Numerical Simulation of the L-glutamic Acid Polymorphic System

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The research detailed in this dissertation expands the field of knowledge in the area of numerical modelling of polymorphic systems and the suspension of polymorphs in stirred tanks. The L-glutamic acid polymorphic system has been the focus of numerous detailed studies in the literature and was chosen to provide a solid foundation on which to develop the numerical model in this treatise. This system consists of two polymorphs, a prismatic shaped $\alpha$ polymorph and a needle-like $\beta$ polymorph.

The analysis of the L-glutamic acid system required a comprehensive study of nucleation, growth and dissolution processes. The rate of nucleation was calculated based on classical nucleation theory, with the birth and spread growth model chosen as the most appropriate method of describing the growth of the $\alpha$ and $\beta$ polymorphs. The kinetic equations developed by Scholl et al. (2006a) were used in the numerical model to calculate the change in size and number of crystals present in the system. A mass transfer dissolution model incorporating the Sherwood correlation was used to describe the dissolution process. An equation was developed to quantify the reduction of solute in the system throughout the transformation process. This work also included the development of a particle death term to account for the loss of $\alpha$ particles due to the dissolution process. This was required to account for the reduction in the zeroth moment in the system. The Method of Moments (MOM) was used to solve the governing dynamic equation. The MOM also formed the basis for the reconstruction of the particle size distribution which had been developed by Hutton (2009) but refined in this
dissertation. The refinement consisted of the development of a more stable and robust method of calculating the intersection of moment iso-lines. This new approach to the reconstruction of particle size distributions (PSDs) was tested and verified for a typical distribution encountered in the L-glutamic acid system.

A series of experiments were performed to verify that the experimental methods used in the literature were accurate and repeatable. The crystal yield was calculated for a range of initial conditions and the results compared to the published data. A very close agreement was achieved with a maximum error of less than 8% present. This verified the applicability of the experimental methods to the system under investigation. Numerically derived solid and solution concentration profiles achieved excellent agreement with published experimental results. A deviation between the experimental and simulated PSDs arose and was attributed to the presence of agglomeration and particle fragmentation within the experimental system. Agglomeration and fragmentation processes had not been considered in the kinetic equations extracted from the literature resulting in discrepancies in the numerical model.

The performance of stirred tank reactors was analysed with particular emphasis on the cloud height and drag in suspensions. The fundamental theory behind suspension modelling and the influence of tank geometry were investigated. The drag induced by non-spherical particles was also examined. An expression for the drag coefficient developed by Loth (2008) was chosen to describe the drag induced by β particles in the suspension.

Numerical modelling of the suspension of L-glutamic acid was performed using Fluent 6.3.26. The RNG k-ε turbulence model was employed to solve the Reynolds-averaged Navier-Stokes equations in conjunction with a multiple reference frame model. An examination of the fluid phase in a stirred tank was carried out and the results successfully compared to the results of Wu and Patterson (1989). The stirred tank model contained a 10% solids volume loading which permitted the use of the Eulerian multiphase model. In order to predict the drag induced by the needle-like β particles it was necessary to supplement the CFD code with the drag coefficient expression. This novel method of predicting the drag of the non-spherical β particles was successfully verified by comparison with the widely validated Wen and Yu drag law within Fluent. The simulation of α and β polymorphic suspensions in a stirred tank were also performed. The solid-liquid interfaces were calculated by monitoring the volume fraction levels at various heights.
throughout the tank. The simulated results were successfully validated against experiments. An average error of 4% and 8% existed for the $\alpha$ and $\beta$ suspensions, respectively. Further simulations were successfully carried out with combined $\alpha$ and $\beta$ suspensions. The homogeneity within the $\alpha$ and $\beta$ suspensions was examined with the bulk of the solid phase providing constant levels of volume fraction indicating good mixing within the tanks. A reduction in the solid volume fractions was apparent near the impeller shaft as the solid-liquid interface was approached. This was attributed to the decrease in velocity around the impeller shaft which reduced the energy available for particle suspension.
This thesis is presented in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Engineering. The substance of this thesis is the original work of the author and due reference and acknowledgement has been made, when necessary, to the work of others. No part of this thesis has previously been accepted for any degree nor has it been submitted for any other award.

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to my parents, family and friends
I would like thank my supervisor Dr. Pat Frawley for all his support, supervision and guidance over the course of my research.

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<td>( \gamma )</td>
<td>Activity coefficient</td>
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\[\begin{array}{l|l}
\kappa & \text{Von Karman’s constant} \\
\lambda & \text{Particle Diameter} \\
\mu & \text{Chemical potential} \quad \text{J/mol} \\
\mu & \text{Molecular volume of solid} \quad \text{m}^3/\text{molecule} \\
\mu & \text{Dynamic viscosity} \quad \text{kg/ms} \\
\mu_t & \text{Turbulent viscosity} \quad \text{kg/ms} \\
\mu_{t0} & \text{Turbulent viscosity without swirl modification} \quad \text{kg/ms} \\
\nu & \text{Kinematic viscosity} \quad \text{m}^2/\text{s} \\
\omega & \text{Rotational velocity} \quad \text{rad/s} \\
\rho & \text{Crystal density} \quad \text{kg/m}^3 \\
\rho_c & \text{Continuous phase density} \quad \text{kg/m}^3 \\
\sigma & \text{Solid-liquid interfacial tension} \\
\sigma & \text{Surface energy} \quad \text{J/m}^2 \\
\tau & \text{Time increment} \quad \text{s} \\
\tau_w & \text{Fluid phase wall shear stress} \quad \text{Pa} \\
\theta & \text{Wetting angle} \\
\end{array}\]

**Subscripts**

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<td>β</td>
<td>β polymorph</td>
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Chapter 1

Introduction

The pharmaceutical industry is one of the largest industries in the world today with the profits being generated in the United States surpassing any other industry in the US. According to the European Federation of Pharmaceutical Industries and Associations (EFPIA), the worldwide pharmaceutical market was worth approximately €484,130 million in 2007. Within the European Union, the pharmaceutical research and development industry required an investment of over €71,000 million which was more than any other industrial sector with 635,000 people employed in the pharmaceutical industry in 2008 in the EU with nearly 20% of these employed in R&D. This illustrates the vast scale and importance of the pharmaceutical industry to the EU.

1.1 Pharmaceutical Development Processes

In order to continue to improve and grow the industry, a lot of emphasis has been placed on the R&D sector. The main focus of research is on the discovery and development of new drugs, with the cost of developing a new single medicine estimated at €1 billion (EFPIA, 2009). The start of the process involves the seeking out and rapidly testing thousands if not millions of compounds. If a compound shows the potential to provide a new method of preventing, treating or alleviating a disease or health problem then the
Chapter 1. Introduction

The process of 'screening' can begin. Screening comprises the testing of the new compound on a bacteria and if successful, testing can begin on infected laboratory animals. The new drug is only tested on humans in a clinical trial if it promises to have a therapeutic advantage over existing drugs or if it is deemed to be safer than existing drugs. This consists of testing the drug on a small group of healthy individuals to determine dosage levels and possible side-effects. If this is successful then further tests are performed on larger groups of people. After this stage, the Food and Drug Administration reviews the drugs performance on the patients before giving approval for commercial use. Once this approval has been granted, the production methods and costs are calculated in order to facilitate mass production of the drug.

The production of the active pharmaceutical ingredients (API) that are used to produce the drugs consist of a number of steps. The raw materials that are required for the manufacturing are tested to ensure that they adhere to the highest quality standards. The solid and liquid materials are mixed in stirred tank reactors under various operating conditions to produce a final slurry. The number of reaction stages necessary is governed by the complexity of the process. A filtration step is used to separate the solid and liquid phases within the slurry and the product is subsequently dried to remove the remaining liquid. To ensure that the correct particle size is produced, a milling stage can be employed. Further testing is performed on the final product to confirm that they meet the required specifications.

The stirred tank reactors mentioned above are commonly used in the chemical process industry to carry out a variety of operations including homogenisation, gas dispersion, heat transfer and crystallisation. Crystallisation is a method used to produce a pure solid particle with specific properties from a solution and it constitutes an extremely important aspect of the pharmaceutical manufacturing processes. Over 90% of all pharmaceutical products, such as tablets, aerosols and suspensions contain drugs in particulate, generally crystalline form. (Shekunov and York [2000]. As a result of this, knowledge of the crystallisation process is paramount. Unfortunately, the crystallisation of APIs, particularly those that posses multiple polymorphic forms, is one of the least understood aspects of the overall process (Lawrence et al. [2004]).
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1.2 Crystallisation and Polymorphism

Crystallisation can be described as a phase that produces a crystalline product from a solution. A solution is a homogeneous, single phase mixture consisting of a solvent (liquid) and a dissolved solute (solid). At a given temperature, there is a maximum amount of solute that can be dissolved in a given amount of solvent. When this point has been reached, the solution is said to be saturated. The amount of solute required to achieve this state at the given conditions is known as the solubility. If the conditions are altered so the solution is no longer able to hold a specific quantity of solute, crystallisation occurs in order to return the solution to its equilibrium state.

There are a number of methods available to induce crystallisation (Cashell, 2003):

- Cooling crystallisation is used if the solubility of a compound increases with temperature. If the solution is cooled, the solubility of the solute is reduced and it subsequently crystallises out of the solution.

- Evaporative crystallisation involves evaporating the solvent which effectively increases the concentration of the solute present. The solute can no longer be supported and therefore crystallises.

- Vacuum crystallisation occurs when the saturated solution is evaporated and cooled by means of applying a vacuum without direct heating or cooling.

- Drowning-out crystallisation involves a decrease in the solubility of a solute due to the introduction of a suitable solvent. This solvent causes the solute to be less soluble in the solution.

- Reactive crystallisation is achieved through the generation of a solute from a chemical reaction. The new solute is less soluble than the reactive components and crystallisation results.

These crystallisation methods can be applied in an industrial or laboratory scale by selection of an appropriate crystalliser in order to produce a specified product.

Some of the crystals that result from the crystallisation process may have a different crystal structure for a given chemical species. Polymorphism is the ability of chemical species to have more than one crystal structure. There are a number of factors that determine which specific crystal form is produced. Primary factors include temperature,
supersaturation and stirring rate and secondary factors are impurity content (Kubota, 2001), solvents (Kitamura and Ishizu, 1998) and additives (Davey et al., 1997). Following the polymorphic process, the crystal has the same chemical composition but the physical properties can be very different. The structure of the crystals dictates the functionality and properties, such as crystal morphology, density and solubility which in turn can affect the performance of a product such as bioavailability of a drug. Therefore, the polymorphic structure needs to be properly controlled.

One example of a well documented polymorphic system is the L-glutamic acid system. L-glutamic acid is an amino acid and has two known polymorphs and can be easily crystallised under reasonable conditions. From an industrial perspective, L-glutamic acid is used in the production of monosodium glutamate (MSG), in the production of drugs for the treatment of neurological disorders and the metabolism of sugars and fats. Over 500,000 tons are produced annually (Ziegler, 2007). The two polymorphs of L-glutamic acid are referred to as $\alpha$ and $\beta$. Generally $\alpha$ is the preferred polymorph as its crystal habit is much more suited to handling in an industrial context (Kitamura, 1989). The $\beta$ polymorph is known to have a needle like structure, resulting in a more fragile crystal, whereas the $\alpha$ polymorph has a prismatic structure.

![Figure 1.1: Prismatic $\alpha$ and needle-like $\beta$ crystals (Cashell et al., 2004)](image)

1.3 Suspension Modelling

Crystallisation processes on an industrial scale are performed in a suspension and for this reason the study of crystallisation requires knowledge of turbulent fluid flow and
theory behind the efficient suspension of particles. The turbulent fluid flow within the tanks greatly influences the crystallisation process due to the potential creation of areas of inhomogeneity within the vessel. This results in less than ideal levels of supersaturation which in turn can reduce the growth and nucleation rates of particles. A lack of knowledge on the influence of the fluid flow may introduce potential deficiencies in the control of the crystallisation process.

The geometric design of a stirred tank is a key aspect in the performance of the mixing and suspension that is required. The turbulent flow present within a tank transports the material around the tank but this flow is dependent on the type of geometric configuration within the tank. Efficient solid suspension is very much determined by the size, positioning and type of impeller chosen, the presence of baffles within a tank and speeds at which the impeller is operated at. A typical stirred tank geometry for two different types of impeller is shown in Figure 1.2

1.3.1 Particle Suspension

One of the issues that can require analysis is the potential settling of particles on the bottom of a stirred vessel. This can arise due to insufficient impeller speeds as well as other geometric and mixture considerations. The speed of the impeller required to suspend all the particles is dependent on the tank geometry and the solid-liquid properties. The mass transfer process is enhanced between liquid and solid if the solids are suspended within a tank, allowing the maximum surface area to be exposed to the liquid and therefore enabling more efficient control over the final crystal product.
Chapter 1. Introduction

1.3.2 Particle Shape

One feature of the solid properties that has not been widely reported in the literature is the significance of the particle shape on the suspension of the solid phase in a solution. This is of special relevance when considering the L-glutamic polymorphic system as the control of the transformation process greatly depends on the levels of supersaturation.
within the vessel and the location of the particles at any given time. This is turn is dependent on the mixing and suspension properties within the tank. The location of the particles is dictated by the turbulent fluid flow and the variation of drag that is induced on the different shapes. The modelling of the particles in tanks has usually been carried out under the assumption that the particles are spherical which is not a reasonable assumption to make in the case of the L-glutamic acid system.

1.4 Problem Outline

The ability to accurately predict and control the polymorphism process is a major issue in the pharmaceutical industry. However, little time has been devoted to the study of industrial crystallisation, reactor design and process modelling [Fevotte 2002]. Inefficiencies within the suspension can produce low quality crystals which may not be of sufficient standard for the final product and poor understanding of the polymorphism process can lead to a lack of control over the crystals produced. The control of the particle size distribution (PSD) is also of considerable importance for the determination of the final quality of the product. If the batch of crystals produced contain a large proportion of fines then the filtration, purity and handling may be impaired. Control over the prediction and reproducibility of the PSD would improve the efficiency of the crystallisation process and improve the final product. Underestimation of the importance of these facets of the crystallisation process can lead to substantial losses of productivity and quality of final product which in turn results in significance financial losses.
1.5 Objectives

The primary aim of this investigation is to develop robust numerical methods for predicting the solvent mediated polymorphic transformation of a crystallisation system, the tracking of the PSD over time and the interaction of various polymorphic forms within a stirred vessel. In order to achieve this the project objectives can be outlines as follows:

- Develop a numerical method to simulate the polymorphic transformation of a crystallisation system
  - Identify the most appropriate numerical method for solution of a crystallisation system.
  - Propose a method of tracking the progression of the PSD
  - Implement the selected modelling approach using the appropriate kinetic equations.
  - Validate the numerical method using measured data in the technical literature in conjunction with experimental results.

- Develop a method to predict the interaction of various polymorphic forms within a stirred tank.
  - Identify the factors that dictate the performance of liquid-solid suspensions.
  - Determine the most suitable CFD modelling approach for predicting turbulent liquid-solid flows in a suspension.
  - Design and construct a stirred tank using the information derived from geometric analysis of various tank configurations.
  - Investigate various existing drag laws to accurately simulate the drag induced by non-spherical particles.
  - Implement the most applicable drag law into the CFD code.
  - Validate the model using experimental data resulting from the constructed stirred tank.
In order to build up an understanding of the crystallisation and polymorphic processes it is first necessary to become familiar with the concepts involved. This chapter has been divided up into a number of sections describing the supersaturation (section 2.1), nucleation (section 2.2), growth (section 2.3), dissolution (section 4.3) and polymorphism (section 2.5) processes. The application of these processes to the L-glutamic acid system is discussed in Chapter 4.

2.1 Supersaturation

A saturated solution is a solution that contains the maximum amount of solute that the solvent can normally dissolve at a given temperature. Thermodynamically this means that the chemical potential of the solute in the solution is the same as the chemical potential of the species in the solid phase \( \text{[Myerson 2002]} \).

\[
\mu_i^{\text{solid}} = \mu_i^{\text{solution}} \tag{2.1}
\]

where \( \mu_i \) denotes the chemical potential of species \( i \) in the solid and solution phase.
If the solution system is disturbed from its equilibrium condition, by a temperature decrease for example, the difference between the new chemical potential and the equilibrium chemical potential can be calculated from

\[ \Delta \mu_i = \mu_{i_{\text{solution}}} - \mu^*_i \]  

(2.2)

Using the Nernst equation to express the chemical potential

\[ \mu_i = \mu_{i_0} - RT \ln(a) \]  

(2.3)

and rewriting equation 2.2 yields

\[ \Delta \mu_i = RT \ln \left( \frac{a_{i_{\text{solution}}}}{a^*_i} \right) \]  

(2.4)

The activity coefficient of species \( i \) is given by:

\[ \gamma_i = \frac{a_i}{c_i} \]  

(2.5)

where \( c_i \) is the molar concentration of species \( i \) in solution. For small concentrations the activity coefficient can be assumed to equal unity and the activity and concentrations may be assumed to be equal [Smith et al., 1996] and equation 2.4 becomes

\[ \frac{\mu_i - \mu^*_i}{RT} = \ln \frac{a_i}{a^*_i} = \ln \frac{c_i}{c^*_i} \]  

(2.6)

Equation 2.6 expresses the supersaturation of the system in dimensionless form. It is more convenient to express the supersaturation in terms of the concentration levels as opposed to the chemical potentials as data regarding the concentration levels for crystallisation systems are more readily available. Data on chemical potential is usually unavailable.

Supersaturation can be expressed as a concentration difference

\[ \Delta c = c_i - c^*_i \]  

(2.7)

and as a ratio of concentrations
Chapter 2. Crystallisation Theory

\[ S = \frac{c_i}{c_i^*} \]  \hspace{1cm} (2.8)

In order for the above supersaturated solution to return to equilibrium it needs to crystallise. Crystallisation can be defined as a phase change in which a crystalline product is obtained from solution.

2.1.1 Metastable Zone

In most crystallisation systems nucleation and growth occur within what is known as the metastable zone (Mullin, 2001). Looking at a plot of concentration versus temperature for a typical solution, Figure 2.1, the concept of supersaturation and the metastable zone can be explained. Starting at point A and proceeding to cool the solution until point B is reached will result in the solution becoming supersaturated. At this stage, the solution is metastable and just needs a small, finite change in environment to initiate crystallisation. This may take a few days to occur as the system is only slightly metastable. If the system is cooled to point C then it becomes more supersaturated and may crystallise in a couple of hours. Cooling can continue until a point where the solution crystallises out more quickly and the solution is no longer stable. The unstable boundary is referred to as the spinodal curve. If the solution operates beyond the spinodal curve the a spontaneous phase separation is probable but not definite. The area between the solubility curve and the spinodal curve is known as the metastable zone.

The metastability of the solution decreases as it becomes more supersaturated. The solution appears to be crystallising out more rapidly as the supersaturation increases but crystals may not immediately form when the solute levels exceed the solubility. The reason why the solute doesn’t crystallise out immediately after achieving supersaturation is down to a concept called nucleation. This process will be discussed in section 2.2.
Chapter 2. Crystallisation Theory

2.2 Nucleation

As described in the previous section, in order for a supersaturated system to regain equilibrium, crystallisation can occur. Before crystals can develop, there must be a number of solid bodies present in the solution to act as centres for crystallisation. These bodies may not be initially present but are necessary for crystallisation to occur. The formation of these bodies is termed nucleation and is discussed in this section.

There are two main mechanisms that describe nucleation, primary nucleation which occurs in the absence of suspended crystals and secondary nucleation which requires suspended solute crystals in order to function. Primary nucleation can also be described as being either homogeneous or heterogeneous, homogeneous being the ideal situation used for several nucleation theories and heterogeneous requiring foreign surfaces, other than the solute crystals, in order to develop. Impurities would be one such example of a foreign body.
2.2.1 Homogeneous Nucleation

Classical nucleation theory developed by [Volmer (1939)] and [Nielsen (1964)] describes how molecules in a supersaturated solution aggregate to form a critical cluster size using a simple addition mechanism.

\[ a_{n-1} + a = a_n \]  
\[ (2.9) \]

where \( a_n \) is cluster of \( n \) sized solute molecules. If the cluster of particles do not reach a critical size then they will dissolve back into solution due to instabilities. If the cluster grows beyond this critical size, it becomes stable and will continue to grow in the solution under supersaturation conditions.

In order to calculate the critical cluster size the Gibbs free energy change for formation of the new phase is calculated [Gibbs (1928)]. The Gibbs free energy change is described as the sum of the free energy change due to the surface generation, \( \Delta G_s \), and the free energy change as a result of the phase transformation \( \Delta G_v \).

\[ \Delta G = \Delta G_s + \Delta G_v \]  
\[ (2.10) \]

If it is assumed that the cluster is spherical, equation 2.10 can be written in terms of the cluster surface area, the surface tension \( \sigma \), the cluster volume and the free energy change due to the phase transformation, \( \Delta G_v \).

\[ \Delta G = 4\pi r^2 \sigma - \frac{4}{3} \pi r^3 \Delta G_v \]  
\[ (2.11) \]

The two terms on the right hand side of equation 2.11 are of opposite sign and depend differently on the cluster radius, \( r \), and therefore will pass through a maximum value of \( \Delta G \) as illustrated in Figure 2.2. The critical nucleus size, \( r_c \), is found by calculating which value of \( r \) produces the maximum change in the free energy, \( \Delta G \). This is achieved through differentiation of the equation 2.11 with respect to \( r \) and setting equal to zero

\[ \frac{\partial \Delta G}{\partial r} = 0 \]  
\[ (2.12) \]

Solving this equation yields
Figure 2.2: Free Energy Diagram showing free energy versus nucleus size

\[ r_c = \frac{-2\sigma}{\Delta G_v} \]  
\[ \Delta G_v = \frac{RT \ln(1 + S)}{V_m} \]

which reveals the critical nucleus radius. According to Myerson [2002], the free energy change required to form a cluster of size \( r \) can also be written as

\[ \Delta G = 4\pi r^2 \sigma - \frac{4\pi r^3}{3V_m} RT(1 + S) \]  

where \( V_m \) is the molecular volume, \( R \) is the gas constant and \( T \) is the temperature. Comparing equations 2.11 and 2.14 allows an expression for \( \Delta G_v \) to be generated

\[ \Delta G_v = \frac{RT \ln(1 + S)}{V_m} \]

Combining equations 2.13 and 2.15 produces an equation for the critical nucleus radius

\[ r_c = \frac{2V_m \sigma}{RT \ln(1 + S)} \]
Particles smaller than $r_c$ will dissolve back into solution and those larger than $r_c$ will continue to grow. It should be noted however that homogeneous nucleation is difficult to observe in practice due to the presence of dissolved impurities as well as physical features such as stirrers, baffles and walls (Myerson, 2002).

### 2.2.2 Heterogeneous Nucleation

If a foreign particle is present in a supersaturated solution then it is generally known that less energy is required for nucleation (Myerson, 2002; Mersmann, 1994). The effective surface energy is lower when a molecule is in contact with a foreign particle and this reduces the free energy barrier required for nucleation to occur as discovered by Volmer (1939). He deduced that the size of the wetting angle of the foreign particle could be related to the free energy as per equation 2.17:

$$\Delta G_{hom} = \phi \Delta G_{het} \quad \text{(2.17)}$$

where $\phi = \frac{1}{4} (2 + \cos \theta)(1 - \cos \theta)^2 \quad \text{(2.18)}$

The energy required for heterogeneous nucleation will always be equal to or less then that for homogeneous nucleation as the wetting angle varies from 0 (no wetting) up to $\pi$ (full wetting).

A study by Liang et al. (2004) examined the effect of stirrer material on the heterogeneous nucleation of L-glutamic acid and concluded that the nucleation process began...
on the surface of the stirrer as opposed to the cooler regions of the tank as is usually assumed. This effect was due to lower free energy required to initiate nucleation on the stirrer.

2.2.3 Secondary Nucleation

A supersaturated solution nucleates at a lower supersaturation when crystals of the solute are already present in the solution or are added. Secondary nucleation is the term used to describe this type of behaviour. There are a number of theories to explain the secondary nucleation phenomena and these are discussed below.

- Initial breeding, also known as dust breeding, has been studied by [Strickland-Constable and Mason (1963)](1963) and is caused by tiny crystallites which form on the surface of the crystal during the growth of the crystals or from fragmentation during storage. These crystallites are larger than the critical cluster size and therefore the nucleation rate is independent of the supersaturation levels in the solution as well as the stirring rate. This phenomena is only important in batch crystallisation processes [Myerson (2002)](2002).

- Needle breeding occurs during high levels of supersaturation as needles are formed and fragment in the solution and act as nucleation sites. This issue can be resolved by reducing the supersaturation or to a lesser extent reducing the agitation levels. Additives may also be introduced into the system in order to negate the formation of dendrites on the crystal surface or change the crystal habit.

- Attrition breeding is a result of microabrasion of the crystals due to high stirring speeds causing the production of crystal fragments that form nucleation sites. It also causes the rounding of the crystals edges and corners and is a function of the crystal hardness, suspension concentration and the retention time.

- Contact breeding is due to crystals contacting other crystals or parts of the crystalliser such as the agitator, baffles and wall. It was found that the probability of the crystals impacting the agitator is proportional to the rotational speed of the
2.3 Growth Models

The crystal growth stage of the process involves the growth of the newly formed nuclei by addition of solute molecules from the supersaturated solution. It encompasses mass transfer of the solute molecules toward the crystal surface and the integration of the molecules into the crystal lattice. The crystals will grow in a layer by layer fashion as the molecules occupy particular sites on the surface. The type of site is chosen based on the most energetically favourable arrangement.

2.3.1 Crystal Growth Models

The growth of a crystal describes the diffusion of solute molecules from the bulk solution toward the surface of a crystal and the addition of molecules onto the face of a crystal causing the formation of a new layer. Diffusion controlled crystal growth occurs when the rate limiting factor is the diffusion of the solute molecules through the bulk fluid toward the crystal surface. Surface integration controlled crystal growth describes how the rate limiting factor is the adsorption of the solute molecules onto the crystal surface and the diffusion of the molecules along the surface until they find the most energetically favourable site in which to remain. There are three potential sites for the molecules to bond to and these are distinguished by the number of bonds between the crystal and the molecules themselves (Figure 2.4). At site A, the molecule attaches at the surface, at site B, the molecule is bonded at the surface and a growing step and at site C, the molecule is attached at three surfaces. Site C is the most energetically favourable site and is therefore the preferred location.
A more detailed description of specific crystal growth models will now be discussed. A theory developed by Volmer (1939) was the first of its kind suggested. It describes how the arrival of solute molecules onto the surface of the crystal merely causes it to lose one degree of freedom and how it continues to travel over the surface (surface diffusion). The molecules will then proceed to link into the crystal structure where the attractive forces are at their greatest and ideally will form a complete step over time. Before a new layer can begin to grow on the surface a ‘centre of crystallisation’ must be created and Volmer suggests that a monolayer island nucleus is formed. This is otherwise known as a two-dimensional nucleus (Figure 2.5).
Kossel (1934) also developed a model based on the theory that an apparently flat crystal surface actually consists of moving layers of monotomic height which may contain one or more kinks. On the crystal surface there will also be loosely adsorbed molecules which can be integrated into the crystal at the kink sites. The kink will continue to move along the surface until the layer is complete. A new step can also be created as a result of surface nucleation. One flaw associated with this model is due to the dependence on high levels of supersaturation for surface nucleation since it is known that many crystal faces can grow at quite high rates at low supersaturation levels (Mullin, 2001).

A solution to this problem was presented by Frank (1949) as he postulated that it is very unlikely that crystals would grow without developing imperfections, such as dislocations, on their surfaces due to the possibility of impurities incorporating themselves into the crystal or stress within the crystal. Most crystals contain these dislocations, the most common type being screw dislocations. These dislocations encourage the growth of crystals without relying on surface nucleation as the growth progresses via continuously growing steps in a manner resembling a spiral staircase. Burton et al. (1951) furthered this theory by developing a kinetic equation relating the curvature of the spiral to the spacing of successive turns and the supersaturation of the solution. This model is known as the Burton Cabrera Frank (BCF) model and is one of a number of layer-growth models.

Another type of layer growth model that is used is the mononuclear model. In this case the growth takes place by the addition of further similar nuclei or through the attachment of growth units to the edge of the nuclei via a surface diffusion flux (Farhadi and Babaheidary, 2002). There are three methods for the formation and spreading of
Chapter 2. Crystallisation Theory

Figure 2.7: Development of growth by the birth and spread mechanism

the nuclei put forward by Ohara and Reid (1973). If the nuclei spreads at an infinite velocity, then the rate determining step in the process is the formation of the surface nuclei and this model is referred to as the mononuclear model. A variation on this is the polynuclear model, which describes how there is a zero spread velocity associated with the nuclei once they nuclei form on the surface. In between the polynuclear and mononuclear models there exists the birth and spread model or nuclei above nuclei (NAN) model describing how the nuclei spread across the surface at a finite velocity. This velocity is also independent of the size of the crystal and the model assumes that nuclei can form anywhere on the surface. A general expression for this model was developed by Ohara and Reid (1973).

\[ R = C_1(S - 1)^{2/3} [\ln(S)]^{1/6} \exp\left[-C_2/T^{2/3} \ln(S)\right] \] (2.19)

Another growth process that can occur in a system that contains a distribution of particle sizes is known as Ostwald ripening. This process describes how the difference in solubilities between small and large particles induces the dissolution of the smaller particles and the subsequent deposition of the solute onto the surface of the larger crystals resulting in an increase in size. The process of Ostwald ripening is driven by the minimisation of the Gibbs free energy through minimising the particle surface area in the system. The effect of size on the solubility of particles is shown using the Gibbs-Thompson equation
\[
\ln \frac{c(r)}{c^*} = \beta V \sigma_s / \rho kT
\] (2.20)

where \(c(r)\) is the solubility of particle of size \(r\), \(c^*\) is the equilibrium solubility, \(\beta\) is the surface shape factor, \(\sigma_s\) is the specific surface energy of solid particles and \(V\) is the molecular volume. Ostwald ripening is important in rapid crystallisation processes involving small particles. It also has the effect of altering the particle size distribution (PSD) in a system that is at an apparent equilibrium with its saturated solution (Myerson, 2002).

### 2.4 Dissolution

Dissolution kinetics are of high importance in the pharmaceutical industry as the bioavailability of drug crystals is highly reliant on the rate of dissolution (Mangin et al., 2006). Their significance is not just confined to the pharmaceutical industry as the kinetics of dissolution makes a large contribution in the world of industrial crystallisation. For example, the dissolution of fine crystals is effective in the production of large crystals with a narrow size distribution (Shan et al., 2002a). This is known as kinetic ripening. Another aspect of industrial crystallisation which requires knowledge of the dissolution process is that of solvent-mediated transformation of crystal polymorphs. This involves the dissolution of metastable crystals in order for the system to achieve its most stable state. This is the feature of dissolution that is of interest in the work presented here.

As it is the case with crystal growth, dissolution can involve two main steps (Koutsoukos and Valsami-Jones, 2004), surface reaction and detachment of the species followed by transfer of this species toward the bulk solution across the diffusion layer which surrounds the crystals and the overall process is controlled by the slower step. Previous work completed in the modelling of the dissolution process has been carried out by a number of authors. In a work published by Noyes and Whitney (1897), the dissolution rate was expressed by assuming that the process is diffusion controlled and involves no chemical reaction. Their equation simply states that the dissolution rate is directly proportional to the difference between the solubility and the solution concentration

\[
\frac{dX}{dt} = \frac{D_{\text{diff}} A(C_s - C)}{\delta}
\] (2.21)

where \(X\) is the mass of crystal dissolved, \(A\) is the surface area of the solid, \(C\) is the con-
centration of the bulk solution, $C_s$ is the concentration in the diffusion layer surrounding the solid, $D_{diff}$ is the diffusion coefficient and $\delta$ is the boundary layer thickness. The above expression was developed further by Lu et al. (1993) during an analysis of the hydrocortisone system. This study also assumed a diffusion controlled process but to simulate the dissolution the authors included a non-spherical geometry for the crystals and a variation of the diffusion layer thickness with the particle size.

A different approach was taken by Shan et al. (2002b) who accounted for both surface reaction and mass transfer toward the bulk solution. In the analysis, the dissolution rate of L-aspartic acid was investigated. In this method, the dissolution rate due to the disintegration of particles on the surface was assumed to equal the diffusion rate of particles toward the bulk solution

$$r = r_d = r_{diff}$$

(2.22)

This produced the following equation:

$$\frac{dC_b}{dt} = \frac{\beta k}{\alpha \rho L_o} C_{all}^{1/3} (C_{all} - C_b)^{2/3} (C_{sol} - C_b)$$

(2.23)

where

$$k = \frac{1}{1/k_d + 1/k_{diff}}$$

(2.24)

and $L_o$ is initial crystal size, $C_{sol}$ is the solubility, $C_b$ is the concentration of solute in the bulk solution and $\alpha$ and $\beta$ are the volume and surface shape factors respectively. $C_{all}$ is the concentration when the initial crystals within the system are completely dissolved.

If $k_d >> k_{diff}$, then equation (2.23) reduces to an equation similar to the Noyes-Whitney type equation. It was concluded by the authors that the process became diffusion controlled at high undersaturation to a surface reaction process at lower levels of undersaturation.
A further dissolution study involving the sucrose system was carried out by Koiranan et al. (1999) and included the development of a simulation assuming that mass transfer was the only process present and the surface reaction process could be ignored. This follows on from the assumption that the dissolution process can be equated to negative growth. The analysis makes use of a Sherwood correlation (Mersmann, 1994) in order to calculate a mass transfer coefficient.

\[
\frac{k_{da}}{D} = \frac{2 + 0.8\left(\frac{\tau L^4}{\nu^3}\right)^{1/5}}{S_{c}^{3/5}}
\]  

where \(D\) is the diffusivity, \(L\) is the crystal length, \(\nu\) is the kinematic viscosity, \(\tau\) is the average power input and \(S_{c}\) is the Schmidt number, \(S_{c} = \nu / D\).

This form of the correlation applies to stirred vessels and fluidised bed crystallisers and the mass transfer is dependent on the crystal size and not the diameter of the vessel. Other crystal dissolution studies incorporating the Sherwood correlation to a satisfactory effect can be found in the literature (Armenante and Kirwan, 1989; Flores et al., 2005).
2.5 Polymorphism

In some crystallisation processes it is not unusual for the first crystalline phase that appears in the system to be metastable (Mullin, 2001). Some metastable phases can transform rapidly to a more stable phase while others can remain in their current state for an extremely long time. It may also be necessary, in the case of some pharmaceutical products, to inhibit the transformation from a metastable phase to a more stable phase in the situation where the metastable phase has more desirable properties for a specific application. This can be achieved by isolating the metastable polymorph itself or by reducing the rate of dissolution of the polymorph by introducing specific impurities that will act as inhibitors (Zhang and Nancollas, 1991).

Figure 2.9: Plot of free energy (G) against reaction (ρ) for crystallization in a dimorphic system (Bernstein et al., 1999)

In a crystallization process involving polymorphism, there may be a number of different crystalline products that arise. Figure 2.9 describes the kinetic factors associated with a polymorphic transformation. $G_o$ represents the Gibbs free energy per mole of a solute in a supersaturated fluid which will crystallize into one of two states, I or II. State II has the lower free energy, $G_{II}$, and as a result it is the more stable form of the crystal. However the relative rates of formation are dictated by the activation energy which implies that state I will form first ($G^*_I < G^*_{II}$) as it is kinetically more favourable and will crystallize according to Ostwald’s law of stages. In order for the crystal to achieve
the state with the lowest free energy, $G_{II}$, a transformation from state $I$ to $II$ will take place (Bernstein et al., 1999).

2.5.1 Solvent-Mediated Transformation

In order to describe the transformation process it is useful to consider the phase diagram for a monotropic system below. In a monotropic system, the transformation of crystals from one form to another can only take place irreversibly in one direction (Cashell, 2003).

Figure 2.10 illustrates two phases in the system, 1 (metastable) and 2 (stable). A solution of composition $x_B$ at the temperature $t_x$ is supersaturated with respect to the stable phase, whereas at $x_i$ both phase could crystallize out of the solution. According to Ostwald’s Law of Stages (Ostwald, 1896), phase 1 precipitates first and the solubility drops to $x_1$. At this point, the solution is supersaturated with respect to phase 2 and may cause the precipitation of crystals of the stable phase. As this continues, the solute concentration will drop again but the solution will become undersaturated with respect to phase 1 and begin to dissolve into solution producing continued supersaturation for phase 2. The dissolution-growth process will continue until all of the metastable phase disappeared and the transformation is complete.

For an enantiotropic system, where the transformations are reversible, the stability of the polymorphs varies depending on the temperature of the solution. In Figure 2.11 it can be seen that form II is stable below the transition temperature whereas from I is stable above $T$. At the transition temperature both forms have equivalent solubilities and reversible transformations can be performed by manipulating the temperatures.

The L-glutamic system that is being studied in this dissertation is monotropic i.e. irreversible.
2.5.2 Polymorphism of L-glutamic acid

A large volume of work has been published on the solvent mediated transformation of the L-glutamic acid polymorphic system. An detailed study into the polymorphic trans-
formation of L-glutamic acid was performed by Kitamura (1989). In this work it was discovered that the transformation will not take place if the solids are left in a dried state. The transformation will only take place in solution and hence the transformation is "solution-mediated". A concentration curve with four distinct stages can be used to describe the crystallization process of L-glutamic acid (Figure 2.12). Beginning with a temperature of 45°C and a solute concentration of 45g/l of solution, a sharp decrease in the concentration takes place due to the nucleation and growth of both the α and β polymorphs. In the second stage, the concentration levels off at the solubility of the α polymorph due to the solution no longer being supersaturated with respect to the α polymorph. It stays at this level until all the α has dissolved back into the solution and while the β is continuing to nucleate and grow. Once all the α has dissolved, the concentration decreases again as the β continues to grow. The fourth part of the process shows the concentration at a constant level, having reached the solubility of the β polymorph.

Figure 2.12: Concentration change during the crystallisation process at 45°C [Kitamura, 1989]
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The same process has been carried out by Kitamura (1989) at 25°C but in this case only α nucleated and grew and the concentration curve decreased to the expected solubility of α. Even the addition of the β crystals to help encourage the transformation from α to β made little difference, leading to the conclusion that the transformation rate is very slow at 25°C. This maybe due to the fact that there is no nucleation and growth of the β polymorph and therefore the concentration levels do not drop enough to initiate the dissolution of α and hence cause the transformation.

The monitoring and modelling of the overall process at 45°C was undertaken by Scholl et al. (2006a). In this work the nucleation, growth and dissolution of the α polymorphs and the nucleation and growth of the β polymorphs were investigated. A series of experiments were carried out at three different initial solute concentrations and the solute and solid concentrations were monitored over time. Using these experimental results, a set of kinetic equations was generated to model the complete process under conditions specified. These kinetic equations are used as the basis for the study performed in this treatise.

An examination of the dissolution of the α polymorph and the growth of the β polymorph was also carried out by Dharmayat et al. (2008) in an effort to calculate the kinetics of the transformation process. The authors employed online X-ray powder diffraction to monitor the phase transformation from the metastable α phase to the stable β phase over a range of initial solids concentrations. The dissolution of the α polymorph was assumed to be diffusion controlled based on the analysis performed. Kinetic equations for the dissolution of α and the growth of β polymorph were also developed based on the experimental results but an investigation into the nucleation process was not included in the study.

Additional studies into the L-glutamic acid system include investigations into the nucleation of the β polymorph upon the surface of the α polymorph (Cashell 2003; Cashell et al. 2004; Ferrari and Davey 2004) and the effect of additives on the growth and transformation within the system (Garti and Zour 1997; Kitamura and Nakamura 2001; Cashell 2003).
2.6 Summary

The analysis of the L-glutamic acid polymorphic system being considered in this treatise requires a knowledge of the processes throughout its evolution. The concept of supersaturation and its role within crystallisation has been studied. The principles behind nucleation, growth and dissolution of crystals as well as the concept of polymorphism have been discussed. The influence of these factors on the polymorphic transformation of L-glutamic acid has been assessed. The theory presented in this chapter provides a foundation for a detailed analysis of the numerical simulation of the L-glutamic acid system.
When studying particulate processes it is necessary to be able to characterise the system in question as no two particles will be exactly the same size. This is achieved using a particle-size distribution (PSD) by choosing a characteristic length as the basis for defining the distribution. A typical example of a PSD is shown in Figure 3.1 which plots the probability of a particle appearing in a specific size class against the range of sizes encountered within the system. The monitoring of these distributions as they evolve over time needs to be examined and this can be achieved using a number of different methods. One of these approaches, the method of moments, is also used as a mechanism to reconstruct the PSD. The different methods used to monitor the development of the PSD’s are discussed in the following chapter as well as a unique approach to reconstructing the PSD.
3.1 Population Balance

The population balance approach is a method used to account for both the size and the number of particles present in a crystalliser [Randolph and Larson 1988]. The number of crystals present in any crystal size class is described by the crystal population density, $n$ and is defined as

$$\lim_{\Delta L \to 0} \frac{\Delta N}{\Delta L} = \frac{dN}{dL} = n \quad (3.1)$$

where $\Delta N$ is the number of crystals in the size range $\Delta L$ per unit volume and $L$ is the characteristic dimension of the particle.

In order to generalise the population balance it is necessary to describe the particle phase space of the system, i.e., the least number of independent coordinates that can fully describe the system. The coordinates can be either internal, which details the properties independent of the position or external, which describes the spatial distribution. A Lagrangian viewpoint of the population balance has the form [Myerson 2002]:

$$\frac{\partial n}{\partial t} + \nabla \cdot (v_e n) + \nabla \cdot (v_i n) = B - D \quad (3.2)$$
where \( t \) is the time, \( v_e \) is the velocity along an external coordinate, i.e., particle velocity; \( v_i \) is the velocity along an internal coordinate, e.g., growth rate \( G = dL/dt \); \( B \) and \( D \) are the birth and death functions respectively.

An Eulerian viewpoint has the form:

\[
\frac{\partial n}{\partial t} + \nabla \cdot (v_e n) + n \frac{d \log V}{dt} = B - D - \sum_k \frac{Q_k n_k}{V}
\]  

(3.3)

with the addition of a system volume term, \( V \) and the flow rate across the system, \( Q_k \).

Using this form of the population balance and assuming that no breakage or agglomeration occurs and there is no flow entering or exiting the system the equation reduces to

\[
\frac{\partial n}{\partial t} + \nabla \cdot (v_e n) = 0
\]  

(3.4)

If the particle length is also chosen as the internal coordinate and the growth is assumed to be size independent then it reduces further to

\[
\frac{\partial n}{\partial t} + G \frac{\partial n}{\partial L} = 0
\]  

(3.5)

which is the generalised population balance used to account for the time-dependent size distributions in a closed system. The first term in equation (3.5) describes the change in number density over time and the second term accounts for the difference in crystals entering and leaving class size \( \partial L \) due to the growth \( G \).

### 3.2 Method of Classes

One of the issues with the population balance is the fact that it is in the form of a partial differential equation and no analytical solution exists and as a result the PSD must first be discretised. One of the methods used to discretise the PSD and solve the population balance equation is the method of classes.

This method was developed by Marchal et al. (1988) and consists of splitting the PSD into a range of class sizes and solving the population balance equation for each class.
The size range is \( L_0, L_1, \ldots L_N \) where \( L_0 \) is the smallest size and \( L_N \) the largest with \( N \) being the number of classes. The width of class \( C_i \) is \( \Delta C_i = L_i - L_{i-1} \) and the characteristic size of class \( C_i \) is \( S_i = (L_{i-1} + L_i)/2 \). If \( N_i(t) \) is the number of crystals of class \( C_i \) at time \( t \) per unit volume of suspension then

\[
N_i(t) = \int_{L_{i-1}}^{L_i} \Psi(L,t) dL
\]  

(3.6)

where \( \Psi \) is the volumetric density function. Following the integration of equation 3.6 between \( L_{i-1} \) and \( L_i \), a differential equation results describing the system over time

\[
\frac{dN_i}{dt} + \frac{1}{V_T} \frac{dV_T}{dt} N_i + \frac{Q_i N_i - Q_e N_e}{V_T} + \frac{G(L_i)}{2\Delta C_{i+1}} N_{i-1} + \frac{G(L_i) - G(L_{i-1})}{2\Delta C_i} N_i - \frac{G(L_{i-1})}{2\Delta C_{i-1}} N_{i-1} = R_{A,i} - R_{B,N}
\]  

(3.7)

The number of class sizes chosen to represent the PSD determines the level of accuracy that is obtained and the main advantage to this method is that the PSD is calculated directly. However, there is a trade off between the level of accuracy achieved and the computational time required to solve the equations. Marchisio et al. (2003b) suggests that the number of size classes required for an accurate solution is of the order of 2000 classes. This number would result in a prohibitively high number of computations for a full simulation. Furthermore, the method of classes would also be unsuitable for introduction into a computational fluid dynamics (CFD) package due to the excessive number for classes required for operation (Marchisio et al., 2003a).

### 3.3 Method of Moments

An alternative approach to the method of classes was proposed by Randolph and Larson (1988) and termed the Method of Moments (MOM). This describes how the PSD can be reduced to a set of numbers (moments) derived from the integrals of the PSD. Each individual moment represents an aspect of the PSD such as total number of particles, total particle length, area and volume. It also avoids having to track individual particles in each class and therefore is a lot less computationally intensive than the method of classes.
The generalised population balance for the system (equation 3.5) is rewritten in terms of the moments using the following technique:

In order to reduce the population density term, \( n \), in equation 3.5 to the moments the following operation must be performed.

\[
m_k = \int_0^\infty n L^k dL
\]

(3.8)

Multiplying equation 3.5 by \( L^k \) and integrating and assuming size independent growth leaves the population balance in terms of moments.

\[
\frac{dm_k}{dt} + \int_0^\infty L^k G \frac{\partial n}{\partial L} dL = 0
\]

(3.9)

Further integration by parts results in equations 3.10 and 3.11

\[
\frac{dm_k}{dt} = \int_0^\infty k L^{k-1} G n(L) dL
\]

(3.10)

\[
\frac{dm_k}{dt} = G k m_{k-1}
\]

(3.11)

For the zeroth moment representing the total number of particles in the system

\[
\frac{dm_0}{dt} = B_0
\]

(3.12)

Equation 3.10 can be simplified to the following moment equations in the case of size-independent growth:

\[
\frac{dm_1}{dt} = m_0 G
\]

(3.13)

\[
\frac{dm_2}{dt} = 2m_1 G
\]

(3.14)

\[
\frac{dm_3}{dt} = 3m_2 G
\]

(3.15)

Using the MOM with the knowledge that the zeroth, first, second and third moments represent the total number of particles, total length, surface area and volume respectively, the mean crystal size can be calculated as follows
and the solids concentration is given by

\[
c_s = \frac{\rho k_v m_3}{M}
\]  

where \(\rho\) is the crystal density, \(k_v\) is the volumetric shape factor that relates the volume of a particle to its characteristic dimension cubed and \(M\) is the molecular weight of the crystal.

For systems utilising size-independent growth and nucleation only, the issue of closure within the population balance is satisfied as the problem is expressed in terms of the lower-order moments. For systems involving aggregation, breakage and size dependent growth this technique is not sufficient for closure of the population balance as the governing equations require discretisation of the PSD using the method of classes before it is integrated with respect to the length coordinate. For this reason the MOM will still be computationally intensive if aggregation and breakage are included in the analysis and therefore another method would be required to formulate a solution.
3.4 Quadrature Method of Moments

For a more detailed and comprehensive analysis of the evolution of the PSD a more thorough approach to the moment technique was developed by McGraw (1997) termed the quadrature method of moments (QMOM). This approach is used when the system in question requires analysis of the agglomeration and breakage that occurs throughout the evolution of the PSD as the MOM is unsuitable for these systems as the governing equations cannot be closed using moment terms exclusively.

The QMOM solves the closure problem by making use of a quadrature approximation

\[ n(L; t) \approx \sum_{i=1}^{N_q} \omega_i(t) \delta[L - L_i(t)] \]  
(3.18)

so that

\[ m_k(t) \approx \sum_{i=1}^{N_q} \omega_i(t)L_i^k(t) \]  
(3.19)

where \( L_i(t) \) and \( \omega_i(t) \) are the abscissas and weights respectively and these can be specified from the lower order moments. To build a quadrature approximation of order \( N_q \) it is sufficient to know \( 2N_q \) moments e.g, the first six moments \( (m_0, ..., m_5) \) would be enough to construct a QMOM approximation of order \( N_q = 3 \). The abscissas and weights are calculated from the moments by constructing a matrix using a product difference algorithm developed by Gordon (1968).

The first step is to construct a matrix \( P \) with components \( P_{ij} \) using the moments of the PSD (McGraw, 1997). The first column of the matrix is as follows

\[ P_{i,1} = \delta_{i1}; \quad i \in 1, ..., 2N_q + 1 \]  
(3.20)

where \( \delta_{i1} = 0 \) for \( i \neq 1 \) and \( \delta_{i1} = 1 \) for \( i = 1 \). The components in the second column of \( P \) are

\[ P_{i,2} = (-1)^{i-1}m_{i-1}; \quad i \in 1, ..., 2N_q + 1 \]  
(3.21)

The remaining components in matrix \( P \) are found using the product difference algorithm.
\[ P_{i,j} = P_{1,j-1}P_{i+1,j-2} - P_{1,j-2}P_{i+1,j-1}; \]
\[ j \in 3,\ldots,2N_q + 1 \text{ and } i \in 1,\ldots,2N_q + 2 - j \] (3.22)

The elements along the top row of the matrix are the only entries required for the calculation of abscissas and weights. The first row elements \( P_{1,1} \) and \( P_{1,2} \) follow equations 3.20 and 3.21 while the remaining entries are obtained using equation 3.22.

A vector \( \phi_i \) is generated by setting the first element equal to zero (\( \phi_1 = 0 \)) and remaining elements computed as follows

\[ \phi_i = \frac{P_{1,i+1}}{P_{1,i}P_{1,i-1}} \quad i \in 2,\ldots,2N_q \] (3.23)

A symmetric tridiagonal matrix is obtained from the sums and products of \( \phi_i \)

\[ a = \phi_{2i} + \phi_{2i-1} \quad i \in 1,\ldots,2N_q - 1 \] (3.24)

\[ b = \sqrt{\phi_{2i+1}\phi_{2i}} \quad i \in 1,\ldots,2N_q - 2 \] (3.25)

The above equation completes the matrix \( J \) where the eigenvalues of \( J \) are the abscissas, \( L_i(t) \), and the weights, \( \omega_i(t) \), are described in terms of the eigenvectors \( \nu_j \)

\[ \omega = \nu_{j1}^2 \] (3.26)

The application of this method will be outlined using a general aggregation-breakage equation from Marchisio et al. (2003b) given below.

\[ \frac{\partial m_k(t)}{\partial t} = B^a_k - D^a_k + B^b_k - D^b_k \] (3.27)

where \( B^a_k, D^a_k, B^b_k \) and \( D^b_k \) are the birth and death terms due to aggregation and breakage respectively. Expanding the death term due to breakage gives the equation below

\[ D^b_k = \int_0^{\infty} L^k b(L)n(L)dL \] (3.28)

In order the integrate a function product, \( W(x)f(x) \) between \( a \) and \( b \), one can approximate the integral as
\[ \int_{a}^{b} W(x)f(x)dx = \sum_{j=1}^{N} \omega_j f(x_j) \]  

(3.29)

where \( W(x), f(x), \omega \) and \( f(x_j) \) are the functions of \( x \), weighting function and \( f(x) \) evaluated at \( x_j \) respectively. Applying this knowledge to equation 3.28 allows us to break up the equation into a function and a corresponding weighting function as follows:

\[ \dot{D}_k = \int_{0}^{\infty} \frac{L^{k}k^{b}(L)n(L)\,dL}{f(L)} \]  

(3.30)

The weighting function used here is chosen to be the number density function and as a result, the quadrature approximation can be expressed in terms of the moments.

As discussed earlier, the abscissas and weights are generated using the eigenvalues and eigenvectors of matrix \( J \), respectively. The death term due to breakage (equation 3.28) is rewritten in terms of the abscissas and weights and hence can be described using the moments of the PSD. A more comprehensive explanation of this method can be found in the literature [Marchisio et al., 2003a; McGraw, 1997; Marchisio et al., 2002].

\[ \frac{\partial m_k}{\partial t} \bigg|_{\text{break-death}} = \sum_{i=1}^{N} \omega_i L^{k} L^{(b)}(L_i) \]  

(3.31)

The QMOM has a number of advantages over the other methods. There are a low number of scalars involved in the computation and therefore it can be incorporated into CFD packages without requiring excessive processing power and as a result less CPU time. Comparing this to the method of classes, which may require thousands of class sizes to produce an accurate solution, the savings are considerable. The amount of memory necessary to store the PSD details at each timestep using the method of classes is also massive. The method of moments requires less CPU time than the method of classes but it is only suitable for situations involving size-independent growth and nucleation. From the above analysis it appears the QMOM is most suitable for tracking the moments of the PSD but in the case of analysis being performed in this treatise the MOM is chosen as breakage and agglomeration have been assumed to be negligible. It should be noted that the 3-node approximation described above works well for monomodal distributions such as a Rosin-Rammler distribution but has an inferior performance with bimodal distributions [Marchisio et al., 2002].
3.5  PSD Reconstruction

The solution to the population balance has an aspect to it that is not ideal. In order to track the PSD throughout the process it needs to be discretised and written in terms of the moments. Unfortunately, the outputs of simulations using the MOM or QMOM are the moments of the PSD and not the PSD itself. This requires a method of regenerating the PSD based on the calculated moment values. This section will discuss a new method recently developed by [Hutton (2009)] and refined by this author. The refinement to the technique consists of an alternative calculation of the intersection of moment isolines. This technique can be used to reconstruct any 2-parameter distribution such as the Rosin-Rammler, Gaussian or Log-Normal distributions.

3.5.1  Number and Volume Based Moments

In the current work the best fit distribution for the system is that of the volume based Rosin-Rammler distribution as defined in equation \( P_v(L) = \frac{n}{\delta n} L^{n-1} \exp \left( -\left( \frac{L}{\delta} \right)^n \right) \) \( (3.32) \)

where \( L, \delta \) and \( n \) are the length coordinate and two distribution defining parameters respectively. The volume based PSD is the volume fraction distribution of the particles whose size is in the range between \( L \) and \( L + \Delta L \). The Rosin-Rammler distribution was chosen to represent the L-glutamic system based on a comparison with the experimental distributions produced by [Scholl et al. (2006a)]. The distributions produced were skewed to toward the larger particle sizes which would not be accurately represented by a Normal distribution. According to [Randolph and Larson (1988)], the Rosin-Rammler distribution is suitable to fitting experimental data to unimodal distributions which are skewed toward larger-sized particles. The Rosin-Rammler distribution was chosen as the basis for the current examination of PSDs based on this knowledge.

Unfortunately, the PSDs that are to be analysed in the simulations are number based and cannot be directly described by a functional form such as the volume based Rosin-Rammler distribution. In order to define the number based distribution accurately the volume based distribution must be calculated first and then transformed into a number based PSD. The conversion from a volume to a number based PSD is achieved using
the following equation.

\[ P_L(L) = \frac{P_V(L)}{\int_0^\infty P_V(L) dL} \] \quad (3.33)

where \( P_V(L) \), \( P_L(L) \) and \( L \) are the volume and number based distributions and the length coordinate respectively.

### 3.5.2 Moment Surface Generation

This method involves the generation of 3-space number based moment surfaces. The limits of these surfaces are created to encompass the values that could be encountered in the modelled system. If the range of values is not sufficiently broad then the technique may not provide an accurate solution. This will become clearer as the technique is discussed further but for this example the range of values of \( n \) and \( \delta \) are 2-8 and 80-300 respectively.

To create the moment surfaces the moments of the distribution are calculated by combining the values of \( L \), \( n \) and \( \delta \) as follows:

\[ m_{V,k} = \int_0^\infty L^{n+k-1} \frac{n}{\delta^n} \exp \left( - \left( \frac{L}{\delta} \right)^n \right) \] \quad (3.34)

\[ m_{L,k} = \int_0^\infty L^k P(L) dL \] \quad (3.35)

where \( m_{V,k} \) is the volume based moment surface and \( m_{L,k} \) is the number based moment surface.

From equation (3.35) the number based moment surfaces may be generated where the \( k^{th} \) moment is plotted as a function of \( n \) and \( \delta \). The 0th moment surface created using this technique is not shown as it is a flat surface spanning the \( m_0 = 1 \) plane.
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Figure 3.2: 1st number Based Moment Surface

Figure 3.3: 2nd number Based Moment Surface
As can be seen in Figures 3.2 - 3.4 each moment surface is unique and this aspect plays a vital part in the resolution of the PSD. The moment surfaces represents the total combined length, surface area and volume of the particles.

### 3.5.3 Moment Isolines

For a given MOM simulation the outputs are represented as moments of the number based size distribution. In order for the PSD reconstruction technique to work the moments must first be normalised with respect to the zeroth moment. This will result in the PSD’s corresponding PDF and is achieved as follows:

\[
m_{k,n} = \frac{m_k}{m_0}
\]  

(3.36)

where \( m_{k,n} \) is the \( k^{th} \) normalised moment.

To extract the PSD the simulated moments must be compared to the moment surfaces. For a given simulated moment, this value is compared to the moment surface. The points where the simulated moments are equal to the moment surfaces trace a line. The line
has been termed the moment iso-line (Hutton, 2009). For each output moment, there exists one moment iso-line. The intersection of the iso-lines on a plot of $n$ and $\delta$ reveals the $n$ and $\delta$ values that result in the best fit PDF for the pre-assumed distribution, in this case the Rosin-Rammler distribution.

To illustrate the method used to reconstruct a PSD using the iso-lines technique, a generalised PSD is chosen with an $n$ value of 4 and $\delta$ value of 95. The first step involves converting the volume based distribution, $P_V(L)$, to a number based distribution $P_L(L)$ using equation 3.33 as shown in Figure 3.5 and 3.6. The moments of the number based distribution are calculated using equation 3.8 and displayed in Table 3.1.

![Volume Based Distribution](image)

**Figure 3.5: Volume Based Distribution**
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![Length Based Distribution](image)

**Figure 3.6: number Based Distribution**

<table>
<thead>
<tr>
<th>Moment Number</th>
<th>PDF Moment Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>$4.6713 \times 10^{-5}$</td>
</tr>
<tr>
<td>2</td>
<td>$3.0681 \times 10^{-9}$</td>
</tr>
<tr>
<td>3</td>
<td>$2.3785 \times 10^{-13}$</td>
</tr>
<tr>
<td>4</td>
<td>$2.0481 \times 10^{-17}$</td>
</tr>
<tr>
<td>5</td>
<td>$1.9024 \times 10^{-21}$</td>
</tr>
</tbody>
</table>

**Table 3.1: Moments of the size distribution**

Using the moment data extracted from the above experiment, the iso-lines were calculated and plotted as per Figure 3.7.

As can be seen, the iso-lines for the first three moments intersect at a point very close to the same location. The reason why the intersection point is not exact is due to the discrete nature the moment surfaces. If the equations for the surfaces where known then an exact intersection point could be located.
The problem of calculating a more precise location for the intersection of the three iso-
lines is as follows. The point where each iso-line intersects one of the other iso-lines is computed. This will produce three points which represent the vertices of a triangle (Figure 3.8). The centroid of this triangle can be easily calculated and this produces the \( n \) and \( \delta \) values that represent the reconstructed PDF. This is a very simple and robust method of finding a more precise location of the intersection of the iso-lines. The validity of this method is illustrated below. An alternative method of calculating this intersection point is discussed elsewhere (Hutton, 2009). The values of \( n \) and \( \delta \) that result from the intersections of the isolines as well as the centroid are given in Table 3.2.

<table>
<thead>
<tr>
<th>Intersection Point</th>
<th>( n )</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>4.0241</td>
<td>94.0562</td>
</tr>
<tr>
<td>1-3</td>
<td>4.0120</td>
<td>94.6185</td>
</tr>
<tr>
<td>2-3</td>
<td>4.0241</td>
<td>94.2972</td>
</tr>
<tr>
<td>Centroid</td>
<td>4.0201</td>
<td>94.3240</td>
</tr>
</tbody>
</table>

Table 3.2: Values of \( n \) and \( \delta \) obtained from iso-line intersections

Using these values of \( n \) and \( \delta \), the first four moments of the reconstructed distribution are calculated. The % error between each of the moments are then calculated in order to verify the accuracy of the iso-line intersection method and the validity of choosing the \( n \) and \( \delta \) values at the centroid to represent the reconstructed PSD.

The moment error between the first and second intersection points are calculated using the following equation

\[
\Delta m_{k,1-2,1-3} = 100 \frac{m_{k,1-2} - m_{k,1-3}}{m_{k,1-2}}
\]  

(3.37)

where \( m_{k,1-2} \) and \( m_{k,1-3} \) represents the \( k_{th} \) moment calculated from the intersection of the the 1st and 2nd isolines and the 1st and 3rd isolines respectively. The same procedure is used to determine the moment errors between the remaining points. The errors for the moments are presented in Table 3.3.

As can be seen from Table 3.3 the difference between the moment values produced by the different intersections and the centroid are minimal. The largest of the errors has an absolute value of 0.89%. The above data suggests that the method used to determine
Intersection Point  |  \( m_1 \) % difference  |  \( m_2 \) % difference  |  \( m_3 \) % difference  \\
--- | --- | --- | ---  \\
1-2 1-3  |  0.02  |  -0.37  |  -0.89  \\
1-2 2-3  |  -0.25  |  -0.51  |  -0.77  \\
2-3 1-3  |  -0.27  |  -0.14  |  -0.12  \\
1-2 Centroid  |  -0.08  |  -0.30  |  -0.55  \\
1-3 Centroid  |  -0.10  |  -0.07  |  -0.33  \\
2-3 Centroid  |  -0.17  |  -0.22  |  -0.21  \\

Table 3.3: % difference between moments from intersection points and centroid

The amount of error between the original moments and the moments determined from the \( n \) and \( \delta \) values and the subsequent moment values provides accurate results.

Using the new \( n \) and \( \delta \) values in the reconstruction of the PDF produces the plot shown in Figure 3.9. As can been seen, this PDF has been reproduced to a high degree of accuracy.

**Initial and Reconstructed Volume Based Size Distributions**

![Initial and Reconstructed Volume Based Size Distributions](image)

Figure 3.9: Original and Reconstructed Volume Based Distributions

The amount of error between the original moments and the moments determined from
the reconstructed PSD are shown in Table 3.4.

<table>
<thead>
<tr>
<th>Moment Number</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Error</td>
<td>0.00</td>
<td>-0.31</td>
<td>0.05</td>
<td>0.62</td>
<td>1.30</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Table 3.4: % error within moments of reconstructed PSD

From the above table it can be seen that the method used to reconstruct a PSD from simulated moments provides accurate and reproducible results.

3.6 Summary

The techniques used to analyse a crystallisation system has been discussed. The population balance approach, used to account for both the size and number of particles in a crystalliser, can be solved using a number of different methods. The method of classes (MOC) is the most straightforward but requires excessive computational effort if a high degree of accuracy is necessary. Alternative methods are the method of moments (MOM), which uses moment values to track the aspects of the PSD in question, and the quadrature method of moments (QMOM) which employs a quadrature method that accounts for both the aggregation and breakage that may occur in a system. The MOM was chosen as the more applicable of these two methods for the study being undertaken in this work due to the assumption of negligible aggregation and breakage within the L-glutamic acid system and the savings in computation requirements achieved as a result. The MOM is also more accurate than the QMOM as it does not require an approximation to represent the PSD whereas the QMOM utilises a quadrature approximation. A technique used to reconstruct the PSD from the moment values has been described and is used throughout the remainder of the dissertation in the analysis of the PSDs encountered in the L-glutamic acid system. The methods employed in this technique have been refined in this treatise in order to ensure more stability in the reconstruction of the PSDs.
Chapter 4

Modelling Methods

The following chapter considers the equations that will describe the polymorphism of L-glutamic acid within the numerical model developed by the author. This will include the kinetic equations such as those of the growth, nucleation and dissolution processes as well as the moment equations used to track the PSD. These equations have been previously fitted to the L-glutamic acid system by Scholl et al. (2006a) under the conditions present in this work. The methods used to estimate the kinetic coefficients are discussed. Following this, a novel method of accounting for the disappearance of the \( \alpha \) polymorphs due to the dissolution process is described.

4.1 Nucleation Kinetics

The kinetic equations for the nucleation of crystals are generally of the form

\[
J = A \exp \left[ -\frac{16\pi \sigma^3 \mu^2}{3kT^3(\ln S)^2} \right]
\]  

(4.1)

as published in the literature (Myerson 2002; Kitamura 1989). This format is the basis for the kinetic equations used in this work.
Scholl et al. (2006b) developed a general method to calculate nucleation kinetics for the L-glutamic acid system using the work of a number of authors as described below. A growth rate correlation developed by Kitamura and Ishizu (2000) was used as a starting point for nucleation kinetics.

\[ G = 1.6 \times 10^{-7} (S_\alpha - 1)^{5/6} \exp\left(\frac{-0.4}{S_\alpha - 1}\right) \]  

(4.2)

This growth rate correlation has been validated by seeded crystallization experiments performed by Scholl et al. (2006b) at low supersaturation \((S_\alpha \leq 4)\).

Equation 4.2 is based on the birth and spread model which has been described in more detail in section 2.3. The growth rate of the L-glutamic crystal was calculated using a single crystal method and involved preparing seed crystals of the \(\alpha\) and \(\beta\) polymorphs and measuring the relationship between crystal size as time elapsed. The results were fitted to equation 4.2.

A general correlation relating the induction time of \(t_i\) of the new phase formation with the nucleation and growth mechanisms was proposed by Kashchiev et al. (1991).

\[ t_i = \left(\frac{3\alpha_v}{\pi JG^3}\right)^{1/4} \]  

(4.3)

where \(\alpha_v\) is the minimum volume fraction of the newly formed solid phase detectable with the measurement device and \(J\) and \(G\) are the nucleation and growth rates respectively. The induction time, \(t_i\), is defined as the time that elapses between the instant the supersaturation is generated and the point at which solid phase particles become detectable (Kim and Mersmann, 2001). Rearranging the above equation provides a method of deducing the nucleation rate of the system.

Scholl et al. (2006b) used equation 4.3 with a growth rate calculated using the Kitamura and Ishizu (2000) and a value of \(\alpha_v\) estimated to be \(10^{-4}\) using a Lasentec Focused Beam Reflectance Measurement (FBRM) probe. The resulting nucleation correlation takes the form
where $A$ and $C$ are kinetic parameters determined using equations 4.2 and 4.3 above.

For the estimation of the kinetics used in this treatise, equation 4.4 was used as a basis and the parameters were calculated using experimental data as described by Scholl et al. (2006a) during unseeded transformation experiments. The experimental data utilised included the evolution of the solute concentration recorded using attenuated total reflection Fourier transform infrared (ATR-FTIR) spectroscopy, the relative polymorphic content tracked by Raman spectroscopy and the average and final particle sizes were measured using particle video microscopy (PVM) and focused beam reflectance measurement (FBRM). The best fit parameters were calculated using the information ascertained from this experiments. The resulting equations describing the primary heterogeneous nucleation of the $\alpha$ polymorph and the primary heterogeneous and surface nucleation of the $\beta$ polymorph are as follows:

\[ J_\alpha = k_{n\alpha} S_{\alpha}^{7/3} \exp \left( -\frac{K_{n\alpha}}{\ln^2 S_{\alpha}} \right) \] 

\[ J_\beta = k_{n\beta} S_{\beta}^{7/3} \exp \left( -\frac{K_{n\beta}}{\ln^2 S_{\beta}} \right) + k_{s\beta} m_2^\beta \exp \left( -\frac{K_{s\beta}}{\ln S_{\beta}} \right) \]

For the $\beta$ kinetics an expression for the surface nucleation is included alongside the primary heterogeneous nucleation in equation 4.6

In order to populate the system with particles as a consequence of the nucleation mechanism, a critical radius needs to be specified which can be used as the length of new nuclei. Multiplying the nucleation rate by this critical radius will produce a value representing the length of new particles entering the system. This also corresponds to the an increase in the 1st moment of the system due to nucleation. The change in the other moments are also calculated accordingly.

\[ \frac{dm_k}{dt} = J r_c^k, \quad k = 1, 2, 3 \]
\[ r_c = \frac{2V_m \sigma}{RT \ln(1 + S)} \]  \hspace{1cm} (4.8)

as previously discussed in section 2.2.1. Values for the interfacial tension were extracted from Mullin (2001) and a specific volume of L-glutamic acid taken to be \( 6.49 \times 10^{-4} \, \text{m}^3 \text{kg}^{-1} \) \( \text{(Lindenberg et al., 2008).} \)

### 4.2 Growth Kinetics

A study produced by Tai et al. (1992) examined a modified two-step model in an attempt to develop kinetic equations that will accurately describe the growth of \( \alpha \) L-glutamic acid. It was found that the Burton Cabrera Frank (BCF) model was able to describe the surface integration step more accurately than the standard two-step model, however the study was not extended to \( \beta \) L-glutamic acid. A subsequent analysis by Kitamura and Ishizu (2000) examined the growth of \( \alpha \) and \( \beta \) L-glutamic acid. In this paper it was discovered that the growth is mainly controlled by surface reaction process and the diffusion of the solute through the boundary layer is negligible. Following on from this, two mechanisms of surface reaction were considered, growth originating from a spiral defect (BCF model) or through two-dimensional nucleation (birth and spread model). The two models can be expressed using the equations below with equation 4.9 representing the BCF model and equation 4.10 representing the birth and spread model.

\[ G = E(S^2/F)\tanh(F/S) \]  \hspace{1cm} (4.9)

where \( E \) and \( F \) are constants that depend on the system and

\[ G = XS^{5/6} \exp(-Y/S) \]  \hspace{1cm} (4.10)

where \( X \) and \( Y \) are constants that change with the system.

The two sets of equations were fit to experimental data by Kitamura and Ishizu (2000) in order to find the constants and this resulted in negative values for kinetic coefficients within the BCF model. However, the parameters produced for the birth and spread model gave strong evidence that the growth of L-glutamic acid adheres to this model and not the BCF model. Subsequent studies have confirmed these findings \( \text{(Scholl et al., 2006b, 2007)} \) and the birth and spread model will be used in the current analysis of the
system as a result.

Scholl et al. (2006a) derived a set of kinetic equations from a series of unseeded transformation experiments performed at various initial concentrations as described in section 4.1. The data produced from the experiments were fit to equation 4.10 and the kinetic parameters estimated. The size independent growth of the \( \alpha \) polymorph is represented by the following equation:

\[
G_\alpha = k_{g\alpha} (S_\alpha - 1)^{5/6} \exp\left(\frac{-K_{g\alpha}}{S_\alpha - 1}\right) \quad (4.11)
\]

Similarly, the size independent growth of \( \beta \) is expressed as follows

\[
G_\beta = k_{g\beta} (S_\beta - 1)^{5/6} \exp\left(\frac{-K_{g\beta}}{S_\beta - 1}\right) \quad (4.12)
\]

The set of parameters producing the best fit of equations 4.5, 4.6, 4.11 and 4.12 are presented in Table 4.1. It should be noted that within this set of equations the potential agglomeration and breakage that may occur in the system are not accounted for as the experiments performed by Scholl et al. (2006a) were conducted under conditions that produced negligible levels of both crystallisation processes.

### 4.3 Dissolution

The dissolution process in the L-glutamic system is assumed to be mass transfer limited as per the studies carried out by Hintz and Johnson (1989) and Lu et al. (1993) as they simulated the dissolution of polydisperse powders. The mass transfer coefficient \( k_{d\alpha} \) is predicted using the Sherwood correlation (Mersmann, 1994).

\[
k_{d\alpha} = \frac{D}{L} \left( 2 + 0.8 \left( \frac{\tau L^4}{\nu^3} \right)^{1/5} Sc^{4/3} \right) \quad (4.13)
\]

where \( D \) is the diffusivity, \( L \) is the crystal length, \( \nu \) is the kinematic viscosity and \( Sc \) is the Schmidt number, \( Sc = \nu/D \). The average power input \( \tau \) is calculated using equation 4.14

\[
\tau = \frac{P_o N^3 D_o^5}{V} \quad (4.14)
\]
Table 4.1: Kinetic Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{nc \alpha}$</td>
<td>$8.0 \times 10^5$</td>
</tr>
<tr>
<td>$K_{nc \alpha}$</td>
<td>$1.0 \times 10^{-1}$</td>
</tr>
<tr>
<td>$k_{ga \alpha}$</td>
<td>$2.5 \times 10^{-7}$</td>
</tr>
<tr>
<td>$K_{ga \alpha}$</td>
<td>$9.0 \times 10^{-2}$</td>
</tr>
<tr>
<td>$k_{va}$</td>
<td>$5.2 \times 10^{-1}$</td>
</tr>
<tr>
<td>$k_{n \beta}$</td>
<td>$5.4 \times 10^4$</td>
</tr>
<tr>
<td>$K_{n \beta}$</td>
<td>$1.5 \times 10^1$</td>
</tr>
<tr>
<td>$k_{s \beta}$</td>
<td>$6.0 \times 10^4$</td>
</tr>
<tr>
<td>$K_{s \beta}$</td>
<td>$1.0 \times 10^{-3}$</td>
</tr>
<tr>
<td>$k_{g \beta}$</td>
<td>$6.5 \times 10^{-8}$</td>
</tr>
<tr>
<td>$K_{g \beta}$</td>
<td>$1.6 \times 10^{-1}$</td>
</tr>
<tr>
<td>$k_{v \beta}$</td>
<td>$1.0 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

where $P_o$, $D_a$, $N$ and $V$ are the power number, impeller diameter, stirrer speed and volume of the crystalliser respectively.

Experiments performed by Scholl et al. (2006a) on the L-glutamic polymorphic system led to the assumption of a mass transfer limited dissolution mechanism with a dissolution rate based on a mass transfer coefficient predicted using the Sherwood correlation described above and in section 4.3.

\[
D_\alpha = k_{da}(1 - S_\alpha) \tag{4.15}
\]

Equation 4.15 was incorporated into the numerical model developed by this author in order to monitor the dissolution of the $\alpha$ polymorph during the polymorphic transformation.
4.4 Mass Balance

The growth of crystals has the effect of reducing the supersaturation of the solution. This is due to the addition of dissolved solute molecules to the surface of the crystals. The rate of this consumption is a function of solution supersaturation, the total particle count and the size distribution. Consider a particle of diameter $L$ growing at rate $G$ with a volume shape factor $k_v$. The volume shape factor relates the actual volume of the particle to the length of one side of a cube of equivalent volume. $V = k_v L^3$ where $V$ is the volume and $L$ is the length of one side of the cube.

The rate of the change of the volume of a single particle due to the growth is given by

$$\frac{dV}{dt} = \frac{dL}{dt} \frac{dV}{dL} = G$$

$$\frac{dV}{dL} = \frac{d}{dL} k_v L^3$$

$$\frac{dV}{dt} = 3k_v L^2 G$$

$$\frac{dm_L}{dt} = 3 \rho k_v L^2 G$$

The overall rate of consumption of solute by all particles is given by

$$\frac{dM}{dt} = \int_0^\infty n(L) \frac{dm_L}{dt} dL$$

$$\frac{dM}{dt} = 3 \rho k_v G \int_0^\infty n(L) L^2 dL$$

$$\frac{dM}{dt} = 3 \rho k_v G m_2$$

The rate of consumption of solute by the crystal phase due to growth is equal to this value due to the conservation of mass. If the system is not undersaturated with respect
to the α polymorph \((S_\alpha \geq 1)\) then the following equation can be used to describe the consumption of solute due to the growth of the crystals

\[
\frac{dc}{dt} = -3\rho (k_{\alpha\alpha}G_\alpha m_{2\alpha} + k_{\alpha\beta}G_\beta m_{2\beta})
\]

(4.24)

Upon initiation of the dissolution process \((S_\alpha < 1)\), when the system becomes undersaturated with respect to the α polymorph, a dissolution term is added to equation 4.24 in order to account for the concurrent dissolution of the α polymorph.

\[
\frac{dc}{dt} = -3\rho (k_{\alpha\alpha}G_\alpha m_{2\alpha} + k_{\alpha\beta}G_\beta m_{2\beta} - k_{\alpha\alpha}D_\alpha m_{2\alpha})
\]

(4.25)

This expanded equation describes the depletion of solute due to the growth of the α and β polymorphs and the dissolution of the α polymorph. Equations 4.24 and 4.25 are incorporated into the numerical model developed in this treatise to simulate the polymorphism of L-glutamic acid.

### 4.5 Dissolution of α Polymorph

Throughout the polymorphic transformation of L-glutamic acid two mechanisms exist concurrently, the growth of the β polymorph and the dissolution of the α polymorph. This process is explained in detail in section 2.5.2. The effect of dissolution on a particle is that of a decrease in size. This size decrease may be related to the surface dissolution velocity as follows

\[
D = -\frac{dL}{dt}
\]

(4.26)

where \(L\) and \(t\) are the particle length and time respectively. The evolution of the PSD can be described by

\[
\frac{dn(L)}{dt} = \frac{dL}{dt} \times \frac{dn(L)}{dL}
\]

(4.27)

\[
\frac{dn(L)}{dt} = -D \frac{dn(L)}{dL}
\]

(4.28)

The discrete form of this equation provides a more intuitive assessment of the effect of dissolution on the PSD. If the crystallisation system is examined under dissolution conditions it can be seen that the number of particles of diameter \(i\) at time \(t\) is equal to
Chapter 4. Modelling Methods

Figure 4.1: PSD shift to left due to dissolution

The number of particles that have dissolved to size $i$ from a larger size less the number of particles which have dissolved to be smaller than $i$.

$$n(i, t) = n(i - D\tau, t - \tau) - n(t - \tau) \quad (4.29)$$

where $n$ is the number density function (NDF), $D$ is the dissolution rate, $\tau$ is the time increment in seconds and $t$ is the actual time in seconds. This equation describes the shift of the PSD toward the smaller particle size at a velocity of $D \, ms^{-1}$.

The example in Figure 4.1 is purely illustrative and describes a system of 1,000,000 particles having progressed by $D\tau$ metres in $\tau$ seconds. This approach was used to simulate the dissolution of the particles resulting in a shift to the left and a subsequent removal of particles from the system as they entered the smallest class size. The approach taken to achieve this requires the use of the method of moments and a reconstruction of the PSD within the simulation during the dissolution process.
Chapter 4. Modelling Methods

The basis of the model developed in this work is the use of the method of moments to track the evolution of the PSD over time. The general form of the equation used to calculate the zeroth moment change at each timestep is as follows:

$$\frac{dm_0}{dt} = B_0$$

(4.30)

where $B_0$ is the nucleation rate and $dm_0$ represents the total number of particles in the system.

This equation proves satisfactory during the supersaturated phase of the system but a problem arises if the system is undersaturated with respect to the $\alpha$ polymorph. This initiates the dissolution process but the zeroth moment doesn’t decrease due to the lack of a death term to describe the removal of particles from the system. Unless a death term is included in the process, the number of particles remains stationary within the simulation. This will be the case even as the other moment values decrease due to the presence of a dissolution rate term during the change in moments calculation. The general solution for the change in moments during the dissolution process was found to be (Hanchen et al., 2007):

$$\frac{dm_k}{dt} = -kD\mu_{k-1}; \; k = 1, 2, 3$$

(4.31)

where $D$ is the dissolution rate. This equation is analogous to equation 3.11 with the dissolution rate having replaced the growth rate but does not account for the reduction of the zeroth moment.

The model being developed in the present work requires that the particles disappear as they reach the smallest class size described by the PSD. This is achieved through a discretisation of the PSD and a subsequent summation of the particles that exist in the smallest class size which is representative of the critical cluster size given by equation 4.8.

This also accounts for the number of particles removed from the system as they disappear completely. As a result of this process, the zeroth moment will reduce accordingly during the dissolution of the $\alpha$ particle. The remaining moments being tracked within the simulation will also decrease in magnitude using equation 4.31. The procedure for the calculation of the death term due to dissolution is given as follows:
• The concentration of the solution is calculated and compared to the solubilities of the polymorph at a given temperature.

• If the $\alpha$ polymorph is supersaturated, the growth and nucleation rates are calculated using the kinetic equations described previously.

• If the system is undersaturated with respect to the $\alpha$ polymorph, the PSD is reconstructed using the comparison between the moment isolines and the moment surfaces (section 3.5).

• The resulting PSD is subsequently discretised into a range of size classes (Figure 4.2).

• Using the discretised PSD, the number of particles in the smallest class are removed from the total number of particles within the system and this represents the death term within the system due to the dissolution process.

![Discretisation of PSD](image)

Figure 4.2: Discretisation of PSD
4.6 Summary

The kinetic equations for the nucleation growth and dissolution have been described. The methods used to derive the equations have been explained and their overall interaction within the system illustrated. A expression used to quantify the reduction of the solute within the system had been developed to account for the processes that are encountered (equation 4.25). The dissolution method presented above is used throughout the simulation process and in conjunction with the kinetic equations, method of moments and PSD reconstruction technique, a robust and accurate simulation of the processes within the L-glutamic acid system can be achieved. A number of simulations are performed using this method and the results are compared to experiments found in the literature. The details of the simulations undertaken and the analysis of the results are described in the next chapter.
In the preceding chapters, the theory and methods used to model the population balance have been presented. Prior to applying these methods to the L-glutamic acid system within this treatise, the validity of the experimental and numerical methods have to be evaluated. In this chapter a series of experiments are performed by the author to validate the experimental method employed in the literature. A number of simulations are also performed using the numerical model and the results compared to published results (Scholl et al., 2006a). Following validation of the numerical method, further simulations are carried out and the model fully tested. The performance of the model is subsequently assessed.

5.1 Solution Procedure

The numerical model developed in this treatise incorporates the kinetic equations described in the previous chapters as well as method of accounting for the dissolution of the α particles during the transformation process. The PSD reconstruction technique developed in Chapter 3.6 is also implemented within the model. The moment changes over time are also tracked and used in the solution process. A diagram outlining the
process used to simulate the polymorphic transformation is shown in Figure 5.1.

The simulation process is outlined as follows:

- The simulation begins by inputting initial moment data representing the form of a typical distribution and this provides the details on the form of the PSD.

- The initial run inputs the rates that are used at time $t = 0$ and this includes calculating the solubilities and the supersaturations of the $\alpha$ and $\beta$ polymorphs. Values for constants that are used throughout the simulation including shape factors, densities and kinetic coefficients are also input (Table 4.1).

- For the remaining timesteps, the growth, nucleation and dissolution rates of the polymorphs are calculated. These values are used to work out the change in the moments over time using equations 3.12 to 3.15.

- The new values of the moments are used in the calculation of the solution concentration and also as the initial values of the moments for the next timestep.

- In the event of dissolution, the change in the zeroth moment of the $\alpha$ polymorph is calculated as discussed in section 4.5.

- Moment values from the simulation are used in the reconstruction of the PSD which involves the comparison of the moment surfaces with the simulated moment values as discussed previously in section 3.5.
Figure 5.1: Numerical Model Flow Chart
5.2 Experimental Verification

A number of experiments were performed by Scholl et al. (2006a) to produce the kinetic equations described in Chapter 4. These experiments have also been performed by this author to determine results that have not been produced previously in the literature in order to fully validate the numerical model. These results include the final crystal yield produced after the complete $\alpha$ to $\beta$ transformation and also the volume based PSD recorded during the initial and final phases of the experiments. The kinetic equations derived by Scholl et al. (2006a) are based on experimental data produced under the conditions described in section 5.2.1. An agreement between the yield produced by the experiments in this treatise and those performed by Scholl et al. (2006a) would lead to the conclusion that the experimental procedure employed is repeatable. This would also imply that the kinetic equations developed in the literature are transferable to the experimental system under investigation in this treatise.

5.2.1 Experimental Procedure

A LabMax reactor system from Mettler-Toledo was utilised in this work to perform the experiments in this work. The reactor is a 1L borosilicate glass jacketed reactor, allowing controlled heating and cooling of solutions. The system is equipped with an overhead motor and a PTFE or glass stirrer, with four blades at a pitch of 45°C, for agitation of solutions.

A Horiba Particle Size Analyser LA-920 was used to determine the PSD of the samples that are removed from the LabMax at specific times throughout the experiments. The Horiba PSA employs Mie scattering theory to measure the radius of particles. A laser is shone at the particles being circulated through the flow cell and the light is scattered from the edge of the particles that pass through the laser. The angle at which the scattering takes place is dependent on the size of the particles. The intensity of the light scattered at different angles is used to determine the relative amounts of different size particles.
For the unseeded experiments, the reactor was prepared using the initial L-glutamic acid solute concentrations, $c_0$ shown in Table 5.1 and applying a constant stirring rate of 300rpm. The vessel was heated at a rate of 2 K/min until a temperature of 80°C was achieved and the solute had fully dissolved. The complete dissolution of the solute was verified using a turbidity probe. The solution was cooled at 1.5 K/min to a temperature of 45°C and maintained at this level throughout the process. The solution remained at these conditions for 24 hours to ensure that the polymorphic transformation of $\alpha$ to $\beta$ was complete. The presence of the $\beta$ particles was confirmed using optical microscopy on a series of samples (Figure 5.4).

<table>
<thead>
<tr>
<th>$c_0$ [g/kg of solvent]</th>
<th>$S_\alpha$ [-]</th>
<th>$S_\beta$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>2.0</td>
<td>2.5</td>
</tr>
<tr>
<td>48</td>
<td>2.2</td>
<td>2.8</td>
</tr>
<tr>
<td>53</td>
<td>2.4</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Table 5.1: Initial Concentrations and Corresponding Supersaturations

The solubilities used to calculate supersaturation levels in the simulations were taken from the experiments published in the literature (Scholl et al., 2006a) (Figure 5.3). The experiments used ATR-FTIR spectroscopy to calculate the solubilities of the two polymorphs that exist within the L-glutamic system. Crystals of each polymorph were added in excess to an undersaturated solution and the liquid phase concentration was monitored until no further change in the concentration could be observed. This procedure was carried out at number of temperatures in the 20º to 60ºC range. Using this
information, the solubilities of the $\alpha$ and $\beta$ polymorphs at 45°C were calculated to be 21.7 and 17g/kg of solvent, respectively.

Figure 5.3: Solubility of the $\alpha$ and $\beta$ polymorphs as a function of temperature

Figure 5.4: Optical micrograph of $\beta$ L-glutamic acid
5.2.2 Yield Determination

In order to verify the accuracy of the published kinetic equations and the results that they produced, a series of experiments were carried out by this author using the conditions outlined previously. The final experimental crystal yield was calculated and these values were compared to the published experimental results (Scholl et al., 2006a).

The method used to calculate the crystal yield is described below:

- 10mL samples were taken from a port in the top of the vessel using a plastic syringe.
- These samples were placed in pre weighed vials.
- The vials were then weighed to determine the mass of the vial plus contents.
- The samples were placed in an oven at 45°C in order to allow the liquid phase to evaporate off.
- Upon evaporation of the liquid phase, the samples were placed in an oven at 80°C to ensure the complete drying of the solid phase.
- Following the drying process, the vials containing the solid phase were weighed once more.
- The mass of the samples were calculated allowing the final yield to be determined using the following equation

\[
\text{Yield} = \frac{\text{Final Mass}}{\text{Initial Mass}} \times 100
\]  

(5.1)

- The sampling process was performed four times at each concentration (Table 5.1) to ensure repeatability and an average value calculated from these results.
5.2.3 Results and Discussion

A number of deductions can be made following the analysis of the crystal yield produced through experiments performed in this work and published experimental results. Examining both sets of results (Table 5.2), it can be seen that the 43 and 48g/kg concentration experiments produced errors of approximately 8% with the 53g/kg experiment varying by only 3%. This is also illustrated graphically in Figure 5.5. The experimental yield for the 48 and 53g/kg samples was also found to decrease. This aberration is reflected in the larger error associated with the 48g/kg sample.

The variation between the results could be attributed to the sampling procedure explained previously. During the sampling of the solution, a pipette is used to remove mixture from the bulk flow in the tank. Ideally the sample would be taken from a completely homogeneous solution in order to match the overall results produced by Scholl et al. (2006a) but this is very unlikely. Due to the mixing within the tank variations in the solution concentrations would exist throughout and it is unlikely that the sample removed would be representative of the overall solution. An effort was made to extract the sample from the bulk of the flow and the procedure repeated numerous times in order to find an average value for the yield. The error between the results was deemed to be acceptable due to the inherent issues arising from the sampling process. Using this information, the experimental procedure employed in Scholl et al. (2006a) was adjudged to be repeatable.

The kinetic equations used in the numerical model were developed based on the experiments performed by Scholl et al. (2006a) and as there was an acceptable agreement between the published experimental results and the results produced in this work, it is reasonable to assume that the results produced within the model can be compared to experiments performed under the conditions outlined in section 5.2.
Table 5.2: Comparison between experimental and published yield at three initial solute concentrations

<table>
<thead>
<tr>
<th>$c_0$ [g/kg of solvent]</th>
<th>Experimental Yield [%]</th>
<th>Scholl Yield [%]</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>64.98</td>
<td>60.47</td>
<td>7.46</td>
</tr>
<tr>
<td>48</td>
<td>69.59</td>
<td>64.58</td>
<td>7.77</td>
</tr>
<tr>
<td>53</td>
<td>68.16</td>
<td>66.04</td>
<td>3.21</td>
</tr>
</tbody>
</table>

Figure 5.5: Experimental and simulated crystal yield

5.3 Model Verification

Using the initial solute concentrations shown in Table 5.1, the unseeded experiments were modelled using the solution procedure described above. ATR-FTIR and Raman spectroscopy were used by [Scholl et al. (2006a)] to track the solute and polymorph profiles and these results were compared to the profiles produced by the numerical model developed in this treatise.

5.3.1 Solid Concentration

Examining Figure 5.7 specifically the 43g/kg initial concentration, there is a rapid initial increase in the mass of $\alpha$ in the solution due to its formation first. The $\alpha$ polymorph
nucleates before the $\beta$ polymorph due to the lower activation energy of the $\alpha$ polymorph as stated in Bernstein et al. (1999). The nucleation rates of the $\alpha$ and $\beta$ polymorphs for the first 30 minutes are shown in Figure 5.6.

![Figure 5.6: Comparison of simulated nucleation rates for the $\alpha$ and $\beta$ polymorphs at an initial solute concentration of 43g/kg](image)

As the transformation of the $\alpha$ to $\beta$ proceeds, a slow gradual increase in the mass of $\beta$ is observed followed by a rapid increase. Approximately seven and a half hours into the process the $\alpha$ polymorph has completely dissolved and only $\beta$ remains with approximately 25g of $\beta$ per kg of solution existing. The same profile is present for the other simulations with 48 and 53 g/kg as the initial solution concentration. Following the process there is approximately 30 and 35 g/kg of $\beta$ present in the solution.
The increase in final mass is a result of the increase in initial concentration levels in the system and hence greater growth and nucleation rates. The mass of the $\beta$ polymorph at the end of the transformation process is greater than the maximum mass of $\alpha$ present. This is due to the lower solubility of the $\beta$ polymorph which allows a larger mass of 

Figure 5.7: Comparison of experimental (Scholl et al., 2006a) and simulated solid concentration profiles
solute to crystallise out of solution. The overall mass produced in the system at the
three initial concentrations is shown in Figure 5.8. There is an immediate increase in
the crystal mass of the system corresponding to the nucleation and growth of the \( \alpha \)
polymorph. The solubility of the \( \alpha \) polymorph is reached initiating the flattening out of
the mass curve. At this stage the transformation from \( \alpha \) to \( \beta \) is occurring. Upon complete
dissolution of the \( \alpha \) polymorph, the mass increases again due to the crystallisation of the
\( \beta \) polymorph until the system also becomes saturated with respect to the \( \beta \) polymorph
and the driving force for crystallisation no longer exists.

![Figure 5.8: Simulated total solid mass](image)

### 5.3.2 Solution Concentration

The monitoring of the solution concentration throughout the nucleation and transforma-
tion processes is shown in Figures 5.9 - 5.11. The concentration levels generated in
the numerical model were compared to the published experimental results produced by
Scholl et al. (2006a). As displayed in Figure 5.9, the solute concentration begins at the
solution concentration of 43 g/kg as expected. An immediate drop in solute concentra-
tion occurs as the nucleation and growth of the crystals begin. After approximately 15
minutes the solution has reached the solubility of the \( \alpha \) polymorph at 21.1 g/kg and
the solution mediated transformation of \( \alpha \) to \( \beta \) begins. This involves the continued
nucleation and growth of the \( \beta \) polymorph and the subsequent dissolution of the \( \alpha \) poly-
morph. At this point the concentration drops again until the solubility of \( \beta \) is attained (17g/kg). This is the final solute concentration as there is no longer a driving force for the nucleation and growth of \( \beta \). The same patterns take place for the solution with initial concentrations of 48 and 53g/kg. It should also be noticed that the intermediate and final plateaus in Figures 5.9-5.11 occur at the solubilities of the two polymorphs.

During the transformation, the solution concentration remains at the solubility of the \( \alpha \) polymorph signifying that the concurrent nucleation and growth of the \( \beta \) polymorph is the rate-determining step and not the dissolution of \( \alpha \). If the \( \beta \) nucleated and grew faster than \( \alpha \) dissolves then a decrease in the solution concentration would be visible as the transformation progresses. There would also be an increase in the overall mass of the solution but as can be seen in Figure 5.8 the mass remains constant until the \( \alpha \) has dissolved. This occurs at approximately 7.5 hours into the overall process.

The rate of dissolution was also monitored throughout the simulations (Figures 5.9-5.11). Up to approximately 2.4 hours into the process dissolution is not observed as the solution concentration has not reached the solubility of the \( \alpha \) polymorph. Upon reaching the solubility of \( \alpha \), the dissolution stage begins. Due to the continuing growth of the \( \beta \) polymorph, solute is still being consumed by the system and this causes the solution concentration to drop below the solubility of the \( \alpha \) polymorph. This results in the \( \alpha \) polymorph becoming undersaturated with respect to the system and initiating dissolution. The dissolution produces a sudden increase in the solute concentration until a saturated solution is once more attained. At the same time the continual growth of the \( \beta \) polymorph is taking place, reducing the solute concentration further and the undersaturated stage is reached once more. This process continues until the complete dissolution of the \( \alpha \) polymorph is achieved after approximately 6.7 hours.

The solution concentration plots displayed in Figures 5.9-5.11 have been shown to agree with the experimental results obtained in the literature. This verifies that the development and application of equation 4.25 in the numerical model was correct. Equation 4.25 had been developed in this treatise to monitor the change in solute that occurs in the system due to the nucleation, growth and dissolution processes.
Figure 5.9: Experimental and simulated solute concentration profiles and dissolution profile for 43g/kg of solvent
Figure 5.10: Experimental and simulated solute concentration profiles and dissolution profile for 48g/kg of solvent
Figure 5.11: Experimental and simulated solute concentration profiles and dissolution profile for 53g/kg of solvent
5.4 Modelling of Polymorphic Transformation

The simulated solid and solution concentration plots in section 5.3 closely match the experimental results published in the literature. The ability of the model to accurately predict important parameters within the L-glutamic acid system leads to the conclusion that further investigations into the system can be performed with confidence. A number of different system parameters are investigated in the following section and the overall applicability of the model the polymorphic system assessed.

The effect of supersaturation on the growth levels of the $\alpha$ and $\beta$ polymorphs can be ascertained from the Figures 5.12 and 5.13 and Table 5.3. The higher initial growth rates can be attributed to the higher supersaturation levels associated with the respective initial solute concentrations (Figure 5.14). The rate of decrease of growth is also illustrated. The greater initial concentration levels results in an elevated supersaturation. This produces a higher growth rate but also causes a faster depletion of solute within the system. A reduction in the supersaturation results until the system achieves equilibrium at the the solubility of the $\alpha$ polymorph in Figure 5.12. This point is reached after approximately 0.25 hours into the process. The $\beta$ growth rate reduces to a constant level at this stage due to the initiation of the polymorphic transformation process but remains at this level due to equivalent supersaturation levels existing in the system. Upon complete dissolution of the $\alpha$ polymorph, the growth rate of the $\beta$ polymorph will reduce further until the system is saturated with respect to $\beta$ and no further driving force exists.

<table>
<thead>
<tr>
<th>$c_0$ (g/kg solvent)</th>
<th>$G_\alpha$ (m/s)</th>
<th>$G_\beta$ (m/s)</th>
<th>$J_\alpha$ (#/m$^3$s)</th>
<th>$J_\beta$ (#/m$^3$s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>$2.25 \times 10^{-7}$</td>
<td>$8.31 \times 10^{-8}$</td>
<td>$3.19 \times 10^6$</td>
<td>$1.66 \times 10^5$</td>
</tr>
<tr>
<td>48</td>
<td>$2.72 \times 10^{-7}$</td>
<td>$9.82 \times 10^{-8}$</td>
<td>$4.35 \times 10^6$</td>
<td>$2.03 \times 10^5$</td>
</tr>
<tr>
<td>53</td>
<td>$3.19 \times 10^{-7}$</td>
<td>$1.13 \times 10^{-7}$</td>
<td>$5.57 \times 10^6$</td>
<td>$2.40 \times 10^5$</td>
</tr>
</tbody>
</table>

Table 5.3: Maximum growth and nucleation rates at three initial solute concentrations

The accuracy of the growth rates for both polymorphs can be verified through examination of the overall process over time. Figures 5.9 - 5.11 illustrated the decrease in solution concentration throughout the process. These plots were generated using solute concentration data produced using equation 4.25 and the results agreed with published
experimental results. Equation 4.25 calculated the change in solute levels due to nucleation, growth and dissolution processes. If the nucleation, growth and dissolution rates produced using the system equations were incorrect, then the resulting change in solute levels would also be incorrect. As a consequence, the solution concentration levels throughout would not agree with the published data. As the solution concentration levels match the experimental data, the nucleation, growth and dissolution rates can be assumed to be accurate.

Figure 5.12: Simulated growth of the $\alpha$ polymorph during initial stage of process
Chapter 5. Model Verification

Figure 5.13: Simulated growth of the $\beta$ polymorph during initial stage of process

Figure 5.14: Difference between the $\alpha$ and $\beta$ supersaturation levels

The average particle size of the $\alpha$ and $\beta$ polymorphs were also tracked throughout the
process, as shown in Figure 5.15 and 5.16 respectively. The average particle diameter, $d_{43}$, is calculated using equation 5.2.

$$d_{43} = \frac{m_4}{m_3}$$  \hspace{1cm} (5.2)

According to the simulations, the size of the $\alpha$ particles increase to a maximum of $80\mu m$ and then starts to decrease due to the dissolution process. Figure 5.17 illustrates an $\alpha$ agglomerate with two particles of approximately $100\mu m$ joined together and $\beta$ crystals removed from the reactor after the complete transformation of the $\alpha$ polymorphs. The $\beta$ particles increase in size at a slower rate than $\alpha$ reflecting the higher nucleation and growth rates of the $\alpha$ polymorph as discussed in 5.3.1. The final average size of the $\beta$ polymorph is calculated to be $250\mu m$.

![Graph showing simulated average particle size of the $\alpha$ polymorph](image.png)

Figure 5.15: Simulated average particle size of the $\alpha$ polymorph
Figure 5.16: Simulated average particle size of the $\beta$ polymorph

Figure 5.17: Optical micrograph of an $\alpha$ agglomerate and $\beta$ crystals

Figure 5.18 depicts the relationship between the rate of surface nucleation of the $\beta$ polymorph and the surface area of the $\alpha$ polymorph. The increase in the initial solute concentration results in a corresponding increase in the mass of $\alpha$ particles produced due to higher growth and nucleation rates (Figure 5.7). This in turn causes an increase in surface area available for the surface nucleation the $\beta$ polymorph. As the $\alpha$ polymorph dissolves back into solution, the surface nucleation of the $\beta$ polymorph is reduced until no more nucleation sites exist.
Figure 5.18: Simulated variation of surface nucleation at different initial solute concentrations
5.5 PSD Determination

The numerical model developed in this work produces a volume based PSD. The validation of this model requires the volume based PSD to be experimentally measured for direct comparison to the simulations. In order to generate a volume based PSD, the experimental procedure described in section 5.2.1 for the unseeded experiments was followed and a number of samples were taken at various intervals throughout the process. Once the nucleation stage began after the solution had reached 45°C, samples were taken after 30 minutes and again following the completion of the polymorphic transformation process after 12 hours. The sampling procedure was as follows:

1. A sample of approximately 25ml was removed from the vessel through the port at the top of the vessel.
2. This sample was then vacuum filtered.
3. The filtrand was quenched in ethanol in order to dissolve any remaining water and to ensure no further crystallisation would take place. L-glutamic acid is completely insoluble in ethanol.
4. The samples were suspended in ethanol to reduce the possibility of agglomerates forming between the sampling and PSD analysis stages.
5. The samples were then analysed using a Horiba PSA employing ethanol as the carrier fluid and the volume based PSD was determined.
6. This procedure was performed four times for each initial solute concentration at both time intervals.

The insolubility of L-glutamic acid in ethanol was verified by monitoring the PSDs produced using the Horiba PSA. There was no shift in the PSDs produced by a sample 5 minutes after the initial analysis within the Horiba PSA (Figure 5.19). This validated the assumption that no dissolution occurred and L-glutamic acid is insoluble in ethanol.
5.5 Results

Using the method described in section 5.5, a series of experimental PSDs were produced for the α polymorph. Results were collated 30 minutes after the initial nucleation was observed, when the α polymorph dominates, and 12 hours into the process when the β polymorph remains. In this section, the experimental PSDs are compared to the PSDs produced using the PSD reconstruction technique described in section 3.6. The plots of the simulated PSDs are in the same format as those produced using the Horiba PSA to allow a direct comparison.

An examination of the PSDs taken 30 minutes into the process reveal the type of distributions associated with the α polymorph (Figure 5.20). The experimental PSDs contain peaks located at approximately 200 μm for the three initial solute concentrations. This is in direct comparison to the numerically derived PSDs which exhibit peaks at a particle diameter of approximately 90 μm. A shift of the experimental PSD toward the right can also be observed as the initial solute concentration has been increased. A much broader PSD is demonstrated by the experimental PSDs relative to the simulated PSDs. This can be attributed to the presence of agglomeration within the experimental
system. Agglomeration shifts the PSD towards the right side of the graph while also flattening the distribution. Further investigation into the experimental process revealed that agglomeration had occurred in the system as shown in Figure 5.17. The possibility of agglomeration occurring between the sampling and PSD analysis stages was minimised by suspending the sample in ethanol following filtration. Agglomerates require a supersaturated environment in order to form and the initial quenching of the sample and subsequent resuspension in ethanol would remove the remaining solute particles necessary to induce agglomeration.

Figure 5.21 displays the PSDs produced 12 hours into the process, following the completion of the polymorphic transformation. At this stage in the process only the $\beta$ polymorph remains. The peaks of the experimental PSDs are positioned at approximately $200\mu m$ for the three initial solute concentrations. A distinct shift of the PSD to the right, as noted in Figure 5.20, was not demonstrated 12 hours into the process. This could be associated with the minimal agglomeration of the $\beta$ particles that exists due to their needle-like shape. It has been shown by Subero-Couroyer et al. (2006) that the agglomeration of elongated particles is more difficult to achieve than the isotropic particles. The simulated PSDs illustrate a peak at approximately $300\mu m$ which differs from the experimental PSDs measured using the Horiba PSA.

The experimental PSD also reveals a larger population of smaller particles than the numerically derived PSD. The disparity between the experimental and simulated PSDs could be accounted for by particle breakage or attrition. There are a number of potential causes of breakage in the system. Particle breakage could be established due to fluid shear in the reactor due to the high stirring rate (Kempkes, 2009). There is also the possibility of breakage occurring due to collisions between particles and also particle-impeller and particle-wall impacts. The high fluid shear levels associated with the circulation of the particles during the particle size analysis in the Horiba PSA may also result in particle breakage. Attrition in crystallisation systems generate small fragments and can be due to fluid turbulence or collisions between particles or parts of the crystalliser (Madras and McCoy, 2007). The attrition process would reduce the overall size of particles in the system and produce a shift of the PSD toward the smaller particles sizes when compared to a system that has negligible attrition. The fragments that are generated would also increase the number of fines present.
An investigation into the particle size analysis technique revealed that the occurrence of particle breakage during the PSD measurement was negligible. After the $\beta$ sample was added into the circulation system, real time distributions were observed and no shift in the distributions existed over a period of 10 minutes. The consistent PSDs produced using the Horiba PSA using this technique lead to the conclusion that any particle breakage or attrition present was generated within the reactor environment.

The PSDs generated within the numerical model used $n$ and $\delta$ values to describe the final form of the distribution. The $n$ and $\delta$ values were calculated using a comparison between the calculated moment surfaces and the simulated moment values. The development and progression of the moment values over time was dependent on the accuracy of the kinetic equations used to describe the system. If inaccuracies were inherent in the kinetic equations used to define the system then a deviation between the predicted and experimental moment values would be expected. This deviation can be attributed the initial assumptions within the model. Previous studies on the L-glutamic acid system under the conditions specified in this treatise only account for the nucleation and growth of the polymorphs. This produced the simulated PSDs of the form shown in Figure 5.20. The presence of agglomeration and breakage or attrition within the reactor would explain the differences between the experimental and simulated PSDs. Further investigation into the experimental process reveals that agglomeration had occurred in the system as shown in Figure 5.17. The presence of agglomeration and particle fragmentation under the conditions specified in this system could not be foreseen based on the previous studies investigated in the literature and consequently, these effects within the system had not been considered in the construction of the kinetic equations in the model.
Figure 5.20: Comparison of experimental and simulated PSDs after 30mins
Figure 5.21: Comparison of experimental and simulated PSDs after 12 hours.
5.6 Discussion

A series of experiments and simulations have been examined in this chapter in order to assess the numerical model developed by this author. The experiments included the calculation of crystal yield within the system under various initial solute concentrations and a comparison with the crystal yield published in the literature. The experimental results for yield produced in this work closely matched those from the literature. This verified the repeatability and accuracy of the experimental method. It also indicated that further experiments performed using this procedure would produce results that could be relied upon for an accurate description of the system.

The numerical model was also tested and compared to published results. This was to ascertain the validity of the kinetic equations chosen to represent the growth, nucleation and dissolution that occur in the L-glutamic acid system. The solid and solution concentrations were calculated and monitored over time within the model and the results successfully matched the experimental results from Scholl et al. (2006a). The average particle sizes of both polymorphs were calculated and compared to optical micrographs of L-glutamic acid samples.

Further simulations were performed and a number of parameters within the system analysed. The growth rates of the $\alpha$ and $\beta$ polymorphs produced by the model were examined and the results generated agree with expected values of growth rate produced by Scholl et al. (2006a). The rate of nucleation of the $\beta$ polymorph on the surface of the $\alpha$ polymorph was also investigated and reasonable values of surface nucleation were obtained. If the rate of surface nucleation and the growth rates of the two polymorphs were incorrect then the resulting moments of the PSD would not be correct. This would cause the simulated solid and solution concentrations to differ from the experimentally determined values. As this is not the case, then the growth rates and surface nucleation rates illustrated in Figures 5.12, 5.13 and 5.18 can be assumed to be accurate under these conditions.

A comparison between the experimental PSDs and those generated using the numerical model (Figure 5.20) reveal a number of discrepancies however. The narrow simulated PSDs differ in size from the broad experimental distributions. An examination of L-glutamic acid samples under the microscope displayed agglomerates of the $\alpha$ polymorph.
which would result in the broad experimental PSDs. During the final stage of the process, PSDs were generated for the $\beta$ polymorph. The experimental PSDs also differed from those produced using the numerical model and this was attributed to the existence of particle breakage or attrition in the reactor. Collisions between the needle-like $\beta$ polymorph and the impeller and reactor walls could greatly reduce the average size of particle present in the tank. Fluid shear due to the turbulent flow in the reactor could also induce breakage or attrition. The presence of agglomeration and fragmentation processes in the system was unexpected and subsequent reexamination of the experimental conditions reinforced the conclusion that the extra processes were characteristic of the system.

An effort was made to account for the presence of agglomeration and breakage with the system, by attempting to introduce kernels into the numerical model. A kernel may be thought of as a rate constant that relates the agglomeration and breakage mechanisms and the process variables. The inclusion of the kernel required using the quadrature method of moments (QMOM) to solve the governing equations and also selecting the most appropriate form of kernel. An empirical agglomeration kernel was chosen as the most suitable form and is shown in equation 5.3

$$k^a = a_1 G^{a_2} \epsilon^{a_3}$$  \hspace{1cm} (5.3)

where $G$ is the growth rate, $\epsilon$ is the energy dissipation rate and $a_1$, $a_2$ and $a_3$ are fitted parameters. Equation 5.3 was successfully used to model the agglomeration of L-glutamic acid within a seeded environment \cite{Lindenberg2008}. The agglomeration kernel, $k^a$ is used in birth and death equations used to described the addition and loss of particles due to agglomeration. As two particles join together to form the agglomerate, the two particles are disappearing from the system and a new particle is being formed. This process is described by the following birth and death equations respectively \cite{Marchisio2003b}

$$B^a(L; t) = \frac{L^2}{2} \int_0^L \frac{k^a[(L^3 - \lambda^3)^{1/3}; \lambda]}{(L^3 - \lambda^3)^{2/3}} n[(L^3 - \lambda^3)^{1/3}; t] n(\lambda; t) d\lambda$$  \hspace{1cm} (5.4)

$$D^a(L; t) = n(L) \int_0^\infty k^a(L, \lambda) n(\lambda; t) d\lambda$$  \hspace{1cm} (5.5)

where $L$ and $\lambda$ are the particle diameters.
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The birth and death of particles due to breakage in the system is described using the following equations

\[ B^b(L; t) = \int_L^\infty k^b(\lambda) \gamma(L|\lambda) n(\lambda; t) d\lambda \]  \hspace{1cm} (5.6)

\[ D^b(L; t) = k^b(L) n(L; t) \]  \hspace{1cm} (5.7)

where \( L \) and \( \lambda \) are the particle diameters, \( k^b \) is the breakage kernel and \( \gamma \) is the fragment distribution function. The fragment distribution function determines the number of particles formed by a rupture of a particle. Vanni (2000); Marchisio et al. (2003b) and Hutton (2009) employed equation (5.8) in the calculation of the fragment distribution. Equation (5.8) assumes the formation of two equal fragments upon breakup.

\[ \gamma = L^k k^2 (3-k)/3 \]  \hspace{1cm} (5.8)

where \( k \) is the moment number. A size independent breakage kernel was chosen in this work to represent the rate of fragment formation as reported previously (Vanni, 2000; Hutton, 2009) and is shown in equation (5.9).

\[ k^b = k^b_0 \]  \hspace{1cm} (5.9)

where \( k^b_0 \) is a fitted constant. The solution of equations (5.4 - 5.7) follows the method described in section 3.4. While trying to calculate appropriate values for the fitted parameters \( a_1, a_2, a_3 \) and \( k^b_0 \) considerable error was encountered within the model and suitable values were unable to be ascertained. An explanation for this issue centred around the initial set of nucleation, growth and dissolution equations that were used to describe the system by Scholl et al. (2006a). When fitting the constants for the kinetic equations, the entire system was assumed to be described exclusively by nucleation, growth and dissolution. The constants were calculated using the experimental data obtained during the unseeded transformation processes and would only apply to systems under those specific conditions and assumptions. In order to obtain accurate system parameters that account for agglomeration and breakage effects as well as growth, nucleation and dissolution, the experimental data would need to be reassessed and the best fit parameters evaluated once more. Unfortunately the time and resources for such an extensive examination of the L-glutamic acid system under these new assumptions could not be achieved within the time constraints of this treatise. Future work in this...
area would need to consider potential agglomeration and particle fragmentation effects within the system.

5.7 Summary

The numerical model developed in this treatise has been presented and analysed in this chapter. A series of experiments and simulations were carried out and the results compared to experimental results published in the literature in order to verify the accuracy of the model. The comparison between the experimental yield produced in this work and the yield published in the literature infer that the experimental procedure was reproducible. The kinetic equations used to describe the crystallisation process provide an accurate description of the L-glutamic acid system up to a certain degree. The solid and solution concentrations are reproduced to a high level. The effect of initial solute concentration on the growth of the $\alpha$ polymorph and surface nucleation and growth of the $\beta$ polymorph is investigated. A description of the PSD could not be properly described using only nucleation and growth. After analysis of the experimental and simulated PSDs it is clear that agglomeration and breakage or attrition effects must be considered in a comprehensive analysis of the process. This aberration within the model has been confirmed using the microscopic pictures of the $\alpha$ agglomerates. An attempt was made to introduce agglomeration and breakage effects into the numerical model but difficulties arising from kinetic parameter estimation prevented an extensive analysis of the these two processes. As a result of this the PSD could not be reproduced accurately. Future work in the area of PSD reconstruction would need to account for the agglomeration and fragmentation processes for successful modelling of the L-glutamic acid system under the conditions specified in this treatise.
The crystallisation process has been successfully modelled in previous chapters and an insight into the polymorphic transformation of L-glutamic acid analysed. Another important aspect of crystallisation systems is the influence of flow within the stirred tank. Stirred tanks are commonly used in the chemical process industry to carry out a number of operations including homogenisation, heat transfer, chemical reactions and solid suspension. As mentioned in Chapter 1.5, areas of inhomogeneous supersaturation with the vessel will reduce the efficiency of the growth and nucleation processes due to the potential for poor mass transfer. The suspension of particles is also an issue as the particles will remain on the bottom of the tank if insufficient impeller speeds are present and the maximum amount of surface area will not be available for mass transfer processes. The speed of the impeller required to suspend all the particles is dependent on the tank geometry and the solid-liquid properties. One other point of interest that has not been widely reported in the literature is the significance of the particle shape on the performance of the suspension and the interaction of different shapes in a suspension as per the L-glutamic acid system. These characteristics of stirred tanks can be analysed in order to produce sufficient mixing levels and suspension of particles. This chapter will introduce the concepts involved in the modelling of suspensions and the influence of the shape of particles on the suspension. Subsequent chapters will focus on the modelling
of the α and β polymorphs within a stirred tank.

### 6.1 Suspension Modelling Approaches

In order to measure the performance of a stirred tank with respect to the solid suspension, it is necessary to know the various different states that are present at different impeller speeds. An increase in the impeller speed provides an increase in turbulent kinetic energy within the tank and results in an increase in suspension height. Kraume (1992) observed the following states of suspension that occur in a solid-liquid stirred tank under different impeller speeds (Figure 6.1):

- For low impeller speeds, the solids remained on the bottom of the vessel.
- As the impeller speed is increased, the solids are lifted up by the recirculating fluid.
- A further increase in the speed results in all the solids being lifted up from the bottom of the tank. At this stage, if the solids are prevented from settling for 1 or 2 seconds, it can be said that complete off-bottom suspension has been achieved. This corresponds to the impeller speed required to just suspend the particles, \( N_{js} \) (Zwietering, 1958). An interface can also be observed between the suspension and clear liquid layer at the top of the vessel.
- An increase in the impeller speed beyond \( N_{js} \) produces an increase in the height of the solid-liquid interface from the bottom of the vessel.
- A homogeneous distribution of the solids throughout the tank can be achieved at further increases in impeller speed.

![Figure 6.1: States of suspension in a stirred vessel (Kraume, 1992)](image)
6.1.1 Just-Suspended Impeller Speed

Many authors have examined the suspension of particulate solids in stirred tanks based on the work produced by Zwietering (1958) who derived an expression for the minimum impeller speed \( N_{js} \) required to just suspend the particles off the bottom of the tank. This critical speed is dependent on the geometry of tank and impeller and the properties of the solid-liquid mixture. The just-suspended speed is given as

\[
N_{js} = S \left( \frac{g(\rho_s - \rho_l)}{\rho_l} \right)^{0.45} X^{0.13} \frac{d_p^{0.2} \nu^{0.1}}{D^{0.85}}
\]

where \( S \) is the Zwietering constant, \( d_p \) is the particle diameter, \( \nu \) is the kinematic viscosity and \( D \) is the impeller diameter. \( \rho_l \) and \( \rho_s \) are the density of the liquid and the solid phases, respectively. \( X \) is the solids loading given by

\[
X = \frac{\rho_s \phi_s}{\rho_l (1 - \phi_s) + \rho_s \phi_s}
\]

where \( \phi_s \) is the solids volume fraction. Under the condition of \( N_{js} \), the entire surface of the solids are exposed to the fluid, ensuring that the maximum surface area is available for crystallisation process. Further analysis based on the theory of \( N_{js} \) has been performed by numerous authors as described in the following section.

Armenante and Nagamine (1998) derived a modified expression for \( N_{js} \) where the Zwietering constant \( S \) is expressed as function of the impeller clearance-to-vessel diameter ratio \( (C/T) \) and impeller diameter-to-tank diameter ratio \( (D/T) \) (Figure 6.2). A number of different impeller types were used in the experiments, including disc turbines, flat blade turbines and six-blade pitched blade turbines (PBT’s). The flow pattern produced by a PBT favours easier suspension than that of a radial impeller. The flow induced by a radial flow impeller sweeps particles along the base of the tank toward the central region below the impeller where it is much more difficult to suspend the particles whereas a PBT drives the particles toward the periphery of the tank where suspension can be achieved more efficiently (Figure 6.3).

This has also been the conclusion of Panneerselvam et al. (2009) who determined that the travel length of the fluid flow is lower for a PBT than a radial flow turbine. The flow field generated by the PBT begins from the tip of the impeller and is driven toward the base of the tank. The flow from the radial turbine travels radially and splits...
into two streams with one above and below the impeller. The lower stream is the only one associated with the suspension of particles and is therefore less efficient than the PBT. The radial flow turbine requires more energy to suspend the particles and hence a greater impeller speed.

The effect of the ratios C/T and D/T on the flow patterns for each impeller were examined and the values for $S$ were calculated. It was found that axial and mixed flow impellers were more efficient than radial flow impellers at suspending the particles at the various clearance heights close to the tank bottom. However, it was discovered that as C/T approached zero, $N_{js}$ may increase.

Figure 6.2: Geometric stirred tank ratios

Figure 6.3: Various impeller flow patterns (Kasat and Pandit, 2005)
The effect of impeller clearance was also examined by Sharma and Shaikh (2003). For an impeller operating close to the tank base, the maximum transfer of energy from the impeller to the particles is achieved and the particles are driven from the stagnant zone below the impeller to their suspended state. If the impeller clearance height is increased, the size of the stagnant zone is also increased and more particles are caught in the area. As a result of the higher impeller position, less energy is imparted to the particles and therefore an increase in the impeller speed is necessary to suspend the particles. A correlation was developed by Sharma and Shaikh (2003) relating $C/T$ and $D/T$ to the $N_{js}$ for four and six-blade PBT’s based on a series of experiments. During the experiments, three regions were identified that describe the effect of impeller clearance (Figure 6.4). In the first region, $N_{js}$ remained constant with $C/T$, in the second region $N_{js}$ becomes a function of $C/T$ and the third region was shown to result in a change in flow pattern produced by the PBT, from axial to radial, and instabilities were introduced into the flow. The authors recommended avoiding this region based on the studies performed.

![Figure 6.4: Effect of impeller clearance on just-suspended speed (Sharma and Shaikh, 2003)](image)

Using this information, a correlation was developed enabling the calculation of $N_{js}$ for PBT’s. Equation 6.1 can be rewritten as follows

$$N_{js} = SD^{-0.85}A$$  \hspace{1cm} (6.3)

where $S$ is the Zwietering constant represented by
\[ S = C_N \left( \frac{D}{T} \right)^{-1.15} \left( \frac{C}{T} \right)^{-0.453} \]  

(6.4)

and

\[ A = \nu^{0.1} \rho_p^{0.2} \left( \frac{g(\rho_s - \rho_l)}{\rho_l} \right)^{0.45} X^{0.13} \]  

(6.5)

The value for \( C_N \) was calculated by Sharma and Shaikh (2003) to be 2.7 for a four-blade 45° PBT operating in the second region of impeller clearance-to-vessel height ratio. This method of calculating \( N_{js} \) has been used in the present treatise to ascertain the minimum impeller speed required to just-suspend the particles in the stirred tank constructed by this author and used in the analysis performed on the suspension of the \( \alpha \) and \( \beta \) particles. The decision to operate in the second \( C/T \) region was taken due to the positive findings by Sharma and Shaikh (2003) who found that efficient suspension was achieved in this area. However, operating in the first region may be detrimental to the suspension considering that if \( C/T \) is too low, \( N_{js} \) may increase (Armenante and Nagamine, 1998). A diagram of the experimental rig used for the suspension analysis is shown in section 8.1.3.

### 6.1.2 Cloud Height

Another criteria that has been used to evaluate the degree of solids suspension in a stirred tank is the height of the homogeneous zone above the suspension. This height is known as the cloud height and has been investigated by a number of authors. An extensive experimental study was performed by Hicks et al. (1997) who looked at the influence of solid properties, impeller type and system geometry on the cloud height. It was found that the cloud height was dependent on the impeller clearance-to-vessel diameter ratio (\( C/T \)), impeller diameter-to-tank diameter ratio (\( D/T \)) and the solids loading but the impeller type and solid properties were not as influential. Bujalski et al. (1999) studied a variety of solid concentrations and particle sizes within a tank and discovered the minimum solid-liquid interface height appeared at the similar rotational speeds and the cloud height for each concentration was almost the same as the speed was increased. The results from the particle size analysis revealed the minimum interface presented itself at approximately 50-60% of the total tank height.
A further examination carried out by Bittorf and Kresta (2003) based on studies analysing three-dimensional wall jets which form along the baffles in a stirred tank. The behaviour of these jets can be described as follows:

- The jet produced by the axial impeller moves up from the the bottom of the tank, the velocity builds to a maximum, $U_{\text{core}}$.
- The velocity drops off as the jet adjusts to its final cross sectional shape.
- The velocity decreases rapidly as the height increases. This region is known as the universal decay region.

The authors developed a model based on the information obtained from the universal decay region, the just-suspended impeller speed, $N_{js}$ and geometric properties of the tank to predict the cloud height, $CH$. Equation 6.6 is applicable to stirred tanks using A310 and HE3 impellers only (Bittorf and Kresta, 2003).

$$CH = \frac{N}{N_{js}} \left[ 0.84 - 1.05 \frac{C}{T} + 0.7 \frac{(D/T)^2}{1 - (D/T)^2} \right]$$ (6.6)

The results produced by equation 6.6 to predict the cloud height were found to agree with the experimental analysis of Hicks et al. (1997). The model of Bittorf and Kresta (2003) was also used by Ochieng and Lewis (2006) in a CFD study of the cloud heights and solids off-bottom suspension. The cloud height was identified by an inflection point along a curve representing the variation of volume fraction with axial distance (Figure 6.5). The CFD predictions of the cloud height (-) compared favourably with the experimental results (○) and illustrate the ability of CFD to accurately predict the cloud height of a suspension.

In the CFD analysis of the suspensions in this treatise the cloud height is calculated and compared with previous results found in the literature using the point of inflection to indicate the sudden decrease in the solids suspension in the stirred tank.

### 6.2 Solid-Liquid Phase Interactions

#### 6.2.1 Flow of Particles

In general, the flow of particles in a tank do not follow the fluid flow exactly. This is mainly due to the particles having inertia and drag forces acting on the particles. The
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Figure 6.5: Location of point of inflection representing cloud height (Ochieng and Lewis, 2006)

The motion of a particle relative to the carrier fluid depends primarily on the Stokes number, which is the ratio of the particle response time, $\tau_p$, to the characteristic time scale of the flow, $\tau_f$, (Crowe et al., 1998):

$$St = \frac{\tau_p}{\tau_f}$$  \hspace{1cm} (6.7)

In the case of a stirred tank, the Stokes number can be defined as the ratio of the particle relaxation time to the time of one impeller revolution

$$St = \frac{\rho_s d_p^2 N}{\rho_f 18 \nu}$$  \hspace{1cm} (6.8)

where $N$ is the impeller speed in revolutions per second. If the Stokes number is much greater than unity, the particles respond slowly to fluctuations in the fluid velocity. A value of $St$ much less than unity implies that the velocity of the particles and the fluid are nearly equal. If $St \approx 1$, the particles are centrifuged radially outward.

For the L-glutamic acid system being examined in this work, an average particle size of $d_p=100\mu m$ and $N=3.5$ rev/s will produce a Stokes number of 0.003. This suggests that the particles almost completely follow the fluid flow throughout the vessel. A small slip velocity between phases also results except where strong swirling flows are encountered.
such as those found in the impeller region. This has been the case in a study performed by Reilly and Marquis (2001) who tracked the slip velocity histories of various particle sizes in a stirred tank. The author used a Lagrangian CFD model to track the flow of the particles over time moving in and out of the impeller region. For particles with small Stokes number, the slip velocities were found to be close to zero but with sharp increases in velocity as the particles passed through the impeller zone. The movement of the particles into the impeller region is identified by the vertical arrows illustrated in Figure 6.6.

![Figure 6.6: Time velocities of the slip velocity of a particle moving in the flow (Reilly and Marquis, 2001)](image)

Due to the increases in the slip velocities as the particles enter the impeller region, the drag of the particles will also be affected in this region. This has a large effect on the calculation of the solid-liquid interactions during the numerical simulation of the flow.

### 6.2.2 Drag of Nonspherical Particles

The simulation of a solid-liquid mixture within a stirred tank requires knowledge of the behaviour of the particles that are present. In a dilute mixture, the particle motion is controlled by the fluid forces, lift and drag, as opposed to a dense flow where particle collisions control the motion (Crowe et al., 1998). A study by Ljungqvist and Rasmuson (2001), using a comparable geometric setup, found that the critical factor in the solid-liquid flow within a stirred tank is the drag force and the lift force can be neglected. Following on from this, the drag force is only considered in the simulations performed within this treatise and the lift force is assumed negligible.

There are many examples of the variation of drag coefficient, $C_D$, with particle Reynolds number, $Re_p$. The Reynolds number is a ratio of fluid inertial forces to viscous forces and is defined as
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\[ Re_p = \frac{\rho_l(v_s - v_l)d_p}{\mu_l} \]  

(6.9)

where \( v_s - v_l \) is the relative velocity of the particle and \( d_p \) is the particle diameter. The density and dynamic viscosity of the working fluid are \( \rho_l \) and \( \mu_l \), respectively. An average particle size of 100\( \mu \)m for the L-glutamic acid system under investigation, with a relative velocity obtained from preliminary simulations, produces a \( Re_p \ll 1 \) which places the particle Reynolds number within the Stokes’ drag regime (Crowe et al., 1998).

Spherical particles suffice as a good approximation for the \( \alpha \) polymorph which is prismatic in shape but has the same volume shape factor as that of a sphere \( (k_v = \pi/6) \) (Lindenberg et al., 2008). In the case of nonspherical particles, less information is found in the literature. Haider and Levenspiel (1989) presented a generalised \( C_D \)-vs.-\( Re \) correlation for nonspherical particles using the concept of sphericity \( (\phi) \), originally developed by Wadell (1934), to account for the particle shape. Sphericity is defined as the ratio of the surface area of a sphere (of equal volume) to that of the original particle.

\[ \phi = \frac{d_{eq}^2}{dL + (d^2/2)} \]  

(6.10)

where \( d_{eq}^2 \) is the equivalent volume sphere diameter, \( d \) is the diameter of the particle and \( L \) is the length of the particle.

The predictions by Haider and Levenspiel (1989) showed relatively poor accuracy for particles with \( \phi < 0.67 \) and subsequent authors tried to improve the correlations. Ganser (1993) introduced two shape factors, \( K_1 \) and \( K_2 \) which are applicable in the Stokes regime \( (Re_p \ll 1) \) and the Newton regime \( (2000<Re_p<300,000) \) respectively. These shape factors are functions of the sphericity and found good agreement for the prediction of drag for systems where the product of the Reynolds number and shape factors, \( ReK_1K_2 \), are less than \( 10^5 \).

Chhabra et al. (1999) collected experimental drag results from 19 independent studies comprising several different particle shapes including needles. Using this data, the authors compared different drag correlations in order to evaluate the best predictive method available. They concluded that the numerical approach taken by Ganser (1993) was found to produce the most accurate results for a variety of shapes and orientations.
and only required values for the sphericity to be known.

An examination of the drag coefficient on slender particles was also performed by Fan et al. (2004) and involved using the orientation of the particles in conjunction with the cylindrical based Reynolds number. One of the conclusions that arose from this work was the eventual horizontal orientation of the particles in a flowing liquid irrespective of the fluid velocity. This conclusion is also presented in the work carried out by Huang et al. (1998) where inertial forces turn a long elliptical particle to a horizontal direction above a critical Reynolds number.

The form of the drag coefficient, $C_D$, that was derived during the experiments performed by Fan et al. (2004) was dependent on the density ratios of the liquid and solid phase as well as the Archimedes number. The Archimedes number describes the relative magnitude of the solid suspension forces of which the gravitational and drag force are included. For a given particle density, smaller particles will have lower values of $Ar$, which means the drag force will be more important ($Ar<100$). This is in agreement with the study completed by Ljungqvist and Rasmuson (2001). The Archimedes number is defined as

$$Ar = \frac{d_p^3}{\nu^2} \left( \frac{g (\rho_s - \rho_l)}{\rho_l} \right)$$

where $\rho_s$ and $\rho_l$ are the solid and liquid densities respectively, $\nu$ is the kinematic viscosity and $d_p$ is the particle diameter. For the L-glutamic acid particles suspended in ethanol, the value of $Ar$ was calculated to be 4.041 for particles of average diameter 100$\mu$m confirming that the drag force is the most important within the suspension being analysed in this dissertation.

A comprehensive analysis of the drag on nonspherical particles of various shapes was obtained by Loth (2008) for a range of particle Reynolds numbers in the Stokes, Newton and intermediate regimes. An alternative method to calculating the sphericity was proposed by approximating the shape of nonspherical particles using the surface and projected area ratios. Each of these areas can be normalised by the surface area of a sphere which has the same volume, i.e.

$$A_{surf}^* = \frac{A_{surf}}{\pi d^2}, \quad A_{proj}^* = \frac{A_{proj}}{\pi d^2}$$

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The surface area ratio will always greater than one since the spherical geometry has the minimum surface area for a given volume, i.e., $A_{surf}^* > 1$. The inverse of the surface area ratio is equivalent to the sphericity defined earlier. Using a cylinder in order to illustrate the method, and defining the length to diameter ratio as

$$E_{cyl} = \frac{L_{cyl}}{d_{cyl}}$$ (6.13)

the surface area ratio and the equivalent volume diameter is given as

$$A_{surf}^* = \frac{2E_{cyl} + 1}{\left(18E_{cyl}^2\right)^{1/3}}$$ (6.14)

$$d = d_{cyl}\left(\frac{3E_{cyl}}{2}\right)^{1/3}$$ (6.15)

The projected area ratio is dependent on the orientation of the particle as well as its shape. Following the previously described studies by Fan et al. (2004) and Huang et al. (1998) it was assumed that the particles fall broadside and this will result in $A_{proj}^* > 1$. A correlation for the relationship between the projected area and surface area ratio was suggested by Leith (1987) for the Stokes regime where $Re_p \ll 1$.

$$f_{shape} = \frac{1}{3} \sqrt{A_{proj}^*} + \frac{2}{3} \sqrt{A_{surf}^*} \quad \text{for } Re_p \ll 1$$ (6.16)

This equation is based on the assumption that one-third of the drag on a sphere is form drag (related to the projected area) and two-thirds is friction drag (related to the surface area) and the form and skin drags are proportional to the spherical diameter. It was found by Leith (1987) that as the objects become less spherical, the flow around the object deviates from the flow around a sphere causing the drag to be higher than that predicted by equation 6.16 but that the expression still approximated the drag to a high degree. This ratio also holds true for nonspherical particles where the aspect ratio is not too high or too small (Clift et al. 1978). This correlation has also be reviewed by Ganser (1993) who indicated that it produced reasonable results for many well-defined shapes.
The drag at high Reynolds numbers in the Newton regime also requires modifications for nonspherical particles. Cylinders and ellipsoids can be approximated by a shape factor similar to equation 6.16.

\[ C_{\text{shape}} \approx 1 + 0.7 \sqrt{A_{\text{surf}}^* - 1} + 2.4(A_{\text{surf}}^* - 1) \text{ for } E > 1 \]  \hspace{1cm} (6.17)

Using the approach taken by Ganser (1993) incorporating shape factors for the Stokes and Newton regimes to calculate the drag in the intermediate regime, Loth (2008) normalised the Reynolds number as

\[ Re_p^* = \frac{C_{\text{shape}} Re_p}{I_{\text{shape}}} \]  \hspace{1cm} (6.18)

and used a dimensionless Clift-Gauvin expression (Clift and Gauvin, 1970) to derive an equation for the drag coefficient of particles of various shapes with an approximately circular cross-section

\[ C_D^* = \frac{24}{Re_p^*} \left[ 1 + 0.15(Re_p^*)^{0.687} \right] + \frac{0.42}{1 + \frac{42500}{(Re_p^*)^{1.16}}} \]  \hspace{1cm} (6.19)

A plot of the normalised Reynolds number \( Re_p^* \) against normalised drag coefficient \( C_D^* \) illustrates how the approximated Clift-Gauvin expression provides a good correlation for the drag coefficient for needles, i.e. the \( \beta \) polymorph of the L-glutamic system. As a result of the level of accuracy produced by the approximation, this form of the drag coefficient is used to represent the drag produced by the interaction of the \( \beta \) polymorph with a liquid within a stirred tank.
Figure 6.7: Plot of normalised drag coefficient and Reynolds number for various particle shapes (Loth, 2008)
6.3 Summary

An analysis of suspension systems has been carried out in this chapter. The behaviour of a suspension due to the influence of geometric properties has been assessed. These properties include the impeller clearance-to-vessel diameter ratio (C/T), impeller diameter-to-tank diameter ratio (D/T) as well as the impeller speed required to just-suspend particles, $N_{js}$. A method of calculating the height of the solid-liquid interface was examined which allows the use of simulation results to predict the cloud height. A comprehensive study of the drag on non-spherical particles was undertaken and a the most applicable correlation for the drag on the needle-like $\beta$ particles of L-glutamic acid chosen (equation 6.19).
Chapter 7

CFD Modelling

The underlying theory of suspensions and mixing in stirred vessels has been discussed in the previous chapter. An efficient geometric setup for a stirred tank was investigated based on previously reported literature. An expression for the drag of the β polymorph was also developed. In order to accurately simulate the role these aspects play in the suspension of L-glutamic acid, a model was developed using the commercial CFD code, FLUENT v6.3.26. The two main areas of examination are the fluid phase and the dispersed phase that exists within the vessel. In order to fully understand the process that takes place within the vessel it is necessary to explain the underlying equations and method used in the analysis. In the present chapter the governing equations of the continuous and dispersed phases, as well as the assumptions made, are presented. The validity of specifying certain criteria while modelling the fluid phase is investigated by comparison of simulated results with published experimental results.

7.1 Fluid Phase Governing Equations

The continuous fluid phase within a stirred tank is modelled in this treatise using the Navier-Stokes equations which are implemented within the FLUENT CFD code. FLU-
ENT uses the finite volume method in its analysis which consists of dividing up the solution domain into control volumes and integrating the governing equations of the flow over these control volumes. Versteeg and Malalasekera (1995) and Ferziger and Peric (1995) both provide a detailed description of this method. The derivation of the governing equations is provided by Drew (1983) but a summary of these equations is presented below.

Equations for mean turbulent fluid motion are achieved using Reynolds decomposition of the Navier-Stokes equations. Reynolds decomposition describes how a velocity components within flow is decomposed into a mean value $U$ and a fluctuating component $u'(t)$.

$$u(t) = U + u'(t) \quad (7.1)$$

This provides time-averaged formulations of the governing equations and this set of equations is known as the Reynolds averaged Navier-Stokes (RANS) equations. The steady state RANS continuity and momentum equations for the fluid phase written in tensor notation are as follows:

$$\frac{\partial}{\partial x_i} (\rho_c U_i) = 0 \quad (7.2)$$

$$\frac{\partial}{\partial x_j} (\rho_c U_j U_i) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu_c \frac{\partial}{\partial x_j} U_i \right) - \frac{\partial}{\partial x_j} \left( \rho_c u'_i u'_j \right) - F_E \quad (7.3)$$

where $\rho_c$ is the continuous phase fluid density, $U$ is the mean fluid velocity, $P$ is the mean pressure and $u'$ is the fluctuating fluid velocity component. When used in particle transport problems these equations are solved in conjunction with the Lagrangian or Eulerian models to simulate the primary fluid phase. The $F_E$ term on the right hand side of equation (7.3) represents the momentum transfer between the fluid and solid phases.

### 7.2 Turbulence Modelling

When using Reynolds decomposition to formulate the time-averaged form of the momentum equation a problem arises due to the presence of the $\rho_c u'_i u'_j$ term in equation

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This term represents the Reynolds stresses and introduces six unknowns into the momentum equation. In order to properly evaluate equations \( 7.2 \) and \( 7.3 \), a solution to the Reynolds stresses must be found. This will provide a closed set of four equations and four unknowns \((U_i, U_j, U_k, P)\). Unfortunately the Reynolds stresses are a function of the flow itself and a turbulence model is required to simulate the effect of turbulence on the mean flow. This allows the closure of the RANS equations.

The \( k-\epsilon \) model is a two-equation turbulence model that can be applied to a wider range of scenarios. It provides a detailed description of turbulence which accounts for the transport of the turbulent properties of the mean flow and diffusion and for the production and destruction of turbulence \( \text{[Versteeg and Malalasekera, 1995]} \). The two equation models have been found to be reasonably successful and are the most widely used in commercial CFD codes \( \text{[Choudhury, 1993]} \). The \( k-\epsilon \) model was developed by \text{Lauder and Spalding, 1972} \) and has been redeveloped and refined over a number of years. \text{Yakhot and Orszag, 1986} \) modified the standard \( k-\epsilon \) model by incorporating renormalisation group (RNG) theory in order to address some of its weaknesses. This improved model is known as the RNG \( k-\epsilon \) model and allows for the determination of model constants analytically whereas the standard model uses empirical constants. This model also allows for the increased accuracy during the simulation of separated flows and curved geometries with only a minimal increase in computational effort \( \text{[Choudhury, 1993]} \). It has also been used by a number of authors for the successfully modelling of fluid flow in stirred tanks \( \text{[Aubin et al., 2004; Sheng et al., 1998; Vlachakis, 2006]} \). The RNG \( k-\epsilon \) was used to model the effect of turbulence on the mean flow in all the simulations in this treatise.

In order to evaluate the Reynolds stresses in equation \( 7.3 \), the shear stress must be related to properties of the fluid phase. This is achieved using the Boussinesq approximation, \( \text{[Versteeg and Malalasekera, 1995]} \), where the Reynolds stresses in equation \( 7.3 \) can be related to the turbulent viscosity by

\[
- \rho_c \bar{u'_i} u'_j = \mu_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} \rho_c k \delta_{ij}
\]

(7.4)

where \( \delta_{ij} \) is the Kronecker delta, \( \mu_t \) is defined as
and $C_\mu$ is taken to be 0.0845 from RNG theory. The Boussinesq approximation relates the mean flow quantities to the Reynolds stresses and allows a solution to the RANS equations.

The transport of the turbulent kinetic energy, $k$, and the rate of dissipation of turbulent energy, $\epsilon$, are accounted for using the RNG $k$-$\epsilon$ model for steady incompressible flow as given by Choudhury (1993).

$$\frac{\partial}{\partial x_j} \left( \rho_c U_j k \right) = \frac{\partial}{\partial x_j} \left( \alpha_k (\mu_c + \mu_t) \frac{\partial k}{\partial x_j} \right) + P_k - \rho_c \epsilon + S_k \tag{7.6}$$

$$\frac{\partial}{\partial x_j} \left( \rho_c U_j \epsilon \right) = \frac{\partial}{\partial x_j} \left( \alpha_k (\mu_c + \mu_t) \frac{\partial \epsilon}{\partial x_j} \right) + C_\epsilon^1 \frac{\epsilon}{k} P_k - C_\epsilon^2 \rho_c \frac{\epsilon^2}{k} - R_\epsilon + S_\epsilon \tag{7.7}$$

where $P_k$ represents the generation of turbulent kinetic energy due to the mean velocity gradients and is defined as

$$P_k = 2S_{ij}S_{ij} (\mu_c + \mu_t) \tag{7.8}$$

where

$$S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \tag{7.9}$$

The rate of strain, $R_\epsilon$, in equation (7.7) is given by

$$R_\epsilon = \frac{C_\mu \rho_c \eta^3 (1 - \eta/\eta_0) \epsilon^2 c^2}{1 + \beta \eta^3 \frac{c^2}{k}} \tag{7.10}$$

where

$$\eta = \sqrt{2S_{ij}S_{ij} \frac{k}{\epsilon}} \tag{7.11}$$

and $\beta=0.012$ and $\eta_0=4.38$. RNG theory is used to calculate the constants of the model. The constants are given by Yakhot and Orszag (1986) as $\alpha=1.39$, $C_{\epsilon 1}=1.42$.
and \( C_{r2}=1.68 \). \( S_k \) and \( S_\epsilon \) are optional user defined source terms.

### 7.2.1 Wall Effects

When modelling the flow in a stirred tank, the turbulent flow emanating from the impeller region dictates the flow throughout the vessel. The solution variables in this region change rapidly so it is necessary to resolve the flow to a high degree. If the computational grid in the impeller region is resolved to a high level, then the viscosity affected near wall region along the impeller can be solved and the turbulent properties assessed. In order to successfully achieve this result, semi-empirical formulae called wall functions are often employed in CFD.

The near wall region can be subdivided into three layers: the viscous sublayer closest to the wall where the flow is almost laminar, the fully turbulent region, and the buffer layer in-between where the flow is transitional [Versteeg and Malalasekera, 1995]. The use of wall functions models the region between the wall and the fully turbulent region. By assuming that the velocity close to the wall is only a function of fluid density, \( \rho_c \), fluid viscosity, \( \mu_c \), wall shear stress, \( \tau_w \), and the distance from the wall, \( y \), it can be shown by dimensional analysis that

\[
\nu^+ = \frac{U}{\mu_r} = f \left( \frac{\rho \mu \tau}{\mu_c} \right) = f(y^+) \tag{7.12}
\]

where \( \mu_r \) is the friction velocity given by

\[
\mu_r = \frac{\frac{1}{2} \tau_w}{\rho_c} \tag{7.13}
\]

Equation 7.12 is commonly referred to as the law of the wall and the function, \( f \), depends on the dimensionless distance from the wall, \( y^+ \). Within the viscous sublayer, \( (y^+ < 5) \), turbulent effects are negligible and it can be assumed that the shear stress is approximately equal to the wall shear stress.

\[
u^+ = y^+ \tag{7.14}
\]
In the log-law layer region \((30 < y^+ < 300)\), both turbulent and viscous effects are important and the log-law can be used to approximate the velocity profile and is given by

\[ u^+ = \frac{1}{\kappa} \ln (E y^+) \]  

(7.15)

where \(\kappa\) denotes Von Karman’s constant to be taken as 0.41 and the wall roughness parameter, \(E\), for smooth walls is 0.98. A turbulent velocity profile near a wall is shown in Figure 7.1.

Figure 7.1: Velocity profile as a function of distance normal to the wall (Ferziger and Peric, 1995)

Two different approaches can be used to determine the near wall velocity profile numerically. The more complex approach, termed near-wall modelling, is to resolve the viscosity-affected region near the wall completely but this requires an excessive computational effort for large geometries. The second approach involves using wall functions similar to the log-law described by equation 7.15 and is less computationally expensive. FLUENT includes a hybrid near-wall modelling method that incorporates both methods described above. The basic premise of this model is to use a near-wall modelling approach if the near wall-mesh is sufficiently fine but wall functions can be used if the grid is coarser. This approach is discussed in detail in Fluent Inc (2006). The hybrid near-wall modelling method will be used in this study. Wall functions have been successfully used on tank walls and an impeller in the modelling of multiphase flows in
Chapter 7. CFD Modelling

7.3 CFD Modelling in Moving Zones

At a basic level, CFD solves fluid flow in a stationary reference frame but problems arise when moving parts, such as rotating bodies and impellers, are introduced into the model. In most cases, an unsteady problem results from the introduction of moving parts but the implementation of certain models can allow the flow to be simulated as a steady-state problem with respect to the moving frame [Fluent Inc, 2006]. In this section a number of these methods are examined. These models vary in their level of complexity, accuracy and speed of solution.

7.3.1 Impeller Boundary Condition (IBC)

This method involves modelling the rotating impeller by imposing empirical boundary conditions on the surface of the impeller blades. It is least demanding of the methods discussed in terms of computing time but relies heavily on experimental data for accuracy. The data required needs to be specific to the stirred vessel under consideration but this negates the ability of CFD to be used in a predictive capacity if detailed experimental setups are needed for each tank. The IBC method is also unable to predict the flow and turbulence in the impeller region and therefore cannot accurately predict important global parameters within the tank such as power and pumping numbers [Brucato et al, 1998].

7.3.2 Single Rotating Reference Frame (SRF)

Rotating equipment can be modelled by applying a radial velocity boundary condition to the fluid and stationary boundaries. While this model offers good stability, simple mesh setup and a reasonable solution time, the main drawback is its inability to model geometries that include stationary baffles or vanes [Fluent Inc, 2006]. Any geometry where the fluid boundaries are not at a fixed radius from the axis of rotation requires the use of a sliding mesh or multiple reference frame model.
7.3.3 Multiple Reference Frame (MRF)

For problems involving multiple moving parts or stationary surfaces which are not revolving, the model domain must be split up into multiple fluid/solid zones with interface boundaries separating the zones. The zones that include the moving parts can be solved using the rotating reference frame whereas stationary zones are solved with the stationary frame equations (Figure 7.2).

The MRF model is a steady state approximation in which the individual cell zones move at different rotational velocities and has been used and validated during the analysis of stirred tanks (Aubin et al., 2004; Montante et al., 2005; Brucato et al., 1998) and also multiphase flows in stirred tanks (Murthy et al., 2007). The MRF model can analyse flows where the impeller-baffle interaction is relatively weak, such as mixing tanks, but is unsuitable for strong rotor-stator interactions where the analysis of the transients are paramount (Fluent Inc., 2006).
7.3.4 Sliding Mesh (SM)

The sliding mesh model is used in situations where a time accurate solution to the mixing is required as opposed to a time averaged solution. It is the most accurate of the rotating reference frame models but also the most computationally demanding [Brucato et al. (1998)]. In the SM model, the fluid zones move against each other at their common interface. The flow is solved using the unsteady solver and the grid is moved on to the next timestep and the process repeated (Figure 7.3).

![Sliding mesh example](https://via.placeholder.com/150)

Figure 7.3: Sliding mesh example [Fluent Inc (2006)]

7.3.5 Comparison of Moving Zone Models

From examining the models described above it is clear that the IBC model will not satisfy the criteria as the experimental values needed are not readily available. The single reference frame model is also inapplicable due to the presence of baffles within the geometry of the stirred vessel in this treatise and the need to simulate difference zones in order to accurately resolve the flow. This leaves a choice of the sliding mesh model or the multiple reference frame model. The SM model had been shown to be more accurate for flows involving high levels of swirling but at a much higher computational cost than the MRF model as stated by Brucato et al. (1998). The studies performed
by [Aubin et al. (2004)] has shown that for a vessel with a comparable geometry to the vessel used in this dissertation, there is little difference between the results produced by both models. This is due to the fact that there exists a sufficient distance between the impeller and baffles such that the flow in the vicinity of the impeller is unaffected by the rest of the tank. A further study carried out by [Deglon and Meyer (2006)] using the MRF model on a stirred tank provided results very similar to those produced by the SM model without incurring excessive computational expense. The similarities between the simulations being developed in this dissertation and the published studies above have led this author to implement the MRF model throughout this work.

### 7.3.6 Modifications for Rotating Flow

When the continuity and momentum equations are solved in a rotating frame of reference modifications must be made within FLUENT to obtain accurate solutions. These solutions can be achieved using either an absolute velocity, $\vec{v}$, or a relative velocity, $\vec{v}_r$, as the dependent variable. They are related by the following equation

$$\vec{v}_r = \vec{v} - (\vec{\Omega} \times \vec{r})$$ (7.16)

where $\vec{\Omega}$ is the angular velocity vector and $\vec{r}$ is the position vector in the rotating frame given by

$$\vec{r} = \vec{x} - \vec{x}_o$$ (7.17)

where $\vec{x}$ is the position in absolute Cartesian co-ordinates and $\vec{x}_o$ is the origin of the zone rotation axis.

Using the absolute velocity formulation, the left hand side of the momentum equation is rewritten as

$$\nabla \cdot (\rho_c \vec{v}_r \vec{v}_r) + \rho_c (\vec{\Omega} \times \vec{v})$$ (7.18)

and the left hand side of the continuity equation becomes

$$\nabla \cdot (\rho_c \vec{v}_r) = 0$$ (7.19)
The RNG $k$-$\epsilon$ model includes an option to account for the effect of swirling flows on the turbulence. This is achieved by modifying the turbulent viscosity appropriately.

\[
\mu_t = \mu_{t0} f\left(\alpha_s, \Omega, \frac{k}{\epsilon}\right)
\]  

(7.20)

where $\mu_{t0}$ is the value of the turbulent viscosity calculated without the swirl modification using equation 7.5. $\alpha_s$ is a swirl constant that assumes different values depending on whether the flow is swirl-dominated or only mildly swirling. FLUENT recommends a default value of 0.07 for the swirling flows described within this dissertation. $\Omega$ is a characteristic swirl number calculated within FLUENT using

\[
\Omega = \frac{\int r\omega \vec{v} \cdot d\vec{A}}{\int \vec{u} \cdot d\vec{A}}
\]  

(7.21)

where $R$ is the hydraulic radius. For swirl numbers less than 0.05, the RNG $k$-$\epsilon$ model provides a marked improvement over the standard $k$-$\epsilon$ model (Fluent Inc, 2006).
7.4 Dispersed Phase

Stirred tanks are widely used in chemical and pharmaceutical industries for mixing and chemical reactions and the flow that occurs in such tanks is usually turbulent (Fan et al., 2005). Another aspect of stirred tanks that are often encountered in an industrial context involve solid-liquid two phase flows. The presence of solid particles makes the flow in the stirred tanks more complicated and numerous models exist in order to resolve the interaction of the solid particles with the fluid phase.

Two different methods for the numerical calculation of multiphase flows exist: the Euler-Lagrange approach and the Euler-Euler approach. The Euler-Lagrange approach includes the assumption that the dispersed second phase occupies a low volume fraction which makes it suitable for modelling of spray dryers and the tracking of some particle laden flows (Corish, 2007). The particles trajectories are individually computed based on the relevant forces acting on the particles. It is unsuitable for applications where the volume fraction of the second phase is not negligible.

For applications where the volume fraction of the second phase exceeds 10%, the Euler-Euler approach is appropriate (Fluent Inc, 2006). This approach assumes the different phases are treated as interpenetrating continua and involves the use of phasic volume fractions. These volume fractions are assumed to be continuous functions of space and time and their sum is equal to unity. Conservation equations are derived for each phase and these equations have a similar structure. The Eulerian multiphase model is used in this treatise to simulate the flow of the $\alpha$ and $\beta$ polymorphs. This section examines the model and outlines the governing equations of particle motion and turbulence modelling that exist within the model. Modifications to the underlying equations within the Eulerian model are performed by this author in order to more accurately simulate the flow of the needle-like $\beta$ crystals that exist within the L-glutamic acid system.

7.4.1 Eulerian Model

When using a multiphase model to simulate the flow and interaction of phases, additional conservation equations must be introduced to account for the extra phases. In the process of adding new conservation equations to the model, the original set must also be modified. This can involve the introduction of volume fractions for the multiple phases as well as mechanisms for the exchange of momentum.
Chapter 7. CFD Modelling

Continuity Equation

The general form of the continuity for phase \( q \) is

\[
\frac{\partial}{\partial t} (\alpha_q \rho_q) + \nabla \cdot (\alpha_q \rho_q \vec{v}_q) = \sum_{p=1}^{n} (\dot{m}_{pq} - \dot{m}_{qp}) \tag{7.22}
\]

where \( \alpha_q \) is the volume fraction for phase \( q \), \( \vec{v}_q \) is the velocity of phase \( q \), \( \dot{m}_{pq} \) describes the mass transfer from phase \( p \) to \( q \) and \( \dot{m}_{qp} \) describes the mass transfer from phase \( q \) to \( p \). Fluent calculates the volume fraction of the primary phase using the continuity equation by solving for each of the secondary phases and applying the condition that the sum of the volume fractions within the solution domain equal one. The continuity equations are identical for both phases.

Momentum equation

The momentum equations for the continuous phase \( l \) and the solid phase \( s \) differ and are given by equations 7.23 and 7.24 respectively (Virdung and Rasmuson, 2007)

\[
\frac{\partial}{\partial t} (\alpha_l \rho_l \vec{v}_l) + \nabla \cdot (\alpha_l \rho_l \vec{v}_l \vec{v}_l) = -\nabla \rho_l \vec{F}_l + \nabla \cdot \tau_l + \rho_l \vec{g} + \alpha_l \rho_l \vec{F}_{\text{vm},l} + \alpha_l \rho_l \vec{F}_{\text{lift},l} + \vec{F}_{\text{dr},sl} \tag{7.23}
\]

\[
\frac{\partial}{\partial t} (\alpha_s \rho_s \vec{v}_s) + \nabla \cdot (\alpha_s \rho_s \vec{v}_s \vec{v}_s) = -\nabla \rho_s \vec{F}_s + \nabla \cdot \tau_s + \rho_s \vec{g} + \alpha_s \rho_s \vec{F}_{\text{vm},s} + \alpha_s \rho_s \vec{F}_{\text{lift},s} + \vec{F}_{\text{dr},sl} \tag{7.24}
\]

where \( \vec{F}_q \) is an external body force, \( \vec{F}_{\text{lift},q} \) is a lift force, \( \vec{F}_{\text{vm},q} \) is a virtual mass force for the \( q^{th} \) phase. \( \vec{F}_{\text{dr},sl} \) is an interaction force between phases and \( p \) is the pressure shared by all phases. \( \vec{v}_{ls} \) is the interphase velocity. The drift velocity \( \vec{v}_{dr} \) in equations 7.23 and 7.24 accounts for the turbulence fluctuations in the volume fraction and when multiplied with the exchange coefficient, the momentum exchange due to the fluctuations are properly accounted for. The phase stress-strain tensor of the \( q^{th} \) phase is given by \( \vec{\tau}_q \) and is explained in detail in Fluent Inc (2006).
7.4.2 Exchange Coefficient

Fluid-solid exchange coefficient

The fluid-solid exchange coefficient term from momentum equations can be written in the following general form:

$$K_{sl} = \frac{\alpha_s \rho_s f}{\tau_s}$$  \hspace{1cm} (7.25)

where $f$ is a drag term and $\tau_s$ is the particle relaxation time defined as

$$\tau_s = \frac{\rho_s d_s^2}{18 \mu_l}$$  \hspace{1cm} (7.26)

where $d_s$ is the diameter of particles of phase $s$. The definition of $f$ is dependent on the exchange-coefficient model that is applied to the particular phase and it consists of a drag function, $C_D$ that differs between exchange coefficient models. Several drag models have been developed to calculate the interphase exchange coefficient such as the models developed by Syamlal and O’Brien (1989), Gidaspow et al. (1992) and Wen and Yu (1966). The Syamlal and Gidaspow models are generally used for fluidised beds whereas the Wen and Yu model is more suited to dilute systems (Wen and Yu, 1966).

The fluid-solid exchange coefficient for the Wen and Yu model is defined as follows

$$K_{sl} = 3 \frac{C_D \alpha_s \rho_l |\vec{v}_s - \vec{v}_l|}{d_s \alpha_l}$$  \hspace{1cm} (7.27)

where

$$C_D = \frac{24}{\alpha_l Re_s} \left[ 1 + 0.15(\alpha_l Re_s)^{0.687} \right]$$  \hspace{1cm} (7.28)

and the relative Reynolds number is given as

$$Re_s = \frac{\rho d_s |\vec{v}_s - \vec{v}_l|}{\mu_l}$$  \hspace{1cm} (7.29)

One of the assumptions made by FLUENT during the calculation of a secondary particulate phase is that the particles being considered are spherical. The work being performed in this treatise requires that the presence of needle-like particles in the system needs to be accounted for by the governing equations. As discussed in section 6.2.2, a drag function encompassing a range of particle shapes was developed by Loth (2008).
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\[ C_D^* = \frac{24}{Re_p^*} \left[ 1 + 0.15(Re_p^*)^{0.687} \right] + \frac{0.42}{1 + \frac{42500}{(Re_p^*)^{1.16}}} \tag{7.30} \]

Equation (7.30) is incorporated into the multiphase model developed in this treatise using user-defined functions (UDF) in order to model the interaction of the continuous liquid phase with the needle-like \( \beta \) secondary phase. The UDF operates as follows:

- The diameter of the \( \beta \) polymorph is input into Fluent.
- The velocity, density, and dynamic viscosity are used to calculate the Reynolds number and slip velocity in each cell.
- The equivalent diameter is calculated using the methods described in section 6.2.2.
- The coefficient of drag, \( C_D \), is generated using the correlation developed by Loth (2008).
- Using the values calculated in the previous steps, the fluid-solid exchange coefficient is determined and input in the momentum equations within Fluent.

This provides an alternative fluid-solid exchange coefficient, \( K_{sl} \), to be included in the Eulerian multiphase model and an accurate resolution of the interaction of the \( \beta \) polymorph in the stirred vessel is achieved. The details of the UDF can be found in Appendix B.

**Solid-solid exchange coefficient**

Throughout the multiphase model there exists interactions between both the fluid and solid phases as described previously. There also exists interactions between the solid phases if multiple solid phases are employed in the model and an exchange coefficient needs to be defined in order to satisfy the momentum equation. The solid-solid exchange coefficient \( K_{ls} \) has been expressed by Syamlal (1987) as

\[ K_{ls} = \frac{3(1 + e_{ls}) (\frac{\pi}{2} + C_{fr,ls} \frac{\pi^2}{6}) \alpha_s \rho_s \alpha_l \rho_l (d_l + d_s)^2 g_{0,ls}}{2\pi (\rho_l d_l^3 + \rho_s d_s^3)} \left| \vec{v}_l - \vec{v}_s \right| \tag{7.31} \]

where \( e_{ls} \) is the coefficient of restitution, \( C_{fr,ls} \) is the coefficient of friction, \( d_l \) is the diameter of particles of solid \( l \) and \( g_{0,ls} \) is the radial distribution coefficient. Further details on the definition of the coefficient of restitution and radial distribution coefficient can be found in Fluent Inc (2006).
7.4.3 Turbulence Models

For turbulence modelling, the presence of multiple phases requires modifications to the RNG $k-\epsilon$ model due to a larger number of terms that need to be solved. Three different multiphase approaches exist within FLUENT as part of the Eulerian multiphase model. The mixture turbulence model is the simplest one and uses mixture properties and velocities for the estimation of $k$ and $\epsilon$. It is suitable for stratified multiphase flows and when the density ratio between the phases is close to one. The per-phase $k-\epsilon$ model, (Shih et al. 1995), solves a set of $k$ and $\epsilon$ equations for each phase. As this model solves two additional equations for each phase it is more computationally expensive than the other turbulence model modifications. The $k-\epsilon$ dispersed turbulence model is the final option available within the Eulerian framework. The basis of this model is the assumption that secondary phase turbulent properties of the flow can be derived from the primary phase turbulent flow properties. This model is applicable where there exists one primary continuous phase and any number of dispersed dilute secondary phases (Fluent Inc. 2006). As the simulations considered in this treatise consist of a dilute secondary solid phase the $k-\epsilon$ dispersed turbulence model was deemed to be most suitable.

7.5 Discretisation

Equations 7.2, 7.3, 7.6 and 7.7 show significant similarities in their formulation. It is useful to write a steady-state transport equation in terms of a general solution variable, $\phi$, that can aid in the solution to the aforementioned equations.

$$\frac{\partial}{\partial x_j}(\rho u_j \phi) = \frac{\partial}{\partial x_j}(\Gamma \frac{\partial \phi}{\partial x_j}) + S_{\phi} \tag{7.32}$$

where $\Gamma$ is the diffusion coefficient and $S_{\phi}$ is the source term. Rewriting equation 7.32 using vector notation gives

$$\text{div} (\rho \phi \mathbf{u}) = \text{div} (\Gamma \text{grad} \phi) + S_{\phi} \tag{7.33}$$

This is the general conservative form of all fluid flow equations and it is used to solve equations 7.2, 7.3, 7.6 and 7.7. The first step in the finite volume method is the integration of equation 7.33 over a three-dimensional control volume $CV$ yielding
Using Gauss’ divergence theorem, which states that the divergence of a vector over a volume is equal to the sum of the vector components normal to the surface over the entire bounding surface $A$, gives

$$\int_A \mathbf{n} \cdot (\rho \phi \mathbf{u}) \, dA = \int_A \mathbf{n} \cdot (\Gamma \, \nabla \phi) \, dA + \int_{CV} S_\phi \, dV \quad (7.35)$$

where $\mathbf{n}$ is the unit vector normal to the surface $A$.

Equation (7.35) can now be applied to each control volume in the computational domain of interest. This involves summation of the convective and diffusive flux through all the faces of the control volume. A graphical representation of a typical 2D grid cell is shown in Figure 7.4. Application of equation (7.35) to one of these cells produces an algebraic equation given by

$$\sum_{f}^{N_{faces}} \rho_f \phi_f \mathbf{u}_f \cdot \mathbf{A}_f = \sum_{f}^{N_{faces}} \Gamma (\nabla \phi)_n \cdot \mathbf{A}_f + S_\phi V \quad (7.36)$$

where $\mathbf{A}_f$ is the face area normal vector and subscripts $f$ and $n$ refer to face values and normal vector components respectively. Equation (7.36) is multidimensional and can be applied to unstructured hexahedral grids and unstructured grids composing of arbitrary polyhedra (Fluent Inc, 2006).
A method is required to calculate the transported scalar $\phi$ at control volume faces $\phi_f$. This is achieved with the use of an appropriate discretisation scheme. The simplest discretisation scheme, known as central differencing, calculates face values using linear interpolation between the centres of adjacent cells. This scheme is relatively straightforward but it fails to account for convective diffusion of the flow and is therefore only applicable for diffusion dominated low Reynolds number flows (Versteeg and Malalasekera, 1995).

In the present work, the Power law discretisation scheme is used to interpolate the face value of a variable $\phi$ using the exact solution to a one-dimensional convection-diffusion equation

$$\frac{\partial}{\partial x} \left( \rho u \phi \right) = \frac{\partial}{\partial x} \left( \frac{\partial \phi}{\partial x} \right)$$  \hfill (7.37)

$\phi$ is shown to vary with $x$ by integrating equation (7.37) to produce the following expression

$$\frac{\phi(x) - \phi_0}{\phi_L - \phi_0} = \frac{\exp(\text{Pe} \frac{x}{L}) - 1}{\exp(\text{Pe}) - 1}$$  \hfill (7.38)

where $\phi_0$ and $\phi_L$ are the values of $\phi$ at $x = 0$ and $x = L$, respectively and $\text{Pe}$ is the Peclet number given by

$$\text{Pe} = \frac{\rho u L}{\Gamma}$$  \hfill (7.39)

The Power law scheme has been found to predict the flow variables around the impeller, in a stirred tank, to a higher degree of accuracy than the standard upwind and hybrid schemes (Sahu and Joshi, 1995).

### 7.5.1 Pressure-Velocity Coupling

In order to solve the continuity and momentum equations (equations 7.2 and 7.3) a pressure gradient term needs to be resolved. The pressure and velocity are intricately coupled and if the correct pressure field is not applied in the momentum equations, the resulting velocity field will not satisfy the continuity equation (7.2). There is no obvious solution to obtaining the pressure field as density cannot be linked to pressure for incompressible flow. This requires the pressure to be treated as a solution variable and must be calculated as part of the solution, despite no specific transport equation for pressure being solved.
This issue is overcome by employing the SIMPLE (Semi-Implicit Method for Pressure Linked Equations) algorithm. The method is described in detail by Pantankar (1980) and only a brief summary is given here. The algorithm describes a relationship linking the velocity and pressure to enforce mass conservation. This is achieved through the introduction of a pressure correction term to the continuity equation given by

$$p = p^* + p'$$

(7.40)

where $p^*$ is the guessed pressure and $p'$ is the pressure correction. The modified continuity equation is termed the pressure-correction equation and produces values of $p'$ in the iterative solution process.

The SIMPLE algorithm had the following solution procedure (Pantankar, 1980):

1. Provide an initial guess of the pressure field $p^*$
2. Solve the momentum equation using the guessed pressure field
3. Solve the pressure-correction equation to obtain the pressure correction $p'$
4. Determine the corrected pressure field using equation (7.40)
5. Use the result of this calculation to provide a new guessed pressure, $p^*$ and repeat the procedure until the correction tends to zero

### 7.6 Fluid Phase Modelling

The turbulent flow within a stirred vessel has been studied by a number of authors including Wu and Patterson (1989) and Armenante et al. (1997) using laser-Doppler velocimetry (LDV). These authors produced mean velocity profiles within the tank which allows a comparison of the numerical modelling of the flow in a tank. The comprehensive experimental study by Wu and Patterson (1989) has been successfully used as a benchmark for numerical analysis previously reported by Deglon and Meyer (2006) and Brucato et al. (1998). The experiments used LDV to produce axial profiles of the mean axial, radial and tangential velocities in a baffled, Rushton turbine agitated vessel. This work has been selected to assess the accuracy of the techniques described in the previous sections and to develop a procedure to produce an accurate simulation of the fluid flow in a stirred tank.
Chapter 7. CFD Modelling

7.6.1 Problem Description

The experiments of Wu and Patterson (1989) were performed in a plexiglass cylindrical tank of diameter (T) equal to 27cm. The tank height is also 27cm. The four baffles in the tank had a width equal to 0.1T and the six-blade Rushton turbine impeller was placed one-third of the way up from the tank bottom. The tank was filled with water being used as the working fluid and was open at the top. A schematic of the tank is shown in Figure 7.5.

![Figure 7.5: Tank and impeller with dimensions](Wu and Patterson, 1989)

The velocity measurements were made in the impeller stream and in the near region around the impeller at 45° with the baffles. The results reported were for data produced at 200rpm which corresponds to an impeller Reynolds number of $3.2 \times 10^4$. The Reynolds number is a ratio of fluid inertial forces to viscous forces and is defined as
\[ Re_y = \frac{\rho ND^2}{\mu} \] (7.41)

where \( N \) is the impeller rotational speed and \( D \) is the impeller diameter. The density and dynamic viscosity of the working fluid are \( \rho \) and \( \mu \), respectively. A summary of the fluid properties and experimental conditions is given in Table 7.1.

<table>
<thead>
<tr>
<th>Model Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>997 kg/m³</td>
</tr>
<tr>
<td>Dynamic Viscosity</td>
<td>0.00089 Pas</td>
</tr>
<tr>
<td>Impeller Velocity</td>
<td>200 rpm</td>
</tr>
<tr>
<td>Impeller Diameter</td>
<td>0.093 m</td>
</tr>
<tr>
<td>Impeller Reynolds Number</td>
<td>( 3.2 \times 10^4 )</td>
</tr>
</tbody>
</table>

Table 7.1: Fluid properties and experimental conditions given by Wu and Patterson [1989]

The stirred tank was modelled and discretised using the Gambit software package. The grid was constructed using size-functions within Gambit to specify the size and growth of cells in from particular areas of the domain. In this case, the initial cells were located around the impeller region allowing a dense grid to be formed in this area and the size of the cells increase as the distance from the impeller increases until the domain has been fully discretised. The initial grid was generated using tetrahedral cells and later converted into polyhedral cells in order to reduce the computational requirements. The conversion from tetrahedral to polyhedral cells can reduce the number of cells in a domain by approximately 3 to 5 times [Fluent Inc. 2006]. The computational grid is shown in Figure 7.6.
Chapter 7. CFD Modelling

The turbulent flow analysis was performed using the FLUENT 6.3.16 software package. A multiple reference frame model (MRF) (section 7.3) was utilised in conjunction with the RNG k-ε model to solve the turbulent quantities (section 7.2). The discretised governing equations are solved using Power law and Upwind schemes. A comparison is made between the two method of discretisation in order to gauge which method produces the more accurate fluid flow prediction. The CFD simulations were solved to a convergence criteria of at least $10^{-3}$. This indicates that the solutions to the transport equations at successive iterations differed by no more than 0.1%.

7.6.2 Grid Independence

In CFD simulations, in order to solve the fluid flow within the computational domain, the geometry is broken up into a number of cells and the governing equations are solved for each cell. This process provides average quantities across each cell in the computational domain. As the number of cells increases and the size of each cell decreases, there is a reduction in numerical error between the values produced by the algebraic equations and the governing equations used to represent the flow. As the number of cells approaches infinity, and solution of the flow, as given by the continuum equations, is resolved.

Unfortunately, a trade off exists between the quantity of cells that can be used to solve the continuum equations and the computational time required to reduce the error between the cell values. A point will exist where an increase in the number of cells will not
produce a significant reduction in the solution error. In order to appraise whether or not a grid level is sufficiently fine, the solution error must be examined. This is achieved by increasing the grid level in a controlled manner and studying the behaviour of important solution variables. This process is known as a grid convergence study.

One particular method of measuring grid convergence was proposed by Roache (1998) which incorporates the use of a Grid Convergence Index (GCI) to quantify the uncertainty of grid convergence. One of the issues associated with the GCI its suitability to unstructured grids. For a grid developed using a geometric refinement, the GCI can be easily applied but for an unstructured grid where the mesh density varies in different regions within the domain, the calculation of the GCI is far from trivial. Due to this issue, the method used in this treatise to analyse the independence of the solution from the grid will involve the monitoring of several important solution variables.

<table>
<thead>
<tr>
<th>Grid size</th>
<th>Number of polyhedral cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>517k</td>
</tr>
<tr>
<td>Medium</td>
<td>627k</td>
</tr>
<tr>
<td>Fine</td>
<td>842k</td>
</tr>
</tbody>
</table>

Table 7.2: Stirred tank model grid sizes

![Figure 7.7: Convergence of turbulent kinetic energy in stirred tank](image)

Figure 7.7 displays the convergence of the turbulent kinetic energy in the stirred tank.
signifying that a stable solution has been achieved within the model. The flow variables examined in this model are the axial velocity and the turbulent kinetic energy at a distance of \( r = 5 \text{cm} \) from the shaft centre and mid-way between two baffles (Figure 7.5). The velocity profiles are normalised by the impeller tip velocity calculated to be 0.0465\( \text{m/s} \) and the vertical height from the centre of the impeller, \( z \), is normalised by the blade width, \( w \). As can be seen from Figure 7.8, the difference between the three sets of grid levels is minimal. Any further grid refinement would not produce a noticeable improvement in the solution prediction. The coarse grid has been used from this juncture onwards in the examination of the flow variables in this case due to its reduced computational expense without the any significant loss of accuracy.

![Figure 7.8: Axial velocity and turbulent kinetic energy profiles on three grid levels](image)

Figure 7.8: Axial velocity and turbulent kinetic energy profiles on three grid levels
Chapter 7. CFD Modelling

Examining Figure 7.9 reveals a $y^+ > 5$ for the coarse grid. This requires the use of the hybrid near-wall modelling method to resolve the near-wall velocity profile as described in section 7.2.1. This method provides an adequate compromise between modelling error and computational effort.

Figure 7.9: Impeller $y^+$ values

7.6.3 Fluid Flow Solution

Figure 7.10 shows the velocity field produced using the RNG k-ε turbulence model. The predicted flow field is dominated by a radial jet emanating from the impeller which produces two major recirculating zones above and below the impeller. The slight asymmetry between the upper and lower recirculating regions can be attributed to the impeller being positioned in the lower third of the tank and not at mid-height.
Chapter 7. CFD Modelling

Figure 7.10: Velocity magnitude vector plot

The mean velocity profiles in Figures 7.11-7.13 illustrate the decrease in the velocities as the radial distance increases. It can be seen that the profiles are not symmetrical about the \( z=0 \) axis but shift upwards as the wall is approached. This pattern is produced due to the impeller not being located at the tank mid-height and variation in boundary condition between the upper and lower surfaces. A no-slip condition is applied to the tank bottom but a symmetry condition exists at the top as suggested by Choi et al. (2004). The results are in line with the findings of Wu and Patterson (1989).

Figure 7.11: Mean radial velocity profiles at various radial positions
Figure 7.12: Mean tangential velocity profiles at various radial positions

Figure 7.13: Mean axial velocity profiles at various radial positions

The results in Figures 7.14-7.16 display the mean radial, axial and tangential velocity profiles at three different radial distances from the centre using two different discretisation schemes. The Power law scheme was compared to the Upwind scheme in order to ascertain the more appropriate method of discretisation. As can be clearly seen, the Power law produces a much higher level of predictability relative to the Upwind scheme. There is a distinct overprediction of the mean tangential and radial velocity components using the Upwind scheme but the Power law provides a more accurate comparison with the experimental results. The mean axial velocity profiles generated using the Upwind scheme produces a negative jet which is opposite to that simulated using the Power law scheme and the experimental results presented by Wu and Patterson (1989). This is
in line with the studies of Sahu and Joshi (1995) who compared various discretisation schemes and found that the Power law scheme was able to predict the flow variables in a stirred tank to higher degree of accuracy. Using the knowledge provided by the comparison of the discretisations schemes and the favourable results produced by using the MRF model with the RNG k-\( \epsilon \) turbulence model, the Power law will be used in conjunction with the MRF and RNG k-\( \epsilon \) turbulence model for the numerical prediction of the flow in a stirred tank in this treatise.
Figure 7.14: Comparison of measured and predicted mean radial velocity for different discretisation schemes
Figure 7.15: Comparison of measured and predicted mean axial velocity for different discretisation schemes
Figure 7.16: Comparison of measured and predicted mean tangential velocity for different discretisation schemes
7.7 Summary

This chapter presented a numerical method for simulation of a continuous fluid and dispersed phase within a stirred tank. The Reynolds-averaged Navier-Stokes equations with some underlying assumptions are employed in the model. Models for evaluating rotating flow were outlined and evaluated. The RNG $k$-$\epsilon$ turbulence model was used to obtain closure of the continuum equations. Required modifications to the fluid and turbulence equations to deal with rotating flow were given and the boundary conditions used in the modelling were explained. The governing equations for the dispersed phase were introduced and the solid-fluid exchange coefficients examined with a focus on the new drag correlation for the needle-like solid phase. The CFD analysis of a standard stirred tank configuration was also performed in order to assess the accuracy of the modelling procedure for the continuous phase. The results provided indicate that approach taken would produce an accurate prediction of the continuous fluid phase.
Chapter 8

Dispersed Phase Modelling

The modelling of two phase flows in a stirred tank is discussed in this chapter. This includes the examination of a novel method of simulating the suspension of needle-like $\beta$ particles. A detailed comparison of drag laws for spherical and non-spherical particles is undertaken. The novel implementation of the non-spherical particle drag law within the CFD code is performed and the validity of the model is assessed through experiment.

8.1 Suspension Experiments

The verification of the CFD simulations required a number of experiments to be performed in the stirred tank employing the same geometric configuration as the tank modelled in the simulations. A specific mass of $\alpha$ and $\beta$ crystals are suspended in the experimental rig and the volume fraction of crystals in the suspension was measured at specific heights in the tank. The optical approach used in the prediction of cloud height in the tanks has been successfully employed by Ochieng and Lewis (2006). This method produced results that were consistent with the CFD predictions. Rieger and Dit (1994) compared a number of different methods for predicting cloud height and also concluded that the visual method was the most effective.
8.1.1 Experimental Procedure

The preparation of the $\alpha$ and $\beta$ crystals (Figure 5.17) required a number of steps. The $\alpha$ crystals were created using the method utilised by Ono et al. (2004). 48g of L-glutamic acid was dissolved in 1000 ml of distilled water at 80°C. The solution was cooled rapidly to 25°C under constant stirring at 300rpm. At 25°C nucleation and growth of the $\alpha$ polymorph was achieved. The resulting precipitate was filtered, rinsed with distilled water and dried. The structure of the crystals was confirmed using scanning electron microscopy (SEM). The $\beta$ crystals were prepared by dissolving 53 g of L-glutamic acid in 1000 ml of distilled water at 80°C, cooling the solution to 45°C at a rate of 1.5K/min and maintaining the solution at this temperature for 10 hours. After this the resulting $\beta$ crystals were filtered, and dried. The two masses of crystals were sieved in order to produce size fractions in the range 90-106$\mu$m and 25-32$\mu$m for the $\alpha$ and $\beta$ polymorphs, respectively.

The suspension analysis of the pure $\alpha$ phase was performed by mixing the crystals with ethanol in the stirred tank shown in Figure 8.1 and the impeller was rotated at speeds from 250-600rpm. The speed required to just-suspend the largest solid particle was calculated to be 199rpm as described in section 8.1.3. Ethanol was used as the fluid phase due to L-glutamic acid crystals being insoluble in ethanol and thus preventing further crystallisation (Jacques et al., 2005). The same procedure was applied to the suspension of the $\beta$ phase.

8.1.2 Volume Fraction Determination

In order to verify the accuracy of the stirred tank simulations, the volume fraction of the suspension was measured at various heights. Four different samples were taken for each increment of impeller rotational speed.

The method used to calculate the volume fraction is described below:

- 5mL samples were taken from stirred tank using a plastic syringe at an approximate radial distance of 6cm from the centre of the tank.
- These samples were placed in pre weighed vials.
- The vials were then weighed to determine the mass of the vial plus contents.
• The samples were placed in an oven at 45°C in order to allow the liquid phase to evaporate off.

• Upon evaporation of the liquid phase, the samples were placed in an oven at 80°C to ensure the complete drying of the solid phase.

• Following the drying process, the vials containing the solid phase were weighed once more.

• The mass of the samples were calculated allowing the volume to be determined using the following equation

\[
\text{Volume Fraction} = \frac{m_{\text{solid}} \rho_l}{m_{\text{solution}} \rho_s}
\]  

(8.1)

8.1.3 Experimental Rig

During the course of this research, the construction of a stirred tank was undertaken in order to implement the findings made from previous literature in the field. A 15cm cylindrical perspex tank, with four equally spaced baffles, was fabricated in order to perform experimental work necessary for the analysis of the L-glutamic acid system (Figure 8.1). A four-blade 45° stainless steel PBT was used to agitate the system. The diameter of the impeller was 5cm corresponding to an impeller diameter-to-tank diameter ratio (D/T) of 0.33 and the shaft diameter was 0.8cm. The baffles were 1.5cm wide corresponding to one-tenth of the tank diameter. The impeller clearance-to-vessel diameter ratio (C/T) was maintained at 0.2. The working fluid was ethanol due to L-glutamic acid crystals being insoluble in ethanol and this prevents any crystallisation processes occurring during the experiments (Jacques et al., 2005). The \( N_{js} \) calculated for this stirred tank using geometric and material properties from Table 8.1 equals 199rpm. The impeller Reynolds number at this speed was calculated to be \( 5.4 \times 10^3 \) using equation 8.2

\[
Re_s = \frac{\rho_l N D^2}{\mu_l}
\]

(8.2)

where \( N \) is the impeller rotational speed in revolution per second and \( D \) is the impeller diameter.
Chapter 8. Dispersed Phase Modelling

Figure 8.1: Configuration of the stirred tank and impeller

<table>
<thead>
<tr>
<th>Tank and Material Properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid Density</td>
<td>1540  kg/m³</td>
</tr>
<tr>
<td>Fluid Density</td>
<td>789   kg/m³</td>
</tr>
<tr>
<td>Kinematic Viscosity</td>
<td>1.52x10⁻⁶ m²/s</td>
</tr>
<tr>
<td>Dynamic Viscosity</td>
<td>0.0012 kg/ms</td>
</tr>
<tr>
<td>Max α Particle Diameter</td>
<td>0.000106 m</td>
</tr>
<tr>
<td>Max β Particle Diameter</td>
<td>0.000032 m</td>
</tr>
<tr>
<td>Impeller Diameter</td>
<td>0.05   m</td>
</tr>
<tr>
<td>Tank Diameter</td>
<td>0.15   m</td>
</tr>
<tr>
<td>Impeller Clearance</td>
<td>0.03   m</td>
</tr>
<tr>
<td>Solids Volume Fraction</td>
<td>10     %</td>
</tr>
</tbody>
</table>

Table 8.1: Material and geometric properties of experimental stirred tank
8.2 Computational Model

The stirred tank used for the multiphase simulations was modelled and discretised using Gambit and incorporating size functions as described in section 7.6.1 in order to create a dense mesh within the impeller region. Once again, the initial grid was generated using tetrahedral cells and converted into polyhedral cells in order to reduce the computational effort. Figure 8.2 shows the coarse grid that was constructed. The number of cells in each of the grids are given in Table 8.2 and a grid independence study was performed on the three grid levels for an impeller rotational speed of 250rpm.

Upon incorporation into Fluent, a multiple reference frame model was used. A Eulerian multiphase model was specified with a 10% solid phase volume fraction. An average particle size of 100µm was used for the grid independence study. The RNG k-ε turbulence model was employed and operated in conjunction with the dispersed turbulence model to account for effect of multiple phases as described in 7.4.3. The hybrid near-wall modelling method was used to resolve the turbulent boundary layer. The Power law and
Upwind schemes were used to solve the governing equations. The stable solution to the simulation was deemed to have been achieved when the turbulent kinetic energy had converged to a final value and a convergence criteria of at least $10^{-3}$ was achieved in all cases. A convergence criteria of $10^{-3}$ would indicate that the solutions to the transport equations varied by no more than 0.1%.

As can be seen in Figure 8.3, the values of the normalised radial velocity for the three different grid levels produce very similar values and any further refinement in the computational grid would not increase the accuracy of the solution. For this reason and in order to minimise the computational effort required, the coarse grid was used from this point onwards in the calculation of the flow variables.
As stated in section 7.2.1, a dense mesh allowed the turbulent boundary layer developing from the impeller to be resolved completely without invoking wall functions. This required that the $y^+$ value of the grid needs to be less than 5. As can be seen in Figure 8.4, which illustrates the $y^+$ for the Coarse mesh, this criteria was fulfilled.

Figure 8.3: Radial velocity profiles on three grid levels

Figure 8.4: Impeller $y^+$ values
8.2.1 Drag Law Comparison

Within the model developed two different drag laws were used in the simulation of the L-glutamic acid polymorphs. The Wen and Yu drag law was invoked to calculate the drag induced by the α particles. The drag law proposed by Loth (2008) and implemented by this author within the CFD code was used to model the needle-like β particles. To this authors knowledge, the simulation of needle-like particles using CFD has not been undertaken previously. In order to examine the variation in the results produced by the drag laws, a comparison was made using equivalent diameters of the particles. Table 8.3 displays the coefficient of drag, $C_D$, produced for a range of equivalent diameters using both drag correlations at a relative solid phase velocity of 0.1m/s.

<table>
<thead>
<tr>
<th>Equivalent Particle Diameter ($\mu m$)</th>
<th>$Re_p$ (-)</th>
<th>$C_D$, Wen and Yu (-)</th>
<th>$C_D$, UDF (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.45</td>
<td>9.97</td>
<td>6.24</td>
</tr>
<tr>
<td>40</td>
<td>2.89</td>
<td>5.75</td>
<td>3.75</td>
</tr>
<tr>
<td>60</td>
<td>4.34</td>
<td>4.26</td>
<td>2.86</td>
</tr>
<tr>
<td>80</td>
<td>5.79</td>
<td>3.49</td>
<td>2.39</td>
</tr>
<tr>
<td>100</td>
<td>7.23</td>
<td>3.01</td>
<td>2.09</td>
</tr>
</tbody>
</table>

Table 8.3: Coefficient of drag, $C_D$, comparison between Wen and Yu and custom UDF

The comparative simulations involved the presence of α particles with an average diameter of 100$\mu m$ and β particles with an average equivalent diameter of 100$\mu m$ within the stirred tank model. The equivalent diameter was calculated as described in section 6.2.2. The Wen and Yu model and Loth correlation were used to calculate the respective drag coefficients. The impeller rotational speed was set to 250rpm to ensure the just-suspended solid requirement was fulfilled. Figure 8.5 and Figure 8.6 illustrate the solid-liquid interfaces for the α and β phases, respectively. The interface height is shown to exist at approximately 45-50% of the overall tank height. A comparison is also shown in Figure 8.7 where volume fractions of the two polymorphs at various heights can be analysed in more detail.
As can be seen in Figure 8.7, the volume fraction remains constant for each polymorph up to approximately 30-40% of the overall height at which point there is a sharp reduction in the volume fraction present. This reflects the diffuse nature of the solids loading until the clear solid-liquid interface is achieved at 50% of the tank height. Both polymorphs have the same equivalent diameter which means they should have coinci-
dent solid-liquid interfaces for the two drag laws. Figure 8.7 shows a negligible variation between the volume fraction plots and proves that the newly employed drag law for the \( \beta \) particles accurately predicts the drag on the needles.

![Figure 8.7: Volume fraction of \( \alpha \) and \( \beta \) phase at three radial distances](image)

Figure 8.7: Volume fraction of \( \alpha \) and \( \beta \) phase at three radial distances
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8.3 Results and Discussion

8.3.1 Suspension of $\alpha$ Particles

A number of simulations were performed in order to model the suspension of the $\alpha$ particles in ethanol with a 10% solids loading. The range of impeller rotational speed was varied from 250rpm to 600rpm. The Wen and Yu drag law was used to model the drag induced by the $\alpha$ particles. Figure 8.8 displays the calculated fluid phase mean velocity magnitude at an impeller rotational speed of 400rpm. The velocity field is illustrated with two clear recirculating zones either side of the impeller shaft which is the typical flow pattern produced with an axial flow impeller.

![Figure 8.8: Velocity magnitude vector plot at 400rpm](image)

The relationship between the impeller rotational speed and the turbulent kinetic energy is illustrated in Figure 8.9. A high level of turbulent kinetic energy is desirable in the lower region of the tank to enhance the suspension of the particles (Srinivasa and Jayanti, 2007). As the rotational speed is increased, a clear increase in the turbulent kinetic energy can be observed. The greater level of turbulent kinetic energy associated with a rotational speed of 400rpm results in an improved level of suspension in the tank. The cloud height is increased from 55 to 67% as the rotational speed is increased from 300 to 400rpm, respectively. This result is presented in more detail in Figure 8.17.
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Figure 8.9: Turbulent kinetic energy profile at 300 and 400rpm in stirred tank for the \( \alpha \) polymorph

Figures 8.10 - 8.15 display the contour plots of the volume fraction at an impeller rotational speed of 300, 400 and 500 rpm and the corresponding calculated volume fractions in the suspension. The volume fraction contours were plotted on a vertical plane through the centre of the stirred tank, mid-way between the baffles. The simulated volume fractions were monitored beginning at a radial distance of 3cm from the centre of the impeller shaft and at 1cm increments thereafter. The experimental volume fraction samples were extracted from the tank at a radial distance of approximately 6cm from the tank centre at four heights as described in section 8.1.2.

An examination of Figure 8.12 clearly shows the qualitative agreement between the multiphase CFD simulations and the corresponding experimental stirred tank image at 400rpm. The solid-liquid interface exists at approximately 65% of the overall height at this rotational speed, as illustrated in both the CFD contour plot and the experimental stirred tank image. Figures 8.8 and 8.12 also illustrate the influence of the flow pattern on the cloud height. The change in direction of the flow from the PBT takes place at approximately 66% of the overall height of the tank and this coincides with the solid-liquid interface in the tank. This is in agreement with the study performed by Oshinowo and Bakker (2002).

The quantitative volume fraction result for an impeller rotational speed of 400rpm is presented in Figure 8.13. The volume fraction remains relatively constant for the majority of the suspension but falls away rapidly near the location of cloud height. The decrease in volume fraction begins at approximately 48% of the overall tank height at
radial distance of 3cm from the centre of the impeller shaft and at 58%, 6cm from the centre. The solid-liquid interface exists at 68% of the overall tank height. A point of inflection is clearly displayed representing the cloud height as described in section 6.1.2.

The experimental volume fraction results extracted from the stirred tank are also displayed in Figure 8.13. Using the experimental method described in section 8.1.1, a close agreement between the volume fraction samples and the simulated volume fraction profile was established throughout. Two samples of solution were taken from the bulk flow below the solid-liquid interface and the remaining samples extracted from the clear solution that existed above the interface.

The volume fraction results obtained underneath the impeller (r=0cm) and at the tank periphery (r=7.4cm) reveal the accumulation of solids in these areas. The volume fraction curves at these radial distances trail off along the tank bottom. This is due to the inability of the fluid to provide complete suspension of all the particles at 400rpm. This is despite the calculation of a just-suspended speed of 199rpm for the particles being studied in this work. This aspect of the suspension is also displayed in Figure 8.16. In order to achieve a more complete suspension of the particles further increases in the impeller rotational speed would be required. This result can be observed in Figure 8.15 where the increase in impeller velocity reduces the volume fraction of solids that exist underneath the impeller and along the periphery of the tank. This feature of the solids suspension had not been originally anticipated based on the analytical approaches described in section 6.1.

A close agreement between the experimental and simulated results is prevalent. Further agreement can be ascertained in Figures 8.11 and 8.15 at impeller rotational speeds of 300 and 500rpm. Additional experimental and simulated results at other rotational speeds are presented in Appendix C.
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Figure 8.10: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 300rpm

Figure 8.11: $\alpha$ solid phase volume fractions at 300rpm
Figure 8.12: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 400rpm

Figure 8.13: $\alpha$ solid phase volume fractions at 400rpm
Figure 8.14: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 500rpm

Figure 8.15: $\alpha$ solid phase volume fractions at 500rpm
The solid-liquid phase interface results from the simulations and experiments are combined in Figure 8.17. The solid-liquid interface height can be clearly seen to increase as the impeller rotational speed increases. This is in line with the suspension theory which states that further increases beyond the just-suspended impeller speed will produce an increase in solid-liquid interface height. A comparison between the simulated and experimental results demonstrates a close agreement. The average error between the predicted and experiment cloud heights was calculated to be 4%. This confirms the reliability of the simulations in the prediction of the solid-liquid interface for the $\alpha$ particles in the stirred tank.
The modelling of a suspension of the $\alpha$ polymorph was successfully performed in the previous section. In order to validate the newly developed user defined function (UDF) for non-spherical particles, the simulation of the $\beta$ polymorph suspension is investigated in this section. A series of suspension experiments at a number of impeller rotational speeds were carried out using the method described in section 8.1.2. The impeller rotational speed was varied from 250rpm to 600rpm in increments of 50rpm. These conditions were used within Fluent to model the suspension of the $\beta$ polymorph. The drag law developed by Loth (2008) was incorporated into Fluent in order to model the drag induced by the $\beta$ particles.

The turbulent kinetic energy developed by the impeller in the tank is illustrated in Figure 8.18. The increase in turbulent kinetic energy emanating from the impeller as the rotational speed is increased from 300 to 400rpm can be clearly seen. This enables a more complete suspension of the $\beta$ particles due to a greater level of energy being available to lift the particles. This is follows on from the analysis of suspensions provided.

Figure 8.17: Simulated and experimental solid-liquid interfaces in stirred tank for the $\alpha$ polymorph

### 8.3.2 Suspension of $\beta$ Particles

The modelling of a suspension of the $\alpha$ polymorph was successfully performed in the previous section. In order to validate the newly developed user defined function (UDF) for non-spherical particles, the simulation of the $\beta$ polymorph suspension is investigated in this section. A series of suspension experiments at a number of impeller rotational speeds were carried out using the method described in section 8.1.2. The impeller rotational speed was varied from 250rpm to 600rpm in increments of 50rpm. These conditions were used within Fluent to model the suspension of the $\beta$ polymorph. The drag law developed by Loth (2008) was incorporated into Fluent in order to model the drag induced by the $\beta$ particles.

The turbulent kinetic energy developed by the impeller in the tank is illustrated in Figure 8.18. The increase in turbulent kinetic energy emanating from the impeller as the rotational speed is increased from 300 to 400rpm can be clearly seen. This enables a more complete suspension of the $\beta$ particles due to a greater level of energy being available to lift the particles. This is follows on from the analysis of suspensions provided.
Analysis of the contour and volume fraction plots (Figures 8.19 - 8.24) associated with the suspension of the needle-like \( \beta \) polymorph provide similar results to those obtained for the prismatic \( \alpha \) polymorph. The novel implementation of the drag law for non-spherical particles produced a good agreement between the experiments and CFD simulations.

Figures 8.21 and 8.22 illustrate the comparison between the simulated and experimental results when an impeller rotational speed of 400rpm was employed. Figure 8.21 reveals the existence of the solid-liquid interface at approximately 65\% of the overall tank height and this agrees with the stirred tank image used in the experiments. A quantitative comparison is presented in Figure 8.22. The initial solid volume fraction remains at 11\% up to approximately 45\% of the height for the at a radial distance of 3cm and 57\% for a radial distance of 6cm. A sharp decrease in the volume fraction is observed until the solid-liquid interface is reached at 72\% of the height. This represents the cloud height for a rotational speed of 400rpm. The accumulation of solids underneath the impeller (\( r=0 \)cm) and along the tank periphery (\( r=7.4 \)cm) is also evident due the increase in volume fraction present in these areas. This also leads to the conclusion that the impeller speed used in this example is insufficient for suspension of all the particles in the tank. This is despite the preliminary analysis performed on suspensions (section 6.1). Further increases in impeller rotational speed would be necessary to lift all the particles from the base of the tank.
A close agreement exists between the experimental volume fractions measured and the CFD predictions at this speed. Inspection of the $\beta$ suspension CFD simulations at alternative rotational speeds also provide a close fit to experiments as displayed in Figures 8.19, 8.20, 8.23 and 8.24.

Figure 8.19: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 300rpm

Figure 8.20: $\beta$ solid phase volume fractions at 300rpm
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Figure 8.21: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 400rpm

Figure 8.22: $\beta$ solid phase volume fractions at 400rpm
Figure 8.23: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 500rpm.

Figure 8.24: $\beta$ solid phase volume fractions at 500rpm.

Figure 8.25 compares the solid-liquid interfaces extracted from experiments and CFD simulations for the $\beta$ polymorph suspension. An increase in rotational speed produces the expected increase in cloud height in all cases. The simulations produce a slight overprediction of the cloud heights but the average error between simulated and experimental results is calculated to be 8%. 

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The overprediction present in the simulations have been attributed to the method used to calculate the equivalent diameter within the UDF. The equivalent diameter of the $\beta$ particles were calculated under the assumption that an average shape and size of needle can be used to describe the entire population of the needles in the suspension. The production of $\beta$ particles for the experimental analysis in this treatise initially required the polymorphic transformation of $\alpha$ to $\beta$. This was followed by sieving the final mass of $\beta$ into size classes in order to reduce the distribution of the particle sizes present in the suspension. The sieving process has a degree of experimental error associated with it and therefore the probability of achieving an ideal particle size fraction is reduced. This contrasted with the development of the UDF which assumed an ideal crystal size and shape in order to calculate an equivalent diameter.

The equivalent diameter calculated in the UDF provided the surface and projected area ratios calculated in equation 6.12. These ratios were combined to develop a shape factor used to normalise the particle Reynolds number in equation 6.18. The resulting normalised Reynolds number was incorporated into the normalised drag coefficient equation.
The drag induced by the $\beta$ particles within the suspension was calculated using this approach.

A comparison between the turbulent kinetic energy profiles in Figures 8.9 and 8.18 revealed a variation in the level of turbulent kinetic energy present around the impeller. This was due to the numerical method used to calculate turbulence within the simulations. The RNG $k-\epsilon$ turbulence model had been employed to describe the turbulent properties of the mean flow. The multiphase models solved in the previous sections also required the effects of the solid phase to be included in the calculation of a solution. In order to account for this, a dispersed phase turbulence model was used in conjunction with the RNG $k-\epsilon$ turbulence model. This provided modifications to the RNG $k-\epsilon$ turbulence model and allowed the influence of the solid phase on the fluid phase to be implemented within the solution. As the needle-like $\beta$ phase consisted of smaller particles than the prismatic $\alpha$ phase, the $\beta$ phase did not influence the continuous phase to the same extent as the $\alpha$ phase. A comprehensive analysis of this numerical method is described in Fluent Inc (2006).

### 8.3.3 Suspension of $\alpha$ and $\beta$ Particles

The previous sections illustrated the ability of the CFD simulations to successfully model the suspension of $\alpha$ and $\beta$ particles at a range of impeller speeds. An examination of a suspension containing both polymorphs was subsequently undertaken using the numerical model validated in the sections 8.3.1 and 8.3.2. The $\alpha$ polymorphs have a diameter of 100$\mu$m throughout and the $\beta$ diameters were examined at 40 and 60$\mu$m. The volume fraction of each polymorph was specified as 5%. The Wen and Yu drag law was used to model the drag induced by the $\alpha$ polymorphs and the custom UDF was used in the modelling of the $\beta$ polymorphs. The simulations were carried out with impeller rotational speeds of 400 and 600rpm. The results of the simulations are displayed in Figures 8.26 and 8.27.

The solid volume fractions illustrated in Figure 8.26 clearly reveal the solid-liquid interface within the suspension. The cloud height exists at approximately 73% of the overall tank height which is slightly higher than the cloud height produced in the exclusively $\alpha$ suspension with particles of diameter 106$\mu$m (Figure 8.13). This slight variation is due to the increase in energy required to suspend the larger particles in the tank. It can be
seen how the increase in impeller speed from 400 to 600rpm increases the cloud height which is in line with the results reported in the previous sections.

Figure 8.27 displays the solid volume fractions when the diameter of the $\beta$ particles was increased to 60$\mu$m. As discussed in section 6.2.2, the approach used to calculate the drag using the UDF involved the calculation of equivalent diameters based on the projected and surface areas of the particles. Using these calculations, a $\beta$ particle with diameter 60$\mu$m would produce an equivalent diameter of approximately 144$\mu$m. As a result this, the energy generated was not sufficient to suspend the larger particles and this results in a reduction in the cloud height when compared to Figure 8.26 at both 400 and 600rpm.

Analysis of Figures 8.26 and 8.27 also reveals a change in the volume fractions as the speed is increased from 400 to 600rpm. The volume fraction in Figure 8.26 is approximately 5.5% but this is reduced to 5.1% as the speed is increased. This is a result of the greater level of suspension within the tank at 600rpm. At 400rpm the full suspension of the solid phase has not been achieved and therefore there is a greater volume of particles in the lower part of the tank. As the speed is increased, a more homogeneous solution results and then volume fraction approaches 5%.

This behaviour is repeated in Figure 8.27. The $\alpha$ and $\beta$ particles have a volume fraction of approximately 5.5 and 6%, respectively. The higher volume fraction of the $\beta$ particles is a result of their larger equivalent diameter and resultant lower level of suspension at 400rpm. The influence of the increased impeller speed is illustrated as the difference between the $\alpha$ and $\beta$ volume fractions is reduced from 0.3 to 0.15% at 600rpm.
Figure 8.26: Numerically derived $\alpha$ and $\beta$ solid phase volume fractions with diameters of $100\mu m$ and $40\mu m$, respectively
8.3.4 Cloud Height Comparison

The following section discusses the comparison between the cloud heights resulting from the work undertaken in this treatise and those results predicted using the model developed by Bittorf and Kresta (2003). The correlation has been previously described in section 6.1.2. The tank configuration used in the analysis is described in Table 8.1. Figure 8.28 illustrates the comparison between both sets of results.
Figure 8.28: Comparison between the experimental cloud heights and those predicted by Bittorf and Kresta (2003).

An approximately linear relationship is shown to exist between the cloud heights calculated from the experiments performed in this dissertation and the impeller rotational speed. A similar relationship exists between the results predicted using the correlation developed by Bittorf and Kresta (2003), however the cloud heights exhibit a greater sensitivity to increases in the impeller speed. The variation between the cloud heights predicted using the model from Bittorf and Kresta (2003) and the experimental results could be due to the difference in impeller used in each example. The predicted results were calculated based on experiments performed using A310 and HE3 impellers with flow patterns that differ with those produced using the PBT employed in this work. The A310 and HE3 impellers produce a pure axial flow which has been shown to suspend particles more efficiently than a PBT (Kasat and Pandit, 2005). The use of the PBT in this treatise required increased impeller speeds to suspend the particles to the same level as the axial impellers employed in Bittorf and Kresta (2003). The development of a more comprehensive cloud height model would need to encompass a greater variety of impeller types and geometric configurations.
8.3.5 Homogeneity

The results discussed so far have focused on the the volume fractions at a selection of radial distances and heights in the tanks. A further investigation in the volume fractions in the bulk of the solid phase and just below the solid-liquid interface is discussed in this section. The analysis was performed on the $\alpha$ suspension at 400rpm but the findings are applicable to both the $\alpha$ and $\beta$ systems at each impeller speed. An examination of Figure 8.13 illustrated a constant solid volume fraction at each radial distance until the curves begin to separate at approximately 40% of the tank height. The solid volume fraction at this height is displayed in Figure 8.29. A negligible variation in the volume fraction exists at this height which signifies the presence of a homogeneous suspension up to this point. A homogeneous mixture within the stirred tank ensures that the maximum crystal surface area is available for mass transfer processes. Good mixing also reduces the variation in the supersaturation levels throughout and increases the efficiency of the crystallisation processes that take place.

Figure 8.30 displays the solid-volume fraction at a height of approximately 55% of the tank height. A comparison with Figure 8.13 reveals that the volume fractions at this height are no longer uniform and vary until approximately 60% of the height. Figure 8.30 illustrates this variation which implies that the mixing levels within this range of suspension heights are not as uniform as those in the bulk of the suspension (up to 40% of the tank height at 400rpm). This poor mixing would lead to uneven supersaturation levels and a reduction in the efficiency of the crystallisation processes. The reason for this variation can be attributed to axial flow pattern in the tank illustrated in Figure 8.8. A noticeable decrease in the velocity is shown at 55% of the tank height in the region around the impeller shaft. This reduction in velocity causes a significant decrease in the energy available for the suspension of the particles in this area. As a result of this, a reduction in the solid volume fraction occurs in this area before a decrease is established in the remainder of the suspension.

Following the comparison between Figures 8.13, 8.29 and 8.30 it has been shown that the bulk of the suspension provides the environment for efficient mass transfer processes. However, a reduction in the suspension homogeneity occurs due to a decrease in the energy available for particle suspension in the area around the impeller shaft. This behaviour is also shown for $\alpha$ suspensions at other impeller speeds and also for the $\beta$ suspension analysis described in section 8.3.2.
Figure 8.29: Volume fraction contour plot in the $\alpha$ suspension at 400rpm at 40% of the overall tank height

Figure 8.30: Volume fraction contour plot in the $\alpha$ suspension at 400rpm at 58% of the overall tank height
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8.3.6 Suspensions and Polymorphism

In the previous sections, the suspension of the $\alpha$ and $\beta$ polymorphs in ethanol within a stirred tank was examined. At the lower impeller rotational speeds, it was discovered that not all the particles in the tank were suspended. This was in contrast to the theoretical analysis of the just-suspended speed undertaken in section 6.1. The inability to achieve complete suspension of the particles would have a large influence of the crystallisation processes that would occur in the tank.

If there was incomplete suspension in a vessel used for crystallisation, there could be a significant effect on the mass transfer processes that occur. As smaller particles are being suspended, the larger ones may be resting on the bottom of the vessel inhibiting their exposure to the solute in the bulk of the solution. This would imply that the overall growth of particles in the tank would drop off considerably once a certain crystal size was achieved as there would be a lack of crystal surface area available for addition of solute molecules.

The rate of dissolution of the metastable $\alpha$ polymorph to the stable $\beta$ polymorph may also be affected by inefficient suspension. The diffusion of molecules from the surface of the crystal to the bulk solution would be reduced if there was a decrease in the surface area exposed to the solution. Particles remaining along the bottom of the tank would decrease the area available for the transfer of molecules. As a result of this the dissolution rate of the $\alpha$ polymorph would decrease and the rate of transformation from $\alpha$ to $\beta$ would subsequently decrease.

The possible presence of agglomeration and particle fragmentation within the L-glutamic acid system was discussed in sections 5.5 and 5.6. Examining the influence of particle suspension on these processes could provide an insight into possible methods of alleviating agglomeration and breakage processes within the tanks. In an effort to achieve complete suspension of the particles, the impeller speed may be increased to such a level as to increase the particle breakage that occurs in the tank. This was one of the conclusions obtained by Hutton (2009) in the analysis of gibbsite precipitation. Increases in impeller speed could also reduce the level of agglomeration that would occur due to higher fluid shear levels. This would reduce the probability of crystals remaining together after collisions in the tank.
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The coupling of the crystallisation modelling within a CFD framework would also provide a detailed insight into the effect of the flow and suspension properties on the crystallisation process. Unfortunately, the implementation of a single model encompassing both crystallisation and suspensions using CFD was beyond the scope of this treatise.

8.4 Summary

In this chapter, the accuracy and validity of employing a non-spherical particle drag law within CFD was examined. A comparison between the Wen and Yu drag law and the non-spherical particle drag law was undertaken for a suspension consisting of equal volumes of $\alpha$ and $\beta$ particles with equivalent diameters. The modelling of the $\beta$ particles required the novel implementation of the drag law for non-spherical particles. A series of CFD simulations were also performed for separate $\alpha$ and $\beta$ suspensions. The solid volume fractions within the suspensions were monitored and the solid-liquid interfaces were calculated. The results generated using the CFD simulations were validated through experiment. The simulation of a combined $\alpha$ and $\beta$ suspension was also carried out and the results analysed. An examination of the volume fractions just below the solid-liquid interfaces revealed reduces levels of homogeneity around the impeller shaft for both the $\alpha$ and $\beta$ suspensions. This was attributed to a reduction in the energy available for particle suspension resulting from decrease in the fluid velocity in this region. The assumptions made in the development of the UDF and the error associated with the experimental analysis provide an explanation for the slight deviation in the simulated and experimental results. However, a very close agreement between the CFD predictions and experiments resulted overall using the newly developed drag law for non-spherical particles. The novel UDF implemented in this work has the capacity to be configured for a wide variety of non-spherical particles.
Summary and Conclusions

The primary aims of the research presented in this thesis were to develop a numerical model which allowed the accurate simulation of the L-glutamic acid polymorphic system and to simulate suspension of non-spherical particles in a stirred tank. This chapter will briefly review the work undertaken in the course of the research and outline the conclusions that arose from the investigation.

9.1 Summary

9.1.1 Polymorphic Transformation

The main crystallisation processes that exist within the L-glutamic acid system are growth, nucleation and dissolution. Initially the nucleation and growth of the $\alpha$ polymorph dominate the process but the subsequent dissolution of the $\alpha$ polymorph allows the final growth of the $\beta$ polymorph. According to previous studies in the area, the nucleation of the L-glutamic acid crystals can be described using classical nucleation theory (Kitamura, 1989). The growth of the crystals may be represented by the birth and spread growth model (Kitamura and Ishizu, 2000; Scholl et al., 2006a, 2007) which describes the formation and spreading of nuclei across the crystal surface. The dissolution of the
α polymorph was based on the method put forward by Koiranan et al. (1999) which assumes that mass transfer is the only process applicable and equates dissolution to negative growth. The Sherwood correlation was used to model the mass transfer from the crystal to the bulk solution. Experimental studies carried out by Scholl et al. (2006a) confirmed that the mass transfer only assumption was valid for the L-glutamic acid system (section 4.3). An equation was also developed (equation 4.25) to account for the overall change in solute within the system due to the nucleation, growth and dissolution processes.

A number of numerical methods can be found in the literature to model the processes described above. This involves calculating a solution to the population balance equation. The Method of Classes was deemed too computationally expensive and the Quadrature Method of Moments (QMOM) was overly complicated for the system of equations that needed to be solved. The assumption of negligible agglomeration and breakage based in previously reported studies negated the need for the QMOM. The Method of Moments (MOM) was chosen due to its ease of implementation, applicability and low computational requirements.

One of the main requirements of the numerical model was to accurately reproduce the volume based particle sized distribution (PSD). The PSD reconstruction method developed by Hutton (2009) was refined in this work and implemented within the code. The reconstruction method involved the generation of moment surfaces and the comparison of simulated length based moments with these surfaces. These comparisons resulted in the construction of moment isolines. The approximate intersections of the isolines were calculated and formed the vertices of a triangle, the centroid of which revealed the $n$ and $\delta$ values which were used to define the PSD. The method was proven to reliably reconstruct the PSD using a combination of predefined moment surfaces and the simulated moments. The maximum error between original and reconstructed moments was calculated to be 2%.

The PSD reconstruction method was used to account for the reduction in the number of α particles in the system due to dissolution of the α polymorph. As the size of the α particles decreased due to dissolution, the PSD shifted toward the smaller particle sizes. The PSD was subsequently discretised and particles present in the smallest class size were removed from the system. The loss of these particles provided the reduction in
the zeroth moment due to the dissolution of the $\alpha$ polymorph. The remaining moments also decreased in magnitude using equation 4.31.

A series of experiments were performed in order to verify that the methods used by Scholl et al. (2006a) to calculate the coefficients within kinetic equations were accurate. The initial experimental conditions incorporated initial solute concentrations of 43, 48 and 53 g/kg of solvent within a solution. The solution was heated to 80°C and cooled to 45°C at which temperature it was maintained. The final yield of the crystals produced experimentally within this dissertation agreed with those of Scholl et al. (2006a) with a maximum error of 7.7%. The numerical model was subsequently tested and the results compared to experimental results in the literature. The solution and solid concentrations were accurately modelled and provided an excellent agreement between the experimental and simulated results. Other process parameters that were monitored within the simulations included the growth and nucleation rates (Table 5.3), average size of the polymorphs and the supersaturation levels throughout the entire process. The processes which determined at what rate the polymorphic transformation occurred was established to be the nucleation and growth of the $\beta$ polymorph and not the dissolution of the $\alpha$ polymorph. If the $\beta$ polymorph nucleated and grew faster than $\alpha$ polymorph dissolved then a decrease in the solution concentration would be visible as the transformation progresses. As the solution concentration remained constant until $\alpha$ had completely dissolved (Figure 5.9), it was concluded that the $\beta$ crystallisation processes were the rate-determining steps.

The numerical model utilising the PSD reconstruction technique generated a narrow distribution which was centered at approximately the 95 $\mu$m particle diameter for the $\alpha$ polymorph. The PSDs produced using the Horiba PSA displayed a much broader distribution indicating the possibility of agglomeration within the system. The presence of $\alpha$ polymorph agglomeration was suspected to have produced the broad experimental PSDs. This was confirmed using with optical microscopy. Narrow simulated PSDs were also presented 12 hours into the process with peaks at approximately 300 $\mu$m. This compared to the broad experimental PSDs with peaks at approximately 200 $\mu$m. The discrepancies between the numerically derived and experimental PSDs after 12 hours were attributed to the existence of particle breakage or attrition within the tank environment. The possibility of particle breakage having occurred during the PSD measurement process was disproved after analysis of the measurement technique used. The recircula-
tion of the L-glutamic acid samples in the Horiba PSA over a period of 5 minutes did not produce any change in the form and position of the PSDs and therefore no breakage could have occurred during the PSD measurement process.

The presence of agglomeration and breakage within the system under the conditions presented had not been considered based on the previously reported literature. An effort was made to incorporate empirical agglomeration and breakage kernels into the numerical model and implement the quadrature method of moments (QMOM) but unfortunately a satisfactory result could not be attained. This was attributed to the inability to add the agglomeration and breakage processes to the model without a complete reevaluation of the kinetic equations presented in the literature. Furthermore, the coefficients of the governing kinetic equations would also need to be refit following an extensive experimental analysis.

9.1.2 Suspension of Polymorphs

The suspension of spherical particles in stirred tanks have been of considerable interest in the process industry with numerous studies undertaken. Experimental analysis of the suspension of non-spherical particles has been studied in detail over the years (Haider and Levenspiel, 1989; Ganser, 1993; Chhabra et al., 1999; Fan et al., 2004; Loth, 2008) but to this authors knowledge the existing literature contained no examination of the numerical modelling of non-spherical particles in a suspension. A comprehensive analysis of this area required an investigation into the factors which influence suspensions.

The efficient suspension of crystals within a stirred tank is dictated by a number of factors including tank geometry, impeller rotational speed and particle size. The just-suspended speed, \(N_{js}\), was found to be dependent on the type of flow pattern produced by the impeller, with the axial flow impeller being the most efficient for the suspension of particles (Armenante and Nagamine, 1998). The solid-liquid interface within the suspension was also found to increase with increasing impeller rotational speed.

It has been shown by a number of authors (Armenante and Nagamine, 1998; Sharma and Shaikh, 2003) that the clearance height from the base of the tank to the impeller is directly related to the impeller rotational speed required to just-suspend the particles.
within the tank. Based on the studies performed by Sharma and Shaikh (2003), it was found that a clearance-to-vessel diameter (C/T) ratio between 0.1 and 0.35 provided a maximum transfer of energy to the particles for a stirred tank employing a 4-blade 45° PBT. Using this information, a stirred tank was designed and built with a C/T of 0.2 and impeller diameter-to-tank diameter ratio (D/T) of 0.33. Four baffles were also constructed in order to greatly reduce the possibility of a vortex developing in the centre of the tank. The $N_{js}$ was calculated to be 199rpm for particles diameters equal to 106µm.

The numerical simulation of the solid and liquid phases in the tank were undertaken using Gambit v2.4.6 and Fluent v6.3.26. An unstructured grid incorporating polyhedral cells was generated in order to reduce the computational effort required for the simulations. The introduction of polyhedral cells reduced the overall cell count in the grid up to a factor of 5 without compromising the accuracy of the solution. The RNG k-ε turbulence model was chosen to obtain closure of the governing fluid flow equations. The multiple reference frame (MRF) model was used to modify the underlying fluid flow equations and the hybrid near-wall modelling method was used to resolve the flow in the near wall region. Following a comparison of Upwind and Power law discretisation schemes within the numerical model, it was established that the Power law produced a more accurate prediction of the axial, radial and tangential velocities in a stirred tank and was subsequently applied to the numerical model thereafter. The modelling of the dispersed phase was achieved using the Eulerian multiphase model. The Wen and Yu drag law was used so calculate the drag coefficient for the α polymorphs. The needle-like β particles required the development of a novel user-defined function (UDF) which was based on the coefficient of drag correlation presented by Loth (2008) (Equation 9.1). Numerous drag laws were investigated but the law drag developed by Loth (2008) was deemed to most applicable to a large selection of non-spherical particles. The equivalent diameters of the non-spherical particles were calculated by approximating the shapes of the particles using the surface and projected area ratios and the normalised particle Reynolds number was subsequently calculated. Equation 9.1 is applicable to Reynolds numbers in the Stokes, Newton and intermediate drag regimes.

\[
C_D^* = \frac{24}{Re_p^*} \left[ 1 + 0.15(Re_p^*)^{0.687} \right] + \frac{0.42}{42500} \left( \frac{1}{Re_p^*} \right)^{1.16} \quad (9.1)
\]

The Wen and Yu drag law was compared to the drag law proposed by Loth (2008) in order to test the performance of the UDF that was to be used in subsequent models. A
stirred tank was modelled with the application of the Wen and Yu and Loth (2008) drag laws to the α and β particles, respectively. The solution phase contained 5% of each polymorph per volume. The α particles had a diameter of 100µm with the β particles possessing an equivalent diameter of 100µm. The volume fractions predicted using the UDF provided a very close agreement to those of the widely validated Wen and Yu drag law. This result illustrated the high degree of accuracy and precision of the UDF developed in this treatise and enabled the subsequent detailed analysis of the α and β suspensions.

The suspension of the α and β polymorphs was modelled using the approach described above and the distribution of the solid and liquid phases investigated at impeller rotational speeds ranging from 250rpm to 600rpm. The stirred tank simulations were also compared to experiments carried out in the rig described in section 8.1.3. The height of the solid-liquid interface was monitored by taking samples from the tank and various heights and calculating the volume fraction present at each height. The results from the α polymorph experiments were compared to the simulations and an excellent agreement was achieved at a wide range of rotational speeds with an average error of 4% resulting (Figures 8.10 - 8.15). The β polymorph suspension was also assessed using CFD and experiments. The solid-liquid interface predicted by the simulations contained an average error of 8% which was attributed to experimental error and assumptions made within the novel UDF. A combined α and β suspension was also successfully simulated and the results analysed. Overall, a very good agreement between simulated and experimental results was achieved for both the α and β suspensions.

The volume fractions in the region just below the solid-liquid interfaces were also examined and revealed a reduced level of homogeneity for both the α and β suspensions. This was in contrast to the volume fraction levels in the bulk of the suspension which remained constant throughout signifying a good level of mixing. This variation was attributed to a reduction in the energy available for particle suspension due to a decrease in the velocity around the impeller shaft at heights just below the solid-liquid interface. The favourable mixing environment in the bulk of the suspension increases the efficiency of the mass transfer processes that take place between the solid and liquid phases which is of great importance in the process industry.
Chapter 9. Summary and Conclusions

9.2 Conclusions

A numerical model was developed to simulate the polymorphic transformation that occurs in the L-glutamic acid system. The suspension of the $\alpha$ and $\beta$ polymorphs in a stirred tank was also investigated. The following conclusions were made during the course of this work.

1. The birth and spread model can be used to model the growth of L-glutamic acid crystals.

2. The Method of Moments can be used to successfully describe the population balance and particle size distributions.

3. A refinement to the PSD reconstruction technique was shown to accurately reproduce the moments of a PSD using a comparison between moment surfaces and simulated moments.

4. A reduction in the number of particles due to the dissolution of the $\alpha$ polymorph can be provided for using a particle death term.

5. The experimental yield generated in this work led to the conclusion that the method used in the subsequent experimental analysis was accurate and repeatable.

6. The solution and solid concentrations produced in the simulations were in excellent agreement with published data.

7. The rate determining step in the transformation process is the nucleation and growth of the $\beta$ polymorph and not the dissolution of the $\alpha$ polymorph.

8. A comparison between experimental and simulated PSDs reveal that the assumption of negligible agglomeration and particle fragmentation within the system under the conditions specified in the literature was flawed.

9. The addition of further crystallisation processes to an existing simulated system cannot be achieved without the refitting of coefficients to the kinetic equations.

10. The conclusions drawn from the single phase flow within the stirred tank are:

   - Using polyhedral cells to construct the computational grid greatly reduced the computational effort required without sacrificing accuracy.
11. The conclusions drawn from the two-phase solid-liquid flow within the stirred tank are:

- The Eulerian multiphase model in conjunction with the RNG k-\( \epsilon \) turbulence model provided an accurate method of simulating the suspension of solids in a stirred tank.
- The drag law proposed by Loth (2008) was successfully implemented into a novel user-defined function (UDF) within the Fluent software. This allowed the successful simulation of non-spherical particle suspensions.
- A comparison between the simulated \( \alpha \) and \( \beta \) solid volume fractions for particles with equivalent diameters produced an excellent agreement using the Wen and Yu drag law and the UDF, respectively.
- The cloud heights for the \( \alpha \) suspension were accurately predicted using CFD in conjunction with the Wen and Yu drag law throughout the range of impeller speeds examined.
- The numerically derived solid volume fractions closely matched the experimental results for both the \( \alpha \) and \( \beta \) suspensions.
- The simulation of a combined \( \alpha \) and \( \beta \) suspension was successfully performed at various impeller rotational speeds.
- The custom drag law for non-spherical particles accurately predicted the cloud height of the \( \beta \) suspension for a range of impeller rotational speeds, validating the accuracy of the UDF.
- Homogeneous mixing was present within the bulk of the suspensions for the range of impeller speeds but a reduced level of mixing existed in the region just below the solid-liquid interface around the impeller shaft.

9.3 Original Contributions

The following are the original contributions to the body of knowledge regarding the numerical modelling of the L-glutamic polymorphic system and the CFD analysis of polymorphic systems.

- A solute concentration balance equation incorporating the nucleation, growth and dissolution mechanisms was employed in the model and allowed the accurate prediction of solution concentration throughout the polymorphic process.
• A particle death term was developed and implemented to account for the reduction in the zeroth moment due to the dissolution of the $\alpha$ polymorph.

• A refinement to the PSD reconstructing technique was implemented to improve the stability of the reconstruction method and this technique was applied to a polymorphic system.

• The disappearance of $\alpha$ particles from the system due to dissolution was modelled using the PSD reconstruction technique.

• It was established that agglomeration and breakage processes were required to accurately model the L-glutamic acid system under the conditions specified in this treatise.

• The development and implementation of a method to model the drag induced by non-spherical particles within CFD was achieved.

• The suspension of $\beta$ polymorphs in a stirred tank were successfully modelled using CFD and validated using experiments.

9.4 Recommendations for Further Work

In the development of the numerical model for the simulation of the polymorphic system, the initial assumptions omitted the agglomeration and breakage processes. This was reflected in the kinetic equations used to model the system and resulted in a deviation between the experimental and simulated PSDs. In order to develop a more comprehensive numerical model, the agglomeration and breakage kernels would be required.

The experiments and simulations examined in this work were carried out in an unseeded system at 45°C but the model could be developed to encompass a greater range of conditions. Seeded experiments under a range of different temperatures would provide a more detailed set of kinetic equations which would result in a greater insight into the L-glutamic acid system. The influence of impurities on the crystallisation processes would be also be of considerable interest. The effect of the impurities could be incorporated into the model following a detailed experimental analysis. A more extensive and detailed numerical model could be generated using the methods described in this treatise.
The suspension of the needle-like $\beta$ particles was performed in this work but further work could be carried out for various other non-spherical particles encountered in the process industry. The methods used in this treatise could be expanded to various other particle shapes in order to develop a more detailed knowledge of particulate systems. There has been detailed experimental work performed on the drag of cubic, octahedral and tetrahedral particles but to this authors knowledge the modelling of these non-spherical particles in suspension has not been investigated. An examination into the scale-up of the non-spherical particle suspensions would also be of considerable interest to the process industry and the implementation of this within CFD would be very useful for future research into the area.

The influence of the the fluid flow on the polymorphic transformation process would also be of relevance to the process industry. This would require the incorporation of the population balance model into the CFD simulations and investigating the mass transfer processes that occur in the crystallisation processes.


Bibliography


Bibliography


Appendix A

POLYMORPHIC TRANSFORMATION CODE

A.1 Main Code

A.1.1 main

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Main Code - Polymorphic Transformation Simulation
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clc
clear all
tic
format long e

%%% Load in initial moment data

load('C:\MATLAB6p5\moment_data\moment_surface_data.mat');

ma = xlsread('C:\MATLAB6p5\Code\initial_moments.xls', 1)
mb = xlsread('C:\MATLAB6p5\Code\initial_moments.xls', 2)
%% Initialise distributions
N_total_a = 1;
ma = N_total_a * ma;
clear ma0 ma1 ma2 ma3 ma4 ma5;

N_total_b = 1;
mb = N_total_b * mb;
clear mb0 mb1 mb2 mb3 mb4 mb5;

%% Specify simulation time

t=1;
t_end = 8*3600;  \% end time
t_inc = 20;  \% time increment
time = (t_inc*[0:t_end/t_inc])';
time(end)=[];

%% Run code once to get initial rates
run constants;
m=ma;  \% calculations apply to alpha moments
run alpha_nuc;  \% calculate nucleation of alpha
run alpha_growth;  \% calculate growth of alpha
run alpha_dissolution;  \% calculate the dissolution of alpha
run beta_nuc;  \% calculate nucleation of beta
run beta_growth;  \% calculate growth of beta
run dmk_dt;  \% calculate initial change in moments
ma(:,t+1) = ma(:,t) + (dma_dt * t_inc);
run soln_conc;  \% calculate the solution concentration

m=mb;  \% calculations apply to beta moments
run beta_nuc;
run beta_growth;
run alpha_nuc;
run alpha_growth;
run alpha_dissolution;
Appendix A. Polymorphic Transformation Code

run dmk_dt;
mb(:,t+1) = mb(:,t) + (dmb_dt * t_inc);
run soln_conc;

%%% Loop code for all time steps to obtain evolution of system
for t=2:t_end/t_inc;

m = ma(:,t);
run alpha_nuc;
run alpha_growth;
run alpha_dissolution;
run beta_nuc;
run beta_growth;
run dmk_dt;

ma(i,t+1) = ma(i,t) + (dma_dt(i,1) * t_inc);

m = mb(:,t);
run beta_nuc;
run beta_growth;
run alpha_nuc;
run alpha_growth;
run alpha_dissolution;
run dmk_dt;
mb(:,t+1) = mb(:,t) + (dmb_dt * t_inc);
run soln_conc;
end

ma(:,end) = []; % last entry set to zero to ensure equal vector length
mb(:,end) = [];
conc_s(end) = []; % set equal to zero as dA/dt is calculated for t=t+1

run mass_calcs; % calculate mass of polymorphs
run PSD_Reconstruction_alpha % generate PSDs
run PSD_Reconstruction_beta

### A.1.2 constants

```
%%% Definition of Constants

%%% Critical size of the nuclei
crt_size_alpha = xlsread('C:\MATLAB6p5\Code\critical_nucleus.xls', 1);
crt_size_beta = xlsread('C:\MATLAB6p5\Code\critical_nucleus.xls', 2);

%%% Initial concentration of solute in solution
conc_s(1) = 43;

%%% Solubilites at 45C
conc_ai(t) = 21.7;
conc_bi(t) = 17;

%%% Universal constants
M = 0.147; % molecular mass (kg/mol)
crystal_density = 1540;

volume_shape_factor_beta = 0.01;
volume_shape_factor_alpha = 0.52;

area_shape_factor_beta = 0.42;
area_shape_factor_alpha = 5.384;

%%% Alpha constants
kn_alpha = 8.0e5; % pre-exponential factor nucleation rate
Kn_alpha = 1.0e-1; % exponential factor nucleation rate
```
Appendix A. Polymorphic Transformation Code

kg_alpha = 2.5e-7; \% pre-exponential factor growth rate  
Kg_alpha = 9.0e-2; \% exponential factor growth rate

%% Sherwood correlation

D = 1.4e-10; \%Mersmann  
L = ma(4,t)/ma(3,t);  
ep = 6.3e-2;  
nu = 1e-6;  
Sc = nu/D;  
kda = (D/L) * (2 + ((0.8 * ((ep * (L^4))/(nu^3))^(1/5)) * (Sc) ^((1/3)));

%% Beta constants

kg_beta = 6.5e-8; \% pre-exponential factor growth  
Kg_beta = 1.6e-1; \% exponential factor growth  
ks_beta= 6.0e4; \% pre-exp factor beta surface nucleation  
Ks_beta = 1.0e-3; \% exp factor beta surface nucleation  
kn_beta = 5.4e4; \% pre-exponential factor growth  
Kn_beta = 1.5e1; \% exponential factor growth

A.1.3 solution concentration

%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Solution Concentration
%%%%%%%%%%%%%%%%%%%%%%%%%%

if t==1; \% for first timestep rate of change of solute is zero  
dA_dt(t,1) = 0;  
else  
\% change in solute due to nucleation, growth and dissolution  
dA_dt(t,1) = - 1000 * 3 * crystal_density
Appendix A. Polymorphic Transformation Code

* (volume_shape_factor_beta * G_beta(t)* mb(3,t) + volume_shape_factor_alpha * G_alpha(t)* ma(3,t) - volume_shape_factor_alpha * Da(t) * ma(3,t));

end

conc_s(t+1,1) = conc_s(t,1) + (dA_dt(t,1)) ; %

A.1.4 dmk dt

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Calculate Rate of Change of Moments
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%% Determine if solution is supersaturated or undersaturated

if Sa(t) > 1 || ma(4,t)*1540*0.52 < 0 %solution supersaturated
  dma_dt(1,1) = J_alpha(t);
else
  %% Calculate PSD and Discretise into Size Classes
  run dmk_dt_PSD_alpha;

  x1 = linspace(1,500,500)';

  for k1=1:length(x1);
    distribution_classes_diss(k1,1) =
      (((x1(k1)/rossin_delta_diss(1))^rossin_n_diss(1))*(rossin_n_diss(1)/x1(k1))
        * exp(-((x1(k1)/rossin_delta_diss(1))^rossin_n_diss(1))));
  end

  %% Calculate shift in distribution due to dissolution
  NDF_classes_diss(:,1) = distribution_classes_diss(:,1) * ma(1,t);
  NDF_classes_diss = NDF_classes_diss * (1-Da(t));

  %% Discretise PSD
  num_ints = length(x1);
APPENDIX A. POLYMORPHIC TRANSFORMATION CODE

Ni(1,1)=NDF_classes_diss(1,1);

NDF_diss=vec2mat(NDF_classes_diss,(length(x1)/100));
NDF_diss=NDF_diss';
B=sum(NDF_diss);
discrete_NDF=B'/5;

%%Deduct the particle in smallest size classes from population
dma_dt(1,1) = -sum(discrete_NDF(1,1)) ;
end

%% Calculate the change in moments
dmb_dt(1,1) = J_beta(t);

%%Overall rate of change of alpha and beta moments
%% due to nucleation, growth and dissolution
for k=1:5;
    dma_dt(k+1,1) = (k * G_alpha(t) * ma(k,t))
    - (k * Da(t) * ma(k,t)) + (crt_size_alpha^k * J_alpha(t));

    dmb_dt(k+1,1) = (k * G_beta(t) * mb(k,t))
    + (crt_size_beta^k * J_beta(t));
end

dma1_dt(t,1) = dma_dt(2,1);
dmb1_dt(t,1) = dmb_dt(2,1);

A.1.5 dmk dt PSD alpha

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Generate PSD For Alpha Dissolution
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Appendix A. Polymorphic Transformation Code

% normalising moments for PDF
for i = 1:6;
    target_array_dmk_dt(i,:) = ma(i,t) ./ ma(1,t);
end

%% Calculating the moment target at each timestep
Moment_1_Target = target_array_dmk_dt(2,:);
Moment_2_Target = target_array_dmk_dt(3,:);
Moment_3_Target = target_array_dmk_dt(4,:);

%% Calculate relative error between target and surface
moment_1_error = abs(Moment_1_Target - moment_1_L);
moment_2_error = abs(Moment_2_Target - moment_2_L);
moment_3_error = abs(Moment_3_Target - moment_3_L);

%% Find moment isolines
for i = 1:length(moment_1_error);
    temp_m = moment_1_error(:,i); % temporary moment array
    [a b] = find(temp_m == min(temp_m)); % location of minimum error
    n_1_value(i,1) = nv(a); % Array of n value across isoline
    del_1_value(i,1) = delv(i); % Array of delta value across isoline

    temp_m = moment_2_error(:,i);
    [a b] = find(temp_m == min(temp_m));
    n_2_value(i,1) = nv(a);
    del_2_value(i,1) = delv(i);

    temp_m = moment_3_error(:,i);
    [a b] = find(temp_m == min(temp_m));
    n_3_value(i,1) = nv(a);
    del_3_value(i,1) = delv(i);
end

%% Determine where isoline cross over
sign_change = abs(n_1_value - n_2_value);
for i = 1:length(moment_1_error);
    if sign_change(i,1)==0
        sign_change(i,1)=NaN;
    end
end
[min_value_sign_change, location]= min(sign_change);

steps=10;

%% Map out number of points either side of approximate intersection
n_1_range=n_1_value(location-steps:location+steps);
del_1_range=del_1_value(location-steps:location+steps);
n_2_range=n_2_value(location-steps:location+steps);
del_2_range=del_2_value(location-steps:location+steps);
n_3_range=n_3_value(location-steps:location+steps);
del_3_range=del_3_value(location-steps:location+steps);

%% Find intersection point
%%finds distance between the lines at each point
for j=1:length(n_1_range);
    min_dist12(j,1) = sqrt((n_1_range(j,1) - n_2_range(j,1))^2);
    min_dist13(j,1) = sqrt((n_1_range(j,1) - n_3_range(j,1))^2);
    min_dist23(j,1) = sqrt((n_2_range(j,1) - n_3_range(j,1))^2);
end

%%finds minimum distance between the lines and calculate row indices
[intersect12 K]= min(min_dist12);
[intersect13 J]= min(min_dist13);
[intersect23 L]= min(min_dist23);

%% Vertices of triangle of intersection points
%% uses row indices to get values of n and delta
%% that equal vertices of triangle
Appendix A. Polymorphic Transformation Code

vertex1 = [del_1_range(K) n_1_range(K)];
vertex2 = [del_2_range(J) n_2_range(J)];
vertex3 = [del_3_range(L) n_3_range(L)];

%%find centroid of triangle to get best fit n and delta values
centroid = [((vertex1(1,1)+vertex2(1,1)+vertex3(1,1))/3)
((vertex1(1,2)+vertex2(1,2)+vertex3(1,2))/3)];

output_n_del_diss = [centroid(1,1) centroid(1,2)];

%% Output n and delta values
rossin_delta_diss = output_n_del_diss(:,1);
rossin_n_diss = output_n_del_diss(:,2);

A.2 α Polymorph

A.2.1 α nucleation

%%%%%%%%%%%%%%%%%%%%
%%% Alpha Nucleation
%%%%%%%%%%%%%%%%%%%%

Sa(t) = conc_s(t)/conc_ai(t); % Calculation supersaturation of alpha

for m=mb %Ignore alpha nucleation if solving beta moments
    J_alpha(t,1) = 0;
end

if Sa(t) <= 1

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Appendix A. Polymorphic Transformation Code

\begin{align*}
J_{\alpha}(t,1) &= 0; \\
\text{else} & \\
J_{\alpha}(t,1) &= (k_{n_{\alpha}}((S_{a}(t))^{-7/3})) \cdot \exp((-K_{n_{\alpha}})/(\log(S_{a}(t)))^2)); \\
\text{end}
\end{align*}

A.2.2 $\alpha$ growth

%%%%%%%%%%%%%%%%%%%%
%%% Alpha Growth
%%%%%%%%%%%%%%%%%%%%

\text{Sa}(t) = \text{conc}_s(t)/\text{conc}_{ai}(t); \text{%) Supersaturation with respect to } \alpha

\text{for } m=mb \quad \% \text{Ignore alpha growth if solving beta moments} \\
\quad G_{\alpha}(t,1) = 0; \\
\text{end}

\text{if } S_{a}(t) \leq 1 \\
\quad G_{\alpha}(t,1) = 0; \\
\text{else} \\
\quad G_{\alpha}(t,1) = k_{g_{\alpha}}((S_{a}(t)-1)^{5/6}) \cdot \exp(-K_{g_{\alpha}}/(S_{a}(t)-1));
\text{end}

A.2.3 $\alpha$ dissolution

%%%%%%%%%%%%%%%%%%%%
%%% Dissolution of Alpha
%%%%%%%%%%%%%%%%%%%%

\text{Sa}(t) = \text{conc}_s(t)/\text{conc}_{ai}(t); \text{%) Supersaturation with respect to } \alpha

\text{if } S_{a}(t) \geq 1 \ || \ \text{ma}(4,t) \cdot 1540 \cdot 0.52 \leq 0 \quad \% \text{solution supersaturated} \\
\quad D_{a}(t,1) = 0; \\
\text{else} \\
\quad D_{a}(t,1) = k_{da}(1-S_{a}(t));
A.3 \( \beta \) Polymorph

A.3.1 \( \beta \) nucleation

\[ S_b(t) = \frac{\text{conc}_s(t)}{\text{conc}_\beta(t)}; \] % Supersaturation with respect to beta

\[ \text{for } m=\text{ma} \] %Ignore alpha nucleation if solving alpha moments
\[ J_{\beta}(t,1) = 0; \]
end

\[ \text{if } S_b(t) \leq 1 \]
\[ J_{\beta}(t,1) = 0; \]
\[ \text{else} \]
\[ J_{\beta\_prim}(t,1) = \frac{\text{kn}_\beta((S_b(t))^{7/3})\exp((-\text{Kn}_\beta)/(\log(S_b(t))^2))}{(\log(S_b(t))^2)}; \]
\[ J_{\beta\_sec}(t,1) = \text{ks}_\beta\text{area\_shape\_factor}_\beta(\text{ma}(3,t)) \times \exp(-\text{Ks}_\beta/\log(S_b(t))) ; \]
\[ J_{\beta}(t,1) = J_{\beta\_prim}(t,1) + J_{\beta\_sec}(t,1); \]
end
Appendix A. Polymorphic Transformation Code

A.3.2 β growth

%%%%%%%%%%%%%%%%%%%%%%%%
%%%% Beta Growth
%%%%%%%%%%%%%%%%%%%%%%%%

Sb(t) = conc_s(t)/conc_bi(t); % Supersaturation with respect to beta

for m=ma %Ignore beta growth if solving alpha moments
    G_beta(t,1) = 0;
end

if Sb(t) <= 1
    G_beta(t,1) = 0;
else
    G_beta(t,1) = kg_beta*((Sb(t)-1)^(5/6))*exp(-Kg_beta/(Sb(t)-1));
end

A.4 PSD Reconstruction

A.4.1 PSD Reconstruction α

%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%% Generate PSD For Alpha
%%%%%%%%%%%%%%%%%%%%%%%%%%%

tic
% Set Target Array-normalises PSD wrt m0 to get PDF
for i = 1:6;
    target_array(i,:) = ma(i,:) ./ ma(1,:);
end
for time_point = 1:length(target_array);

% Calculating the moment target at each timestep

Moment_1_Target = target_array(2, time_point);
Moment_2_Target = target_array(3, time_point);
Moment_3_Target = target_array(4, time_point);

% Calculate relative error between target and surface

moment_1_error = abs(Moment_1_Target - moment_1_L);
moment_2_error = abs(Moment_2_Target - moment_2_L);
moment_3_error = abs(Moment_3_Target - moment_3_L);

% Find moment isolines

for i = 1:length(moment_1_error);
    temp_m = moment_1_error(:, i); % temporary moment array
    [a b] = find(temp_m == min(temp_m)); % location of minimum error
    n_1_value(i, 1) = nv(a); % Array of n value across isoline
    del_1_value(i, 1) = delv(i); % Array of delta value across isoline

    temp_m = moment_2_error(:, i);
    [a b] = find(temp_m == min(temp_m));
    n_2_value(i, 1) = nv(a);
    del_2_value(i, 1) = delv(i);

    temp_m = moment_3_error(:, i);
    [a b] = find(temp_m == min(temp_m));
    n_3_value(i, 1) = nv(a);
    del_3_value(i, 1) = delv(i);
end

% Determine where isoline cross over

    sign_change = abs(n_1_value - n_2_value);
for i = 1:length(moment_1_error);
    if sign_change(i,1)==0
        sign_change(i,1)=NaN;
    end
end
[min_value_sign_change, location]= min(sign_change) ;

%% Map out number of points either side of approximate intersection
steps=10;
n_1_range=n_1_value(location-steps:location+steps);
del_1_range=del_1_value(location-steps:location+steps);
n_2_range=n_2_value(location-steps:location+steps);
del_2_range=del_2_value(location-steps:location+steps);
n_3_range=n_3_value(location-steps:location+steps);
del_3_range=del_3_value(location-steps:location+steps);

%% Find intersection point

%% finds distance between the lines at each point
for j=1:length(n_1_range);
    min_dist12(j,1) = sqrt((n_1_range(j,1) - n_2_range(j,1))^2);
    min_dist13(j,1) = sqrt((n_1_range(j,1) - n_3_range(j,1))^2);
    min_dist23(j,1) = sqrt((n_2_range(j,1) - n_3_range(j,1))^2);
end

[intersect12 K]= min(min_dist12);
[intersect13 J]= min(min_dist13);
[intersect23 L]= min(min_dist23);

%% Vertices of triangle of intersection points
%% uses row indices to get values of n and delta
%% that equal vertices of triangle

vertex1 = [del_1_range(K) n_1_range(K)];
vertex2 = [del_2_range(J) n_2_range(J)];
vertex3 = [del_3_range(L) n_3_range(L)];

%% find centroid of triangle to get best fit n and delta values

centroid = [((vertex1(1,1)+vertex2(1,1)+vertex3(1,1))/3)
            ((vertex1(1,2)+vertex2(1,2)+vertex3(1,2))/3)];

output_n_del(time_point,:) = [centroid(1,1) centroid(1,2)];

%% PSD Reconstruction

rossin_delta(:,time_point) = output_n_del(time_point,1);
rossin_n(:,time_point) = output_n_del(time_point,2);

%% use n and delta values to generate PSD

x = linspace(1,250,250)’;
for k=1:250; % number of intervals in x
    distribution_classes(k,time_point) =
        ((x(k)/rossin_delta(time_point))^rossin_n(time_point))
        * (rossin_n(time_point)/x(k))
        * exp(-(x(k)/rossin_delta(time_point))^rossin_n(time_point)));
end

NDF_classes(:,time_point) = distribution_classes(:,time_point) * ma(1,t);
end
Appendix A. Polymorphic Transformation Code

A.4.2 PSD Reconstruction $\beta$

%%% Generate PSD For Beta
%-----------------------------------------------

tic
%% Set Target Array-normalises PSD wrt m0 to get PDF

for i = 1:6;
    target_array(i,:) = mb(i,:) ./ mb(1,:);
end

for time_point = 1:length(target_array);

%%Calculating the moment target at each timestep

Moment_1_Target = target_array(2,time_point);
Moment_2_Target = target_array(3,time_point);
Moment_3_Target = target_array(4,time_point);

%%Calculate relative error between target and surface

moment_1_error = abs(Moment_1_Target - moment_1_L);
moment_2_error = abs(Moment_2_Target - moment_2_L);
moment_3_error = abs(Moment_3_Target - moment_3_L);
%% Find moment isolines

for i = 1:length(moment_1_error);
    temp_m = moment_1_error(:,i); % temporary moment array
    [a b] = find(temp_m == min(temp_m)); % location of minimum error
    n_1_value(i,1) = nv(a); % Array of n value across isoline
    del_1_value(i,1) = delv(i); % Array of delta value across isoline

    temp_m = moment_2_error(:,i);
    [a b] = find(temp_m == min(temp_m));
    n_2_value(i,1) = nv(a);
    del_2_value(i,1) = delv(i);

    temp_m = moment_3_error(:,i);
    [a b] = find(temp_m == min(temp_m));
    n_3_value(i,1) = nv(a);
    del_3_value(i,1) = delv(i);
end

%% Determine where isoline cross over

    sign_change=abs(n_1_value-n_2_value);

    for i = 1:length(moment_1_error);
        if sign_change(i,1)==0
            sign_change(i,1)=NaN;
        end
    end

    [min_value_sign_change, location]= min(sign_change) ;

%% Map out number of points either side of approximate intersection

    steps=10;

    n_1_range=n_1_value(location-steps:location+steps);
    del_1_range=del_1_value(location-steps:location+steps);
n_2_range = n_2_value(location-steps:location+steps);
del_2_range = del_2_value(location-steps:location+steps);
n_3_range = n_3_value(location-steps:location+steps);
del_3_range = del_3_value(location-steps:location+steps);

%% Find intersection point
%% finds distance between the lines at each point
for j=1:length(n_1_range);
    min_dist12(j,1) = sqrt((n_1_range(j,1) - n_2_range(j,1))^2);
    min_dist13(j,1) = sqrt((n_1_range(j,1) - n_3_range(j,1))^2);
    min_dist23(j,1) = sqrt((n_2_range(j,1) - n_3_range(j,1))^2);
end

%% finds minimum distance between the lines and calculate row indices
[intersect12 K] = min(min_dist12);
[intersect13 J] = min(min_dist13);
[intersect23 L] = min(min_dist23);

%% Vertices of triangle of intersection points

%% uses row indices to get values of n and delta
%% that equal vertices of triangle
vertex1 = [del_1_range(K) n_1_range(K)];
vertex2 = [del_2_range(J) n_2_range(J)];
vertex3 = [del_3_range(L) n_3_range(L)];

%% find centroid of triangle to get best fit n and delta values
centroid = [((vertex1(1,1)+vertex2(1,1)+vertex3(1,1))/3)
    ((vertex1(1,2)+vertex2(1,2)+vertex3(1,2))/3)];

output_n_del(time_point,:) = [centroid(1,1) centroid(1,2)];

%% PSD Reconstruction
rossin_delta(:,time_point) = output_n_del(time_point,1);
rossin_n(:,time_point) = output_n_del(time_point,2);

%%use n and delta values to generate PSD
x = linspace(1,250,250)’;
for k=1:250; % number of intervals in x
    distribution_classes(k,time_point) =
        ((x(k)/rossin_delta(time_point))^rossin_n(time_point))
        * (rossin_n(time_point)/x(k))
        * exp(-((x(k)/rossin_delta(time_point))^rossin_n(time_point)));
end

NDF_classes(:,time_point) = distribution_classes(:,time_point) * mb(1,t);
end
The following subroutines were derived from Fluent Inc (2006) to provide a method of resolving the drag induced on slender particles.

```
UDF for calculating the drag of non-spherical particles

#include "udf.h"
#include "math.h"
define pi 3.1415
define E_cyl 10

DEFINE_EXCHANGE_PROPERTY(custom_drag, cell, mix_thread, s_col, f_col)
{
double d; /*equivalent sphere diameter*/
Thread *thread_f, *thread_s;

double x_vel_f, x_vel_s, y_vel_f, y_vel_s, abs_v, slip_x, slip_y;
```
double rho_f, rho_s, mu_f, reyp, void_f, vfac, fdrgs, c_d, k_g_s;
double n_diam, A, B, C, vrs, vol, s_area, s_area_equiv_sphere;
double sphericity, k1, k2, double A_surf, Astar_surf, sphere_sa;
double A_proj, Astar_proj, f_shape, c_shape, reyps;

/* find the threads for the fluid (primary) and solids (secondary phases) */

/* fluid phase*/
thread_f = THREAD_SUB_THREAD(mix_thread, s_col);

/* solid phase*/
thread_s = THREAD_SUB_THREAD(mix_thread, f_col);

/* find phase velocities and properties*/

n_diam = C_PHASE_DIAMETER(cell, thread_s);
x_vel_f = C_U(cell, thread_f);
y_vel_f = C_V(cell, thread_f);
x_vel_s = C_U(cell, thread_s);
y_vel_s = C_V(cell, thread_s);

slip_x = x_vel_f - x_vel_s;
slip_y = y_vel_f - y_vel_s;

rho_f = C_R(cell, thread_f);
rho_s = C_R(cell, thread_s);

mu_f = C_MU_L(cell, thread_f);

/*compute slip*/
abs_v = sqrt(slip_x*slip_x + slip_y*slip_y);

void_f = C_VOF(cell, thread_f); /* fluid vol frac*/

vol = (2.5*pi*pow(n_diam,3)); /* Volume of cylinder*/
Appendix B. FLUENT Subroutines

d = pow((6*vol/pi),(1./3)); /* equivalent sphere diameter*/

reyp = (rho_f*abs_v*d)/mu_f; /* calculate Reynolds Number*/

/*-----------------------------------Loth 2008-----------------------------------*/
sphere_sa = pi*pow(n_diam,2);

A_surf = 21*pi*pow(n_diam,2)/2; /* Surface area of cylinder*/

/* Astar_surf = A_surf/sphere_sa; */ /* for cylinder*/

Astar_surf = ((2*E_cyl)+1)/pow((18*pow(E_cyl,2)),1./3);

A_proj = n_diam*(10*n_diam); /* projected area of cylinder falling broadside*/

Astar_proj = A_proj/(0.25*pi*pow(n_diam,2));

f_shape = ((1./3)*pow(Astar_proj,0.5)) + ((2./3)*pow(Astar_surf,0.5));

c_shape = 1.+(0.7*pow((Astar_surf-1),0.5)) + (2.4*(Astar_surf-1.));

reyps = (c_shape*reyp)/f_shape;
/*-----------------------------------------------*/

/* compute drag and return drag coeff, k_g_s*/

c_d = (24/(void_f*reyps))*((1+0.15*(void_f*pow(reyps,0.687)))) + (0.42/(1+(42500/(pow(reyps,1.16))))); /* Loth 2008*/

k_g_s = (0.75 * c_d * (1-void_f) * void_f * rho_f * abs_v * pow(void_f,-2.65))/d; /* Wen and Yu*/

return k_g_s;

}
Appendix C

Volume Fraction Results

C.1 $\alpha$ Volume Fractions

The volume fraction plots for the $\alpha$ polymorph are presented in the following section. Impeller rotational speeds of 250, 350, 450 and 550rpm are illustrated.
Appendix C. Volume Fraction Results

Figure C.1: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 250rpm

Figure C.2: $\alpha$ solid phase volume fractions at 250rpm
Appendix C. Volume Fraction Results

Figure C.3: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 350rpm

Figure C.4: $\alpha$ solid phase volume fractions at 350rpm
Figure C.5: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 450rpm

Figure C.6: $\alpha$ solid phase volume fractions at 450rpm
Figure C.7: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 550rpm

Figure C.8: $\alpha$ solid phase volume fractions at 550rpm
C.2 $\beta$ Volume Fractions

The volume fraction plots for the $\alpha$ polymorph are presented in the following section. Impeller rotational speeds of 250, 350, 450 and 550rpm are illustrated.

Figure C.9: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 250rpm

Figure C.10: $\beta$ solid phase volume fractions at 250rpm
Appendix C. Volume Fraction Results

Figure C.11: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 350rpm

Figure C.12: $\beta$ solid phase volume fractions at 350rpm
Appendix C. Volume Fraction Results

Figure C.13: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 450rpm

Figure C.14: $\beta$ solid phase volume fractions at 450rpm
Figure C.15: Volume fraction contour plot and experimental rig photograph illustrating solid-liquid interface at 550rpm

Figure C.16: $\beta$ solid phase volume fractions at 550rpm