null space excitation of desired control vector.

In the open loop analysis, it can be seen that by changing the flight conditions, static $B_c$ does not fulfil the virtual demand exactly and its elements need to be updated. The most influential state variable in this study was found to be the angle of attack. The variations in the angle of attack in this open loop simulation were taken to conform to the model used, which is a commercial aircraft. This analysis comes under research objective 3 (i.e. to update control derivatives for control allocation) in section 1.4. It is seen in the closed loop analysis that, when the aircraft has encountered a failure (i.e. a jammed surface), the successful reallocation of remaining control surfaces was done through a control allocation scheme with the same baseline control law. It is observed that the closed loop stability is not affected by this damage, as this jamming is considered to be an input disturbance which does not affect the closed loop poles of the system. It was shown that for this case of jamming that there was sufficient control authority available for this type of aircraft undergoing this hard manoeuvre, which is exploited using control allocation. Another important test in the case of jamming is that the aircraft is still able to retrim by the remaining control surfaces. It was checked that for this particular jamming in the aircraft, it was still retrimmable for all possible jamming of this control surface (i.e. $\delta_{ail} = \pm 20$ deg). These observations conforms to the second objective of the thesis (i.e. to use control redundancies to compensate for a failure without changing the baseline control law). How centre of gravity could be used as a redundant pitch attitude control was investigated. The limitation was the rate constraint on the movement of the c.g., or, in other words, how fast fuel can be moved in the aircraft. In the second branch of control allocation the solution is made as close as possible to the desired values (i.e. control surfaces to be zero and c.g. positions to be forward or aft). In both cases of desired c.g. positions (i.e. c.g. forward or aft) control allocation has shifted the c.g. towards aft with different progression of c.g. movement slopes. This satisfies the
secondary objective in control allocation (i.e. minimisation of the drag on the aircraft), which is programmed in the second branch of control allocation. This study relates to the fifth objective of the thesis (i.e. to change the centre of gravity by moving fuel to have an additional pitch attitude control of the aircraft). The effects of control surface prioritisisation were studied in the two variables problem. It was shown that the weighted least squares are affected in the case of the weighting matrix as the identity matrix and has given an infeasible solution. In the case of the non identity matrix but positive definite matrix a feasible solution is found. The geometrical interpretation is also given for this case. This has proven a considerable effect in the case of the two variable problem of weighted least squares problem. This could be another area for further research (i.e. to update the weighting matrix in control allocation). Identifying the control derivatives of $B_c$ in the case of fault/failure is required in some cases. In the case of a commercial aircraft, the load factor is an important variable to be considered. The excitation signal for system identification should not degrade the aircraft response and the aircraft should be within the load factor limits. This methodology of system identification by exciting the aircraft in the null space of $B_c$ has given a decorrelated regressor matrix which is a necessary condition for least squares, and also the aircraft performance is not degraded. This study is effectively the fourth objective of the thesis (i.e. to identify control derivatives for control allocation).
6  Application of Evolutionary Computing in Control Allocation

6.1 Introduction

This chapter is based on the issue H (i.e. actuator dynamics) identified in chapter 2. This issue deals with the interaction of control allocation and actuator dynamics and has been dealt with by very few researchers. What was not considered in most control allocation algorithms is the fact that the control surfaces are manipulated by either hydraulic or electric actuators, and constitute a dynamic system which cannot produce infinite accelerations. In other words, if a control was initially at rest, and later commanded to move at its maximum rate in some direction for a specified amount of time, it would gradually build up speed until it reached the commanded rate. The final position of the control would therefore not be the same as that calculated using the commanded rate and the time during which it was instructed to move (Bolling 1997). In this thesis work, a method, which post-processes the output of a control allocation algorithm, is developed to compensate for actuator dynamics. The method developed is solved for a diagonal matrix of gain corresponding to individual actuators. This matrix is then multiplied with the commanded change in control effector settings as computed by the control allocator and actuators dynamics interactions. The basic premise of this method is to post process the output of the control allocation algorithm to overdrive the actuators so that at the end of a sampling interval the actual actuator positions are equivalent to the desired actuator positions (Oppenheimer and Doman 2004). The overdriving of the actuators is done by multiplying the change in commanded signal with the identified gain matrix which is called the compensator. This identification is done by using soft computing technique (i.e. genetic algorithms). The simulation setup including control allocator block, compensator and actuator rig makes a nonlinear set up. During the identification of the compensator using this setup by soft computing, the likelihood of the solution being a global minimum is high as compared to other optimisation techniques. The main contribution is to design a compensator using an evolutionary computing technique (i.e. genetic algorithms) to compensate the interaction between control allocation and
actuator dynamics. It should be mentioned that in this method the model of the actuator does not need to be known. The simulation setup consists of excitation signals, the control allocation block, the compensator and the actuators rig.

When designing control allocation typically the actuator dynamics are ignored because the bandwidth of the actuators is larger than the frequencies of the rigid body modes of the aircraft. Figure 6.1 shows a control allocator with actuator dynamics neglected. If there is a case in which actuator frequencies are comparable with the bandwidth of the rigid body modes then the actuator dynamics cannot be neglected, as shown in Figure 6.2.

In this case the output of the control allocator should match the output of the actuator dynamics. In reality the output of the control allocation is attenuated due to the presence of non-negligible
actuator dynamics. The loss of the gain from the CA output signal is compensated by the scheme shown in Figure 6.3. In the second order dynamics of the actuator the rate could be estimated using a Kalman filter if the rate sensing is not available. The Kalman filter is an efficient recursive filter that estimates the state of a dynamic system from a series of noisy measurements. The matrix of gains as shown in Figure 6.3 is tuned offline using genetic algorithms (GA). The structure of the compensator is taken from (Oppenheimer and Doman 2004).

Section 6.2 describes the interaction of first order actuator dynamics and control allocation and the structure of the compensator is established in this section for first order actuator dynamics. Similarly, in section 6.3 the structure of the compensator is established for second order actuator dynamics. In section 6.4 tuning of the compensator parameters using genetic algorithm is described. In section 6.5 simulation and results for a tuned compensator are shown for a range of first and second order actuator dynamics. Finally, in section 6.6 some conclusions are established. The author has published the following paper in this area:
Control allocation with actuator dynamics for aircraft flight controls, ” Ahmad, H., Young, T. M., Toal, D., Omerdic, E., 7th AIAA Aviation Technology, Integration and Operations Conference (ATIO), September 2007, Belfast, Northern Ireland.

6.2 First-Order Actuator Dynamics Interaction

In this section, the effects of first-order actuator on the system are shown in Figure 6.2 (Oppenheimer and Doman 2004). Let the dynamics of a single actuator be represented by a continuous time first order transfer function of the form

\[
\frac{u(s)}{\tilde{u}_{cmd}(s)} = \frac{a}{s + a} \tag{6.1}
\]

The discrete time solution to the first-order actuator dynamic equation for one sample period is given by

\[
u(kT + T) = e^{-aT} u(kT) + \int_{kT}^{kT+T} e^{-a(kT+T-\tau)} \tilde{u}_{cmd}(\tau) d\tau
\]

where \( T \) is the sampling time. This result does not depend on the type of hold because \( u \) is specified in terms of its continuous time history, \( \tilde{u}_{cmd}(t) \) over a sample interval (Franklin et al. 1998). The most common hold element is zero-order hold (ZOH) with no delay, i.e.

\[
\tilde{u}_{cmd}(\tau) = \tilde{u}_{cmd}(kT), \quad kT \leq \tau \leq kT + T \tag{6.3}
\]

Performing substitution

\[
\gamma = kT + T - \tau \tag{6.4}
\]

in Eq. (6.2) yields

\[
u(kT + T) = e^{-aT} u(kT) + \int_{0}^{T} e^{-ay} \tilde{u}_{cmd}(kT) dy \tag{6.5}
\]
Defining

\[
\Phi = e^{-aT}
\]

\[
\Gamma = \int_0^T e^{-\gamma} d\gamma
\]  

(6.6)

Eq. (6.5) can be written as a difference equation of standard form

\[
u(k + 1) = \Phi u(k) + \Gamma \bar{u}_{cmd}(k)
\]

(6.7)

The signal \(\bar{u}_{cmd}(k)\), is held constant over each sampling period. The command to actuator is given by

\[
\bar{u}_{cmd}(k) = \Delta u_{cmd}(k) + u(k)
\]

(6.8)

The command increment change in actuator position over one sample as shown in Figure 6.4 is defined by

\[
\Delta u_{cmd}(k) \triangleq u_{cmd}(k) - u(k)
\]

(6.9)

Figure 6.4: Command increment change in actuator position with gain matrix M equal to Identity matrix I of dimension (11X11)
where \( u_{\text{cmd}} \) is the actuator command coming from the control allocator. Since the effector commands are held constant for one sample period then \( \Delta u_{\text{cmd}}(k) \) appear to be a step command from the measured position \( u(k) \). Substituting Eq. (6.8) in Eq. (6.7) gives

\[
u(k + 1) = \Phi u(k) + \Gamma(\Delta u_{\text{cmd}}(k) + u(k)) \tag{6.10}
\]

If \( \Gamma < 1 \), the increment command signal from the control allocation algorithm, \( \Delta u_{\text{cmd}}(k) \) is attenuated by the actuator dynamics, thus \( u(k + 1) \neq u_{\text{cmd}} \). The objective is to find the gain \( M \) that changes the output of the control allocation algorithm such that \( u(k + 1) = u_{\text{cmd}} = \Delta u_{\text{cmd}}(k) + u(k) \) (Oppenheimer and Doman 2004). Hence

\[
u(k + 1) = \Phi u(k) + \Gamma(M\Delta u_{\text{cmd}}(k) + u(k)) \tag{6.11}
\]

The gain \( M \) is tuned by using the genetic algorithm in section 6.4.2. If there is a bank of first order actuator dynamics, then the gain \( M \) is chosen to be a diagonal matrix \( M \) of dimensions (11x11), as shown in Figure 6.3 and Figure 6.4.

### 6.2.1 Example showing effect of first-order actuator dynamics

Let us consider an example with

\[
B_c = 10^{-3} \begin{bmatrix}
-4.2 & 4.2 & -5.0 & 5.0 & 0 & 0 & 0 & 0 & 0 & 2.6 & 0.7 \\
-0.9 & -0.9 & -2.9 & -2.9 & -9.4 & -9.4 & -6.9 & -6.9 & -80.5 & -7.7 & 5.8 \\
-0.2 & 0.2 & -0.1 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Position in (deg) and rate in (deg/s) constraints are defined as follows:

\[
u_{\text{min}} = [-20 -20 -12 -12 -23 -23 -23 -12 -25 -25]^T \tag{6.12}
\]

\[
u_{\max} = [20 20 15 15 17 17 17 3 25 25]^T
\]
\[
\dot{\rho}_{\text{min}} = [-45 -45 -45 -45 -37 -37 -37 -0.5 -50 -50]^T \\
\dot{\rho}_{\text{max}} = [45 45 45 45 37 37 37 0.5 50 50]^T
\] (6.13)

First the time response of control allocation without actuator dynamics is shown in Figure 6.5 and Figure 6.6. It can be seen that if the actuators are fast enough to cater for the rigid body modes, there is no need to consider the actuator dynamics and hence one to one mapping between the control allocator and control surfaces is sufficient. This would not be the case with the non aerodynamic actuators, so actuator dynamics cannot be ignored. It can be seen from the results shown in Figure 6.7 and Figure 6.8 how the actuator dynamics affects the outcome of the control allocator. It can also be seen how the control allocator command is attenuated. The first-order actuator dynamics used for this example are given as

\[
\frac{u(s)}{\tilde{u}_{\text{cmd}}(s)} = \frac{0.6128}{s + 0.6128}
\] (6.14)

Figure 6.5: Block diagram with desired demand produced by the control allocator and compared with the actual demand when there is no actuator dynamics included.\(^1\)

\(^1\) cf. (from Latin confer) means compare
Figure 6.6: Comparison of results with no dynamics of actuator involved

Figure 6.7: Block diagram with desired demand produced by the control allocator and compared with the actual demand when there is actuator dynamics included
In the following the second order actuator dynamics are parameterised for the design of the compensator.

**6.3 Second-Order Model Dynamics Interaction**

In this section, the effects of second – order actuator on the system is shown in Figure 6.2 (Oppenheimer and Doman 2004). Let the dynamics of a second actuator be represented by a continuous time second order function of the form

\[
\frac{u(s)}{\hat{u}_{cmd}(s)} = \frac{k}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]  

(6.15)
The state space representation of this transfer function is given

\[
\begin{bmatrix}
\dot{u}(t) \\
\ddot{u}(t)
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-\omega_n^2 & -2\zeta \omega_n
\end{bmatrix} \begin{bmatrix} u(t) \\
\dot{u}(t)
\end{bmatrix} + \begin{bmatrix} 0 \\
1
\end{bmatrix} \bar{u}_{cmd}(t)
\]

(6.16)

\[
\begin{bmatrix}
\dot{u}(t) \\
\ddot{u}(t)
\end{bmatrix} = A \begin{bmatrix} u(t) \\
\dot{u}(t)
\end{bmatrix} + B \bar{u}_{cmd}(t)
\]

\[
\begin{bmatrix}
\dot{u}(t) \\
\ddot{u}(t)
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix} u(t) \\
\dot{u}(t)
\end{bmatrix} = C \begin{bmatrix} u(t)
\end{bmatrix}
\]

(6.17)

The discrete time solution to the second-order actuator dynamic Eq. (6.16) to Eq. (6.17) for one sample period is given by

\[
\begin{bmatrix}
u(kT + T) \\
\dot{u}(kT + T)
\end{bmatrix} = e^{AT} \begin{bmatrix} u(kT) \\
\dot{u}(kT)
\end{bmatrix} + \int_{kT}^{kT+T} e^{A(kT+\gamma)B} \bar{u}_{cmd}(\gamma) d\gamma
\]

(6.18)

where \( T \) is the sampling time. This result does not depend on the type of hold because \( \bar{u}_{cmd} \) is specified in terms of its continuous time history, \( \bar{u}_{cmd}(t) \) over a sample interval (Franklin et al. 1998). A zero-order hold (ZOH) with no delay, i.e.

\[
\bar{u}_{cmd}(\tau) = \bar{u}_{cmd}(kT), \ kT \leq \tau \leq kT + T
\]

(6.19)

Performing substitution

\[
\gamma = kT + T - \tau
\]

(6.20)

In Eq. (6.18) yields

\[
\begin{bmatrix}
u(kT + T) \\
\dot{u}(kT + T)
\end{bmatrix} = e^{AT} \begin{bmatrix} u(kT) \\
\dot{u}(kT)
\end{bmatrix} + \int_{0}^{T} e^{A\gamma} B \bar{u}_{cmd}(kT) d\gamma
\]

(6.21)

Defining,
\[ \Phi = e^{AT} = \begin{bmatrix} \Phi_{1,1} & \Phi_{1,2} \\ \Phi_{2,1} & \Phi_{2,2} \end{bmatrix} \] (6.22)

\[ \Gamma = \int_0^T e^{AY} d\gamma B \]

\[ \begin{bmatrix} u(kT + T) \\ \dot{u}(kT + T) \end{bmatrix} = \Phi \begin{bmatrix} u(kT) \\ \dot{u}(kT) \end{bmatrix} + \Gamma \ddot{u}_{cmd}(kT) \] (6.23)

The first state variable \( u(kT + T) \) equation can be written as

\[ u(k + 1) = [\Phi_{1,1} \quad \Phi_{1,2}] \begin{bmatrix} u(kT) \\ \dot{u}(kT) \end{bmatrix} + \ddot{u}_{cmd}(kT) \int_0^T k \Phi_{1,2}(\gamma) d\gamma \] (6.24)

Parametrizing Eq. (6.24) will give

\[ u(k + 1) = C_1 u(k) + C_2 \dot{u}(k) + C_3 \ddot{u}_{cmd}(k) \] (6.25)

where \( C_1 = \Phi_{1,1}, C_2 = \Phi_{1,2} \) and \( C_3 = \int_0^T k \Phi_{1,2}(\gamma) d\gamma \).

The objective is to find \( M \) to modify the \( \Delta u_{cmd}(k) \), as shown in Figure 6.3 such that \( u(k + 1) = u_{cmd}(k) \).

\[ \Delta u_{cmd}(k) + u(k) = C_1 u(k) + C_2 \dot{u}(k) + C_3 (M \Delta u_{cmd}(k) + u(k)) \] (6.26)

Solving for \( M \) gives (Oppenheimer and Doman 2004)

\[ M = \frac{\Delta u_{cmd}(k) + (1 - C_3 - C_1) u(k) - C_2 \dot{u}(k)}{C_3 \Delta u_{cmd}(k)} \] (6.27)

These parameters \( C_1, C_2 \) and \( C_3 \) are tuned using genetic algorithm optimisation. Here it is assumed that the positions and rate of change of actuators are available. If there is a bank of second order actuator dynamics then \( M \) is chosen to be a diagonal matrix \( M \) of dimension (11X11).
In the following a stochastic evolutionary algorithm technique was discussed and applied to tune the parameters for the compensator design in section 6.4.

### 6.4 Tuning of Compensator to Mitigate Interaction Using GAs

A brief introduction to Genetic Algorithms is included in appendix A for readers not so familiar with the topic.

The idea is to combine the design objective in the form of a cost function that is to be optimised using an optimizer such as a Genetic Algorithm. Where the cost function includes the time domain objective; the tracking error is transformed into the integrated square of error between the commanded signal and actual output $u$ of the actuators. In addition there is another design objective, exception handling (e.g. division by zero) and this is also included in the cost function. The schematic is shown in Figure 6.9.

![Figure 6.9: Cost function error generated by the simulation](image)

Numerically the cost function for tracking error is given by
where the $t_{end}$ is the simulation run time.

Numerically the cost function for exception handling is given by

$$E_{\text{excep}} = \begin{cases} \text{penalty if exception generated} \\ 0 \text{ if no exception} \end{cases}$$

where the penalty is assigned as a large number 1010 so that the individual generating this exception would most likely not be selected in the next generation because of having very low fitness. Numerically the combined cost function is given as

$$J = E_{\text{track}} + E_{\text{excep}}$$

This cost function is then minimised to tune the parameters for the compensator. In the next section GA based optimisation details are given.

### 6.4.1 GA based optimisation

Genetic Algorithms are a part of Evolutionary Computing which is a rapidly growing area of Artificial Intelligence. “GA take up the process of evaluating the relative fitness of the individuals of a large population called genes, to select for a new generation, and mimic mutations and crossing over (mixing genes from two parent genes to form offspring genes), the so called evolution phenomenon” (Lindenberg 2002). Unlike biological evolution, in GA the gene controls some other processes like compensator parameters in this work, and is evaluated by comparing properties of the process instead of simply computing some function on the gene space (Lindenberg 2002). Genes used in GA are encoded as bit strings, and their fitness (Lindenberg 2002) is a relation described by a real valued fitness function $f$ on the set of bit strings $H = \{ b_0, ..., b_{n-1} \}$ such that gene $a$ is fitter than gene $b$ if $f(b) < f(a)$.

Optimisation using GA begins with a set of solutions (represented by chromosomes) called the population. Solutions from one population (based on some selection criteria) are taken and used
to form a new population. The flow chart of illustration of GA is shown in Figure 6.10. After selection of an encoding method (binary encoding for this case) and fitness function (cost function value for an underlying gene), the algorithm proceeds in the following steps (Lindenberg 2002)

- Select an initial population indexed by $P$ at random from some subset of $H$
- Repeat the following
  - evaluate the fitness of all the $b_P$ and use them to assign selection probabilities $r_P$ to $b_P$
  - select a new population, dropping genes of low fitness and duplicating fit ones, keeping index set $P$ (the size of population)
  - apply the genetic operators of mutation and crossing over (after forming couples)

This is repeated until some condition (for instance maximum number of generations or improvement of the best solution) is satisfied. The main advantage (Leigh 2004) of GA over other optimizers is their parallelism, GA is travelling in a search space using more individuals so they are less likely to get stuck in a local minima. The most important attributes of GA are
mutation and cross over. A good cross over rate is expected to take better parts of parent genes to
the next generation. Mutation on the other hand changes the individuals and if it is kept to a safe
low level it helps the population to avoid falling in local minima. This makes GA different from
other optimisers, and particularly suitable for non-convex optimisation problems like the
compensator parameter optimisation in this research. The main disadvantage linked with GA is
the higher computation time and required resources, but this can be avoided if there is a
possibility to stop the GA anytime in the routine. Also with the ever increasing processing power
of computers over time this constraint diminishes

6.4.2 Optimizing routine using GA

Numerically the optimizing problem is given as –Find $\mathbf{M}$ by minimizing $J$”.

$$
\min_{\mathbf{M}} J 
$$

(6.31)

where $\mathbf{M}$ is a diagonal gain matrix of dimension (11X11). The GA optimising routine is
formulated by using the MATLAB Genetic Algorithm Direct Search Toolbox. A flow chart
representation of the optimisation routine is shown in Figure 6.11.
Figure 6.11: Flow chart for tuning compensator parameters using GA

The complete process shown in Figure 6.10 can be summarised as:

- The GA main function calls the evaluation function, giving searched parameters to calculate compensator parameters
- The evaluation function calculates compensator parameters and calls the simulation model giving the parameters for the compensator
- The simulation model runs the simulation for the given compensator parameter (i.e. individual of population) and returns the value of error between $u_{cmd}$ and actual $u$
- The evaluation function calculates the cost function value for given errors and returns to the main GA function
- This is repeated for the total number of genes in one generation (population), and then one generation completes, and so the remaining generations are iteratively completed
- The above process is repeated until the cost function attains convergence, or the maximum number of generations is reached.
In the next section the simulation results are given to show how the compensator mitigates the interaction between the control allocation and actuator dynamics.

6.5 Simulation Results

During simulation, a mixture of actuator dynamics was used. In the case of redundant control surfaces diagonal gain matrices were tuned by the GA. The control surfaces were approximated by the transfer functions as shown in Table 6.1.

Table 6.1: Aerosurfaces actuator dynamics (Esteban and Balas 2004)

<table>
<thead>
<tr>
<th>Control surfaces</th>
<th>Number of control surfaces</th>
<th>Transfer functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ailerons</td>
<td>4</td>
<td>$\frac{s^2 + 22.19s + 270}{s^2 + 22.19s + 270}$</td>
</tr>
<tr>
<td>Elevators</td>
<td>4</td>
<td>$\frac{0.6128}{s + 0.6128}$</td>
</tr>
<tr>
<td>Stabilizer</td>
<td>1</td>
<td>$\frac{0.0087}{s + 0.0087}$</td>
</tr>
<tr>
<td>Rudders</td>
<td>2</td>
<td>$\frac{270}{s^2 + 22.19s + 270}$</td>
</tr>
</tbody>
</table>

The virtual control signal, $\mathbf{v}$, consists of chirps of amplitude 0.1, 0.15, 0.1 ($\text{rad/s}^2$) in roll, pitch and yaw angular accelerations respectively. The frequencies of chirps ranged from 0.1– 1 Hz in 20 seconds. In the processing of the GA routine exception handling is carried out to avoid breaking the GA optimisation process. For example if there is an individual (i.e. gains in diagonal matrix) in the population that gives division by zero that would break the simulation. This is dealt with in an exception handling block, which will give a penalty to that individual without breaking the simulation. In the next generation that individual would not be selected.
Simulations are done with compensation (Figure 6.12 and Figure 6.13) and without compensator (Figure 6.14 and Figure 6.15). As can be seen clearly from the results with no compensation there is serious attenuation and mismatch, but as soon as the compensation is turned on, $B_x u = v$ is achieved because sufficient control authority exists.

Deviations in the case of no compensation case means that the desired control surface positions coming out of the control allocator are different from the actual position of control surfaces. This interaction between the control allocator and the actuator dynamics results in serious consequence if the bandwidths of the actuators are not high or, in other words, the actuators are slow.

![Diagram](image_url)

**Figure 6.12:** Implementation scheme for compensator when the compensator is switched on
Figure 6.13: Desired angular accelerations ($v$) and actual angular acceleration ($B_c u$) in rad/s$^2$ when compensation is on

Find $\min_{u} |v - u|_{\infty}$ subject to $B_c u = v$, $\underline{u} \leq u \leq \overline{u}$

Control allocator

Compensator

Sampler

Zero order hold

Actuators rig

Compare

cf. Fig.6.15

Actuators rig

$M_{1\times1} = I_{1\times1}$

$\Delta u_{cmd}$

$u_{cmd}$

$\overline{u}$

$\underline{u}$

$u$

11X1

11X1

11X1

11X1

$B_c u$
6.6 Conclusions

This chapter details the application of genetic algorithms for the design and tuning of a compensator to alleviate the effects of control allocation and actuator dynamics interaction. The effects of non-negligible actuator dynamics have been investigated first. It was observed that, for the Boeing 747-200, the actuator dynamics cannot be ignored if the excitations are in the range of 0.1 to 1 Hz, which normally depends on the pilot dynamics. Another observation suggests that the bandwidths of the actuators are smaller than the rigid body modes of the aircraft and should not be neglected. The benefit of using a soft-computing methodology for tuning the compensator gains is to avoid the optimisation converging to a local minima and it is seen that the likelihood of the genetic algorithms converging to local minima solution is less as compared to other
techniques. In this methodology the model of the actuator is not needed to be known because this methodology was designed to be used on the actuator rig. In the case of the second order actuator, the rates should be either measured or observed. GAs are used offline and the band limited chirps are used as the excitation signal in the simulation. However, in the real system a band limited pseudo-random binary signal (PRBS) for this type of identification process could be used as an excitation signal rather than chirp because the later gives cyclic loading on the actuator, which could be problematic.
7 Discussion and Conclusions

7.1 Introduction

This chapter summarises the work described in the thesis, lists and discusses the main contributions and proposes suggestions for further work.

7.2 Summary of Thesis

Chapter 1 (Introduction) describes the need for redundancies in flight controls and aircraft actuators (control surfaces and thrusters). It is argued that redundancies are the backbone for any fault tolerant control system. The idea of control allocation in aircraft is strongly tied up with the notion of redundant controls. Most air crashes may be significantly contributed to by pilot mismanagement or misallocation of the available resources in the event of faults/failures. Chapter 1 also sets the scene for and outlines the presentation of the detail in the remaining chapters in this thesis.

Chapter 2 (Literature Review) introduces, thoroughly discusses, analyzes and proposes research objectives based on the literature in the area of control allocation focused on three major applications (i.e. aerospace, marine and automotive). It also classifies control allocation into a framework of sixteen issues. These issues are carefully selected based on the contributory effort importance of contribution of reviewed research papers. In addition to this, for any problem formulation which has overlaps with others, the most salient issues in this overlap are also discussed and analyzed. The novelty of the work presented in this thesis is based on research questions resulting from the detailed analysis and conclusions drawn from review of the secondary research available in the public domain. The contributions of the author have thus resulted from the detailed work undertaken to investigate, propose, develop and test solutions to address the key research questions that are identified in the review.
Chapter 3 (Civil Aircraft Model (B747-200)) provides insight into the uniqueness of the nonlinear aircraft model used for the control law and control allocation design. A nonlinear dynamic model of the aircraft is presented with some explanations of nonlinear stability and control derivatives to show the complexity of the model. The distinctiveness of the model comes from the aerodynamic moments used in calculating the total moment applied to the equation of motion of the aircraft. In the aerodynamic model the effects of aeroelasticity on the non-dimensional moment makes it unique for the design of the control law and control allocation. The model used in this study is a derivative of the Simulink model FTLAB747 version 6.5 developed by (Esteban and Balas 2003). This software is an upgraded version of two previous programs (i.e. Delft University Aircraft Simulation Modelling and Analysis Tool, DASMAT and Flight Lab 747, FTLAB747) developed in Delft University (van der Linden 1998). In this research the author has modified trimming and linearisation routines of this model to cater for the needs in the design of the control law and control allocation.

Chapter 4 (Control Allocation) describes the application of the active set methods to solve the multi-branch quadratic program for civil aircraft. Two types of objective function (i.e. a linear objective function and a quadratic objective function) are used to explain the properties of the optimal and feasible solution for the constrained optimisation problem. The constraints used in this optimisation are both the _inequality_ and _equality_ constraint. The modular design strategy is implemented in this chapter where the control law is designed separately from the control allocation module. Control allocation is simply the distribution of the virtual demands in terms of angular accelerations from the output of the control law module. In the active set method the idea of updating the working set \( W \) is presented graphically to enhance the understanding of the algorithm. A multi-branch quadratic objective constrained control allocation problem is used. In the second branch (the control sufficiency branch) a secondary objective namely the desired position of control surfaces is optimised. This specific secondary objective is very important for aircraft flying and handling qualities and thus for certification of the aircraft. Another factor in using the secondary objective is to exploit the system identification for the estimation of control derivatives in the null space of the control effectiveness matrix. This multi-branch control
allocation is solved in a sample time of 0.02 seconds during the closed loop simulation. Based on the work undertaken in this chapter, the author has published a paper titled

“Compensation of jammed control surface of large transport aircraft by control reconfiguration,” Ahmad, H., Young, T. M., Toal, D., Omerdic, E., Mediterranean Conference on Control and Automation, July 2007, Athens, Greece.

Chapter 5 (Fault-Tolerant Control Allocation) describes the implementation of the basic steps employed by the author in fault tolerant control allocation and these are summarised and discussed in the bulleted list below.

- **Fast update of control effectiveness matrix**: In the chapter graphical descriptions are used to illustrate how a fault progressively goes to failure and then how the control allocation actually reallocates the controls to compensate for the failure. Normally, the linear control allocation problem is solved in each iteration. However, if the control effectiveness matrix is static then the desired virtual demand cannot be produced exactly if the operating conditions are changed. The control effectiveness matrix is more sensitive to operating condition in the lateral and directional dynamics (i.e. control derivative corresponding to the 1st and 3rd row of $\mathbf{B}_c$) than in longitudinal dynamics. The open loop analysis has shown the effects of changing the angle of attack on the performance of control allocation in tracking the desired virtual demand (i.e. non-dimensional rolling moment). The fast automatic update of $\mathbf{B}_c$ based on approximating the relationship between the control surfaces and their effectiveness with a quadratic function and treating all other terms in the non-dimensional moment equation as constant because they do not depend on the control surfaces. This has given a faster way to update the $\mathbf{B}_c$.

- **Fault accommodation in control allocation**: The fault tolerant control allocation is implemented on the closed loop simulation of the nonlinear aircraft at a sampling frequency of 50 Hz. The column corresponding to failure is removed from $\mathbf{B}_c$ and the control allocation is performed on the remaining control surfaces.
• **Centre of gravity as redundant pitch control:** Fuel management will react to decision information on allocating the centre of gravity (by changing the fuel forward and aft to the trim tank) from the control allocation module. The author has published a paper in this area titled


• **Effect of weighting on control allocation:** The control prioritisation selection may improve the solution in control allocation. It was observed how the solution is found by changing the weighting on the control variables when the weighted pseudo inverse is used. In a two-dimensional problem a simple mathematical approach was suggested to calculate a weighting matrix when one control is saturated. This approach seems to work well when the objective function is strictly convex (i.e. $l_2$ norm objective function) This study is only done offline to the specific two-dimensional problem. To generate a weighting matrix update law for online implementation future research must be considered

• **System identification by null space excitation of desired control vector:** In the second branch of control allocation, the desired control vector is randomly perturbed in the null space of $\mathbf{B}_c$ which gives a highly decorrelated regressor matrix which is required for a least squares estimate. The method is applied offline to the input output data collected during the simulated flight with the excitation of the desired vector in the null space of $\mathbf{B}_c$. There are two limitations of this method; first there must be redundancy in the flight controls and the second is that the measurement of angular acceleration of the aircraft must be available or it must be estimated. In the first limitation, if the damage for example is so severe such that there are no lateral/directional control redundancies available then this method of identification will fail to work. The application of this offline identification method to the author's knowledge was for the first time implemented on civil aircraft in this research work. The author has published a paper in this area titled
Chapter 6 (Application of Evolutionary Computing in Control Allocation) details the usage of genetic algorithms for the design and tuning of a compensator to alleviate the effects of control allocation and actuator dynamics interaction. The comparison between static control allocation with no actuator dynamics and with actuator dynamics is given in this chapter. First and second order actuator dynamics are used in this design approach. The interaction of these dynamics with the control allocator and the design of a compensator to mitigate the effects of those interactions are presented. The benefit of using this method is that in the case of first order time lag dynamics of the actuators there is no need to know the dynamics of the actuators themselves. But in the case of a second order system, the rates are required in compensation hence for the estimation of these rates a Kalman filter could be used (which requires the model of the actuators). The author has published a paper in this area titled

“Control allocation with actuator dynamics for aircraft flight controls,” Ahmad, H., Young, T. M., Toal, D., Omerdic, E., 7th AIAA Aviation Technology, Integration and Operations Conference (ATIO), September 2007, Belfast, Northern Ireland.

**7.3 Realisation of Research Objectives**

This section discusses accomplishments in the realisation of the research objectives that were set out in section 1.4. Following are given the objectives of this research work with brief comment on achievement of same:

1. **Using a modular approach to design and implement a control law and control allocation for the nonlinear civil aircraft model.** This approach was successfully implemented and applied to the nonlinear civil aircraft model and has produced satisfactory results within the bandwidth of flight control. The application of the active set methods to the author’s knowledge was the first time implemented on a civil aircraft in this research work.
2. Using control redundancies to compensate for a failure without changing the baseline control law. Damage is introduced to the aircraft (model) in the experiments within this thesis under the assumption that there is a good fault detection and identification system available. Then control allocation based on the active set methods is again applied and has produced satisfactory results. Lateral and longitudinal damages are considered. The longitudinal damage is based on the EAL 1862 air crash accident in Amsterdam.

The objectives 1 and 2 are described in the following peer reviewed conference proceedings: “Compensation of jammed control surface of large transport aircraft by control reconfiguration,” Ahmad, H., Young, T. M., Toal, D., Omerdic, E., Mediterranean Conference on Control and Automation, July 2007, Athens, Greece.

3. Fast update of control derivatives for control allocation: This has been realised by using the information of aerodynamic coefficients to estimate the control derivative in $B_c$. Open loop analysis is given for the effects of changing operating condition on control allocation with static and dynamic $B_c$.

4. Identification of control derivatives by exploiting control redundancies: This has been realised by randomly perturbing the desired control vector in the second branch of the multi-branch quadratic control allocation problem. This has not affected the actual trajectory of the aircraft and has produced the necessary condition for the regressor matrix (i.e. decorrelation) for the least squares estimate. This procedure has been adopted only in an offline fashion. Online implementation remains to be investigated in future work. This outcome of the research has been published in the following peer reviewed conference proceedings: “System identification using null space excitation in control allocation,” Ahmad, H., Young, T. M., Toal, D., Omerdic, E., 11th Mechatronics Conference, June 2008, University of Limerick, Ireland.

5. Centre of gravity movement by moving fuel to yield additional control of pitch attitude of the aircraft. By integrating the fuel management system and the flight control allocation blocks the position of c.g. can be changed through the transfer of fuel to and from trim tanks. In this scenario, there is a limitation of rate of change of centre of gravity. Another very important phenomenon observed in this study is that during manoeuvre, if the desired c.g. position is selected forward, the longitudinal tracking of the aircraft for this specific scenario
is damped and improves the handling qualities of the aircraft. This outcome of the research has been published in a peer reviewed conference proceedings:


6. **Study of the effects of the desired control vector and control surfaces prioritisation in control allocation.** A two dimensional result has been presented for the weighted pseudo inverse solution in this thesis. Further investigation is recommended in future work.

7. **Control allocation and actuator dynamics interaction:** A compensator has been designed and tuned to mitigate the effects of control allocation and control surface dynamics using genetic algorithms. The problem is solved at the sampling frequency 50 Hz and can thus be implemented in real time. This outcome of the research has been published in the following peer reviewed conference proceedings:

“Control allocation with actuator dynamics for aircraft flight controls,” Ahmad, H., Young, T. M., Toal, D., Omerdic, E., 7th AIAA Aviation Technology, Integration and Operations Conference (ATIO), September 2007, Belfast, Northern Ireland.

### 7.4 Practical Implementation

Following are some future practical implementation of contributions:

- **Control allocation and actuator dynamics interaction:** This objective which is thoroughly discussed in chapter 6 can have practical implementation. Consider an aeroplane which has undergone maintenance and some of the aerosurface actuators are replaced and the compensator for the control allocation and actuator dynamics interaction needs to be tuned. The following procedure could be adopted to tune the compensator: First of all the bandwidth of the chirp signal corresponding to the range of the virtual demand is selected as an input to the system. Now for the first order actuator dynamics the compensator is designed straight away using the methodology given in section 6.4 by having a push button and the aircraft is still in the hanger.
In the case of the second order dynamics the rate of change actuator position is needed which is either estimated or provided by the actuator manufacturer as a measurement. Then the same procedure is adopted as discussed in section 6.4. The interesting thing about this method is that if the position and velocities of the actuators are known then the compensator is designed for all of the actuator simultaneously with a push button.

### 7.5 Future Work

Some directions for future investigations are described in the following:

- How to prioritise the controls or, in other words, how to schedule the control weighting matrix? This suggests how to maintain the positive definiteness of the weighting matrix, while changing the solution of the control allocation problem. In the classical way this matrix is selected to be a constant diagonal matrix. In (Golub and Van Loan 1996) the column and row weighting is described. The row weighting was done as the function of uncertainty of the observed values.

- Analysis of the potential for selection of a ‘desired control vector’ which is detrimental to the aircraft flying and handling qualities. To check the flying and handling qualities with control allocation on board in the light of metrics regarding airworthiness of the aircraft with control allocation. It was observed that the desired vector tends to improve the flying and handling qualities of the aircraft. This could be an area for further research. The flying and handling qualities of the aircraft should be assessed with control allocation in the loop, especially during the takeoff and landing conditions.

- Identification of the damaged aircraft model is the most important area which needs to be covered in future research. As the control law and control allocation designs are based on modelling, so the identification of an accurate model of the damaged aircraft with as little uncertainty as possible is very important for the successful implementation of the control law and control allocation design. Online identification of the control effectiveness matrix when the aircraft has encountered a failure is an important question to be resolved in the future research work.
• In this thesis, c.g. was used as an additional pitch attitude control in control allocation. This approach has opened many research questions, like —What is the rate of change of c.g. position, which is dependent on how fast the fuel can flow between the forward and aft fuel tanks?”. This question opens a new area of research in the domain of fuel pumping system design. In this research work only the longitudinal redundancy (i.e. c.g. moving forward and aft) was considered, but effectively the lateral redundancy could also be exploited in further future work.

• Non-monotonic behaviour of the control to moment curve will produce undesirable results if this nonlinearity is not taken in to account for in control allocation properly. This makes the problem definition non-convex in nature which could be a future research question.
References


8 Appendices

8.1 Appendix A: Genetic Algorithms

Genetic algorithms are a part of evolutionary computing, which is a rapidly growing area of artificial intelligence. Genetic algorithms are inspired by the so called theory of evolution; the same principle seems to work for problem solving. Problems are solved by an evolutionary process resulting in a best (fittest) solution (survivor) - in other words, the solution is evolved. This theoretical brief introduction about GAs are based on tutorial given in http://www.obitko.com/tutorials/genetic-algorithms/

8.1.1 Genes and Chromosome

Since the idea is inspired by the building blocks of living things genes and chromosomes, so a brief description of genes and chromosome is worth mentioning here. Chromosomes are strings of DNA and serve as a model for the whole organism. A chromosome consists of genes, blocks of DNA. In each cell there is the same set of chromosomes. Each gene encodes a particular protein. Basically, it can be said that each gene encodes a trait, for example colour of eyes. Possible settings for a trait (e.g. blue, brown) are called alleles. Each gene has its own position in the chromosome. This position is called locus.

8.1.2 Mutation and Crossing over

During reproduction, recombination (or crossover) first occurs. Genes from parents combine to form a whole new chromosome. The newly created offspring can then be mutated. Mutation means that the elements of DNA are a bit changed. These changes are mainly caused by errors in copying genes from parents. The fitness of an organism is measured by success of the organism in its life (survival).

8.1.3 Problem Solving with GA

The algorithm begins with a set of solutions (represented by chromosomes) called the population. Solutions from one population are taken and used to form a new population. This is
motivated by a hope, that the new population will be better than the old one. Solutions which are then selected to form new solutions (offspring) are selected according to their fitness - the more suitable they are the more chances they have to reproduce. This is repeated until some condition (for example number of populations or improvement of the best solution) is satisfied.

1. Start
   Generate random population of n chromosomes (suitable solutions for the problem)

2. Fitness
   Evaluate the fitness \( f(x) \) of each chromosome \( x \) in the population

3. New Population
   Create a new population by repeating following steps until the new population is complete

4. Selection
   Select two parent chromosomes from a population according to their fitness (the better the fitness, the bigger the chance to be selected)

5. Crossover
   With a crossover probability cross over is done on the parents to form new offspring (children). If no crossover was performed, offspring are exact copies of parents

6. Mutation
   With a mutation probability, mutate new offspring at each locus (position in chromosome).

7. Accepting
   Place new offspring in the new population

8. Replace
   Use new generated population for a further run of the algorithm

9. Test
   If the end condition is satisfied, stop, and return the best solution in current population
   Loop

10. Go to step 2
8.1.4 GA Parameters

Following are the GA parameters:

8.1.4.1 Chromosome Encoding

Chromosome should in some way contain information about the solution that it represents. The most common way of encoding is a binary string. A chromosome then could look like this:

<table>
<thead>
<tr>
<th>Chromosome 1</th>
<th>1101100100110110</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chromosome 2</td>
<td>1101110000111110</td>
</tr>
</tbody>
</table>

Each chromosome is represented by a binary string. Each bit in the string can represent some characteristics of the solution. Another possibility is that the whole string can represent a number; this is what was used in the GA optimizer used in this thesis. There are many other ways of encoding. The encoding depends mainly on the problem.

8.1.4.2 Binary Encoding

In binary encoding, every chromosome is a string of bits – 0 or 1.

<table>
<thead>
<tr>
<th>Chromosome A</th>
<th>101100101100101011100101</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chromosome B</td>
<td>11111100001100000111111</td>
</tr>
</tbody>
</table>

Binary encoding gives many possible chromosomes even with a small number of alleles. On the other hand, this encoding is often not natural for many problems and sometimes corrections must be made after crossover and/or mutation.
8.1.4.3 Crossover:

Crossover operates on selected genes from parent chromosomes and creates new offspring. The simplest way to do this is to choose randomly some crossover point and copy everything before this point from the first parent and then copy everything after the crossover point from the other parent. Crossover can be illustrated as follows: ( | is the crossover point):

<table>
<thead>
<tr>
<th>Chromosome 1</th>
<th>1101</th>
<th>00100110110</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chromosome 2</td>
<td>1110</td>
<td>11000011110</td>
</tr>
<tr>
<td>Offspring 1</td>
<td>1101</td>
<td>11000011110</td>
</tr>
<tr>
<td>Offspring 2</td>
<td>1101</td>
<td>00100110110</td>
</tr>
</tbody>
</table>

There are other ways of making crossover, for example, one can choose more crossover points. Crossover can be quite complicated and depends mainly on the encoding of chromosomes. The specific crossover made for a given problem can improve performance of the genetic algorithm.

8.1.4.4 Mutation

After a crossover is performed, mutation takes place. Mutation is intended to prevent all solutions in the population falling into a local optimum of the solved problem. The mutation operation randomly changes the offspring resulting from crossover. In the case of binary encoding we can switch a few randomly chosen bits from 1 to 0 or from 0 to 1. Mutation can be then illustrated as follows:

<table>
<thead>
<tr>
<th>Original offspring 1</th>
<th>1101111000011110</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original offspring 2</td>
<td>1101100100110110</td>
</tr>
<tr>
<td>Mutated offspring 1</td>
<td>1100111000011110</td>
</tr>
<tr>
<td>Mutated offspring 2</td>
<td>1101101100110100</td>
</tr>
</tbody>
</table>
8.1.4.5 Elitism

Generating populations only from two parents may cause loss of the best chromosome from the last population. This is true, and so elitism is often used. This means, that at least one of a generation's best solution is copied without changes to a new population, so the best solution can survive to the succeeding generation.
8.2 Appendix B: List of Publications

- “Compensation of jammed control surface of large transport aircraft by control reconfiguration,” Ahmad, H., Young, T. M., Toal, D., Omerdic, E., Mediterranean Conference on Control and Automation, July 2007, Athens, Greece.

- “Control allocation with actuator dynamics for aircraft flight controls,” Ahmad, H., Young, T. M., Toal, D., Omerdic, E., 7th AIAA Aviation Technology, Integration and Operations Conference (ATIO), September 2007, Belfast, Northern Ireland.
