Modeling of magnetic properties and alternate current losses in high-T_c superconductors using a power law voltage–current characteristic

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This article presents numerical diffusion models which describe the electromagnetic properties and alternate current (ac) losses of a high-T_c superconducting tape carrying transport current. The models simulate the tape as a highly nonlinear conductor with equivalent resistivity being a function of J, the local current density, as deduced from a power law E–J characteristic of the material. The power law E–J expression can be adjusted by varying both the power, α, and the value of the critical current density, J_c, to accurately fit the current–voltage characteristic of a given high-T_c superconducting tape, as obtained from direct current measurements. Results of various field quantities, in the interior of the tape, and losses for a range of ac currents and frequencies are presented and discussed in conjunction with the critical state, flux creep, and flux flow theories of high-T_c materials. [S0021-8979(99)25008-6]

I. INTRODUCTION

The steady maturing of high-temperature superconductors (HTSs), and silver-sheathed BISCCO tapes in particular, over the last few years has intensified efforts for the development of large-scale HTS power applications. It has been widely recognized that power cables, fault current limiters, motors, and transformers are the most promising applications, and nearest to realization. In appraising the economic and technical viability of such applications, few key issues have to be considered and assessed. The most important of these issues are the response of the superconductor to alternate current (ac) transport currents and the level of the ac losses in operation. The knowledge of ac losses in the superconducting tape is essential when designing a HTS power device. In order to evaluate ac losses, one has to work out the current and various field profiles within the HTS tape carrying the transport current.

Classically, the current/field profiles and, consequently, ac losses within a superconductor are estimated using Bean's critical state model.[1] This model describes the superconducting material with a step-function-like E–J characteristic whereby $E=0$ until $J=J_c$, where J_c is the critical current density. Unfortunately, in HTSs the critical state model loses its accuracy due to thermally activated flux creep and macroscopic inhomogeneities. This results in smooth current–voltage characteristics with ill-defined J_c. Therefore, design of advanced HTS power application requires a more sophisticated model which takes these features into account.

We previously reported a numerical diffusion model for the computation of ac losses in HTS tapes.[2,3] In that model, the flux creep is formulated in terms of a nonlinear diffusion of magnetic flux through the HTS tape. The tape is modeled as a nonlinear conductor whose equivalent resistivity is derived from an E–J characteristic which is, in turn, deduced from the Anderson–Kim model describing the flux creep.[4] However, experiments have indicated that most HTS materials, particularly BISCCO tapes, show power law current–voltage characteristics throughout a wide measurement range.[5] Hence, in this current work we apply a power law E–J characteristic, as suggested by Rhyner,[6] to simulate HTS tapes carrying transport currents. This article reports the development, and presents results, of numerical computational models which work out the current and various field profiles within a HTS tape carrying a given ac transport current. Losses are then deduced using a Poynting vector method.

II. NONLINEAR DIFFUSION MODEL

A. One-dimensional H formulation

As a first step to macroscopically model a HTS tape carrying an ac transport current, a semi-infinite one-dimensional ribbon configuration is used. In this model, a HTS tape of thickness $2b$ with the center plane of its cross-section at $y=0$, carries a current $I_m \sin \omega t$ per unit width in the z direction, and the axis of the ribbon, is simulated. Using Maxwell's equations, a one-dimensional diffusion equation is formulated in terms of a single component of magnetic field strength H_x:

$$\frac{\partial}{\partial y}\left(\rho \frac{\partial H_x}{\partial y}\right) = \mu_0 \frac{\partial H_x}{\partial t}, \tag{1}$$

where ρ is the equivalent local resistivity. Using a power law characteristic, the relationship between E and J can be expressed in a form similar to that suggested by Rhyner:[6]

$$E=E_c(J/J_c)^{\alpha}, \quad \rho=\rho_c(J/J_c)^{\alpha-1}. \tag{2}$$

Here J_c is the critical current density, E_c the 100 $\mu\mathrm{V\,m}^{-1}$ criterion used to define J_c in HTSs, and $\rho_c=E_c/J_c$. The power law contains the linear and the critical state extremes as limiting cases ($\alpha=1$ and $\alpha\rightarrow\infty$, respectively). The governing diffusion equation is completed by substituting Eq. (2) into (1). A simple time-stepping implicit finite-difference algorithm, combined with successive over-relaxation (SOR) process, was used to determine its solution.[2,3]

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However, for certain cases this H-formulation model has been found to suffer from a numerical instability which has been attributed to the extreme nonlinearity of the system in which local resistivity can change by several orders of magnitude within one cycle.[3] In fact, a dimensional asymptotic analysis[3] has shown that for large values of α, the solution converges to the critical state solution of Bean,[1] whereby the current is either the critical current or zero in the interior of the tape. Consequently, very small variations in H_x will result in huge variations in the solution, hence making the H formulation a very difficult computational problem. The asymptotic analysis has also identified E as the field quantity which diffuses in slower manner, and hence causes much smaller variations in the solution compared to H.

B. One-dimensional *E* formulation

In terms of E, the diffusion equation governing the system takes the form:

$$\frac{\partial^2 E_z}{\partial y^2}=\mu_0\frac{\partial(\sigma E_z)}{\partial t}, \tag{3}$$

where σ is the equivalent local conductivity which can easily be deduced from the Rhyner formula (2) as $\sigma=\sigma_c|(E_z/E_c)|^{(1/\alpha)-1}$, where $\sigma_c=J_c/E_c$. Substituting for σ into Eq. (3) yields:

$$\frac{\partial^2 E_z}{\partial y^2}=\mu_0\frac{\partial}{\partial t}\left(\sigma_c\left|\frac{E_z}{E_c}\right|^{\frac{1}{\alpha}-1}E_z\right) \tag{4}$$

with $\partial E_z/\partial y=0$ at $y=0$, and $\partial E_z/\partial y=-\mu_0\partial H_x/\partial t=\omega\mu_0 H_m\cos\omega t$ at $y=b$. Dimensional analysis has shown that, in contrast to the H formulation, this formulation has the benefit of reducing the problem to Laplace's rather than a highly nonlinear elliptic equation in regions of very high conductivity.[3]

To solve the problem the system was configured using two equally spaced staggered grids. The magnetic field H_x was modeled on the first grid whose first node coincides with the surface of the tape. The electric field E_z was taken at the second grid whose nodes are located between the nodes of the first grid. This is followed by spatial discretization of Eq. (4) using simple central differences within which the boundary conditions are fed. For the time stepping, a simple explicit Euler method was employed to solve for E_z values at the interior points as (replacing E_z by E for convenience):

$$E_{i,k+1}=\frac{E_c^{1-\alpha}}{\sigma_c^{\alpha}}\left\{\left|\sigma_c E_c^{1-\frac{1}{\alpha}}|E_{i,k}|^{1/\alpha}\mathrm{sign}(E_{i,k}) + R(E_{i+1,k}-2E_{i,k}+E_{i-1,k})\right|\right\}^{\alpha}\mathrm{sign}(\Omega_k), \tag{5}$$

where

$$\Omega_k=\mathrm{sign}\left\{\sigma_c E_c^{1-\frac{1}{\alpha}}|E_{i,k}|^{1/\alpha}\mathrm{sign}(E_{i,k}) + R(E_{i+1,k}-2E_{i,k}+E_{i-1,k})\right\},$$

$R=\Delta t/\mu_0\Delta y^2$, Δt the time step, Δy the spatial step and the two address integers i and k are space and time counters, respectively. The stability of the solution was ensured by continuously satisfying the condition $\Delta t \leqslant \mu_0\sigma_c(E_{i.k}/E_c)^{(1/\alpha)-1}\Delta y^2/2$. Having solved for all nodal values of E_z, the magnetic field H_x is computed by performing the necessary integration using the midpoint rule.

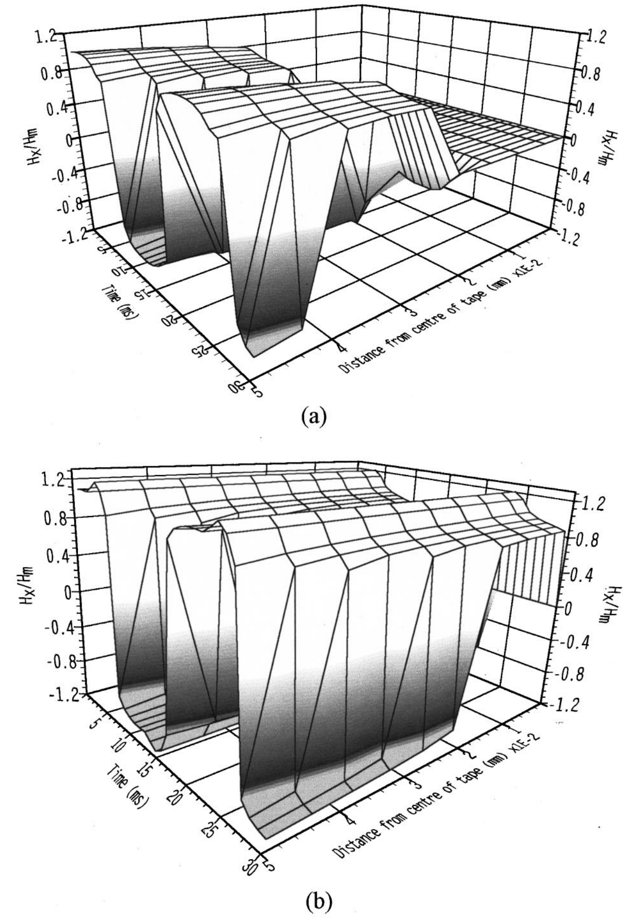

FIG. 1. Current density profiles inside the HTS tape when (a) $J_0=500$ A mm^{-2}, and (b) $J_0=1200$ A mm^{-2}.

III. RESULTS AND DISCUSSION

Using the above model, Fig. 1 shows the current density profiles inside a HTS tape of 0.1 mm thickness, for two values of peak imposed current density, $J_0=H_m/b$, at 50 Hz. The assumptions here are $J_c=1000$ A mm^{-2} and $\alpha=20$. It is noted that case (a), where $J_0=500$ A mm^{-2}, corresponds to a partially penetrated tape, whereas when $J_0=1200$ A mm^{-2}, as in case (b), a complete saturation of the tape is indicated. For both cases the current is either almost equal to J_c or zero, as predicted by the dimensional analysis.[3] The conclusion here is that the problem has a solution structure similar to that of Bean's critical state model. However, the variation of loss density as a function of J_0 and frequency, as shown in Fig. 2, reveals more analysis. It is found that the loss per unit surface of the tape is proportional to $f^p J_0^q$, where $p\approx 0.91$and $q\approx 3.2$. As J_0 approaches J_c, the loss density shows different behavior ($p\approx 0.6$ and $q\approx 5$). In contrast, the critical state gives $p=1$ and $q=3$. The conclusion is that self-field ac losses fall into two different regimes. At low $\partial H/\partial t$ imposed by low currents, the loss mechanism is mainly hysteretic similar to the critical state. As $\partial H/\partial t$ increases, the period of the imposed current exceeds the time

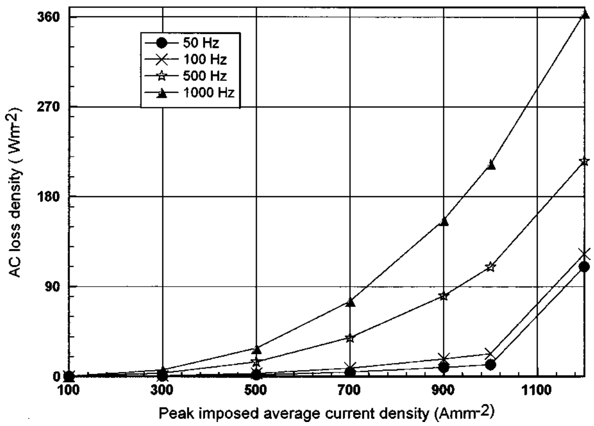

FIG. 2. ac losses as function of J_0 at various power frequencies.

constant of flux creep and the losses become dominated by a resistive mechanism.[4] This is also indicated in Fig. 3, in the shape of the wave forms of the electric field at the surface of the tape which also indicate a good qualitative agreement with experimental data.[5]

IV. TWO-DIMENSIONAL DIFFUSION MODEL

The benefits of the reformulation discussed in Sec. II B can also be exploited in two dimensions. In this case, a HTS tape of a rectangular cross-section $2a \times 2b$, centered at $x = 0$ and $y = 0$, carrying an ac current $I_m \sin \omega t$ in the z direction. The diffusion equation can again be formulated in a similar fashion to that described in Sec. II B, as:

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = \mu_0 \frac{\partial}{\partial t}\left(\sigma_c \left| \frac{E}{E_c} \right|^{\frac{1}{\alpha}-1} E \right). \tag{6}$$

One quadrant of the problem needs to be solved, and is assumed to be surrounded by an air space bounded by two surfaces $x = c$ and $y = d$, as shown in Fig. 4. The indicated boundary conditions on the four surrounding planes, have been derived from the field equation $curl\,\mathbf{E} = -\partial \mathbf{B}/\partial t$. To obtain the values of the tangential magnetic field components, tape aspect ratio a/b is taken to be very large, such that $c \gg a$ and $d \gg b$. With this arrangement, the current can be assumed to be uniformly distributed across the cross-sectional area of the tape, and the values of H_x and H_y can be computed using formulas derived from Biot-Savart law.[7] The diffusion Eq. (6) and Laplace's equation that governs the behavior of the air space, are then discretized. The whole problem is then solved using a time-stepping combination of an implicit method with an SOR process for the air nodes, and a simple explicit Euler method for the interior of the tape. A computer code for this algorithm is currently being optimized.

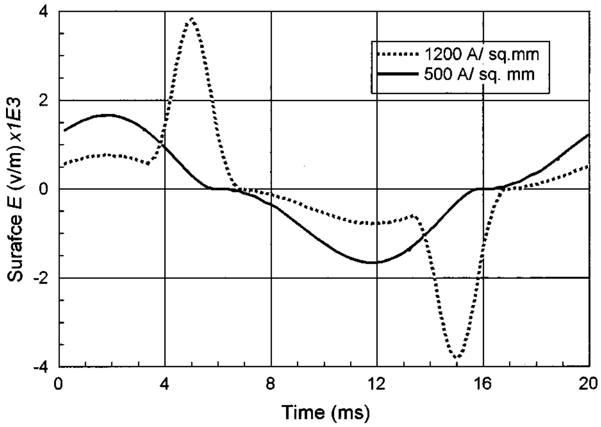

FIG. 3. Surface electric field E_z for J_0=500 A mm^{-2} and J_0=1200 A mm^{-2}, both at 50 Hz (wave form for J_0 = 500 A mm^{-2} has been scaled up by a factor of 10 for clarity).

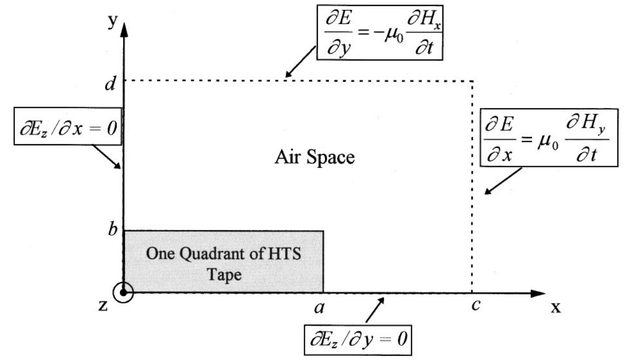

FIG. 4. A schematic diagram of the configuration of the two-dimensional E-formulation diffusion problem for a HTS tape, of a rectangular cross-section.

V. CONCLUSIONS

We have presented two computational models which simulate the electromagnetic properties and estimate the loss in HTS tapes carrying ac transport currents. The models are based on a diffusion process and use a power law E–J expression, similar to that suggested by Rhyner, to characterize the HTS tape. Results have demonstrated that the one-dimensional E-formulation works very well, and overcomes the problems encountered with a previous H formulation. The conclusion here is that the E formulation is more robust in dealing with the extreme nonlinearity of this problem. Also, by choosing the appropriate parameters based on sample direct current (dc) measurements, the power law E–J characteristic can accurately simulate the behavior of a given HTS tape in both the flux-creep and the resistive flux-flow regimes. The computational results, particularly those of the ac losses, show that Bean's critical state provides sufficient-predictions for currents below J_c at low frequencies. However for higher currents and frequencies, the ac loss shows different behavior from the hysteretic nature, showing a good qualitative agreement with experimental data. Hence the developed diffusion models seem to offer better descriptions of HTS materials over wider ranges of parameters. A two-dimensional model has recently been developed, and its implementation and results are being investigated at present.

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