Case Study: The Effectiveness of Virtual Manipulatives in the Teaching of Primary Mathematics

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<tr>
<td>HCI</td>
<td>Human Computer Interface</td>
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<tr>
<td>ICT</td>
<td>Information Communication Technology</td>
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<td>ITS</td>
<td>Intelligent Tutoring System</td>
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<tr>
<td>NCTE</td>
<td>National Centre for Technology in Education</td>
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<tr>
<td>NLVM</td>
<td>National Library of Virtual Manipulatives</td>
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Declaration

I hereby declare that this thesis represents my own work and has not been submitted, in whole or in part, by me or another person, for the purpose of obtaining any other university award. I agree that this thesis may be made available by the College to future students.

Catriona Lane
14th March 2010
Abstract

Case Study: The Effectiveness of Virtual Manipulatives in the Teaching of Primary Mathematics by Catriona Lane

This case study aimed to examine how effective virtual manipulatives are in the teaching of primary mathematics. Concrete manipulatives are an expensive investment for primary schools and as a result there is often not enough manipulatives for all pupils to use. Virtual manipulatives can be obtained free of charge once a pupil has access to a computer that has an internet connection. This case study involved dividing a third class group into three groups; each group used a different form or combination of mathematical manipulative. One group used virtual manipulatives, one group used concrete manipulatives and one group used a combination of concrete and virtual manipulatives. Qualitative and quantitative data collection tools were used to elicit the effectiveness of each tool. Areas such as mathematical dialogue, motivation and the learning benefits for low achievers were also examined. The case study found that pupils using both virtual manipulatives and concrete manipulative had the greatest increase in test score results. Due to the lack of funding in primary schools, combining the use of virtual manipulatives and concrete manipulatives could solve teachers’ problems surrounding the lack of concrete manipulatives, as well as benefiting learners.
Chapter 1- Introduction

1.1 Introduction & Background

‘The new technology will not replace teachers, textbooks or the classroom. It will supplement them by creating new combinations of opportunities and help to put pupils’ learning in the very centre’

(The Minister for Schools and Adult education 2000 cited in Freeman et al 2001)

Today’s young people have been born into a technological age. Information technology has infiltrated every aspect of society and digital competence is now seen as one of the eight proficiencies required for lifetime learning. Citizens must be educated in the use of information communication technology (ICT) so they can participate fully in this digital age. Educators must strive towards ‘the ultimate goal of transforming our classrooms into 21st century e-learning environments’ (Department of Education and Science 2008, p.7).

In 1997, Ireland was ranked in the third division with regards to its readiness for the information age. In response to this the Irish Government launched IT 2000. The purpose of IT 2000 was to integrate ICT into Irish primary and secondary schools. IT 2000 aimed for pupils to become more computer literate so as to equip themselves with the necessary skills for the information society we live in. It also aimed to provide professional development for teachers so they could become confident in using ICT in their teaching. However in order for the aims of IT 2000 to be accomplished, action was first required in the area of ‘classroom resources, infrastructure, teacher skills’ development and support, policy and research’ (Freeman et al 2001). In an effort to monitor the implementation of IT 2000, the National Centre for technology in Education (NCTE) was set up in 1998 (Freeman 2001).

Since the launch of the IT 2000 programme the incorporation of ICT into education has been increased significantly. The benefits of ICT in the classroom are extensive. According to the Strategy Group Report into Investing Effectively in Information and
Communications Technology in Schools (2008-2013), ICT ‘provides richer, more immediate, world-relevant educational resources and opportunities. When used well, ICT ‘enriches learning and enhances teaching. It invigorates classroom activities and is a powerful motivational tool that encourages learners to progress in more personalised and self-directed ways.’ (2008, p.11).

Despite the learning benefits associated with ICT and the support which the Department of Education has given ICT, the transition of ICT into the classroom has been a slow process. The focus of ICT in the classroom must shift from a ‘technology provision to a focus on its deliberate use by the learner’ (Department of Education and Science 2008, p.11). ICT classroom use must be centred on interactivity and increased user generated information (Department of Education and Science 2008).

1.2 Research Topic

1.2.1 Research Question
The purpose of this case study was to explore the effectiveness of virtual manipulatives in the teaching of primary mathematics. The research aimed to examine and determine the following:

- How effective are virtual manipulatives in the teaching of primary mathematics?
- Are concrete manipulatives and virtual manipulatives more effective when used together?
- Are pupils more motivated when using virtual manipulatives or when using concrete manipulatives?
- Do virtual manipulatives allow for more or less mathematical dialogue than concrete manipulatives?
1.2.2 Research Context
This study was conducted in a middle sized rural primary school in the south of Ireland. The participants were a group of 3rd class pupils of mixed gender. The 1999 Primary Mathematics Curriculum emphasises the importance of a ‘hands on’ approach in the teaching of mathematics and recommends the use of manipulatives in the teaching of mathematics. In accordance with curriculum guidelines, the primary school chosen for this case study uses manipulatives regularly in the teaching of mathematics; however virtual manipulatives had not been used in the school before this study (Department of Education and Science 1999).

1.3 Relevance of the Study
The National Centre for Technology in Education (NCTE) was set up in 1998 with the aim of introducing ICT into the Irish education system (NCTE 2009). The core aims of the NCTE are to:

• Allow pupils the chance to develop computer literacy
• To provide pupils with skills necessary for the information society we live in.
• To provide teachers with the support necessary to develop their professional skills.
• To allow ICT to become part of the learning environment.

(NCTE 2009)

In the primary mathematics classroom, pupils often use concrete manipulatives to reinforce their learning. Many of these mathematical manipulatives are available on the internet, these are called virtual manipulatives. Virtual manipulatives can be accessed in all classrooms, once there is a computer and an internet connection in the classroom.

In order to develop the aims of the NCTE and allow ICT to become part of the everyday classroom environment, educators need to make use of the digital learning tools available to them and ensure learners are exposed to all forms of digital tools available. In sight of
the recommendations of the NCTE, this research study set out to explore and evaluate the effectiveness of virtual manipulatives in the primary mathematic classroom.

1.4 Research Methodology

This research study adopted a case study approach. According to Opie a case study concentrates on a real situation in a setting which is familiar to the researcher (2004). Data was collected using both qualitative and quantitative data collection methods. Pupils undertook teacher designed pre-tests and post-tests, and standardised pre-test and post-test. Teacher observation and pupil questionnaires were also used in order to determine the effectiveness of virtual and concrete manipulatives. The participants were divided into three groups, one group using virtual manipulatives, a second group using concrete manipulatives and third group using both concrete and virtual manipulatives. The data collected from each group was then compared and contrasted so as to elicit the effectiveness of the learning tools.

1.5 Structure of the Research Study

Chapter 1 is an introduction to the thesis. It provides background information to the research being undertaken. It also discusses the rationale and significance of the study.

Chapter 2 reviews the literature of education and is divided into seven sections. Section one introduces the section. Section two reviews the learning theories of behaviourism and constructivism, motivation and learning styles. Section three explores computer based learning theories and how computer based learning can motivate the learner and facilitate mathematical dialogue. Section four reviews the mathematics curriculum and ICT in education. Section five looks at the advantages and disadvantages of using manipulatives in the teaching of mathematics Section six examines the benefits of using virtual manipulatives in the teaching of mathematics and defines the different forms and
representations of virtual manipulatives. Section seven reviews the main aspects of human computer interface according to the theories of Alessi & Trollip and Mayer. These theories are then discussed in light of the virtual manipulatives used in this study.

Chapter 3 looks at two research approaches which are generally used in educational research; case study research and action research and explains the reasons why case study research was chosen as the research approach for this study. The data collection tools used in this study are described. Next the areas of reliability and validity of research is explored. Ethical issues are also considered. Chapter 3 concludes with a timeline of the research undertaken.

Chapter 4 presents the findings resulting from the case study research. The findings are presented in four sections which are based on the research questions. Firstly the findings relating to the effectiveness of virtual manipulatives in the teaching of mathematics are outlined. Secondly the effectiveness of concrete manipulatives and virtual manipulatives when used together to teach mathematics are presented. Thirdly the findings relating to the difference in motivation between pupils using virtual manipulatives and pupils using concrete manipulatives are outlined and finally the findings relating to the quantity of mathematical dialogue that pupils engaged in when using virtual manipulatives and the quantity of dialogue that pupils engaged in when using concrete manipulatives is outlined.

Chapter 5 firstly presents an overview of the study and outlines the key findings of the case study. The findings are then compared, contrasted and analysed with the literature which was outlined in Chapter 2. The analysis is done in five sections; benefits of virtual manipulatives, benefits of using a combination of concrete manipulatives and virtual manipulatives, pupil motivation, mathematical dialogue and problems that arose during the case study.

Chapter 6 presents the recommendations and suggestions for further research in the area of virtual manipulatives and in the combined use of virtual and concrete manipulatives.
Chapter 2-Literature Review

2.1 Introduction

This chapter will examine behaviourist and constructivist learning theories, learner styles and motivation. This chapter will also discuss the effect learning theories have on computer based learning and how computer based learning affects pupil motivation. This chapter will also look at the role of computer based learning in the primary education system in Ireland and more specifically how computer based learning, especially virtual manipulatives, can benefit mathematical learning.

2.2 Learning Theories

2.2.1 Behaviourism

Behaviourism is the view that learning can be measured through observable behaviour. J.B. Watson was the father of behaviourist ideas. He defined learning as a series of stimuli and observable responses (Forrester and Jantzie 1998; Santrock 2004). The behaviourist concept of classical conditioning is illustrated in the example of Pavlov’s dog. In this case, Pavlov’s dog began to associate a neutral stimulus with an unconditioned stimulus (Santrock 2004).

Edward Thorndike was another influential behaviourist psychologist. He introduced the ‘Law of effect’. This suggests that if behaviours are followed by positive outcomes they will increase in frequency and if behaviours are followed by negative outcomes they will decrease in frequency. Thorndike’s curiosity in connectionism directed him to the principle of associative shifting. This theory suggests that behaviour may be linked with a new stimulus if the initial response remains with slow changes in the stimulus. B.F.
Skinner built further on the ideas of Thorndike and developed the theory of operant conditioning. Operant conditioning occurs when the results of a behaviour cause changes in the likelihood of the behaviour reoccurring. (Forrester and Jantzie 1998; Santrock 2004).

Behaviourist learning theories suggest that learning should be teacher centred. Behaviourist learning theories are still visible in teaching today. Teachers still use direct instruction, exams and reward and punishment systems in their teaching (Forrester and Jantzie 1998).

### 2.2.2 Constructivism

In the second half of the twentieth century, psychologists began to see behaviourist theories as outdated and too teacher directed. As a result of these new ideas, constructivism was born (Forrester and Jantzie 1998). Constructivists differ to behaviourists as they believe that learners can actively construct their own knowledge and understanding.

John Dewey was the pioneer of constructivism. He introduced the idea of pupils as active learners. Jean Piaget shared Dewey’s views. He believed that children actively construct their own knowledge and understanding through discovery learning and problem solving. Piaget also believed learners used a schema or a plan for organising and interpreting information. There are two processes that learners use to change and adapt their schema, these processes are assimilation and accommodation. Assimilation is a when a pupil integrates new information with existing information. Accommodation occurs when a child adjusts to new knowledge. In this way children make sense of the world around them (Santrock 2004).

Vygotsky shared many of Piaget’s views on constructivism. However unlike Piaget he believed that learning had social and cultural origins. Vygotsky believed learning takes place through interaction and co-operation with others. He also suggests that a ‘zone of
proximal development’ (ZPD) exists in the learning process. The ZPD refers to a range of tasks that are too difficult for a child to master on their own but can be mastered with assistance or guidance. Closely linked to the ZPD is the idea of scaffolding. This is a technique where the level of support given to a child to complete a task changes as their competence increases. Vygotsky also believed that learners should be encouraged to use dialogue during the scaffolding process. While engaging in dialogue the learner can reflect and discuss his/her understanding of the task at hand and thus begin constructing his/her own knowledge around the concept (Santrock 2004).

2.2.3 Learning Styles

The term learning style was first coined by R. Dunn in 1960. Learning styles relate to the methods or preferences that learners engage in, while learning something new. Learning styles also refer to the factors which influence how pupils view, interact and respond to learning environments (Funda Dag and Gecerb 2009).

There are many different styles of learning.

- Visual learners learn best when pictures, diagrams and other visual aids are used in learning.
- Auditory learners remember what they hear more so than what they see.
- Sensory learners gather new information using their senses.
- Intuitive learners learn unconsciously through their perceptions.
- Inductive learners gather information starting with specific knowledge and move to more general knowledge.
- Deductive learners learn by moving from general knowledge to specific knowledge.
- Active learners perform best in an environment which allows them to actively engage with the knowledge being taught.
- Reflective learners need time to consider and think about the new knowledge and would not perform well in an environment that does not include time for reflection.
• Sequential learners learn in a step by step fashion, mastering skills and knowledge in a linear manner.
• Global learners perform well when they are given the freedom to solve problems themselves and generally learn sporadically.

(Felder 2002)

When learning environments are adapted to suit the learning style of pupils, learning amongst pupils is increased (Babadoan 2000 cited in Funda Dag and Gecerb 2009). When learning environments do not cater for pupils’ learning style, poor performance and teacher frustration can occur. As so many different learning styles exist it is impossible to cater for every style, however teachers should try and use a variety of teaching styles in their teaching so as to appeal to different learning styles (Felder 2002).

2.2.4 Motivation

‘Motivation involves the processes that energise, direct and sustain behaviour’

(Santrock 2004, p.414)

Pupils become more motivated when they:
• are given choices.
• become engrossed in a learning task (that is at their level).
• receive incentives that are not used for control.
• are praised.

There are two main forms of motivation; intrinsic motivation and extrinsic motivation. Extrinsic motivation arises when a pupil completes a task because of the reward that will be obtained on completion. This form of motivation is linked to behaviourist theories. Intrinsic motivation occurs when pupils wish to complete an activity for the sake of completing it. This form of motivation is related to constructivist theories (Santrock 2004). Current research into motivation favours intrinsic motivation over extrinsic motivation. Extrinsic motivation is completely dependent on external factors which are
out the learners’ control (Wigfield & Eccles, 2000; Hennessey and Amabile 1998 cited in Santrock 2004).

There are two sources of intrinsic motivation. Firstly intrinsic motivation may be related to free will and personal choice. When pupils are allowed choices with regard to their learning, they learn more effectively. Secondly there is the intrinsic motivation of optimal experience and flow. According to Mihaly Czikszentmihalyi optimal flow occurs when ‘alienation gives way to involvement, enjoyment replaces boredom, helplessness turns into a feeling of control, and psychic energy works to reinforce the sense of self, instead of being lost in the service of external goals’ (Mihaly Czikszentmihalyi 1990 cited in Forrester and Jantzie 1998).

2.3 Computer based learning theories

2.3.1 Early Computer based learning theories
Early computer based learning theories (CBLT) were based on the behaviourist theories of Thorndike and Skinner. These programmes were based on drill and practice methods and divided instruction into small steps, teaching isolated facts and rewarding correct responses. The methods used in behaviourist computer programmes are extremely structured and involved repetition and practice. These programmes give the learner little ownership over what was being learnt (Valdez et al 2000).

2.3.2 Constructivist computer based learning
Today’s education system requires learners to problem solve and use critical thinking skills, constructivist approaches allow for such learning. Behaviourist theories cannot cater for such learner control. Constructivist approaches can be seen in many computer based learning programmes. Constructivist programmes allow learners to interact with the material they are learning and thus actively construct knowledge and understanding
for themselves. Many programmes also provide support or scaffolding for the learner (Strommen 1992).

One of the broadest studies carried out in the field of information technology in education was the Apple Classroom of Tomorrow (ACOT). This study took place over a ten year period. The results of the study indicated that the introduction of computers into the classroom lead to a decrease in didactic teacher-directed activities and an increase in constructivist approaches (Apple Computer Inc. 1995). These approaches include allowing the learner to be in control of the pace, direction, and content of his or her learning. This freedom and ownership that computers allow the learner mirror the key principles of constructivism (Forrester and Jantzie 1998).

2.3.3 How computers affect social interaction and mathematical dialogue

According to Vygotsky social interaction and co-operation are important aspects of the constructivist learning process. Such interaction is possible with computers as the computer can act as a catalyst for social interaction (Clements 1999). Different types of computer usage encourage different types of interaction. Open-ended programmes allow the learner to be in control of his/her learning and foster collaboration. Programmes based on drill and practice; promote turn taking and competition amongst users. The physical environment in which the computers are located also has an effect on interaction. When computer users are located close together mathematical dialogue is encouraged (Clements 1999).

2.3.4 Motivation and computer based learning.

Computer based learning increases pupil’s motivation in a number of ways. Firstly computers allow for learning to occur in a multimedia environment. A multimedia environment allows the real world to be brought to the learner. The learner can experience sound, images, animation and video. This allows the learner to connect to the real world and in turn can motivate the learner (Forrester and Jantzie 1998).
Secondly computers allow for instantaneous feedback. The learner is not waiting for the teacher to correct their work. Immediate feedback provides the learner with praise if they have solved the problem correctly and corrects the learner or invites the learner to try again if they have not completed the task correctly. This type of feedback is instantaneous and allows the learner to continue with their work immediately. This provides extrinsic motivation for the learner (Forrester and Jantzie 1998).

Computers also allow for self-directed learning. Self directed learning allows the pupils to focus on aspects that interest them and allows them to link new knowledge with existing knowledge. This allows the learner to become intrinsically motivated (Forrester and Jantzie 1998).

2.4 Primary Mathematics and ICT

2.4.1 The Primary Curriculum and ICT

In 1997, ‘IT 2000 - A Policy Framework for the New Millennium’ was published by the Department of Education and Science. In order to oversee the administration of this policy the National Centre for Technology in Education (NCTE) was established in 1998. The NCTE is responsible for the ICT in schools programme.

The core objectives of this programme are as follows:

- To allow pupils to become computer literate so they can participate fully in the information society we live in.
- To support teachers in using ICT as a teaching and professional development tool.

(NCTE 2009)

The next step taken by the Department of Education and Science was the Schools IT 2000 project. The aim of this project was to make sure every pupil in primary and secondary education would become a competent computer and internet user. This was succeeded by the ‘Blueprint for the Future of ICT in Education Programme’ in 2000. This project sought to increase capital provided to schools for ICT, increase internet access in
schools, integrate ICT into the curriculum and provide professional development for teachers. In conjunction with this programme a capital investment of €107.92 million was made by the Government. The Schools Internet Access Scheme coincided with these schemes and in 2004 the Computer Networking Grants were given to schools. From 1998-2004 a total of €157 million was invested by the Department of Education and Science into ICT in schools (NCTE 2009).

In 2008, the minister’s strategy group report ‘Investing Effectively in Information and Communications Technology in Schools, 2008-2013’ was published. This report outlined the following investment aims for ICT in schools:

• Provision of adequate ICT structures in every school.
• Support for innovative ways of integrating ICT into the curriculum.
• Professional development for teachers.
• Provision of digital content and tools which are relevant to the curriculum.
• Provision of reliable broadband internet access to all schools.

(Department of Education and Science 2008)

In 2009, the Government issued a further grant of €22 million to primary schools for the purpose of upgrading their ICT infrastructure. These grants are currently being used by primary schools to update their existing ICT infrastructure. This funding must firstly be used to equip each classroom with a wireless mouse, keyboard and a fixed digital projector. When this equipment has been provided, the funding may be used for purchasing other ICT hardware components (Department of Education and Science 2009).

Since the introduction of IT 2000 in 1997, the Department of Education and Science have supported the use of ICT in schools through ICT grants and projects. The NCTE which was established by the Department of Education provides teachers with the support and knowledge necessary to integrate ICT into their teaching in a manner that will maximise learner benefits (NCTE 2009). Through the initiatives of the Department of Education
and the NCTE, the use of ICT in schools has increased greatly since 1998 (National Policy and Advisory Development Committee 2001).

2.4.2 The Mathematics Curriculum and ICT

In this section we shall consider the guidelines for primary mathematics, as laid out in the 1999 curriculum. The author will also consider how ICT can be a useful tool in the teaching of primary mathematics.

The 1999 Primary Curriculum introduced new constructivist approaches to the teaching of mathematics. The mathematics’ curriculum became more ‘child centred’ as a result of this curriculum. The 1999 mathematics’ curriculum recommends a ‘hands on’ approach to learning mathematics and encourages that children are allowed to ‘manipulate, touch, and see objects’ in order to gain an understanding of mathematical concepts (Department of Education and Science 1999b: 3). The curriculum also values ‘sociocultural theory’ (Department of Education and Science 1999b: 4). This theory mirrors the beliefs of Vygotsky and sees learning as a product of social interaction both at home and at school (Department of Education and Science 1999b).

The curriculum guidelines recommend the use of ICT as a learning tool. According to the curriculum, computers can improve the way the curriculum is taught (Department of Education and Science 1999b). The curriculum suggests that ICT can stimulate the learner and provide extension work for children of all abilities and can challenge learners by engaging them in higher order problem solving activities. The curriculum guidelines also suggest ICT is used for group and paired maths activities which can stimulate mathematical discussion. This can ensure that the social interaction and co-operation can be made possible through the use of digital learning. In essence ICT is seen in the curriculum as a valuable learning tool which can facilitate ZPD, scaffolding, social interaction and dialogue (Department of Education of Science 1999b). However the primary curriculum guidelines caution against the overuse of digital media, especially when the learner has achieved mastery in a particular area (1999b).
2.5 Manipulatives

2.5.1 Defining manipulatives

In order for children to construct their own meaning of a mathematical concept their first encounter with such a concept should be of a concrete nature. Piaget, Montessori and Frobel all agree that it is by doing that children learn best (Clements 1999). Furthermore Piaget's work (1952) pointed out that children’s thinking moves from concrete, to pictorial and lastly to abstract thinking. Educators facilitate this type of ‘hands on’ learning through the use of concrete materials (Clements 1999).

The Irish primary mathematics curriculum agrees with the theories of Piaget and states that mathematical concepts should be first introduced using concrete materials. This allows learners to build understanding, prior to formal symbolic instruction. In mathematics we refer to the concrete materials as manipulatives. Common classroom manipulatives include; base ten blocks, Cuisenaire rods, unifix cubes and fraction circles (Department of Education and Science 1999b).

The aim of manipulatives is to build integrated concrete ideas. Integrated concrete knowledge is built as pupils learn to combine actions performed on physical objects and abstraction into strong meaningful interconnected mental structures. When using manipulatives, teachers should ensure they use meaningful representations in which the ‘objects and actions available to the student parallel the mathematical objects and actions’ they want students to learn (Clements 1999). Successful manipulatives are meaningful to the pupil and allow the learner to make connections between mathematical concepts (Clements 1999).
2.5.2 Benefits of mathematical manipulatives

Mathematical manipulatives provide many learning benefits for pupils. Let us consider the benefits that the research into manipulatives has highlighted.

Suydam and Higgins have undertaken a vast amount of research into the benefits of manipulatives. The cumulative findings of their research in the use of manipulatives suggest that:

- Manipulatives should be used frequently.
- Simple materials should be chosen for use as manipulatives.
- Manipulatives should be used in conjunction with exploratory methodologies.
- Symbols should be used to record findings.

(Suydam & Higgins 1976)

Suydum and Higgins’ research also highlighted that the use of manipulatives heighten the probability of increasing achievement and are important in providing a solid foundation for development of mathematical concepts (1976).

Between 1954-1984, Sowell conducted sixty studies into the benefits of using manipulatives in mathematics. His results indicated that mathematical success is increased through the long term use of manipulatives. The studies also indicated that pupils’ attitudes were more positive when the pupils were using manipulatives that had been provided by a knowledgeable teacher (Dorward 2002).

In 1998 Moyer undertook a study of middle grade students who were given free access to manipulatives in classes where the teachers were either control orientated or autonomy orientation. Research from this study indicates that manipulatives have the ability to:

- change students’ motivation during mathematical activities.
- change students’ attitudes towards mathematics.
- assist students to develop mathematical concepts.
- increase students’ problem solving skills.

(Moyer 1998)
The study also suggests that pupils make connections with mathematical concepts during concrete experiences involving manipulatives. The success of the experience is dependent on how meaningful these connections are (Durmus and Karakink 2006).

Researchers Martin, Lukong and Reeves undertook a study into the role of manipulatives in geometry and arithmetic tasks. Their research suggests that manipulatives aid learning when ‘physically distributed learning occurs’ (PDL) (2007). They define PDL as the way manipulatives enable pupils to adapt to and interpret the environment (Martin et al 2007). The results of this study show that the pupils using manipulatives were more able to adjust to their environment and learn by interacting freely with it than those not using manipulatives (Martin et al 2007).

The research findings from the studies discussed in this section suggest manipulatives are an invaluable learning tool in the teaching of mathematics.

2.5.3 Problems associated with manipulatives

Despite the learning benefits that are associated with manipulatives, manipulatives do not always lead to successful learning. Mathematical concepts cannot be automatically transferred from manipulatives (Clements and McMillen 1996). Therefore it is important that teachers consider carefully the way in which manipulatives represent mathematical ideas.

Despite the benefits of the kinesthetic learning that is experienced by pupils when using virtual manipulatives, manipulatives do not ensure understanding. Occasionally the external actions that are used on the manipulatives do not match the internal mental activity. In order to ensure that the external and internal actions match, one must reflect on the actions taken. Reflection may take the form of paired, group or whole class discussion, in which the pupil can verbalize their understanding and build on their knowledge (Clements 1999).
2.5.4 Recommendation for the use of manipulatives

In order for manipulatives to benefit learners, manipulatives that permit children to use informal methods should be selected. During the use of manipulatives pupils must be in control of and be allowed to think of their own solutions.

Manipulatives should never be used in a rote manner. After the pupils have interacted freely with the manipulatives, pupils should be encouraged to reflect and explain their solutions. In this way the learner can understand and analyse errors (Clement 2000).

2.6 Virtual Manipulatives

2.6.1 Defining virtual manipulatives

Recent additions to the realm of manipulatives are virtual manipulatives. Virtual manipulatives fall under the realm of dynamic learning systems (Heid 2005). Dynamic learning systems allow for the development of learners’ cognitive skills and promote discovery learning. Moyer, Bolyard and Spikell undertook a study in 2001 in search of a definition for virtual manipulatives and the unique properties they possess. As a result of their study they define a virtual manipulative as an ‘interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge’ (Moyer et al 2002, p.373). Virtual manipulatives are modeled on traditional classroom manipulatives such as base ten blocks and fraction circles (Moyer et al 2002). Virtual manipulatives are moved and manipulated using the computer mouse.

Virtual manipulatives have many unique properties. The most notable property for educators is that virtual manipulatives may be obtained free of charge by schools. As they are available online, virtual manipulatives may also be accessed by pupils from home. Virtual manipulatives can be physically altered by the pupils, for example when using virtual tangrams, colours of shapes may be changed. This is not possible when using concrete manipulatives. Often older students feel they are too mature to use concrete
materials in their learning; however they view virtual manipulatives in a different light and see them as being more ‘sophisticated’ (Moyer et al 2001, p.186). When using virtual manipulatives teachers need not worry about not having enough manipulatives for each pupil, once each pupil or pair of pupils has access to a computer, they have access free of charge to the virtual manipulatives. In addition virtual manipulatives are cleared away instantaneously with the click of a mouse.

2.6.2 Benefits of virtual manipulatives

The amount of research undertaken on the benefits of virtual manipulatives is quite limited and therefore ‘a judgment about their potential uses in mathematical instruction is entirely speculative’ (Reimer and Moyer 2005: 8). Many researchers have pondered if research supporting physical manipulatives can be transferred to virtual manipulatives. The answer that Clements and McMillen give to this question is that ‘concrete is quite literally, in the mind of the beholder’ (1996, p. 272).

Suh and Moyer undertook a study to investigate how tutorials based on virtual manipulatives could encourage the development of fraction sense (Suh et al 2005). In this study the use of virtual manipulatives showed the greatest benefits amongst low achievers. This study also found that the use of the virtual manipulatives allowed pupils to test out their answers without receiving judgemental feedback. The programme used in this particular study had built in teacher prompts which assisted the students in understanding the mathematical concept being taught. Thus the virtual manipulatives provided a safe, non-judgemental, scaffolded learning environment (Suh et al 2005).

Suh and Moyer’s study also indicated that following the use of virtual manipulatives children’s representation fluency, in particular that of visual learners, was increased. This occurred as the visual and symbolic formats were linked on screen. This linkage allowed pupil’s to make connections between both forms of representations. The environment also appealed visually to the learners and helped sustain their attention.
Clements suggests that computer manipulatives can help to increase the motivation and attention of learners (1999). In the study by Hoyles, Healy and Sutherland, pairs of learners working on computers were observed. The observation found that very little ‘off-task’ talking occurred (Hoyles et al 1991). Thus computer manipulatives may also help to increase motivation and attention of learner (Clements 1999).

Virtual manipulatives can also encourage problem solving and conjecture. As pupils test their ideas using virtual manipulatives they can easily move from ‘empirical to logical thinking’ (Clements 1999). Some types of manipulatives are centred on Vygotsky’s theory of scaffolded learning and provide graded support for the learner as he/she requires it. Virtual manipulatives can also reduce the cognitive load of the learner using online cognitive tools. In this way the pupils working memory is freed up to process information (Le Fevre et al 2005).

Research undertaken by Clements on the use of virtual manipulatives suggests that computer representations of manipulatives can be more ‘manageable, clean, flexible and extensible’ than concrete manipulatives (1999). Virtual manipulatives do not need to be stored or tidied away and can be easily extended unlike their concrete counterparts (Clements 1999). Another advantage of virtual manipulatives is that teachers need not worry about manipulatives going missing. In a study done by Moyer on the use of manipulatives in middle grade classes, teachers indicated their main pre-research concern was the fear that manipulatives may become mislaid (2008). This fear, which no doubt encapsulates many teachers, is not an issue when using virtual manipulatives.

In a study of pupils from grade 2 to grade 5, using base ten blocks, it was found that students who used a combination of virtual and concrete manipulatives had a greater gain between pre-test and post-test scores than students who used only concrete manipulatives or only virtual manipulatives. This suggests that more effective mathematical learning occurs when a combination of virtual manipulatives and concrete manipulatives are used (Terry 1995 cited in Moyer et al 2008).
2.6.3 Different forms and representations of virtual manipulatives

There are currently two forms of virtual manipulatives; static visual representations and dynamic visual representations. Static visual representations are pictorial representations of concrete manipulatives and are not true virtual manipulatives as you cannot interact with them (move/flip) as you would with concrete manipulatives (Moyer et al 2002). Dynamic visual representations however are visual representations that can be manipulated just like their concrete counterparts. This type of representation is a true virtual manipulative (Moyer et al 2002).

In addition to the different representations of virtual manipulative there are also different types of virtual manipulative. microworlds, intelligent tutoring systems and applets are example of some virtual manipulatives available to learners (Suh 2005).

A microworld is ‘an interactive, exploratory learning environment of a small subset of a domain that is immediately understandable by a user and also intrinsically motivating to the user’ (Rieber 2005, p.12). The microworld can be altered by the user in order to test theories about the domain. One of the most well-known microworlds is Logo. Logo is a programme developed by Papert in the 1980’s. The function of this programme was to allow pupils to learn mathematics naturally as they would any other language (Rieber 2005).

There are three common traits that exist in all microworlds:

- They allow more people (beginning at a younger age) the opportunity to understand the concepts of complex systems.
- They are based on qualitative understanding, involving concrete materials.
- They attempt to lessen the divide between learning and doing.

(Rieber 2005)

An intelligent tutoring system (ITS) is a ‘computer based instructional system’. This computer based instruction is modeled on a personal human tutor. It uses content models which allow for real life meaningful learning opportunities. ITS’s can adapt their content to suit the needs and ability of the learner, thus allowing for individualized learning. An
ITS also enables the user to have control over their learning and allows for interaction. However a major disadvantage of an ITS is the expense and difficulty involved in creating it (Murray 1999). An example of an ITS is the Pump algebra tutor (PAL). This was piloted in 1992-1993 in Pittsburgh’s Langley High School, where it taught 25 class periods. In 1995-1996 PAL taught 750 students in six high schools. Pupils using PAL performed 10-15% better on standardised tests than those who had human tutors (Corbett and Anderson 1997).

An applet is a standalone Java application available on the internet. This type of application is most effective when situated in a real life context (Reimer and Moyer 2005). Linked representation similar to that used in tutoring systems is a feature of some applets (Suh 2005). Applets should be easy to navigate, have clear direct instruction and be visually appealing to the learner. The content of these applets should be focused and at an appropriate level for the user. Robustness is an important feature of an applet as it needs to withstand an array of inputs from the user. A mathematical applet is known as a mathlet (Roby 2001).

There are many websites which provide learners with free applets/mathlets. The National Library of Virtual Manipulatives (NLVM) and the Mathplayground website provided the virtual manipulatives used in this study.

The NLVM was created in 1999. It provides a digital library of Java applets and activities for primary and secondary school pupils. The virtual manipulatives are graded and divided into five sections:

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data analysis and probability

(Utah State University 2009)
The Mathplayground website was created in 2002 by Colleen King. It allows children to learn mathematics in a fun way. It covers a wide range of mathematical topics and provides mathematical manipulatives, games, puzzles, problems and videos for mathematical learners. It provides the learner with virtual manipulatives based on number, shape and space and data (King 1999).

2.6.4 Recommendations for the use of virtual manipulatives.

The first factor one should consider when choosing a virtual manipulative is that it should be an appropriate representation of its concrete counterpart. An appropriate representation allows the learner to use concrete thinking skills to construct an understanding of a mathematical concept (Clements 2000). Multiple representation of the same concept can also benefit the learner as they demonstrate different aspects of the same concept (Durmus and Karakink 2006).

Another factor to consider is learner control. In order to allow optimal learning when using virtual manipulatives, pupils should be in control of their learning and be allowed to think of their own solutions to problems (Clement 2000). Mayer’s research suggested that learners had a better transfer of knowledge when they were in control of the pace of their learning (2001). When this type of exploratory, discovery learning occurs the teacher’s role becomes that of a facilitator (Clements 2000).

After the learner engages in a task involving virtual manipulatives, the learner should be encouraged to reflect and explain their solutions. In this way the learner can build understanding and analyse errors in a constructivist manner. Co-operative learning such as paired learning is beneficial when using virtual manipulatives. This encourages mathematical discussion. Following an activity, further reflection and discussion should be facilitated in groups or as a class unit. For optimal learning benefits, class discussion should ideally take place around a large screen on which the manipulative can be displayed (Clement 2000).
2.7 Human Computer Interface

2.7.1 Definition of human computer interface

Unlike a human teacher, computers are unable to convey affective information such as change in tone of voice, facial expressions or body language. Nonetheless, a computer can present information in a more advanced manner using graphics, text, and sound. Human computer interface (HCI) design refers to the way in which the tools of graphics, text, sound and controls, which allow the user to navigate and use a programme, are presented to the learner. The way in which these tools are used to present information to the user and the way in which the user interacts with the computer programme can affect the pupils’ learning (Zelevansky 1995).

2.7.2 Alessi and Trollip’s guidelines to successful HCI

Alessi and Trollip outline four steps which they believe lead to successful multimedia instruction. The four steps are:

- presentation of information
- guiding the user
- practice
- assessment

(Alessi and Trollip 2001)

The presentation of information is a crucial element in HCI. The learner’s perception and attention are closely linked to the presentation of information (Alessi and Trollip 2001). There are four main ways of presenting information; text, graphics, sound and video. The presentation of information in more than one way allows the learner to encode information. The format and presentation of text on the screen is very important. Graphics are excellent at grasping the learner’s attention, but should only be used where necessary and not as decoration. Video is also a valuable tool; however learners should be given control over play and viewing options. Sound can be beneficial in reinforcing
information and redirecting pupil’s attention, however similar to videos, pupils should be given control over play options (Alessi and Trollip 2001).

Let us consider how Alessi and Trollip’s principles relate to the virtual manipulatives used in this case study. The virtual manipulatives used in this study are available at the following web addresses:

- www.mathplayground.com (Mathplayground)
- http://nlvm.usu.edu/ (NLVLM)

The time virtual manipulatives used in the study were provided by the NLVM and the 2D shape virtual manipulatives were provided by Mathplayground. All virtual manipulatives used in the study are presented in a text and graphics format and do not include sound or animation. As recommended in Alessi and Trollip’s guidelines for multimedia presentations, a similar text layout and format is used in each virtual manipulative (2001).

Alessi and Trollip’s next step to successful multimedia design is guiding the user. This principle suggests that a help option should be available to the user throughout the programme. The instructions should be clear and easy to follow (Alessi and Trollip 2001). The NLVM have an ‘instructions’ and ‘back’ button that are present on each page. These buttons are labelled with a yellow symbol and white text and are position on a toolbar at the top of the page. The back button brings the user back to the main menu. The instructions button allows the user to scroll through instructions which appear at the right hand side of the screen, whilst at the same time viewing the virtual manipulative they are using.
The Mathplayground website has three buttons at the bottom of the virtual manipulatives screen: copy, clear, and help. The buttons are blue with white text and are embedded in a purple background. The help button displays instructions in the virtual manipulatives screen. When the help button is right clicked, the instructions appear and when the button is right clicked again, the instructions disappear and are replaced with the previous virtual manipulatives screen. Unlike the NLVM, Mathplayground does not display the instructions and virtual manipulatives at the same time and on the same screen.

Alessi and Trollip’s final two steps for multimedia instruction are practice and assessment. Both the NLVM and Mathplayground allow for repeated practice using the virtual manipulatives. Assessment however is not available from either the NLVM or Mathplayground.
2.7.3 Mayer’s principles of multimedia design

Mayer outlines seven principles for multimedia design. Mayer tested these principles using both transfer of knowledge and retention of knowledge tests (2001).

Mayer’s first principle states that multimedia works better when pictures and words are presented together (2001). In addition, Mayer suggests that learning is enhanced when related words and pictures are positioned close together on the screen. The manipulatives in the study are based on 2D shape and time, therefore words are not a predominant feature of either virtual manipulative. 2D shape manipulatives take the form of geoboards and tessellation boards. The 2D virtual manipulatives provided by the Mathplayground website only uses words for labelling buttons. In addition to the way words were used in the 2D shape virtual manipulatives, the time based virtual manipulatives provided by the NLVM use words to ask the user to perform a task and to give the pupil feedback on a task they have just completed. If the pupil has performed the task correctly, ‘correct’ appears in blue writing, however if the pupil has performed the task incorrectly ‘try again’ appears in red writing. Questions and pictures are located close together in NLVM (Mayer 2001). If we consider the example inserted below the question is written clearly at the top of the screen close to the clock. The corresponding time is written to the left and is also close to the clock. The feedback text is at the top of the screen in red.

![Figure 2.3: Example of feedback provided by the NLVM](image_url)
Mayer’s third principle suggests that pupils learn better when text and pictures are displayed at the same time rather than one after the other. As can be seen from the earlier examples, this principle is adhered to by NLVM and Mathplayground.

Mayer’s fourth principle, the coherence principle, states that pupils learn more efficiently when unnecessary words and pictures are disregarded. This theory is supported by Alessi and Trollip (2001). They suggest graphics can easily grasp the attention of the learner and therefore unnecessary and ornamental graphics should be avoided (Alessi and Trollip 2001). The NLVM uses a simple concise presentation with little unnecessary text. Mathplayground only uses text to label, however the virtual manipulative is embedded in a screen that can be scrolled up or down and has a lot of unnecessary text on it.

The Modality principle suggests it is more effective to use animation and narration than animation and text together. This principle does not apply to the manipulatives used from the NLVM or Mathplayground, as animation is not present in the manipulatives used. When using animation and text the learner has to process both modes of information through their auditory channel, and this overloads the learner. Animation and narration however use two different channels; the visual channel and the auditory channel.
In addition to these principles, Mayer also suggest that students` learn better from animation and narration alone than animation, narration and text (2001). This principle does not apply as the virtual manipulatives used in this study do not incorporate animation.

Mayer’s final principle indicates that multimedia has greater benefits for low knowledge learners than high knowledge learner (2001). This principle will be considered in greater detail in the analysis stage of this case study.

2.8 Conclusion

This chapter explored how learning theories especially constructivism supports the use of ICT in the primary mathematics’ classroom. It also explored the areas of learning styles, motivation and dialogue and their role in pupils’ learn. The importance of manipulatives and virtual manipulatives in the teaching of mathematics was also outlined. Finally the HCI designs of the virtual manipulatives chosen for use in this study were examined. The next chapter will explore the research approach and the data collection methods which were chosen for this study.
Chapter 3 Methodology

3.1 Introduction

This chapter will examine the processes of inquiry which are used in educational research. Following this the researcher will justify the type of inquiry and the data collection methods chosen for this research study.

3.2 Statement of Problem/Research Question

The purpose of this study was to examine the benefits of virtual manipulatives in the teaching of primary mathematics. This study aimed to investigate the following:

- How effective are virtual manipulatives in the teaching of primary mathematics?
- Are concrete manipulatives and virtual manipulatives more effective when used together?
- Are pupils more motivated when using virtual manipulatives or when using concrete manipulatives?
- Do virtual manipulatives allow for more or less mathematical dialogue than concrete manipulatives?

3.3 Research Setting

3.3.1 Physical setting

The physical setting for this study was the researcher’s classroom. The classroom is a modern room, rectangular in shape, yellow in colour and is bright and airy. The classroom has two computers with broadband internet connection.
3.3.2 The target population

The target population for this study was a 3rd class group from a rural primary school in the south of Ireland. There are 25 children in the class, 11 boys and 14 girls. The majority of the class have computers at home and are familiar with computers. The children occasionally use the class computers and have basic computer skills. This target group was chosen as the researcher is teaching in the school and could easily access the group.

3.4 Research Approach

3.4.1 Historical background of research approaches

Action research is a relatively new research approach. For much of the 20th century positivist and postpositivist research was done in an out of context artificial setting. In the last quarter of the 20th century, this type of research setting was criticised and researchers opted to undertake research in more natural settings. Kurt Lewin was one practitioner who contributed greatly to the theory of action research. His theory on the use of a spiral approach in action research and his ideas on the involvement of the practitioner in the research, form the basis for postpositivist action research (Willis 2007).

Unlike action research case studies have a long history in social sciences and were used in the early days of sociology to study a particular group of people or a certain programme (Lichtman 2006). Case studies originated from a variety of sources:

- Clinical methods of doctors.
- Case-work method used by social workers.
- Techniques of historians and anthropologists.
- Qualitative researcher’s descriptions.

(Hammersley 1989 cited in Hitchcock and Hughes 1995)

Despite the long history of case studies in the social sciences, in 1937 researchers at Columbia University began to question case study research and believed research should
employ more scientific methods. This belief became widespread amongst sociologists. Nonetheless in the 1960’s, researchers became concerned with the limitations of quantitative research and case studies began to regain popularity once more (Lichtman 2006).

3.4.2 Defining action research

Action Research is defined as ‘on-the-spot’ research designed to deal with a concrete problem located in an immediate situation’ (Cohen et al 2000, p.223). This type of research is attractive to practitioner-researchers who have recognised a problem and wish to examine and improve their practice. The task is not finished when the research project ends, the practitioners continue to review and evaluate their work (Bell 1993). In this way action research allows for a reflective cycle of research (Opie 2004).

3.4.3 Defining case study research

Case studies ‘evolve around the in-depth study of a single event or series of linked cases over a defined period of time’ (Hitchcock and Hughes 1995, p.319). A case study focuses on an authentic situation in a setting which is familiar to the researcher (Opie 2004). They are characterised by their detailed chronological description of events and are generally inductive and heuristic in nature (Merriam 1988).

The initial step in the case study process is defining the case study. Once the case study has been defined the researcher must decide whether to adopt a single-case study or a multiple-case study. The researcher also has the option of using theory development as a basis for the case study. This involves extending or challenging what the existing research states. First time researchers can benefit greatly from using theory development. However the experienced researcher could find themselves limited in their ability to make new discoveries when using theory development (Yin 2006).
When the researcher has decided on the type of case study he/she will engage in, the researcher must select a case on which to base his/her case study. When a case has been selected, the researcher must choose the data collection methods which will be employed in the study. Good case studies use multiple sources of evidence. Case study evidence can include both qualitative and quantitative data (Yin 2006).

Case studies can be classified by the way the data is collected during the case study. For example; a descriptive case study is characterised by the descriptive nature of the data collected and an interpretive case study collects data that needs to be interpreted by the researcher. Case studies may also be distinguished by the subjects they are examining for example; ethnographic, situation, historical, sociological etc.

3.4.4 Limitations of a case study

Case study research can be a valuable research approach however there are limitations to case study research. Often case studies focus on a single situation and therefore generalisation of findings is not possible (Bell 1993). This creates problems with regard to the reliability, credibility and validity of the study.

Another difficulty that case studies present for the researcher is that they require the researcher to enter into the environment that is being studied. Sometimes entry can be problematic for a researcher. The researcher must also gain the acceptance of the participants and establish a friendly and co-operative relationship with participants (Willis 2007).

Postpositivists have very little belief in case study research except for when it is a predecessor to quantitative research. Postpositivists advocate that a case study is only credible when it contains scientific or mathematical data and not observations or recollections (Willis 2007). Many case studies are ethnographic in nature and do not contain quantitative methods of data collection and therefore if one were to concur with postpositivist views on case studies, much case study research could be discredited.
(Willis 2007). Despite these criticisms of case study research, the case study approach is still popular amongst practitioner-researchers in the field of education.

### 3.4.5 Justifying a case study

The researcher has chosen to adopt a case study approach for this study. This approach was chosen as it allows an individual researcher the opportunity to study one situation in detail over a limited period of time (Bell 1993).

For the purpose of this study a target population which was familiar to the researcher was chosen. The target population was a third class group in a primary school setting. The researcher is a teacher in this primary school and could easily access this target group. The researcher wished to examine one particular learning aspect; if virtual manipulatives are an effective mathematical learning tool.

Due to the time limitation of this study the researcher chose to use only one target group and thus undertook a single case study. As the researcher was familiar with the target population and only one specific aspect of their learning was being investigated, the researcher decided that a case study approach was the most effective and feasible tool for the research study.
3.5 Data Collection Tools

3.5.1 Introduction to research methods

There are two main research methods used when collecting data; qualitative research methods and quantitative research methods.

Quantitative research methods were first ‘developed in the natural sciences to study natural phenomena’, however today they are also used in the social sciences and in education (Berry 2006). Quantitative research involves gathering statistical facts and examining the relationship between sets of facts (Bell 1993). Quantitative data, as it is statistical in nature, can be easily measured and patterns discovered in the research can be accurately generalized as they are based on a large quantity of material. Quantitative research uses scientific methods of data collection such as testing, surveys and structured observation (Hitchcock and Hughes 1995).

Qualitative research is defined as ‘any kind of research that produces findings not arrived at by means of statistical procedures or other means of quantification’ (Strauss and Corbin 1990, p.17). It focuses on ‘individuals’ perceptions of the world’ and ‘seeks depth rather than breath’ (Bell 1993, p.6). Qualitative research therefore focuses on smaller groups rather than a representative sample of the whole population. Qualitative research looks at the reasons why people act, think and understand in the manner they do (Ambert et al 1995). It generally involves an inductive approach which allows the researcher to acquire new insights into situations. Qualitative research uses naturalistic approaches of inquiry such as; ‘open-ended questionnaires, in-depth interviews, ethnographic studies and participant observation’ (Ambert et al 1995, p. 880).
3.5.2 Testing

Testing is a method of quantitative research. There are two types of tests; parametric tests and non-parametric tests. Parametric tests are designed for use on a wide population and enable the researcher/teacher to compare the results of a class/group with the whole population and make inferences about the results. However one clear disadvantage is that the tests are not tailored specifically to the students needs. Non-parametric tests on the other hand are designed for a specific population. They do not make assumptions about a population. A teacher designed test is generally non-parametric and a teacher designed test can be modified to suit a certain class or group. This type of test allows teachers to gain feedback quickly on students’ performances (Cohen et al 2000).

Tests are also categorised as norm-referenced tests or criterion referenced tests. Norm referenced tests ‘compare students’ achievement relative to other students’ achievement’ whereas criterion referenced tests require ‘a student to fulfil a certain set of criteria’, like for example a driving test (Cohen et al 2000, p. 318). Criterion testing allows the researcher to find out exactly what a student has learned and what he/she can do. Criterion testing also requires a level of mastery learning. Norm referenced testing, on the other hand, informs the researcher of how one student’s knowledge or skills compares to other students. Domain referenced testing is a recent ‘outgrowth of criterion referenced tests’ (Cohen et al 2000, p. 319). In this form of testing the domain is laid out clearly and test items are taken from the full domain. The student’s knowledge of the whole domain is inferred by the pupil’s performance (Cohen et al 2000).

Teacher designed tests are an important part of primary assessment (Department of Education and Science 1999). The first stage one undertakes when constructing a test is to decide on the purpose of the test. Pre-testing elicits the target populations’ prior knowledge. A test that occurs during a programme is known as a formative test. It monitors progress and identifies strengths and weaknesses. Diagnostic testing is used when pupils are experiencing difficulties during a programme. It identifies the strengths, weakness and difficulties that pupils have with the programme. Summative testing
measures a pupil’s achievement after a programme of study has been completed (Cohen et al 2000).

The first stage in constructing a test is identifying test speculations. These include the objectives of the test, learning outcomes, areas of the programme to be included in the test and the number of items/questions on the test. The test objectives should be specific and concise and represent the learning outcomes of the programme equally.

The next stage of construction is selecting the contents of the test. In order to ensure equal representation of a programme in a test an ‘item analysis’ is necessary (Cohen et al 2000, p.323). Grounloud and Linn make suggestions on what should be considered when undertaking an ‘item analysis’:

- Ensure the test item suits the learning objective.
- Ensure the test item is clear to the learner.
- How will achievement be indicated?
- What will be required to indicate mastery in each test item?
- What will the result of the test be?

(Cohen et al 2000)

Piloting the test is an important step in creating a test. It allows the teacher to become aware of any errors or problem areas in the test prior to using the test formally.

For the purpose of this case study, teacher designed testing and standardised testing was used to gather data. Testing was chosen as it provided the researcher with numerical data which could be analysed.

At the beginning of this research study, the pupils’ Sigma T level 2 results from June 2009 were reviewed. The Sigma T is a standardised, parametric, norm referenced, commercially produced series of mathematics tests. The Sigma T was first developed in 1991 by the Curriculum Development Unit (CDU) in Mary Immaculate College of Education. It has been updated regularly and most recently in 2007 to support the Primary
School Curriculum. The Sigma T series of tests cater for pupils from 1st class to 6th class (Wall and Burke 2007). The target population’s Sigma T level 2 results were used to allocate pupils into three groups, ensuring that each groups consisted of pupils of corresponding mathematical ability. Each group used different forms or combinations of manipulatives. By ensuring that all three groups consisted of pupils of corresponding mathematical ability, the reliability of the results was increased.

In addition to using the Sigma T results to organise the target population into groups, the Sigma T was also used in a pre-test/post-test manner in order to elicit the skills and knowledge acquired during the research period. Therefore the Sigma T was administered again after the four week research period had lapsed. Each group’s Sigma T results were compared and analysed in order to identify the benefits of virtual manipulatives. The topics taught during the research period belonged in the subsections of Shape & Space and Measures. The Sigma T results from these two sections were also extracted. These results allowed the researcher to elicit the knowledge and skills acquired by each group, in these subsections during the research period.

In addition to the Sigma T, the researcher used teacher designed pre-tests and post-tests. This assessed in a more specific way, the knowledge and skills that each pupil acquired during the four week period. In the case of the teacher designed tests, domain referenced testing was used. The test items were chosen from the entire topic and the pupil’s knowledge of the whole domain was inferred from their test score results. The teacher designed tests were piloted on another class grouping prior to their administration.

3.5.3 Observation

The method of qualitative research used in this case study was observation. Observation is a method of collecting data through field research. It can lead to a deeper understanding for the researcher because it allows the researcher to observe things that the participants themselves may not be aware of, or may be reluctant to discuss during an interview (Patton 1990). Observation also allows the researcher to witness less unpredictable situations (Cohen et al 2000). Observational research gains data on the
physical setting, human setting (i.e. the way people are organised), interactional setting (type of interaction that the subjects engage in verbal/non verbal) and the programme setting (pedagogy used, curriculum etc.) (Morrison 1993 cited in Cohen et al 2000). Observation is recommended in the Primary Curriculum as an important assessment tool and is a common assessment strategy in the primary classroom (Department of Education and Science 1999).

There are three main types of observation; structured observation, semi-structured observation and unstructured observation. In structured observation the researcher will have done a large amount of preparation and will have already decided on the focus of the observation prior to beginning the observational period. Structured observation allows for the collection of numerical data using an observational schedule with distinct categories. There are concerns regarding structured observation as it is largely based on behaviourist theories and does not take motivation or intention into consideration (Cohen et al 2000). A semi structured observation on the other hand takes less time to prepare and will have a less structured approach. An unstructured observation is undertaken when the researcher has not yet decided on a focus for the research and will have to undertake the observation before the focus becomes apparent.

Observational research can also be categorised by the way the researcher engages with their subjects. There are five categories of researcher participation; non-participation, passive participation, moderate participation, active participation and complete participant (Mertens 2005). Any form of participant observation can be ‘subjective’, ‘biased’ and ‘impressionistic’ (Cohen et al 2000, p.313). It can give the researcher a great insight into the target population, however the underlying danger is that the researcher may begin adopting the customs and viewpoints of those being observed and thus the research becomes unreliable and loses it validity (Cohen et al 2000).

For the purpose of this study, semi-structured observations were be used. The researcher recorded the time spent on-task using an on-task/off-task observational record sheet. The researcher also used the observational record sheet to record how often the pupils
interacted with each other and the reason interaction occurred (made a comment, encouraged a peer, asked a question etc.) (Appendix G). In addition, any other incidents or occurrences that the researcher deemed to be valuable to the study were recorded as incidental observational notes.

The type of observation employed during this study took the form of participant observation. This form of observation was chosen as the researcher was also teaching the pupils and therefore participation was necessary. By engaging in this form of observation, the teacher/researcher was able to offer assistance or prompts where necessary, while at the same time observing the pupils.

### 3.5.4 Questionnaires

There are two main reasons for using questionnaires:

- To find out information that cannot be found out in any other way.
- To examine the effect of an intervention when a response cannot be determined in any other way.

The first step that should be considered when constructing a questionnaire is why you want the information. The researcher must also consider if he/she has permission to use the questionnaire on the target population (McNiff et al 1996).

There are two types of questions that can be included in a questionnaire; closed questions and open questions. Closed questions generally invite the subject to choose an answer from a limited range of specified answers. These types of questions restrict the users’ answers. However they allow the researcher to chart information more easily. Open questions are more beneficial to the researcher as they invite the respondent to articulate their views more extensively. Open questions however require more space for an answer than closed questions. They also take longer to examine because they are so broad. However both types of questions plant ideas in the respondents heads prior to answering
and because of this pupils' responses have been influenced. Questionnaires should always be piloted prior to use (Mc Niff et al 1996).

Questionnaires were used in this study to find out how the students felt about using virtual manipulatives and/or concrete manipulatives. Questionnaires were chosen as it allowed the pupils to voice their feelings about the use of manipulatives in a confidential manner without judgement from teachers or peers.

Two questionnaires were designed for use in this case study. One questionnaire was based on the use of virtual manipulatives and the second questionnaire was based on the use of concrete manipulatives (Appendix H). Group A completed the questionnaire based on virtual manipulatives and group C completed the questionnaire based on concrete manipulatives. Group B used both concrete and virtual manipulatives so they completed both questionnaires. Both questionnaires consisted of three closed questions and two open questions. Questionnaires were piloted on another class group prior to being used on the target population. Before pupils began the questionnaires, the questions were read with the pupils so as to ensure the pupils understood what was required for each question.

3.6 Validity, reliability and triangulation

3.6.1 Validity

Validity informs us if an item measures or describes what it was meant to measure or describe (Bell 1993). There are two forms of validity; internal validity and external validity. Internal validity occurs when somebody examines data collected in a study and finds that the data is consistent and can be replicated in another setting. In an effort to increase the validity of a study, researchers often divide their target group into two groups; a control group and an experimental group. The control group uses the traditional learning tools and the experimental group uses the learning tools being studied. The groups are both tested on the content of the programme before beginning the programme and are tested again after the program has been completed. This common method, used
by researchers to increase validity can often have the opposite effect. The pre-test may have an impact on the post-test results. It is difficult also to know if the pre-test or the variable (new way of teaching) has affected the pupils’ learning. In addition if the study is to be generalised, is it possible to generalise in a setting where pre-testing is not the norm. However, if the pre-test is not given it is questionable whether the results of the study can be generalised as the pre-test could have been an important factor in the pupils’ learning (Willis 2007). Another important consideration when using this pre-test/post-test model is the time frame available to the researcher. If the period of research is short, the subjects will still have the first test in their memory and the post-test may be affected by this. However if the research period is too long, the participants attitudes and ideas may have changed. Therefore the pre-test/post-test method should be considered carefully before deciding to use it as a means of increasing reliability (Opie 2004). This case study has used the pre-test/post-test model to examine the learning that occurred during the research period.

External validity depends on whether the results obtained in one study can be transferred to another setting. In order for a theory to be transferable there must be some similarities between the situation in which the theory was tested and the situation to which the theory was transferred (Hoepfl 1997). Partial external validity can occur, however it is not possible for a study to have complete external validity as one cannot be sure what could arise in an unknown setting (Opie 2004).

3.6.2 Reliability

Reliability ‘is the extent to which a test or procedure produces similar results under constant conditions on all occasions’ (Bell 1993, p. 64). In other words reliability occurs when the data collection method is reliable (Opie 2004, Bell 1993). Therefore in theory, another researcher could employ the same data collection methods and expect to obtain the same data. One way in which to gain reliability is to use the pre-test/post-test method that was discussed above.
3.6.3 Triangulation

Triangulation is defined as ‘the use of two or more methods of data collection in the study of some aspect of human behaviour’ (Cohen et al 2000, p. 112). Triangulation according to Lichtman is a method of making ‘qualitative research more objective and less subjective-in other words more scientific’ (2006, p.194). When a researcher has used two or methods of data collection, and results from both methods correspond, the researcher can have confidence in his/her results (Cohen 2000). Credibility of research can therefore be enhanced through triangulation (Hoepfl 1997). Triangulation however is not without its disadvantages and does not guarantee reliability (Patton 1990).

There are many types of triangulation. Methodological triangulation involves the use of different data gathering methods (Willis 2007). There are two forms of methodological triangulation; ‘within methods’ triangulation and ‘between methods’ triangulation. ‘Within methods’ triangulation happens when a study is duplicated and reliability is being confirmed (Smith 1975 cited in Cohen et al 2000). ‘Between methods’ triangulation occurs when more than one method is used in order to gain a particular outcome (Cohen et al 2000, p.114).

Despite the benefits of triangulation, sometimes a study conducted extensively using one research tool could prove to be more reliable and valid than a study using multiple research tools (Willis 2007).

‘Between Methods’ triangulation is the approach used in this study. This approach is evident in the quantitative and qualitative research tools that are used. Parametric and non-parametric testing were used prior to and after each topic. Observation was used throughout the case study to observe the pupils’. In addition to testing and observation, pupil questionnaires were also used. These inform the researcher about the pupils’ attitudes to and experience with concrete manipulatives and virtual manipulatives. The results from these three methods allow for triangulation and thus enhance the validity and reliability of the case study.
3.6.4 A critical friend
A critical friend is somebody who can ‘engage with another person in a way which encourages talking with, questioning and even confronting the trusted other in order to examine planning for teaching, implementation and its evaluation’ (Hatton & Smith 1995, p. 4 cited in Opie 2004). In the case of the practitioner as researcher, a critical friend can be very beneficial to the research process. A critical friend can act as a mentor in discussing and evaluating the research. The role of a critical friend is to:

- confirm the research is recorded accurately.
- assist the researcher in giving a truthful and correct account of the research.
- offer evaluation to the practitioner.
- offer moral support and feedback on the research.

A critical friend is generally an available colleague with similar values to the researcher. He/she would need to have an interest in the research or be involved in the research being undertaken and finally he/she should be able to provide critical feedback to the researcher (McNiff et al 1996).

In the case of this study the researcher chose a third class teacher to act as a critical friend for the duration of this research study. The experience of another third class teacher benefited the researcher greatly, especially when devising teacher designed test, questionnaires and observational record sheets.
3.7 Criteria

‘Criteria are the signs by which something is judged’
(McNiff et al 1996, p.68)

Criteria in research is decided on by researchers using their idea of what is ‘good’, thus criteria are dependent on those who set them. The standards of judgements used in the criteria are also dependent on personal views. Thus the nature of criteria is highly subjective.

When one submits work as part of a grade bearing programme, the work can be assessed based on the quality of the written work rather than the quality of the research it describes. This places the student at a disadvantage because the marker’s criteria for a good quality piece of work are quite different to that of the researcher. Work may also be judged on pre-set criteria that was not the intended criteria of the researcher or may be judged by a researcher with different values or different standards of judgment (McNiff 1996 et al).
3.8 Ethics

‘Ethics has to do with the application of moral principles to prevent harming or wronging others, to promote the good, to be fair’

The ethics of research is concerned with the level of honesty of the researcher when analysing and reporting results (Greenfield 2002). Lichtman suggests four ethical principles that a researcher should apply to their research. The principles suggest participants should:

- be protected from any physical or psychological harm.
- not be deceived in any way by the researcher.
- have their privacy protected.
- consent to partaking in the research.

(Lichtman 2006)

Lichtman’s first principle deals with protecting the participants of a study from physical or psychological harm. This study is located in the target population classroom. This is a safe familiar environment for all participants.

Lichtman’s second principle states that participants should not be deceived in any way. This principle is often difficult to adhere to, as in certain circumstances, covert research may be the only safe way for a researcher to gain entry to a setting that he/she wishes to study. These settings may be violent or involve prejudice of some kind. Therefore some cases of covert research can be justified as the findings of the research may allow for improvements in social justice (Opie 2004). Nonetheless covert research still involves the researcher deceiving his/her subjects (Greenfield 2002).

The last two principles outlined by Lichtman advise researchers to gain participant consent prior to undertaking research or referring to participants in a research report.
This is important especially when the participants are children as parental consent/teacher consent is necessary.

In addition to Lichtman’s principles, an ethical researcher should consider the effect their research will have on the participants. If the research is being undertaken for the sole purpose of answering a personal question posed by the researcher and has no benefit for the participants, it would seem unethical to proceed with the study.

One must also consider the ethical issues concerned with using a control group. If the researcher believes the tools the experimental group are using are superior and offer greater benefits to pupils than the tools the control group are using, it would appear unethical that the control group should not also be exposed to the same tools available to the experimental group (Opie 2004).

In this study the researcher has conformed to all four of Lichtman’s principles. The school principal gave his consent (on behalf of the Board of Management) to the research study. The parents of the target population received a letter which informed them of their children’s participation in the study. The children were also fully aware of their involvement in the study. The pupils’ identity was not revealed in the study’s report. In addition the control and experimental group both had the opportunity to use the tools they did not have access to during the study after the study was completed. This eliminated any possible benefits one group might have gained over another group as a result of the learning tools used in this study.
3.9 Timeline

The time frame for this case study was eleven months. The proposal for the case study was submitted in April 2009. Following this, changes were made to the original proposal and the proposal was resubmitted in May 2009. Following the submission of the final proposal, the author began research into the area of education theories; computer based learning theories, mathematical learning, concrete materials and virtual manipulatives. This stage of the research (literature review) continued until September 2009.

In September 2009, the author began the methodology stage of the research. The researcher explored different research approaches and data collection methods and decided what approach and methods would best suit this research study.

The practical research period of the case study began in November 2009 and lasted four weeks. During this four week period the researcher taught two mathematical topics (time and 2D shape). The class was divided into three groups. These groups were organized using the class’ Sigma T results. In this way each group consisted of pupils of corresponding mathematical ability. Group A used virtual manipulatives only, group B used a combination of concrete and virtual manipulatives and group C used virtual manipulatives only. As there were 25 pupils in the class, it was not possible to have an equal number of pupils in each group. Group A had 9 pupils and group B and group C had 8 pupils each. In addition each group was made up of pupils of varying mathematical ability.

Prior to beginning the first topic (2D shape) the researcher used a teacher designed test to elicit the prior knowledge of the pupils. Following this the pupils were taught eight lessons on 2-D shapes and participated in reinforcement sessions involving virtual manipulatives, concrete manipulatives or both. All group participated in an equal amount of reinforcement sessions using manipulatives. The researcher observed the pupils during the tasks involving concrete manipulatives and/or virtual manipulatives. Following the completion of the topic the pupil received a teacher designed post-test. The same format was followed for the second topic (time). When both topics were completed in early
December, the pupils were retested with the Sigma T and completed a questionnaire based on their experiences when using virtual manipulative and/or concrete manipulatives.

Following the practical research period, the test results, observational records and questionnaire comments were documented (Chapter 4). The researcher then compared, contrasted and analyzed the finding in light of the literature presented in Chapter 2 (Chapter 5). Finally the researcher documented the recommendation and suggestions for further research in the area of mathematical manipulatives (Chapter 6).

3.10 Conclusion

This chapter has outlined the research approach and methodologies that this study engaged in. The study will take the form of a case study and will utilize both qualitative and quantitative data collection methods. The next chapter will present the results which arose from the data collection methods of testing, observation and pupil questionnaires.
Chapter 4 Findings

4.1 Introduction

4.1.1 Overview
This case study set out to explore the benefits of virtual manipulatives in the teaching of primary mathematics. This chapter details the findings of this case study. Data was collected from standardised and teacher designed tests, observation and questionnaires. These three methods of data collection allowed the researcher to triangulate data and increase the validity and reliability of the research.

The results emerging from the case study are presented in four sections, which are based on the research questions outlined in Chapter 3. Section one will look at how effective virtual manipulatives were in comparison to concrete manipulatives in the teaching of the mathematics to the target audience. Section two will deal with how effective concrete manipulatives and virtual manipulatives were when used together to teach mathematical concepts. Section three will look at the difference in motivation between pupils using virtual manipulatives and pupils using concrete manipulatives. Section four will look at the quantity of mathematical dialogue the pupils engaged in when using virtual manipulatives and when using concrete manipulatives.

4.1.2 The participants
The participants of this study were a third class group in a rural school in the south west of Ireland. The 3rd class group were of mixed mathematical ability and mixed gender. The class was divided into three groups (Group A, B, C). Each group consisted of pupils of mixed ability and gender. Group A used virtual manipulatives only to reinforce their learning. Group B used both virtual manipulatives and concrete manipulatives to reinforce learning and group C used concrete materials only to reinforce their learning.
4.2 The effectiveness of virtual manipulatives in teaching of primary mathematics

4.2.1 Introduction
In order to examine how effective virtual manipulatives are in the teaching of primary mathematics, one must look at the learning that occurred when;

- pupils were using virtual manipulatives
- pupils were using concrete manipulative
- pupils were using a concrete manipulatives and virtual manipulatives.

The standardised testing and teacher designed test scores collected from each group can give us an accurate indication of the learning that occurred.

4.2.2 Learning benefits indicated through standardised test results
Pupils in all three groups were tested with the Sigma T Level 2 before the case study began and again after the four week research period had ended.

Table 4.1 presents the overall results of the Sigma T tests which were completed in full both before and after the research period. The learning that occurred can be elicited from the increase in score each group recorded between the pre-test and post-test. Group A, who used virtual manipulatives to reinforce their learning, had an average increase in score of +11%. Group B, who used both virtual and concrete manipulatives to reinforce their learning and had an average increase in score of +8% and group C who used concrete manipulatives in their learning had an average increase in score of +6%. Group A recorded the highest average increase in score and group C recorded the lowest average increase in score.
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Table 4.1: Sigma T Level 2 Pre-test and Post-test Results

Table 4.2 presents the results of the Sigma T subsections of shape and space and measures. The scores for these two sections have been extracted as the topics taught
during the case study are part of these strands in the Primary Mathematics Curriculum. Time is part of the measures’ strand and 2D shape belongs to the strand of shape and space.

The following results are from the Sigma T subsection on shape and space. Group A showed had an increase in score of +25 %, which was the lowest average increase in score. Group B had an average increase in score of +31% which was the highest average increase in score and group C had the second highest increase in score with an increase of +29%.

In the subsection based on measures, group A had an average increase of +8% which was the lowest average increase in scores. Group B had an average increase in score of +12%. This was the greatest average increase in test score results in the area of measures. Group C had an average increase of +11%, which was second highest average increase in score.
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| Group B|          |           |                   |          |           |                   |
| 75     | 75       | 0         | 65                | 78       | 7         |                   |
| 75     | 100      | 25        | 53                | 76       | 23        |                   |
| 75     | 100      | 25        | 65                | 71       | 6         |                   |
| 50     | 100      | 50        | 53                | 65       | 12        |                   |
| 50     | 75       | 25        | 65                | 70       | 5         |                   |
| 25     | 100      | 75        | 47                | 61       | 14        |                   |
| 50     | 75       | 25        | 41                | 47       | 6         |                   |
| 50     | 75       | 25        | 35                | 59       | 24        |                   |
| Average increase in score | 31 | Average increase in score | 12.1 |

| Group C|          |           |                   |          |           |                   |
| 50     | 100      | 50        | 53                | 71       | 18        |                   |
| 75     | 100      | 25        | 59                | 82       | 23        |                   |
| 100    | 100      | *         | 59                | 65       | 6         |                   |
| 75     | 75       | 0         | 47                | 59       | 12        |                   |
| 50     | 100      | 50        | 59                | 65       | 6         |                   |
| 50     | 75       | 25        | 71                | 82       | 11        |                   |
| 50     | 75       | 25        | 53                | 60       | 7         |                   |
| 25     | 50       | 25        | 47                | 55       | 8         |                   |
| Average increase in score | 28.6 | Average increase in score | 11.4 |

**Table 4.2: Sigma T Subsection Results**

*Note: Where pupils achieved 100% in the pre-test, their results were not used in calculating the overall average increase for that group.*
4.2.3 Learning benefits indicated from the teacher designed test results
In addition to the Sigma T, the researcher used teacher designed tests. Before the teaching period based on the topic of 2D shape began, the pupils were tested with a teacher designed test. The pupils were retested with the same test after the period had ended. This measured the learning that occurred during the teaching period. The same procedure was carried out with the teacher designed tests based on time.

Table 4.3 displays the pupils’ scores before and after the topic of 2D shape was taught. Group A had an average increase in score of +20% in the post-test. Group B had an average increase of +26% in the post-test. Group C had an average increase of +24% in the post-test. The results indicate that Group A had the lowest average increase in scores in the post-test and group B had the highest average increase in score in the post-test.
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Table 4.3: Teacher designed test results-2D shape
Table 4.4 illustrates the pupils’ pre-test and post-test scores from the teacher designed test based on time. Group A had an average increase of +14% in the post-test. Group B had an average increase of +13% in the post test and Group C had an average increase of +10% in the post test. In contrast to the 2D shape teacher designed test results, Group A had the highest average increase in score and group C had the lowest average increase in score.
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Table 4.4: Teacher designed test-Time
4.2.4 Time spend on task when using virtual manipulatives

In addition to the quantitative data presented, the researcher also gathered qualitative data through observation and questionnaires. Observational data was collected using observational checklists. Small groups of two or three pupils were observed for periods of five minutes while engaging in tasks using virtual or concrete manipulatives. These observational records indicate that pupils spent on average 20% more time on task when using virtual manipulatives than when using concrete manipulatives.

Figure 4.1: Time spend on-task and off-task when using concrete manipulatives and when using virtual manipulatives
4.2.5 Cognitive levels of learning when using virtual manipulatives and concrete manipulatives.

From work samples collected during the study, it became evident that when using virtual manipulatives pupils were allowed more choice as regards the tools they used. For example virtual tessellation activities allowed pupils to use endless amount of shapes and allowed pupils to change the colour of shapes. Virtual geoboards allowed pupils to use colour to fill in their shapes after they had created them with virtual elastics. These choices are not available when using concrete manipulatives.

This increase in learner choice led to increased independence and freedom. This in turn led to increased creativity. This was observed in the work produced by virtual manipulative users.

Figure 4.2: Example of virtual tessellation
4.2.6 Low achievers and virtual manipulatives
For the purpose of this study, pupils who scored below 50% in the pretest were regarded as low achievers and pupils who scored above 50% were regarded as average to high achievers. In the overall Sigma T results, low achievers in Group A recorded an average increase of +24 in the post-test and average/high achievers had an average increase of +8. Low achievers in group B, recorded an average increase of +23 and average/high achievers recorded an average increase of +6. Low achievers in group C had an average increase in score of +4, while average/high achievers had average increase of +6. In groups A and B, low achievers recorded a greater increase in grades than their average/high achieving peers.

In the Sigma T subsection, Shape and Space, low achievers in group A had an average increase of +25 and average/high achievers had an average increase in score of +29. In group B low achievers had an average increase in score of +75 and their average/high achieving counterparts recorded an average increase of +29. In group C, low achievers had an average increase of +25 and average/high achievers had an average increase of +35. Low achievers in group B had a higher increase in score than average/high achievers. However in group A and C, average/high achieving pupils recorded higher score increases than low achieving peers.

In the Sigma T subsection based on Measures, low achievers in group A recorded an average increase of +10, while the average/high achievers recorded an average increase of +7. In group B, low achievers recorded an average increase in score of +15 and average/high achievers recorded an average increase of +11. In group C low achievers recorded an increase of +10 and average/high achievers recorded an increase of +12. Low achieving learners in group A and B recorded greater average score increases than their high achieving counterparts in the Sigma subsection based on measures. High/average achieving achievers in group C had a greater score increase than low achievers.

In the teacher designed test based on 2D shape, all pupils scored above 50% in the pretest. In the teacher designed test based on time, low achievers in group A had an
average increase in score of +23 and average/high achievers had an increase of +11. In group B low achievers recorded an increase of +19 and high achievers had an increase of +11. In group B low achievers recorded an average increase of +11 and high achievers recorded an increase of +9. Thus, in the teacher designed test based on time, low achievers in all groups recorded a greater increase in score than their average/high achieving counterparts.

4.3 The effectiveness of using a combination of concrete manipulatives and virtual manipulatives in the teaching of primary mathematics

4.3.1 Introduction

In order to investigate if concrete manipulatives and virtual manipulatives are more effective when used together, we must compare the test results achieved by group B, who used both concrete and virtual manipulatives, to the test results achieved by group A and group C.

4.3.2 Test results

Table 4.1 presents the overall pre-test and the post-test Sigma T results. Group B had an average increase of +9% in their results, which was lowest average increase in score. Group A had the highest average increase in score (+11).

Table 4.2 displays the results pupils achieved in the Sigma T subsections of shape and space and measures. From table 4.2 we see that Group B had an average increase of +11 in the subsection of measures and +36 in the subsection of shape and space. Group B recorded the highest average increase in score in the shape and space and measures subsection.

The teacher designed test results based on 2D shape presented in table 4.3 indicate that Group B had the highest average increase in score in post-test (+26). Table 4.4 indicates
that in the teacher designed post-test based on time, group A had the greatest average increase in score (+14) and Group B had the second greatest increase in score (+13).

Group B had the greatest increase in score in both Sigma T subsections (shape and space, and measures) as well the teacher designed test based on 2D shape. Group B recorded the second highest average increase in score in the overall Sigma T results and in the teacher designed test based on time.

4.4 Pupil motivation when using manipulatives

4.4.1 Introduction

Pupil’s motivation cannot be measured using quantitative data; therefore it was necessary to collect qualitative data by using the data collection method of observation. The data recorded during the observations were used to examine the motivation pupils experienced when using virtual manipulatives and concrete manipulatives.

4.4.2 Internal and External Motivation

Observational records indicated that when pupils were engaging in reinforcement tasks using virtual manipulatives they spent more time on task than pupils using concrete manipulatives. As shown in figure 4.1, pupils using virtual manipulatives spent, on average, 20% more time on task than those using concrete manipulatives.

The observation records also indicate that pupils using virtual manipulatives worked more independently than pupils using concrete manipulatives. Pupils using concrete manipulatives engaged more frequently with others. They regularly sought encouragement and praise from others and expressed a sense of achievement when their work was shared with their peers. This interaction was less frequently observed when pupils were using virtual manipulatives.
4.4.3 Feedback and Motivation
User feedback was a built-in component of the time based virtual manipulative used in this study. When pupils were using the time based virtual manipulatives, the user received immediate feedback when they completed a task. However pupils using concrete materials (i.e. physical clocks) had to wait for the teacher to check they formed the correct time on the clocks. This waiting period decreased pupils’ motivation for the next task. In the questionnaires completed by each group after the case study, 20% of pupils in group C and B responded that they got bored waiting for the teacher to check their work. This was not an issue with virtual manipulatives as the waiting period did not occur. Therefore pupils did not go off task and motivational levels were maintained.

4.4.4 Independence and Motivation
From the pupil questionnaires undertaken at the end of the research period, the majority of pupils from Group A and B stated they enjoyed the independence that virtual manipulatives allowed them.

4.5 Mathematical dialogue
The observational schedule used indicates that increased interaction occurred when pupils were using concrete manipulatives. Pupils tended to interact and engage less in mathematical dialogue when they were using virtual manipulatives. Pupils who were engaging in tasks involving concrete manipulatives often made comments relevant to the task, asked questions to clarify understanding, listened to others and made constructive and helpful suggestions. Pupils worked more collaboratively when using concrete manipulatives. This type of interaction was less frequently observed when pupils were using virtual manipulatives. Pupils using virtual manipulatives were more likely to ask the teacher for assistance rather than a peer. Whereas pupils using concrete manipulatives were more likely to ask a peer for help rather than the teacher. Observation also indicated
that pupils using concrete manipulatives frequently engaged in personal dialogue to ‘talk
themselves’ through problems. Pupils who used virtual manipulatives were not observed
to engage in this type of personal dialogue.

4.6 Conclusion

This chapter presented detailed findings from the research that was carried out. The
findings were gathered using standardized testing; teacher designed testing, observation
and pupil questionnaires. In Chapter 5 these findings will be analysed in light of the
literature outlined in Chapter 2.
Chapter 5 Discussion

5.1 Introduction

5.1.1 Outline of the chapter

In Chapter 4, the findings of the case study were presented. This chapter will evaluate the findings in more detail and discuss the findings in relation to the existing literature which was presented in the literature review in Chapter 2.

5.1.2 Overview of research undertaken

This case study set out to examine the benefits of virtual manipulatives in the teaching of primary mathematics. The study took place in a middle sized primary school in the Southwest of Ireland. The target population was a group of 3rd class pupils. The pupils had basic keyboard skills and 60% of pupils claim to use computers regularly at home.

In order to compare and contrast the learning benefits of virtual manipulatives with the learning benefits associated with concrete manipulatives, and with concrete manipulatives and virtual manipulatives combined, the target population where placed into three groups: group A, group B and group C. Group A used only virtual manipulatives to reinforce their learning. Group B used virtual manipulatives and concrete manipulatives to reinforce their learning and group C used concrete manipulatives only to reinforce their learning. The pupil’s Sigma T results were used to assign pupils to groups. This ensured that each group consisted of pupils of corresponding mathematical ability. All groups had an equal number of opportunities to engage in manipulative activities. Group B used concrete manipulatives 50% of the time and virtual manipulatives 50% of the time and engaged in the same amount of manipulative tasks as the other two groups. In order to examine the differences in learning between each group, test results, observational notes and questionnaire comments from each group have been compared and analysed.
5.1.3 Key Findings

The key findings of the research study were as follows:

- Pupils using a combination of virtual and concrete manipulatives had on average a higher increase in test results than those using concrete manipulatives or virtual manipulatives.
- Pupils using virtual manipulatives only engaged less frequently in mathematical dialogue than those using concrete manipulatives only.
- Pupils using virtual manipulatives only engaged in less ‘off task’ time than those using concrete manipulatives only.
- Pupils using virtual manipulatives only were observed to be more intrinsically motivated.
- Pupils using concrete manipulatives only were observed to be more extrinsically motivated.

This chapter aims to determine if the findings which have arisen from this case study are consistent with current theories in this area. The findings will be analysed under the following headings:

- Benefits of virtual manipulatives.
- Benefits of using a combination of concrete manipulatives and virtual manipulatives.
- Pupil motivation.
- Mathematical dialogue.
- Problems that arose during the case study.
5.2 Benefits of virtual manipulatives

5.2.1 Introduction
In chapter two the learning theories of behaviourism and constructivism were discussed. The 1999 Primary Mathematics Curriculum was built on constructivist philosophies and emphasised a ‘hands on’ approach to learning mathematics. The curriculum asserts that children learn best when they ‘manipulate, touch, and see objects’ (Department of Education and Science 1999, p.3). Concrete manipulatives allow for this ‘hands on’ approach but as the literature emphasizes virtual manipulatives can also do this as well (Clements 1999; Moyer et al 2001). As highlighted in Chapter 2 ACOT, which was one of the broadest studies carried out in the field of information technology in education indicated that the use of computers in the classroom lead to an increase in constructivist approaches (Apple Computer Inc. 1995).

As the research suggest virtual manipulatives can provide for an increase in constructivist approaches and allow learners to actively build knowledge and understanding for themselves as well as being in control of the pace and direction of their own learning. The freedom that virtual manipulatives allow the learner mirror constructivist philosophies (Forrester and Jantzie 1998).

5.2.2 Considerations when using virtual manipulatives
During this case study the researcher adhered as much as possible to Suydam & Higgins’ suggestions for the use of manipulatives as outlined in Chapter 2:

- Manipulatives should be used frequently.
- Simple materials should be chosen for use as manipulatives.
- Manipulatives should be used in conjunction with exploratory methodologies.
- Symbols should be used to record findings.

(1977)
Another recommendation that was considered when choosing manipulative materials was that the manipulatives chosen were appropriate representations of their physical counterpart (Clements 2000). These suggestions assisted the researcher in choosing appropriate virtual manipulatives for use in this case study.

5.2.3 The effectiveness of virtual manipulatives in reinforcing mathematical concepts

The overall effectiveness of virtual manipulatives is indicated by group A’s test results. Group A recorded the highest increase in score in the Sigma T post-test (+12). However in the Sigma T subsection results, group A recorded the lowest increase in score in the subsection based on shape and space and the subsection based on time. The increase in score in the overall Sigma T test results was not reflected in either of the subsections covered in this case study.

In the teacher designed test based on 2D shape, pupils in group A recorded the lowest increase in score of all groups (+20%). In the teacher designed test based on time, pupils in group A recorded the highest increase in score of all the groups (+14%). These results suggest that virtual manipulatives may work less effectively than concrete manipulatives or a combination of virtual manipulatives and concrete manipulatives in the consolidation of 2D shape.

Group A achieved the highest average increase in score in the teacher designed test based on time however they achieved the lowest average increase in score in the sigma T subsection results based on measures. These results contradict each other. However the Sigma T subsection based on measures included more topics than time so pupils’ grades could reflect pupils’ knowledge in other areas of the measures strand. Due to these inconclusive results, the effectiveness of virtual manipulatives in the teaching of time is still unknown. However, the test results do not indicate that virtual manipulatives are more effective than concrete manipulatives or a combination of concrete manipulatives and virtual manipulatives.
5.2.4 Learning benefits for low achievers

A research study undertaken by Suh and Moyer in 2005 indicated that virtual manipulatives showed the greatest learning benefits amongst low achievers (Suh et al 2005). In order to investigate if Suh and Moyer’s theory is true in relation to this case study, pupils with a grade of 50% or under were categorised as low achievers and pupils with a grade of over 50% were categorised as average/high achievers.

As was outlined in Chapter 4, low achievers in Group A had a higher increase in test score results than their average/high achieving peers in the overall Sigma T post-test, in the Sigma T subsection based on time and in the teacher designed test based on time. Low achievers in group A did not have a greater increase in score than their high achieving counterparts in the Sigma T subsection based on 2D shape. The results from the teacher designed test based on 2D shape are not applicable as all pupils scored over 50% in the pre-test. The findings from this case study correlate with the theory of Suh and Moyer and indicate that virtual manipulatives may have greater learning benefits for low achievers (Suh et al 2005).

5.2.5 Learner freedom

Optimal learning occurs when learners are in control of the pace of their learning (Clements 2000). In addition learners have a better transfer of knowledge when they are in control of their own learning (Mayer 2001).

During this case study, pupils using virtual manipulatives were observed to have more control and freedom over their learning than those using concrete manipulatives. Questionnaires responses also indicated that pupils using virtual manipulatives enjoyed the freedom and control they had over their own learning. Responses such as these were absent from questionnaires completed by concrete manipulative users. When pupils were participating in tasks that involved concrete manipulatives, the pupils were more reliant on the teacher for instruction and to inform them of the next task. On the other hand, pupils using virtual manipulatives had to simply double click the computer mouse when
they required a new task/problem. They were not dependent on anybody to allow them to continue with their activity. The teacher’s role in relation to pupils using virtual manipulatives became that of a facilitator.

Virtual manipulatives also allow the learner to have more freedom over the properties of manipulatives. Some virtual manipulatives allow the users to change the properties of the manipulative. During this case study, pupils who completed virtual tessellation activities were able to change the colour of the shapes they were using. Concrete manipulatives do not allow pupils the freedom to change the properties of manipulatives. In group B’s questionnaire responses this property was cited as a reason why pupils preferred using virtual manipulatives to concrete manipulatives.

5.2.6 Scaffolded learning

Many virtual manipulatives provide a certain level of scaffolding for the learner and provide cognitive online tools that assist the learner (Karadag 2007). The scaffolding process was not a main component of the virtual manipulatives chosen for use in this case study. However the time based virtual manipulatives used in this case study did provide a certain level of scaffolding for the learner. The virtual manipulatives allowed the researcher to enable and disable certain functions of the programme, so the task matched the ability of the learner. Although the virtual manipulatives chosen for use in this study did not give the learner assistance when they were having difficulty with a task, they allowed the learner endless attempts at solving problems.

5.2.7 Creative thinking

The primary mathematics’ curriculum suggests that ICT can challenge learners by engaging them in higher order problem solving activities (Department of Education and Science 1999b). From work samples collected during the study, it was observed that pupils engaged in an increased amount of higher order and creative thinking when using virtual manipulatives as opposed to concrete manipulatives. Increased motivation was observed when pupils encountered problems which allowed them to use higher order and
creative thinking skills. Virtual manipulatives appeared to provide a greater platform than concrete materials for such thinking.

5.2.8 Classroom management

Another key benefit of virtual manipulatives is that they can be cleared away with the click of a mouse (Clements 1999). Concrete manipulatives are not as easy to clear away and during this case study the collection of concrete manipulatives such as geoboards, elastic bands, shapes and clocks, proved to be quite laborious and time consuming. The teacher also had to be careful that pupils did not lose manipulative pieces. This is an important issue due to the financial cost of replacing materials. In addition virtual manipulatives do not need to be stored away, whereas concrete manipulatives require large amounts of storage space. It is evident that there are many more classroom management issues associated with concrete manipulatives than with virtual manipulatives.

5.2.9 Financial Issues

The literature emphasises that virtual manipulatives are available free of charge once one has a computer and access to the internet (Moyer et al 2001). Although this is true one has to also consider the ratio of computers to pupils. Unfortunately the school in which this case study was set did not have access to a computer room. Therefore the research had to take place in the classroom, which was only equipped with 3 computers. The ratio of pupils to computers made the virtual manipulative activities very time-consuming as only three pupils could work on a virtual manipulative activity at one time. If virtual manipulatives were to replace concrete manipulatives in a primary school, it would not be feasible without the use of a computer room or a large number of classroom computers.
5.3 The effectiveness of concrete manipulatives and virtual manipulatives in the teaching of primary mathematics

The literature states that multiple representation of the same concept can benefit the learner as they demonstrate different aspects of the same concept (Durmus and Karakink 2006). The use of both virtual and concrete manipulatives can allow for different representations of the same concept to be presented to the learner. Pupils who used both virtual manipulatives and concrete manipulatives to reinforce their learning recorded on average the greatest increase in test results

- Highest increase in score in both Sigma T subsection results (shape & space +36%, time +12%).
- Highest increase in score in the teacher designed test based on 2D shape (+26%).
- Second highest increase in score in the overall Sigma T results (+9%).
- Second highest increase in score in the teacher designed test based on time (13%).

The test results from this case study indicate that on average pupils learnt best when they used a combination of concrete and virtual manipulatives.

5.4 Pupil motivation

5.4.1 Introduction

The literature refers to two forms of motivation; extrinsic motivation and intrinsic motivation. Extrinsic motivation is linked to behaviourist learning theories and occurs when an external factor such as a reward motivates a person to complete a task. Intrinsic motivation occurs when somebody completes a task for the sake of completing it (Santrock 2004).
5.4.2 Multimedia and motivation

The literature suggests that computer based learning provides a multimedia environment which allows the real world to be brought to the learner. This connection with the real world motivates the learner (Forrester and Jantzie 1998). The virtual manipulatives used in this case study used both sound and images. This gave the learner an authentic virtual representation of the concrete manipulative and allowed a connection with the real world and in turn increased pupil motivation.

5.4.3 Feedback

Another factor which was observed to increase motivation amongst virtual manipulative users was the instantaneous non judgemental feedback that virtual manipulatives provided for the learner (Moyer and Suh 2005). Forrester and Jantzie state that instantaneous feedback provides external motivation for the learner (1998). During this case study pupils using concrete manipulatives such as clocks (which were used to reinforce their learning of time), had to wait their turn to show their work to the teacher. The teacher then checked if they completed the task correctly. Pupils using virtual clocks did not have to endure this waiting period. When virtual manipulative users had finished a task they simply clicked an on-screen button that instructed the programme to check the virtual manipulative task was completed correctly. Feedback was instantaneous and non-judgmental (Forrester and Jantzie 1998). This extrinsically motivated the learner. Pupils using concrete manipulatives did not experience such immediacy in teacher feedback. Concrete manipulative users tended to become bored and unmotivated while waiting for teacher feedback. When teacher praise and feedback was given, extrinsic motivation occurred, however this was adversely affected by the waiting period encountered.
5.4.4 Self-directed Learning

Forrester and Jantzie suggest that self-directed learning is an important intrinsic motivational factor (1998). Virtual manipulative and concrete manipulative users were self-directed for the majority of their activities that were based on shape and space. The teacher outlined the task and the pupils were allowed to interpret the task on their own and complete the activity. The activities based on time did not allow for as much freedom. Pupils’ questionnaire responses indicated that they enjoyed the activities based on 2D shape more than the activities based on time. Pupils were also observed to be more enthusiastic and more motivated when engaging in tasks based on the topic of 2D shape.

5.4.5 Time on-task and off-task

The observational records suggest that as pupils using virtual manipulatives spent more time ‘on task’ than those using concrete manipulatives. The observational records recorded any time that a pupil engaged in off-task talking or appeared to be distracted. This finding correlates with the literature. Observational studies undertaken by Hoyles, Healy and Sutherland indicated that pupils using virtual manipulatives displayed very little time off-task (1991). The absence of ‘off task’ talking or other ‘off-task’ activities allows the pupils to increase their time on-task and this in turn increases their motivation and learning (Clement 1999). Observational records suggest the pupils using concrete manipulatives engaged in more ‘off task’ talking than those using virtual manipulatives. This suggests virtual manipulative users spent more time on-task than concrete manipulatives users.
5.5 Mathematical dialogue

5.5.1 Mathematical dialogue and virtual manipulatives

Vygotsky states that social interaction and co-operation are important aspects of the constructivist learning process. Computer based learning activities such as those based on virtual manipulatives can act as a catalyst for social interaction. Virtual manipulatives allow the learner to be in control of his/her learning and foster collaboration (Clements 1999). Despite what the literature states, the observational data collected during this study indicated that pupils using concrete manipulatives engaged in more dialogue than those using virtual manipulatives. Concrete manipulative users tended to make comments about the task they were completing and make helpful suggestions more frequently than their counterparts who were using virtual manipulatives. Collaboration was also observed more frequently in groups of pupils who were using concrete manipulatives. In addition pupils using concrete manipulatives were more likely to ask a peer to assist them than a teacher, whereas those using virtual manipulatives were more likely to ask a teacher than their peers. Therefore the evidence collected during this case study does not correlate with Clement’s theory that virtual manipulatives can act as a catalyst for social interaction (1999).

5.5.2 Mathematical dialogue and constructing knowledge

Clements argues that the use of manipulatives does not automatically mean the pupil understands the concept being taught (1999). Pupils must reflect on the task at hand so the internal and external actions can be matched. Reflection can occur as paired, group or whole class discussion, in which the pupil’s can verbalize their understanding and build on their knowledge.

During this study pupils were encouraged to discuss with their peers, the activity they were undertaking. As suggested by Clements, pupils engaged in teacher directed group
discussion after the activity had been completed so they could reflect on the learning that
had taken place (2000). This reflection allowed pupils to analyse and understand errors
(Clements 2000).

5.5.3 The physical environment and mathematical dialogue
The physical environment in which the computers are located also has an effect on
interaction. When computer users are located close together mathematical dialogue is
more likely to occur (Clements 1999). The physical environment in which this case study
was conducted was the researcher’s classroom. The computers used were located together
in one corner of the classroom. Despite the close proximity of the computer users,
mathematical dialogue and collaboration occurred less frequently amongst virtual
manipulative users than concrete manipulative users.

5.6 Problems that arose during study

5.6.1 Availability of computers
One problem highlighted in the study was the lack of computers. There were only three
computers available to the researcher. Group A and B both needed to use virtual
manipulatives to reinforce their learning and because of this the computer pupil ratio was
3:17. This resulted in the study being very time consuming for the researcher.

5.6.2 The learning curve associated with virtual manipulatives
Another problem which arose during this case study was that children who did not have
good mouse skills found it difficult to use virtual manipulatives that required fine
accurate mouse movement, for example the virtual geoboards. Some pupils had great
difficulty using the mouse to move elastic bands on the geoboards. This caused great
frustration for the pupils and as a result their motivation and time off-task increased. In
order to allow these pupils to engage more fully with the task, the researcher changed the mouse setting to a lower speed setting in the accessibility toolbar. This allowed the user to have more control and required less accuracy from the user when he/she was using the mouse. Despite this change, a small number of pupils still had difficulty with the fine motor skill involved when using this virtual manipulative.

It was observed that pupils using the virtual geoboards often spent up to three times longer doing an activity than those using the concrete materials. A small number of pupils commented in their questionnaire responses, that they found virtual manipulatives difficult to use. On the other hand pupils using concrete manipulatives did not record any difficulties with the use of materials in their questionnaire responses.

5.6.3 Problems associated with case study research
Bell stated that as case study research generally focuses on one particular situation, findings cannot be generalised (1993). This is true of this case study, as it focuses solely on one particular group of third class pupils in a rural school in the south west of Ireland. In addition only two mathematical topics were examined; measures and shape & space. The narrow scope of this case study both in relation to the setting of the study and the mathematical areas examined, negatively affects the reliability, credibility and validity of the case study results. It is therefore necessary to undertake a more widespread study to validate the results.
5.7 Conclusion

This chapter has discussed the findings of this case study in light of the literature which was reviewed in Chapter 2. The findings of this case study suggest that a combination of virtual manipulatives and concrete manipulatives had on average greater learning benefits than concrete manipulatives only or virtual manipulatives only. Virtual manipulatives had greater learning benefits for low achievers than average/high achievers. Virtual manipulatives users enjoyed more freedom and creativity in their learning than concrete manipulative users. Pupils using virtual manipulatives enjoyed immediate feedback and as a result stayed on task longer than pupils using concrete manipulatives. Pupils using concrete manipulatives engaged in more mathematical dialogue and collaboration than pupils using virtual manipulatives. Pupils using concrete manipulatives appeared to be largely extrinsically motivation and pupils using virtual manipulatives appeared to be largely intrinsically motivated. Chapter 6 will conclude by recommending areas which would merit further research.
Chapter 6-Conclusions and Recommendations

6.1 Introduction

This study aimed to examine the effectiveness of virtual manipulatives in the teaching of primary mathematics in a rural primary school in the south of Ireland. The target population was a mixed gender and mixed ability 3rd class group. This chapter will provide a short summary of the research findings and will give recommendations for further research in the area of virtual manipulatives.

6.2 Outcomes of the case study

The purpose of this case study was to examine the benefits of virtual manipulatives in the teaching of primary mathematics. The objectives of the case study were as follows:

- How effective are virtual manipulatives in the teaching of primary mathematics?
- Are concrete manipulatives and virtual manipulatives more effective when used together?
- Are pupils more motivated when using virtual manipulatives or when using concrete manipulatives?
- Do virtual manipulatives allow for more or less mathematical dialogue than concrete manipulatives?

The findings based on these objectives indicated that:

- Virtual manipulatives do not appear to be more effective than concrete manipulatives in the teaching of primary mathematics.
- Concrete manipulatives and virtual manipulatives are generally more effective when used together.
- Concrete manipulative users appear to be more extrinsically motivated than virtual manipulative users and virtual manipulative users appear to be more intrinsically motivated than concrete manipulative users.
Concrete manipulative users were observed to engage in more dialogue and collaboration than virtual manipulative users.

In addition to these finding, the following discoveries were also made:

- The use of virtual manipulatives had greater learning benefits for low achievers than average/high achievers.
- Pupils using virtual manipulatives spent less time ‘off task’ than those using concrete manipulatives.

### 6.3 Recommendations for the use of virtual manipulatives in the teaching of primary mathematics

The section outlines some recommendations for the use of virtual manipulatives in the teaching of primary mathematics.

- An important consideration when using virtual manipulatives is the computer to pupils’ ratio. In order for virtual manipulatives to become an integral tool in the mathematics lesson, a computer room or a large quantity of classroom computers should be available to the teacher.

- Concrete manipulative users engaged in more dialogue with their peers than virtual manipulative users. In response to this finding, it would be beneficial for learners to know what other prerequisites are necessary to encourage dialogue between computer users.

- One way of possibly increasing the mathematical dialogue and collaboration of virtual manipulative users is to introduce an interactive whiteboard or a computer and data projector into the mathematics lesson. An interactive whiteboard is a large touch screen board which is connected to a computer and data project. It
displays the computer screen’s content on the touch screen panel. This would allow pupils to work on virtual manipulative tasks in groups and could increase mathematical dialogue and collaboration.

- Another way of possibly increasing mathematical dialogue is to allow pupils to work in pairs using virtual manipulatives.

- This study indicated that concrete manipulatives and virtual manipulatives benefit the learner more than virtual manipulatives alone. It would be beneficial for mathematics educators to learn more about the benefits of using both concrete and virtual manipulatives to assist learners to consolidate mathematical knowledge.

- Some pupils had difficulty with the fine motor skill involved in the geoboard virtual manipulative activity. A graphic tablet could assist pupils with fine motor difficulties. A graphic tablet is a handheld screen that is connected to the computer and allows the user to draw what they wish to input into the computer using a special pencil. The user’s input is then transferred and displayed on the computer screen. This tool would allow the user more accuracy.

6.4 Conclusion

This study has examined the effectiveness of virtual manipulatives in the teaching of primary mathematics in one learning environment. The study has made some interesting findings based on the area of manipulative use in the primary mathematics classroom. However as this case study only focused on two mathematical topics in one class setting over a four week period, further research is needed to validate the findings of the study.
Bibliography


Engineering Education, 78(7), 674–681


Perspective’, paper presented at 12th International Conference of the Society for
Information Technology & Teacher Education, Charlottesville, VA, USA, March 2001.

Funda, D. and Gecerb, A. (2009), ‘Relations between online learning and learning

Green, J.L., Camilli, G., and Elmore, P.B., eds. (2006) Handbook of Complimentary
Association.


Guba, E.G. and Lincoln, Y.S., (1994) ‘Competing paradigms in Qualitative Research’ in
Denzin, N.K. and Lincoln, Y.S. eds., Handbook of Qualitative Research, Thousand Oaks:
SAGE Publications, 105-117.

Hammersley, M., ed. (1986) Case studies in Classroom Research, Milton Keynes: Open
University Press.

Future’ in Masalski, W.J. and Elliot. P.C. eds., Technology-Supported Mathematics
Learning Environment, Reston: The National Council of Teachers of Mathematics, Inc.


Appendices

Appendix A: Letter to Parents

For the Attention of Third Class Pupils
Third class pupils have been invited to take part in a case study which will begin next week. The study will involve short practical mathematical activities based on the mathematical lesson being taught by the class teacher. Pupils’ previous test results may be used anonymously to measure their progress in these activities. Each pupil will engage in approximately two/three 15 minute practical activities each week. These activities will involve concrete materials and/or computer based materials.

Kind Regards,
Catriona Lane
Appendix B - Example of teacher designed pre-tests based on time
1. What time does each clock show?

(a) (b) (c) (d) (e)

2. Show these times on the following clocks.

(a) (b) (c) (d) (e)

3. Write the time shown on the first clock. Show and write the time half an hour later on the second clock.

(a) (i) (ii) (b) (i) (ii)

4. Write the time shown on the first clock. Show and write the time 2 hours later on the second clock.

(a) (i) (ii) (b) (i) (ii)

5. What time does each clock show?

(a) (b) (c) (d) (e)

6. Show and write the time a half an hour earlier than each of the times in question 5 above.

(a) (b) (c) (d) (e)

7. Show and write the time one and a half hours later than each of the times in question 5 above.

(a) (b) (c) (d) (e)

8. Write these times as digital times.

(a) $\frac{1}{2}$ past 4 (b) $\frac{1}{2}$ past 6 (c) $\frac{1}{2}$ to 8 (d) 25 past 2 (e) 25 to 9 (f) 10 to 6

9. Write these digital times as analogue times.

(a) 7:15 → $\frac{1}{2}$ past (b) 8:45 → (c) 10:40 → (d) 11:25 → (e) 12:35 → (f) 1:50 →

10. A match started at $\frac{1}{2}$ past 3. Eric arrived at 3:40. How many minutes late was he? mins.

11. Ann is waiting to see her favourite programme on TV at 7:50. If it is now 10 minutes past 7, how many minutes has she to wait? mins.
Time 1

1. What time does each clock show?
   (a)  (b)  (c)  (d)  (e)
   [Clock images]

2. Show these times on the following clocks.
   (a)  (b)  (c)  (d)  (e)
   [Clock images]

3. Write the time shown on the first clock. Show and write the time half an hour later on the second clock.
   (a) (i)  (ii)
   [Clock images]

4. Write the time shown on the first clock. Show and write the time 2 hours later on the second clock.
   (a) (i)  (ii)
   [Clock images]

5. What time does each clock show?
   (a)  (b)  (c)  (d)  (e)
   [Clock images]

6. Show and write the time a half an hour earlier than each of the times in question 4 above.
   (a)  (b)  (c)  (d)  (e)
   [Clock images]

7. Show and write the time one and a half hours later than each of the times in question 4 above.
   (a)  (b)  (c)  (d)  (e)
   [Clock images]

8. Write these times as digital times.
   (a)  (b)  (c)  (d)  (e)  (f)
   [Digital time images]

9. Write these digital times as analogue times.
   (a)  (b)  (c)  (d)  (e)  (f)
   [Analogue time images]

10. A match started at 4:30. Eric arrived at 3:40. How many minutes late was he? 20 mins.

11. Ann is waiting to see her favourite programme on TV at 7:50. If it is now 10 minutes past 7, how many minutes has she to wait? 40 mins.
Appendix C: Examples of teacher designed post-tests based on time
Time 1

1. What time does each clock show?
   (a) 2 o'clock  
   (b) 3:00  
   (c) 6:30  
   (d) 1:15  
   (e) 9:45

2. Show these times on the following clocks.
   (a) 2:00  
   (b) 3:00  
   (c) 6:30  
   (d) 1:15  
   (e) 9:45

3. Write the time shown on the first clock. Show and write the time half an hour later on the second clock.
   (a) (i) 2:00  
   (ii) 2:30  
   (b) (i) 3:00  
   (ii) 3:30  
   (c) (i) 6:30  
   (ii) 7:00

4. Write the time shown on the first clock. Show and write the time 2 hours later on the second clock.
   (a) (i) 2:00  
   (ii) 4:00  
   (b) (i) 3:00  
   (ii) 5:00  
   (c) (i) 6:30  
   (ii) 8:30

Time 2

1. What time does each clock show?
   (a) 2 o'clock  
   (b) 3:00  
   (c) 6:30  
   (d) 1:15  
   (e) 9:45

2. Show these times on the following clocks.
   (a) 2:00  
   (b) 3:00  
   (c) 6:30  
   (d) 1:15  
   (e) 9:45

3. Write the time shown on the first clock. Show and write the time half an hour later on the second clock.
   (a) (i) 2:00  
   (ii) 2:30  
   (b) (i) 3:00  
   (ii) 3:30  
   (c) (i) 6:30  
   (ii) 7:00

4. Write the time shown on the first clock. Show and write the time 2 hours later on the second clock.
   (a) (i) 2:00  
   (ii) 4:00  
   (b) (i) 3:00  
   (ii) 5:00  
   (c) (i) 6:30  
   (ii) 8:30
Time 1

1. What time does each clock show?
(a) 2 o'clock  (b) 3 past 2  (c) 3 past 7  (d) 8 to 12  (e) 3 past 6

2. Show these times on the following clocks.
(a) 3 past 3  (b) 3 past 3  (c) 2 to 1  (d) 12 o'clock  (e) 5 to 9

3. Write the time shown on the first clock. Show and write the time half an hour later on the second clock.
(a) 3 past 3  (b) 4 past 2  (c) 5 to 8  (d) 12 o'clock  (e) 11 to 7

4. Write the time shown on the first clock. Show and write the time 2 hours later on the second clock.
(a) 3 past 3  (b) 4 past 2  (c) 5 to 8  (d) 12 o'clock  (e) 11 to 7

5. What time does each clock show?
(a) 3 past 7  (b) 5 past 2  (c) 5 to 5  (d) 10 to 7  (e) 5 to 1

6. Show and write the time a half an hour earlier than each of the times in question 5 above.
(a) 2 to 6  (b) 4 to 12  (c) 4 to 5  (d) 9 to 7  (e) 4 to 11

7. Show and write the time one and a half hours later than each of the times in question 5 above.
(a) 4 to 8  (b) 5 to 10  (c) 5 to 11  (d) 10 to 9  (e) 10 to 12

8. Write these times as digital times.
(a) 3 past 4  (b) 2 past 6  (c) 5 to 8  (d) 25 past 2  (e) 5 to 6

Time 1 (contd.)

9. Write these digital times as analogue times.
(a) 7:15  (b) 8:45  (c) 10:40  (d) 11:25  (e) 12:35  (f) 1:50

10. A match started at 3 past 2. Eric arrived at 3:40.
How many minutes late was he? 20 mins.

11. Ann is waiting to see her favourite programme on TV at 7:50. If it is now 10 minutes past 7, how many minutes has she to wait? 10 mins.
Appendix D: Examples of teacher designed pre-test based on 2 D shape

1) Name the shapes below

<table>
<thead>
<tr>
<th>Name of shape</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2) Name these shapes.

- Circle
- Oval
- Semi-circle

3) Put an x on the shape in question 2 that has a straight side.

4) Shapes that fit together leaving no spaces between them are called __________.

5) Can you see any shapes in your classroom? Name three.
   - Shape: Rectangle, Object: Plastic
   - Shape: Circle, Object: Clock
   - Shape: Square, Object: Book
1) Name the shapes below.

- **a**
- **b**
- **c**
- **d**
- **e**
- **f**

2) Name these shapes.

- Circle
- Oval
- Window

3) Put an x on the shape in question 2 that has a straight side.

4) Shapes that fit together leaving **no spaces** between them are called ___

5) Can you see any shapes in your classroom? Name three.
   - Shape: **Circle**
     - Object: Chair
   - Shape: **Rectangle**
     - Object: Table
   - Shape: **Square**
     - Object: Window
1) Name the shapes below.

<table>
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<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
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</table>

2) Name these shapes.

Circle, Oval, Triangle

3) Put an x on the shape in question 2 that has a straight side.

4) Shapes that fit together leaving no spaces between them are called **secant lines**.

5) Can you see any shapes in your classroom? Name three.
   - Shape: Triangle  Object: The wall trim
   - Shape: Circle  Object: Lamp
   - Shape: Square  Object: Book end
Appendix E: Examples of teacher designed post-tests based on 2D shape

2-D Shapes

1) Name these shapes. Put a circle around the shape that has a straight side.

2) Can you see any shapes in your classroom? Name three.

3) When shapes fit together leaving no spaces between them, it is called ________.

4) 

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</table>

20

3.75

2
2-D Shapes

1) Name these shapes. Put a circle around the shape that has a straight side.

- circle
- circle
- oval

2) Can you see any shapes in your classroom? Name three.

- Shape: rectangle, Object: blackboard
- Shape: square, Object: black, red
- Shape: circle, Object: clock

3) When shapes fit together leaving no space between them, it is called ________

4) 

<table>
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<th>b</th>
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Total points: 20
2-D Shapes

1) Name these shapes. Put a circle around the shape that has a straight side.

- circle
- circle
- oval

2) Can you see any shapes in your classroom? Name three.
- Shape: rectangle  Object: paper
- Shape: square  Object: desktop
- Shape: circle  Object: clock

3) When shapes fit together leaving no spaces between them, it is called tessellating.

4) Table:

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Total:

- 15
- 12
- 18
### SIGMA-T LEVEL 2 Form A

**Mathematics Level Indicator:**  

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### SIGMA-T LEVEL 2 Form B

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Appendix G: Teacher Observational Records

### Observational Schedule

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<th>Makes comments relevant to the task</th>
<th>Ask questions to clarify understanding</th>
<th>Learns from others</th>
<th>Makes constructive and helpful suggestions or offers options</th>
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Appendix H: Questionnaires

**Questionnaire**

1. Did you enjoy using the geoboards/shapes/clocks? _____

2. Did they help you to learn more about shapes?
   ______________________________
   ______________________________
   ______________________________

3. Did they help you to learn more about time?
   ______________________________
   ______________________________
   ______________________________

4. Name on thing you enjoyed about using the manipulatives to learn about time/shape?
   ______________________________
   ______________________________
   ______________________________

5. Any comments or suggestions?
   ______________________________
   ______________________________
   ______________________________
Questionnaire

1. Did you enjoy using the geoboards/shapes/clocks on the computer? _____

2. Did they help you to learn more about shapes?
   ______________________________________________
   ______________________________________________
   ______________________________________________

3. Did they help you to learn more about time?
   ______________________________________________
   ______________________________________________
   ______________________________________________

4. Name on thing you enjoyed about using the computer to learn about time/shape?
   ______________________________________________
   ______________________________________________
   ______________________________________________

5. Any comments or suggestions?
   ______________________________________________
   ______________________________________________
   ______________________________________________