A Numerical and Experimental Investigation of the Mean and Turbulent Characteristics of a Wing-tip Vortex in the Near-field

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Declaration

I herewith declare that I have produced this thesis without the prohibited assistance of third parties and without making use of aids other than those specified. Reference and acknowledgement, where necessary, has been made to the work of others. This thesis has not previously been presented in identical or similar form to any other Irish or foreign examination board.

The thesis work was conducted from 2010 to 2014 under the supervision of Dr. Philip Griffin and Dr. Trevor Young at the University of Limerick.

Signed: Micheál Seán O’Regan  Date

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Abstract

Turbulent wing-tip vortices are an extremely important fluid dynamics phenomena for their negative impact in several applications. Despite the many numerical and experimental studies conducted on this particular flow, there are still parameters that require further research to advance the current understanding and provide a benchmark for future prediction methods and computational studies.

In this study, the near-field (up to three chord lengths) development of a wing-tip vortex is investigated at two angles of attack (five and ten degrees) using experimental and numerical methods. The experimental study was conducted a priori to the numerical simulations to provide a base case and inlet boundary conditions for the numerical models. The vortex shed from a straight rectangular wing with squared tips was investigated to identify the main mechanisms involved in the near-field roll up of the vortex. The combination of experimental measurement techniques, such as hot-wire anemometry and a five-hole pressure probe, gave great insight into the behaviour of the mean and turbulent characteristics of the vortex during roll up and near-field formation. The experimental measurements revealed both wake-like and jet-like axial velocity profiles depending on the angle of attack and the presence of a secondary counter rotating vortex just behind the wing (x/c = 0) for both angles of attack. The vortex was also characterized by high levels of vorticity in the core and a circulation parameter that increased with downstream distance. Turbulence levels in the vortex were found to be highest on the core periphery just behind the wing (x/c = 0) but decayed with downstream distance.
in the core of the vortex due to the relaminarizing effect of the core solid body rotation.

The numerical investigation utilised finite volume flow solver Star-CCM+ and consisted of Steady and Unsteady Reynolds Averaged Navier-Stokes (RANS/URANS) modelling using a Reynolds stress model, Large Eddy Simulation (LES) and the application of a vorticity confinement model (VC) to the URANS and LES equations. The RANS and URANS with VC models predicted the mean flow reasonably well for an angle of attack of five degrees, whereas the mean flow for an angle of attack of ten degrees and the turbulence magnitudes for both angles of attack were greatly under-predicted. The LES with VC model had the best agreement with experiment, reproducing the principal features observed in the experimental measurements. The LES with VC model correctly predicted the wake-like and jet-like axial velocity profiles, the presence of a secondary counter rotating vortex and turbulent quantities in close agreement with those of experiment. The LES with VC model predicted the axial velocity and the root mean square (rms) of the streamwise turbulent fluctuations to within 3% and 15% of the experimental results and the core radius to be the same size as experiment at the last measurement location of $x/c = 3$ for an angle of attack of five degrees.
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<td>$\vec{a}$</td>
<td>arbitrary vector quantity</td>
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<tr>
<td>$A$</td>
<td>hot-wire/film calibration constant</td>
<td></td>
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<td>Reynolds number $= \frac{\rho U_{\infty} c}{\mu}$</td>
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<td>K</td>
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<td>streamwise turbulence intensity</td>
<td>%</td>
</tr>
<tr>
<td>$TU_y$</td>
<td>spanwise turbulence intensity</td>
<td>%</td>
</tr>
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<td>$TU_{\infty}$</td>
<td>freestream turbulence intensity</td>
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<td>$u, v, w$</td>
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<td>m/s</td>
</tr>
<tr>
<td>$u', v', w'$</td>
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<td>$U, V, W$</td>
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<td>$U_{\theta_{\text{max}}}$</td>
<td>maximum circumferential velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$U_{\infty}$</td>
<td>freestream velocity</td>
<td>m/s</td>
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**NOMENCLATURE**

\[ \vec{v}_i \] velocity vector along circulation integration path \[ \text{m/s} \]

\[ u'v', v'w', w'u' \] Reynolds stresses in cartesian coordinates \[ \text{m}^2/\text{s}^2 \]

\[ u'^2, v'^2, w'^2 \] turbulent normal stresses in cartesian coordinates \[ \text{m}^2/\text{s}^2 \]

\[ x, y, z \] dimensionless cartesian coordinates \[-\]

**Greek Symbols**

\[ \alpha \] angle of attack \[ \text{deg} \]

\[ \alpha_p \] pitch angle \[ \text{deg} \]

\[ \beta \] yaw angle \[ \text{deg} \]

\[ \gamma \] ratio of specific heat capacities/adiabatic index \[-\]

\[ \Gamma \] circulation \[ \text{m}^2/\text{s} \]

\[ \Gamma_w \] theoretical wing circulation \[ \text{m}^2/\text{s} \]

\[ \delta_{ij} \] kronecker delta \[-\]

\[ \Delta \] LES spatial filter cut-off width \[ \sqrt[3]{\Delta x \Delta y \Delta z} \] \[ \text{m} \]

\[ \Delta \bar{\Delta}_i \] segment length along circulation integration path \[ \text{m} \]

\[ \Delta x, \Delta y, \Delta z \] grid spacing \[ \text{m} \]

\[ \varepsilon \] turbulent kinetic energy dissipation rate \[ \text{m}^2/\text{s}^3 \]

\[ \varepsilon_{ij} \] dissipation term of Reynolds stress model \[-\]

\[ \theta \] five-hole probe cone angle \[ \text{deg} \]

\[ \theta_{eff} \] effective angle \[ \text{deg} \]

\[ \mu \] dynamic viscosity \[ \text{Pa} \cdot \text{s} \]

\[ \mu_t \] turbulent viscosity \[ \text{Pa} \cdot \text{s} \]

\[ \mu_{sgs} \] SGS viscosity \[ \text{Pa} \cdot \text{s} \]

\[ \nu \] kinematic viscosity \[ \text{m}^2/\text{s} \]

\[ \Pi_{ij} \] pressure-strain interaction term of Reynolds stress model \[-\]

\[ \rho \] density of air \[ \text{kg/m}^3 \]

\[ \sigma_k \] Reynolds stress model constant = 0.82 \[-\]
<table>
<thead>
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<td>LES filtered function</td>
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<td>LES unfiltered function</td>
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<td>$\Psi$</td>
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<tr>
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<td>rotation term of Reynolds stress model</td>
</tr>
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<td>$\Omega_k$</td>
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Chapter 1

Introduction

This chapter gives a brief introduction into the flow-field around a finite wing and the formation and importance of wing-tip vortices. After a brief description of the flow-field around a finite wing and an overview on the formation of trailing vortices, theories are given to explain the formation of wing-tip vortices. The importance of wing-tip vortices and their application is then given, followed by the current objectives of this study and the layout of the thesis.

1.1 The Flow-field around a Finite Wing

The flow-field around a three-dimensional finite wing differs from that around a two-dimensional aerofoil in a number of ways. If one were to simply look at the streamlines projected in a longitudinal plane perpendicular to the span the flow would look qualitatively like the flow around a two-dimensional aerofoil (Figure 1.1a). However, that projected view misses an important aspect of the three-dimensional flow; that is, that the streamlines of the three-dimensional flow have a significant out of plane or spanwise component as seen in Figure 1.1b. This cross-stream velocity field develops in conjunction with a pressure field that is also non-uniform in the spanwise direction and persists for long distances downstream of the wing. It is clear that the flow is characterized by downward flow between the wing-tips and upward flow outboard of the tips, outboard flow below the wing and inboard flow above the wing. The flow that develops persists for long distances downstream, whereas velocity disturbances
decrease immediately in two-dimensional flow. The wing is flying through the air that is already moving downward between the wingtips and is thought of as flying in a downdraft or downwash of its own making. The three-dimensional downwash is seen as a downward shift in the apparent angle of attack of each aerofoil section of the wing and is often called the induced angle of attack. The effect of the induced angle of attack, integrated over the span, always corresponds to a reduction in the angle of attack of the wing. As a result of the downwash the total apparent lift vector is tilted backwards and this backward component is called the induced drag or vortex drag. In Figure 1.1b, it is evident that the spanwise velocity components behind the wing are in an outboard direction below the wing and in an inboard direction above the wing. This mismatch, or jump, in the spanwise velocity constitutes a vortex sheet that is shed from the trailing edge and convected downstream.

![Flow-field illustrations around a three-dimensional wing taken from McLean (2012)](image)

**Figure 1.1:** Flow-field illustrations around a three-dimensional wing taken from McLean (2012)

### 1.2 Wing-tip Vortices: A Brief Overview of Their Formation

Frederick Lanchester in his book *Aerodynamics* (Lanchester, 1907) was the first to mention the idea of vortices that trail downstream of the wing-tips. He knew that
1.2 Wing-tip Vortices: A Brief Overview of Their Formation

a vortex filament could not end in space as proposed by Helmholtz theorem and he theorized that the vortex filaments which constituted the two wing-tip vortices must cross the wing along its span – this is the first mention of the horseshoe vortex concept (illustrated in Figure 1.2, where $U_\infty$ is the free-stream velocity and $b$ is the wing span). The first practical theory for predicting the aerodynamic properties of a finite wing was developed by Ludwig Prandtl and his colleagues at Göttingen, Germany, during the period 1911-1918.\(^1\) Such theory was very important in understanding the role of vortices in the generation of lift and the reason for their formation. This particular model involves replacing the finite wing and wake with a system of vortices which imparts to the surrounding air a motion similar to the actual flow. The system can be divided into three main parts which together form a vortex ring: the starting vortex, the trailing vortex system and the bound vortex system (Houghton and Carpenter, 2003). The first two vortex systems are observable physical quantities, whereas the bound vortex system is a hypothetical arrangement of a number of vortices which replace the physical wing and where the vorticity of each vortex filament is associated with the span-wise gradient of the wing circulation distribution. This bound vortex segment represents the boundary layer of the upper and lower surfaces of the wing and it is the source of the lift and drag through pressure and shear stress distributions. The distribution of vortex filaments eventually merge into the trailing vortex system downstream of the wing. Due to its shape the resulting system which models the wing and its wake is called a horseshoe vortex (Figure 1.2b). Based on Helmholtz theorems \(^2\), which are valid for inviscid and incompressible flows, the vortex filament replacing the wing will continue in the wake as two infinitely long free trailing vortices and the strength of the bound vortex is equal to the strength of the trailing vortices. Despite the fact that viscosity plays a crucial role during vortex formation, roll up and evolution in the wake, the model is capable of accurate predictions of the lift generated by the wing.

McLean (2012) described the vortex wake as originating at the trailing edge of a wing but that it could not be the actual origin of the vorticity in the wake as vortex lines cannot end at a solid surface with a no-slip condition. Therefore, the

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\(^2\)For an explanation of Helmholtz theorems see Anderson (1991) p. 323.
1. INTRODUCTION

vorticity in the wake must originate in the viscous or turbulent boundary layers on the upper and lower surfaces of the wing. When viewed in terms of net vorticity, the shed vortex sheet is actually a continuation of the bound vorticity associated with the lift of the wing. For a finite wing, the lift per unit span decreases in the outboard direction along the span and the circulation and total bound vorticity must also decrease. The vorticity representing this loss in total strength cannot just disappear and is shed from the trailing edge into the flowfield, supplying the vorticity that forms the vortex wake. The vortex wake consists of a wake shear layer that is a real physical shear layer but is often idealised as a vortex sheet for simplicity. The vortex lines in this sheet leave the trailing edge of the wing and follow the general direction of the flow downstream. In the case of the ideal thin vortex sheet, the lines are aligned with the mean of the velocity vectors above and below the sheet and within a couple of wingspans downstream, the sheet generally rolls up towards its outer edges to form two distinct vortex cores. These vortex cores are often referred to as wing-tip vortices but McLean (2012) believes this definition leads to some confusion. He states that while it is true that the vortex cores line up not very far inboard of the wing-tips, the term “wing-tip vortices” implies that the wing-tips are the source of all of the vorticity. The truth is that the vorticity that feeds into the cores comes from the entire span of the trailing edge and not just the wing-tips.

Another popular way of looking at the formation of wing-tip vortices is the pressure field interpretation. It is mentioned in Anderson (1991) that the physical
mechanism for generating lift on a wing is the presence of high pressure on the bottom surface and low pressure on the top surface (see Figure 1.3). The net imbalance of this pressure distribution creates the lift force and the by-product of this pressure imbalance causes the flow near the wing-tips to curl around the wing-tips migrating from the high-pressure region underneath to the low-pressure region on top. As a result, on the top surface there is a span-wise component of flow from the tip toward the wing root and on the bottom surface there is a span-wise component of flow from the root toward the tip as shown in Figure 1.3. The leaking flow from around the wing-tips establishes a circulatory motion, which trails downstream of the wing and forms what is known as a trailing vortex.

![Figure 1.3: Pressure field schematic showing exaggerated streamline curvature over a finite wing (adapted from Anderson (1991))](image)

**1.3 The Importance and Applications of Wing-tip Vortices**

Wing-tip vortices are observed to occur on aircraft wings, wind turbine blades, helicopter rotor blades and the propellers of a ship – examples of which can be seen
1. INTRODUCTION

in Figure 1.4. One of the most important and researched areas is that of wing-tip vortices shed from commercial aircraft. The study of aircraft wing-tip vortices first came to prominence in the 1970’s with the introduction of large scale jumbo jets such as the Boeing 747. Several research papers emerged during this period, which focused on characterising the vortex wakes of these aircraft as evidenced by the Symposium on Aircraft Wake Turbulence and it’s Detection held in Seattle, Washington in 1970. Some of the studies published around this time include those of Grow (1969), Donaldson and Bilanin (1975), Chigier and Corsiglia (1972), Corsiglia et al. (1973). Large aircraft, like the Boeing 747, shed wing-tip vortices that possess an extremely high level of energy, which is proportional to the weight of the aircraft. These energetic structures can induce a significant rolling moment on trailing aircraft, creating a potential safety hazard for commercial aviation as they often persist for several minutes or nautical miles. It is because of this hazard that the airworthiness regulatory authorities, such as the Federal Aviation Administration (FAA) and the European Aviation Safety Agency (EASA) as well as the International Civil Aviation Organization (ICAO) require that aircraft following instrument flight rules have a minimum separation distance on take-off and landing. Wing-tip vortices are also a source of vibration and noise in wind turbines and contribute to the power losses experienced by turbines in the wake of another turbine rotor. The development and behaviour of wind turbine wakes has been the focus of many investigations (Espana et al., 2012; Grant et al., 2000; Whale et al., 2000). In addition, investigations by Yu et al. (1997), Yu (2000) and Beaumier and Delrieux (2005) have shown that tip vortices shed from helicopter rotor blades can induce unsteady pressure fluctuations on following blades, which cause unwanted vibrations and aerodynamic noise. The interaction of the wake with the blade, which is known as blade-vortex interaction (BVI) has a significant effect on the aerodynamics and structural dynamics of a helicopter rotor system. Marine propellers suffer from what is called tip vortex cavitation. It is caused by the low pressure present in a tip vortex and is the first type of cavitation to appear with intense noise. Propeller noise is especially important for navy ships as it is often used as a source of underwater detection. Park et al. (2009) mentioned that it is necessary to understand the generation and behaviour of a tip vortex in the near field for the efficient design of low noise marine propellers. A greater
understanding of the physics behind vortex formation and decay gained through numerical and experimental research is therefore essential to the efficient design of vortex generating surfaces.

(a) Tip vortices generated by the rotors of a Boeing 234 helicopter (Oregonianlive, 2013)  
(b) Tip vortices generated by a Boeing 757 at Gatwick Airport (RAAF, 2010)  
(c) Tip-vortex cavitation on a marine propeller (Maris and Balibar, 2000)  
(d) Wind turbine tip vortices generated during a wind tunnel test (Rapin, 2010)

Figure 1.4: Examples of wing-tip vortices

1.4 Objectives

The following objectives were established for this research study:

- The first objective is to design and manufacture a suitable wing-tip vortex test facility to enable cross-stream (yz) measurements to be taken in the
vortex for up to three chord lengths downstream of the wing. The test facility will incorporate an automated traverse mechanism to enable high resolution measurements in the wind tunnel test section.

- The next objective is to investigate experimentally, the mean and turbulent characteristics of a wing-tip vortex in the near-field (up to three chord lengths downstream). The wing-tip vortex is to be generated from a rectangular NACA 0012 half-wing for a Reynolds number of $3.2 \times 10^5$ at two angles of attack ($\alpha = 5^\circ$ and $10^\circ$). The mean flow and turbulent characteristics will be measured using a five-hole probe and x-wire anemometer respectively.

- After the experimental investigation, the objective is to numerically model the aforementioned wing-tip vortex by solving for the Steady and Unsteady Reynolds Averaged Navier-Stokes equations (RANS/URANS) using a Reynolds stress turbulence model for closure of the equations and Large Eddy Simulation (LES) using a Wales SGS model. Inflow boundary conditions will be set using results measured from experiment and measurement grids identical to experiment will be used to extract the data. The mean flow and turbulent characteristics will be validated against the findings of the experimental investigation.

- The final objective of this research study is to assess the suitability of the vorticity confinement model of Steinhoff et al. (2005) in effectively containing the downstream vorticity when applied to the momentum equations of the URANS and LES equations.

1.5 Thesis Outline

The remaining chapters of this thesis are structured as follows:

Chapter Two is a literature review on the current understanding of wing-tip vortices through numerical and experimental research.

Chapter Three describes the governing equations of the turbulence modelling approaches taking in the numerical modelling of the wing-tip vortex.
Chapter Four gives details on the experimental apparatus used in this work, the calibration and operation of experimental equipment and the procedures and techniques used to acquire the data.

Chapter Five describes the experimental results acquired by the five-hole probe and x-wire anemometer, which includes both mean flow and turbulence measurements.

Chapter Six describes the numerical modelling of the wing-tip vortex and gives details on the mesh generation and turbulence models employed. A detailed analysis of the results obtained from the numerical simulations is given at the end of the chapter.

Chapter Seven draws conclusions from the conducted research and suggests possible future work.
Chapter 2

Literature Review

This chapter contains a review of existing literature concerning both numerical and experimental research on wing-tip vortices. It gives an insight into the current understanding of wing-tip vortices and the different techniques used to study them. This chapter is divided into two subsections: (1) Experimental investigations of wing-tip vortices in the near-field and (2) Numerical investigations of wing-tip vortices in the near-field.

2.1 Experimental Investigations of Wing-tip Vortices in the Near-field

The near-field or near wake of a wing-tip vortex extends from the trailing edge to some chord lengths downstream of the wing. Although different vortex life definitions exist, the near-field is generally recognized as being between zero and ten chord lengths downstream of the generating wing (Giuni, 2013), (Bailey et al., 2006), (Chow et al., 1997b). Figure 2.1 shows a typical zone classification for the life of a vortex, where $x/c$ is the non-dimensional distance used to indicate the location of the vortex ($x$ is downstream distance and $c$ the chord length of the wing).

The vortex rollup takes place in the near-field and secondary structures such as the wing wake vorticity and secondary flap and tail vortices are entrained into the main wing-tip vortex. In the immediate near-field (zero to three chord lengths),

\[ x/c \in [0, 3] \]
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The flow is highly three-dimensional and the tip-vortex displays characteristic asymmetry and high turbulence levels.

One of the earliest near-field investigations of a wing-tip vortex was by Grow (1969) who used a five-hole probe and vorticity meter to investigate the wing-tip vortices shed from wings of different aspect ratio and taper ratio. He found that the circulation and maximum tangential velocity of the vortex increased with an increase in aspect ratio and taper ratio and that within one chord 90% of the measured circulation had entered the wing-tip vortices.

A number of early papers investigating wing-tip vortices were conducted by the authors Chigier and Corsiglia (1971). They surveyed four chord lengths downstream of a tip vortex generated from a NACA 0015 wing using a triple hot-wire anemometer. The tests were conducted at an angle of attack of 12° and a Reynolds number of $9.53 \times 10^5$. A large circumferential velocity of $0.42U_\infty$ was recorded at the mid-chord ($x/c = 0.5$) of the wing and decayed to $0.24U_\infty$ by rollup. They also discovered a large axial velocity excess in the core of the vortex which reached a maximum of $1.4U_\infty$ at the quarter chord ($x/c = 0.25$) and decayed to $1.2U_\infty$ by the last measurement location of $x/c = 4$. The authors conducted a second study (Chigier and Corsiglia, 1972) on a Convair 990 wing and a rectangular NACA 0015 wing, which covered the complete near-field. Effects of the variation of angle of attack from $4^\circ$ to $16^\circ$ and the variation of axial distance up to 12 chord lengths downstream of the models were investigated. Measurements of the peak axial velocity in the rectangular wing vortex core revealed that for angles of attack of $8^\circ$ or less an axial velocity deficit occurred but at angle of attack of $12^\circ$ an axial velocity excess was present. An axial velocity deficit occurred for all angles of attack for the Convair 990 wing. Axial and crossflow

![Figure 2.1: Vortex life schematic (length scale is taken from Giuni (2013))](image-url)
2.1 Experimental Investigations of Wing-tip Vortices in the Near-field

Velocities were found to decrease with angle of attack and the turbulence intensity increased linearly with angle of attack. Maximum turbulence intensity values of 11% and 12% were recorded for the Convair 990 model and rectangular wing model respectively. A third study by the authors (Corsiglia et al., 1973) utilised a rapid scanning technique with a triple-wire anemometer to eliminate the effects of wandering in the mid-field. They measured mean flow and turbulent characteristics up to 31 span lengths downstream of the NACA 0015 wing. Maximum tangential velocity was found to be higher than measurements taken in the previous study and was found to decrease with downstream distance. Measured axial velocities reached a value of $0.8U_\infty$ with no appreciable downstream variation, whereas the core radius increased with downstream distance.

Lee and Schetz (1985) conducted a study on a swept NACA 0012 wing using a five-hole pressure probe and one component hot-wires. The study investigated the effect of Reynolds number on the mean flow and turbulent characteristics for up to nine chord lengths downstream. The vortex was found to be axisymmetric and rolled up by six chord lengths downstream. In addition, it was found that increasing Reynolds number led to a decrease in the axial velocity deficit and core turbulent intensity but an increase in maximum swirl velocity.

One of the first non-intrusive investigations was carried out by Higuchi et al. (1987) with the hope of predicting the near-field roll-up of the tip vortex. Laser Doppler Velocimetry (LDV) was used to measure the mean flow up to four chords downstream of an elliptic wing. It was found that the vortex core was inherently asymmetric and that the size of the core increased with angle of attack and downstream distance but decreased with Reynolds number. A slight increase in vortex circulation with downstream distance was observed but the vortex was found to contain only 45% of the theoretical mid-span circulation.

McAlister and Takahashi (1991) conducted a comprehensive study on a rectangular NACA 0015 wing using laser doppler velocimetry (LDV) to measure the mean flow characteristics. A number of conditions were varied, which included aspect ratio, chord size, chord Reynolds number, tip shape, presence or absence of a boundary layer trip and downstream position. Pressure and velocity measurements were recorded for angles of attack ranging from $4^\circ$ to $12^\circ$ and for Reynolds numbers between $1 \times 10^6$ and $3 \times 10^6$. A maximum core axial velocity of $1.5U_\infty$
and a maximum tangential velocity of $0.84U_{\infty}$ were recorded just behind the wing at a location of $x/c = 0.1$ at $12^\circ$ angle of attack. Single parameters were varied and all other parameters were held constant to determine the effect of each. An increase in chord size was found to increase tangential and axial velocities, whereas increasing Reynolds number was found to decrease the tangential and axial velocities. A square tip wing was found to produce a secondary vortex, which was eliminated by the introduction of a round lateral tip.

Bandyopadhyay et al. (1991) carried out the first investigation that sought to identify organized motions in a trailing vortex and understand their role in the production and dissipation of turbulence in a vortex up to 40 chord lengths downstream. The vortex was shed from a flow aligned cylinder and two oppositely loaded aerofoils in a low Reynolds number ($15 \times 10^3$ to $25 \times 10^3$) flow subjected to a range of free-stream turbulence ($0.032\%$ to $1.48\%$). The Rossby number (axial velocity defect/maximum tangential velocity) was found to have a stronger effect on the turbulence structure than that due to the vortex core Reynolds number (circulation/viscosity). The conventional view that the vortex core was a benign solid body of rotation was rejected by the findings of this study. Instead, the authors suggested the core had a wavelike character with intermittent patches of turbulent and partially relaminarized fluid in the core and the exchange of momentum between the outer turbulent region and the core is carried out by organized motions.

Shekarriz et al. (1993) conducted a towing tank study on a rectangular wing for Reynolds numbers between $3.6 \times 10^4$ and $2.2 \times 10^5$. They found that the vortex rollup was almost complete at the trailing edge of the wing and that less than 66% of the root circulation was entrained into the tip vortex. A number of secondary vortices were observed outside of the main tip vortex, which dominated tangential velocity profiles. A core axial velocity deficit, which increased with angle of attack was found for all cases with Reynolds numbers less than $10^5$. The velocity distribution and the overall circulation of the tip vortex were found to become increasingly unsteady as the Reynolds number was reduced and angle of attack increased.

The first study to investigate in-depth the effects of vortex wandering and its contribution to Reynolds stress fields was that by Devenport et al. (1996).
2.1 Experimental Investigations of Wing-tip Vortices in the Near-field

They found that vortex wandering amplitudes were small (typically 1% chord) and that they increased with downstream distance but decreased with angle of attack. They developed a vortex wandering correction, which was later used by Bailey and Tavoularis (2008), and concluded that the vortex core was laminar and velocity fluctuations experienced there are inactive motions produced as the core is buffeted by turbulence from the surrounding wake. In addition, it was found that turbulence stress-fields high-pass filtered to remove contributions from wandering and inactive motions show true turbulence levels to be constant in the core and much less than those in the surrounding wake.

One of the most important experimental wing-tip vortex studies in the near-field was conducted by Chow et al. (1997b) (see also Chow et al. (1997a)), which has been used to validate the majority of numerical investigations in the last 10 years (Churchfield and Blaisdell, 2009, 2013; Craft et al., 2006; Dacles-Mariani et al., 1995; Kim and Rhee, 2005; Revell and Iaccarino, 2006; Uzun et al., 2006). They investigated the mean flow and turbulent characteristics of a wing-tip vortex generated from a NACA 0012 half-wing at a chord Reynolds number of $4.6 \times 10^6$. A number of experimental measurement techniques were used, such as seven-hole pressure probe measurements, triple-wire anemometry, surface pressure measurements and oil flow visualization. An axial velocity excess of $1.77U_\infty$ and a maximum crossflow velocity of $1.072U_\infty$ were measured in the vortex core just behind the wing at a location of $x/c = 0.125$. The study also recorded detailed turbulence measurements with all components of the Reynolds shear stress tensor being measured. The flow was found to be highly turbulent in the near-field (24% rms velocity), which decayed with stream-wise distance due to the stabilising effect of the solid body rotation of the vortex core mean-flow. One of the most important findings of the study was that the Reynolds shear stresses were not aligned with the mean strain rate (see Figure 2.2), suggesting that an isotropic-eddy-viscosity-based turbulence model would be unsuccessful at numerically modelling the turbulence in a wing-tip vortex.

Ramaprian and Zheng (1997) investigated a wing-tip vortex up to three chord lengths downstream of a rectangular NACA 0015 wing. They used non-intrusive LDV to measure all three of the instantaneous components of velocity at two angles of attack ($5^\circ$ and $10^\circ$). It was found that the vortex core was dominated
by axial vorticity, which was maximum at the centre and nearly zero at the outer part of the vortex. The axial vorticity was found to gradually increase with downstream distance due to the continuous trapping of the shear layer and the vortex was observed to become nearly axisymmetric by about two chord lengths downstream. They also found that the time averaged flow structure within the inner part of the vortex quickly attained a well-known universal state.

Sousa and Pereira (2000) also used the non-intrusive LDV technique to investigate a trailing vortex issued from a blade with flow separation for a Reynolds number of $1.5 \times 10^4$ at an angle of attack of $9.5^\circ$. Mean and turbulent measurements were recorded for up to four chord lengths downstream of the vortex. Two regions of axial velocity deficit were found downstream of the blade, one region in the wake resulting from the flow separation over the blade and the other region in the tip vortex. The deficit in the tip vortex exhibited a mild decay, whereas

Figure 2.2: Contours of mean strain rate and respective Reynolds stress (Chow et al., 1997b)
2.1 Experimental Investigations of Wing-tip Vortices in the Near-field

the deficit in the wake was rapidly dissipated. In contrast to the findings of Ramaprian and Zheng (1997), axial vorticity was found to decrease with downstream distance (see Figure 2.3), which is most likely caused by the predominance of viscous/turbulent diffusion over the continuous trapping of vorticity from the shear layer.

The study by Anderson and Lawton (2003) sought to make a correlation between the vortex strength (circulation) and the axial velocity in a trailing vortex. They investigated the vortex generated by a NACA 0015 wing section using three different Reynolds numbers ranging from $0.75 \times 10^6$ to $1.25 \times 10^6$. The axial velocity and circulation parameter were found to be linearly proportional to the wing angle of attack and were therefore proportional to each other. The linear relationship between the circulation and angle of attack is consistent with

![Figure 2.3: Contours of mean axial vorticity at four axial stations downstream of the blade: (a) $x = 0.5$; (b) $x = 1.0$; (c) $x = 2.0$; (d) $x = 4.0$ (Sousa and Pereira, 2000)]](image-url)
2. LITERATURE REVIEW

Prandtl’s lifting-line theory.

Heyes et al. (2003) investigated the rate of roll-up of the vortex sheet in the near-field of a wing-tip vortex generated from a NACA 0012 wing section. Measurements were made up to 10 chord lengths downstream at a chord Reynolds number of \(10^5\). They found that about 50% of the total circulation in the wing wake is shed from the separated flow around the tip. The vorticity in the sheet shed from the wing was observed to have rolled up at about one chord length downstream, while the circulation around the rolled-up region increased to around 80% of the total shed circulation within ten chord lengths downstream. A similar study was carried out around the same time by Birch et al. (2003), which sought to add to the understanding of a tip vortex by conducting an investigation into the roll-up and near-field behaviour of a vortex shed from a high lift cambered wing at a Reynolds number of \(3.25 \times 10^5\). Measurements were made with a miniature seven-hole pressure probe and particle image velocimetry (PIV) up to two and a half chord lengths downstream at angles of attack ranging from 2° to 18°. The tip region was found to be dominated by multiple secondary vortex structures and the rollup was almost complete at the trailing edge. Vortex characteristics such as tip vortex strength and the maximum tangential and core axial velocities significantly increased with the angle of attack. Depending on the angle of attack, the axial velocity exhibited either a wake-like or jet-like pattern. A subsequent study by Birch et al. (2004) investigated the structure and induced drag of a tip vortex generated by the same high lift cambered wing as used by Birch et al. (2003) and a rectangular NACA 0015 wing section. Measurements were made in cross-stream planes for two chord lengths downstream of the wing at an angle of attack of 10° and a chord Reynolds number of \(2.01 \times 10^5\). For both wings, the tip region was dominated by the strong interaction between the multiple secondary vortices and the main vortex. Roll up was observed to be complete by half a chord length and the overall circulation of the tip vortex for both wings remained nearly constant up to two chord lengths downstream. The vortex circulation and peak values of vorticity were found to increase linearly with angle of attack, which was in keeping with Prandtl’s lifting line theory and similar to the observations by Anderson and Lawton (2003). As expected, the cambered wing was found to generate much higher induced drag than the symmetrical NACA 0015 wing. The
2.1 Experimental Investigations of Wing-tip Vortices in the Near-field

induced drag of the NACA 0015 wing accounted for 20% of the total drag on the three-dimensional wing.

Similar to the earlier study by Bandyopadhyay et al. (1991) was an experimental study of a trailing vortex immersed in an external grid-generated turbulence field carried out by Beninati and Marshall (2005). They sought to add to the knowledge of how a vortex interacts with external turbulence, citing a lack of literature on the problem (despite its importance) as motivation. The study focused on examining the distribution of the turbulent and wave energy in spectral space in order to determine the influence of different length-scale oscillations on the turbulent kinetic energy and Reynolds stress. Turbulence from the blade and hub boundary layer is advected into the vortex core and persists with significant intensity downstream of the blade. For cases with and without the presence of external turbulence, the total turbulent kinetic energy in the core was found to reach an asymptotic state which didn’t change beyond $x/c = 14$. In the presence of external turbulence the vortex core was found to exhibit a bending wave far downstream with a dominant wavelength of between one and three times the core diameter. The bending wave on the vortex core appeared to result directly from the response of the vortex to buffeting from the external turbulence. Waves of this wavelength were found to contribute significantly to the transverse components of the turbulent kinetic energy and the Reynolds shear stresses far downstream in the vortex flow. Another two comprehensive studies on the effect of free-stream turbulence on a vortex were carried out by Bailey et al. (2006) and Bailey and Tavoularis (2008), where they investigated the effects of free-stream turbulence on the rollup, formation and development of a wing-tip vortex in the near-field. A four-sensor hot-wire probe was used to measure the instantaneous velocity vectors in cross-stream planes within ten chord lengths downstream of a NACA 0012 wing. Three free-stream turbulence levels were tested, and in all cases the tip vortex was found to form three smaller vortices but the turbulence in its core increased with increasing free-stream turbulence. The vortex trajectory was unaffected by free-stream turbulence but the wing wake that was rolling up around the vortex had a decrease in curvature as free-stream turbulence increased. They also observed an increased rate of decay of the vortex peak circumferential velocity and an increase in vortex core radius with increasing free-stream turbulence.
The development of maximum circumferential velocity ($U_{\theta_{\text{max}}}$ normalized by the local free-stream velocity $U_o$) and maximum core radius ($r_{\theta_{\text{max}}}/c$ normalized by the wing chord length $c$) with downstream distance can be seen in Figure 2.4 fitted with a power law and the wandering corrections of Devenport et al. (1996).

Gerontakos and Lee (2006) investigated a near-field tip vortex behind a swept NACA 0015 wing using a miniature seven-hole probe. The investigation focused on the effects of angle of attack and downstream distance on the behaviour and variation of the tangential velocity, axial velocity and vorticity distributions of the tip vortex. The tangential, axial velocity and vorticity distributions were found...
2.2 Numerical Investigations of Wing-tip Vortices in the Near-field

to become axisymmetric with downstream distance. There was no appreciable change in the level of vorticity contours with downstream distance, whereas the core axial velocity was observed to vary between $0.64U_\infty$ and $0.86U_\infty$. The tangential velocity decreased with downstream distance and increased with angle of attack, while the inboard shifting and core radius of the vortex were found to have no dependence on the angle of attack.

A recent experimental study on a vortex in the near-field was conducted by Giuni and Green (2013), where flow visualization and Particle Image Velocimetry (PIV) were used to investigate the vortex formation on a squared and rounded tip wing. The vortex was generated from a NACA 0012 with a squared and rounded tip at a Reynolds number of $3 \times 10^3$ for the flow visualization and $7.4 \times 10^3$ for the PIV measurements. Two different sources of vortex fluctuation, which contributed to the wandering of the vortex were identified for the two wing-tip geometries: the interaction of secondary vortices moving around the primary vortex for the square tip wing and the rolling up of the vorticity sheet for the rounded tip wing. Turbulence arising from secondary structures and shear layers is entrained into the vortex and high levels of turbulence were measured in the vortex core in the early wake. Measurements in the wake of the wing at zero angle of attack showed a distinctive counter rotating vortex pair.

2.2 Numerical Investigations of Wing-tip Vortices in the Near-field

Although experimental investigations are essential to understanding the behaviour of a wing-tip vortex and provide useful physical data, they can be extremely time consuming and complex to set up. Properly validated numerical investigations, on the other hand, can provide a lot of details about the flow-field in a reasonable amount of time. The recent improvements in computational power have made the idea of numerically modelling a wing-tip vortex a more appealing proposition. However, the numerical modelling of such a fluid flow phenomena does have its challenges as outlined by Craft et al. (2006): The flow over the outboard part of the wing develops into a highly skewed, three-dimensional boundary layer
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that, as it detaches, rolls up to form the strong, nearly axisymmetric trailing vortex. Therefore, the whole wing boundary layer on both the pressure and suction surfaces has to be computed accurately to provide the correct initial conditions for the vortex. In addition, the turbulence model chosen has to be capable of resolving accurately the complex wing boundary layer as well as the free-shear rotational flow downstream of the wing. It is perhaps, for this reason, that there are relatively few numerical studies reported of the development and roll-up of a conventional wing-tip vortex.

One of the first numerical studies of a wing-tip vortex in the near-field was conducted by Srinivasan et al. (1988). They used thin layer Navier-Stokes and the Euler equations together with a Baldwin-Lomax algebraic turbulence model to numerically model the tip vortex from four different wing planform shapes in subsonic and transonic flows. The subsonic flow investigation involved computing the flow over a rectangular NACA 0015 wing at an angle of attack of 5° and a Reynolds number of $2 \times 10^6$. The subsonic simulation reproduced the qualitative behaviour of the experimental tip vortex formation, including the changes in the tip flow separation and the vortex lift off due to the rounded caps was also in agreement with experiment. However, the square tip simulations failed to predict the correct magnitudes for the suction peaks under the vortex and the vortex flow in the wake of the wing was poorly predicted. The next major numerical investigation was conducted by Dacles-Mariani et al. (1995) in conjunction with the experimental study of Chow et al. (1997). The investigation looked at the three-dimensional flow over a NACA 0012 wing for an angle of attack of 10° and a Reynolds number of $4.6 \times 10^6$. They utilised a modified version of the one-equation Baldwin-Barth turbulence model as the Baldwin-Lomax model of Srinivasan et al. (1988) was deemed to have severe limitations when applied to vortical flows. Good agreement was achieved between the computed and measured mean velocity fields, whereas the turbulence modelling was not as accurate as desired. One of the major findings of the joint numerical/experimental study was that the Reynolds stresses were not aligned with the mean strain rate and as a result any turbulence model which utilises a constant or eddy viscosity approach is not likely to be fully successful at modelling a wing-tip vortex.
2.2 Numerical Investigations of Wing-tip Vortices in the Near-field

The realisation that the flow physics of a wing-tip vortex were not likely to be captured accurately with linear eddy viscosity turbulence models led to a number of studies utilising more advanced turbulence models and larger grids. One such study was that of Kim and Rhee (2005), which focused on assessing the capability of modern computational fluid dynamics in predicting the flow features of a wing-tip vortex. A feature adaptive mesh refinement technique was evaluated along with the impact of turbulence modelling. Four turbulence models were evaluated: a one-equation Spalart-Allmaras model with the correction proposed by Dacles-Mariani et al. (1995) applied, a realizable $k - \varepsilon$ (RKE) model, an SST-Mentor $k - \omega$ model and a Reynolds stress transport model. The Reynolds stress transport model was found to produce the best result with the Reynolds shear stress component $\overline{v'w'}$ exhibiting the same four leaf clover pattern observed by Chow et al. (1997b). In addition, the modified Spalart Allmaras model predicted the mean flow remarkably well, whereas, the $k - \varepsilon$ (RKE) and SST-Mentor $k - \omega$ model performed rather poorly, substantially under-predicting the strength of the tip vortex. It was also concluded that a 1.4 million cell mesh with feature adaptive mesh refinement performed similarly to a globally refined 2.3 million cell mesh. Another recent study by Craft et al. (2006), examined a non-linear eddy viscosity model and a two-component limit (TCL) Reynolds stress transport model for their ability to compute the wing and wake region without modification to the model. A linear eddy-viscosity $k - \varepsilon$ model was also used as a reference. Both the linear and non-linear eddy viscosity models were found to perform poorly as they predicted a rapid decay of the core centre velocity with downstream distance. The TCL Reynolds stress model was found to perform better and was the only model to reproduce the principal features found in the experimental measurements. However, the TCL model still had a much faster decay rate of the turbulent stress field than in the experiment. The striking difference between the predicted axial velocity among the three models is shown in Figure 2.5. The authors also found that the vortex core was especially sensitive to grid density and spatial order of the numerical scheme. Although not the same as performing grid refinement, using a high-order numerical scheme improves the solution accuracy without as much computational cost as adding more cells.
The extensive grid requirements of modelling a wing-tip vortex have been reported in several numerical investigations (see Dacles-Mariani et al. (1995), Craft et al. (2006), Churchfield and Blaisdell (2009)), clearly highlighting the computational expense of predicting such a flow. Added to this the solution of a seven-equation Reynolds stress turbulence model further increases the computational expense. In an attempt to bridge the gap between the computational expense of a Reynolds stress transport model and the weaknesses of the two-equation eddy viscosity models, Revell and Iaccarino (2006) assessed the performance of a new turbulence model which accounted for the stress-strain misalignment effects in a turbulent flow. The new model consisted of a parameter $C_{as}$, which accounted for the stress-strain misalignment by projecting the six equations of the Reynolds stress onto a single equation. This third equation was then incorporated into the SST-Menter $k - \omega$ model, and was termed the SST-$C_{as}$ model. A standard SST-Menter model was used as a reference. The standard SST model was found to
exhibit a rapid decay of the axial velocity with downstream distance and at a location of $x/c = 0.24$ the core value was already much lower than the experimental value of Chow et al. (1997b) at the same location. The SST-$C_{as}$ model predicted a higher core axial velocity at $x/c = 0.24$; however, it was still a lot less than the experimentally predicted axial velocity. The main difference between the two models was the fact that the SST-$C_{as}$ model correctly predicted the axial velocity overshoot in the centre of the vortex at $x/c = 0.44$ and $x/c = 0.67$, whereas the standard SST model predicted a rapid decay. Although some features were predicted correctly, the turbulence models tested had a number of issues to be resolved and the turbulence modelling was very much in the infancy stage.

Another numerical study that examined advanced RANS turbulence models was conducted by Churchfield and Blaisdell (2009), who tested four turbulence models for their ability to simulate a wing-tip vortex in the near-field. The models assessed were the Spalart-Allmaras model in both standard form (SA) and with the Spalart-Shur correction for rotation and streamline curvature (SA-RC) (Spalart and Shur, 1997), the SST-Menter $k – \omega$ model and the Rumsey-Gatski $k – \varepsilon$ algebraic Reynolds stress model (ASM). The SA-RC model was most accurate in predicting the mean flow of the vortex as it predicted the centreline axial velocity to within 0.6% of the experimental value. The SA and SST models were found to perform the most poorly in the highly rotational vortex flow and the ASM model performed better than the SA and SST models but not as well as the SA-RC for the mean flow. However, the ASM model was the only model to correctly predict the lag of some of the Reynolds stress components behind the corresponding strain-rate components, which is true of experiment.

Although advanced RANS turbulence models can predict with reasonable accuracy the mean flow of a tip-vortex (Churchfield and Blaisdell, 2009; Craft et al., 2006; Dacles-Mariani et al., 1995) they struggle to predict the magnitudes of the turbulence quantities and the lag of the Reynolds stress components behind the corresponding strain rate components (Churchfield and Blaisdell, 2009). As a result, LES studies of a wing-tip vortex have become more popular and feasible in recent years (the increase in computational power available to research institutions is also partly responsible). One such study was that by Jiang et al. (2008) who conducted an LES study based on the experiment of Chow et al.
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(1997b). The investigation revealed the presence of a secondary vortex, which was counter-rotating with respect to the primary wing-tip vortex. It was found that the primary tip-vortex was originally turbulent but rapidly relaminarized after shedding in the near-field. The turbulence inside the primary tip vortex was not created by the vortex itself but by the turbulent shear layer and interaction between the primary and secondary vortex structures (similar to the experimental study by Devenport et al. (1996)). An earlier LES study by Youssef et al. (1998) also found that large scale structures of the spiral wake were responsible for the formation of turbulent fluctuations in the vortex core. Another relatively new technique in the numerical simulation of wing-tip vortices is the concept of vorticity confinement, which was developed by Steinhoff and his co-workers at the University of Tennessee (Hu et al., 2000; Kimbrell, 2012; Lynn, 2007; Steinhoff et al., 2005). The basic technique consists of adding a force-term to the momentum equations, which acts normal to the vorticity, thereby convecting it back towards the centre of the vortex as it diffuses away. It was demonstrated that vortices could be captured and maintained for long distances without dissipation on uniform Cartesian grids. Advantages of the vorticity confinement model was that it gave results comparable to other LES simulations and that it could be solved on relatively coarse computational grids using low-order discretization schemes (Kimbrell, 2012; Lynn, 2007).

2.3 Contribution of the Current Objectives to Unresolved Issues

As outlined in the literature review sections 2.1 and 2.2, a large number of experimental and numerical studies have been carried out in the near-field of a wing-tip vortex. However, there are still questions that remain unanswered, modelling techniques that require further investigation and flow physics that need better understanding. The main points that the objectives of this investigation look to address that have not been fully covered in the literature are as follows:

- There have been very few comprehensive experimental near-field investigations looking at both the mean flow and turbulent characteristics of a
2.3 Contribution of the Current Objectives to Unresolved Issues

wing-tip vortex. This is evidenced by the fact that nearly all numerical studies in the last 10 years have based their simulations on the experiment of Chow et al. (1997b). The experimental study in this work will serve as a future reference or benchmark for numerical modellers of wing-tip vortices.

- Although the work by Chow et al. (1997b) was an extensive investigation into the mean and turbulent characteristics of a wing-tip vortex, it was conducted at only one condition (one angle of attack and one Reynolds number) and only focused on the formation and roll up of the vortex in the very near-field (less than one chord length downstream). The current investigation aims to conduct a significant experimental study into both the mean and turbulent characteristics of a wing-tip vortex for up to three chord lengths downstream, which will greatly improve the understanding of a wing-tip vortex from initial roll up to the vortex becoming steady and axisymmetric further downstream.

- Although advanced RANS turbulence models, such as a Reynolds stress transport model or modified eddy viscosity models, have shown some promise at reproducing the principal features of a vortex for the mean flow, they struggle at correctly predicting the turbulence structure of the vortex. The focus of the numerical modelling in this investigation is to evaluate advanced turbulence modelling techniques such as LES and vorticity confinement for their ability to accurately predict both the mean and turbulent characteristics of a wing-tip vortex. URANS and RANS simulations are also conducted for comparison. It is expected that the numerical simulations conducted in this investigation will help gain a better understanding of the vortex characteristics and the limits and capabilities of state of the art numerical modelling.

- In order to develop effective wing-tip modifications such as drag reducing blended winglets, more studies are needed to gain a better understanding of the flow physics and vortex formation in the near-field. The dynamics of the initial roll up and its subsequent development in the near-field have not been the subject of intense investigation and this study aims to contribute
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to the understanding of the fundamental processes involved in the roll up of a wing-tip vortex in the near-field to improve the design of such devices.

- Whether or not the core of a vortex is laminar or turbulent is still a matter of debate and the effect of turbulence on the mean flow and the early development of wing-tip vortices is still unclear (Birch, 2012). More detailed studies of the turbulence structure within a vortex are needed to address this question.
Chapter 3

Numerical and Experimental Theory

The turbulence modelling strategies are outlined in this chapter along with a detailed explanation of the time averaging and spatial averaging of the governing equations of incompressible flow. The Reynolds stress transport model and vorticity confinement model are also outlined.

3.1 Governing Equations of Incompressible Flow

The governing equations of unsteady incompressible flow are discussed in this section. Only the conservation of mass and the Navier-Stokes momentum equations are outlined as the energy equation was not invoked for the numerical simulations conducted in this study. The derivations of the Navier-Stokes equations from first principles are not given and the reader is directed to the work of Versteeg and Malalasekera (2007) for a complete derivation.

3.1.1 Continuity Equation in Three Dimensions

The continuity or mass flow equation for an incompressible flow in three dimensions is:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (3.1)
\]
3. NUMERICAL AND EXPERIMENTAL THEORY

or in vector form:

\[ \nabla \cdot \vec{u} = 0 \]  \hspace{1cm} (3.2)

3.1.2 Momentum Equation in Three Dimensions

The Navier-Stokes momentum equations for unsteady incompressible flow in three dimensions where there are no additional body forces are:

**x-direction**

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \]  \hspace{1cm} (3.3)

or in vector form:

\[
\frac{\partial u}{\partial t} + \nabla \cdot (u\vec{u}) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla \cdot (\nabla u) \]  \hspace{1cm} (3.4)

**y-direction**

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \]  \hspace{1cm} (3.5)

or in vector form:

\[
\frac{\partial v}{\partial t} + \nabla \cdot (v\vec{u}) = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \nabla \cdot (\nabla v) \]  \hspace{1cm} (3.6)

**z-direction**

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \]  \hspace{1cm} (3.7)

or in vector form:

\[
\frac{\partial w}{\partial t} + \nabla \cdot (w\vec{u}) = -\frac{1}{\rho} \frac{\partial p}{\partial w} + \nu \nabla \cdot (\nabla w) \]  \hspace{1cm} (3.8)
3.2 Reynolds-Averaged Navier-Stokes (RANS) Modelling

Reynolds averaging involves applying the principles of time-averaging to the equations of fluid motion, i.e. Continuity and Navier-Stokes momentum equations. The fluid in this study is assumed incompressible with no thermal interaction and no additional body forces present; therefore, the process of Reynolds averaging the equations of fluid motion is somewhat simplified (outlined in the first section). The theory behind the Reynolds Stress Transport Model (RSM) (the model used in this investigation) is then discussed.

3.2.1 Reynolds Averaging of Unsteady Fluid Flow Equations

The governing equations of unsteady incompressible flow (Eqs. 3.1 - 3.7) apply to both laminar and turbulent flows. They can be solved directly for laminar flow as there are four equations and four unknowns $u, v, w$ and $p$. However, in turbulent flows the vortical motions lead to velocity fluctuations about the mean value and turbulent shearing stresses known as the Reynolds stresses. These effects are incorporated into the equations using the time-averaging principle.

The first step in the time-averaging process is to replace the instantaneous flow variables $\vec{u} = (u, v, w)$ and $p$ in all four equations with the sum of mean and fluctuating components:

\[
\vec{u} = \vec{U} + \vec{u}', \quad u = U + u', \quad v = V + v', \quad w = W + w', \quad \text{and} \quad p = P + p'
\]

Before time averaging the equations of fluid motion it is necessary to know some of the properties of time-averages of scalar and vector flow variables. Table 3.1 adapted from Versteeg and Malalasekera (2007) summarises the properties of time averages of arbitrary scalar properties $\varphi$ and $\psi$ and a vector quantity $\vec{a}$:
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Table 3.1: Properties of time-averages of scalar and vector flow variables

Fluctuating and mean values of $\varphi$ and $\psi$:

$$\overline{\varphi'} = \overline{\varphi} = 0, \overline{\Phi} = \Phi \quad \text{and} \quad \overline{\Psi} = \Psi$$

Integrals and derivatives of $\varphi$ and $\psi$:

$$\frac{\partial \varphi}{\partial s} = \frac{\partial \Phi}{\partial s} \quad \text{and} \quad \int \varphi ds = \int \Phi ds$$

Summations and products of $\varphi$ and $\psi$:

$$\overline{\varphi + \psi} = \Phi + \Psi, \overline{\varphi \psi} = \Phi \Psi + \overline{\varphi' \psi'}, \overline{\varphi \Psi} = \Phi \Psi, \overline{\varphi' \psi} = 0 \quad \text{but} \quad \overline{\varphi' \psi'} \neq 0$$

Vector derivatives and combinations with scalars:

$$\nabla \cdot \vec{a} = \nabla \cdot \vec{A}, \nabla \cdot (\varphi \vec{a}) = \nabla \cdot (\varphi \vec{A}) = \nabla \cdot (\Phi \vec{A}) + \nabla \cdot (\varphi' \vec{a}')$$

$$\quad \text{and} \quad \nabla \cdot (\nabla \varphi) = \nabla \cdot (\nabla \Phi)$$

Time-Averaged Continuity Equation for Turbulent Flow

The velocity in the continuity or mass flow equation is expressed in terms of a mean and fluctuating component:

$$\nabla \cdot \vec{u} = 0 \Rightarrow \nabla \cdot \vec{U} + \nabla \cdot \vec{u}' = 0 \quad (3.9)$$

Then applying time-averaging the equation becomes:

$$\overline{\nabla \cdot \vec{U}} + \overline{\nabla \cdot \vec{u}'} = 0 \quad (3.10)$$

From Table 3.1, the time-average of the mean velocity $\nabla \cdot \vec{U}$ is simply the mean velocity $\nabla \cdot \vec{U}$ and the time-average of the fluctuating component $\nabla \cdot \vec{u}'$ is zero. The time-averaged continuity equation now becomes:

$$\nabla \cdot \vec{U} \overline{= \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0} \quad (3.11)$$
3.2 Reynolds-Averaged Navier-Stokes (RANS) Modelling

Time-Averaged Momentum Equations for Turbulent Flow

The exact same procedure can be applied to the instantaneous Navier-Stokes momentum equations and their turbulent flow equivalents can be obtained. These are commonly referred to as the Reynolds Averaged Navier-Stokes (RANS) equations. Applying time-averaging to the \( x \)-direction momentum equation by replacing the instantaneous velocities and pressures with the sum of a mean and fluctuating component gives:

\[
\frac{\partial(U + u')}{\partial t} + \nabla \cdot [(U + u')(\bar{U} + \bar{u}')] = -\frac{1}{\rho} \frac{\partial(P + p')}{\partial x} + \nu \nabla \cdot [\nabla(U + u')] \tag{3.12}
\]

Expanding the differential terms inside the brackets gives:

\[
\frac{\partial U}{\partial t} + \frac{\partial u'}{\partial t} + \nabla \cdot [U \bar{U} + U \bar{u}' + u' \bar{U} + u' \bar{u}'] = -\frac{1}{\rho} \frac{\partial P}{\partial x} - \frac{1}{\rho} \frac{\partial p'}{\partial x} + \nu [\nabla \cdot (\nabla U) + \nabla \cdot (\nabla u')] \tag{3.13}
\]

and time-averaging the above equation yields:

\[
\frac{\partial U}{\partial t} + \frac{\partial u'}{\partial t} + \nabla \cdot [U \bar{U} + U \bar{u}' + u' \bar{U} + u' \bar{u}'] = -\frac{1}{\rho} \frac{\partial P}{\partial x} - \frac{1}{\rho} \frac{\partial p'}{\partial x} + \nu [\nabla \cdot (\nabla U) + \nabla \cdot (\nabla u')] \tag{3.14}
\]

Applying the principles of time-averaging from Table 3.1 to each transport term in the previous equation gives:

**Unsteady Term:**

\[
\frac{\partial U}{\partial t} + \frac{\partial u'}{\partial t} = \frac{\partial U}{\partial t} \tag{3.15}
\]

**Convective Term:**

\[
\nabla \cdot [U \bar{U} + U \bar{u}' + u' \bar{U} + u' \bar{u}'] = \nabla \cdot (U \bar{U}) + \nabla \cdot (u' \bar{u}') = \nabla \cdot (U \bar{U}) + \nabla \cdot (u' \bar{u}') \tag{3.16}
\]
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Pressure Gradient Term:

\[-\frac{1}{\rho} \frac{\partial P}{\partial x} - \frac{1}{\rho'} \frac{\partial P'}{\partial x} = -\frac{1}{\rho} \frac{\partial P}{\partial x} = -\frac{1}{\rho} \frac{\partial P}{\partial x}\]  \hspace{1cm} (3.17)

Diffusion/Viscous Term:

\[\nu[\nabla \bullet (\nabla U) + \nabla \bullet (\nabla \bar{u}')] = \nu \nabla \bullet (\nabla U) = \nu \nabla \bullet (\nabla U)\]  \hspace{1cm} (3.18)

Re-assembling the remaining terms the time-averaged \(x\)-direction momentum equation now becomes:

\[\frac{\partial U}{\partial t} + \nabla \bullet (U \bar{U}) + \nabla \bullet (\bar{u}' \bar{u}') = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \nabla \bullet (\nabla U).\]  \hspace{1cm} (3.19)

Performing the same procedure on the corresponding \(y\) and \(z\)-direction Navier-Stokes Momentum Equations yields their turbulent versions:

\(y\)-direction

\[\frac{\partial V}{\partial t} + \nabla \bullet (V \bar{U}) + \nabla \bullet (\bar{v}' \bar{u}') = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \nabla \bullet (\nabla V)\]  \hspace{1cm} (3.20)

\(z\)-direction

\[\frac{\partial W}{\partial t} + \nabla \bullet (W \bar{U}) + \nabla \bullet (\bar{w}' \bar{u}') = -\frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \nabla \bullet (\nabla W)\]  \hspace{1cm} (3.21)

The above equations resemble quite closely the original instantaneous Equations 3.3 - 3.7 except that the flow variables are now represented by mean flow values and new convective terms with fluctuating velocity i.e. \(\nabla \bullet (u' \bar{u}'), \nabla \bullet (v' \bar{u}')\) and \(\nabla \bullet (w' \bar{u}')\). These extra terms represent the additional momentum that is transported by the turbulent vortical motions in the flow. Normally the time-averaged Navier-Stokes momentum equations are represented such that the turbulent momentum
3.2 Reynolds-Averaged Navier-Stokes (RANS) Modelling

Transfer terms appear on the right hand side with the pressure and viscous stress terms. The additional convective terms are multiplied by density to represent them as stresses. Rearranging each expression and expanding out the new stress terms gives:

\( x \)-direction

\[
\frac{\partial U}{\partial t} + \nabla \cdot (U \vec{U}) = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \nabla \cdot (\nabla U) - \nabla \cdot (\bar{u}'\bar{u}')
\]

(3.22)

\[
\Rightarrow \frac{\partial U}{\partial t} + \nabla \cdot (U \vec{U}) = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \nabla \cdot (\nabla U) + \frac{1}{\rho} \left[ \frac{\partial (-\rho u'^2)}{\partial x} + \frac{\partial (-\rho u'v')}{\partial y} + \frac{\partial (-\rho u'w')}{\partial z} \right]
\]

(3.23)

\[
\Rightarrow \frac{\partial U}{\partial t} + \nabla \cdot (U \vec{U}) = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \nabla \cdot (\nabla U) + \frac{1}{\rho} \left[ \frac{\partial \tau'_{xx}}{\partial x} + \frac{\partial \tau'_{xy}}{\partial y} + \frac{\partial \tau'_{xz}}{\partial z} \right]
\]

(3.24)

\( y \)-direction

\[
\frac{\partial V}{\partial t} + \nabla \cdot (V \vec{U}) = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \nabla \cdot (\nabla V) - \nabla \cdot (\bar{v}'\bar{u}')
\]

(3.25)

\[
\Rightarrow \frac{\partial V}{\partial t} + \nabla \cdot (V \vec{U}) = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \nabla \cdot (\nabla V) + \frac{1}{\rho} \left[ \frac{\partial (-\rho u'v')}{\partial x} + \frac{\partial (-\rho u'^2)}{\partial y} + \frac{\partial (-\rho u'w')}{\partial z} \right]
\]

(3.26)

\[
\Rightarrow \frac{\partial V}{\partial t} + \nabla \cdot (V \vec{U}) = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \nabla \cdot (\nabla V) + \frac{1}{\rho} \left[ \frac{\partial \tau'_{yx}}{\partial x} + \frac{\partial \tau'_{yy}}{\partial y} + \frac{\partial \tau'_{yz}}{\partial z} \right]
\]

(3.27)
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\[ \begin{align*}
&\frac{\partial W}{\partial t} + \nabla \cdot (W \mathbf{U}) = -\frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \nabla \cdot (\nabla W) - \nabla \cdot (\mathbf{w} \mathbf{u}^\prime) \\
\Rightarrow &\frac{\partial W}{\partial t} + \nabla \cdot (W \mathbf{U}) = -\frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \nabla \cdot (\nabla W) + \frac{1}{\rho} \left[ \frac{\partial (-\rho \mathbf{u}^\prime \mathbf{w}^\prime)}{\partial x} + \frac{\partial (-\rho \mathbf{v}^\prime \mathbf{w}^\prime)}{\partial y} + \frac{\partial (-\rho \mathbf{w}^\prime \mathbf{w}^\prime)}{\partial z} \right] \\
\Rightarrow &\frac{\partial W}{\partial t} + \nabla \cdot (W \mathbf{U}) = -\frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \nabla \cdot (\nabla W) + \frac{1}{\rho} \left[ \frac{\partial \tau^\prime_{xx}}{\partial x} + \frac{\partial \tau^\prime_{yy}}{\partial y} + \frac{\partial \tau^\prime_{zz}}{\partial z} \right] 
\end{align*} \tag{3.29} \]

These new stress terms have arisen from the time averaging process and are known as Reynolds Stresses. There are nine Reynolds stresses, three of which are normal stresses:

\[ \tau^\prime_{xx} = -\rho \mathbf{u}^\prime \mathbf{u}^\prime, \quad \tau^\prime_{yy} = -\rho \mathbf{v}^\prime \mathbf{v}^\prime \quad \text{and} \quad \tau^\prime_{zz} = -\rho \mathbf{w}^\prime \mathbf{w}^\prime \]

The other six Reynolds stresses act tangentially (i.e. as shear stresses):

\[ \tau^\prime_{xy} = \tau^\prime_{yx} = -\rho \mathbf{u}^\prime \mathbf{v}^\prime, \quad \tau^\prime_{yz} = \tau^\prime_{zy} = -\rho \mathbf{v}^\prime \mathbf{w}^\prime \quad \text{and} \quad \tau^\prime_{xz} = \tau^\prime_{zx} = -\rho \mathbf{u}^\prime \mathbf{w}^\prime \]

The Reynolds stresses are often expressed as a turbulent stress tensor $\tau^\prime_{ij}$ where the normal stresses are the diagonal elements with the shear stresses symmetric about the diagonal.

\[ \tau^\prime_{ij} = -\rho \mathbf{u}^\prime i \mathbf{u}^\prime j = -\rho \begin{pmatrix}
\mathbf{u}^\prime \mathbf{u}^\prime & \mathbf{u}^\prime \mathbf{v}^\prime & \mathbf{u}^\prime \mathbf{w}^\prime \\
\mathbf{v}^\prime \mathbf{u}^\prime & \mathbf{v}^\prime \mathbf{v}^\prime & \mathbf{v}^\prime \mathbf{w}^\prime \\
\mathbf{w}^\prime \mathbf{u}^\prime & \mathbf{w}^\prime \mathbf{v}^\prime & \mathbf{w}^\prime \mathbf{w}^\prime
\end{pmatrix} \]

There are now four equations (time-averaged continuity and the three RANS equations in $x$, $y$ and $z$ space), four mean unknowns ($U, V, W$ and $P$) and six turbulence unknowns (Reynolds stress components). In order to close the problem, it is necessary to have as many equations as unknowns, therefore the Reynolds stresses need to be expressed in terms of the mean velocities ($U, V, W$) so that there are four equations and four unknowns. This is essentially the purpose of turbulence modelling.
3.2 Reynolds-Averaged Navier-Stokes (RANS) Modelling

3.2.2 Reynolds Stress Transport Model (RSM)

The Reynolds Stress Transport Model is also known as the Second Moment Closure Model was originally developed by Launder et al. (1975) and is the most complex RANS based turbulence model. It does not rely on the Boussinesq eddy viscosity hypothesis and instead solves transport equations for each of the six Reynolds stresses as well as an additional equation for the turbulent dissipation $\varepsilon$.

Reynolds Stress Transport (RST) Equation

The starting point for the RSM model is the exact form of the Reynolds stress transport equation, which is derived by multiplying the instantaneous Navier Stokes equations by a fluctuating property such as velocity and time-averaging the result. The full Reynolds stress transport equation from Versteeg and Malalasekera (2007) is:

\[
\begin{align*}
\frac{\partial}{\partial t} \left( \rho u'_i u'_j \right) &+ \frac{\partial}{\partial x_k} \left( \rho u_k u'_i u'_j \right) = \frac{\partial}{\partial x_k} \left[ \rho u'_i u'_j u'_k + p(\delta_{kj} u'_i + \delta_{ki} u'_j) \right] \\
&+ \frac{\partial}{\partial x_k} \left[ \frac{\mu}{\kappa} \left( u'_i u'_j \right) \right] - \rho \left( \frac{u'_i u'_k}{\partial x_k} \frac{\partial u'_j}{\partial x_j} + \frac{u'_j u'_k}{\partial x_k} \frac{\partial u'_i}{\partial x_i} \right) + p \left( \frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right) \\
&- 2\rho \Omega_k \left( \frac{u'_j u'_m e_{ikm} + u'_i u'_m e_{jkm}}{\partial x_k} \right) - 2\mu \left( \frac{\partial u'_i}{\partial x_k} \frac{\partial u'_j}{\partial x_k} \right) \\
&\equiv \text{Unsteady Term} \hspace{1cm} C_{ij} \equiv \text{Convection} \\
&\hspace{1cm} D_{T,ij} \equiv \text{Turbulent Diffusion} \\
&\hspace{1cm} D_{L,ij} \equiv \text{Molecular Diffusion} \\
&\hspace{1cm} P_{ij} \equiv \text{Stress Production} \\
&\hspace{1cm} \Pi_{ij} \equiv \text{Pressure Strain-Interaction} \\
&\hspace{1cm} \Omega_{ij} \equiv \text{Rotation} \\
&\hspace{1cm} \varepsilon_{ij} \equiv \text{Dissipation} \\
\end{align*}
\]

(3.31)

Of the terms in the RST equation only the unsteady, convective $C_{ij}$, molecular diffusion $D_{L,ij}$ and rotational terms do not require modelling. However, the turbulent diffusion $D_{T,ij}$, dissipation $\varepsilon_{ij}$ and pressure-strain $\Pi_{ij}$ terms need to be
modelled to close the equation set. The pressure-strain term is the most challenging and important term to model and there are two different pressure-strain treatments available in the finite volume flow solver used for this investigation (Star-CCM+).

The original form of the Reynolds stress transport model was known as the Launder-Reece-Rodi (LRR) model and the stress transport equation is summarised as follows:

\[
\frac{\partial}{\partial t} \left( \rho u_i' u_j' \right) + C_{ij} = D_{T,ij} + D_{L,ij} + P_{ij} + \Pi_{ij} + \Omega_{ij} + \varepsilon_{ij} \quad (3.32)
\]

The model solves six versions of the above equation, one for each of the unique Reynolds stresses. The dissipation of turbulence is computed from the same \( \varepsilon \) equation as used in the standard \( k-\varepsilon \) model:

\[
\frac{\partial (\rho \varepsilon)}{\partial t} + \nabla \cdot (\rho \varepsilon \vec{U}) = \nabla \cdot \left[ \left( \mu + \mu_t \frac{\sigma_\varepsilon}{\varepsilon} \right) \nabla \varepsilon \right] + C_{1\varepsilon} \frac{\varepsilon}{k} 2 \mu_t S_{ij} S_{ij} - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} \quad (3.33)
\]

Where the model constants are set as: \( C_{1\varepsilon} = 1.44, C_{2\varepsilon} = 1.92 \) and \( \sigma_{\varepsilon} = 1 \)

As is the case with the \( k-\varepsilon \) model, this equation cannot be applied down to the wall, so the LRR Reynolds stress model must use wall functions to resolve the near-wall flow. The turbulent viscosity is calculated using the same equation as the Standard \( k-\varepsilon \) model:

\[
\mu_t = \rho C_{\mu} \frac{k^2}{\varepsilon} \quad (3.34)
\]

Where \( C_{\mu} = 0.09 \)

The computable terms of the RST equation are the convection, production, rotation and molecular (laminar) diffusion and they can be solved in their exact form.
3.2 Reynolds-Averaged Navier-Stokes (RANS) Modelling

Convective Term

\[ C_{ij} = \frac{\partial}{\partial x_k} \left( \rho U_k u'_i u'_j \right) = \nabla \cdot \left( \rho \vec{u}' \vec{u}' \vec{U} \right) \]  \hspace{1cm} (3.35)

Stress Production Term

\[ P_{ij} = \rho \left( u'_i u'_k \frac{\partial u'_j}{\partial x_k} + u'_j u'_k \frac{\partial u'_i}{\partial x_k} \right) \]  \hspace{1cm} (3.36)

Rotation Term

\[ \Omega_{ij} = -2 \rho \Omega_k \left( \overline{u_k u_m e_{ikm}} + \overline{u'_i u'_m e_{jkm}} \right) \]  \hspace{1cm} (3.37)

Molecular Diffusion Term

\[ D_{T,ij} = \frac{\partial}{\partial x_k} \left[ \frac{\mu}{\sigma_k} \frac{\partial}{\partial x_k} \left( u'_i u'_j \right) \right] = \nabla \cdot \left[ \frac{\mu}{\sigma_k} \nabla \left( u'_i u'_j \right) \right] \]  \hspace{1cm} (3.38)

Where \( \Omega_k \) is a rotational vector and \( e_{ijk} \) is an operator with a value of 0 or ±1.

To obtain a solvable form of Equation 3.32 we need models for the turbulent diffusion, dissipation rate and pressure-strain terms on the right hand side.

The turbulent diffusion \( D_{T,ij} \) term can be modelled with the assumption that the rate of transport of Reynolds stresses is proportional to gradients of Reynolds stresses. Commercial CFD codes often implement this version:

\[ D_{T,ij} = \frac{\partial}{\partial x_k} \left[ \frac{\mu_t}{\sigma_k} \frac{\partial}{\partial x_k} \left( u'_i u'_j \right) \right] = \nabla \cdot \left[ \frac{\mu_t}{\sigma_k} \nabla \left( u'_i u'_j \right) \right] \]  \hspace{1cm} (3.39)

With \( \sigma_k = 0.82 \) and the turbulent viscosity \( \mu_t \) given in Equation 3.34.
The dissipation rate $\varepsilon_{ij}$ is modelled by assuming isotropy of the small dissipative eddies:

$$\varepsilon = \frac{2}{3} \varepsilon \delta_{ij}$$

(3.40)

Where $\varepsilon$ is the dissipation rate of turbulent kinetic energy given in Equation 3.33 and $\delta_{ij}$ is the Kronecker delta which is given as $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$.

The pressure-strain interactions constitutes one of the most important terms of Equation 3.32, but one of the most difficult to model accurately (Versteeg and Malalasekera, 2007). Their effect on the Reynolds stresses is caused by two distinct physical processes: (i) a slow process that reduces anisotropy of the turbulent eddies due to their mutual interactions and (ii) a rapid process due to interactions between turbulent fluctuations and the mean flow strain that produce the eddies such that the anisotropic production of turbulent eddies is opposed.

The pressure-strain term in this investigation was modelled using the Linear-Pressure Strain Model of Gibson and Launder (1978). The Linear Pressure-Strain Model expresses the pressure-strain term as a sum of a number of components:

$$\Pi_{ij} = \Pi_{ij,1} + \Pi_{ij,2} + \Pi_{ij,w}$$

(3.41)

$\Pi_{ij,1}$ is the slow pressure-strain or return to isotropy term and is modelled by:

$$\Pi_{ij,1} = -C_1 \rho \frac{\varepsilon}{K} \left[ u'_i u'_j - \frac{2}{3} k \delta_{ij} \right]$$

(3.42)

With $C_1 = 1.8$

$\Pi_{ij,2}$ is the rapid pressure-strain term and is modelled by:

$$\Pi_{ij,2} = -C_2 \left[ (P_{ij} + \Omega_{ij} - C_{ij}) - \frac{2}{3} \delta_{ij} P \right]$$

(3.43)
3.2 Reynolds-Averaged Navier-Stokes (RANS) Modelling

With $C_2 = 0.6$

$P_{ij}$, $\Omega_{ij}$ and $C_{ij}$ are the turbulence production, rotation and convection terms from Equation 3.32.

$P$ represents the production terms derived from the mean strain rates:

$$P = -\rho \overline{u'_i u'_k} \frac{\partial U_j}{\partial x_k}$$

(3.44)

$\Pi_{ij,w}$ is the wall reflection pressure-strain term which accounts for the anisotropy of the normal Reynolds stresses near the wall of boundary layer flows. It dampens normal stresses perpendicular to the wall and enhances stresses parallel to the wall, which is similar to experimental observations. The wall-reflection term is calculated as follows:

$$\Pi_{ij,w} = C_{1w} \varepsilon_k \left[ \overline{u'_i u'_m n_k n_m \delta_{ij}} - \frac{3}{2} \overline{u'_i u'_k n_j n_k} - \frac{3}{2} \overline{u'_j u'_k n_i n_k} \right] k^{1.5} C_l \varepsilon y$$

(3.45)

$$+ C_{2w} \left[ \Pi_{km,2} n_k n_m \delta_{ij} - \frac{3}{2} \Pi_{ik,2} n_j n_k - \frac{3}{2} \Pi_{jk,2} n_i n_k \right] k^{1.5} C_l \varepsilon y$$

With $C_{1w} = 0.5$, $C_{2w} = 0.3$ and $C_l = 2.5$, $n$ terms are normal vectors and $y$ is normal distance from the wall.

As the RSM models use the same $\varepsilon$-equation as in the standard $k - \varepsilon$ model, it is incapable of resolving flow down to the wall. It is therefore a high Reynolds number model that uses wall-functions to resolve flow in the near-wall region. In this investigation a high $y^+$ wall treatment was selected in Star-CCM+ and the centroid of the wall adjacent cell was kept within $30 < y^+ > 300$. 

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3.3 Large Eddy Simulation (LES)

Large Eddy Simulation (LES) is a turbulence modelling method where the large eddies (those above a threshold size) are directly resolved and the higher frequency smaller eddies are modelled using Sub-Grid Scale (SGS) models. LES requires substantially finer meshes than RANS based models and needs to run for a sufficiently long flow time to achieve a statistically stable solution. It is therefore much more computationally demanding than RANS but much less than that of Direct Numerical Simulation (DNS).

3.3.1 Spatial Filtering of Unsteady Fluid Flow Equations

The equations for LES are obtained through spatial filtering of the continuity and Navier-Stokes equations as opposed to Reynolds averaging. The starting point for LES is usually the selection of a filtering function and cut-off width, and the purpose of filtering in LES is to split the continuity and momentum equations into a retained part and a rejected part. The spectrum of turbulent fluctuations in Figure 3.1 illustrates the scales that are filtered. The design of the filter and its cut-off width $\Delta$ determine what is retained and what is rejected (smaller eddies). The spatial filtering is then performed on the unsteady incompressible flow equations, giving rise to a Sub-Grid-Scale (SGS) stress term that needs to be modelled to solve for the smaller self-similar turbulent eddies.

In LES, a spatial filtering operation is defined by means of a filter function $G(x,x',\Delta)$ as follows:

$$
\overline{\phi}(x, t) \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(x,x',\Delta)\phi(x', t)dx'_1dx'_2dx'_3
$$

(3.46)

Where $\overline{\phi}(x, t) =$ filtered function, and $\phi(x, t) =$ original (unfiltered) function and $\Delta =$ filter cut-off width.

In this section the overbar indicates spatial filtering and not time-averaging. Spatial filtering is an integral in three-dimensional space as indicated by Equation 3.46, unlike time-averaging which is an integral with respect to time. The most common types of filtering function used in LES are:
Top-hat or box filter

\[ G(x, x', \Delta) = \begin{cases} \frac{1}{\Delta^3} & |x-x'| \leq \Delta/2 \\ 0 & |x-x'| > \Delta/2 \end{cases} \]  

(3.47)

Gaussian filter

\[ G(x, x', \Delta) = \left( \frac{\gamma}{\pi \Delta^2} \right)^3 \exp \left( -\gamma \frac{|x-x'|^2}{\Delta^2} \right) \]  

(3.48)

Where a typical value for \( \gamma = 6 \)

Spectral cut-off

\[ G(x, x', \Delta) = \prod_{i=1}^{3} \frac{\sin[(x_i - x'_i)/\Delta]}{(x_i - x'_i)} \]  

(3.49)

The top-hat filter is used in finite volume implementations of LES such as the one in this investigation. The Gaussian is generally used in finite difference
3. NUMERICAL AND EXPERIMENTAL THEORY

Computational Fluid Dynamics (CFD) LES code and spectral cut-off filters are difficult to implement in general-purpose CFD but can be useful for the resolution of eddies at certain frequencies. The cut-off width $\Delta$ is an indicative measure of the size of the eddies that are retained in the computations and can have any size, but it is usually taken to be of the same order of the grid size. In three-dimensional computations with grid cells of different length $\Delta x$, width $\Delta y$ and height $\Delta z$ the cut-off width is often taken to be the cube root of the grid cell volume:

$$\Delta = \sqrt[3]{\Delta x \Delta y \Delta z}$$ (3.50)

The top-hat filtering function is then applied to the incompressible continuity and momentum Equations 3.1–3.8:

**Continuity**

$$\nabla \cdot \bar{u} = 0$$ (3.51)

**x-direction**

$$\frac{\partial \pi}{\partial t} + \nabla \cdot (\bar{u} \bar{u}) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla \cdot (\nabla \bar{u})$$ (3.52)

**y-direction**

$$\frac{\partial \pi}{\partial t} + \nabla \cdot (\bar{v} \bar{u}) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla \cdot (\nabla \bar{v})$$ (3.53)

**z-direction**

$$\frac{\partial \bar{w}}{\partial t} + \nabla \cdot (\bar{w} \bar{u}) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla \cdot (\nabla \bar{w})$$ (3.54)
3.3 Large Eddy Simulation (LES)

In a similar way to the RANS equations the convective term \( \nabla \cdot (\bar{\phi} \bar{u}) \) in the momentum equations is modified slightly in the spatial averaging process. The term is expressed as follows:

\[
\nabla \cdot (\bar{\phi} \bar{u}) = \nabla \cdot (\bar{\phi} \bar{u}) + \left[ \nabla \cdot (\bar{\phi} \bar{u}) - \nabla \cdot (\bar{\phi} \bar{u}) \right] \tag{3.55}
\]

This modified convection term is then substituted into Equations 3.52–3.54 to yield the LES momentum equations:

**x-direction**

\[
\frac{\partial \bar{u}}{\partial t} + \nabla \cdot (\bar{u} \bar{u}) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} + \nu \nabla \cdot (\nabla \bar{u}) - \left[ \nabla \cdot (\bar{u} \bar{u}) - \nabla \cdot (\bar{u} \bar{u}) \right] \tag{3.56}
\]

**y-direction**

\[
\frac{\partial \bar{v}}{\partial t} + \nabla \cdot (\bar{v} \bar{u}) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} + \nu \nabla \cdot (\nabla \bar{v}) - \left[ \nabla \cdot (\bar{v} \bar{u}) - \nabla \cdot (\bar{v} \bar{u}) \right] \tag{3.57}
\]

**z-direction**

\[
\frac{\partial \bar{w}}{\partial t} + \nabla \cdot (\bar{w} \bar{u}) = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial z} + \nu \nabla \cdot (\nabla \bar{w}) - \left[ \nabla \cdot (\bar{w} \bar{u}) - \nabla \cdot (\bar{w} \bar{u}) \right] \tag{3.58}
\]

Similar to the Reynolds stresses arising out of the time-averaging process, the LES stress terms are a consequence of spatial averaging. This term can be considered as a divergence of a set of stresses \( \tau_{ij} \).

\[
\nabla \cdot (\bar{u} \bar{u}) - \nabla \cdot (\bar{u} \bar{u}) = \frac{\partial \tau_{ij}}{\partial x_j} \tag{3.59}
\]

\[
= \frac{1}{\rho} \left[ \frac{\partial (\rho \bar{u} \bar{u})}{\partial x} - \frac{\partial (\rho \bar{v} \bar{u})}{\partial y} + \frac{\partial (\rho \bar{w} \bar{u})}{\partial z} \right]
\]
3. NUMERICAL AND EXPERIMENTAL THEORY

Where \( \tau_{ij} = \rho \overline{u_i u_j} - \rho \overline{u_i} \overline{u_j} = \rho \overline{u_i u_j} - \rho \overline{u_i} \overline{u_j} \)

The stress terms given by \( \tau_{ij} \) on the right of the filtered equations are termed the sub-grid-scale (SGS) stresses—a term which requires modelling. The LES SGS stresses contain in addition to the Reynolds stress terms other contributions, which are revealed when a flow variable \( \phi(x,t) \) is decomposed into a filtered variable \( \overline{\phi}(x,t) \) above the cut-off \( \Delta \) and \( \phi'(x,t) \) below the cut-off:

\[
\phi(x,t) = \overline{\phi}(x,t) + \phi'(x,t)
\]  

(3.60)

This approach allows the decomposition of the SGS term into:

\[
\tau_{ij} = \rho \overline{u_i u_j} - \rho \overline{u_i} \overline{u_j} = (\rho \overline{u_i u_j} - \rho \overline{u_i} \overline{u_j}) + \rho \overline{u_i u_j} + \rho \overline{u_i} \overline{u_j} + \rho \overline{u_i} \overline{u_j} \]  

(3.61)

The three components of the SGS stress terms are:

- Term (i): Leonard stresses \( \overline{L_{ij}} \).
- Term (ii): Cross stresses \( \overline{C_{ij}} \).
- Term (iii): SGS Reynolds stresses \( \overline{R_{ij}} \).

The Leonard stresses \( \overline{L_{ij}} \) arise in the spatial averaging process as \( \overline{\phi} \neq \overline{\phi} \) and do not appear in time averaging as \( \overline{\phi(t)} = \overline{\Phi} = \Phi \). The cross stresses \( \overline{C_{ij}} \) are due to interactions between the SGS eddies and the resolved flow. The SGS Reynolds stress term arises due to convective momentum transfer due to interactions of the SGS eddies. Similar to RANS based CFD where a turbulence model was required to model the Reynolds stresses, LES requires an SGS turbulence model to compute the SGS term in the LES transport equations. Smagorinsky (1963) suggested that the smallest turbulent eddies are nearly isotropic and for that reason the Boussinesq hypothesis is used in SGS models to compute the turbulence stresses as a single entity from:

\[
\tau_{ij} = \overline{L_{ij}} + \overline{C_{ij}} + \overline{R_{ij}} = -2\mu_{SGS} \delta_{ij} + \frac{1}{3} \tau_{ij} \delta_{ij} = -2\mu_{SGS} \left[ \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right] + \frac{1}{3} \tau_{ii} \delta(ij)
\]  

(3.62)
3.3 Large Eddy Simulation (LES)

Where $\mu_{SGS}$ is the sub-grid-scale viscosity-analogous to $\mu_t$ in RANS models.

Most mainstream CFD codes offer the following SGS models:

- Smagorinsky-Lilly (1963/1966)
- Dynamic Smagorinsky-Lilly (1991)

The WALE SGS model was used to compute the SGS stresses in this investigation as it was less computationally demanding and reasonably insensitive to the value of the model coefficient when compared to the Smagorinsky Subgrid Scale model. In the WALE model the SGS viscosity is computed using the following:

$$
\mu_{sgs} = \rho \Delta^2 S_w
$$

(3.63)

Where the term $S_w$ is a deformation parameter defined by:

$$
S_w = \frac{\left[ S_{ij}^d S_{ij}^d \right]^\frac{3}{2}}{\left[ S_{ij}^d S_{ij}^d \right]^\frac{3}{2} + \left[ S_{ij}^d S_{ij}^d \right]^\frac{5}{2}}
$$

(3.64)

The tensor $S_{ij}^d$ is computed from the vorticity and strain-rate tensors:

$$
S_{ij}^d = S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj} - \frac{1}{3} \delta_{ij} \left[ S_{mn}S_{mn} - \Omega_{mn}\Omega_{mn} \right]
$$

(3.65)

The cut-off width is calculated using a new coefficient $C_w = 0.544$

$$
\Delta = \begin{cases} 
C_w V^\frac{4}{3} & \text{Length scale unlimited} \\
\min \left[ \kappa d, C_w V^\frac{1}{3} \right] & \text{Length scale limited} 
\end{cases}
$$

(3.66)
3.4 Vorticity Confinement Model

The vorticity confinement model used in this work was introduced by Steinhoff et al. (2005) and refined by Löhner (2009). The basic approach of the vorticity confinement model is the addition of a forcing term $f_\omega$ to the momentum equation in all directions. An example of the modified momentum equation in the x-direction is:

$$\frac{\partial u}{\partial t} + \nabla \cdot (u\ddot{u}) = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla \cdot (\nabla u) + f_\omega$$  \hspace{1cm} (3.67)

where the forcing term taken from Löhner (2009) is:

$$f_\omega = -\varepsilon \rho (\hat{n} \times \bar{\omega})$$  \hspace{1cm} (3.68)

where $\bar{\omega}$ is the vorticity, $\varepsilon$ is a user defined constant, $\rho$ is the fluid density and $\hat{n}$ is the unit vector that is aligned with:

$$\hat{n} = \frac{\nabla |\bar{\omega}|}{|\nabla |\bar{\omega}|}$$  \hspace{1cm} (3.69)
Chapter 4

Experimental Apparatus and Procedure

This chapter gives a detailed description of the experimental apparatus, procedures and systems utilised for this investigation. A description of the design and manufacture of hardware components necessary to complete the experimental investigation is given and the measurement procedures, coordinate systems and the generation of free-stream turbulence are also discussed.

4.1 Wind Tunnel Wing-tip Vortex Test Facility

4.1.1 Non-return Medium Speed Wind Tunnel

The experiments were conducted in the University of Limerick’s non-return medium speed wind tunnel. The tunnel has test section dimensions of $1m \times 0.3m \times 0.3m$ and a residual turbulence intensity of 0.5%, as measured by hot-wire anemometer (O’Regan et al., 2014). Airflow through the tunnel is supplied by a centrifugal compressor driven by a 70kW electric motor. The motor speed can be varied to achieve flow velocities ranging from 0 – 110 m/s, and the flow is conditioned prior to entering the test section by a series of honeycomb meshes in the tunnel settling chamber to remove any disturbances introduced by the compressor.
4. EXPERIMENTAL APPARATUS AND PROCEDURE

4.1.2 Wind Tunnel Test Section

A new wind tunnel test section was built for this investigation, which incorporated an adjustable wing model and cross-stream (yz) measurement slots downstream of the wing. The tunnel section was constructed from 40mm × 5mm angle section mild steel. The wing model with chord length of 140mm was mounted on its quarter chord on a circular disk at the side of the tunnel, which allowed the wing angle of attack to be adjusted in increments of 5°. The ceiling of the test section was manufactured with five cross-stream measurement slots corresponding to stream-wise distances $x/c = 0$ to $x/c = 4$. The slots were covered in a silicone membrane (see Figure 4.2) and secured in place with polycarbonate “letterbox” sections. This enabled the probe to be automatically traversed in the yz direction and helped to maintain the pressure in the tunnel section.
4.2 Multi-hole Pressure Probe

4.2.1 Overview

The three-dimensional mean velocity measurements, described in Chapter 5, were acquired using a multi-hole (five-hole) pressure probe. Multi-hole pressure probes are a cost effective, robust and accurate method for resolving three-dimensional velocity vector and fluid properties in an unknown flow-field (Johansen et al., 2001). In fact, multi-hole pressure probes are the only probes that can provide the local value of all three components of the velocity and exploit the fact that the static pressure varies over a solid surface immersed in the flow. The pressure ranges from the maximum value, which is equal to the stagnation pressure, to low values of the order of the base pressure in the wake of the body. Measuring the pressure at specific points over the body of a probe can provide all the necessary information on the velocity components and in-field pressures. This often requires a tedious and exhaustive calibration with several hundred calibration points. The probes must be inserted into the flow at the point where a measurement is required, and is therefore known as a point measurement instrument. However, multi-hole pressure probes have a number of advantages over other mean velocity measurement techniques, some of which are:
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1. *Cost and complexity:* When recording mean flow characteristics, multi-hole pressure probes are relatively inexpensive and easy to use when compared to triple-hot wire anemometry, Laser Doppler Velocimetry (LDV) or Stereoscopic Particle Image Velocimetry (SPIV). Triple hot-wire anemometry requires expensive calibration equipment and a tedious and complex calibration. Three-component LDV and SPIV systems can be up to 50 times more expensive than a multi-hole probe system and require complex set up procedures.

2. *Flow-field resolution:* The multi-hole pressure probe is capable of resolving the three components of the mean velocity field and the static and dynamic pressure. LDV and SPIV measurement techniques cannot measure the static and dynamic pressures of the flow.

3. *Robustness:* Multi-hole pressure probes are extremely robust and only require a single calibration (unless damaged), whereas triple-hot wire anemometers are very fragile and require complex calibrations before each use. LDV and SPIV also comprise delicate optical equipment and cameras that can get damaged quite easily. The multi-hole pressure probe can also be used in contaminated or near wall flows, which are conditions than can often preclude the use of other methods.

4.2.2 Principle of Operation of a Multi-hole Pressure Probe

When a body is inserted in the flow of any fluid, the pressure distribution over its surface varies from a maximum at the stagnation point to lower values that are often lower than the static pressure far upstream. For bluff bodies, the maximum pressure is equal to the total pressure \( p_o \), which is the sum of the static pressure \( p_\infty \), and the dynamic pressure far from the body, as given by:

\[
p_o = p_\infty + \rho U_\infty^2 / 2 \tag{4.1}
\]

Where \( U_\infty \) and \( \rho \) are the free-stream velocity and fluid density respectively.
4.2 Multi-hole Pressure Probe

The lowest pressures on a bluff body are generally found near the regions where the inclination of the surface of the body is nearly parallel to the free-stream. This does not always happen as flow separation alters the local pressure distribution and introduces adverse pressure gradients in regions where the slope of the surface of the body is decreasing but not quite zero. A common example would be the fact that laminar flow over a circular cylinder separates at about $80^\circ$ rather than $90^\circ$ where the tangent to the surface is parallel to the free-stream velocity.

The principle of multi-hole probe measurements is based on the fact that if a bluff body is immersed in a stream, the pressure at specific points on its surface is related to the direction and magnitude of the free-stream velocity.

To demonstrate the principle, a three-hole pitch probe consisting of a circular cylinder with pressure taps 1, 2 and 3 along the meridional angles at $\theta = 0, 45^\circ$ and $-45^\circ$ (as shown in Figure 4.3) is considered. If the free-stream is normal to the axis of the cylinder, the pressure reading at $\theta = 0$ will be the stagnation pressure and the readings along the other two pressure taps will be equal to each other. If the probe is inserted in a two-dimensional field inclined with respect to the probe axis with an angle of attack $\alpha$, then pressure measurements at the points $\theta = 0, 45^\circ$ and $-45^\circ$ can return the free-stream velocity magnitude $U_\infty$, the angle of attack, as well as the static and dynamic pressure. This is now demonstrated analytically.

![Figure 4.3: Cylindrical pitch/magnitude three-hole probe (adapted from Telionis et al. (2009))](image-url)
4. EXPERIMENTAL APPARATUS AND PROCEDURE

For incompressible flow, the pressure \( p \) and velocity \( U \) at a point on a body are related to the pressure \( p_\infty \) and velocity \( U_\infty \) far from the body, by Bernoulli’s equation:

\[
p_\infty + \frac{\rho U_\infty^2}{2} = p + \frac{\rho U^2}{2}
\]  

(4.2)

For a circular cylinder, the potential flow solution gives the velocity on the cylinder as

\[
U(\theta) = 2U \sin \theta
\]  

(4.3)

where \( \theta \) is the angular distance from the point of stagnation to the point of interest. It is now possible to use Equations 4.2 and 4.3 to solve for the pressure at the three pressure taps as follows:

\[
p_\infty + \frac{\rho U_\infty^2}{2} = p(45^\circ - \alpha) + 2\rho U_\infty^2 \sin^2(45^\circ - \alpha)
\]  

(4.4)

\[
p_\infty + \frac{\rho U_\infty^2}{2} = p(\alpha) + 2\rho U_\infty^2 \sin^2(\alpha)
\]  

(4.5)

\[
p_\infty + \frac{\rho U_\infty^2}{2} = p(45^\circ + \alpha) + 2\rho U_\infty^2 \sin^2(45^\circ + \alpha)
\]  

(4.6)

The pressure at ports 1, 2, and 3 can now be measured and Equations 4.4–4.6 can be solved for the unknowns \( p_\infty, U_\infty \) and \( \alpha \), which are the local value of the static pressure, the local magnitude of the velocity and the angle of attack. The concept is now to design a probe with a tip shape that will involve measurable variations of the local pressure which can be related to the direction and magnitude of the local velocity vector. Pressure taps are placed along the tip of the probe at locations where the local pressure will have a measurable variation associated with the magnitude and direction of the flow.

Analytical relations connecting the pressure values on the probe tip to the local velocity, and static and dynamic pressure can be derived for any probe tip shape but minute errors in the machining of the probe tip and the location of pressure taps introduce measurement errors that can only be eliminated by careful calibration. A multi-hole probe calibration requires inserting the probe
in a known uniform flow field and traversing it along yaw and pitch angles and measuring the corresponding pressures.

4.3 Five-hole Pressure Probe Measurements

4.3.1 Overview

The detailed three-dimensional velocity components ($u$, $v$ and $w$) were acquired by traversing an Aeroprobe L-shaped probe (see Figure 4.4) along cross-stream measurement grids at four downstream locations ($x/c = 1, 2, 3$ and $4$) using a high precision traverse mechanism (described in Section 4.6). Measurement grids of varying density were used to record the cross-stream velocity fields (discussed in more detail in Section 4.3.3). The measurement grids were taken for a chord Reynolds number of $3.25 \times 10^5$ at two angles of attack ($5^\circ$ and $10^\circ$). All measurement grids were recorded at a free-stream turbulence intensity of 0.5%.

Figure 4.4: The Aeroprobe L-shaped five-hole probe (adapted from Aeroprobe (2008))
4. EXPERIMENTAL APPARATUS AND PROCEDURE

4.3.2 Five-hole Pressure Probe Acquisition System

The five-hole pressure probe had previously been used with a pressure scanner system whereby each of the five port pressures would be manually recorded via a Furness Controls manometer. It was decided to automate the pressure measurement process for the purpose of this investigation due to the large number of grid points required for the crossflow planes downstream of the wing-tip vortex. Five Honeywell DUXL05D differential pressure transducers were used together with a National Instruments 6210 DAQ device and a Labview virtual instrument to simultaneously log the five differential port pressures at each grid measurement location. The differential port pressure being the difference between the free-stream static pressure and the total pressure at the selected probe port. The five Honeywell pressure transducers had a measurement range of 1245 Pa, which corresponded to a flow velocity range of 0 – 45 m/s. They were calibrated against a Furness Controls manometer to determine their peak voltage output and zero offset. The pressure transducers had a linear response to increasing pressure and this was verified by plotting the maximum and minimum pressures and their corresponding voltage output. The transducers had a typical measurement accuracy of 0.1% full scale, which was the recommended transducer accuracy for use with the Aeroprobe five-hole probe (Aeroprobe, 2008). The differential pressures were converted to the three dimensional velocity components ($u$, $v$ and $w$) and total and static pressures using Aeroprobe’s Multi-probe data reduction software.

4.3.3 Measurement Grids and Coordinate Systems

Initial square measurement grids of 121 points with spacing $\Delta z = \Delta y = 5$ mm were surveyed to pinpoint the location of the vortex and the flow areas of interest. Subsequently, more dense measurement grids of 324 points with spacing $\Delta z = \Delta y = 2$ mm were measured around the vortex core to yield more detailed flow-field information (see Figure 4.5). The measurement grids were mapped out using the four degree of freedom traverse mechanism described in Section 4.6. The five port pressures from the probe were recorded at each of the 324 grid points using the automated pressure acquisition system (described in Section 4.3.2). The five-hole probe, when used in the perpendicular stem orientation utilised a different
coordinate system to that of the wind tunnel (see Figure 4.6); therefore the measurements had to be transformed after data reduction. All recorded data was plotted using the wing coordinate system shown in Figure 4.5a and post processed using Matlab technical computing software.

![Coordinate System Diagram](image)

**Figure 4.5:** Schematics of the experimental measurement grids

### 4.3.4 Calibration of a Five-hole Pressure Probe

The five-hole probe was calibrated by inserting the probe into a flow-field of known magnitude and direction. The probe is then mounted on a traversing system capable of pitching and yawing the probe. While the flow magnitude and direction remains constant, the probe is pitched and yawed through a set of
known angles relative to the flow direction, keeping the location of the probe tip fixed. For each set of angles all the probe port pressures are recorded. A typical calibration system would consist of two stepper motors that can vary the probe angles pitch ($\alpha_p$) and yaw ($\beta$) within a specified range. Using high stepping resolutions the entire calibration domain is covered by recording up to several thousand discrete calibration points to describe every possible angle inclination for the probe. There are two typical angle systems used. When a straight probe is calibrated the recorded angles are the cone ($\theta$) and roll ($\phi$) angles. When an L-shaped probe is calibrated the angles recorded are the pitch ($\alpha_p$) and yaw ($\beta$) angles. The two different systems of angles are used to describe low and high angularity flow for the calibration technique which is now presented.

The following calibration technique was developed by Johansen et al. (2001) and was an advancement of the sectoring technique originally developed by Bryer and Pankhurst (1971). The algorithm was developed for application on both five and seven-hole probes in a compressible flow and forms the basis for the calibration technique of the Aeroprobe five-hole probe. The algorithm utilises low-angle and high-angle notation with pitch ($\alpha_p$) and yaw ($\beta$) angles (see Figure 4.7a) used for the low-angle flow regime ($\pm 20^\circ$) and cone ($\theta$) and roll ($\phi$) angles (see Figure 4.7b) used for the high angle flow regime ($20^\circ - 55^\circ$). The conversion from the pitch and yaw angles to the cone and roll angles is then made using the

\[ \text{Conversion formula} \]

Figure 4.6: Five-hole probe coordinate system
4.3 Five-hole Pressure Probe Measurements

following equations:

\[ \theta = \cos^{-1}(\cos\alpha_p\cos\beta) \]  \hspace{1cm} (4.7)

\[ \phi = \tan^{-1}\left(\frac{\sin\alpha_p}{\tan\beta}\right) \]  \hspace{1cm} (4.8)

(a) Yaw and pitch angle definitions     (b) Cone and roll angle definitions

**Figure 4.7:** Calibration angle definitions (taken from Johansen et al. (2001))

The low-angle and high-angle sectoring technique is utilised in the algorithm and each sector is identified by the port that senses the highest pressure for all possible flow inclinations within that sector. Figure 4.8 shows the numbering and sector designation for a typical five-hole probe. The low-angle flow regime is identified when pressure port 1, the central port, records the highest pressure and high angle flow regime is identified when the highest pressure is recorded in one of the peripheral ports 2-5 (port \(i\)). The velocity vector at any measurement location can be described by four variables. For low angle flow, these variables are the pitch angle \(\alpha_p\), yaw angle \(\beta\), total pressure coefficient \(A_t\) and static pressure coefficient \(A_s\). For high-angle flow, the variables are the cone angle \(\theta\), roll angle \(\phi\), and coefficients \(A_t\) and \(A_s\). These quantities are then determined as functions of
4. EXPERIMENTAL APPARATUS AND PROCEDURE

four non-dimensional pressure coefficients. For a five-hole probe in the low angle flow regime the following coefficients are defined:

\[ b_{\alpha p} = \frac{(p_2 + p_4 - p_5 - p_3)}{2q} \]  \hspace{1cm} (4.9)

\[ b_\beta = \frac{(p_5 + p_2 - p_3 - p_4)}{2q} \]  \hspace{1cm} (4.10)

\[ A_t = \frac{(p_1 - p_o)}{q} \]  \hspace{1cm} (4.11)

\[ A_s = \frac{q}{(p_o - p_s)} \]  \hspace{1cm} (4.12)

where the indicated dynamic pressure \( q \) is defined as:

\[ q = p_1 - \frac{(p_2 + p_3 + p_4 + p_5)}{4} \]  \hspace{1cm} (4.13)

For a five-hole probe operating in the high angle flow regime (sectors 2-5) the following coefficients are used:

\[ b_\theta = \frac{(p_i - p_1)}{q} \]  \hspace{1cm} (4.14)

\[ b_\phi = \frac{(p^+ - p^-)}{q} \]  \hspace{1cm} (4.15)

\[ A_t = \frac{(p_i - p_o)}{q} \]  \hspace{1cm} (4.16)

\[ A_s = \frac{(p_i - p_s)}{q} \]  \hspace{1cm} (4.17)

where the indicated dynamic pressure \( q \) is defined as:
4.3 Five-hole Pressure Probe Measurements

\[ q = p_i - \frac{(p^+ + p^-)}{q} \]  \hspace{1cm} (4.18)

In the previous definitions \( p_i \) corresponds to the port that registers the highest pressure. Looking into the probe tip \( p^+ \) and \( p^- \) are the pressures measured by the peripheral holes adjacent to \( p_i \) (\( p^+ \) being in the clockwise direction and \( p^- \) in the anti-clockwise direction).

![Figure 4.8: Five-hole probe tip numbering and sector designation (adapted from Telionis et al. (2009))](image)

4.3.5 Five-hole Pressure Probe Data Reduction Procedure

When the five-hole probe is inserted into an unknown flow the five port pressures are recorded and the non-dimensional coefficients are calculated depending on what sector records the highest pressure. The data reduction algorithm then searches for the closest points to the test values using a method which selects test points within that sector and points from adjacent sectors depending on whether the test point lies close to the boundary between sectors. A procedure then checks to see if the closest calibration points form a triangle and the points are altered...
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until the points form a triangle around the test point. This is required in order to obtain a well behaved polynomial surface to allow for interpolation. Four interpolation surfaces are then created and a least squares surface fit is carried out to calculate the two non-dimensional coefficients $b_\alpha$ and $b_\beta$ and to interpolate for the two required angles. The total and static pressures are calculated from the non-dimensional coefficients $A\alpha$ and $A\beta$. Since the calibration technique is valid for both compressible and incompressible flow, the Mach number is used to calculate the velocity magnitude. The Mach number is calculated from the total and static pressure using the following expression from Johansen et al. (2001):

$$M = \sqrt{\frac{5}{7} \left( \left( \frac{p_o}{p_s} \right)^{\frac{7}{5}} - 1 \right)}$$

The velocity magnitude can now be calculated using the speed of sound relationship:

$$\overline{U} = M\sqrt{\gamma RT}$$

Where $\gamma$, $R$ and $T$ are the adiabatic index, molar gas constant and absolute temperature in Kelvin.

The Cartesian velocity components $u, v$ and $w$ are now calculated using the yaw-pitch vector resolution equations:

$$u = \overline{U}\cos\alpha_p\cos\beta$$

$$v = \overline{U}\sin\beta$$

$$w = \overline{U}\sin\alpha_p\cos\beta$$
4.4 Constant Temperature Anemometry

4.4.1 Overview

The experimental turbulence measurements presented in Chapter 5 were acquired using constant temperature anemometry (CTA). CTA establishes the fluid velocity of a flow from the convective heat loss from a heated wire or surface mounted film. Capable of measuring flow velocities from a few cm/s to supersonic, a CTA provides instantaneous flow velocity information with very high temporal and spatial resolution. Despite the fact hot wires are an intrusive point measurement technique (which can be effected by vortex wandering), they have benefits over non-intrusive turbulent measuring techniques (e.g. Laser Doppler Velocimetry), which include:

1. *Cost and complexity:* CTA systems are a lot less expensive than LDV systems, which require substantial expense in optics and image processing software. A CTA system only requires the hardware to digitise the signal, the sensors (hot-wire and hot-film) and a computer to store and process the raw data. Hot-wire probes are relatively straight forward to calibrate and use, whereas the alignment of emitted and reflected beams on LDV systems can be complex.

2. *Size:* Hot-wires and hot-films are usually 5 to 10µm in diameter. In contrast, LDV measurement volumes are 50µm by 250µm (Bruun, 1995). Hot wires are therefore more suited to measuring near-wall fluid phenomena and turbulent flow close to the surface.

4.4.2 Operation of the TSI IFA 300 Constant Temperature Anemometer

The operation of a CTA system requires the appropriate hardware, flow sensor, and computer and software to store and process the acquired data. The CTA system used in the experimental part of this work was a TSI IFA 300 system with a Dell Optiplex GX620 computer and THERMALPRO software. The operation of a constant temperature anemometer has been addressed comprehensively in
Bruun (1995) and will only be described here in the context of the CTA system used for this investigation. The block diagram in Figure 4.9 shows the basic circuit of the constant temperature bridge. Like most constant temperature anemometer systems, a Wheatstone bridge is used to maintain the sensor at a given operating resistance. For most applications the 10Ω resistor above the sensor is used but for higher power applications a 2Ω resistor above the sensor can be used. The IFA 300 bridge uses the SMARTTUNE technology, which constantly monitors the bridge voltage and feeds a signal back to the amplifier circuit. As a result the bridge does not require tuning for frequency response regardless of the type of sensor used or the length of the cable. SMARTTUNE also prevents oscillations which may damage the sensor.

![Figure 4.9: Basic circuit of the IFA 300 constant temperature bridge (adapted from TSI (2010))](image)

The IFA 300 system includes signal conditioners which can be used to increase the bridge voltage gain to use the entire ±5V signal range. High pass filters of 0.1Hz, 1Hz and 10Hz can be used when velocity fluctuation measurements are needed since the mean velocity information is removed from the signal. Thirteen low pass filter settings are available from 10Hz to 1MHz, which can be used to remove unwanted high frequency signals and to eliminate aliasing.
4.4.3 Frequency Optimisation

A finite time is required for the heat transfer from the sensor to change as a result of fluctuations in the flow velocity. It is therefore necessary to determine the highest frequency of flow event to which the sensor can respond. This is of particular importance in turbulence investigations which require a measurement system with a very high frequency response. The dynamic response of the anemometer system may be determined either by perturbing the flow in which the sensor resides or imposing an electronic disturbance on the measurement system. This electronic disturbance can take the form of either a square or sine wave.

The TSI IFA 300 CTA system incorporates a square wave generator with an input frequency of 1kHz. The square wave test relies on the assumption that heating and cooling of the sensor by varying the fluid velocity is thermodynamically identical to the heating and cooling of the sensor by varying the heating current. However, convective cooling is a surface phenomenon, whereas electrical heating occurs volumetrically within the sensor material. As a result, the two can only be considered as an approximation of each other. The injected square wave simulates a repeated step change in velocity causing the sensor current to rise and then return to its original value. Figure 4.10 shows the response of a CTA system for an injected square wave on both a hot-wire and hot-film sensor.

From Figure 4.10a, it can be seen that the response signal for a hot-wire anemometer exhibits approximately 15% undershoot relative to the maximum amplitude $h$. If $\tau_w$ is defined as the time from the start of the injected pulse until the signal has decayed to 3% of its maximum value, then the cut-off frequency $f_c$ for the system can be evaluated from $f_c = (1.3\tau_w)^{-1}$, which represents the maximum measurable frequency by the hot-wire. Figure 4.10b shows the response signal for a hot-film anemometer after being subjected to a square wave input. For the hot-film anemometer $\tau_f$ is defined as the time from the start of the injected pulse until the signal returns to its original value and the maximum measurable frequency is obtained from $f_c = (\tau_f)^{-1}$. Bruun (1995) mentions that the frequency response and damping of the system increases with increased fluid velocity, therefore the square wave input test should be carried out at the fluid
velocity under investigation. The frequency response of the 20μm hot-film was determined to be approximately 50kHz.
4.4.4 X-probe Holder Mount

The x-probes were required to be calibrated and used in a perpendicular stem orientation with the mean flow at $45^\circ$ to both wires of the sensor. A probe holder mount was designed in order to facilitate this requirement (see Figure 4.11). An extension coupling for the stepper motor was also designed in order for the probe to traverse the required distance in the $z$-direction.

Figure 4.11: The x-probe holder $45^\circ$ mount and stepper motor coupling

4.5 X-probe Measurements

4.5.1 Overview

Detailed measurements of the mean ($u$ and $v$) and the root mean squared (rms) fluctuating velocities ($\sqrt{\bar{u}'^2}$ and $\sqrt{\bar{v}'^2}$) were acquired by traversing an x-probe through the vortex core centre for Reynolds numbers of $3.25 \times 10^5$ ($\alpha = 5^\circ$ and $10^\circ$) and $1.8 \times 10^5$ ($\alpha = 5^\circ$) at turbulence levels of 0.5% and 4.3% respectively. The measurements made at a Reynolds number of $1.8 \times 10^5$ and a free-stream turbulence intensity of 4.3% were to assess its effect on the turbulence characteristics in the vortex core. All measurements were taken with a TSI 1240-T1.5 5$\mu$m x-wire and a TSI 1240-20 20$\mu$m x-film. The measurements were taken with both
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wires of the x-probe aligned at 45° to the mean flow using the custom made x-probe mount seen previously in Figure 4.11. The x-probe was traversed through the vortex core centre in 1mm increments. The vortex core centre identified as being the point of minimum crossflow velocity, which was deduced from the five-hole probe measurements. The fragility of the wires on the x-probe prohibited them from touching the surface of the wing; therefore the probe was aligned with the wing-tip reference point using a laser sheet generated by a Picotronic LD532 class 1M laser (see Figures 4.12c and 4.12d). Prior to acquiring the x-probe measurements it was necessary to calibrate the x-probe sensors. A description on the calibration of an x-probe and the procedure carried out for this investigation is outlined next.

(a) X-probe in flow with turbulence intensity of 0.5%
(b) X-probe in flow with turbulence intensity of 4.3%
(c) Laser alignment of x-probe and wing-tip
(d) Side view of x-probe and wing-tip laser alignment

Figure 4.12: X-probe measurement and laser alignment setup
4.5 X-probe Measurements

4.5.2 Calibration of an X-Probe Anemometer

X-wire/film anemometry is a useful tool for determining the instantaneous velocity components in two directions. The calibration of an x-wire/film anemometer using the THERMALPRO software constitutes a velocity voltage calibration and a yaw calibration to determine the directional response of the wires. THERMALPRO uses the well known effective angle method of calibration, which is described in more detail by Browne et al. (1989), Bruun et al. (1990) and Bakken and Krogstad (2004).

A velocity versus voltage \( (U \text{ versus } E) \) calibration for the inclined anemometer should be carried out in a low turbulence intensity flow of known velocity and the x-wire should be orientated such that the mean flow velocity is at 45° to both wires of the sensor. A set of between 10 and 30 points should be obtained across the entire velocity range of interest to determine a relationship between the mean anemometer voltage and the mean velocity in each direction of the \( x \). The velocity voltage relationship is often to assumed to be of the form suggested by King (1914):

\[
E^2 = A + BU^n \quad (4.24)
\]

Bruun (1995) recommended values of \( n \) in the range 0.4–0.45 as optimum for a 5\( \mu \)m diameter tungsten wire and the calibration constants \( A \) and \( B \) in Equation 4.24 can be evaluated experimentally from a linear fit of \( E^2 \) and \( U^n \). The best fit to the calibration data of this investigation was provided by a 4th order polynomial of the form:

\[
U = a_1 + a_2E + a_3E^2 + a_4E^3 + a_5E^4 \quad (4.25)
\]

where the constants were determined by a least squares best fit.

For the yaw angle calibration, the mean flow velocity is fixed at the velocity under investigation and the probe is rotated through a number of positive and negative yaw angles. For this investigation the probe was yawed ±30° in increments of 6° using a stepper motor. The effective angle of each yaw angle is
obtained and then the mean of all the effective angles is taken to be the effective angle of the wire. The adopted calculation procedure of an effective angle is as follows:

- The mean flow velocity $U_1$ is fixed to a value representative of the flow under investigation and the anemometer output voltage $E_1$ is noted.

- The probe is yawed through an angle $\theta_{yaw}$ and the new anemometer output voltage $E_2$ is noted. Assume that a positive yaw angle, $\theta_{yaw}$, is used such that the angle between $U_1$ and the wire is decreased (see Figure 4.13a).

- It is assumed that the effective angle of the wire in its new position has been increased by $\theta_{yaw}$. Let $U_2$ be the velocity corresponding to this voltage $E_2$, as obtained from the velocity against voltage calibration results, that is, $U_2$ is the velocity which, if it were used on the wire in its un-yawed position, would give the same anemometer output voltage $E_2$ as is obtained from the yawed wire (see Figure 4.13b).

\[ \theta_{yaw} (+ve) \]
\[ \text{un-yawed wire output} = E_1 \]
\[ \text{yawed wire output} = E_2 \]
\[ U_1 \]
\[ \theta_{yaw} \]  
\[ U_2 \]
\[ \text{un-yawed wire output} = E_2 \]
\[ \text{n' n} \]

Figure 4.13: (a) Position of yawed and un-yawed wire; (b) equivalent velocity on un-yawed wire to produce same output as yawed wire (adapted from Browne et al. (1989))

- The anemometer output voltage is now the same for both cases and the proposition is made that the effective cooling velocity over the wire using
4.5 X-probe Measurements

$U_1$ with the wire in the yawed position is the same as the effective velocity $U_2$ with the wire in its unyawed position. This yields the following relationship:

$$U_1 f(\theta_{eff} + \theta_{yaw}) = U_2 f(\theta_{eff})$$ (4.26)

- An equation to represent $f(\theta)$ must now be selected. The most commonly used is the form introduced by Hinze (1959):

$$f(\theta) = (\cos^2 \theta + k^2 \sin^2 \theta)^{0.5}$$ (4.27)

where the $k^2$ term is intended to account for the effects of longitudinal cooling of the wire. Although $k^2$ varies with yaw angle, an average value of $k^2$ is usually found to reproduce reasonably accurate results and this is the approach usually taken. Substituting Eq. 4.27 into Eq. 4.26 yields the following equation:

$$U_2 [\cos^2(\theta_{eff}) + k^2 \sin^2(\theta_{eff})]^{0.5}$$ (4.28)

- Using a selected value for $k^2$, Eq. 4.28 can be solved to obtain the effective angle of the wire for the particular yaw angle obtained. This process is repeated for positive and negative yaw angles and the mean of the effective angles is taken to be the effective angle of the wire.

4.5.3 X-probe Calibration Procedure

The TSI 1240 crossflow x-wire and x-film probes were calibrated in a perpendicular stem orientation in the test section of the medium speed wind tunnel, which had a background turbulence intensity of 0.5%. The probe to be calibrated was fixed to the automated traverse via a custom made probe holder mount (see Figure 4.11), which allowed the sensor wires maintain a 45° angle with the mean flow. The x-probe was connected to the two channels on the TSI IFA 300 unit using 5-metre coaxial cables. A shorting probe was used to determine the resistance
4. EXPERIMENTAL APPARATUS AND PROCEDURE

in the cable and probe holder and the operating temperature was set to 250°C (the maximum recommended by the manufacturer). The reference velocity in the tunnel test section was measured with a pitot-static probe and a Furness Controls manometer. A built-in type-T thermocouple was used to monitor the fluid temperature so that variations in fluid temperature could be compensated for. A voltage versus velocity calibration was carried out with 17 points acquired across the velocity range of interest and a 4th order polynomial curve fit of the voltage versus velocity calibration was applied. A yaw angle calibration was then carried out at the velocity of the flow under investigation, where the probe was yawed ±30° in increments of 6° and the THERMALPRO software derived an optimum value of the yaw coefficient $k$ for the x-probe.

4.6 Traverse Mechanism

4.6.1 Overview

Acquiring detailed mean ($u$, $v$ and $w$) and fluctuating velocities ($\sqrt{\bar{u'}^2}$ and $\sqrt{\bar{v'}^2}$) meant that the x-probe and five-hole probe needed to be accurately traversed in the cross-stream direction ($yz$). This was achieved by modification of a previously constructed traverse mechanism (see Section 4.6.2), as described by Whelan (2010). The traverse mechanism was firmly mounted on top of the wind tunnel test section using rubber damping mounts to eliminate vibration. The traverse positioned the probes in the tunnel test section through slots sealed with silicone membrane on top of the test section (see Figure 4.14). Linear motion was achieved through the use of THK linear screws with leads of 40mm, 5mm and 8mm for the $x$, $y$ and $z$ axes and Astrosyn stepper motors with an angular resolution prior to micro stepping of 1.8°. Using the micro stepping settings the linear displacement per pulse sent to the motor of the three axes was decreased to 50µm, 10µm and 6.25µm for the $x$, $y$ and $z$ axes respectively. A resolution of 10µm or less was deemed sufficiently adequate for probe positioning in the cross-stream plane. The previous traverse mechanism Whelan (2010) was not capable of precision movement in the $y$-direction which was a necessary requirement for this investigation. The following section describes the work undertaken
for modification of the traverse mechanism to allow for precision movement in the $y$-direction.

![Figure 4.14: Traverse mechanism with five-hole probe mounted on tunnel test section](image)

### 4.6.2 Traverse $Y$-direction Motion

The previous traverse mechanism (Whelan, 2010) consisted of two stepper motor driven screws capable of precision movement in the $x$ and $z$-directions and yaw movement of the probe about the $z$ axis. The cross-stream measurement grids used in this study required accurate probe positioning in the $y$ and $z$-directions. Therefore, the traverse mechanism was modified to incorporate precision movement in the $y$-direction. The lead distance of the linear screw determined the distance travelled along the screw shaft for one complete revolution of the screw shaft. The previous traverse had a positional accuracy of 6.25$\mu$m in the $z$-direction and it was required that the $y$-direction would have a positional accuracy of 10$\mu$m or less. The linear THK screw is driven by an Astrosyn stepper motor with an angular resolution prior to micro stepping of 1.8°. The conver-
4. EXPERIMENTAL APPARATUS AND PROCEDURE

Conversion of angular displacement of the motor shaft into linear travel is given by the following equation:

\[ x_\theta = \frac{l}{360} \times \theta_s \]  

(4.29)

where \( x_\theta \) is the distance travelled per angular step, \( l \) is the lead of the linear screw and \( \theta_s \) is the angular step of the motor.

Using the recommended THK ball screw BNT-1808-3.6 and linear screw of lead 8mm from Whelan (2010), the linear screw would translate prior to micro stepping 40\( \mu \)m per angular step.

As it was required to have a positional accuracy of 10\( \mu \)m or less in the cross-stream direction, micro-stepping was used to enhance the resolution. Fourteen micro step settings are available on the Astrosyn P403-A motor driver. To achieve a 10\( \mu \)m resolution, equation 4.29 was rearranged:

\[ \theta_s = \frac{x_\theta \times 360}{l} \]  

(4.30)

By setting \( x_\theta = 10\mu m \) and \( l = 8 \times 10^{-3} \), it follows that

\[ \theta_s = 0.45^\circ \]  

(4.31)

The angular step of 0.45\(^\circ\) translates to a micro-step setting of 800 steps per 360 degrees of revolution of the motor. The ball screw BNT 1808-3.6 and lead screw TS18-08 were therefore adequate for a positional accuracy of 10\( \mu \)m in the \( y \)-direction, provided a micro-step setting of 800 steps per revolution was used. The BNT 1808-3.6 ball screw was bolted directly to the underside of the main base plate (see Figure 4.15a). The base plate was bored and tapped about its centre position to allow for even distribution of the payload being moved by the ball screw. The lead screw was attached to the guide rails by two SKF bearings housed within the rails. The bearings chosen were SKF 7201 BEP angular contact bearing and a SKF 1201 ETN9 self aligning bearing. The internal diameter of the bearings was 12mm which required the ends of the 18mm diameter lead screw to be ground down to fit. The CAD model of the traverse mechanism modification can be seen in Figure 4.15a.
4.6 Traverse Mechanism

Figure 4.15: (a) CAD modification to existing traverse mechanism to enable $y$ direction motion and (b) complete CAD model of traverse mechanism (Whelan, 2010)
4.7 Free-stream Turbulence Generation

4.7.1 Grid Generated Turbulence

The residual turbulence intensity in the tunnel test section was measured as 0.5% using an x-wire anemometer (O'Regan et al., 2014). The elevated free-stream turbulence level was achieved by placing a square mesh grid at a distance of 52mm upstream from the wing leading edge. The free-stream turbulence level was raised to determine its effect on the turbulent characteristics of the wing-tip vortex. The grid used had a bar and mesh width diameter of 0.9mm and 5mm, which resulted in a freestream turbulence level of 4.3% at the leading edge of the wing. Turbulence attenuates as it is convected from the grid, and according to Batchelor (1959) the turbulence can be assumed to be approximately homogeneous and isotropic within 10 mesh widths (50mm) downstream.

The free-stream turbulence grid was designed to give the required turbulence level at the leading edge of the wing based on a correlation between the turbulence intensity at various distances downstream of the generating grid. The following correlation, by Roach (1987), is used to represent the decay of isotropic free-stream turbulence with downstream distance:

\[ T_{u_\infty}(\%) = 80 \left( \frac{x}{d} \right)^{-\frac{5}{7}} \]  

(4.32)

where \( x \) represents the distance downstream of the grid and \( d \) the diameter of the bar.

The validity of Equation 4.32 was tested by comparing the experimentally measured x-wire turbulence intensities with the predicted values at four locations downstream of the grid (Figure 4.16).

It is clear from Figure 4.16 that the grid generated turbulence is not isotropic after 10 mesh lengths downstream of the grid as the turbulence intensity in the \( x \) and \( y \) directions are not equal. The predicted turbulence intensity is seen to have good agreement with the measured \( T_{u_x} \) value at the final downstream location of \( x/c = 3 \).
Figure 4.16: Comparison between the measured and predicted values of $T_u\infty$ at four downstream locations

## 4.8 Experimental Uncertainty

The mean velocity measurements were based on an average of 2000 pressure readings sampled at a rate of 1kHz at every grid point. The sampling rate and number of samples were chosen based on a time sampling independence analysis. The averaged pressure readings were reduced to velocity components using Multi-probe data reduction software. The five-hole probe when used with pressure transducers accurate to 0.01\% full scale (such as the Honeywell DUXL05D transducers used in this investigation) has an accuracy of 0.4\° for flow angles and 0.8\% for velocity magnitude within the angle range ±55\°. The calibration of the x-probe anemometer involved taking 17 points covering the full velocity range of 0 – 50 m/s. The mean errors associated with the x-probe calibration are less than 4\% for velocities between 0 and 3 m/s, less than 2\% for velocities between 3 m/s and 17 m/s and less than 1\% for velocities greater than 17 m/s. The automated traverse had positioning accuracy of less than 10µm for $y$ and $z$ and 1.2\° for rotation. The free-stream velocity was recorded with a Furness Controls FC012 manometer and a pitot-static probe. The manometer was calibrated prior to experiment by Furness Controls Limited according to UKAS calibration standards and had
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a measurement range of 0 –199mmH₂O (0–56.6m/s) with a stated accuracy of ±1% of the range (±0.4m/s).
Chapter 5

Experimental Investigation of a Wing-tip Vortex

This chapter presents the results of the experimental investigation of the wing-tip vortex in the near-field. The chapter is divided into two main sections: the mean flow results measured by the five-hole probe and the turbulence results measured by the x-wire anemometer. The results are presented for a Reynolds number of $3.25 \times 10^5$ at two angles of attack ($5^\circ$ and $10^\circ$) and a free-stream turbulence level of 0.5%. An elevated free-stream turbulence level of 4.3% was introduced at a Reynolds number of $1.8 \times 10^5$ and an angle of attack of $5^\circ$ to assess the effects of external turbulence on a vortex. The elevated free-stream turbulence was introduced at a lower Reynolds number due to blockage effects caused by the turbulence grid at the higher Reynolds number. All mean flow measurements were recorded at a free-stream turbulence level of 0.5%.

5.1 Mean Flow Measurements

5.1.1 Axial Velocity

The axial velocity in the core of a wing-tip vortex is generally affected by two primary mechanisms: (1) the momentum deficit caused by the boundary layer on the wing and (2) the axial development of crossflow velocities, which introduces a favourable pressure gradient and accelerates the inner vortex core flow. The
5. EXPERIMENTAL INVESTIGATION OF A WING-TIP VORTEX

former increases the vortex core pressure leading to a reduction in core velocity. Therefore, the two mechanisms have opposite effects on the core axial velocity. Whichever mechanism is more dominant usually determines whether there is an axial velocity excess or deficit in the core of the vortex as illustrated by Figures 5.1-5.4 in the text.

Coarse Measurement Grids

As mentioned previously in Section 4.3.3, initial coarse measurement grids of size $50\text{mm} \times 50\text{mm}$ with spacing $\Delta z = \Delta y = 5\text{mm}$ and consisting of 121 measurement points were recorded to deduce the approximate location of the wing-tip vortex and the flow areas of interest. The contours of axial velocity obtained from the coarse measurement grids at each of the four downstream locations (i.e. $x/c = 0, 1, 2$ and $3$) for $\alpha = 5^\circ$ can be seen in Figure 5.1. The axial velocity at $x/c = 0$ had a maximum deficit of $0.5U_\infty$ in the wing shear layer identified by the thin region with high velocity deficit. There is no vortical structure apparent at $x/c = 0$ and the low velocity fluid below the wing is seen to migrate towards the upper surface of the wing. The crossflow velocity vectors (superimposed to aid in identifying the core centre as the point of minimum crossflow velocity) are nonaxisymmetric indicating that the vortex roll-up was still in its infant stage. A vortical structure emerges above the wing at $x/c = 1$ with a maximum axial velocity deficit of $0.9U_\infty$. The maximum axial velocity deficit is seen to occur in the vortex core centre (point of minimum crossflow velocity indicated by the vectors), which by now have attained a certain degree of axisymmetry. The high pressure shear layer from underneath the wing appears to be feeding the spiral sheet of the vortex as it rolls up above the wing. The axial velocity structure of the vortex appears relatively axisymmetric at $x/c = 1$ and had moved upward and inboard a distance of $0.03c$ and $0.05c$ respectively. The core axial velocity decreased slightly at $x/c = 2$ to $0.88U_\infty$ and had now moved inboard by $0.08c$. At the last measurement location of $x/c = 3$ the axial velocity increased slightly to $0.9U_\infty$ and the vortex structure appeared to be axisymmetric and moved inboard by a distance of $0.1c$.

Contours of axial velocity obtained from the coarse measurement grids for $\alpha = 10^\circ$ are shown in Figure 5.2. A large degree of asymmetry is evident in
5.1 Mean Flow Measurements

Figure 5.1: Contours of normalized axial velocity \( U_x/U_\infty \) overlaid with crossflow velocity vectors \((jv + kw)/U_\infty\) for \( \alpha = 5^\circ \) and \( Re = 3.25 \times 10^5 \) (coarse measurement grids)

the axial velocity structure at \( x/c = 0 \) for \( \alpha = 10^\circ \), which is similar to the \( \alpha = 5^\circ \) case at the same location. However, the velocity vectors do appear more axisymmetric when compared to the \( \alpha = 5^\circ \) case suggesting that the migration of fluid from the high pressure region to the lower pressure region occurs earlier along the tip. At \( x/c = 1 \), the axial velocity exhibited regions of excess and deficit across the core region. The vortex core centre exhibited a velocity excess of 1.08\( U_\infty \), whereas the fluid surrounding the vortex core centre exhibited a deficit condition with a maximum value of 0.92\( U_\infty \) recorded at the top and inboard side of the vortex as shown in 5.2b. The crossflow velocity vectors attain their
maximum value in the velocity deficit containing fluid and are thought to be responsible for the acceleration of the inner core flow and the velocity excess present in the core centre (Chow et al., 1997b). The deficit containing wake feeds the deficit containing spiral that surrounds the vortex core centre, which has moved inboard by approximately $0.07c$ at $x/c = 1$. The axial velocity structure maintained a similar pattern at $x/c = 2$ with the velocity excess in the core centre surrounded by the velocity deficit containing fluid. The maximum axial velocity excess decreased slightly to $1.07U_\infty$ and the axial velocity deficit increased to $0.95U_\infty$. The crossflow velocity vectors appear to become more axisymmetric and similar to the $x/c = 1$ location, the vectors with maximum magnitude are observed at the edge of the vortex core in the velocity deficit containing fluid. The vortex centre location has moved further inboard and now lies at a distance of $0.1c$ of the chord from the tip. The maximum axial velocity excess decreases further at the last measurement location of $x/c = 3$ with a value of $1.04U_\infty$ recorded in the vortex core centre. The maximum velocity deficit in the region surrounding the core centre decreased slightly to $0.93U_\infty$ and had moved more inboard. However, the vortex core location remained relatively constant at a distance of $0.1c$. The roll up process does not seem to be fully complete at $x/c = 3$ as the shear layer underneath the wing still appears to be feeding into the vortex. The velocity vectors are by now deemed axisymmetric at this location but no reasonable axisymmetry is observed for the axial velocity structure. The coarse measurement grids helped gain a valuable insight into the axial velocity structure of the vortex and its downstream variation. The general trends observed from the measurements taken on the coarse grids were a good indication of the location of the vortex region that would be investigated with fine measurement grids. The region chosen for further investigation using fine measurement grids can be seen in Figures 5.1d and 5.2d. The refined region extended a distance of $0.2c$ from the wing-tip in both the $y$ and $z$ directions as the region of axial velocity deficit extends beyond the coarse measurement grid as shown in Figure 5.2d.
5.1 Mean Flow Measurements

![Figure 5.2: Normalized axial velocity \( U_x/U_\infty \) overlaid with crossflow velocity vectors \((jv+kw)/U_\infty\) for \( \alpha = 10^\circ \) and \( Re = 3.25 \times 10^5 \) (coarse measurement grids)](image)

Refined Vortex Measurement Grids

A fine measurement grid concentrated on the vortex region was defined based on the results obtained from the coarse measurement grids. The fine measurement grids allowed for the vortex at each measurement location to be surveyed in much greater resolution. The dimensions of the fine measurement grids were 34mm × 34mm consisting of 324 points with spacing \( \Delta z = \Delta y = 2\)mm (see Figure 4.5b).

The axial velocity contours obtained from the fine measurement grids for \( \alpha = 5^\circ \) are shown in Figure 5.3. The wing-tip vortex is in the early stages of roll
5. EXPERIMENTAL INVESTIGATION OF A WING-TIP VORTEX

up at $x/c = 0$ as there is no axisymmetric axial velocity structure evident and the crossflow velocity vectors move in a downward rather than circular motion. The crossflow velocity vectors are at a minimum directly above the wing-tip where the axial velocity reaches a value of $0.67U_\infty$. There are two separate areas of axial velocity deficit separating from the wing tip at a location of $0.03c$ along the wing. This would suggest that there are two separate vortices formed at $x/c = 0$, a feature that is not apparent in the results from the coarse measurement grids at the same location. Further downstream at $x/c = 1$, the two axial velocity structures appear to have coalesced and the crossflow velocity vectors exhibit a more circular motion. The vortex has moved inboard by $0.08c$ and the axial velocity deficit in the vortex core centre has increased to $0.78U_\infty$. Entrainment of fluid from the shear layer into the vortex is still evident suggesting that roll up is not fully complete at $x/c = 1$. The axial velocity vortex structure is somewhat axisymmetric at $x/c = 2$ with the velocity reaching a minimum at the centre of the vortex and increasing slowly with radial distance from the centre. The axial velocity deficit in the core centre increases slightly to $0.82U_\infty$ and the vortex moves slightly more inboard to a distance of $0.1c$. The vortex appears detached from the shear layer underneath the wing suggesting that the shear layer is no longer being entrained into the vortex. At the last measurement location of $x/c = 3$, the vortex appeared to capture fluid from the separated wing boundary layer and the velocity deficit remained at $0.82U_\infty$. The crossflow velocity vectors are uniform and axisymmetric, and the vortex has moved inboard by $0.11c$.

The axial velocity contours obtained from the fine measurement grids for $\alpha = 10^\circ$ are shown in Figure 5.4. At $x/c = 0$, a vortex type structure is evident just above the wing with a maximum axial velocity excess of $1.21U_\infty$ occurring in the core region, which is in contrast to the measurements taken on the coarse grid, where no vortical structure or axial velocity excess is present at $x/c = 0$. The maximum velocity excess in the core centre was $1.13U_\infty$. The crossflow velocity vectors exhibit a mostly circular rotational pattern but look somewhat disorganized just above the wing tip. The momentum deficit shear layer fluid is seen to migrate from underneath the wing and surround the still forming vortex. At $x/c = 1$, the axial velocity in the vortex core centre decreases to $1.05U_\infty$ at an inboard distance of $0.06c$ with the vortex core surrounded by a velocity deficit.
5.1 Mean Flow Measurements

Figure 5.3: Normalized axial velocity $U_x/U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (fine measurement grids)

region with a maximum deficit of $0.81U_\infty$ on the inboard side of the vortex. The crossflow velocity vectors at $x/c = 1$ gained more axisymmetry and reached a maximum in the axial velocity deficit region at the outer boundary of the vortex core. At $x/c = 2$, the axial velocity in the vortex core centre remained constant at $1.05U_\infty$ at a distance of $0.07c$. However, the axial velocity deficit region had now shifted from around the vortex periphery to the inboard half of the vortex, effectively splitting the vortex into two regions (the outboard side having a velocity excess and the inboard side having a velocity deficit). The maximum velocity deficit increased slightly to $0.85U_\infty$ and the crossflow velocity vectors appeared to have become more disorganized, which may be caused by the
conflict in axial velocity values between the two regions. The axial velocity in the vortex core centre increased slightly to $1.06U_\infty$ at $x/c = 3$ and the crossflow velocity vectors exhibited a symmetric circular pattern. The axial velocity deficit region at the inboard side of the vortex reduced in size, which may indicate the two regions had begun to coalesce. Furthermore, the maximum velocity deficit in this region decreased to $0.75U_\infty$.

Figure 5.4: Normalized axial velocity $U_x/U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (fine measurement grids)
5.1 Mean Flow Measurements

5.1.2 Crossflow Velocity

The crossflow velocity vectors \((v \text{ and } w)\) and the crossflow velocity magnitude \(\sqrt{v^2 + w^2}\) for two angles of attack \((5^\circ \text{ and } 10^\circ)\) and at four downstream locations \((x/c = 0, 1, 2 \text{ and } 3)\) are presented in this section. The results obtained from both the coarse and fine measurement grids are given. The crossflow velocity of a tip vortex has a direct effect on the core axial velocity as it tends to accelerate the inner core flow, often to values above the free-stream velocity. The larger crossflow velocities encountered at high angles of attack may explain why an axial velocity excess occurs above a certain angle of attack and an axial velocity deficit occurs below this threshold angle of attack.

Coarse Measurement Grids

The crossflow velocity vectors \((v \text{ and } w)\) for \(\alpha = 5^\circ\) recorded on the coarse measurement grids are shown in Figure 5.5. The vectors are normalized by free-stream velocity and clearly show the clockwise rotation of the vortex structure. The crossflow vectors at \(x/c = 0\) do not appear axisymmetric and the exact location of the vortex core centre is unclear. The largest vectors are seen on the inboard side of the vortex where they exhibit a strong downward motion. The crossflow vectors at \(x/c = 1\) exhibit a more axisymmetric shape and the largest vectors are more evenly distributed around the rotational vortex but have decreased in magnitude. The rotational structure also appears to have moved inboard by 0.036\(c\). The crossflow vectors at \(x/c = 2\) and \(x/c = 3\) resemble a stable rotational structure with the magnitude of the vectors remaining relatively constant and the vortex progressively moving more inboard by \(x/c = 3\), where it was now at an inboard distance of 0.1\(c\).

The contours of crossflow velocity magnitude calculated from the resultant of the \(v\) and \(w\) vectors \(\sqrt{v^2 + w^2}\) for \(\alpha = 5^\circ\) are shown in Figure 5.6. At \(x/c = 0\), the crossflow velocity magnitude attains its maximum value of 0.29\(U_\infty\) on the inboard side of the vortex and the core centre location (minimum velocity magnitude) is unidentifiable from the plot. The crossflow velocity structure was nonaxisymmetric and the wake below the wing had near zero crossflow velocity. The maximum crossflow velocity decreased to 0.21 at \(x/c = 1\) with the crossflow
5. EXPERIMENTAL INVESTIGATION OF A WING-TIP VORTEX

![Normalized crossflow velocity vectors](image)

(a) $x/c = 0$  
(b) $x/c = 1$  
(c) $x/c = 2$  
(d) $x/c = 3$

**Figure 5.5:** Normalized crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (coarse measurement grids)

The crossflow velocity structure becoming more axisymmetric and the rotational fluid from beneath the wing feeding into the spiral surrounding the vortex. The crossflow velocity magnitude increased to $0.27U_\infty$ at $x/c = 2$, which may be attributed to the continuous trapping, by the vortex, of the wing shear layer. The crossflow velocity then decreased slightly to $0.25U_\infty$ at $x/c = 3$, where the structure of the crossflow velocity magnitude had still not become fully axisymmetric.

The crossflow velocity vectors for $\alpha = 10^\circ$ recorded on the coarse measurement grids are shown in Figure 5.7. The crossflow velocity vectors at $x/c = 0$ are noticeably larger and more axisymmetric than the crossflow velocity vectors for $\alpha = 5^\circ$ at the same location. The increased crossflow velocity could play a role
5.1 Mean Flow Measurements

Figure 5.6: Normalized crossflow velocity magnitude $U_c/U_\infty$ for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (coarse measurement grids)

in the vortex becoming axisymmetric quicker than the vortex for $\alpha = 5^\circ$. The migration of high pressure fluid underneath the wing to the low pressure region on top also occurs closer to the leading edge of the wing at higher angles of attack, which allowed the vortex a greater distance to form before the trailing edge ($x/c = 0$). The vortex core centre appeared to be at an inboard distance of $0.035c$ at $x/c = 1$. As the vortex progressed downstream it was seen to migrate further inboard with the vortex core centre at a distance of $0.07c$ and $0.1c$ at $x/c = 1$ and $x/c = 2$ respectively. Furthermore, the vortex became more axisymmetric and the velocity vectors remained relatively constant. The vortex core centre remained at an inboard distance of $0.1c$ at the last measurement location of $x/c = 3$ and
the velocity vectors decreased in size.

\begin{figure}[h]
\centering
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{fig5-7a.png}
\caption{$x/c = 0$}
\end{subfigure}
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{fig5-7b.png}
\caption{$x/c = 1$}
\end{subfigure}
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{fig5-7c.png}
\caption{$x/c = 2$}
\end{subfigure}
\begin{subfigure}{0.4\textwidth}
\centering
\includegraphics[width=\textwidth]{fig5-7d.png}
\caption{$x/c = 3$}
\end{subfigure}
\caption{Normalized crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (coarse measurement grids)}
\end{figure}

The contours of crossflow velocity magnitude for $\alpha = 10^\circ$ show a marked difference than the contours for $\alpha = 5^\circ$ as the crossflow velocity minimum, which identifies the vortex core centre is clearly identifiable in Figure 5.8. The maximum crossflow velocity magnitude of $0.62U_\infty$ at $x/c = 0$ is over twice the maximum crossflow velocity measured at $x/c = 0$ for $\alpha = 5^\circ$. A region of high crossflow velocity is seen to surround the vortex centre with the maximum crossflow velocity occurring on the inboard side of the vortex. At $x/c = 1$, the region of high crossflow velocity is more evenly distributed around the vortex centre and the
5.1 Mean Flow Measurements

maximum crossflow velocity magnitude has decreased slightly to $0.53U_\infty$ on the inboard side of the vortex. At $x/c = 2$, the maximum crossflow velocity magnitude remained at $0.53U_\infty$, where the region of high crossflow velocity seemed to progress inboard beyond the area surveyed with the coarse measurement grids. At $x/c = 3$, the crossflow velocity magnitude had decreased to $0.28U_\infty$, but it is suspected that the coarse measurement grids did not capture all of the vortex as it has progressed inboard by a distance of $0.1c$. The refined measurement grids are presented next and extend further inboard along the wing to fully capture the salient flow features.

Figure 5.8: Normalized crossflow velocity magnitude $U_c/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (coarse measurement grids)
5. EXPERIMENTAL INVESTIGATION OF A WING-TIP VORTEX

Refined Vortex Measurement Grids

Figure 5.9 shows the crossflow velocity vectors for $\alpha = 5^\circ$, which were recorded on the fine measurement grids. The increased measurement point density gives a more detailed insight into the flow-field and the vortex core centre can be located much more accurately. The vectors at $x/c = 0$ are disorganized and nonaxisymmetric with the core centre location appearing just above the wing tip. A strong downward motion of the velocity vectors is observed on the inboard side of the vortex, which is similar to the vectors obtained on the coarse measurement grid. This would suggest that as the high pressure flow underneath the wing migrates to the lower pressure region on top, it accelerates as it curls around the wing tip and attains its maximum velocity on its downward path. The vectors attain a more circular motion at $x/c = 1$ and the vortex appeared to shift from side to side indicated by the two circulatory structures present in Figure 5.9b. The crossflow velocity vectors decreased slightly at $x/c = 1$. The vectors at $x/c = 2$ appear more organized and axisymmetric and seem to follow a uniform circular pattern. The vectors have not decayed significantly and maintain a strong downward motion on the inboard side of the vortex. The vortex core centre has moved inboard by 0.08$c$ at $x/c = 2$ and it continued to progress inboard to a distance of 0.11$c$ by $x/c = 3$. The crossflow velocity vectors reveal that the vortex is a stable and axisymmetric structure (at least in terms of cross-stream velocity characteristics) by $x/c = 3$.

The contours of crossflow velocity magnitude for $\alpha = 5^\circ$ are shown in Figure 5.10. At $x/c = 0$, the maximum crossflow velocity of $0.34U_\infty$ occurs in a region of high crossflow velocity on the inboard side of the vortex. This is in agreement with the observations made on the coarse measurement grids and the fine grid vector plot at the same location. The crossflow velocity structure was nonaxisymmetric, which suggests that the flow is undergoing a highly three dimensional mixing and that the roll up of the vortex is in it’s infancy stage. The vortex centre indicated by the minimum crossflow velocity magnitude is observed to be just above the wing tip. The maximum crossflow velocity magnitude decays to $0.25U_\infty$ at $x/c = 1$ and the region of high crossflow velocity now encompasses a much a larger area than at $x/c = 0$. The crossflow velocity structure does gain some
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Figure 5.9: Normalized crossflow velocity vectors \( (jv + kw)/U_\infty \) for \( \alpha = 5^\circ \) and \( Re = 3.25 \times 10^5 \) (fine measurement grids)

Axisymmetry at \( x/c = 1 \) and the two patches of minimum crossflow velocity magnitude at 0.058c and 0.086c reiterate the earlier observation of the vortex shifting from side to side. The maximum crossflow velocity magnitude increases to 0.27\( U_\infty \) at \( x/c = 2 \) and the region of high crossflow velocity is seen to migrate to the inboard side of the vortex like at \( x/c = 0 \). The maximum crossflow velocity magnitude decayed to 0.25\( U_\infty \) at \( x/c = 3 \) and the region of high crossflow velocity stayed on the inboard side of the vortex. The continuous trapping of the shear layer is thought to be the main reason for the fluctuation of the maximum crossflow velocity magnitude and its location.
The crossflow velocity vectors recorded on the fine measurement grids for \( \alpha = 10^\circ \) are shown in Figure 5.11. At \( x/c = 0 \), the vectors show a vortex in the early roll up stage with the largest vectors appearing on the periphery of the vortex structure. It is difficult to pinpoint the exact location of the vortex core centre as there are a number of vectors with a near-zero velocity. The lowest of which is seen to occur at an inboard distance of 0.028c. Similar to Figure 5.9, vectors with maximum velocity occur at the top and inboard side of the vortex. Further downstream at \( x/c = 1 \), the crossflow velocity vectors become more axisymmetric and the vortex core centre is clearly identifiable at an inboard distance of 0.057c. The vectors become disorganized and nonaxisymmetric at
5.1 Mean Flow Measurements

$x/c = 2$ suggesting that the vortex is shifting from side to side perhaps due to further entrainment of the wing shear layer at this location. The vortex regains it’s axisymmetry at $x/c = 3$, by which point the vortex core centre has moved inboard a distance of $0.1c$.

![Normalized crossflow velocity vectors](image)

**Figure 5.11:** Normalized crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (fine measurement grids)

The contours of crossflow velocity magnitude for $\alpha = 10^\circ$ are shown in Figure 5.12. The crossflow velocity magnitude at $x/c = 0$ attains a maximum value of $0.74U_\infty$ near the lower boundary of the vortex at an inboard distance of $0.03c$. This value is over twice the maximum crossflow velocity recorded for $\alpha = 5^\circ$. Apart from the isolated point of maximum crossflow velocity magnitude, a region
5. EXPERIMENTAL INVESTIGATION OF A WING-TIP VORTEX

of high crossflow velocity is seen to occur in the upper portion of the vortex. This region of high velocity indicates the vortex is clearly in the early stages of roll up as it is nonaxisymmetric and one sided. The vortex core centre location at $x/c = 0$ is located at an inboard distance of 0.044c. At $x/c = 1$, the maximum crossflow velocity magnitude decreases to 0.57$U_\infty$ and is located on the inboard side of the vortex. The region of high crossflow velocity now surrounds the vortex core centre located at 0.072c and the size of the crossflow velocity magnitude structure has increased considerably. Further downstream at $x/c = 2$ and $x/c = 3$, the maximum crossflow velocity magnitude remains at 0.57$U_\infty$ on the inboard side of the vortex. The vortex core centre had moved inboard a distance of 0.11c by the last measurement location of $x/c = 3$.

5.1.3 Vortex Circulation

The tip vortex circulation was calculated for both angles of attack (5° and 10°) at all downstream locations ($x/c = 0, 1, 2$ and 3) from the velocities recorded on the fine measurement grids using the following equation:

$$\Gamma = \sum_i \bar{v}_i \cdot \Delta \bar{l}_i$$  \hspace{1cm} (5.1)

where $\bar{v}_i$ and $\Delta \bar{l}_i$ are the velocity vector and segment length along the square path surrounding the vortex, respectively.

The vortex circulation is presented in terms of a circulation parameter normalized by the product of the free-stream velocity and chord length. Table 5.1 gives the circulation parameters at each measurement location for both angles of attack. The vortex circulation for $\alpha = 5^\circ$ was at a minimum of 0.065 at $x/c = 0$ after which it increased to 0.086 at $x/c = 1$ before finally reaching its maximum value of 0.09 at $x/c = 3$. The fact that the vortex strength is increasing with downstream distance would imply that the vortex is continually trapping the wing shear layer as it progresses downstream. As expected, the circulation was seen to increase for $\alpha = 10^\circ$, attaining a value of 0.243 at $x/c = 0$ and reaching its maximum of 0.266 at $x/c = 2$ before decreasing slightly to 0.252 at $x/c = 3$. This would be in agreement with Prandtl’s lifting line theory that states the circulation
5.1 Mean Flow Measurements

![Normalized crossflow velocity magnitude](image)

**Figure 5.12**: Normalized crossflow velocity magnitude $U_c/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (fine measurement grids)

The strength of a trailing vortex is proportional to the strength of the bound wing vorticity. A theoretical two-dimensional wing circulation $\Gamma_w$ can be determined using the following relationship:

$$L' = \rho U_\infty \Gamma_w = \frac{1}{2} C_i \rho U_\infty^2 c$$  \hspace{1cm} (5.2)

which yields the following wing circulation parameter

$$\frac{\Gamma_w}{U_\infty c} = 0.5C_l$$  \hspace{1cm} (5.3)
5. EXPERIMENTAL INVESTIGATION OF A WING-TIP VORTEX

Using $C_t$ values of 0.5 and 0.93 for 5° and 10° angle of attack (O’Regan et al., 2014) resulted in wing circulation parameters of 0.25 and 0.46 for $\alpha = 5^\circ$ and $10^\circ$ respectively. Therefore, looking at 5.1 it is clear that the tip vortex contained only 36% and 54% of the wing circulation for 5° and 10° angle of attack at $x/c = 3$.

**Table 5.1:** Non-dimensionalized circulation parameter $\Gamma / U_\infty c$

<table>
<thead>
<tr>
<th>Angle of attack $\alpha$</th>
<th>Downstream location $x/c$</th>
<th>Circulation parameter $\Gamma / U_\infty c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5°</td>
<td>0</td>
<td>0.065</td>
</tr>
<tr>
<td>5°</td>
<td>1</td>
<td>0.086</td>
</tr>
<tr>
<td>5°</td>
<td>2</td>
<td>0.085</td>
</tr>
<tr>
<td>5°</td>
<td>3</td>
<td>0.09</td>
</tr>
<tr>
<td>10°</td>
<td>0</td>
<td>0.243</td>
</tr>
<tr>
<td>10°</td>
<td>1</td>
<td>0.244</td>
</tr>
<tr>
<td>10°</td>
<td>2</td>
<td>0.266</td>
</tr>
<tr>
<td>10°</td>
<td>3</td>
<td>0.252</td>
</tr>
</tbody>
</table>

5.1.4 Mean Streamwise Vorticity

The mean streamwise vorticity $\omega_x$ was obtained by calculating the curl of the mean velocity components ($v$ and $w$). It is reasonable to assume that $\omega_x$ is the only dominant vorticity in the vortex core region for $x/c < 3$ according to Ramaprian and Zheng (1997) and therefore is the only vorticity component presented.

**Coarse Measurement Grids**

The mean streamwise vorticity contours for $\alpha = 5^\circ$ are shown in Figure 5.13. At $x/c = 0$ the negative vorticity attains a maximum of $-0.049$ in the centre of the vortex just above the wing at an inboard distance of 0.035c. The vorticity structure appears to attain axisymmetry at an earlier stage than the axial and crossflow velocity structures. Further downstream, the maximum negative vorticity levels decrease slightly to $-0.053$ at $x/c = 2$ before finally increasing.
again to −0.049 at the last measurement location of $x/c = 3$. The fluctuation in maximum vorticity may be explained by the fact that there are a number of processes taking place during vortex roll up which affect the peak vorticity. The first process, involves the continuous trapping, by the vortex of vorticity from the shear layer, which tends to increase the vorticity in the vortex core. The second process is the viscous diffusion of the vorticity, which would normally decrease the peak vorticity. The combined effect is sometimes a small but gradual increase in vorticity with streamwise distance, which is seen to happen from $x/c = 0$ to $x/c = 3$. The point of maximum vorticity had progressed inboard a distance of 0.1c by $x/c = 3$.

The mean streamwise vorticity contours for $\alpha = 10^\circ$ are shown in Figure 5.14. The streamwise vorticity at $x/c = 0$ encompassed a much larger region than at the same location for $\alpha = 5^\circ$ and the maximum negative vorticity increased by 83% to −0.090 in the vortex core centre. Furthermore, the vorticity structure is reasonably axisymmetric at $x/c = 0$. Further downstream, the maximum vorticity gradually increases in a similar manner to the vorticity structure for $\alpha = 5^\circ$. It reaches a value of −0.11 at $x/c = 1$ before increasing to it’s peak value of −0.12 at $x/c = 2$ and then decreased back to −0.11 at $x/c = 3$. Other than the vorticity evident in and around the vortex core, the remainder of the wing wake looked to have little vorticity. The fluctuation of the maximum vorticity with downstream distance is attributed to the processes that are taking place during roll up as mentioned previously. The location of maximum vorticity had progressed inboard a distance of 0.1c by $x/c = 3$, the same distance as the $\alpha = 5^\circ$ case.

**Refined Vortex Measurement Grids**

The mean streamwise vorticity contours for $\alpha = 5^\circ$ are shown in Figure 5.15. The vorticity structure is considerably smaller in size in comparison to the vorticity structures recorded on the coarse measurement grids. At $x/c = 0$, a maximum negative vorticity of −0.057 occurred at an inboard distance of 0.058c, whereas the minimum in crossflow velocity occurred at 0.014c. It is believed that because the vortex is in the early stages of roll up, the point of maximum vorticity and point of minimum crossflow velocity were misaligned. Two small regions of positive
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Figure 5.13: Normalized streamwise vorticity $\omega_{xc}/U_{\infty}$ overlaid with crossflow velocity vectors $(jv+kw)/U_{\infty}$ for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (coarse measurement grids)

Vorticity are also noted at $x/c = 0$, which suggests the presence of secondary counter rotating vortices. Further downstream at $x/c = 1$, the maximum vorticity decreased to $-0.037$ and has moved inboard a distance of $0.07c$. The maximum vorticity remained at $-0.037$ up to $x/c = 3$ at which point it had progressed inboard a distance of $0.11c$. The location of the maximum vorticity from $x/c = 1$ to $x/c = 3$ coincided with the point of minimum crossflow velocity suggesting that the vortex roll up was almost complete. The vorticity structure became axisymmetric at $x/c = 2$ and remained that way until $x/c = 3$. The reduction in vorticity with streamwise distance was in contrast to the vorticity recorded on
the coarse measurement grids. It is thought that the coarse measurement grids did not capture enough data to accurately calculate the vorticity or that viscous diffusion played a larger part than the trapping of vorticity from the shear layer.

The mean streamwise vorticity contours for $\alpha = 10^\circ$ are shown in Figure 5.16. At $x/c = 0$, a maximum negative vorticity of $-0.069$ occurs in the vortex core at an inboard distance of $0.04c$ and the point of maximum vorticity and minimum crossflow velocity do not coincide (similar to the vorticity for $\alpha = 5^\circ$ at the same location). This represents a 23% reduction in peak vorticity from that recorded on the coarse measurement grids at $x/c = 0$. This reduction is
5. EXPERIMENTAL INVESTIGATION OF A WING-TIP VORTEX

Figure 5.15: Normalized streamwise vorticity $\omega_{xc}/U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (fine measurement grids)

attributed to the large spacing of the coarse measurement grids that captured an insufficient number of points for an accurate calculation of the vorticity. The region of negative vorticity at $x/c = 0$ seems to surround the point of minimum crossflow velocity, which suggests that the vorticity is in the process of roll up into the vortex core. A small region of positive vorticity is also seen to occur just under the wing, which indicates the presence of a small secondary counter rotating vortex. The vorticity at $x/c = 1$ became more axisymmetric with the negative vorticity concentrated in the core of the vortex. The maximum vorticity decreased slightly to $-0.065$ at $x/c = 1$ and remained constant up to $x/c = 3$
(similar to the vorticity for $\alpha = 5^\circ$). The location of the maximum vorticity had progressed inboard a distance of $0.1c$ by the last measurement location $x/c = 3$.

Figure 5.16: Normalized streamwise vorticity $\omega_x c/U_\infty$ overlaid with crossflow velocity vectors $(j v + k w)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (fine measurement grids)

### 5.2 Turbulence Measurements

#### 5.2.1 RMS Fluctuating Velocities

The root mean squared (rms) fluctuating velocities ($\sqrt{\bar{u}'^2}$ and $\sqrt{\bar{v}'^2}$) acquired with the x-probe anemometer and normalized by the free-stream velocity $U_\infty$
are presented in this section. The measurements were taken on a line through the vortex core, which was determined from the point of minimum crossflow velocity recorded on the fine measurement grids presented previously. The normalized rms velocities are plotted against $y/d_c$, which is the inboard distance non-dimensionalized by the vortex core diameter.

**High Reynolds Number ($3.25 \times 10^5$)**

The following normalized $u'$ and $v'$ rms velocities were recorded at a free-stream turbulence level of 0.5% for $\alpha = 5^\circ$ and $10^\circ$. The $u'$ and $v'$ rms velocities for $\alpha = 5^\circ$ are shown in Figure 5.17. As expected the non-dimensional turbulent rms velocities at $x/c = 0$ are nonaxisymmetric as the vortex is still in the early stages of roll up. The maximum $u'$ rms and $v'$ rms values are 0.05 and 0.056 respectively. Both velocities are nearly equal in magnitude on the outboard side of the vortex core, whereas they become more skewed on the inboard side. This would suggest that the turbulence is approximately isotropic in one half of the vortex core at $x/c = 0$. The isotropic turbulence perhaps being created by the stabilizing effect of the rotational flow. At $x/c = 1$, it is clear that the turbulence structure has a attained a large degree of axisymmetry with the maximum $u'$ rms and $v'$ rms velocities of 0.046 and 0.049 occurring at the vortex core centre and decreasing towards the periphery of the vortex core. The rms velocities in both directions are in very good agreement on both sides of the vortex suggesting that the turbulence has now attained a high degree of isotropy in the core. The slight decrease in the maximum rms velocities at $x/c = 1$ is believed to be attributed to the damping effect of the core solid body rotation. The rms velocities became slightly skewed at $x/c = 2$ with the maximum $v'$ rms velocity increasing to 0.058 and the maximum $u'$ rms velocity decreasing to 0.041 on the inboard side of the vortex. This would suggest that the surrounding spiral wake had perhaps entrained some additional turbulent shear layer fluid, which then impacted on the core turbulence axisymmetry and isotropy. The maximum $u'$ and $v'$ rms velocities decreased once more at $x/c = 3$ to 0.028 and 0.043 with the turbulence structure retaining some symmetry with both rms velocity maxima occurring at the vortex core centre. Both rms velocities were of similar magnitude at the outer edge of the vortex core at $x/c = 3$. 
5.2 Turbulence Measurements

Figure 5.17: Normalized rms velocities $\sqrt{\bar{u}'^2/U_\infty}$ and $\sqrt{\bar{v}'^2/U_\infty}$ across the vortex core for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (broken line represents vortex core centre)

The $u'$ and $v'$ rms velocities for $\alpha = 10^\circ$ are shown in Figure 5.18. The maximum values of the $u'$ and $v'$ rms velocities at $x/c = 0$ were 0.131 and 0.103, which was an increase of 162% and 83% on the maximum values for $\alpha = 5^\circ$ respectively. This is expected due to the higher crossflow velocities and larger circulation being rolled up into the vortex, thereby creating larger fluctuations in the vortex core. The rms velocities are seen to be more axisymmetric than the velocities for $\alpha = 5^\circ$ at $x/c = 0$, although the symmetry is slightly skewed by a peak in the velocities on the inboard core boundary. The velocities are also in relatively good agreement with each other at $x/c = 0$. However, at $x/c = 1$
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the maximum $u'$ and $v'$ rms velocities had decreased by 70% and 58% to 0.04 and 0.044 respectively. These values are less than the rms velocities recorded for $\alpha = 5^\circ$ at the same location. This could be attributed to the higher crossflow velocities present for $\alpha = 10^\circ$, which would have dampened out the turbulent fluctuations at a faster rate. The rms velocities still have a degree of axisymmetry but the maximum velocities do not occur in the vortex core centre. The values of both rms velocities are in best agreement at the outer boundaries of the vortex core. The $u'$ and $v'$ rms velocities continued to decrease further downstream with maxima of 0.022 and 0.024 occurring at opposite sides of the vortex at $x/c = 2$. The rms velocities seemed to level out at $x/c = 2$ with the values at the centre and outer boundaries close in magnitude, which would suggest the turbulence had reached an isotropic state. Furthermore, the rms velocities have a degree of axisymmetry and are in relatively good agreement at $x/c = 2$. The maximum $u'$ and $v'$ rms velocities increased slightly to 0.024 and 0.03 at $x/c = 3$ and the two sets of velocities became more skewed suggesting the turbulence in the core had become anisotropic.

Low Reynolds Number ($1.8 \times 10^5$)

The following normalized $u'$ and $v'$ rms velocities were recorded at free-stream turbulence levels of 0.5% (no grid) and 4.3% (square mesh grid) at an angle of attack of $5^\circ$. The $u'$ and $v'$ rms velocities for a turbulence intensity of 0.5% (no grid) for $\alpha = 5^\circ$ and $Re = 1.8 \times 10^5$ are shown in Figure 5.19. The maximum $u'$ and $v'$ rms velocities at $x/c = 0$ are 0.027 and 0.025 respectively. The magnitude of the velocities are significantly less than at the same location for the higher Reynolds number of $3.25 \times 10^5$, which is expected as the lower crossflow velocities and vorticity lead to less velocity fluctuations. The velocities are seen to decrease linearly from a maximum at the outboard side of the vortex to a minimum at the inboard side and no axisymmetry was observed in the rms velocity structure at $x/c = 0$. The rms velocities resembled a somewhat axisymmetric structure at $x/c = 1$ with the maximum $u'$ rms velocity decreasing to 0.019 and the $v'$ rms velocity remaining constant at 0.025. The location of the velocity maxima was on the inboard side of the vortex with both maxima occurring at the same point. The velocities decreased linearly from their maxima near the vortex centre.
5.2 Turbulence Measurements

Figure 5.18: Normalized rms velocities $\sqrt{\bar{u}'^2}/U_\infty$ and $\sqrt{\bar{v}'^2}/U_\infty$ across the vortex core for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (broken line represents vortex core centre)

to the outer edges of the vortex core. At $x/c = 2$, the $u'$ rms velocity decreased slightly to 0.018, however, the $v'$ rms velocity increased to 0.03. A similar trend was observed at the same location for $\alpha = 5^\circ$ at the higher Reynolds number of $3.25 \times 10^5$, suggesting that the vortex entrains some additional shear layer fluid at the same location for both Reynolds numbers. Both the $u'$ and $v'$ rms velocities decreased to 0.017 and 0.028 at the last measurement location of $x/c = 3$ and the rms velocity structure became less axisymmetric.

The $u'$ and $v'$ rms velocities for the elevated free-stream turbulence level of 4.3% at angle of attack of 5$^\circ$ are shown in Figure 5.20. At $x/c = 0$ The maxi-
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Figure 5.19: Normalized rms velocities $\sqrt{u'^{2}}/U_{\infty}$ and $\sqrt{v'^{2}}/U_{\infty}$ acquired at a free-stream turbulence level of 0.5% across the vortex core for $\alpha = 5^\circ$ and $Re = 1.8 \times 10^5$ (broken line represents vortex core centre)

Maximum $u'$ and $v'$ rms velocities of 0.024 and 0.025 were very similar to the no-grid measured velocities at the same location. One marked difference between the two turbulence levels was the shape of the rms velocity structure across the core as the structure for the higher turbulence level is more axisymmetric. At $x/c = 1$ the $u$ rms velocity decreased significantly to 0.018, whereas the $v'$ rms velocity only decreased slightly to 0.024. This trend was very similar to that observed for the no-grid case. The shape profiles of the rms velocities through the vortex core at $x/c = 1$ were very similar to the profiles recorded in the no-grid case. Similar
5.2 Turbulence Measurements

to the no-grid case and high Reynolds number case, at \( x/c = 2 \), the maximum \( u' \) rms velocity decreased to 0.016 and the maximum \( v' \) rms velocity increased to 0.026. The maximum \( u' \) and \( v' \) rms velocities decreased to 0.015 and 0.026 at \( x/c = 3 \), which was similar to the no-grid case, however, the peak rms velocities occurred on the inboard side of the vortex for the 4.3% free-stream turbulence case.

![Normalized rms velocities](image)

**Figure 5.20:** Normalized rms velocities \( \sqrt{\bar{u}'^2}/U_\infty \) and \( \sqrt{\bar{v}'^2}/U_\infty \) acquired at a free-stream turbulence level of 4.3% across the vortex core for \( \alpha = 5^\circ \) and \( Re = 1.8 \times 10^5 \) (broken line represents vortex core centre)
5. EXPERIMENTAL INVESTIGATION OF A WING-TIP VORTEX

5.2.2 Reynolds Stresses

The $\overline{u'v'}$ Reynolds stress component normalized by the square of the free-stream velocity $U^2_\infty$ was measured on a line taken through the vortex core. The Reynolds stress is plotted against the non-dimensional distance $y/d_c$ in a similar manner to the plots for the fluctuating rms velocities.

High Reynolds Number (3.25 $\times$ $10^5$)

The $\overline{u'v'}$ Reynolds stress was recorded at a free-stream turbulence level of 0.5% for $\alpha = 5^\circ$ and $10^\circ$.

The $\overline{u'v'}$ Reynolds stress component for $\alpha = 5^\circ$ is shown in Figure 5.21. Looking at Figure 5.21a, it is clear the vortex core contains regions of opposite signed stress at $x/c = 0$. The Reynolds stress reaches its maximum value of 0.00074 at a distance of 0.25$d_c$ and two negative peaks of $-0.00065$ and $-0.00069$ are seen to occur at $-0.16d_c$ and $0.5d_c$. Further downstream at $x/c = 1$ the peak Reynolds stress value decreased significantly to $-0.00021$ at a distance of 0.42$d_c$. At $x/c = 1$, the Reynolds stress changed sign at the vortex core (indicated by the broken line) with a peak stress value of 0.00019 occurring at a distance of $-0.14d_c$. At $x/c = 2$, the maximum Reynolds stress increased to $-0.00026$ at a distance of $0.06d_c$ with a positive peak of 0.0002 also occurring at 0.18$d_c$. The increase in the value of the maximum Reynolds stress agrees with the earlier observed increases in peak rms velocities at this location. The reason behind the increase in turbulent activity is thought to be due to the re-energising of the vortex from additional shear layer fluid. The Reynolds stress decreased significantly at $x/c = 3$ with a maximum value of $-0.00009$ measured at $-0.125d_c$. The reduction in the Reynolds stress with downstream distance is most likely due to the damping effects of the rotational vortex core.

Looking at the $\overline{u'v'}$ Reynolds stress component for $\alpha = 10^\circ$ in Figure 5.22, it is clear that the magnitudes of the Reynolds stresses are much larger than those for $\alpha = 5^\circ$. The maximum Reynolds stress at $x/c = 0$ was 0.0025 located just outside the vortex core centre at a distance of 0.05$d_c$. This value represented an increase of 237% over the maximum Reynolds stress for $\alpha = 5^\circ$ at the same location. There is also a peak negative stress of $-0.001$ at 0.33$d_c$ in the vortex core at
5.2 Turbulence Measurements

Figure 5.21: Normalized Reynolds stress component $\overline{uv''}/U_\infty^2$ acquired at a free-stream turbulence level of 0.5% across the vortex core for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (broken line represents vortex core centre)

$x/c = 0$. There is a significant decrease in the peak Reynolds stress at $x/c = 1$ with a maximum value of 0.0004 recorded at $0.25d_c$ and a peak negative value of $-0.0003$ recorded on the edge of the vortex core at $-0.43d_c$. The Reynolds stress changed sign at the vortex core centre (a similar trend noted for $\alpha = 5^\circ$ at the same location). The maximum Reynolds stress decreases further at $x/c = 2$ to a value of $-0.00016$ at $-0.5d_c$, which is in contrast to the increase of turbulent activity for $\alpha = 5^\circ$ at the same location. Furthermore, at $x/c = 2$, the Reynolds stress is observed to go from a negative stress to a positive stress in a linear...
5. EXPERIMENTAL INVESTIGATION OF A WING-TIP VORTEX

fashion as seen in Figure 5.22c. The Reynolds stress increases at $x/c = 3$ with a maximum value of $-0.0024$ occurring at $-0.5d_c$ (the stress again changed sign as it passed through the centre of the vortex core). The increase in Reynolds stress at this location suggests that the vortex captured some shear layer fluid, which increased the turbulent fluctuations in the core.

![Figure 5.22](image)

**Figure 5.22:** Normalized Reynolds stress component $u'v'/U_\infty^2$ acquired at a free-stream turbulence level of 0.5% across the vortex core for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (broken line represents vortex core centre)

**Low Reynolds Number (1.8 \times 10^5)**

The $u'v'$ Reynolds stress was recorded at a free-stream turbulence level of 0.5% and 4.3% for $\alpha = 5^\circ$ and $Re = 1.8 \times 10^5$. The $u'v'$ Reynolds stress component
obtained at both free-stream turbulence levels across the vortex core is shown in Figure 5.23. The Reynolds stress structure for $x/c = 0$ was nonaxisymmetric and disorganized for both the 0.5% and 4.3% free-stream turbulence levels. The maximum Reynolds stress for the 0.5% and 4.3% turbulence levels occurred at either side of the vortex centre in the negative stress region with values of $-0.00033$ and $-0.00030$ at $-0.41d_c$ and $0.25d_c$ respectively. At $x/c = 1$, the maximum Reynolds stresses for the 0.5% and 4.3% turbulence levels decreased to $-0.00008$ and $-0.00017$ just outside the vortex core at $-0.071d_c$ and $0.071d_c$. Furthermore, there was no apparent axisymmetry in the Reynolds stress structure at $x/c = 1$. At $x/c = 2$, the maximum Reynolds stress increased slightly to $-0.00009$ for the 0.5% turbulence level and decreased to $-0.00015$ for the 4.3% turbulence level. The maximum stresses occurred at distances of $0.125d_c$ and $0.0625d_c$ for the 0.5% and 4.3% turbulence levels respectively. The Reynolds stress for both turbulence levels decreased at $x/c = 3$ with values of $0.00007$ and $-0.00012$ recorded for the 0.5% and 4.3% turbulence levels respectively. The maximum Reynolds stress was higher for the 4.3% turbulence case at $x/c = 1, 2$ and 3, which perhaps would indicate that free-stream turbulence increased the vortex core turbulence during roll up in the near-field.

5.3 Summary

The axial velocity in the core of the wing-tip vortex was found to contain an axial velocity deficit (wake-like) for $\alpha = 5^\circ$ and an axial velocity excess (jet-like) for $\alpha = 10^\circ$ (Re = $3.25 \times 10^5$) – a feature also noted by Birch et al. (2004). This also confirms the statement by Chow et al. (1997a), that the angle of attack is the key parameter that influences the core axial velocity (Reynolds number and Aspect Ratio (AR) were also found to influence the core axial velocity). As the vortex progressed downstream the core axial velocity slowly returned towards free-stream velocity for both the wake-like and jet-like cases. The vortex was found to contain regions of axial velocity excess and deficit at all downstream locations for $\alpha = 10^\circ$. The deficit region surrounded the core axial velocity excess at $x/c = 1$ and then moved to the inboard portion of the vortex at $x/c = 2$ and 3. This phenomenon is similar to the findings of Shekarriz et al. (1993) who stated
that it is possible to find axial momentum deficit in one portion of the vortex and momentum excess in another due to the dependence of the axial pressure gradient on the radial distance from the vortex core centre.

The crossflow velocity vectors were found to be nonaxisymmetric just behind the wing at \( x/c = 0 \) for \( \alpha = 5^\circ \) and \( 10^\circ \), a consequence of the highly three dimensional mixing of the high and low pressure fluid. The crossflow velocity magnitude reached a minimum at the vortex core centre and slowly increased to a maximum at the edge of the vortex core for both angles of attack and all
downstream locations (a similar observation was made by Chow et al. (1997b)).

The crossflow velocity magnitude increased significantly with angle of attack and the maximum velocity magnitude for $\alpha = 10^\circ$ at $x/c = 0$ was 100% greater than the maximum for $\alpha = 5^\circ$ at $x/c = 0$. The vortex strength or circulation increased with downstream distance for both angles of attack, most likely owing to the continuous trapping of shear layer vorticity as it progressed downstream. The vortex strength also increased with angle of attack, which was in agreement with Prandtl’s lifting line theory. Only 36% and 54% of the theoretical wing circulation was rolled up into the tip vortex for $5^\circ$ and $10^\circ$ by the last measurement location of $x/c = 3$, whereas studies by Higuchi et al. (1987) and Shekarriz et al. (1993) found that the tip-vortex contained approximately 45% and 66% of the theoretical wing circulation.

The maximum value of vorticity was found to occur at the point of minimum crossflow velocity after $x/c = 0$. The vorticity magnitude was found to decrease with downstream distance for $\alpha = 5^\circ$, whereas a slight increase in vorticity was observed for $\alpha = 10^\circ$ (similar to the studies by Ramaprian and Zheng (1997) and Birch et al. (2004)). The continuous trapping of the shear layer vorticity usually tends to increase the vorticity magnitude as the vortex progresses downstream, whereas viscous diffusion leads to a decrease in vorticity. The more dominant process usually determining the correlation between downstream distance and vorticity magnitude. The presence of a small secondary counter rotating vortex was observed at $x/c = 0$ for both $\alpha = 5^\circ$ and $10^\circ$ by the presence of a patch of positive signed vorticity. The maximum $u'$ and $v'$ rms velocities for $\alpha = 5^\circ$ were found to occur at the outer edge of the vortex core at $x/c = 0$ but further downstream the maximum rms velocities were observed in the vortex core centre (similar to the observations reported by Chow et al. (1997b)). The maximum $u'$ and $v'$ rms velocities for $\alpha = 10^\circ$ occurred at the edge of the vortex core at $x/c = 0$ but the maximum rms velocities were observed in the vortex core centre for $x/c = 1$ and $x/c = 3$. The maximum rms velocities for $x/c = 2$ occurred halfway between the outer edge of the core and the core centre. The turbulence in the vortex core is thought to be attributed to the buffeting of the core by the surrounding wake (Devenport et al., 1996) and was found to decay
with streamwise distance for both angles of attack due to the stabilizing effect of the vortex solid body rotation (Chow et al., 1997b).

The $u'v'$ Reynolds stress component exhibited two regions of opposite signed stress for both angles of attack, which was similar to the Reynolds stress in the studies by Chow et al. (1997b) and Churchfield and Blaisdell (2009). The $u'v'$ Reynolds stress component decayed with downstream distance in a similar manner to the turbulent rms velocities. The vortex was subjected to an elevated free-stream turbulence level of 4.3% at a Reynolds number of $1.8 \times 10^5$ to examine the effect of external turbulence on the vortex.

The rms velocities in the vortex core for the elevated free-stream turbulence level of 4.3% exhibited a very similar shape profile to the rms velocities recorded in the no-grid case (0.5% turbulence) and the magnitudes of the rms velocities were similar for both turbulence levels. The Reynolds stresses for the elevated free-stream turbulence level of 4.3% were slightly higher than the no-grid turbulence level of 0.5%, which might indicate that the free-stream turbulence increased the vortex core turbulence during roll up in the near-field (similar to the study by Bailey et al. (2006)).
Chapter 6

Numerical Modelling of a Wing-tip Vortex

This chapter describes the numerical modelling of the wing-tip vortex, including the geometry creation, mesh generation and turbulence modelling approach taken in this investigation. A comprehensive discussion and presentation of the numerical results is then given, where the numerical modelling results are validated against experimental measurements.

6.1 Modelling Overview

The numerical simulations were carried out using finite volume CFD software STAR-CCM+ version 7.06 (a commercial CFD meshing and flow solver supplied by CD-adapco) on a high-performance Dell Precision T7600 computer with 32GB of RAM and two 8-core Xeon E5-2665 processors capable of hyper-threading, which enabled a total of 32 parallel processes to compute the simulation. The wing model used to generate the vortex was a NACA 0012 half wing with a 0.14m chord and 0.15m semi-span. Two angles of attack were simulated (5° and 10°) at a chord Reynolds number of $3.25 \times 10^5$. Similar to the experimental study, four downstream locations were investigated ($x/c = 0, 1, 2$ and $3$) in the simulation.
6. NUMERICAL MODELLING OF A WING-TIP VORTEX

6.2 Geometry Creation

The NACA 0012 wing model was created using three-dimensional modelling software ProEngineer Wildfire 4.0. The wing coordinates for the NACA 0012 profile were obtained from the University of Illinois Applied Aerodynamics Group Airfoil Coordinates Database UIUC (2010). The coordinates were imported into Pro-Engineer and extruded as a three-dimensional solid part. The solid part was saved as an IGES file and imported into STAR CCM+ as a surface mesh. The completed solid wing geometry can be seen in Figure 6.1a. A control volume was then created around the wing geometry that resembled the wind tunnel test section used for experimentation (see Figure 6.1b). The exact dimensions of the experimental test section could not be replicated due to convergence difficulties. The wind tunnel wall boundaries above and below the wing section were checked for pressure continuity to ensure that there was no adverse wall interference due to their proximity. As the wall boundaries were situated over a chord length away from the wing section there was minimal change in pressure on the boundaries. It was therefore concluded that wall interference effects were negligible and no correction had to be applied.

6.3 Mesh Generation

6.3.1 Overview

In order to solve the governing partial differential equations of fluid flow, the fluid domain has to be discretised into smaller volumes (cells) on which the numerical solver can solve the flow conditions. The numerical meshing approach and the different meshing requirements are highly dependent on the type of flow simulation used. The meshing strategy for the numerical investigation of a wing-tip vortex in the near-field is outlined in this section.

6.3.2 Surface Mesh

A surface mesh of the imported wing geometry and the control volume was generated before the volume mesh. The surface remesher applied a delaunay trian-
6.3 Mesh Generation

(a) Wing Geometry and Coordinate System

(b) Numerical Control Volume

Figure 6.1: Schematics of the wing geometry and control volume

gulation on the surface and improved the surface quality. The surface remesher is always employed unless a discretised surface mesh is imported. The surface mesh was refined over the wing in order to capture the required geometry details and achieve a smooth curvature over the leading and trailing edges. The grid reference parameter “base size” was set to the chord length of the wing (0.14m) and the minimum surface and target surface sizes were set to 1% of the base size (1.4mm). This resulted in a smooth curvature over the wing leading and trailing edges surfaces, whereas initial size settings resulted in choppy and rough curvature over these edges. The final uniform wing surface mesh used for the simulations in this investigation is shown in Figure 6.2a.
6. NUMERICAL MODELLING OF A WING-TIP VORTEX

Figure 6.2: Smooth surface mesh curvature on wing geometry

6.3.3 Polyhedral Volume Mesh

The volume mesh represents the geometry and flow in three-dimensional space. The volume mesh cell type and the sizing of the volume mesh are critical to the accurate resolution of the flow gradients and non-physical solutions can result from coarse or inappropriate meshes.

An unstructured polyhedral mesh with prism layer mesh elements was chosen for the simulations in this investigation. A polyhedral mesh was chosen as it provides a balanced solution for complex mesh generation problems and is less computationally demanding than a tetrahedral mesh. In addition, the polyhedral
cells model swirling flow more accurately than tetrahedral and cartesian cells as the faces of a polyhedral cell are orthogonal to the flow regardless of flow direction (Maxwell, 2014).

### 6.3.4 Prism Layer Mesh Elements

The prism layer mesh model was used with the core volume polyhedral mesh to generate orthogonal prismatic cells next to wall boundaries. The prism layer cells are aligned with the flow and allow high aspect ratio cells to be used, which allow a high resolution in the wall normal direction whilst keeping a lower resolution in the flow direction. The effect of using prism layer cells in the mesh effectively minimises numerical diffusion and improves the accuracy of the computation. A total of seven prism layers with a stretching ratio of 1.5 were generated within the boundary layer of the wing. It is crucial to have good control of the prism layer thickness with respect to the local mesh size and it is generally recommended that the last prism layer thickness should be between 30 – 50% of the first cell thickness (Maxwell, 2014). The prism layers on the wing and the thickness of the prism layers with respect to the local mesh size for this investigation can be seen in Figure 6.3.

### 6.3.5 Mesh Refinement and Independence Analysis

Initially, a coarse grid consisting of $2 \times 10^6$ cells was created to determine the vortex trajectory and core location. The steady simulation was run for 1000 iterations to become fully converged, before the vortex trajectory and core location were determined. The vortex core was visualised in STAR-CCM+ using the parallel vectors operator to implement the Eigenvector method for vortex core extraction proposed by Roth and Peikert (2006). They indicated that a rotational motion is present when the Jacobian (velocity gradient matrix) has a conjugate complex pair of Eigenvalues. A volumetric control was then created around the vortex after the trajectory of the core was extracted (see Figure 6.4a). After the volumetric control was created a number of simulations were run for each turbulence modelling approach to determine mesh independence. Mesh independence
was assessed separately for the RANS/URANS and LES simulations. Five different mesh densities were assessed for the RANS/URANS simulations using the Reynolds stress model for $\alpha = 5^\circ$. Turbulent and mean flow characteristics in the vortex core at $x/c = 0$ were examined for convergence. As seen in Table 6.1, the finest mesh produced minimal change in the mean and turbulent characteristics when compared to the second finest mesh, and it was decided, based on computational resources available, that the second finest mesh of $5.7 \times 10^6$ cells would be used for the RANS and URANS simulations.
Table 6.1: RANS mesh configurations (α = 5°)

<table>
<thead>
<tr>
<th>Mesh No.</th>
<th>Number of cells</th>
<th>Number of cells across core</th>
<th>Axial velocity deficit $x/c = 0$</th>
<th>Max. vertical velocity component $w (x/c = 0)$</th>
<th>Max. Reynolds stress $u'v'$ $(x/c = 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2 \times 10^6$</td>
<td>8</td>
<td>$0.84U_\infty$</td>
<td>$0.23U_\infty$</td>
<td>0.00012</td>
</tr>
<tr>
<td>2</td>
<td>$3.2 \times 10^6$</td>
<td>9</td>
<td>$0.83U_\infty$</td>
<td>$0.23U_\infty$</td>
<td>0.00016</td>
</tr>
<tr>
<td>3</td>
<td>$4.6 \times 10^6$</td>
<td>10</td>
<td>$0.82U_\infty$</td>
<td>$0.24U_\infty$</td>
<td>0.00025</td>
</tr>
<tr>
<td>4</td>
<td>$5.7 \times 10^6$</td>
<td>11</td>
<td>$0.76U_\infty$</td>
<td>$0.22U_\infty$</td>
<td>0.00025</td>
</tr>
<tr>
<td>5</td>
<td>$7.6 \times 10^6$</td>
<td>13</td>
<td>$0.78U_\infty$</td>
<td>$0.21U_\infty$</td>
<td>0.00035</td>
</tr>
</tbody>
</table>

Three different mesh densities and two time-steps were simulated for $\alpha = 10^\circ$ to assess convergence for the LES modelling. Similar to the RANS analysis, both turbulent and mean flow characteristics in the vortex core at $x/c = 0$ were examined for convergence. Looking at Table 6.2, it is clear that the increase in mesh density from the most coarse mesh of $2 \times 10^6$ cells to the second finest mesh of $3.3 \times 10^6$ cells produced the most significant change in the characteristics examined, specifically the $u'$ rms and the axial velocity excess. Decreasing the time-step size from 0.002s to 0.001s for the mesh size of $3.3 \times 10^6$ cells and increasing the mesh size from $3.3 \times 10^6$ cells to $6.2 \times 10^6$ cells produced minimal change in the vortex core characteristics. It was decided that the most dense mesh of $6.2 \times 10^6$ cells and a time-step of 0.001s would be used for the LES simulations.

The volumetric control contained over 70% of the total cell count and the chosen mesh sizes for the RANS/URANS and LES simulations contained 11 cells across the estimated vortex core diameter. The mesh density in the volumetric control for the RANS/URANS mesh of $5.7 \times 10^6$ can be seen in Figure 6.4b.


6. NUMERICAL MODELLING OF A WING-TIP VORTEX

Table 6.2: LES mesh and time-step configurations ($\alpha = 10^\circ$)

<table>
<thead>
<tr>
<th>Mesh Config.</th>
<th>Number of cells</th>
<th>Time-step (s)</th>
<th>Axial velocity excess ($x/c = 0$)</th>
<th>Max. crossflow velocity magnitude ($x/c = 0$)</th>
<th>Max. $u'$ rms $\sqrt{\bar{u}'^2}$ ($x/c = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2 \times 10^6$</td>
<td>0.002</td>
<td>$1.13U_\infty$</td>
<td>$0.61U_\infty$</td>
<td>0.32</td>
</tr>
<tr>
<td>2</td>
<td>$3.3 \times 10^6$</td>
<td>0.002</td>
<td>$1.17U_\infty$</td>
<td>$0.61U_\infty$</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>$3.3 \times 10^6$</td>
<td>0.001</td>
<td>$1.16U_\infty$</td>
<td>$0.60U_\infty$</td>
<td>0.26</td>
</tr>
<tr>
<td>4</td>
<td>$6.2 \times 10^6$</td>
<td>0.001</td>
<td>$1.15U_\infty$</td>
<td>$0.61U_\infty$</td>
<td>0.24</td>
</tr>
</tbody>
</table>

6.4 Turbulence Modelling

6.4.1 RANS/URANS

The RANS and URANS simulations were run with a segregated flow solver, which solved the flow equations (one for each component of velocity and one for pressure) in a segregated or uncoupled manner. The segregated flow model was suited to constant density incompressible flows and used a second order discretization scheme for the URANS simulation. The segregated flow model uses a Rhie-and-Chow-type pressure velocity coupling combined with a SIMPLE-type algorithm. A full Reynolds stress turbulence model was utilised for closure of the URANS equations as it has been shown to predict the lag of the Reynolds stress behind the mean strain rate (Churchfield and Blaisdell, 2009), which is one the shortcomings of even advanced eddy viscosity models such as the $k$-$\omega$-SST Menter. The inherent weaknesses of eddy viscosity based models at modelling complex rotational and swirling flows is well documented (Czech et al., 2005; Dacles-Mariani et al., 1995; Kim and Rhee, 2005). Attempts have been made to escape from these weaknesses by adding correction terms for rotation and streamline curvature (Czech et al., 2005; Spalart and Shur, 1997), and these rotation corrected models have been shown to predict the mean flow quite accurately (Churchfield and Blaisdell, 2009). Unfortunately, none of these rotation corrected eddy viscosity models were available in the STAR-CCM+ flow solver package. The
6.4 Turbulence Modelling

(a) Volumetric control geometry

(b) Mesh density in volumetric control for mesh size of $5.7 \times 10^6$ cells

Figure 6.4: Mesh refinement using volumetric control

pressure-strain term (Equation 3.41) of the Reynolds stress model was modelled using the linear-pressure strain approach of Gibson and Launder (1978). By solving for all the components of the Reynolds stress tensor, this model naturally accounts for effects such as anisotropy due to swirling motion, streamline curvature and rapid changes in strain rate. The model solves equations for the six unique Reynolds stress components and one for the isotropic turbulent dissipation $\varepsilon$ resulting in the solution of seven equations. It is also more capable at predicting turbulence anisotropy when compared to eddy viscosity models like the $k$-$\omega$-SST Menter. Time was non-dimensionalised by multiplication of the freestream veloc-
ity \((U_\infty = 34m/s)\) divided by the chord length \((c = 0.14m)\). A non-dimensional time-step \((t^*)\) of 0.48 was chosen for the URANS simulations, which was 50% of the maximum allowable time-step of 0.97. The maximum allowable time-step is known as the large eddy turnover time or integral time scale and was estimated from Tropea (2002). Time-step convergence was judged after 20 iterations (see Figure 6.5) and the flow solutions for both angles of attack were run for over 20 flow through times (FTT). Field functions recorded the mean data after 10 FTT when the flow was judged to be steady.

![Figure 6.5: Convergence of inner iterations for a single time-step of 0.48sU_\infty/c for the URANS with VC case for \(\alpha = 10^\circ\)](image)

**6.4.2 Vorticity Confinement**

The rapid dissipation of vortices has been a significant challenge for CFD modellers (Churchfield and Blaisdell, 2009, 2013; Craft et al., 2006; Dacles-Mariani et al., 1995; Kim and Rhee, 2005). The large number of grid points needed to resolve a vortex downstream of the generating wing often limits the numerical simulation of a wing-tip vortex to just a few chord lengths downstream. In order to avoid the rapid dissipation of vortices, Steinhoff and his co-workers introduced the concept of vorticity confinement (Hu et al., 2000; Kimbrell, 2012; Lynn, 2007;
6.5 Large Eddy Simulation

Steinhoff et al., 2005), which was described previously in Section 2.2. The vorticity confinement model refined by Löhner (2009) for use on unstructured grids is the model implemented in STAR-CCM+ for the URANS and LES cases in this investigation.

6.5 Large Eddy Simulation

The Large Eddy Simulation model utilized a segregated flow solver with a bounded central convection scheme. Implicit unsteady time discretization was chosen so that a CFL (Courant-Friedrichs-Lewy) number greater than one could be used, which allowed for a coarser mesh and larger time steps to be used in the simulation. The filtered Navier-Stokes equations were closed with the Wall Adapting Local Eddy-viscosity (WALE) Subgrid Scale Model from Nicoud and Ducros (1999) as it has a number of advantages compared to the Smagorinsky Subgrid Scale model. The WALE Subgrid Scale Model detects all the relevant structures for the kinetic energy dissipation as the spatial operator consists of a mixing of both the local strain and rotation rates. In addition, the eddy-viscosity naturally goes to zero in the vicinity of a wall so that neither constant adjustment nor a damping function are needed to compute wall bounded flows (Nicoud and Ducros, 1999). A non-dimensional time-step of 0.24 was chosen for the LES simulations based on the computer resource limitations and the large eddy turnover time-step of 0.97.

6.6 Mean Velocity Results

The numerical mean velocities were extracted at four downstream locations ($x/c = 0, 1, 2$ and $3$) at a Reynolds number of $3.25 \times 10^5$ for two angles of attack ($5^\circ$ and $10^\circ$) using three different turbulence modelling approaches (RANS, URANS and LES). The contours of the mean velocities for each case are given and discussed first, followed by a section on experimental validation, where the velocities taken on a line through the vortex core is compared to experimentally obtained measurements.
6. NUMERICAL MODELLING OF A WING-TIP VORTEX

6.6.1 Axial Velocity

RANS Reynolds Stress Model

The axial velocity contours obtained from the RANS Reynolds stress model for \( \alpha = 5^\circ \) are shown in Figure 6.6. There is no axisymmetric axial velocity vortex structure apparent at \( x/c = 0 \) and the axial velocity contour is dominated by the thin momentum deficit shear layer behind the wing trailing edge, which had a maximum axial velocity deficit of \( 0.27U_\infty \). Similarly, no axisymmetric axial velocity vortex structure was observed at \( x/c = 0 \) for the experimental case as seen in Figure 6.6a. However, the crossflow velocity vectors show the rotational motion of a vortex structure and the vortex core centre appears to occur at an inboard distance of \( 0.016c \). The RANS model predicted the roll up of the vortex quickly as the axial velocity at \( x/c = 1 \) exhibited an axisymmetric vortex structure with a maximum axial velocity deficit of \( 0.86U_\infty \). Entrainment of the separated shear layer fluid is still evident suggesting the roll up is not yet complete. The vortex has moved inboard a distance of \( 0.058c \) at \( x/c = 1 \). At \( x/c = 2 \), the vortex was almost completely axisymmetric and the maximum axial velocity deficit increased to \( 0.88U_\infty \) in the vortex core centre. The vortex had moved inboard a distance of \( 0.072c \) and the addition of shear layer fluid was almost complete. At the last measurement location of \( x/c = 3 \) the axial velocity deficit in the vortex core centre decreased to \( 0.82U_\infty \), perhaps due to the addition of momentum deficit shear layer fluid. The vortex had moved inboard a distance of \( 0.086c \) and the velocity vectors and axial velocity structure were completely axisymmetric.

The axial velocity contours predicted by the RANS model for \( \alpha = 10^\circ \) are shown in Figure 6.7. Similar to the \( \alpha = 5^\circ \) case, there was no axial velocity vortex structure observed at \( x/c = 0 \). The crossflow velocity vectors appeared to show the vortex core centre at an inboard distance of \( 0.028c \). The majority of the axial velocity contour was dominated by the momentum deficit shear layer just behind the trailing edge, where it had a maximum axial velocity deficit of \( 0.54U_\infty \). The presence of an axial velocity deficit of \( 0.95U_\infty \) in the vortex core centre at \( x/c = 1 \) was in sharp contrast to the experimentally measured axial velocity excess in the core for \( \alpha = 10^\circ \) at all downstream locations. The axial velocity structure appeared to be in the process of rollup at \( x/c = 1 \) with a large
region of axial velocity deficit being entrained into the vortex. A small region of axial velocity excess below the wing was also observed with a maximum velocity excess of $1.03U_\infty$. The crossflow velocity vectors showed the vortex core centre to occur at an inboard distance of $0.072c$. At $x/c = 2$, the axial velocity deficit in the vortex core centre decreased to $0.91U_\infty$ and the axial velocity structure became axisymmetric. The decrease in axial velocity deficit is thought to be due to a deficiency in the RANS modelling of the vortex rollup where the small region of the axial velocity excess is not predicted. The vortex had progressed inboard by a distance of $0.086c$ at $x/c = 2$. The axial velocity deficit in the vortex core centre decreased dramatically at $x/c = 3$ to a value of $0.62U_\infty$. The vortex also became
more diffuse and the crossflow velocity vectors appeared to decrease significantly implying that the RANS predicted vortex had lost much of its rotational motion by $x/c = 3$.

![Contour plots](image_url)

**Figure 6.7:** Contours of normalized axial velocity $U_x/U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (RANS model)

### URANS Reynolds Stress Model with Vorticity Confinement

The axial velocity contours predicted by the URANS model with VC for $\alpha = 5^\circ$ are shown in Figure 6.8. Much like the RANS predicted axial velocity, the only major feature present in the axial velocity structure at $x/c = 0$ was the thin region of momentum deficit shear layer fluid behind the trailing edge, which had
Mean Velocity Results

a maximum deficit of $0.65U_\infty$. The axial velocity structure at $x/c = 1$ suggested the vortex was still undergoing roll up but the axial velocity structure did attain more axisymmetry. The maximum axial velocity deficit in the vortex core centre was $0.95U_\infty$ at an inboard distance of $0.058c$. At $x/c = 2$, the axial velocity structure was asymmetric and the velocity deficit in the vortex core centre remained at $0.95U_\infty$ at an inboard distance of $0.086c$. Similar to the RANS case at $x/c = 2$, the vortex still appeared to be capturing some wing shear layer fluid. At the last measurement location $x/c = 3$, the maximum axial velocity deficit in the vortex core centre remained at $0.95U_\infty$, which was in contrast to the RANS case for $\alpha = 5^\circ$ where the axial velocity deficit decreased at the last measurement location.

The axial velocity contours predicted by the URANS model with VC for $\alpha = 10^\circ$ are shown in Figure 6.9. The axial velocity at $x/c = 0$ is again much the same as the RANS case with the thin shear layer region having a maximum axial velocity deficit of $0.66U_\infty$. The axial velocity structure at $x/c = 1$ exhibited a region of axial velocity deficit on the inboard periphery of the vortex core and a region of axial velocity excess emanating from below the trailing edge of the wing. The maximum axial velocity excess below the wing was $1.03U_\infty$ and the maximum axial velocity deficit at the periphery of the vortex core was $0.95U_\infty$. The vortex core centre had an axial velocity value of $1.0U_\infty$. At $x/c = 2$, the region of axial velocity deficit had shifted into the core region of the vortex and the outboard periphery of the vortex core was surrounded by the region of axial velocity excess. The maximum axial velocity excess had decreased to $1.01U_\infty$ and the maximum axial velocity deficit had increased to $0.97U_\infty$. The axial velocity deficit in the vortex core centre was $0.98U_\infty$. The axial velocity structure did not exhibit a lot of axisymmetry at $x/c = 2$, which was in contrast to the RANS case for $\alpha = 10^\circ$ at the same location. The URANS vortex core centre had also moved inboard a distance of $0.086c$ at $x/c = 2$. A more axisymmetric axial velocity structure was evident at $x/c = 3$ with the axial velocity reaching a minimum in the centre of the vortex and gradually increasing towards the outer periphery. The maximum axial velocity deficit in the core of the vortex decreased slightly to $0.97U_\infty$ at $x/c = 3$, which is in sharp contrast to the large decline in axial velocity for the RANS case for $\alpha = 10^\circ$ at the same location. The large discrepancy in
core axial velocity deficit is thought to be attributed to the vorticity confinement model preserving the vortex and preventing excessive downstream diffusion for the URANS case. The crossflow velocity vectors also appeared larger than the RANS case at $x/c = 3$, which shows the vorticity confinement model enabled the vortex to maintain its circular motion for a greater distance downstream of the wing.
6.6 Mean Velocity Results

Figure 6.9: Contours of normalized axial velocity $U_x/U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (URANS model with VC)

LES and Wales SGS Model with Vorticity Confinement

The axial velocity contours predicted by the LES with VC are shown in Figure 6.10. Similar to the previous RANS and URANS simulations, no clear axisymmetric vortex structure is evident in the axial velocity for $x/c = 0$. However, in contrast, the shear layer inboard of the wing-tip behind the trailing edge is observed to approach stagnation or zero velocity. The vortex core centre had an axial velocity deficit of $0.64U_\infty$ at the wing-tip. Further downstream at $x/c = 1$, 

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the axial velocity structure reveals a vortex undergoing roll up. A deficit region with a maximum velocity deficit of $0.85U_\infty$ formed in the vortex core and a further deficit containing region with a maximum velocity deficit of $0.89U_\infty$ surrounded the vortex periphery. The maximum axial velocity deficit in the vortex core centre increased to $0.91U_\infty$ at an inboard distance of $0.044c$. The axial velocity structure gained more axisymmetry at $x/c = 2$ with the velocity deficit region now fully entrained in the core of the vortex. The vortex core was also surrounded by two patches of momentum deficit shear layer fluid below the trailing edge of the wing. The maximum axial velocity deficit in the two patches outside the core and the vortex core centre was $0.92U_\infty$ and $0.91U_\infty$ respectively. The vortex core had moved inboard a distance of $0.058c$ by $x/c = 2$ and was still capturing momentum deficit shear layer fluid. The patches of velocity deficit fluid coalesced with the main vortex core structure at $x/c = 3$ and the axisymmetry of the vortex core was disrupted as it merged with the surrounding velocity deficit fluid. Three small patches of momentum deficit shear layer fluid can also be seen inboard of the main vortex. This is in sharp contrast to the RANS and URANS simulations at $x/c = 3$, both of which had axisymmetric structures with minimal addition of shear layer fluid. The maximum velocity deficit in the vortex core centre increased to $0.94U_\infty$ at a distance of $0.072U_\infty$.

The LES with VC was the only numerical model to correctly predict the experimentally measured jet-like axial velocity excess in the core of the vortex for $\alpha = 10^\circ$. On inspection of Figure 6.11 an axial velocity structure is evident at $x/c = 0$ with an axial velocity excess region in surrounding the vortex core centre. The vortex core centre had a maximum axial velocity excess of $1.15U_\infty$ at an inboard distance of $0.03c$. The LES predicted axial velocity structure at $x/c = 0$, closely resembles the axial velocity structure of the experimental case at the same location (see Figure 5.4a). Further downstream at $x/c = 1$, the vortex structure comprises of an axial velocity excess region in the core of the vortex surrounded by three patches of velocity deficit shear layer fluid, which is in close agreement with the experimental case seen in Figure 5.4b. The maximum axial velocity excess in the vortex core centre reduced slightly to $1.13U_\infty$ at an inboard distance of $0.058c$ and the axial velocity deficit in the three patches surrounding the vortex core varied between $0.91U_\infty$ and $0.96U_\infty$. The axial velocity excess
6.6 Mean Velocity Results

Figure 6.10: Contours of normalized axial velocity $U_x/U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (LES with VC)

in the core centre decreased further at $x/c = 2$ to $1.1U_\infty$ with a single region of axial velocity deficit on the upper periphery of the vortex core having a maximum velocity deficit of $0.91U_\infty$. The axial velocity excess in the vortex core centre increased to $1.12U_\infty$ at $x/c = 3$ at a distance of $0.086c$. The axial velocity deficit region remained at the vortex core periphery with a maximum axial velocity deficit of $0.92U_\infty$. The region of axial velocity deficit also being observed for the experimental case at $x/c = 3$. 
6. NUMERICAL MODELLING OF A WING-TIP VORTEX

Figure 6.11: Contours of normalized axial velocity $U_x/U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (LES with VC)

Experimental Validation

The axial velocity for $\alpha = 5^\circ$ exhibited a wake like profile for the numerical and experimental cases at all downstream locations as shown in Figure 6.12. At $x/c = 0$, the LES predicted axial velocity deficit of $0.73U_\infty$ was in best agreement with the experimentally measured deficit of $0.67U_\infty$ in the vortex core centre, whereas the RANS and URANS simulations predicted axial velocity deficits in the core centre of $0.7U_\infty$ and $0.76U_\infty$. The maximum axial velocity deficit in the vortex core centre increased for all cases at $x/c = 1$ with the RANS and URANS values increasing to $0.85U_\infty$ and $0.91U_\infty$ and LES and experimental
values increasing to $0.91U_\infty$ and $0.78U_\infty$ respectively. At $x/c = 2$, the RANS and URANS predicted axial velocity deficits increased to $0.87U_\infty$ and $0.92U_\infty$, whereas the LES axial velocity deficit remained constant at $0.91U_\infty$. The experimental velocity deficit in the core centre increased to $0.83U_\infty$, meaning that the RANS predicted core centre velocity deficit was in best agreement at $x/c = 2$. At $x/c = 3$ the RANS predicted core centre velocity deficit decreased to $0.82U_\infty$, which meant that it was in best agreement with the slightly decreased experimental value of $0.81U_\infty$. The URANS and LES predicted core centre velocity deficits increased to $0.93U_\infty$ and $0.94U_\infty$.

Figure 6.12: Normalized axial velocity $U_x/U_\infty$ across the vortex core for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$
The axial velocity profiles for $\alpha = 10^\circ$ are shown in Figure 6.13. At $x/c = 0$, the LES predicted core centre axial velocity excess of $1.17U_\infty$ was in best agreement with the experimental core centre axial velocity excess of $1.13U_\infty$, whereas the RANS and URANS simulations predicted a core centre axial velocity deficit of $0.98U_\infty$. The LES predicted core axial velocity decreased to $1.09U_\infty$ at $x/c = 1$ and was again in best agreement with the reduced experimental core axial velocity of $1.06U_\infty$. The RANS axial velocity decreased to $0.95U_\infty$ at $x/c = 1$, whereas the URANS core axial velocity increased to $1.0U_\infty$. The LES core axial velocity excess increased slightly to $1.11U_\infty$ at $x/c = 2$, whereas the URANS core axial velocity remained at $0.98U_\infty$ and the RANS predicted core velocity decreased to $0.92U_\infty$. The experimental core centre axial velocity at $x/c = 2$ was $1.05U_\infty$. At the last measurement location of $x/c = 3$, the core axial velocity excess remained constant for the LES and URANS simulations, whereas the RANS predicted core centre velocity decreased significantly to $0.62U_\infty$. There was a slight increase in the experimental core centre velocity excess as it rose to $1.06U_\infty$, with the LES predicted core centre velocity again the closest.

6.6.2 Crossflow Velocity

RANS Reynolds Stress Model

The contours of crossflow velocity magnitude for $\alpha = 5^\circ$ are shown in Figure 6.14. At $x/c = 0$, the maximum crossflow velocity of $0.28U_\infty$ occurs in a region of high crossflow velocity on the upper boundary of the vortex core. The crossflow velocity structure was non-axisymmetric, which is in agreement with the experimental crossflow velocity magnitude and is a consequence of the vortex undergoing roll up. The vortex centre was observed to be slightly inboard of the wing tip at a distance of $0.016c$. The maximum crossflow velocity magnitude decreased to $0.23U_\infty$ at $x/c = 1$ and the region of high crossflow velocity had now shifted to the outboard periphery of the vortex core. The crossflow velocity structure did become more axisymmetric at $x/c = 1$ and the vortex centre had now moved inboard a distance of $0.058c$. The maximum crossflow velocity magnitude decreased further at $x/c = 2$ to $0.21U_\infty$ and the region of high crossflow velocity was now observed at the top of the vortex core like at $x/c = 0$. The maximum crossflow
velocity magnitude decreased significantly at $x/c = 3$ to $0.17U_\infty$ and the region of high crossflow velocity grew substantially. Therefore, the vortex core diameter increased, which is thought to be due the insufficient grid resolution in the vortex core and the second-order numerical convection scheme used. It is proposed that the vortex will rapidly diffuse after $x/c = 3$ when using this modelling approach.

The contours of crossflow velocity magnitude for $\alpha = 10^\circ$ are shown in Figure 6.15. At first glance, it is clear that the crossflow velocity vortex structure is much larger than that for $\alpha = 5^\circ$, which is similar to the experimental crossflow
Figure 6.14: Normalized crossflow velocity magnitude $U_c/U_\infty$ for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (RANS model)

velocity. At $x/c = 0$, the maximum crossflow velocity of $0.48U_\infty$ occurred at the upper boundary of the vortex core. The crossflow velocity structure was more axisymmetric than for $\alpha = 5^\circ$ but still exhibited the skewed vortex shape characteristic to early roll up. The vortex core centre occurred inboard of the wing tip at a distance of $0.03c$. The maximum crossflow velocity magnitude decreased to $0.42U_\infty$ at $x/c = 1$ and the region of high crossflow velocity now surrounded a significant portion of the vortex core region. The overall crossflow velocity structure also became more axisymmetric at $x/c = 1$. The maximum crossflow velocity magnitude decreased further at $x/c = 2$ to $0.39U_\infty$ and the crossflow velocity structure has now axisymmetric. In a similar manner to the crossflow velocity
6.6 Mean Velocity Results

for \( \alpha = 5^\circ \), the maximum crossflow velocity magnitude decreased significantly at \( x/c = 3 \) to 0.29\( U_\infty \) and as a result the vortex diameter expanded and became more diffuse.

\[ \text{Figure 6.15: Normalized crossflow velocity magnitude } U_c/U_\infty \text{ for } \alpha = 10^\circ \text{ and } Re = 3.25 \times 10^5 \text{ (RANS model)} \]

\textbf{URANS Reynolds Stress Model with Vorticity Confinement}

The contours of crossflow velocity magnitude for \( \alpha = 5^\circ \) are shown in Figure 6.16. The crossflow velocity vortex structure is much smaller at all downstream locations than those for the RANS model. At \( x/c = 0 \), the maximum crossflow velocity of 0.27\( U_\infty \) occurred in a region of high crossflow velocity on the upper
boundary of the vortex core in a similar way to the RANS predicted crossflow velocity at the same location. Following the same trend as the RANS model, the crossflow velocity structure at $x/c = 0$ was nonaxisymmetric with the vortex core centre observed to be slightly inboard of the wing tip at a distance of $0.016c$. The maximum crossflow velocity magnitude decreased to $0.24U_\infty$ at $x/c = 1$ and the region of high crossflow velocity had now split into two smaller regions at the inboard and outboard periphery of the vortex core. The maximum crossflow velocity magnitude decreased further at $x/c = 2$ to $0.22U_\infty$ and the region of high crossflow velocity was now observed at the top of the vortex core similar to the RANS model at the same location. In contrast to the findings of the RANS model was the slight decrease in maximum crossflow velocity to $0.21U_\infty$ at $x/c = 3$. There was also no large increase in vortex core diameter at $x/c = 3$. The vorticity confinement model is thought to be responsible for conserving the crossflow velocity at $x/c = 3$ and preventing the vortex core from becoming less tightly wound and more diffuse.

The contours of crossflow velocity magnitude for $\alpha = 10^\circ$ are shown in Figure 6.17. Similar to the RANS and experimental cases, the URANS crossflow velocity vortex structure was observed to increase in size for $\alpha = 10^\circ$. At $x/c = 0$, the maximum crossflow velocity of $0.48U_\infty$ occurred in a region of high crossflow velocity on the upper boundary of the vortex core. The vortex was in its nonaxisymmetric state during roll up and the vortex core centre was at an inboard distance of $0.03c$. The maximum crossflow velocity magnitude decreased to $0.44U_\infty$ at $x/c = 1$ and the region of high crossflow velocity had now split into two smaller regions at the inboard and outboard periphery of the vortex core in the same way as the $\alpha = 5^\circ$ case at the same location. The maximum crossflow velocity magnitude decreased further at $x/c = 2$ to $0.41U_\infty$ and the region of high crossflow velocity now surrounded a large portion of the vortex core. There was no decrease in the maximum crossflow velocity magnitude or increase in vortex core diameter at $x/c = 3$, which was attributed to the vorticity confinement model preventing the excessive diffusion of the vortex.
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Figure 6.16: Normalized crossflow velocity magnitude $U_c/U_\infty$ for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (URANS model with VC)

LES and Wales SGS Model with Vorticity Confinement

The contours of crossflow velocity magnitude for $\alpha = 5^\circ$ are shown in Figure 6.18. The crossflow velocity vortex structure is seen to be much smaller than the RANS and URANS cases at all downstream locations. At $x/c = 0$, similar to the RANS and URANS cases, the maximum crossflow velocity of $0.34U_\infty$ occurred in a region of high crossflow velocity on the upper boundary of the vortex core. The crossflow velocity structure at $x/c = 0$ was non-axisymmetric with the vortex core centre occurring at the wing tip. The maximum crossflow velocity magnitude decreased to $0.30U_\infty$ at $x/c = 1$ and the region of high crossflow velocity had now
6. NUMERICAL MODELLING OF A WING-TIP VORTEX

![Figure 6.17: Normalized crossflow velocity magnitude $U_c/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (URANS model with VC)](image)

Figure 6.17: Normalized crossflow velocity magnitude $U_c/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (URANS model with VC)

The vortex core region was also observed to decrease in size at $x/c = 3$, which is thought to be caused by the increase in crossflow velocity causing the vortex to become more tightly wound.
6.6 Mean Velocity Results

Figure 6.18: Normalized crossflow velocity magnitude $U_c/U_\infty$ for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (LES with VC)

The contours of crossflow velocity magnitude for $\alpha = 10^\circ$ are shown in Figure 6.19. Similar to $\alpha = 5^\circ$, the crossflow velocity vortex structure is smaller than the RANS and URANS cases at all downstream locations. At $x/c = 0$, the maximum crossflow velocity of $0.70U_\infty$ occurred in the high crossflow velocity region on the inboard portion of the vortex core. The vortex core centre was located at an inboard distance of $0.03c$. The maximum crossflow velocity magnitude decreased to $0.60U_\infty$ at $x/c = 1$ and the region of high crossflow velocity had now split into three smaller regions at the upper, inboard and outboard periphery of the vortex core. The maximum crossflow velocity magnitude increased to $0.67U_\infty$ at $x/c = 2$ in just a single region of high crossflow velocity on the upper periphery of
the vortex core. The increase in crossflow velocity magnitude caused the vortex core diameter to decrease as happened at $x/c = 3$ for $\alpha = 5^\circ$. The maximum crossflow velocity remained at $0.67U_\infty$ to $x/c = 3$ but occurred in the outboard portion of the vortex with three regions of high crossflow velocity now evident as was the case at $x/c = 1$.

![Normalized crossflow velocity magnitude](image)

**Figure 6.19:** Normalized crossflow velocity magnitude $U_c/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (LES with VC)

**Experimental Validation**

The crossflow velocity magnitude exhibited a characteristic v-shape pattern across the vortex core for both $\alpha = 5^\circ$ and $\alpha = 10^\circ$. The crossflow velocity magnitude
6.6 Mean Velocity Results

taken on a line across the vortex core for $\alpha = 5^\circ$ is seen in Figure 6.20. The maximum crossflow velocity magnitude recorded at $x/c = 0$ for the RANS and URANS simulations was $0.24U_\infty$, whereas the LES and experimental maximum crossflow velocities were greater at $0.32U_\infty$ and $0.27U_\infty$. The RANS maximum crossflow velocity decreased slightly to $0.23U_\infty$ at $x/c = 1$, whereas the URANS maximum crossflow velocity remained constant at $0.24U_\infty$. The LES and experimental maximum crossflow velocities decreased to $0.25U_\infty$ and $0.22U_\infty$ at $x/c = 1$. At $x/c = 2$, the RANS and URANS maximum crossflow velocities both decreased to $0.18U_\infty$ and $0.19U_\infty$, whereas the LES and experimental maximum crossflow velocities increased significantly to $0.30U_\infty$ and $0.27U_\infty$ respectively. The RANS maximum crossflow velocity decreased again at $x/c = 3$ to $0.16U_\infty$, whereas the URANS maximum crossflow velocity increased to $0.20U_\infty$. The LES maximum crossflow velocity increased to $0.32U_\infty$ and the experimental maximum crossflow velocity decreased again to $0.24U_\infty$.

The crossflow velocity magnitude taken on a line across the vortex core for $\alpha = 10^\circ$ is seen in Figure 6.21. At $x/c = 0$, the maximum crossflow velocity magnitude for the RANS and URANS simulations was $0.46U_\infty$ and the LES maximum crossflow velocity of $0.61U_\infty$ was in best agreement with the experimental maximum crossflow velocity of $0.54U_\infty$. The RANS and URANS maximum crossflow velocities decreased slightly at $x/c = 1$ to $0.41U_\infty$ and $0.43U_\infty$ respectively. The LES maximum crossflow velocity decreased to $0.57U_\infty$ and was in best agreement with the experimental maximum of $0.54U_\infty$, which had remained constant. The v-shaped crossflow velocity profile across the vortex core exhibited a high degree of symmetry at $x/c = 1$ (see Figure 6.21b). At $x/c = 2$, the RANS and URANS maximum crossflow velocities decreased to $0.38U_\infty$ and $0.41U_\infty$, whereas the LES maximum crossflow velocity increased to $0.60U_\infty$ and the experimental maximum crossflow velocity again remained at a constant value of $0.54U_\infty$. The profile of the LES crossflow velocity across the vortex core at $x/c = 2$ agreed extremely well with the experimental crossflow profile (see Figure 6.21c). It had the same vortex core diameter and inboard centre location with a similar velocity gradient towards the outer edges of the core. The RANS maximum crossflow velocity decreased significantly at $x/c = 3$ to $0.24U_\infty$ (similar to the axial velocity for $\alpha = 10^\circ$ at the same location). The LES maximum crossflow velocity increased
Figure 6.20: Normalized crossflow velocity $U_c/U_\infty$ across the vortex core for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ to $0.66U_\infty$ at $x/c = 3$, whereas the maximum URANS and experimental crossflow velocities stayed constant at $0.41U_\infty$ and $0.54U_\infty$. The increase in the LES crossflow velocity could be attributed to the entrainment of more circulatory fluid into the core of the vortex as it progresses downstream. The capturing of wing-shed vorticity downstream of the wing has the ability to maintain the vortex strength and keep the velocity relatively constant within a few chord lengths downstream.
6.6 Mean Velocity Results

Figure 6.21: Normalized crossflow velocity $U_c/U_\infty$ across the vortex core for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$.

6.6.3 Vortex Circulation

The tip vortex circulation was calculated using Equation 5.1 given previously in section 5.1.3. The numerical and experimental circulation parameters for $\alpha = 5^\circ$ and $10^\circ$ are shown in Figure 6.22. For $\alpha = 5^\circ$, the RANS, URANS and LES circulation parameters were 0.105, 0.109 and 0.084 at $x/c = 0$ respectively. The RANS, URANS and LES circulation parameters decreased to 0.100 and 0.101 and 0.077 at $x/c = 1$. The RANS circulation parameter decreased again to 0.096...
at $x/c = 2$ before increasing to a maximum of 0.104 at $x/c = 3$. The URANS circulation parameter increased to 0.105 at $x/c = 2$ and reached a maximum value of 0.108 at $x/c = 3$. Furthermore, the LES circulation remained at 0.077 at $x/c = 2$ and then increased slightly to 0.079 at $x/c = 3$. The experimental circulation parameter was at a minimum of 0.065 at $x/c = 0$ after which it increased to 0.086 at $x/c = 1$ before reaching its maximum of 0.09 at $x/c = 3$. The LES circulation was in best agreement with the experimental circulation for $\alpha = 5^\circ$ at all downstream locations. There was a large increase in circulation for $\alpha = 10^\circ$, with values of 0.230, 0.235 and 0.214 calculated for the RANS, URANS and LES cases at $x/c = 0$. The experimental circulation parameter of 0.243 at $x/c = 0$, meant the URANS circulation parameter was in best agreement. The RANS circulation decreased to 0.225 at $x/c = 1$ and 0.224 at $x/c = 2$ before decreasing to 0.216 at the last measurement location of $x/c = 3$. In contrast, the URANS circulation decreased to 0.224 at $x/c = 1$ but increased to 0.230 at $x/c = 2$ and 0.233 at $x/c = 3$. The LES circulation followed a similar trend decreasing to 0.211 at $x/c = 1$, remaining at 0.211 to $x/c = 2$ and then increasing to 0.216 at $x/c = 3$. The increase in circulation is attributed to the fact that the vortex roll up was not complete and there was a slow addition of vorticity from the shear layer arriving from the inboard regions of the wing. The experimental vortex circulation increased slightly to 0.244 $x/c = 1$ before reaching a maximum of 0.266 at $x/c = 2$ and decreasing to 0.252 at $x/c = 3$. The URANS circulation parameter was in closest agreement with the experimental circulation parameter for $\alpha = 10^\circ$.

6.6.4 Mean Streamwise Vorticity

RANS Reynolds Stress Model

The mean streamwise vorticity contours for $\alpha = 5^\circ$ are shown in Figure 6.23. At $x/c = 0$, a maximum negative vorticity of $-0.032$ occurred at an inboard distance of 0.016$c$ at the point of minimum crossflow velocity. Further downstream at $x/c = 1$, the maximum vorticity decreased to $-0.028$ and had moved inboard a distance of 0.058$c$. The maximum vorticity decreased further to $-0.024$ at $x/c = 2$ and $-0.012$ at $x/c = 3$ at which point it had progressed inboard a
6.6 Mean Velocity Results

Figure 6.22: Variation of normalized vortex strength $\Gamma/U_\infty c$ with downstream distance

The mean streamwise vorticity contours for $\alpha = 10^\circ$ are shown in Figure 6.24. The size of the vorticity structure is much larger than for $\alpha = 5^\circ$ despite the fact that the vorticity magnitudes are not much greater. At $x/c = 0$, a maximum negative vorticity of $-0.037$ occurred at an inboard distance of $0.03c$ just above the point of minimum crossflow velocity. In contrast to the vorticity for $\alpha = 5^\circ$, the maximum vorticity increased to $-0.041$ at $x/c = 1$ and had moved inboard a distance of $0.072c$. The maximum vorticity returned to a value of $-0.037$ at $x/c = 2$ but dramatically decreased at $x/c = 3$ to a value of $-0.012$ (similar to $\alpha = 5^\circ$). The diffusion of vorticity also lead to a large increase in the vortex core diameter at $x/c = 3$, which was similar to the large increase in axial velocity and crossflow velocity seen previously at the same location. The location of the maximum vorticity coincided with the point of minimum crossflow velocity for all
downstream locations except $x/c = 0$ and like the vorticity for $\alpha = 5^\circ$ there were no secondary vorticity structures present at any of the four downstream locations. The excessive numerical diffusion is also seen to be greatest for $\alpha = 10^\circ$.

**URANS Reynolds Stress Model with Vorticity Confinement**

The mean streamwise vorticity contours for $\alpha = 5^\circ$ are shown in Figure 6.25. At $x/c = 0$, a maximum negative vorticity of $-0.028$ occurred just above the wing tip as the vortex underwent roll up. This value was slightly less than the RANS predicted vorticity of $-0.032$ at $x/c = 0$. The maximum vorticity remained at $-0.028$ up to $x/c = 1$ but decreased slightly to $-0.024$ at $x/c = 2$. 

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**Figure 6.23:** Normalized streamwise vorticity $\omega_x c/U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (RANS model).
6.6 Mean Velocity Results

![Normalized streamwise vorticity $\omega_x c / U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (RANS model)](image)

Figure 6.24: Normalized streamwise vorticity $\omega_x c / U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (RANS model)

The same values were recorded for the RANS vorticity at $x/c = 1$ and $x/c = 2$. In sharp contrast to the RANS simulations the vorticity at $x/c = 3$ did not decrease and remained at $-0.024$ at which point it had progressed inboard a distance of $0.1c$. The conservation of the vorticity at $x/c = 3$ meant the vortex core diameter also remained constant. The vorticity confinement model is thought to be responsible for the conservation of vorticity at $x/c = 3$ although it did not seem to effect the results prior to this location. The location of the maximum vorticity coincided with the point of minimum crossflow velocity for all downstream locations except $x/c = 0$ and similar to the RANS simulations there were no secondary vorticity structures present at any of the four downstream
locations. The vorticity structure became axisymmetric at $x/c = 2$ and retained that shape until $x/c = 3$, which was also in agreement with the observations for the RANS simulations.

The mean streamwise vorticity contours for $\alpha = 10^\circ$ are shown in Figure 6.26 and closely resemble those of the RANS simulation except at $x/c = 3$. The maximum vorticity at $x/c = 0$ reaches a value of $-0.045$ just outboard of the wing tip at $-0.012c$. The maximum vorticity remained at $-0.045$ for all remaining downstream locations but was found to occur in the centre of the vortex from $x/c = 1$ to $x/c = 3$. This was in contrast to findings of the RANS
simulations, where there was a large decrease in vorticity and increase in vortex core diameter at the last measurement location of $x/c = 3$. This highlights the ability of the vorticity confinement model to preserve the vortex for longer distances downstream but seemed to have minimal effect on the overall vortex shape or values before $x/c = 3$.

![Normalized streamwise vorticity $\omega_x c / U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (URANS model with VC)](image)

**Figure 6.26:** Normalized streamwise vorticity $\omega_x c / U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (URANS model with VC)

**LES and Wales SGS Model with Vorticity Confinement**

The mean streamwise vorticity contours for $\alpha = 5^\circ$ are shown in Figure 6.27. Looking at the vorticity plot for $x/c = 0$, the maximum negative vorticity of
-0.045 occurs at an inboard distance of 0.044c but not at the point of minimum crossflow velocity as the vortex is in the early stages of roll up. There also appears to be a small region of positive vorticity with a value of 0.016 just below the region of negative vorticity suggesting the presence of a small counter rotating vortex. The maximum vorticity decreased slightly to -0.041 at $x/c = 1$ but then increased to -0.045 at $x/c = 2$ and -0.053 at $x/c = 3$. The slight increase in the vorticity with downstream distance is a consequence of the process whereby wing shear layer vorticity is continually entrained into the vortex with downstream distance. The vorticity structure became axisymmetric at $x/c = 1$ and retained that symmetry to $x/c = 3$.

![Figure 6.27](image)

**Figure 6.27:** Normalized streamwise vorticity $\omega_x c/U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (LES with VC)
The mean streamwise vorticity contours for $\alpha = 10^\circ$ are shown in Figure 6.28. At $x/c = 0$, the region of negative vorticity was much larger than for $\alpha = 5^\circ$ with a maximum vorticity of $-0.061$ occurring at a distance of $0.044c$ albeit not at the centre of the vortex. Similar to the vorticity for $\alpha = 5^\circ$, a small region of positive vorticity with a maximum vorticity of 0.02 is observed outside the region of negative vorticity. A similar trend to the vorticity for $\alpha = 5^\circ$ is observed as the maximum negative vorticity decreased slightly to $-0.057$ at $x/c = 1$ but increased to $-0.061$ at $x/c = 2$ and to $-0.065$ at $x/c = 3$. However, in contrast to the vorticity for $\alpha = 5^\circ$, the region of positive vorticity persists beyond $x/c = 0$ and is seen to rotate around the periphery of the vortex core further downstream. Furthermore, the maximum positive vorticity increased to 0.024 at $x/c = 1$ after which it decreased to 0.016 at $x/c = 2$ and 0.008 at $x/c = 3$. The vorticity structure became axisymmetric at $x/c = 2$ and retained that symmetry to $x/c = 3$.

**Experimental Validation**

The mean streamwise vorticity exhibited a jet-like pattern across the vortex core for both $\alpha = 5^\circ$ and $\alpha = 10^\circ$. The mean streamwise vorticity taken on a line across the vortex core for $\alpha = 5^\circ$ is seen in Figure 6.29 represented on an inverted y-axis for clarity (the negative symbol represents the direction of the vorticity). At $x/c = 0$, the RANS, URANS and LES simulations were $-0.028$, $-0.024$ and $-0.024$, whereas the experimental maximum vorticity was much greater at $-0.057$. As the vortex progressed downstream the numerical and experimental vorticity values were in better agreement. The RANS maximum vorticity remained at $-0.028$ at $x/c = 1$, whereas the URANS and LES maximum vorticity increased to $-0.028$ and $-0.041$. However, the experimental maximum vorticity was observed to decrease substantially to $-0.032$ at $x/c = 1$. The line shape also shows that the vorticity structure attained a large degree of axisymmetry for all cases at $x/c = 1$. At $x/c = 2$, the RANS and URANS maximum crossflow velocities both decreased to $-0.02$ and $-0.024$, whereas the LES and experimental maximum crossflow velocities increased to $-0.045$ and $-0.037$ respectively. It is clear from the vorticity line plot that the LES vorticity peak did not progress inboard as much as the other cases. The RANS maximum vorticity decreased substantially
6. NUMERICAL MODELLING OF A WING-TIP VORTEX

Figure 6.28: Normalized streamwise vorticity $\omega_{x}c/U_{\infty}$ overlaid with crossflow velocity vectors $(jv + kw)/U_{\infty}$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (LES with VC)

at $x/c = 3$ to $-0.012$, whereas the URANS maximum vorticity remained at $-0.024$. The preservation of the URANS vorticity is attributed to the application of the vorticity confinement model. In addition, the LES maximum vorticity increased to $-0.053$, whereas the experimental maximum vorticity remained at $-0.037$.

The mean streamwise vorticity taken on a line across the vortex core for $\alpha = 10^\circ$ is seen in Figure 6.30 represented on the same inverted y-axis as mentioned previously. At $x/c = 0$, the RANS, URANS and LES simulations were $-0.037$, $-0.028$ and $-0.057$, whereas the experimental maximum vorticity was the greatest at $-0.074$ (similar to $\alpha = 5^\circ$). Similar to the vorticity for $\alpha = 5^\circ$,
6.6 Mean Velocity Results

Figure 6.29: Normalized streamwise vorticity $\omega_x c/U_\infty$ across the vortex core for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$

the numerical and experimental vorticity values were in better agreement as the vortex progressed downstream. The RANS and URANS maximum vorticity increased to $-0.041$ and $-0.045$ at $x/c = 1$, whereas the LES maximum vorticity remained at $-0.057$. However, the experimental maximum vorticity was seen to decrease to $-0.065$ at $x/c = 1$. Furthermore, the vorticity taken on a line through the core became more axisymmetric for all cases at $x/c = 1$. At $x/c = 2$, the RANS maximum vorticity decreased to $-0.037$, whereas the URANS maximum vorticity remained at $-0.045$. The LES maximum vorticity increased slightly and the experimental maximum vorticity decreased slightly with both reaching the
same value of $-0.061$. All the cases gained more axisymmetry at $x/c = 2$, with the peak vorticity occurring at the same location for all cases. Similar to the $\alpha = 5^\circ$ case, the RANS maximum vorticity decreased substantially at $x/c = 3$ to a value of $-0.012$, whereas the URANS maximum vorticity remained at $-0.045$. The vorticity confinement model had a large role to play in the conservation of vorticity for the URANS case at $x/c = 3$. In addition, the LES and experimental maximum vorticity increased to $-0.061$ and $-0.07$.

![Normalized streamwise vorticity $\omega_xc/U_\infty$ across the vortex core for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$](image)

**Figure 6.30:** Normalized streamwise vorticity $\omega_xc/U_\infty$ across the vortex core for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$
6.6.5 Vortex Trajectory, Size and Core Centre Location

The vortex core centre location was determined from the point of minimum cross-flow velocity and the radius of the vortex estimated by calculating the distance from the core centre to the point of maximum crossflow velocity. Admittedly, for a wing-tip vortex that is still in the early stages of roll up, this definition is a bit arbitrary but it has been deemed adequate as a general descriptor by Chow et al. (1997b). Figure 6.31 shows the estimated core radii and trajectory for all downstream locations for $\alpha = 5^\circ$. The RANS and URANS vortex core centre points were at the same location at $x/c = 0$, although the URANS had a slightly larger core radius of $0.064c$. The LES and experimental vortices also had the same core centre location but the experimental vortex had a slightly larger core radius of $0.05c$. The numerical and experimental vortices had all progressed inboard from the tip by $x/c = 1$. The core centres of the RANS and URANS vortices were at an inboard distance of $0.057c$ and both vortices had a radius of $0.057c$. The LES and experimental vortices were both of similar size at $x/c = 1$ with radii of $0.05c$ and $0.042c$ respectively. However, the core centre of the experimental vortex had moved inboard a distance of $0.071c$, whereas the LES vortex core centre was at a distance of $0.042c$. The inboard progression of the vortex core continued with the RANS and URANS core centres moving inboard a distance of $0.071c$ and $0.085c$ by $x/c = 2$. The core centres of the LES and experimental cases had also progressed inboard a distance of $0.057c$ and $0.085c$. The URANS and RANS vortices both had a radii of $0.057c$ and the LES and experimental vortices both had a radii of $0.035c$. At the last measurement location of $x/c = 3$, the size of the RANS and URANS vortices grew considerably to radii of $0.078c$ and $0.064c$, whereas the radii of the LES and experimental vortices remained the same at $0.035c$. The experimental and URANS vortices had moved the furthest inboard to a distance of $0.1c$, whereas the LES and RANS vortices had moved inboard by $0.071c$ and $0.085c$. The size of the LES predicted vortex was in best agreement with the experiment vortex size at all downstream locations but did not progress as far inboard as the experimental or RANS and URANS cases.

Figure 6.32 shows the estimated vortex core radii and trajectory at all downstream locations for $\alpha = 10^\circ$. At $x/c = 0$, all vortices had moved in-board by the
6. NUMERICAL MODELLING OF A WING-TIP VORTEX

Figure 6.31: Downstream development of the vortex trajectory, size and core centre location for $\alpha = 5^\circ$

same distance of 0.028$c$. The LES and experimental vortex had the same core radius of 0.064$c$, whereas the RANS and URANS vortices had a slightly larger core radius of 0.071$c$. The URANS vortex core radius remained constant at 0.071$c$ until the last measurement location of $x/c = 3$, whereas the RANS vortex core radius remained constant to $x/c = 1$ after which it increased to 0.078$c$ at $x/c = 2$ and increased significantly to 0.121$c$ at $x/c = 3$. The LES vortex core radius reduced to 0.042$c$ at $x/c = 1$, then increased to 0.05$c$ at $x/c = 2$ and increased again at $x/c = 3$ to 0.057$c$. The experimental vortex core radius reduced to 0.057$c$ at $x/c = 1$ and reduced again to 0.05$c$ at $x/c = 2$ after which it remained
constant to $x/c = 3$. The numerical and experimental vortices progressed inboard with downstream distance and by $x/c = 3$, the RANS and LES vortices had shifted inboard $0.085c$, whereas the URANS and experimental vortices had shifted inboard a distance of $0.1c$. The reduction in vortex core radii for the LES and experimental cases might be explained by the fact that the vortex became more tightly wound as it progressed downstream. This argument is supported by the fact that the crossflow velocity did not decay significantly with downstream distance for the LES and experimental cases. The crossflow velocity decreased with downstream distance for the RANS and URANS cases and hence their radii rapidly increased and remained the same with downstream distance.

6.7 Turbulence Results

6.7.1 RMS Fluctuating Velocities

The RANS and URANS turbulence modelling approaches failed to predict the turbulence in the tip-vortex and they predicted rms fluctuating velocities of the order of $10^{-6}$. As a result, only the LES turbulence results are presented in this section. The rms fluctuating velocities ($\sqrt{\overline{u'}^2}$ and $\sqrt{\overline{v'}^2}$) normalized by the free-stream velocity $U_\infty$ are presented for $\alpha = 5^\circ$ and $10^\circ$ and $Re = 3.25 \times 10^5$.

LES and Wales SGS Model with Vorticity Confinement

The contours of $u'$ rms velocity for $\alpha = 5^\circ$ can be seen in Figure 6.33. At $x/c = 0$, the vortex core region did not contain a high level streamwise turbulence; however, the shear layer that ultimately wraps up into the wing-tip vortex had a peak $u'$ rms velocity of 0.15 recorded at an inboard distance of 0.18c. At $x/c = 1$, the maximum $u'$ rms velocity decreased significantly to 0.05 and again was seen to occur in the detached wing shear layer. Low levels of $u'$ rms velocity were also seen to occur in the outer boundary of the vortex core where the crossflow velocity was at a maximum. The maximum $u'$ rms velocity decreased again at $x/c = 2$ attaining a value of 0.019 underneath the wing at an inboard distance of 0.1c. The $u'$ rms velocity was highest in the detached wing shear layer with significant levels also seen to occur around the periphery of the vortex. The $u'$ rms velocity
Figure 6.32: Downstream development of the vortex trajectory, size and core centre location for $\alpha = 10^\circ$

continued to decrease at $x/c = 3$ attaining a value of 0.015 outboard of the wing-tip in the shear layer fluid that was rolling into a spiral surrounding the vortex. Overall, there were no significant levels of turbulent $u'$ rms velocity in the core of the vortex at all downstream locations.

The contours of $v'$ rms velocity for $\alpha = 5^\circ$ can be seen in Figure 6.34. Similar to the $u'$ rms velocity, the maximum $v'$ rms velocity of 0.048 occurs in the wing shear layer at $x/c = 0$. The magnitude of the $v'$ rms velocity being only 34% of the $u'$ rms velocity at the same location. The maximum $v'$ rms velocity decreased to 0.017 at $x/c = 1$ and was now seen occur in the centre of the vortex. Furthermore,
the \( v' \) rms velocity structure had now attained a vortex type structure with the maximum fluctuations seen to occur in the centre and found to decrease towards the outer core periphery. The \( v' \) rms vortex structure was observed to be quite axisymmetric at \( x/c = 1 \) and was in sharp contrast to the \( u' \) rms velocity where no vortex like structure was apparent. Surprisingly, the maximum \( v' \) rms velocity was seen to increase at \( x/c = 2 \) and \( x/c = 3 \) to 0.021 and 0.032 in the core centre, which suggests that a significant portion of the turbulence in the core is created by the vortex itself. The roll up of turbulent shear layer fluid seems to have ceased by the last measurement location of \( x/c = 3 \).

Figure 6.33: Normalized \( u' \) rms velocity \( \sqrt{u'^2/U_\infty} \) overlaid with crossflow velocity vectors \( (jv + kw)/U_\infty \) for \( \alpha = 5^\circ \) and \( Re = 3.25 \times 10^5 \) (LES with VC)
6. NUMERICAL MODELLING OF A WING-TIP VORTEX

![Normalized velocity plots for different x/c ratios](image)

(a) $x/c = 0$
(b) $x/c = 1$
(c) $x/c = 2$
(d) $x/c = 3$

**Figure 6.34:** Normalized $v'$ rms velocity $\sqrt{v'^2}/U_{\infty}$ overlaid with crossflow velocity vectors $(jv + kw)/U_{\infty}$ for $\alpha = 5^\circ$ and $Re = 3.25 \times 10^5$ (LES with VC)

The LES contours of $u'$ rms velocity for $\alpha = 10^\circ$ can be seen in Figure 6.35. Similar to the $u'$ rms velocity for $\alpha = 5^\circ$, there was no significant streamwise turbulence in the vortex core region at $x/c = 0$ and the peak $u'$ rms velocity of 0.076 occurred in the shear layer emanating from underneath the wing at an inboard distance of 0.19$c$. At $x/c = 1$, the maximum $u'$ rms velocity decreased to a value of 0.028 and occurred near the location of maximum crossflow velocity. This trend continued further downstream as the maximum $u'$ rms velocity remained on the outer boundary of the vortex core with values of 0.026 and 0.027 at $x/c = 2$ and $x/c = 3$ respectively. Similar to the $u'$ rms velocity for $\alpha = 5^\circ$, no significant levels of $u'$ rms velocity were recorded in the vortex core at any of the four
downstream locations.

![Normalized u' rms velocity](image)

**Figure 6.35:** Normalized $u'$ rms velocity $\sqrt{u'^2}/U_\infty$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (LES with VC)

The LES contours of $v'$ rms velocity for $\alpha = 10^\circ$ are shown in Figure 6.36. Similar to the $v'$ rms velocity for $\alpha = 5^\circ$, the vortex core does not contain a high level of $v'$ rms turbulence at $x/c = 0$. A maximum $v'$ rms value of 0.05 was recorded in the wing shear layer just above the trailing edge at an inboard distance of 0.18$c$. The turbulent shear layer is seen to wrap into the forming tip vortex at $x/c = 1$ and two patches of high $v'$ rms velocity are seen to occur near the centre and the outer edge of the vortex core. The maximum $v'$ rms velocity attaining a value of 0.034 near the centre of the vortex core and a value of 0.033 at the edge of the vortex core. Similar to the $u'$ rms velocity for $\alpha = 5^\circ$, the
maximum $v'$ rms velocity increased to 0.069 at $x/c = 2$ and 0.071 at $x/c = 3$, which is most likely attributed to the addition of some shear layer fluid at which re-energises the vortex and additional turbulent fluctuations.

![Normalized $v'$ rms velocity](image)

Figure 6.36: Normalized $v'$ rms velocity $\sqrt{v'_{\text{rms}}^2/U_\infty}$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$ for $\alpha = 10^\circ$ and $Re = 3.25 \times 10^5$ (LES with VC)

**Experimental Validation**

The downstream development of the maximum $u'$ and $v'$ rms velocities for the numerical and experimental cases is shown in Figure 6.37. The experimental maximum $u'$ rms velocities for $\alpha = 5^\circ$ were observed to steadily decrease with downstream distance having values of 0.05, 0.046, 0.041 and 0.028 at $x/c = 0, 1, 2$ and 3 respectively. The LES maximum $u'$ rms velocities for $\alpha = 5^\circ$ also decreased
with downstream distance but at a faster rate. The maximum $u'$ rms velocity at $x/c = 0$ was 0.15, which was three times the experimental maximum at the same location. The LES maximum $u'$ rms velocity decreased significantly at $x/c = 1$ to 0.05, which was in good agreement with the experimental maximum of 0.046. The LES maximum $u'$ rms velocity continued to decrease and attained values of 0.021 and 0.015 by $x/c = 2$ and $x/c = 3$ respectively. In contrast, the experimental and LES maximum $v'$ rms velocities for $\alpha = 5^\circ$ were to seen to increase and decrease with downstream distance. The maximum experimental $v'$ rms velocity at $x/c = 0$ was 0.056, which was in good agreement with the LES maximum value of 0.047. At $x/c = 1$, the experimental maximum $v'$ rms velocity decreased slightly to 0.049, whereas the LES maximum $v'$ rms velocity decreased significantly to 0.017. Both the experimental and LES maximum $v'$ rms velocities increased at $x/c = 2$ to 0.058 and 0.021 respectively. At the last measurement location of $x/c = 3$, the experimental maximum $v'$ rms velocity decreased to 0.043, whereas the LES maximum $v'$ rms velocity increased to 0.032.

The experimental maximum $u'$ rms velocities for $\alpha = 10^\circ$ were observed to increase and decrease with downstream distance. The experimental maximum $u'$ rms velocity at $x/c = 0$ was 0.13, whereas the LES maximum $u'$ rms velocity was only 0.076. The experimental maximum $u'$ rms velocity decreased to 0.043 and 0.022 at $x/c = 1$ and $x/c = 2$ and the LES maximum $u'$ rms velocity decreased to 0.028 and 0.026 at $x/c = 1$ and $x/c = 2$. The latter value being in close agreement with the experimental value. The experimental and LES maximum $u'$ rms velocities both increased to 0.024 and 0.027 respectively at the last measurement location of $x/c = 3$ with both in close agreement. The experimental and LES maximum $v'$ rms velocities exhibited the same increasing and decreasing pattern with downstream distance for $\alpha = 10^\circ$ as the $u'$ rms velocities. The maximum experimental $v'$ rms velocity at $x/c = 0$ was 0.103, which was nearly double the experimental maximum $v'$ rms velocity recorded for $\alpha = 5^\circ$ at the same location. The LES maximum $v'$ rms velocity at $x/c = 0$ was 0.05. The experimental and LES $v'$ rms velocities decreased to 0.044 and 0.034 at $x/c = 1$, where they were in good agreement. At $x/c = 2$, the experimental maximum $v'$ rms velocity decreased to 0.024, whereas the LES maximum $v'$ rms velocity increased to 0.069. The large increase in LES $v'$ rms velocity would suggest that the vortex entrained
some additional turbulent fluid, whereas the decrease in the experimental maximum $v'$ rms velocity would suggest that the solid body rotation of the vortex is damping out turbulent fluctuations in the vortex core. At the last measurement location of $x/c = 3$, the LES and experimental maximum $v'$ rms velocities both increased to 0.071 and 0.029 respectively.

![Graphs showing development of maximum normalized rms velocities](image)

(a) $\alpha = 5^\circ$  
(b) $\alpha = 10^\circ$

**Figure 6.37:** Development of the maximum normalized rms velocities $\sqrt{u'^2}/U_\infty$ and $\sqrt{v'^2}/U_\infty$ with downstream distance

### 6.8 Summary

The RANS and URANS with VC turbulence modelling approaches predicted the wake-like axial velocity deficit for $\alpha = 5^\circ$ reasonably well for all downstream locations. However, the RANS and URANS models both failed to predict the jet-like axial velocity excess for $\alpha = 10^\circ$. The LES with VC was the only turbulence modelling approach to correctly predict both the jet-like and wake-like axial velocity profiles that were evident in the experimental measurements for $\alpha = 10^\circ$ and $\alpha = 5^\circ$. The LES predicted axial velocity excess was in very good agreement with the experimentally measured velocity excess being only 3% greater than experiment at $x/c = 1$ for $\alpha = 10^\circ$. The LES with VC was also the only modelling approach to correctly predict the axial velocity deficit region in the vortex core.
6.8 Summary

periphery for $\alpha = 10^\circ$, which was observed in the experimental axial velocity measurements.

All turbulence modelling approaches predicted the characteristic near-zero crossflow velocity in the vortex core centre, which increased to a maximum at the outer periphery of the vortex core. The RANS model exhibited the most diffuse crossflow velocity structure, whereas the LES had the most concentrated or tightly wound crossflow velocity structure. The vorticity confinement model did prevent excessive diffusion of the vortex for the URANS model at $x/c = 3$ but was found to have minimal effect on the mean velocity characteristics upstream. The crossflow velocity taken on a line through the vortex core revealed the LES to have the best agreement with experiment for $\alpha = 10^\circ$, whereas the LES crossflow velocity for $\alpha = 5^\circ$ was slightly skewed.

The LES circulation was in best agreement with the experimental circulation for $\alpha = 5^\circ$ at all downstream locations but the URANS model had the best agreement for $\alpha = 10^\circ$. The general magnitude of the circulation parameter for $\alpha = 10^\circ$ was in close agreement with the circulation parameter of 0.26 from Birch et al. (2004), who investigated a wing-tip vortex generated by a NACA 0015 wing at $\alpha = 10^\circ$.

The LES with VC model predicted a region of positive vorticity at $x/c = 0$ for $\alpha = 5^\circ$ and at all downstream locations for $\alpha = 10^\circ$, which was similar to the LES investigation by Jiang et al. (2008), who also noted the presence of a secondary counter rotating vortex in the wake of a NACA 0012 wing. This corroborated the findings of the experimental vorticity, where a secondary counter rotating vortex was present in the flow at $x/c = 0$ for $\alpha = 5^\circ$ and $\alpha = 10^\circ$. The presence of a secondary vorticity structure correlated with the presence of the axial velocity deficit region surrounding the vortex core for $\alpha = 10^\circ$. The vorticity on a line taken through the vortex core revealed the characteristic vorticity maximum at the centre, which decreased to zero at the vortex core periphery. The LES predicted vorticity was again in best agreement with the experimental vorticity for $\alpha = 5^\circ$ and $10^\circ$.

The vortex core radius was much larger for $\alpha = 10^\circ$ than for $\alpha = 5^\circ$ (a characteristic also noted by Birch et al. (2003)) for all numerical and experimental cases. The LES predicted vortex core radius was in best agreement with experiment at
all four downstream locations and both angles of attack. The LES predicted core radius was the same as experiment at $x/c = 3$ for $\alpha = 5^\circ$ and 14% greater at $x/c = 3$ for $\alpha = 10^\circ$. However, the LES vortex did not progress inboard as far as the experimental vortex. The RANS and URANS predicted core radii were much greater than experiment but were in better agreement in terms of inboard movement of the vortex.

The RANS and URANS numerical models predicted fluctuating rms velocities that were orders of magnitude less than experiment ($\approx 10^{-6}$) and could not be represented legibly on graphs or plots. Churchfield and Blaisdell (2009) also found that a number of advanced RANS turbulence models failed to accurately capture the turbulence quantities in the core of a wing-tip vortex. The LES with VC model was the only turbulence modelling approach to predict turbulence values within an order of magnitude of the experimental values. The LES with VC model predicted little or no $u'$ rms velocity in the core of the vortex for $\alpha = 5^\circ$ and $\alpha = 10^\circ$, which was similar to the LES study by Youssef et al. (1998). Instead, the maximum $u'$ rms velocities occurred in the detached shear layer or on the periphery of the vortex core. The maximum $u'$ rms velocities were seen to decrease with downstream distance for both angles of attack. In contrast, the $v'$ rms velocities were much smaller in magnitude than the $u'$ rms velocities. The maximum $v'$ rms velocities occurred in the detached shear layer at $x/c = 0$ but occurred in the vortex core centre thereafter for both angles of attack. The $v'$ rms velocities increased with downstream distance after $x/c = 1$ suggesting that vortex turbulence production was much stronger in the spanwise direction.
Chapter 7

Conclusions and Future Research

This chapter summarizes the main findings of the research with reference to the stated objectives (Section 1.4) and the unresolved issues described in Section 2.3. The dissemination of the conducted research is outlined and recommendations are made for future numerical and experimental research of wing-tip vortex flow.

7.1 Conclusions

A wing-tip vortex test facility has been designed and manufactured, which enabled high resolution cross-stream (yz) measurements for up to three chord lengths downstream of the generating wing. The wing-tip vortex test facility consisted of a custom built test section with cross-stream (yz) measurement slots (see Figure 4.2) and an automated traverse mechanism with positional accuracy of 10µm and 6.25µm in the y and z directions respectively. The wing-tip vortex test facility enabled a comprehensive experimental investigation to be conducted.

The mean flow and turbulent characteristics of a wing-tip vortex in the near-field (up to three chord lengths downstream) have been studied experimentally using a five-hole pressure probe and x-wire anemometer, and numerically using a number of turbulence modelling approaches. These include solving for the Steady and Unsteady Reynolds Averaged Navier-Stokes (RANS/URANS) equations using a full Reynolds stress turbulence model, Large Eddy Simulation (LES) using a Wales SGS model and the application of a vorticity confinement model to the
7. CONCLUSIONS AND FUTURE RESEARCH

URANS and LES momentum equations. The experimental and numerical measurements were taken on cross-stream planes perpendicular to the free-stream velocity at four downstream locations in the wake ($x/c = 0, 1, 2$ and $3$). The effects of several parameters on the mean and turbulent characteristics have been addressed, which has generated a broader understanding of the physics of the flow. In particular, a symmetric NACA 0012 rectangular wing model with an aspect ratio of $1$ has been tested at two angles of attack ($\alpha = 5^\circ$ and $10^\circ$) and a Reynolds number of $3.25 \times 10^5$. The effect of an elevated level of free-stream turbulence (4.3%) on the turbulent characteristics in the vortex core has also been examined for a Reynolds number of $1.8 \times 10^5$. The detailed experimental measurements and the advanced numerical modelling techniques used to model the wing-tip vortex represent a unique feature of this investigation and advance the current state of understanding of the roll-up and formation of a wing-tip vortex in the near-field.

The experimental measurements revealed that the core of the tip vortex contained an axial velocity deficit (wake-like) for $\alpha = 5^\circ$ and an axial velocity excess (jet-like) for $\alpha = 10^\circ$. The vortex was also found to contain both regions of axial velocity excess and deficit at all downstream locations for $\alpha = 10^\circ$. The vortex was found to be highly nonaxisymmetric at $x/c = 0$ for both angles of attack, after which it became axisymmetric by $x/c = 2$. The crossflow velocity magnitude reached a minimum at the vortex core centre and slowly increased to a maximum at the edge of the vortex core. The maximum crossflow velocity magnitude increased significantly with angle of attack and was 100% greater for $\alpha = 10^\circ$ at $x/c = 0$ than for $\alpha = 5^\circ$ at the same location. The strength of the vortex increased with downstream distance and angle of attack. However, the vortex was found to contain only 36% and 54% of the theoretical wing circulation for $5^\circ$ and $10^\circ$ angle of attack by $x/c = 3$. The vorticity magnitude was found to decrease with downstream distance for $\alpha = 5^\circ$ but increased gradually for $\alpha = 10^\circ$ due to the continuous trapping of the detached shear layer. The presence of a secondary counter rotating vortex was observed at $x/c = 0$ for $\alpha = 5^\circ$ and $10^\circ$ by the presence of a small patch of positive signed vorticity. The maximum turbulent $u'$ and $v'$ rms velocities for $\alpha = 5^\circ$ and $10^\circ$ occurred at the edge of the vortex core at $x/c = 0$ but were found to occur in the vortex centre further downstream.
7.1 Conclusions

The $u'$ and $v'$ rms velocities were both found to decay with downstream distance. The $u'v'$ Reynolds stress exhibited two regions of opposite signed stress for both angles of attack and was also found to decay with downstream distance. The elevated free-stream turbulence level of 4.3% had minimal effect on the turbulent rms velocities in the vortex core but the $u'v'$ Reynolds stress was observed to increase.

The RANS and URANS with VC models predicted the wake-like axial velocity deficit for $\alpha = 5^\circ$ reasonably well but failed to reproduce the jet-like axial velocity observed in the experiment. The LES with VC was the only turbulence modelling approach to correctly predict both the jet-like and wake-like axial velocity profiles that were evident in the experimental measurements. For $\alpha = 10^\circ$, the LES predicted axial velocity excess was in very good agreement with experiment, being only 3% greater than experiment at $x/c = 1$. The LES with VC was also the only turbulence modelling approach to correctly predict the axial velocity deficit region in the vortex core periphery for $\alpha = 10^\circ$. The LES predicted circulation was in best agreement with experiment for $\alpha = 5^\circ$, but the URANS model had the best agreement for $\alpha = 10^\circ$. The LES with VC was the only turbulence modelling approach to correctly predict the presence of a secondary vorticity structure for $\alpha = 5^\circ$ at $x/c = 0$ and at all downstream locations for $\alpha = 10^\circ$. The LES predicted vorticity was again in best agreement with the experimental measurements for both angles of attack. The LES predicted core radius was the same as experiment at $x/c = 3$ for $\alpha = 5^\circ$ and only 14% greater at $x/c = 3$ for $\alpha = 10^\circ$. The RANS and URANS predicted core radii were much greater than experiment but were in better agreement in terms of inboard movement. The RANS and URANS models struggled to predict the magnitudes of the turbulence quantities and the turbulent $u'$ and $v'$ rms velocities were of the order of $10^{-6}$. The LES with VC model on the other hand predicted values that were reasonably close to experiment. The LES predicted maximum $u'$ and $v'$ rms velocities within 15% and 35% of experimental values at $x/c = 2$ and $x/c = 3$ for $\alpha = 10^\circ$ and 5$^\circ$ respectively. Overall the LES with VC model was found to be the most accurate in terms of mean flow, vortex core size and turbulence within the core of the vortex. The RANS predicted vortex was found to be very diffuse after a distance of $x/c = 3$ and both the RANS and URANS with VC models failed to accurately
predict the jet-like axial velocity excess in the core of the vortex for $\alpha = 10^\circ$ and
the magnitudes of the turbulence quantities. The vorticity confinement model
was found to preserve the excessive diffusion of the vortex at $x/c = 3$ but had
minimal effect on the magnitudes of the mean flow characteristics when used with
the URANS model.

7.2 Dissemination of Work

Journal Publications

O’Regan, M.S., Griffin, P.C. and Young, T.M. (2014) ‘Numerical and Experimental
Investigation of the Mean and Turbulent Characteristics of a Wing-tip Vortex
in the Near-field’, Proceedings of the Institution of Mechanical Engineers. Part

O’Regan, M.S., Griffin, P.C. and Young, T.M. (2014) A Vorticity Confinement
Model Applied to URANS and LES Simulations of a Wing-tip Vortex in the

Conference Publications

O’Regan, M.S., Griffin, P.C. and Young, T.M. (2013) ‘Wingtip Vortices in the
Near-field–A Numerical and Experimental Investigation’, 4th CEAS Air and
Space Conference, Linköping, Sweden, 16-19 September, 2013, Linköping, Swe-
den: Linköping University Electronic Press, 150-159.

O’Regan, M.S., Griffin, P.C., McNicholas, G. and Young, T.M. (2012) ‘Exper-
imental/Numerical Investigation of a Wingtip Vortex in the Near-field’, 30th
AIAA Applied Aerodynamics Conference, New Orleans, Louisiana, USA, 25-28
June, 2012, Reston VA: AIAA.
7.3 Future Research

Presentations


7.3 Future Research

The contributions of this work have created a better understanding of the physics involved in the roll up and formation of a wing-tip vortex in the near-field. The results of the experimental and numerical investigations will serve as a benchmark for future prediction methods and numerical investigations. Although the experimental work conducted was considerable, there are a number of experimental parameters and non-intrusive experimental measurement techniques to be further explored. Similar experiments on other wing-geometries, Reynolds numbers and angles of attack are advised to create a better understanding and identify similarities and correlations in the vortex behaviour under different conditions.

Between four and ten chord lengths downstream of a wing-tip vortex is a region that has not received considerable study in recent years and it would be worthwhile to conduct further research in this region to advance the current knowledge on how the vortex behaves after initial formation (one to three chord lengths). Non-intrusive experimental measurement techniques such as PIV or SPIV have not been extensively used to study the near-field of a wing-tip vortex and could yield useful instantaneous flow-field data on the roll up and near-field formation of the vortex.

This work has exposed the inability of RANS based turbulence models at accurately predicting the magnitudes of the turbulence quantities in the vortex core. If the goal of further study is to determine the effect of the turbulence on the
mean flow and resolve the turbulence accurately, advanced turbulence modelling techniques such as Detached Eddy Simulation (DES) or Large Eddy Simulation (LES) is advised. Computational limitations in this work meant that a relatively large grid spacing and time step was used for the LES with VC simulations. It is advised to conduct further LES simulations utilising a more dense grid and smaller time-step advancement. Adaptive mesh refinement techniques are also suggested as a means of controlling the mesh density in the vortex and conserving computational resources.
References

Aeroprobe (2008), Five and Seven-hole Probe Manual, Aeroprobe Corporation, Christiansburg, VA 24073, United States.


Beaumier, P. and Delrieux, Y. (2005), ‘Description and validation of the onera computational method for the prediction of blade-vortex interaction noise’, Aerospace Science and Technology 9, 31–43.


REFERENCES


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Appendix A

Matlab Data Processing Scripts

The following Matlab scripts were used for the post-processing of numerical and experimental data and for the creation of the plots presented in the results sections.

Axial Velocity Contour Plots

The following script was used to plot the contours of normalized axial velocity \( \frac{U_x}{U_\infty} \) overlaid with crossflow velocity vectors \( (jv + kw)/U_\infty \).

```matlab
% Axial velocity contour plots
[x,y] = meshgrid(-.04:0.014:.2,-.04:0.014:.2);
data = xlsread('axialvelocity0chord')
z = data(1:18,24:41);
figure
contourf(x,y,z,50);
shading flat
xlabel('Y/C');ylabel('Z/C')
hd= colorbar;
figureHandle = gcf;
caxis([0.4 1.0]);
set(0,'DefaultAxesFontSize', 14)
set(0,'DefaultAxesFontWeight', 'bold')
set(findall(figureHandle,'type','text'),'fontSize',14,...
    'fontWeight','bold')
set(get(hd,'title'),'String','U_x/U_\infty','fontSize',14,...
     'fontWeight','bold');
set(gcf,'color','white')
```
APPENDIX

```matlab
line('XData', [0 .2], 'YData', [0 0], 'LineStyle', '--', 'LineWidth', 2, 'Color','k');
hold on
data = xlsread('crossflow0chord','Sheet2');
x = data(1:325,1);
y = data(1:325,2);
u = data(1:325,3);
v = data(1:325,4);
quiver(x,y,u,v,'color','k')
```

Crossflow Velocity Vector Plots

The following script was used to plot the normalized crossflow velocity vectors 
\((jv + kw)/U_\infty\).

```matlab
% Cross flow velocity vectors plots
[x,y] = meshgrid(-.04:0.014:.2,-.04:0.014:.2);
data = xlsread('crossflow0chord','Sheet2');
x = data(1:325,1);
y = data(1:325,2);
u = data(1:325,3);
v = data(1:325,4);
figure
quiver(x,y,u,v,'Color','k')
axis([- .05 .21 - .05 .24])
xlabel('Y/C');ylabel('Z/C')
str1 = {'0.5U/\textit{infty}'};
text(0.074,0.22,str1,'Fontweight','bold','fontsize',14)
line('XData', [0 .23], 'YData', [0 0], 'LineStyle', '--', 'LineWidth', 2, 'Color','k');
```

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Crossflow Velocity Magnitude Contour Plots

The following script was used to plot the contours of normalized crossflow velocity magnitude $U_c/U_\infty$.

% Crossflow velocity magnitude contour plots
[x,y] = meshgrid(-.04:0.014:.2,-.04:0.014:.2);
data = xlsread('crossflow0chord','Sheet3')
z = data(1:18,4:21);
figure
contourf(x,y,z,50);
shading flat
xlabel('Y/C');ylabel('Z/C')
hd=colorbar;
figureHandle = gcf;
set(0,'DefaultAxesFontSize', 14)
set(0,'DefaultAxesFontWeight', 'bold')
set(findall(figureHandle,'type','text'),'fontSize',14,...
    'fontWeight','bold')
set(get(hd,'title'),'String','U_c/U_\infty','fontSize',14,...
    'fontWeight','bold');
set(gcf,'color','white');
line('XData', [0 .2], 'YData', [0 0], 'LineStyle', '--', ...
     'LineWidth', 2, 'Color','k');

Streamwise Vorticity Magnitude Contour Plots

The following script was used to plot the contours of streamwise vorticity magnitude $\omega_x$ overlaid with crossflow velocity vectors $(jv + kw)/U_\infty$.

% Streamwise vorticity contour plots
[x,y] = meshgrid(-.04:0.014:.2,-.04:0.014:.2);
data = xlsread('vorticity0chord');
V = data(1:18,6:23);
U = data(24:41,6:23);
[curlx,cav]= curl(U,V);
xlswrite('vorticitycomponents.xls', curlx, 1, 'B1')
hold on
APPENDIX

data = xlsread('vorticitycomponents');
z = data(1:18,1:18);
figure
contourf(x,y,z,50)
shading flat
xlabel('Y/C');ylabel('Z/C')
hd= colorbar;
figureHandle = gcf;
set(0,'DefaultAxesFontSize', 14)
set(0,'DefaultAxesFontWeight', 'bold')
set(findall(figureHandle,'type','text'),'fontSize',14,...
    'fontWeight','bold')
set(get(hd,'title'),'String', '\textomega (1/s)',
     'fontSize',... 14,'fontWeight','bold');
set(gcf,'color','white')
line('XData', [0 .2], 'YData', [0 0], 'LineStyle', '--', ...
     'LineWidth', 2, 'Color','k');
hold on
data = xlsread('crossflow0chord','Sheet2');
x = data(1:325,1);
y = data(1:325,2);
u = data(1:325,3);
v = data(1:325,4);
quiver(x,y,u,v,'color','k')

Vortex Trajectory, Size and Core Centre Location Plots

The following script was used to plot the estimated vortex core size and inboard
location as it progressed downstream.

% Vortex size, core centre location and trajectory plots
function circle(x,y,r)
\%x and y are the coordinates of the center of the circle
\%r is the radius of the circle
\%0.01 is the angle step, bigger values will draw the circle
\%faster but there might be imperfections (not very smooth)
hold on
c=140;
x = 2/c;
y = 4/c;
r = 8/c;
ang=0:0.00001:2*pi;
xp=r*cos(ang);
yp=r*sin(ang);
plot(x+xp,y+yp,'LineStyle','-.','LineWidth',2,'color','k');
x = plot(x,y,'b *');
hAnnotation = get(x,'Annotation');
hLegendEntry = get(hAnnotation','LegendInformation');
set(hLegendEntry,'IconDisplayStyle','off')
hold on
x = 2/c;
y = 4/c;
r = 9/c;
ang=0:0.00001:2*pi;
xp=r*cos(ang);
yp=r*sin(ang);
plot(x+xp,y+yp,'LineStyle',':','LineWidth',2,'color','b');
x = plot(x,y,'b *');
hAnnotation = get(x,'Annotation');
hLegendEntry = get(hAnnotation','LegendInformation');
set(hLegendEntry,'IconDisplayStyle','off')
hold on
x = 0/c;
y = 2/c;
r = 6/c;
ang=0:0.00001:2*pi;
xp=r*cos(ang);
yp=r*sin(ang);
plot(x+xp,y+yp,'LineStyle','--','LineWidth',2,'color','r');
x = plot(x,y,'r *');
hAnnotation = get(x,'Annotation');
hLegendEntry = get(hAnnotation','LegendInformation');
set(hLegendEntry,'IconDisplayStyle','off')
hold on
x = 0/c;
y = 2/c;
r = 7/c;
ang=0:0.00001:2*pi;
xp=r*cos(ang);
yp=r*sin(ang);
APPENDIX

```matlab
plot(x+xp,y+yp,'LineStyle','-','LineWidth',2,'color','g');
x = plot(x,y,'g *');
hAnnotation = get(x,'Annotation');
hLegendEntry = get(hAnnotation,'LegendInformation');
set(hLegendEntry,'IconDisplayStyle','off');
xlabel('Y/C'); ylabel('Z/C');
legend('RANS','URANS','LES','Exp. Fhp','Location','NorthEast');
line('XData',[0 .22], 'YData',[0 0], 'LineStyle','--', 'LineWidth',2, 'Color','k');
axis([-.05 .21 -.05 .22]); box on
set(0,'DefaultAxesFontSize', 14); set(0,'DefaultAxesFontWeight', 'bold');
set(findall(figureHandle,'type','text'),'fontSize',14,...
'fontWeight','bold');
end
```

X-probe Anemometer Data Processing

The following script converted the x-probe anemometer output files into known turbulent quantities.

```matlab
% Post processing hot-wire data
% Range will change depending on grid resolution
for i = 1:15
f = load(['XCZERO', num2str(i),'.dat']);
fsample=20000;
dt=1/fsample;
N=131072;
T=N*dt;
t=0:dt:T;
t=t(1:131072);
U=f(:,2);
V=f(:,3);
Vmean=mean(V);
v1=V-Vmean;
v1=v1(1:131072);
RMSy=(mean(v1.^2)).^0.5;
```

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Umean=mean(U);
ul=U-Umean;
ul=ul(1:131072);
RMSu=(mean(ul.^2)).^0.5;
TUxy =((mean(ul.^2) + mean(v1.^2))/2).^0.5)/Umean*100;
RStress = mean(ul.*v1);
Tauxy = RStress*(-1.225);s
%Writes each output variable to a column in an excel file
xlswrite('XCZERO.xls', TUxy, 1, ['A',num2str(i)]
xlswrite('XCZERO.xls', RMSu, 1, ['B',num2str(i)]
xlswrite('XCZERO.xls', Umean, 1, ['C',num2str(i)]
xlswrite('XCZERO.xls', RMSy, 1, ['D',num2str(i)]
xlswrite('XCZERO.xls', Vmean, 1, ['E',num2str(i)]
xlswrite('XCZERO.xls', RStress, 1, ['F',num2str(i)]
xlswrite('XCZERO.xls', Tauxy, 1, ['G',num2str(i)]
end