Hygro-thermal Residual Stresses in Unsymmetrical Multi-Stable Composite Laminates

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Declaration

The substance of this thesis is the original work of the author and due reference and acknowledgement has been made, where necessary, to the work of others. No part of this thesis has already been submitted for any degree and is not being concurrently submitted in candidature for any degree.

Candidate: ___________________ Supervisor 1: ___________________ Supervisor 2: ___________________

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Date: ___________________ Date: ___________________ Date: ___________________
To John, Patricia and Tania,

For all your love and support
Abstract

In this study, an approach to predict and analyse the effects of moisture ingress on residual stresses in multi-stable composite laminates is developed. Residual stresses are a common consequence of the manufacturing process of composite laminates (e.g. formed thermally, following cool-down from manufacture). Imbalance in these stresses about the mid-plane can lead to warping, and so composite laminates are usually restricted to symmetrical lay-ups. In certain cases, unbalanced residual stresses can be used advantageously, such as in novel morphing structures by use of multi-stable parts. These are parts which feature two or more stable shape configurations, which are obtainable through a force application. With energy only being required to alternate between shapes, multi-stable laminates have been proposed as morphing aerodynamic surfaces for aerospace and wind-energy applications. In these cases (and others in which the laminates are sensitive to residual stresses such as thin plates, ply drop off and bonded repairs) a thorough understanding of the residual stresses (both as-manufactured and in-service) is required.

The residual stresses in fibre-resin composites are known to be sensitive to environmental effects, which can be encountered under in-service conditions. One such effect is moisture absorption, which alters the residual stress state of a laminate through matrix swelling and plasticisation. These changes may lead to a change in the laminate’s shape, and in the case of multi-stable laminates, a change in the multi-stable behaviour. Applications based upon these unsymmetrical laminates therefore require consideration to moisture effects at a design stage.

In this work, a combined numerical/experimental approach is presented whereby the macro-scale through-thickness residual stresses of dry and saturated unsymmetrical composite laminates can be predicted and analysed. A range of unsymmetrical laminates were manufactured from carbon-fibre reinforced plastic (unidirectional continuous fibres pre-impregnated in a polymer-resin matrix), featuring both square cross-ply and tailored (i.e. featuring local variations in lay-up and/or thickness – representative of laminates that would be used in complex applications) laminate configurations. Following manufacture, the dry laminate shapes were measured, with, in the case of the tailored laminates, laser scanning – a full-field, non-contact surface
measuring technique. Three-dimensional continuum based finite element models were created (using the software Abaqus) to simulate the thermal deformation of the laminates. The models were benchmarked using analytical approaches, and subsequently calibrated to match the experimentally measured laminate shapes by means of equivalent orthotropic thermal expansion coefficients, negating the need to account for individual residual stress contributors. Subsequently, laminates were immersed in water until saturation, and the change in shape due to matrix swelling was measured. The numerical models were then adapted to take into account moisture induced matrix swelling by use of the analogy between thermal expansion and moisture induced swelling. Subsequently, the variation in shape and residual stress distribution in the laminates following moisture saturation could be analysed. Using laser scanning to measure the tailored laminate shapes allowed for a detailed analysis of the full-field variation between numerically-predicted and experimentally-measured laminate shapes.

Using this analysis technique, macro-scale through-thickness residual stress profiles were extracted for each of the cross-ply and tailored laminate configurations. It was found that peak residual stresses can drop by over 70% following moisture saturation resulting in a significant loss of curvature. Likewise, laminate potential energy can drop by over 90%, impacting upon the laminates multi-stable behaviour.
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Table of contents

Abstract ................................................................................................................................ i
Table of contents .................................................................................................................. v
Nomenclature ................................................................................................................... viii
List of Figures .................................................................................................................... x
List of Tables...................................................................................................................... xv

Chapter 1  Introduction.................................................................................................... 1
  1.1  Background ........................................................................................................... 1
  1.2  Objectives and Methodology ............................................................................... 10
  1.3  Overview of Thesis ............................................................................................. 11
  1.4  Dissemination ...................................................................................................... 12

Chapter 2  Literature Review ......................................................................................... 14
  2.1  Introduction ......................................................................................................... 14
  2.2  Applications of Multi-stable Laminates .............................................................. 14
  2.3  Predicting Cured Laminate Shapes ..................................................................... 21
  2.4  Manufacturing Effects ......................................................................................... 37
  2.5  Moisture Absorption Effects ............................................................................... 43
  2.6  Measuring Laminate Shapes and Residual Stresses ............................................ 47
  2.7  Summary ............................................................................................................. 49

Chapter 3  Analytical Solution ....................................................................................... 51
  3.1  Introduction ......................................................................................................... 51
  3.2  Predicting Multi-stable Shapes of Unsymmetrical Composite Laminates........ 52
  3.3  Analytical Tool for Shape Prediction .................................................................. 57
  3.4  Summary ............................................................................................................. 58

Chapter 4  Experiments ................................................................................................ 60
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>60</td>
</tr>
<tr>
<td>4.2</td>
<td>Manufacturing of Composite Laminates</td>
<td>60</td>
</tr>
<tr>
<td>4.3</td>
<td>Material Characterisation</td>
<td>64</td>
</tr>
<tr>
<td>4.4</td>
<td>Unsymmetrical Laminates</td>
<td>67</td>
</tr>
<tr>
<td>4.4.1</td>
<td>Cross-ply Laminates</td>
<td>67</td>
</tr>
<tr>
<td>4.4.2</td>
<td>Tailored Laminates</td>
<td>69</td>
</tr>
<tr>
<td>4.5</td>
<td>Moisture Absorption</td>
<td>74</td>
</tr>
<tr>
<td>4.6</td>
<td>Summary</td>
<td>77</td>
</tr>
<tr>
<td>Chapter 5</td>
<td>Numerical Model</td>
<td>79</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>79</td>
</tr>
<tr>
<td>5.2</td>
<td>Model Description</td>
<td>79</td>
</tr>
<tr>
<td>5.3</td>
<td>Solution Strategy for Cross-ply Laminates</td>
<td>80</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Dry Laminate Shapes</td>
<td>80</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Saturated Laminate Shapes</td>
<td>83</td>
</tr>
<tr>
<td>5.4</td>
<td>Solution Strategy for Tailored Laminates</td>
<td>84</td>
</tr>
<tr>
<td>5.4.1</td>
<td>Numerical model development</td>
<td>84</td>
</tr>
<tr>
<td>5.4.2</td>
<td>Comparison of Measured and Predicted Tailored Laminate Shapes</td>
<td>86</td>
</tr>
<tr>
<td>5.4.3</td>
<td>Tailored Laminates: Refined approach</td>
<td>91</td>
</tr>
<tr>
<td>5.4.4</td>
<td>Saturated Tailored Laminate Shapes</td>
<td>100</td>
</tr>
<tr>
<td>5.5</td>
<td>Summary</td>
<td>105</td>
</tr>
<tr>
<td>Chapter 6</td>
<td>Extracted Through-Thickness Stress Profiles</td>
<td>107</td>
</tr>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>107</td>
</tr>
<tr>
<td>6.2</td>
<td>Cross-ply Laminates</td>
<td>107</td>
</tr>
<tr>
<td>6.3</td>
<td>Tailored Laminates</td>
<td>112</td>
</tr>
<tr>
<td>6.3.1</td>
<td>Procedure used to obtain stress profiles</td>
<td>112</td>
</tr>
<tr>
<td>6.3.2</td>
<td>Dry Stress Profiles</td>
<td>113</td>
</tr>
<tr>
<td>6.3.3</td>
<td>Saturated Stress Profiles</td>
<td>126</td>
</tr>
<tr>
<td>6.4</td>
<td>Summary</td>
<td>138</td>
</tr>
</tbody>
</table>
Chapter 7  Discussion .................................................................................................. 140
  7.1 Thermal Expansion and Swelling Coefficients ............................................. 140
  7.2 Modelling and Experimental Approach ........................................................ 144
  7.3 Sensitivity of Residual Stresses to Modelling Parameters ............................. 148
  7.4 Predicting the Transient Response using a Piecewise Approach ...................... 149

Chapter 8  Conclusions and Future Work................................................................. 153
  8.1 Conclusions ....................................................................................................... 153
  8.2 Recommendations for Future Work ................................................................. 156

References .............................................................................................................. 159

Appendix A: Matlab Code for Multi-stable Shape Calculator ......................... A-1
Appendix B: Comparison of Experimental and Numerical Laminate Shapes ......... B-1
Appendix C: Mechanical Testing .............................................................................. C-1
Appendix D: Cured and Saturated Laminate Shapes .............................................. D-1
Nomenclature

**Upper case letters**

- $A_{ij}$: Extensional stiffness terms
- $B_{ij}$: Coupling stiffness terms
- $D_{ij}$: Bending stiffness terms
- $E$: Young’s modulus
- $G$: Shear modulus
- $L_i$ ($i=x,y$): Side length along $x$-axis or $y$-axis
- $M_i^T$ ($i=x,y$): Thermally induced moment resultant
- $N_i^T$ ($i=x,y$): Thermally induced axial force resultant
- $Q_{ij}$: Reduced laminate stiffness terms
- $\tilde{Q}_{ij}$: Transformed laminate stiffness terms
- $T$: Temperature

**Lower case letters**

- $a$: Curvature along $x$-axis
- $b$: Curvature along $y$-axis
- $c$: Value related to analytical solution, defined by equation 3.10
- $c^*$: Moisture content
- $d$: Value related to analytical solution, defined by equation 3.11
- $h$: Out-of-plane deflection
- $l_c$: Chord length
- $p$: Partition length (for tailored laminates)
- $r$: Radius of curvature
- $t_l$: Laminate thickness
- $t_e$: Element thickness
- $t_p$: Ply thickness
- $u^0$: Laminate mid-plane deflection along $x$-axis
- $v^0$: Laminate mid-plane deflection along $y$-axis
- $w$: Laminate out-of-plane deflection
- $w^*$: Normalised variation between experimental and numerical out-of-plane co-ordinate
- $w_e$: Experimentally measured out-of-plane co-ordinate
- $w_n$: Numerically predicted out-of-plane co-ordinate
Greek letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_L$</td>
<td>Material longitudinal thermal expansion coefficient</td>
</tr>
<tr>
<td>$\alpha_T$</td>
<td>Material transverse thermal expansion coefficient</td>
</tr>
<tr>
<td>$\beta_L$</td>
<td>Material longitudinal moisture-induced swelling coefficient</td>
</tr>
<tr>
<td>$\beta_T$</td>
<td>Material transverse moisture-induced swelling coefficient</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Corrected variation between experimental and numerical out-of-plane co-ordinate</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Strain</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Direct stress</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Laminate total potential energy</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Laminate strain energy density</td>
</tr>
</tbody>
</table>
List of Figures

Figure 1-1 Possible cured shapes of unsymmetrical cross-ply laminates..........................3
Figure 1-2 Concept for variable camber trailing edge, featuring locally unsymmetrical and multi-stable lay-up (green areas) surrounded by a symmetrical lay-up (purple areas). From Mattioni et al. (2007)...............................9
Figure 2-1 Various intermediate swept-back positions for a multi-stable (variable sweep) wingbox structure. From Mattioni et al. (2006). .............................................15
Figure 2-2 Concept for a morphing winglet utilising a tailored laminate construction, with (a) planform and lay-up of the winglet; (b) manufactured laminate in extended configuration; (c) manufactured laminate in deployed configuration. Adapted from Mattioni et al. (2007). ............16
Figure 2-3 Integration of multi-stable laminates into bi-stable helicopter blade flap. From Daynes and Weaver (2013). ..................................................................................17
Figure 2-4 Multi-stable air inlet: (a) closed state and; (b) open state. Adapted from Daynes et al. (2011). .............................................................................................................18
Figure 2-5 Examples of tailored laminate configurations (a) with piecewise ply variation; and (b) with uni-directional strips embedded within a unidirectional ply. Adapted from Potter and Weaver (2004).................................20
Figure 2-6 Room temperature shapes of a square cross-ply laminate with a [0/90] lay-up, predicted by Hyer (1981a). .................................................................23
Figure 2-7 Lay-up sequence of bi-stable tailored laminate. From Mattioni et al. (2008). .......................................................................................................................32
Figure 2-8 (a) Variable stiffness morphing laminate based on curvilinear fibres; (b) Straight-fibre laminate. From Sousa et al. (2013). .........................................................35
Figure 2-9 Design curve of MFC actuated bi-stable plates. From Portela et al. (2008). .........................................................................................................................45
Figure 3-1 Curvature definitions for square cross-ply laminates.....................................52
Figure 3-2 Example of the predicted multi-stable shapes of a square cross-ply laminate, using extended CLT. .......................................................................................58
Figure 4-1 Bagging technique used in the manufacture of the CFRP laminates............61
Figure 4-2 Measured cure cycle for the manufactured laminates, showing temperature as a function of time. Locations 1 and 2 refer to temperature readings above and below a laminate at one end of tool-plate, with Locations 3 and 4 being taken above and below a laminate at the other end of the tool-plate. ................................................................................................................62
Figure 4-3 Curvature observed in symmetrical [0] laminates. Note: peel-ply is still bonded to bottom laminate surface, but showed no influence to laminate shape once removed. ....................................................................................63
Figure 4-4 Transverse and longitudinal CFRP specimens used in dilatometry testing...............................................................................................................................65
Figure 4-5 Thermal straining of HTA 6376 material measured using dilatometry .......65

Figure 4-6 Cured shape of the manufactured cross-ply laminates.................................68

Figure 4-7 Unsymmetrical cross-ply laminate: (a) cured cylindrical shape with generator about the y-axis and (b) measured parameters to calculate curvatures. ...........................................................................................................68

Figure 4-8 Typical configuration of manufactured tailored laminates, featuring partition along the x-axis. In all cases, \( l_x = 200 \text{ mm} \), \( l_y = 100 \text{ mm} \). Only cross-ply (0° or 90°) ply orientations were used.......................................................69

Figure 4-9 Lay-up configuration and cured shapes of tailored Laminate 1-4. ..............71

Figure 4-10 Lay-up configuration and cured shapes of tailored Laminate 5 and 6. .......72

Figure 4-11 Laser scanning experimental setup, depicting: (a) computer system with David-Laserscanner software; (b) hand-held, flat-line laser; (c) Trust HD webcam; (d) scanning background; and (e) object being scanned. Note: Laser is shown clamped for illustrative purposes. During scanning, the laser is swept by hand....................................................................................73

Figure 4-12 Moisture absorption over time for the cross-ply laminates. Averaged values are used for each lay-up family. Circled values are considered outliers and not used. At saturation, \( c^{*}_{[90_2/0_2]} = 0.69 \text{ wt.\%}^{-1} \), \( c^{*}_{[90_3/0_1]}, [90_3/0_2] = 0.75 \text{ wt.\%}^{-1} \). ........................................................................................75

Figure 4-13 Average curvatures (a) of cross-ply laminates in dry and saturated conditions. The percentage loss in curvature is given in red. .............................76

Figure 4-14 Moisture absorption over time for the tailored laminates. ..........................77

Figure 5-1 Finite element modelling strategy for cross-ply laminates with: (a) sensitivity of through-thickness residual stresses to element aspect ratio (all stresses are in MPa); (b) FE strategy showing geometrical imperfection; (c) constant element size used with three elements per ply thickness. .............................................................................................................82

Figure 5-2 Sensitivity of stress profiles to mesh aspect ratio. ........................................85

Figure 5-3 Calculation of variation between measured and predicted laminate shapes. Note: distorted areas highlighted with circles result from the supporting mechanism used in experiments being included in measurements. .................................................................87

Figure 5-4 Variation between experimentally measured and numerically predicted shapes. The cured shapes of the manufactured laminates are given for reference. Note: Values on \( w^* \) axis highlight minimum and maximum values of \( w^* \). .........................................................................................................................88

Figure 5-5 Stress-free shape of Laminate 1 showing initially curved state, along the x-axis only. .................................................................................................................93

Figure 5-6 Laminate 1, variation between experimentally-measured and numerically-predicted laminate shapes: (a) FE prediction, configuration A; (b) FE prediction, configuration B; (c) variation between the numerical and experimental shape, configuration A; and (d) variation between the numerical and experimental shape, configuration B. .................................................................94
Figure 5-7 Laminate 2, variation between experimentally-measured and numerically-predicted laminate shapes: (a) FE prediction, configuration A; (b) FE prediction, configuration B; (c) variation between the numerical and experimental shape, configuration A; and (d) variation between the numerical and experimental shape, configuration B. ............................................. 95

Figure 5-8 Laminate 3, variation between experimentally-measured and numerically-predicted laminate shapes: (a) FE prediction, configuration A; (b) FE prediction, configuration B; (c) variation between the numerical and experimental shape, configuration A; and (d) variation between the numerical and experimental shape, configuration B. ............................................. 96

Figure 5-9 Laminate 4, variation between experimentally-measured and numerically-predicted laminate shapes: (a) FE prediction; (b) variation between the numerical and experimental shape. ..................................................................................................................... 97

Figure 5-10 Laminate 5, variation between experimentally-measured and numerically-predicted laminate shapes: (a) FE prediction, configuration A; (b) FE prediction, configuration B; (c) variation between numerical and experimental shape, configuration A; and (d) variation between the numerical and experimental shape, configuration B. ............................................. 98

Figure 5-11 Laminate 6, variation between experimentally-measured and numerically-predicted laminate shapes: (a) FE prediction; (b) variation between the numerical and experimental shape, configuration. ......................... 99

Figure 5-12 Calibration of numerical model of Laminate 1, saturated condition. The subscript \( e \) refers to the experimentally-measured shape, the subscript \( n \) refers to the numerically-predicted shape, the subscript \( d \) refers to the dry laminate, and the subscript \( s \) refers to the saturated laminate. .................... 102

Figure 6-1 Laminate through-thickness stress distribution: (a) laminate orientation with curvature developed along the x-axis and (b) Laminate cross section showing example of resulting through-thickness normal stresses \( \sigma_{xx} \) and \( \sigma_{yy} \). ..................................................................................................................... 107

Figure 6-2 Through-thickness residual stress profiles \((\sigma_{xx}, \sigma_{yy})\) of manufactured laminates \([90_2/0_2], [90_3/0_1]\) and \([90_3/0_2]\) for dry and saturated laminate conditions (all stresses are in MPa). .................................................................................... 109

Figure 6-3 Through-thickness residual stresses for Laminate 1 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) cured shapes; (c) \( \sigma_{xx} \), Shape A; (d) \( \sigma_{yy} \), shape A; (e) \( \sigma_{xx} \), Shape B; (f) \( \sigma_{yy} \), Shape B ..................................................................................................................... 114

Figure 6-4 Through-thickness residual stresses for Laminate 2 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) cured shapes; (c) \( \sigma_{xx} \), Shape A; (d) \( \sigma_{yy} \), shape A; (e) \( \sigma_{xx} \), Shape B; (f) \( \sigma_{yy} \), Shape B ..................................................................................................................... 116

Figure 6-5 Through-thickness residual stresses for Laminate 3 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) cured shapes; (c) \( \sigma_{xx} \), Shape A; (d) \( \sigma_{yy} \), shape A; (e) \( \sigma_{xx} \), Shape B; (f) \( \sigma_{yy} \), Shape B ..................................................................................................................... 118
Figure 6-6 Through-thickness residual stresses for Laminate 4 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) cured shape; (c) $\sigma_{xx}$; (d) $\sigma_{yy}$ .............................. 120

Figure 6-7 Through-thickness residual stresses for Laminate 5 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) cured shapes; (c) $\sigma_{xx}$, Shape A; (d) $\sigma_{yy}$, shape A; (e) $\sigma_{xx}$, Shape B; (f) $\sigma_{yy}$, Shape B................................................. 122

Figure 6-8 Through-thickness residual stresses for Laminate 6 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) cured shape; (c) $\sigma_{xx}$; (d) $\sigma_{yy}$ ................................. 124

Figure 6-9 Saturated through-thickness residual stresses for Laminate 1 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) dry and saturated shapes; (c) $\sigma_{xx}$, extracted at Positions 1 and 2; (d) $\sigma_{yy}$, extracted at Positions 1 and 2; (e) $\sigma_{xx}$, extracted at Positions 3 and 4; (f) $\sigma_{yy}$, extracted at Positions 3 and 4.................................................... 128

Figure 6-10 Saturated through-thickness residual stresses for Laminate 2 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) dry and saturated shapes; (c) $\sigma_{xx}$, extracted at Positions 1 and 2; (d) $\sigma_{yy}$, extracted at Positions 1 and 2; (e) $\sigma_{xx}$, extracted at Positions 3 and 4; (f) $\sigma_{yy}$, extracted at Positions 3 and 4.................................................... 130

Figure 6-11 Saturated through-thickness residual stresses for Laminate 3 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) dry and saturated shapes; (c) $\sigma_{xx}$, extracted at Positions 1 and 2; (d) $\sigma_{yy}$, extracted at Positions 1 and 2; (e) $\sigma_{xx}$, extracted at Positions 3 and 4; (f) $\sigma_{yy}$, extracted at Positions 3 and 4.................................................... 132

Figure 6-12 Saturated through-thickness residual stresses for Laminate 4 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) dry and saturated shapes; (c) $\sigma_{xx}$, extracted at Positions 1 and 2; (d) $\sigma_{yy}$, extracted at Positions 3 and 4................................. 134

Figure 6-13 Saturated through-thickness residual stresses for Laminate 5 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) dry and saturated shapes; (c) $\sigma_{xx}$; (d) extracted $\sigma_{yy}$ ...... 135

Figure 6-14 Saturated through-thickness residual stresses for Laminate 6 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) dry and saturated shapes; (c) $\sigma_{xx}$, extracted at Positions 1 and 2; (d) $\sigma_{yy}$, extracted at Positions 1 and 2; (e) $\sigma_{xx}$, extracted at Positions 3 and 4; (f) $\sigma_{yy}$, extracted at Positions 3 and 4. Note: Due to the varying laminate thickness a different scale is used on the z axis of (c) – (f)............. 137

Figure 7-1 Predicted residual stress profiles for $[90_2/0_2]$, $[90_3/0_1]$ and $[90_3/90_2]$ square laminates using piecewise diffusion (top and bottom plies saturated). Values refer to the ‘diffusion’ stress profile (all stresses are in MPa)......................................................... 151

Figure C-1 Specimens used for tensile tests. Three longitudinal, four transverse and two ±45° specimens were used................................................................. C-2

Figure C-2 Stress-strain results for the longitudinal specimens................................. C-3
Figure C-3 Stress-strain results for the transverse specimens.................................C-3
Figure C-4 Stress-strain results for the ±45° specimens ........................................C-4

Figure D-1 Laser scan data, Laminate 1 with: (a) Shape A; (b) Shape B; (c) Shape B, saturated; (d) cross-sectional profile, Shape A, at y=-50 mm; (e) cross-sectional profiles, Shape B, at y=-50 mm; (f) cross-sectional profile, Shape A, at x=100 mm; (g) cross-sectional profile, Shape B, at x=100 mm. Contour plot depicts out-of-plane deflection (z). ..............................................D-2

Figure D-2 Laser scan data, Laminate 2 with: (a) Shape A; (b) Shape B; (c) Shape B, saturated; (d) cross-sectional profile, Shape A, at y=-50 mm; (e) cross-sectional profiles, Shape B, at y=-50 mm; (f) cross-sectional profile, Shape A, at x=100 mm; (g) cross-sectional profile, Shape B, at x=100 mm. Contour plot depicts out-of-plane deflection (z). ..............................................D-3

Figure D-3 Laser scan data, Laminate 3 with: (a) Shape A; (b) Shape B; (c) Shape B, saturated; (d) cross-sectional profile, Shape A, at y=-50 mm; (e) cross-sectional profiles, Shape B, at y=-50 mm; (f) cross-sectional profile, Shape A, at x=100 mm; (g) cross-sectional profile, Shape B, at x=100 mm. Contour plot depicts out-of-plane deflection (z). ..............................................D-4

Figure D-4 Laser scan data, Laminate 4 with: (a) Shape B; (b) Shape B, saturated; (c) cross-sectional profiles, at y=-50 mm; (d) cross-sectional profiles at x=100 mm. ........................................................................................................D-5

Figure D-5 Laser scan data, Laminate 5 with: (a) Shape A; (b) Shape B; (c) Shape B, saturated; (d) cross-sectional profile, Shape A, at y=-50 mm; (e) cross-sectional profiles, Shape B, at y=-50 mm; (f) cross-sectional profile, Shape A, at x=100 mm; (g) cross-sectional profile, Shape B, at x=100 mm. Contour plot depicts out-of-plane deflection (z). ..............................................D-6

Figure D-6 Laser scan data, Laminate 6 with: (a) Shape B; (b) Shape B, saturated; (c) cross-sectional profiles, at y=-50 mm; (d) cross-sectional profiles at x=100 mm. ........................................................................................................D-7
List of Tables

Table 2-1 Maximum change in curvature for a ±5% variation in each design variable. Adapted from Brampton et al. (2013). ................................................................. 32
Table 2-2 Laminate level imperfections which can influence geometry of manufactured CFRP laminates. From Ochinero and Hyer (2002) ......................... 39
Table 4-1 Ply properties of HexPly HTA 6376. .......................................................... 66
Table 4-2 Moisture content ($c^*$) at saturation of the tailored laminates. .................. 77
Table 5-1 Material properties used in numerical models. ............................................. 81
Table 5-2 Predicted curvatures of dry laminates after calibration ($\alpha_L = 5.5\times10^{-6}$ K$^{-1}$). ................................................................. 83
Table 5-3 Predicted curvatures of saturated laminates after calibration ($\beta_T = 3.65\times10^{-3}$ wt.%$^{-1}$) ................................................................. 84
Table 5-4 Calibrated equivalent $\alpha_L$ used in numerical models. .......................... 87
Table 5-5 $\alpha_L$ values required to describe laminate shapes for the initial and refined model ................................................................. 100
Table 5-6 Strain energy (SE) for tailored laminates in dry and saturated states ....... 104
Table C-1 Test matrix for tensile tests ................................................................. C-2
Table C-2 Material property results ................................................................. C-6
Chapter 1  Introduction

1.1 Background

The use of Carbon Fibre Reinforced Plastic (CFRP) in engineering applications has become increasingly popular in recent times, particularly when applied to high performance structures where its high strength and low weight make it an excellent candidate material. As such, it is used extensively in manufacturing sectors such as aerospace, motorsport and wind energy. A popular method of manufacturing composite parts involves using pre-preg plies (sheets of parallel carbon fibres already impregnated in a polymer resin), which are stacked to create a laminate. Stacking these plies (i.e. ‘laying up’) in a particular order, with a particular fibre orientation, allows engineers to tailor the properties of the laminate to suit the loading that a structure will experience in service. Once stacked, the laminate is cured in an autoclave at an elevated temperature and pressure. This cure at an elevated temperature (e.g. 178°C) and the subsequent cool-down to room-temperature results in the development of residual stresses, caused by the unequal thermal expansion of the composite’s constituent materials (i.e. the carbon fibre and resin). These stresses can be significant and, if ignored, can have a detrimental effect on a component (e.g. leading to unexpected failure or warping of a component). Optimisation of engineering components requires consideration of the combined effect of residual and applied stresses as a result (Withers and Bhadeshia 2001).

It is possible to use the thermally induced residual stress state beneficially to create laminates that display a multi-stable property. Multi-stable structures are described by Coburn et al. (2013) as being “elastic objects that exhibit more than one equilibrium configuration, only requiring actuation between stable states. These adaptive structures can provide stiffness and strength whilst allowing appreciable shape change (in a highly non-linear way)”. In general, this property is achieved during the manufacturing process, by tailoring the stress state within the laminate in such a way as to have multiple configurations for which the total potential energy is locally minimized (Eckstein et al. 2013). One of the most popular methods of manufacturing multi-stable laminates uses the difference in the Coefficients of Thermal Expansion (CTE) between the longitudinal (0°) and transverse (90°) direction of a CFRP ply coupled with an unsymmetrical lay-up sequence. When two adjacent plies are stacked with different orientations, the ply’s orthogonal CTE values lead to the development of residual
stresses between the two plies. Depending upon the symmetry of the laminate’s lay-up sequence, these residual stresses may lead to warping from the mould shape.

The orientation of individual plies about the laminate’s mid-plane determines the laminate’s symmetry, with the mid-plane being a plane that lies mid-way through the laminate thickness. Having the orientation of each ply mirrored about the mid-plane (e.g. \([\theta^\circ/-\theta^\circ/-\theta^\circ/\theta^\circ]\)) leads to a symmetrical lay-up sequence, which in turn results in the development of residual stresses which are symmetrical about the mid-plane. As any moments about the mid-plane caused by the residual stresses are balanced, the cured laminate will maintain its moulded shape (barring any manufacturing imperfections or environmental effects). In general, symmetric lay-up sequences are preferred as it is easier to predict the part shape and they don’t display any coupling effects under load. Such coupling can be seen through the laminate’s stiffness matrix (e.g. bend-stretching, leading to bending of the laminate in response to an in-plane load) and is a generally unwanted consequence of unsymmetrical lay-up sequences.

Multi-stable laminates using thermal residual stresses purposely use an unsymmetrical lay-up to achieve the multi-stable property. The unsymmetrical lay-up coupled with the drop in temperature following cure results in the laminate warping from its moulded shape. By meeting certain conditions (e.g. lay-up, length-to-thickness ratio), it is possible to achieve multi-stability. An example of this behaviour is shown in Figure 1-1. A flat laminate with an unsymmetrical lay-up sequence is prepared (Figure 1-1(a)). This is then cured in an autoclave, where it consolidates at an elevated temperature, maintaining the flat mould shape (Figure 1-1 (b)). Once removed from the autoclave and cooled down to room temperature, the laminate will warp due to the unsymmetrical lay-up sequence. Depending on the geometry (particularly, the length-to-thickness ratio), the laminate will either obtain a saddle shape (Figure 1-1(c)), or be multi-stable (Figure 1-1(d), (e)). For a square \([0_2/90_2]\) laminate, the two multi-stable shapes obtainable are cylindrical, with their generators lying on two mutually perpendicular axes and with opposite curvature. It is possible to alternate between these shapes by applying a bending moment, resulting in a ‘snap-through’ event.
Introduction

Figure 1-1 Possible cured shapes of unsymmetrical cross-ply laminates.

Normally, Classical Laminate Theory (CLT) can be used to predict the room temperature shapes of such laminates. For a cross-ply laminate (e.g. the [0\,/90\,\,2\,/90\,\,2] lay-up shown in Figure 1-1), CLT predicts a ‘saddle’ shape, with equal and opposite curvatures about both the $x$ and $y$ axes (see Figure 1-1(c)). This is contrary to the observed multi-stable shapes (Hyer 1981b). To this end, the CLT was extended by Hyer (1981a) to include non-linearity in the mid-plane strain formulation, which is required due to the high out-of-plane deformation achieved. This is coupled with a Rayleigh-Ritz approach, using assumed laminate shape functions, to minimise the laminate’s potential energy, resulting in a prediction of the multi-stable laminate shapes. This theory has been widely used as a basis by subsequent authors in predicting the behaviour of multi-stable laminates.
Recent research efforts aim to understand the behaviour of multi-stable shapes, under such areas as shapes obtainable, snap-through behaviour and environmental effects (see for example, Pirrera et al. (2010), Tawfik et al. (2006), Sousa et al. (2013), Mattioni et al. (2009)) with the aim of exploiting the behaviour for use in novel adaptive or ‘morphing’ structures.

Several innovative engineering applications have been proposed, from morphing air inlets (Daynes et al. 2011) and conduits (Dano 1997), to morphing aircraft wings (Falcão et al. 2009, Mattioni et al. 2006), helicopter blades (Daynes and Weaver 2013) and wind turbine blades (Daynes and Weaver 2012). Morphing structures are able to change shape seamlessly to maximise the structures efficiency for a variety of different conditions, such as all stages of flight for an aircraft (Thill et al. 2008). Traditionally, the use of morphing structures has been hindered by the conflicting requirements of adequate load bearing capability and low activation loads. Any ‘brute force’ approaches, whereby energy is used to deform the structure and then hold a particular shape configuration, require a constant load to be applied, resulting in a potentially heavy and complex actuation system. In this regard, multi-stable laminates can offer an attractive solution, as energy is only required during the snap-through process, with the laminate maintaining the new shape configuration without any additional load or energy (Davis et al. 2008). The associated reduction in actuator requirements may render morphing structures feasible. Additionally, the laminate can be used internally as part of a mechanism for adapting shapes, or as a seamless morphing ‘skin’ – which is advantageous in aerodynamic applications (Thill et al. 2008). The use of composite materials (traditionally, CFRP) further enhances their attractiveness due to the high strength and low weight of the material. The potential for such a design has been well documented (e.g. Turnock et al., 2009).

In general, the research concerning multi-stable composites has focused upon the ‘as-cured’ condition, immediately after the laminate has been manufactured. Any application in an adaptable structure also requires knowledge of the behaviour of such laminates when exposed to in-service conditions. For example, a morphing laminate used in an aeronautical application can be exposed (and thus needs to be resistant) to different weather conditions, fuel/chemical spillage and local damage during the course of its use (Kikuta 2003). Consequently, the effect of these factors needs to be understood for each potential application, to judge if the adaptable structure’s behaviour
is compromised in any way. One particular environmental condition that is known to affect the residual stress state (and thus potentially the multi-stable behaviour) of composite laminates is moisture absorption (Benkeddad et al. 1996, Benkhedda et al. 2008). Moisture absorption in CFRP laminates can occur following exposure to ambient humidity or due to direct exposure to water. As such, the possibility of moisture absorption during service is quite high, and so advanced consideration is required at a design stage of an adaptable structure to take these effects into account.

The changes to the residual stress state in a CFRP laminate is caused by swelling of the matrix (polymer) material following exposure to moisture. This swelling reduces the thermal contraction of the matrix which is developed following cool-down after the curing cycle. This is coupled with a degradation in material properties due to matrix plasticisation (Choi et al. 2001). This results in a reduction in the residual stresses, and thus in the case of unsymmetrical laminates, manifests itself as a reduction in the curvature developed. This loss of curvature obviously impacts upon the ability of multi-stable laminates to be part of a morphing structure.

The swelling induced by moisture absorption is a matrix material based phenomenon, and involves processes of moisture penetrating the laminate (diffusion), followed by swelling. Diffusion in the laminate is a complex phenomenon, and is described by Youssef et al. (2009) as follows: "there exists the thermally agitated motion of chain segments providing penetrant-scale transient gaps (free volume) in the polymer matrix allowing penetrants to diffuse into the bulk of the material."

A simplified explanation involves molecular scale voids, which penetrant molecules (i.e. water) are able to occupy. With enough thermal energy, the water molecules can ‘jump’ to the next void, leaving a vacant void for the next molecule to occupy. This movement progresses through the laminate thickness, resulting in diffusion (Crank and Park 1968). On top of that, the existence of any imperfections or damage within the matrix (e.g. channels or cracks) can lead to transport of water through the laminate. The resulting swelling is described by Adamson (1980) as follows. The swelling that results is due to the presence of the water molecule within the matrix material. Specifically, “because the water molecule is polar, it is capable of forming hydrogen bonds with hydroxyl groups, thereby disrupting inter-chain hydrogen bonding with the net effect of increasing the inters-segmental hydrogen bond length“.
It has been suggested that water molecules can exist in the polymer material in two states: bound and unbound to the polymer molecule (Adamson 1980). Bound molecules are responsible for swelling as they interrupt inter-chain hydrogen bonding, and are immobilized. The unbound molecules are relatively free to travel through the free-volume voids, and do not contribute to swelling. Finally, Adamson (1980) states that it is generally assumed that the network structure of cross-linked epoxy resins is homogenous. However, optical microscopy reveals the existence of a two-phase network structure. This two-phase network structure comprises highly cross-linked micro-gel particles (or micelles) embedded in a less highly cross-linked matrix. Because of their high density, the micelles are less easily penetrated by water than the surrounding matrix.

Predicting and analysing the behaviour of residual stresses due to hygro-thermal loading is a very complex task, and as such leads to generalized approaches. A brief description of the approaches and the inherent assumptions will be given. Firstly, as previously mentioned, the thermally induced residual stress state is due to the different thermal expansion characteristics of the constituent materials of CFRP; that is, the carbon fibre and the resin matrix material. This leads to residual stresses being developed at two different scales. At a macro-scale level, the different thermal contraction behaviour between adjacent plies orientated differently leads to interlaminar stresses. These stresses equilibrate over macro-scale length scales, and are referred to as ‘type 1’ stresses (Withers and Bhandeshia 2001). At a micro-scale level, the different thermal expansion behaviour between the fibre and the resin of the composite material results in stresses which equilibrate over much smaller distances. These stresses are formed both transverse to the fibre direction, as well as a parallel to the fibre direction. The net result of these two stress types is a complex multi-scale tri-axial stress state. Adding to the complexity is the inherent imperfect nature of a composite material, with fibre waviness, variations in local fibre volume fraction, fibre distribution, matrix cracking and so forth all impacting upon the residual stress state – at both stress scales. Measuring and accounting for such effects in modelling is extremely challenging, and so modelling techniques generally make assumptions in this area.

A commonly used theory to examine such deformation behaviour (and thus, residual stresses) is bimetallic strip theory (Timoshenko 1925). In this theory, two homogenous and isotropic materials with different thermal expansion and mechanical properties are
bonded perfectly to each other to form a strip, before being subjected to a temperature change. The through-thickness residual stresses imposed upon both materials and the resulting deformation of the strip can be calculated by reducing the problem to equivalent axial loads and bending moments that act on each section. Although the thermally induced stresses within a composite are much more complex, the analysis of thermal deformation of composite laminates takes a similar approach to that presented by Timoshenko, in that each ply is reduced to homogenised orthotropic material properties (including CTEs). CLT then idealises the problem in the same way as Timoshenko, by creating a composite structure comprising these homogenous layers to calculate the thermally induced residual stresses following cure (Jones 1998, Hyer 1998). This use of homogenous material properties focuses on macro-scale inter-lamina stresses, and ignores the micro-scale stresses.

The approach is then adapted to account for the moisture swelling response of a laminate. The swelling characteristics are homogenised to a ply level, leading to orthotropic values for the material’s Moisture Swelling Coefficient (MSC) (Hyer 1998). These are $\beta_L$ (longitudinal) and $\beta_T$ (transverse). In the longitudinal direction, the fibre is often assumed to be impermeable, and thus doesn’t undergo swelling due to moisture (i.e. $\beta_L$ is assumed to be zero). The resulting swelling coefficient ($\beta_T$) is then used in conjunction with the ply’s moisture content to determine the moisture induced strain of the ply. This is analogous to the study of thermal expansion effects, whereby a coefficient (CTE) is multiplied by the temperature to calculate the thermal straining.

When applied to studying the hygro-thermal stresses of composite laminates, the described modelling approach has some important aspects which should be considered. Firstly, the complex, tri-axial and multi-scale stress state is not precisely reproduced, as, most significantly, no micro-scale stresses are considered. The assumption is widely accepted when studying cured laminate shapes, and is integral to theoretical approaches which are widely used to predict the stress state and cured shapes of composite laminates (i.e. CLT). Furthermore, any other contributions to residual stress (such as chemical shrinkage of the matrix material or manufacturing effects (Parlevliet et al. 2006)) are generally ignored. Similarly, such assumptions may have implications when moisture diffusion is considered. It has been suggested that moisture uptake is a function of residual stress state (Youssef et al. 2009). In this instance, the micro-scale stress state can play an important role. Local areas of increased tension, for example,
could lead to an increase in free volume available for moisture uptake, due to stress induced matrix cracking (Parlevliet et al. 2007) providing diffusion paths. This can lead to increased levels of local moisture content, and thus, different levels of moisture induced straining. Combined with the inhomogeneous nature of the matrix material (as previously described), the moisture induced swelling can be expected to vary from location to location within the composite. In addition, at a ply level there can exist several imperfections (such as resin rich areas) which will likewise lead to a locally different response to moisture. Therefore, the assumption that moisture swelling effects can be homogenised to a ply level (through orthotropic MSC values) is a generalisation that needs to be kept in mind during any analysis of moisture effects of stress states.

Multi-stable laminates are highly sensitive to the residual stress state, as it is responsible for both the shape configurations and the multi-stable behaviour itself. Thus, morphing structures using multi-stable laminates are sensitive to the moisture state of the multi-stable laminate. This requires that moisture ingress be considered at a design stage of such structures, to ensure that the shape configurations aren’t adversely affected, that the snap-through activation loads are still acceptable, and that the multi-stable property is still present. Accounting for moisture effects is not trivial, however. Firstly, material specific moisture properties (for diffusion, saturation, and swelling) are required. These are challenging to obtain and not widely available in the literature. Secondly, the effectiveness of the general approach to account for moisture swelling (using swelling coefficients and moisture contents) is not known when multi-stable laminates are considered. The analysis is further complicated by the fact that several concepts for morphing structures use a single continuous laminate, which feature a locally multi-stable section. An example is shown in Figure 1-2, for a variable camber trailing edge. The structure is one continuous laminate, with a change in lay-up to incorporate a multi-stable and unsymmetrical section (green areas).
Introduction

![Figure 1-2 Concept for variable camber trailing edge, featuring locally unsymmetrical and multi-stable lay-up (green areas) surrounded by a symmetrical lay-up (purple areas). From Mattioni et al. (2007).](image)

Such laminates feature a change in residual stresses along their length caused by the change in lay-up. Their behaviour following moisture ingress is not known, particularly at the partition between symmetrical and unsymmetrical lay-ups. These laminates may feature, for example, locally resin rich areas at the boundary between different lay-up partitions (Sousa et al. 2013), which would consequently be subject to increased moisture induced swelling.

It is clear that studying the effects of moisture ingress on composite laminates is a complex problem, as several intricate factors come into play (transient diffusion, matrix homogeneity, matrix damage, stress-diffusion coupling, ply and laminate level imperfections, etc.). An all-encompassing study which considers all these elements is prohibitive both in terms of time and complexity, as a multi-scale and highly coupled modelling technique would be required, along with a significant number of experimentally obtained parameters. To this end, an approach is presented in this research work whereby basic experimental data (in the form of unsymmetrical laminate curvatures) are used in conjunction with numerical models (using Finite Element (FE) software) to analyse the effects of moisture ingress on unsymmetrical and multi-stable composite laminates. The study focuses on dry and saturated states only, to capture the bounds of the possible effects of moisture on multi-stable laminates. A number of complex lay-ups are considered, both in the form of cross-ply and tailored laminates configurations. Tailored laminates feature local variations in lay-up and/or thickness, and are representative of what may be used in morphing applications. Finally, the consequence of moisture ingress is presented in the form of changes in the through-
thickness residual stresses between dry and saturated states. The study also includes detailed full-field comparisons of predicted and experimentally measured tailored laminate shapes, which is made possible by using laser-scanning as a surface measurement technique. To the best of the author’s knowledge, such a detailed analysis of the response of unsymmetrical multi-stable laminates to moisture induced loading has not been presented before.

1.2 Objectives and Methodology

The objectives of this research work are:

1) To experimentally measure the cured and saturated unsymmetrical laminate shapes for both cross-ply and tailored multi-stable laminate configurations, and record the loss of curvature in the laminates following moisture saturation.

2) To create an analytical tool, based on existing analytical methods, to predict the multi-stable laminate shapes of square cross-ply laminates.

3) To develop a numerical model (using a three-dimensional continuum finite element model) that can predict multi-stable laminate shapes for cross-ply laminates in both dry and saturated states, and which is benchmarked against the analytical tool.

4) To use the experimentally measured laminate shapes to calibrate the numerical model, and obtain equivalent material property values to reproduce the dry and saturated laminate shapes.

5) To refine the numerical model for tailored laminates, to predict the multi-stable dry and saturated shapes.

6) To compare in-depth the correlation between numerically predicted and experimentally measured tailored laminate shapes.

7) To use the numerical model to extract and analyse residual stress profiles for the cross-ply and tailored laminates in both the dry and saturated conditions.

8) To use the developed model to analyse the through-thickness residual stresses, and the changes in the stresses and laminate potential energy resulting from moisture ingress, in both cross-ply and tailored composite laminates.

The methodology of the approach is as follows:

A range of cross-ply multi-stable laminates were manufactured and their curvatures were measured. Subsequently, the laminates were immersed in water until gravimetric
measurements indicated that saturation had been reached. The saturated curvature of the laminates was then recorded. A numerical model was developed to predict the cured shape of the unsymmetrical laminates, which was subsequently calibrated to reproduce the ‘dry’ laminate curvatures, resulting in an equivalent value of the materials longitudinal CTE ($\alpha_L$). Subsequently, the model was adapted to incorporate the experimentally observed loss in curvature due to saturation, which results in an equivalent value for the materials transverse MSC ($\beta_T$). The model was then used to extract and analyse the through-thickness residual stress profiles in both the dry and saturated laminate conditions.

Subsequently, the ability of the approach to predict the behaviour of tailored laminates (i.e. laminates featuring local changes in lay-up and/or thickness, similar to morphing concepts) was analysed. A range of tailored composite laminates were manufactured, and their dry shapes recorded by using laser scanning (a full-field non-contact surface measuring method). Such an approach was used as tailored laminates feature varying curvatures along their length. Also, this allows for a detailed full-field comparison of measured shapes and those numerically predicted, for use in calibrating the numerical model. The residual stress profiles at each lay-up partition were extracted from numerical models. For a majority of the laminates analysed, this was done for all multi-stable shapes. The change in stresses caused by the tailored laminate configuration could also be investigated.

Finally, the change in the shapes of the tailored laminates following moisture saturation was incorporated in numerical models, and the change in residual stresses and total laminate potential energy between dry and saturated was analysed.

1.3 Overview of Thesis
In Chapter 2, a review of relevant publications is presented. These publications focus upon numerical and theoretical approaches used to predict multi-stable laminate shapes, concepts presented using multi-stable laminates, and approaches used to incorporate moisture effects into the prediction of multi-stable laminate behaviour. Methods of measuring laminate shapes and residual stresses will also be briefly presented.

Chapter 3 presents important theory to predict the shapes of unsymmetrical multi-stable cross-ply laminates. This theory is then incorporated into a calculator (using the
mathematical software Matlab) which is used to predict the shapes of multi-stable laminates. This is subsequently used to benchmark numerical models.

Chapter 4 describes the experiments undertaken in this research work. The manufacturing method used for all the CFRP laminates investigated in this work is presented. A material characterisation study, for the HTA 6376 material used in this study, is then presented, with includes measures of the materials transverse CTE ($\alpha_T$) and elastic properties. Finally, the methods used to measure the laminates (curvatures for the cross-ply laminates and full-field shape measurement for the tailored laminates) are presented.

In Chapter 5, the numerical model used to predict the cured and saturated shapes of the cross-ply and tailored laminates is presented. The results of the calibration process (resulting from a comparison of measured and predicted shapes) are given, in the form of equivalent values for the materials $\alpha_L$ and $\beta_T$.

Chapter 6 presents the extracted through-thickness stress profiles for each of the cross-ply and tailored laminates analysed. These are extracted for both the cross-ply and the tailored laminates, in both the dry and saturated laminate configurations.

Chapter 7 discusses the approaches used in this study (numerical and experimental), to demonstrate the strengths and limitations of the approach.

Finally, Chapter 8 presents key conclusions of the work undertaken, along with some recommendations for future work.

1.4 Dissemination

The work presented in this thesis has been disseminated in the following ways:

Journal papers


This paper contains a description of an approach used to analyse dry and saturated residual stress profiles, by using experimentally-measured laminate shapes to calibrate numerical model, from which residual stress profiles can be extracted.
2. Telford R, Katnam KB, Young TM. Analysing thermally induced macro-scale residual stresses in tailored morphing composite laminates. *Composite Structures* 2014 (published online, DOI: 10.1016/j.compstruct.2014.06.013)

An extension of the previous paper, this work deals with the through-thickness stress profiles of tailored composite laminates. Full-field shape comparisons between experimentally measured and numerically predicted shapes are used to calibrate models and analyse dry residual stresses in tailored laminates.

**Conference Presentations**

Chapter 2  Literature Review

2.1  Introduction
A review of the literature relating to multi-stable laminates will be presented. Firstly, concepts using multi-stable laminates as part of a morphing structure will be given, with a view towards understanding what is required to make such applications feasible. In particular, it is shown that moisture absorption can significantly alter the behaviour of multi-stable laminates and must be considered at a design stage. Then, existing theoretical and numerical approaches to predict the cured shapes of multi-stable laminates will be reviewed, to assess the capabilities, flexibility and overall assumptions of such techniques. As thin laminates are of interest, manufacturing induced warping will also be discussed. As the shapes of multi-stable laminates are of interest in this study, the contribution of these effects to the laminate shapes and the ability for models to incorporate the behaviour will be assessed. Finally, the environmental effects (specifically, those experienced under in-service conditions and which may alter the behaviour of a multi-stable laminate) will be discussed. Methods of including such effects (specifically, moisture absorption) will be emphasised. Finally, the methods to measure laminate shapes and residual stresses will be given. Once again, these are discussed in the context of understanding the general approaches used to model multi-stable laminates and the limitations of these approaches.

2.2  Applications of Multi-stable Laminates
Several concepts for morphing structures utilising multi-stable laminates have previously been proposed. The potential advantage offered by multi-stable laminates in morphing aeronautical and underwater vehicles is described by Turnock et al. (2009) as “the ability to have multiple shapes without continually supplying power” to “adjust ... the vehicle wetted shape [allowing for] optimal propulsion efficiency to be achieved at multiple operation speeds and vehicle attitudes”.

A solution to create a variable sweep wing based on multi-stable composites was presented by Mattioni et al. (2006) and is shown in Figure 2-1. In this case, a wing-box structure was proposed with two multi-stable spars connected by truss ribs.
This is approached as a viable solution to the conflicting requirements of being “structurally compliant and sufficiently rigid to limit aeroelastic divergence”. Existing solutions using a pivot point about which the wing rotates are subject to large stress concentrations as all the aerodynamic loads of the wing act entirely upon this point. The spars are constructed using a [0/90] lay-up (with CFRP material) for the shear web, onto which the spar caps (5 layers of 0° unidirectional fibres) are bonded. The wing-box structure is then able to snap to different sweep angles, due to the unsymmetric lay-up of the spar web. The load bearing capability (lift induced bending stresses) is increased by the formation of a curvature with a generator along the spar length, which increases the moment of inertia of the structure. The lay-up configuration is an example of a tailored laminate – one which features local variations in lay-up or thickness. A finite element model (based on shell elements for the spars) was able to correctly predict the multi-stable configurations of the structure when compared against a demonstrator. Apart from some initial static loading (at a quarter of the spar’s semi-span, to simulate an elliptical lift distribution) indicating the need for further structural refinement, no further details on the next steps towards making this concept feasible are provided. One such issue may be that of environmental effects and/or chemical ingress. As part of a wing structure, it is likely that fuel tanks and/or other fuel related plumbing will be in close proximity to the morphing spar. Indeed, large aircraft commonly feature ‘wet wings’, where the internal structure – including the wing spar – forms the aircraft’s fuel tank. Consequently, any spillages or leakages could be absorbed by the laminate which may cause a change in its behaviour. Likewise, aircraft based applications could experience a large range of temperature and humidity conditions, both of which could affect the residual stress state and thus would require consideration.

Mattioni et al. (2007) continued with concepts for two further aeronautical based applications. By using piecewise variations in lay-up, the authors presented a multi-
stable winglet as well as a variable camber trailing edge section with both numerical models and experimental demonstrators being presented. The winglet concept featured an inboard section with an eight ply thick symmetrical lay-up, while the outboard (multi-stable) section featured an unsymmetrical $[0_{4}/90_{4}]$ lay-up (Figure 2-2(a)).

In the first configuration (Figure 2-2(b)), the winglet is deployed flat as an extension of the wing, while in the second configuration (Figure 2-2(c)), the winglet is canted upwards to create a blended winglet. The authors allege that altering between such configurations may improve the aircraft’s aerodynamic efficiency at various stages of flight.

The multi-stable variable camber concept comprises a continuous laminate extending from the top skin, around the trailing edge, to the bottom skin. This concept is similar to present day flaps used on aircraft as high-lift devices during take-off and landing. Unsymmetric sections (see Figure 1-2) on the upper and lower skin are used to obtain the multi-stable configurations. For the configuration presented, these patches snap through at different loads: approximately 20N for the upper and 30N for the lower skin. The use of locally unsymmetrical sections offers the ability of adding additional multi-
stable configurations, and thus increasing flexibility (e.g. increasing the number of variable camber or deployed flap configurations available). As with the variable sweep concept, significant research is required to develop the concept into a viable morphing concept (e.g. impact of environmental effects). In addition, both of these concepts utilise tailored laminate configurations. This involves a transition in residual stresses from different lay-ups. This may influence the multi-stable laminates behaviour in terms of cured shape and snap-through loading. More importantly, the combination of moisture ingress and the influence of the neighbouring symmetrical lay-up have not been studied and could alter the multi-stable shapes or snap-through loads. This requires modelling approaches that take the neighbouring lay-up into account.

The use of stiffness tailoring to achieve multi-stability is discussed by Daynes et al. (2008) and Daynes and Weaver (2013). Stiffness tailoring involves pre-stressing an element (e.g. a ply) of a structure during manufacture. Subsequent removal of the pre-stress results in a stress redistribution, which can be tailored to invoke multi-stability. This can be done by modifying the mid-plane strains (in-plane pre-stressing) or by modifying bending strains (out-of-plane pre-stressing). As an example, a bi-stable helicopter blade flap demonstrator was presented, as shown in Figure 2-3.

![Figure 2-3 Integration of multi-stable laminates into bi-stable helicopter blade flap. From Daynes and Weaver (2013).](image)

Bi-stability was achieved by using six glass fibre reinforced plastic bi-stable laminates in a trailing edge section. The laminates featured 1.1% pre-strain to the fibres on the outer edges of the laminate. This configuration provided “sufficient stiffness to withstand aerodynamic loading whilst still achieving a 10° flap deflection”. Such a concept may require a study of the stresses within the multi-stable laminates, due to the several loading conditions identified (centrifugal and bending loads). Any loading
could, in theory, affect the multi-stable behaviour as well as lead to failure (due to the interaction of residual plus applied stresses). This requires consideration at a design stage, and so modelling techniques which can predict such stresses would be of interest.

A multi-stable air inlet was also presented, which uses a multi-stable section which can snap open and closed (see Figure 2-4).

![Multi-stable air inlet: (a) closed state and; (b) open state. Adapted from Daynes et al. (2011).](image)

This demonstrator is an example of an out-of-plane pre-stressed member, with a “sinusoidal variation of bending stiffness along its length using composite ply drop-offs and build-ups”. Lastly, a morphing wing proof-of-concept that features twist morphing is presented. The concept uses “prestressed spars to suppress the wing’s torsional stiffness” – and be open to twisting. Further detail of the morphing air inlet is provided by Daynes et al. (2011), including a working demonstrator and an analytical solution to predict the multi-stable shapes and snap through behaviour based on the theory developed by Dano and Hyer (1998) with a modified shape function assumption. This is benchmarked against a FE model. The behaviour of the developed inlet matches the snap through loads criteria, whereby the system is sufficiently stiff so as not to snap through due to aerodynamic loads (in this instance, a +/- 3 kPa simulated dynamic pressure). However, traditional actuating systems (such as electrically actuated jack screws or pneumatic pistons) were discounted by the authors due to their weight. In addition, piezoelectric patches don’t provide the required actuating strain, while shape memory alloys feature a time delay during cool-down, making them unsuitable choices. The authors therefore proposed a pneumatic bladder which could be inflated/deflated (using a compressed air source) to actuate the air inlet between states.
Bowen et al. (2007) studied the response on cantilever and unsupported laminates from the applied strain of piezoelectric actuators. The actuators were bonded to the laminate surface using an epoxy adhesive and held flat during cure. The work demonstrated, through experiments, the ability of piezoelectric patches to actuate the various shapes of multi-stable composites. The response to an applied load was also investigated. The work concludes that modelling of such a system requires knowledge “of the piezoelectric material, the unsymmetric composite, and the interface between the materials”. A modelling strategy (either numerical or analytical) is not suggested, but would, when expanded to include environmental effects discussed previously, require the ability to account for such effects on the bonding, the actuator, and the multi-stable laminate itself.

The previous paragraphs describe some early concepts and demonstrators using multi-stable laminates for morphing applications. Other researchers have investigated the ability to maximise the potential of multi-stable laminates, in areas such as improving and maximising the multi-stable shapes available, alternate manufacturing methods (i.e. not based upon thermal deformation following cure at elevated temperatures), and tailoring the lay-ups to improve their response to other loadings (e.g. by having a locally symmetrical lay-up throughout. Early studies have focused on simple laminates to develop an understanding of the fundamental behaviour of multi-stable laminates. These shape configurations are not generally useful in morphing applications. Consequently, authors have looked at being able to control the deformation to produce useful shapes.

Potter and Weaver (2004) looked at creating multi-stable lay-up configurations that are based on symmetrical lay-up sequences. This was done by using tailored laminate configurations, in which individual plies within a symmetrical laminate featured varying fibre orientations (as in Figure 2-5) and dissimilar materials (carbon-fibre and glass-fibre).
This tailoring takes place at the laminate mid-plane, maintaining laminate symmetry throughout. This approach allows for a virtually unlimited number of multi-stable shape configurations (as an unlimited number of transverse strips may be used) and also eliminates coupling issues that arise from unsymmetrical laminates. Additionally, a manufacturing effect was used to manipulate the residual stress state to promote multi-stability. The difference in thermal expansion/contraction between the laminate and the tool plate onto which it is cured leads to interfacial stress between the composite and the aluminium. The authors were able to exploit this stress by curing a ‘H’ shaped unidirectional laminate (in plan-view), and subsequently cutting off the legs. The stresses within the remaining section equilibrate following this procedure in such a way as to result in multi-stability. An analytical solution describing this behaviour was also presented. It is concluded that strategic use of such tailored laminates enables a virtually “unlimited number of stable states” to be available. Even though laminate symmetry/balance is maintained, they “clearly have asymmetries or discontinuities in their structure”. Before any applications using this technology become viable, it is stressed that a “much better understanding of these phenomena than is currently available” is required. While the shapes obtained may not be particularly suitable for morphing structures (due to their small out-of-plane deflections), this work demonstrates another of the several possibilities available for creating multi-stable laminates. With advanced manufacturing techniques (e.g. steerable tow-placement) it is possible to precisely control manufacturing and thus residual stresses of lay-ups, further optimising the morphing structure’s performance. However, the analysis of such laminates becomes increasingly challenging as the lay-ups become more complex. In
addition, although the lay-up is symmetrical, the impact of environmental effects may not be symmetrical (e.g. moisture ingress through one surface only). As such, these lay-up configurations require particular attention to such affects.

Application-based studies into the characteristics required to maximise the out-of-plane deflection (i.e. curvature) developed by unsymmetrical laminates were presented by Gigliotti et al. (2005, 2009). The studies focused on \([0\_m/90_n]\) laminates, and focused on the effect on curvature of the ratio of longitudinal to transverse layers, as well as the difference in elastic and thermo-elastic properties of the laminate. It was found that the magnitude of the curvature is dependent upon the difference in the longitudinal and transverse CTEs, while the position of the maxima (i.e. the ratio of transverse to longitudinal layers required to obtain the maximum) is dependent upon the elastic differences. Coburn et al. (2013) then investigated tri-stability (i.e. three stable shape configurations) that arises – not from thermal residual stresses – but from the geometry of the laminate which, in this case, is due to the laminate being cured with an initial curvature. The advantages of this form of multi-stability include “negligible hygrothermal effects and the base stable state matching the shape of the tooling or mould allowing close control of manufactured tolerances”. Additionally, “significantly larger actuation loads to transition between stable states” is required for curvature based multi-stability. This is due to mid-plane stretching being required for a doubly curved shell to deform, requiring significant energy. Finally, the existence of a third stable shape configuration increases the flexibility of multi-stable structures. An experimental demonstrator was manufactured, and analytical and numerical (FE based) modelling techniques used to predict curvatures. The importance of the shape functions assumed in analytical models (in this case, constant curvature) is shown to be important, as “the bending boundary layer length, assumed negligible in the analytical model was evidenced in the FEA and demonstrator by a region of rapid curvature change towards the shell edge”.

2.3 Predicting Cured Laminate Shapes

Significant interest in multi-stable composites first started in the early 1980’s with a series of publications by Hyer (1981a, 1981b, 1982). Hyer noted that in some cases (length-to-thickness ratio, lay-up sequence), the shape of an unsymmetrical laminate did not match that predicted by CLT. Traditionally, CLT is used to predict the cured shapes of unsymmetrical laminates. In this theory, ply properties are reduced to homogenised
orthogonal linear-elastic constants. From these, stiffness and compliance matrices are compiled. The geometrical response to temperature changes (in terms of a mid-plane curvature) can then be evaluated. In the case of multi-stable laminates (e.g. using a square cross-ply construction), multiple cylindrical shapes are obtained following cure, with a snap through action being required to alternate between them. This is in contrast to the saddle shape that is predicted by CLT. Subsequently, substantial research effort has focused on correctly predicting the shapes and snap-through behaviour of multi-stable laminates, with the purpose of applying the technology to morphing applications.

The behaviour was first described by Hyer (1981b), who presented the cured shapes of a range of unsymmetrical laminates with different lay-ups and geometry and noted, in particular, that a square \([0_2/90_2]\) laminate did not cure to the saddle shape predicted by CLT, but rather cured to a cylindrical shape with a multi-stable property. To describe this behaviour, Hyer (1981a) developed theory based on polynomial approximations for displacements, the minimization of potential energy using the Rayleigh-Ritz method and extending CLT by accounting for geometric nonlinearities. The nonlinearity was introduced in the form of non-linear mid-plane strains, which were chosen due of the fact that the out-of-plane deformation exhibited by unsymmetrical laminates can be many orders of magnitude larger than the laminate thickness (Hamamoto and Hyer 1987). This theory involved approximating the displacement of a laminate during cooling, as a function of unknown coefficients. These unknown coefficients were then evaluated by using the Rayleigh-Ritz method to minimise the potential energy of the laminate. This theory was limited to square cross-ply laminates, and was used to predict the existence of multi-stable behaviour and the cured shapes obtainable of such laminates. For example, the dependency on the multi-stable behaviour on the laminate side length was demonstrated, as shown in Figure 2-6.
Figure 2-6 Room temperature shapes of a square cross-ply laminate with a [0\_2/90\_2] lay-up, predicted by Hyer (1981a).

The behaviour depicted in Figure 2-6 can be described as follows. For a square cross-ply laminate, the cured shape in the form of curvature is described along the x and y axes as a function of side-length. Initially, for side lengths under roughly 35 mm, a single saddle shape solution exists with equal and opposite curvatures developing across the x and y axes. At a critical side length (denoted by point B), the solution bifurcates and several shape solutions are possible, denoted by branches B-C, B-D and B-E. Branches B-C and B-E are equal and opposite, denoting two cylindrical multi-stable solutions, with equal and opposite curvatures being generated on orthogonal axes. Branch B-D remains as a saddle shape. To verify if a solution is stable (and thus which of the above branches is physically realizable), a second variation of the potential energy is computed, which must be positive definite to denote a stable solution. From this, it was shown that the saddle shape (branch B-D) is unstable and thus not realizable in practice. The single data point shown in Figure 2-6 shows the correlation between the predicted curvature and that of an experimentally measured laminate, which the author
describes as “fair”. More importantly, however, the authors state that the “character of
the measured shapes, i.e. cylindrical or saddle, compares well with the predictions”. The
fundamental behaviour of such laminates appeared to have been captured, and ways in
which to improve the accuracy of the solution were suggested. These included effects
such as “moisture absorption, viscoelastic relaxation, or any other mechanism that alter
the internal stress state of the laminate”, which were all felt to be important. Indeed, any
external perturbation that is unsymmetrical about the mid-plane (curing, moisture
absorption) was deemed likely to result in changes to the laminate behaviour, such as
more force being required to snap the laminate from one shape to the other. Thus, it was
recognised that while the fundamentals of the behaviour appeared to have been
captured, the behaviour of multi-stable laminates is not limited to thermally induced
residual stresses developed during manufacture. Additionally, assumptions taken (for
example, assumed shape functions and in-plane shear strains being zero) limited the
accuracy and applicability of the work.

Hyer continued to develop the approach to make it more accurate and more applicable
to all unsymmetrical laminates (and not just those in the cross-ply family). Firstly, the
assumed spatial form of the mid-plane strains were modified to make them less
restrictive (Hyer 1982). This was done as he felt the initial assumptions may have been
too restrictive, but was shown to be unnecessary. The temperature-curvature
relationship of cross-ply laminates was also explored by Hamamoto and Hyer (1987),
using the existing theory. This predicts the room temperature curvatures of an
unsymmetrical laminate, and the existence of a bifurcation temperature after which a
multi-stable solution exists (similar to what is shown in Figure 2-6). It was shown that
for a ‘perfect’ laminate (i.e. no spatial variation in material properties, perfectly aligned
plies, constant laminate thickness, etc.) a sharp bifurcation point exists. This bifurcation
point is the temperature at which the predicted shape transitions from a single saddle
shape solution to a multi-stable cylindrical solution. Experimentally measured laminate
curvatures (as a function of temperature) further verified the requirement of geometric
nonlinearities to describe the behaviour of unsymmetrical laminates (as non-linear
temperature-curvature relationships existed). However, it was found that “after the
bifurcation temperature, the experimental results did not follow the predicted
temperature-curvature relation as closely as at the lower temperatures”. Further
investigations attributed this behaviour to imperfections within the laminate.
Specifically, the idealised ‘perfect’ laminate used in theoretical calculations was felt to
be unrepresentative of physical laminates. A demonstration of the effect of varying laminate thicknesses (varying +/- 1% from the nominal ply thickness) on the predicted curvatures showed similar behaviour to that observed in experiments. The sharp bifurcation point disappeared, and was replaced by “limit point behaviour”. It was concluded that “an imperfection … could very well be responsible for the lack of good correlation between theory and experiment over the entire temperature range, particularly near the bifurcation temperature”. This demonstrates the importance of the general modelling approach, which assumes homogenous orthotropic materials properties as well as ‘perfect’ ply and laminate constructions. This may not be representative of the actual laminate, leading to inconsistencies between predictions and experimental observations. Other factors (including environmental, such as moisture absorption; or manufacturing, such as tool-plate interaction) can influence the residual stress state of a manufactured laminate, reducing correlation between experiments and predictions. Hyer also assumed that the stress-free temperature (i.e. the temperature during the cure cycle at which residual stresses begin to form in the laminate) is equal to the dwell temperature during cure. That is, residual stresses begin to form and the laminate begins to warp the instant that the temperature in the autoclave drops. This may not necessarily be the case. Due to the exothermic nature of the curing process, the temperature of the laminate during consolidation may be higher than the programmed cure temperature. Additionally, matrix contraction is not solely a thermal effect. Cure shrinkage occurs due to a rearrangement of polymer molecules into a more compact mass as the curing reaction proceeds. These are factors which may affect the formation of residual stresses and thus should be kept in mind.

Jun and Hong (1990) extended the theory further to account for in-plane shear strain, which was shown to be significant for intermediate length-to-thickness ratios. They developed the theory further to account for generally unsymmetrical laminates, by including further terms in the assumed displacements (Jun and Hong 1992). Schlecht et al. (1995) compared Hyer’s theory against FE model predictions. Shell element based FE models comprising 196 8-node bilinear shell elements were used, as part of the FE analysis programme MARC. A good correlation between Hyer’s developed theory, FE shape predictions, and limited experimental data was reported, with both the curvatures and the correct snap-through behaviour being predicted. It was shown that FE methods were better suited at predicting experimentally-observed edge effects, which are not accounted in theoretical calculations due to the assumption of constant curvature. This
was reiterated in a further study, where the analysis was extended to circular multi-
stable laminates (Schlecht and Schulte 1999). The experimentally measured laminates
were manufactured using a hot-press system, and the material properties (elastic and
CTEs) were experimentally measured as a function of temperature. Single values of
CTE were presumed, despite non-linearity being evident over the cure temperature
range. It is worthwhile noting that predictions and experiments showed good
correlation. This may be due to the symmetric curing process using a press.

Dano and Hyer (1996) then modified the theory of Hamamoto and Hyer (1987) to
account for simple applied forces, by considering the changes in potential energy
induced by externally applied forces. Comparisons with experiments showed good
correlation between the predicted and measured snap-through forces. The unloaded
laminate’s curvature was recorded by tracing an edge on paper. Such measurements
weren’t possible when applying an external load, and so comparisons between
measured mid-plane strains and those predicted theoretically were used. These showed
good correlation at forces below the snap-through force. It was also stressed that a
Rayleigh-Ritz based approach “can provide a good response in an average, or overall
sense. … On the other hand, strains, local curvatures, and effects near the edges and
corners of the laminate may not be so accurately modelled with a Rayleigh-Ritz
approach”.

Dano and Hyer (1998) continued the development of theory to predict the shape of
unsymmetrical laminates with arbitrary lay-up angles. Contrary to previous approaches,
which use approximations of displacements, this study used approximations to the strain
fields in the expression for the total potential energy. As before, the Rayleigh-Ritz
approach was applied. Shapes predicted using this theory were compared against
experimentally measured shapes. Experimental measurements were then fitted into a
polynomial equation from which curvatures were extracted. An FE model was used
(100 4-node shell elements) to benchmark the theoretical approach. In the case of a [-
60/30] lay-up, a non-constant curvature was predicted during FE analysis. In both the
experiments and the theory, constant curvature values are used. FE models don’t use
assumed displacement functions for the laminates, and so when increasingly more
complex laminates are being analysed, the use of FE methods becomes more accurate.
However, some knowledge of the deformed shape is still required (e.g. through
experiments) so that the FE solution can be coaxed towards a particular shape
configuration. This can be done, for example, with a geometric imperfection. The response of generally unsymmetrical laminates to applied loads was then described by Dano and Hyer (2002). A challenge in correctly predicting the laminate behaviour during snap-through was identified by the authors. Analytic formulation up to this point was based upon assumed constant curvatures across the laminate lengths. When coupled with a mechanism to snap the laminate from one shape to another, the interaction of the mechanism causes the laminate to locally deform, changing the local curvature. The predictions of snap-through forces showed good correlation, but if a detailed analysis of the snap-through event is required, this approach may have limitations. It is interesting to note that in this work, the authors used identical assumed material properties to their previous studies, except for the longitudinal expansion coefficient which was increased from $0.5094 \times 10^{-6}/°C$ to $0.621 \times 10^{-6}/°C$. It appears, therefore, that an element of curve fitting was used, which will indirectly account for other contributors to the residual stress state. This is not addressed in their works and so is only surmised.

Hufenbach et al. (2001) continued to advance the theory by adding variables to the potential energy formulation which took into account moisture-induced swelling (through classical MSCs and moisture concentration) and chemical shrinkage. These were accounted with reduced lamina stiffness terms, and so were homogenised to each ply. This is useful in terms of analysing academically the effects of moisture, but doesn’t lend itself to a transient moisture diffusion analysis easily, where there may be a through-thickness gradient in the moisture concentration. The work concentrated on obtaining the most optimal lay-up to obtain a design requirement (e.g. a specific curvature), by accounting for hygro-thermal and chemical shrinkage stress contributors, and so no experimental data on the effect of moisture on laminate shapes was presented. The approach by Hufenbach et al. (2001) was further expanded to include a failure analysis, due to the potentially high residual stresses developed (Hufenbach and Gude 2002).

A further development of Hyer’s theory was undertaken by Ren et al. (2003), who expanded the theory to include initially curved laminate shapes (i.e. cured on curved tool-plate). A shell based model (using Abaqus FE software, S4 elements). Experimental results for three cross-ply laminates ([0/90]), [90/0/90/0], [90/0/90/0]) cured on a mould with a 150 mm curvature were presented and compared against analytical and FE models. In almost all cases a good correlation was observed. However, the last
laminate ([90₂/0₂]) was not solved numerically due to convergence issues, stemming from “significant geometric non-linearity”. Note, Abaqus recommends the use of reduced integration elements for bending cases, such as the S4R element. Otherwise, good agreement was reported against experiments.

A study of the curvatures developed by cross-ply laminates was carried out by Gigliotti et al. (2004), who compared FE predictions with Hyer’s theory (neglecting in-plane shear) for different laminate configurations. FE analysis was conducted using Abaqus software. A shell model comprising S4R elements was used. In cases where laminate symmetry resulted in a saddle shape being predicted, a geometric imperfection was introduced to bias the solution towards a particular cylindrical shape. The earlier observation regarding in-plane shear by Jun and Hong (1990) was confirmed in FE simulations, where high shear strain peaks were found to occur locally, particularly for laminates with intermediate length-to-thickness ratios. Additionally, finite element models showed that the inclusion of imperfections (changing the relative thickness of the 0 and 90 layers) results in a loss of a sharp bifurcation point (as was previously noted by Hamamoto and Hyer (1987) using Rayleigh-Ritz models). Finally, a study on the effect of aspect ratio on the curvatures of the laminates was undertaken (again, using FE and Rayleigh-Ritz solution strategies). In this case, a difference in the bifurcation behaviour between FE models and Rayleigh-Ritz solutions was observed. The FE models behaved as if an imperfection was introduced, with a loss in the sharp bifurcation behaviour appearing. The Rayleigh-Ritz approach continued to predict a sharp bifurcation point. The more pronounced this affect (i.e. for laminates with higher aspect ratios), the larger the difference in predicted curvature between FE and Rayleigh-Ritz based techniques. Furthermore, anticlastic effects (producing a quasi-saddle shape) were once again observable experimentally and reproducible using FE models, and not with the Rayleigh-Ritz approach. Finally, it is worth noting that the authors used the experimentally measured curvatures of [0₈/90₈] and [0₂/90₂] laminates manufactured from two different materials to determine the stress free temperature to use in models. This was done by reheating the laminates until a flat shape was observed. In the case of the HTA/913 material the stress free temperature corresponded to the curing temperate ($T_{\text{cure}}$) of 120°C. With the AS4/8552 composite system, a stress free temperature of 188°C was required, compared to a $T_{\text{cure}}$ of 180°C. This was attributed to “the presence of non-thermoelastic sources of residual stress”, which could include chemical shrinkage of the matrix material, as was previously pointed out by Hyer. This work
showed that with increasingly complex geometries (in this case, increasing aspect ratio), the FE and ECLT/Rayleigh-Ritz based predictions begin to diverge, both in terms of predicted curvature and in the behaviour around the bifurcation point. FE predictions are compared to experimentally measured curvatures (as a function of temperature) for high aspect ratio (AR) laminates (AR=10), and show good correlation. However, the authors state that “further work is necessary to validate model predictions for other geometrical arrangements”.

Tawfik et al. (2006) proposed a FE approach to predict the cured multi-stable shapes of cross-ply laminates and the corresponding snap-through loads by means of a buckling analysis and addition of a geometric imperfection using the FE software Abaqus. The approach involved the following:

1. An initial eigenvalue buckling analysis was conducted on the ‘perfect’ structure to establish probable collapse modes and to verify that the mesh discretised those results accurately.
2. A second analysis was performed with imperfections in the geometry being introduced by addition of the buckling modes to the ‘perfect’ geometry using Abaqus’ *IMPERFECTION option.
3. A geometrically nonlinear load-displacement analysis of the structure containing the imperfections was performed using the modified Riks method.

This approach is advantageous in that the buckling analysis reveals the probable cured shapes of the laminate, negating precise knowledge of the probable cured shapes (and a manually introduced geometric imperfection to ‘bias’ the solution to that shape). The authors use the out-of-plane displacements from FE analysis to obtain a curvature-versus side-length relationship which is compared to that presented by Hyer (1981a). Their results (which include a geometric imperfection) compared favourably to that of the analytic solution, as well as some limited experimental data also presented by Hyer (1981a). Snap-through load results were also presented for laminates with varying aspect ratio (from AR = 1-8). It was seen experimentally that for square cross-ply laminates, the load required to alternate from the first multi-stable shape to the second (snap-through) is equal to the load required to alternate from the second shape back to the first (snap-back). However, when dealing with rectangular laminates (AR>1) the loads required for snap-through and snap-back differ. Comparisons between the loads
required to snap between shapes in experiments and in FE models revealed a difference ranging from 3.78% to 46.2%, with FE results always over-predicting. A possible reason for this is attributed to the boundary conditions in experiments which are not reproduced in models. In experiments, the laminate deforms during loading and flattens out, changing the contact areas between the laminate and the loading plunger, as well as between the laminate and the plate on which it rests. A comparison against an ECLT/minimum potential energy approach is not presented.

Portela et al. (2008) analysed the snap-through forces of a multi-stable laminate, with the purpose of evaluating a potential actuating system. In this case, after the first multi-stable shape had been predicted by using a geometric imperfection, the second shape was obtained by simulating the snap through. A shell based numerical model was used to predict snap through forces involved, which measured favourable against experimental data presented by Potter et al. (2007b). A secondary bifurcation that was observed in experiments was also reproduced in modelling.

Eckstein et al. (2013) included thermal gradients through the laminate thickness along with temperature dependent material properties, utilizing the model developed by Dano and Hyer (1998). Thermal gradients were included as previous studies had indicated that the assumption of constant material properties could lead to errors when predicting curvatures over a significant temperature range. For the analysis of shapes at room temperature, this effect is somewhat minimised by averaged material property values over the range from cure temperature to room temperature (e.g. in CTE values).

The inclusion of thermal gradients enables the use of multi-stable laminates to be actuated passively (e.g. to shape change as a function of temperature), as well as to account for transient thermal effects that may be present under in-service conditions. It was found that sufficiently large thermal gradients can trigger a bifurcation into obliquely orientated cylinders, i.e. other snap through shape configurations are possible when thermal gradients are considered. Comparisons with a FE model demonstrated that good correlation overall. In cases where obliquely orientated cylinders are predicted, agreement is worse on account of the assumed shape functions of the analytical model. The FE model used was based on shell elements, and used thermal gradients (presumably, through the use of thermal conductivity). The approach developed here (utilizing thermal gradients) can offer an ability to predict the effect of moisture diffusion in transient states by using the analogy between moisture...
diffusion/swelling and heat transfer/thermal expansion. Indeed, if including thermal
gradients can lead to additional multi-stable shapes, it is possible that a transient
moisture diffusion case could lead to similar results.

To further develop analytical models, Pirrera et al. (2010) combined high-order non-
dimensional Ritz approximations with path-following algorithms. Although previous
classical low-order Ritz models based on Hyer’s approach were able to capture the
stable shapes of multi-stable laminates with reasonable accuracy, it was felt that the
"inherently poor conditioning properties of Ritz approximations of slender structures"
mean that capturing other behaviour can be improved upon. The authors compared their
developed approach against FE models, with showed a discrepancy in snap-through
load of under 5%. Additionally, an experimentally observed (by Potter et al. (2007b))
but never analytically described phenomenon whereby the snap-though phenomena is
characterised by two separate snap-through events was captured. Likewise, the loss of
bifurcation described by Gigliotti et al. (2004) for various plan-form geometries and
aspect ratios was also predicted. The use of a path-following strategy allows for the
correct equilibrium points to be found when complex bifurcation paths (e.g. due to the
presence of secondary instabilities such as local buckling) are involved. FE simulations
have to deal with cumbersome trial-and-error analyses as a result.

A sensitivity analysis of the analytical model developed by Dano and Hyer (1998) was
presented by Brampton et al. (2013). This sensitivity of the curvatures of multi-stable
laminates to material properties and moisture absorption was considered. A ± 5%
variation in material properties was applied, and the effect on the predicted curvature
was given. Selected results are summarised in Table 2-1.

Note: These results are the maximum change observed for a range of length-to-
thickness ratios from 0 to 250.
Table 2-1 Maximum change in curvature for a ±5% variation in each design variable. Adapted from Brampton et al. (2013).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Room temperature value</th>
<th>Percentage change in curvature with ±5% change in variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>+5% (%)</td>
</tr>
<tr>
<td>$E_{11}$ (GPa)</td>
<td>146</td>
<td>2.16</td>
</tr>
<tr>
<td>$E_{22}$ (GPa)</td>
<td>11.7</td>
<td>-2.18</td>
</tr>
<tr>
<td>$v_{12}$</td>
<td>0.308</td>
<td>0.27</td>
</tr>
<tr>
<td>$G_{12}$ (GPa)</td>
<td>6.95</td>
<td>-0.01</td>
</tr>
<tr>
<td>$a_\ell$ (°C^{-1})</td>
<td>2.48 x 10^{-6}</td>
<td>0.52</td>
</tr>
<tr>
<td>$a_T$ (°C^{-1})</td>
<td>2.65 x 10^{-5}</td>
<td>-5.55</td>
</tr>
<tr>
<td>Single ply thickness, $t_p$ (mm)</td>
<td>0.25</td>
<td>5.32</td>
</tr>
</tbody>
</table>

The sensitivity of curvature to material properties can, therefore, be significant. The material properties of a manufactured specimen can vary from those of the manufacturer supplied material data, depending upon storage life and manufacturing method. Therefore, a variation between the predicted shapes of unsymmetrical laminates and those observed experimentally can be expected when using the manufacturer's material data.

Mattioni et al. (2008) investigated multi-stable tailored laminate configurations. These are laminates which feature a local change in lay-up and/or thickness (as shown in Figure 2-7), and can contain a locally multi-stable section.

![Figure 2-7 Lay-up sequence of bi-stable tailored laminate. From Mattioni et al. (2008).](image)

The lay-ups varied between 4, 8 and 12 ply thick, and featured a symmetrical lay-up in one half ([0/90]$_s$, [0$_s$/90$_s$]$_s$, or [0$_s$/90$_s$]$_s$) coupled to an unsymmetrical part ([0$_s$/90$_s$], [90$_s$/0$_s$]$_s$, or [90$_s$/0$_s$]$_s$).
A finite element approach to predict the cured shapes and buckling behaviour of a tailored laminate was discussed; the solution strategy involving the use of a non-linear static (*STATIC) step or a pseudo-dynamic non-linear (*STATIC, STABILISE) step to obtain the stable shape configurations. The authors noted that a static analysis will always converge to one equilibrium shape, while a dynamic solution will converge to another. This is due to the kinetic energy contribution to potential energy that is accounted for in the dynamic step but not the static step, leading to different temperature/potential energy paths. The pseudo-dynamic scheme employs the use of artificial dampening, which can then be controlled to obtain either stable solution by use of damping coefficients. These coefficients do affect the equilibrium configuration, and so must be chosen carefully to ensure their affects are minimal. This technique was further used by Zhang et al. (2014) and Daynes et al. (2011) in simulations of the snap-through process of multi-stable laminates. The latter authors noted that the use of artificial damping can have a small influence on the predicted snap-through/snap-back forces from simulations (e.g. -186.48 N snap-through versus 181.26 N snap-back for the morphing air inlet studied). A comparison between shapes predicted by numerical models and the measured experimentally was presented. The comparisons were done along the free edges of the laminate width, and along a line parallel to the laminate’s length, at the centre. The maximum difference for the eight-layered laminate was given as 8.5%.

Mattioni et al. (2009) extended analytical theory to account for the piecewise variation of lay-up in the planform. A rectangular laminate comprising two square sections (one with a symmetrical lay-up and one with an unsymmetrical lay-up) was described analytically. The theory developed by Dano and Hyer (2002) was extended by: (a) modifying the displacement fields to account for the non-constant curvature, specifically, the out-of-plane displacements which are "regarded as the result of the product of two parabolas along the principal directions (i.e. parabolic variation of the curvatures)"; and (b) formulating the strain energy for each substructure separately, before summing them together to obtain the energy for the complete structure. Continuity in the displacements across the boundary between the two different regions is maintained by imposing boundary conditions. The modified approach is initially compared to Hyer’s model for a square plate, as well as a FE model by comparison of the bi-stable shape. This revealed that the modified theory model did display a slight parabolic shape at the laminate’s edge, whereas Hyer's model predicts an almost
perfectly straight edge. The edge effect predicted by FE model was almost flat in the central part and slightly curved towards the corners. This comparison against FE modelling shows that the modified analytical solution was still unable to accurately predicted edge affects. This is due to the assumed displacement functions being chosen to obtain a “good approximation of the overall deformation and not the local effects close to the boundary”.

The effectiveness of the solution to predict tailored laminate shapes was then compared against FE and experimental results, for an 8-ply thick laminate. The experimental process (including the measurement of laminate shapes) is not described in detail. Nonetheless, the comparison shows that modelling (both numerical and FE based) can capture the correct multi-stable configuration of the tailored laminate. A larger mismatch between experiments and modelling was observed for the 4-ply thick laminate, with models (both numerical and analytical) acting overly stiff. This was attributed to the reduced number of degrees of freedom of the analytical model, and "presumably due to the mesh density chosen as a compromise between accuracy and computational efficiency" for FE models. Additionally, a slight curvature along the length of the symmetrical region of the experimental laminate was observable, which does not appear to have been captured by models. This demonstrates the increased difficulty in predicting tailored laminate shapes, as compliance between the two sections needs to be correctly predicted. Detailed shape analysis is therefore required to be able to ascertain which aspects of the tailored laminate shapes the numerical and analytical models are capable of predicting correctly.

The effectiveness of FE models and analytical models appear to be roughly equal in this study. The assumed shape functions/degrees of freedom appear to limit the analytical model, whereas the FE techniques involved (pseudo-dynamic, as described previously by Mattioni et al. (2007) involves the use of a damping factor which much be chosen carefully not to influence the result. However, moving to more complex configurations, FE methods would appear to have an advantage. The summation of potential energy for each lay-up region involves an increase in the number of unknown coefficients that need to be solved (twenty for a square plate to forty four for the two lay-up sections). Increasing the number of partitions further can result in a considerable number of coefficients needing to be solved for in a non-linear set of equations, requiring
considerable computational effort. To this end, FE solutions begin to display an advantage in terms of user-friendliness.

Sousa et al. (2013) compared the cured shapes and snap-through behaviour of two tailored laminate configurations by using FE modelling. Additionally, a first ply failure analysis was conducted. The tailored laminate configurations were based on that presented by Mattioni et al. (2008, 2009) with a rectangular four ply laminate consisting of two square sections of different lay-up sequences (in this case $[0/90]$, and $[0_2/90_2]$). It is hypothesised that the abrupt transition from one lay-up configuration to the next will lead to stress concentrations in the resin-rich regions where the fibre discontinuities occur. To this end, a concept was proposed whereby the change in lay-up sequence (and thus, fibre orientation) is not abrupt and discontinuous, but rather gradual and continuous. This is possible in practice by means of tape laying technologies, where fibres can be steered around curvilinear paths. Two roughly equivalent models (one of a straight fibre tailored laminate, one using curvilinear fibre paths) were analysed using the FE software Abaqus, using eight hundred linear quadrilateral elements of the type S4R. It was found that the curvilinear based model featured similar snap-through shapes to the straight fibre model. However, due to the inherent differences in lay-up configurations (see Figure 2-8) a direct comparison of the shapes and snap-through loads was not discussed.

Figure 2-8 (a) Variable stiffness morphing laminate based on curvilinear fibres; (b) Straight-fibre laminate. From Sousa et al. (2013).

A failure index analysis showed that (a) the predicted failure mode is by matrix cracking for both configurations; and (b) that the location of the first ply failure changes (from the transition between lay-ups in the discontinuous fibre case, to a free edge on the
laminate's width in the curvilinear model). The overall failure index is higher for the curvilinear model, yet the failure index at the middle of the laminate (where the peak failure index occurs in the discontinuous fibre model) appears reduced.

However, the authors point out that the resin-rich area between lay-up transitions in the straight fibre model may be a point of failure in straight fibre models that is not accounted for in this work, which they aim to address in the future. A detailed view of the interaction of stresses between transitions (depending on lay-ups, the transition length involved, etc.) may reveal peculiar behaviour which may impact upon potential applications. Even though no experimental laminates were manufactured, the work demonstrates another manufacturing technique to create multi-stable laminates, taking advantage of the flexibility and accuracy of fibre steering techniques.

It was shown by Daynes et al. (2008) that it is possible to create a bi-stable laminate by employing mechanical pre-stressing to symmetrical laminates during cure. Following cure, the mechanical load was removed, resulting in the stresses readjusting, and buckled laminate shapes. An analytical model (validated by FE models) was used to describe the behaviour to identify optimal conditions to maximise bi-stable effects. Apart from eliminating bending–stretching coupling, the advantages of using pre-stressed symmetrical laminates as opposed to thermally stressed unsymmetrical laminates include a reduced sensitivity to moisture effects. As the laminate is symmetrical, equal moisture diffusion across the bottom and top laminate surfaces will result in balanced moisture induced straining, and thus no changes in shape will occur. However, an adjustment of the residual stresses would still occur, which would possibly affect the multi-stability of the laminate. Also, morphing aircraft skins are given as a potential application of this technology. In such a case, it is very possible that unsymmetrical moisture diffusion will occur, as one surface of the laminate is exposed to the atmosphere while the other surface forms part of the internal aircraft structure. In such a case, moisture ingress will also affect the laminate shape. As such, these laminate types could still require a study on moisture ingress effects.

Eckstein et al. (2014) expanded on the theory presented by Dano and Hyer (1998) to account for an initially curved plate at the stress free temperature, whereby the change in curvature from an initially curved state is included in the computation of potential energy. A passive means of morphing is explored by focusing on temperature dependent shapes/multi-stability. The inclusion of an initially curved state gave rise to multi-mode
deformation as a function of temperature. An analytical model and FE simulations (using a shell model) were compared against experiments (using a [0\(\pi/90_3\)] laminate), and a good agreement was shown across the temperature range investigated. An over-prediction of the twist curvature \(k_{xy}\) of the analytical solution compared to experiments at temperatures where the oblique cylindrical shapes exist (80 to 180 °C) was attributed to the simple strain functions utilized in formulating the shear strain. The analytical solution acted overly stiff as "artificial stiffness imposed by the limited membrane stretching degrees of freedom makes it more energetically favourable for the laminate to store strain energy via bending deformation relative to stretching, and thus curvature magnitudes are over predicted". The analytical solution accounted for temperature dependent properties (elastic and thermal expansion) by utilising experimentally measured values at temperatures from 30 to 180°C. The material’s CTE values were measured between 30°C and 180°C using dilatometry, and found to range from 23.4 \(\times 10^{-6}\)/°C to -2.3 \(\times 10^{-6}\)/°C for \(\alpha_T\), and -2.3 \(\times 10^{-6}\)/°C to 2.3 \(\times 10^{-6}\)/°C for \(\alpha_L\). Finally, using the same procedure described by Gigliotti et al. (2003) (where the residual curvature above the cure temperature is taken to be due to non-thermoelastic effects) the non-thermoelastic strain (e.g. due to chemical shrinkage during the cure) was found to be negligible in the fibre direction and -6.9 \(\times 10^{-4}\) in the transverse direction.

The continual advancements in the predictions of the buckling behaviour of multi-stable laminates has culminated in the ability to tailor systems for to obtain a specific desired multi-stable state (Pirrera et al. 2013).

### 2.4 Manufacturing Effects

In many cases, the differences between expected and measured laminate shapes are attributed to either environmental effects or manufacturing imperfections (Hamamoto and Hyer 1987, Kaushik and Raghavan 2010, Eckstein et al. 2014). Factors such as these are felt to be important in the case of multi-stable laminates, as they may affect the residual stress state to such an extent that a critical threshold is reached and the multi-stable behaviour disappears (Hyer (1981a)).

Manufacturing composite laminates from pre-preg materials results from several distinct manufacturing processes, each of which can introduce variability from the optimal laminate configuration (i.e. a ‘perfect’ laminate). Such variability can stem from numerous factors, which can be introduced into the manufacture of the pre-preg material, the laminate lay-up phase, or during the cure cycle. This variability can give
rise to local changes in residual stress state, arising from a locally different thermal expansion/contraction during the cool-down. Any asymmetry of these effects about the laminate mid-plane can subsequently lead to warping from the moulded laminate shape.

It is worth considering the development of residual stresses during the cure cycle, as this can provide insight into when manufacturing effects can occur. Gigliotti et al. (2003) looked at the development of residual stresses during the cure cycle of an unsymmetrical composite ([0/90] lay-up) by interrupting the cure cycle at specific points and measuring the curvature. The specimens were manufactured using the AS4/8552 composite system. A standard curing cycle was used, with certain specimens being removed and quenched at a certain time during the cure cycle. The results indicated that:

1. Once the elevated temperature is reached, the degree of cure of the composite material advances quickly. After 10-20 minutes at the elevated cure, the vitrification point is reached and all specimens displayed the same degree of room-temperature curvature. Beyond this point, the laminate may be considered cured and significant thermo-elastic residual stresses will form with temperature changes.

2. The stress free temperature for specimens (*i.e.* following cure, the temperature at which all residual curvature is removed) was found to be above the cure temperature, and constant following the vitrification point.

3. The temperature/curvature relationships for each of the specimens indicate that a) the relationship is linear, and b) the transverse expansion coefficient (*α_T*) is constant during the cure cycle when below the glass transition temperature (*T_g*).

It would appear, therefore, that any theoretical approaches that consider process induced residual stresses being ‘locked’ into the laminate following consolidation of the resin would require the stress to be built up early in the cure cycle.

Some of these factors which can contribute to dimensional variability are described by Ochinero and Hyer (2002), and are summarised in Table 2-2. These effects lead to changes in the thermally induced residual stress state, and thus can lead to differences between predicted and observed laminates shapes.
Table 2-2 Laminate level imperfections which can influence geometry of manufactured CFRP laminates. From Ochinero and Hyer (2002).

<table>
<thead>
<tr>
<th>Imperfection</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ply thickness</td>
<td>Variation in ply thickness, both within a single ply and between different plies</td>
</tr>
<tr>
<td>Fibre waviness</td>
<td>Fibre misalignment within a ply, resulting in fibres not being straight within the ply, <em>i.e.</em> they form either short or long wavelength ‘S’ patterns</td>
</tr>
<tr>
<td>Uneven resin distribution</td>
<td>Variation in volume fraction, both within a ply and ply-to-ply</td>
</tr>
<tr>
<td>Broken fibres</td>
<td>Fibre damage leading to local variations in residual stress distribution</td>
</tr>
<tr>
<td>Misalignment between plies</td>
<td>Ply orientation errors introduced during lay-up, common tolerance given as ± 3-5°.</td>
</tr>
<tr>
<td>Ply shifting</td>
<td>Relative movement between two plies before consolidation, resulting in ply misalignment</td>
</tr>
</tbody>
</table>

Indeed, variations in the pre-preg material can be expected before any processing takes place. Potter et al. (2007a) investigated fibre waviness and variability in mass properties of a pre-preg material. A sinusoidal fibre waviness pattern was found, with the misalignment angle reaching 3.8°. Likewise, some variation in the pre-preg materials density (mass/unit area) is possible, both from batch-to-batch and amongst different areas of pre-preg. The authors report that a previous study indicated that four-ply thick and one metre long specimens manufactured from material either at the edge or the mid-width of the pre-preg roll feature different degrees of warping away from the tool-plate once cured. Specimens cured with material from the edge of the pre-preg deflected 7mm above the tool-plate, while the other deflected just 1 mm, which is attributed to the variation in pre-preg mass properties. This effect may be compounded by variations in volume fraction of the material that occurs during cure. In instances whereby the resin bleed during cure occurs over the top surface of the laminate, a resin rich area can form at the bottom of the laminate (which is in contact with the tool-plate). This causes a volume fraction gradient through the laminates thickness, and results in deviation from the mould shape. This was measured by Radford (1995) and shown to induce warping in moulded specimens, due to unbalanced thermal contraction about the laminate’s mid-plane.
At a process level, several studies have tried to characterise and reproduce through modelling the mismatch between laminates and the mould shape they were cured on. This task is made challenging by the difficulty in experimentally isolating a single effect, with the modification of one parameter having possible inadvertent effects on others (Stefaniak et al. 2012). Investigated contributions include gradients in temperature and/or the degree of cure, changes in the shape of the tool part, fibre/tool-plate interaction (Cho and Roh 2003, Kaushik and Raghavan 2010, Wisnom et al. 2006, Stefaniak et al. 2012). Curvature is even linked to the laminates geometry, with longer/thinner symmetrical laminates producing larger curvatures (Twigg et al. 2004a).

Of the several manufacturing parameters which may influence laminate shapes, tool-part interaction was found to be the predominant reason for the deformation of flat plates (Stefaniak et al. 2012, Kaushik and Raghavan 2010). Due to the different thermal expansion properties of the tool-plate and the pre-preg material, a degree of slippage will occur between the laminate’s bottom surface and the tool-plate during the temperature ramp up phase of the cure cycle (Radford 1995). This, coupled with the high pressure forcing the laminate onto the tool-plate results in friction between the two components, which can lead to significant shear being formed at the laminate’s bottom surface (Ersoy et al. 2005, Twigg et al. 2004a, 2003). The coarseness of the surface in contact with the laminate was shown to have a significant effect on the induced curvature by Stefaniak et al. (2012). In addition, the use of release film between the laminate surface and the tool-plate was found to enable the transfer of shear stresses into the laminate. Their results also indicate that pre-preg composition (i.e. the resin/fibre system used) can influence the curvature induced during processing.

Twigg et al. (2003) used strain measurements from a thin tool-plate to characterise the interfacial shear stresses that arise between the tool and the part during cure. The strains were monitored at different locations along the laminate length, under different pressure and tool-plate conditions (release film or release agent). It was shown that the behaviour between the part and the tool can be characterised as being ‘sliding-friction’ or ‘bonded’, depending upon the interfacial shear stress. The use of release film removes tool-part adhesive bonding, but can still transfer shear stresses. Even at low stages of cure the fibres can begin to develop stresses. The interfacial shear stress was shown to increase with degree of cure, meaning that the cure cycle employed (e.g. temperature ramp-rate) may influence the final part geometry. The results from this show the complexity of the curing process when taking tool-part interaction into consideration.
Incorporating such effects into modelling would require a thorough characterisation of the manufacturing process being used.

One approach to numerically reproduce the effects of tool-plate interaction is to model an elastic shear layer between the tool and the part. The elastic modulus of this layer can be adjusted to match an experimentally observed curvature, thus tailoring the amount of shear stress transfer between the tool and part. Twigg et al. (2004b) used this approach to conduct a parametric study encompassing laminate properties, geometry and interface shear. The study required, amongst other things, extensive experimental characterization of the material under different temperature and cure states. This method was found to have difficulties in predicting the behaviour of laminates over different part lengths. The complex experimental and modelling parameters make this an unattractive approach to incorporate manufacturing effects in models of multi-stable laminates, whose focus is generally in the buckling behaviour.

To account for slippage effects, Kaushik and Raghavan (2010) measured the static and dynamic friction coefficients between a steel plate (as used in autoclave processing) and glass fibre/epoxy fabric pre-preg, by measuring the pull-out force required for a laminate sandwiched between the two plates. The two plates used simulated the tool-plate, and autoclave heating and pressure application. The authors obtained four different sets of coefficients corresponding to different degrees of cure. This data was then used by Zeng and Raghavan (2010) in a thorough study on the process induced deformation of an aircraft part. An aircraft aft strut fairing, measuring roughly 1.8 m in length was manufactured using E-glass fabric along with sandwich material at certain locations, in a steel ‘female’ mould. The deformation of this manufactured specimen was used as a benchmark to compare FE results against. The FE model took into account material changes during the cure, thermal gradients, friction between the part and the tool-plate, and thermal deformation of the tool-plate. This required an extensive experimental campaign, to obtain values of the material’s physical, thermal, rheological and mechanical properties as a function of temperature and degree of cure. Of all the effects studied, the authors concluded that “the simulation suggests that the two components of tool-part interaction that contribute to warping are change in shape of the tool and part, and the process-induced stress caused by constrained deformation of the tool and part”. Accounting for friction is complex, however, and in this case required (on top of complex experimental data) a user sub-routine to be implemented in the FE
software Abaqus. This doesn’t lend itself to the numerical analysis of multi-stable composite structures, which already have complex buckling behaviour to address. While the overall surface profile of the aircraft strut was predicted well, absolute correlation against the manufactured demonstrator was not reached. This is with a very detailed numerical model that included many separate inputs. As such, it is evident that reproduced manufacturing effects is complicated and still not completely understood.

Analytical models have been proposed to reproduce the process induced shear at the laminate’s bottom surface. An approach to address slippage effects was presented by Cho and Roh (2003), who modified the theory of Hamamoto and Hyer (1987) to include an experimentally observed shearing between the bottom laminate surface and the tool plate onto which it is cured. The shearing was calibrated using experimental observations, and resulted in a shear stress profile through the laminate’s thickness. It was found that accounting for improved correlation between predictions and measurements, when compared to Hyer’s theory. However, it is unclear as to how the CTE values were obtained, and thus if manufacturing effects were indirectly considered through curve fitting. The side-length curvature profile depicted for the modified theory shows equal and opposite curvature values for opposing multi-stable shapes. If a shearing was reproduced on one surface and one surface only, then it stands to reason that the curvature of the second multi-stable shape should be different to the first. It was concluded that the effect of non-thermo-elastic related curvature (i.e. due to effects other than elastic thermal stresses) contributed to the curvatures by an order of 5%. Twigg et al. (2004a) proposed a numerical model which included both tool-part slippage as well as inter-ply slippage (which may relieve some of the residual stresses). Using FE models, it was concluded that inter-ply slippage may generally occur for the bottommost two plies only (Twigg et al. 2004b). They concluded that part warpage can be estimated to be a function of processing pressure, part length and part thickness.

It is clear that several manufacturing factors can influence the cured shape of thin composite laminates. Including these effects is a research topic in itself, with several studies focusing on identifying the key aspects of the behaviour. Several of these factors are directly related to the manufacturing set-up (e.g. the material, surface, and geometry of the tool-plate) and thus accounting for such effects can require data to be obtained for the specific manufacturing set-up being used. Therefore, including their effects is not trivial, as values obtained from literature may be largely unsuitable. For example,
should the (generally flat) tool-plate deform due to the combined effects of thermal expansion and its mechanical attachments any structure, this can lead to the tool-plate warping. Therefore, the assumption of an initially flat laminate shape at the curing temperature may be incorrect. Including the deformation, if any, of the tool-plate is specific to the exact manufacturing process being used and so must be measured.

With this in mind, and with the complexity of the experimental and numerical aspects involved, the direct inclusion of such effects into the study of multi-stable unsymmetrical laminates is largely unfeasible, and as such can lead to differences between measured and predicted multi-stable laminate shapes. As such, these factors have generally been ignored; particularly as the fundamental behaviour of multi-stable laminates is being explored, as opposed to precise shape predictions. However, as laminate shapes are of particular importance in this study, methods of measuring and incorporating certain manufacturing effects have been reviewed.

### 2.5 Moisture Absorption Effects

Moisture absorption is known to affect residual stresses, and thus the curvatures of unsymmetrical laminates. The traditional approach to account for moisture swelling (strain) through modelling is in the use of swelling coefficients and moisture content, which are analogous to thermal straining caused by thermal expansion coefficients and temperature changes. As applications involving multi-stable laminates rely heavily on both the shape and multi-stable property, adjustment of the residual stresses due to moisture ingress requires attention to further validate the potential of technology.

Portela et al. (2008) included the effects of moisture straining in numerical models of multi-stable laminates to predict the resulting loss of curvature and change in snap-through forces. They predicted the loss in curvature following saturation of a [0/90] laminate to be roughly 70%, indicating a near total loss of the ‘as manufactured’ curvature. The analysis of moisture effects was conducted as follows: FE models using shell elements were created, and the thermal straining reproduced (as has been described in section 2.3). To simulate the stress relief due to moisture, the thermal expansion coefficients used in models were modified to take into account the moisture induced swelling. The total strain applied to each lamina was reproduced, summing the individual contributions of thermal effects and moisture effects, using the following relationship (Portela et al. 2008):
\[
\begin{pmatrix}
\alpha^L_k \\
\alpha^T_k \\
\alpha_k
\end{pmatrix} = \begin{pmatrix}
\alpha^L_0 \\
\alpha^T_0 \\
0
\end{pmatrix} + \begin{pmatrix}
\beta^L_k \\
\beta^T_k \\
\beta_k
\end{pmatrix} \frac{c^*}{\Delta T} \tag{2.1}
\]

Where: \( k \) refers to the \( k \)th ply; \( \alpha^L_k \) and \( \alpha^T_k \) are the equivalent expansion coefficients encompassing both thermal and moisture effects; \( \alpha^L \) and \( \alpha^T \) are the longitudinal and transverse ply CTE values respectively; \( \beta^L \) and \( \beta^T \) or the coefficients of moisture expansion; \( \Delta T \) is the applied temperature field and \( c^* \) is the moisture concentration, given by:

\[
c^* = \frac{\text{Weight of moist material} - \text{Weight of dry material}}{\text{Weight of dry material}} \times 100 \tag{2.2}
\]

To proceed with the analysis of moist materials, values for the \( \beta^L \), \( \beta^T \) and \( c^* \) were required. In the fibre direction, \( \beta^L \) was taken to be zero, as the fibre doesn’t absorb moisture, while \( \beta^T \) was estimated to be 0.005 wt.%\(^{-1}\) from Herakovich (1998). The authors assumed an equilibrium moisture content \( c^* \) of 0.6 wt.% (by weight) from Springer (1981). According to Etches et al. (2009), the moisture content of \( c^* = 0.6\% \) is equivalent to the saturation content of a laminate exposed to 35% relative humidity environment. This would require roughly 70 days to reach saturation (Portela et al. 2008).

In addition, a drop of 67% in the load required to initiate snap-through was experimentally recorded, demonstrating the effect of moisture ingress on multi-stable laminate behaviour. The snap-through load was underestimated by 7.49% when compared to that of experiments, when using a moisture content 0.2 wt.% (it is not clear if this was experimentally measured or estimated). It must be assumed that this value represents a laminate stored in a humid condition that has reached saturation, as a transient value of \( c^* \) would lead to unequal moisture concentration through the laminate thickness and thus alter the laminate’s behaviour. Nonetheless, this is a relatively low moisture content demonstrating that a larger loss in snap-through load may be experienced when larger values of moisture content are considered.

The difficulty in examining moisture effects is highlighted in this work, and that is obtaining actual material data for \( \beta^T \) and \( c^* \). These values are material dependent, and may vary as a function of the material, as well as “the relative humidity of the
environment, the nature of the fluid, the matrix material and geometry”. Thus, while this analysis gives insight into the effects of moisture absorption on multi-stable, the values of MSC and $c^*$ specific to the laminate in question are required when a detailed analysis specific to a particular composite material is sought. Nonetheless, taking the predicted loss in curvature and the measured loss in snap-through forces it is clear that multi-stable laminates in morphing structures need to consider moisture effects. Indeed, the authors subsequently incorporated a Macro Fibre Composite (MFC) actuator into the analysis and concluded that moisture effects can affect the choice of actuator. This actuator type is directly bonded to the surface of the laminate and can therefore influence the laminate’s shape. It was shown that the suitability of using such an actuating system drops significantly once moisture ingress is considered. This is due to a conflicting requirement of using an actuator which is large and strong enough to actuate the multi-stable laminate in the dry condition, yet small enough not to interfere with the moist shape to such a degree that the multi-stable behaviour is removed. This shown by Figure 2-9, where a small range of actuator sizes (‘working combination’) is denoted as being suitable for both dry and moist laminate states.

Figure 2-9 Design curve of MFC actuated bi-stable plates. From Portela et al. (2008).

Morphing structures could very well be subjected to such conditions, leading to a higher moisture content than the 0.2% used by Portela et al. (2008) above. This would result in even larger changes in curvature and snap-through forces.
It has been suggested by Youssef et al. (2009) that moisture diffusion in fibre-matrix composite materials is coupled to the stress state. Indeed, results from stressed and unstressed graphite-epoxy composites show an increase in diffusivity and saturation moisture content as a function of stress (Comyn 1985). For instance, depending on the local stress state of the matrix, any pores or channels available for water transport may be expanded (tensile stress state) or closed (compressive state). That is, the process of diffusion in influenced by the stress state of the laminate. Fahmy and Hurt (1980) noted that the diffusion coefficient of a pure epoxy matrix increases when it is submitted to a uni-axial tensile load, and decreases when subjected to a compressive load. Additionally, the saturation moisture content is also affected by the stress state. This can be attributed to an increase in water transport (and thus storage) through the material, due to an increase in the 'free volume'. This leads to potentially a strong coupling between stress and moisture. The stress state of the resin (both at the micro and macro scale) can thus influence moisture absorption and moisture swelling. Consequently, predicting the transient moisture diffusion effects can require a complex modelling technique, with difficult to obtain parameters. For example, Youssef et al. (2009) presented a coupled moisture-stress model, whereby the local diffusion and moisture content were dependent on the stress state, which in turn was dependent on the moisture. To undertake such an analysis requires knowledge of the moisture diffusion and saturation properties of both strained and unstrained resin – not trivially obtained values. Nonetheless, comparison against an uncoupled model revealed potentially significant changes in the stress state when a coupled moisture/stress state was considered.

Gigliotti et al. (2007) expanded CLT to take into account transient states of moisture absorption by employing Fick’s law for moisture diffusion. Moisture absorption in composite materials is generally accepted to follow Fick’s law, which describes the diffusion process (i.e. moisture uptake as a function of time) by means of an initially linear diffusion coefficient, followed by a non-linear segment which levels off when saturation is reached (Comyn 1985). An experimental analysis was undertaken whereby the moisture diffusion in an unsymmetrical laminate was monitored during absorption by use of a number of witness specimens. Following saturation, the laminates were subsequently dried, and the laminate’s shape was periodically recorded and used to compare against models. From this transient drying stage, a moisture swelling coefficient was inferred, which can subsequently be used to infer detail on the internal
stress state. However, the comparisons against experiments are based upon the reversibility of the moisture diffusion process. Some elements of moisture diffusion are not reversible, such as matrix cracking (Katnam et al. 2010, Comyn 1985). Indeed, a large increase in the diffusion coefficients has been reported following several successive cycles of absorption and desorption, indicating a damage mechanism which influence moisture uptake (Comyn 1985). Also, desorption was not directly measured, but based on the absorption data of witness specimens, which was then used as a parameter in curve fitting. In addition, the model used an assumed uncoupled diffusion process (Fickian), with no stress dependency being considered. This was later used to infer a swelling coefficient, with no direct measure of the diffusion front being made. Should this method be used with other lay-ups (with different stress states, and thus desorption processes) it is possible that different swelling coefficients could arise. As such, using these assumptions may lead to errors in the obtained swelling coefficient. Removing the transient element of the analysis (i.e. exploring saturated laminates) would remove these assumptions as well as creating a simpler experimental set-up. The general premise of the technique is extremely useful in dealing with the issue of obtaining swelling coefficients, which are difficult to measure or obtain from published works.

2.6 Measuring Laminate Shapes and Residual Stresses
Jeronimidis and Parkyn (1988) and Cowley and Beaumont (1997) examined thermally induced residual stresses in thermoplastic based fibre-matrix composites. The through-thickness residual stresses predicted by CLT were compared against experimentally measured stresses. Experimental methods included the first ply failure method (where the residual stresses in a ply and the applied tensile stress during testing are compared to the tensile strength of the material) and measuring the curvatures of unsymmetrical laminates. In the latter case, it was deduced that “if the theory can predict the curvatures accurately, it can be assumed that the residual stress levels are also accurate and that appropriate thermoelastic properties have been used”. A generally good correlation between CLT predictions and experiments was found, particularly when the curvatures of unsymmetrical laminates were used, leading the authors to conclude that residual stresses can be predicted using CLT. In the experimental results presented by Cowley and Beaumont (1997), it was noted that calculated stresses (from curvature measurements) were initially 50% lower than those predicted. This stress relaxation was attributed to the milling process generating localised heating and accelerating the stress
relaxation process. No mention is made of moisture absorption, which is likely to occur from humidity, and would also result in stress relaxation. Following four heating cycles above the glass transition temperature ($T_g$) the stresses had reached predicted values. Gigliotti et al. (2006, 2007) used an optical method (fringe projection method) to measure laminate shapes for further analysis using CLT. The fringe projection method relies upon a periodic pattern of white and black lines which are projected on an object and captured by a CCD video-camera. Due to its non-contact nature, distortion of the laminate during measurements is reduced. The full-field nature also eliminates any assumptions regarding the final shape, such as that a square cross-ply laminate obtains a cylindrical shape with constant curvature. This is particularly advantageous when measuring laminates with complex geometries, for which assumptions as to the shape can’t be made. Multi-stable cross-ply laminates generally form cylindrical shapes, and so measurements have the laminate’s chord/deflection have been widely used to calculate the laminate’s curvature, for comparison against modelling (e.g. by Etches et al. (2009)). This is a simple method, and doesn’t require any sophisticated experimental set-up or procedure.

Eckstein et al. (2014) measured the shapes of square multi-stable laminates by using dial gauges, with the laminate clamped via a bolt (6mm) passing through the laminate centre. The effect (if any) of this hole on the extracted laminate shapes is not discussed, possibly as the fundamental behaviour of multi-stable laminates was of interest (and not an exact shape comparison against modelling). Using such a technique to measure saturated laminate shape may not be the most ideal solution as including a hole in the centre of the laminate may affect the diffusion dynamics at that area, where the modelling approach generally ignores edge effects (especially when thin plates are considered). Finally, using this technique, the authors were able to obtain four discrete points against which to compare measured shapes against predictions. This method was chosen (instead of deflection/chord measurements to calculate curvatures) due to twisting in the laminates. Dano and Hyer (1998) used a similar approach to measure laminate shapes. A grid of tracer fibres (which were embedded in the flat laminate configuration) provided 121 points at which deformed laminate co-ordinates could be obtained. These co-ordinates were measured using a dial gauge moved by hand and an “automated shape-measuring instrument”. While giving a lot of data points to compare against modelling, such an experimental set-up could be tedious, especially when multiple laminates are being investigated.
### 2.7 Summary

The analysis of multi-stable laminates has largely focused on understanding their fundamental behaviour, by utilising analytical and numerical modelling techniques. As such, key phenomena such as the shapes and snap-through forces of cured laminates have been investigated, and modelling techniques (both analytical and numerical) advanced to account for these. With a good understanding of the fundamental behaviour, potential morphing engineering applications have been proposed and concepts presented for the aerospace sector. These concepts often involve tailored laminate configurations, ones in which a laminate features a change in lay-up sequence to seamlessly accommodate a multi-stable section into the surrounding structure. Mechanically pre-loaded concepts have also been presented, allowing symmetry in the lay-up sequence to be preserved.

Before such technologies can advance to that stage, the effect of moisture ingress must be understood. Moisture ingress has been shown to severely affect the multi-stable shapes and snap-through forces of multi-stable laminates. Unsymmetrical diffusion can also affect multi-stable laminates employing symmetrical lay-ups. Accounting for moisture ingress effects is made difficult by the complex mechanisms at play, which makes it difficult to obtain material values required for modelling.

Likewise, the ‘as-cured’ laminate shapes are influenced by a number of complex processing factors, which require numerous complex experimental data to be acquired before implementing them in modelling. This is a research topic in and of itself and, even though manufacturing can significantly influence the cured shape of multi-stable laminates, this area is generally excluded from the analysis of cured laminate shapes.

Modelling approaches (both analytical and numerical) generally assume ply-level homogenised and orthotropic material properties, including CTEs and MSCs. This leads to prediction of residual stresses at a macro-scale level, ignoring complex micro-scale stresses and processes. Also, obtaining material specific CTE and MSC values remains difficult and are not widely available in literature.

Analytical methods to predict multi-stable laminate responses offer the advantage of being able to predict multi-stability, while numerical methods (FE) generally required knowledge of the multi-stable shapes *a priori*. Conversely, FE allows greater flexibility
in the analysis of finer details (such as edge effects) and more complex situations, such as interactions with local structures, applied loading and complex geometries.

Unsymmetrical laminates have been used in combination with analytical techniques to infer the residual stress state of dry and saturated laminates in the past. Monitoring the shape change of a laminate during desorption has led to a value of the material’s swelling coefficient being obtained. Using a simpler experimental process (monitoring saturated laminate shapes) and removing transient stages of analysis (and associated assumptions as to the absorption/desorption process) could lead to a simpler experimental set-up as well as a more robust value of the material swelling coefficient being obtained for modelling. Extending the analysis to tailored laminates would benefit from the use of contactless optical methods of shape measurement, which would remove any assumptions pertaining to the laminate shapes.

The ability of these macro-scale residual-stress prediction techniques to correctly predict the hygro-thermal behaviour of a range of multi-stable laminates is not generally covered in the literature. Experimental comparisons tend to focus on single lay-up families, and so it is unknown if the values obtained (e.g. for the MSC) can be used to describe a wide range of lay-ups. This is particularly true for tailored laminates, with very little detail on the hygro-thermal response being available. As these laminate types are a prime candidate for morphing applications, the ability to predict their response to hygro-thermal loading is a key research area.
Chapter 3  Analytical Solution

3.1 Introduction

A numerical calculator was created, based on the theory presented by Hyer (1981a), using the mathematical program Matlab. The calculator allows the user to predict the existence of the multi-stable property of laminates and the corresponding cured shape(s). The theory used is applicable to unsymmetrical cross-ply laminates, and uses the laminate geometry, lay-up sequence, the difference between cure and room temperatures ($\Delta T$) and material properties (orthotropic and linear elastic) as an input. Such a tool is useful as it can predict the stability of the shapes of multi-stable laminates, without prior knowledge from experiments. This is particularly advantageous over FE methods, which can require ‘coaxing’ to get the desired result. Without this coaxing, unstable laminate shapes can be predicted (Dano and Hyer 1998). A description of the tool is provided in Appendix A.

The definitions of curvatures used in this study are shown in Figure 3-1.
Figure 3-1 Curvature definitions for square cross-ply laminates.

The side lengths along the $x$ and $y$ axes are given the symbols $L_x$ and $L_y$ respectively, while the curvatures along the $x$ and $y$ axes are given by the symbols $a$ and $b$.

### 3.2 Predicting Multi-stable Shapes of Unsymmetrical Composite Laminates

The theory developed by Hyer (1981a) will be presented here. Although this theory has been extended since, it remains the basis for predicting multi-stable shapes, and also represents the theory used to benchmark the numerical analysis (Chapter 5). Normally, CLT is used to predict the cured shape of composite laminates. Using a series of simplifying assumptions, individual ply properties are used to calculate the stress, strain and deformation behaviour of the complete laminate under different loading conditions (e.g. thermal loading). The theory is linear, and includes the assumption that the strain-displacement relationship is linear. This particular assumption was modified by Hyer to explain the multi-stable behaviour observed for some unsymmetrical laminates.
Specifically, geometric nonlinearities were included in the form of von Kármán nonlinear terms in the strain-displacement relationships (Reddy 2003). This was done as: (a) multi-stable laminates display two room temperature shapes, thus ruling out linear extensions to the theory (which would predict one solution); and (b) since the out-of-plane deflections of the multi-stable laminates were many times greater than the laminate thickness, it was felt that geometric nonlinearities were important to consider.

To predict the multi-stable shapes, the potential energy of the laminate is minimised. This was done using the Rayleigh-Ritz approximation method, and is applied as follows. The total potential energy of the system is formulated, and displacement functions are assumed to vary as a function of a number of undetermined parameters. A set of simultaneous equations minimizing the total potential energy with respect to these parameters are performed, from which solutions to the parameters are obtained (Zienkiewicz and Taylor 2000, Reddy 2003).

Applied to the analytical approach being presented here, a set of displacement functions were assumed, according to the saddle and cylindrical shape configurations observed experimentally. These functions were combined with the non-linear strain-displacement relationships, resulting in four coefficients \((a-d)\) to be determined. The laminate potential energy \((\Omega)\) is then formulated using these terms. The problem then becomes one of finding solutions for \(a-d\) so that the first variation of the laminate potential energy is equal to zero.

To summarise, predicting the cured shape of square cross-ply laminates can be broken down into the following steps:

1. Extend CLT by adding a geometric non-linearity.
2. Assume functional forms for the laminate mid-plane displacements, \(u^0(x,y)\), \(v^0(x,y)\), along with the laminate out-of-plane displacements \(w(x,y)\). The assumed shape functions include four yet-to-be-determined coefficients, \(a\), \(b\), \(c\) and \(d\).
3. Use these terms to calculate the laminate mid-plane strains, \(\varepsilon^0_x\), \(\varepsilon^0_y\).
4. Substitute these terms into the laminate’s thermal potential energy \(\Omega\).
5. Minimise the potential energy to solve for \(a-d\).

From classical laminate theory, the laminate strains can be defined as:

\[
\varepsilon_{xx} = \varepsilon^0_x - Z \frac{\partial^2 w}{\partial x^2} \quad (3.1)
\]
The non-linear mid-plane strains can be obtained as follows (Reddy 2003):

\[
\varepsilon_x^0 = \frac{\partial u^0}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2
\]

(3.4)

\[
\varepsilon_y^0 = \frac{\partial v^0}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2
\]

(3.5)

\[
\varepsilon_{xy}^0 = \frac{1}{2} \left( \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} + \left( \frac{\partial w}{\partial x} \right) \left( \frac{\partial w}{\partial y} \right) \right)
\]

(3.6)

The total potential energy within the laminate is given by:

\[
\Omega = \int_{V_{ol}} \omega dVol
\]

(3.7)

where \(\omega\) (the strain energy density) is given by:

\[
\omega = \frac{1}{2} \tilde{Q}_{11} \varepsilon_{xx}^2 + \tilde{Q}_{12} \varepsilon_{xx} \varepsilon_{yy} + 2\tilde{Q}_{66} \varepsilon_{xy}^2 + \frac{1}{2} \tilde{Q}_{22} \varepsilon_{yy}^2 - \left( \tilde{Q}_{11} \alpha_x + \tilde{Q}_{12} \alpha_y \right) \varepsilon_{xx} \Delta T - \left( \tilde{Q}_{12} \alpha_x + \tilde{Q}_{22} \alpha_y \right) \varepsilon_{yy} \Delta T
\]

(3.8)

The out-of-plane deflection \(w\), mid-plane displacement in the \(x\) direction \(u^0\), and the mid-plane displacement in the \(y\) direction \(v^0\), are assumed to be of the form:

\[
w(x, y) = \frac{1}{2} (ax^2 + by^2)
\]

(3.9)

\[
u^0(x, y) = cx - \frac{a^2 x^3}{6} - \frac{ab xy^2}{4}
\]

(3.10)

\[
v^0(x, y) = dy - \frac{b^2 y^3}{6} - \frac{ab x^2 y}{4}
\]

(3.11)

The functional form of \(w(x, y)\) (Eqn. 3.9) is a reasonable approximation of the situation in Figure 3-1, where different values of \(a\) or \(b\) can produce the shapes shown.

Using Eqns. 3.10 – 3.11 and 3.4 – 3.6, the mid-plane strains take the form:

\[
\varepsilon_x^0 = c - \frac{ab y^2}{4}
\]

(3.12)
The problem now turns to finding minima for the potential energy, which is done by finding solutions for \(a-d\) when the first variation of \(\Omega\) is zero, i.e.

\[
\delta \Omega = \left( \frac{\partial \Omega}{\partial a} \right) \delta a + \left( \frac{\partial \Omega}{\partial b} \right) \delta b + \left( \frac{\partial \Omega}{\partial c} \right) \delta c + \left( \frac{\partial \Omega}{\partial d} \right) \delta d \equiv 0
\]

Referring to Figure 3-1, the laminate geometry at the elevated temperature is defined by:

\[
\begin{align*}
-\frac{L_x}{2} & \leq x \leq \frac{L_x}{2} \\
-\frac{L_y}{2} & \leq y \leq \frac{L_y}{2} \\
-\frac{t_l}{2} & \leq z \leq \frac{t_l}{2}
\end{align*}
\]

where \(t_l\) is the laminate thickness.

So \(\Omega\) takes the form (from Eqn. 3.7):

\[
\Omega = \int_{x=-L_x/2}^{L_x/2} \int_{y=-L_y/2}^{L_y/2} \int_{z=-t_l/2}^{t_l/2} \omega(a, b, c, d; \bar{Q}_{ij}, \alpha_x, \alpha_y, \Delta T, x, y, z) \, dx \, dy \, dz
\]

Continuing with obtaining solutions for \(a-d\), Eqns. 3.9, 3.12 – 3.14 are introduced into Eqns. 3.1 – 3.3. These are then substituted into the strain energy function, Eqn. 3.8, before performing the spatial integration in Eqn. 3.17.

The first derivative of \(\Omega\) may now be assessed and set to equal zero.

\[
\delta \Omega = f_1(a, b, c, d) \delta a + f_2(a, b, c, d) \delta b + f_3(a, b, c, d) \delta c + f_4(a, b, c, d) \delta d
\]

Eqn. 3.18 leads to four coupled and non-linear equations, which can be solved simultaneously.

These are:

\[
f_1(a, b, c, d) = -C_1 c b + C_2 a b^2 + 2C_3 a b - B_{11} c + D_{11} a - C_4 c b + 2C_5 a b^2 - C_6 d b + D_{12} b - C_7 d b + C_8 a b^2 + C_9 b^2 + \left( \frac{l_z^2}{4 b^2} \right) N_x^T b + M_x^T + \left( \frac{l_z^2}{4 b^2} \right) N_y^T b = 0
\]
Analytical Solution

\[ f_2(a, b, c, d) = -C_1 ac + C_2 a^2 b + C_3 a^2 - C_4 ac + 2C_5 a^2 b + D_1 a - C_6 da - C_7 da + \]
\[ C_8 a^2 b + 2C_9 ab - B_{22}d + D_{22} b + \left(\frac{L^2}{48}\right) N^T_x a + \left(\frac{L^2}{48}\right) N^T_y a + M^T_y = 0 \]  

(3.20)

\[ f_3(a, b, c, d) = A_{11} c - C_1 ab - B_{11} a + A_{12} d - C_4 ab - N^T_x = 0 \]  

(3.21)

\[ f_4(a, b, c, d) = A_{12} c - C_6 ab - B_{22} b + A_{22} d - C_7 ab - N^T_y = 0 \]  

(3.22)

where:

\[ C_1 = \frac{A_{11}L^2_y}{48} \quad C_2 = \frac{A_{11}L^4_y}{1280} \]  

(3.23)

\[ C_3 = \frac{B_{11}L^2_y}{48} \quad C_4 = \frac{A_{12}L^2_y}{48} \]

\[ C_5 = \frac{A_{12}L^2_x}{2304} \quad C_6 = \frac{A_{12}L^4_y}{48} \]

\[ C_7 = \frac{A_{22}L^2_y}{48} \quad C_8 = \frac{A_{22}L^4_y}{1280} \]

\[ C_9 = \frac{B_{22}L^2_x}{48} \]

where \( A_{ij} \) and \( B_{ij} \), and \( D_{ij} \) are the laminate extensional stiffnesses, bending-extension coupling stiffnesses, and the bending stiffnesses respectively, as defined in CLT (e.g. (Jones 1998)).

Note: Eqn. 3.20 presented here is a corrected version of that presented by Hyer (1981a), which features an error.

The \( \bar{Q}_{ij} \) are standard reduced stiffness terms, while \( N^T_x, N^T_y, M^T_x, M^T_y \) are the thermally induced force/moment resultants.

\[ N^T_x = \Delta T \int_{-t/2}^{t/2} (\bar{Q}_{11} \alpha_x + \bar{Q}_{12} \alpha_y) \, dz \]  

(3.24)

\[ N^T_y = \Delta T \int_{-t/2}^{t/2} (\bar{Q}_{12} \alpha_x + \bar{Q}_{22} \alpha_y) \, dz \]  

(3.25)

\[ M^T_x = \Delta T \int_{-t/2}^{t/2} (\bar{Q}_{11} \alpha_x + \bar{Q}_{12} \alpha_y) \, z \, dz \]  

(3.26)

\[ M^T_y = \Delta T \int_{-t/2}^{t/2} (\bar{Q}_{12} \alpha_x + \bar{Q}_{22} \alpha_y) \, z \, dz \]  

(3.27)
Solving Eqns. 3.19 – 3.22 gives five approximate values for $a, b, c$ and $d$. Depending on the solution, these values will be either in the form of one real number and four imaginary numbers (corresponding to a single saddle shape), or three real numbers and two imaginary numbers (corresponding to two multi-stable solutions, and one unstable saddle solution). A stability analysis on each of the shape solutions obtained (to verify if the predicted shape represents a stable or unstable shape) can be carried out by carrying out a second variation of the potential energy, which in turn must be positive definite to represent a stable solution.

The presented theory has been the fundamental basis of subsequent research into multi-stable composite laminates. One of the limitations remains, however, that the solution is based on the Rayleigh-Ritz method – a variational method which obtains a solution based upon an initial approximation (in this case, the shape functions described in Eqns. 3.9 – 3.11). The solution is therefore constrained to this assumed shape. For square cross-ply laminates that feature cylindrical multi-stable shapes, the assumed shape function is a reasonable description of what is observed experimentally. Other laminate geometries or lay-ups required extensions to the theory to account for these.

3.3 Analytical Tool for Shape Prediction

Using the theory developed by Hyer (1981a) and presented in section 3.2, a calculator was created using the mathematical programming software Matlab (7.10). The calculator uses inputted values for temperature change, laminate geometry, lay-up and material properties to calculate from standard composite mechanics the reduced stiffness matrix ($Q_{ij}$), transformed reduced stiffness matrix ($\tilde{Q}_{ij}$), thermal moments and axial force resultants ($N_{x,y}^T, M_{x,y}^T$), extensional stiffnesses ($A_{ij}$), bending-extensional coupling stiffnesses ($B_{ij}$) and bending stiffnesses ($D_{ij}$). These are then used to solve Eqns. 3.19 – 3.22 above, to obtain solutions for $a, b, c$ and $d$. Solving these equations was done using the ‘symbolic toolbox’ in Matlab, which allows for the use of unknown coefficients in formulas. An example of an output is illustrated in Figure 3-2, showing the two multi-stable shapes predicted for a square cross-ply laminate with a $[0_2/90_2]$ lay-up. Both shapes are cylindrical, and differ in the orientation and generator axis of the curvature.
The calculator was validated in two separate steps. Firstly, the stiffness matrices ($Q_{ij}$, $A_{ij}, B_{ij}, D_{ij}$) and the force and moment resultants ($N_x^T, M_x^T$) were compared to examples given in literature (e.g. Jones (1998)). Additionally, through e-mail correspondence, Prof. M.W. Hyer sent the values for $Q_{ij}, N_x^T, N_y^T, M_x^T$ and $M_y^T$ that were used to predict the curvatures in his published work. This allowed for the reproduction of the figures presented in Hyer (1981a). A description of the developed tool is given in Appendix A.

Being based on the theory presented in Section 3.2, this tool is subject to the same assumptions and limitations (mainly, square cross-ply laminates). It is used in this study to benchmark the curvatures of square cross-ply laminates predicted by numerical models. Several advancements have been made to this theory to increase its applicability (for example, to general unsymmetrical laminates with arbitrary lay-ups). However, for the purpose at hand (benchmarking numerical models) this theory is adequate. Numerical models can then be adapted to predict the shapes of more complex laminate lay-ups and geometries.

3.4 Summary

An analytical approach for predicting the possible shapes, as well as the stability of those shapes, of square cross-ply laminates is presented. The theory is based upon the extension of classical laminate theory by incorporating non-linear strain-displacement relationships, followed by the minimization of the laminate’s total potential energy by means of the Rayleigh-Ritz method.

This theory was incorporated into a mathematical tool used to predict the multi-stable shapes of square cross-ply laminates, using laminate geometry, lay-up, material...
properties and the difference between cure and room temperature to predict laminate shapes. This tool was benchmarked by using values obtained from the developer of the theory (M. W. Hyer), validating it for further use against numerical models.
Chapter 4  Experiments

4.1  Introduction
A large range of experiments were conducted, focusing on material characterisation, the dry and saturated shapes of cross-ply and tailored laminates, and the full-field surface measurements using laser scanning.

As the shapes of unsymmetrical laminates are central to this study, a thorough description of the manufacturing process used for all laminates will be provided. Also included will be a description of the observed process induced curvature (i.e. curvature induced independently of laminate symmetry), which needs to be considered when comparing experimentally measured shapes against those predicted by modelling. Subsequently, a description of the material characterisation will be given. The characterisation was conducted to obtain material properties required for modelling, which included elastic properties (Young’s Modulus, shear modulus, etc.) as well as the material’s linear transverse CTE value (α_T).

Next, the unsymmetrical laminates (cross-ply and tailored) used in this study will be presented. The lay-ups used and the corresponding cured shapes will be given, along with the methods used to measure their shapes.

Finally, a description of the moisture absorption experiments will be given, along with the gravimetric data taken during immersion.

4.2  Manufacturing of Composite Laminates
All laminates used in this study were manufactured using HTA-6376 CFRP material. This is a pre-preg, unidirectional, fibre-resin material and is manufactured by HEXCEL. The material was stored and subsequently manufactured according the manufacturer’s recommended guidelines. The procedure for manufacturing a laminate involved cutting the necessary plies from a roll of the CFRP material, followed by stacking the plies together using hand lay-up techniques. The laminates were then cured in an autoclave (TC1000LHTHP, LBBC, UK) under 7 bar of pressure and at a temperature of 178 °C. During the cure, the laminates were placed within an air-tight bagging system, which is used to draw a vacuum and force the laminate onto the mould (in this case, a flat tool-plate), as shown in Figure 4-1. The bagging material covers the laminates being cured.
and is sealed to the tool-plate surface by using tacky-tape. This creates the air-tight environment within the bag. In combination with the pressure applied within the autoclave, this vacuum results in an equal force across the laminate surface, which is held during the cure. The release film on top of the tool-plate ensures that the laminate doesn’t adhere to the tool-surface during the cure. The peel ply then transfers a rough surface onto the laminate being manufactured; this can be useful in gripping the laminate during mechanical testing. Finally, the breather material is a thick, porous material which allows gases to travel from all areas under the bagging to the vacuum pump. This enables any air/gases within the laminate to be extracted during the cure cycle.

**Figure 4-1 Bagging technique used in the manufacture of the CFRP laminates.**

The consolidation process is then as follows. The laminate is sealed in the bagging at room temperature, and a vacuum is drawn. This forces the laminate to the flat shape of the tool-plate, which will be maintained during the cure. The tool-plate assembly (including the laminate and bagging) is then inserted into the autoclave, which is sealed. The autoclave ramps the temperature up to the curing temperature (178 °C) and pressure (7 bar). During the ramp up, and after the temperature exceeds the material’s glass transition temperature, the resin becomes viscous. At this stage, the constituent materials of the laminate (fibre and resin) undergo thermal expansion/contraction, according to their own CTEs. No stresses are built up at this stage, as the resin is free to flow and adjust during the straining. At the same time, it is worth remembering that the steel tool-plate itself has undergone thermal expansion. At the elevated temperature, the resin material consolidates and hardens, ‘locking’ the constituent materials together. Following consolidation, the temperature is ramped down to 60 °C and the pressure is released, after which the cure cycle is considered complete. The vacuum that was drawn on the laminates is still present within the bag, and thus the laminates are still forced flat against the tool-plate. Finally, the tool-plate assembly is removed from the autoclave, and the laminates are extracted from the bagging after the assembly has cooled down to
room temperature. During the entire cool-down process, the fibre, resin, and tool-plate all undergo different amounts of thermal straining. In the case of the fibre and resin, this leads to the development of micro- and macro-scale stresses building up. As the cure cycle is an integral part of the curing process, the temperature recorded during the cure cycle of some of the laminates used in this project is given in Figure 4-2. These readings were taken at locations above and below two 4-ply thick laminates, at either end of the tool-plate. The readings were recorded using thermocouples which were plugged directly into the autoclave to record the data. The purpose of this was to see if there were any significant differences in temperature during the cure at different locations within the autoclave, which could lead to inconsistencies in the manufactured laminates. Apart from showing no significant differences in temperature at each location (importantly, during the ramp up), the profile also demonstrates the curing profile used for the laminates.

Figure 4-2 Measured cure cycle for the manufactured laminates, showing temperature as a function of time. Locations 1 and 2 refer to temperature readings above and below a laminate at one end of tool-plate, with Locations 3 and 4 being taken above and below a laminate at the other end of the tool-plate.

It is worth discussing, at this stage, some of the manufacturing factors that influence the shape of the composite laminates. As the cured shapes of the composite laminates are used in calibration, it is important to bear in mind additional contributors to the shapes (and thus potentially the residual stress state).

Figure 4-3 shows the shape of two unidirectional laminates following cure. The two laminates shown are of equal [0₄] lay-up sequence, and manufactured using the same materials and methods as all the laminates used in this research. The image was taken...
immediately following cool-down to room temperature. The white peel-ply on the bottom surface had yet to be removed, and the laminates are on the same tool-plate that they were cured on.

![Curvature observed in symmetrical [0 4] laminates. Note: peel-ply is still bonded to bottom laminate surface, but showed no influence to laminate shape once removed.](image)

It is quite clear that a significant curvature is formed in the laminates, with a concave curvature being consistently formed away from the tool-plate. Even though thermally induced macro-scale residual stresses should not be present (due to the unidirectional nature of the laminate), the curvature developed is significant. The reasons for this were not fully understood, prompting a further range of laminates to be manufactured to gain insight into the effect. What can be deduced is as follows: the curvature induced during manufacturing is consistently concave to the tool-plate; the curvature developed is consistent with the bottommost plies contracting along the fibre direction; the curvatures developed by laminates with identical lay-ups and orientations on the tool-plate are consistent and identical. The consistency of the curvatures – in terms of magnitude and direction – points away from laminate level imperfections (ply misalignment, variations in thickness or volume fraction, etc.) and towards a process induced effect (e.g. laminate/tool-plate friction, tool-plate deformation). One possible explanation is that the tool-plate CTE value is more similar to that of the resin (and thus, of the laminate in the transverse direction) than that of the fibre. During the ramp up to curing temperature, the fibres and the tool-plate strain at different rates. Due to the high pressure (7 bar) on the laminate surface, it is possible that the difference in strain between the tool-plate and the fibres lead to a tensile stress state being formed along the fibre length of the bottommost fibres. When the resin is viscous, the stress is not transferred through the laminate thickness, and so remains in the bottommost fibres.
Experiments

only. The stress is maintained during the consolidation phase of the resin, meaning that the bottommost fibres are locked into a state of tension when compared to the fibres on the upper laminate surface. Once removed from the tool, through-thickness stresses are introduced resulting in a state of curvature along the fibre length, as observed in Figure 4-3. Such interaction has been described before (Wisnom et al. 2006, Stefaniak et al. 2012, Kaushik and Raghavan 2010). Nonetheless, this is put forward here as a possibility only – no experiments verifying this behaviour have been conducted. In fact, all the aforementioned authors describe that several other influences are present and so a combination of various factors are possible (see, specifically, Zeng and Raghavan (2010)). The end result is, however, that the manufacturing method introduces a curvature which may, affect the correlation between measured and predicted shapes.

4.3 Material Characterisation

To improve conformity between modelling and experiments, a material characterisation study was undertaken, with specimens to be used in mechanical testing being manufactured simultaneously with the cross-ply laminates. The in-plane tensile and shear properties of the material in a dry state were experimentally obtained by following ASTM D3039M (2000) and ASTM D3518M (2001), the results of which showed good correlation to previous testing of the material published by O’Higgins et al. (2011). A complete description of the test methods, calculations used to obtain the material properties and the material properties obtained are given in Appendix C. Subsequently, the thermal expansion behaviour of the CFRP material was measured using dilatometry. A summary of the results obtained for use as material properties in modelling is given in Table 4-1.

The thermal expansion behaviour of the material was also required. As has been previously described, the general modelling approach for determining macro-scale stresses in fibre/matrix composites uses ply level orthotropic CTE values, in longitudinal and transverse ply directions. To obtain these values, the thermally induced strains, $\epsilon_T$, as a function of temperature change ($\Delta T$) was required. Dilatometry was used to measure the transverse thermal expansion coefficient ($\alpha_T$) using a Netzsh DIL 402C dilatometer. The thermally induced strain was measured across a temperature range of 25°C to 175°C, from which the expansion coefficient was obtained. One unidirectional laminate [0°/45°] (measuring 80 mm x 75 mm) was manufactured from which six transverse and six longitudinal samples were cut using a diamond-tipped saw.
Composite cutting machine. The specimens used are shown in Figure 4-4. As water is used as a coolant during cutting, the specimens were then dried for 24 hours at 50°C before being measured. Three of the longitudinal samples and one of the transverse samples were found to be outside dimensional tolerances for testing, and so they were not used.

![Figure 4-4 Transverse and longitudinal CFRP specimens used in dilatometry testing.](image)

The remaining eight samples were tested, and their thermal straining behaviour is shown in Figure 4-5.

![Figure 4-5 Thermal straining of HTA 6376 material measured using dilatometry](image)

The longitudinal expansion coefficient ($a_L$), which is dominated by the expansion/contraction behaviour of carbon fibres, gave highly non-linear and irregular results, especially when averaged between 25 °C to 175 °C. Therefore, the results were not used in this study. These irregular results may be due to the very small value CTE of the fibre (typically zero, Toho Tenax Europe (2011)), which makes experimental
measurement challenging. Note: due to the scale used in Figure 4-5, the inconsistencies amongst the three longitudinal specimens aren’t obvious.

The thermal straining of the transverse specimens showed a generally linear relationship with temperature change. As the temperature change of interest in modelling is from the curing temperature to room temperature, the strain behaviour from 175°C to 30°C was used to obtain a linear CTE value (by means of the slope of the average slope of the five transverse specimens in Figure 4-5). This resulted in a value for $\alpha_T$ of $2.86 \times 10^{-5}$ °C$^{-1}$.

A summary of the experimentally obtained material properties is given in Table 4-1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$E_{11}$ (GPa)</th>
<th>$E_{22}$ (GPa)</th>
<th>$\nu_{12}$</th>
<th>$G_{12}$ (GPa)</th>
<th>$\alpha_T$ (K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude</td>
<td>135.64</td>
<td>10.14</td>
<td>0.29</td>
<td>5.86</td>
<td>$2.86 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Note: $E_{11}$ and $E_{22}$ refer to the longitudinal and transverse elastic modulus, respectively; $\nu_{12}$ the in-plane Poisson’s ratio; $G_{12}$ the in-plane shear modulus, and $\alpha_T$ the transverse CTE.

Another popular method for determining the thermal expansion characteristics of CFRP involves using strain gauges. Strain gauges are devices used to measure the strain of an object. Vishay Micro-Measurements (2005) gives a comprehensive description of using strain gauges for thermal expansion testing which is applicable to testing composite materials. The experiment involves concurrent measurement of the thermal straining of the test specimen and another specimen made from a material with a very well-known expansion profile. The second (reference) material should have similar thermal straining characteristics to the material being tested, and the difference in thermal expansion between the two materials is then calculated. This is done by installing identical strain gauges on both the test material and the reference material. Both specimens are then heated together in the same chamber and the resulting strains are recorded. The difference between the strain measured from the reference material and the test material can then be used to determine the thermal expansion of the test material. It must be noted that because the result of interest is the difference in strain results between the reference and test materials, it is important to reduce as much as possible any differences between readings from the two strain gauges. This means that the strain gauges should be matched as much as possible (by, for example, choosing a pair from an identical batch). Additionally, all positioning, adhesion, soldering and wiring between the two specimens should be kept as identical as possible. Otherwise, the strain results obtained may include false readings due to the gauges installation.
While this procedure is suitable for thermal expansion tests on the CFRP material, and can possibly be even more suitable than dilatometry (for example, in obtaining the longitudinal expansion coefficient), it was decided not to proceed with this technique. While simple in theory, it is a precise experiment that requires a very precise experimental set-up in order to get valid results. Experimental errors may arise from adhesion, installation differences between reference and test specimens, and alignment errors in relation to the composites principal directions. The dilatometer results were deemed sufficient as: (a) the thermal straining of the transverse specimens provided consistent results, and (b) a measure of the longitudinal expansion coefficient was not required, as it was left open for calibration in numerical models (as will be described in Chapter 5).

4.4 Unsymmetrical Laminates

The lay-ups and shape measuring techniques for the unsymmetrical laminates used in this study will be described. Two groups of unsymmetrical laminates were manufactured. The first group (section 4.4.1) comprised square cross-ply laminates with three different lay-ups families. The second group (section 4.4.2) comprised the tailored laminates, which feature changes in lay-up and/or thickness along their lengths.

4.4.1 Cross-ply Laminates

To measure the change in out-of-plane deflection following moisture saturation in the unsymmetrical laminates, twenty cross-ply laminates (180 mm x 180 mm) were investigated, with four laminates of a [90₂/0₂] lay-up, eight of a [90₃/0₁] lay-up and eight of a [90₃/90₂] lay-up. The cured shapes of the laminates are shown in Figure 4-6.
After manufacture, the out-of-plane deflection and the chord length of the laminates were measured to calculate the curvature \( (a, \text{mm}^{-1}) \), as shown in Figure 4-7. This method has been used to measure curvatures of unsymmetrical laminates (e.g. by Choi et al. (2001)) and was chosen due to the simplicity of the experimental set-up. As multiple laminates were manufactured per family, a single averaged curvature value was obtained for each lay-up family. This value was later used to reproduce the cured shapes of the laminates in modelling.

An interesting observation was made following the manufacture of the cross-ply laminates. The laminates with lay-ups of \([90_3/0_2]\) and \([90_5/0_1]\) were orientated on the tool-plate slightly differently during the cure. For each lay-up, a total of eight specimens were manufactured. Four of these specimens were cured upside-down with respect to
the other four. In effect, this means that for the [90/0]_1 family, four of laminates were in fact cured as [0/90]_3. Likewise for the [90/0]_2 family, four of the laminates were cured as [0/90]_3 lay-ups. In essence, the difference meant that for four of the laminates, the thicker layer (i.e. the layer comprising three plies) was in contact with the tool-plate, while for the other four, the thinner layer (comprising either one or two plies, depending on the lay-up) was in contact with the tool-plate. All other aspects of manufacturing were kept identical. In both lay-up cases, what resulted were different curvatures between the two groups of four laminates. Amongst each group of four, the curvatures were very similar with very little variation. Therefore, the surface that is in contact with the tool-plate can have an effect on the cured shape. This was especially prevalent amongst the [0/90]_3 and the [90/0]_2 laminates. This behaviour further demonstrates the process induced curvature. Although the laminate lay-up with respect to the tool-plate was different for each family (with, for example, the [90/0]_1 family consisting of both [90/0]_1 and [0/90]_1 lay-ups), a single lay-up designation will be used to describe the family. These designations are [90/0]_1 (comprising both the [90/0]_1 and [0/90]_1 lay-ups) and [90/0]_2 (comprising both the [90/0]_2 and [0/90]_2 lay-ups). This was done as there is no influence from the tool-plate included in models, and so there is no distinction between, for example, the [90/0]_1 and [0/90]_1 lay-ups. As such, an average value of curvature was calculated for each lay-up family. The curvature of the dry (as manufactured) laminates will be given along with the saturated curvature in section 4.5.

4.4.2 Tailored Laminates

Six different tailored lay-up configurations were manufactured. Each configuration featured either 0° or 90° ply orientations, with partitions along the length (x-direction) of the laminate (see Figure 4-8, showing a single partition).

![Figure 4-8 Typical configuration of manufactured tailored laminates, featuring partition along the x-axis. In all cases, l_x = 200 mm, l_y = 100 mm. Only cross-ply (0° or 90°) ply orientations were used.](image)
The cross-section of each laminate is presented in Figure 4-9 and Figure 4-10 along with the corresponding cured shape. For convenience, each lay-up and multi-stable shape configuration will be referred to by the corresponding numbers and letters given in Figure 4-9 and Figure 4-10 (i.e. Laminate 1-6, Shape A or B). It should be noted that Laminate 4 displayed an additional multi-stable shape after manufacture. However, the second shape configuration was barely stable and could not be maintained during shape measurements. This second shape is therefore not explored any further. Likewise, Laminate 6 featured some complex buckling modes that were not symmetric about the x-axis, which could also be due (in part) to manufacturing imperfections, and so Shape A was selected to compare against numerical models.
Figure 4-9 Lay-up configuration and cured shapes of tailored Laminates 1-4.
Figure 4-10 Lay-up configuration and cured shapes of tailored Laminate 5 and 6.

The cured shapes at room temperature feature varying curvature along their lengths because of the tailored and unsymmetrical nature of the laminates used in this work. In order to make detailed comparisons against numerical models, a full-field non-contact shape measuring technique was required. This was done using laser scanning, a technique whereby the distortion of a flat laser line is captured and used to record the surface of an object. The experimental set-up used in this study is shown in Figure 4-11.
Figure 4-11 Laser scanning experimental setup, depicting: (a) computer system with David-Laserscanner software; (b) hand-held, flat-line laser; (c) Trust HD webcam; (d) scanning background; and (e) object being scanned. Note: Laser is shown clamped for illustrative purposes. During scanning, the laser is swept by hand.

A computer system with specialist software (David-Laserscanner 3.9.1) (a) is used in conjunction with a high-definition webcam (c) to obtain a live stream of the object being scanned (e). The scanning background (d) features a 90° corner, as well as a certain pattern which is sized according to the object being scanned, and is required for calibration of the camera. The background must subsequently be retained during laser scanning. The object (e) is placed in front of the background, and the flat laser-line (b) is swept over the object and the computer system observes the distortion of the laser line and uses this information to reproduce the surface being scanned. This results in (amongst other information) a point cloud of co-ordinates describing the surface. In the case of the tailored laminates scanned, there were typically over 8,000 points in the point cloud. The quantity of data, and the fact that they were obtained in a non-contact manner, makes this a very useful tool for recording the shapes of composite laminates. For a detailed description of the technique, the reader is referred to Winkelbach et al. (2006).

The accuracy of the laser-scanning technique is stated to be 0.5% of the object size (DAVID 3D Solutions 2013). To determine the accuracy of the experimental set-up used in this work, a calibration plate with patterns of known dimensions (item (e) in Figure 4-11) was scanned. The plate was manufactured using identical methods and
materials to those of the laminates, so as to give an identical scanning surface. The plate was scanned and a point cloud obtained. Using this point cloud, the dimensions of the patterns were compared against the actual dimensions. It was found (due to the resolution of the point cloud) that an error of 2 mm is possible when calculating in-plane distances, depending upon the point cloud spacing. However, the important metric required in this work is the accuracy of the coordinates (specifically, in the out-of-plane direction) of each point in the point cloud. As the calibration plate was flat (save for a slight warping), the variation in the measured out-of-plane co-ordinates was checked. The maximum variation was between +1.4/-1.8 mm. This was deemed acceptable as: (a) the calibration plate was not completely flat and (b) the advantages of this technique (simplicity, full-field, non-contact) made it the most attractive shape measuring technique. Therefore, the technique was used to record the dry, and later the saturated, shapes of the tailored laminates.

The laminate shapes obtained from laser scanning (for dry and saturated laminates) are given in full in Appendix-D.

4.5 Moisture Absorption

Moisture absorption tests were conducted separately for the cross-ply laminates and for the tailored laminates. Before immersion, all laminates were heated in an oven for 12 hours at 50°C to remove any moisture absorbed from the atmosphere. Following recording of the dry laminate shapes, the laminates were immersed in water, with less than one week having passed since manufacture.

The cross-ply laminates were immersed in water at room temperature until mass measurements showed no further uptake in moisture (i.e. saturation had been achieved). Three mass measurements per laminate were made (which were then averaged) using an Ohaus Explorer analytical balance (resolution of 100 μg). The laminates were immersed in a plastic container with distilled water. Thread was used to create partitions across the container, which supported the laminates vertically in the container. This allowed for moisture to be absorbed equally across both laminate surfaces. Average saturation moisture contents (as a percentage of mass of the dry laminates, c* (wt.%)) were calculated, along with the corresponding curvature, for each lay-up family. These values were later used to calibrate numerical models to reproduce the saturated laminate shapes.
The procedure for drying was as follows:

1. The laminates were all removed and placed on a working surface.
2. The laminates were dried one-by-one with micro-fibre cloth.
3. The laminates were allowed to dry further in the atmosphere for another 30 minutes. This was done to ensure that the panels were completely dry (so there were no partially wet patches) and also allowed the panels to reach room temperature (important in shape measurements, which are dependent on temperature).
4. The laminates were weighed, with three weight results being averaged.

Following this, the laminates could then have their shape profiles measured.

The moisture uptake for the cross-ply laminates is shown in Figure 4-12.

![Figure 4-12 Moisture absorption over time for the cross-ply laminates. Averaged values are used for each lay-up family. Circled values are considered outliers and not used. At saturation, $c^*_{[90_2/0_2]} = 0.69 \text{ wt.}\%$, $c^*_{[90_1/0_1]}, [90_3/0_2] = 0.75 \text{ wt.}\%$. The circled area in Figure 4-12 shows that the final mass readings featured a large drop. No change in laminate curvature was noted between these points and those preceding them. As such, these points were taken as being outliers, and were ignored. The preceding points were used to obtain the saturation moisture content, which would later be used to obtain an equivalent moisture swelling coefficient in numerical models.

The average curvature for each lay-up family is given in Figure 4-13. As can be seen, following saturation each family lost 71-72% of the ‘dry’ curvature. It is also worth noting that the $[90_2/0_2]$ laminates lost their multi-stable property following moisture ingress, pointing at a significant change to the residual stress/strain energy state.
To accelerate the diffusion process, the tailored laminates were stored in water at an elevated temperature (60 °C). This was done using a Burco water boiler. The laminates were stored upright, standing on a raised platform away from the boiler’s heating element. The platform featured uprights around its circumference across which thread was stretched to create partitions (similar to the previous tests). Temperature logging data over a fifteen day period (with a sampling rate of once per 11.5 min) showed that the boiler was able to maintain the water temperature to within ±2 °C. Three mass measurements were taken for each laminate, which were then averaged. As an improvement in the experimental procedure, the use of a calibration mass was adopted and used consistently for this phase of testing. As with the cross-ply laminates, the multi-stable shape configuration was lost following a short period of immersion. After 125 hours of immersion, the property disappeared for all laminates.

The moisture uptake graph is shown Figure 4-14 for the tailored laminates. It appears that saturation was achieved relatively quickly, (approximately 480 hours, or 20 days). The immersion was continued for another 38 days. This was done in an effort to guarantee saturation and that all moisture effects had reached steady-state.

Figure 4-13 Average curvatures (a) of cross-ply laminates in dry and saturated conditions. The percentage loss in curvature is given in red.
Experiments

Figure 4-14 Moisture absorption over time for the tailored laminates.

The moisture contents ($c^*$) at saturation for the tailored laminates are given in Table 4-2.

Table 4-2 Moisture content ($c^*$) at saturation of the tailored laminates.

<table>
<thead>
<tr>
<th>Laminate</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^*$ (wt.%)</td>
<td>0.79</td>
<td>0.78</td>
<td>0.78</td>
<td>0.75</td>
<td>0.77</td>
<td>0.83</td>
</tr>
</tbody>
</table>

4.6 Summary

A range of unsymmetrical laminates were manufactured using HTA 6376 CFRP pre-preg material. The curvatures of the laminates were recorded in the dry and saturated states, and gravimetric measurements were used to obtain the saturated moisture contents. In the case of the tailored laminates, laser scanning was used as a non-contact full-field surface measuring technique, and resulted in a point-cloud of roughly 8,000 points which could be used for direct comparison to the shapes obtained through modelling. A manufacturing induced curvature was observed, whereby symmetrical laminates (which should be free of macro-scale stresses) featured a curvature following manufacture. This curvature was orientated away from the tool-plate, as if the bottommost plies had being contracted in the longitudinal direction only. In addition, depending on which surface of the laminate was in contact with the tool-plate, the unsymmetrical laminates obtained two different curvatures. The manufacturing method employed used hand lay-up techniques to manufacture the laminates, followed by
Experiments

autoclave curing at 178°C and 7 bar pressure. Temperature readings taken at either end of the tool-plate during manufacture revealed no significant temperature gradients during the cure cycle. Following shape measurements, the laminates were immersed in distilled water; the cross-ply laminates at room temperature and the tailored laminates at 60°C. The saturated moisture contents ranged from 0.69-0.75 wt.% for the cross-ply laminates and 0.75-0.83 wt.% for the tailored laminates. Following saturation, the multi-stable property was lost in all laminates.
Chapter 5  Numerical Model

5.1 Introduction
A description of the finite element model used to extract through-thickness residual stresses will be described in this section, along with the calibration procedure used in the numerical prediction of laminate shapes. The basis of the approach involves using measured laminate shapes to calibrate material CTE values used in numerical modelling. The procedure used for cross-ply laminates differs in certain areas to that used for the tailored laminates, and so each of these cases will be treated separately in sections 5.3 and 5.4 respectively. Following calibration, a value of $\alpha_L$ used in modelling is obtained. It is important to stress that this is not a direct measure of the material’s CTE. Rather, it is a modelling parameter used to reproduce the cured laminate shapes. As such, it should be considered as an equivalent CTE value. In the case of the tailored laminates, a full field comparison between experimentally measured and numerically predicted shapes is presented, highlighting local differences between the two. The correlation between models (along with a difficulty in obtaining a converged solution for one lay-up) resulted in the addition of a manufacturing imperfection into the numerical models of the tailored laminates. This imperfection is of the form of a slight initial curvature at the laminate’s stress free state, reproducing an experimentally observed effect.

After numerical prediction of the dry laminate shapes, the process is repeated for the saturated laminate shapes, to obtain a calibrated equivalent value of the materials MSC ($\beta_T$). Certain difficulties arose at this stage for the tailored laminates, as will be described in section 5.4.4.

Subsequently, the dry and saturated numerical models could be used to extract through-thickness residual stresses, as will be described in chapter 6.

5.2 Model Description
Previous studies of multi-stable laminates focused on capturing certain fundamental aspects of their behaviour, such as predicting the multi-stable shape configurations or the snap-through forces involved. In this study, the change in through-thickness residual stresses due to moisture ingress was of interest. Three-dimensional continuum elements
were chosen, based on the ability of such elements to capture a wide range of detail. For example, should it be required to include process induced stresses in an analysis, the material behaviour in the through-thickness direction can be of great importance and thus does not lend itself to shell based approaches (Zeng and Raghavan 2010). Furthermore, this approach lends itself to transient moisture diffusion analyses by means of the analogy between moisture diffusion and heat transfer (Shen and Springer 1976, Katnam et al. 2010, Katnam et al. 2011). Finally, as through-thickness stresses are of particular interest, this approach allows for multiple integration points in the through-thickness direction from which to obtain stresses. This comes with an increase in computational cost. To the best of the author’s knowledge, this approach has not been used in the past for predicting the shapes of multi-stable laminates.

The finite element (FE) models were developed and implemented using the commercially available software Abaqus/Standard (Version 6.11). A coupled stress-temperature analysis was used to incorporate thermal expansion/contraction during cool-down (i.e. for the current study, from 178 °C to 21 °C) and predict the induced residual stresses in the dry unsymmetrical laminates. Similarly, as moisture ingress induces matrix swelling, the thermal expansion coefficients used in modelling were adapted to take into account the moisture induced swelling.

5.3 Solution Strategy for Cross-ply Laminates

5.3.1 Dry Laminate Shapes

Models representing each of the families of the square cross-ply unsymmetrical lay-ups used in experiments (i.e. [90/0₂], [90/0₁] and [90/0₂]) were developed to simulate the thermal deformation resulting from the curing process, which arise from the difference in orthotropic ply CTEs only. These models would subsequently be adapted to simulate moisture induced swelling. For each model, an initially flat three-dimensional solid was created to the nominal dimensions of the manufactured laminates (i.e. side-lengths of 180 mm x 180 mm, with a thickness of 0.5 mm and 0.625 mm for 4 ply thick and 5 ply thick laminates, respectively). In order to coax the solution into a quasi-cylindrical shape (as opposed to a saddle shape) a geometric imperfection was added in the form of unequal side-lengths, 181 mm x 179 mm (Gigliotti et al. 2004). The model was then partitioned into separate layers, each representing a ply 0.125 mm thick. The orientation of the individual ply was defined by applying a local material orientation to each ply. Orthotropic material properties (largely obtained through experiments) were then
assigned, including linear elastic properties and linear thermal expansion properties (Table 5-1).

Table 5-1 Material properties used in numerical models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$E_{11}$ (GPa)</th>
<th>$E_{22}$ (GPa)</th>
<th>$v_{12}$</th>
<th>$v_{13}^*$</th>
<th>$v_{23}^*$</th>
<th>$G_{12}$ (GPa)</th>
<th>$G_{13}^*$ (GPa)</th>
<th>$G_{23}^*$ (GPa)</th>
<th>$\alpha_T (K^{-1})$ (x $10^{-5}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude</td>
<td>135.64</td>
<td>10.14</td>
<td>0.29</td>
<td>0.29</td>
<td>0.45</td>
<td>5.86</td>
<td>5.86</td>
<td>4.4</td>
<td>2.86</td>
</tr>
</tbody>
</table>

Note: * denotes values estimated from Kelly (2005)

A structured mesh was generated, comprising three-dimensional continuum elements (C3D8R). The elements feature linear formulation and reduced integration. Three-dimensional solid elements can suffer from a phenomenon called ‘shear locking’ when subjected to bending problems (Abaqus-Inc. 2011). Shear locking refers to the overly-stiff behaviour of such elements in bending problems and results from the incorrect deformation of the element under bending, which introduces extraneous shear strains (Abaqus-Inc. 2011, Brown 1997, Fung and Tong 2001). This shear ‘absorbs’ strain energy, resulting in the element reaching equilibrium with artificially small nodal displacements (Brown 1997). The bending behaviour of the laminates in this study is obviously important, and so the possibility of shear locking needs to be considered. This can be achieved by using non-linear, reduced integration elements. Non-linear elements can be computationally expensive, and as such the thickness of each ply in numerical models was reproduced using three linear elements with reduced integration in the through-thickness direction, giving rise to the C3D8R elements used.

The element size was held constant across all models. As through-thickness residual stresses are of interest, a fine mesh was used in the through-thickness direction with three elements through the thickness of each ply. This gave an element thickness of 41.66 μm. To decide upon a suitable element aspect ratio, the sensitivity of the through-thickness stress profiles extracted from the model to the element aspect ratio was investigated. The through-thickness stress profiles of a [90/0] laminate comprising elements with an Aspect Ratio (AR) (length/width to thickness ratio) of 15 and 125 were compared (Figure 5-1(a)). A difference of less than 1% was observed in the extracted stresses. The number of elements required to maintain an AR of 15 was, however, very high (approximately 1 million for the [90/0] lay-up, requiring substantial computation cost). An AR of 100 was chosen as a compromise between maintaining as low an aspect ratio as possible, while reducing the element count and
thus computational costs. The modelling approach, including the meshing scheme and geometric imperfection used to create a cylindrical solution is shown in Figure 5-1.

Figure 5-1 Finite element modelling strategy for cross-ply laminates with: (a) sensitivity of through-thickness residual stresses to element aspect ratio (all stresses are in MPa); (b) FE strategy showing geometrical imperfection; (c) constant element size used with three elements per ply thickness.

An encastre boundary condition was used to restrain a node at the geometric centre of the top surface of the laminate in both rotations and displacements. The line through the cross section of the laminate was restrained from displacement along the \( x \) and \( y \) axes.

This eliminates rigid body motion, yet allows for thermal expansion in the through-thickness direction. Finally, the temperature drop between cure and room temperatures (\textit{i.e.} -157°C) was incorporated using predefined fields. A geometric non-linear analysis was used to consider the large out-of-plane deformations (using \textit{nlgeom}).

The process then involved calibrating on a trial-and-error basis the \( \alpha_L \) value used to reproduce the experimentally measured curvatures (see Figure 4-13). In this case, a single value for \( \alpha_L \) (of \( 5.5 \times 10^{-6} \text{K}^{-1} \)) was sufficient to describe the curvatures of all three laminate families to within \( \pm 10\% \), as shown in Table 5-2. Finally, the curvatures predicted numerically were compared against those predicted by the analytical Matlab model. Good agreement was obtained, as shown in Table 5-2.
Table 5-2 Predicted curvatures of dry laminates after calibration ($\alpha_L = 5.5 \times 10^{-6}$ K$^{-1}$).

<table>
<thead>
<tr>
<th>Family</th>
<th>Experimental (mm$^{-1}$)</th>
<th>Numerical (normalised)</th>
<th>Analytical (normalised)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[90$_2$/0$_2$]</td>
<td>0.0056</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>[90$_3$/0$_1$]</td>
<td>0.0122</td>
<td>0.90</td>
<td>0.89</td>
</tr>
<tr>
<td>[90$_3$/0$_2$]</td>
<td>0.0069</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

5.3.2 Saturated Laminate Shapes

With the cured shapes of the laminates modelled, the changes introduced due to moisture saturation were subsequently incorporated. This process was practically identical to the previous analysis, whereby an orthotropic hygro-thermal strain was applied to the model to capture the experimentally observed shapes of saturated laminates. The finite element models used to predict the cured shape were modified for this process. The model geometry, boundary conditions, mesh and solution procedure were unchanged from the earlier analysis. The material properties used were modified in two ways. Firstly, material degradation due to moisture ingress was taken into account. The elastic material properties were modified by using data from Selzer and Friedrich (1997) and Ogi et al. (1999). As different saturation moisture contents were obtained for the different families of laminates ($c^* = 0.69$ wt. % for the 4 ply thick laminates, and $c^* = 0.75$ wt. % for the 5 ply thick laminates), two sets of degraded material properties were calculated by linear interpolation. Secondly, the material’s CTE values were modified to replicate moisture induced swelling. The strains induced due to moisture are analogous to thermal expansion, whereby the CTE value used in modelling is equivalent to the swelling coefficient, and the temperature change is equivalent to the moisture content (as a weight percentage). The swelling coefficient was assumed to be orthotropic, with different values describing the material’s longitudinal and transverse directions. Due to the high stiffness of the (assumed impermeable) fibres, which dominate the longitudinal direction, the swelling coefficient in this direction ($\beta_L$) was assumed to be zero (Ogi et al. 1999). Therefore, the longitudinal ply hygro-thermal straining is equal to the thermal expansion. This leaves the transverse ply hygro-thermal straining open to calibration, to reproduce the shapes of the saturated laminates. A swelling coefficient ($\beta_T$) was obtained by using Eq. 5.1, with $\beta_T$ being the transverse swelling coefficient, $c^*$ being the moisture content, $\alpha_T$ being the transverse CTE value, and $\Delta T$ being the difference between cure and room temperatures.
The results of this calibration process are shown in Table 5-3, with the swelling coefficient being calibrated to a value of $\beta_T = 3.65 \times 10^{-3}$ wt.%$^{-1}$. As before, this single swelling coefficient was used to describe the curvatures of all the manufactured laminates, within a $+14%/-16%$ margin.

Table 5-3 Predicted curvatures of saturated laminates after calibration ($\beta_T = 3.65 \times 10^{-3}$ wt.%$^{-1}$).

<table>
<thead>
<tr>
<th>Family</th>
<th>$c^*$ (wt.%$^{-1}$)</th>
<th>Curvature Experimental (mm$^{-1}$)</th>
<th>Curvature Numerical (normalised)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[902/02]</td>
<td>0.69</td>
<td>0.0016</td>
<td>1.14</td>
</tr>
<tr>
<td>[903/01]</td>
<td>0.69</td>
<td>0.0034</td>
<td>1.07</td>
</tr>
<tr>
<td>[903/02]</td>
<td>0.75</td>
<td>0.002</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Note: Curvatures obtained from numerical models are normalised against experimental measured curvatures.

5.4 Solution Strategy for Tailored Laminates

5.4.1 Numerical model development

The numerical model developed for the tailored laminates was largely identical to that used for the cross-ply laminates, with a coupled temperature-stress analysis being used to reproduce the thermal deformation following cool-down from cure temperature. As before, the models were developed as flat three-dimensional solids to the nominal dimensions of the manufactured laminates ($l = 200$ mm, $w = 100$ mm, ply thickness = 0.125 mm). The models were then partitioned to create each ply as well as the tailored sections. A local material orientation was then assigned to each ply according to the ply stacking sequence for that section. Once again, the orthotropic and linear-elastic material properties were used to represent the HTA 6376 material (Table 5-1), and the temperature change (-157 °C) was applied by using predefined fields. As before, $nlgeom$ was used due to the large out-of-plane deformations.

† This value was incorrectly reported as $3.65 \times 10^{-4}$ wt.%$^{-1}$ in Telford R, Katnam KB, Young TM, The effect of moisture ingress on through-thickness residual stresses in unsymmetric composite laminates: A combined experimental–numerical analysis, Composite Structures 2014; 107:502–511.
In this instance, three-dimensional quadratic continuum elements of type C3D20R were used. This is a change from those used for the cross-ply laminates (linear, type C3D8R), and were required as the bending stiffness of the models was found to be critical in this analysis; otherwise the correct laminate configuration would not be predicted. This was found to be especially important for Laminates 4 and 5, which feature a tapered change in lay-up and a change in the curvature orientations along their length. Three elements were used in the through-thickness direction of each ply in order to accurately capture the through-thickness residual stresses. A structured mesh comprising elements with a fixed element AR of 60:1 was used to alleviate the solution from being overly stiff, while maintaining an element count that does not result in an overly computationally expensive analysis. A sensitivity check was conducted by comparing extracted stress profiles (using Laminate 1) with AR values of 40:1, 60:1 and 80:1. Although a change was noted when reducing the AR from 80:1 to 60:1, no further change was noted when reducing it from 60:1 to 40:1 (Figure 5-2). Thus, an AR of 60:1 was chosen. This results in element dimensions of 2.5 mm (length, width) and 0.0417 mm (thickness).

At this stage, the predicted laminate shapes could be compared to those measured experimentally. As a preliminary step, this was done for just one of the multi-stable shape configurations obtainable for each of the lay-ups. The configuration featuring the largest out-of-plane deformation was chosen, as it would give the best insight into the model’s capability in predicting the full-field deformation of the laminates. At this stage, it was not possible to obtain a converged solution for Laminate 5. As initial
Numerical Model

attempts to obtain convergence did not resolve the issue, this laminate was temporarily set aside. Following shape comparisons (which will be shown in section 5.4.2), the models were modified and a converged solution for Laminate 5 was obtained. These results will be given in section 5.4.3.

5.4.2 Comparison of Measured and Predicted Tailored Laminate Shapes

As described in section 4.4.2, the shapes of the dry and saturated tailored laminates were obtained using a laser scanning technique. This gave a full-field point cloud which could be used for detailed comparisons of experimental and numerical shapes. This was achieved by subtracting the out-of-plane co-ordinates of the numerical models from those of the experimentally measured shapes to obtain the variation between the two.

The process for this is described in Figure 5-3. The raw shapes (i.e. as extracted directly from experiments and models) were first transformed to a common coordinate system for comparison, using the mathematical analysis software Matlab (see ‘experimental’ and ‘numerical’ shapes, Figure 5-3). A fine grid comprising $x$ and $y$ coordinates was then superimposed on both the experimental and numerical shapes. From this, an out-of-plane $z$-coordinate was linearly interpolated from surrounding points in the point cloud. The purpose of this step was to have identical $x$ and $y$ positions for the experimental and numerical shapes at which out-of-plane deflections can be compared. At this stage, the outer 5 mm of the laminates were ignored, so as to remove any edge effects that result from laser scanning. The $z$-coordinates of the numerically predicted shape ($w_n$) were then subtracted from those of the experimentally measured shape ($w_e$), giving a full field view of the out-of-plane variation between the two shapes. The variation is presented as a percentage of the maximum out-of-plane deflection measured for each of the experimental laminates ($w^*$ in Eq. 5.2). The full field variation (in terms of $w^*$) was then plotted, as shown in Figure 5-3.

$$w^* = \left( \frac{w_e - w_n}{w_{e,\text{max}}} \right) \times 100$$ (5.2)
In order to support the laminates during laser scanning, a small fixture was used which allowed the laminates to stand upright, without being distorted due to clamping. This fixture was scanned during the shape measurement process, and included in the scanned shapes of the laminates. This is responsible for the distorted areas highlighted by circles in Figure 5-3. Similar features are present in all variation plots to follow and should be ignored. For reference, the tools developed to compare dry laminate shapes (and later, saturated shapes) are given in Appendix B.

The calibration process then involved minimising the difference (or variation) between experimentally observed and numerically predicted shapes. This was done for Laminate 1-4 and Laminate 6. As previously described, Laminate 5 featured convergence issues and so the analysis of the laminate was postponed (the analysis is continued in Section 5.4.3). The material’s $\alpha_L$ value used in modelling was then altered to minimise the variation between plots. The result of this process was the calibrated equivalent CTE value for $\alpha_L$. The process was done for each lay-up family, resulting in a range of equivalent CTE values for $\alpha_L$ being required, as given in Table 5-4.

### Table 5-4 Calibrated equivalent $\alpha_L$ used in numerical models.

<table>
<thead>
<tr>
<th>Laminate</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_L \times 10^6$ K$^{-1}$</td>
<td>6.5</td>
<td>1</td>
<td>12</td>
<td>7.2</td>
<td>6</td>
</tr>
</tbody>
</table>
Figure 5-4 Variation between experimentally measured and numerically predicted shapes. The cured shapes of the manufactured laminates are given for reference. Note: Values on $w^*$ axis highlight minimum and maximum values of $w^*$.

In all cases, the numerical models were able to predict the correct tailored laminate shape configuration; that is, the correct curvature orientation for each of the tailored segments. However, of particular interest is the ability of the model to correctly predict the complete profile of the laminates, such as where the laminate over/under predicts the out-of-plane curvature. The largest variation range for all the laminates investigated in this work was found to be -6.6/+24.9 for Laminate 3 (Figure 5-4(c)). While it is possible to reduce this further by means of further calibration of the $\alpha$ value used, it is clear that
entirely eliminating the variation will not be achievable by this means. Indeed, this is apparent for all the laminates investigated. One reason for this is that the laminates manufactured were thin in nature (maximum of four plies thick, 0.5 mm) and thus are very susceptible to manufacturing effects and imperfections (Ochinero and Hyer 2002, Twigg et al. 2004a). It is known from experience that the manufacturing method used to create the laminates introduces a warp in thin laminates, bowing away from the tool plate. As some of the multi-stable shape configurations of the manufactured laminates featured relatively small out-of-plane deflections, the manufacturing effects become more pronounced in the cured laminate shapes. While the calibrated CTE is intended to minimise the need to account for manufacturing effects individually in modelling, it becomes less effective when dealing with laminates which feature large opposing curvatures, such as laminates 4, or laminates with a very low out-of-plate deflection, such as Laminate 2. In the case of Laminate 4, a curvature exists along the $x$-axis, which transitions to an inverse curvature along the $y$-axis. The effect due to manufacturing tends to warp the laminate away from the tool-plate, and thus will increase one of those curvatures, and decrease the other (something which a calibrated CTE value does not take into account). Therefore, it is expected that this laminate in particular will feature variation with respect to the experimentally measured shape. The way in which manufacturing affects each laminate differently is further demonstrated by the fact that a range of equivalent CTE values were required to numerically reproduce the laminate shapes (Table 5-4).

The variation between experimental measurements and numerical predictions for Laminate 1 and Laminate 3 are both very similar. Initially, the numerically predicted out-of-plane deformation is under-estimated along the $x$-axis. Mid-way through the first partition ([90$_3$/0$_1$] for Laminate 1 and [90$_2$/0$_2$] for Laminate 3) the out-of-plane deflection is subsequently over-predicted. It appears that the first partition is acting overly stiff in models (i.e. an increase in curvature is required in this region). Doing this (by decreasing the equivalent CTE value) would, however, degrade the correlation for the second partition of the laminate. This behaviour may be due to the manufacturing effects already noted in this section. A possible solution to increase the accuracy of predicted shapes would be to use separate equivalent CTE values for each partitioned section, thus separating manufacturing effects and including them in two separate calibrated values of $\alpha_L$. 
Overall, the maximum variation is between -6.7/+14.55 for Laminate 1 and -6.6/+24.9 for Laminate 3. As this study is limited to cross-ply laminates, no twist curvature is developed in numerical models. Close examination of the manufactured shape across the width of Laminate 1 (parallel to the \(y\)-axis) shows that the laminate is slightly twisted, a consequence of manufacturing imperfections that is hard to avoid. This leads to an increase in the minimum/maximum variation. However, the curvature across the \(y\)-axis appears to be well captured.

The profile of Laminate 2 also shows signs of manufacturing imperfections (see Figure 5-4(b)). As the laminate is symmetrical about the \(y\)-axis, it is expect that the variation would be mirrored about the centre of the laminate. Instead, the variation is greater on the right hand side of the laminate than on the left. In addition, this section is curved, showing that the manufactured laminate featured a curvature in this (symmetrical) lay-up section. However, when considering the low out-of-plane deflection of this laminate, the overall variation is low (-12.5/+0.4\%). The curvature of the centre \([0_2/90_2]\) section is marginally underestimated, as shown by the curved section in the variation plot. Further calibration (by reducing \(a_L\)) would improve the correlation between experiments and the numerical prediction.

The variation of Laminate 4 shows a range of -0.6/+22.2. Initially, the longitudinal out-of-plane deflection is slightly underestimated. This area corresponds to the \([90_3/0_1]\) lay-up. The bi-stable region exists in the second partition, with a \([90_2/0_2]\) lay-up. Along this section is where the curvature alternates between being generated parallel to the \(y\)-axis to parallel to the \(x\)-axis. This was a region that was particularly sensitive to overall stiffness of the model; otherwise, only the other multi-stable shape (where the curvature generated in this section remained parallel to the \(y\)-axis) could be obtained. The largest variation appears at the right-hand most edge of the laminate, with the curvature being under predicted in this section. Further calibration of the equivalent \(a_L\) value used may improve overall correlation, but it is clear that all variation will not be removed by this means. This may be due to manufacturing induced warping increasing the curvature generated along the \(y\)-axis while decreasing the curvature generate along the \(x\)-axis, as previously described.

Being only two plies thick over most of the laminate, Laminate 6 is particularly flexible and very sensitive to residual stresses. The two sections of symmetrical lay-up (\([0_2]\)) are reacting to residual stresses imposed due to the tailored laminate configuration and any
Numerical Model

manufacturing imposed residual stresses. At the first \([0_2]\) section, the laminate is largely deforming under the effect of the \([0_2/90_2]\) lay-up which neighbours it. Subsequently, the \([0_2/90_2]\) is in a multi-stable configuration whereby a cylinder along the \(x\)-axis is formed. Coupled with the last partition \((0/90)\) these partitions control the shape of the laminate. The curvatures along these two partitions appear to be captured well. However, the laminate did feature multiple buckling modes, with the first partition featuring a twist curvature. This is shown by the raised corner at the bottom left corner of Figure 5-4(e). This behaviour was not captured in numerical models (due to the inherent symmetry) and thus appears as a contributor to the variation.

It is worth noting that this particular tailored configuration features a large number of snap-through configurations available, due to the bi-stable region being able to alternate shape in two separate steps. It is possible to snap one half of the region through to the second shape obtainable, with the first half remaining in the original configuration (similar to what has been reported by Potter et al. (2007b)).

5.4.3 Tailored Laminates: Refined approach

Due to the spread in CTE values required to describe the tailored laminate shapes, the model was refined to take into account the manufacturing induced curvature that is observed in symmetrical laminates (see section 4.2). As previously explained, this curvature may affect each lay-up partition differently, resulting in difficulty in eliminating the variation between experimental and numerical shapes. In particular, as the laminates in this study feature multi-stable shapes, it is possible that the manufacturing imperfection adds to the curvature of one multi-stable shape configuration, and detracts from the other.

However, without knowing precisely the source(s) of this curvature, an effective implementation into the numerical solution becomes challenging as well as time consuming. Other authors have created numerical and analytical models to analyse manufacturing effects, such as tool-plate/laminate friction (e.g. see for instance Cho and Roh (2003) and Zeng and Raghavan (2010)). These analyses are complicated, and require a range of complex experiments to obtain values (e.g. friction coefficients) for modelling. In addition, accounting for these would then substantially increase the complexity of the numerical model. Therefore, a compromise solution was used. The end result of the manufacturing effects (an additional curvature) was reproduced, so that the stress-free shape (i.e. that at the elevated curing temperature following
Numerical Model

consolidation) was not perfectly flat, but featured a slight curvature. From there, the laminates were cooled to room temperature, during which regular thermal deformation occurred. The purpose of this was twofold. Firstly, the contribution of this manufacturing effect on the variation between predicted and numerical shapes only could be investigated. Secondly, as there were difficulties in obtaining a converged solution for Laminate 5, it was postulated that including this manufacturing imperfection would help resolve this issue.

The process was carried out as follows. A symmetrical [0/90] strip was manufactured and cured at the same time and conditions as the tailored laminates. As expected, this strip deformed away from the tool-plate, consistent with the bottom most ply contracting in its longitudinal fibre direction. During the laser-scanning of the tailored laminates dry shapes, this laminate was also scanned to record its curvature. A finite element model (using the same mesh and construction as the tailored laminates) was developed. A thermal strain was applied to the bottom-most ply only, in its longitudinal direction. This straining was calibrated until the experimentally measured deformed shape of the strip was obtained.

This thermal straining could then be applied to numerical models. The analysis was broken into two steps. Firstly, the manufacturing effect was applied, followed by the temperature drop to room temperature. In the first step, the bottommost ply had the calibrated thermal straining from the symmetrical strip applied. This gave an initial curvature. By use of result files generated by Abaqus, the second analysis step then used the deformed shape as a starting point, with all stresses being removed. This gave a stress-free initially curved shape. The material properties were changed to those of the HTA material (Table 5-1) by defining a field variable, and the analysis continued as before. It is important to reiterate that the stresses were removed in moving from the first analysis step to the second. It is probable that the curvature induced during manufacture is due, in part, to stresses being formed in the laminate. The profile of these stresses through the laminate thickness is not known. Therefore, the stresses induced following the first analysis step described above were removed, as otherwise they would contribute to the through-thickness stress profiles of the laminates in the cured state. This is, of course, a rough approximation of what is actually occurring to the laminate during cure. In the context of the approach developed in this work, it is therefore a first step into assessing the importance of considering manufacturing effects, by evaluating
its effect on the correlation between numerical models and experiments, and the calibrated CTE values required to reproduce those shapes. A check of the through-thickness residual stress profiles for a particular CTE showed very little change in stresses between a model starting from an initially curved shape compared to one starting from a completely flat shape. The approach used to apply the thermal strain was chosen to roughly simulate the contraction of the bottom ply (in the fibre direction only) to create an initially curved shape, while still accounting for the different flexural stiffness of the laminates, which feature different lay-up configurations. By using this approach, Laminate 5-Shape A (which did not converge using the “initial” approach described earlier) was now able to converge.

![Stress-free shape of Laminate 1 showing initially curved state, along the x-axis only.](image)

The calibration process to obtain an equivalent $\alpha_L$ continued as before. This time, all multi-stable shape configurations shown in Figure 4-9 and Figure 4-10 were predicted (with the exception of Laminate 6, Shape B and C). With additional shapes being studied, convergence issues once again arose with the solution. This was solved by applying artificial damping ($\text{static, stabilise}$), as previously described by Mattioni et al. (2008). The pseudo-dynamic scheme employs the use of artificial dampening to dampen local instabilities. These coefficients affect the equilibrium configuration, and so must be chosen to be as small as possible so as not to affect the predicted shape. In the simulations used in this study, the artificial energy due to damping never exceeded 2.3% of the total internal energy of the laminate, meaning that its influence on the final solution was minimal.

With these numerical models later being adapted to predict the saturated shapes of the laminates, each multi-stable shape was calibrated separately. As residual stress profiles were desired, it was felt that it was important to do so, so as to obtain the most representative residual stress profiles in the dry profiles (against which the saturated
shape profiles could later be compared). The results of the calibration, in terms of the variation between experimentally-measured and numerically-predicted shapes are presented in Figure 5-6 to Figure 5-11, for Laminates 1-6. The corresponding equivalent $\alpha_L$ value for each shape configuration is also provided.

![Figure 5-6 Laminate 1, variation between experimentally-measured and numerically-predicted laminate shapes: (a) FE prediction, configuration A; (b) FE prediction, configuration B; (c) variation between the numerical and experimental shape, configuration A; and (d) variation between the numerical and experimental shape, configuration B.](image)

The equivalent $\alpha_L$ value of $5.5 \times 10^{-6} \text{ K}^{-1}$ was able to describe both laminate shapes to within $-17/+19\%$ (Figure 5-6 (c) and (d)). As this laminate has the highest out-of-plane deformation, it is the easiest to calibrate with a single $\alpha_L$ value, as other contributors to the curvature (such as manufacturing imperfections) are dominated by the large curvature developed by the laminate asymmetry. Also, a small degree of twist curvature is evident as the variation for both Shape A and Shape B is at a minimum/maximum at opposite corners. It is evident that the inclusion of the manufacturing imperfection hasn’t entirely eliminated the variation between experimental and numerical shapes, with the difficulty being in trying to eliminate variation in both lay-up partitions.
Looking at the curvature evident in the variation plot (Figure 5-7 (c)), it appears as the curvature at the front of the laminate ([0/90] lay-up) is under-predicted, while the curvature at the second lay-up partition ([0/90] lay-up) is over-predicted. Again, one possibility to reduce the variation further is to treat each lay-up partition separately during the calibration process.

**Figure 5-7** Laminate 2, variation between experimentally-measured and numerically-predicted laminate shapes: (a) FE prediction, configuration A; (b) FE prediction, configuration B; (c) variation between the numerical and experimental shape, configuration A; and (d) variation between the numerical and experimental shape, configuration B.

With a relatively small out-of-plane deflection (as shown in Figure 5-7 (a) and (b)), Laminate 2 is particularly difficult to calibrate, especially as the maximum out-of-plane deflection is used as a denominator during calibration parameter (see \( w_{e, \text{max}} \), Figure 5-3), meaning a small value of \( w_{e, \text{max}} \) can lead to a large variation. Thus, a range of -0.3/+24.5% for Shape A and -28.9/-1.8% for Shape B represents a small difference between predicted and measured laminate shapes in absolute terms. However, the difficulty in predicting such laminate configurations is demonstrated by the difference in CTE values required for both shapes. As before, starting with an initially curved
shape did completely eliminate the variation in both shape cases. In addition, the curvature of Shape A is over-predicted, even though a high value of $\alpha_L$ was used (remember, a high value of $\alpha_L$ translates to a smaller difference between $\alpha_T$ and $\alpha_L$, resulting in less curvature). Conversely, the curvature of Shape B was under-predicted, necessitating a lower CTE value. This result demonstrated that the inclusion of an initial curvature didn’t result in a single value for $\alpha_L$ being able to describe both multi-stable shapes. The high value of $\alpha_L$ required to describe Shape A is surprising as the stress-free curvature was negative in orientation compared to the final curvature reached. This means that during the cool-down phase, additional deformation is required to eliminate the stress-free curvature, before the final curvature can be reached.

![Figure 5-8 Laminate 3, variation between experimentally-measured and numerically-predicted laminate shapes: (a) FE prediction, configuration A; (b) FE prediction, configuration B; (c) variation between the numerical and experimental shape, configuration A; and (d) variation between the numerical and experimental shape, configuration B.](image)

The variation between shapes of Laminate 3, Shape A (Figure 5-8 (c)) shows that the curvature at the front of the laminate (which has a $[90_2/0_2]$ lay-up) is initially under-predicted, leading to a variation of $-5/+20\%$. Subsequently, the variation increases in the neighbouring $[0/90]_s$ section. This laminate configuration demonstrates a subtlety of the
calibration process. In Shape A (Figure 5-8 (c)) the initial curvature is under-predicted. This results in a longer chord length (the distance from each corner of the laminate along the x-axis). As both the numerical and experimental shapes are pinned at the front of the laminate, the numerical laminate (which has a longer chord due to the under-predicted curvature) extends further along the x-axis. This manifests itself in a higher variation in the second half of the laminate.

For Shape B, the variation reaches values of -21/+16%, and is at its highest at the [0/90], partition. The principal curvature for this shape is along the y-axis. This appears to have been well captured, as the variation is relatively flat along the y-axis. Along the x-axis, however, the variation increases. This is particularly evident at the second partition, featuring a [0/90], lay-up. Due to the symmetry, this section would be flat were it not for the tailored laminate configuration. Therefore, any curvature developed along this section is due to the interaction with neighbouring unsymmetrical section. It appears that the numerical model didn’t entirely capture the interaction in this section. This could be due to the numerical model itself or due to a manufacturing imperfection.

![Figure 5-9 Laminate 4, variation between experimentally-measured and numerically-predicted laminate shapes: (a) FE prediction; (b) variation between the numerical and experimental shape.](image)

The variation of Laminate 4 with the updated model (Figure 5-9(b)) shows an improvement over that of the basic model (Figure 5-4(d)), with the variation ranging from +2.2/+16% for the refined model, versus -0.6/+22.2% for the initial model. Additionally, the CTE required dropped from 7.2×10^-6 K^-1 (initial model) to 3.5×10^-6 K^-1 (updated model). This could be due to the fact that this laminate features a change in
curvature along its length, and thus the curvatures are influenced to different degrees by the manufacturing effect discussed earlier. As this effect is now included, it was possible to capture both the developed curvatures with a smaller variation. The new value for the equivalent CTE value is more in line with the actual materials $\alpha_L$ value, which is expected to be close to zero due to the low CTE of carbon fibres (Toho Tenax Europe 2011). It may indicate that including such an effect is effectively removing one of the manufacturing influences in this particular lay-up case.

![Figure 5-10](image.png)

**Figure 5-10** Laminate 5, variation between experimentally-measured and numerically-predicted laminate shapes: (a) FE prediction, configuration A; (b) FE prediction, configuration B; (c) variation between numerical and experimental shape, configuration A; and (d) variation between the numerical and experimental shape, configuration B.

The variation of Laminate 5 shows the difficulty in predicting the complete profile of tailored laminates, and the care that must be taken when conducting shape comparisons with laminates that feature both positive and negative curvatures. This laminate features three partitions, of $[0_2/90_1]$, $[0_2/90_2]$ and $[0_2/90_3]$ lay-up. The variation of Shape A (Figure 5-10(c)) shows that initially, the curvature is quite well predicted, with the variation being close to zero. Subsequently, the laminate’s curvature changes sign and generator axis. It should be pointed out that, counter-intuitively, the curvature in this
section is actually under-predicted – as with a curvature in this orientation, a higher variation indicates that the numerically predicted shape has a lower curvature to that measured experimentally. The transition between both of the curvatures (facilitated by the \([0_2/90_2]\) lay-up) shows a sharp change in variation. This indicates that in this section, the numerical model has difficulty in reproducing the behaviour of the experimental laminate. All in all, the variation ranged from \(0/\pm 18\%\), a reasonable result for a complex laminate. With Shape B the area with the high curvature is well predicted, with the variation being close to zero. The variation subsequently increases, which appears to be due to the curvature along the \(y\)-axis being under-predicted at the last lay-up section (\([0_2/90_3]\) lay-up). This leads to the variation reaching a maximum of \(+25\%\). The shape variations were obtained using just one equivalent CTE value to describe both laminate shapes.

Figure 5-11 Laminate 6, variation between experimentally-measured and numerically-predicted laminate shapes: (a) FE prediction; (b) variation between the numerical and experimental shape, configuration.

The variation of Laminate 6 is similar to that previously observed for the basic model case (Figure 5-4(e)). In this instance, however, the variation has been changed from \(-15.5/\pm 9\%\) to \(-9.5/\pm 9\%\), a slightly smaller range. The required value of \(\alpha_L\) reduced from \(6 \times 10^{-6} \text{ K}^{-1}\) to \(5 \times 10^{-6} \text{ K}^{-1}\). One of the difficulties in reproducing the shape of Laminate 6 is shown in the variation plot (Figure 5-11(b)). The point of highest variation (marked in red) is not repeated at the other side of the laminate (\(y = 0\)). This indicates that there is some local twist present in the laminate. Indeed, this section is particularly thin (\([0/90]\) lay-up) and can have locally buckled regions. Looking at the front edge of the variation plot (along the \(y\)-axis with \(x = 0\)), this local buckling can be seen as the variation increases and decreases several times. The rest of the laminate is reasonably well
Numerical Model

predicted, with the last lay-up section ([0₂]) showing the largest difference between experiments and modelling. The variation here is curved along the x-axis, and indicates that some difference between the measured and predicted curvature. As with Laminate 3, any curvature formed in this section is due to the interaction with the neighbouring unsymmetrical [0₂/90₂] section. This mismatch could be caused by a shortcoming in the model when trying to predict this behaviour, or it could be due to a manufacturing effect; the cause is not clear. Nonetheless, the overall variation is relatively low at -9.5/+9%.

As a further assessment of the ability of the refined model to improve correlation between predictions and experiments, the \( \alpha_L \) values required to describe the laminate shapes for both the initial and updated models are given in Table 5-5. It can be seen that by including the manufacturing imperfection (in the form of an initially curved laminate shape) the range in CTEs required to describe laminate shapes dropped from 1-12 \( \times 10^6 \) K\(^{-1} \) to 3.5-9.5\( \times 10^6 \) K\(^{-1} \). This result is subjective, as the calibration process contains an element of interpretation. To verify this result (and to make the calibration process more robust in general) it is recommended that the variation be minimised by reducing the root mean square of all the variation plots. This would give a better indication of whether or not the inclusion of manufacturing effects significantly improves the equivalent \( \alpha_L \) values required. Nonetheless, the reduction in the range of values of \( \alpha_L \) required is encouraging.

<table>
<thead>
<tr>
<th>Laminate (Shape)</th>
<th>1(A)</th>
<th>2(B)</th>
<th>3(A)</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial model ( \alpha_L ) (x10(^6)) K(^{-1} )</td>
<td>6.5</td>
<td>1</td>
<td>12</td>
<td>7.2</td>
<td>6</td>
</tr>
<tr>
<td>Refined model ( \alpha_L ) (x10(^6)) K(^{-1} )</td>
<td>5.5</td>
<td>4</td>
<td>9.5</td>
<td>3.5</td>
<td>5</td>
</tr>
</tbody>
</table>

5.4.4 Saturated Tailored Laminate Shapes

Following the calibration of the dry laminate shapes, moisture swelling effects were implemented. As done previously for the square cross-ply laminates (and as described in Eqn. 5.1) the thermally induced straining applied to each ply in the model is altered to account for both thermal straining (using the calibrated CTE values) and moisture straining (using experimentally measured moisture contents unique to each lay-up configuration and a calibrated swelling coefficient \( \beta_T \)). The calibration of \( \beta_T \) then focused on reproducing the change between dry and saturated shapes. As a variation
between models and experiments in the dry configuration was evident (see Figures 5-6 to 5-11), it was decided to compare the loss in out-of-plane deflections following saturation of numerical models and experimentally measured laminates. This was done as shown in Figure 5-12. Comparison of the tailored laminate shapes is a complicated task. The CTE values used in modelling take into account both the thermal strain (following cure) and the moisture swelling of the material. The thermal strain is taken into account by using the calibrated $\alpha_L$ values from Table 5-5. It is known that using these calibrated $\alpha_L$ values leads to a degree of variation when compared to experiments. Therefore, their involvement in the modelling the saturated laminate behaviour may lead to misleading results. As such, the contribution of the ‘dry’ model was reduced by comparing the loss in out-of-plane deflection ($w^*$) of both the numerical and experimental laminates.

As an example, a comparison of the numerically-predicted and experimentally-measured shape of Laminate 1 in the saturated condition is given in Figure 5-12(a). Considering the under-predicted curvature at the front of the laminate, it may appear that the swelling coefficient was too high, resulting in the numerical laminate shape having too low a curvature. However, direct comparison of the loss of $w^*$ observed experimentally and predicted numerically produces Figure 5-12(b). With the variation from the dry laminate shape removed, it is clear that the numerical model is over predicting, and thus a higher swelling coefficient is required. After some rearranging of terms, the strategy of comparing the loss in $w^*$ leads to the formula for $\delta$ given in Figure 5-12(b), where it can be seen that the variation between numerical and experimental shapes in the dry condition is subtracted from the variation in the saturated condition.

The swelling strains induced in the models could then be altered until the change in shape observed experimentally was reproduced.
Figure 5-12 Calibration of numerical model of Laminate 1, saturated condition. The subscript $e$ refers to the experimentally-measured shape, the subscript $n$ refers to the numerically-predicted shape, the subscript $d$ refers to the dry laminate, and the subscript $s$ refers to the saturated laminate.

At this stage, some difficulties arose in applying this procedure. Firstly, following saturation, the multi-stable shape configurations featuring the largest out-of-plane deflections were lost, resulting in the alternate multi-stable shape configuration with a relatively small out-of-plane deflection remaining (see Appendix-D, Figs. D-1 – D-6, which show the saturated laminate shapes obtained from laser-scan data). This meant that the change in out-of-plane displacements following moisture saturation was of a small magnitude. This was not ideal, as a large change in out-of-plane displacements would lend itself better to trying to capture the effect of moisture swelling through modelling.

In addition, the saturated laminate shapes featured twisting or additional warping that was not evident in the dry laminate shapes. The causes of these effects are not immediately obvious. The laminates were immersed for a long period of time to ensure equal moisture ingress to all parts of the laminate, and so the effect of moisture ingress should be consistent throughout the laminate. One possible reason to explain this behaviour is that in the dry state, the stresses are dominated by the macro-scale thermally induced stresses (reaching 100 MPa in places). When these are significantly reduced (often by over 80%, as will be shown later) by moisture ingress, it is possible that other stresses begin to dominate, leading to changes in the laminate shapes.
residual stress state within the saturated laminate is developed due to a large number of factors: thermal stresses, moisture stresses, at both a macro and a micro scale, along with imperfections in ply thickness, volume fraction, fibre waviness and so forth, leading to local changes in stresses. As the developed numerical models only deal with macro-scale stresses, any effect that the other factors contribute are not considered. Additionally, moisture diffusion can be uneven through fibre-resin composite materials, due to factors such as stress-dependent diffusion, micro-cracks/voids (including moisture induced damage), and fibre-resin delamination. These factors are clearly not taken into account in the numerical model, which has homogenised material properties to the scale of a composite ply.

With no twisting obtained in the numerical models (as expected), comparisons of the predicted shapes against those experimentally measured for the purpose of calibration of the swelling coefficient wasn’t feasible. Therefore, the following strategy was used. Through simulations, it was known that a swelling coefficient between $3.5 \times 10^{-3}$ wt.\%$^{-1}$ and $4.5 \times 10^{-3}$ wt.\%$^{-1}$ would correctly predict the change of curvature of all the laminates, bar Laminate 2. Also, Laminate 1 featured a relatively high curvature due to the $[90/0]$ partition, which was reduced significantly following saturation. Therefore, this laminate would be used for calibration, and the swelling coefficient obtained is applied to all other laminate families, except for Laminate 2. The result of the calibration process for Laminate 1 is shown in Figure 5-12, with a MSC of $3.8 \times 10^{-3}$ wt.\%$^{-1}$ being used. This MSC was within the range required for all laminates, and so when used to obtain residual stress profiles, should give a representative result. Laminate 2 required a different swelling coefficient, and so this was calibrated as well as possible, and found to be $2 \times 10^{-3}$ wt.\%$^{-1}$. As before, the MSC value obtained is an equivalent value and not a direct measure of the material’s MSC.

One way to potentially reduce the range of swelling coefficients required would be to remove the variations evident in the ‘dry’ laminate state. The change in laminate geometry following moisture saturation is of interest, which could be analysed using the laser scanning data of the dry laminate shapes to create a geometrically perfect model of the dry laminate, from which moisture swelling can be implemented directly. This could also lead to a more representative value of the material’s MSC being obtained, as the thermally-induced straining (present due to the summing of the thermal straining and the moisture-induced straining in saturated laminate models) is removed.
Finally, the strain energy of the laminates in the dry and saturated conditions was extracted from numerical models. The strain energy (using the ALLIE field output in Abaqus) is an important metric in the study of multi-stability, as it is minimised between different multi-stable shape configurations. As such, the loss in strain energy following moisture absorption may lead to a change in the multi-stable behaviour. Indeed, in this study, it was found that all laminates lost their multi-stable behaviour following moisture ingress. The total strain energy (SE) (for the entire laminate) is given for each of the laminate families in Table 5-6, for both the dry and saturated laminate conditions. As can be seen, the strain energy drops dramatically following saturation, to below 10% of its original value. This could have a significant impact upon the multi-stable property of the laminate, depending upon the ‘path’ of the potential energy as a function of moisture content. Specifically, if the potential energy as a function of moisture content path possesses bifurcation points at which the shape configuration can change from one to the next. This would require a more detailed study into the transient laminate response as a function of moisture absorption, and no conclusions are drawn in this area other than to say that the SE drops significantly as a function of moisture, leading to potentially large changes in the multi-stable behaviour.

Table 5-6 Strain energy (SE) for tailored laminates in dry and saturated states.

<table>
<thead>
<tr>
<th>Laminate</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape (A,B)</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>SE (mJ)</td>
<td>Dry</td>
<td>357</td>
<td>373</td>
<td>199</td>
<td>499</td>
<td>356</td>
</tr>
<tr>
<td>Saturated</td>
<td>10.0</td>
<td>7</td>
<td>32</td>
<td>26</td>
<td>27</td>
<td>4</td>
</tr>
<tr>
<td>Remaining SE (%)</td>
<td>2.9</td>
<td>1.4</td>
<td>5.4</td>
<td>7.5</td>
<td>5.5</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Finally, it is interesting to note that the shape configuration obtained by the saturated laminate is always that which possesses the highest SE in the laminate’s dry state. This is simply presented as an observation, and so needs to be verified by further study. Nonetheless, with multi-stable laminates being observed to lose their multi-stable property with moisture absorption, the potential energy may give an indication at an early stage of what the saturated laminate shape configuration will be. This could be particularly useful in morphing applications based on morphing laminates.
5.5 Summary

The numerical modelling technique to predict cured and saturated laminate shapes was presented. The models were created using the FE software Abaqus, and simulated the thermal deformation due to ply level orthotropic expansion coefficients coupled with a drop from cure temperature to room temperature. The models used three-dimensional continuum elements, with three elements per ply through-thickness to capture the residual stress profiles. Reduced integration was used along with a minimised element aspect ratio (of 100:1 and 60:1 for the cross-ply and tailored laminates respectively) in order to aid in alleviating shear-locking. With cross-ply laminates, an artificial imperfection (in the form of unequal side-lengths) was implemented into the model to coax the solution into a cylindrical shape. Controlling the simulation to obtain the different multi-stable shapes of the tailored laminates was achieved by means of altering the cool-down of certain plies to bias the solution to a certain shape. In addition, the tailored laminate models required the use of non-linear elements (type C3D20R) to correctly predict the multi-stable shape configurations.

The modelling procedure then involved altering the material’s longitudinal CTE value until the experimentally-measured shape was reproduced. This produced an equivalent value for $\alpha_L$ which, it should be stressed, is not a direct measure of the materials CTE value, but rather encompasses many additional contributions to residual stresses which contribute to the final laminate shape. Correlation between experimentally measured laminate shapes showed that a single equivalent value was able to reproduce the cured laminate curvatures within a range of ±10%.

The tailored laminates required a range of CTE values (from $1-12 \times 10^{-6}$ K$^{-1}$) being required to describe the laminate shapes. In one instance (Laminate 5) a converged solution was not obtained. In addition, a full-field comparison against numerical models revealed that absolute correlation would not be achieved, with the deflection of certain laminate partitions being over/under predicted with respect to the neighbouring partitions. In an effort to improve correlation, a manufacturing effect was implemented into models whereby a curvature that has been observed in manufactured symmetrical laminates was reproduced in tailored laminates. This resulted in the range of CTE values required (ranging from $3.5-9.5 \times 10^{-6}$ K$^{-1}$) being reduced, when compared to the initial models. In addition, a converged solution for Laminate 5 was obtained. Reproducing all multi-stable shape configurations with this modelling approach resulted
in a range in equivalent $\alpha_L$ values of $3.5-13 \times 10^{-6} \text{ K}^{-1}$ being required. It must be remembered that the minimization of shape contains an element of human interpretation, and so subsequent comparisons could include more robust variation measuring techniques, such as root mean square values.

The saturated laminates displayed a significant loss in peak residual stresses, typically over 66% for both tailored and cross-ply laminates, and in some instances exceeding over 90%. In the cross-ply laminate case, a swelling coefficient of $3.65 \times 10^{-3} \text{ wt.}\%^{-1}$ in numerical models was capable of describing the experimentally-measured laminate curvatures to a range of -16/+14%.

Due to the saturated shapes of the tailored laminates obtaining the multi-stable shape configuration with the smallest out-of-plane deformation, the changes in curvatures due to moisture absorption were small. This lead to difficulties in reproducing the loss of curvature in numerical models. As a result, a MSC of $3.8 \times 10^{-3} \text{ wt.}\%^{-1}$, which was calibrated using Laminate 1, was used for the majority of the laminates. Laminate 2 required a different value for the MSC of $2 \times 10^{-3} \text{ wt.}\%^{-1}$. This range in calibrated MSC values could be due to differences between laminates, or as a consequence of the modelling technique used. One hypothesis is that following moisture saturation the thermally induced macro-scale residual stresses no longer dominate the laminate’s stress state, causing other residual stresses to dominate the laminate shapes.

Finally, extraction of the laminate’s strain energy before and after moisture saturation shows that a very large change in strain energy is possible, with the laminates losing over 90% of the strain-energy following moisture saturation. This large change in strain energy can be seen in practice by the loss in the multi-stable laminate property.
Chapter 6  Extracted Through-Thickness Stress Profiles

6.1 Introduction
Following calibration, the numerical models were used to extract through-thickness residual stresses. This was done separately for the cross-ply family of laminates (section 6.2) and the tailored laminates (section 6.3).

The stresses that are extracted are direct stresses (identified as S11 and S22 in Abaqus). An example of the extracted stresses is presented in Figure 6-1, which shows the typical stress profiles of a cross-ply laminate in a cylindrical shape configuration. In this particular example, the stresses along the x-axis are responsible for the curvature developed. The bottommost plies are transversely orientated, and their contraction during cool-down from curing temperature is resisted by the longitudinal layers above. This gives rise to a state of tension in the transverse layers. This tension is counteracted by the longitudinal layers which are thus in a state of compression. This leads to the example of through-thickness stress profiles shown in Figure 6-1(b).

6.2 Cross-ply Laminates
The through-thickness stresses of the cross-ply laminates resulted in cylindrical shapes, similar to that which is shown in Figure 6-1(a). The co-ordinate system is kept consistent with this figure – that is, the curvature is developed along the x-axis.
Extracted Through-Thickness Stress Profiles

(generated along the $y$-axis), in a positive $z$ direction. This requires the top layer to be made up of longitudinal plies, while the bottom layer is made up of transverse plies. With the cylindrical multi-stable solutions of these laminates, the stresses in the $x$ and $y$ direction ($\sigma_{xx}$ and $\sigma_{yy}$ respectively) subsequently take a form similar to those depicted in Figure 6-1(b).

Once the dry and saturated laminate shapes were reproduced with the finite element models, the through-thickness residual stress profiles were extracted. The path feature in Abaqus was used to describe a line through the laminate’s thickness, along which stress components for $\sigma_{xx}$ and $\sigma_{yy}$ were extracted. This was done at a point away from any edge or boundary condition effects. Using the calibrated finite element model, the normal residual stress profiles for the experimentally investigated laminate lay-ups $[90_2/0_2]$, $[90_3/0_1]$ and $[90_3/0_2]$ were extracted and are shown in Figure 6-2(a)-(c) respectively. The normal stresses along the $x$-axis ($\sigma_{xx}$) are displayed on the left and the normal stresses along the $y$-axis ($\sigma_{yy}$) are displayed on the right.
Figure 6-2 Through-thickness residual stress profiles ($\sigma_{xx}$, $\sigma_{yy}$) of manufactured laminates [90/0]$_2$, [90/0]$_1$ and [90/0]$_2$ for dry and saturated laminate conditions (all stresses are in MPa).

By looking at the [90/0]$_2$ laminate (see Figure 6-2(a)), the effect of the laminate reaching a minimum potential energy state (i.e. curing to a quasi-cylindrical shape as opposed to a saddle shape) on the through thickness stresses can be seen. Instead of having similar stress profiles in $\sigma_{xx}$ and $\sigma_{yy}$ (corresponding to a saddle shape), both the profiles and the stress peaks are significantly different. The stress profile along the x-axis ($\sigma_{xx}$) shows a complicated stress distribution, varying from tension at the top
surface to compression roughly mid-way through the longitudinal layer, before returning to tension in the transverse layers. Additionally, the peak stresses along the $x$-axis are substantially higher than those along the $y$-axis, with the maximum tensile and compressive stress along the $x$-axis being $-105.2/+67.5$ MPa, while the peak stresses along the $y$-axis are $-36.2/+35.4$ MPa. The dominance of the longitudinal layers can be seen in the stress profiles.

In Figure 6-2(b), one of the $0^\circ$ plies is replaced with a $90^\circ$ ply to produce the $[90_3/0_1]$ laminate. The overall curvature produced increases due to the change in ratio of $0^\circ/90^\circ$ plies. Focusing on $\sigma_{xx}$ (dry state), the peak stresses in the longitudinal layer are reduced when compared to the $[90_2/0_2]$ laminate, from $67.5$ MPa to $38.5$ MPa, and $-105.2$ MPa to $-82.5$ MPa. This is to be expected due to the reduction in the ratio of $0^\circ/90^\circ$ plies. Interestingly, the peak tensile stress in the transverse layers remains largely unchanged at $25$ MPa. The slope of the stress profile in the transverse layers itself changes, with a transition occurring from tensile stress to compressive stress. Due to the high stiffness of the fibres in the longitudinal plies, the (almost zero) thermal strain of the longitudinal plies appears to dominate the transverse layer by imposing a strain. This strain at the interface between longitudinal and transverse layers remains largely unaffected by an increase in transverse plies, leading to a relatively constant peak tensile stress being generated. The profile through the thickness of the transverse layers then readjusts to accommodate the change in lay-up, and reaches a state of compression at the bottom surface. This compression also results in a reduction in the peak compressive stress in the longitudinal layer due to stress equilibrium through the thickness of the laminate. It is interesting to note that the same behaviour is evident in the $\sigma_{xx}$ profiles, with peak stresses in the transverse layers maintaining roughly $36$ MPa. These stress peaks are higher than those (roughly $25$ MPa) achieved in the transverse layers along the $x$-axis.

Looking at $\sigma_{yy}$ in Figure 6-2(b), the tensile stress peaks increase slightly compared to the $[90_2/0_2]$ laminate, while the compressive stress peaks are reduced substantially (e.g. reduced from $-36.2$ MPa to $-17$ MPa). It is worth remembering that along the $y$-axis, the top ply is orientated in a transverse direction, while the bottom three plies are orientated longitudinally. Therefore, there are now three longitudinal plies whose thermal strain is dominating and driving the behaviour of the one transverse ply. A small increase in peak tensile stresses in the transverse layers is observed, due to the additional longitudinal ply. In this instance, a similar peak stress to that observed in Figure 6-2(a)
is expected, as the fibres in the longitudinal layers still impose a similar strain to the transverse layers, leading to a similar stress peak being required to maintain equilibrium and causing the stress peaks in the longitudinal layers to reduce as a result.

In Figure 6-2(c), the stress profiles for a [90₃/0₂] laminate are presented. These differ from the previous two examples as both thickness and the ratio of 0°/90° plies are different. The addition of a 0° ply, compared to Figure 6-2(b), results in higher $\sigma_{xx}$ stress peaks in the longitudinal layers. In the 90° plies, the peak tensile stress of 24.4 MPa once again remains similar to that of $\approx 2$ MPa observed in the [90₂/0₂] and [90₃/0₁] laminates. However, the slope of the profile changes once again, and this time does not transition into compression (unlike Figure 6-2(b)). The $\sigma_{yy}$ profile shows similar behaviour compared to that which was previously observed in Figure 6-2(b), with the peak stress values in the transverse layer largely unchanged and the stress profile of the longitudinal layer changing to accommodate the different number of plies.

The dominance of the longitudinal plies can be seen in these stress profiles. When subjected to bending moments, the $\sigma_{xx}$ profile within the longitudinal layers is largely linear (bar two small regions at the top and bottom boundaries). This linear stress behaviour is akin to that in an isotropic beam subjected to pure bending. The slope of this line changes according to the ratio of longitudinal plies to transverse plies. Subsequently, the peak stresses that are generated in the longitudinal layer changes as a function of the slope. The majority of the changes in the stress profiles then occurs in the less stiff (and thus more compliant) transverse layers, where the profiles adjust to maintain stress equilibrium through the laminate thickness.

The changes in stress profiles due to moisture saturation can also be seen in Figure 6-2. As expected, a reduction in stresses is seen, as the contraction undergone by the transverse layers during curing is reduced due to moisture induced swelling. The magnitude of the reduction in residual stresses observed is very substantial, with peak stress reduced to around 30% of the original stress values in both the $\sigma_{xx}$ and $\sigma_{yy}$ cases. In order to accommodate this reduction in stresses, the slope of the profiles changes. The through-thickness coordinates ($z$) at which the stresses alternate from tension to compression does not change. As residual stresses lead to the development of strain energy, which is responsible for the bifurcation of multi-stable laminates, this large reduction in stresses may lead to a change in the bifurcation behaviour of such laminates due to moisture absorption.
6.3 Tailored Laminates

6.3.1 Procedure used to obtain stress profiles

The dry and saturated tailored laminate stress profiles will be presented in sections 6.3.2 and 6.3.3 respectively. The initial results, in section 6.3.2 present dry stress profiles obtained using the refined model, from an initially curved stress-free shape. These profiles will be used to describe the effect of the tailored laminate lay-up sequence on laminate shapes, including the changes in stress near the boundary between two lay-ups. Where possible, stress profiles for both multi-stable shape configurations will be presented. Finally, in section 6.3.3, the change in stresses due to moisture saturation will be presented.

The process used to extract residual stresses in the tailored laminates was different to that used for the cross-ply laminates. Element integration points were used to directly obtain the stress components (S11 and S22) required to produce the through-thickness stress profiles. This is a modification to the procedure that was used in the cross-ply case, which used the path feature of Abaqus to obtain stresses. This was done to eliminate stress averaging at the nodes, which may influence the stresses obtained – particularly at the interface between plies orientated differently. The use of integration points was possible due to the structured mesh scheme used in modelling. With each element containing two integration points in the through-thickness direction, and with three elements being used per ply thickness, this results in six integration points per ply being available to extract stresses. These stresses can then be plotted as a function of laminate thickness to identify the through-thickness stress profiles. Finally, depending on the ply orientation, the stresses may need to be transformed from the local “material” orientation system to the global “laminate” orientation system.

The through-thickness residual stress profiles for the laminates were extracted at certain discrete locations within each laminate (either at the middle of a partitioned section or near a boundary between two different lay-ups). These are marked in each of the cross-sectional figures and are taken at points in the middle of the laminate width. The stresses obtained are along both the laminate length (x-axis, i.e. \( \sigma_{xx} \)) and the laminate width (y-axis, i.e. \( \sigma_{yy} \)). In some instances, local curvature where a stress profile has been extracted may not be obvious in the accompanying images. For further details on the shape configuration at the points in the laminates from which the stresses are extracted, the reader is referred to Appendix-D, where – in addition to the complete laser-scanned...
shapes in the dry and saturated configurations – cross-sectional views of the laminates are also provided.

6.3.2 Dry Stress Profiles

The effect of the tailored laminate configuration on residual stresses will be discussed by observing changes in the stress profiles due to neighbouring lay-up partitions. As the stress profiles themselves are typical of what has been observed in the cross-ply cases, the discussion will focus on the changes imparted on the stress profiles by the laminate partitions. By studying the stress profiles of the laminates it is evident that a tailored laminate configuration impacts upon the residual stresses of a partition comprising a certain lay-up.

To facilitate interpreting the extracted stresses, each figure contains the following details. Firstly, the cross-section of the laminate is given, with the positions (numbered 1-4) at which the through-thickness stresses are extracted being marked (Figure 6-3(a)). The partition length \(p\) refers to the length of the sections with different lay-up configurations, which remains constant for each laminate. Four locations are used to extract the residual stresses. Locations 1 and 4 are at the middle of the first and second lay-up sections respectively, and thus represent ‘steady-state’ stresses away from edge-effects or lay-up boundaries. Locations 2 and 3 are on either side (by one element length of 2.5 mm) of the boundary, and are used to show the transition in stresses from one lay-up configuration to the next. As Laminates 4-6 contain several different lay-up partitions, the stresses at the boundaries between lay-up partitions aren’t presented for these laminates. Secondly, the FE predicted shapes are given in Figure 6-3(b), to link the extracted stresses for Shapes A and B to a shape configuration. Lastly, the extracted stresses (\(\sigma_{xx}\) and \(\sigma_{yy}\)) are presented for both shape configurations in Figure 6-3 (c)-(f).
Figure 6-3 Through-thickness residual stresses for Laminate 1 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) cured shapes; (c) \(\sigma_{xx}\), Shape A; (d) \(\sigma_{yy}\), shape A; (e) \(\sigma_{xx}\), Shape B; (f) \(\sigma_{yy}\), Shape B.

The stress profiles at the four different positions give an indication of the change in stresses brought about by the partition in lay-up sequences. Specifically, the stresses at the centre of the lay-up partitions (Positions 1 and 4) are taken at an area free of significant stress gradients, and thus may be considered as ‘steady-state’ stresses. At Positions 2 and 3, the stresses are close to the boundary between two lay-up partitions and thus may be subject to a gradient in stresses. Considering Shape A, it appears in this instance that the transition zone between stresses is small, as there is a minimal
difference in stresses between Positions 1 and 2, and Positions 3 and 4. The transition in stress is most evident in the $\sigma_{yy}$ case. This may be due to the fact that the curvature of shape A – both in terms of direction and generator axis – is identical in both lay-up partitions, and so the changes in stresses are easily accommodated. The stresses extracted for Shape B (Figure 6-3(e) and (f)) shows the effect of changing the curvature orientation on the stresses at the transition between partitions. At Position 3 ([90/0] lay-up, close to the partition) the stresses are similar to those of the [90/0] partition, at Positions 1 and 2. This shows the transition in shapes from the first partition to the second, with the stresses in Position 3 showing a similar profile (and thus curvature) to the [90/0] partition. Further along the laminate, at Position 4, the curvature has changed generator axis and orientation, leading to a completely different stress profile. The $\sigma_{yy}$ stress profiles show similar behaviour, where the steady-state and transitional stresses are significantly different in terms of both profile and peak stresses. The transition is more noticeable, with a gradual change in stresses being observed across all four partitions.

With the curvature of the [90/0] partition in Shape B being developed along the $y$-axis, the $\sigma_{yy}$ stress profile becomes quite similar to the $\sigma_{xx}$ stress profile for the same partition in Shape A. The peak stresses, however, are not equal, going from -114/+75 MPa in Shape A to -53/+31 MPa in Shape B, despite the identical lay-ups. This is of course, due to the interaction between the lay-ups in the tailored laminate configuration, and demonstrates the impact of such a configuration on residual stresses.

The peak tensile stress in the transverse layers is similar in both shape configurations, with a maximum of 36 MPa (Position 1, Shape A). This is a significant fraction of the material’s transverse tensile strength (55 MPa, see Table C-2, Appendix-C) and thus shows the significance of the developed residual stresses.
With Laminate 2, a more significant change in stresses is observed when alternating between Shape A and Shape B. Positions 1 and 2 are in the symmetrical [0/90/0] lay-up partition of the laminate. Therefore, the shape developed at this region in largely influenced by the neighbouring unsymmetrical region. In Shape A, it appears that the influence of the unsymmetrical region is minimal, with this section remaining largely flat (Figure 6-4(b)). With Shape B however, this region becomes curved, a consequence
of the shape configuration achieved by the unsymmetrical region. In fact, this region appears to be showing evidence of a saddle shape configuration, which may be due to the tailored laminate configuration.

With Shape A, the stress profiles at the transition (Positions 2 and 3) are quite similar to the steady-state stresses shown at Positions 1 and 4. As before, this behaviour is not repeated in Shape B, as significant differences in the stresses at the partition boundary are observed. This is most evident in the $\sigma_{xx}$ stress profiles (Figure 6-4(c)), particularly at Positions 3 and 4, where the multi-stable partition is located. Close examination of the cured shape of Shape B (Figure 6-4(b)) shows a small curvature along the $x$-axis near the lay-up partition. This curvature is seen in the stress profile at Position 2, with the top and bottom plies (both longitudinally orientated) having significantly different stress profiles. Likewise, at Position 1 the stresses in the longitudinal layers show that local curvature is developed. Interestingly, as this partition is symmetrical in lay-up, the curvatures are due to the interaction with the neighbouring partition. Whereas in Shape A this section was curved with a generator along the $y$-axis, in Shape B this section is curved with a generator along the $x$-axis. The change in stress state in this partition affects the stresses in the symmetrical lay-up region, with the peak tensile stresses ($\sigma_{xx}$) in the symmetrical region increasing in magnitude from $-11/+23$ MPa in shape A to $-28/+36$ MPa in shape B. The change in tensile stress occurs in the transversely orientated ply, which has a relatively low tensile strength. Therefore, change in tensile stress can be considered to be significant and demonstrates the importance of considering residual stresses developed in tailored composite laminates, particularly for structural applications.

The $\sigma_{yy}$ stress profiles (Figure 6-4(d),(f)) shows the compliance of the symmetrical [0/90/0] region to the unsymmetrical [02/902] region. In shape A, the [0/90/0] region has a small curvature in the $y$ direction. However, in shape B, a curvature is induced, adding the equivalent of an external bending moment to this section, and altering its stress profile.

The adjustment in residual stresses is demonstrated in this laminate by the stress profile at Position 3, Figure 6-4(c). At the top ply, the stress profile changes, showing that in this case the stresses required more than 2.5mm to reach a steady state following the change in thickness.
Figure 6-5 Through-thickness residual stresses for Laminate 3 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) cured shapes; (c) $\sigma_{xx}$, Shape A; (d) $\sigma_{yy}$, shape A; (e) $\sigma_{xx}$, Shape B; (f) $\sigma_{yy}$, Shape B.

The cured shapes of Laminate 3 (Figure 6-5(b)) show a peculiarity induced by the tailored configuration. As seen previously, it is expected that [90°/0°] square cross-ply laminates will obtain a quasi-cylindrical shape configuration, with practically no anticlastic curvature observable. This is demonstrated in the [90°/0°] section of Shape A. Careful examination of Shape B, however, shows that a secondary curvature is developed orthogonal to the principal curvature. The curvature is slight but noticeable.
Examination of the stress profiles shows that at the centre of the laminate, this behaviour is not prevalent, as the stress profile is typical of what would be expected along the straight edge of a cylindrical laminate (Figure 6-5(c), Position 1). Continuing along to Positions 2 and 3, it can be seen that a curvature is developed near the lay-up boundary, that continues into the symmetrical [0/90], partition.

The stresses of the multi-stable [90/0] partition are denoted by Positions 1 and 2 in Figure 6-5(c)-(f). As well as the different stress profiles in this section for Shape A and Shape B, it can also be seen that Shape A has the highest peak stresses, with values of -95/+63 MPa (Position 1 Figure 6-5(e)).

Shape B shows the largest changes in stress profiles at each of the 4 Positions, particularly in the $\sigma_{xx}$ profiles (Figure 6-5(e)). The stresses near the lay-up partitions are quite different to the steady state stresses, showing that the transition length (where the stresses equilibrate from those in the first lay-up partition to those of the second partition) is relatively long when compared to that of Shape A. Also, the peak stresses imposed on the symmetrical section increase from -26/+27MPa (Shape A) to -34/+36 MPa (Shape B).
Laminate 4 features a staggered change in lay-up, similar to what may occur in some scarf repairs. Additionally, a bi-stable region exists which allows for the curvature to transition from being along the $x$-axis to the $y$-axis. The large changes in residual stresses due to the tailored lay-up can be seen here. The $[90_2/0_2]$ partition facilitates a change in curvature, from being generated along the $x$-axis (first partition) to being generated along the $y$-axis. As such, both the $\sigma_{xx}$ and $\sigma_{yy}$ profiles are practically identical, being mirrored due to the 90° degree offset in ply orientations (Position 2, Figure 6-6(c),(d)).

This laminate is a good example of the effect that the tailored laminate lay-up sequence has on residual stresses. The lay-up sequence at Position 3 is $[90_1/0_3]$. When looking at the $\sigma_{yy}$ stress profiles at this position, the ply orientations are rotated by 90°, and so the lay-up sequence becomes $[0_1/90_3]$. This is the same lay-up as at Position 1, along the $x$-axis. Therefore, the $\sigma_{yy}$ stress profile at Position 1 (Figure 6-6(c)) can be compared to the (inverted) $\sigma_{yy}$ stress profile at Position 3 (Figure 6-6(d)). A brief comparison shows...
that the peak stresses are not equal (e.g. the peak stresses in the longitudinally aligned plies are -109/+67 MPa at Position 1 and -91/+32 MPa at Position 3). Additionally, the stresses in both the longitudinal and the transverse layers take on a different form. For example, in the transverse layer at Position 1 (bottom three plies) the stresses alternate from tensile stress to compressive stress. At Position 3, the σ_yy stress profile shows that the transverse layer (upper three plies) stay in tension throughout.

A look at the cured shape (Figure 6-6(b)) shows that the areas where Positions 1 and 3 are located feature different curvatures, with the first partition (Position 1) having a greater curvature than that of Position 3, resulting from the higher stresses at Position 1. The difference is caused by the fact that the change in lay-up takes place along the x-axis. As such, whenever the curvature is generated along the x-axis (as at Positions 3), it directly influences the shape of neighbouring sections. This can be seen by the curvature, and thus the stresses induced at Position 4 (Figure 6-6(d)). Along this axis, the stress profiles at a particular lay-up partition are formed not just from the thermal straining at that partition, but also from the bending stiffness of neighbouring partitions.
Figure 6-7 Through-thickness residual stresses for Laminate 5 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) cured shapes; (c) $\sigma_{xx}$, Shape A; (d) $\sigma_{yy}$, shape A; (e) $\sigma_{xx}$, Shape B; (f) $\sigma_{yy}$, Shape B.

Laminate 5 also features an alternating curvature along its length, when in the Shape A configuration (Figure 6-7(b)). At the $[0_{2}/90_{3}]$ section, the curvature has a generator along the $y$-axis. The neighbouring $[0_{3}/90_{2}]$ section then changes both the direction and the generator axis of the curvature, resulting in the curvature seen in the $[0_{2}/90_{1}]$ section. This is not seen in Shape B, where the principal curvature in the $[0_{3}/90_{2}]$ section is now generated entirely along the $x$-axis. The difference in the two shapes is most pronounced in the $[0_{2}/90_{3}]$ section which sees a drastic change in peak stresses. In Shape A, the peak...
\( \sigma_{xx} \) values ranged from -115/82 MPa, while in Shape B they ranged from -50/29 MPa (Position 1, Figure 6-7(c), (e)). The majority of the changes occur in the longitudinal plies, with the transversely orientated plies seeing an increase of 10 MPa between Shape A and Shape B. The large compressive stress experienced in the longitudinal layers of Position 1, Shape A is not absorbed by the transverse layers. Rather, the stress profile at this position readjusts, culminating in the stresses in the longitudinal layers reverting back to tension at the laminate’s bottom surface.

As was seen with the cross-ply laminates, the peak tensile stresses in the transverse appear to be almost equal at each of the Positions 1-3. Depending on the local shape, the stress profile of the transverse layers may adjust, but the peak stresses themselves remain very similar. This could be due to the longitudinal layers dominating the behaviour of the laminate.

The \( \sigma_{xx} \) stress profiles of Shape B (Figure 6-7(e)) give insight into the dominance of the fibre-orientated layers. The transverse plies (the upper three) all feature roughly identical peak tensile stresses. This could stem from the fibre controlling the strain at the interface, and thus imparting a certain stress into the transverse layers. The stresses of the longitudinal layers then adjust to balance this stress. As the laminate is at its thickest at Position 1, this balancing compressive stress required is at its highest at this point, followed by that at Position 2 and then Position 1.
Extracted Through-Thickness Stress Profiles

Figure 6-8 Through-thickness residual stresses for Laminate 6 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) cured shape; (c) $\sigma_{xx}$; (d) $\sigma_{yy}$.

The residual stress profiles of Laminate 6 (Figure 6-8) once again show the interaction between the unsymmetrical sections and the neighbouring symmetrical sections. In this instance, the two unsymmetrical sections (with $[0_2/90_2]$ and $[0/90]$ lay-ups) are neighboured by unidirectional $[0_2]$ sections, which, due to their unidirectional nature, are free of thermally induced residual stresses. Thus, through-thickness stresses evident at these locations (Positions 1 and 3, Figure 6-8(c),(d)) are due to the neighbouring sections. The unsymmetrical regions resulted in the laminate obtaining a curvature generated along the $x$-axis. This curvature is maintained in the unidirectional regions, helped in part by the low flexural stiffness in these sections along the $y$-axis. This bending of the unidirectional sections results in the through-thickness stresses shown at Position 1 and 3, in Figure 6-8(d). Looking at Figure 6-8(b), where the laminate is predominantly flat along the $x$-axis. However, by examining the $\sigma_{xx}$ it appears that there are still some bending stresses being imparted on the unidirectional sections along the $x$-axis as well (Figure 6-8(c)).
The range of laminates studied in this work allows for additional observations to be made on the behaviour of tailored laminates. For example Laminates 1 and 3 feature a similar construction, with a [90\textsubscript{2}/0\textsubscript{2}] partition neighbouring a [90\textsubscript{0}/0\textsubscript{1}] partition (Laminate 1) and a [0/90\textsubscript{s}] partition (Laminate 3). In the Shape A configuration, the stress $\sigma_{xx}$ profiles show that the peak stresses in the [90\textsubscript{2}/0\textsubscript{2}] partition is higher for Laminate 1 than for Laminate 3 (see Figure 6-3(c) and Figure 6-5(c) respectively, Positions 4 and 1). The difference between the two laminates can be reduced to the substitution of a transverse ply for a longitudinal ply in the partition neighbouring the [90\textsubscript{2}/0\textsubscript{2}] section. As the longitudinal ply will not thermally contract to the same extent as the transverse ply, it therefore reduces the thermal straining of the bordering ply. This can be seen directly in the reduced stresses in the lower half (i.e. transverse plies) of stress profiles, which causes a reduction in the stresses counteracted by the longitudinal plies. Extending the comparison between the [90\textsubscript{2}/0\textsubscript{2}] partition of Laminates 1 and 3 to the $\sigma_{yy}$ stresses, it can be seen that the tailored configuration impacts upon these stresses as well. The peak stresses in Laminate 1 (Figure 6-3(d), Position 4) are much higher than those in Laminate 3 (Figure 6-5(d), Position 1). However, this particular observation is a consequence of the laminate geometries investigated in this work. Along the $y$-axis, there are no changes in lay-up configuration and changes in residual stresses. The curvatures developed along neighbouring partitions directly influence each other. Therefore, it is expected that a change in the neighbouring lay-up will have a large effect on the [90\textsubscript{2}/0\textsubscript{2}] partition, particularly when going from a symmetrical (Laminate 3, where no anticlastic curvature is imposed upon the [90\textsubscript{2}/0\textsubscript{2}] section) to an unsymmetrical (Laminate 1) lay-up.

It must be remembered that these stresses are extracted at the middle of the laminate width. To maintain equilibrium, these changes (which are only observed in the longitudinal plies and thus not balanced by the transverse plies) are reacted by other stresses when moving away from the middle of the laminate along the $y$-axis. A similar effect may be seen in the stress profile of Laminate 6 at Positions 1 and 3 (Figure 6-8(c)). Close examination of the $\sigma_{xx}$ stress profiles at these positions show a slight non-linearity in the top ply. These stresses are not balanced through the laminate thickness at Positions 1 and 3. Rather, the unbalanced stresses at Position 1 are balanced by those at Position 3. This behaviour appears to be confined to the stress profiles orthogonal to the stresses generating the primary curvature in the laminates ($\sigma_{yy}$ in Figure 6-8(d)).
It is interesting to note that the stress profiles are significantly affected by the tailored laminate configuration. This can be seen directly from the stresses in the $[0_2/90_2]$ region of each laminate. Each laminate features a $[0_2/90_2]$ (or a $[90_2/0_2]$) section which is influenced differently by surrounding lay-ups, with the peak residual stresses varying drastically among the laminates. For example, even when the curvature formed is of the same orientation (as in Shape A of Laminates 1 and 3) the peak stresses are significantly different (from $\sigma_{xx} = -114/+75$ MPa in Laminate 1, to $\sigma_{xx} = -95/+63$ MPa in Laminate 3). This seems obvious, in that each tailored configuration has different curvatures, resulting from different residual stresses. The implications of this though are significant. In design cases where residual stresses are critical (such as morphing parts, which depend upon residual stresses for their multi-stable property), the interaction between the laminate and the surrounding lay-ups/structure can have a significant impact upon residual stresses.

It is worth pointing out that the dominance of the longitudinal layers (and thus the fibres) is once again observable in the stress profiles. Looking at $\sigma_{xx}$ profiles, the maximum stress in the transverse layers (matrix dominated) of all laminates and multi-stable shape configurations ranges from 17-38MPa. The stress profiles in the transverse layers generally don’t change drastically; rather, the stress values vary by a few MPa. It appears, therefore, that the thermally induced strain of the highly stiff fibres dominates the strain (and thus the stress) at the interface between the longitudinal and transverse layers. This stress is then held more-or-less constant through the transverse layer, depending on the geometry of the laminate at that point. The stresses of the longitudinal layers, which have a high tensile strength, can vary drastically – in both profile and peak values – depending upon the local geometry. Even though high stresses are sometimes achieved (nearing 100 MPa at times), the stress in the transverse layers never exceeds the tensile strength of the matrix material. However, the peak stresses in the transverse layers are a high proportion of the materials tensile strength in that direction (73 MPa, O’Higgins et al. (2011)). This could require consideration to the stresses resulting from external loading when such a laminate is part of a morphing structure.

6.3.3 Saturated Stress Profiles

The stress profiles presented in this section compare the stress profiles of the dry and saturated laminate shapes. The modelling technique used is as described in section 5.3.2 for each laminate. Once again, the laminate cross section depicting the lay-up at each
position that stresses are extracted is provided (Figure 6-9(a)). Secondly, the numerical solutions for the dry and saturated laminate shapes, from which the stress profiles are extracted, are shown (Figure 6-9(b)). The through-thickness stress profiles $\sigma_{xx}$ and $\sigma_{yy}$ are then given at Positions 1 and 2 (Figure 6-9(c),(d)), and at Positions 3 and 4 (Figure 6-9(e),(f)). The dry multi-stable shape configurations chosen represent the most similar shape configuration to the saturated laminate shape.
The stress profiles show the significant loss in through-thickness residual stresses following moisture saturation. For example, the peak $\sigma_{xx}$ at Position 1 dropped from -97 MPa to -18 MPa in the longitudinal layer, and from 35 MPa to 10 MPa in the transverse layer (Figure 6-9(c)). A proportionally larger drop is experienced in the $\sigma_{yy}$ stresses at
the same position, from $\sigma_{yy} = -17/+36$ MPa in the dry case to $\sigma_{yy} = -4/+6$ MPa in the saturated case. At all four positions, the loss in peak stresses (both compressive and tensile) is in the range of 71-86%, showing the substantial loss in thermally induced residual stresses following moisture saturation. The only exception to this range is the loss in compressive stresses ($\sigma_{xx}$) at Position 4. The profile at this position changes form, and consequently the loss in the peak compressive stress is 57%.

The fact that the form of the stress profiles are identical (bar those at Position 4) is a consequence of adapting the ‘dry’ numerical model to predict the ‘saturated’ shape, which will most likely result in an identical shape being predicted, with a reduced deformation. This is an important point to consider, as it demonstrates one of the difficulties in modelling the saturated laminate behaviour of tailored laminates. In numerical studies of multi-stable laminates, prior knowledge of the desired laminate shape is needed before simulations can be conducted. This is particularly true with the modelling approach employed here, where the laminate shapes are ‘biased’ to the desired solution. With very small out-of-plane deflections being observed with the saturated laminates, it is very difficult to ascertain the precise shape configuration of the laminate in order to bias the numerical solution. Further compounding the issue are the additional ‘unknowns’ in the saturated shape (e.g. the twist curvature exhibited by some of the laminates). This creates an extra difficulty in correctly predicting the residual stress profiles in the saturated case. Because of this, the numerical model of the dry shape most closely resembling the experimentally observed saturated laminate shape is modified to obtain the numerically predicted saturated laminate shapes. This means that the predicted saturated shape configuration will be largely identical to that of the dry shape. This may lead to errors in the extracted through thickness stresses, without precise knowledge of the exact saturated laminate configuration.

Nonetheless, the saturated laminates are almost flat, a consequence of the very low stress state. As such, although the exact stress profiles are assumed to be similar in form to those of the dry laminates, the low stress-magnitudes are indicative of the overall stress state, and can be used to obtain the large loss of residual stresses between the dry and saturated laminates. This is ultimately what is of interest in this study, as the loss in residual stresses can be used to ascertain information into the multi-stable behaviour of the laminate.
Like Laminate 1, the stress profiles of Laminate 2 show a significant reduction in peak stresses following moisture saturation. For example, the peak stresses at Position 1 went from $\sigma_{xx} = -28/+36$ MPa to $\sigma_{xx} = -11/+10$ MPa. However, the behaviour of Laminate 2 in the saturated condition differs to that of Laminate 1 in that some of the stress profiles...
change form. Most notably, this occurs with the $\sigma_{yy}$ profiles of at Positions 3 and 4 (Figure 6-10(f)). In this instance, the stresses are reversed (from tension to compression and vice versa) between the dry and saturated states. This would indicate a change in the curvature in this section, from positive to negative. This change is only along the $y$-axis; the stresses in the $x$-axis (Figure 6-10(e)) don’t show this behaviour. The stresses are very small, so any curvature that is developed is very small. This makes it difficult to compare against the experimentally-measured saturated laminate shape to verify if the curvature is reversed in this section, especially when the twisted saturated laminate shape is considered. However, as already mentioned, the stresses (and developed curvature) are small, and so the physical significance of this finding is limited; both the experimentally-measured and numerically-predicted shapes are predominantly flat along the $y$-axis at this position. The changes in the form of the stress profiles lead to a large range in the percentage reduction in the peak stresses between the dry and saturated laminates, with a loss of between 34% and 95% being recorded (peak compressive stress at Position 4, $\sigma_{xx}$ and $\sigma_{yy}$ respectively).

The difference between the ‘steady state’ stresses and those near the boundary between lay-ups (denoted by Positions 2 and 3, Figure 6-10(c)-(f)) shows that, even though the residual stresses are small, the transition zone can still be further than 2.5 mm from the lay-up partition. This is particularly true for the $\sigma_{yy}$ stress profiles.
The stress profiles of Laminate 3 showed a generally consistent response to saturation, in that each stress profile was reduced substantially, but the form of the stress profile does not change. The only exceptions are the $\sigma_{xx}$ profiles at Positions 1 and 2 (Figure 6-11(c)). The loss in stresses at all other Positions was in the range of 71-79%.
are some small differences between the steady state stresses (Positions 1 and 4) and the stresses near the partition boundaries (Positions 2 and 3). The $\sigma_{xx}$ profiles at Positions 1 and 2 only lost 50-53% of their peak compressive stresses. At Position 2, it can be seen that the longitudinal plies (uppermost) transitioned into tension when saturated, which is in contrast to the dry stress profile. Also, the peak compressive stresses in the dry condition at these Positions are relatively low (-35 MPa at Position 1, -42 MPa at Position 2). The profile in the longitudinal sections is different to those in Laminate 1 which develop a high curvature, as these alternate between high compressive and tensile stresses. In this laminate, the curvatures along the $x$-axis remains small, leading to the stresses in longitudinal layers remaining low. Subsequently, the loss in stress between dry and saturated states is also low.

Finally, the change in stresses at the transition of the boundary shows some peculiar behaviour. In the dry state, the $\sigma_{xx}$ profiles at Positions 1 and 2 shows that the top ply (longitudinal orientation) is entirely in compression. When saturated, the upper surface of the laminate alternates into tension (+ 7 MPa), but only at Position 2. This means that in the saturated state, the top surface of the laminate alternates between compression (Position 1, -1 MPa) to tension (Position 2, +7 MPa) and back into compression (Positions 3 and 4, -15 MPa and -6 MPa respectively). These are small stress values and so may not be precise indications of the stress state of the manufactured laminates, but shows the peculiar behaviour that is possible following saturation.
The stress profiles of Laminate 4 continue to show the large drop in stresses due to moisture absorption, with most profiles seeing a drop in peak stresses of 51-81%. The form of most of the profiles doesn’t adjust between the dry and saturated states, with the $\sigma_{xx}$ stress profiles at Positions 3 (Figure 6-12(c)) being the most obvious exception.

What is interesting about this profile is that the peak compressive stress at the interface
between the longitudinal plies (top three) and the transverse ply (bottommost) doesn’t change. In the transverse ply, the stresses are reduced as expected, due to swelling. To adjust for this, the stresses in the longitudinal plies readjust their form, yet maintain the peak compressive stress value to that of the dry state. This results in a very low change in the peak compressive residual stress of only 15% (Position 3, dry and saturated, Figure 6-12(e)). The difference in stress profiles may indicate that the curvature configuration at Position 3 undergoes a change between the dry and saturated states. This observation is not repeated, however, for the dry and saturated $\sigma_{yy}$ profiles at the same position (Figure 6-12(f)), which are identical in form.

Figure 6-13 Saturated through-thickness residual stresses for Laminate 5 with (a) laminate cross-section showing positions of extracted through-thickness stresses; (b) dry and saturated shapes; (c) extracted $\sigma_{xx}$; (d) extracted $\sigma_{yy}$.

As expected, Laminate 5 experienced a large loss in residual stresses following moisture saturation (with peak stresses reducing by 54-92%). The behaviour of the stress profiles
is as before: the form of the stress profiles doesn’t change; the stress values themselves are reduced. What is particularly interesting, however, is the fact that the saturated $\sigma_{xx}$ stress profiles are largely identical at Positions 1, 2 and 3 (Figure 6-13(c)). This behaviour could be linked to the two continuous longitudinally orientated plies at the bottom of the laminate, which sets the laminate apart from all the other laminates investigated in this work. These two longitudinal plies still dominate, which (as described before) applies a strain at the interface between the longitudinal and transverse layers. With the relatively small mismatch in straining between the longitudinal and transverse plies (due to the moisture induced swelling), the overall stress state in the transverse layers is very small. This results in a nearly identical stress profile in throughout the longitudinal layers. What is also interesting to note is that at the bottom surface, the longitudinal plies alternate between compressive and tensile stresses (Figure 6-13(c)) in the saturated case only. Any time this behaviour is observed, it appears to occur in the longitudinal plies only (see Laminates 2-4). It appears therefore that the interface between the longitudinal and transverse layers introduces the stress peak in the transverse layers (due to the strain of the stiff longitudinal plies dominating), which introduces a relatively constant tensile stress in the transverse plies. The longitudinal plies then adjust to obtain equilibrium in the through-thickness stresses.
The change in stresses in Laminate 6 shows some differences to earlier cases. As expected, most of the stress profiles displayed a sharp loss in peak stresses (of 65-82%).
However, other profiles did not act as expected. The $\sigma_{xx}$ stress profiles at Position 2 (Figure 6-14(c)) show that after saturation, the stress in the transverse layers reduced to just 3.9-4.3 MPa. The longitudinal layers then adjusted, and (unlike in the dry case) reached a state of tension at the bottom surface. The same stresses at Position 1 (unidirectional lay-up) show a marked decrease in the curvature that was induced into this section. The loss of curvature at Positions 1 and 2 is also shown by the $\sigma_{yy}$ stress profiles (Figure 6-14 (d)), which are responsible for the primary curvature observed in the dry laminate shape. Following saturation, the stress transverse layers at Position 1 never exceeds 1.8MPa, while at Position 2 it never exceeds 3.6 MPa. As a result, the curvatures are reduced substantially in these sections.

At Position 3, both the $\sigma_{xx}$ and the $\sigma_{yy}$ stress profiles display unusual behaviour. Firstly, the $\sigma_{xx}$ stress profile (shown in Figure 6-14(e)) shows that the slope of the stress profile doesn’t change much following saturation. Rather, the stress profile is shifted slightly by a net tensile stress. It is unclear why this happens, but it points at some increased transient stress behaviour along the $x$-axis, as this stress must be cancelled by an equivalent compressive stress in the same plane. The $\sigma_{yy}$ stress profile then behaves differently, as it undergoes both a change in slope, as well as an addition net tensile stress state. In fact, the stress at this position is entirely tensile, increasing from 0.3 MPa to 2.9 MPa. As this section is unidirectional, the stress is caused by the neighbouring partitions. This behaviour would seem to indicate that one of the neighbouring partitions is attempting to stretch along the $y$-axis, and as such is imposing a tensile state on this partition.

### 6.4 Summary

The dry and saturated through-thickness residual stress profiles were extracted. Following saturation, a large drop in peak stresses (often over 70%) was recorded. In most cases, the residual stress profile itself didn’t change, other than the changes in peak stresses. With the tailored laminates, this could be in part due to the dry laminate models being adapted to predict the saturated laminates, resulting in largely identical shape configurations.

The transition from one stress state to another at the lay-up boundary of the tailored laminates was investigated by extracting the stresses one element length (of 2.5mm) to each side of the partition between neighbouring lay-ups. This was done for Laminates 1-3. In the cases of Laminate 1 and Laminate 3 the changes in $\sigma_{xx}$ stresses across the
boundary are almost imperceptible when compared to the ‘steady-state’ stresses, taken from the middle of each partition. It appears, therefore, that the transition zone from one stress state to the next is shorter than one element length. Laminate 2 featured a large change in profiles (both peak stresses and profiles) near the boundary. It was also found that the $\sigma_{yy}$ stresses close to the boundary feature a larger transition zone, with a more significant change in stresses observable when compared to the steady-state stresses. The transition from one lay-up often introduced a net change in stress (either compressive or tensile) along the longitudinal plies.

The effect of the tailored laminate configuration can also be seen in the $\sigma_{xx}$ stresses at the $[0_2/90_2]$ lay-up section which all laminates possess. Even in laminates with the same curvature orientation being formed in this lay-up section, large differences in stresses can be seen (e.g. from $\sigma_{xx} = -114/+75$ MPa in Laminate 1, to $\sigma_{xx} = -95/+63$ MPa in Laminate 3). This shows that the tailored lay-up configuration can significantly impact upon residual stresses, and thus any applications which are sensitive to residual stresses can be affected by this lay-up strategy.

In all cases, the stress profiles appear to be dominated by the longitudinal layers (which can feature large variations in peak stresses or stress profiles), with the transverse layers reacting to the applied strain. This results in a small range of tensile stresses in the transverse layers, from 17-38 MPa. This represents up to 52% of the materials tensile strength in the transverse direction.
Chapter 7  Discussion

7.1 Thermal Expansion and Swelling Coefficients

The developed approach relies on calibrated equivalent values for the materials CTE and MSC. Applied to square cross-ply laminates, a single equivalent \( \alpha_L \) value was able to describe shapes of the cross-ply laminates. This was helped by the fact that the curvatures of each multi-stable shape of the \([90/0]_2\) laminates – which should be equal but weren’t – were averaged, potentially minimising any manufacturing induced contributions to the curvature. A somewhat similar finding was not made for tailored laminates, with a range of \( \alpha_L \) values required to describe the laminate shapes. This was despite the fact that significant effort was made to maintain consistency between the manufactured laminates. For each study (involving both the cross-ply and the tailored laminates) all laminates were manufactured (including cutting, lay-up and curing) together and under the same conditions. Coupled with identical curing conditions (including orientation on the tool-plate), this resulted in consistent laminate shapes being manufactured. This would suggest that imperfections introduced during the lay-up were kept to a minimum.

The multiple CTE values required when modelling tailored laminates is contrary to what is expected after examining the literature, where good correlation between predicted and measured laminate shapes is generally presented. It is worth pointing out that studies in the literature deal with either one, or a small number of different lay-up combinations (e.g. Hyer (1981a), Mattioni et al. (2007), Gigliotti et al. (2007)), against which comparisons of experimental and numerical shapes are done (e.g. square cross-ply laminates only). With the tailored laminates, a large range of lay-ups were considered, making it harder to use a single CTE value to describe all lay-up families. Also, in some instances in the literature, no data is given as to the source of the CTE values used in models, and as such these studies could be employing curve-fitting themselves (e.g. Hyer (1981a), Dano and Hyer (1998), Gigliotti et al. (2004)). Nonetheless, two aspects are considered to be important with regards to multiple CTE values being necessary to describe the tailored laminate shapes. Firstly, the manufacturing process itself. As noted before, the orientation of the laminate on the tool-plate affects the developed curvature, and some ‘spring-out’ is also to be expected.
With the square cross-ply laminates, these manufacturing effects were minimised by averaging the developed curvatures, and so a single CTE value was able to describe the behaviour. For the tailored laminates, however, a more complex curvature is formed, and thus the effects of manufacturing may affect the curvature at each partition differently, leading to difficulties in precisely correlating experimental and numerical results. Including a crude representation of the manufacturing-induced curvature appears to have reduced the range in equivalent $\alpha_L$ values required. However, the variation was not completely eliminated. One possibility to reduce the variation between laminates is treating each tailored partition separately during calibration.

Another possibility for the variation between experimental and numerical shapes is the inherent difference between models and manufactured laminates. These differences come down to construction (fibre-resin composite in practice, idealised to homogenised orthotropic plies in model) and assumptions (imperfect laminate in practice, idealised as being perfect in modelling). The imperfections could take the form of varying volume fraction, varying ply thicknesses, fibre waviness, local debonding/matrix damage and ply misalignment to name a few. In addition, the tailored laminate configuration can lead to other imperfections, such as resin rich areas near the partition boundary (Sousa et al. 2013). With additional experimental data, such as changes in the through-thickness volume fraction or the existence of resin-rich areas at the lay-up partition, the model presented in this study can be easily adapted to consider such aspects.

To this end, some measure of local in-plane strain (by embedded optical-fibre or strain gauges) could be useful. Such a measure could be compared against the developed numerical model to further validate the result, as well as to give an indication of the actual CTE value of the material. However, measuring strains developed during cure can lead to additional complexities. For example, should embedded strain sensors be used, the thermal behaviour and the interaction between the instrument and the surrounding material would need careful consideration, as would any changes in the local stress field caused by the presence of the embedded sensor. Additionally, measuring strains near the partition between different lay-ups using sensors may be challenging, as stress gradients exist over a potentially small distance. Consequently, the use of a non-contact, optical based measurement technique allows for the full-field macro-scale stresses to be predicted in a non-intrusive manner with a relatively simple experimental set-up. It is interesting to note that the equivalent CTE value obtained to
describe the cross-ply laminates is within the range of those obtained for the tailored laminates. The cross-ply laminates required a single CTE value of \(5.5 \times 10^{-6} \, \text{K}^{-1}\), while the tailored laminates required a CTE range of \(3.5-13 \times 10^{-6} \, \text{K}^{-1}\). In fact, the CTE value required in the tailored laminate case compares favourably with the average of the CTEs required to describe the tailored laminate \((6.6 \times 10^{-6} \, \text{K}^{-1})\).

To describe the saturated laminate shapes of both the cross-ply laminates and the tailored laminates, three different values were obtained for the MSC. These were \(3.65 \times 10^{-3} \, \text{wt.\%}^{-1}\) for the cross-ply laminates and either \(2 \times 10^{-3} \, \text{wt.\%}^{-1}\) (Laminate 2) or \(3.8 \times 10^{-3} \, \text{wt.\%}^{-1}\) (Laminates 1, and Laminates 3-6) for the tailored laminates. The range required for the majority of the tailored laminates correlates well with the swelling coefficient obtained for the cross-ply laminates, even though the test conditions were different (elevated water temperature for the tailored laminates). This gives some confidence that the approach developed to obtain an equivalent MSC for modelling can be applied to other families of laminates to predict their behaviour, which may be useful during the design of multi-stable structures. However, further analysis is required to verify this point, as Laminate 2 (tailored laminate) needed a particularly low MSC value, and the majority of the other laminates had their MSC values calibrated to within a range.

The approach for predicting saturated laminate behaviour depends on two values: the moisture content and the swelling coefficient. From this, the moisture straining is calculated. When applied to tailored laminates, a single value for the moisture content leads to the assumption that moisture ingress is equal amongst all the lay-up partitions. As seen in the cross-ply case, laminates of different lay-ups may feature different saturated moisture contents. It is possible, therefore, that there is unequal moisture induced swelling in each of the lay-up partitions. One possible reason for this behaviour is stress dependent moisture diffusion, which is an area which requires additional research effort to determine its influence, if any, on the moisture diffusion and saturated moisture content on composite laminates. In the context of the work presented here, the shape comparison technique may be used to see whether or not calibrating the moisture induced straining in each lay-up partition separately leads to any clues as to the moisture induced behaviour of the laminates. Also, edge effects were ignored during moisture tests. Laminates 2, 5 and 6 in the tailored family featured a change in thickness along their lengths. This change in thickness may provide a locally resin rich area where
the new ply is introduced, as well as an additional edge for moisture ingress to occur (and thus open to capillary diffusion along the fibre interface, see Comyn (1985)). Though ignored due to the thin-plate nature of the laminates investigated, moisture ingress at the laminate edges can take on a different form as moisture permeates along the fibre length, for instance. When a change in thickness along the laminate length is considered, this may lead to locally different moisture swelling at regions well within the laminate boundary (and thus away from outer-edge effects). This is another area in which experiments and modelling differ, and could potentially lead to a lack of correlation between the two.

It should also be noted that an element of human interpretation is present in the calibration process of the equivalent CTE and MSC values. The variation between experimental and numerical models was minimized in an iterative fashion, with the result from an initial CTE value being used to obtain an estimate for a subsequent model. The variation was judged based on the peak values for each model, as well as the area of the laminate that was over/under predicted which is open to interpretation. To remove this subjective interpretation, it is suggested that future studies include a calculation of the root-mean-square values as part of the calibration technique. As the data processing and calculations to produce the variation plots are done through Matlab, including such a calculation should be relatively straightforward.

Finally, it must be also be stressed that the calibrated thermal expansion and moisture induced swelling coefficients used during the analysis should always be considered as equivalent, and not true absolute values. Although the possibility of using unsymmetrical laminates to determine CTEs and swelling coefficients has been noted (by Gigliotti et al. (2007), who investigated one lay-up configuration), in this study a range of laminates with different lay-ups were used, resulting in a range of CTE values being required to describe the deformation behaviour of all the laminates. The margin required could be reduced by controlling the manufacturing process further to eliminate laminate imperfection (e.g. eliminating unbalanced tool plate interactions by curing the laminates with a plate on its top surface). However, it is clear following this study that when analysing thin unsymmetrical laminates, manufacturing effects and/or imperfections can lead to significant differences between measured and predicted laminate shapes. As through-thickness residual stresses were of interest in this study, the experimentally observed laminate shapes had to be reproduced as closely as
Discussion

possible, and thus a range of CTE values was required. On top of that, it must be remembered that the exact residual stress state (in both dry and saturated conditions) is not reproduced. The use of homogenised orthotropic material properties aids in the simplicity of the approach, but neglects any fibre-resin interaction (i.e. only macro-scale stresses are predicted).

### 7.2 Modelling and Experimental Approach

The objective of the numerical model approach presented was to analyse moisture ingress effects on multi-stable composites applied to unsymmetrical laminates in structures (e.g. morphing structures). As such, flexibility has been implemented into the model developed to allow for the study of a wider range of laminate geometries and lay-ups. The use of FE models allows for the analysis of complex laminate geometries and lay-ups following moisture saturation. While analytical approaches have evolved to be able to predict and describe the behaviour of complex laminates (Mattioni et al. 2009), they don’t retain the flexibility of numerical models when applied to the design of a morphing structure. As such, a three-dimensional model, comprising solid elements (as opposed to shell elements), allows for detailed analysis of morphing structures. For example, the effects of additional mechanical loads, mechanical fastening, damage and so forth can be analysed using this modelling approach; all important factors during the design of morphing structures using multi-stable laminates.

It was found that special consideration is needed when analysing the force resultants of the laminates, which can be calculated by summing the areas encapsulated by the residual stress profiles. These should (to obey force equilibrium requirements) be sum to zero. Calculating the force resultants from the stress profiles shows that in some instances, a degree of imbalance is present, which may be due to discretization errors in Abaqus. However, summing the nodal forces across cross-sections shows that equilibrium is in fact maintained. Therefore, any imbalance at a point is countered by a corresponding imbalance along the cross-section of the laminate.

When applied to tailored laminates, the approach developed was used to predict the different through-thickness stress profiles at different lay-up partitions, which can be applied to analyse the residual stresses in morphing applications at a structural level. But the numerical model used is relatively simple and could allow the analysis of more
general composite structural situations which are sensitive to residual stresses (e.g. co-cured composite laminate assemblies, laminates with ply-drop conditions etc.). However, the laminates investigated in this study feature very high length/width-to-thickness ratios, and as such can be considered to be thin-plates. Solid continuum elements can be overly stiff in these instances (due, for example, to shear-locking effects). Indeed, when linear elements (C3D8R) were used during the analysis of the tailored laminates, the solution was found to be overly stiff for Laminates 4 and 5, and would not accurately predict the change in curvature along the length of the laminate. As such, an effort needs to be made to reduce the element aspect ratio as much as possible, while keeping computational effort down. This may lead to a computationally expensive analysis being required to predict laminate shapes. The high mesh density is useful in providing detail (e.g. many integration points from which to extract through-thickness stresses), but adds considerably to the time and computational costs required (particularly at the calibration stage). With this in mind, it is recommended that when a continuum model is best suited to the analysis, a shell element based model is used as well during the calibration phase. These models are substantially less computationally expensive and can expedite considerably the calibration process. For this to be effective, however, a comparison of continuum and shell based modelling approaches (applied to multi-stable laminates) is required.

It must be noted that the process of predicting saturated laminate shapes through numerical methods is not trivial. Firstly, the experimentally observed behaviour – whereby a laminate loses its multi-stable behaviour and becomes mono-stable – was not readily observable in numerical models. With square cross-ply laminates, the shape following saturation was still cylindrical (not saddle shaped, as could be expected for a mono-stable laminate with such a composition). In modelling terms, the existence of a cylindrical shape with a square [90\textdegree/0\textdegree] laminate implies multi-stability, as both multi-stable shapes are equal and opposite geometrically and in terms of potential energy. It is therefore possible that an additional ‘piece of the puzzle’ needs to be taken into account, on top of moisture swelling/moisture concentration approaches. This could be, for example, stress dependent moisture absorption. As the laminates in this study were subjected to moisture uptake in one of its multi-stable shapes (and thus in one particular stress configuration, it is possible that the shape configuration used during moisture uptake can alter the moisture affects.
In the tailored laminate configurations, additional difficulties came into play. As before, only a single laminate shape was observed following saturation (i.e. the multi-stable property disappeared), and yet, in some cases, both multi-stable shape configurations were obtainable during modelling of the saturated shapes. Also, some laminates appeared to have deviated from original multi-stable shapes. In this study, Laminates 2, 4 and 6 all showed saturated shapes which appear different to those originally attainable, with, for instance, a heavy twisting being observed. The reason for this is unknown. All the laminates used in this study were particularly thin (maximum 0.625 mm thick), and thus are particularly susceptible to manufacturing imperfections. It is possible therefore that thicker laminates, which would most probably be used in practice, would not be so sensitive to moisture effects as to change shape configurations. Likewise, it is unlikely that laminates used in morphing applications would be subjected to the moisture concentrations reached in this study, as they would not be immersed in water for extended periods of time. This is, however, an observation that can significantly alter the applicability of a certain lay-up configuration to moisture effects and thus deserves attention.

Another difficulty in predicting saturated laminate shapes is that the stress state is reduced drastically from that of the dry laminate state, and often becomes close to zero. As such, other stresses, which were largely insignificant when the high residual stresses of dry laminates were considered, become increasingly dominant. A numerical/analytical model, which considers only orthotropic CTE/moisture induced macro-scale residual stresses, becomes less capable in predicting the through-thickness stress distribution. As such, it may be worthwhile to consider that models taking moisture effects into account may predict the change in residual stresses more accurately than the final stress state of the laminate. In terms of studying the multi-stable behaviour of laminates, this may be sufficient, as the change in potential energy state is of particular interest.

The modelling approach developed has maintained a level of flexibility, with the specific purpose of allowing for the analysis of transient diffusion states. In reality, multi-stable laminates in morphing applications would rarely reach a fully saturated state with the moisture contents achieved in this study. It is logical therefore that a transient moisture case is of more interest. In particular, it is known from the experimental results of this study that the multi-stable behaviour can disappear.
following moisture absorption. From experience, it has been noticed that this loss of multi-stability can occur following absorption from the atmosphere, in conditions less aggressive as what could be encountered under in-service conditions. This is an important factor that must be considered before applying multi-stable laminates to morphing structures.

Such an analysis could be done numerically using an identical model to that presented here, but with elements that include a temperature degree of freedom (e.g. C3D20RT). Following the simulated cure, a transient analysis – this time using the analogy between thermal conduction and moisture diffusion – can be carried out. Subsequently, a stability analysis could be undertaken whereby the loss of multi-stability during a transient moisture absorption stage could be analysed. This is particularly important as the loss of multi-stability was not reproduced in this study. Based on observation of the saturated laminate shape, the dry multi-stable shape most closely resembling the saturated shape was chosen as a basis for the saturated model. Therefore, the single saturated shape of the laminates needs to be known a priori, and can’t, using the current approach, be predicted. In this instance, an analytical approach (such as that undertaken by Eckstein et al. (2013)) can be explored to help determine the possible multi-stable shape configurations. Out of interest, it was noted that for all laminates whose multi-stable shapes were considered (Laminates 1-3, and 5) the saturated shape was always most closely related to Shape B of each laminate, with the curvature being generated along the laminate’s longitudinal axis. This shape corresponds with the highest potential energy state, and could be used to give an indication as to what the saturated multi-stable shape will be.

This point also leads on to another important consideration. In this study, it was assumed that the saturated laminate shape is related to one of the multi-stable shapes, and as such the dry numerical models were adapted to account for moisture effects. It was seen in experiments that some laminates displayed unexpected behaviour, such as twist curvatures, which made comparisons of experimental and numerical shapes difficult. The cause of this unexpected behaviour is unknown, and as such, the possibility that the saturated laminate shape is not completely related to the dry laminate shape should not be ruled out. To gain further insight into this behaviour, some further experimental research is required on the dry and saturated shapes of tailored laminates. Unfortunately, the lay-ups used in this study didn’t lend themselves to quantifying the
change in shape due to moisture saturation, as the saturated shape was related to the multi-stable shape with an already low out-of-plane deflection. Therefore, only a minimal change in out-of-plane deflection was recorded following saturation. Therefore, should such an analysis be conducted, it could be beneficial to use multi-stable lay-ups and geometries chosen to maximise the loss of curvature following saturation for both multi-stable shape configurations. This would allow for a better understanding of the changes in shape due to moisture absorption, and possibly confirming whether or not the saturated shape is related to one of the dry multi-stable shapes.

The possible changes in shape (and thus stress state) following diffusion may be related to a number of factors. Firstly, it has been suggested that diffusion is a residual stress dependent parameter (Youssef et al. (2009), Comyn (1985)), and so the rate of diffusion through the laminate will be influenced by the residual stress state. This tri-axial stress state varies across different length scales and as a function of stacking sequence, and could lead to moisture ingress not being mirrored about the laminate mid-plane – possibly affecting the saturated laminate shapes and multi-stable behaviour.

The difficulty with such an exercise remains in comparing experimental and numerical results. From experiments, a diffusion coefficient can be estimated (e.g. using a Fickian based analysis). This would allow for the comparison of experimental and numerical results as a function of time. For example, after five days, it is known that the laminate moisture content reached a certain value, and using the estimated diffusion coefficient, it is estimated that the moisture had penetrated a certain amount into the laminate thickness. However, no direct measure of the penetration is made, for a direct comparison against numerical simulations. This could lead to misleading results (e.g. regarding the loss of multi-stability).

7.3 Sensitivity of Residual Stresses to Modelling Parameters
Comparisons between numerical and experimentally-measured shapes showed that a degree of variation between the two will always be evident, regardless of calibration. This can be due, in part, to differences between the numerical model and the manufactured laminate. In general, most of the material properties required for numerical modelling (Young’s modulus, Poisson’s ratio etc.) are readily available from manufacturer’s data sheets or experiments. The equivalent αL value, however, is calibrated as part of this approach to match the experimentally observed laminate
shapes. In addition, with hand lay-up techniques being used in the manufacture of the laminates, a degree of ply misalignment can be expected. Both these parameters ($\alpha_L$ and fibre alignment) may influence the extracted residual stress profiles. As such, the sensitivity of residual stresses to changes in the $\alpha_L$ value and fibre orientation was explored. This was done by using the model of Laminate 1 from the tailored group as a baseline (initial model, with a flat stress-free shape) and altering the value of $\alpha_L$ used, as well as introducing a ply misalignment. The changes to the residual stresses caused by these altered parameters could then be assessed.

In the case of $\alpha_L$, two separate simulations were carried out. Firstly, the value of $\alpha_L$ used was reduced from $6.5 \times 10^{-6}$ to $6.0 \times 10^{-6}$. This increased the difference between $\alpha_T$ and $\alpha_L$, resulting in greater residual stresses. The change in residual stresses was relatively small, with the largest change observed being a 2.5 MPa increase in the peak $\sigma_{xx}$ value in the $[90_2/0_2]$ partition. Secondly, both the values of $\alpha_L$ and $\alpha_T$ were altered, with the difference between the two ($\alpha_T - \alpha_L$) being kept identical to that of the baseline case. The revised values were $6.0 \times 10^{-6}$ for $\alpha_L$ and $2.81 \times 10^{-5}$ for $\alpha_T$. Comparisons against the baseline case showed that there were no changes in the extracted stress profiles. It would appear, therefore, that the difference between the two values is particularly critical when using this approach to extract through-thickness residual stresses.

To take fibre misalignment into account, a third simulation was carried out where a $+5^\circ$ ply misalignment was introduced into the longitudinally oriented plies (to produce a $[90_3/5_1], [90_2/5_2]$ tailored laminate). The misalignment is in the region of what may be expected with hand lay-up techniques (Ochiner and Hyer 2002). The change in residual stresses caused by the misalignment was low, in the order of 1 MPa. However, the variation in shape showed the significance of ply imperfections, with the variation going from $+14.5\%/-6.7\%$ in the baseline case, to $+9.4\%/-11.2\%$ in the misaligned ply case. As the variation between shapes is used as a calibration parameter, such an imperfection in the manufactured laminate could result in further calibration in the equivalent $\alpha_L$ value, which would, as shown previously, result in different stress profiles being extracted.

7.4 Predicting the Transient Response using a Piecewise Approach

Investigating the response of unsymmetric laminates during the diffusion process (i.e. before saturation is reached) is challenging due to the complexities of the diffusion process. In order to hypothesize the laminate behaviour during the diffusion process, a
diffusion model was created, whereby the top and bottom layers of the laminate were modelled as having saturated properties (\textit{i.e.} saturated material properties and fully swollen), while the innermost layers were modelled as having ‘dry’ properties. This piecewise approach has obvious limitations, due to the fact that there is no transient diffusion front used. Nonetheless, this method may be of use in gaining an insight into the changes that occur during the diffusion process. The resulting through-thickness stress profiles are shown as dotted lines in Figure 7-1(a-c) for [90_2/0_2], [90_3/0_1] and [90_3/0_2] lay-ups respectively.
Figure 7-1 Predicted residual stress profiles for [90/0₂], [90/0₁] and [90/90₂] square laminates using piecewise diffusion (top and bottom plies saturated). Values refer to the ‘diffusion’ stress profile (all stresses are in MPa).

For comparison, the stress profiles for the dry and saturated cases are also presented, and are identical to before. The effect of the discontinuous change from ‘dry’ laminate conditions at the inner layers to the ‘saturated’ laminate condition at the outer layers is seen by the stress response at the interfaces. It is interesting to note that in all cases
(with the exception of $\sigma_{yy}$ in Figure 7-1(b)) an increase in peak stresses is predicted for the transverse layers when compared to the stress profile of the dry laminate. As the material properties of the longitudinal layers remain unchanged following moisture absorption, the thermal strain imposed by these fibres remains largely unchanged. As the properties of the bottom-most transverse layers (which are modelled as being saturated) are now degraded, the inner transverse layers are now forced to absorb additional stresses to maintain equilibrium. For the $\sigma_{yy}$ profile in Figure 7-1(b) no substantial increase in transverse stresses are observed. Along this axis, there is only one transverse layer, which is now fully saturated. Therefore, it cannot absorb higher stresses and the stress profile remains very similar to that of the saturated laminate case. The stress profiles in the longitudinal layers (plies 2, 3 and 4) readjust to balance the stresses in the rest of the laminate. The ‘step’ observed in the stresses when transitioning from ply 3 to ply 4 appear to be due to anticlastic effects, due to the compressive $\sigma_{xx}$ stresses induced in ply 4. This trend is repeated in Figure 7-1(a),(c).

In order to provide an insight into the diffusion process, the deflections obtained for the laminates in Figure 7-1 were compared against experimental data collected during the moisture absorption process of these families of laminates. It was noted in experiments that the loss of curvature is a predominantly linear function of the moisture absorbed. Therefore, for a 4 ply laminate, one expects that after the top and bottom plies are saturated (i.e. half the laminate is saturated), the laminate’s deflection would correspond to that experimentally observed half way through the diffusion process. This premise held true for only the [90/2/0] laminate. This may be due to the fact that diffusion is a stress dependent process, and thus is not equal through the bottom and top surfaces (whose differences in stress states are evident in the through-thickness stress distributions). Of the three families of laminates tested, the [90/2/0] had the most similar stress state for both $\sigma_{xx}$ and $\sigma_{yy}$, thus perhaps leading to this piecewise approach correctly predicting the deflection after 50% saturation.
Chapter 8  Conclusions and Future Work

8.1  Conclusions

An approach was presented whereby the macro-scale through-thickness residual stress states of unsymmetrical multi-stable composite laminates can be obtained. The approach uses experimentally measured laminate shapes in both dry and saturated conditions to calibrate numerical models, from which residual stresses can be extracted.

The modelling approach used required a single equivalent CTE value ($\alpha_L$) of $5.5 \times 10^{-6}$ $K^{-1}$ to describe the curvatures of the three families of cross-ply laminates investigated, to within ±10%. In the case of the tailored laminates, a range of equivalent $\alpha_L$ values was required. Inclusion of an observed processing induced curvature reduced the range required to $3.5 - 13 \times 10^{-6}$ $K^{-1}$, for all laminates and multi-stable configurations. However, full-field comparisons between the numerically-predicted and experimentally-measured shapes revealed that inclusion of the process induced curvature did not eliminate the variation between the two. Probable causes for the range include manufacturing imperfections and the assumptions associated with the modelling approach (e.g. only macro-scale stresses being considered). Additionally, an element of human interpretation is present during the calibration process, and so a more robust shape comparison technique, such as root mean square values, could be used. This range of equivalent CTE values demonstrates the difficulty in accurately predicting the shapes.

The changes in the macro-scale residual stresses following moisture saturation of cross-ply and tailored laminates were presented. The residual stress profiles of dry composite laminates showed that the peak $\sigma_{xx}$ in the transverse layers varied by a small amount between different lay-up configurations (ranging from 17 MPa to 38 MPa). As a result, the stresses in the longitudinal layer adjust, resulting in large differences in peak stresses between laminates. For example, in the cross-ply laminates the peak stresses in the longitudinal layers ranged from -82.5 to -105 MPa (minimum) and 24 to 67 MPa (maximum).

The tailored laminate configuration was shown to affect the residual stresses developed. All laminates featured sections with a $[0_2/90_2]$ (or a $[90_2/0_2]$) lay-up, which, depending on the neighbouring partition, featured peak stresses in the dry state ranging from $\sigma_{xx} =$ -
Conclusions and Future Work

114/+75 MPa in Laminate 1 to $\sigma_{xx} = -95/+63$ MPa in Laminate 3. In most cases, the zone in which the $\sigma_{xx}$ profiles transitioned from one lay-up to the next was less than one element length (of 2.5 mm) either side of the lay-up partition. The $\sigma_{yy}$ profiles extracted near the lay-up partition showed that transition length is longer than 2.5 mm. This was prevalent in laminates that featured a curvature generated along their rectangular lengths, and so was caused by the curvature of neighbouring regions directly impacting upon residual stresses.

A study of the sensitivity of the extracted residual stress profiles and the variation (using Laminate 1 from the tailored laminate series) between models and experimentally measured shapes showed that the peak residual stresses increased by a small amount (maximum of 2.5 MPa increase) following a $0.5 \times 10^{-6}$ K$^{-1}$ reduction in the $\alpha_L$ value used. Altering both the $\alpha_L$ and $\alpha_T$ value used in modelling, keeping the difference between the two identical resulted in no change in residual stresses. This leads to the conclusion that it is the difference between the values, and not the absolute values used, that is important when extracting residual stresses. Finally, a 5° ply misalignment (representative of possible misalignments during hand lay-up) in the longitudinally orientated plies showed a change in residual stresses of less than 1 MPa, but changed the variation between the experimentally-measured and numerically-predicted shapes from $+14.5\%/-6.7\%$ to $+9.4\%/-11.2\%$. This could lead to further calibration, with different calibrated CTE values being obtained, which could have a small impact on residual stresses.

It was found that peak residual stresses were reduced by over 70% following saturation. This led to a reduction in the total strain energy of the laminates of over 94%. The changes in residual stress (and thus curvature) and the strain energy could have a significant impact on a multi-stable laminate’s suitability as part of a morphing structure.

The laminates studied were immersed in water until saturation, obtaining a moisture concentration of between 0.69 and 0.83 wt.%. Reaching saturation may take a long time under regular operating conditions (e.g. through humidity in the air) and thus the behaviour of multi-stable laminates under diffusion conditions needs to be considered, to quantify the effects on multi-stability under transient moisture absorption conditions.
For the cross-ply laminates a MSC value of 3.65x10^{-3} wt.%^{-1} was capable of describing all the saturated laminate curvatures within a range of -16/+14%. Using this MSC value, a piecewise diffusion case was hypothesised where the top and bottom plies were modelled as being saturated. This showed that peak stresses in the transversely orientated plies may increase during moisture diffusion.

With the tailored laminates, difficulties such as twisting following saturation resulted in difficulty in calibrating a MSC value for each laminate. Consequently, a swelling coefficient of 3.8x10^{-3} wt.%^{-1}, obtained following calibration of the model for Laminate 1, was applied to Laminates 3-6. This MSC was known to be within the range required for these laminates. Laminate 2 required an MSC of 2x10^{-3} wt.%^{-1}. The range in MSC values could be due, in part, to differences in the moisture uptake behaviour between laminates (local moisture content, damage, etc.). The computation of the calibrated MSC value uses the calibrated value for $\alpha_L$ as an input, and so is subject to the variation in shapes between experimental measured and numerically predicted shapes in the dry condition. The calibrated CTE and MSC values obtained from the cross-ply laminates are within the range of those required for the tailored laminates. As such, the possibility of using the values obtained from simple cross-ply laminates and applying them to more complex tailored laminates remains open, with the caveat that further refinement of the equivalent CTE values used would lead to increased accuracy in the extracted shapes and residual stress profiles.

The modelling approach showed that the flexural stiffness of the models becomes increasingly important when complex laminates are being considered. The cross-ply laminate models used linear elements (type C3D8R) with an aspect ratio of 100:1, and were benchmarked against the existing analytical approach. To reproduce the multi-stable shapes of the tailored laminates required non-linear elements (type C3D20R), along with a reduced element aspect ratio of 60:1.

The combined experimental/numerical approach presented is useful in extracting residual stress profiles and understanding saturated laminate behaviour. One limitation is that knowledge of the stable shape configurations needs to be known a priori. This is particularly relevant for the saturated laminate shapes, as experimentally one of the multi-stable shape configurations was not attainable, while in numerical models both shape configurations were occasionally obtainable. The use of analytical models which
Conclusions and Future Work

use the minimization of potential to predict multi-stable shape configurations is useful in this regard.

8.2 Recommendations for Future Work

The ultimate goal of researching moisture absorption on multi-stable composites is to understand its effects under in-service conditions. As it is known from observations in this study that the multi-stable property can disappear following moisture saturation, a transient analysis of moisture diffusion in multi-stable composites is deemed to be a very important area for future investigation. Saturated moisture contents (as was investigated in this work) may not be achieved in the majority of morphing applications proposed. However, moisture absorption through the atmosphere is very likely to occur and is difficult to avoid. Therefore, the shapes and multi-stable property of such laminates can be altered under in-service conditions, leading to certain laminate configurations not being suitable for certain applications. Such a study is however very complex and requires further knowledge on the moisture effects on multi-stable composites. A step-by-step approach is proposed, consisting of the following elements.

The shape configuration attained by multi-stable laminates needs to be understood in more detail. With the laminates studies in this research work, the saturated tailored laminate shapes occasionally displayed twisting or curvatures which differed from the initial ‘dry’ multi-stable shapes. This could be due to an additional effect that hasn’t been considered, such as laminate-level imperfections dominating the now low residual stress state, or it could be due to the laminate itself obtaining a new shape configuration following saturation. The implications of this are significant when numerically analysing multi-stable shapes as (with the methods used in this study) the saturated laminate shape needs to be known in advance to coax the model into the correct shape. To answer this, it is recommended that some similar tailored laminates to the ones used in this study are manufactured, but with larger dimensions. This is to better capture the change in out-of-plane deflections following moisture saturation. The multi-stable property should be maintained so as to maintain the minimum-potential-energy aspect, which can be used to determine if the saturated shape is of the same configuration as one of the two multi-stable shapes, or whether a new third configuration is obtained. Understanding this point will give further clarity for a transient moisture diffusion analysis, as the dry and saturated shapes are better defined. Subsequently, a simulation of the multi-stable property following saturation could be carried out. One method used
Conclusions and Future Work

Previously to explore the multi-stable property in multi-stable laminates is to apply concentrated forces at certain laminate locations, to see if the laminate ‘snaps-through’ to the alternative shape. A similar approach could be applied to the numerical model of the saturated laminate, to see whether it (incorrectly) snaps through to an alternate multi-stable shape configuration; one which is not obtainable through experiments. As it was possible in this study to obtain both multi-stable shapes in the saturated case, it is possible that the numerical model will incorrectly predict this behaviour. The result of this study could be benchmarked against the analytical solution suggested by Eckstein et al. (2013).

Due to the differences between the predicted and experimentally measured laminate shapes, any subsequent comparison of the saturated laminate shapes must take the initial variations in the dry shape configuration into account. This is particularly relevant in the study presented here, which uses the equivalent dry CTE values as a parameter during the prediction of saturated laminate shapes. As such, removing the dry laminate variance would benefit the analysis by isolating moisture swelling as the only contributor to the saturated shape. This could, perhaps, be done by using experimental data (in the form of point clouds of the dry laminate shapes from laser scanning) to reconstruct the dry laminate shapes in FE models. From there, the moisture swelling effects can be added and the predicted saturated shape compared directly to that measured experimentally. This would be particularly useful when studying the change in shapes due to transient diffusion, as the small changes in laminate shapes could be compared without needing to consider the initial variation in the dry laminate shape.

Subsequently, transient effects may be implemented into FE models by further use of the analogy between moisture diffusion/swelling and heat transfer/thermal expansion. Elements with a temperature degree of freedom (e.g. C3D20RT) could be used in the model to simulate heat transfer (moisture diffusion). The rate of diffusion can be obtained from experiments by measuring the slope of the mass-uptake/time graph, as described by Shen and Springer (1976), while a calibrated value for the moisture swelling coefficient can be obtained as in this study. It must be noted that this ignores micro-scale residual stresses as well as stress dependent diffusion. However, the analysis must start somewhere, and the loss of multi-stability (if predicted) can be compared against experiments. In addition, simulations similar to in-service conditions could be carried out, whereby the two surfaces of the multi-stable laminates experience
different rates of absorption, similar, say, to what a morphing wind-turbine blade using multi-stable laminates as part of the blade skin could experience. The unequal diffusion will lead to additional asymmetry in the laminate, and thus further effect multi-stable shapes and residual stresses. This is also an important consideration for mechanically induced multi-stable laminates, which use symmetrical lay-ups with some plies pre-loaded to introduce residual stresses and multi-stability.

With regard to the modelling approach, two recommendations are suggested. Firstly, in the interests of reducing the computational time during the calibration process, a comparison shell and continuum based models could be conducted. Should the shell approach be able to predict identical curvatures to that of continuum based models, shell based models can be used to quickly calibrate the equivalent CTE/MSC values required in this study. This would be a substantial saving in time over continuum based models, which require a large number of elements due to the requirement to have a low element aspect ratio. Secondly, to remove human interpretation during the calibration process, it is suggested that the total variation between predicted and measured laminate shapes be measured mathematically, for example, by using root mean square values. This method is often used in the comparisons between predicted and measured profiles, and would give rise to a more robust shape comparison technique.
References


References


References


Appendix A: Matlab Code for Multi-stable Shape Calculator

The analytical tool developed to predict the cured shapes of shapes of square cross-ply unsymmetrical laminates is presented. This theory presented in Hyer (1981a) is used and implemented in the Matlab script (“curvcalc.m”).

Table A - 1 Description of variables used in analytical tool (“curvcalc.m”).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Laminate curvature, x-axis</td>
</tr>
<tr>
<td>A</td>
<td>A Matrix terms (CLT)</td>
</tr>
<tr>
<td>alphax</td>
<td>Material’s longitudinal expansion coefficient</td>
</tr>
<tr>
<td>alphay</td>
<td>Material’s transverse expansion coefficient</td>
</tr>
<tr>
<td>b</td>
<td>Laminate curvature, y-axis</td>
</tr>
<tr>
<td>B</td>
<td>B Matrix terms (CLT)</td>
</tr>
<tr>
<td>c</td>
<td>Variable associated with solution</td>
</tr>
<tr>
<td>Ci (i=1-9)</td>
<td>Equation defined in Hyer (1981a)</td>
</tr>
<tr>
<td>d</td>
<td>Variable associated with solution</td>
</tr>
<tr>
<td>dT</td>
<td>Difference between cure and room temperatures</td>
</tr>
<tr>
<td>D</td>
<td>D Matrix terms (CLT)</td>
</tr>
<tr>
<td>E1</td>
<td>Material’s longitudinal Young’s modulus</td>
</tr>
<tr>
<td>E2</td>
<td>Material’s transverse Young’s modulus</td>
</tr>
<tr>
<td>f1</td>
<td>Equation defined in Hyer (1981a)</td>
</tr>
<tr>
<td>f2</td>
<td>Equation defined in Hyer (1981a)</td>
</tr>
<tr>
<td>f3</td>
<td>Equation defined in Hyer (1981a)</td>
</tr>
<tr>
<td>f4</td>
<td>Equation defined in Hyer (1981a)</td>
</tr>
<tr>
<td>G12</td>
<td>Material’s shear modulus</td>
</tr>
<tr>
<td>int</td>
<td>Parameter to set spacing of point cloud to describe laminate shape</td>
</tr>
<tr>
<td>Lx</td>
<td>Laminate side length</td>
</tr>
<tr>
<td>Mx</td>
<td>Moment resultant, x-axis</td>
</tr>
<tr>
<td>My</td>
<td>Moment resultant, y-axis</td>
</tr>
<tr>
<td>Nx</td>
<td>Axial force resultant, x-axis</td>
</tr>
<tr>
<td>Ny</td>
<td>Axial force resultant, y-axis</td>
</tr>
</tbody>
</table>
### Table A - 1, continued

<table>
<thead>
<tr>
<th>Q</th>
<th>Reduced stiffness matrix terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qb</td>
<td>Transformed stiffness matrix terms</td>
</tr>
<tr>
<td>t</td>
<td>Ply thickness</td>
</tr>
<tr>
<td>V12</td>
<td>Poisson’s ratio of material</td>
</tr>
</tbody>
</table>

```matlab
dT= alphax= alphay= Lx= Ly=Lx

E1 = E2 = v12 = v21 = (v12*E2)/E1
G12 =teta = [0 0 90 90]
i=length(teta);
t= T=t*i;
int= %sets spacing in point cloud

Q11 = E1/(1-(v12*v21));
Q22 = E2/(1-(v12*v21));
Q12 = (v12*E2)/(1-v12*v21);
Q66 = G12;

Q=[Q11 Q12 0; Q12 Q22 0; 0 0 Q66]
compliance=inv(Q);

for n = 1:i;
    teta1=teta(1,n);
    Qb11(n) = Q11* (cosd(teta1)^4)+(2*Q12+4*Q66) *(sind(teta1)^2*(cosd(teta1))^2)+Q22*(sind(teta1)^4);
    Qb12(n) = (Q11+Q22-4*Q66)*((sind(teta1))^2)*((cosd(teta1))^2)+Q12*(sind(teta1)^4+cosd(teta1)^4);
    Qb22(n) = Q11* ((sind(teta1))^4)+(2*Q12+4*Q66)*((sind(teta1))^2)*((cosd(teta1))^2)+Q22* ((sind(teta1))^4);
    Qb16(n) = (Q11-Q22-2*Q66)*((sind(teta1))^3)+(Q12-Q22+2*Q66)*((sind(teta1))^3)*((cosd(teta1))^4);
    Qb26(n) = (Q11-Q22-2*Q66)*((sind(teta1))^3)*((cosd(teta1)))+(Q12-Q22+2*Q66)*((sind(teta1))^3)*((cosd(teta1))^3);
    Qb66(n) = (Q11+Q22-2*Q12-2*Q66)*((sind(teta1))^2)*((cosd(teta1))^2)+Q66* (sind(teta1)^4+cosd(teta1)^4);
```

---

_A-2 | Page_
Qb(:,:,n)=[Qb11(n) Qb12(n) Qb16(n); Qb12(n) Qb22(n) Qb26(n); Qb16(n) Qb26(n) Qb66(n)]
end

z= (-T/2:t:T/2);

for n=1:i
    a11(n) = (Qb11(n)*(z(n+1)-z(n)));
    a12(n) = (Qb12(n)*(z(n+1)-z(n)));
    a16(n) = (Qb16(n)*(z(n+1)-z(n)));
    a22(n) = (Qb22(n)*(z(n+1)-z(n)));
    a26(n) = (Qb26(n)*(z(n+1)-z(n)));
    a66(n) = (Qb66(n)*(z(n+1)-z(n)));

    b11(n) = (Qb11(n)*(z(n+1)^2-z(n)^2));
    b12(n) = (Qb12(n)*(z(n+1)^2-z(n)^2));
    b16(n) = (Qb16(n)*(z(n+1)^2-z(n)^2));
    b22(n) = (Qb22(n)*(z(n+1)^2-z(n)^2));
    b26(n) = (Qb26(n)*(z(n+1)^2-z(n)^2));
    b66(n) = (Qb66(n)*(z(n+1)^2-z(n)^2));

    d11(n) = (Qb11(n)*(z(n+1)^3-z(n)^3)); % Note: same notation as used in
dano matlab calculator (initialisation.m)
    d12(n) = (Qb12(n)*(z(n+1)^3-z(n)^3));
    d16(n) = (Qb16(n)*(z(n+1)^3-z(n)^3));
    d22(n) = (Qb22(n)*(z(n+1)^3-z(n)^3));
    d26(n) = (Qb26(n)*(z(n+1)^3-z(n)^3));
    d66(n) = (Qb66(n)*(z(n+1)^3-z(n)^3));
end

A11 = sum(a11);
A12 = sum(a12);
A16 = sum(a16);
A22 = sum(a22);
A26 = sum(a26);
A66 = sum(a66);

B11 = 0.5*sum(b11);
B12 = 0.5*sum(b12);
B16 = 0.5*sum(b16);
B22 = 0.5*sum(b22);
B26 = 0.5*sum(b26);
B66 = 0.5*sum(b66);

D11 = (1/3)*(sum(d11));
D12 = (1/3)*(sum(d12));
D16 = (1/3)*(sum(d16));
D22 = (1/3)*(sum(d22));
D26 = (1/3)*(sum(d26));
D66 = (1/3)*(sum(d66));

A = [A11 A12 A16; A12 A22 A26; A16 A26 A66]
B = [B11 B12 B16; B12 B22 B26; B16 B26 B66]
D = [D11 D12 D16; D12 D22 D26; D16 D26 D66]

for n=1:i; %i=number of plies
Appendix A: Matlab Code for Multi-stable Shape Calculator

\[
\begin{align*}
Nx(n) & = dT \cdot ((Qb11(n) \cdot \alpha x \cdot z(n+1)+Qb12(n) \cdot \alpha y \cdot z(n+1)) - (Qb11(n) \cdot \alpha x \cdot z(n)+Qb12(n) \cdot \alpha y \cdot z(n))); \\
Ny(n) & = dT \cdot ((Qb12(n) \cdot \alpha x \cdot z(n+1)+Qb22(n) \cdot \alpha y \cdot z(n+1)) - (Qb12(n) \cdot \alpha x \cdot z(n)+Qb22(n) \cdot \alpha y \cdot z(n))); \\
Mx(n) & = dT \cdot ((Qb11(n) \cdot \alpha x + Qb12(n) \cdot \alpha y) \cdot (z(n+1)^2) - (Qb11(n) \cdot \alpha x + Qb12(n) \cdot \alpha y) \cdot (z(n)^2)); \\
Mx(n) & = dT \cdot ((Qb12(n) \cdot \alpha x + Qb22(n) \cdot \alpha y) \cdot (z(n+1)^2) - (Qb12(n) \cdot \alpha x + Qb22(n) \cdot \alpha y) \cdot (z(n)^2));
\end{align*}
\]

end;

\[
\begin{align*}
Nx & = \text{sum}(Nx) \\
Ny & = \text{sum}(Ny) \\
Mx & = \text{sum}(Mx)
\end{align*}
\]

\[
\begin{align*}
C1 & = \frac{(A11 \cdot Ly^2)}{48} \\
C2 & = \frac{(A11 \cdot Ly^4)}{1280} \\
C3 & = \frac{(B11 \cdot Ly^2)}{48} \\
C4 & = \frac{(A12 \cdot Lx^2)}{48} \\
C5 & = \frac{(A12 \cdot Lx^2 \cdot Ly^2)}{2304} \\
C6 & = \frac{(A12 \cdot Ly^2)}{48} \\
C7 & = \frac{(A22 \cdot Lx^2)}{48} \\
C8 & = \frac{(A22 \cdot Ly^4)}{1280} \\
C9 & = \frac{(B22 \cdot Lx^2)}{48}
\end{align*}
\]

\[
\begin{align*}
s & = \text{solve}(f1, f2, f3, f4, c, d); \\
a & = \text{double}(S.a); \\
b & = \text{double}(S.b); \\
c & = \text{double}(S.c); \\
d & = \text{double}(S.d); \\
for & \ j = 1: \text{length}(a); \\
if & \ \text{isreal}(a(j)) == 1;
\end{align*}
\]

\[
\text{for} \ x = 0: \text{int} : Lx; \\
\text{for} \ y = 0: \text{int} : Ly;
\end{align*}
\]

\[
\begin{align*}
u & = \text{double}(x + (c(j) \cdot x - ((a(j) \cdot x^3)/6) - ((a(j) \cdot b(j) \cdot x^2 \cdot y^2)/4)))); \\
v & = \text{double}(y + (d(j) \cdot y - ((b(j) \cdot y^3)/6) - ((a(j) \cdot b(j) \cdot x^2 \cdot y)/4)))); \\
w & = \text{double}(.5*((a(j) \cdot x^2) + b(j) \cdot y^2)); \\
if & \ y == Ly;
\end{align*}
\]
n=0;
end
end
end
end

plot3(u(:, :, 1), v(:, :, 1), w(:, :, 1), 'r+');
xlabel('x (mm)');
ylabel('t (mm)');
zlabel('z (mm)');
title( );
Appendix B: Comparison of Experimental and Numerical Laminate Shapes

The Matlab based tool for comparing experimentally measured shapes (in the form of a point cloud from laser scanning) against numerical shapes (comprising co-ordinates of the nodes of the laminate’s surface) are presented. Three separate steps are involved:

1. A Python script is used to extract the nodal co-ordinates of the nodes comprising the surface of the laminate predicted using Abaqus.
2. The numerical and experimental laminate shapes in the dry condition are compared, resulting in $w^*$. 
3. The numerical and experimental laminate shapes in the saturated condition are compared, resulting in $\delta$.

B-1. Extracting Model Surface Co-ordinates

A Python script is used to extract the co-ordinates of the nodes of the bottom surface of the numerical laminates. The script identifies the required nodes through the nodeset “BOTTOM”, which is defined during the construction of the model. The output (in the terms of nodal co-ordinates) is then exported into a text file (“cords.txt”)

```python
from abaqus import *
from abaqusConstants import *
import visualization
import odbAccess
import os

# find current viewport
currentViewport = session.viewports[session.currentViewportName]
# assign odb file from current viewport
odbFile = currentViewport.displayedObject
# get file name and path
odbFileNameFull = odbFile.path
#split into separately name and path
```

Appendix B: Comparison of Experimental and Numerical Laminate Shapes

B-2. Comparing Dry Laminate Shapes
The dry and saturated laminate shapes are compared using the Matlab script “DryShapeComp.m”. Before using the script, the raw data (in the form of co-ordinates)
Appendix B: Comparison of Experimental and Numerical Laminate Shapes

is formatted into columns (for $x$, $y$ and $z$ co-ordinates) using Microsoft Excel. Two files are prepared, one comprising the experimental data and one comprising numerical prediction data.

```matlab
DryShapeComp.m

a='.xlsx' % experimental data
b='.xlsx' % numerical data

%Import Experimental data and then Abaqus data
x0 = xlsread(a,'Data','A:A');
y0 = xlsread(a,'Data','B:B');
z0 = xlsread(a,'Data','C:C');

x1 = xlsread(b,'Data','A:A');
y1 = xlsread(b,'Data','B:B');
z1 = xlsread(b,'Data','C:C');

A0=TriScatteredInterp(x0,y0,z0);
A1=TriScatteredInterp(x1,y1,z1);

%----------------Calculate dimensions of grid, then generate grid
xmax1=max(x0);
xmax2=max(x1);
if xmax1>xmax2;
xmax=xmax1;
else
   xmax=xmax2;
end
ymax1=max(y0);
ymax2=max(y1);
if ymax1>ymax2;
   ymax=ymax1;
else
   ymax=ymax2;
end

xi1=5:0.5:(95);
xi2=5:0.5:(ymax-5);
[dx,dy]=meshgrid(xi1,xi2);
iz0=A0(dx,dy);
iz1=A1(dx,dy);

dz=iz1-iz0 %Subtract shape

%------------------Get rid of 'NaN' values in dz
k=size(dz)
m=k(1)
n=k(2)
r=1
for i=1:m
   s=1
   for j=1:n;
      if isnan(dz(i,j))==0;
         DX1(r,s)=dx(i,j);
      end
   end
end
```

B-3 | Page
Appendix B: Comparison of Experimental and Numerical Laminate Shapes

DY1(r,s)=dy(i,j);
DZ1(r,s)=dz(i,j);
s=s+1
end
r=r+1
end

%-----------------------------Eliminate clamp area
mesh(dx,dy,dz);
xlabel('X (mm)')
ylabel('Y (mm)')
zlabel('Z (mm)')
title('Experiments')
axis equal

view(60,45)
datacursormode on
dcm_obj=datacursormode
pt1=getCursorInfo(dcm_obj)%breakpoint
pt1=struct2cell(pt1)
dcm_obj=datacursormode
pt2=getCursorInfo(dcm_obj)%breakpoint
pt2=struct2cell(pt2)
dcm_obj=datacursormode
pt3=getCursorInfo(dcm_obj)%breakpoint
pt3=struct2cell(pt3)

S=(pt3{2}(3)-pt2{2}(3))/(pt3{2}(2)-pt2{2}(2))
c=pt3{2}(3)-(S*pt3{2}(2))

k=size(dy)
m=k(1)
n=k(2)
t=1

for i=1:m;
    for j=1:n
        if ((pt1{2}(1))<dx(i,j)) && (dx(i,j)<(pt2{2}(1))) &&
           (dy(i,j)>(pt2{2}(2)))
            dz(i,j)=S*(dy(i,j))+c;
            t=t+1
        end
    end
end

iz0max=max(iz0) % returns maximum of each column (i.e. returns an
iz0max=max(iz0max)
iz0min=min(iz0)
iz0min=min(iz0min)
sizeiz0=size(iz0);
iz0max=iz0max-iz0min

for i=1:sizeiz0(1);
    for j=1:sizeiz0(2);
        dzl(i,j)=(dz(i,j)/iz0max)*100; %Abaqus deflection minus
        Experimental deflection, normalised against maximum exp deflection
    end
end
end
dz=dz1

xmax=max(dx);
xmax=max(xmax)
xmin=min(dx);
xmin=min(xmin)

ymax=max(dy);
ymax=max(ymax)
ymin=min(dy);
ymin=min(ymin)

dzmin=min(dz);
dzmin=min(dzmin)
dzmax=max(dz);
dzmax=max(dzmax)

%-------------------Plot

mesh(dx,dy,dz);
xlabel('x (mm)')
ylabel('y (mm)')
zlabel('w*')
%colormap(gray)
axis equal
set(gca,'XTick',[])
set(gca,'YTick',[])
set(gca,'ZTick',[])
%caxis([-10, 9])
colorbar

view(60,45)

c=colorbar;
set(c,'Position', [0.94 0.01 0.015 0.9])

xlabh = get(gca,'XLabel');
set(get(gca,'XLabel'),'Rotation',305);
set(get(gca,'XLabel'),'Position', [80 -25 0])

xlabh = get(gca,'YLabel');
set(get(gca,'YLabel'),'Position', [130 90 0])

xlabh = get(gca,'ZLabel');
set(get(gca,'ZLabel'),'Position', [-5 -10 0])
B-3. Comparing Saturated Laminate Shapes

The saturated shape comparison tool is similar to that of the dry shapes, with the difference that both dry and saturated data are read, for both the experimentally-measured and numerically-predicted laminate shapes.

SatShapeComp.m

```matlab
% Import Experimental data and then Abaqus data
xa = xlsread(a,'Data','A:A');
ya = xlsread(a,'Data','B:B');
za = xlsread(a,'Data','C:C');
xb = xlsread(b,'Data','A:A');
yb = xlsread(b,'Data','B:B');
zb = xlsread(b,'Data','C:C');
xc = xlsread(c,'Data','A:A');
yc = xlsread(c,'Data','B:B');
zc = xlsread(c,'Data','C:C');
xd = xlsread(d,'Data','A:A');
yd = xlsread(d,'Data','B:B');
zd = xlsread(d,'Data','C:C');

Aa=TriScatteredInterp(xa,ya,za); %exp dry
Ab=TriScatteredInterp(xb,yb,zb); %exp sat
Ac=TriScatteredInterp(xc,yc,zc); %num dry
Ad=TriScatteredInterp(xd,yd,zd); %num sat

% Calculate dimensions of grid, then generate grid from experiments
ymax=max(ya);
ymax=max(yb);
if ymaxa>ymaxb;
    ymax=ymaxa;
else
    ymax=ymaxb;
end

tia=5:0.5:(95); %X grid size
tib=5:0.5:(ymax-5); %Y grid size

[dx,dy]=meshgrid(tia,tib);
iza=Aa(dx,dy); %Z of experimental dry
izb=Ab(dx,dy); %Z of experimental saturated
izc=Ac(dx,dy); %Z of numerical dry
izd=Ad(dx,dy); %Z of numerical saturated

% Calculate variations, then decide on normalising strategy
dze=izb-iza; % Saturated shape minus dry shape, Exp
dzn=izd-izc; % Saturated shape minus dry shape, Num
```
dzD = dzn-dze % Numerical saturated shape minus Experimental saturated shape. i.e. Correlation of both minus variation in dry shapes

% No normalising
% dzd=dzD

% Normalising by experimental loss of deflection
dzE=abs(dze)
maxdze=max(dzE)
maxdze=max(maxdze)
dzd = (dzD/maxdze)*100 % normalising against change in shape between saturated and dry, experimental

% Normalising by max experimental saturated shape deflection
% izbmax=max(izb) % returns maximum of each column (i.e. returns an array)
% izbmax=max(izbmax)
% izbmin=min(izb)
% izbmin=min(izbmin)
% sizeizb=size(izb);
% izbmax=izbmax-izbmin
% dzd = (dzD/izbmax)*100

subplot(1,3,1); mesh(dx,dy,dze);
xlabel('X (mm)')
ylabel('Y (mm)')
zlabel('Z (mm)')
title('Experiments')
axis equal
colorbar
view(60,45)

subplot(1,3,2); mesh(dx,dy,dzn);
xlabel('X (mm)')
ylabel('Y (mm)')
zlabel('Z (mm)')
title('Numerical')
axis equal
colorbar
view(60,45)

subplot(1,3,3); mesh(dx,dy,dzd);
xlabel('X (mm)')
ylabel('Y (mm)')
zlabel('Z (mm)')
title('Difference')
axis equal
colorbar
view(60,45)
Appendix C: Mechanical Testing

This Appendix contains details on the mechanical testing undertaken as part of the material characterisation. The methodology follows that presented by ASTM D3039M (2000) and ASTM D3518M (2001), in which tensile tests are conducted on a variety of specimens with different ply orientations. By analysing the stress-strain plots that result, calculations can be made to obtain mechanical properties. The tests determined orthogonal elastic properties, parallel and transverse to the fibre direction. The elastic properties that were measured included $E_{11}$, $E_{22}$ and $E_{33}$ (Young’s modulus in the longitudinal, transverse, and out-of-plane directions respectively), $S_{11}$ and $S_{22}$ (ultimate tensile strength in the longitudinal and transverse directions respectively), $\nu_{12}$, and $\nu_{21}$, (in plane Poisson’s ratio) and $G_{12}$ (in plane shear modulus). Also, the ultimate tensile strengths were also obtained.

C-1. Elastic properties

In order to obtain material properties required for numerical modelling, tensile tests were conducted on specimens of the HTA-6376 material. The specimens were cured in the same fashion as all the laminates used in this study, and according to the description in section 4.2. The specimens used in testing are described in Figure C-1. All tests were conducted using a Tinius Olsen tensile tester. Vishay strain gauges of type L2A-06-125-LW-350 (350 ohm resistance) were used to obtain strain data, which was recorded (along with load data from the load cell) using a System 6000 data acquisition unit. To aid in gripping, aluminium tabs were glued using cyanoacrylate to the specimens. These tabs were 2 mm in thickness, and were to the dimensions shown in Figure C-1. Due to limited availability of strain gauges, the number of specimens tested was limited to three for the longitudinal specimens, four for the transverse specimens, and two for the $\pm45^\circ$ specimens.
Figure C-1 Specimens used for tensile tests. Three longitudinal, four transverse and two ±45° specimens were used.

The material properties sought are given in Table C-1 Test matrix for tensile tests, including details on which specimen type was used in tests to obtain the each property.

Table C-1 Test matrix for tensile tests

<table>
<thead>
<tr>
<th>Material property</th>
<th>$E_{11}$</th>
<th>$E_{22}$</th>
<th>$E_{33}$</th>
<th>$v_{12}$</th>
<th>$G_{12}$</th>
<th>$v_{21}$</th>
<th>$S_{11}$</th>
<th>$S_{22}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen type (See Fig. C-1)</td>
<td>L</td>
<td>T</td>
<td>T</td>
<td>L</td>
<td>±4°</td>
<td>T</td>
<td>L</td>
<td>T</td>
</tr>
</tbody>
</table>

Note: $S_{11}$ and $S_{22}$ (material’s ultimate tensile strength in the longitudinal and transverse directions respectively) were measured but not required for modelling.
Appendix C: Mechanical Testing

Figure C-2 Stress-strain results for the longitudinal specimens

Figure C-3 Stress-strain results for the transverse specimens
Appendix C: Mechanical Testing

Figure C-4 Stress-strain results for the ±45° specimens

The ultimate tensile strength ($S_{11}$ for the material’s longitudinal direction and $S_{22}$ for the material’s transverse direction) was given by Eqn. C-1.

$$P^{tu} = \frac{P^{max}}{A}$$  \hspace{1cm} (C-1)

where:

$P^{tu} =$ the Ultimate tensile strength (MPa)

$P^{max} =$ the maximum force before failure (N)

$A =$ cross-sectional area.

The Young’s moduli ($E_{11}$ and $E_{22}$ for the longitudinal and transverse material directions respectively) were given by Eqn. C-2.

$$E = \frac{\Delta \sigma}{\Delta \varepsilon}$$  \hspace{1cm} (C-2)

where:

$E =$ tensile modulus of elasticity, GPa

$\Delta \sigma =$ difference in applied tensile stress between two chosen strain points

$\Delta \varepsilon =$ difference between two chose strain points.

The in-plane shear response was determined as follows. Firstly, the shear stress at each data point (logged from experiments) was calculated according to Eqn. C-3.
where:

\( \tau_{12} = \frac{P_i}{2A} \)  \hspace{1cm} (C-3)

\( \tau_{12} \) = shear stress

\( P_i \) = load at the \( i \)'th data point

\( A \) = cross sectional area.

Subsequently, the shear modulus of elasticity was calculated according to Eqn. C-4. This was done over a 3980 \( \mu \varepsilon \) strain range, with a lower strain point of 1510 \( \mu \varepsilon \) being used.

\[ G_{12}^{\text{chord}} = \frac{\Delta \tau_{12}}{\Delta \gamma_{12}} \]  \hspace{1cm} (C-4)

where:

\( G_{12}^{\text{chord}} \) = shear chord modulus of elasticity, GPa

\( \Delta \tau_{12} \) = difference in applied shear stress between the two chosen shear strain points

\( \Delta \gamma_{12} \) = difference between the two chosen shear strain points (nominally 0.004)

Poisson’s ratio is determined using Eqn. C-5.

\[ \nu = -\frac{\Delta \varepsilon_t}{\Delta \varepsilon_l} \]  \hspace{1cm} (C-5)

where:

\( \nu \) = Poisson’s ratio

\( \Delta \varepsilon_t \) = difference in lateral strain between the two chosen longitudinal strain points

\( \Delta \varepsilon_l \) = difference between the two longitudinal strain points.

The averaged results for the testing are given in Table C-2.
### Table C-2 Material property results

<table>
<thead>
<tr>
<th></th>
<th>$E_{11}$ (GPa)</th>
<th>$S_{11}$ (GPa)</th>
<th>$E_{22}$ (GPa)</th>
<th>$S_{22}$ (MPa)</th>
<th>$G_{12}$ (GPa)</th>
<th>$\nu_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean Value</strong></td>
<td>135.64</td>
<td>1.96</td>
<td>10.14</td>
<td>55</td>
<td>5.86</td>
<td>0.29</td>
</tr>
<tr>
<td><strong>Standard Deviation (%)</strong></td>
<td>2.34 (2.9%)</td>
<td>0.13 (6.4%)</td>
<td>0.18 (1.8%)</td>
<td>3.39 (6.17%)</td>
<td>0.18 (3%)</td>
<td>0.01 (3.2%)</td>
</tr>
</tbody>
</table>

Note: the standard deviation as a percentage of the mean value is given in brackets
Appendix D: Cured and Saturated Laminate Shapes

The shapes of the tailored laminates were measured using laser scanning (see section 4.4.2). The results of the scanning will be provided in this appendix. This will include a three-dimensional view of the laminates (for both dry multi-stable shapes, where applicable) and for the saturated shape configuration. Due to the complex shapes obtained, the cross-sectional profile of the tailored laminates will also be presented. These profiles are extracted along the laminate's $x$ and $y$ axes, and are taken mid-way across the laminate's length/width. This data can be used to further facilitate comparisons between numerical predictions/experimental comparisons, as well as give further detail on the local shape configuration at each location where residual stresses are extracted.

A brief description of the detail provided in Figures D-1 to D-6 will be given, by using Figure D-1 (for Laminate 1) as an example. Firstly, the laminate’s two multi-stable shapes, as well as the saturated laminate shape, are given in Figure D-1 (a) – (c). These plots are generated directly from laser scan data, using the mathematical software Matlab to create a surface between points in the point cloud. Subsequently, the cross-sectional views (taken mid-way through the laminate’s width/length) are given in Figure D-1 (d) – (g). In the case of Shape B, the saturated cross-sectional profile is also given.
Figure D-1 Laser scan data, Laminate 1 with: (a) Shape A; (b) Shape B; (c) Shape B, saturated; (d) cross-sectional profile, Shape A, at $y=-50$ mm; (e) cross-sectional profiles, Shape B, at $y=-50$ mm; (f) cross-sectional profile, Shape A, at $x=100$ mm; (g) cross-sectional profile, Shape B, at $x=100$ mm. Contour plot depicts out-of-plane deflection ($z$).
Figure D-2 Laser scan data, Laminate 2 with: (a) Shape A; (b) Shape B; (c) Shape B, saturated; (d) cross-sectional profile, Shape A, at $y=-50$ mm; (e) cross-sectional profiles, Shape B, at $y=-50$ mm; (f) cross-sectional profile, Shape A, at $x=100$ mm; (g) cross-sectional profile, Shape B, at $x=100$ mm. Contour plot depicts out-of-plane deflection ($z$).
Figure D-3 Laser scan data, Laminate 3 with: (a) Shape A; (b) Shape B; (c) Shape B, saturated; (d) cross-sectional profile, Shape A, at $y=-50$ mm; (e) cross-sectional profiles, Shape B, at $y=-50$ mm; (f)
cross-sectional profile, Shape A, at $x=100$ mm; (g) cross-sectional profile, Shape B, at $x=100$ mm. Contour plot depicts out-of-plane deflection ($z$).

Figure D-4 Laser scan data, Laminate 4 with: (a) Shape B; (b) Shape B, saturated; (c) cross-sectional profiles, at $y=-50$ mm; (d) cross-sectional profiles at $x=100$ mm.
Figure D-5 Laser scan data, Laminate 5 with: (a) Shape A; (b) Shape B; (c) Shape B, saturated; (d) cross-sectional profile, Shape A, at y=-50 mm; (e) cross-sectional profiles, Shape B, at y=-50 mm; (f) cross-sectional profile, Shape A, at x=100 mm; (g) cross-sectional profile, Shape B, at x=100 mm. Contour plot depicts out-of-plane deflection (z).
Figure D-6 Laser scan data, Laminate 6 with: (a) Shape B; (b) Shape B, saturated; (c) cross-sectional profiles, at \( y = -50 \) mm; (d) cross-sectional profiles at \( x = 100 \) mm.