An Investigation into the Nature of Mathematics Textbooks at Junior Cycle and their Role in Mathematics Education

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Abstract

This research study is aimed at improving the quality of the mathematics textbooks available for Junior Cycle students. It is widely agreed that there is room for improvement with regard to the quality of mathematics at both Junior and Senior Cycle level in Ireland. One such area which can be improved is the effectiveness of the resources available in both Junior and Senior Cycle mathematics classrooms. While the TIMSS report (Valverde et al., 2002) has explored textbooks on an international scale, minimal research (minor role in TIMSS Report) has been carried out on Irish mathematics textbooks. Considering the level of responsibility shouldered by mathematics textbooks, there is an obvious gap in mathematics education research.

The aim of this study is to investigate the quality of the mathematics textbooks currently in use at Junior secondary school level in Ireland. This is achieved by investigating, extending and applying suitable methodological tools for textbook analysis. Ultimately the aim of this research is to improve the quality of teaching and learning of mathematics at Junior Cycle level which should feed directly into improving the quality of mathematics at Senior Cycle. This will be achieved by first measuring the quality of the current Junior Cycle mathematics textbooks and then highlighting the role of improved textbooks in students’ conceptual understanding. At present in Ireland, there is a move away from the more traditional didactical approaches to teaching and learning towards teaching and learning for understanding. This move towards teaching and learning for understanding is as a result of the new curriculum initiative - Project Maths, which requires students to understand and apply mathematics. This change in focus within mathematics classrooms across Ireland highlights the need for teachers to be more aware of students’ conceptual development and hence influenced the author as this research study evolved.

The study presented here provides a theoretical framework for a complete analysis of mathematics textbooks which will allow for an in-depth analysis of any mathematics textbook. This research builds on the work of international studies such as the TIMSS report and established frameworks for mathematics textbook analysis such as Morgan (2004), Mikk (2000) and Rivers (1990) to create a single framework for a complete mathematics textbook analysis. This study also highlights key design features of mathematics textbooks which are significant in students’ conceptual development. The author’s investigation into the quality of the current Junior Cycle mathematics textbooks is the first large scale study of its kind in Ireland. This research not only identifies the quality of the current mathematics textbooks but it also highlights key design features which impact on students’ understanding of mathematics.
Author’s Declaration

This thesis is presented in fulfilment of the requirements for the degree of Doctorate of Philosophy. It is entirely my own work and has not been submitted to any other University or higher education institution, or for any other academic award in this University. Where use has been made of the work of other people it has been fully acknowledged and fully referenced.

Signature
Lisa O’Keeffe,
March, 2011
To my parents, without your patience and support this would not have been possible.
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Chapter 1

Introduction

1.1 Introduction

Over the past twenty years changes in practice and teaching methodologies have led to concerns regarding the quality of mathematics textbooks. Robitaille and Travers (1992) express the view that textbook content and how such textbooks are used impact directly on students’ learning. Schoenfeld (1988: 163) argues that while good teaching might compensate for the inadequacies of the textbook “there is evidence to suggest ... that it does not”. While it is widely accepted that the curriculum is central to influencing the choice and treatment of subject matter in mathematics classrooms, one of the key factors in implementing this content is the mathematics textbook (Schmidt, McKnight, Valverge, Houang, and Wiley, 1997). Many researchers (Skemp, 1982; Van Dormolen, 1986; Pimm, 1997; Hiebert and Carpenter, 1992; Dowling, 1996; Orton, 2004) have looked specifically at some of the key concerns with mathematics textbooks, however, few researchers have examined the textbook as a whole with the exception of the Third International Mathematics and Science Study (TIMSS) (Valverde, Bianchi, Wolfe, Schmidt, and Houang, 2002).

Textbooks are widely accepted as a commonly used mathematical resource for teaching and learning. Studies such as Hiebert, Gallimore, Garnier, Givvin, Jacobs, Hollingsworth, Chui, Wearne, Smith, Kersting, Manaster, Tseng, Etterbeck,
1.1. Introduction

Manaster, Gonzales, and Stigler (2003) found that in many countries teachers rely heavily on the mathematics textbook (e.g. 91-100% of their sample). A preliminary study conducted by the author identified that over 75% of Irish secondary school teachers in her sample use just one textbook on a daily basis (O’Keeffe, 2007). The National Council for Curriculum and Assessment (NCCA) also suggests that over reliance on mathematics textbooks in an Irish context may well be a contributory factor to the low uptake of higher level mathematics in state examinations (only 16% of the student cohort opted for Higher Level Leaving Certificate mathematics in 2009 compared to 64% opting for higher level English). In 2006 the NCCA consulted with a number of interested parties, such as 2nd and 3rd level students and teachers to identify and discuss recommendations for enhancing proficiency in mathematics in Ireland (NCCA, 2006). As part of this report the NCCA suggested a number of changes, one of these being to improve the textbooks and available resources. Over 90% of those involved in the 2006 NCCA consultations felt strongly that an improved textbook could play an effective role in student learning (NCCA, 2006). Since the mathematics textbook is a dominant classroom resource, and in some cases the only classroom resource, the need for effective mathematics textbooks is undeniable. However, improving or in fact affirming the quality of the current Irish mathematics textbooks requires the use of textbook analysis.

The purpose of this research study is to examine the quality of the mathematics textbooks currently in use at Junior secondary school level in Ireland. At present in Ireland, there is a move away from the more traditional didactical approaches to teaching and learning towards teaching and learning for understanding. This move towards teaching and learning for understanding is as a result of the new curriculum initiative - Project Maths, which requires students’ to understand and apply mathematics. This change in focus within mathematics classrooms across Ireland highlights the need for educators to be more aware of students’ conceptual development and hence influenced the author as this research study evolved.
1.2. Background to Research

Many researchers acknowledge the need for textbook analysis and its place in mathematics education. As this research progressed it became evident that the well known and widely used models of textbook analysis were insufficient to complete the task of establishing the quality of the current mathematics textbook, specifically as regards their impact on students’ conceptual development in mathematics. For this reason a number of amendments and adaptations to models of textbook analysis were applied throughout the research process.

1.2 Background to Research

The author, having conducted exploratory research as a final year undergraduate project, identified an overwhelming expectation for the mathematics textbook to embrace all aspects of teaching and learning mathematics. Despite this expectation the NCCA (2005a) has noted that other than a minor inclusion in the TIMSS\(^1\) Report on textbooks (Valverde, G., L. Bianchi, R. Wolfe, W. Schmidt and R. Houang,2002) no Irish specific study has been carried out on mathematics textbooks. The importance of effective textbooks can never be underestimated. Textbooks have emerged as a dominant mathematics classroom resource, not only internationally but also nationally. Previous research carried out by the author has shown that 76.9% of teachers from twenty schools surveyed in the south of Ireland use only a mathematics textbook in every class (O’Keeffe, 2007). With the textbooks assuming such an important role, it is vital that they be analysed to establish their quality and ensure their effectiveness.

Taking all this information into account the next logical step is to look at the Irish mathematics textbook and its role in our education system. Irish students have performed poorly in the PISA\(^2\) studies as reported in 2001, 2003 and again in 2006, and are currently ranked in the middle of the OECD\(^3\) countries for mathematical

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\(^1\)TIMSS: Trends in International Mathematics and Science Study

\(^2\)PISA: OECD Programme for International Student Assessment

\(^3\)OECD: Organisation for Economic Co-operation and Development
1.2. Background to Research

literacy (Cosgrove, Shiel, Sofroniou, Zastrutzki, and Shortt, 2005). Along with the PISA reports two curriculum mapping exercises have been carried out to determine the true position of Ireland’s mathematical education system. The first being a ‘test-curriculum rating’ which involves measuring Irish students’ expected ‘curriculum familiarity’ with the concepts, contexts and formats of PISA, based on an analysis of the Junior Certificate examination papers at Higher, Ordinary and Foundation levels (Cosgrove, Oldham, and Close, 2005) and the second of which mapped the 2003 Junior Certificate and 1974 Intermediate Certificate against the PISA three-dimensional framework (Close and Oldham, 2005). The initial curriculum mapping exercise highlighted the discontinuity between the senior primary school curriculum and the lower secondary school textbooks. This discontinuity has been noted by Smyth, McCoy, and Darmody (2004) as affecting the students’ transition from primary to secondary education. Close (2006) reinforces these findings when he states that the current Junior Cycle curriculum does not motivate students to learn mathematics. The mathematics textbook is the closest connection students have with the curriculum and this curriculum is found wanting. These findings reinforce the need for a mathematics textbook specific study. A common thread emerging from all of these studies indicates that Ireland is currently teaching mathematics in post primary schools through a didactic style of teaching with little or no emphasis on problem solving (NCCA, 2005a). Sadly our textbooks are contributing to this situation. However the new mathematics curriculum, Project Maths, should prove to be a step in the right direction.

Mathematics textbooks are designed to be used to assist and complement teaching and learning and not to act as a crutch. However, textbooks are being relied upon far too much, as indicated by the author’s prior research and international experience (Cockroft, 1982; Smith, 2004; Valverde et al., 2002). The textbooks, it seems, are contributing to the practice of ‘teaching to the exam’. The textbooks focus entirely on what the NCCA report calls ‘vertical learning’. Vertical learning
1.2. *Background to Research*

can be described as the manipulation of symbols and the process of transferring this mathematical knowledge into the real world (horizontal learning, on the other hand, refers to the real world situations and the applications of mathematical concepts). The danger with teaching via vertical learning only is that the process of transferring the mathematical symbols back into real world situations is often too difficult for students. Without sufficient knowledge, students tend to fall at this hurdle leaving them short in terms of their mathematical learning (Freudenthal, 1973).

Another topical area for analysis, is the issue of conceptual development. The new mathematics curriculum in Ireland places a high emphasis on teaching and learning for understanding. Conceptual development can be described as “learning that changes an existing conception” (Orey, 2001: 1). Incorporating conceptual development into education can enhance a student’s ability to overcome misconceptions and make them better prepared to deal with more difficult concepts. In order for conceptual development to occur in the classroom, teachers of mathematics must work on trying to remove misguided assumptions and preconceptions. According to Merenluoto and Lehtinen (2004) conceptual development fails when students lack sufficient prior knowledge and also when traditional teaching methods are relied on. Problem solving is another key concern in mathematics education. It has been described by many as the essence of all mathematics (Orton, 2004). However as the NCCA (2005a) noted, an over emphasis in Irish classrooms on procedures is a barrier to developing problem solving skills. Much research has been carried out since Polya (1945) on how best to embed problem solving into the classroom activity. Many researchers, such as Schoenfeld (1992) have examined this area and have determined that problem solving cannot be taught as a separate unit. Problem solving needs to be successfully integrated within the teaching and learning activities of textbooks in order to avoid the failures of some of the ‘problem solving reforms’ as mentioned by Cockcroft (1982) and Neyland (1995).
1.2. Background to Research

1.2.1 Personal Motivation

The need for an Irish specific mathematics textbook study cannot be ignored. There is widespread agreement (Graybeal and Stodolsky, 1986; Horsley and Laws, 1992; Mikk, 2000; NCCA, 2006; Sewall, 1992; Valverde and Schmidt, 1998), that good textbooks impact positively on student learning. The textbook can also form part of the solution to other important issues in mathematics education such as student attitudes, transitions from primary to secondary and secondary to tertiary education and take-up rates for Higher Level mathematics. Students have a multitude of other information sources available to them; consequently if their textbooks are dull, they will be unwilling to study them. Interesting and exciting textbooks can develop curiosity and interest in the subject. Students’ experiences of mathematics textbooks will undoubtedly impact their ability to learn individually from textbooks (Robinson, 1981). Creative and innovative textbooks will encourage an improved attitude towards mathematics, while also assisting students with their own individual learning from a textbook; a skill which is greatly required in Higher Education (HE).

The author is a member of the mathematics education research group based at the National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL). The brief of the NCE-MSTL is to research issues and national priorities in mathematics and science education in Ireland. In the context of the introduction of the new national mathematics curriculum, Project Maths, this research study addresses a major national issue regarding the development of new mathematics textbooks. The author’s intention is to provide a knowledge base for understanding the role and quality of the textbooks in Irish Junior Cycle mathematics education, while also providing an evidence base for addressing changes and improvements in future mathematics textbooks.

Additionally, having spent a number of years learning from and teaching with the
1.2. Background to Research

Irish mathematics textbooks, the author felt the need for improved textbooks. From personal experiences as a student, at both second and third level, the author found that the purpose of many of the Irish mathematics textbooks, particularly at secondary level, is to provide a bank of questions and exercises. In practice little help can be found in the Irish school mathematics textbook in clarifying key concepts and approaches to understanding a topic. Also there is minimal information in Irish mathematics textbooks about the background or purpose of any of the mathematical topics. Like many Irish students the author has also experienced poor mathematics teaching. Ineffective teachers tend to rely heavily on the textbook and their knowledge outside of the textbook is often limited. This confines the classroom and homework experiences of students to just the textbook. The mathematics textbooks were never designed to act as the only mathematics resource and therefore would often be found wanting, boring and off-putting. This study will highlight the quality of the current Irish mathematical Junior Cycle textbooks and will look at ways of improving the quality of mathematics textbooks.

1.2.2 Outline of Irish Secondary Education System

Irish students begin primary school at approximately aged five and remain in primary education for eight years. Students then enter secondary education at aged 12/13. The Irish secondary education system is broken into two examination cycles, the first of which is called the Junior Cycle and is examined by the Junior Certificate examination. The Junior Cycle comprises the first three years of secondary school and the Junior Certificate examination is a state examination which is conducted in June of the third year. The second educational cycle is the Senior Cycle and this is examined by the Leaving Certificate examination. The Leaving Certificate examination results are the basis on which college places are offered to Irish students. The Senior Cycle is two years and can be commenced immediately after the Junior Certificate year or students can opt for Transition year. Transition year is a non academic year and is not compulsory. Within the
1.3. Scope and Significance of Research

Junior and Leaving Certificate examinations there are three different levels, Higher, Ordinary and Foundation Level. Irish mathematics textbooks typically cater for all three levels, the first textbook for first year of both Junior and Senior Cycle mathematics covers the entire Ordinary Level course content a second textbook is introduced in the second year to the Higher Level students. Hence the mathematics textbook series involved in this research each comprise of two textbooks, the first which is aimed at all mathematics students and the second which caters for Higher Level mathematics.

The textbooks used in this research study are coded as follows: TBS represents Textbook Series. In total there are four textbook series A, B, C and D. Within each textbook series there are two textbooks, the first which is aimed at Ordinary Level mathematics and which all students use in their first year of secondary school and the second which is aimed at Higher Level students of mathematics in their second and third year of secondary school.

TBS A1 - Textbook Series A Book 1
TBS A2 - Textbook Series A Book 2
TBS B1 - Textbook Series B Book 1
TBS B2 - Textbook Series B Book 2
TBS C1 - Textbook Series C Book 1
TBS C2 - Textbook Series C Book 2
TBS D1 - Textbook Series D Book 1
TBS D2 - Textbook Series D Book 2

1.3 Scope and Significance of Research

In Ireland every student is exposed to mathematics for eight years at primary level and though mathematics is not compulsory 96% of the student cohort take
mathematics for a further five or six years, all the way to their Leaving Certificate examination (Chief Examiner, 2005). While mathematics is not strictly compulsory, it is virtually compulsory as it is a requirement for entry into the vast majority of third level institutions. While large numbers of the student cohort are taking mathematics as a subject the percentages of students taking higher level mathematics remain low. In 2008 17% of students taking mathematics opted for higher level, only 16.2% of students took higher level in 2009 and in 2010 this dropped marginally to 16%. The low percentages opting for higher level are a direct result of the low numbers of students taking the higher level Junior Certificate examination. In 2008, 2009 and 2010 only 43% of students opted for the higher level Junior Cycle examination. The NCCA’s recommended uptake level for higher level Junior Certificate mathematics is 60%. In conjunction with the low uptakes of higher level mathematics the NCCA (2005a) review into mathematics at post primary level highlights concerns over the low level of mathematics knowledge and skills demonstrated by students and the lack of understanding about basic concepts and a reliance on rote learning. Poor teaching and over-reliance on textbooks is a significant contributory factor and needs to be addressed at this level.

In conjunction with these findings a study conducted by Ní Ríordáin and Hannigan (2009) found that 48% of teachers in their survey who are teaching mathematics in Irish post-primary schools have no qualification in mathematics teaching. Ní Ríordáin and Hannigan refer to these teachers as out-of-field teachers. Out-of-field teachers populate the lower levels of secondary school classrooms and hence are responsible for a great deal of the teaching of Junior Cycle mathematics. A preliminary study carried out by the author also identified that 75% of the teachers in her sample (all first year mathematics teachers) use one textbook for all their daily teaching and planning (O’Keeffe, 2007).

The importance of presenting information in an effective manner has been high-
lighted by Mobley (1968: 6), who stated that the way in which “information is presented is as important as the information itself”. In order to learn, students need to be actively involved in the learning task. They need to be able to relate to the topic at hand and to carry out tasks in a suitable environment. It is therefore obvious that textbooks play a vital role in shaping, not only the knowledge, but the opinions and beliefs of students with regard to mathematics. The study presented here provides a theoretical framework for a complete analysis of mathematics textbooks which will allow for an in-depth analysis of any mathematics textbook. This research builds on the work of international studies such as the TIMSS and established frameworks for mathematics textbooks analysis such as Morgan (2004), Mikk (2000) and Rivers (1990) to create one framework for a complete textbook analysis. This study also highlights key design features of mathematics textbooks which are significant in students conceptual development.

The author’s investigation into the quality of the current Junior Cycle mathematics textbooks is the first large scale study of its kind in Ireland. This research not only identifies the quality of the current mathematics textbooks but it also highlights key design features which impact on students’ understanding of mathematics. This study is timely in the context of the introduction of the new national mathematics curriculum, Project Maths, which is underway and will be of benefit to mathematics teachers and students in this context. This research project also provides valuable insights into methodological tools of textbook analysis and key design features of mathematics textbooks which impact on students’ conceptual development.

1.4 Research Problem and Aims of the Research

The motivation for this research study originated from the author’s own experiences as both a teacher and a student in Irish secondary schools. The initial investigation focused on an application of the TIMSS textbook analysis (Valverde et al., 2002)
however it soon became clear that such a framework was insufficient at providing a definitive overview of each of the textbooks. Further review of the literature identified language as being a significant factor in both the teaching and learning of mathematics. At this juncture the author added the element of language analysis to the overall framework for textbook analysis. This additional element enabled the author to engage in a detailed, in-depth investigation into the quality of the current mathematics textbooks in Ireland (Phase two). The review of the literature also uncovered a number of textbook features which have the potential to enhance student learning, in particular student motivation and comprehension. This literature review and other considerations also informed the creation and design of a model chapter in fraction addition incorporating valued design features, which were calculated to improve students’ conceptual development (Phase 3).

The research problem is to investigate the quality of the Junior Cycle mathematics textbooks currently in use in Ireland and to identify key features which impact on students’ learning and understanding. The primary aims of this research study are:

- To review the literature in relation to methods of mathematics textbook analysis, the role of the textbook in mathematics education nationally and internationally and the key concerns in mathematics education which can be impacted on by the mathematics textbook.

- To investigate the quality of the Junior Cycle mathematics textbooks currently in use in Ireland.

- To establish the most effective methodological tools for a complete textbook analysis.

- To identify key textbook design features which impact on students’ conceptual development.
1.5 Research Questions

Each phase of this research project relied upon and was based on the findings or outcomes of the previous phase and were each guided by the following research questions:

Phase One - Exploratory Research

- What role do mathematics textbooks play in mathematics education i.e. in the teaching and learning of mathematics?
- What are the main areas of concern in mathematics education which are impacted by the content and structure of mathematics textbooks?
- Which of the available methods of textbook analysis are the most applicable and relevant to mathematics textbook research?

Phase Two

- Are the current Junior Cycle mathematics textbooks an effective resource for teaching and learning?
- What is the significance and impact of language considerations as interpreted in textbooks for both the teaching and learning of mathematics?
- Do the current textbooks address the key areas of concern in mathematics education such as language deficiencies and poor problem solving skills?

Phase Three

- Can improving the textbooks directly improve students’ understanding of mathematics and their conceptual development?

1.6 Research Design

This research follows three main phases which are presented and outlined in Figure 1.1.
### 1.6. Research Design

<table>
<thead>
<tr>
<th><strong>Phase 1:</strong></th>
<th><strong>Final Year Dissertation</strong></th>
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<tr>
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<td><strong>Exploratory Research</strong></td>
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<td></td>
<td><strong>Design of Methodology</strong></td>
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<td><strong>Data Collection Instruments</strong></td>
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<td><strong>Preliminary Investigation</strong></td>
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<td><strong>Literature and Relevant Background Review</strong></td>
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<td><strong>Coordination of Suitable Theoretical Frameworks</strong></td>
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<td></td>
<td><strong>Content – Rivers Matrix &amp; TIMSS</strong></td>
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<td></td>
<td><strong>Structure – Mikk’s Structure Analysis Grids &amp; TIMSS</strong></td>
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<td></td>
<td><strong>Expectation – Rivers Matrix &amp; TIMSS</strong></td>
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<td></td>
<td><strong>Language – Morgan’s Functional Grammar Analysis</strong></td>
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<th><strong>Phase 2:</strong></th>
<th><strong>Data Collection from Textbook Analysis</strong></th>
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<tr>
<td></td>
<td><strong>Content Analysis</strong></td>
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<td></td>
<td><strong>Structure Analysis</strong></td>
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<td></td>
<td><strong>Expectation Analysis</strong></td>
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<td><strong>Language Analysis</strong></td>
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<th><strong>Phase 3:</strong></th>
<th><strong>Design &amp; Create Research Materials</strong></th>
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<td></td>
<td><strong>Compilation of Key Design Features</strong></td>
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<td></td>
<td><strong>Create Model Chapter</strong></td>
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<td></td>
<td><strong>Create Two-Tier Diagnostic Test</strong></td>
</tr>
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<td></td>
<td><strong>Implementation of Model Chapter and Two-Tier Diagnostic Test</strong></td>
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<td></td>
<td><strong>Implementation into First Year Mathematics Classrooms</strong></td>
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<td></td>
<td><strong>Data Collection &amp; Evaluation</strong></td>
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<td></td>
<td><strong>Data Analysis</strong></td>
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<td><strong>Pre-test</strong></td>
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<td><strong>Post-test</strong></td>
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**Figure 1.1: Outline of this Study**

The initial framework for the textbook analysis is based on the TIMSS framework for textbook analysis (Valverde et al., 2002), however the literature review made the author acutely aware of the relevance and significance of language to mathematics.
1.6. Research Design

and hence to mathematics textbooks. For this reason an extra branch was included in the TIMSS framework - The Language Analysis. The overall framework for this textbook analysis study now comprises four main elements. Suitable theoretical frameworks for analysis were applied to each element of this textbook analysis study.

1.6.1 Phase One:

Phase one incorporates a preliminary analysis of the Junior Cycle textbooks in Ireland, uncovering the most widely used in Irish mathematics classrooms. Following this analysis a comprehensive review of available literature was carried out to deepen the insight into mathematics education in Ireland, and to establish the role of the textbook as the classroom resource in Ireland and worldwide. Phase one allowed for the creation of a detailed methodology, which guided the development of a suitable framework for research in Phase two. The research design also accommodates the review and selection of relevant text analysis tools, each of which was employed in phase two; (TIMSS (2002), Rivers (1990), Mikk (2000) and Morgan (2004)).

1.6.2 Phase Two:

Phase two entails the mathematics textbook analysis. Data from the textbook analysis is obtained by applying the previously selected textbook analysis tools to Irish mathematics textbooks. Textbooks were sourced based on their frequency of use in schools, that is the three most commonly used textbooks were identified and selected.
1.6. Research Design

Theoretical Framework for Phase two

The author acknowledges the complexity of mathematics textbook analysis and the variety of analysis tools available. However, the author was required to make certain decisions regarding research methods specific to this study. Such decisions include identifying the most effective methods of mathematics textbook analysis which would provide an in-depth analysis of the textbooks with an added focus that would provide for design considerations, language and conceptual development. The following table (Table 1.1) outlines the theoretical frameworks adopted in this study. Figure 1.2 gives the theoretical outline of phase two of this research project.

<table>
<thead>
<tr>
<th>Theoretical Frameworks (TF)</th>
<th>Function</th>
<th>Significance*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morgan (2004)</td>
<td>Strengthens textbook analysis by providing a fourth element for analysis</td>
<td>4. Language Analysis</td>
</tr>
</tbody>
</table>

*The data collected from each phase of textbook analysis (Content, Structure, Expectation and Language) is interpreted and analysed in terms of impact on students’ conceptual learning.
1.6.3 Phase Three:

The author applied the design specification, developed from the findings of the textbook analysis, to the construction of a model chapter with a focus on conceptual understanding and problem solving. This model chapter was centered on the topic of adding fractions. The author has identified fractions as an aspect of mathematics which is significant to mathematics in its entirety and is widely accepted as containing difficult concepts to learn and more importantly to understand. Researchers such as Curry et al. (1996) have also noted that students of all ages find fraction manipulation difficult. In her work, (Gill, 2006) identified that students beginning third level service mathematics courses exhibited an obvious rote learning of fraction manipulation and hence an inability to manipulate algebraic fractions. For these reasons and due to informal feedback from students at second and third level with regard to how difficult they perceive the concepts of fraction manipulation, the author has chosen fraction addition to be the central focus of the model chapter. The design for this model chapter evolved from a combination...
1.6. *Research Design*

of four well established relevant frameworks, Project Maths and the Junior Cycle curriculum, Adult Numeracy Network (Curry, Schmitt, and Waldron, 1996), Adult Based Education (M.D.E., 2005)\(^4\) and the Pisa Mathematical Cycle (O.E.C.D., 2006)\(^5\). Following a pilot, review and amendment phase this model chapter was trialled in a small scale intervention in three secondary schools. The model chapter was used in conjunction with a two tier diagnostic test to ascertain its effectiveness in enhancing student conceptual development. This two-tier diagnostic test was based on the work of Treagust (1988). The quantitative data collected from this phase of the research was analysed using SPSS version 16.0.

**Theoretical Framework for Phase Three**

Table 1.2 outlines the theoretical frameworks adopted in this study. Figure 1.3 gives the theoretical outline of this phase of the research project.

<table>
<thead>
<tr>
<th>Phase 3</th>
<th>Theoretical Frameworks (TF)</th>
<th>Function</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Chapter</td>
<td>JC Syllabus &amp; Project Maths</td>
<td>Provides structure fraction addition content</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ANN framework</td>
<td>Adult Numeracy Network</td>
<td>Curry et al. (1996)</td>
</tr>
<tr>
<td></td>
<td>Two-Tier Diagnostic Test</td>
<td>Test Instrument which measures Conceptual Understanding</td>
<td>Treagust (1988)</td>
</tr>
<tr>
<td></td>
<td>RMARS</td>
<td>Test Instrument which measures Anxiety</td>
<td>Plake and Parker (1982)</td>
</tr>
</tbody>
</table>

\(^4\)M.D.E. - Massachusetts Department of Education  
\(^5\)O.E.C.D. - Organisation for Economic Co-operation and Development
Phase 3

Model Chapter

JC Syllabus & Project Maths → Fraction content → Currey et al, 1996
Teaching of fraction, with a focus on attainment of concepts and applications

ANN Framework → Adult Numeracy Network → M.D.E., 2005
Teaching of fraction, with a focus on problem solving and decision making

ABE Framework → Adult Based Education → O.E.C.D., 2006
Typical of an RME approach and focuses on applications

Pisa Mathematical Cycle for Problem Solving

Evaluation

Two-Tier Diagnostic Test → Treagust, 1988
Tier 1 focuses on answering a mathematics question. Tier 2 focuses on the concept required to answer the question correctly.

RMARS (Anxiety Inventory) → Parker & Plake, 1982
Identifies students’ anxiety levels before and after the teaching intervention

Figure 1.3: Theoretical Outline for the Development and Evaluation of the Model Chapter
1.7 Overview of Thesis Chapters

Chapter 2: comprises a review of literature relevant to textbook analysis in mathematics education and key concerns in mathematics education which can be impacted by the mathematics textbook. These key concerns are discussed in relation to mathematics teaching and learning progressing to their impact on mathematics textbooks and the way mathematics textbooks impact on them. This chapter outlines the role and beliefs in today’s society about mathematics, it examines considerations of effective teaching of mathematics and highlights key concerns such as problem solving and conceptual development.

Chapter 3: explores the background issues directly associated with textbooks in mathematics education. This chapter outlines classroom resources for mathematics teaching and learning in general and their use in student learning. Throughout this chapter the author highlights how mathematics textbooks impact on student learning; the need for a textbook analysis study in general and identifies the variables to be analysed and highlights the role of textbooks in both the teaching and learning of mathematics.

Chapter 4: examines and explains the theoretical processes applied to all elements of this research study, describing all elements relating to the research design and research methodologies employed in this study. The research was guided by an in-depth literature review (chapter 2 and chapter 3), which provides the theoretical frameworks and methodologies. Throughout this study a number of choices and decisions were made by the author which are outlined in this chapter. Chapter 4 provides the theoretical frameworks which underpin this research while also outlining each analysis tool employed. Methods of data collection and analysis are also discussed and explained and issues of validity and reliability together with ethical considerations are discussed.
1.7. *Overview of Thesis Chapters*

**Chapter 5:** presents the findings from phase two of this research study and provides a comprehensive analysis of the mathematics textbooks involved in this study. The data presentation and preliminary analysis and commentary follow the structure outline of the textbook analysis. These findings are discussed in detail in chapter 7.

**Chapter 6:** provides the key textbook design features which impact on student learning and presents and comments on the findings from phase three, the creation and trial of a model chapter. These findings are discussed in more detail in chapter 7.

**Chapter 7:** discusses the findings presented in chapters 5 and 6. This study is shaped by three main phases; phase one incorporates a comprehensive review of available literature detailing mathematics education in Ireland and classroom resources worldwide, phase two encompasses the textbook analysis and phase three entails the creation and small scale trial of a model chapter which promotes conceptual development. The discussion of the findings from each phase of this research study is framed by the research questions which were set out for each phase of this study (section 1.5 and 4.4.3) and are discussed with the initial project aim in mind. Ultimately the aim of this research is to improve the quality of teaching and learning of mathematics at Junior Cycle level through the use of improved mathematics textbooks.

**Chapter 8:** concludes the thesis by drawing together the overall conclusions of this study; by discussing the contribution of this research nationally and to the wider community of mathematics education; and making recommendations and suggestions for future research.
1.8 Conclusion

This chapter presents the reasoning, motivation and purpose for conducting this research project. It identifies the objectives and research questions while also providing an insight into the author’s beliefs about the significance of a project of this type. This chapter highlights the need for such a research study to be conducted in Ireland and the relevance of this study in the context of the new national mathematics curriculum. This chapter also outlines the theoretical frameworks employed at each phase of this research study and the overall outline of the methodology employed. Following this an overview of each chapter is provided. Chapter two proceeds to examine in greater detail the background literature relevant to textbook analysis in mathematics education and key concerns in mathematics education which can be impacted by the mathematics textbook.
Chapter 2

Issues in Mathematics Education and Factors that Impact on Mathematics Textbooks

2.1 Introduction

The purpose of this chapter is to explore mathematics as a subject and the teaching of mathematics in order to establish significant areas which impact directly on the mathematics textbook. This chapter outlines the role and beliefs in todays society about mathematics, the theoretical considerations for effective mathematics teaching and learning and the impact of problem solving and the language of mathematics on teaching and learning. These issues have a direct bearing on school mathematics and students’ experiences of school mathematics and impact directly on resources for teaching and learning such as the school mathematics textbooks.

2.2 Mathematics as a Subject

This section outlines how mathematics is perceived as a subject and the connections between this perception and school mathematics. It also identifies the role of beliefs surrounding school mathematics. Both the role of school mathematics and the general perception of school mathematics impact directly on mathematics textbooks.
2.2. *Mathematics as a Subject*

Mathematics textbook authors should be aware of the stereotypes associated with mathematics and ensure that they do not reinforce negative opinions.

There are many explanations and definitions of what exactly mathematics is. Many dictionary definitions describe mathematics as the study of numbers and relationships of numerical and operational systems. One common definition is

“mathematics is what mathematicians do” (Orton and Wain, 1994).

However the reality is that mathematics is more than this, it is alive and present in everyday life, having huge impact on the work of meteorologists, scientists, geologists and many more.

Mathematics is often clearly described as a subject, however no single definition of mathematics as a subject, can be provided. Such a definition would need to relate to a wide range of interested parties and situations. However, one definition which is most intriguing and explains why such a variety of definitions and beliefs about mathematics exist is the definition that:

“mathematics changes over time, for each thoughtful mathematician within a generation often formulates a definition according to his lights.”

(Orton and Wain, 1994: 10 -11)

This suggests that mathematics and its focus changes over time, according to the environment it is applied to. Therefore many varieties of definitions exist to explain mathematics as a subject. Hence finding the place of school mathematics within such varied definitions is complex.

As far back as 1928, D.E. Smith (cited in Orton and Wain, 1994) created a list which he felt provided sufficient reason for the inclusion of mathematics in the school curriculum:
2.2. *Mathematics as a Subject*

- Every educated person should know what mathematics means to society and to our race, what its greatest uses are,
- It has high value as a mental discipline,
- It has intrinsic interest and value of its own - it has its own beauty and magic,
- It possesses truth which, in an ever changing world, is external and enduring,
- It enables us to understand our place in a world which contains such contrasts as the infinite and the infinitesimal,
- It came into being through the yearning to solve the mysteries of the universe and still works for us in this way,
- The history of mathematics is the history of the human race.

(Orton and Wain, 1994: 12)

Over the years many educationalists and mathematicians have amended and edited this list in order to create what they felt was a more modern viewpoint. However, considering that the definition of mathematics varies greatly, and is based upon personal beliefs, it is no wonder that all amendments and updates of the list have significant differences; with emphasis being placed on different specifics and even some areas being omitted. Agreeing on one set list is virtually impossible. A concise summary of these lists was created by Orton and Wain (1994) in which they considered the most frequented and predominant aspects of the modifications and creations. Their summarisation is as follows:

- Mathematics is useful,
- Mathematics is important in our lives and its place needs to be understood,
- Mathematics trains the mind,
- Mathematics is a powerful means of communication,
- Mathematics is enjoyable and has aesthetic value.

(Orton and Wain, 1994: 13)

While there is a need to understand what mathematics as a subject really is, it is vital for this understanding to comply with how the role of mathematics is viewed within the education system.
2.2. Mathematics as a Subject

2.2.1 Role of Mathematics in Education

Mathematics is essential and its importance in education is unquestionable. The role of mathematics in life is a complex one as is that of mathematics in schools. Mathematics can be described simply in terms of counting or measuring and is seen to provide basic languages and techniques for everyday life. On the other hand it can also be seen as abstract, logical and intellectual. Mathematics in education is a combination of both of these (NCCA, 2005a).

The purpose of school mathematics is to educate students’ towards an understanding of mathematics, thus enabling them to utilise mathematics in real life. With regards to the term ‘understanding of mathematics’, Skemp (1976) provides a transparent meaning to understanding. He describes understanding as having two distinct meanings: Instrumental Understanding and Relational Understanding. Instrumental understanding is often defined as ‘rules without reason’. This type of understanding is what is commonly the outcome of traditional mathematics teaching, whereby the student is given a formula or a set of rules and required to learn them off by rote learning, in order to apply them without ever really knowing the ‘why’. In contrast to this, relational understanding involves knowing what to do and why one would need to do it. It involves understanding why a process makes sense and being able to make connections between interrelated ideas and concepts. Despite an abundance of research in support of rejecting the traditional approach to the teaching of mathematics, teaching for instrumental understanding is ever-present. Relational understanding better equips students to cope with mathematics in the real world and also allows mathematics to be the purposeful activity that it is intended to be, where the learner can see and understand a finished product or outcome. The NCCA (2005a) describes mathematics as theoretical and practical, suggesting that mathematics should provide students with the knowledge of ‘how’ and ‘why’. This should therefore provide students with a basis of mathematics to

1NCCA represents National Council for Curriculum and Assessment
2.2. Mathematics as a Subject

assist them in everyday life.

Another purpose of mathematics education is to enhance industry, both economically and socially. School mathematics can also be seen as preparing students for third level mathematics, which in turn is simply a preparation for real-life mathematics. Currently education systems see mathematics as both a theoretical and practical entity. However all areas of mathematics cannot be covered, it would simply be impossible. Thurston (1990) describes mathematics using the analogy of a banyan tree,

“Mathematics isn’t a palm tree, with a single long straight trunk covered with scratchy formulas. It’s a banyan tree with many interconnected trunks and branches - a banyan tree that has grown to the size of a forest, inviting us to climb and explore.” (NCCA, 2005a: 23)

Therefore students are provided with a basis in many of the areas of mathematics which the curriculum designers feel are of most significance, such as algebra, trigonometry and geometry. The detail of each topic delivered by the curriculum and the depth of study varies from topic to topic.

2.2.2 Beliefs Surrounding Mathematics

A teacher’s preferred approaches to teaching can rest entirely upon a their personal beliefs about mathematics. During pre-service training, teachers are exposed to a number of approaches and models of teaching. However, once in the classroom they are free to apply whichever model they believe in. This is often the reason for failed reforms, as to fully change the face of mathematics would require teachers to change their deepest beliefs about teaching and learning (Battista, 1994). Ernest (1988) presented a paper to the 6th International Congress of Mathematical Education in which he discusses the practice of teaching mathematics. In it he suggests some key elements which influence mathematics teaching:
2.2. *Mathematics as a Subject*

- the teacher’s mental contents or schemas, particularly the system of beliefs concerning mathematics and its teaching and learning;
- the social context of the teaching situation, particularly the constraints and opportunities it provides; and
- the teacher’s level of thought processes and reflection. (Ernest, 1989: 249)

Along with this Ernest created a list of components which are significant in understanding how a teacher’s beliefs can impact on student learning. Ernest described how concepts and views of the nature and processes of mathematics are used to predict teaching outcomes. Ernest’s model (Figure 2.1) of the relationship between beliefs and their impact outlines how a teacher’s mental espoused model of learning can be reflected in their classroom experiences and practices.

![Figure 2.1: Relationships between Beliefs, and their Impact on Practice](Ernest, 1988: 252)

A teacher’s mental model is based on his/her own beliefs but is also subject to constraints within the school setting. This suggests that a teacher’s interaction,
2.2. Mathematics as a Subject

communication and resource utilisation is all a reflection of his/her own beliefs.

While Ernest focused on teachers’ beliefs within mathematics many other educationalists have analysed the significance of teacher beliefs for education in general, but all reached similar conclusions, that teacher beliefs impact significantly on their teaching. For example, Kagan (1992: 12) describes teacher beliefs as being “instrumental in determining the quality of interaction”. He also states that beliefs may well provide a clear distinction between good and bad teachers while also reflecting a teacher’s personal growth.

When examining the role of beliefs of teachers and students in the classroom one must also consider the impact of beliefs on learning. Ignacio, Blanco-Nieto, and Barona (2006) highlighted four key beliefs with greatest influence; beliefs about mathematics as an object, beliefs about oneself, beliefs about mathematics teaching, and beliefs about social context. Each of which is connected directly with attitudes and emotions towards learning mathematics. No clear definition of belief can be created without consideration of attitude. For this reason it is difficult to distinguish between the two. However Pehkonen (2003, cited in Kislenko, Grevholm, and Lepik, 2007) provides a base definition for attitudes; involving what people think, feel and behave towards an object. Thinking requires the cognitive domain and feeling needs the affective domain. Both influence the behaviour of the learner. He also provides a definition of beliefs which he describes as overlapping between the cognitive and affective domain making it unavoidable to consider beliefs without considering attitudes.

Learners’ beliefs about learning mathematics have been the source of many complex and detailed studies (e.g. Ernest (1989), Kislenko et al. (2007)). The purpose of such studies is to determine the role of beliefs in student learning. Researchers have applied instruments such as the ‘Fennema and Sherman’ scale (Fennema and Sherman, 1976) (which is constantly being applied and adapted for research) to
2.3. Teaching and Learning Mathematics

discover the attitudes present towards mathematics. Students often view mathematics, despite its importance, as irrelevant, boring and abstract (Kislenko et al., 2007). They have accepted the perception of mathematics as requiring a ‘special talent’. The frank dislike of mathematics by the general population is a sharp reminder of the failure of school mathematics.

Learning outcomes experienced by students are directly linked to their own beliefs about mathematics (Kislenko et al., 2007). The perception of mathematics as a talent and the beliefs about mathematics voiced by family, peers and siblings have often lead students to undermine their own ability before attempting to even tackle mathematical problems. This in turn creates an experience of frustration, discouragement and anxiety - reducing the students’ confidence and self-esteem. Failure in mathematics can be directly linked to self esteem and confidence (Ignacio et al., 2006). All too often students blame their own lack of ability as a reason for failure rather than admitting fear of succeeding. Ignacio also noted that often when success is experienced by students they lay credit for this with their teacher, peers or luck - anything but believing in their own ability.

2.3 Teaching and Learning Mathematics

It is a common belief that learning mathematics is difficult. Mathematics classrooms are diverse by nature which makes finding common ground for research on teaching and learning problematic. While teaching and learning are interlinked one must first separate each area and try to analyse it with respect to the other. Firstly, one must look at mathematics teaching and effective teachers of mathematics in order to determine how teachers can impact on learning. There are many criteria which describe an effective teacher, many of which are provided from the perspective of a student such as enthusiastic, helpful etc, but effective teachers need more than personality if they are to impact positively on learning outcomes.
2.3. *Teaching and Learning Mathematics*

Research has yet to find any characteristic which can definitely and directly be connected with student learning, however Neyland (1995: 34-48) describes eight approaches to teaching mathematics, all of which come with their own positive and negative attributes.

- **New Maths,**
- **Behaviourist Approach,**
- **Structuralist Approach,**
- **Formative Approach,**
- **Integrated Environment Approach,**
- **Problem Solving Approach,**
- **Cultural Approach,**
- **Social Constructivist Approach.**

**‘New Maths’:** The new maths theory (piloted in New Zealand in the 1970’s) is a drastic attempt to improve student’s learning, while providing school mathematics with a foundation and concept unity. New math represents mathematics as a coherent, logical and organised subject. However, it fails to recognise the important pedagogical concerns of mathematics such as providing a link between topics, and as such was never fully accepted.

**Behaviourist Approach:** The behaviourist approach is based on educational psychology rather than mathematics. This approach draws together the work of many educational psychologists such as Skinner, Bloom and Gagne and it proposes to organise mathematics into precise steps to optimise learning. However this does not encourage the construction of new ideas and also fails to examine misconceptions.

**Structuralist Approach:** This approach combines mathematical and psychological theory. It relies on the teacher introducing the essential structures and
2.3. Teaching and Learning Mathematics

processes of mathematics and allows the students to use these as frameworks to
develop their own mathematical thinking. This too has shortcomings, in that
not enough emphasis is placed on students creating their own structures and also
embodiments were often too confusing.

**Formative approach:** The formative approach is learner centred, provides learn-
ing opportunities and is built on Piaget’s idea of constructing one’s own knowledge.
However, its short-comings are similar to those experienced by the integrated en-
vironment approach, in that it requires unrealistic quantities of individual teacher
time.

**The Integrated Environmentalist Approach:** This approach relies on using
statistics, modelling thematic units and project work. However, like the formative
approach it too requires unrealistic amounts of individual teacher time.

**Problem Solving Approach:** This was brought to the fore in 1980 when the
NCTM\(^2\) published ‘Agenda for Action’ followed closely by Cockcroft’s report in
1982. This approach, it was believed, would provide teachers and students with ac-
cess to mathematical processes. It was intended that this approach would highlight
the reasons for doing mathematics, would place an emphasis on strategies - not
rules, and encourage the creative and developmental aspects of mathematics. Sadly
this approach also had failures in that problem solving was considered by teachers
as a stand alone unit separate from all other aspects of the mathematics curriculum.
One reason for this is that teachers did not fully understand how to approach the
content and textbooks presented problem solving through a behaviourist approach
failing to provide support for teachers.

**Cultural Approach:** This approach suggests providing a link from the students’
mathematics to their lives and cultures. However this would require a complete

\(^2\)NCTM = National Council of Teachers of Mathematics
2.3. Teaching and Learning Mathematics

rewrite of the curriculum.

**Social Constructivist Approach:** This approach is based on the Vygotskian perspective. Students would be actively engaged, implicit and explicit uses of mathematics would be discovered and existing mathematical knowledge would be used as a basis for understanding new mathematics (Neyland, 1995). The Social Constructivist approach was far-fetched in that it would require the teacher to have a solid understanding of the perspective approaches, attitudes and processes of the entire mathematical community. Despite this flaw it did however provide a basis for research, combining a number of areas of each approach to assist with the pedagogical approaches to teaching mathematics.

The most significant of these approaches is the Problem Solving Approach. It had the greatest potential for positively impacting on student learning. However the textbook was one of the main factors contributing to its failure in that they failed to adapt accordingly. In the Cockroft Report (Cockroft, 1982), there is reference to the increase in popularity of a ‘Problem Solving’ approach in the early 1980’s, however, Cockroft notes that despite the change in curriculum focus and the obvious acceptance of this approach, textbooks never followed suit. Since the Cockroft Report many curriculum initiatives have applied a problem solving approach or an adapted problem solving approach. One such initiative is currently underway in Ireland at present. Project Maths, a new mathematics curriculum which was implemented nationwide in September 2010, incorporates many of the ideals of the problem solving approach such as focusing on problem solving strategies in favour of rules and procedures.

Another area fundamental to learning outcomes is teacher pedagogical knowledge. The next section highlights the key areas and considerations regarding teacher knowledge and its impact on student learning.
2.3. Teaching and Learning Mathematics

2.3.1 Pedagogical Knowledge

One area of research which has proven to have significance for student learning is that of teachers’ subject knowledge. Teachers communicate mathematics based on their own understanding of it (Skemp, 1976). This idea has been widely researched for over twenty years and without question command of subject matter knowledge is essential in the teaching of any subject. However while two teachers may have similar mathematical understanding and knowledge backgrounds, their respective attitudes and beliefs will undoubtedly inform the approaches and emphases they place on certain aspects of mathematics (Ball, 2000) and hence will impact on how they use and interpret a mathematics textbook. Mathematical competency is one of many factors influencing effective teaching. Mathematical competence in teaching is related to how one understands and applies the pedagogical considerations which impact on learning.

Lappan and Theule-Lubienski (1994, cited in Cooney, 1999) considered this idea of pedagogical knowledge combined with mathematical competency and concluded that teachers need a basis of knowledge in three areas; knowledge of the mathematics, knowledge of their students and knowledge of the pedagogy of mathematics. Fajemidagba (1998) found that competent mathematical teachers when teaching a new topic, provide a history, an explanation of why and where within mathematics it is applicable and apply simple and coherent counting and measuring practises initially. A teacher who is pedagogically competent can follow such a format and will be proficient in providing students with a rationale for any terms, definitions and algorithms. This correlates directly with Cooney (1999) supporting his idea of creating a constructivist orientated approach to teaching, where the medium and the message are inseparable suggesting that one must look at teaching as a whole. That is, one must do more than simply provide rules and formulae. Such research is significant in the context of mathematics textbooks in that textbooks may be used to fill the gaps for non-competent teachers, for example, the textbook
2.3. Teaching and Learning Mathematics

can provide details on applications and history of new topics. A recent study in Ireland by Ní Ríordáin and Hannigan (2009) identified that nearly 50% of the teachers currently teaching mathematics in Ireland are out-of-field teachers, i.e. they do not have any formal qualifications in mathematics teaching. Such teachers are commonly found teaching lower level secondary school mathematics and may benefit more from ‘effective textbooks’.

Student experiences and exposures vary greatly; one student’s perception is not necessarily identical to another’s. The values held and emphases used by the teacher will vary and influence student exposure. Similarly, one teacher’s expected outcomes can often have no resemblance to a neighbouring teacher’s outcomes in the same subject. Reasons such as these provide barriers to research on teaching and learning theories, and explain why one may experience difficulties when attempting to generalise mathematical education research (Berliner, 2000). Along with this one must consider ‘constructivism’, described by Snowman and Biehler (2006) as being fundamental to human learning. Constructivism is how one interprets information and it is unique to each individual. Constructivism is compatible with situated learning theory. They both support the idea of students actively participating in their own learning, specifically in problem solving and critical thinking; both of which are imperative in mathematics education.

Metacognition is also an important factor in learning which can clarify the process of learning mathematics. Metacognition is one’s awareness of the process of learning, a simple process of monitoring one’s progress and making the necessary changes and adaptations to consistently work towards intended goals (Winn and Snyder, 1998). While metacognition is critical to effective teaching and learning, many teachers and learners alike fail to recognise metacognition and the role it plays in education. Finally, Skemp’s (1976) work on the psychology of learning mathematics has a bearing on these considerations especially his classification
2.3. Teaching and Learning Mathematics

of understanding as instrumental and relational. Instrumental understanding is often defined as ‘rules without reason’ and results from traditional mathematics teaching. Relational understanding involves knowing what to do and why to do it, understanding why a process makes sense and being able to make connections between interrelated ideas and concepts. These, when used in isolation from each other can prevent students from learning mathematics. Skemp (1976: 91) describes them as “two effectively different subjects being taught under the same discipline”.

Having considered general issues related to teaching and learning mathematics, the next step is to further analyse a topical issue in teaching and learning mathematics which is conceptual development in mathematics.

2.3.2 Conceptual Development

Conceptual change can be described as “learning that changes an existing conception” (Orey, 2001: 1). It requires adjusting existing knowledge by shifting or reconstructing it in order to fundamentally change the existing concept to accept a new concept. Incorporating conceptual change into education can enhance a student’s ability to overcome misconceptions and make them better prepared to deal with more difficult concepts. Vosniadou (1994) emphasised that the easiest form of conceptual change is the addition of new information to existing knowledge. For example after learning the multiplication of natural numbers the conceptual change which would encourage development would be facing the problem of multiplication of fractions (Vosniadou, 1994). There is a need for conceptual development when new knowledge is incompatible with a student’s existing knowledge. Students create presuppositions very early on in their learning which influence their beliefs. When they face new knowledge it may not fall into line with their self-created beliefs so they need to create a synthetic model which combines the new theory with their individual beliefs and understandings (Biza, Souyoul, and Zachariades, 2005).
2.3. Teaching and Learning Mathematics

Introducing conceptual development, the process where students are encouraged to question and reshape existing knowledge, would allow students to combine new and existing knowledge. This can be achieved by reshaping and modifying their existing synthetic models so that all knowledge (new and old) can assist in the formation of their understanding and beliefs, thus developing students’ concepts of pattern recognition and abstract operations, (Snowman and Biehler, 2006). In order for conceptual development to occur in the classroom, teachers of mathematics must work on trying to remove misguided assumptions and preconceptions. According to Merenuoto and Lehtinen (2004), conceptual development fails when students lack sufficient prior knowledge. When this occurs undetected student preconceptions very often create misconceptions. Preconceptions and misconceptions are difficult to change, meaning teachers can only help students advance in their learning by assisting them in creating and understanding new conceptions. In addition, there is a need for the affective, social and contextual factors which influence a classroom to be considered as they all contribute greatly to a student’s conceptual development. This incorporates the pedagogical knowledge of teaching - a teacher needs to combine his/her constructivist approaches with metacognition to allow the learner to actively reorganise their knowledge. Duit (1999) suggests applying Piaget’s cognitive conflict strategy, (which fosters Posner et al’s theory of conceptual change) in order to create a learning environment which truly captures the meaning of conceptual development (Kleine, Jordan, and Harvey, 2005).

There have been many instructional models for conceptual change but Nussbaum and Novick (1984, cited in Orey, 2001) analysed each of these and created four goals common to all models which will help teachers guide students towards reconstructing their concepts:

---

3 Cognitive conflict occurs when there is a discrepancy between what the student believes to be true and what he/she is experiencing. Piaget’s strategies for cognitive conflict involve creating situations where the students’ existing conceptions are made explicit and then directly challenged in order to create a state of ‘cognitive conflict’.
2.3. **Teaching and Learning Mathematics**

1. First one must reveal the students’ preconceptions,
2. Then these preconceptions must be evaluated,
3. Create a conceptual conflict using the given preconceptions,
4. Then encourage the students to take onboard this conflict and use it to create a new concept.

For practices such as conceptual development to occur naturally in a classroom there needs to be a move from the old traditional style of teaching mathematics, hence changes need to be brought about in the mathematics classroom.

### 2.3.3 Changes in the Mathematics Classroom

Greer and Mulhern (1992) provide an interesting explanation for the lack of change in the teaching of mathematics. They conclude that despite teachers knowing and wanting constructivist learning and active engagement of students to occur, these ideals are rarely enacted. Consequently, teachers’ dependency on didactic teaching approaches is limiting student learning in the classroom. Teachers are afraid to change their teaching methods as they believe that these methods are successful in attaining better grades. However, with the growth in mathematics education research, classroom norms have changed and the following practices have emerged as good practice:

- Active learning
- Cooperative learning
- Technology supported learning

**Active learning:** involves the ‘doing’ of mathematics by the student. It incorporates significantly more than ‘pen and paper’; it encourages students to visualise, to predict, to model and to create where possible. This approach to classroom learning allows students not only to express their understanding but also to learn
2.3. Teaching and Learning Mathematics

how to learn. Hirsch (1992) describes how an active learning approach to learning the Sine Rule in trigonometry allows students to work with actual triangles, to measure and to calculate. In this way, the proof comes alive for the students and the theory behind it is learnable.

Cooperative learning: involves the students working in groups - a far cry from the traditional, silent and individualised classrooms. This approach models the way mathematics is applied in today’s working world (cooperatively) and, according to Hirsch (1992), appeals to the students’ intuitive side and encourages an open environment for questioning and discussion.

Technology supported learning: is gaining popularity rapidly even though the transition from research to classroom practice is slow. Technology in schools is improving annually, as more and more funding is allowed for improving technology. In 2000 Twining outlined that 100% of Irish secondary schools now have computer access but only 66% of these have broadband internet access (Twining, 2000). While the figures improved for 2003, the O.E.C.D. (2003) reported that Ireland was still below average in terms of student to computer ratios (9:1 v’s 6:1) and availability of broadband internet on school computers (67% v’s 78%). The NCTE (2005) recommends that computers be integrated into the general classrooms and not be confined to designated computer rooms. They also recommend that all computers should be networked and broadband enabled. A survey in Ireland reported by Shiel and O’Flaherty (2005) identifies that in 2005 the student to computer ratio in post primary school in Ireland was 7:1, which is an improvement from the 2002 figure of 7.4:1 and they also noted that only 4% of the computers in post primary schools are located in general teaching classrooms. Technology removes the mundane and abstract stigma of many algebraic manipulations while also permitting instruction to become more diversified and individualised.
2.3. Teaching and Learning Mathematics

**Developmental learning:** This approach reflects how students acquire mathematical knowledge. It considers how students develop knowledge based on what they already know, providing teachers with a framework for creating lesson plans and ideas.

Changes in classroom practice may be largely connected with changes in mathematics textbooks due to the dominant position of mathematics textbooks in classrooms worldwide (Hiebert et al., 2003). Perhaps, only when there is a movement towards changes in classroom practices (e.g. move towards effective problem solving) will effective teaching become common practice in mathematics education. The public image of mathematics is having a detrimental effect on the attitudes and decisions of students regarding mathematics. This public image is based on the face of the mathematics classroom. Moving away from the traditional ‘chalk and talk’ approach to a more student friendly environment may help to remove many of the negative descriptions associated with mathematics such as “difficult”, “cold”, “abstract”, “remote”, “inaccessible”, “anxiety ” and “fear”, enhancing the possibility of effective learning.

2.3.4 How Children Learn Mathematics

Researchers tend to look at how students learn specific areas/topics within mathematics. For example, there are numerous articles on how a child learns place value or arithmetic. Detailed research of an overview of how children approach and learn mathematics in general is limited. However it is clear that even before children enter the education system they are equipped with mathematical knowledge (Resnick, 1989). In fact, Hiebert (1984) suggests that children enter school with relatively good problem solving abilities and lose this ability somewhere along the way. Children are naturally curious, making them intrinsically motivated and they have an informal problem solving ability (Baroody and Ginsburg, 1990). However this knowledge base and mathematical understanding does not necessarily transfer
2.3. Teaching and Learning Mathematics

to the way mathematics is taught in schools. Children’s initial knowledge base is dependant on the concept of counting and thus they often see mathematics as a process of using this calculation, for example using their counting methods to assist in solving products, difference, etc. (Baroody and Ginsburg, 1990).

Limitations also exist in that, the ever-present knowledge base of the child is not complete or logical. This is where school mathematics finds its role. One of the functions of school mathematics is to assist children with the process of assimilation whereby the children can digest and interpret new information using their existing knowledge as a base for conceptual development. However a teacher cannot decide for the child when he/she needs to assimilate information, this can only occur when the child is actively engaged both intellectually and emotionally in any situation which will naturally excite their curiosity (Davis and McKnight, 1980).

Children learn by combining form and understanding (Skemp, 1971). From the point of view of learning mathematics, form can be seen as the “syntax of the system” (rules or patterns associated with mathematics) and understanding the “semantics of the system” (meaning and mathematical developments) (Hiebert, 1984: 498). While children can easily attain these independent of each other they must be combined for learning to occur. The idea of creating this combination can be traced as far back as 1949. The three main links to providing this connection are (all of which should be evident in the mathematics textbook):

1. Develop a meaning for the symbols learned (a connection with the real world)
2. Linking procedural rules and conceptual knowledge (most complex, as the amount of knowledge increases this becomes more difficult to ascertain)
3. Checking the rationality of solutions (take a step back and consider how rational the approach and answer are).

(Hiebert, 1984)
2.4. Problem Solving

Perhaps this is where the collaboration of children’s learning and school mathematics teaching falls apart - at the beginning. Research has indicated that children enter schools with reasonable mathematical ability and strategies, yet some where along the line the children opt to “abandon their analytical approach and solve problems by selecting a memorised algorithm based on a relatively superficial reading of a problem” (Hiebert, 1984: 496).

2.3.5 Conclusion

This section outlines the main theoretical implications for learning mathematics. It identifies the processes by which students learn and ways of teaching which can impact of these processes. In addition, this section also outlines a number of considerations which are significant for mathematical textbook research such as conceptual development. The process of how children learn mathematics is fundamental to mathematics education, in order to teach effectively one must be aware of how students learn. It is important to realise that the teacher of mathematics is situated in a most influential position as regards students’ learning in mathematics.

2.4 Problem Solving

2.4.1 Introduction

The problem solving approach has been identified in section 2.3 as an effective approach to teaching mathematics. This section outlines the types of problems, methods of teaching problem solving and how teachers themselves can adapt to incorporate a problem solving approach in their teaching.

2.4.2 Problem Solving

A problem can be defined in many ways. For the purpose of school mathematics Kilpatrick (1985: 2) provides the following definition of a problem:
“a problem is defined generally as a situation in which a goal is to be attained and a direct route to the goal is blocked..[which] usually requires the presence of a person who has the problem....[and an] activity of a motivated subject”.

Problem solving became the theme of mathematics education in the 1980’s both in the USA and the UK, so strong was the belief in problem solving that the NCTM (1980: 1) stated that it “must be the focus of school mathematics”. Problem solving has caused much debate since this, with many reforms and revisions trying to adapt and improve it in the school situation. According to Schoenfeld (1992) one of the key failures as regards problem solving is encapsulated in the knowledge that it is problems that have been ever present in the school curriculum not problem solving, and therein lies the issue.

The role of problem solving is unclear, Webster (1979, cited in Schoenfeld, 1992) provided two definitions for problem solving. The first describes problem solving as anything in mathematics that is required to be done or requires the doing of something. This would suggest that routine practices and exercises would fall into the category of problem solving. However, his second definition suggests problem solving is a question that is perplexing or difficult. These conflicting definitions appear on the same page yet are worlds apart. Further reinforcing this confusion associated with problem solving, Schoenfeld (1992) provides a list of categories of goals established for a third level problem solving course. This list varies from portrayal of problem solving goals, to thinking creatively and to learning standard techniques.

The purpose of problems can be easily understood, students need to move away from routines and rules in order to express their thoughts as they understand them (NCTM, 1989). Problem solving can equip students, not only with mathematical knowledge, but also with the ability to communicate mathematics. Schoenfeld
2.4. *Problem Solving*

(1992: 338) provides three purposes for problems, some of which are evident in classrooms and some which should be:

1. Problem Solving as Context,
2. Problem Solving as a Skill,
3. Problem Solving as an Art.

**Problem Solving as a Context** is arguably the most predominant use of problems. It sees problem solving as a justification for teaching mathematics. It involves providing real life applications for routine exercises in order to highlight that mathematics is used in real life. These type of problems are used as recreation at the end of units/chapters and describe themselves as offering motivation for mathematics as fun. However this can also been seen as a break from ‘real’ mathematics, portraying problem solving as a separate entity from mathematics. The last type of ‘problem’ found in this category are those which are used for practice, the vast majority of school mathematical problems are used simply for the practice of routines and procedures which do not require much critical or analytical thinking.

**Problem Solving as a Skill.** This view dates back at least as far as 1901 to Thorndike and Woodworth who believed that if one could learn reasoning skills they would improve their ability to perform reasoning with problems. This view regards problem solving as a value in its own right, as one single skill. Frequently in school mathematics problem solving is considered as a number of skills combined but never as one single skill in itself.

**Problem Solving as an Art** is a strong contrast to the above two purposes of problems. This approach portrays problem solving as the “heart of mathematics if not mathematics itself” (Schoenfeld, 1992: 338). It supports the ideas expressed by Halmos (1980), when he stated that the purpose of mathematics is to solve
2.4. *Problem Solving*

problems. Halmos suggests that mathematics is problem solving. This is reinforced by Orton (2004: 25) when he described problem solving as the “real essence of mathematics”.

Schoenfeld (1992: 348) also identified five aspects of cognition which are crucial to effective problem solving. Each aspect comes with its own values and merits, and together they can provide a greater insight into the task of teaching and learning problem solving:

1. Knowledge Base,
2. Heuristics (Problem Solving Strategies),
3. Self Regulation (Monitoring and Controlling),
4. Beliefs and Affects,
5. Practices.

**Knowledge Base**: encompasses how information is found and stored in the brain. It is believed that in order to benefit from one’s knowledge base one must construct representations, thus indicating that what one already knows is what is important to solving problems, not what one does not know. A knowledge base consists of informal and intuitive knowledge, facts, definitions, algorithmic procedures, routine procedures, competencies and knowledge of rules. The following diagram (Figure 2.2) indicates the processes involved for problem solving.

The knowledge base allows one to classify problems into types. This categorisation occurs without formulating a solution and one can then use this categorisation along with their body of information to formulate the problem and move towards a solution.
Heuristics: The notion of Heuristics in mathematics education dates back to Polya (1945). His book *How to Solve it* focuses on the creation of strategies to enhance the ability to solve problems. Studies such as that completed by Kantowski (1977) correlates student use of heuristics with performance. This is supported by Heller and Hungate (1985) who suggest that the key to teaching problem solving is to get students discussing the problem. This can be done by providing them with guided practice and emphasising the need for qualitative understanding and specific procedures. The mathematics textbook can facilitate this activity with such guided practice. Textbooks present problem solving in two different ways, in separate sections or sprinkled throughout the text. Problem solving is the heart of mathematics (Halmos, 1980), therefore it cannot be dealt with in isolation in mathematics textbooks. When teaching problem solving it is important that teachers remember that heuristics cannot be learned, if an attempt is made to teach heuristics then they become algorithms. Heuristics are created when students are given sufficient time and support to tackle and solve problems. Teaching problem
2.4. Problem Solving

solving is a difficult task and it places three separate demands on the teacher.

1. Mathematically: teachers must perceive student approaches and whether or not they are useful

2. Pedagogically: teachers must decided when to intervene and what suggestions to offer without providing solutions

3. Personally: teachers must be prepared to admit that they may not know all the answers. This requires experience, confidence and self awareness.

(Burkhardt, 1988)

Self Regulation: involves the process of monitoring and assessing progress as you work. Studies carried out by Schoenfeld (1992) demonstrate how experts divide their attention while solving a problem. The graph obtained by Schoenfeld indicates that the major focus should be placed on reading and analysing the problem throughout the time allocated.

Figure 2.3: Time Line Graph of a Mathematician Working a Difficult Problem (Schoenfeld, 1992: 356)

Along with this graph Schoenfeld (1992) provides a table of teaching actions for problem solving which are designed to enhance a teacher’s ability to encourage self regulation.
### Teaching actions for problem solving

<table>
<thead>
<tr>
<th>Teaching Action</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BEFORE</strong></td>
<td></td>
</tr>
<tr>
<td>1. Read the problem... Discuss words or phrases students may not understand</td>
<td>Illustrate the importance of reading carefully; focus on special vocabulary</td>
</tr>
<tr>
<td>2. Use whole-class discussion to focus on importance of understanding the problem</td>
<td>Focus on important data, clarification process</td>
</tr>
<tr>
<td>3. (Optional) Whole-class discussion of possible strategies to solve a problem</td>
<td>Elicit ideas for possible ways to solve the problem</td>
</tr>
<tr>
<td><strong>DURING</strong></td>
<td></td>
</tr>
<tr>
<td>4. Observe and question students to determine where they are</td>
<td>Diagnose strengths and weaknesses</td>
</tr>
<tr>
<td>5. Provide hints as needed</td>
<td>Help students past blockages</td>
</tr>
<tr>
<td>6. Provide problem extensions as needed</td>
<td>Challenge early finishers to generalize</td>
</tr>
<tr>
<td>7. Require students who obtain a solution to &quot;answer the question&quot;</td>
<td>Require students to look over their work and make sure it makes sense</td>
</tr>
<tr>
<td><strong>AFTER</strong></td>
<td></td>
</tr>
<tr>
<td>8. Show and discuss solutions</td>
<td>Show and name different strategies</td>
</tr>
<tr>
<td>9. Relate to previously solved problems or have students solve extensions</td>
<td>Demonstrate general applicability of problem solving strategies</td>
</tr>
<tr>
<td>10. Discuss special features, e.g. pictures</td>
<td>Show how features may influence approach</td>
</tr>
</tbody>
</table>

(Adapted from Lester, Garofalo, & Kroll, 1989, P. 26)

**Figure 2.4: Teacher Actions for Problem Solving**
(Schoenfeld, 1992: 358)

The role of the teacher is demanding and difficult, in that it is to monitor and facilitate each student. Many teachers fail to encourage determination and often once frustration sets in or a mistake has been made they will offer a solution to
2.4. Problem Solving

students. However according to the PISA Report, (O.E.C.D., 2003)\textsuperscript{4} in Asian countries (Asian countries scored highest on mathematical tests) it is normally accepted that the more time and thinking involved in solving a problem the more learning has occurred.

Beliefs and Affects: One of the main student beliefs is that if you understand a problem it should take you approximately two minutes to solve, if it takes longer than this - give up (Schoenfeld, 1989). This idea contradicts the main belief present among the top scoring countries in the PISA study (O.E.C.D., 2003), that struggling with a problem evokes understanding. The following table from Schoenfeld (1992: 69) outlines the most common student beliefs:

\begin{center}
\begin{tabular}{l}
\textbf{Typical student beliefs about the nature of mathematics}\\
\hline
\textbullet Mathematics problems have one and only one right answer.\\
\textbullet There is only one correct way to solve any mathematics problem -- usually the rule the teacher has most recently demonstrated to the class.\\
\textbullet Ordinary students cannot expect to understand mathematics; they expect simply to memorize it, and apply what they have learned mechanically and without understanding.\\
\textbullet Mathematics is a solitary activity, done by individuals in isolation.\\
\textbullet Students who have understood the mathematics they have studied will be able to solve any assigned problem in five minutes or less.\\
\textbullet The mathematics learned in school has little or nothing to do with the real world.\\
\textbullet Formal proof is irrelevant to processes of discovery or invention.\\
\end{tabular}
\end{center}

Figure 2.5: Student Beliefs
(Schoenfeld, 1992: 359)

\textsuperscript{4}O.E.C.D represents the Organisation for Economic Cooperation and Development
2.4. Problem Solving

Student beliefs are directly linked to determination and effort. Teachers are crucial in the development process of student beliefs. Thompson (1985: 288-290) interviewed two mathematics teachers and discovered that the students of one teacher performed better than the other. He believes that the teacher’s attitudes contributed to this. The first teacher believed she was present to transmit her knowledge to the students, while the second teacher, whose students where achieving higher grades, believed in the discovery and verification of mathematics through the process of discussion and exploration and by encouraging her students to seek answers and not to give up. The second teacher’s approach to mathematics teaching allowed for effective problem solving to take place whereas the first teacher was restricting the students to routine problems and procedures.

Practices: Beliefs and attitudes can instigate bad classroom practices. In order to enhance classroom practices for problem solving Schoenfeld (1992: 87) recommends:

- an open and comfortable classroom atmosphere,
- modelling problem solving behaviours, exploring and experimenting,
- encouraging student explanations,
- allowing for multiple strategies,
- and presenting real life problem situations.

Based on the work of Polya (1945) and then Schoenfeld (1992), many researchers created their own frameworks for solving problems. The following framework was included in the PISA Report (O.E.C.D., 2003: 170-171) and its purpose is to highlight the processes involved in solving a problem:
2.4. Problem Solving

Step 1: Understand the Problem
This involves understanding the text, diagrams and any relevant information to the problem, one must draw references and relate information.

Step 2: Characterise the Problem
This involves identifying relevant and irrelevant variables, constructing hypothesis, retrieving and organising information, considering and evaluating contextual information.

Step 3: Representing the Problem
This involves the construction of tables or graphs, verbal or symbolic representations and the ability to shift between representations.

Step 4: Solving the Problem
This involves making a decision, designing or analysing a system, proposing or diagnosing a solution.

Step 5: Reflecting on the Solution
This involves examining solutions, looking for clarification, evaluating solutions from different perspectives in order to reconstruct or justify the solution.

Step 6: Communicating the Solution
This involves selecting the appropriate media and representations to express and communicate the solution.

2.4.3 Types of Problems

The PISA Report (O.E.C.D., 2003) also created three classifications of problems to assist with their research:

Decision Making Problems:
Decision Making problems require students to understand a situation which involves a number of constraints and alternatives. In order to solve such a problem one must comprehend the information provided, identify the relevant features, create a representation and make a decision.

System and Analysis Problems:
System and analysis problems require a student to analyse a complex situation in order to understand its logic or to design a system which works to achieve desired goals. In order to solve such a problem one must analyse or
2.4. Problem Solving

design, identify related variables and discover how they interact, evaluate, justify and communicate the solution.

Trouble Shooting Problems:

Trouble Shooting problems require students to comprehend the main features of a problem and diagnose a fault, while also demanding an understanding of the logic of the mechanism in question. In order to solve such problems one must understand how the procedure/device works, identify the critical features, create or apply representations, diagnose the problem, propose a solution and execute the solution.5

Similarly, problems can be considered ‘Routine Problems’ or ‘Non - Routine Problems’, (Diaz and Poblete, 2000). The selection of problems which fall into the category ‘Routine Problems’ can be further categorised into Real, Realistic, Fantasy and Purely Mathematical.

Real Context Problems

Include the type of problems which are created in a real environment and which involve or engage the student.

Realistic Context Problems

Include the type of problems which have the possibility of being reproduced, involves a stimulation of reality.

Fantasy Context Problems

Include the type of problems which are not based on reality and are the product of imagination.

Purely Mathematical Context Problems

Include the type of problems which are exclusively mathematical and relate only to mathematical objects.

Non Routine Problems are those which cannot be answered using routine procedures, each of these can also be classified as Real, Realistic, Fantasy and Purely Mathematical.

5A more detailed description of each problem type is outlined in Appendix A which is a table of problem types (O.E.C.D., 2003: 172).
2.4. Problem Solving

Mathematical. These classifications are echoed by Orton (2004) when he categorises problems into four types: Routine Problems, Novel Problems (which can be likened to non-routine problems), Word Problems (which encompasses realistic problems) and Real Life Applications (identical to real context problems). All of these classifications of problems reflect the work of Polya (1945). In the early 1980’s in his book *Mathematical Discovery: on understanding, learning and teaching problem solving*, Polya consolidates much of his work (which dates back to the 1940’s) to give brief summaries of each type of problem. Polya describes problems as having one of four characteristics:

- Rule under your nose
- Application with some choice
- Choice of a combination
- Approaching a research level.

These characteristics are the basis on which the Schoenfeld and the OCED classifications are justified and developed.

2.4.4 Teaching Problem Solving

“we do not have a final vision of what problem solving is and how to teach it, but we are much more keenly aware of the complexity of both.”

(Kilpatrick, 1985)

The teaching of problem solving is as topical in the world of research as problem solving itself. There is an abundance of research on effective teaching as outlined in the next section. However many researchers of problem solving felt the need to include reference to the teaching styles and methods which they have found to enhance conceptual learning with particular regard to problem solving. Kilpatrick (1985) suggests there are two approaches to teaching problem solving, a top-down approach or a bottom-up approach. The top-down approach encompasses the idea
2.4. Problem Solving

of heuristics provided by Polya (1945). This approach recommends using guided practice and open discussions to encourage students to develop their problem solving skills. The Bottom-Up approach focuses very much on the concepts and the skills involved in problem solving. It places more weight on the pedagogical and cognitive aspects of making a skill automatic. However, both approaches have limitations; practice alone is not sufficient and memorisation is only effective for a minimal number of problems. In saying that, the research presented by Kilpatrick (1985) does indicate that a Top-Down approach may be more effective because the main limitation to problem solving is instruction (which is a large part of the bottom-up approach) and cooperation is a contributory factor to student learning. Group problem solving sessions and discussions help to clarify problems for students. Papert (1975) credits the educationalists Dewey, Montessori and Piaget with the honing in on the idea that children learn by doing. This supports a Top-Down method of teaching problem solving as Polya’s heuristics were initially designed to prompt student thinking.

Polya likened problem solving to a practical skill such as swimming, believing that it could be learned by following four guidelines: prepare, imitate, practice and verify. Similarly, the following figure, provided by Orton (2004: 87) indicates that the processes required for solving problems can be obtained by following guidelines.
When teaching problem solving it is important for teachers of mathematics to remember that there are many obstacles to learning how to solve problems. One of the main obstacles, as outlined by Orton (2004), is fixation. Fixation refers to the initial false assumptions that learners are prone to create. Orton (2004: 87) describes fixation as follows, “if insight is the essential element in intelligent
2.4. Problem Solving

problem solving then fixation is its arch enemy”. Other obstacles such as over motivation, limitations on the ‘human information processing system’ are also outlined. Humans, unlike computers, are limited to one process at a time, with each process being passed through the short term memory. Short term memory has only the ability to hold seven units of information (Miller, 1956). This creates another obstacle in that, while long term memory seems to have unlimited capacity, there are difficulties in transferring information from the short term to the long term memory.

2.4.5 Teachers of Mathematics

“Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn well” (NCTM, 2000: 16).

Like everyone else, teachers learn from experience (Maher and Alston, 1990). However, where there is little or no guidance teachers can often create pedagogical theories which are based on poor and disappointing teaching. Teacher effectiveness relies upon the interaction of subject matter and pedagogy. Research on effective teaching has indicated that due to the variety of criteria and characteristic changes with regard to context, it is rather difficult to generalise effective teaching. However as Kyriacou (1986) highlights this must not prevent further research but encourage researchers to specify exactly the context and focus of their research. Teachers need to capitalise on learning opportunities as they arise in the classroom. Students’ explanations and questions will exhibit their understandings and beliefs about a topic (Cobb and Yackel, 1996). These learning opportunities are not reserved solely for the students. The teacher also needs to avail of any opportunities to learn, as the notion that the teacher has all the knowledge and must simply decide how to transmit it no longer stands. Teachers need to work within a constructive pedagogy and their role needs to move towards that of a facilitator. Ernest (1988),
2.4. Problem Solving

provided a view of the three main roles a teacher can assume:

<table>
<thead>
<tr>
<th>TEACHER’S ROLE</th>
<th>INTENDED OUTCOMES</th>
</tr>
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<tbody>
<tr>
<td>1. Instructor:</td>
<td>Skills mastery with correct performance</td>
</tr>
<tr>
<td>2. Explainer:</td>
<td>Conceptual understanding with unified knowledge</td>
</tr>
<tr>
<td>3. Facilitator:</td>
<td>Confident problem posing and solving</td>
</tr>
</tbody>
</table>

Facilitating learning is far more effective than lecturing and creates an environment where students are engaged in individual and collaborative learning. In turn this will arouse students’ natural curiosity and encourage them to see mathematics as a personal challenge. This will allow them to believe mathematics is achievable and makes sense, thus providing opportunities to discuss issues or problems as they arise. This is reinforced by Wilson, Cooney, and Stinson (2005), who provide a list of four key characteristics of an effective teacher. This list infers that a teacher must promote mathematical understanding, have good subject matter knowledge, engage and motivate students and efficiently manage time and the classroom.

Bulger, Mohr, and Walls (2002) provides four key points (which he refers to as aces) to enhance teaching.
Ace 1: Outcomes: students need to have clear outcome goals to focus their attention. Outcomes will direct students and provide the teachers with a framework for effective teaching and a measure for assessing student learning.

Ace 2: Clarity: To be an effective teacher, one must provide clarity of instruction to students through the provision of explicit directions and explanations. Teachers need to engage in effective instructional practice by providing various methods of delivery to enhance students’ abilities and opportunities to understand. This combines conceptual development and the idea of scaffolding.

Ace 3: Engagement: This encourages students to be active participants in their own learning and in the construction of their own knowledge (based on Piaget’s theories). Effective teaching engages students in order to facilitate their learning of knowledge, skills and processes.

Ace 4: Enthusiasm: Effective teachers need to demonstrate a love and interest
in the topic they are teaching. Enthusiasm is contagious and it reflects a teacher’s competence and confidence in themselves and their teaching. Enthusiasm can be introduced easily by being actively involved in and around the classroom, using students’ names and by reinforcing participation.

2.4.6 Conclusion

This section identifies how to classify problem types for analysis and how teachers and textbooks should present and approach problems. Having reviewed the literature of mathematics as a subject in the classroom, how students learn this mathematics and how teachers can be effective in their role of facilitating this learning it is necessary to examine one more aspect of mathematics education which has a significant impact on the teaching and learning environment which is language.

2.5 Mathematics Language

2.5.1 Introduction

This section outlines mathematics as a language in general, it identifies key considerations for textbook research with regard to language, vocabulary, symbols and reading mathematical texts.

2.5.2 Mathematics Language

Pimm (1997) suggests that learning mathematics can be likened to learning a foreign language. Mathematics is a language and speaking mathematically requires speaking the language patterns of mathematics. The language of mathematics was created simply to allow mathematics to be fully expressed. Bullock (1994) supports this by pointing out that Newton had to invent calculus in order to develop and express his ideas. The language of mathematics has existed since Euclid’s time (300 BC). What makes this language so special (and often so difficult) is that
2.5. Mathematics Language

it is not an extension of any other language but has its own syntax, grammar, vocabulary and rules (Bullock, 1994). However, there are many aspects of this mathematical language which can hinder learning (Orton, 2004). Kane (1968) states that mathematical English can be described as a mixture of ordinary English and formal symbol systems. This ties in with Noonan (1990) and Chapman (1993) and their definitions of mathematical English. Both Chapman and Noonan support Kane’s idea, that hidden within mathematical English is mathematical vocabulary, ordinary English vocabulary and symbols. To consider the presence of ordinary English within this language we must remember that it occurs in two forms.

The first form is simply English as itself with no hidden meanings, for example, the word ‘method’ in English and in mathematics has the same meaning. The second form occurs when ordinary English is present as part of the mathematical vocabulary but has no connection to its English meaning, for example the word ‘mean’ in English would constitute interpretation of something that was not nice however, in mathematics the word can be defined as an average. Tapson (2000) reinforces this complexity of the vocabulary in mathematics when he identifies the word ‘conjugate’ as having eleven different meanings. Similarly Skemp (1982) talks about two levels of mathematical language; the ‘deep structures’ and the ‘surface structures’. Deep structures refer to the ideas which teachers/textbooks try to communicate, while surface structures are the symbol systems which represent the ideas.

The language of mathematics is hugely important to textbook research. Textbook authors need to be aware not only of the forms of mathematical language and vocabulary but they must also consider the symbols and the implications they have for student learning. Symbols add to the complexity of mathematics. Noonan (1990) identifies the confusion that symbols can cause when reading mathematical texts as they need to be identified with regard to their meaning and context.
2.5. *Mathematics Language*

Pimm (1997: 141) identifies four types of symbols which exist within the language of mathematics:

- **Logograms:** Logograms are specific to mathematics and are symbolic representations for whole words (only used within mathematical contexts), for example \( \infty \) is a logogram.

- **Pictograms:** Pictograms offer visual diagrams which are image representations of words, for example \( \triangle \) for triangle.

- **Punctuation Symbols:** Punctuation symbols are ordinary English punctuation marks which have mathematical meaning and representations, for example \( : \) denotes ratio \((a : b)\) or defines a function \( f : A \to B \).

- **Alphabetic Symbols:** Alphabetic symbols, usually Roman or Greek, are typically used to represent unknown variables, for example \( \Theta \) or \( \Phi \).

Given the multitude of specific words and symbols encompassed by the language of mathematics it is convenient that it can be broken down into four categories of language outlined as follows:

- **Research Mathematics:** The spoken mathematics of the professional mathematician

- **Inquiry Mathematics:** Used by mathematically literate adults

- **Journal Mathematics:** The language of mathematical publications and papers

- **School Mathematics:** The discourse of the standard mathematics classroom. (Morgan, 1998: 11)

### 2.5.3 The Mathematics Register

Considering the categories of mathematical language provided by Morgan (1998), one would be correct to assume that within the language of mathematics there exists a school mathematics register. Register refers to the type of language that is used in a specific context. According to Halliday (1973: 195) a register describes ‘a
set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings’. The mathematics register refers to specific uses of natural language for mathematical purposes. According to Setati (2002: 9) “Part of learning mathematics is acquiring control over the mathematics register - learning to speak, read, and write like a mathematician”. Chapman (1993) gives a detailed account of this register and its relevance to student learning. According to Chapman this register is ‘highly specialised’ and it includes words which are appropriated and refined from the English language (mean, obtuse, show), words which are borrowed from other languages (Latin words: subtract, binary - French words: domain, cone - Greek words: isosceles, pi) and words which are combined to create new words (e.g. histogram is created from the Latin word historia and the French word gramme). This mathematics register contains not only vocabulary and technical terms but also words, phrases and methods of argument which would be required in specific school mathematics situations, conveyed through the use of natural language (Pimm, 1987).

Noonan (1990) and Meaney (2005) had a similar way of defining the words found within the school register. Noonan believed that they could be categorised into three sections:

- Mathematical English,
- Ordinary English,
- Words with Dual Reading.

Mathematical English refers to those words which occur only in mathematics, such as parallelogram. Ordinary English includes all words which the students might encounter in any other reading material, words which they may know and use in their everyday lives. Dual Reading refers to all words which may be known to the student as ordinary English but have a specific mathematical meaning, for example ‘product’, ‘mean’.
Meaney also categorised the school mathematics register into three sections:

- Maths-Specific Language,
- Everyday Language,
- Symbolic Language.

Mathematics specific language refers to those words which occur only in mathematics, such as circumference (similar to Noonan’s mathematical English). Everyday language is the same as Noonan’s category of Ordinary English and Symbolic Language refers to all mathematics specific symbols such as $\pi$.

Noonan (1990) makes reference to the fact that changes to mathematical teaching styles and a move away from the traditional ‘chalk and talk’ method of teaching has created an increased pressure for students. They are “expected to be able to learn mathematics by reading and by carrying out activities which are described in writing” (Noonan, 1990: 57). Furthermore, many studies have been carried out which highlight the significance of the school mathematics register to student learning. In a study carried out by Marks, Doctorow, and Wittrock (1974), they replaced 15% of the words in a text with more commonly used words and presented the text to six hundred 6th grade students. They found that comprehension was increased from 47% to 73%. Bell (1970) created a list of basic (minimum requirements) vocabulary for school mathematics. This list contains three hundred and sixty five words. It is estimated that students have to learn approximately one hundred new words per school year (Orton, 2004).

A similar study carried out by Glynn and Britton (1986) focused on analysing frequency of words, sentence length, study aims, emphasising headings, questions for actualising and prior knowledge. They found that all of the above played a vital role in students’ ability to acquire the knowledge, the time spent reading the text and the mental effort it took for them to do so. Another study carried out by
2.5. Mathematics Language

Klare (1963) found that suitable readability levels proved to increase effectiveness of text in over 68% of the cases they investigated.

2.5.4 Reading Mathematical Texts

“The readability of texts has become a major concern in education in recent years” (Orton, 2004: 160).

Research such as that carried out by Schoenfeld (1992) and Marks et al. (1974) highlight the significance of reading mathematics for learning and engaging in mathematics hence it is necessary that mathematics textbooks are readable and encourage reading. While it is difficult to differentiate between mathematical and school texts, other than the typical language differences, school texts should clearly depict one main difference to distinguish them from mathematical texts. School mathematical texts should be written for the purpose of learning so it should go without saying that the focus of textbooks should be on the pedagogical aspects of mathematics. However Stray (1994) notes that while the purpose of a textbook is to provide a pedagogical approach to mathematics, the pedagogy is often marginal to the intentions of the textbook publishers as priorities of a publishing company differ greatly from the priorities of a teacher.

Language plays a vital role in school mathematics and more importantly in school mathematical texts. Reading a mathematical text requires one to learn, without hindrance, from the vocabulary present. However the levels of reading ability vary greatly amongst students. This poses a problem for school mathematics textbooks. Pitching the readability at a level suitable for the intended readers poses a problem as not everyone is capable of reading at the average reading levels. Aiken (1972) explains that students with a higher readability level tend to do better than others. The reason for this, he suggests, is not that they are any better at mathematics but they can read and understand what is being asked of them better. Students need to be able to read a text in order to learn from it.
2.5. Mathematics Language

Balas (2000) says that the authors of Living and Learning Mathematics, give stories and strategies for supporting mathematical literacy and they suggest that students can become mathematically literate in the same way as they would become literate in reading. Reading mathematics is identical to reading a new language. The students not only have to read the text but they must also interpret the symbols and words presented and then translate the meaning of these with regard to the appropriate context (Noonan, 1990). Pimm (1997) determined that students often read (or hear) the ordinary English and the mathematical English is left behind. A study carried out by Lim and Clements (2002) found that 88% of their sample had difficulty understanding and translating algebraic word problems and symbols into vocabulary and language that they understood.

Much of the linguistic research, such as that carried out by Hodge and Kress (1993); Fairclough (1989); Fowler and Kress (1979), is based on Halliday’s (1973) ideas of the functions of language. Similarly, Morgan (1996) recommends that in order to analyse the language of a text one must follow a linguistic approach based on the ideas of the functions of language put forward by Halliday (1973). Both Morgan and Halliday suggest that all mathematical text should consider the ideational, interpersonal and textual functions of language. The ideational function refers to the way language is expressed and the way in which objects are named. The interpersonal refers to the forms of relationships presented by the language, while the textual function refers to argument of the text and the messages that are communicated. Considering this it is relevant to note that Noonan (1990) claims there are two main difficulties associated with reading mathematical texts, ideas not being clearly explained and the difficulty of actually reading the mathematics. These problems are directly correlated with Halliday’s principles of language. How the ideational, interpersonal and textual functions are presented in a text will have implications for the reader. Difficulties reading mathematical text can occur in six different ways:
2.5. *Mathematics Language*

**Words** According to both Chapman (1993) and Noonan (1990) the types of words and the categories which they fall into can create reading problems specifically with regard to words with dual meaning.

**Syntax** Long sentences are more difficult to read than shorter ones. In addition, the order of words is significant in that sentences which use the passive, subordinate clauses and comparatives are much more complex to read than those extended by adjectives, phases or people.

**Diagrams** Diagrams can cause confusion as students need to determine their relevance to the text, i.e. are they purely for decorative purposes, are they related but not essential or are they essential and to be read as a part of the text.

**Symbols** Unlike words, symbols are not based on sounds, therefore when learning symbols students need to link the symbol to the words which correspond to it. However, symbols may often have many interpretations in spoken language, therefore students must read a symbol according to its appropriate meaning and context. For example ‘+’ can be read as add, plus, more than or together with.

**Rhetorical Questions** They generally signal what is to come in the next section or chapter. However according to Noonan (1990) they cause reading issues in that students are unsure what to do with them.

**Page Layout** According to Noonan (1990: 60) “comprehension of mathematics texts for children improves when there are illustrations”. Dowling (1996) tells us how useful diagrams and pictures can be in captivating students’ attention and helping them move from horizontal to vertical learning. Therefore, while considering that diagrams may inhibit reading, carefully placed and presented diagrams will benefit the overall motivation to read the text. A page should be pleasant to look at and easy to follow (Noonan, 1990; Shuard and Rothery, 1984).

While Noonan identifies a list of common factors which hinder student reading, it is also important to look at the factors/methods which will enhance students’ mathematical reading. Many studies have been carried out to discover what can be done to change student approaches to reading mathematics. One such study is provided by Campbell, Schlumberger, and Pate (2001). They provide an approach devised by Dr. A. Schlumberger and Dr. J. Keller which aims to facilitate a student’s ability to read and understand mathematics textbooks. They provide four elements which can help students:
1. **Pre-reading:** This involves reading the texts and highlighting words in italics. Students should also question themselves about what the main topic is.

2. **During Reading:** Students need to be aware that textbooks follow the pattern of: statement, example and explanation and that this layout is repeated throughout the text.

3. **Practice:** Students need to work through a couple of problems which encourage them to use the rules and examples they have just studied.

4. **Review:** Review is attained through homework.

### 2.5.5 Mathematics Text vs Mathematics Textbook Writings

There is little research available on the analysis of academic mathematical texts however it is evident that academic text is typically formal, impersonal and includes an absence of human activity. The statement that mathematics is a language supports many mathematician’s views of mathematics as a system of symbols (Rotman, 2006). This symbolic representation of academic mathematics text contributes to the formal and impersonal nature of academic mathematics text. Strube (1989) identifies the use of a distant authoritative voice, the use of the passive voice and the absence of reference to the author or the reader as characteristics of academic science text. Halliday (1973) also notes that excessive use of nominal rather than verbal expressions contributes to the impersonal nature of academic mathematics text.

In her doctoral studies, Morgan (1995) analysed students’ own written mathematical texts. In her work she identified a number of research studies which highlight the absence of mathematics writing in school mathematics. She also identified that even when teachers committed themselves to increasing the writing opportunities for students’ in their classrooms they confined these opportunities to recounting information. Pimm (1987) considered students’ own written mathematical texts in his discussion of the language of mathematics. He describes that students’
2.5. *Mathematics Language*

own written mathematics text typically consist of recordings of generalisations and symbols, which according to Laborde (1990) follows a format and structure similar that presented by the school mathematics textbook. Ernest (1993) suggests that one of the main features of students’ own written mathematical texts is the presence of a sequence leading to an answer, which he suggests is influenced by students’ expectations that school mathematics requires them to find an answer.

The repetitive manipulation of symbols is common in school mathematics. However, Ernest (1993) notes that the range of symbols is limited to arithmetic and elementary algebra symbols. School mathematics textbooks vary greatly from academic mathematics text in terms of subject matter and the relationship between the author and the reader. Like academic texts, school textbooks contain symbols, graphs, tables, diagrams, plans and pictures (Shuard and Rothery, 1984). Shuard and Rothery (1984) suggest that the extent to which graphs, tables etc in school mathematics textbooks are mathematical varies immensely from those in academic mathematics text. According to Ernest (1993: 8) school mathematics textbooks differ from academic text in that they have a rhetorical style which include:

- Use a restricted technical language and standard notation,
- Use spare, minimal overall forms of expression,
- Use certain forms of spatial organisation of symbols, figures and text on the page,
- Avoid deixis (pronouns or spatio-temporal locators),
- Employ standard methods of computation, transformation or proof.

Questions, instructions and practice exercises are also typical of school mathematics textbooks and they serve to involve the student as an active participant in the mathematics and to impart new knowledge on the students (Shuard and Rothery, 1984). The dominance of instructions, worked examples and a repetitive structure is a clear distinction between academic mathematics text and school
2.6. Conclusion

mathematics textbooks. Morgan (1995) also notes that the forms of language used in school mathematics textbooks varies between textbooks and hence analysing such language is complex.

2.5.6 Conclusion

This section identifies the main features of language which will impact on a students’ ability to read a mathematical text. It also highlights the importance of reading text to a student’s mathematical learning and the impact this has for textbook research. The distinction between students’ written mathematical text and mathematics textbook writing is an important distinction for the author.

2.6 Conclusion

This chapter addresses background issues which have implications for research on mathematical textbooks. It begins by identifying the role of school mathematics and the significance of research in the teaching and learning of mathematics. It then proceeds to identify the importance of problem solving and language for students’ mathematical and conceptual development, identifying key concerns which can be impacted on by the mathematics textbook. The next chapter will look at resources in mathematics in general, the textbook, textbook analysis and the current situation in Ireland.
Chapter 3

Textbooks in Mathematics Education

3.1 Introduction

This study is concerned with mathematics textbook analysis and its influence on mathematics education in Ireland. The purpose of this chapter is to explore the background issues directly associated with textbooks in mathematics education. This chapter outlines classroom resources for mathematics in general and their use in student learning, examines the literature on mathematical textbooks and textbook analysis. It also explores the Irish mathematics curriculum with specific regard to mathematics textbooks.

3.2 Mathematics Classroom and Resources

3.2.1 Introduction

This section outlines the use of resources in the mathematics classroom. It identifies the most dominant classroom resource for mathematics and details its role in the teaching and learning of mathematics.

3.2.2 Mathematics Classroom and Resources

“What lies between the resource and school mathematics practice is their use in practice - their transparency” (Adler, 2000: 211)
3.2. Mathematics Classroom and Resources

One of the key issues outlined by Otte (1983, cited in Bishop, Kietal, Kilpatrick, and Laborde, 1996: 389) is making mathematical thinking visual and active. Felix Klein (in his book *Elementary Mathematics from an Advanced Standpoint*) speaks about the need to make mathematics a living organism, for it to become alive. Students must be encouraged to become involved in their mathematics. For this to occur students need to have more than ‘pen and paper’ - it requires resources. Also, there is a need to remove the monotony and stigma attached to the lack of success with mathematics. Mathematics is often seen as dull and reserved for the talented. This is not the real story. Bullock (1994: 739) believes that this idea of mathematics is often used as an excuse “not to learn and not to teach”.

Over twenty five years ago a mathematical commission of inquiry was carried out in the UK and chaired by Dr. W.H. Cockcroft (1982). The Cockroft report gives two main recommendations which can be easily applied to every classroom; a list of basic provisions such as three dimensional objects, glue, books etc and a list of outcomes which all mathematics teaching should allow for. These findings, though dated are still relevant as they are simple and straightforward, inexpensive solutions to classroom resources.

As mathematics is a hierarchal subject there is a need for effective learning to take place. Effective learning will help students to feel comfortable in continuing on with mathematics and, according to Cockroft (1982), resources have a role to play in effective teaching. A recent survey in Ireland by the I.N.T.O. (2005)\(^1\) indicated that there is still an overwhelming need to gather resources for mathematics teaching.

Initiatives can be seen in Irish primary school mathematics, where schools can choose to involve themselves in projects such as: Shared Maths/Maths for fun and Maths Recovery, giving students a chance to experience mathematics from a different perspective (I.N.T.O., 2006). Initiatives such as these support Rogoff’s

\(^1\)INTO = Irish National Teachers Organisation
3.2. Mathematics Classroom and Resources

(1990) idea of varying the interaction between teachers and students as variety “influences the nature of cognitive development”. Furthermore Kamii (1995) and Nicholls (1983) (cited in Wood, 1996: 87) also believe that the environment created is fundamental to learning and children are discouraged and unmotivated when doing monotonous and repetitive tasks. One such initiative towards removing this monotony is Realistic Mathematics Education (RME)\(^2\). RME grew out of the work of Hans Freudenthal, who spent thirty years developing this idea, which has greatly influenced mathematics education in several countries (Freudenthal, 1973).

According to Freudenthal (1973), mathematics must be connected to reality and be relevant to society in order to be of any human value. Mathematics is more than just subject matter to be transmitted - it is a human activity. RME focuses on mathematics as an activity rather than a closed system. It involves whole class and one to one teaching where students are seen as individuals with their own learning path (Heuvel-Panhuizen, 2001). In order to fully incorporate real life mathematics and a combination of individual and whole class teaching into the classroom it is necessary to have a good supply of mathematical resources in place.

Having a supply of resources however does not necessarily mean that they will be used. Graybeal and Stodolsky (1986), like many researchers, found that the textbook is the most dominant mathematical resource. However, along with this there are many other resources widely available. Graybeal and Stodolsky (1986: 11) compared the use of some resources in mathematical activities with their use in social science activities. They found the following:

\(^2\)RME sees mathematics as being connected to reality and relevant to society.
3.2. *Mathematics Classroom and Resources*

- Blackboard (16% of mathematical activities vs. 4% social science activities)
- Manipulative (10% vs. 0%)
- Worksheet (5% vs. <1%)
- Games (3% vs. <1%)
- Reference materials (1% vs. 20%)
- Non-Text maps (<1% vs. 4%)
- No materials (16% vs. 8%)
- Other material (12% vs. 16%)

Other materials in mathematics consist of 66% scripted problems or tables, 10% student made materials and 8% real life objects.

Resources are widely available in mathematics and can be of great benefit, however mathematics teachers fail to take advantage of them relying instead on the available textbooks. Jita (1998) provides five kinds of resources that interact to shape the classroom practices of successful science teachers; human resources (teachers, students, parents), knowledge (of science, science education, and the transformative agenda), time, a sense of mission and commitment and textual materials. All five of these resources are also of great significance to mathematics teaching. Adler (2000: 213) also provides a list of material resources which are specific to the mathematics classroom. Adler’s list include the following:

- Technologies; blackboard, calculator, computer, copier
- School mathematics materials; textbooks, other texts, cuisenaire rods, geoboards, computer software
- Mathematical objects; proofs, number lines, magic squares
- Everyday objects; money, newspapers, stories, calculators, rulers

According to both Jita (1998) and Adler (2000) the quality of each resource should be ensured in order to maximise their effectiveness in teaching and learning. With this in mind it is necessary to examine the role of the most commonly found resource in any mathematics classroom - the textbook.
3.2.3 Role of Textbooks in Teaching and Learning

The role of the textbook varies greatly from classroom to classroom and teacher to teacher, however Gelfman, Podstrigich, and Losinskaya (2004) provide a basic outline for the intermediary role of the textbook:

- To teach and encourage students to construct new knowledge,
- To balance detail and precision of information,
- To provide logical and consistent mathematical systems,
- To bring about new questions,
- To provide students with active, creative, many sided information.

Textbooks can be defined simply as books which are written for the purpose of teaching and/or learning. However Vaneck (1995) suggests that a textbook is a book which can meet the requirements of all educational tasks. He describes how a textbook should meet the recommendations of the curriculum, encompassing and elaborating curriculum content while acting as a guide for students’ learning.

Textbooks are the closest thing students have to working from the curriculum and the purpose of these textbooks is to assist with student learning. Despite such an obvious relationship between the textbook and the student there is limited evidence which outlines how students actually use their textbooks. In the context of education in general and mathematics education in particular research which highlights textbook use is limited to how teachers use their textbooks. Textbooks are a vital ingredient of successful learning. The importance of their role can never be over exaggerated. Mathematics teachers have been found to rely on textbooks for at least 90% of their teaching time (Mikk, 2000). This statistic can only highlight the need for good textbooks. Sewall (1992) goes so far as to say that it is almost impossible to achieve a high level of education without the use of textbooks. According to Valverde and Schmidt (1998) the major failing of textbooks occurs when teachers try to cover every aspect of it, hindering the
3.2. *Mathematics Classroom and Resources*

application of suitable methodologies for teaching and learning. Horsley and Laws (1992) claim that notion of teachers not using textbooks effectively cannot be correct if there are good textbooks in place. The purpose of the textbook is to help and motivate students to learn. Mikk (2000: 17), highlights the need for exciting, imaginative textbooks; “students have many sources of information available, if their textbooks are dull, they are unwilling to study them. Interesting and enthusiastic textbooks develop curiosity and interest in the subject”.

Research has shown that illustrations can enhance motivation in relation to student learning (Carney and Levin, 2002). The following table indicates the results of two such studies (Table 3.1). These studies indicate that information which is seen as well as heard is better retained.

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<tr>
<td></td>
<td>% retained</td>
<td>% retained</td>
</tr>
<tr>
<td>Information one has heard</td>
<td>15%</td>
<td>20%</td>
</tr>
<tr>
<td>Information one has seen</td>
<td>25%</td>
<td>30%</td>
</tr>
<tr>
<td>Information one has seen and heard</td>
<td>65%</td>
<td>70%</td>
</tr>
</tbody>
</table>

Building on this research, Dowling (1996) indicates that the way in which information is presented further increases retainment. Illustrations are a simple and attractive method of including mathematical information.

The first illustrated textbook was created in 1658 by J.A. Comenius and since then illustrations have become more and more important in their new role in student learning. The importance of illustrations in textbooks can be briefly summarised by considering the dual code theory. The dual code theory indicates that information can be better acquired when presented in two modes, i.e. in written text and
in images/illustrations (Paivio, 1986). Similarly Mikk (2000: 270) states that the effect of repeating information via illustration serves to deepen impressions. Mikk (2000: 272-3) provides a table of information obtained from the authors of textbooks. In it they provide (in numerical order) their reasons for the inclusion of illustrations in their textbooks. The function group category is given in parenthesis beneath each reason (Table 3.2).

Table 3.2: Textbook Authors’ Reasons for the Inclusion of Illustrations

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<th>Author:</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<tbody>
<tr>
<td>Williams (1986)</td>
<td>Recall (Information)</td>
<td>Aid Study (Information)</td>
<td>Correct Misconceptions (comprehension)</td>
</tr>
<tr>
<td>Duchastel (1980)</td>
<td>Attention (Motivation)</td>
<td>Presentation (Information)</td>
<td>Memorising (Motivation)</td>
</tr>
<tr>
<td>Teyman (1985)</td>
<td>Describing (Information)</td>
<td>Instruction (Comprehension)</td>
<td>Pleasure (Motivation)</td>
</tr>
<tr>
<td>Peeck (1994)</td>
<td>Attention (Motivation)</td>
<td>Describing (Information)</td>
<td>Interpretation (Comprehension)</td>
</tr>
</tbody>
</table>

This table indicates that the most common reasons for the inclusion of illustrations, provided by textbook authors, is the provision of information and student motivation. The emergence of these categories is supported by Mikk (2000) who concludes that the first function of all textbooks, is to involve students. Illustrations can do this extremely effectively, especially with coloured illustrations depicting people.
3.2. Mathematics Classroom and Resources

Dowling (1996) tells how useful diagrams and illustrations can be in captivating student attention and assisting them to move from horizontal to vertical learning. Illustrations also serve to assist students in understanding complicated information while also supporting the students’ thinking processes. Illustrations foster a student’s transfer of knowledge and allow for comparisons and contrasts to be easily made.

![Diagram: Illustrations in Textbooks (Dowling, 1996)]

Figure 3.1: Illustrations in Textbooks (Dowling, 1996)

However, illustrations need to have a purpose. Noonan (1990) indicates how diagrams can add confusion if they serve no obvious purpose. When it comes to reading a text which has unnecessary illustrations students get confused while trying to determine the purpose of the diagram. Bishop et al. (1996: 382-3) stress
that all too often pictures are put in textbooks simply to help sell them rather than having any real purpose.

3.2.4 Conclusion

This section highlights the role of resources in good mathematics teaching. It identifies the textbook as the most commonly found and used classroom resource, while also highlighting the effect a textbook (in particular textbook illustrations) can have on motivation. This section also presents some reasons for the inclusion of illustrations by a number of textbook authors.

3.3 Mathematical Textbooks

3.3.1 Introduction

This section explores the history, selection and role of the mathematical textbook. It identifies the current status of the mathematics textbook while also highlighting what its purpose is and should be.

3.3.2 Mathematical Textbooks

‘Curriculum materials have the potential to influence classroom instruction, yet analysis of curriculum guides is a relatively unexplored field of study’ (Graybeal and Stodolsky, 1986).

The initial problem, which is evident, for Irish textbooks is that despite their consistent presence in classrooms very little research has been carried out on them. The TIMSS Report (2002) carried out a study across 48 countries analysing mathematics and science textbooks. Irish textbooks formed a small part in this study but apart from this the NCCA (2005a) has stated that there have been no studies of Irish mathematical post primary textbooks to date.

Lockhart (2002: 2) voiced his opinion of the mathematics problem, one which many will agree with but few will admit, “Everyone knows something is wrong”. There
3.3. Mathematical Textbooks

seems to be a breakdown in students' learning of mathematics. As Lockhart also notes, the only people who truly understand this breakdown are those closest to it - the students. Furthermore Lockhart also describes textbooks as “unreadable monstrosities”. Textbooks are described by Mulryan (1984)\(^3\) as being by far the most used and depended upon aid for teaching mathematics. She also indicates that from as young as aged eight the usual situation in Irish mathematics classrooms is initial clarification or explanation of a topic by the teacher followed by textbook work. She also noted that a Northern Ireland report (Mulryan, 1984: 63) stated that the textbook is used “sensibly and with moderation in grades five and seven and excessively in all other grades” (grade five correlates with 4th class in the Irish education system). Inspectors who visited fifty-six primary schools were asked to provide reasons for poor mathematics achievement at primary level and a substantial amount cited overdependence on textbooks and workbooks as the main cause (Mulryan, 1984). Much anecdotal evidence suggests that students rely on their textbooks just as heavily as teachers. For example Apple (1986) estimates that students use their mathematics textbooks for 75% of their class time and over 90% of their homework time. Similarly, Haggarty and Peppin (2002) suggest that the mathematics textbook is used by students as a book of exercises and Valverde et al. (2002) noted that English students of mathematics equate success in mathematics with completing the exercises in their mathematics textbook. From studies such as these we can see that textbooks are the predominant resource in all mathematics classrooms. The significant role of such a resource is huge, yet research fails to validate its effectiveness.

A consultative conference on education (primary education) directed by the Education Committee in Ireland (2005) speaks about how the purpose of mathematics education is to ‘kindle a lively interest’ in mathematics. It also states that the

\(^3\)The work of Mulryan (1984) focuses on an Irish based study of primary mathematics textbooks.
3.3. Mathematical Textbooks

teacher should act as a guide and the textbook should act as a source of ideas rather than dictating the content and pace of mathematics learning. As far back as 1986, Beilinson talks about the need for textbooks to consider links between subjects and much more recent studies have echoed this, such as the TIMSS Report (2002). These studies also indicate the need for textbooks to co-ordinate with the use of other educational aids. While many subject syllabi have embraced this idea, the Junior Cycle Mathematics syllabus itself only mentions calculators and mathematical tables in terms of additional resources.

3.3.3 History of Textbooks

The practice of using textbooks is, according to Robinson (1981), as old as the practice of writing. The word textbook appeared in the 1830's long after ‘collocation’ textbook (Love and Pimm, 1996). According to Walbesser (1973) the first arithmetic textbook was written by Isaac Greenwood in 1729; ‘Arithmetick, Vulgar and Decimal’. The sequencing of this textbook was as follows:

- Present a rule,
- Provide example which uses the rule,
- Exercises for students to apply the rule,
- Formal proof of the rule.

While Walbesser was referring to the situation present in the seventeen hundreds, the reality is that little has changed. This process of “rule - example - practice is still with us today” (Walbesser, 1973: 63). Walbesser also informs of the indifference of publishers with regard to visual appeal of the textbooks, it was 1834 before drawings appeared in American mathematics textbooks. Modern textbooks contain diagrams and appealing covers and are generally well illustrated.

Lockhart (2002: 16) quotes Bertrand Russell when he speaks of how little change is evident in mathematics,
3.3. Mathematical Textbooks

“I was made learn by heart: ‘The square of the sum of two numbers is equal to the sum of their squares increased by twice their product’ - I had not the vaguest idea what this meant and when I could not remember the words, my tutor threw the book at my head, which did not stimulate my intellect in any way”.

Walbesser noted an emphasis on the practice of rule-example-practice in his research more than thirty years ago and despite his research being dated little has changed with regard the process of rule-example-practice or in the development of textbook appeal.

3.3.4 Selection of Textbooks

“Given the ubiquitous role of textbooks the decision of textbook adoption committees has serious implications for what mathematics students learn and how they learn it” (Tarr, Reys, Barker, and Billstein, 2006).

Textbook selection is specific from school to school; however it is recommended that a number of mathematical texts should be selected and used by every teacher. Mikk (2000), recommends that more than one textbook be available for student use throughout the classroom. Students vary greatly, as do their abilities. Some students will engage with a textbook with limited explanations and concise laws and data. However, many students will need a more detailed background of the subject, laws to be broken down into everyday language that they will understand and be provided with the data and references to support the problem/method/question. Not only will textbook choices enhance learning experiences it will also open the opportunity for mathematical discussion and student-to-student communication about the various mathematical texts. Stodolsky (1983) offers the following guidelines which can be applied to assist with textbook selection:

1. What types of group arrangements are suggested?
2. What kinds of instructional formats are emphasised?
3.3. Mathematical Textbooks

3. What kinds of student behaviours are expected?
4. What levels of student cognitive processes are sought?
5. What types of materials are recommended?
6. Do instructional recommendations vary between subjects?

Mikk (2000) points to a recommended textbook structure which teachers can use when selecting textbooks. Mikk (2000) suggests that each textbook unit should comprise an introduction, main text and a conclusion.

<table>
<thead>
<tr>
<th>Introduction:</th>
<th>Main Text:</th>
<th>Conclusion:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motivation,</td>
<td>Sub Units Differentiated,</td>
<td>Important Ideas,</td>
</tr>
<tr>
<td>Study Aims,</td>
<td>Cohesive Ties,</td>
<td>Unsolved Problems,</td>
</tr>
<tr>
<td>Advance Organisers,</td>
<td>Organisation,</td>
<td>Connections with</td>
</tr>
<tr>
<td>Prior Knowledge</td>
<td>Content Relations,</td>
<td>other Units</td>
</tr>
<tr>
<td></td>
<td>Generalisations,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Emphasising</td>
<td></td>
</tr>
</tbody>
</table>

The introduction should be short, motivate students, state the learning goals, recall previous knowledge and provide advance organisers. The main body should present the facts and ideas, differentiate between concepts and provide connections. While the conclusion should reinforce the important ideas and make connections with other units. Motivation can be enhanced by including examples and relevance to everyday life, by highlighting the importance of mathematics and the presence of mathematics in daily life. By identifying firstly the aims and objectives of the mathematics class, one can find a textbook which will answer Stodolsky’s questions in a way which will meet the intended aims while also considering the relevance of the structure of the unit.

3.3.5 Role of the Mathematics Textbook

“I do not believe mathematics education is as bad as the textbooks which are used” (Freudenthal, 1973: 159).
3.3. Mathematical Textbooks

Horsley and Laws (1992), claim that the statement about good teachers not using textbooks would not be correct if there were good textbooks in place. The purpose of the textbook is to help and motivate students to learn. Mikk (2000: 17) highlights the need for exciting, imaginative textbooks, “Students have many sources of information available, if their textbooks are dull, they are unwilling to study them. Interesting and enthusiastic textbooks develop curiosity and interest in the subject”. Gelfman et al. (2004: 3) consolidated much related research to create the following outline of the role of a good textbook. A textbook should:

- Contain logical, consistent and exact presentation of accepted mathematical knowledge.
- Teach students to construct new knowledge.
- Provide a balance of detailed and precise information.
- Be micro or macro (Tseitlin, 1988)
  - Macro: Provides detailed information, explanations of notions and methods (diverse in content and structure, should be used alongside additional texts)
  - Micro: needed for the processes of teaching, provides statements, directions rules, glossary etc. (micro texts have certain linguistic structure)
- Be measured on the basis that answers bring forward new questions.

Textbooks should provide students with active, creative and many sided usage of information while encompassing a combination of macro and micro styles. How a textbook is presented in a classroom will impact significantly on how students will view it, therefore if a teacher regards a textbook as extremely useful and helpful, its status within the classroom will be high.

3.3.6 Status of Textbook

Some textbooks direct students towards rote learning, “crippling their reasoning” (Mikk, 2000). Statements such as this give textbooks a bad name. Robinson (1981) states that one of the reasons for such little research on textbooks is the
3.3. Mathematical Textbooks

bad name they have received in the press, which is damaging their status. Graybeal and Stodolsky (1986) carried out an investigation on instructional practice in mathematics and social science. This research provides useful statistics such as the percentage of resources used in mathematical activities, the percentage of group and subgroup activities and types and it also identifies the presence of student interaction. However, the most interesting findings are those associated with the textbook:

- Mathematical texts consist of a short demonstration (approx 43 words) followed by approx 18 examples/exercises.
- The primary use of mathematics textbooks is as a source of problems/exercises. (Graybeal and Stodolsky, 1986)

This supports Robinson’s (1981) assertion, ‘a textbook which is not read cannot teach anything’. Considering the abundance of research indicating that any classroom material needs to appeal to those it is aimed at, it is bewildering to assume that teachers may ignore research in favour of mundane textbooks.

3.3.7 Purpose of Textbook

According to Gelfman et al. (2004) the functions of a modern textbook are; governing, developing, communicating and expressing, as well as functions of individualisation and differentiation of teaching. These functions seem to cover every aspect of teaching. Teaching time is valuable but limited. Teachers spend approximately 90% of their preparation time with the textbook (Mikk, 2000). However, in the classroom, teachers face the dilemma of needing to divide their time between approximately 30 students while concentrating on maximising each individual’s learning outcomes. Good textbooks can play an important role in assisting the teacher with this. According to Zuev (1983, cited in Mikk, 2000), good textbooks can support self assessment. They can do this with the provision of keys to help direct students towards correct procedures in various questions and problems.
3.4. Mathematical Textbook Analysis

However, the NCCA suggests that expectations for the textbook such as Gelfman’s are unrealistic as the textbook’s purpose is to supplement a teaching programme (NCCA, 2005a). The textbook was never intended to replace face to face interactions therefore it cannot be held accountable for the governing and developing of a student’s mathematical education. The textbook is in a key position, (being the most relied upon mathematical resource), to impact on how a teacher governs and develops student mathematical experiences. Good textbooks will impact positively providing support for both teachers and students. However, one purpose of a textbook which makes it indispensable is that it prepares students for further learning; third level education requires students to learn a significant amount themselves from textbooks and text materials (Robinson, 1981). Students’ experiences of mathematical textbooks at post primary level will undoubtedly impact their ability to learn individually from textbooks at third level.

3.3.8 Conclusion

This section provides the background to mathematical textbooks by highlighting their history and role in mathematics education. It identifies a lack of available support for the selection of textbooks while also noting how the status of the textbook can impact on student learning. Finally it provides an overview of what the intended and actual purpose of the textbooks is. The next section builds on this literature to identify effective instruments for textbook analysis.

3.4 Mathematical Textbook Analysis

3.4.1 Introduction

This section identifies the key textbook features for analysis and the various test instruments available for this analysis.
3.4.2 Mathematical Textbook Analysis

“Textbook analysis dates back to 900 AD when Talmudists counted words and ideas in texts” Mikk (2000: 77).

The importance of analysing textbooks is well established in mathematics education. However, according to the NCCA (2005a), excluding the TIMSS Report, no Irish mathematical textbooks have been analysed to date. While no Irish research exists many international studies have been conducted. These researchers have identified the key areas involved in textbook analysis. Pingel (1999) speaks about analysis from two perspectives, a didactic/methodological approach and a content analysis approach. Mikk (2000) applies analysis of optimal text comprehension, text acquisition and cloze tests to textbook analysis.

Mikk’s findings show that a text or textbook is coherent when students can understand 90% of it. The TIMSS report (2002) carried out a number of tests to determine the effectiveness of textbooks, and like Mikk’s analysis, the TIMSS main focus was on analysis of:

- Structure,
- Content,
- Expectation. (Valverde et al., 2002)

Robinson (1981: 22) describes textbook analysis as researching the four main characteristics of well written textbooks; structure, coherence, unity and audience appropriateness.

**Structure:** how ideas and topics are put together. According to Robinson (1981: 22) structure affects the amount and kind of knowledge acquired.

- Goetz and Armbruster (1980) state that collective discourse is easier to learn and remember than isolated sentences or lists. They state that
better organised texts are easier to remember “the better the structure
of the text, the more likely the reader is to remember the information
and to engage in the higher level cognitive process” Robinson (1981:
22).

**Coherence:** the interweaving of ideas and logical connection of information.

- Robinson indicates that coherence begins with the title and introduction
  and they are significant to comprehension. Paragraphs without headings
  make no sense to students (Robinson, 1981: 23). Headings can also
  act as advance organisers (short verbal or visual prior to paragraph).
  Advance organisers do two things:
  (1) give a paragraph a structure
  (2) activate students’ prior knowledge to create a learning scheme or
  structure.

- Holliday (1976) carried out a number of studies which supported the
effectiveness of visual displays and diagrams with regards learning from
texts.

**Unity:** refers to how a text addresses a single topic, there is a need for closure
and completeness and it should not stray unnecessarily.

- This is often why summaries are effective, they do not include any
distractions.

**Audience Appropriateness:** incorporates both prior knowledge and readability.

- Relevant topic knowledge which considers the word knowledge of the
  student is strongly related with comprehension.

The educational value of a textbook is crucially important. A textbook is some-
ting which students will be reading on a daily basis and any messages, no matter
3.4. Mathematical Textbook Analysis

how small or innocent, are open for interpretation by a young impressionable mind. The process of developing, editing and publishing of a textbook should incorporate a number of people. Mikk (2000) suggests that this ‘working team’ should comprise a subject specialist, a teacher, an education psychologist, an illustrator and a text specialist in order for all considerations to be embraced.

Good textbooks need to consider content, value forming aspects, motivational elements, accessibility, illustrations, study guides etc; they must encourage a thirst for knowledge. Vygotsky (1956, cited in Mikk, 2000: 69) talks about the ‘zone of proximal development’. He identifies the need for textbooks to direct students to such a zone, one where there is optimal learning. Difficult tasks cause frustration and tasks which are considered too easy have little influence on students’ progress. There is a need for textbooks to encourage students to work in this zone. Research can help ensure textbooks are focused on reaching such a goal.

There is an abundance of research on how to effectively analyse a textbook. However if one was to summarise all the key factors the following four emerge:

1. Structure
2. Content
3. Language & Readability
4. Expectation

Within each of these categories of analysis there are numerous sub-headings and related topics.

3.4.3 Structure

Textbook structure adds to or takes from textbook comprehension. Succession and connections between text elements need to be analysed carefully. Halliday and Hasan (1993 cited in Mikk, 2000: 94) broke text cohesion/structure down into 5 parts; reference (pronominal, comparatives, articles), substitution (nominal,
3.4. Mathematical Textbook Analysis

clausal, etc.), ellipsis (nominal, verbal, etc.), conjunction (additive, temporal etc.), lexical (same item, general item, etc.).

Mikk (2000: 99) illustrates by way of a matrix table how one can easily analyse the structure of a text and record diagrammatically how frequent ideas/topics appear and therefore connections are visualised. In order for a structure to impact positively there are a number of key issues which need to be incorporated. These were analysed in the TIMSS report as ‘Physical Scale’. While the structure of the knowledge within a textbook is vital the physical structure will determine whether the intended audience will even consider the text. It includes many aspects such as those outlined by Valverde et al. (2002); Area and framing, Elements (pictorial, verbal, design), Colour and Non colour, Information levels, Unification and Separation.

3.4.4 Content

Textbook content influences the selections and emphases applied by teachers and students, consequently impacting on learning outcomes (Mulryan, 1984). Rivers (1990) discusses four aspects of content analysis which are similar to those outlined by Gerbner (1969). She created four subheadings in the area of content analysis:

- **Motivational factors** - which includes historical notes, scientist and mathematician biographies, career information, applications and photographs,

- **Comprehension cues** - focuses on colour and graphics,

- **Technical Aids** - includes all material related to calculators and computers,

- **Philosophical Position** - emphasis and predominant philosophy.

These subheadings are easily identified for analysis and their role in effective teaching is transparent. Wittlin (1978), whose work on museum exhibits was connected with science textbook analysis by Robinson (1981) insists that the very
3.4. Mathematical Textbook Analysis

first objective of any textbook must be to attract student attention. Then the focus switches to presenting the message clearly and comprehensibly, and finally maintaining attention (Robinson, 1981). The following table outlines Wittlin’s recommendations, which need to be considered in content analysis especially with regard to motivational factors. According to Wittlin (1978) one must first attract attention (Initial Arousal), then present a clear and concise message (Attending Message reception) and finally maintain attention (Maintenance of Attention).

<table>
<thead>
<tr>
<th>Initial Arousal</th>
<th>Attending Message Reception</th>
<th>Maintenance of Attention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Danger:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Underestimation</td>
<td>Overestimation</td>
<td>Monotony</td>
</tr>
<tr>
<td>To Avoid Danger:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relevance, Interest,</td>
<td>White Space, Signal Noise, Planned Redundancy, Planned Redundancy,</td>
<td>Change modality</td>
</tr>
<tr>
<td>Dissonance, Sensory,</td>
<td>Integration of multiple channels, Hierarchal organisation</td>
<td>Insert questions</td>
</tr>
<tr>
<td>Appeal, Appeal to effect</td>
<td>under key areas</td>
<td>Vary senses used</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Drama of Issues</td>
</tr>
</tbody>
</table>

Figure 3.2: Factors to Consider for Textbook Analysis
Wittlin (1978)

3.4.5 Expectation

Expectation is the third element of the TIMSS framework for textbook analysis. Performance expectations are embedded throughout textbooks and will impact significantly on how students choose to deal with the topics presented. For example if the focus of a mathematics textbook is on repetition and practice then a student will subconsciously look to replicate a previous method as soon as he/she encounters a question, without attempting to use any problem solving
skills. The most basic consideration of expectation is that students and teachers alike will read and understand the material presented (Valverde et al., 2002). The TIMSS Report (Valverde et al., 2002) identified expectation as crucial to textbook analysis and identified 19 different expectations that can be placed on students throughout textbook chapters. The Rivers Matrix (Rivers, 1990) also contains an expectation component. It looks to examine the presence of emphases and philosophies throughout textbooks. Both the emphasis and philosophy have a direct bearing on the student expectations put forward by textbooks.

### 3.4.6 Language and Readability

**Language**

Students should be able to communicate mathematics, both ‘verbally and in written form’ (NCTM, 1989). However, it seems that students are expected to acquire this communicative ability by osmosis. That is, they must acquire it themselves from textbooks or notes. Also, much of the notation and symbols used in these textbooks may not be conducive to learning. Mulryan (1984) describes primary textbooks as having an excessive vocabulary load, variability of word meaning, insufficient repetition of mathematical terms and inadequate vocabulary control.

How to use mathematical language is not something that is taught in Irish mathematics education, yet the significance of the language of mathematics to learning is acknowledged. When analysing mathematical textbook language Newall (1990) found a number of features in textbook language such as discourse type (narration, description etc), coordinators (connectors between sentences) and semantic structures. These features can provide a basis to inform textbook analysis of mathematical language. Mulryan (1984) provides three subheadings for language analysis; word signifiers, notational signs and graphical signs. These areas can be analysed, with consideration of Newall’s language features, under the following headings:
3.4. Mathematical Textbook Analysis

- **Word signifiers:**
  General vocabulary: word signs used regularly in daily life e.g. and, from.
  Mathematical terms: term with specific mathematical meaning, there are two types technical or special:
  Technical vocabulary: word signs peculiar to math e.g. Heptagon, multiple.
  Special vocabulary: word signs used in daily life which have different mathematical meaning e.g. match, set, group figure.
  Abbreviations: shortened or abbreviated technical words such as cm, km, HCF etc.
  Letters: alphabetical letters which represent numbers, lines.

- **Notational signs:**
  Notation signs: Hindu- Arabic number systems or signs such as >.

- **Graphical signs:**
  Pictorial/diagrams symbols: pictures/graphs which demonstrate mathematical principles.

The most significant feature of language is of course the words used. In a study carried out by Marks et al. (1974), they replaced 15% of the words in a text with more commonly used words and presented the text to 600 6th grade students. They found that comprehension was increased from 47% to 73%. Word length also has a significant impact on student learning, the longer the word the more information there is contained in it, making it more difficult to fully understand. Mathematical terms need to be explained, and their meanings need to be understood by the reader. The development of thinking can be divided into three stages, active, figurative and abstract. This would suggest that when learning/teaching the focus should be on being actively involved and engaged in ideas and where possible physical objects. The visual centre in our brains is approximately thirty times bigger than the audio centre hence it is often easier to understand something which is visual.

Sentence complexity is known as syntactic complicacy and involves sentences and paragraphs. To comprehend a sentence one must first remember it, long sentences cannot be remembered easily thus making them complicated. Luria (1975) analysed sentence complexity and found that the following causes confusion: inversion
(a later event being mentioned before earlier one for example if a textbook men-
tions simultaneous equations with three unknowns before simultaneous with two
unknowns), multi-meaningful phrases, subordinate connections, distant construc-
tions, triplet comparisons and double negation.

A study carried out by Glynn and Britton (1986) focused on analysing frequency of
words, sentence length, study aims, emphasising headings, questions for actualising
and prior knowledge. They found that all of the above played a vital role in
students’ ability to acquire the knowledge, their time spent reading the text and
the mental effort it took for them to do so. Another study carried out by Klare
(1963) found that suitable readability levels proved to increase effectiveness of text
in over 68% of cases they investigated.

**Readability**
The term readability refers to a number of factors which influence the reader,
including interest and motivation, legibility of the print, complexity of the words
and sentences in relation to the ability of the reader. Interest and motivation are
a textbook as “a book that no-one would read unless they had to”. This idea
is reinforced by Wiest (2003) who highlights the significance of reader interest to
readability levels of a textbook, the more interest the book can evoke from the
reader the deeper the level of comprehension and understanding attained. Wiest
also talks about including novel or demanding stimuli in favour of simple stimuli
since engaging students in fantasy demands higher comprehension levels.

Davy 1987 (cited in Mikk, 2000: 79) noted that textbooks with familiar words are
easier to understand. Mikk (2000: 79) used an electro-oculograph to fix the eye
movements of students while reading a passage and found that they spent more
‘time and fixations’ on unfamiliar words. Word frequency assessment (Cloze Test)
is the most common method for assessing familiarity of words. This can then
create a frequency dictionary, for the students in the subject area, to create a list of commonly used words. Some researchers such as Roos (1993) believe that this method gives a better indication of readability than standard readability tests. Wiio (1968, cited by Mikk (2000: 81)) devised a modification ratio which can be used as an indication of text complicacy. Many formulae such as this have been created and modified over the years, and it is recommended that to ensure an accurate result more than one formula should be applied to your text.

There are many varieties and adaptations of readability formulae, the most commonly used formulae are the Flesch Reading Ease and Flesch - Kincaid Grade level (TxReadability, 1998).

**Flesch Reading Ease**

- The Flesch Reading Ease gives an output from 1 - 100. The higher the output the easier a text is to read.

- The Flesch Reading Ease Formula = 206.835 - (1.015 × ASL) - (84.6 × ASW).
  - Where ASL is the average sentence length, i.e. the number of words in the whole text divided by the number of sentences.
  - ASW is the average number of syllables per word, i.e. the syllable count for the whole text divided by the word count for whole text.

**Flesch - Kincaid Grade level**

- The Flesch -Kincaid Grade level gives a grade readability result. The output value will indicate which grade level the text is most suitable for.

- The Flesch -Kincaid Grade Formula = (0.39 × ASL) + (11.8 × ASW) - 15.59.
  - Where ASL is the average sentence length, i.e. the number of words in the whole text divided by the number of sentences.
  - ASW is the average number of syllables per word, i.e. the syllable count for the whole text divided by the word count for whole text.

However, these readability tests are designed to analyse English-language paragraphs. Much mathematics research, which involves readability measurement, uses the above tests as a basis of comparison of readability levels but none of these
readability tests can effectively and accurately measure the actual readability of a mathematical text. Mathematical texts as previously stated combine ordinary English with mathematical English and symbols (Taylor and Hargreaves, 1999). This varied information embedded in mathematical text is unlikely, according to Thomas (1997), to ever be fully understood by such English-language based readability tests. This raises the issue for a need to create a mathematically focused test for readability. Many studies have indicated the obvious lack of correlation between standard readability scores, problem solving performance and comprehension (Paul, Nibbelink, and Hiiver, 1986; Hembree, 1992; Wiest, 9967) . Despite the direct connection between readability and problem solving (a highly topical element in many countries including Ireland) “research in reading and mathematics continues to attract little attention” (Thomas, 1997: 39).

### 3.4.7 Conclusion

Robinson (1981: 9) believes that textbook research can only be purposeful if it can impact on “schooling outcomes”. He also provides guidelines and suggestions for an ideal textbook. According to Robinson (1981) ideal textbooks should:

- Include titles which suggest how content might be organised,
- Have a natural flow from idea to idea,
- Not be littered with unnecessary details,
- Ensure that important information stands out,
- Include summaries,
- Provide a clear sense of where going, where have been and where are now.
3.5 Mathematics Education in Ireland: Issues for Consideration

3.5.1 Introduction

This section outlines the history of the mathematics curriculum in Ireland, the role the textbook has within this curriculum and it also identifies the concerns, trends and recommendations put forward by NCCA (2005a) with regard to mathematics education in Ireland.

3.5.2 Irish Mathematics Curriculum and Syllabus

The NCCA (2005a) refers to the Irish mathematics curriculum as lacking. There is a need for it to cater to students’ abilities, needs and interests and a focus on student development and learning. These recommendations are part of the 2005 review of mathematics in post primary education. In addition to this the NCCA claims an increase in mathematical motivation among Junior Cycle mathematics students based on their survey (NCCA, 2005a). This increase in motivation is credited to an increase in the use of real life mathematics, games and puzzles. However this increase in motivation is not endorsed by research. While teachers are documented as experiencing and noticing this increased motivation there is no solid research to support their claim. The teachers that were interviewed accredited this motivation to a ‘hands on approach’ to learning (NCCA, 2005a). This ‘hands on approach’ as outlined by the teachers is, to date, not reinforced with data outlining how the mathematics was integrated, how the range of abilities were considered, how textbooks were used or what language was used etc. Without this data the opinions of teachers and researchers alike can never be validated. Similarly in 2003, the revised Junior Cycle curriculum recommends that teachers take part in career development (C.P.D)\(^4\), that the need for new innovative teaching methodologies be addressed and that mathematical understanding needs to be increased. No

\(^4\)C.P.D represents Continued Professional development

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3.5. Mathematics Education in Ireland: Issues for Consideration

reference is made in any subsequent publications about the achievement of these aims and recommendations, although they are being addressed in some measure by the Project Maths development team.

3.5.3 The Irish Mathematics Curriculum: Background Issues and Recent Concerns

The following paragraphs provide a brief overview of the history of the Irish Mathematics Curriculum in the context of the international setting.

The 1980’s, in the USA, became known as the problem solving decade. This decade saw the NTCM develop the ‘Standards of Mathematics’ which ranked problem solving as the most important goal of mathematics education. At the same time Ireland was developing the new Junior Cycle curriculum, syllabi A, B and C. At the end of the eighties (1989) syllabi A, B and C were renamed Higher, Ordinary and Ordinary alternative, but no significant revisions or amendments of these syllabi were introduced at this stage. Then in 1990 the Mathematics Course Committee was set up to analyse the impact of the new Junior Cycle Curriculum and in 1992 the committee were asked to re-examine the Junior Certificate curriculum. The main problems identified in both these reports were:

- While the Higher Level curriculum was shortened in 1987 it still called for coverage of far too much material.
- The geometry syllabus needed revision.
- Calculators were used for examinations but not in the classroom,
- Examination paper layout needed to be revised.

Following the review of the Junior Certificate mathematics curriculum the committee was again asked in 1994 to critique the Junior Certificate curriculum but this time the committee was given freedom to make amendments. However the drawback this time was that it
Mathematics Education in Ireland: Issues for Consideration

“was specified that the outcomes of the review would build on current syllabus provision and examination approaches rather than leading to a root and branch change of either. Thus, once more, the syllabuses were to be revised rather than fundamentally redesigned” (NCCA, 2003: 6).

The committee had already established the main problems and after consulting with groups such as the Irish Mathematics Teachers’ Association, they established that the areas they had previously outlined were indeed the main issues. Following this a draft of a revised syllabus was submitted and in May 1998 was approved by the NCCA but not enforced. The first examinations of the Junior Cycle syllabi under the titles of Higher and Ordinary Level were held in 1992 and in 1995 the alternative level of mathematics was renamed for aesthetic reasons to Foundation Level (NCCA, 2000). Following this in 1999, the revision of the Primary School Curriculum began and of all the subjects within the new Primary Curriculum, mathematics was one of first to be implemented.

Understandably, as the prerequisite knowledge of new second level students would now be altered, pressure mounted to update the Junior Cycle Curriculum. In 2000, after more than ten years in place without review the recommendations of the mathematics course committee were finally considered and the Junior Cycle Mathematics Syllabi were revised and first examined in 2003 (NCCA, 2003).

In 2005 a review of curriculum and a consultation discussion paper was published (NCCA, 2005a). This led to the decision to create and implement Project Maths, which was implemented nationally in September 2010. Project Maths was piloted in twenty four schools in Ireland prior to its implementation during the period 2008-2010, with mathematics teachers in these schools exploring a new range and availability of classroom resources and teaching methodologies. One of the main aims of Project Maths is to provide teaching and learning plans, teacher guides and student worksheet, online exemplification and a range of assessment
3.5. Mathematics Education in Ireland: Issues for Consideration

materials. As each strand was piloted in the twenty four secondary schools the cooperating teachers were required to evaluate the resource materials and provide feedback, the intention being to improve the quality of teaching and learning in mathematics. It is intended that students’ mathematical learning will be enhanced by this new curriculum through teaching for understanding and an increased focus on applications and relevance to daily life. Students pursuing this new curriculum will face their first full nationwide examination in 2012 for Leaving Certificate (Senior Cycle state examination) and 2013 for Junior Certificate (Junior Cycle state examination). The author is aware that no textbooks, specifically designed for Project Maths, have yet been developed. This is as result of a conscious decision by the NCCA and is addressed in Chapter 8 (page 308).

3.5.4 Curriculum and Textbook

Looney (2003) (cited in NCCA, 2005a) noted that teachers believe the textbook is more influential than the curriculum in terms of making decisions about classroom teaching. The link between textbook and curriculum is evident when considering curriculum content, context and antecedents. Regardless of context the consideration of curriculum needs to take into account a variety of factors, such as streaming, mixed ability teaching, teaching styles and practices. Factors such as these are changeable. Likewise content is not only varied from school to school but also amongst teachers. The understandings and representations of the intended, implemented and attained curriculum can be hugely diverse. Curriculum antecedents, similarly, cannot be generalised as family, school, student and teacher backgrounds are characteristics which all play an influential role in the curriculum application. The only resource/link available to help control these variations is the textbook. The textbook is in a position to incorporate or at least take account of all of these factors at classroom level.

Greer and Mulhern (1992) outline changes to the curriculum content, the most
3.5. *Mathematics Education in Ireland: Issues for Consideration*

significant being the introduction and impact of ICT\(^5\). ICT is represented in the curriculum and its importance in mathematics is widely researched. However in order for ICT to be fully appreciated in the mathematics classroom it needs to be endorsed by teachers and classroom materials; specifically it needs to feature textbooks.

Between 1955 and 1960 due to a push towards ‘modern mathematics’ many countries revised their curricula. At this time England placed an emphasis on applications of mathematics. However ten years later, in the English textbooks mathematics applications were hardly mentioned (Cockroft, 1982). Since the sixties much research has been carried out on the teaching of mathematics but few studies have incorporated the textbook, despite its influential and dominant role. In the 1960’s, the USMES\(^6\) project in the US changed the focus of mathematics teaching to a more problem solving centered approach. Following its failure (it placed too many demands on teachers without sufficient support) the SPODE\(^7\) group decided to change the focus of mathematics in the UK to encompass a wide range of applications of mathematics and in doing so provided sample lesson plans. However they failed to provide adequate research to map the success or otherwise of their project. The Cockroft Report (1982) reported on the position of mathematics in the UK and encouraged teachers of mathematics to place greater emphasis on applications. (Cockroft, 1982: 283) encouraged a broad range of teaching strategies including:

- Exposition by teaching,
- Discussion,
- Practical work (must be appropriate),
- Consolidation and practice of skills,
- Problem solving,

\(^5\)ICT = Information and Communication Technology
\(^6\)USMES = Unified Sciences and Mathematics for Elementary Schools (a US project)
\(^7\)The SPODE Group (a UK initiative)
3.5. Mathematics Education in Ireland: Issues for Consideration

- Investigating work.

In order for such practices to become widespread they need to be endorsed by teachers and supported by good textbooks.

3.5.5 Mathematics in Ireland: Trends, Concerns and Recommendations

The textbook is acknowledged in literature as a vital ingredient in the successful teaching and learning of mathematics, and is highly valued by mathematics teachers in their practice. The NCCA discussion paper (1995) outlines four main trends in Irish mathematics education; a lack of awareness of the importance of problem solving and realistic mathematics education; an overemphasis on procedures which is to the detriment of understanding; students exhibit low levels of mathematical skill and there is a poor uptake of Higher Level mathematics.

There is an abundance of research regarding the importance of problem solving in the learning of mathematics (Schoenfeld, 1992; Orton, 2004; O.E.C.D., 2006). Ireland is no different from international mathematics education communities as regards the importance it places on problem solving, both at primary and secondary level. However, as the NCCA (2005a) noted, the over emphasis in Irish classrooms on procedures is a barrier to developing problem solving skills. In order to solve problems students should be encouraged to ‘play’, to question, to model, to criticise and to investigate anything which challenges the question posed.

Another trend in Irish Education, according to the NCCA, is that the focus is entirely on succeeding in state examinations. Students should be encouraged to develop their own mathematical abilities and not be confined by practices dictated by ‘teaching to the examination’. A combination of the above factors leads to the final worrying reality - low uptake of Higher Level mathematics. In 2009 and 2010 only 16% of the student cohort opted for Higher Level Leaving
3.5. *Mathematics Education in Ireland: Issues for Consideration*

Certificate mathematics compared to 64% opting for Higher Level English (SEC, State Examinations Commission, 2010). Mathematics, as mentioned previously, is perceived as only for those with ‘talent’. The failure of the Irish education system to make mathematics interesting and alive for students and the pressures of an examination focused classroom turns many capable students away from Higher Level mathematics.

These trends give rise to many concerns, some of which are also outlined in the NCCA discussion paper (NCCA, 2005b):

**Concerns:**

1. Post primary curriculum in Ireland does not make reference to RME or modelling,

2. Pre-service teachers’ own experience of mathematics is very often traditionally taught mathematics,

3. Primary curriculum is currently far more in line with considering different methodologies and approaches to teaching and learning. A recent study by the I.N.T.O. (2005) indicated that the vast majority of Primary teachers feel they are successful at teaching mathematics and there is evidence of a variety of teaching methodologies being employed,

4. Primary curriculum provides mathematics as a creative activity as well as a process of managing and communicating information. This creativity is lost at Junior Cycle,

5. Modelling is traditionally for applied mathematics as teachers find it can be time consuming as it is so far removed from the drill and practice approach of routine skills and procedures.

Having considered these trends and concerns the NCCA proceeds with a list of recommendations. These recommendations highlight the need for a change in the learning culture in Irish mathematics classrooms and the need to break the cycle of teaching and learning for the ‘exam’. They acknowledge the need to bridge the gaps from primary mathematics to Senior Cycle mathematics, at the primary Junior Cycle and the Junior Cycle Senior Cycle interfaces. The NCCA encourages an emphasis on problem solving and modelling in mathematics in order to enhance
3.5. Mathematics Education in Ireland: Issues for Consideration

student understanding. These recommendations also espouse the case to improve the take up rates of Higher Level mathematics at both Junior and Senior Cycle. According to the NCCA (2006) a new range of mathematics textbooks are required which place an emphasis on explanations, theory and applications, “a reform of post-primary mathematics toward a more problem-solving orientation will, it could be argued, necessitate a radical overhaul of mathematics textbooks” NCCA (2006: 179). This is a major challenge for the mathematics education community in Ireland including the textbook publishers.

3.5.6 Conclusion

These recommendations by the NCCA are specific to Irish mathematics education. The first of these recommendations is to change the learning culture. A change such as this would require enormous amounts of time, effort and commitment by teachers. However one simple tool can be very effective in implementing change - the textbook, as it sets the tone of the learning culture. The second recommendation speaks about consistency and obvious sequencing from primary level to Leaving Certificate. Again the textbook is hugely significant, a collaboration between textbook writers from each level could assist with such a change. The next two recommendations request a change in focus towards applications and modelling. As noted previously by Neyland (1995) the typical failing of such a focus is that teachers do not have sufficient resources to support them. A change in textbook focus would provide such support. In order to increase uptake of Higher Level Leaving Certificate there needs to be a significant improvement of the teaching methods and resources available at Junior Cycle level. Once new teaching methods are in place at Junior Cycle level the benefits should filter up through the system. Changes such as these would also satisfy the sixth recommendation, breaking the cycle of learning for the examinations, as teaching and learning would become the main classroom focus.
3.6. Conclusion

There is a need for new textbooks, textbooks which have logical structure and good sequencing, that provide background information and applications; textbooks that focus on challenging the student and encouraging investigation and problem solving and textbooks which encourage and evoke learning.

3.6 Conclusion

This chapter addresses background issues for textbook research in mathematics education. It lays the foundations for the author’s investigation into mathematical textbooks by identifying the key areas for analysis and important researchers in each area. It reinforces the importance of the textbook to mathematics education and identifies a niche in Irish mathematics education research for such work. This chapter and the previous chapter serve to inform the author and the reader on the frameworks and methods which direct and inform the research methodology, analyses and interpretation in this study.
Chapter 4

Methodology

4.1 Introduction

The purpose of this chapter is to examine and explain the theoretical methods applied to all elements of this research study. This research started as a process of textbook analysis which involved both a qualitative and quantitative study of Junior Cycle mathematics textbooks. Following this analysis, which included language analysis, a model chapter was designed and created. This model chapter was piloted, amended and then trialled on a small scale within a number of first year mathematics classrooms. The research was guided by and referenced to an in-depth literature review (chapter 2 and chapter 3), which provided the theoretical frameworks and methodologies. Throughout this study a number of choices and decisions were made by the author in terms of deciding on and applying of appropriate methods and methodologies. The author’s background as a mathematics teacher was the main influencing factor in such decisions. All decisions and choices made were conducted with the final outcome in mind, that is improving the quality of mathematics teaching and learning.

4.2 Research Paradigms

The methodological aim of research is to measure and analyse relationships between variables (Lincoln and Guba, 2000). Within this methodological aim one must
4.2. *Research Paradigms*

ensure that suitable research paradigms are in place. Denzin and Lincoln (2000: 33) refer to paradigms as a framework for interpretation, which is guided by “a set of beliefs and feelings about the world and how it should be understood and studied.” They also outline three categories of beliefs; ontology, epistemology and methodology. Ontology refers to the assumption of reality, epistemology is the knowledge about this reality and methodology concerns the ways in which we gain such knowledge. Quantitative and qualitative paradigms are discussed based on the above categories, ontology, epistemology and methodology.

Qualitative research is defined by Denzin and Lincoln (2000: 3) as

“a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world ... into a series of representations, including fieldnotes, interviews, conversations, photographs, recordings and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them.”

The qualitative paradigm is also known as the interpretative, constructivist, or postpositivist approach. At a methodological level the qualitative paradigm places an emphasis on processes and meanings (Lincoln and Guba, 2000), with the intention of describing and understanding a phenomena from a participant’s point of view. Lincoln and Guba (2000) identify that there are five key research paradigms; positivism, interpretivism, critical post-modernism, constructivism and participatory. According to Robson (1993) the most widely used research paradigms in educational research are positivism and interpretivism.

The quantitative research paradigm is also known as the traditional, experimental
4.2. Research Paradigms

or positivist approach and is concerned with answering questions about relationships between measured variables with the intention of explaining and predicting (Leedy and Ormond, 2001). Quantitative research is independent of the researcher and is usually carried out under experimental settings rather than in natural settings. As researchers in the quantitative paradigm are concerned with unnatural environments it is necessary for educational researchers to remove bias, remain detached and uninvolved with participants. In general within a quantitative study the variables need to be clearly defined and measurable in order to increase validity and the results can be converted to numbers.

4.2.1 Positivist Vs Interpretist

According to Krauss (2005) in the positivist paradigm the object of a study is independent from the researchers, that is knowledge is discovered and verified through direct observations or measurements and facts are established by taking apart a phenomenon in order to examine it. In the interpretist paradigm, the researchers interact with the subjects of study in order to obtain data and knowledge is obtained in context and is time dependant (Krauss, 2005).

Positivist and interpretist paradigms describe their data, construct explanations from their data and speculate about outcomes. The need to use these approaches is reflective of the type of data collection; qualitative data collection or quantitative data collection. Table 4.1 provides a brief overview of the comparisons between the positivist and interpretist approaches. For this research study a positivist approach was used in conjunction with quantitative data collection and an interpretist approach was applied with qualitative data collection (Leedy and Ormond, 2001).
### Table 4.1: Positivist Research Paradigms vs Interpretist Research Paradigms

<table>
<thead>
<tr>
<th></th>
<th>Positivist</th>
<th>Interpretist</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Philosophical Inquiry</strong></td>
<td>• Reality is independent from observers</td>
<td>• Reality is constructed by individuals who participate in it</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• It is constructed differently by different individuals because individuals build their understandings of the world through experience.</td>
</tr>
<tr>
<td><strong>Research Design</strong></td>
<td>• Inquiry focus is on general trends.</td>
<td>• Inquiry focus is on the study of multiple social realities</td>
</tr>
<tr>
<td></td>
<td>• Features of the environment remain constant</td>
<td>• The study of individuals’ interpretations of social reality occur at the local, immediate level Generalisation of case study findings are made on a case-by-case basis.</td>
</tr>
<tr>
<td></td>
<td>• The researcher must first define a population of interest, select a representative of that population, generalise findings obtained the sample to the larger population using the statistical techniques.</td>
<td>• The focus is on the transferability instead of generalisation.</td>
</tr>
<tr>
<td><strong>Data Collection and Design</strong></td>
<td>• Use of mathematics to represent and analyse features i.e. particular features can be isolated and conceptualised as a variable</td>
<td>• Focuses on the study of individual cases and making verbal descriptions of observations</td>
</tr>
<tr>
<td></td>
<td>• Variables can be expressed as a numerical scale</td>
<td>• Requires the searching through data bit by bit and then inferring that certain events or statements are instances of the same underlying themes or patterns</td>
</tr>
<tr>
<td></td>
<td>• Can involve hypothesis testing</td>
<td></td>
</tr>
<tr>
<td><strong>View of Causality</strong></td>
<td>Mechanistic causality</td>
<td>Individuals’ interpretation of situations</td>
</tr>
</tbody>
</table>

(Summary of the work of Krauss (2005) and Leedy and Ormond (2001))

#### 4.2.2 Qualitative, Quantitative and Mixed Methods

Qualitative and quantitative approaches to research have dominated research culture and it is only in relatively recent times that researchers are opening to the idea of mixed methodologies. Reichardt and Cook (1979) state that the time
4.2. Research Paradigms

has come to stop building walls between research methods and to begin building bridges. Quantitative and qualitative research have been described as scientific and humanistic, positivistic and humanistic, and positivistic and interpretive (Clarke, 2003: 38). Individually, both these methods have their own merit. However, a combination of these methods provides a greater insight into research. Hence a mixed method approach was applied to this research study. A mixed method approach to research combines quantitative and qualitative at the methodology stage of research. The application of a mixed method approach allows the strengths of both methods to be exploited while minimising the limitations. Bryman (1996: 105-107) provides a framework for the mixing of qualitative and quantitative research methods. He outlines how a mixed method approach can enhance the research process by ensuring triangulation and providing methods to answer difficult research questions. The following table (Table 4.2) summarises the occurrence of mixed method application in this research project. It also demonstrates where mixing occurs at both the data collection and data analysis levels.

<table>
<thead>
<tr>
<th>Phase:</th>
<th>Data Collection</th>
<th>Data Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Qualitative</td>
<td>Quantitative</td>
</tr>
<tr>
<td>Phase 1: Preliminary Analysis</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Literature Review</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Phase 2: Content Analysis</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Structure Analysis</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Language Analysis</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Readability Analysis</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Phase 3: Student Interviews</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Diagnostic testing</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
4.3 Research Design

The methodology of any research project is crucial, the chosen methodology controls the study by the way it dictates how data is collected and analysed. There are a number of considerations to be outlined before deciding on the best research methodology. Research is a cyclical pattern which creates gaps and insights which warrant further investigation. Having discovered these gaps, the most important thing is to apply the appropriate methods of analysis. In order to ensure the correct methodology the following tools of research must be considered; inquiry, approach and design (Cohen, Manion, and Morrison, 2000). The process of inquiry involves identifying research gaps through a review of the literature and one must consider alternative knowledge claims, the stages of inquiry and suitable methods. Following this a research problem is highlighted which determines what research approach is required; qualitative, quantitative or a mixed method perspective. Translating each method into practice, following Creswell’s (2003) design process which is outlined below, determines the design process of the research:

- Questions,
- Theoretical Lens,
- Data Collection,
- Data Analysis,
- Write Up,
- Validation.

4.3.1 Theoretical Frameworks and Research Design

A theoretical framework is a conceptual model which outlines how the researcher makes sense of the various relationships and factors significant to a problem (Sekaran, 2000). That is, a theoretical framework links key literature and conceptualises a problem that can be observed, tested or measured. Radhakrishna, Leite, and
4.3. Research Design

Baggett (2003) classified research designs into three categories: descriptive, descriptive-correlational, and experimental. Selection of the research design is dependant on the goals of the research and the available literature. For this study a mixture of descriptive, descriptive correlational and experimental research designs are employed. Descriptive research identifies the incidence, distribution, and characteristics of a situation (Radhakrishna, Yoder, and Ewing, 2007). Descriptive research is employed in phase two of this research study, it identifies the key variables which are then studied in depth using descriptive correlational research. Descriptive correlational research explains or predicts relationships, it is employed in phase two of this research to provide a more in-depth analysis of the variables identified by descriptive research. Experimental research involves the use of a control group, a random assignment and manipulation (Radhakrishna et al., 2007). The purpose of experimental research is to identify if the intervention or treatment made a difference. Experimental research, which builds on descriptive and descriptive correlational research, is employed in phase three of this research study.

4.3.2 Research Problem

Research can be described as a process of collecting and analysing data with the view to increasing understanding. In Ireland, mathematics is deemed to be one of the more difficult subjects in school and is traditionally taught through memorising rules, facts and formulae with little or no emphasis on understanding (NCCA, 2006). This process of teaching alienates many students and significantly increases the failure rates for mathematics. From previous research the author is aware that the textbooks are the most significant and widely used resource in Irish mathematics classrooms. However no research has ever validated the effectiveness of these textbooks or simply analysed them with the view of correlating the research (developed to increase students’ mathematical learning) with the approaches and methods presented in the textbooks.
4.4. Description of the study

This research will focus on formal research with the view to developing and deepening understanding as regards mathematics textbooks and their role in education, especially their role in Irish education. The purpose of this research is to investigate the quality of the current Junior Cycle mathematics textbooks and it is the first large scale study of its kind in Ireland. This research not only identifies the quality of the current mathematics textbooks but it also highlights key design features which impact on students’ understanding of mathematics.

4.4 Description of the study

This study comprises a tri-phase approach, the characteristics of which will be discussed. The relationship between the mathematics textbook and student learning is a complex one, with limited Irish based research available in this area. This research focus is specific to an Irish context and is concerned with improving the teaching and learning of mathematics in Ireland.

4.4.1 Outline of Phases

This research follows three main phases which are presented and outlined in figure 4.1.
4.4. Description of the study

![Diagram of the study phases and methods](image)

**Phase 1: Final Year Dissertation**
- Exploratory Research
- Preliminary Investigation
- Literature and Relevant Background Review
- Coordination & Adaptation of Suitable Theoretical Frameworks
- Content – Rivers Matrix & TIMSS
- Structure – Mikk’s Structure Analysis Grids & TIMSS
- Expectation – Rivers Matrix & TIMSS
- Language – Morgan’s Functional Grammar Analysis

**Data Collection Instruments**
- Textbook Analysis
- Language Analysis
- Expectation Analysis
- Structure Analysis
- Content Analysis

**Phase 2: Data Collection from Textbook Analysis**
- Design & Create Research Materials
- Compilation of Key Design Features
- Create Model Chapter
- Create Two-Tier Diagnostic Test
- Implementation of Model Chapter and Two-Tier Diagnostic Test
- Implementation into First Year Mathematics Classrooms
- Data Collection & Evaluation
- Pre-test
- Post-test

**Phase 3: Design & Create Research Materials**
- Piloting of Model Chapter and Two-Tier Diagnostic Test
- Create Model Chapter
- Create Two-Tier Diagnostic Test
- Implementation of Model Chapter and Two-Tier Diagnostic Test
- Implementation into First Year Mathematics Classrooms
- Data Collection & Evaluation
- Data Analysis

**Overall Findings & Contributions**

Figure 4.1: Outline of the Study and Main Phases
4.4. Description of the study

**Phase 1:** Phase one incorporates a preliminary analysis of the Junior Cycle mathematics textbooks in Ireland, identifying the most widely used in Irish mathematics classrooms. Following this analysis a comprehensive review of available literature was carried out to deepen the insight into mathematics education in Ireland, and to establish the role of the textbook as the classroom resource worldwide. Phase one allowed for the creation of a detailed methodology, which guided the development of a suitable framework for research in Phase 2. The research design also accommodates the review and selection of relevant text analysis tools, each of which was employed in phase 2; (TIMSS (2002), Rivers (1990), Mikk (2000) and Morgan (2004)).

**Phase 2:** Phase 2 entails the mathematics textbook analysis. Data was obtained from the Irish Junior Cycle mathematics textbooks by means of textbook analysis, that is applying the previously selected text analysis tools. Textbooks were sourced based on their frequency of use in schools, that is the three most commonly used textbooks were identified and selected. Identification of such textbooks required the author to initially contact the largest textbook provider in the country. However, for competition reasons they declined to provide sales figures or any details with regard to their sales (Appendix B). Hence the author selected these textbooks based on more than one hundred phone calls to school textbook providers throughout the country. A list of these school textbook providers was obtained from the golden pages website and the three main textbooks which emerged are employed in this study.

The initial framework for the textbook analysis is based on the TIMSS framework for textbook analysis which was reported in detail by Valverde et al. (2002). However, from the literature review the author is acutely aware of the relevance and significance of language to mathematics learning and hence to mathematics textbooks. For this reason an extra branch was included in the TIMSS framework -
4.4. Description of the study

The Language Analysis. The overall framework for the textbook analysis engaged in this study now comprises four main elements which are:

- Content,
- Structure,
- Expectation,
- Language.

Theoretical Framework for Phase two

Suitable theoretical frameworks for analysis were employed for each element of this textbook analysis study and are outlined in Table 4.3 and Figure 4.2.

<table>
<thead>
<tr>
<th>Theoretical Frameworks (TF)</th>
<th>Function</th>
<th>Significance*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morgan (2004)</td>
<td>Strengthens textbook analysis by providing a fourth element for analysis</td>
<td>4. Language Analysis</td>
</tr>
</tbody>
</table>

*The data collected from each phase of textbook analysis (Content, Structure, Expectation and Language) is interpreted and analysed in terms of impact on students’ conceptual learning.
4.4. Description of the study

The author acknowledges the complexity of mathematics textbook analysis and the variety of analysis tools available. However, the author was required to make certain decisions regarding research methods specific to this study. Such decisions include identifying the most effective methods of mathematics textbook analysis which would provide for an in-depth analysis of the textbooks with an added focus that would provide for design considerations, language and conceptual development (Table 4.3). Figure 4.2 outlines the theoretical frameworks adopted in this study.

### Figure 4.2: Theoretical Outline for Textbook Analysis

**Phase 3:** The author applied the design specification, developed from the findings of the textbook analysis, to the construction of a model chapter with a focus on conceptual understanding and problem solving. This model chapter was centered around the topic of adding fractions. Fractions were selected based on the author’s own experiences as a teacher, from informal feedback from students of all ages and a number of studies which highlight the difficulties that students’ experience with fraction manipulation. The design for this model chapter evolved from a
4.4. Description of the study

combination of four well established relevant frameworks, Project Maths and the Junior Cycle curriculum, Adult Numeracy Network (Curry et al., 1996), Adult Based Education (M.D.E., 2005) and the Pisa Mathematical Cycle (O.E.C.D., 2006). Following a pilot, review and amendment phase this model chapter was trialled in a small scale intervention in three secondary schools. The model chapter was used in conjunction with a two tier diagnostic test to ascertain its effectiveness in enhancing student conceptual development. This two-tier diagnostic test was based on the work of Treagust (1988). The quantitative data collected from this phase of the research was analysed using SPSS version 16.0.

Theoretical Framework for Phase Three

Table 4.4 outlines the theoretical frameworks adopted in this phase of the study. Figure 4.3 gives the theoretical outline of this research project.

<table>
<thead>
<tr>
<th>Phase 3</th>
<th>Theoretical Frameworks (TF)</th>
<th>Function</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Chapter</td>
<td>JC Syllabus &amp; Project Maths</td>
<td>Provides structure &amp; fraction addition content</td>
<td>Dept. of Education <a href="http://www.projectmaths.ie">www.projectmaths.ie</a></td>
</tr>
<tr>
<td></td>
<td>ANN framework</td>
<td>Provides information on teaching and learning fraction addition</td>
<td>Adult Numeracy Network Curry et al. (1996)</td>
</tr>
<tr>
<td></td>
<td>ABE Framework</td>
<td>Provides information on teaching and learning fraction addition</td>
<td>Adult Based Education M.D.E. (2005)</td>
</tr>
<tr>
<td>Evaluation</td>
<td>Two-Tier Diagnostic Test</td>
<td>Test Instrument which measures Conceptual Understanding</td>
<td>Treagust (1988)</td>
</tr>
<tr>
<td></td>
<td>RMARS</td>
<td>Test Instrument which measures Anxiety</td>
<td>Plake and Parker (1982)</td>
</tr>
</tbody>
</table>
4.4. Description of the study

Figure 4.3: Theoretical Outline for the Development and Evaluation of the Model Chapter
4.4. Description of the study

4.4.2 Research Aims

Before identifying specific research questions it is first necessary to remind oneself of the overall aim and focus of this study.

- To review the literature in relation to methods of mathematics textbook analysis, the role of the textbook in mathematics education nationally and internationally and the key concerns in mathematics education which can be impacted by the mathematics textbook.

- To investigate the quality of the Junior Cycle mathematics textbooks currently in use in Ireland.

- To establish the most effective methodological tools for a complete textbook analysis.

- To identify key textbook design features which impact on students’ conceptual development.

4.4.3 Research Questions

The initial investigation focused on an application of the TIMSS textbook analysis (Valverde et al., 2002) (Phase 1 and 2). However, it soon became clear that such a framework was inadequate in a number of ways and could not provide a definite status for each of the textbooks. Further review of the literature, identified language as being a significant factor in both the teaching and learning of mathematics. At this juncture the author added an element of language analysis to the overall framework for textbook analysis. This additional element enabled the author to engage in a detailed, in-depth investigation into the standard of the current mathematics textbooks in Ireland (Phase 2). The review of the literature also uncovered a number of textbook features which enhance student learning, in particular student motivation and comprehension. The literature review also informed the creation and design of a model chapter in fraction addition which
4.4. Description of the study

lent itself to establishing the role textbooks can play in students’ conceptual development (Phase 3).

Each phase of this research project relied upon, and was based on, the findings or outcomes of the previous phase and each was guided by the following research questions:

**Phase One - Exploratory Research**

- What role do mathematics textbooks play in mathematics education i.e. in the teaching and learning of mathematics?

- What are the main areas of concern in mathematics education which are impacted by the content and structure of mathematics textbooks?

- Which of the available methods of textbook analysis are the most applicable and relevant to mathematics textbook research?

**Phase Two**

- Are the current Junior Cycle mathematics textbooks an effective resource for teaching and learning?

- What is the significance and impact of language considerations as interpreted in textbooks for both the teaching and learning of mathematics?

- Do the current textbooks address the key areas of concern in mathematics education such as language deficiencies and poor problem solving skills?

**Phase Three**

- Can improving the textbooks directly improve students’ understanding of mathematics and their conceptual development?
A number of theoretical frameworks were employed throughout in phases two and three of this research study. Figure 4.4 provides an overview of the theoretical frameworks applied in this study.

Figure 4.4: Theoretical Frameworks
4.5. Theoretical Frameworks employed in this Study

4.5.1 Phase one:

This study originated from an undergraduate final year project which was completed by the author. The final year project consisted of research in the area of Junior Cycle mathematics textbooks. During the initial project it became evident that no Irish research was previously carried out to determine the effectiveness of the Irish mathematics textbooks. From here a further review of the relevant and available literature was carried out during the academic year 2007/2008. The aim of phase one is to determine the importance of the textbook and the quality of textbooks that should be in place for effective mathematics teaching and learning. This phase also informs the methodology to be employed in the overall study. The main outcome of this phase consists of a literature review.

Phase One - Research Questions

- What role do mathematics textbooks play in mathematics education i.e. in the teaching and learning of mathematics?
- What are the main areas of concern in mathematics education which are impacted by the content and structure of mathematics textbooks?
- Which of the available methods of textbook analysis are the most applicable and relevant to mathematics textbook research?

4.5.2 Theoretical Frameworks Phase Two - Textbook Analysis

Phase two consists of the textbook analysis and data collection. The mathematics textbooks selected for this study are the current Junior Cycle mathematics textbooks. Data is collected from the textbooks via textbook analysis which is applied to determine the quality of each of these textbooks. Comparisons are widely used at this stage of the research. Following this analysis a list of guidelines and recommendations for future textbooks are created. Based on these guidelines
4.5. *Theoretical Frameworks employed in this Study*

A sample chapter (model chapter) based on fraction addition is designed for the sole purpose of highlighting how effective the key design features are in developing students’ conceptual understanding. The purpose of phase two is to answer the following main research questions:

**Phase Two - Research Questions**

- Are the current Junior Cycle mathematics textbooks effective at assisting teaching and learning?
- What is the significance and impact of language considerations for both the teaching and learning of mathematics?
- Do the current textbooks address the key areas of concern in mathematics education such as language deficiencies and poor problem solving skills?

**Textbook Analysis:**

Textbook analysis was carried out under four main headings: Structure, Content, Language and Expectation.

**Structure:** This area of analysis includes both the structure of knowledge and information within the textbook and the make up of the textbook. The structuring of knowledge can be analysed with consideration to cross referencing, in the index and throughout the chapters. The author also examined the textbook under a number of headings such as Block Type, Content Structure and Physical Scale, as identified by the TIMSS framework. Data was recorded diagrammatically using a method devised by Mikk (2000).

**Content:** Content is directly related to motivation and comprehension. This analysis was based on River’s (1990) findings. Rivers used a four point measure of analysis to determine motivational factors, comprehension cues, technical aids and the philosophical position of the textbook. Motivational factors can be identified by
4.5. *Theoretical Frameworks employed in this Study*

the presence of historical notes, scientists and mathematician biographies, career information, applications and photographs. Comprehension cues are evident in the presence of colour and graphics. Technical aids refer to the inclusion of calculator and computer material, while the philosophical position highlights any emphases or points of view created by the author.

Active learning and problem solving have emerged from international studies, as a prominent area for consideration in textbook analysis. In the content analysis, the author focuses on the presence of active learning and the quality of problems presented to the audience (Appendix C). Problem solving is a common area of weakness for Irish mathematics students (PISA, 2001), hence the author analysed the approach outlined by the mathematics textbook with regard to problem solving.

**Expectation:** The analysis of student expectation portrayed by the textbook played a large role in the TIMSS study. A similar approach is applied here, combined with the River’s Matrix (Rivers, 1990) method of analysis, in order to determine the overall expectations of students.

**Language & Readability:** The language analysis in this research study is based on the worked of Halliday (1973) and Morgan (2004). There is limited research on the language analysis of mathematical text, particularly in conjunction with text analysis as a whole. Halliday created a framework for analysing functional grammar in the seventies. In 2004, Morgan applied this framework to students’ own written mathematical compositions. For the purpose of this study, Morgan’s method of analysis was applied faithfully and then extended to complement this research. The extension of the framework allowed the author to successfully draw conclusions with regard to the language present within the texts and language analysis of mathematical texts in general. (Refer to Chapter 6 on Language analysis for a more in-depth and detailed treatment of this phase of the research.)
4.5. *Theoretical Frameworks employed in this Study*

Table 4.5 summarises the theoretical frameworks for textbook analysis and the element of analysis for which each framework was significant. The numbers in the significance column relate to the element of the overall textbook analysis framework for which that particular aspect is relevant.

<table>
<thead>
<tr>
<th>Theoretical Frameworks (TF)</th>
<th>Function</th>
<th>Significance*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morgan (2004)</td>
<td>Strengthens textbook analysis by providing a fourth element for analysis</td>
<td>4. Language Analysis</td>
</tr>
</tbody>
</table>

*The data collected from each phase of textbook analysis (Content, Structure, Expectation and Language) is interpreted and analysed in terms of impact on students’ conceptual learning.

**TIMSS (2002)**

The most well known, international textbook study was conducted by TIMSS and reported by Valverde et al. (2002). TIMSS involved an in-depth analysis of 630 mathematics and science textbooks. The TIMSS analysis comprised of three key elements; Content, Structure and Expectation. In order to effectively and systematically complete this research study the authors had to establish how best to present their results. The TIMSS method of both analysis and presentation forms a large part of this researcher’s framework for textbook analysis.
4.5. *Theoretical Frameworks employed in this Study*

**Rivers (1990)**

In 1990 Janelle Rivers undertook a two-part analysis of first year algebra textbooks in South Carolina. Phase 1 of her study focused on a comparison of five textbooks, and in-depth analysis of the

- Motivational Factors,
- Comprehension Cues,
- Technical Aids,
- Philosophical Orientation.

She also looked at the cost and sales figures for each textbook. Part 2 of her study was based on the changes made to each textbook based on the NCTM \(^1\) standards. Part 1 of the Rivers study is directly applicable to this research study with the first three of the Rivers elements (listed above) overlapping and reinforcing the data analysis and collection of the TIMSS content analysis. The fourth and final element supports the TIMSS expectation analysis.

**Mikk (2000)**

Mikk (2000) encompasses many issues and concerns of mathematics textbook analysis related to the use, evaluation and analysis of textbooks. In his research Mikk illustrates by way of a matrix table how one can easily analyse the structure of a text and record diagrammatically how frequent ideas/topics appear and therefore connections are visualised (Appendix D) (Mikk, 2000: 99). This method of structure analysis is combined with that of TIMSS to strengthen the data collection for structure analysis.

**Morgan (2004)**

Having spent a number of years as a mathematics teacher Morgan completed her PhD in 1995 which focused on the analysis of discourse of written mathematical

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\(^1\)NCTM = National Council of Teachers of Mathematics
4.5. *Theoretical Frameworks employed in this Study*

reports. Following on from here Morgan has continued her research in the area of language and mathematics. One of the key areas emerging from Morgan’s work is the analysis of written mathematical text, however she focuses on student’s own mathematical writings. Her work is primarily based on Halliday’s functional grammar analysis. This functional grammar analysis comprises three main elements (see section 5.5 for a detailed explanation of each):

- Ideational Function,
- Interpersonal Function,
- Textual Function.

This framework, created by Halliday (1973) which was applied to mathematics by Morgan (2004), is utilised in this research study to analyse the language of the mathematics textbooks. This is a novel application of the Halliday/Morgan framework.
4.5. *Theoretical Frameworks employed in this Study*

Figure 4.5 provides a brief overview of the theoretical frameworks applied in phase two of this research study.

![Diagram of Theoretical Frameworks](image)

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Figure 4.5: Theoretical Frameworks - Phase 2
4.5. Theoretical Frameworks employed in this Study

4.5.3 Theoretical Frameworks Phase Three

Phase three consists of the development of a model chapter (Phase 3a)(Appendix E) the development of a two tier diagnostic test instrument (Phase 3b) and the implementation of both of these in Irish secondary schools.

The design and creation of the model chapter is detailed in section 6.4, the following six steps give a brief outline of each stage of development:

Step 1
Identify the topic of interest - fraction addition.
[The author’s own experiences of teaching at both second and third level in conjunction with the literature review are the main deciding factors in choosing the topic of fraction addition.]

Step 2
Create a topic based concept map (Appendix F).
[This concept map includes careful consideration for the Irish Junior Cycle mathematics curriculum.]

Step 3
Consideration for the Key Design Features.
(The key design features which have a significant impact on student comprehension and motivation are most applicable.)

Step 4
Create first draft of the model chapter.

Step 5
Review of the Model Chapter by an expert panel.
[This expert panel is comprised of both second and third level educators. The model chapter is amended and redrafted based on the recommendations provided by the expert panel.]

Step 6
The Model Chapter is piloted with a group of sixth class students near the end of the school year.
[Sixth class students were chosen for this pilot study as these students will be going directly into first year the following September (when the testing phase is expected to begin). Student questionnaires are distributed to obtain student feedback. Semi-structured interviews are also conducted to follow up on this feedback. The final draft of the model chapter is then finalised.]
4.5. *Theoretical Frameworks employed in this Study*

The author has identified fractions as an aspect of mathematics which is significant to mathematics in its entirety and is widely accepted as containing difficult concepts to learn. The authors own experiences as an educator and informal feedback from students of all ages reinforce the selection of the topic fraction addition. The model chapter was designed and created based on the key findings from phase two of this research study combined with a number of key frameworks, as outlined in Table 4.4. This model chapter was then piloted, reviewed, amended and trialled on a small scale in three secondary schools in Ireland.

As outlined by Remillard (2000) teachers need teacher centered guidelines or text material to support student mathematics textbooks. Hence the author amended the student model chapter in this study to create a more detailed teacher textbook. This teacher textbook is provided in Appendix G.

<table>
<thead>
<tr>
<th>Phase 3 Theoretical Frameworks (TF)</th>
<th>Function</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Chapter JC Syllabus &amp; Project Maths</td>
<td>Provides structure &amp; fraction addition content</td>
<td>Dept. of Education <a href="http://www.projectmaths.ie">www.projectmaths.ie</a></td>
</tr>
<tr>
<td>ANN framework</td>
<td>Provides information on teaching and learning fraction addition</td>
<td>Adult Numeracy Network Curry et al. (1996)</td>
</tr>
<tr>
<td>ABE Framework</td>
<td>Provides information on teaching and learning fraction addition</td>
<td>Adult Based Education M.D.E. (2005)</td>
</tr>
</tbody>
</table>

The selected frameworks (provided in Table 4.6) are based on the teaching and learning of numeracy as there is no specific framework currently in existence appropriate to this chapter. The dictionary definition of numeracy is a ‘mastery of the basic symbols and processes of arithmetic’: 1. Numbers, 2. Addition, 3. Subtraction, 4. Simple, 5. Multiplication, 6. Simple Division, 7. Simple Weights and Measure, 8. Money Counting and 9. Telling Time.
4.5. *Theoretical Frameworks employed in this Study*

These mathematical processes, as listed above, have many mathematical skills common to them, the ability to work with fractions being one. Fractions, while obviously an important component of any mathematics are “frequently mentioned as hard topic in school math” by both adults and children (Curry et al., 1996: 36). The four basic mathematical skills, (numbered 2 - 5 above), are all relevant to and involved with fraction manipulation. Numbers 6 - 8 above rely heavily on fraction and decimal manipulation, for example when working with weight and measure.

A proficiency in the manipulation of fractions and related operations is necessary for the successful execution of much of the Junior Cycle mathematics curriculum. Currently in Ireland, Project Maths is a new curriculum initiative which will replace the current eight strand Junior Cycle syllabus with a five strand curriculum approach. The first two strands of this new curriculum will be implemented in September 2010 and its five strands are as follows:

- Statistics and Probability
- Geometry and Trigonometry
- Numbers
- Algebra
- Functions.

While ‘fractions’ are not explicitly stated as an individual strand, fraction manipulation is a central part of each. For example statistics involves collecting and working with data which may not always be confined to whole number manipulation,

“A Junior Cert class consists of five 13 year-olds, eight 14 year-olds and seven fifteen year-olds.

(i) Find the mean age of the group”

(Murphy, 2001: 158).
The foundational principles of probability are based on fraction and decimal manipulation. There is an inherent relationship between fractions and probabilities. Exact probabilities can be computed hence the occurrence of an event can be determined as a fraction of possible outcomes (Van de Walle, 1994). With Geometry and Trigonometry, fractions are inherent in working with the unit circle, right angles triangles, area and volume and much more with many formulas being derived as fractions, for example the formula for the slope of a line is

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

“Find the slope of the line which contains the points \((-1, 3)\) and \((4, 1)\)”

(Humphrey, 2001: 280).

The third strand, ‘Numbers’, involves working with natural and rational numbers, therefore it has a direct link to fraction manipulation. For example:

“Write each of the following fractions in its simplest form:

(i) \(\frac{35}{42}\)  (ii) \(\frac{60}{96}\)  (iii) \(\frac{24}{72}\)  (iv) \(\frac{30}{48}\)  (v) \(\frac{175}{217}\)

(Maxwell and Evans, 2000: 36).

The algebra strand requires a knowledge of fractions for the manipulation of algebraic fractions and for skills such as solving equations, working with indices and factorisation.

“Solve the equation: \(\frac{x + 1}{4} - \frac{x}{3} = \frac{1}{12}\)”

(O’Driscoll, 2001: 19).

Fractions also play a role in the last strand, Functions, with one significant area being the making of estimations from a graphical function, drawing these graphs and for solving the equations.

Having considered the relevance of fractions to Junior Cycle mathematics, the author created a framework to assist with the design of a fractions chapter suitable for a Junior Cycle mathematics textbook.
4.5. *Theoretical Frameworks employed in this Study*

Phase 3 is also concerned with the development of the two tier diagnostic test instrument, the framework for this text instrument (section 6.4) is adapted from the work of Treagust (1988) with science education diagnostic tests, which have previously been proven to be effective in measuring conceptual development. Following the sourcing of the three schools required for this study, (those which use the textbooks associated with this study) three teachers in each school were selected to facilitate this study. All of these teachers are teaching a first year mathematics class. Two teachers in each school replaced the usual textbook with the model chapter while the remaining teacher continued to teach with the usual textbook, acting as the control group. Each teacher involved teaches the same topic - fraction addition. All students involved are subject to pre and post testing. The application of these pre and post tests is as follows:

**Step 1** Selection of three schools based solely on textbook choice. For the purpose of this study, three schools each using one of the types of textbooks involved in the analysis in phase two are required. Within each school, teachers teaching first year mathematics (must be at the same level) are required to take part.

**Step 2** The teachers in the test group teach with the model chapter while the teacher in the control group uses the usual textbook. Both the test and control class group work on the same topic within the same time-frame but only the test group students are provided with a copy of the model chapter.

**Step 3** Prior to beginning the intervention the students are subject to a pre-test. This pre-test identifies the level of understanding students have prior to learning the section. In conjunction to this two-tier diagnostic pre-test the students also complete a Revised Mathematics Anxiety Rating Scale (RMARS) test (pre-test) (Plake and Parker, 1982) (see Appendix H).

**Step 4** This stage of the research is the intervention phase, the teaching of fraction addition with either the model chapter or the usual textbook. Following this
the same two-tier diagnostic and RMARS test are applied (post-test).

**Step 5** Pre and post test results are compared and analysed using SPSS version 16.0.

4.6 Data Collection and Analysis - Phase Two

Data was collected from the Junior Cycle mathematics textbooks by means of textbook analysis. As previously outlined (see section 4.5) the overall framework for textbook analysis applied to this study comprises four key elements; Content, Structure, Expectation and Language.

4.6.1 Data Collection Instruments

Figure 4.6 provides an overview of the data instruments employed in phase two of this research study.

![Data Collection Instruments](image_url)

Figure 4.6: Overview of Data Collection Instruments - Phase 2
Content

The content data of each textbook was collected using a counting method as outlined in the TIMSS combined with the Rivers Matrix. The Rivers Matrix provides four key headings for data collection; Motivation, Comprehension, Expectation and Philosophical Orientation. The entire textbook was subject to data collection.

Structure

The structure data of each textbook was collected using a counting method as outlined in the TIMSS combined with the structure analysis grids provided by Mikk (2000) (see Appendix E). The entire textbook was subject to data collection.

Expectation

The expectation data of each textbook was collected using a counting method as outlined in the TIMSS combined with the Rivers Matrix. The Rivers Matrix outlines expectations that one would expect to find within a mathematics textbook. Expectation grids were used as a means of presenting this data (see section 5.4). The entire textbook was subject to data collection.

Language

Morgan’s functional grammar analysis methods were applied to this study for the language data collection. The first page of each chapter was selected for language analysis as it was determined that this page consistently contained more language than randomly selected pages in each of the textbooks. A counting method was again used for data collection.

4.6.2 Analysis of Data

Data analysis for phase two of this research primarily employed an interpretist approach. The data in phase two of this research is collected and stored quantitatively however, the analysis of this data also required interpretation. Judgements
are made based on underlying themes and patterns that are evident, in conjunction with the literature, from data collection.

4.7 Data Collection and Analysis - Phase Three

Data was collected, in phase three, before and after the model chapter was trialled on a small scale in three secondary schools. Within each secondary school three first year mathematics classes participated in this study, two of these class groups acted as test groups and the remaining class acted as a control group. All students were subject to a pre and post two-tier diagnostic test (Treagust, 1988) and a questionnaire designed to measure their anxiety levels (Plake and Parker, 1982). Prior to commencing this small scale trial a pilot study was conducted. This pilot study was conducted to identify any flaws in the test instrument and also to obtain student feedback about the model chapter.

The two-tier diagnostic tests are often described as conceptual diagnostic tests. These tests aim to assess a students' conceptual understanding. These tests are area specific tests and are designed and created in such a way that a student must have a clear understanding of a concept in order to select the correct response. These two-tier diagnostic tests were created and applied by Treagust in 1988. He used these tests to determine the conceptual understanding of students in biology with regard to photosynthesis and respiration. The first tier of these tests consists of a content question with two or three answer options. The second tier of these tests contains a set of three to five justifications for the answer provided in the first part. These justifications include the correct answer and a number of distracters. The Revised Mathematics Anxiety Rating Scale (RMARS) was used in conjunction with the two-tier diagnostic test in both pre and post testing. The RMARS provides a measure of anxiety, the higher the result the more anxious an individual is. The RMARS was selected as a data collection instrument as fraction manipulation is widely acknowledged as a source of anxiety for students and hence
4.7. Data Collection and Analysis - Phase Three

an anxiety scale was the most suitable for this phase of the research.

4.7.1 Data Collection Instruments

Pilot Study

An expert panel which comprised ten experts in the field of mathematics education from primary, secondary and tertiary education participated in the expert panel evaluation. These experts were provided with a copy of the model chapter and asked to evaluate it (Appendix I). In conjunction with this the model chapter was implemented as a pilot study with a sixth class primary school group and their teacher. They provided feedback about the model chapter via a self designed questionnaire (Appendix J).

Model Chapter Evaluation

In order to measure the effectiveness of the model chapter it was necessary to introduce an assessment type which could identify whether learning and more importantly understanding took place. The two-tier diagnostic test, created by Treagust (1988) has been identified as measuring whether or not conceptual development was achieved. These tests have been employed primarily in the sciences to date.
Figure 4.7: Framework for the Two-Tier Diagnostic Test Instrument (Treagust, 1988)

Figure 4.7 outlines the methodology applied when creating the two-tier diagnostic test instrument (Treagust, 1988). There are three key stages involved in the devolvement of such a test instrument. Stage one involves defining the content.
4.7. *Data Collection and Analysis - Phase Three*

Fraction addition is identified as the topic for the model chapter (see section 6.4), the level of knowledge required for fraction addition in first year mathematics was identified and a concept map around the topic was created using C-Map Tools (Appendix F). The second stage in developing this test instrument requires a review of literature and semi structured interviews with students to determine their misconceptions and actual levels of knowledge. These semi structured interviews were conducted with sixth class pupils in May 2009 as these pupils would be entering first year mathematics in September 2009 (i.e. these students would be in first year when the model chapter was being implemented). The final stage of developing this test instrument involved creating the actual two-tier diagnostic test (Appendix K) based on the outcomes of the concept map and the semi-structured interviews. This test instrument was then drafted, piloted, reviewed and amended.

The two-tier diagnostic test was implemented pre and post the teaching period. Students were giving one mark for a correct answer on the two-tier diagnostic test and one mark for a correct concept on the two-tier diagnostic test. Data was recorded using SPSS. The Revised Mathematics Anxiety Rating Scale (RMARS) was devised by Plake and Parker (1982) (Appendix H) and serves to determine the level of anxiety students have about mathematics. The RMARS was implemented in the secondary schools in conjunction with the two-tier diagnostic tests. The numerical score that a student obtained in the pre and post test is recorded in SPSS.

Table 4.7 provides an outline of the theoretical frameworks involved in the data collection process of phase three.
4.7. *Data Collection and Analysis - Phase Three*

Table 4.7: Theoretical Frameworks Supporting Phase Three (Evaluation)

<table>
<thead>
<tr>
<th>Phase 3</th>
<th>Theoretical Frameworks (TF)</th>
<th>Function</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluation</td>
<td>Two-Tier Diagnostic Test</td>
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</tr>
<tr>
<td></td>
<td>RMARS</td>
<td>Test Instrument which measures Anxiety</td>
<td>Plake and Parker (1982)</td>
</tr>
</tbody>
</table>

### 4.7.2 Analysis of Data

Data resulting from the two-tier diagnostic test was analysed using the statistical package SPSS version 16. Students were coded numerically to ensure anonymity. Each students’ data was entered into SPSS using these codes. Missing data was also coded to ensure that only the data from students who took both the pre and post test was utilised in computing and comparing the total differences, i.e. a student being absent for the post test does not give him scores of zero as these would impact on the data set. Initially all the data was stored in one SPSS file, however using ANOVA (a measure of variance which compares mean values for two or more groups) it became evident that the schools involved in this study were statistically different. Hence three separate data sheets were created, one for each school in order to allow the data from each school to be analysed independently.

Initial analysis was carried out on each individual class group using paired t-tests. Paired t-tests compare the mean scores of continuous variables. In this study the continuous variable is the two-tier diagnostic test. Paired t-tests identify the changes over time between the pre and post tests for each class independently (see section 6.5). A second method of analysis, independent t-tests, considers the changes between class groups. Independent t-tests compare mean differences between two different groups, allowing the author to identify significant changes between the test and control groups (see section 6.5). Data resulting from the RMARS test was also analysed using the statistical package SPSS version 16 (see
4.8. Validation and Reliability

The difference in scores between the pre and post test indicate whether a positive or negative change (or no change) occurred over a period of time. High results indicate that a student has high anxiety levels about mathematics at the time of testing. Paired t-tests then identify whether the changes between pre and post testing are statistically significant or not.

4.8 Validation and Reliability

According to Cohen et al. (2000) validity is vital to effective research. Invalid and unreliable information serves no purpose. Fink (1995) states that research is valid if it measures what it purports to measure. Validity and reliability are particularly sensitive as regards data collection and analysis. It is of utmost importance that only valid and reliable research methods are employed.

4.8.1 Validity

Levels of validity are strengthened by the selection of suitable research methodology. The author of this research employs qualitative and quantitative research methods combined in a mixed method approach. The methods of analysis applied to the textbooks are similar to those applied by international textbook researchers such as those used in the TIMSS Report (2002). Finally the sample was selected as representative of the population of specific textbook users. Several types of validity are considered to ensure it is maintained (Cohen et al., 2000).

Internal Validity:

Internal validity ensures that the “findings accurately describe the phenomena” (Cohen et al., 2000: 107) and conclusions resulting from the analysis can be sustained by the data collected. Internal validity is ensured by repetitive testing and also the integration of a number of reputable frameworks for analysis e.g. content structure is analysed using the TIMSS (2002) framework and also the Rivers Matrix (1990).
4.8. **Validity and Reliability**

**External Validity:**
External validity refers to how data is collected and analysed and can be generalised for the wider population. External validity is ensured through sample selection, the sample in this case is selected by school textbook choice. Therefore results can be generalised by textbook choice only. The biggest national bookseller was contacted in the hope of receiving their sales figures for mathematics Junior Cycle textbooks to allow the author to easily identify the most commonly used. However, this bookseller declined to provide any data and so the author selected 100 bookshops at random and rang each one, asking which mathematics textbook they sold more of. This information enabled the author to establish which textbooks were the most commonly used in Ireland.

**Content Validity:**
Content validity ensures the author is fair and comprehensive with respect to the research problem. This research contains an exploratory stage which identified the initial gap in the research.

**Construct Validity:**
Construct validity ensures that the author’s understanding is in line with what is generally accepted. To ensure this the author has used frameworks based on reputable sources such as TIMSS.

**4.8.2 Reliability**
Consistent reliability ensures that if the same research study was conducted again it would yield similar results. Multiple checks are built into the process of data collection and analysis to ensure reliability. Reliability must be considered in conjunction with accuracy. Adequate levels of reliability are ensured by using consistency and replicability with regards test procedures within each research element. All participants were subject to the same method of data collection.
4.8. *Validity and Reliability*

and analysis. A mixed method approach to analysis was applied to this research therefore both quantitative and qualitative reliability were considered.

**Quantitative Reliability:**

Quantitative reliability is concerned primarily with precision and accuracy. In order to ensure quantitative reliability the author must ensure consistency in tools and methods of research testing. Definitions were established prior to commencing textbook analysis to ensure that findings were categorised correctly for each textbook, also each element of the textbook analysis framework was conducted on each textbook in one sitting to remove the possibility of confusion.

**Qualitative Reliability:**

In order to ensure qualitative reliability the author must consider stability of observations and parallel forms. To ensure qualitative reliability the frameworks applied for textbook analysis were sourced from reputable studies and applied in the same manner to each textbook. Open ended questions were used in all questionnaires to allow for a greater insight into the opinions of those working with the textbooks on a daily basis. This type of data must be analysed carefully and only the facts provided by the respondent can be considered, no assumptions can be made.

**4.8.3 Researcher Distance**

As previously mentioned the idea of this research stems from the author’s experiences as both a teacher and a student in the Irish secondary school system. Leedy and Ormond (2001) state that research is a systematic process of collecting and analysing data with the aim of gaining a greater understanding into a particular problem/situation. Thus the author is acutely aware that she has her own biases, assumptions and expectations. However, the author maintains researcher distance by acknowledging the presence of such bias, assumptions and expectations. Along
with this the author has also applied a mixed method approach and an overlap of theoretical frameworks allowing for strong triangulation. Hence in following this process and by applying professional standards, to all aspects of the research, researcher distance is achieved, allowing for greater confidence in these research findings.

4.9 Ethics

Within any discipline which involves working with human subjects there are a number of common and ethical implications (Leedy and Ormond, 2001). Having recognised such ethical implications, ethical approval was sought and obtained from the University of Limerick Ethics Committee (ULREC 08/22). Ethical considerations and concerns were adhered to throughout this research study.

- Participation was voluntary
- The right to withdraw was present and known to participants.
- Parental/guardian approval was sought prior to investigation where participants were less than eighteen years of age.
- Each participant was provided with a clearly worded information sheet which described the nature of the research and well as requirements of the participant.
- Individual codes were applied to all documentation to ensure confidentially and anonymity.
- The data is used only for research purposes.
- The data is stored according to UL Ethics regulations.

4.9.1 Quantitative Study

Having obtained ethical approval from ULREC and prior to the distribution of questionnaires permission to conduct this study was obtained from the school principals and the participating teachers and students. Principals and teachers were asked to sign a consent form outlining the study. As the students involved in
4.10. Conclusion

this study are under the age of eighteen, the students were asked to take a letter, which describes both the research and implications of participation, home to their parents to read and sign. Participation was voluntary with participants retaining the right to withdraw at any time. University guidelines in obtaining and storing student data were followed and analysis was carried out other highest standards using the statistical package (SPSS). Prior to submitting all data into SPSS each student was coded for confidentiality purposes.

4.9.2 Qualitative Study

Ethical issues in interviewing were also considered when undertaking stage two of the framework for the devolvement of a two-tier diagnostic test instrument in phase 3. Students involved in the pilot study were asked to participate in the semi structured interviews, signed permission forms were again required from each of the participants’ parents. The researcher again guaranteed confidentiality and students were reminded that they may withdraw at any stage.

4.10 Conclusion

This chapter outlines the different phases of research employed by the author for this study. It identifies the tri-phase approach which informs the overall design of the study. The general outline of the tri-phase approach is presented along with the theoretical frameworks employed at each stage of analysis. In order to systematically analyse and conclude findings a mixed method approach is adopted. A detailed description of this approach and of each element of the research design is provided while acknowledging the author’s concerns with regard researcher distance, validity, reliability and ethics. The findings, as a result of this methodology, are presented in the proceeding chapter.
Chapter 5

Textbook Analysis (Phase 2): Presentation of Findings

5.1 Introduction

This study is concerned with applying standard textbook analysis techniques to Junior Cycle (lower secondary) textbooks in Ireland. The author extends this analysis by applying an analytical tool adapted from the language analysis of written texts (student’s own written texts), functional grammar analysis (Morgan, 2004). The language analysis tool enables the researcher to develop an overall view of the textbook in terms of student learning while also allowing for a better understanding of the difficulty of the mathematical language as encountered by students using textbooks. This overall study is organised into three phases, phase one develops the literature review, phase two is concerned with all of the detail of the textbook analysis and phase three involves the creation and trial of a model chapter based on the hybrid model combining features of TIMSS with Morgan/Halliday’s functional grammar analysis.

The purpose of this chapter is to present and comment on the findings from phase two, phase two incorporates the TIMSS textbook analysis and Morgan’s functional grammar analysis. This method of textbook analysis is applied to the four most commonly used mathematics textbook series at Junior Cycle in Ireland. This is
5.2. Textbook Content Analysis

the first investigation of this type in Ireland and the purpose of this phase of
the research is to examine the quality of the current Junior Cycle mathematics
textbooks. The textbooks are analysed under four headings, incorporating the
TIMSS and Morgan’s functional grammar analysis;

1. Content,
2. Structure,
3. Expectation,
4. Language.

These four headings with their relevant frameworks for analysis allow for a detailed
evaluation of the selected textbooks. The data presentation and preliminary
analysis and commentary follow the structure outlined above.

5.2 Textbook Content Analysis

The content of each textbook is analysed using TIMSS content analysis combined
with the ‘River’s Matrix’, (Rivers, 1990). Data is collected from the textbooks
using the TIMSS method and is then analysed and recorded via the Rivers Matrix.
The Rivers Matrix separates content into four separate areas of focus; Motivation,
Comprehension, Technical Aids, Philosophical Orientation. Each section creates
four tables of comparison presenting the data for each textbook.

5.2.1 Motivation

According to Mikk (2000: 245) new information evokes student interest. However
he also points out that including new information can often intimidate and evoke
fear in students thus causing a decline in interest. Mikk suggests that textbooks
should include approximately 30-40% new information and no more if the wish is
to engage and motivate students. The reality is that the majority of textbooks cur-
rently comprise approximately 70-80% new information (Mikk, 2000). Motivation
can be increased by including:
5.2. *Textbook Content Analysis*

- **Historical Data:** make reference to mathematical discoveries and applications.

- **Practical Implications:** provide concrete examples as opposed to just theoretical explanations.

- **Inclusion of Problems:** such as problematic situations which motivate students; the textbook should include questions aimed at reproducing, analysing and applying information and unanswered questions, provide options before solutions.

- **Humour:** directly address the reader, use proverbs, riddles, jokes etc.

- **Figurative Representations:** provide illustrations and mental images.

- **Narrate about People:** (interlinked with historical data) provide information on mathematics, narrate about modern people for realistic problems.

The above sections provided by Mikk (2000) are considered along with the Rivers Matrix for analysis (Rivers, 1990). The Rivers Matrix identifies the following subsections for textbook motivation analysis; Historical Notes, Biographies, Career Information, Applications and Photographs.

The framework used for this analysis is the Rivers Matrix however, Mikk’s theories are used for reference (Appendix D). ‘Historical Data’ is referred to as historical notes. The data collected on ‘Practical Implications’ and the ‘Inclusion of Problems’ will be outlined in the structure analysis therefore, this Matrix only presents the total number of problems present. ‘Figurative Representations’ refer to all mental images and illustrations, a more in-depth analysis of figures and graphs is also outlined in section 5.3.2, comprehension. This matrix only indicates the presence of photographs (real life images) throughout each textbook. Finally the last section, ‘Narrate about People’ is considered under the headings of Biographies and Career Information.
5.2. Textbook Content Analysis

Table 5.1: Textbook Content Analysis - Motivation

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Historical Notes</th>
<th>Biographies</th>
<th>Career Information</th>
<th>Photos</th>
<th>Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>498</td>
</tr>
<tr>
<td>TBS A2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>266</td>
</tr>
<tr>
<td>TBS B1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>536</td>
</tr>
<tr>
<td>TBS B2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>309</td>
</tr>
<tr>
<td>TBS C1</td>
<td>12</td>
<td>22</td>
<td>0</td>
<td>22</td>
<td>542</td>
</tr>
<tr>
<td>TBS C2</td>
<td>14</td>
<td>12</td>
<td>0</td>
<td>14</td>
<td>283</td>
</tr>
<tr>
<td>TBS D1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>701</td>
</tr>
<tr>
<td>TBS D2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>254</td>
</tr>
</tbody>
</table>

Rivers (1990) used this matrix as an indicative measure for motivation in her textbook analysis. As is evident in the above table (Table 5.1), the majority of the textbooks neglect to include historical notes, biographies, career information or photographs/real life pictures, each of which contributes to make a textbook interesting. According to Mikk (2000) textbooks need to be interesting in order to compete with the plethora of stimuli in today’s modern world of technology. The Rivers matrix appears to indicate a high presence of problem solving within each of the textbooks. In section 5.3 (structure analysis) these problems are categorised into problem type where it becomes evident that the bulk of these problems fall into the category of ‘routine problems’, which are merely dressed up exercises (Appendix D).

5.2.2 Comprehension

Comprehension can be greatly influenced by the colour, layout and inclusion of graphics within a text (Dowling, 1996). Table 5.2 indicates the use of font, graph-line and page background colour. The use of font colour is very limited across all eight textbooks, when colour is used it is dependent on page background colour and has no meaning or consistency in terms of highlighting key points. The page background colour evident in all textbooks varies based on page number and does not have any connection to content type. The graph-line colour is also only varied
5.2. *Textbook Content Analysis*

based on page background colour.

<table>
<thead>
<tr>
<th><strong>Textbook</strong></th>
<th><strong>Page Background Colour:</strong></th>
<th><strong>Font Colour:</strong></th>
<th><strong>Graph-line Colour:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>White, Yellow</td>
<td>Black</td>
<td>Black</td>
</tr>
<tr>
<td>TBS A2</td>
<td>White, Blue</td>
<td>Black</td>
<td>Black</td>
</tr>
<tr>
<td>TBS B1</td>
<td>White, Blue, Yellow</td>
<td>Black, Blue</td>
<td>Red</td>
</tr>
<tr>
<td>TBS B2</td>
<td>White, Blue, Yellow</td>
<td>Black, Blue</td>
<td>Red</td>
</tr>
<tr>
<td>TBS C1</td>
<td>White, Blue, Red, Red, Orange</td>
<td>Blue, Orange</td>
<td>Blue, Orange</td>
</tr>
<tr>
<td>TBS C2</td>
<td>White, Green, Purple</td>
<td>Black, White, Green</td>
<td>Black, Green, Purple</td>
</tr>
<tr>
<td>TBS D1</td>
<td>White, Green</td>
<td>Black</td>
<td>Black</td>
</tr>
<tr>
<td>TBS D2</td>
<td>White, Orange</td>
<td>Black</td>
<td>Black</td>
</tr>
</tbody>
</table>

The following two graphs highlight the differences in page background and use of colour throughout each textbook. As is evident in Figure 5.1, yellow, blue and black are the most commonly used textbook colours with grey not far behind. However only pale dull shades of each colour as opposed to bright vibrant colours are found in each textbook. Many researchers, (Mikk, 2000; Noonan, 1990; Rivers, 1990; Dowling, 1996) have highlighted the need for relevant, bright, attractive illustrations. Not only do graphics assist with students’ understanding of the mathematical topic/problem they also assist in grabbing and holding a student’s attention (Mikk, 2000).
5.2. Textbook Content Analysis

Figure 5.1: Textbook Content Analysis - Comprehension (Colour)

![Use of Colour for Shaded diagrams](image)

Figure 5.2: Textbook Content Analysis - Comprehension (Graphics)

Figure 5.2 represents the distribution of graphics throughout each of the textbooks. The number of graphics within each textbook is relatively high whereas the number
5.2. Textbook Content Analysis

of graphics assisting real life problems and real life graphics themselves (graphics which depict real life situations) are extremely low.

5.2.3 Technical Aids

By encouraging and evoking the use of technical aids, computers and calculators, textbooks can broaden the learning experience for a student. Due to the rapid evolution of technology, incorporating the use of ICT into mathematics teaching and learning is essential (Mackie and Scott, 1988) yet few mathematics teachers take this opportunity to maximise learning outcomes. Duffy and O’Donoghue (1992) suggest that many Irish Mathematics teachers neglect technology, as the novelty of computer based learning has worn off. The following table indicates the number of chapters in each textbook which make reference to calculator or computer based work.

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Calculator Reference</th>
<th>Computer Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>TBS A2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>TBS B1</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>TBS B2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>TBS C1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>TBS C2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>TBS D1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TBS D2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

This table (Table 5.3) speaks for itself in terms of the failure of textbooks to incorporate technology. There is an abundance of research outlining how ICT can enhance the learning experience of students (Mooij and Smeets, 2001; Oldknow and Taylor, 2003); however none of these textbook series took this into consideration. There are a small number of references to calculators within the textbooks however with an abundance of online resources and software packages available which can enhance student learning these findings are extremely low.
5.2. Textbook Content Analysis

5.2.4 Philosophical Orientation

The philosophical orientation of a textbook considers the aims, the nature of the learning and the teaching of the mathematics. Ernest (1994) discusses the new directions in mathematical philosophy and the key elements of modern mathematics which influence this, along with areas which are often ignored by mathematical philosophers. Simply put, Ernest’s work can be summarised as follows; the philosophy of mathematics considers; the language of a text, the concepts, the prior knowledge assumed and the conversation engaged in. These ideas combine with those of Rivers (1990). She provides a list of possible emphases and philosophies evident in textbooks. The predominant emphasis considers the aims and nature of teaching and learning mathematics and thus provides a basis for selecting the predominant philosophy of a textbook.

**Emphasis:**

1. Utility and motivation
2. Career goals & immediate reinforcement
3. Proficiency & logic
4. Not stated - inference
5. Retention & depth of understanding
6. Language of mathematics & active participation
7. Critical thinking & communication.

**Philosophy:**

a Reconstructionism - Focus on society and change.

b Pragmatism - Practical approach to problems, problem solving central focus.

c Perennialism - Focuses on the intellect and importance of understanding.

d Realism - Focus on observable facts and information, no fantasy only actual experience.

e Existentialism - Emphasis is entirely on the individual, very much student based learning.
5.2. Textbook Content Analysis

A textbook can have any number of emphases evident throughout but only one predominant philosophy. Emphasis number 5 (retention and depth of understanding) is not included in the following table as the author felt that as a combined emphasis it is not present. This is due to the lack of educational weight on understanding throughout each textbook, despite an obvious focus on retention. Also, the textbook analysis determined that none of Rivers’ philosophies sat comfortably with any of the textbooks, however the closest is C, perennialism where the focus is on the intellect which is true, but understanding is not given the same importance as procedure and method throughout each textbook (Rivers, 1990; Valverde et al., 2002).

Table 5.4: Textbook Content Analysis - Philosophical Orientation

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Emphasis</th>
<th>Predominant Philosophy</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>3</td>
<td>C</td>
</tr>
<tr>
<td>TBS A2</td>
<td>3</td>
<td>C</td>
</tr>
<tr>
<td>TBS B1</td>
<td>3</td>
<td>C</td>
</tr>
<tr>
<td>TBS B2</td>
<td>3</td>
<td>C</td>
</tr>
<tr>
<td>TBS C1</td>
<td>3 + 6</td>
<td>C</td>
</tr>
<tr>
<td>TBS C2</td>
<td>3 + 6</td>
<td>C</td>
</tr>
<tr>
<td>TBS D1</td>
<td>3 + 6</td>
<td>C</td>
</tr>
<tr>
<td>TBS D2</td>
<td>3 + 6</td>
<td>C</td>
</tr>
</tbody>
</table>

As a summation of content in each textbook, the following graph indicates the percentage of chapters dedicated to each topic for all the textbooks. The topic list is from the NCCA Project Maths proposed Junior Cycle curriculum. This new curriculum was implemented in all secondary schools in Ireland in September 2010 and separates the Junior Cycle into five strands: statistics and probability, geometry and trigonometry, number, algebra and functions. The distribution of content in the new curriculum is significantly altered in comparison to the old mathematics curriculum. Hence it is reasonable to expect that the distribution of content in the current mathematics textbooks also needs to be altered as the current mathematics textbooks were not created with the Project Maths curriculum in mind. The following graph (Figure 5.3) demonstrates how inadequately
5.3. Textbook Structure Analysis

prepared the current textbooks would be to deal with the new curriculum. For example, statistics now makes up one fifth of the new mathematics curriculum but the current textbooks devote less than 10% of their content to statistics.

Figure 5.3: Percentage of Chapters Dedicated to each Project Maths Curriculum Strand

5.3 Textbook Structure Analysis

The structure of each textbook is analysed using a combination of the TIMSS framework which focuses on the physical scale of a textbook and a method of analysis devised by Mikk (2000), whereby the structure of a text is recorded diagrammatically. Employing both frameworks the author analysed the structure of each textbook and created a table for comparison (Appendix L). The structure of the textbooks is analysed under a number of headings Block Type, Content Structure and finally Expectation. Following this, the physical structure of each textbook is also examined. The results are indicated in the following graphs.

5.3.1 Block Type

The category ‘block type’, as outlined in the TIMSS report (2002), consists of various types of structure elements (found in the mathematics textbooks). The block types this study looks at are: Narration, Definitions, Graphics, Exercises, Examples.
5.3. Textbook Structure Analysis

Narration:
According to Mikk and Kukemelk (2010), narrative texts facilitate comprehension. Alexander and Jetton (1996) also state that narrative texts can evoke student interest. The following graph (Figure 5.4) indicates the number of lines of narration present in each textbook. In order to ensure validity, prior to analysis the author detailed the constraints of the analysis which are that any line which is over half the page width is counted as a line and anything less than this is neglected. This constraint is applied to all eight textbooks to ensure consistency. As can be seen from Figure 5.4, a high presence of narration does not necessarily reflect the presence of instructional narration. Instructional narration is defined as any text which contains/evokes a direct action. For example: ‘Do Step 1 initially then draw the diagram’. The TBS A series of textbooks has the highest rate of narration but as evident in Figure 5.4 it does not present the greatest rate of instructional narration.

Figure 5.4: Textbook Structure Analysis - Narration and Instructional Narration
5.3. *Textbook Structure Analysis*

**Definitions:**

Wu (cited in McCrory, 2006) describes definitions as being the building blocks of learning mathematics. Definitions are considered to be any statement which is highlighted, in bold and boxed off from the rest of the text. Only those definitions which are identified and obviously separated from the main text are counted. Many of the textbooks provide definitions which are integrated into the textual instruction or explanation. Integrated definitions are not counted because according to Kirkness and Neill (2009) new words and concepts should be highlighted and bold face. Kirkness and Neill (2009) also argue that definitions provide the technical vocabulary for textbooks. Therefore definitions should stand out from the main body of text. TBS C1 & 2 have a greater number of definitions highlighted and singled out in their textbooks than any of the other series. The following graph (Figure 5.5) outlines the presence of definitions.

![Figure 5.5: Textbook Structure Analysis - Definitions](image-url)
5.3. Textbook Structure Analysis

Graphics:
The TIMSS analysis refers only to related graphics, however the graphics in each of these textbook series could be linked to some aspect of the text. Therefore the following data is a representation of all graphics present in the textbooks (Figure 5.6). The first section of Figure 5.6 indicates the number of graphics present and the second section indicates how many of these graphics are related to a real life problem or exercise. The percentage of graphics associated with real life problems is exceptionally low. Further analysis of textbooks (see content structure analysis in section 5.3.2) indicates that this figure is a reflection of the limited presence of problems within all textbook series.

Figure 5.6: Textbook Structure Analysis - Graphics
5.3. *Textbook Structure Analysis*

**Exercises:**

The textbook structure analysis also examines the presence of exercises and real life exercises present in the textbook. The following graph (Figure 5.7) indicates the presence of exercises and also how many of these exercises are real life problems or exercises (Appendix M). The average number of exercises per text is approximately 2,391. As one can see, three of the textbooks fall very short of this average, TBS A2, TBS D2 and TBS C2. However if the author is to consider the average number of exercises across a textbooks series (approximately 4,782), TBS A and TBS C fall short of this average.

![Figure 5.7: Textbook Structure Analysis - Exercises](image)

The following table (Table 5.5) is based on the data collected on exercises within the textbook series combined with that of problem solving. TBS C1 and C2 have the highest rate of real life problems with just over 20% present in both, independently and combined (see Table 5.5).
5.3. *Textbook Structure Analysis*

<table>
<thead>
<tr>
<th>Textbook</th>
<th>% of which are Real Life Problems:</th>
<th>Textbook Series:</th>
<th>% of which are Real Life Problems:</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>20.84%</td>
<td>TBS A1 &amp; 2</td>
<td>19.86%</td>
</tr>
<tr>
<td>TBS A2</td>
<td>18.27%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TBS B1</td>
<td>15.07%</td>
<td>TBS B1 &amp; 2</td>
<td>14.60%</td>
</tr>
<tr>
<td>TBS B2</td>
<td>13.78%</td>
<td>TBS C1 &amp; 2</td>
<td>22.74%</td>
</tr>
<tr>
<td>TBS C1</td>
<td>23.58%</td>
<td>TBS D1 &amp; 2</td>
<td>16.42%</td>
</tr>
<tr>
<td>TBS C2</td>
<td>21.29%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TBS D1</td>
<td>17.58%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TBS D2</td>
<td>13.93%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Examples:**

Similar to the previous section, textbook structure analysis determines the presence of examples and the number of which are real life activities or problems (Figure 5.8). The average number of examples per textbook is approximately 237 with all, except TBS B1, TBS D1 and TBS C1 falling short of this average. The average number of examples per textbook series is approximately 473 examples, TBS A is the only series to fall significantly short of this with TBS C marginally behind the average.

![Figure 5.8: Textbook Structure Analysis - Examples](image-url)
5.3. *Textbook Structure Analysis*

Table 5.6 is based on the data collected on the presence of examples throughout the textbook series, combined with that of problem solving. TBS C1 and C2 again indicate the highest rate of real life examples.

**Table 5.6: Textbook Structure Analysis - Examples**

<table>
<thead>
<tr>
<th>Textbook</th>
<th>% of which are Real Life Problems:</th>
<th>Textbook Series:</th>
<th>% of which are Real Life Problems:</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>16.19%</td>
<td>TBS A1 &amp; 2</td>
<td>16.12%</td>
</tr>
<tr>
<td>TBS A2</td>
<td>16%</td>
<td>TBS B1 &amp; 2</td>
<td>16.21%</td>
</tr>
<tr>
<td>TBS B1</td>
<td>17.51%</td>
<td>TBS C1 &amp; 2</td>
<td>20.82%</td>
</tr>
<tr>
<td>TBS B2</td>
<td>14.22%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TBS C1</td>
<td>20.96%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TBS C2</td>
<td>20.62%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TBS D1</td>
<td>16.26%</td>
<td>TBS D1 &amp; 2</td>
<td>15.28%</td>
</tr>
<tr>
<td>TBS D2</td>
<td>12.5%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Average of Worked Examples:**

TBS A provides a total of one worked example per 11.46 exercises. TBS D is the only textbook series with a higher ratio of examples to exercises than TBS A, with one worked example per 11.55 exercises. TBS B has one worked example to 9.94 exercises while TBS C has the lowest ratio with one worked example per 7.79 exercises.

**Physical Scale:**

“Big books can contain more content but their size means it takes longer to read them and they are less likely to be covered entirely” (Valverde et al., 2002: 34). According to Valverde et al. (2002) the average number of pages per lower secondary school mathematics textbook is 225. Table 5.7 outlines page area and textbook size and weight.
5.3. Textbook Structure Analysis

Table 5.7: Textbook Structure Analysis - Physical Features

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Dimensions</th>
<th>Weight</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>17cm x 23.5cm</td>
<td>0.822kg</td>
<td>441 (38 pages of answers)</td>
</tr>
<tr>
<td>TBS A2</td>
<td>17cm x 23.5cm</td>
<td>0.567kg</td>
<td>320 (23 pages of answers)</td>
</tr>
<tr>
<td>TBS B1</td>
<td>19cm x 25cm</td>
<td>0.935kg</td>
<td>420 (43 pages of answers)</td>
</tr>
<tr>
<td>TBS B2</td>
<td>19cm x 25cm</td>
<td>0.708kg</td>
<td>442 (43 pages of answers)</td>
</tr>
<tr>
<td>TBS C1</td>
<td>17.5cm x 23.5cm</td>
<td>0.794kg</td>
<td>438 (13 pages of answers)</td>
</tr>
<tr>
<td>TBS C2</td>
<td>17.5cm x 23.5cm</td>
<td>0.697kg</td>
<td>394 (20 pages of answers)</td>
</tr>
<tr>
<td>TBS D1</td>
<td>21cm x 16cm</td>
<td>0.992kg</td>
<td>439 (34 pages of answers)</td>
</tr>
<tr>
<td>TBS D2</td>
<td>16cm x 21cm</td>
<td>0.552kg</td>
<td>226 (20 pages of answers)</td>
</tr>
</tbody>
</table>

The lightest of these textbook series is TBS A with a combined weight of 1.389kg, which is just 0.102 kg lighter than TBS C. The dimensions of TBS A and TBS C are also very similar with TBS A being the smallest textbook by a margin of 0.5cm in the width, however TBS C devotes the least amount of pages to answer pages.

5.3.2 Content Structure

The following grids outline the structure of the content throughout the textbooks. A list of topics, adapted from the TIMSS analysis, 1 - 27 outlined in the Table 5.8, are selected as a basis for analysing content structure. These topics are then identified throughout each chapter in the textbooks, with black marks identifying their presence on the grids below. The topic is only considered as being present if the chapter clearly indicates its use or it is a large part of the chapter. These grids clearly indicate where each topic is present throughout each textbook. These grids can be used in conjunction with the textbook series to identify if cross-referencing is correctly indicated. Each grid has 27 rows, each of which represents a topic unit (presented in Table 5.8). The number of columns in each grid is dependent on the number of chapters in each textbook (horizontal axis). So for example, in the first grid (TBS A1) we can see that topic 1, Whole Number Operations, is present in chapter 2 (column 2) and chapter 10 (column 10).
### 5.3. Textbook Structure Analysis

#### Table 5.8: Content Structure Labelling

<table>
<thead>
<tr>
<th>1</th>
<th>Whole Number Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Operations</td>
</tr>
<tr>
<td>3</td>
<td>Properties of Operations</td>
</tr>
<tr>
<td>4</td>
<td>Fractions &amp; Decimals</td>
</tr>
<tr>
<td>5</td>
<td>Common Fractions</td>
</tr>
<tr>
<td>6</td>
<td>Decimal Fractions</td>
</tr>
<tr>
<td>7</td>
<td>Negative number, Integers &amp; their Properties</td>
</tr>
<tr>
<td>8</td>
<td>Exponents, Roots &amp; Radicals</td>
</tr>
<tr>
<td>9</td>
<td>Number Theory</td>
</tr>
<tr>
<td>10</td>
<td>Estimation &amp; Number Sense</td>
</tr>
<tr>
<td>11</td>
<td>Rounding &amp; Significant Figures</td>
</tr>
<tr>
<td>12</td>
<td>Units</td>
</tr>
<tr>
<td>13</td>
<td>Perimeter, Area &amp; Volume</td>
</tr>
<tr>
<td>14</td>
<td>Geometry: Position, Visualisation &amp; Shape</td>
</tr>
<tr>
<td>15</td>
<td>2D Coordinate Geometry</td>
</tr>
<tr>
<td>16</td>
<td>2D Geometry Basics</td>
</tr>
<tr>
<td>17</td>
<td>2D Geometry, Polygons &amp; Circles</td>
</tr>
<tr>
<td>18</td>
<td>3D Geometry</td>
</tr>
<tr>
<td>19</td>
<td>Transformations</td>
</tr>
<tr>
<td>20</td>
<td>Congruence &amp; Similarity</td>
</tr>
<tr>
<td>21</td>
<td>Constructions</td>
</tr>
<tr>
<td>22</td>
<td>Proportionality Concepts</td>
</tr>
<tr>
<td>23</td>
<td>Patterns, Relations &amp; Functions</td>
</tr>
<tr>
<td>24</td>
<td>Data Representation &amp; Functions</td>
</tr>
<tr>
<td>25</td>
<td>Uncertainty &amp; Probability</td>
</tr>
<tr>
<td>26</td>
<td>Validation &amp; Justification</td>
</tr>
<tr>
<td>27</td>
<td>Structuring &amp; Abstracting</td>
</tr>
</tbody>
</table>
5.3. **Textbook Structure Analysis**

Topics 9 and 25, number theory and uncertainty and probability do not appear across either textbook in textbook series A. Congruence and similarity only appears in TBS A2. Also the following topics only appear once in each textbook: Common fractions, negative number, integers & their properties, exponents, roots & radicals, units, perimeter, area & volume, 2D geometry, polygons & circles, 3D geometry and constructions. The most common topic across both textbooks in TBS A is topic 2, operations.
Again for TBS B, topics 9 and 25, number theory and uncertainty and probability do not appear across either textbook. TBS B1 also does not consider rounding & significant figures and validation & justification, while TBS B2 does not include common fractions, estimation & number sense and units. The following list of topics only appear once in each textbook negative number, integers & their properties, perimeter, area & volume, 2D geometry basics, 3D geometry and congruence & similarity. The most common topic across both textbooks in TBS B is topic 2, operations. Patterns, relations & functions is also rather frequent in TBS B2.
Again for TBS C, topics 9 and 25, number theory and uncertainty and probability do not appear across either textbook. TBS C1 also does not consider topic 12, units, and topic 26, validation & justification while TBS C2 does not include topic 5, common fractions. The following topics only appear once in each textbook; decimal fractions, exponents, roots & radicals, 2D geometry, polygons & circles, 3D geometry, transformations and congruence & similarity. Again the most common topic across both textbooks is operations.
Topics 9 and 25, number theory and uncertainty and probability, are also omitted from this textbook series. However, as is evident from Figure 5.12, TBS D2 depicts a much sparser topic coverage than previous textbooks with only 6 topics appearing more than once across the textbook. Topics 2 and 3 are the most common across TBS D2, operations and properties of operations. Estimation and number sense does not appear in TBS D1 and common fractions, exponents, roots & radicals, units, 3D geometry, transformations, congruence & similarity, proportionality concepts. The most common topic found in TBS D1 is also operations.
These grids indicate the distribution of content throughout the eight textbooks involved in this study. The following table (Table 5.9) summarises the above grids: (see Appendix N for the breakdown of the distribution of content through the Textbook)

Table 5.9: Distribution of Content

<table>
<thead>
<tr>
<th>Content Units (Table 5.4)</th>
<th>Total % Coverage</th>
<th>Minimum Coverage*</th>
<th>Maximum Coverage*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.35%</td>
<td>0.59%</td>
<td>4.12%</td>
</tr>
<tr>
<td>2</td>
<td>49.42%</td>
<td>2.94%</td>
<td>10%</td>
</tr>
<tr>
<td>3</td>
<td>20.59%</td>
<td>1.77%</td>
<td>4.12%</td>
</tr>
<tr>
<td>4</td>
<td>17.06%</td>
<td>0.59%</td>
<td>3.53%</td>
</tr>
<tr>
<td>5</td>
<td>2.94%</td>
<td>0%</td>
<td>0.59%</td>
</tr>
<tr>
<td>6</td>
<td>5.88%</td>
<td>0%</td>
<td>1.18%</td>
</tr>
<tr>
<td>7</td>
<td>8.82%</td>
<td>0.59%</td>
<td>2.35%</td>
</tr>
<tr>
<td>8</td>
<td>5.88%</td>
<td>0.59%</td>
<td>1.18%</td>
</tr>
<tr>
<td>9</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>10</td>
<td>5.88%</td>
<td>0%</td>
<td>1.18%</td>
</tr>
<tr>
<td>11</td>
<td>8.82%</td>
<td>0%</td>
<td>3.53%</td>
</tr>
<tr>
<td>12</td>
<td>4.71%</td>
<td>0%</td>
<td>1.18%</td>
</tr>
<tr>
<td>13</td>
<td>5.88%</td>
<td>0.59%</td>
<td>1.18%</td>
</tr>
<tr>
<td>14</td>
<td>15.29%</td>
<td>0.59%</td>
<td>2.94%</td>
</tr>
<tr>
<td>15</td>
<td>17.65%</td>
<td>1.18%</td>
<td>4.12%</td>
</tr>
<tr>
<td>16</td>
<td>10.59%</td>
<td>0.59%</td>
<td>2.35%</td>
</tr>
<tr>
<td>17</td>
<td>7.06%</td>
<td>0.59%</td>
<td>1.77%</td>
</tr>
<tr>
<td>18</td>
<td>4.71%</td>
<td>0.59%</td>
<td>0.59%</td>
</tr>
<tr>
<td>19</td>
<td>5.29%</td>
<td>0.59%</td>
<td>1.18%</td>
</tr>
<tr>
<td>20</td>
<td>3.53%</td>
<td>0%</td>
<td>0.59%</td>
</tr>
<tr>
<td>21</td>
<td>8.82%</td>
<td>0.59%</td>
<td>2.35%</td>
</tr>
<tr>
<td>22</td>
<td>7.06%</td>
<td>0.59%</td>
<td>1.77%</td>
</tr>
<tr>
<td>23</td>
<td>29.41%</td>
<td>1.77%</td>
<td>6.47%</td>
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<td>2.94%</td>
</tr>
<tr>
<td>25</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>26</td>
<td>7.06%</td>
<td>0%</td>
<td>1.77%</td>
</tr>
<tr>
<td>27</td>
<td>21.18%</td>
<td>1.77%</td>
<td>5.29%</td>
</tr>
</tbody>
</table>

* The minimum coverage column represents the minimum percentage cover of a topic in any one textbook, while the maximum coverage column represents the maximum percentage cover.
5.3. **Textbook Structure Analysis**

From this table we can clearly see that ‘Operations’ (Topic 2) is the topic most common across all eight textbooks. This topic also has the lowest minimum percentage of coverage and the highest of maximum coverage. ‘Patterns, Relations and Functions’ (topic number 23) has the next highest maximum percentage of coverage across all eight textbooks. Topics numbered 5, 6, 9, 10, 11, 12, 20, 25 and 26 (see Table 5.8) do not appear in all eight textbooks. One reason for the absence of the above list of topics could simply be that the topic may have been considered too difficult for the first book of the textbook series or too easy to be included in the second book. This however does not fully explain the absence of these topics. The majority of the topics above (such as validation and justification) are key elements for all areas of mathematics and should be present throughout most, if not all, textbook chapters. One exception is topic number 25 ‘Uncertainty & Probability’, which does not appear on the current Junior Cycle mathematics curriculum and hence would not be expected to be present in the Junior Cycle mathematics textbooks.

**Problem Solving:**

The Pisa Report (2003) categorised problems into three types; decision making, system and analysis and trouble shooting. Decision making requires that students understand the situation, system and analysis requires students to analyse a situation and trouble shooting requires students to understand the main feature and diagnose a fault. Diaz and Poblete (2000) made similar characterisations when they broke problems into two categories; ‘Routine Problems’ or ‘Non - Routine Problems’, each of which can be further broken down into four types Real, Realistic, Fantasy and Purely Mathematical. While Orton (2004) categorises problems into four types: Routine Problems, Novel Problems (which can be likened to non-routine problems), Word Problems (which encompasses realistic problems) and Real Life Applications (identical to real context problems). Each of these classifications echo features of the others however the work of Diaz and Poblete
5.3. Textbook Structure Analysis

(2000) is the most applicable, with each definition being very distinct from the other ensuring consistency throughout data collection. According to Diaz and Poblete (2000) routine problems can be described as dressed-up exercises while non-routine problems are those which cannot be solved by following a routine procedure. Real Context Problems, includes the type of problems which are created in a real environment and which involve or engage the student. Realistic Context Problems are the type of problems which have the possibility of being reproduced, involves a stimulation of reality. Fantasy Context Problems are the type of problems which are not based on reality and are the product of imagination while Purely Mathematical Context Problems are exclusively mathematical and relate only to mathematical objects.

For the purpose of structure analysis the author created a table of analysis of problem types. This data enabled the author to distinguish between an exercise and a problem. The first graph separates problems into two types, routine problems and non-routine problems as defined by Diaz and Poblete (2000).

Figure 5.13: Review of Problem Type

Figure (5.13) indicates that the types of problems present across all the textbook series are primarily routine problems, while the students may think they are
5.3. Textbook Structure Analysis

engaging in problem solving they are in fact working mostly with ‘dressed up’ exercises. TBS A1 includes the most non-routine problems, however the numbers are extremely low.

As outlined in chapter two, routine and non-routine problems can be broken down into real, fantasy, realistic or pure mathematical problems based on the work of Diaz and Poblete (2000). Figure 5.14 shows the break down of the routine and non-routine problems into their subcategories. The above figure would suggest that TBS A1 might be the best source of problems (out of the eight textbooks involved in this study). However, because routine problems have a much higher frequency than non-routine problems it is necessary to look at the type of routine problems being presented to the students. While real, pure and fantasy problems can be useful in learning, the bulk of research (from as far back as Freudenthal (1973)) indicates that real and realistic problems are the most influential and beneficial to students. Here we can see that TBS A1 has the second least amount of realistic problems present. TBS C1 has by far the greatest number of realistic problems present.

![Figure 5.14: Classification of Problems](image)

Alternatively problems can be broken into three categories (O.E.C.D., 2003), decision making problems, system and analysis problems and trouble shooting prob-
5.4. *Textbook Expectation Analysis*

Performance expectations are embedded throughout the textbooks and impact significantly on how students choose to deal with the topics presented. The most basic consideration of expectation analysis is that students and teachers alike will read and understand the material presented (Valverde et al., 2002). The following table indicates the nineteen performance expectations (TIMSS Report 2002) considered for this analysis. The grids, similar to those obtained in content structure, demonstrate black marks in each chapter where the expectation is present.

![Classification of Problems](image_url)

Figure 5.15: Classification of Problems

5.4 **Textbook Expectation Analysis**

Performance expectations are embedded throughout the textbooks and impact significantly on how students choose to deal with the topics presented. The most basic consideration of expectation analysis is that students and teachers alike will read and understand the material presented (Valverde et al., 2002). The following table indicates the nineteen performance expectations (TIMSS Report 2002) considered for this analysis. The grids, similar to those obtained in content structure, demonstrate black marks in each chapter where the expectation is present.
### Table 5.10: Textbook Expectation Analysis

<table>
<thead>
<tr>
<th></th>
<th>Representing</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Recognising Equivalents</td>
</tr>
<tr>
<td>2</td>
<td>Recalling Mathematical Objects &amp; Properties</td>
</tr>
<tr>
<td>3</td>
<td>Using Equipment</td>
</tr>
<tr>
<td>4</td>
<td>Performing Routine Procedures</td>
</tr>
<tr>
<td>5</td>
<td>Using More Complex Procedures</td>
</tr>
<tr>
<td>6</td>
<td>Formulating &amp; Clarifying Problems or Situations</td>
</tr>
<tr>
<td>7</td>
<td>Solving</td>
</tr>
<tr>
<td>8</td>
<td>Predicting</td>
</tr>
<tr>
<td>9</td>
<td>Verifying</td>
</tr>
<tr>
<td>10</td>
<td>Developing Algorithms</td>
</tr>
<tr>
<td>11</td>
<td>Generalising</td>
</tr>
<tr>
<td>12</td>
<td>Conjecturing</td>
</tr>
<tr>
<td>13</td>
<td>Justifying &amp; Proving</td>
</tr>
<tr>
<td>14</td>
<td>Using Vocabulary &amp; Notation</td>
</tr>
<tr>
<td>15</td>
<td>Relating Representations</td>
</tr>
<tr>
<td>16</td>
<td>Describing/Discussing</td>
</tr>
<tr>
<td>17</td>
<td>Critiquing Using Vocabulary &amp; Notation</td>
</tr>
<tr>
<td>18</td>
<td>Solving Problems</td>
</tr>
</tbody>
</table>
TBS A1 does not expect students to verify (expectation 10), conjecture (expectation 13) or to critique using vocabulary or notation (expectation 18) while the only expectation absent from TBS A2 is expectation 18, critiquing using vocabulary & notation. The most common expectation for both textbooks in this textbook series are expectations 8 and 19, solving and solving problems. This would suggest that this textbook places a high value on problem solving, however previous analysis on problem types (section 5.4.2) would suggest otherwise.

Figure 5.16: Textbook Expectation Analysis - TBS A
The only expectation absent from both textbooks in TBS B is expectation 18, critiquing using vocabulary & notation, however expectation 14, justifying & proving, is absent from TBS B1. There is little weight on predicting across both textbooks with this expectation (9) only appearing once in each. The most common expectations emerging from both textbooks in this textbook series are expectations 3 and 5, recalling mathematical objects & properties and performing routine procedures.
Figure 5.18: Textbook Expectation Analysis - TBS C

Expectations 9 and 18, predicting and critiquing using vocabulary & notation, are absent from both textbooks in textbook series C, with describing/discussing is also absent from TBS C2. The most common expectations across both textbooks in this textbook series are representing, formulating & clarifying problems or situations, solving and problem solving. Expectation 3, recalling mathematical objects & properties, is a common expectation in TBS C1 with expectation 6, using more complex procedures emerging strongly from TBS C2.
Figure 5.19: Textbook Expectation Analysis - TBS D

Expectation 18, critiquing using vocabulary & notation is absent from both textbooks in TBS D, while expectations 2 and 13, recognising equivalents and conjecturing, are absent from TBS D2. The most commonly found expectations across TBS D are 5, 7, 8, and 19; performing routine procedures, formulating & clarifying problems or situations, solving and problem solving.

These grids indicate the distribution of expectation throughout the eight textbooks involved in this study (see Appendix O for a numeric breakdown of the expectation grids). The follow table summarises the above grids:
5.4. Textbook Expectation Analysis

Table 5.11: Distribution of Expectation

<table>
<thead>
<tr>
<th>Content Units (Table 5.6)</th>
<th>Total % Coverage</th>
<th>Minimum Coverage*</th>
<th>Maximum Coverage*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64.12%</td>
<td>4.71%</td>
<td>10.59%</td>
</tr>
<tr>
<td>2</td>
<td>18.24%</td>
<td>0%</td>
<td>6.47%</td>
</tr>
<tr>
<td>3</td>
<td>68.82%</td>
<td>3.53%</td>
<td>15.3%</td>
</tr>
<tr>
<td>4</td>
<td>2.06%</td>
<td>1.18%</td>
<td>5.3%</td>
</tr>
<tr>
<td>5</td>
<td>81.77%</td>
<td>5.88%</td>
<td>15.88%</td>
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<td>6</td>
<td>34.71%</td>
<td>1.76%</td>
<td>6.47%</td>
</tr>
<tr>
<td>7</td>
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<td>12.94%</td>
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<tr>
<td>8</td>
<td>71.18%</td>
<td>6.47%</td>
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<td>9</td>
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<td>0.59%</td>
</tr>
<tr>
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<td>18.24%</td>
<td>0%</td>
<td>5.29%</td>
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<tr>
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<td>14.12%</td>
<td>0.59%</td>
<td>3.53%</td>
</tr>
<tr>
<td>12</td>
<td>34.12%</td>
<td>2.94%</td>
<td>5.88%</td>
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<td>13</td>
<td>4.71%</td>
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<td>3.53%</td>
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<td>5.88%</td>
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<td>0%</td>
</tr>
<tr>
<td>19</td>
<td>68.82%</td>
<td>5.29%</td>
<td>14.12%</td>
</tr>
</tbody>
</table>

* The minimum coverage column represents the minimum percentage cover of a topic in any one textbook, while the maximum coverage column represents the maximum percentage cover.

Expectation number 5 (Performing Routine Procedures, Table 5.10) has the greatest percentage of textbook cover, it is present in 81.77% of the textbook chapters. This is not surprising considering that the NCCA reports for the past number of years have been discussing an over-emphasis on routine procedures which relates to poor teaching. Expectations 2, 9, 10, 11, 14, 17 and 18 (see Table 5.11) are present only in some of the textbooks, as can be seen in the above table the minimum coverage for these is 0%. These expectations include predicting, verifying, justifying and proving, common words associated with mathematics. Words that the textbooks should be encouraging students to use. Other expectations which comprise over
50% textbook coverage include; representing, formulating and clarifying problems or situations, solving and solving problems. A strong presence of solving problems is coming across from this data, however one must remember results from the analysis of problem types (Figure 5.13 and Figure 5.14). The majority of problems present in the textbooks are routine problems. Routine problems do engage the above expectations but not to the extent that a non-routine problem would.

## 5.5 Textbook Language Analysis

Researchers have identified that mathematics is a complicated, diverse but unique language. Bullock (1994) reinforced the significance of this language when highlighting the fact that Newton had to invent calculus in order to develop and express his ideas and in 1987 Pimm compared the learning of mathematics to the learning of a foreign language. Language analysis and its significance has been widely researched for a number of years and has formed a significant part of mathematical research from the early 1990’s, with, for example, the work of Halliday (1973); Skemp (1982); Van Dormolen (1986); Pimm (1987); Noonan (1990); Chapman (1993); Dowling (1996); Mikk (2000); Morgan (2004); Orton (2004), with Mikk and Morgan focusing particularly on the role of language in mathematics texts or textbooks. For the purpose of this study the author draws primarily upon the work of Halliday (1973) and Morgan (2004).

Halliday’s research provides the basis for much language analysis in many different subject areas, focusing on the functional aspects of language. He outlines this functional aspect as the way in which language is used, the purpose that it serves and the way in which a reader can achieve these purposes. One of the reasons Halliday outlines for following this line of investigation is to “establish general principles relating to the use of language”. For this reason Halliday’s functional grammar analysis is applicable to this study, as the author is seeking to not only analyse the language present in mathematical text but also to research the
overall effectiveness of the language for teaching and learning. Halliday’s functional
grammar analysis is based on three elements:

- Ideational Function,
- Interpersonal Function,
- Textual Function.

The **Ideational Function** looks at the nature of the activity, in particular at the
structure and logic of relationships within a text (any written material). Halliday
(1973: 38) describes it as “the categories of one’s experience of the world and how
they interpret this experience”.

The **Interpersonal Function** examines the social and personal relationships between
the author and others while establishing the expression of the author’s authority
and the relationship between the author and reader. Halliday (1973: 41) defines
his interpersonal function as “including all forms of the speaker’s intrusion into the
speech situation and speech act”.

The **Textual Function** makes the language relevant for its intended purpose. Halli-
day (1973: 42) points out that it “distinguishes a living message from a mere entry
in a grammar or a dictionary”.

Halliday developed his functional grammar analysis for language in general and
the language of mathematics never featured as a stand alone unit within his work.
However, in 2004 Morgan applied this functional grammar analysis to mathematics
to students’ own written mathematical texts. She describes how the ideational
function can look specifically at the mathematics and the mathematical activities
presented while the interpersonal function highlights sources of concern in the
mathematics language such as the use of the word “we” (identified by Pimm (1987)
5.5. *Textbook Language Analysis*

as a cause for concern). The textual function identifies the formation of argument in a mathematical text and any message portrayed via reports, descriptions or narratives.

Morgan’s framework (2004) for mathematics language analysis can be applied effectively to any mathematical text. The framework originated with the work of Halliday (1973) and is not mathematically specific, however, Morgan herself used this framework for analysing students’ own mathematical writings and in this research study it is applied to school mathematics textbooks. Morgan’s framework, while applied in its entirety, differs from the analysis in this study with regards to the interpretation element. The function of the language analysis tool is to work in conjunction with the other elements of textbook analysis to provide an overall view of the impact of the mathematics textbook on student learning. The three key areas of the framework provide an individual outlook at each stage and combine to give an overall view of the textbooks in question.

The *Ideational Function* serves to identify the main process that the text provides, in doing so the framework considers the role of the author, which is the interpersonal function and concludes to provide an outcome which directly informs the textual analysis.

The *Interpersonal Function* serves to establish ownership and in particular the author’s position, which has a number of implications for the reader. This analysis overlaps with the ideational particularly in terms of the presence of pronouns and use of the passive sentences. The outcome provided by the interpersonal analysis also directly feeds into the textual analysis.

Finally, the *Textual Analysis* draws on the work of the ideational and interpersonal analysis in order to effectively analyse and hence establish the purpose and role of the mathematics text and more importantly the language within the text.
5.5. *Textbook Language Analysis*

These elements of analysis can be linked directly with Shuard and Rothery’s (1984) measures for improving written materials. Shuard and Rothery (1984) provide a list of measures which can improve written mathematics material. This list suggests a more detailed focus on the use of vocabulary and syntax, symbols, graphics and the text as a whole. They believe that written material should teach vocabulary, but the vocabulary used should be simple as a formal style of writing is out of place in text which is aimed at students. In conjunction with this vocabulary should be repeatedly used and only introduced if it is necessary. Consistency is also vital with the introduction of new vocabulary and the introduction of new words should be planned so only a limited number are introduced at a time. Words with dual meaning, such as ‘product’, need careful consideration and a glossary of new terms should be included. Shuard and Rothery (1984: 133-134) also recommend that the present tense is used and that passive sentences and rhetorical questions are avoided.

Shuard and Rothery (1984) point out that the presence of symbols is necessary for the language of mathematics but that their excessive use is unnecessary for the reader. Interpretation of symbols can often prove difficult. Skemp (1971) highlights the impact context can have on the interpretation of a symbol. Also, consistency of use and interpretation minimises students’ misconceptions. Shuard and Rothery (1984: 134) make the following recommendations for the use of symbolism in written mathematics text:

- The concept to be symbolised needs to be understood before the symbol is introduced.
- Only those symbols which are necessary should be introduced.
- All symbols must be included in a glossary of terms.
- Like with the introduction of new vocabulary, introducing new symbols should be planned carefully.
- Careful consideration must be given to any combinations of symbols and how their meaning may be affected by their position.
- Avoid the use of “verbose” text when explaining symbols.

Shuard and Rothery’s suggestions for the use of graphics throughout a text anticipate those of Dowling (1996) and Morgan (2004). Shuard and Rothery (1984) agree that there is a place in text for graphics; they also note that pictures are more often found in text aimed at young children while graphs and tables replace these pictures for older children. According to Shuard and Rothery (1984: 135) the purpose of pictures is to assist in teaching young students about relationships in mathematics. This method of teaching a student how to perceive seems to be left entirely to the teacher when it comes to older students despite the complementary role text material can play in assisting such teaching. Shuard and Rothery (1984: 47) also note that diagrams function in three ways; they can be essential, related or decorative. In order to ensure that students can get the maximum benefit from graphics the following recommendations, for graphics in written text, are outlined; graphics should be placed in position which makes it easy to relate the graphic to the text, the text should highlight essential graphics and consistency of styles is important along with the need for graphics to be clear and simple (Shuard and Rothery, 1984: 135).

The recommendations for vocabulary and syntax, symbols and graphics all directly relate to the framework for analysis created by Halliday (1973) and that which is applied to mathematics text by Morgan (2004). Each of the elements outlined by Shuard and Rothery (1984) impact directly on the interpersonal and ideational functions of the proposed framework for this language analysis. The final element that Shuard and Rothery (1984) highlight is ‘the text as a whole’. This also links directly with the framework for language analysis as it combines the recommendations for the vocabulary and syntax, symbols and graphics together for the purpose of providing clarity and flow to a mathematics text. Similarly the textual function of the language analysis considers the text as a whole and the key areas which contribute to the effectiveness of a mathematics textbook for teaching
5.5. Textbook Language Analysis

and learning.

The three key areas for language analysis in this study are:

- Ideational Function,
- Interpersonal Function,
- Textual Function.

The three textbook series involved in the previous elements of this study are again used to determine the effectiveness of this analytical tool for analysing the language of a mathematics text. Each textbook series contains two textbooks and a text selection from each is used in this phase of research. This text selection is the first page of each chapter within each textbook. This page is selected as within each textbook it exhibited the greatest quantity of language. The framework is applied to each textbook selection and the following sections outline the findings for each element of this framework.

5.5.1 Ideational Function

In order to implement Halliday’s ideals effectively Morgan (2004) developed the following table to assist with the ideational function analysis:

<table>
<thead>
<tr>
<th>Human</th>
<th>Object</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>Specific</td>
<td>Basic or Derived</td>
</tr>
<tr>
<td>Material</td>
<td>Mental</td>
<td></td>
</tr>
<tr>
<td>Mental</td>
<td>Relational</td>
<td></td>
</tr>
<tr>
<td>Relational</td>
<td>Verbal</td>
<td></td>
</tr>
<tr>
<td>Behavioural</td>
<td>Existential</td>
<td></td>
</tr>
</tbody>
</table>

The first column indicates the six main types of processes identified by Halliday (1973), which indicate the various types of activities present in any text. According
to Halliday (1973) the most commonly found processes are material, mental and relational. Morgan (1995) also indicates that the frequency of each of the types of processes present in a text indicates the nature of the mathematical activity. Morgan (1995) also suggests that the role of human beings within a mathematical text, i.e. the extent of the presence within the text and the sort of processes they are involved in, is crucial in identifying the nature of any mathematical text. The use of passive voice removes the active presence of human activity. Morgan suggests that the passive voice actually obscures agency.

In order to ensure consistency the following definitions were established prior to commencing data collections:

<table>
<thead>
<tr>
<th><strong>Human</strong></th>
<th>This refers to the role of human beings in the text and can be further categorised at specific (individuals, author, the reader or a third party) or general (typically addressed as you or one)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Object</strong></td>
<td>Basic objects refer to basic shapes and numbers and objects derived from these such as products, factors, lengths etc. Relational Objects refer to patterns and formulae while representational objects include graphs, diagrams and tables.</td>
</tr>
<tr>
<td><strong>Notation</strong></td>
<td>The most significance element of notation to the ideational function is the use of passive sentence, which directly impacts the role of human involvement in mathematics.</td>
</tr>
</tbody>
</table>

(Morgan, 2004: 83)

Each sentence within the selected text is categorised into one of the Halliday’s six process categories. In order to complete this categorisation the author needs to be aware of elements which comprise a sentence, for example:

*John gave me a book yesterday*

In this sentence the process is defined by the verb, gave. John me and the book are all participants or actors, with John and me being human participants and yesterday is the circumstance of this sentence. The process determines the kind of
activity which is expressed and the types and manner of the participants. There are six types of processes, as identified by Halliday (1973), each of which is explained in the following paragraphs at the appropriate point.

When applying the ideational function to any mathematics textbook it is important to remember a number of key research concerns. The main process evident (of Halliday’s six processes) identifies the type of involvement and engagement that the reader will have with the textbook, for example a high presence of material processes are indicative of active learning being encouraged. A high presence of mental encourages thinking, relational encourages students to develop concepts and construct relationships, verbal encourages the students the focus on the meaning of the text, and behavioural should create connections while existential highlights the importance of facts.

The following table outlines the data collected (number of instances where each process occurs) using the framework for language analysis for the Ideational element. Due to the presence of six new terms (material mental, relational, verbal, behavioural and existential) in conjunction with six different mathematics texts the author will present the data firstly as a table of results and following this the data is summarised into categories of processes to allow a flow of text for the reader. The elements which occur significantly within each textbook are discussed. According to Halliday (1973), one would expect that the majority of the text would be discussed within the first three processes (material, mental and relational) with little or no reference to the final three processes (verbal, behavioural and existential). However, as you will see the reality does not fully reflect Halliday’s expectations.

the mental process and a high presence of the relational process as the presence of the relational process is a direct contrast to procedure. One would also expect to find a low presence of passive sentences (they should be non-existent in the verbal process) and that specific human references are not dominated by “we”. There should also be effective, plentiful and consistent use of objects across all three categories and processes, particularly the relational process.
### Table 5.13: Data from the Analysis of the Ideational Function

<table>
<thead>
<tr>
<th>Notation</th>
<th>Human</th>
<th>Object</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>General</td>
<td>Specific</td>
</tr>
<tr>
<td><strong>TBS A1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>1</td>
<td>68</td>
</tr>
<tr>
<td>Mental</td>
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<td>9</td>
</tr>
<tr>
<td>Relational</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Verbal</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Behavioural</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Existential</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td><strong>TBS A2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>1</td>
<td>28</td>
</tr>
<tr>
<td>Mental</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Relational</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Verbal</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Behavioural</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Existential</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td><strong>TBS B1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>3</td>
<td>29</td>
</tr>
<tr>
<td>Mental</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Relational</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Verbal</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Behavioural</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Existential</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td><strong>TBS B2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>3</td>
<td>23</td>
</tr>
<tr>
<td>Mental</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Relational</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Verbal</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Behavioural</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Existential</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td><strong>TBS C1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>10</td>
<td>85</td>
</tr>
<tr>
<td>Mental</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Relational</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Verbal</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>Behavioural</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Existential</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td><strong>TBS C2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>7</td>
<td>113</td>
</tr>
<tr>
<td>Mental</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Relational</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>Verbal</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>Behavioural</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Existential</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>
5.5. Textbook Language Analysis

Material Process

The material process encapsulates the concept of “doing”. Every material process must involve an actor and either a participant or a goal. Morgan (1995) believes that a high presence of material processes suggest that a text’s focus is on “doing” mathematics. A high presence of passive sentences is in direct conflict with the material process. Passive sentences remove the presence of human activity and effective implementation of the material process requires human participation, i.e. the activity must have a purpose, e.g. *If you take a cylindrical can, such as one containing soft drink, and use a string to measure its circumference ...*(TBS A1)

Textbook Series A - Textbook 1 (TBS A1)

<table>
<thead>
<tr>
<th>Notation</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive Sentences</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 5.14: Data from the Analysis of the Material Process for TBS A1

<table>
<thead>
<tr>
<th>Notation</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive Sentences</td>
<td>14</td>
</tr>
</tbody>
</table>

The material process indicates a high presence of ‘doing’, however the large number of passive sentences takes away from this. Passive sentences remove the presence of human activity and effective implementation of the material process requires human participation. Pimm (1987) noted that the over use of the pronoun ‘we’ may have a negative effect, particularly for any reader who does not feel confident enough to accept the responsibility being passed onto them by the author. There are 14 passive sentences related to the material function which makes up 20.6% of the total number of passive sentences in this text selection. Examples of such sentences are:
5.5. *Textbook Language Analysis*

“Negative numbers are always written with the negative sign, -, before them, e.g., -3, -5, -8.” (pg.39)

“The circle on the right is divided into 4 equal parts.” (pg.48)

As is evident from the Table 5.14 there is only 1 general human reference and 68 specific. Of the 68 specific references 48 of these are expressed using the pronoun ‘we’, the pronoun ‘I’ does not appear. Examples of specific references are:

“ If you examine the three collections of items below you will notice that the items in each collection are very similar.” (pg.7)

“ On the right we have a straight line passing through the points a and b.” (pg.105)

With regard to the object element of the ideational function, all pictures, tables and definitions fall into either the basic or representational categories.

**Textbook Series A - Textbook 2 (TBS A2)**

Table 5.15: Data from the Analysis of the Material Process for TBS A2

<table>
<thead>
<tr>
<th></th>
<th>Human</th>
<th>Object</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>General</td>
<td>Specific</td>
<td>Basic or Derived</td>
</tr>
<tr>
<td><strong>TBS A2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>1</td>
<td>28</td>
<td>7</td>
</tr>
</tbody>
</table>

The greatest material representation in this textbook is the number of specific human references, with 28 in total. Of these 28 references the pronoun ‘we’ dominates with 20 instances, the pronoun ‘I’ does not appear. The over use of the pronoun ‘we’ indicates the author sharing responsibility. Also the majority of objects lie in the material process with 7 basic, 1 relational and 9 representational objects.
Table 5.16: Data from the Analysis of the Material Process for TBS B1

<table>
<thead>
<tr>
<th></th>
<th>Human</th>
<th>Object</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>General</td>
<td>Specific</td>
<td>Basic or Derived</td>
</tr>
<tr>
<td>TBS B1</td>
<td>3</td>
<td>29</td>
<td>27</td>
</tr>
</tbody>
</table>

Again, in this text there are a large number of passive sentences. However, there is a much broader selection of pronouns, indicating that author is more open to giving, taking and sharing responsibility.

There are 18 passive material sentences, almost 25% of the total present in this text selection. Examples of these are:

“Plus 3 is written as 3. Minus 2 is written as -2.” (pg.12)

“By covering the quantity required, D, S or T, any of the three formulas above can be found by inspection.” (pg.68)

The majority of human references lie within the specific category, 29 out of a total of 33. Of these 29 specific references 12 of them are comprised of the pronoun ‘we’, 7 are ‘you’, 6 are ‘your’ and 4 are ‘our’, the pronoun ‘I’ does not appear. Examples of some of these are:

“The scale will stay ‘balanced’ provided we do the same thing to both sides.” (pg.54)

“Therefore, our solution x = 2 and y = 5 is correct.” (pg.178)

“In the question, you will always be given a set of inputs, x, called the domain.” (pg.358)

Over half of the objects also lie within the Material Process, 27 of these are used as basic knowledge additions, 12 are used to establish relations and 10 for representation.
Table 5.17: Data from the Analysis of the Material Process for TBS B2

<table>
<thead>
<tr>
<th></th>
<th>Human</th>
<th>Object</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>General</td>
<td>Specific</td>
<td>Basic or Derived</td>
</tr>
<tr>
<td>TBS B2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>3</td>
<td>23</td>
<td>20</td>
</tr>
</tbody>
</table>

There are fewer passive sentences present in this text selection than in the previous one (both a part of the same textbook series). The use of pronouns however is more restricted in this text selection than its counterpart, with a high presence of the pronoun ‘we’.

There are 12 passive material sentences, just over 35% of the total number of passive sentences in this text selection. Examples of these are:

“Ratios remain the same when both are multiplied by the same number.” (pg.104)

“After the fourth century, due to such oppression, little or no progress was made the field of mathematics until the Renaissance.” (pg.220)

The majority of human references in this text selection also lie within the specific category. Of the 23 specific references present 19 of them contain the pronoun ‘we’; the pronoun ‘I’ does not appear. Examples of some of these are:

“Two equations are necessary if we are to be able to find the values of x and y that satisfy both equations.” (pg.24)

“You can use your calculator to help you find the largest possible square number that will divide exactly into the number under the square root symbol.” (pg.254)
5.5. *Textbook Language Analysis*

Two-thirds of the objects also lie within the Material Process, with 20 of these in the category of basic/derived, 11 in the relational category and 19 in the representational category.

**Textbook Series C - Textbook 1 (TBS C1)**

Table 5.18: Data from the Analysis of the Material Process for TBS C1

<table>
<thead>
<tr>
<th>Notation</th>
<th>Human</th>
<th>Object</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>General</td>
<td>Specific</td>
<td>Basic or Derived</td>
</tr>
<tr>
<td>TBS C1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>10</td>
<td>85</td>
<td>5</td>
</tr>
</tbody>
</table>

The material process should indicate an active presence, in this textbook there is a high rate of human specific references within this process. This text selection has a total of 85 references, containing a broad range of pronouns. Of the 85 specific references there are only 16 instances of the pronoun ‘we’. Here the pronoun ‘I’ does appear but only in the context of speech. Also the pronouns ‘he’ and ‘you’ are also quite frequent. Examples of some of these are:

“. If Bert starts at zero, goes up 4 storeys and then goes down 7 storeys, he ends up at -3.” (pg.42)

“. As soon as we start to break objects into bits, we move into the world of fractions.” (pg.54)

“For example, if you need a new part for your car you would go to a 'motor factors' shop.” (pg.207)

**Textbook Series C - Textbook 2 (TBS C2)**

This text selection, like TBS C1, has a large number of the human specific material process - 113. Unlike the previous texts the most commonly used pronoun in this text selection is ‘he’. The use of the pronoun ‘we’ is much more limited than previous sections with only 8 instances of the pronoun, also the pronoun ‘I’ does
appear but only 7 times and always in the context of speech. Examples of some of these are:

“ If I am not Irish, then I am not from Cork.” (pg.1)

“ When he was four he corrected his father’s calculations of a wage packet.” (pg.21)

“ \[ \frac{355}{13} = 3.14159292 \ldots \text{ but we often use 3.14 or even 3.} \]” (pg.155)

“ He invented the T-test and later a statistical graph called T-distribution.

” (pg.155)

**Mental Process**

The mental process refers to thinking, feeling and perceiving. Mental processes are distinguished from the material because the presence of the actor is referred to as he or she and the second actor may be a fact or object and is typically indicated in simple present tense form. Morgan (1995) also suggests that a high presence of the mental process displays mathematics as a pre-discovered entity, *e.g. You can think of a function as a number machine that changes one number (input) into another number (output) according to some rule (TBS B1).*

**Textbook Series A - Textbook 1 (TBS A1)**

The mental process is concerned with all thinking, feeling and perceiving. The low number of passive sentences within this category is a positive trait as it suggests that all of the mental processes are encouraging human participation.
5.5. *Textbook Language Analysis*

Table 5.20: Data from the Analysis of the Mental Process for TBS A1

<table>
<thead>
<tr>
<th>Human</th>
<th>Object</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>Specific</td>
<td>Basic or Derived</td>
</tr>
<tr>
<td>TBS A1</td>
<td>Mental</td>
<td>0</td>
</tr>
</tbody>
</table>

There are only 3 passive sentences related to the mental function, example of such sentences are:

- “These are known as negative numbers.” (pg.39)

- “It can be seen from the bar chart that 14 travel by bus, 12 by car, 8 by train, 16 walk and 10 cycle.” (pg.123)

The majority of mental object processes fall into the representational category and all the human references are specific.

- “We know from old records that the Egyptians were interested in geometry more than 4500 years ago.” (pg.105)

- “Similarly, if we want to multiply two unknown numbers x and y...” (pg.71)

*Textbook Series C - Textbook 1 (TBS C1)*

Table 5.21: Data from the Analysis of the Mental Process for TBS C1

<table>
<thead>
<tr>
<th>Human</th>
<th>Object</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>Specific</td>
<td>Basic or Derived</td>
</tr>
<tr>
<td>TBS C1</td>
<td>Mental</td>
<td>0</td>
</tr>
</tbody>
</table>

The majority of objects lie within the mental process in the representational category and this is primarily due to the textbook structure and layout. The object presented, consistently, at the start of each chapter triggers an everyday experience of activity which is related to the mathematics presented.
Similar to TBS C1 the presence of an object at the beginning of each section is representational of the mathematics which is about to be presented.

**Relational Process**

The relational process is about “being”, where one individual/fact or object assists in identifying another. A high presence of relational process indicates that mathematics is based on the relationships between objects (Morgan, 1995). Much research discusses the procedural function and the minimal impact it has on critical thinking, the relational function being a much more highly valued educational process. Van Dormolen (1986) makes distinctions between the procedural and relational functions, noting that levels of language coexist with levels of knowledge at both the procedure and relational function, i.e. language is not confined to procedural thinking as stated by Freudenthal (1973),

e.g. *Simultaneous linear equations are a pair of such equations* (TBS A2)

**Textbook Series A - Textbook 2 (TBS A2)**

Table 5.23: Data from the Analysis of the Relational Process for TBS A2

<table>
<thead>
<tr>
<th></th>
<th>General</th>
<th>Specific</th>
<th>Basic or Derived</th>
<th>Relational</th>
<th>Representational</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A2</td>
<td>0</td>
<td>0</td>
<td>4 (=,121)</td>
<td>3</td>
<td>3 (=, 13)</td>
<td>2</td>
</tr>
</tbody>
</table>

TBS A2’s main use of the equal to sign is within basic or derived statements with 121 of these found. 13 instances of the equal to sign used for representational
5.5. *Textbook Language Analysis*

purposes are present. (The data in brackets represents the use of the equal to sign within the text selection Table 5.23). Inconsistent and often incorrect use of the equal to sign can cause confusion for the reader.

**Textbook Series B - Textbook 1 (TBS B1)**

Table 5.24: Data from the Analysis of the Relational Process for TBS B1

<table>
<thead>
<tr>
<th>Human</th>
<th>Object</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>General</td>
</tr>
<tr>
<td>TBS B1</td>
<td>Relational</td>
<td>0</td>
</tr>
</tbody>
</table>

The relational process plays a large role with regard to objects in this textbook, with a total of 27 objects falling into its category. Of these 27 objects 21 are basic, with 1 used for relational purposes and the remaining 5 are representational. The data in brackets in Table 5.24 represents the use of the equal to sign within the text selection, there again the equal to sign serves a number of purposes; to define, show and relate. This is both misleading and confusing for the reader.

*e.g.*

**Define** frequency = the number of times that something occurs (pg.124)

\[
\text{Speed} = \frac{\text{Distance}}{\text{Time}} \quad (pg.68)
\]

**Show/Represent** letter = number \hspace{1cm} LHS = RHS (pg.54)

\[
P = \text{the Principal} \quad (pg.100)
\]

**Relate** 1 hour = 60 minutes (pg.61)

\[
\text{per annum} = \text{per year} \quad (pg.89)
\]
5.5. Textbook Language Analysis

Textbook Series B - Textbook 2 (TBS B2)

Table 5.25: Data from the Analysis of the Relational Process for TBS B2

<table>
<thead>
<tr>
<th>Human Notation</th>
<th>Object Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General</strong></td>
<td><strong>Specific</strong></td>
</tr>
<tr>
<td>TBS B2 Relational</td>
<td>0</td>
</tr>
</tbody>
</table>

The relational process focuses on the being and relating of objects or actors. In this text the remaining one-third of objects lie within the relational process with 10 in both the basic and relational categories and a the remaining 5 which are used for the purpose of representation. The data in brackets in Table 5.25 represents the use of the equal to sign within the text selection. Inconsistent use of the equal to sign can be detrimental to students’ conceptual understanding.

Textbook Series C - Textbook 2 (TBS C2)

Table 5.26: Data from the Analysis of the Relational Process for TBS C2

<table>
<thead>
<tr>
<th>Human Notation</th>
<th>Object Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General</strong></td>
<td><strong>Specific</strong></td>
</tr>
<tr>
<td>TBS C2 Relational</td>
<td>0</td>
</tr>
</tbody>
</table>

The relational process focuses on the being and relating of objects or actors, in this text there are a total of 14 human specific references in the relational category. This is slightly higher than previous texts which is positive in terms of the reader being encouraged to use relations to make connections. Also, the frequency of the equal to sign is much lower than in previous texts.

Verbal Process

The verbal process is concerned with the exchange of meaning, it involves the process of putting forward meaning,
5.5. Textbook Language Analysis

e.g. 'I bags the biggest quarter', *says* the least mathematical but greediest member of the family. (TBS C1)

Textbook Series A - Textbook 1 (TBS A1)

Table 5.27: Data from the Analysis of the Verbal Process for TBS A1

<table>
<thead>
<tr>
<th></th>
<th>Human</th>
<th>Object</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>General</td>
<td>Specific</td>
<td>Basic or Derived</td>
</tr>
<tr>
<td>TBS A1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verbal</td>
<td>1</td>
<td>10</td>
<td>0 (=.9)</td>
</tr>
</tbody>
</table>

There are 25 passive sentences in the verbal process, almost 37% of the total passive sentences for this text selection. Passive sentences remove the purpose of human involvement, this would suggest a high rate of obscured agency within this textbook. Some examples of these passive sentences are:

“ Notice that the numbers of students who travel by the different methods are shown on vertical line (or axis). ” (pg.123)

“ In the right-angled triangle shown, the 90 angle is indicated by the mark ” (pg.358)

Textbook Series B - Textbook 1 (TBS B1)

Table 5.28: Data from the Analysis of the Verbal Process for TBS B1

<table>
<thead>
<tr>
<th></th>
<th>Human</th>
<th>Object</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>General</td>
<td>Specific</td>
<td>Basic or Derived</td>
</tr>
<tr>
<td>TBS B1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verbal</td>
<td>1</td>
<td>6</td>
<td>0 (=.10)</td>
</tr>
</tbody>
</table>

The verbal process contains over 50% of the passive sentences found for this text selection. The high presence of passive sentences removes human presence, combined with the verbal process passive sentences can completely alienate the human presence in mathematics and depict mathematics as a unattainable or even
5.5. *Textbook Language Analysis*

worse as an entity to be learned off as opposed to being understood. Here are some examples of the 37 verbal passive sentences found within this text selection:

“The set of numbers that are put into a function is called the ’domain’.” (pg.291)

“The power or index simply tells you how many times a number is multiplied by itself.” (pg.200)

“A set can also be shown with a Venn diagram (set diagram).” (pg.151)

**Textbook Series B - Textbook 2 (TBS B2)**

Table 5.29: Data from the Analysis of the Verbal Process for TBS B2

<table>
<thead>
<tr>
<th></th>
<th>Human</th>
<th>Object</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>General</td>
<td>Specific</td>
<td>Basic or Derived</td>
</tr>
<tr>
<td>TBS B2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verbal</td>
<td>0</td>
<td>4</td>
<td>0 (=,26)</td>
</tr>
</tbody>
</table>

The verbal process indicates the expression of meaning, the most common feature within the verbal process in this text selection are passive sentences, containing 28 out of a total of 44. Here are some examples of the 28 verbal passive sentences found within this text selection:

“A function is also called a ’mapping’ or simply a ’map’” (pg.197)

“The original position of a figure is called the ’object’.” (pg.372)

**Textbook Series C - Textbook 1 (TBS C1)**

Table 5.30: Data from the Analysis of the Verbal Process for TBS C1

<table>
<thead>
<tr>
<th></th>
<th>Human</th>
<th>Object</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>General</td>
<td>Specific</td>
<td>Basic or Derived</td>
</tr>
<tr>
<td>TBS C1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Verbal</td>
<td>0</td>
<td>5</td>
<td>0 (=,5)</td>
</tr>
</tbody>
</table>
5.5. *Textbook Language Analysis*

The verbal process, the putting forward of meaning, for this text selection is dominant in terms of passive sentences, 40% of the total lie within this category. Here are some examples of the 14 verbal passive sentences:

“The objects in the set are called members or elements.” (pg.1)

“The spotted region is called a segment.” (pg.258)

**Behavioural Process**

The behavioural process shares many characteristics with both mental and material process but is removed from these processes by the fact that it relies on a physical or mental process which involves two actors,

*e.g.* *Nowadays the operation of all computer circuits is based on Boolean algebra.* *(TBS C1).*

**Existential Process**

The existential process refers to the existence or presence of an occurrence or fact,

*e.g.* *There are four types of factors* *(TBS B2).*

**Conclusions of the Ideational Function Analysis**

The ideational function serves to identify where the ownership of responsibility and authority lie. This is achieved by engaging the six processes identified by Halliday. Of these six processes Halliday pinpoints three of these as the most significant for language analysis;

- Material,
- Mental,
- Relational.

With regard to mathematics education the three main ideational processes have a clear and definite purpose.
Material - suggests that mathematics is process of ‘doing’ (Morgan, 2004). A high presence of material processes throughout any mathematics textbook would indicate an encouragement of the reader to actively participate in and create mathematics. As indicated by the findings of this analysis a number of the textbooks have a high rate of passive sentences within this category which are counterproductive, 20.6% of the passive sentences in TBS A1 are in the material process. Also prevalent use of the pronoun “we” can impact negatively on students’ learning. Pimm (1987) noted how over use of the pronoun “we” is particularly negative for students who lack confidence. Also Fortanet (2004) speaks about how lecturers often use “we” as a means of building up a rapport but that in fact they are rhetorical indicators. Researchers such as Gerofsky (1999) and Svinicki and Dixon (1987) highlight the usefulness of rhetorical questions in pedagogic interactions between the teacher and student and reflective practices. However many researchers refer to rhetorical questions as non-questions (Davis, 1997). Noonan (1990) suggests that the use of rhetorical questions in school texts can inhibit student learning as they cause confusion. Also Kamio (2001) highlights the close relationship that the pronoun “we” suggests exists between the reader and the author which reflects directly back to Pimm’s (1987) idea of the negative presence the use of “we” has on under confident students.

Mental - suggests that mathematics is pre-discovered (something that just exists) by mathematicians (Morgan, 2004). A high presence of mental process throughout any mathematics textbook would indicate a certain level of knowledge requirements and ability. It is directly in contrast to the material process as instead of doing and creating mathematics the reader is being encouraged to know and think about mathematics. The low number of passive sentences within the mental process is a positive trait for all textbooks involved in this study. However, as you can see from Table 5.13 the presence of ‘objects’ within this category is very low. The use of objects within the mental process is necessary for visual learners. Studies
5.5. *Textbook Language Analysis*

carried out by Pinto (1998) and Pinto and Tall (1999) depicted that some students learn mathematics by extracting meaning (beginning with the formal definition and constructing properties by logical deduction), which supports Dubinsky’s theory that learners convert a process into a mental object. They also found that some students learn by giving meaning, refining and reconstructing their existing imagery until it is in a form that can be used to construct a formal theory. According to Pinto and Tall (2002) learners of this type exhibit a thinking process which resembles that of mathematicians who use broad problem-solving strategies. This would suggest that an increase in the presence of objects throughout the mental process would enable more students to learn effectively from the textbook.

**Relational** - suggests that mathematics is about relationships between objects (Morgan, 2004). A high presence of relational processes throughout any mathematics textbooks would indicate an effort to the reader to make connections and establish relationships between concepts and knowledge thus creating an understanding of mathematics. As previously stated researchers such as Van Dormolen (1986)) highlight the significance of the relational as opposed to the procedural function in terms of educational value. However, from Table 5.13 it is clear that the most prevalent element of the relational function is the use of the ‘equal to’ sign in define specifics within mathematics. All the textbooks involved in this study demonstrate a high frequency of use of the equal to sign to define, in comparison a much lower rate for showing or relating mathematics. This high frequency of the use of the equal to sign to define basic objects hinders the effect of the relational process and in fact suggests a much more procedural process might be in place.

5.5.2 **Interpersonal Function**

The interpersonal function focuses on determining the authority and ownership presented by the text. Morgan (2004) also looks at the presence of symbolism, specialist vocabulary and imperatives. The presence of personal pronouns throughout
5.5. **Textbook Language Analysis**

the text indicates with whom the ownership and authority of the text lie. The use of ‘I’ and ‘we’ advocates the author’s personal involvement while also assuming the reader’s interest. Directly addressing the reader suggests not only his/her interest levels but also a close relationship between the author and reader. While the use of the passive voice is indicative of the nature of the text (Ideational Function) it also highlights the author’s position on the usefulness of the presence of human activity.

Table 5.31: Framework for Analysis of the Interpersonal Function

<table>
<thead>
<tr>
<th>Ownership</th>
<th>Informal Sentences</th>
<th>Specialist words</th>
<th>Symbolism</th>
<th>Imperatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>me</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>you</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>we</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>my</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mine</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>your</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>their</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>our</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Pimm (1987) also reinforces Morgan’s beliefs about the use of imperatives. Imperatives are directly associated with the reader while also supporting a claim from the author that he/she is a member of the mathematical community. This community uses specialist vocabulary, thus allowing the author to project an authoritative voice. However, the author establishing himself/herself as a member of the mathematical community also allows for the reader to belong to the mathematical community and who, as such, becomes a colleague. Morgan (1995) states that, while a collegial relationship is assumed in academic writing, tensions exist within school texts between conventional familiar language for students and assessment driven mathematical language. Morgan (1995) also noted how children’s own
written texts (which were presented at a meeting of the British Society for Research into Learning Mathematics) are viewed in a negative light as they expressed an authoritative position. Ironically an authoritative position is in fact the widely accepted authors’ position in many school mathematics textbooks.

In this phase of language analysis data collection entailed a count of the various pronouns as provided in Table 5.31. For the purpose of this study informal sentence refers to any chat like sentence within the textbook. The use of pronouns within a textbook shares responsibility of the text with the reader and aims to establish a relationship between the reader and the author. An overuse of the pronoun ‘we’ however can alienate students, particularly those who lack confidence (Pimm, 1997). An even spread of the use of pronouns would remove any unnecessary pressure from students while shared ownership would remain intact. The presence of the informal sentences, specialist words, symbolism and imperatives are reflective of the type of expectation the author has of the reader. Informal sentences relax the tone of the textbook, unexplained or excessive use of specialist words can create confusion. The use of imperatives within the textbook can also be misleading. Rotman (2006) distinguishes between inclusive and exclusive imperatives. Inclusive imperatives ask the reader to be a thinker and exclusive imperatives ask the reader to be a scribbler. One would expect an even mix of inclusive and exclusive imperatives as mathematicians are thinkers and scribblers (Herbel-Eisenmann and Wagner, 2005). Also, the over use of specialist words and symbols can often be confusing for students and careful consideration should be applied to how and when new symbols and vocabulary should be introduced.

The position of the textbook author varies insignificantly between the two textbooks within each series, for that reason the interpersonal analysis can consider the textbooks in terms of their series (only three textbook series involved in this section of the analysis).
As you can see from Table 5.32 the author does not demand ownership of the text, in fact he is attempting to share responsibility with the reader (Pimm, 1987). The use of the pronoun ‘we’ also suggests the author’s personal involvement and the reader’s assumed interest (Morgan, 2004). It also alludes to the author asserting himself as an authoritative member of the mathematical community. The presence of the pronoun ‘you’ supports the close relationship implied by the high presence of the pronoun ‘we’, directly involving the reader, and again assuming their interest in the material presented.

The use of imperatives, specialist vocabulary and symbolism all support an author’s claim that he belongs within the mathematics community, combined with the author’s willingness to share responsibly. This also suggests that the author believes the reader is also a member of this community and as such is a colleague of sorts. The low level of informal sentences present in this textbook supports this view of the author and reader belonging to the mathematical community.
5.5. Textbook Language Analysis

Of the 37 and 31 imperatives present, 10.8% and 3.23% are inclusive imperatives respectively. This suggests the author sees the reader more as a scribbler than a thinker.

Textbook Series B

The data for TBS B (Table 5.33) shows little deviation from the findings for TBS A. However the main difference is a much more even distribution of the use of the pronouns ‘you’, ‘we’ and ‘your’. The use of ‘you’ and ‘your’ directly implicates the reader and again assumes their interest. Of the 42 and 58 imperatives present, 9.5% and 10.3% represent inclusive imperatives, suggesting that this textbook series sees the reader more as a scribbler than a thinker.

Table 5.33: Analysis of the Interpersonal Function - TBS B

<table>
<thead>
<tr>
<th>Ownership</th>
<th>TBS B1</th>
<th>TBS B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>me</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>you</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>we</td>
<td>17</td>
<td>21</td>
</tr>
<tr>
<td>my</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>mine</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>your</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>their</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>our</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>Informal Sentences</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Specialist words</td>
<td>317</td>
<td>312</td>
</tr>
<tr>
<td>Symbolism</td>
<td>197</td>
<td>309</td>
</tr>
<tr>
<td>Imperatives</td>
<td>42</td>
<td>58</td>
</tr>
</tbody>
</table>

Textbook Series C

The focus within this textbook series is very much about personal involvement and sharing of responsibility with almost the same number of instances of the use of the pronoun ‘we’ and ‘you’. TBS C deviates a great deal from the two previous
textbooks series in terms of the presence of informal sentences, has a total of 63 ‘chat like’ sentences, which infer a relaxed and comfortable relationship (Table 5.34). This is supported by the reduction of the number of specialist words and symbols and the presence of only 12 imperatives in the text section from both of these textbooks.

**Conclusion of Interpersonal Function Analysis**

The interpersonal function examines the relationship that the text suggests exists between the author and the reader. The position of the author impacts directly on the position of the reader and any assumptions or expectations assumed by the author will also directly affect the reader. While the author’s position does not significantly differ within a textbook series hence while the interpersonal function looks at the series as whole the author does however present his own definite position and with this he/she make assumptions and expectations about the reader.

As outlined by Morgan (2004) any academic mathematical text will have a higher occurrence of the pronoun ‘we’ and the author’s position is very much that he/she
5.5. Textbook Language Analysis

is an established member of the mathematical community. However, school mathematics texts, while they are designed for an academic purpose, differ greatly from academic texts in terms of intention. The intention of a school mathematics text is to support teaching and learning. Morgan (2004) also talks about how this intention difference is a source of tension. She outlines how a focus on assessment can often force a need for academic features to demand attention while the student themselves may need a focus on familiarity in order to construct and create their own knowledge, a feature far removed from academic texts. School texts differ in terms of the following; subject matter, the relationship between the author and reader, a more limited use of symbolism and technical language and a substantial graphical element (Shuard and Rothery, 1984; Morgan, 2004).

According to Shuard and Rothery (1984) the main goals of a school text are to teach concepts, skills and problem solving strategies, provide opportunity to practice, revise, and test these while developing ones mathematical language. These goals differ greatly from those of a academic text thus a school text should not be directly comparably to an academic text and text authors should be aware of the implications of not being too concerned about who the intended reader will be.

5.5.3 Textual Function

For the textual analysis Morgan draws on the work of Van Dormolen (1986) to assist when analysing the themes present in the texts. This work is completed in conjunction with examining the consistency of the structure and also the presence of reasoning within the text.
Identifying the overall theme put forward by a text can only be considered in conjunction with the ideational and interpersonal aspects. By constructing a thematic overview of a text one can then conclude on the nature of the discourse and its relevance to its intended purpose. Morgan (1995) suggests that due to the reasoning/deductive nature of mathematics one would expect to uncover a focus on logical reasoning and the progression of argument. For example ‘Hence’,

<table>
<thead>
<tr>
<th>Theme (all school text)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Separators:</strong></td>
</tr>
<tr>
<td>Layout</td>
</tr>
<tr>
<td>Paragraphs</td>
</tr>
<tr>
<td><strong>Consistency in Structure</strong></td>
</tr>
<tr>
<td><strong>Reasoning:</strong></td>
</tr>
<tr>
<td><strong>Conjunctions:</strong></td>
</tr>
<tr>
<td>due to</td>
</tr>
<tr>
<td>as a result of</td>
</tr>
<tr>
<td>despite</td>
</tr>
<tr>
<td>even though</td>
</tr>
<tr>
<td>since</td>
</tr>
<tr>
<td>so that</td>
</tr>
<tr>
<td>in order</td>
</tr>
<tr>
<td>why</td>
</tr>
<tr>
<td><strong>Logical Reasoning:</strong></td>
</tr>
<tr>
<td>basis for</td>
</tr>
<tr>
<td>aim</td>
</tr>
<tr>
<td>because</td>
</tr>
<tr>
<td>here</td>
</tr>
<tr>
<td>therefore</td>
</tr>
<tr>
<td>thus</td>
</tr>
<tr>
<td>and so</td>
</tr>
<tr>
<td>by + verb</td>
</tr>
<tr>
<td>Next</td>
</tr>
<tr>
<td>First (verb)</td>
</tr>
<tr>
<td>Then</td>
</tr>
<tr>
<td><strong>Reasoning + Imperative:</strong></td>
</tr>
<tr>
<td>Next</td>
</tr>
<tr>
<td>First (verb)</td>
</tr>
<tr>
<td>Then</td>
</tr>
</tbody>
</table>
5.5. Textbook Language Analysis

‘Therefore’, ‘By’ would suggest deductive argument, while ‘First’, ‘Next’, ‘Then’, etc. suggest recalling and recounting. Also reasoning presented in the form of conjunctions may be viewed as a sort of vagueness about the intended outcome. Conjunctions are a characteristic of speech as opposed to written text and inexplicit conjunctions will cause difficulty for the reader (Martin, 1989).

The thematic structures do not significantly vary between the two textbooks within each series, for this reason the textual analysis can also be considered for both textbooks together. The data collection for Separators (Table 5.35) looks solely at consistency of styles, while the three remaining sections, Conjunctions, Logical Reasoning and Reasoning + Imperative are all based on a count. The in-built word find function in Microsoft word is used to assist with such counts. The overall theme of each of these textbooks is school mathematics (as expected), Van Dormolen (1986) defines the school mathematics theme as one with a focus on listing material with a repetitive structure. The repetitive structure in these texts is primarily evident in the approach to examples and exercises. Van Dormolen (1986) further clarifies the thematic structure into three more categories; Problems and Exercises, Generalisations and Rules followed by Problems and Exercises (strictly separated), and a Mixture of both.

Textbook Series A

TBS A falls into the third category, mixture of problems and exercises with generalisations and rules, new material is presented and followed by examples then exercises then more material etc. The presence of ‘Thus’ throughout the text selection indicates a deductive process, reinforcing what Morgan (2004) outlined would be expected in any mathematical text. However there are slightly more references to ‘Then’ which indicates recall and recount. Overall there is a mixture of importance of deduction and recalling, with slightly more weight placed on the processes of recall and recount.
### 5.5. Textbook Language Analysis

#### Table 5.36: Analysis of the Textual Function - TBS A

<table>
<thead>
<tr>
<th>Theme (all school text)</th>
<th>3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TBS A1</td>
<td>TBS A2</td>
</tr>
<tr>
<td><strong>Separators:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layout</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Paragraphs</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Consistency in Structure</td>
<td>Consistent</td>
<td>Consistent</td>
</tr>
<tr>
<td><strong>Reasoning:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Conjunctions:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>due to</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>as a result of</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>despite</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>even though</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>since</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>so that</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>in order</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>why</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td><strong>Logical Reasoning:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>basis for</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>aim</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>because</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>here</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>therefore</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>thus</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>and so</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>by + verb</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>Next</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>First (verb)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Then</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td><strong>Reasoning + Imperative:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Next</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>First (verb)</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Then</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

**Textbook Series B**

The overall theme of TBS B, like TBS A is school mathematics and it too falls into the third category of “Mixture”.

The presence of ‘Therefore’, ‘By + a verb’ and ‘Then’ are regularly found through-
5.5. *Textbook Language Analysis*

out the text selection. ‘Therefore’ and ‘by’ are indicative of a deductive process while ‘then’ conveys recall and recount. Like TBS A there is a mixture of importance placed on deduction and recall. Also the higher number of reasoning + imperative combinations indicates the active participation by the reader, they are being directly told to do some activity.

Table 5.37: Analysis of the Textual Function - TBS B

<table>
<thead>
<tr>
<th>Theme (all school text)</th>
<th>TBS B1</th>
<th>TBS B2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Separators:</strong></td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Layout</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Paragraphs</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Consistency in Structure</td>
<td>Consistent</td>
<td>Consistent</td>
</tr>
<tr>
<td><strong>Reasoning:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Conjunctions:</strong></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>due to</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>as a result of</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>despite</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>even though</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>since</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>so that</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>in order</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>why</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Logical Reasoning:</strong></td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>basis for</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>aim</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>because</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>here</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>therefore</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>thus</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>and so</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>by + verb</td>
<td>14</td>
<td>7</td>
</tr>
<tr>
<td>Next</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>First (verb)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Then</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

**Reasoning + Imperative:**

<table>
<thead>
<tr>
<th></th>
<th>TBS B1</th>
<th>TBS B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Next</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>First (verb)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Then</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>
5.5. *Textbook Language Analysis*

**Textbook Series C**

TBS C again breaks the mould created by the two previous textbook series. The initial and most significant distinction between this textbook series and the previous ones are that not only has it a school mathematics theme but it equally has a narrative theme. The focus here is on drawing in the student’s attention prior to the introduction of new material. For this reason it differs greatly in the textual function analysis. The three most commonly found adverbs/conjunctions are ‘here’, ‘therefore’ and ‘then’, all of which are used in the logical reasoning of actual events or histories. Unlike previous textbook series the focus of this series is very much concerned with the provision of the information.
### Table 5.38: Analysis of the Textual Function - TBS C

<table>
<thead>
<tr>
<th>Theme (all school text)</th>
<th>Narrative TBS C1</th>
<th>Narrative TBS C2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Separators:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layout</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Paragraphs</td>
<td>Consistent</td>
<td>Consistent</td>
</tr>
<tr>
<td><strong>Consistency in Structure</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Reasoning:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Conjunctions:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>due to</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>as a result of</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>despite</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>even though</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>since</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>so that</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>in order</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>why</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td><strong>Logical Reasoning:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>basis for</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>aim</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>because</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>here</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>therefore</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>thus</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>and so</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>by + verb</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Next</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>First (verb)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Then</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td><strong>Reasoning + Imperative:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Next</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>First (verb)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Then</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
5.5. Textbook Language Analysis

Conclusion of Textual Function Analysis
The textual function serves to identify the theme of a text, thus examining the discourse and language with the purpose of determining its relevance to the targeted audience. The audience for the texts involved in this study would be 12-15 year old students, so one would not expect the same textual outcomes as a text intended for adults or mathematics students. Morgan (2004) speaks about three themes that can emerge, particularly from a school mathematics text:

- Deductive Reasoning,
- Logical Reasoning,
- Recall and recount.

The intention of a mathematics text should be deducible from the theme it presents. The intention of a school mathematics text is to support teaching and learning therefore texts of this nature should exhibit a combination of all of the above characteristics. However the weight placed on each characteristic is how the theme is uncovered. For example an effective school mathematics text will not focus on recall and recount as recall and recount do not serve as an effective teaching methodology. Shuard and Rothery (1984) conclude that a school text should not be impersonal, in fact everywhere a text provides exercises or problems for a reader they are in fact involving the reader. However they also note that while the reader may appear in this case to be an active participant the text may in fact be contributing to procedural emphasis particularly when the text fails to use exercises to lead to new knowledge and development towards the solving of mathematical problems.

5.5.4 Readability Analysis

Readability is analysed using the two most common readability tests: Flesch Reading Ease and Flesch Kincaid Grade Level and an online Lexile Measure. The Lexile
5.5. *Textbook Language Analysis*

framework for reading was developed by psychometricians at MetaMetrics, Inc., which is a private educational measurement company based in the USA. A lexile measurement can be obtained for individual students along with mathematical texts. These measures are for children and can range from a low of 5L to a high of 2000L, with 5L being rated as a beginner reader. These lexile measures allow for students to be matched with texts that have a same lexile measure, ensuring that text difficulties are suitable for the reader (MetaMetrics, 2010).

The aforementioned readability tests can only provide a comparison of texts and are insufficient in analysing mathematical texts due to the complexity of the mathematical language present. The lexile measure, while it does measure the reading value of a text needs to be compared to that of individual students. The following table outlines typical reader and text measures by grade in the USA. This table can only be used as a guideline and cannot be considered as exact values for a typical Irish situation.

Table 5.39: Typical Reader and Texts Measures by Grade

<table>
<thead>
<tr>
<th>Grade</th>
<th>Reader Measures (Interquartile Range, Mid Year)</th>
<th>Text Measures (from Lexile Map)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25th - 75th percentile</td>
<td>25th - 75th percentile</td>
</tr>
<tr>
<td>1</td>
<td>up to 300L</td>
<td>200L to 400L</td>
</tr>
<tr>
<td>2</td>
<td>140 to 500L</td>
<td>300L to 500L</td>
</tr>
<tr>
<td>3</td>
<td>330L to 700L</td>
<td>500L to 700L</td>
</tr>
<tr>
<td>4</td>
<td>445L to 810L</td>
<td>650L to 850L</td>
</tr>
<tr>
<td>5</td>
<td>565L to 910L</td>
<td>750L to 950L</td>
</tr>
<tr>
<td>6</td>
<td>665L to 1000L</td>
<td>850L to 1050L</td>
</tr>
<tr>
<td>7*</td>
<td>735L to 1065L</td>
<td>950L to 1075L</td>
</tr>
<tr>
<td>8*</td>
<td>805L to 1100L</td>
<td>1000L to 1100L</td>
</tr>
<tr>
<td>9*</td>
<td>855L to 1165L</td>
<td>1050L to 1150L</td>
</tr>
<tr>
<td>10</td>
<td>905L to 1195L</td>
<td>1100L to 1200L</td>
</tr>
<tr>
<td>11 and 12</td>
<td>940L to 1210L</td>
<td>1100L to 1300L</td>
</tr>
</tbody>
</table>

*7th and 8th grade are closely related to first and second year in Irish secondary schools respectively.
### 5.5. Textbook Language Analysis

#### Table 5.40: Textbook Readability Analysis - Counts

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Words</th>
<th>Characters</th>
<th>Paragraphs</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>3146</td>
<td>13276</td>
<td>282</td>
</tr>
<tr>
<td>TBS A2</td>
<td>2556</td>
<td>10072</td>
<td>194</td>
</tr>
<tr>
<td>TBS B1</td>
<td>5524</td>
<td>23664</td>
<td>370</td>
</tr>
<tr>
<td>TBS B2</td>
<td>4799</td>
<td>21390</td>
<td>568</td>
</tr>
<tr>
<td>TBS C1</td>
<td>3290</td>
<td>14567</td>
<td>187</td>
</tr>
<tr>
<td>TBS C2</td>
<td>2534</td>
<td>11831</td>
<td>124</td>
</tr>
</tbody>
</table>

#### Table 5.41: Textbook Readability Analysis - Averages

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Sentences per paragraph</th>
<th>Words per sentence</th>
<th>Characters per word</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>1.5</td>
<td>13.5</td>
<td>3.9</td>
</tr>
<tr>
<td>TBS A2</td>
<td>2.1</td>
<td>11.9</td>
<td>3.7</td>
</tr>
<tr>
<td>TBS B1</td>
<td>2.4</td>
<td>11.5</td>
<td>4.1</td>
</tr>
<tr>
<td>TBS B2</td>
<td>1.5</td>
<td>12.7</td>
<td>4.2</td>
</tr>
<tr>
<td>TBS C1</td>
<td>2.3</td>
<td>13.1</td>
<td>4.2</td>
</tr>
<tr>
<td>TBS C2</td>
<td>2.7</td>
<td>16.4</td>
<td>4.5</td>
</tr>
</tbody>
</table>

#### Table 5.42: Textbook Readability Analysis - Readability

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Passive Sentences</th>
<th>Flesch Reading Ease</th>
<th>Flesch-Kincaid Grade Level</th>
<th>Lexile Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>27%</td>
<td>74.9</td>
<td>6.1</td>
<td>1070L</td>
</tr>
<tr>
<td>TBS A2</td>
<td>12%</td>
<td>75.9</td>
<td>5.5</td>
<td>1740L</td>
</tr>
<tr>
<td>TBS B1</td>
<td>20%</td>
<td>67.6</td>
<td>6.6</td>
<td>1130L</td>
</tr>
<tr>
<td>TBS B2</td>
<td>15%</td>
<td>66.6</td>
<td>7.0</td>
<td>1380L</td>
</tr>
<tr>
<td>TBS C1</td>
<td>15%</td>
<td>69.2</td>
<td>6.8</td>
<td>960L</td>
</tr>
<tr>
<td>TBS C2</td>
<td>16%</td>
<td>57.4</td>
<td>9.2</td>
<td>1110L</td>
</tr>
</tbody>
</table>

In the passive voice, the subject of the sentence is neither a do-er or a be-er, passive sentences do not encourage action and should be avoided where possible in favour of active sentences (Quirk and Greenbaum, 1993). Flesch Reading Ease test provides a score which can be correlated with a suitable grade level for which the text is most suitable, the higher the score the easier to read a text is. A score
of 60 - 70 would mean that the text has a suitable readability level for 8th - 9th graders (equivalent to second and third year Junior Cycle). This would suggest that book one in each series should be from 70 - 80, (first year students) and book two 60 - 70 (second to third year students). Table 5.42 indicates that TBS A1 has a readability level which is more suitable for older students than TBS A2. This is supported by the Flesch-Kincaid Grade Level readability test. Flesch-Kincaid Grade Level results indicate the grade for which the text is most suitable, grade five would be the equivalent of 5th class in Irish primary schools, grade six - 6th class, grade seven - first year in Irish secondary schools, grade eight - second year and grade nine would be the equivalent of third year. As stated in the review of literature these results do not give an actual readability for the text (as it is a mathematical text and combines ordinary English with mathematical English) but does provide a comparison.

5.6 Conclusion

In summary, a number of key textbook features have emerged as significant to student learning. The textbooks in this sample are analysed to identify the presence of such features and hence predict their significance to the development of students’ mathematical concepts. In section 5.2 it emerged that the textbooks in this sample do not include motivational, comprehension and technical cues. This is reflected by the lack of real life figures and graphics, the dull and inconsistent use of colour and the neglect to reference technology of any sort. Also, the philosophical orientation put forward by all textbooks is a focus on intellect, whereas the emphasis evident is primarily ‘Proficiency and Logic’. Understanding and conceptual development appear to be secondary considerations after recall, recount and practice of mathematics routines. This is supported in section 5.4 where it emerged that the most common expectations presented by all of the textbooks are representing, performing routine procedures, formulating and clarifying problems and situation,
5.6. **Conclusion**

solving (mostly exercises and routine problems). Also in section 5.5 it emerged that there is a lack of language which encourages logical reasoning.

It is evident from section 5.3 that there is a lack of instructional narration across all textbooks, with definitions and key terms being lost in the main body of the textbooks. Researchers such as Shuard and Rothery (1984), Dowling (1996) and Mikk (2000) reiterate the importance of graphics to student learning, however they all point out that any graphics within a textbook need to have a specific purpose. Unnecessary graphics serve only to distract student attention and clutter up textbooks pages. Across all textbooks involved in this study graphics are plentiful but their role in assisting real life problems is minimal. Also the ratio of exercises to problems is at least 5:1 and even greater for the ratio of mathematical examples to real life examples. Cross referencing is not indicated in the index or throughout any of the textbooks.

In section 5.5 it became evident that there is a high rate of passive sentences across all textbook series, with TBS C presenting the lowest rate. There is an overuse of the pronoun, ‘we’ which is a characteristic of academic texts and there is a lack of connection between the language present and the objects used. This lack of connection demonstrates a lack of weight on visual learning and on the use of objects within the textbooks, suggesting an emphasis on the procedural as opposed to the relational aspects of mathematics. There is a lack of consistency of the use of the ‘equal to’ sign which contributes to student misconceptions. This confusion is adduced by the high rate of specialist words and symbolism and the lack of a glossary or dictionary. The low rate of the use of inclusive imperatives also suggests that the textbook authors see the readers as only scribblers and not as thinkers. Also, the language present in TBS A and B suggest a focus on deduction and recall whereas TBS C places more emphasis on the provision of information. Only one of the textbook series (TBS C) has a focus on the use of narration.
Chapter 6

The Role of the Textbook in Conceptual Development (Phase 3): Presentation of Findings

6.1 Introduction

This overall study is concerned with applying and extending standard textbook analysis techniques to Junior Cycle (lower secondary) textbooks in Ireland. This textbook analysis serves to highlight the quality of the current Junior Cycle mathematics textbooks while also identifying key features of mathematics textbooks which impact on students’ learning. The overall study is organised into three phases; phase one develops the literature review, phase two is concerned with the detail of the textbook analysis and phase three involves the creation and trial of a model chapter based on the hybrid model combining features of TIMSS with Morgan/Halliday’s functional grammar analysis.

The purpose of this chapter is to highlight and apply the key textbook design features which impact on student learning and to present and comment on the findings from phase three; the creation and trial of the model chapter. The model chapter is created from the ideals which emerged from phase 2 (findings of which are presented in chapter 5) as key textbook design features and is then piloted and trialled in a number of first year mathematics classrooms (lower level secondary
6.2. Textbook Design Features

A number of features have emerged from this research study which can enhance student experiences of mathematics textbooks. The intention of a school mathematics textbook is to assist with students’ learning, therefore it is vital that any textbook features which can enhance students’ understanding should be highlighted. The framework for textbook analysis employed in this study comprised four key elements; Content, Structure, Expectation and Language. For the purpose of organisation the following key textbook design features are presented in Table 6.1 using this framework structure.

The key content considerations are those features which support the mathematical content. The inclusion of these features serves to enhance students’ comprehension and motivation. The inclusion of technical cues and the consideration and awareness of philosophy all serve to engage and support the students in their learning. The structural considerations identify the key features which can help students to use their mathematics textbook more effectively, engage the students in problem solving and allow them to see the purpose of mathematics outside of the textbook chapter. The expectation considerations can facilitate oral and written communication of mathematics while encouraging critiquing and questioning which can help students to understand mathematics. The language considerations will encourage students to read the textbook and allow for a flow to this reading. Language considerations can facilitate with relational understanding, create connections and representations, encourage active thinking and participation while also encouraging communication.
6.2. **Textbook Design Features**

<table>
<thead>
<tr>
<th>Content Considerations:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Motivation</td>
<td>- Historical notes</td>
</tr>
<tr>
<td></td>
<td>- Applications and problem solving</td>
</tr>
<tr>
<td></td>
<td>- Photographs and figurative representations</td>
</tr>
<tr>
<td></td>
<td>- Biographies, career information and narration about people</td>
</tr>
<tr>
<td>Comprehension</td>
<td>Consistent use of colour</td>
</tr>
<tr>
<td></td>
<td>- Attractive but not overpowering colours</td>
</tr>
<tr>
<td></td>
<td>- Graphics to be connected to the text</td>
</tr>
<tr>
<td></td>
<td>- Only purposeful graphics</td>
</tr>
<tr>
<td></td>
<td>- Graphics to be used to assist problem solving</td>
</tr>
<tr>
<td>Technical Cues</td>
<td>- References to the internet and helpful websites</td>
</tr>
<tr>
<td></td>
<td>- References to supportive software such as Geogebra</td>
</tr>
<tr>
<td>Philosophical Orientation</td>
<td>- Clear emphasis and philosophy</td>
</tr>
<tr>
<td>Structure Considerations:</td>
<td></td>
</tr>
<tr>
<td>Narration</td>
<td>- High presence of narration</td>
</tr>
<tr>
<td></td>
<td>- Lower presence of instructional narration</td>
</tr>
<tr>
<td>Definitions</td>
<td>- Highlighted as being significant</td>
</tr>
<tr>
<td>Graphics</td>
<td>- Connected to text</td>
</tr>
<tr>
<td></td>
<td>- Connected to problem solving (as outlined in content considerations)</td>
</tr>
<tr>
<td>Problem Type</td>
<td>- Not just a book of exercises</td>
</tr>
<tr>
<td></td>
<td>- High ratio of worked examples to exercises</td>
</tr>
<tr>
<td></td>
<td>- High percentage of problems among exercises</td>
</tr>
<tr>
<td></td>
<td>- Some higher order problems are necessary (Non- routine problems)</td>
</tr>
<tr>
<td></td>
<td>- Mix of real, realistic, pure and fantasy problems</td>
</tr>
<tr>
<td>Content</td>
<td>- Obvious connections between content</td>
</tr>
<tr>
<td></td>
<td>- Cross chapter links outlined in index</td>
</tr>
<tr>
<td></td>
<td>- Emphasise key topics such as validation and justification</td>
</tr>
<tr>
<td>Physical Scale</td>
<td>- Small and light</td>
</tr>
<tr>
<td></td>
<td>- Limit page waste on answer pages</td>
</tr>
</tbody>
</table>
6.3 Assessing the Role of the Mathematics Textbooks in Students’ Conceptual Development

| Expectation Considerations: | - Encourage critiquing  
|                            | - Oral and written use of vocabulary  
|                            | - Emphasis on understanding |
| Language Considerations:    | - Avoid passive sentences  
|                            | - Avoid rhetorical questions  
| Ideational                 | - Use active verbs to encourage participation  
|                            | - Use objects to represent and relate the mathematics  
|                            | - Encourage thinking via mental processes  
|                            | - Focus on relating mathematics |
| Interpersonal              | - Textbook should be in present tense  
|                            | - Choose pronouns based on intended reader  
|                            | - Explain symbols clearly  
|                            | - Consistent use of symbols  
|                            | - Include informal sentences  
|                            | (dependant on intended reader)  
|                            | - Limited use of imperatives  
|                            | - Plan the introduction of new words/symbols  
|                            | - Use logical reasoning conjunctions to facilitate reading |
| Textual                    | - Establish textbook intent  
|                            | - Be consistent with structure  
|                            | - Be narrative (encourage reading)  
|                            | - Include a glossary or dictionary  
|                            | - Encourage oral and written use of vocabulary |

6.3 Assessing the Role of the Mathematics Textbooks in Students’ Conceptual Development

The following figure (Figure 6.1) from section 4.4 outlines the approach to phase three of this research study. Phase three includes the creation and design of a model chapter. This model chapter is centered around the topic of fraction addition. The theoretical framework for this model chapter consists of a combination of four well
6.4. The Model Chapter

established relevant frameworks which are outlined in section 4.5. Following pilot, review and amendment phases this model chapter is implemented in three secondary schools. The model chapter is used in conjunction with a two tier diagnostic test to ascertain the role of textbooks in student conceptual development. This two-tier diagnostic test is based on the work of Treagust (1988). The quantitative data collected from this phase of the research is analysed using ‘Statistical Package for the Social Sciences’ (SPSS) version 16.0.

![Figure 6.1: Outline of Phase 3](image)

**Figure 6.1: Outline of Phase 3**

6.4 The Model Chapter

In order to assess the importance of the role of the textbook in Junior Cycle Mathematics teaching and learning, the author created a chapter based on her research findings. The author has identified fractions as an aspect of mathematics which is greatly significant to mathematics in its entirety and is widely accepted as containing difficult concepts to learn and more importantly to understand. Vincent and Stacey (2008) also identify the lack of problem solving in the fraction unit
6.4. The Model Chapter

of Australian Textbooks as problematic. For these reasons and due to informal feedback from students at second and third level with regard to how difficult they perceive the concepts of fraction manipulation, the author has chosen this aspect to be the central focus of the model chapter (Appendix E & F).

The model chapter is implemented in three different secondary schools. These secondary schools are selected based on their textbook choice, one school using each textbook series involved in this research is required. Within each secondary school the three first year mathematics classes in each school are selected to participate in this study (each school only had three class groups in first year). Of the three first year mathematics classes participating in this study two first year classes act as test groups and one first year class acts as the control group. One teacher within each school acts as the main point of contact. The number of students in each school varies. School one has 66 students in first year, school 2 has 62 students in first year and school three has 67 students. A total of 195 students took part in this study. The first year students involved in this study are of mixed gender and ability (streaming had not occurred in any of the schools). The model chapter replaced the original textbook in each test group for the duration of the topic ‘fraction addition’. Each control group continues to use the original textbook as normal. In total, there are six test groups and three control groups in this phase (phase 3) of the research study.

Prior to commencing this small scale trial the author met with the contact teacher in each school. The teacher of each control group is advised to continue teaching as normal and the teachers of each test group are asked to implement the fraction addition framework (page 234) as best as possible within their teaching. The test group teachers are also asked to withdraw the usual mathematics textbook from use until after the post testing is completed. No further restrictions or requirements are placed on the test group teachers as the author felt that a textbook should not
6.4. The Model Chapter

make demands on a teacher.

6.4.1 Frameworks which Informed the Overall Framework for the Model Chapter

Before creating a framework for the model chapter, the author needed to examine a number of existing frameworks. These frameworks will first be outlined in order to provide the necessary pre-requisite knowledge for understanding the author’s framework.

Project Math

Following the OECD’s Programme for International Student Assessment which ranked Irish teenagers 16th out of 30 (O.E.C.D., 2003), the Irish Minister for Education at the time, Mary Hanafin, supported a proposal to encourage the uptake of Higher Level Leaving Certificate mathematics. This proposal recommended that students would obtain higher college entry points for Leaving Certificate Higher Level mathematics in comparison to other Leaving Certificate subjects. Her successor, Batt O’Keeffe, agreed with the need for change but felt that the introduction of an improved Junior Cycle curriculum would directly influence the uptake of Leaving Certificate Higher Level mathematics. He put his weight behind ‘Project Maths’. Project Maths was piloted in twenty four schools in Ireland in 2008, with mathematics teachers in these schools exploring a new range and availability of classroom resources and teaching methodologies.

One of the current aims of Project Maths is to provide teaching and learning plans, teacher guides and student worksheets, online exemplification and a range of assessment materials. As each strand was introduced into the schools teachers are required to evaluate the resource materials and provide feedback, with the intention being to improve the quality of teaching and learning in mathematics. It is intended that students’ mathematical learning will be enhanced through better teaching, an increased focus on applications and relevance to daily life. The
importance of analysing, interpreting and presenting mathematical information is also key, with problem solving emerging as essential. In conjunction with the new curriculum, Project Maths also seeks to create a ‘Bridging Framework’ to assist with linking the Primary curriculum to the Junior Cycle curriculum and the Junior Cycle curriculum to the Senior Cycle curriculum. The content and intentions of Project Maths in combination with the current Junior Cycle curriculum informs the content selection for the model chapter. Also the change of focus of this new curriculum which seeks to improve students’ problem solving skills directed the author towards the PISA mathematical cycle for problem solving.

**Adult Numeracy Network (ANN)**

ANN is an American initiative which created a framework for adult numeracy. The aim of this framework is to provide the mathematical skills and knowledge that adults need to be equipped with for their future lives. The ANN framework (Curry et al., 1996) provides a rationale for the inclusion of fractions in adult numeracy education. It also highlights some of the aspects of mathematics which they felt teachers need to consider prior to teaching any mathematical topic, such as the importance of teaching in context. This rationale ties in with the beliefs of Benn (1996) and Kaiser (2005). Curry et al. (1996) state that whole number computation skills are necessary for mathematical communication and attainment, but only when taught in conjunction with estimation, fractions, decimals, percentages and ratios, all of which are recognised as being equally necessary for mathematical proficiency.

Curry et al. (1996) suggests that relevant mathematics assists in better attainment of concepts and that directly applying mathematics to real life and the workplace gives the mathematics greater meaning and makes it easier to understand. Benn (1996) concurs, stating that teaching mathematics out of context results in many students finding the subject difficult and boring. Similarly the work of Freudenthal
6.4. *The Model Chapter*

(1973), anticipates the ideas expressed by the ANN framework and these have found expression and widespread recognition of RME\(^1\). Teaching realistic and applicable mathematics is far more beneficial to the learner than out of context mathematics (Freudenthal, 1973). Also, with great significance to this particular study, Curry et al. (1996) stated that textbook mathematics and word problems bear little resemblance to what adults consider real life mathematics. This overlaps with the experiences of students who, according to Mikk (2000), feel there is a need for imaginative textbooks. Preliminary research carried out by the author indicates an overwhelming 76.92% of teachers felt that the textbook choice had a significant impact on the students’ learning. A worrying 88.48% of teachers felt that current textbooks can alienate students (O’Keeffe, 2007).

With regard to problem solving, ANN suggests building up a repertoire of strategies and tools while also drawing on real world experiences and practices for examples of problem solving. Communication is also necessary within all aspects of mathematics although vitally so in problem solving, for understanding and for connecting students’ ideas. The ANN framework (Curry et al., 1996) provides the following guidelines for teaching numeracy:

- Focus on real life situated learning and student centered approaches to teaching
- Interdisciplinary: reading and writing should be an inclusive part of learning mathematics
- Teach concepts before rules
- Link new mathematics to previous learning
- Embed mathematics content and skills within the problem solving and decision making tasks
- Integrate reasoning and problem solving into all teaching
- Increase focus on communication

\(^1\)Realistic Mathematics Education
6.4. *The Model Chapter*

- Use group work and group discussions
- Teach in context, building on number sense.  
  (Curry et al., 1996)

**Adult Based Education (ABE) curriculum Framework**

The Massachusetts Department of Education (M.D.E.) created an Adult Based Education curriculum framework (ABE) in 2005. In this framework they outlined the underlying conditions required for numeracy eloquence.

“Numerate behaviour involves managing a situation or solving a problem in a real context by responding to information about mathematical ideas that is represented in a range of ways and requires activation of a range of enabling knowledge, behaviours and processes”  
(M.D.E., 2005: 7)

With this in mind they created a framework, whose core concepts are similar to those presented by the ANN framework. ABE’s core concepts are as follows (M.D.E., 2005):

- **Problem Solving:** According to ABE, when teaching any topic, problem solving is an essential skill which encourages conceptual development and enables the learner to reach their own solutions. It also provides the tools necessary to create and generalise problem solving strategies which can be applicable to a wide range of significant and relevant problems. ABE recommends that, where possible, problem solving tools should be real objects, calculators, computers, and measurement instruments.

- **Reasoning:** ABE suggests that teaching with mathematical reasoning encourages the learner to validate their own thinking and intuition, to ask questions and to question concepts. This will provide learners with confidence in their own mathematical ability.

- **Decision Making:** Mathematical decision making encourages the learner to apply their knowledge of concepts and procedures. Therefore, they assist
6.4. The Model Chapter

the teacher in foreseeing any misconceptions in conceptual development, thus minimising learners’ misconceptions. The ABE framework states that decision making promotes thinking about the procedures required to complete a task/problem.

- **Communication:** Communication allows for interaction and for learners to define, reflect and clarify their beliefs and thoughts about mathematical concepts and ideas. Discussion and reflection also allows for confident decision making.

- **Connections:** Making connections with real life is essential to enhancing student learning. ABE clearly states that allowing the learner to view mathematics from a realistic, connected approach enables them to apply mathematical thinking and modelling in order to solve problems.

Clearly, the focus is again on problem solving and decision making. Emphasis is placed on integrating reasoning, creating connections and using communication. For each curriculum strand, ABE provides a detailed table to assist with the teaching of each strand. The tables are based on the headings below (M.D.E., 2005):

<table>
<thead>
<tr>
<th>Standard</th>
<th>Benchmark: At this level an adult will be expected to:</th>
<th>Enabling Knowledge and Skills</th>
<th>Examples of Where Adults Use It</th>
</tr>
</thead>
</table>

Figure 6.2: ABE Framework

**PISA Mathematical Cycle for Problem Solving**

The PISA framework (O.E.C.D., 2006) (based on that proposed in O.E.C.D., 2003) provides the final link for creating the framework for the model chapter. This particular framework is based on an assumption of mathematics as a human
activity and elaborated as Realistic Mathematics Education (RME), and is referred to as mathematising. O.E.C.D. (2003: 26) summarise mathematising into a five step process:

1. Start with a problem situated in reality,
2. Organise it according to mathematical concepts,
3. Make assumptions, generalise and formalise,
4. Solve the problem,
5. Make sense of solutions in terms of the real situation.

This framework is typical of an RME approach and focuses on applications rather than ‘word story problems’.

Figure 6.3: The Mathematical Cycle

The graphical summarisation of the framework (Figure 6.3) illustrates how students are required to combine both vertical learning and horizontal learning\(^2\) to arrive

\(^2\)Treffers (1987) formulated the idea of two types of mathematisation, explicitly in an educational context, and distinguished “horizontal” and “vertical” mathematisation. In horizontal mathematisation, the students come up with mathematical tools, which can help to organise and solve a problem located in a real-life situation. Vertical mathematisation is the process of reorganisation within the mathematical system itself, like, for instance, finding shortcuts and discovering connections between concepts and strategies and then applying these discoveries.

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6.4. The Model Chapter


Framework for Model Chapter (Fraction Chapter)

Having considered the importance of the role of fractions in mathematics, specifically in Junior Cycle mathematics, the author chose to design a textbook chapter dedicated solely to fraction addition. The framework for the model chapter (Figure 6.4) is based upon the intentions of Project Maths, the Adult Numeracy Network (ANN) framework for teaching and learning numeracy (Curry et al., 1996), the Adult Based Education (ABE) framework (M.D.E., 2005) and the PISA Mathematical Cycle for Problem Solving (O.E.C.D., 2006).

At a first glance at this framework (Figure 6.4) it appears that fractions and solutions are feeding into the framework. The motivation behind this is based on the PISA Mathematical Cycle (O.E.C.D., 2006) which indicates that once a solution has been obtained it should feed back into the mathematical cycle and create more questions/problems. The reintroduction of solutions back into the cycle ensures a connection between the ‘mathematical world’ and the ‘real world’.

The inner cycle links directly with the Project Maths curriculum. One of the current aims of Project Math is to move the teaching focus towards a more problem based approach, intending to increase the presence and quality of applications and communication within the mathematics classroom. These three key areas (the inner circle) are the main focus of teaching and therefore the framework will direct the chapter to ensure that direct references are made to each of these approaches.

The outer circle dictates how to achieve the requirements of the inner circle. This outer circle is based on the evidence from the ANN framework (Curry et al., 1996), the ABE framework (M.D.E., 2005) and the PISA Mathematical Cycle (O.E.C.D., 2006).
Figure 6.4: Framework for Fractions Chapter
6.4. The Model Chapter

**Benchmark** refers to setting achievable yet challenging goals. While not explicitly mentioned in any of the aforementioned frameworks it is a common expectation which emerges for all.

**Group work and discussions** are vital for mathematical communication, ANN (Curry et al., 1996) emphasises the role of group work and discussion in creating mathematical connections and for linking students’ ideas. Similarly ABE (M.D.E., 2005) states that group work and discussion allows for student to student interaction. This helps students to voice, clarify and define their mathematical ideas while also assisting with the mathematising process (O.E.C.D., 2006), moving between the ‘mathematical’ and the ‘real’ world.

**Reading and Writing** is strongly connected with communication. Writing mathematics is common practice in many mathematical classrooms however it is usually limited to completing/working on exercises. Curry et al. (1996) are referring to reading and writing when they speak about interdisciplinary i.e. mathematics is written and/or read only in the context of other subjects such as science.. Writing one’s mathematical ideas and beliefs should be an inclusive part of mathematical learning. Also, writing one’s mathematical beliefs and concepts will assist with step 2 of the PISA Mathematical Cycle (O.E.C.D., 2006), organising one’s thoughts according to mathematical concepts.

**Linking new content to prerequisite knowledge** will create the basis on which to start the PISA Mathematical Cycle (O.E.C.D., 2006) and will also assist with step 5: making sense of solutions. Students need to be able to connect new mathematics with what they already know in order to develop conceptually (Orey, 2001). Curry et al. (1996) felt that creating such links is vital to all mathematical learning. Linking knowledge allows students to reason out problems and make decisions about how to approach the mathematics, all of which are essential in the ABE framework (M.D.E., 2005).
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‘Embed Content and Skills’ is referred to in each of the existing frameworks. Teaching mathematics in context is vital in order to achieve the inner circle. Curry et al. (1996) connect the work of Benn (1996) and Kaiser (2005) when they discuss the relevance of teaching in context for student learning. According to Curry et al. (1996), teachers should be aware of the need for contextual mathematics and should directly apply the mathematics, where possible, to the lives of the students. Similarly, the PISA framework refers to mathematics as being likened to a human activity, which relies on content being embedded into the appropriate context (as in RME).

Content and Structural Considerations

Having created the framework for the model chapter there are a number of key content and structural considerations which also need to be outlined. The model chapter content is based on a problem solving approach. This type of approach began featuring in mathematics education in the 1980’s with both the American curriculum initiative, ‘Agenda for Action’ (NCTM, 1980) and the British curriculum initiative (Cockroft, 1982) highlighting the significance of teaching with a problem based approach. A ‘Problem Solving’ approach to teaching provides students with a reason for doing the mathematics, an emphasis on strategies as opposed to rules and encourages the creative and developmental aspects of mathematics (Neyland, 1995). This ‘Problem Solving’ approach also aims to consider the students’ conceptual development by ensuring the required prior knowledge is stated and options for revision are included, by encouraging and presenting active, co-operative, technology based and developmental learning (Hirsch, 1992). Also the chapter will ensure it is learnable material by considering the work of Hiebert (1984) whose research on how children learn mathematics identified three significant factors:
6.4. *The Model Chapter*

1. Make connections to the Real World,
2. Create links between rules and conceptual knowledge,
3. Reflect on calculations and solutions.

In line with the recommendations outlined by Robinson (1981) for an ideal textbook, this chapter will consider the following six points:

- Include titles which suggest how content might be organised,
- Have a natural flow from idea to idea,
- Not be littered with unnecessary details,
- Ensure that important information stands out,
- Include summaries,
- Provide a clear sense of where we are going, where we have been and where we are now.

**Structure**

According to Goetz and Armbruster (1980) discourse is easier to attain when it is collective. Robinson (1981) provides a lot of research on the recommended textbook structure. His main findings (relevant to this study) focus on the importance of coherence and unity within a text. In order to ensure coherence it is vital to have a logical flow of content and the inclusion of summaries will assist with providing content unity.

**Content**

Content within a text plays a vital role in student learning. Rivers (1990) highlights the impact of content and content structure on students’ motivation and comprehension. In order to maximise student motivation, this chapter will include historical and biography references, (where applicable), career information, applications to real life and photographs. The inclusion of colour and graphics will enhance comprehension, while encouraging the use of technical aids broadens the students’ learning experience.
6.4. The Model Chapter

Illustrations

As noted previously, illustrations can positively impact on student motivation but can be detrimental to reading if merely for decorative purposes (Noonan, 1990). Dowling (1996) has done much research on illustrations and diagrams. He recommends that diagrams be used in dual mode with the problems/exercises, that they should be simple and easy to read, coloured and realistic (real if possible). Figure 6.5 outlines why and how illustrations should be included within a textbook.

![Figure 6.5: Illustrations within a Text (Dowling, 1996)](image_url)
6.4. The Model Chapter

Language Considerations

The presence and organisation of titles throughout a textbook, or in this case a chapter, refers directly to the language applied. This chapter is designed for first year secondary school mathematics students therefore the school mathematical register is most applicable. According to Balas (2000), how one reads mathematics has a significant impact on how ones learns mathematics. As mathematics is a language in its own right this chapter will encourage the reading of mathematics. In order to maximise student learning outcomes the findings of Noonan (1990) and Lim and Clement (2002) are applied:

- In order to minimise the difficulties associated with words of dual meaning (mathematical meaning is different to the ordinary English meaning) vocabulary lists will be included.
- Noonan (1990) made reference to the impact of the syntax of a mathematical text and states that short sentences cause less confusion to reading than long sentences.
- Diagrams, while they have major motivational advantages, can disrupt reading if not relevant.
- Symbols cannot be ‘sounded out’ like words so therefore a list of symbols and their meanings will also be included.
- Rhetorical questions serve only to confuse students therefore will be omitted.
- The page layout should not be cluttered, should be appealing and bright.

The importance of fractions and their role in numeracy and mathematics in general is unchallenged. The above frameworks, though not explicitly devoted to the teaching of fractions, have highlighted the significance and indeed necessity of a good knowledge base in topic areas and hence have been adapted, combined and modified to enable the author to create a framework suitable for the teaching of fractions (Figure 6.3). This framework in conjunction with the key design features is used to create and design the model chapter. The model chapter is then implemented in a small scale teaching intervention in three secondary schools.
6.4. The Model Chapter

6.4.2 Outline of Schools

School 1
School 1 is catholic co-educational fee paying day school with a student population of 450 in the Senior School (secondary school). The teachers and students of first year mathematics in School 1 use TBS C1.

School 2
School 2 is a co-educational fee paying secondary school with a student population of over 500 students. The teachers and students of first year mathematics in School 2 use TBS B1.

School 3
School 3 is an all-boys public secondary school with a student population of 410. The teachers and students of first year mathematics in School 3 use TBS A1.

6.4.3 The Two-Tier Diagnostic Test Instrument

In conjunction with the model chapter a two tier conceptual development diagnostic test is designed to identify the presence of conceptual understanding. This diagnostic test is applied prior to and after teaching with either the model chapter or the usual textbook. The length of teaching time varied between school but was approximately 3 weeks. The author did not introduce time constraints as a textbook should not make such demands on either teachers or students. The intention is for this model chapter to play a similar role to the usual textbook.

The diagnostic test has a total of 16 questions, each with two elements. The first section of each question requires the students to complete a mathematical problem/exercise, while the second section of each question is concerned with the conceptual knowledge required to complete the mathematical process. A binary system is used when correcting each question. A correct answer or concept would score a student 1 mark for each element and an incorrect answer or concept would
6.5. Comparison of School Mathematical Performance

result in a zero score. These scores then combine to give an overall mark (described in further writings as the overall element of the diagnostic test) for each question, a combination of 1,1 (both correct) would score 1 mark and a combination of 1,0 (correct answer and incorrect concept) or 0,1 (incorrect answer and correct concept) would score zero. Therefore a student could obtain a maximum of 16 marks for the answer element, 16 marks for the concept element and 16 marks overall.

6.4.4 Mathematics Anxiety Inventory

The Revised Mathematics Anxiety Rating Scale (RMARS) was devised by Plake and Parker (1982) (Appendix H) and serves to determine the level of anxiety students have about mathematics. The RMARS was designed to be implemented as a pre and post test. The difference in scores between the pre and post test indicate whether a positive or negative change (or no change) has occurred over a period of time. High results indicate that a student has high anxiety levels about mathematics at the time of testing.

6.5 Comparison of School Mathematical Performance

Table 6.2 outlines the baseline scores for each school prior to the intervention. This table indicates the mean scores of the students within each school in the pre diagnostic test i.e. the starting point of the students within each school.

Using ANOVA it became evident that there is a statistically significance difference between the schools conceptual development starting points and therefore in order to ensure valid results it is necessary to analyse the data on a school by school basis. These conceptual differences are demonstrated in Figure 6.6 (the starting positions for the conceptual elements vary significantly).
6.5. *Comparison of School Mathematical Performance*

Table 6.2: Baseline for each School

<table>
<thead>
<tr>
<th>School</th>
<th>Starting Point for concept element</th>
<th>Starting Point for answer element</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
<td>8.73 (SD = 2.66)</td>
<td>10.02 (SD = 2.82)</td>
</tr>
<tr>
<td>School 2</td>
<td>8.47 (SD = 2.91)</td>
<td>10.24 (SD = 2.82)</td>
</tr>
<tr>
<td>School 3</td>
<td>7.62 (SD = 2.62)</td>
<td>8.5 (SD = 2.775)</td>
</tr>
</tbody>
</table>

Figure 6.6: Baseline for Each School - Conceptual Element

The purpose of the box plots, in this chapter, is to provide a visual explanation of the changes evident in each school or class. The emphasised black line in each box indicates the median values. The lower end point of the whisker represents the minimum value and the upper end of the whisker represents the maximum value. Any outliers (any data which deviates significantly from the median) are
Comparison of School Mathematical Performance

represented by the circles outside the boxes, each circle has a number associated with it which represents an individual student. The lower and upper limits of the box represent the 25th and 75th percentiles respectively.

The diagnostic test mean scores for each school are compared to give an overall view on how the three schools involved in this study scored on the answer element and conceptual understanding element.

Table 6.3: Mean Differences for each School

<table>
<thead>
<tr>
<th>School</th>
<th>Mean difference for concept element (SD)</th>
<th>Mean difference for answer element (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
<td>1.79 (SD = 2.48)</td>
<td>1.95 (SD = 2.11)</td>
</tr>
<tr>
<td>School 2</td>
<td>1.63 (SD = 2.58)</td>
<td>1.48 (SD = 2.59)</td>
</tr>
<tr>
<td>School 3</td>
<td>1.74 (SD = 1.86)</td>
<td>2.42 (SD = 2.21)</td>
</tr>
</tbody>
</table>

From Table 6.3 it is evident that at first glance each school exhibits an increase of between 1 and 3 questions, that is the students within each school got, on average, one to three more questions correct in the post diagnostic test. While these numbers may seem low, when taken in their context (the short duration of the intervention (three weeks) and the baseline values for each school which indicate that the students are already getting between 8.5 and 10.2 answers correct) these values are very positive.

Each school indicates a general improvement in student performance for the mathematical answer, the conceptual understanding and both (overall) elements of the diagnostic test (Table 6.3). For example in School 1 students’ diagnostic tests results indicates that the students improved between pre and post tests by approximately 1.95 in the answer element and 1.79 concept elements. However, to get a full insight into the situations occurring within the schools it is necessary
6.5. Comparison of School Mathematical Performance

to take each school separately to examine the overall improvements and more importantly differences between the test and control groups.

6.5.1 School 1

Table 6.4 gives an indication of the starting point for the students within each class in School 1. There is no statically significant difference between each class group with regard to their baseline scores for the answer (\( p = 0.628 \)) and concept element (\( p = 0.601 \)) and hence it is acceptable to examine these class groups collectively.

<table>
<thead>
<tr>
<th></th>
<th>Sample Size</th>
<th>Starting Point for concept element</th>
<th>Starting Point for answer element</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1A (Control)</td>
<td>18</td>
<td>9.14 (SD = 2.53)</td>
<td>9.59 (SD = 2.75)</td>
</tr>
<tr>
<td>Class 1C</td>
<td>21</td>
<td>8.76 (SD = 2.68)</td>
<td>10.42 (SD = 3.008.32)</td>
</tr>
<tr>
<td>Class 1D</td>
<td>22</td>
<td>8.32 (SD = 2.81)</td>
<td>10.05 (SD = 2.77)</td>
</tr>
</tbody>
</table>

Table 6.5 details the mean changes between the pre and post diagnostic tests for students within School 1. As expected there is a general increase in student performance for all elements of the diagnostic test (the norm following any period of teaching). As is evident from Table 6.5 the conceptual element for each class group increased by between 1.17 and 3 concepts, i.e. students are answering, on average, 1.17 to 3 more concepts correctly in the post test compared with the pre test.

<table>
<thead>
<tr>
<th></th>
<th>Sample Size</th>
<th>Mean difference for concept element</th>
<th>Mean difference for answer element</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1A (Control)</td>
<td>18</td>
<td>1.78 (SD = 1.89)</td>
<td>1.55 (SD = 2.35)</td>
</tr>
<tr>
<td>Class 1C</td>
<td>18</td>
<td>2.94 (SD = 2.37)</td>
<td>1.78 (SD = 2.04)</td>
</tr>
<tr>
<td>Class 1D</td>
<td>18</td>
<td>1.17 (SD = 1.89)</td>
<td>1.35 (SD = 1.76)</td>
</tr>
</tbody>
</table>
6.5. *Comparison of School Mathematical Performance*

Figure 6.7, provides a more detailed look at the conceptual element median differences within each class.

From Figure 6.7 it is evident that Test Group 1 (1C) excelled in terms of increasing their conceptual development. The difference between pre and post scores are depicted in this graph, the zero marks on this graph represent no difference between post - pre scores. Therefore, it is evident that all students are performing better in the post tests than in the pre tests, which is expected from any teaching period. Paired-t tests identified that the increases in conceptual development for the control group (1A) and both test groups are statistically significant. One explanation for the statistically significant increase in the conceptual understanding of the students in the control may be due to the nature of the textbook used. This school uses
6.5. *Comparison of School Mathematical Performance*

TBC C for all its first year mathematics. TBS C emerged as better than the other textbooks (textbook analysis findings chapter 5) in terms of its use of problems and language.

Further analysis of the conceptual development of the class groups within School 1 identified that there is a statistically significant difference between test group 1 and the control group \((p = 0.047)^*\), i.e. test group 1 are out-performing the control group in terms of conceptual understanding. However, there is not a statistically significant difference between test group 2 and the control group \((p = 0.339)\). One factor for test group 2 not being statistically different than control group is due the nature of the textbook being used in the control group in this school, external factors can also affect the data such as the teacher and classroom environment.

![Figure 6.8: Median Difference in Answer Element - School 1](image)

Figure 6.8 provides a more detailed look at the answer element mean differences

\(^*p < 0.05\) is statistically significant development
6.5. Comparison of School Mathematical Performance

within each class. From Figure 6.8 it is evident that each class group appears to performing at a similar level in terms of their answers. This would suggest that a text which includes a greater emphasis on the concepts behind the mathematics (inclusion of the design features outlined in section 6.2) does not jeopardise a student’s ability to do well on traditional tests. The paired t-tests conducted on each class individually indicate that each class group exhibits a statistically significant increase in the answer element of the diagnostic test, again this would be expected from a teaching period. The further analysis, with the use of independent t-tests, highlights that the differences between the test and control groups for the answer element are not statistically significant.

6.5.2 School 2

School 2 also demonstrates an increase in student performance for all elements of the diagnostic test. Table 6.6 gives an indication of the starting point for the students within each class. Again, using one way ANOVA it is evident that there is no statistically significant difference between each class group with regard to their baseline scores for the answer (p = 0.494) and concept element (p = 0.140).

Table 6.6: Baseline Score for School 2

<table>
<thead>
<tr>
<th>Class 1Q (Control)</th>
<th>Sample Size</th>
<th>Starting Point for concept element</th>
<th>Starting Point for answer element</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>8.06 (SD = 3.51)</td>
<td>9.24 (SD = 2.64)</td>
</tr>
<tr>
<td>Class 1P</td>
<td>18</td>
<td>9.56 (SD = 1.72)</td>
<td>10.11 (SD = 2.19)</td>
</tr>
<tr>
<td>Class 1T</td>
<td>20</td>
<td>7.8 (SD = 3.01)</td>
<td>10.15 (SD = 2.82)</td>
</tr>
</tbody>
</table>

Table 6.7 gives the breakdown of the mean difference for all the answer and concept elements of the diagnostic test. As is typical of any teaching period there is a general increase in students answer and conceptual elements between pre and post testing. However, the mean answer difference for the control group is 0,
6.5. *Comparison of School Mathematical Performance*

this suggests that the control group (1Q) answered the same amount of questions correct following the teaching period.

Table 6.7: Mean Score for School 2

<table>
<thead>
<tr>
<th>Class</th>
<th>Sample Size</th>
<th>Mean difference for concept element</th>
<th>Mean difference for answer element</th>
</tr>
</thead>
<tbody>
<tr>
<td>1Q (Control)</td>
<td>13</td>
<td>1.62 (SD = 3.13)</td>
<td>0.00 (SD = 2.04)</td>
</tr>
<tr>
<td>1P</td>
<td>16</td>
<td>0.81 (SD = 1.11)</td>
<td>2.00 (SD = 2.16)</td>
</tr>
<tr>
<td>1T</td>
<td>17</td>
<td>2.77 (SD = 2.49)</td>
<td>1.65 (SD = 2.57)</td>
</tr>
</tbody>
</table>

Figure 6.9 provides a more detailed look at the conceptual element median differences within each class.

![Figure 6.9: Median Difference in Conceptual Element - School 2](image)

From Figure 6.9 it is evident that Test Group 1 (1T) excelled in terms of increasing conceptual development. It is also evident from Figure 6.9 that all students are
6.5. Comparison of School Mathematical Performance

performing better in the post tests than in the pre tests, which is expected from any teaching period (the minimum values for each class group which is indicated by the lower limit of each box, are greater than zero). Paired-t tests identify that the increases in conceptual development for the control group (1Q) are not statistically significant (p = 0.087) whereas both test groups exhibit statistically significant increases in conceptual understanding (for 1P p = 0.01 and for 1T p = 0.000)∗. Further analysis of the conceptual development of the class groups within School 2 identify that there is not a statistically significant difference between test group 1 and the control group or test group 2 and the control group. As previously mentioned external factors can also affect the data such as the teacher and classroom environment.

From Figure 6.10 it is evident that test group 2 excelled in terms of the answer element of the diagnostic test.

The paired t-tests conducted on each class individually indicate that each test group exhibits a statistically significant increase in the answer element of the diagnostic test however the control group do not exhibit statistically significant increases. The further analysis, with the use of independent t-tests, highlights that the differences between the test group 1 and the control groups for the answer element are not statistically significant. The difference between the control group and test group 2 are however statistically significant.

6.5.3 School 3

Similar to the schools previously mentioned, school 3 exhibits an increase in student performance for all elements of the diagnostic test. Table 6.8 gives an indication of the starting point for the students within each class. Prior to the teaching period the students are getting on average between 7 and 8 concept elements of the diagnostic correct. Using one way ANOVA it is evident that there is no statistically

∗p < 0.05 is statistically significant development
significant difference between each class group with regard to their baseline scores for the answer (p = 0.571) and concept elements (p = 0.441).

Table 6.8: Baseline Score for School 3

<table>
<thead>
<tr>
<th>Class</th>
<th>Sample Size</th>
<th>Starting Point for concept element</th>
<th>Starting Point for answer element</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 111 (Control)</td>
<td>22</td>
<td>7.68 (SD = 2.4)</td>
<td>8.73 (SD = 2.69)</td>
</tr>
<tr>
<td>Class 112</td>
<td>22</td>
<td>7.5 (SD = 3.14)</td>
<td>8.36 (SD = 2.97)</td>
</tr>
<tr>
<td>Class 113</td>
<td>22</td>
<td>7.18 (SD = 2.44)</td>
<td>8.14 (SD = 2.6)</td>
</tr>
</tbody>
</table>

Table 6.9 gives the breakdown of the mean difference for all elements of the diagnostic test; Answer and Concept. The students in class 111 (control group) are demonstrating a mean difference of 0.75 concepts and 2.1 answers. Positive results are also evident across both test groups.
As with Schools 1 and 2, the difference between post and pre tests of all three elements are then tested for statistical significance using paired t-tests.

![Figure 6.11: Median Difference in Conceptual Element - School 3](image)

Figure 6.11 provides a more detailed look at the conceptual element median differences within each class. From Figure 6.11 it is evident that Test Group 1 and 2 are outperforming the control group in terms of increasing conceptual development. It is also evident from Figure 6.11 that all students are performing better in the
6.5. *Comparison of School Mathematical Performance*

post tests than in the pre tests, which is expected from any teaching period (the minimum values for each class group which is indicated by the lower limit of each box, are greater than zero). Paired-t tests identify that the increases in conceptual development for the control group (111) are not statistically significant (p = 0.069) whereas both test groups exhibit statistically significant increases in conceptual understanding (p < 0.001 for both)*.

Further analysis of the conceptual development of the class groups within School 2 identifies that there is a statistically significant difference between test group 1 and the control group (p = 0.006) and also between test group 2 and the control group (p = 0.011)*. This again suggests that the student experiences of using the model chapter helped them to develop their own conceptual understanding of adding fractions.

From Figure 6.12 it is evident that each class group answers more questions correctly in the post diagnostic test as the minimum values for each class group are greater than zero (with the exception of two outliers in test group 2). The paired t-tests conducted on each class individually indicate that each test and control group exhibits a statistically significant increase in the answer element of the diagnostic test. Further analysis, with the use of independent t-tests, highlights that the differences between the each test group and the control group for the answer element are not statistically significant. This suggests that a text which includes a greater emphasis on the concepts behind the mathematics does not jeopardise a student’s ability to do well on traditional tests.

* *p < 0.05 is statistically significant development*
6.6. Comparison of Anxiety Levels

The RMARS Mathematics anxiety test Plake and Parker (1982) was applied both pre and post to the intervention. Using the RMARS scale a figure can be obtained (for comparative purposes) which indicates a student’s level of anxiety. Using this method of analysis it is expected that the levels of post anxiety would be less than pre anxiety, therefore one would expect that the post - pre difference would yield low or negative numbers. Before examining the data from post - pre tests it is important to know the initial anxiety levels of the students within each school. Students can score a maximum of 120 on this anxiety scale. On average the students on each school are just below the mid-way mark (Table 6.19) in terms of maximum anxiety (based on RMARS).

Table 6.11 indicates the anxiety changes within each school. The lower the mean
6.6. Comparison of Anxiety Levels

Table 6.10: Baseline Anxiety Level for each School

<table>
<thead>
<tr>
<th>School</th>
<th>Starting Point (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
<td>47.79 (14.72)</td>
</tr>
<tr>
<td>School 2</td>
<td>54.07 (17.99)</td>
</tr>
<tr>
<td>School 3</td>
<td>50.02 (14.14)</td>
</tr>
</tbody>
</table>

The better as one would expect anxiety levels to be reduced following the intervention. A more detailed look within each school gives a better overall view on the changes to anxiety created by this intervention.

Table 6.11: Mean Score for each School’s Anxiety Differences

<table>
<thead>
<tr>
<th>School</th>
<th>Mean Difference (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
<td>-4.15 (8.84)</td>
</tr>
<tr>
<td>School 2</td>
<td>-5.45 (10.14)</td>
</tr>
<tr>
<td>School 3*</td>
<td>-8.05 (11.75)</td>
</tr>
</tbody>
</table>

* The data for School 3 is not normally distributed (whereas School 1 and 2 are) therefore the median value is a more accurate measure of centrality, for School 3 the Median difference is -5 and the IQR is 13.5.

From Table 6.11 it would appear that each school is demonstrating a reduction in anxiety of between 4.15 and 5.45. Further analysis of each schools anxiety levels provides a more detailed account these reductions.

6.6.1 School 1

While Table 6.12 provides the baseline for each school it is necessary to highlight the baseline scores of each individual class within the school as this research examines the schools independently of each other. Table 6.12 exhibits the baseline anxiety levels for each class group which average at 40.33 to 52.18.

Table 6.13 gives the breakdown of the mean changes to anxiety levels within each class. From this table it becomes clear that the control group (1A) exhibit lower
6.6. Comparison of Anxiety Levels

Table 6.12: Baseline Score for Anxiety School 1

<table>
<thead>
<tr>
<th>Starting Point</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1A (Control)</td>
<td>50.5 (SD = 15.38)</td>
</tr>
<tr>
<td>Class 1C</td>
<td>40.33 (SD = 9.65)</td>
</tr>
<tr>
<td>Class 1D</td>
<td>52.18 (SD = 14.72)</td>
</tr>
</tbody>
</table>

reductions (-2.75) in their anxiety in comparison to the two test groups (-4.79 and -4.86).

Table 6.13: Mean Score for Anxiety Changes in School 1

<table>
<thead>
<tr>
<th>Mean Difference</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Post Anxiety - Pre Anxiety</td>
<td></td>
</tr>
<tr>
<td>Class 1A (Control)</td>
<td>-2.75 (SD = 11.75)</td>
</tr>
<tr>
<td>Class 1C</td>
<td>-4.79 (SD = 9.19)</td>
</tr>
<tr>
<td>Class 1D</td>
<td>-4.86 (SD = 4.81)</td>
</tr>
</tbody>
</table>

The paired t-tests however indicate that the anxiety level changes for within the control group (p = 0.308) are not statistically significant while the test groups gives mean changes which are found to be statistically significant (1C gives p = 0.036 and 1D gives p = 0.000). This suggests the class groups exposed to the model chapter are both demonstrating a greater (and statistically significant) reduction in anxiety compared to the control group.

6.6.2 School 2

Table 6.14 provides the initial anxiety levels within each class for School 2. The anxiety levels for class group 1Q (control group) are much higher than the other two class groups with (49.2 and 50.67).

Table 6.14: Baseline Score for Anxiety School 2

<table>
<thead>
<tr>
<th>Starting Point</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1Q (Control)</td>
<td>62 (SD = 19.45)</td>
</tr>
<tr>
<td>Class 1P</td>
<td>49.2 (SD = 15.93)</td>
</tr>
<tr>
<td>Class 1T</td>
<td>50.67 (SD = 17.99)</td>
</tr>
</tbody>
</table>
6.6. Comparison of Anxiety Levels

Table 6.15 gives the breakdown of the mean changes to anxiety levels. From this table it becomes clear that the control group (1Q) exhibits greater reductions (-7.79) in their anxiety in comparison to the two test groups (-2.94 and -5.35), one reason for this would be the high initial levels of anxiety. The higher initial levels would allow more room for reductions in anxiety. The paired t-tests however indicate that the anxiety level changes for the control group (1Q) are statistically significant (p = 0.01). However, one of the test groups (1P) does not provide statistically significant reductions (p = 0.296) while the other test group (1T) is borderline (p=0.05).

Table 6.15: Mean Score for Anxiety Changes in School 2

<table>
<thead>
<tr>
<th>Mean Difference</th>
<th>Post Anxiety - Pre Anxiety</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1Q (Control)</td>
<td>-7.79 (SD = 8.79)</td>
</tr>
<tr>
<td>Class 1P</td>
<td>-2.94 (SD = 11.22)</td>
</tr>
<tr>
<td>Class 1T</td>
<td>-5.35 (SD = 10.39)</td>
</tr>
</tbody>
</table>

6.6.3 School 3

Table 6.16 provides the initial anxiety levels within each class for School 3, which average from 45.77 to 53.18.

Table 6.16: Baseline Score for Anxiety School 3

<table>
<thead>
<tr>
<th>Starting Point</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 111 (Control)</td>
<td>45.77 (SD = 12.06)</td>
</tr>
<tr>
<td>Class 112</td>
<td>53.18 (SD = 17.01)</td>
</tr>
<tr>
<td>Class 113</td>
<td>51.09 (SD = 11.8)</td>
</tr>
</tbody>
</table>

Table 6.17 gives the breakdown of the mean changes to anxiety levels. From this table it is evident that each class depicts a reduction of over 7 points in their anxiety level. The control group exhibits the smallest reduction of -7.43. Further analysis of these reduction scores, using paired t tests, indicate that the reduction evident
6.7. Conclusion

in the control group of -7.43 is not statistically significant \((p=0.135)\), one reason for this might be the high standard deviation evident \(SD = 15.2\). In comparison to this, the test groups 112 and 113 exhibit reductions of -8.05 and -8.89 which are both statistically significant \((p= 0.000 \text{ for } 112 \text{ and } p= 0.001 \text{ for } 113)\). This suggests that both the tests groups using the model chapter have significant decreases in their anxiety following the teaching period.

<table>
<thead>
<tr>
<th>Mean Difference</th>
<th>Post Anxiety - Pre Anxiety</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 111 (Control)</td>
<td>-7.43 ((SD = 15.2))</td>
</tr>
<tr>
<td>Class 112</td>
<td>-8.05 ((SD = 9.38))</td>
</tr>
<tr>
<td>Class 113</td>
<td>-8.89 ((SD = 9.72))</td>
</tr>
</tbody>
</table>

Table 6.17: Mean Score for Anxiety Changes in School 3

6.7 Conclusion

The data collected from each school is analysed independently, as the ANOVA tests found there are statistically significant differences between each school. The paired t-tests, utilised in this phase of the research, identify the changes between pre and post tests for each class independently. The test groups within each school exhibited statistically significant increases \((p < 0.05)\) in their conceptual development following the teaching period with the model chapter. In contrast to this only one of the control groups demonstrated statistically significant changes in their conceptual development. The control group (in School 1) who exhibit a statistically significant increase in conceptual development are using TBS C. The findings in phase 2 of this research study suggest that TBS C exhibits a greater consideration for who its intended reader is.

Independent t-tests are conducted to compare the increases in conceptual development between test and control groups. The findings from the independent t-tests indicate that some test groups did statistically better than the control groups but
6.7. Conclusion

some did not. This is because there are many external factors which could have contributed to such findings which are beyond the control of this study e.g. teacher, classroom environment.

The RMARS anxiety tests identified that all class groups had a reduction in anxiety following the teaching period. In School’s 1 and 3 only the test groups had statistically significant decreases in anxiety and in School 2, one test group and the control group demonstrated statistically significant reductions in anxiety. The paired t-test findings for the answer elements of the two-tier diagnostic test exhibit statistically significant increases between pre and post test for all class groups. This suggests that while the students who worked with the model chapter exhibited significant increases in their conceptual understanding of adding fractions their ability to answer traditional mathematics questions is not jeopardised.
Chapter 7

Discussion of Findings

7.1 Introduction

The purpose of this research study is to examine the quality of the mathematics textbooks currently in use at Junior secondary school level in Ireland. This is achieved by investigating, extending and applying suitable methodological tools for textbook analysis. Ultimately the aim of this research is to improve the quality of teaching and learning of mathematics at Junior Cycle level which should feed directly into improving the quality of mathematics at Leaving Certificate. This will be achieved by first measuring the quality of the current Junior Cycle textbooks and then highlighting the role of improved textbooks in students’ conceptual understanding. At present in Ireland, there is a move away from the more traditional didactical approaches to teaching and learning towards teaching and learning for understanding. This move towards teaching and learning for understanding is as a result of the new curriculum initiative - Project Maths, which requires students to understand and apply mathematics. This change in focus within mathematics classrooms across Ireland highlights the need for teachers to be more aware of students’ conceptual development and hence influenced the author as this research study evolved.

An investigation was carried out by the author to examine the quality of the current mathematics textbooks which involved both a qualitative and quantitative study.
7.1. Introduction

of Junior Cycle mathematics textbooks. Following this analysis, which included language analysis, a model chapter was designed and created. This model chapter was piloted, amended and then tested in a number of first year mathematics classrooms where the role of improved textbooks for students' conceptual development was tested using quantitative analysis. The research was guided by and referenced to an in-depth literature review (chapter 2 and chapter 3), which provided the theoretical frameworks and methodologies.

This project is located within an Irish context and no previous research of this type has been undertaken nationally. While similar international studies have been conducted the intent and focus of this project is unique. This chapter discusses the key findings which have emerged from this research study (presented in chapters 5 and 6). This study is shaped by three main phases; phase one incorporates a comprehensive review of available literature detailing mathematics education in Ireland and classroom resources worldwide, phase two encompasses the textbook analysis and phase three entails the creation, design and small scale trial of the model chapter. The discussion of the findings from each phase of this research study are framed by the research questions which were set out for each phase (sections 1.5 and 4.4.3).

Phase One - Exploratory Research

The following questions emerged from the author’s preliminary investigation and her own experiences as a teacher.

- What role do mathematics textbooks play in mathematics education i.e. in the teaching and learning of mathematics?
- What are the main areas of concern in mathematics education which are impacted by the content and structure of mathematics textbooks?
- Which of the available methods of textbook analysis are the most applicable and relevant to mathematics textbook research?
Phase Two - Textbook Analysis

- Are the current Junior Cycle mathematics textbooks an effective resource for teaching and learning?
- What is the significance and impact of language considerations as interpreted in textbooks for both the teaching and learning of mathematics?
- Do the current textbooks address the key areas of concern in mathematics education such as language deficiencies and poor problem solving skills?

Phase Three - Evaluation of Role of Textbook in Student’s Conceptual Development

- Can improving the textbooks directly improve students’ understanding of mathematics and their conceptual development?

Comparisons will be made between the literature and the key findings from this research study. This project provides, not only a number of effective measures for textbook analysis, but also a framework for a complete textbook analysis. Thus, this project can provide conclusions and recommendations for further investigation, textbook creation and textbook selection. The discussion of findings is organised under the research questions from each of the three main phases of this research project; phase one (review of the literature), phase two (textbook analysis) and phase three (identify the role of textbooks in conceptual understanding).

7.2 Key Findings for Phase 1

Phase one comprises a comprehensive review of available literature which was carried out to deepen the insight into mathematics education in Ireland and to establish the role of the textbook as a classroom resource worldwide. Phase
7.2. Key Findings for Phase 1

one contributed to the creation of a detailed methodology by providing insights into textbook analysis, which guided the development of a suitable framework for research in Phase Two. The discussion of phase one is guided by the phase one research questions.

7.2.1 Key Findings for Research Question 1

“What role do mathematics textbooks play in the teaching and learning of mathematics?”

Apple (2000: 183) quotes A. Graham Down of the Council for Basic Education when he states that

“Textbooks, for better or worse, dominate what students learn. They set the curriculum, and often the facts learnt, in most subjects. For many students, textbooks are their first and sometimes only early exposure to books and to reading.”

While there is an abundance of research detailing the significance and extent of the teachers’ use of the mathematics textbook, one must rely on anecdotal evidence to clarify how students use their mathematics textbook. However, such evidence and research suggests that despite the changing face of education, (with the introduction of new technologies and the internet), the mathematics textbook is still the most dominant classroom resource worldwide (Hiebert et al., 2003; O’Keeffe, 2007; Mikk, 2000). Remillard (1996) suggests that mathematics textbooks have two roles, to organise the content and to teach the content. Later work by Remillard which studied how teachers learn from textbooks identified that textbooks which are written for students do not improve a teacher’s learning (Remillard, 2000) and hence one can assume that a textbook written for teachers will not be an ideal learning resource for students. Haggarty and Peppin (2002: 568) noted that Kietel et al. (1980) suggest that the textbook is “one of the most important orientations
Researchers such as Sewall (1992); Valverde and Schmidt (1998); Horsley and Laws (1992) all support the need for effective mathematics textbooks. According to Mikk (2000) the purpose of the textbook is to help and motivate students to learn and hence mathematics textbooks should be exciting and imaginative, “students have many sources of information available, if their textbooks are dull, they are unwilling to study them. Interesting and enthusiastic textbooks develop curiosity and interest in the subject” (Mikk, 2000: 17). Nationally the NCCA also echoes such findings, with the 2006 consultation report identifying the need for improved textbooks in order to enhance national proficiency (NCCA, 2006). Therefore, both nationally and internationally, researchers alike agree that the textbook plays a central role in mathematics classrooms.

It is a common belief that mathematics in Ireland is still taught via traditional methods of teaching which require heavy use of the mathematics textbook (NCCA, 2005a). Such traditional teaching styles require the textbook to play the role of the dominant classroom resource. A preliminary study carried out by the author identified that 75% of the teachers in her sample use only one textbook for all their classroom planning and teaching (O’Keeffe, 2007). A recent report into the teaching of mathematics, conducted by Ní Riordáin and Hannigan (2009), identifies that almost 50% of teachers currently teaching mathematics in Ireland do not have a teaching qualification in mathematics (out-of-field teachers). The impact of the role of the mathematics textbook as a dominant classroom resource is more significant in the context of so many out-of-field teachers. Such out-of-field teachers are not aware of the pedagogy specific to mathematics and may be inclined to remain reliant on the textbook.
7.2. Key Findings for Phase 1

7.2.2 Key Findings for Research Question 2

“What are the main areas of concern in mathematics education which have a significant impact of the content and structure of mathematics textbooks?”

There are a number of key areas of concern in mathematics education with regard to student learning. However the researcher is particularly interested in important issues which involve mathematics textbooks or key concerns which can be influenced by mathematics textbooks e.g.

1. How students learn mathematics (Enhancing conceptual development),
2. The use of problem solving,
3. Language and reading concerns,
4. Teacher competency.

1. How students learn mathematics (Enhancing conceptual development)

According to Baroody and Ginsburg (1990); Resnick (1989) and Hiebert (1984) children enter the education system with existing mathematical knowledge and with relatively good informal problem solving abilities. Despite the fact that children are naturally curious they lose these problem solving abilities as they move through the education system (Baroody and Ginsburg, 1990). Baroody and Ginsburg (1990) suggest children’s initial knowledge base is dependent on the concept of counting and thus they often see mathematics as a process of using this for calculation, for example using their counting methods to assist in solving products, difference, etc. This method does not necessarily transfer to the way mathematics is taught in schools or presented in textbooks. To counteract this loss of problem solving ability, mathematics textbooks need to consider the work of Skemp (1971) and Hiebert (1984) who agree that children learn by combining
7.2. Key Findings for Phase 1

form and understanding\(^1\). Effective combinations of form and understanding will enhance students’ conceptual development. Conceptual development is a topical area of student learning, described as “learning that changes an existing conception” (Orey, 2001: 1). For conceptual development to have occurred students need to have no misconceptions about previous knowledge. This highlights the importance of prerequisite knowledge to student learning, underlining the need to include significant mathematics knowledge in mathematics textbooks.

2. The use of problem solving

Problem solving has been identified in section 2.2 as an effective approach to teaching mathematics. According to NCTM (1989) students need to move away from routines and rules in order to express their thoughts as they understand them. Problems can be classified as non-routine or routine problems. Non routine problems are those which cannot be answered using routine procedures, each of these can also be classified as Real, Realistic, Fantasy and Purely Mathematical, and provide maximum opportunity for student thinking, discussions and learning to occur. Problem solving can equip students, not only with mathematical knowledge, but also the ability to communicate mathematics. The current Junior Cycle mathematics textbooks are flooded with practice exercises and routine problems, however, non-routine problems are very scarce. Amendments to the type of problems present in the textbooks and the way in which problems are presented is an easy way to encourage and improve problem solving skills.

3. Language and reading concerns

According to researchers such as Kane (1968), Noonan (1990), Chapman (1993) and Orton (2004) there are many aspects of mathematical language which can hinder learning. The language of mathematics comprises ordinary English, math-

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\(^1\)Form is the “syntax of the system”, the rules or patterns associated with mathematics and understanding refers to the “semantics of the system”, meaning and mathematical developments (Hiebert, 1984: 498)
7.2. Key Findings for Phase 1

Mathematical English and words with dual meaning (Noonan, 1990), all of which can cause reading difficulties. Similarly Skemp (1982) talks about two levels of mathematical language; the ‘deep structures’ of mathematics which refers to the ideas that teachers and textbooks attempt to communicate and the ‘surface structures’ which are the symbolic systems which represent these ideas. Reading a mathematical text requires one to learn, without hindrance, from the vocabulary present. That is, students need to be able to read and understand the various types of English and symbolism that is presented to them. Noonan (1990) highlighted that one of the main obstacles to reading a mathematics text is the difficulty in understanding/interpreting the language used. These problems are directly correlated with Halliday’s principles of language; how the ideational, interpersonal and textual functions are presented in a text will have implications for the reader.

Research such as that carried out by Glynn and Britton (1986), Marks et al. (1974), Noonan (1990) and Schoenfeld (1992), highlights the significance of reading mathematics for learning and engaging in mathematics. As highlighted in chapter 2 it is essential that mathematics textbooks are readable and encourage reading and that textbook authors are aware of the forms of mathematical language and vocabulary that they use. Textbook authors must also consider the ways in which they use and introduce symbols and be aware that every decision they make has implications for student learning.

4. Teacher competency

According to Ball (2000) teachers’ attitudes and beliefs will undoubtedly inform the approaches and emphases they place on certain aspects of mathematics. Similarly teachers’ dependency on didactic teaching approaches limits student learning in the classroom. Hirsch (1992) identifies four areas of good practice which can be incorporated into both the classroom and the textbook; Active learning, Cooperative learning, Technology supported learning and Developmental learning.
7.2. Key Findings for Phase 1

The work of Fajemidagba (1998) and Cooney (1999) highlights the role of the textbook in bridging the gaps in teacher competency. Fajemidagba (1998) identified that competent teachers are in a position to include additional information such as the history or purpose of a topic which is reinforced by Cooney (1999) when he suggests that one must look at teaching as a whole. For example when teaching the rules and procedures of a topic it is essential that one also considers the context and applications of the topic. In fact the distinction between Project Maths and the old Junior Cycle mathematics curriculum is the idea of ‘teaching in context’ with a focus on applications. Such research is significant for mathematics textbooks in that textbooks can fill the gaps for non competent teachers. For example, the textbook can provide details on applications and history of new topics.

7.2.3 Key Findings for Research Question 3

“Which of the available methods of textbook analysis are the most applicable and relevant to mathematics textbook research?”

While no Irish research has been conducted on mathematics textbooks for secondary schools (with the exception of a minor inclusion in the TIMSS Report (2002)) many international studies have been conducted. Such studies have identified the key areas which are involved in textbook analysis and more particularly in mathematics textbook analysis. The methods of mathematics textbook analysis utilised in this research study are primarily based on the work of TIMSS (2002), Rivers (1990) and Mikk (2000), however, the work of many other researchers (as identified in chapter 3) has informed or enhanced this research in some aspect. There is an abundance of research on how to effectively analyse a textbook. However if one were to summarise all the key factors the following four would emerge as fundamental:
7.2. Key Findings for Phase 1

1. Structure
2. Content
3. Expectation
4. Language & Readability

Methods of Textbook Structure Analysis

Textbook structure adds or takes from textbook comprehension, hence the succession and connections between text elements need to be analysed carefully. Mikk (2000: 99) illustrates by way of a matrix table (Appendix D) how one can easily analyse the structure of a text and record diagrammatically how frequently ideas/topics appear and therefore connections are visualised. The TIMSS examined the structure of mathematics textbooks under the term ‘Physical Scale’. While Robinson’s work, as we have seen, focuses on four areas of textbook analysis which all fall directly under the wing of structure analysis: structure, coherence, unity and audience appropriateness.

Having considered all of the above research the author set up grids for textbook analysis (see Appendix L) which provided for detailed data collection under the following headings:

- Block Type,
- Narration,
- Definitions,
- Graphics,
- Exercises,
- Examples,
- Physical Scale.

The author used this classification for structure analysis within her research. Following on from this, structure grids were established to signify the content structure throughout each textbook (See section 5.4).
7.2. Key Findings for Phase 1

Methods of Textbook Content Analysis

According to Mulryan (1984) the content within a mathematics textbook influences the selections and emphasis applied by teachers and students (Mulryan, 1984). Hence, analysing the textbook content is an essential part of mathematics textbook analysis. In her work with algebra textbooks, Rivers (1990) provided for four aspects of content analysis (which are similar to those outlined by Gerbner (1969)). Rivers’ four areas of content analysis are **motivational factors** which include historical notes, scientist and mathematician biographies, career information, applications and photographs; **comprehension cues** which focus on colour and graphics; **technical aids** which include all material related to calculators and computers and **philosophical orientation** which emphasises the predominant philosophy of the textbook.

Wittlin’s work on the content of museum exhibitions had somewhat similar ideas. Wittlin (1978) identified that in order to maximise interest in a museum exhibit, the exhibit first had to attract attention, then provide the information and finally it had to keep attention. Robinson (1981) noted that Wittlin’s work can be transported to the ideas behind textbook content analysis as the very first objective of any textbook must be to attract student attention. Then the focus moves to presenting the message clearly and comprehensibly, and finally to maintaining students’ attention.

The TIMSS analysed textbook content in terms of topic coverage. The author adopted a suitable and effective measure for content analysis based on a combination of the work conducted by TIMSS and Rivers (1990). The overall framework for content analysis is based on the River’s Matrix which consists of four key areas; Motivational Factors, Comprehension Cues, Technical Aids and Philosophical Position. The method of data collection using grids is based on the TIMSS method of textbook analysis (see Appendix N for the content analysis grid).
7.2. Key Findings for Phase 1

Methods of Textbook Expectation Analysis

As outlined in section 3.4.5 performance expectations are embedded throughout a textbook and they impact significantly on how students chose to deal with the topics presented. The TIMSS Report (Valverde et al., 2002) identified that expectation analysis is also an essential element of mathematics textbook analysis and hence identified nineteen different expectations that one could find in a mathematics textbooks.

The expectation component of the Rivers Matrix (Rivers, 1990) identifies the presence of emphases and philosophies throughout a mathematics textbook. Both the emphasis and philosophy have a direct bearing on the student expectations put forward by textbooks. A combination of the work of TIMSS and Rivers (1990) provides the method for expectation analysis for this study. Nineteen performance expectations (outlined by the TIMSS Report 2002) are considered in conjunction with the final element of the River’s Matrix, (See Appendix P for expectation analysis grids).

Methods of Textbook Readability Analysis

Despite the direct connection between readability and problem solving (a highly topical element in many countries including Ireland) “research in reading and mathematics continues to attract little attention” (Thomas, 1997: 39). The term readability refers to a number of factors which influence the reader e.g. interest and motivation, legibility of the print, complexity of the words and sentences in relation to the ability of the reader. Interest and motivation are especially significant to a textbook.

The following readability formulae were applied in this research study:
7.2. Key Findings for Phase 1

1. Flesch Reading Ease,
2. Flesch - Kincaid Grade level,
3. Lexile Measure\(^2\).

However, these readability tests are designed to analyse English-language paragraphs. Much mathematics research, which involves readability measurement, uses the above tests as a basis of comparison of readability levels but none of these readability tests can effectively and accurately measure the actual readability of a mathematical text. This consideration influenced the author in her preparation and planning to find suitable measures for readability of mathematical texts - Language Analysis.

Methods of Textbook Language Analysis

Students should be able to communicate mathematics, both ‘verbally and in written form’ (NCTM, 1989). Mulryan (1984) cited that Irish primary mathematics textbooks have an excessive vocabulary load, variability of word meaning, insufficient repetition of mathematical terms and inadequate vocabulary control. In conjunction with this how to use mathematical language or how to read a mathematics textbook is not something that is taught in Irish mathematics education, yet the significance of the language of mathematics to learning is acknowledged. Language analysis and its significance has been widely researched for a number of years and has been well represented in mathematics education research from the early 1990’s, with, for example, the work of Halliday (1973), Skemp (1982), Van Dormolen (1986), Pimm (1987), Noonan (1990), Chapman (1993), Dowling (1996), Mikk (2000), Morgan (2004) and Orton (2004), with Mikk and Morgan focusing particularly on the role of language in mathematics texts or textbooks. For the purpose of this study the author drew primarily upon the work of Halliday (1973) and Morgan (2004).

\(^2\)The lexile framework for reading was developed by psychometricians at MetaMetrics,Inc., which is a private educational measurement company based in the USA.
7.3. Key Findings for Phase 2

Halliday’s research provides the basis for much language analysis in many different subject areas, focusing on the functional aspects of language. He outlines this functional aspect as the way in which language is used, the purpose that it serves and the way in which a reader can achieve these purposes. Halliday’s functional grammar analysis is based on three elements:

- The *Ideational Function* - identifies the way in which language is used.
- The *Interpersonal Function* - identifies the relationships presented in the text.
- The *Textual Function* - identifies the overall theme put forward by a text.

Halliday’s intended purpose for his functional grammar analysis is for language in general and the language of mathematics never featured as a stand alone unit within his work. However, in 2004 Morgan applied this functional grammar analysis to mathematics research (Morgan, 2004). The author analysed textbook language using Morgan’s adaptation of Halliday’s framework of functional grammar.

7.3 Key Findings for Phase 2

Phase two is concerned with all of the detail of the textbook analysis, the findings of which are presented and commented on in chapter 5. The discussion of phase two is guided by its associated research questions.

7.3.1 Key Findings for Research Question 1

“Are the current Junior Cycle mathematics textbooks an effective resource for teaching and learning?”

Much research into the teaching of mathematics has noted that the only measure of effective teaching is whether or not learning has occurred (Clark, 1993; Sullivan, 2001). In order to effectively comment on teaching and learning the author will
be discussing key findings which have emerged from the literature as significant to student learning. Four mathematics textbooks series were initially included in this research study, however almost one year into the research one of these textbooks was taken off the market and for this reason the author felt that it could no longer play a role in this study. For this reason the discussions will focus on the three textbook series which were involved in the entire research study, TBS A, B and C.

While in general some positives have emerged from the textbook analysis, the majority of the findings appear to have a negative impact on student learning. The presence of motivational and comprehension features are very low across all the textbooks series. Motivation and its significance for learning have been widely researched for over 30 years. According to researchers such as Yunus and Wan-Ali (2009) and Pintrich and Schunk (2002), students need motivation in order to do well in school. The motivational aspects of mathematics textbooks are vital considering the amount of direct contact that students have with their mathematics textbook. Fajemidagba (1998) found that competent mathematics teachers provide a history, an explanation of why and where within mathematics a topic is applicable and apply simple and coherent counting and measuring practises initially, such features of good teaching should also be evident in good textbooks.

TBS C presents the highest rate of motivational features. However, as you can see from Table 7.1, none of these textbook series give much consideration to the identified motivational aspects. While TBS C undoubtedly devotes more textbook space to motivational features, all these findings are alarmingly low. According to Robinson (1981) the presence of background knowledge and additional information leads to increased understanding, none of which is evident across TBS A and B.

The use of colour across all six textbooks is plentiful but is not consistent and tends to be only in dull shades. According to Noonan (1990) student comprehension can be directly connected to the use of illustrations and this idea is echoed by Dowling
7.3. Key Findings for Phase 2

Table 7.1: Textbook Content Analysis - Motivation

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Historical Notes</th>
<th>Biographies</th>
<th>Career Information</th>
<th>Photos</th>
<th>Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>498</td>
</tr>
<tr>
<td>TBS A2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>266</td>
</tr>
<tr>
<td>TBS B1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>536</td>
</tr>
<tr>
<td>TBS B2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>309</td>
</tr>
<tr>
<td>TBS C1</td>
<td>12</td>
<td>22</td>
<td>0</td>
<td>22</td>
<td>542</td>
</tr>
<tr>
<td>TBS C2</td>
<td>14</td>
<td>12</td>
<td>0</td>
<td>14</td>
<td>283</td>
</tr>
<tr>
<td>TBS D1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>701</td>
</tr>
<tr>
<td>TBS D2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>254</td>
</tr>
</tbody>
</table>

(1996) who tells how useful diagrams and pictures can be in captivating students’ attention and helping them move from horizontal to vertical learning. While colour consistency is neglected in each textbook the use of graphics is prevalent. Graphics are plentiful across all the textbook series which is evident from Figure 7.1. However, their role in problem solving is limited, with only TBS A1, C1 and C2 demonstrating some connections between graphics and problem solving.

Figure 7.1: Textbook Content Analysis - Comprehension (Graphics)

Graphics, the use of graphics and the type of graphics have been the subjects of much research (Rivers, 1990; Dowling, 1996; Noonan, 1990; Mikk, 2000). Polya
7.3. Key Findings for Phase 2

(1945); Schoenfeld (1992) and the O.E.C.D. (2003) all discuss the role of graphics in problem solving, while Rivers (1990); Newall (1990); Valverde et al. (2002) all promote the use of graphics for student learning in general. Graphics play a vital role in the learning of mathematics. However, as noted by Dowling (1996) unnecessary graphics which are not directly related to the text will actually hinder learning and only serve as a distraction. While all the graphics present in each textbook series are in some way connected to the text very few of these graphics are used to assist problem solving. Further analysis of the textbooks reveals that this low level of graphics assisting real life problems may in fact be a better reflection of the problem types present in the textbooks as opposed to use of graphics.

![Figure 7.2: Review of Problem Type](image)

All textbooks demonstrated a very high presence of exercises, the average number of exercises per textbook is 2,391. Further analysis reveals that approximately 20% of these exercises can be classified as problems. However, a detailed analysis on the problem types discovered that the majority of these problems are routine problems (Figure 7.2). Teachers and students may think they are engaging in problem solving when in fact they are not. Routine problems are described by Diaz and Poblete (2000) as ‘dressed up exercises’ and while they do have an educational purpose they are not a replacement for problem solving.
7.3. Key Findings for Phase 2

Greer and Mulhern (1992) noted that one of the main changes to mathematics curriculums and classrooms worldwide was the introduction of ICT and the internet. This reflects one of the key teaching practices put forward by Hirsch (1992), 'Technology Supported Learning'. The use of ICT and software packages such as Geogebra allow students to be come engaged in their mathematics (Mackie and Scott, 1988) yet few mathematics teachers take this opportunity to maximise learning outcomes. Duffy and O’Donoghue (1992) suggest that the novelty, (for Irish Mathematics teachers) of computer based learning has worn off. Technology removes the mundane and abstract nature of many algebraic manipulations while also permitting instruction to become more diversified and individualised, however the presence of technical cues is omitted from each of the textbooks series which have no reference to or recommend uses for ICT (Table 7.2).

Table 7.2: Textbook Content Analysis - Technical Aids

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Calculator Reference</th>
<th>Computer Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>TBS A2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>TBS B1</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>TBS B2</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>TBS C1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>TBS C2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>TBS D1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TBS D2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The block type of a textbook is defined by a number of characteristics such as the use of narration, definitions, graphics, exercises and examples. Narration is distinguished by two categories, narration and instructional narration. The levels of narration throughout each of the Irish Junior Cycle mathematics textbooks is high, with low instructional narration.
Figure 7.3: Textbook Structure Analysis - Narration and Instructional Narration

The basic expectation of a textbook is that it will be read, therefore one would assume that all textbooks should encourage the reader to read. However, this does not appear to be the case with two of these textbook series, TBS A and B. While the levels of narration appear to be high across all textbooks further analysis shows that only one of the textbook series, TBS C, has a narrative focus. A narrative focus will encourage students to read the textbook, “students have many sources of information available, if their textbooks are dull, they are unwilling to study them” (Mikk, 2000: 17).

According to Schoenfeld (1992) a good mathematics knowledge base consists of informal and intuitive knowledge, facts, definitions, algorithmic procedures, routine procedures, competencies and knowledge of rules. This suggests that students need to be aware of definitions. Wu (cited inMcCory, 2006) describes definitions as the building blocks of mathematics. Recent research reinforces the author’s belief that definitions are important but in order to be effective need to stand out from the text in a consistent manner (Kirkness and Neill, 2009). However, TBS C is the
7.3. Key Findings for Phase 2

only textbook series which gives such considerations to definitions (Figure 7.4).

![Figure 7.4: Textbook Structure Analysis - Definitions]

Findings from the expectation analysis uncovered that all of these textbook series place a large emphasis on performing routine procedures (Figures 5.16 to 5.19 in section 5.5). This relates to the poor teaching and poor performance reported from Ireland (O.E.C.D., 2006; NCCA, 2006). According to a number of NCCA reports Irish mathematics teachers focus entirely on routine procedures and place little or no value on the concepts of understanding, communicating, validating and justifying mathematics. This is reflected in the Irish textbooks, the authors of which are all mathematics teachers, by the dominance of the expectation to perform routine procedures and failure to expect students to predict, verify, conjecture, justify, prove, critique, describe and discuss their mathematics.

The failure of the current textbooks to facilitate and help student learning is reinforced by the findings from the language analysis. Key features of language such as the effective use of definitions (as mentioned above), inclusion of a glossary and avoiding passive sentences and rhetorical questions are neglected by these textbook series. The use of passive and rhetorical sentences are particularly harmful to
under confident students. Passive sentences remove the need for human agency and can enhance the abstract nature of mathematics as opposed to the applicable nature of mathematics. TBS A and B demonstrate a number of counterproductive characteristics such as the inclusion of a high rate of passive sentences in the material process. The material process, according to Morgan (2004), focuses on the doing of mathematics; however passive sentences remove the active element of the process. Both of these textbook series also overuse the pronoun ‘we’, which is a trait of academic text. Fortanet (2004) states that overuse of the pronoun ‘we’ is a rhetorical indicator and many researchers such as Gerofsky (1999); Svinicki and Dixon (1987) have highlighted the confusion that rhetorical questions can cause, particularly in school texts. Students are unsure how to read and deal with rhetorical questions and indicators and often choose simply to ignore them. Such features in a textbook do not encourage students to engage with the mathematics and as previously mentioned can be particularly harmful to the students with the greatest need for good textbooks.

Table 7.3: Summary of Language Analysis Findings; Passive Sentences and The Pronoun ‘we’

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Passive Sentences</th>
<th>Pronoun ‘We’</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>68</td>
<td>63 out of 80</td>
</tr>
<tr>
<td>TBS A2</td>
<td>17</td>
<td>23 out of 37</td>
</tr>
<tr>
<td>TBS B1</td>
<td>73</td>
<td>17 out of 46</td>
</tr>
<tr>
<td>TBS B2</td>
<td>44</td>
<td>21 out of 48</td>
</tr>
<tr>
<td>TBS C1</td>
<td>34</td>
<td>21 out of 63</td>
</tr>
<tr>
<td>TBS C2</td>
<td>20</td>
<td>9 out of 24</td>
</tr>
</tbody>
</table>

Studies carried out by Pinto (1998) and Pinto and Tall (1999) indicate that some students learn mathematics by extracting meaning (beginning with the formal definition and constructing properties by logical deduction) and some students learn by giving meaning, refining and reconstructing their existing imagery until it is in a form that can be used to construct a formal theory. According to Pinto and Tall (2002) learners who learn by giving meaning engage in a thinking process...
7.3. Key Findings for Phase 2

which resembles that of mathematicians who use broad problem-solving strategies. This would suggest that an increase in the presence of objects throughout the mental process would enable more students to learn effectively from the textbook. However the use of objects throughout each textbook is limited (as noted early with the use of graphics). The language analysis uncovered the failure of these textbooks to use objects to relate and represent mathematics. In fact the most common ‘object’ found in these textbooks is the “equal to” sign. Researchers such as Van Dormolen (1986) have noted how the relational aspects of mathematics are much more beneficial to student learning than the procedural, however, the high use of the equal sign across all textbooks is reflective of a procedural approach to mathematics.

Table 7.4: Summary of Language Analysis Findings; Objects

<table>
<thead>
<tr>
<th>Textbooks</th>
<th>Basic or Derived</th>
<th>Relational</th>
<th>Representational</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>27 (154)*</td>
<td>11 (9)</td>
<td>25 (0)</td>
</tr>
<tr>
<td>TBS A2</td>
<td>11 (127)</td>
<td>4 (0)</td>
<td>12 (13)</td>
</tr>
<tr>
<td>TBS B1</td>
<td>54 (135)</td>
<td>13 (18)</td>
<td>15 (5)</td>
</tr>
<tr>
<td>TBS B2</td>
<td>30 (159)</td>
<td>21 (40)</td>
<td>24 (3)</td>
</tr>
<tr>
<td>TBS C1</td>
<td>8 (46)</td>
<td>0 (17)</td>
<td>33 (1)</td>
</tr>
<tr>
<td>TBS C2</td>
<td>2 (40)</td>
<td>5 (8)</td>
<td>16 (0)</td>
</tr>
</tbody>
</table>

*The data in brackets represents the use of the equal sign within the text selection.

Finally, students need to read the mathematics text in order to learn from it, therefore every effort should be made by textbook authors to encourage students to read the textbook. The intention of a school mathematics text is to support teaching and learning. Morgan (2004) outlines how a focus on assessment can often force a need for academic features to be present in school texts, however the student themselves may need a focus of familiarity in order to construct and create their own knowledge, a feature far removed from academic texts. The lack of informal sentences across TBS A and B is reflective of such pressures. The limited use of pronouns in TBS A and B is also a characteristic of academic text.
7.3. Key Findings for Phase 2

and demonstrates a lack of familiarity for its intended reader. Shuard and Rothery (1984) conclude that a school text should not be impersonal, in fact everywhere a text provides exercises or problems for a reader they are involving the reader. Also, the lack of a glossary of terms, highlighted definitions and opportunities to discuss and communicate mathematics inhibits a reader’s flow of text. Students need to understand the language used in the text but they also need to know and use the correct mathematical terms. In a study carried out by Marks et al. (1974), they replaced 15% of the words in a text with more commonly used words and presented the text to 600, 6th grade students. They found that comprehension was increased from 47% to 73%. This reinforces the need for a textbook to promote communication because students need to use mathematical vocabulary in order to understand and familiarise themselves with it.

Morgan (2004) speaks about three themes that can emerge, particularly from a school mathematics text; deductive reasoning, logical reasoning and recall and recount, with recall and recount being the weakest of these emphases from a learning perspective. The intention of a school mathematics text is to support teaching and learning therefore texts of this nature should exhibit a combination of all of the above characteristics. For example an effective school mathematics text will not focus on recall and recount as recall and recount does not serve as an effective teaching methodology. However each of the textbooks series A, B and C demonstrate low levels of usage of the conjunctions and verbs which Morgan connects with logical reasoning (Table 7.5).
### 7.3. Key Findings for Phase 2

#### Table 7.5: Summary of Language Analysis Findings; Textual Function

<table>
<thead>
<tr>
<th>Textbooks</th>
<th>Reasoning Conjunctions</th>
<th>Logical Reasoning</th>
<th>Reasoning + Imperative</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>5</td>
<td>23</td>
<td>3</td>
</tr>
<tr>
<td>TBS A2</td>
<td>4</td>
<td>39</td>
<td>2</td>
</tr>
<tr>
<td>TBS B1</td>
<td>1</td>
<td>44</td>
<td>4</td>
</tr>
<tr>
<td>TBS B2</td>
<td>3</td>
<td>23</td>
<td>3</td>
</tr>
<tr>
<td>TBS C1</td>
<td>8</td>
<td>24</td>
<td>0</td>
</tr>
<tr>
<td>TBS C2</td>
<td>5</td>
<td>22</td>
<td>1</td>
</tr>
</tbody>
</table>

In conclusion it is fair to say that there is room for much improvement in the current Irish Junior Cycle mathematics textbooks. Textbooks are intended for students and so should be written for students. The focus of a school mathematics textbook needs to be centered around student learning and they should include all aspects of research which serve to enhance conceptual understanding.

#### 7.3.2 Key Findings for Research Question 2

“What is the significance and impact of language considerations as interpreted in textbooks for both the teaching and learning of mathematics?”

Studies such as that carried out by Noonan (1990) have identified an increased pressure on students to be able to read and understand mathematics language. Furthermore, many studies such as Marks et al. (1974), Glynn and Britton (1986) and Bell (1970) highlight the significance of the school mathematics register to student learning. It is estimated that students have to learn approximately one hundred new words per school year (Orton, 2004). However, none of the textbooks involved in this study include a glossary or dictionary of terms. Definitions were kept to a minimum and in many cases were embedded through the text and not isolated or highlighted.
7.3. *Key Findings for Phase 2*

Another study carried out by Klare (1963) found that suitable readability levels proved to increase effectiveness of text in over 68% of the cases they investigated. The readability levels of each of these textbooks were compared using ordinary English readability tests (Table 7.6). While these figures do not give an actual readability measure (due to the complexity of mathematical English, see section 5.5.4 and 2.5) they do offer a comparison between textbooks.

Table 7.6: Summary of Language Analysis Findings; Readability

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Passive Sentences</th>
<th>Flesch Reading Ease</th>
<th>Flesch-Kincaid Grade Level</th>
<th>Lexile Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>27%</td>
<td>74.9</td>
<td>6.1</td>
<td>1070L</td>
</tr>
<tr>
<td>TBS A2</td>
<td>12%</td>
<td>75.9</td>
<td>5.5</td>
<td>1740L</td>
</tr>
<tr>
<td>TBS B1</td>
<td>20%</td>
<td>67.6</td>
<td>6.6</td>
<td>1130L</td>
</tr>
<tr>
<td>TBS B2</td>
<td>15%</td>
<td>66.6</td>
<td>7.0</td>
<td>1380L</td>
</tr>
<tr>
<td>TBS C1</td>
<td>15%</td>
<td>69.2</td>
<td>6.8</td>
<td>960L</td>
</tr>
<tr>
<td>TBS C2</td>
<td>16%</td>
<td>57.4</td>
<td>9.2</td>
<td>1110L</td>
</tr>
</tbody>
</table>

The textbooks A1, B1 and C1 are intended for first and second year mathematics students (grade 7 and grade 8) while A2, B2 and C2 are aimed at second and third year students (grades 8 and 9). A score from between 60 to 70 in the Flesch Reading Ease would indicate that a textbook is suitable for readers in the 8th and 9th grades (second to third year students). The Flesch Reading Ease and Flesch-Kincaid Grade level both indicate the TBS A1 is in fact more difficult to read than TBS A2. The Lexile Measure maps an individual’s reading levels to a text, the average 1st year student would require a textbook with a readability of somewhere between 950L and 1100L and the average second year student would need a textbook with a readability between 1000L and 1150L (see section 5.5.4). According to the Lexile Measure the majority of the readability levels are too high with the exception of TBS A1, C1 and C2 which fall into the correct category. As stated in the review of literature these results do not give an actual readability for the text (as it is a mathematical text and combines ordinary English with mathematical English) but does provide a comparison.
7.3. Key Findings for Phase 2

With research such as that carried out by Schoenfeld (1992) and Marks et al. (1974) highlighting the significance of reading mathematics for learning and engaging in mathematics it is essential that mathematics textbooks are readable and encourage reading. School mathematical texts should be written for the purpose of promoting student learning so it should go without saying that the focus of textbooks should be on the pedagogical aspects of mathematics. Stray (1994) notes that while the purpose of a textbook is to provide a pedagogical approach to mathematics, the pedagogy is often marginal to the intentions of the textbook publishers as priorities of a publishing company differ greatly from the priorities of an educator. However despite such findings this research highlights that only one textbook series has a narrative focus, TBS C. This inevitably means that only the students using TBS C (which is not the most commonly found Junior Cycle mathematics textbook in Ireland) are presented with a textbook which is encouraging them to read mathematics.

According to Aiken (1972), students with a higher readability level tend to do better than others. He suggests that the most common reason for this is not that the students with higher readability levels are any better at mathematics but that they can read and understand what is being asked of them better. Students need to be able to read a text in order to learn from it. Informal sentences can also help to increase students’ motivation to read by presenting information in a relaxed comfortable way that students are familiar with. In conjunction with this excessive uses of specialist vocabulary and symbolism hinders a student’s comprehension of the text. While specialist words and symbols are necessary for the language of mathematics excessive use of these in a textbook is unnecessary for the reader. Pimm (1987) also reinforces Morgan’s beliefs about the use of imperatives. Morgan (1995) states that imperatives, which are directly associated with the reader, support a claim from the author that he/she is a member of the mathematical community.
7.3. Key Findings for Phase 2

In his work, Rotman (2006) distinguishes between inclusive and exclusive imperatives. He suggests that inclusive imperatives ask the reader to be a thinker while exclusive imperatives ask the reader to a scribbler. Of the imperatives in the textbooks involved in this study no more than 11% of the imperatives in each textbook can be considered inclusive imperatives (Table 7.7).

Table 7.7: Summary of Language Analysis Findings; Interpersonal Function Findings

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Informal Sentences</th>
<th>Specialist Words</th>
<th>Symbols Symbols</th>
<th>Imperatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>7</td>
<td>208</td>
<td>117</td>
<td>37 (10.8%)</td>
</tr>
<tr>
<td>TBS A2</td>
<td>3</td>
<td>125</td>
<td>216</td>
<td>31 (3.23%)</td>
</tr>
<tr>
<td>TBS B1</td>
<td>2</td>
<td>317</td>
<td>197</td>
<td>42 (9.5%)</td>
</tr>
<tr>
<td>TBS B2</td>
<td>2</td>
<td>312</td>
<td>309</td>
<td>58 (10.3%)</td>
</tr>
<tr>
<td>TBS C1</td>
<td>63</td>
<td>135</td>
<td>115</td>
<td>6</td>
</tr>
<tr>
<td>TBS C2</td>
<td>9</td>
<td>61</td>
<td>23</td>
<td>6</td>
</tr>
</tbody>
</table>

* The data in brackets represents the percentage of imperatives which are inclusive imperatives.

Balas (2000) suggests that students can become mathematically literate by practicing reading mathematics, as he suggests that reading mathematics is identical to reading a new language. Noonan (1990) and Pimm (1997) both reinforce Balas’ ideas when they both identify the difficulty that students have reading and understanding new symbols and words in the correct context. Similarly the work of Lim and Clements (2002) identifies that students have difficulty translating algebraic problems into language that they understood. The high frequencies of specialist words and symbols evident in TBS A and B (Table 7.7) will hinder a student’s flow of reading. These findings combined with the lack of dictionary or glossary of terms discourage students from reading the text as once they encounter a word or symbol that they do not know they have no point of reference within the textbook.
7.3. Key Findings for Phase 2

As identified in section 2.5.4 much of the linguistic research is based on Halliday’s (1973) ideas of the functions of language. Both Morgan and Halliday suggest that all mathematical text should consider the ideational, interpersonal and textual functions of language. In conjunction with this Noonan (1990) provides six factors which can cause difficulty for students’ reading a mathematics text; words, syntax, symbols, rhetorical questions and page layout. For example, long sentences are more difficult to read than shorter ones. In addition, the order of words is significant in that sentences which use the passive, sub-ordinate clauses and comparatives are much more complex to read than those extended by adjectives or phases. Unlike words, symbols are not based on sounds, therefore when learning symbols students need to link the symbol to the words which correspond to it. However, symbols may often have many interpretations in spoken language, therefore students must read a symbol according to its appropriate meaning and context. For example ‘+’ can be read as add, plus, more than or together with. Rhetorical questions can cause difficulties for students as according to Noonan (1990) students are unsure what they are to do with them. Finally the page layout can cause reading difficulties in a number of ways. If a page is too cluttered, if the text is not supported by diagrams where diagrams can easily be of benefit and if the flow of the text is difficult to follow then a student’s ability to read the text effectively is reduced.

This research question serves to identify the main features of language which will impact on a students’ ability to read a mathematical text. It also highlights the importance of reading text to a student’s mathematical learning and the impact this has for textbook research.
7.3.3 Key Findings for Research Question 3

“Do the current textbooks address the key areas of concern in mathematics education such as language deficiencies and poor problem solving skills?”

The significance of problem solving for the teaching and learning of mathematics is widely accepted. Problem solving became the theme of mathematics education in the 1980’s both in the USA and the UK. The NCTM (1980: 1) felt so strongly about the importance of problem solving in mathematics that they stated that it “must be the focus of school mathematics”. As identified in section 2.4.2 problem solving has been the source of many discussions since this with researchers such as Schoenfeld (1992) identifying that one of the key failures with regard to problem solving is its absence from the school curriculum. Schoenfeld (1992) acknowledges an effort by the school curricula to include problem solving however he notes that it is problems that are present in the mathematics classrooms not problem solving.

The current Junior Cycle mathematics textbooks in Ireland, demonstrate a similar problem as that noted by Schoenfeld (1992) - the absence of problem solving. From Table 7.8 it is evident that despite the large presence of exercises in these textbooks, problem solving only plays a minor role.

Table 7.8: Textbook Structure Analysis - Exercises

<table>
<thead>
<tr>
<th>Textbook</th>
<th>% of which are Real Life Problems:</th>
<th>Textbook Series:</th>
<th>% of which are Real Life Problems:</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>20.84%</td>
<td>TBS A1 &amp; 2</td>
<td>19.86%</td>
</tr>
<tr>
<td>TBS A2</td>
<td>18.27%</td>
<td>TBS B1 &amp; 2</td>
<td>14.60%</td>
</tr>
<tr>
<td>TBS B1</td>
<td>15.07%</td>
<td>TBS C1 &amp; 2</td>
<td>22.74%</td>
</tr>
<tr>
<td>TBS B2</td>
<td>13.78%</td>
<td>TBS D1 &amp; 2</td>
<td>16.42%</td>
</tr>
<tr>
<td>TBS C1</td>
<td>23.58%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TBS C2</td>
<td>21.29%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TBS D1</td>
<td>17.58%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TBS D2</td>
<td>13.93%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7.3. Key Findings for Phase 2

Figure 7.5 highlights the presence of routine problems (routine problems can be described as dressed-up exercises) and non-routine problems (non-routine problems are those which cannot be solved by following a routine procedure) as defined by Diaz and Poblete (2000). This indicates that the types of problems present across all the textbook series are primarily routine problems, while the students may think they are engaging in problem solving they are in fact working mostly with ‘dressed up’ exercises. TBS A1 includes the most non-routine problems, however the numbers are extremely low.

As outlined in chapter two, routine and non-routine problems can be broken down into real, fantasy, realistic or pure mathematical problems based on the work of Diaz and Poblete (2000). Figure 7.6 shows the break down of the routine and non-routine problems into their subcategories. However, because routine problems have a much higher frequency than non-routine problems it is necessary to look at the type of routine problems being presented to the students. While real, pure and fantasy problems can be useful in learning, the bulk of research (from as far back as Freudenthal (1973)) indicates that realistic problems are the most influential and beneficial to students. Here we can see that while TBS A1 has the greatest number of non-routine problems it has the second least amount of realistic problems present. TBS C1 has by far the greatest number of realistic problems.

Figure 7.5: Review of Problem Type
7.3. Key Findings for Phase 2

Across all of the textbook series involved in this research study, problem solving fails to play a vital role in the textbooks and one could go as far as to say it is in fact neglected. The majority of problems encountered by the students using these textbooks are what is described as ‘dressed up’ exercises and while they have some educational value they will not encourage students to think critically about mathematics or enhance students’ problem solving abilities. None of the textbooks include any method or guidance on how to approach or solve problems, in fact how to deal with problem solving is also neglected. Shuard and Rothery (1984) note that if a textbook fails to use exercises to lead to new knowledge and development towards the solving of mathematical problems then the textbooks is simply demonstrating a procedural emphasis.

There are many key language considerations which have emerged from mathematics education research. The first and most obvious of these is that students must be able to read their textbook. They are “expected to be able to learn mathematics by reading and by carrying out activities which are described in writing” (Noo-nan, 1990: 57). Studies carried out by Marks et al. (1974); Glynn and Britton (1986); Klare (1963), have highlighted the significance of careful planning and consideration about the vocabulary used in mathematics textbooks. According
to Bell (1970) students are required to learn approximately one hundred new words each school year. Mulryan (1984) also noted that Irish primary school students are subjected to a heavy load of new mathematics vocabulary. The mathematics textbooks could facilitate learning by encouraging the communication of mathematics using the mathematics specific terms and by the inclusion of a glossary of terms or dictionary, none of which is present in the current Junior Cycle mathematics textbooks.

Frequency of words, sentence length, study aims, emphasising headings, questions for actualising and prior knowledge all impact on student learning (Glynn and Britton, 1986). The amount of time students spend reading the text and the mental effort it takes them to do so is also significant. However, only one of these textbook series, TBS C, has a narrative focus. Findings from the textbook language analysis suggest that the some of the textbook language features will actually hinder student learning. The over use of the ‘we’ pronoun and the inclusion of passive sentences all serve to confuse the students and block the flow of reading. The low presence of mental processes across all textbooks also suggests that the intended reader (the students) are not encouraged to think about mathematics. This is reinforced by the use of exclusive and inclusive imperatives. Inclusive imperatives ask the reader to be a thinker and exclusive imperatives ask the reader to be a scribbler. TBS A and B present no more than 11% inclusive imperatives in their textbooks (TBS A1 10.8% of imperatives are inclusive, TBS A2 3.23%, TBS B1 9.5% and TBS B2 10.3%) which serves to discourage students from actually thinking and reading about the mathematics.

The use of graphics within the current Junior Cycle mathematics textbooks does not serve to enhance the flow of reading as graphics are rarely used to relate or represent mathematics. The high rates of specialist vocabulary and symbolism evident in TBS A and B also hinder a student’s reading of these mathematics
7.3. Key Findings for Phase 2

textbooks. Without the presence of a glossary of terms or dictionary, and with so few highlighted definitions present in these textbooks, student’s reading will be blocked each time they encounter such words and symbols. Shuard and Rothery (1984) and Morgan (2004) both agree that a more limited use of symbolism and technical language and a substantial graphical element is more beneficial for school mathematics textbooks.

The intention of a mathematics textbook is to support teaching and learning and hence this should be deducible from the overall theme of the textbook (Shuard and Rothery, 1984). The limited use of deductive and logical reasoning conjunctions and verbs is indicative of an emphasis on recall and recount in the current Junior Cycle mathematics textbooks with is reflective of the author placing an emphasis on the procedural aspect of mathematics. Such finders are in direct contrast to the work of Shuard and Rothery (1984).

The textbooks involved in this study vary slightly from each other, however, the model chapter created and implemented in phase three of this research study differs greatly in terms of paragraphs, words per sentence and the presence of passive sentences. In the passive voice, the subject of the sentence is neither a do-er or a be-er, passive sentences do not encourage action and should be avoided where possible in favour of active sentences (Quirk and Greenbaum, 1993). The following tables provide a brief comparison of the findings (outlined in chapter 5) of word counts and averages for the current Junior Cycle Textbooks and the model chapter developed in phase three of this research.
### 7.3. Key Findings for Phase 2

**Table 7.9: Textbook Readability Analysis - Counts**

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Words</th>
<th>Characters</th>
<th>Paragraphs</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>3146</td>
<td>13276</td>
<td>282</td>
</tr>
<tr>
<td>TBS A2</td>
<td>2556</td>
<td>10072</td>
<td>194</td>
</tr>
<tr>
<td>TBS B1</td>
<td>5524</td>
<td>23664</td>
<td>370</td>
</tr>
<tr>
<td>TBS B2</td>
<td>4799</td>
<td>21390</td>
<td>568</td>
</tr>
<tr>
<td>TBS C1</td>
<td>3290</td>
<td>14567</td>
<td>187</td>
</tr>
<tr>
<td>TBS C2</td>
<td>2534</td>
<td>11831</td>
<td>124</td>
</tr>
<tr>
<td>Model Chapter - Fractions</td>
<td>3589</td>
<td>15407</td>
<td>980</td>
</tr>
</tbody>
</table>

**Table 7.10: Textbook Readability Analysis - Averages**

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Sentences per paragraph</th>
<th>Words per sentence</th>
<th>Characters per word</th>
</tr>
</thead>
<tbody>
<tr>
<td>TBS A1</td>
<td>1.5</td>
<td>13.5</td>
<td>3.9</td>
</tr>
<tr>
<td>TBS A2</td>
<td>2.1</td>
<td>11.9</td>
<td>3.7</td>
</tr>
<tr>
<td>TBS B1</td>
<td>2.4</td>
<td>11.5</td>
<td>4.1</td>
</tr>
<tr>
<td>TBS B2</td>
<td>1.5</td>
<td>12.7</td>
<td>4.2</td>
</tr>
<tr>
<td>TBS C1</td>
<td>2.3</td>
<td>13.1</td>
<td>4.2</td>
</tr>
<tr>
<td>TBS C2</td>
<td>2.7</td>
<td>16.4</td>
<td>4.5</td>
</tr>
<tr>
<td>Model Chapter - Fractions</td>
<td>1.4</td>
<td>4.8</td>
<td>4.3</td>
</tr>
</tbody>
</table>

Across all of the textbook series research findings and outcomes with regard language considerations appear to be neglected. The students encounter many mathematical words and symbols without any dictionary or glossary for these terms. They are faced with long sentences, which combined with the difficult terminology makes reading the textbooks extremely difficult. Only one of the textbook series (TBS C) considers the importance of narrative flow, in TBS A and B students are not encouraged to actually read their textbook. Campbell et al. (2001) suggest that students should be encouraged to pre-read, read during mathematics class, practice mathematics and review their mathematics. However, there is no evidence in TBS A and B to suggest that reading the textbook is important, with limited evidence in TBS C.
7.4 Key Findings for Phase 3

7.4.1 Key Findings for Research Question 1

“Can improving the textbooks directly improve students’ understanding’ of mathematics and their conceptual development?”

Phase three of this research study involved the creation of a model chapter and its implementation into three secondary schools. The intention of the model chapter is to highlight the impact that changes to the mathematics textbook have on students’ conceptual understanding. The model chapter was created (on the topic of fractions) based on the review of literature and the research findings presented in chapter 5. This model chapter was piloted and subjected to an expert panel review before the final draft was completed. In conjunction with the model chapter the author also established a two-tier diagnostic test instrument in order to ascertain if changes in students’ conceptual development occurred due to the model chapter. As a side issue the author also applied RMARS, a mathematics anxiety inventory devised by Plake and Parker (1982). The intentions of the RMARS questionnaire were supplementary to those of the two-tier diagnostic test instrument.

Table 7.11: Mean Conceptual Differences for each School

<table>
<thead>
<tr>
<th>School</th>
<th>Mean difference for concept element</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>School 1</td>
<td>1.79</td>
<td>2.48</td>
</tr>
<tr>
<td>School 2</td>
<td>1.63</td>
<td>2.58</td>
</tr>
<tr>
<td>School 3</td>
<td>1.74</td>
<td>1.86</td>
</tr>
</tbody>
</table>

The above table (Table 7.11) outlines the overall difference between pre and post tests for each school. At first glance it appears that all students involved in this study experienced a positive increase in their conceptual development (Table 7.11).
7.4. Key Findings for Phase 3

This is expected as all students were subjected to teaching, the only factor that was controlled by this study was the textbook. The original textbook was replaced by the model chapter in all of the test groups but the control groups used their original textbook as normal. The two-tier diagnostic test was designed to assess whether students actually understand the answer they provide to a mathematics question. The first tier of the diagnostic question comprises a number of answer options for each question and the second tier consists of possible explanations for the chosen answer (Appendix K). This two-tier diagnostic test was taken by the students immediately prior to and post working on the topic of fraction addition. However, a more detailed examination of the data within each school indicates that the students within the test groups in each school out performed the students within the control groups in terms of their increase in conceptual development.

Figure 7.7: Median Difference of Conceptual Element - School 1

Figure 7.7 provides a visual explanation of the change in the students’ conceptual development in School 1. From Figure 7.7 it is evident that each group...
7.4. *Key Findings for Phase 3*

demonstrated an increase in their conceptual understanding following the teaching period (the zero mark on the vertical axis indicates no change between pre and post testing). It is also evident that Test group 1 exhibited a much greater increase than the control group. A closer look at the data analysis (paired t-tests) identifies that all groups, both test and control, in School 1 exhibit a statistically significant increase between pre and post tests with regard to conceptual development.

Similar findings were exhibited in School 2. Figure 7.8 presents the median difference in conceptual understanding between pre and post tests. From Figure 7.8 it is not immediately obvious that test group 2 out performed the control group. However, data analysis (paired t-tests) identifies that both tests groups exhibited a statistically significant increase in conceptual deployment whereas the control group did not.

![Figure 7.8: Median Difference of Conceptual Element - School 2](image-url)
7.4. Key Findings for Phase 3

Finally in School 3, Figure 7.9 demonstrates the mean difference between pre and post tests in terms of conceptual development. The test groups both exhibit greater increases in conceptual development (both exhibit statistically significant changes) compared with the control group (not statistically significant).

Figure 7.9: Median Difference of Conceptual Element - School 3

Further statistical analysis of this data (with paired t-tests) outlines that the test groups in all three schools exhibit statistically significant increases in their conceptual development. In contrast to this only the control group in school 1 had statistically significant increases in conceptual development (class group which uses TBS C). The only factor that was controlled within each of the test groups was the textbook. These results indicate that changes to the mathematics textbook, which reflect research studies such as this one, can have a positive impact on student’s conceptual understanding.
Similarly the findings from the RMARS pre and post tests suggest students’ anxiety levels may be decreased when their conceptual understanding is increased. All groups which demonstrated a statistically significant increase in their conceptual understanding also indicated a statistically significant decrease in their anxiety levels. All test and control groups showed a positive change in anxiety levels. Within School 1 and School 2 the paired t-tests however indicate the anxiety level changes for within the control groups are not statistically significant (p = 3.08 for School 1 and p = 0.296). The control group in School 3 demonstrated a statistically significant change in anxiety levels (p= 0.037), while each test group in all three Schools also exhibited a positive statistically significant change in anxiety levels. This would suggest that effective changes to the mathematics textbook can increase a student’s conceptual understanding while also decreasing anxiety levels about mathematics.

7.5 Conclusion

In general some positive findings have emerged from the textbook analysis. However, the majority of the findings suggest that there is a need to improve the current Junior Cycle mathematics textbooks. Some of the key issues emerging from this study include the low presence of motivational factors, inconsistency in comprehension cues, omittance of technical cues, emphasis on routine problems, minimal narrational focus, emphasis on performing routine procedures, neglect of key language considerations such as glossary of terms, inconsistent style of definitions, passive sentence, over reliance on exclusive imperatives and a high frequency of specialist words and symbols.

There are six instances where students have difficulty reading mathematics texts; the words presented, the syntax used, unnecessary diagrams, excessive use of symbols, the presence of rhetorical questions and the layout of the page. The majority of these six instances were identified as concerns in this textbook analysis study,
7.5. Conclusion

with the exception of unnecessary diagrams. The final phase of this research study identified the key design features which would improve the quality of the Junior Cycle mathematics textbooks. These design features were central to the design of the model chapter which was then trialled on a small scale, to identify if improving the quality of the mathematics textbooks would impact on students’ conceptual development. In conclusion, all of the students using the model chapter exhibited an increase in their conceptual understanding of fraction addition following the teaching period.
Chapter 8

Thesis Contribution and Future Research

8.1 Introduction

The dominance of the mathematics textbook as a classroom resource is evident worldwide hence good quality mathematics textbooks are essential. This research study examined the quality of the mathematics textbooks currently in use at Junior secondary school level in Ireland. In the context of the introduction of the new national mathematics curriculum, Project Maths, this research study addresses a major national issue regarding the development of new mathematics textbooks. To date, there is no evidence based research of this type in Ireland. Therefore this research is both important and timely in an Irish context.

The key findings emerging from this thesis are presented and discussed in chapters 5, 6 and 7. Each research question identified significant insights into the quality of the current Junior Cycle mathematics textbooks and key textbooks features which can enhance this quality. In this chapter the author summarises the main findings of the research and acknowledges the contributions that this research has made in the field of mathematics education nationally and internationally. Possible directions for future research are also outlined.
8.2 Summary

The preliminary analysis conducted in phase one of this research study identified the need for an in-depth analysis of mathematics textbooks in Ireland. A review of the relevant literature identified suitable theoretical frameworks for mathematics textbook analysis. As TIMSS is the most established textbook analysis study to date the initial textbook analysis framework mirrored that of TIMSS. However, the TIMSS framework alone was not sufficient to complete an overall analysis which examined the textbook as a whole. Hence, a number of suitable frameworks and methods of data collection were merged with TIMSS to provide an overall theoretical framework for textbook analysis comprising four key elements; Content, Structure, Expectation and Language.

The author then proceeded with the textbook analysis using the frameworks and data collection instruments identified in phase one of the research.

This textbook analysis identified that the presence of motivational, comprehension and technical cues are minimal throughout the Irish Junior Cycle mathematics textbooks. This is reflected by the lack of real life figures and graphics, the dull and inconsistent use of colour and the neglect to reference technology of any sort. Students’ understanding and conceptual development appear to be secondary considerations after recall, recount and practice of mathematics routines. This is supported by the expectation findings which indicate that the most common expectations presented by all of the textbooks are representing and performing routine procedures. Also the ratio of exercises to problems is at least 5:1 and even greater for the ratio of mathematical examples to real life examples.

The Flesch Reading Ease and Flesch - Kincaid Grade level readability tests suggest that the readability levels of TBS A1 is higher than TBS A2, despite the second book be aimed at older students. The Lexile Measure suggests that the readability
8.2. Summary

Levels of all textbooks are too high with the exception of TBS A1, C1 and C2 with are suitable. This also highlights the inability of such tests to provide accurate information about the readability of mathematics textbooks.

There are six instances where students have difficulty reading mathematics texts; the words presented, the syntax used, unnecessary diagrams, excessive use of symbols, the presence of rhetorical questions and the layout of the page. The majority of these six instances were identified as concerns in this textbook analysis study, with the exception of unnecessary diagrams. The language analysis also identified that there is a lack of instructional narration across all textbooks, with definitions and key terms being lost in the main body of the textbooks. There is a high rate of passive sentences across all textbook series and an overuse of the pronoun, ‘we’. There is also a lack of consistency of the use of the ‘equal to’ sign which encourages student misconceptions. Confusion is adduced by the high rate of specialist words and symbolism and the lack of a glossary or dictionary. The majority of these findings (presented in chapter 5) highlight the need for improved textbooks which give greater consideration to who its intended reader is. Effective use of language and problem solving are instrumental in enhancing students’ conceptual development yet their presence and use within the current Junior Cycle mathematics textbooks in Ireland is limited. TBS C emerged as better than the other textbook series with regard to consideration for its intended reader and its use of language and problem solving. However, there is still much to be improved even in TBS C.

A number of key textbook design features emerged from phase two of this research study which impact on student learning. In her attempt to test these key design features the author created a model chapter. This model chapter was implemented into three secondary schools to determine if these key design features impacted positively on students’ conceptual development. Data collection was conducted by
means of a two-tier diagnostic test instrument and identified that all students using
the model chapter exhibited statistically significant increases in their conceptual
development.

8.3 Contribution

This research study highlights the value of textbook analysis in providing detailed
insights into the obstacles that mathematics textbooks are presenting to students
on a daily basis. The purpose of a mathematics textbook is to support teaching
and learning and they should in no way hinder students’ learning. While some
international research has been conducted, this localised study allows for further
more in-depth analysis of the Irish problem. The significant overall contributions
emerging from this research study include the following:

• This investigation puts a spotlight on the quality of the Junior Cycle math-
  ematics textbooks currently being used in Ireland using a theoretical lens
  comprising four key elements; Content, Structure, Expectation and Lan-
  guage. By means of the methodological tools identified in phase one of
  this research, data was obtained from the current Junior Cycle mathematics
  textbooks. These textbooks appear to give little consideration to who its
  intended reader may be and to key concerns in mathematics education such
  as the use of language and problem solving. The main focus of the current
  mathematics textbooks in Ireland is on ‘recall and recount’, emphasising
  only the procedural aspects of mathematics. Little attention is given to
  enhancing student understanding with excessive amounts of practice exercises
  present. This research study gives a better understanding of the advantages
  or shortcomings of the current Junior Cycle curriculum.

• Leading on from the above contribution, it is obvious that there is a need
to improve the quality of these textbooks. This research is timely and is
8.3. Contribution

particularly significant to an Irish context at present. In September 2010 a new mathematics curriculum initiative was implemented in Ireland. This new curriculum aims to improve the levels of understanding among students though the use of applications and problem solving centered mathematics. Such a curriculum change requires a vast change in classroom practices and teaching methodologies nationwide. The Cockroft Report identified that one of the reasons that the problem solving curriculum of the 1980’s didn’t take off in the UK was due to the textbooks not changing sufficiently to support both teachers and students (Cockroft, 1982). The research presented here highlights that the current textbooks are not adequate at efficiently supporting teachers and students in the old curriculum and hence will need major revisions in order to effectively support the new curriculum, Project Maths. Failure to make such revisions may contribute to a situation similar to that of the UK in the 1980’s, where sound curriculum initiatives failed due to poor textbooks and traditional methods of teaching. This research study provides an evidence-based starting point for writing the next generation of mathematics textbooks.

- In her research the author identified a need for a framework for mathematics textbook analysis that will provide an overall analysis of the textbook as a whole. Many mathematics textbook analysis studies isolate areas of the textbook such as language or structure for their analysis. However, one cannot draw conclusions about the textbook as a whole without analysing the textbook as a whole. Hence, the author devised a single framework for mathematics textbook analysis which encompasses all areas of the mathematics textbooks. This framework consists of four key elements; Content, Structure, Expectation and Language. Data can be collected for each element of the framework separately but interpreted in conjunction with the other three elements, thus allowing for concrete conclusions to made with regard
8.3. **Contribution**

to the textbook as a whole.

- This research also identified a number of key design features which will enhance the quality of any mathematics textbook. These key design features will allow textbooks to be created or modified on evidence based criteria that are applicable worldwide. The key design features should be embedded throughout a mathematics textbook in order to encourage students to engage with and read their mathematics textbook. Such engagement and more importantly reading of the mathematics textbook allows students to gain command of the mathematical language and will encourage them to actively participate in their own learning, contributing to increased conceptual development at secondary level.

- This research, by adapting the work of Morgan (2004) and Halliday (1973), provides an effective measure of the language in mathematics textbooks. Many researchers have created complex and difficult to implement methods of language analysis such as the Dale-Chall list or time consuming non mathematics specific methods such as ‘Cloze Tests’. The method for mathematics textbook language analysis developed by the author can be applied to any mathematical text, does not require a large testing phase with students and can be used in conjunction with other methods of textbook analysis. The three phases of the language analysis (ideational, interpersonal and textual) ensure than no area of language is omitted from the analysis allowing for a detailed, in-depth analysis of mathematical language in its given context.

- All elements of the methodology developed and employed in this localised Irish context are portable and may be used in other settings provided due care is exercised.
8.4 Limitations

As with most research there are some limitations that the researcher needs to be aware of.

It was necessary for the author to make certain decisions pertaining to the methods of data collection in phase two of this research study, the textbook analysis. The author’s judgement was required in order to categorise the data collected in phase two. In order to ensure consistency a number of definitions were created prior to commencing data collection. These definitions ensured that the treatment of text was consistent across all textbooks series, for example: ‘realistic problems are all those which can be applied directly to a real life situation’ and ‘a narrated line of text is counted as one line if it is over half a page in width, anything less than this is neglected’.

Independent t-tests were employed to statistically compare the test and control groups in phase three. However, a number of factors outside of the control of this study can effect the data such as the teacher, the classroom environment etc. For this reason the main data analysis methods employed in phase three are paired t-tests. Paired t-tests compare an individual’s pre and post scores. However, these scores can only compare within each class group and do not statistically compare between test and control groups.

8.5 Recommendations and Future Research

8.5.1 Recommendation

Collaboration with textbook publishers is necessary to ensure that new textbooks are improved using evidence based research. The author recommends that the process of textbook creation/production cannot be done effectively by one individual and should be conducted by a panel of experts. In an Irish context this
panel of experts should include representatives from the Department of Education, the NCCA, publishers, researchers, text designers, educational psychologists and a committee of mathematics teachers comprising primary, secondary and tertiary teachers.

8.5.2  Future Research

This investigation identified the quality of the Junior Cycle mathematics textbooks currently in use in Ireland and is a positive step towards enhancing the quality of mathematics classroom resources. The author has demonstrated that there is a need for textbook analysis to be conducted and evidence based improvements within textbooks can encourage positive changes in student learning. Some suggestions for future research are outlined below:

- The author has introduced a tool to achieve a measure of mathematics specific langue analysis. This method of mathematics textbook language analysis is based primarily on the work of Morgan (2004). Further research is necessary to refine this tool and allow for more extensive mathematics textbook analysis. For example this research should set out to determine effective ratios of exercises to problems, the balance between mathematics specific vocabulary and non specific vocabulary and the point where the use of symbolism becomes excessive.

- There is an urgent need for further research into how teachers and students actually use mathematics textbooks in the classroom. Evidence on how teachers and students use textbooks will ensure that recommendations or changes and improvements in mathematics textbooks are related to the textbooks’ intended use, thus allowing for further improvement of mathematics textbooks and student learning. Such research will highlight if students are actually reading their mathematics textbooks or being encouraged to read their mathematics textbooks. It may well be the case that students are not
8.6. Final Comment

using their textbooks in an effective manner and hence further research will provide some valuable insights.

- It is also important that issues impacting on students’ conceptual development be understood in greater detail. There is a need for research to identify other obstacles to conceptual development such as the impact of teaching methodologies and the classroom environment and how these interact with textbook use and design.

8.6 Final Comment

This thesis reports on the quality of the current Junior Cycle mathematics textbooks while establishing an overall theoretical framework for textbook analysis comprising Content, Structure, Expectation and Language Analysis. This research also identifies key design features which can be implemented in any mathematics textbook. The idea for this research originated from author’s own experiences as a student and teacher in the Irish education system and her desire to highlight the role of improved mathematics textbooks in enhancing students’ conceptual development. This investigation has demonstrated that the current Junior Cycle mathematics textbooks are limited and need major revision in order to support an ambitious new curriculum initiative such as Project Maths. This research also provides a framework for textbook analysis as a whole and for the language analysis of a mathematics textbook. The task that lies ahead is in publicising the findings from this textbook study to ensure that new textbooks created for Project Maths are effective in their role of supporting mathematics teaching and learning.

Project Maths was implemented nationwide in September 2010. At present there are no textbooks supporting this new mathematics curriculum despite significant pressure from publishers and teachers. The NCCA made a deliberate decision not to make any recommendations in regard to textbooks for Project Maths. In
8.6. *Final Comment*

order for Project Maths to succeed it should not be defined by the mathematics textbook as previous curriculum initiates have. There is an opportunity here for evidence based research such as this research to highlight the quality of the current mathematics textbooks and to promote key design features which can impact on student learning.
Bibliography


TxReadability (1998). *How are the Flesch-Kincaid and Flesch Reading Ease scores calculated?*


Appendices

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Appendix A

Framework for Analysis of Problem Type
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Routine Problems</th>
<th>Non-Routine Problems</th>
<th>Real</th>
<th>Realistic</th>
<th>Fantasy</th>
<th>Pure</th>
<th>Decision Making</th>
<th>System &amp; Analysis</th>
<th>Trouble Shooting</th>
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Appendix B

Textbook Sales Company
Response
From: Eason Internet Shop [mailto:sales@easons.com]
Sent: 01 July 2008 08:46
To: Lisa.OKeeffe
Subject: Re:

Good Morning.

Thank you for your email query. We are unable to release any information on sales figures in Eason shops.

Sincerely,
Eason Internet Shop

----- Original Message ----- 
From: Lisa.OKeeffe
To: sales@easons.com
Sent: Monday, June 30, 2008 2:40 PM

I am a PhD research student at the University of Limerick and am currently undertaking postgraduate research on the mathematics textbooks. As part of my research I intend on analysing the Junior Cycle textbooks. I am seeking your help in identifying the most popular Junior Cycle textbooks. As one of Irelands leading schoolbook suppliers I was wondering if you could provide me information on the sales of mathematics textbooks from last year; this information would enable me to determine the most popular textbooks. Any information which you would provide would be confidential and used only as part of this study.

I would appreciate nay help which you can offer,
I can be contacted at this email or by phone.

Regards,

Lisa O’Keeffe

Lisa O'Keeffe,
Department of Mathematics & Statistics,
University of Limerick,
Limerick
Email: lisa.okeeffe@ul.ie
Phone: 086-1214924
Appendix C

Problem Type - Data
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<thead>
<tr>
<th>Textbook:</th>
<th>Routine Problems</th>
<th>Non-Routine Problems</th>
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<th>Fantasy</th>
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<th>Decision Making</th>
<th>System &amp; Analysis</th>
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Appendix D

Framework for the Content Analysis
### Content Analysis

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D-III Framework for the Content Analysis
Appendix E

Model Chapter
Introduction

In primary school you will have encountered at least four different sets of numbers:

- Whole Numbers (counting)
- Ordinary Fractions ($\frac{1}{2}$, $\frac{1}{4}$)
- Decimals (0.10, 0.5)
- Percentages (50%)

In this section you will look at the fraction number system in a little more detail, helping you to learn how fractions behave and how to add them. Fractions are an important part of everyday life. Most people use fractions on a daily basis, you too will have encountered fractions in your life. For example: sharing out sweets (quarter of a bag of jellies), time (quarter past 2), distance (running half 1km) etc.

Aim

The aim of this section is to learn how to add fractions.

Prior Knowledge

In primary school you will have studied addition, subtraction, multiplication and division of whole numbers. From 1st to 6th class you will have looked at the following:

(1st and 2nd class)
What is a Fraction?
Introduction to Halves.

(3rd and 4th class)
Introduction to Equivalent Fractions.
Compare and Order Fractions.

(5th and 6th class)
Addition and Subtraction of Fractions.
Finding the Lowest Common Multiple.

Fractions in the Real World

Hundreds of jobs require a good knowledge base in fractions.

An example of some would be:

- Management
- Medical Profession
- Production
- Sales
- Administration
- Farming
- Construction
A fraction is a number so, like all numbers, fractions can be added, subtracted, divided and multiplied. A fraction is written as one number above another, separated by a fraction bar.

**Here are some of the fractions which you will have encountered before:**

<table>
<thead>
<tr>
<th>A circle divided into 2 halves:</th>
<th>A circle divided into 3 thirds:</th>
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</thead>
<tbody>
<tr>
<td>Each piece is $\frac{1}{2}$</td>
<td>Each piece is $\frac{1}{3}$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>A circle divided into 4 quarters:</th>
<th>A circle divided into 5 fifths:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each piece is $\frac{1}{4}$</td>
<td>Each piece is $\frac{1}{5}$</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>A circle divided into 6 sixths:</th>
<th>A circle divided into 7 sevenths:</th>
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</thead>
<tbody>
<tr>
<td>Each piece is $\frac{1}{6}$</td>
<td>Each piece is $\frac{1}{7}$</td>
</tr>
</tbody>
</table>

Before starting this unit why not do some quick revision questions:

**Revision Question 1.**

(a) If a pizza is divided into eight equal slices. What fraction does each slice represent and how are they represented in mathematical notation?

(b) If a pizza is divided into two unequal sized parts, is each part a $\frac{1}{2}$?
Unit 1: Proper, Improper & Mixed Fractions

In this Unit we will look at three types of fractions: Proper Fractions, Improper Fractions and Mixed Fractions. On the bottom right hand side of this page there is a blue box which indicates the objectives of this unit. As you achieve each of the objectives, cross them off the list.

Before beginning this section let’s have a quick reminder of the specialist vocabulary that you need to know:

**Vocabulary List:**

**Denominator:** A denominator is the number below the ‘ —’ (fraction bar) in a fraction. It tells how many equal parts the ‘whole’ is divided into.

**Numerator:** A numerator is the number above the ‘ —’ (fraction bar) in a fraction. It tells how many of these equal parts the fraction represents.

**Did you know:**

Our modern method of writing fractions was first used by the Indian civilisation in the 6th Century A.D. They used a number system called *brahmi* which had nine numbers from 1—9 and a 0. While they were the first people to write fractions as one number above another they did not use the line (fraction bar) in between. The Arabs adapted this Indian method of writing fractions by adding the line to separate the numbers. However it wasn’t until the 17th century that this method, that we currently use, reached Europe.

**Objectives**

You will be able to:

- recognise and label fractions.
- identify between proper, improper and mixed fractions.
- change improper fractions to mixed fractions.
- change mixed fractions to improper fractions.
Proper Fractions

A Proper Fraction can be described as being a piece of a whole object. For example consider the strip shown below. It is divided up into four equal segments. One of these segments is shaded in red. Therefore we can say that one of the four segments is shaded red. One in four can be written as $\frac{1}{4}$, therefore $\frac{1}{4}$ of the strip is red.

\[
\begin{array}{cccc}
\phantom{\text{Red}} & \phantom{\text{Red}} & \phantom{\text{Red}} & \text{Red}
\end{array}
\]

$\frac{1}{4}$ is an example of a proper fraction. Proper fractions are those whose numerator is less than the denominator.

For example in the fraction $\frac{3}{7}$, the numerator is 3 and the denominator is 7. Therefore it is a proper fraction. The fraction $\frac{3}{7}$ tells us that the ‘whole’ is divided into 7 equal parts and the fraction $\frac{3}{7}$ represents three of these parts.

Exercise 1.1:
Q. 1.
What fraction of the following diagrams are shaded?

(a)
\[
\begin{array}{cccc}
\phantom{\text{Red}} & \phantom{\text{Red}} & \phantom{\text{Red}} & \text{Red}
\end{array}
\]

(b)
\[
\begin{array}{ccccccc}
\text{Blue} & \text{Blue} & \text{Blue} & \phantom{\text{Red}} & \phantom{\text{Red}} & \phantom{\text{Red}} & \phantom{\text{Red}}
\end{array}
\]
Q. 2.
(a) Draw a whole object with \(\frac{1}{3}\) shaded in.
(b) Draw a whole object with \(\frac{3}{8}\) shaded in.
(c) Draw a whole object with \(\frac{4}{7}\) shaded in.

Reminder:
Proper Fraction
Proper fraction means part of a whole. For example, consider the cookie shown here. Someone has taken a bite out of it. The part they have bitten off is a fraction. This fraction is a proper fraction because it is a piece of the whole cookie.

Did you know:
The fraction bar (also called the vinculum) seemed to cause a few problems when it came to printing or typing. The main problem was that writing a fraction with the fraction bar took up three lines. Therefore this lead to the introduction of the diagonal fraction bar. The earliest known usage of the diagonal fraction bar is in 1718, when Thomas Twining listed quantities of tea using the diagonal fraction bar (e.g. 1/4 pound green tea).
An Improper Fraction represents more than a whole object (more than 1). If you look at an improper fraction you will notice that the numerator is larger than the denominator. Therefore it can be described as being a top heavy fraction. Consider the strip shown. One full strip is divided into four equal segments, here we have one full strip ($\frac{4}{4}$) shaded plus one extra segment ($\frac{1}{4}$). This means that in total we have five shaded segments. Using the definitions from the last section we know that the denominator tells how many pieces the whole is divided into (4), and the numerator tells how many of these pieces we are referring to (5). This gives the fraction $\frac{5}{4}$.

$\frac{8}{3}$ is an example of an improper fraction. Improper fractions are those whose numerator is greater than the denominator. For example in the fraction $\frac{9}{5}$, the numerator is 9 and the denominator is 5. Therefore it is an improper fraction. The fraction $\frac{9}{5}$ tells us that the ‘whole’ is divided into 5 equal parts and the fraction $\frac{9}{5}$ represents nine of these parts.

**Exercise 1.2:**

Q. 1.
What fraction of the following diagrams are shaded? Write your answers as improper fractions.

(a)
Q. 2.
(a) Draw a diagram which represents $\frac{4}{3}$.
(b) Draw a diagram which represents $\frac{5}{2}$.
(c) Draw a diagram which represents $\frac{7}{6}$.

**Reminder:**

**Improper Fraction**

An Improper fraction represents more than one, it is top heavy. For example, consider the cookies shown here. There are two and a half cookies. Break each cookie up into halves - now you have 5 half cookies. The numerator represents how many half cookies we have which is 5. Therefore the fraction representing the cookies is $\frac{5}{2}$. This fraction is an improper fraction because it represents one whole object plus more!

**Did you know:**

Any career in the scientific or engineering field will require fractions. Without fractions there would be no cell phones, TVs, stereos, video games, microwaves, computers, or virtually anything electronic. There would be no bridges, tunnels, roads, skyscrapers, automobiles, ships, planes, rockets or most things mechanical.
A Mixed Fraction is very similar to an improper fraction; it too can be described as being a more than 1. However we write mixed fractions in a different way. Mixed fractions are comprised of whole number and a proper fraction. For example, in the strip below, which we previously represented as \( \frac{5}{4} \), we can see that there is one whole object and a \( \frac{1}{4} \) of a whole object, that is \( 1\frac{1}{4} \). Therefore all improper fractions can be written as mixed fractions and all mixed fractions can be written as improper fractions.

\[
\begin{array}{cccc}
\text{Whole} & \text{Number} & \frac{3}{4} & \text{Proper Fraction} \\
1 & & & \\
\end{array}
\]

\( 2\frac{1}{5} \) is an example of an improper fraction. Mixed fractions are those which combine a whole number plus a proper fraction. For example in the fraction \( 1\frac{3}{4} \), the whole number part is 1 and the proper fraction part is \( \frac{3}{4} \). Therefore it is mixed fraction. The fraction \( 1\frac{3}{4} \) tells us that the we have one whole object and three-quarters of the same object.

Exercise 1.3:
Q. 1.
What fraction of the following diagrams are shaded? Write your answers as mixed fractions.

(a)
Q. 2.
(a) Draw a diagram to represent the mixed fraction $3\frac{1}{2}$.
(b) Draw a diagram to represent the mixed fraction $2\frac{3}{4}$.
(c) Draw a diagram to represent the mixed fraction $1\frac{2}{5}$.

**Reminder:**

**Mixed Fraction**

A mixed fraction is made up of whole objects plus a fraction of the whole object. For example, if you have 6 and a half cookies, the fraction which represents the cookies is $6\frac{1}{2}$. This fraction is a mixed fraction because it is made up of whole objects plus a proper fraction.

**Did you know:**

Few occupations use numbers as much as carpenters and architects. They need to measure everything they do. Carpenters have to know how to use fractions easily and determine length, width, and depth. Architects truly need to have a handle on fractions and ratios when doing their initial drawings. Also Interior Decorators have to know the square footage they are covering. They have to be able to measure, add and subtract fractions, and stay within their budget.
Equivalent fractions refer to two or more fractions that represent the same amount; two (or more) fractions which are written differently but are of equal value. Consider the two strips shown. The first strip has six segments in total with two of these shaded, \(\frac{2}{6}\). The second strip (which is the same size) has three segments in total with one of these shaded, \(\frac{1}{3}\). From this diagram we can see that \(\frac{2}{6}\) represents the same amount as \(\frac{1}{3}\), therefore \(\frac{2}{6}\) and \(\frac{1}{3}\) are equivalent fractions. You have already been introduced to some equivalent fractions in 3rd and 4th class. The aim of this section is to remind you of the equivalent fractions you have already encountered. You will work more with equivalent fractions in Unit 3.

Equivalent Fractions have the same value even though they may look differently. For example \(\frac{1}{2}\) and \(\frac{2}{4}\) are equivalent, the pizza shown here is half an entire pizza. This half pizza is divided into two quarters. The two quarters of the pizza are the exact same amount of the pizza as one half.
$\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$ are all equivalent fractions.

$\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}$ are all equivalent fractions.

For a quick revision of equivalent fractions why not check out the following websites:

www.learningplanet.com
www.aaamath.com
Vocabulary List:

**Proper Fraction:** A proper fraction is a fraction that is less than one. For a fraction to be less than one, the numerator must be less (of smaller value) than the denominator, e.g. $\frac{2}{3}$.

**Improper Fraction:** An improper fraction is a fraction that is greater than one. For a fraction to be greater than one, the numerator must be greater than (of bigger value) than the denominator, e.g. $\frac{5}{3}$.

**Mixed Fraction:** Numbers that consist of a whole number part and a fraction part are mixed fractions. All improper fractions can also be written as mixed fractions, e.g. $3\frac{1}{2}$.

All mixed fractions can be written as improper fractions and all improper fractions can be written as mixed fractions. Let’s consider the fraction strip shown here: Here we have two shaded strips and a third strip which has 2 segments shaded. We can write the shaded section as an improper fraction by finding the denominator (how many parts the whole is divided into) and the numerator (how many of these parts are shaded in): $\frac{12}{5}$.

Or we can write this as a mixed fraction by finding how many whole objects there are and what the remaining proper fraction is. Here we have $2\frac{2}{5}$. Therefore we can assume that $\frac{12}{5}$ is equal to $2\frac{2}{5}$.
Changing a Mixed Fraction into an Improper Fraction

Let’s take the fraction $2\frac{2}{5}$. We want to write this as an improper fraction.

The first thing we need to do is to decide what denominator our improper fraction will have. This is already given to us in the mixed fraction as we must continue with the existing denominator, which is 5. Now all we do is figure out what our numerator will be. We can do this by looking at the diagram. (If you don’t have a diagram, why not draw your own!). Count up the number of segments you have shaded and this will give you the numerator, which is 12.

\[
\begin{array}{cccccc}
\text{Diagram 1} & \text{Diagram 2} & \text{Diagram 3} & \text{Diagram 4} & \text{Diagram 5} & \text{Diagram 6} \\
\end{array}
\]

Therefore the improper fraction is $\frac{12}{5}$.

Let's take the fraction $\frac{9}{5}$. We need to write this as a mixed fraction.

First thing we need to do is to decide how many whole parts our fraction represents. We can see this very easily by drawing a diagram of our improper fraction.

\[
\begin{array}{cccccc}
\text{Diagram 1} & \text{Diagram 2} & \text{Diagram 3} & \text{Diagram 4} & \text{Diagram 5} & \text{Diagram 6} \\
\end{array}
\]

Now we can see that there is one whole strip shaded in, therefore the whole part of the mixed fraction is 1. Also there are another four out of five segments shaded in which gives the proper fraction part of the mixed number $\frac{4}{5}$.

Therefore the mixed fraction is $1\frac{4}{5}$. 

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E-XIV
Exercise 1.4:
Q. 1.
What fraction of the following strips are shaded? Give your answers as both Improper and Mixed fractions.

(a) 
(b) 

Q. 2.
Here are some practice questions on converting between Mixed fractions and Improper fractions. Try drawing your own diagrams to help you with the answers.

Rewrite the following as improper fractions:
(a) $1\frac{2}{3}$  
(b) $3\frac{7}{7}$  
(c) $1\frac{4}{9}$

Rewrite the following as mixed fractions:
(d) $\frac{6}{5}$  
(e) $\frac{11}{7}$  
(f) $\frac{9}{1}$

Did you know:
Lots of jobs require fractions.
Hairdressers need fractions for mixing colours, chefs require them for making recipes, pharmacists, nurses and doctors use them for creating and administering medicines. Organisations such as NASA use fractions, for example a team from the University of Tennessee used fractions less than 100 nanometres (billionths of a metre) in size for examining lunar dust.
Unit 2: Adding Fractions with the same Denominator

**Vocabulary List:**

**Proper Fraction:** A proper fraction is a fraction which is less than one. e.g. $\frac{2}{3}$

**Improper Fraction:** An improper fraction is a fraction which is greater than one. e.g. $\frac{5}{3}$

**Mixed Fraction:** Numbers which consists of a whole number part and a fraction part are mixed fractions. All improper fractions can be written as mixed fractions, e.g. $3\frac{1}{2}$

When using addition we know that we can only add like terms to like terms i.e. we can only add apples to apples, oranges to oranges etc. So for example, if you have a fruit bowl with 4 apples and 3 oranges and a friend has a fruit bowl with 2 apples and 5 oranges then all together you will have 6 apples and 8 oranges.

With fractions we consider those with the same denominators as ‘like’ fractions. For example all the quarters are like fractions, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$, $\frac{5}{4}$, $\frac{6}{4}$, …

This means that we can only add quarters to other quarters.

This rule for number addition, *we can only add like terms to like*, is also the standard rule for adding fractions.

Fraction addition is only possible between like fractions, *i.e.* fractions must have the same denominator to be added together.
With this rule in mind let’s look at an example:

\[
\begin{array}{c}
\frac{3}{5} + \frac{1}{5} = ?
\end{array}
\]

Now consider that the denominator only tells us how many parts our whole is divided into (i.e. what type of fraction we are working with) and the numerator tells how many parts of the fraction we are interested in. Therefore when we are adding fractions we are looking to find out how many pieces of the whole we end up with after adding the fractions, so we add the numerators. Using the diagrams can help us to see this.

\[
\frac{3}{5} + \frac{1}{5} = \frac{3 + 1}{5} = \frac{4}{5}
\]

Example 2.1:

Your younger sister buys a bag of gummy bears and she empties out the bag to count them. There are 72 sweets in the bag. As she counts them she puts the ones she has counted to her left-hand side and without her noticing you first take 5 sweets and then when you get a little braver you take 16.

What fraction of the bag did you first take?

\[
\begin{align*}
\text{Total number of sweets} & = 72 \\
\text{Number of sweets you took} & = 5 \\
\text{Total fraction of sweets you took} & = \frac{5}{72}
\end{align*}
\]
What fraction of the bag did you take the second time?

Total number of sweets = 72
Number of sweets you took = 16
Total fraction of sweets you took = \( \frac{16}{72} \)

What fraction of sweets did you take in total?

\[
\frac{5}{72} + \frac{16}{72} = \frac{5 + 16}{72} = \frac{21}{72}
\]

These fractions are ‘like fractions’ because the denominators are the same. Therefore to add these fractions we just the numerators.

Did you know:
Designers also need to know a lot about fractions. Graphical designers require fractions on a daily basis for dimensional measuring. While for fashion designers, fractions are a necessity for scaling patterns and designs. Fractions are also needed in the world of sport for statisticians to create statistics and averages for games/players. Musicians and dancers also use fractions for beat and tempo.
### Practice Exercises 2.1:

**Q. 1.**

Add the following:

(a) \( \frac{5}{17} + \frac{3}{17} \)  
(b) \( \frac{5}{11} + \frac{3}{11} \)  
(c) \( \frac{2}{9} + \frac{5}{9} \)  
(d) \( \frac{1}{11} + \frac{8}{11} \)  

### Exercises 2.1:

**Q. 1.**

**Groupwork**

What fraction is your group of the entire class?

What fraction of your group have blonde hair?

What fraction of your group wear glasses?

What fraction of your group has a red school bag?

What fraction of the class have blonde hair?

What fraction of the class wear glasses?

What fraction of the class have red school bags?

What fraction of your group have nokia mobile phones?

What fraction of your group support Irish rugby?

What fraction of your group likes shopping?

What fraction of your group have pets?

What fraction of your group play sport?

What fraction of your group have no brothers or sisters?

**Q. 2.**

You have €400 in savings. You spend €80 on a new Irish rugby jersey, you spend €150 buying a new mobile phone, and €70 on a wii fit game. What fraction of your savings did you spend?
Q. 3.
Michelle went to the movies with Sharon on Saturday night. They bought a large popcorn. Michelle ate one quarter of the popcorn and Sharon ate one quarter of the popcorn. How much of the popcorn did they eat?

Q. 4.
James has created a list of countries he would like to go to, there are 97 countries on his list. He has been to 11 countries out of the 97 countries and his cousin Seán has been to 31 of these countries. Together, what fraction of James’ list of countries have they visited?

Q. 5.
A large bar of Cadbury Dairymilk has 20 pieces. If my sister ate 8 squares, and I ate 6 squares, what fraction of the bar is left over?

Q. 6.
Tom finishes $\frac{2}{9}$ of his homework during a study period. He finishes $\frac{3}{9}$ of his homework before dinner. What fraction of his homework will he have to do after dinner to complete all of his homework?

Reminder
Fraction Addition:
Fractions can only be added if they are ‘like fractions’, that is they must have the same denominator. For example $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$
Unit 3: Adding Fractions with different Denominators

**Vocabulary List:**

**Equivalent Fraction:** Equivalent fractions represent the same amount of a whole object, e.g. \( \frac{1}{2} \) is equivalent to \( \frac{2}{4} \).

**LCD - Lowest Common Denominator:** is the lowest common multiple of the denominators in question (i.e. the lowest number which is a multiple of the denominators you are working with) e.g. The LCD of \( \frac{1}{2} \) and \( \frac{1}{3} \) is the lowest common multiple of 2 and 3 which is 6. LCD = 6.

When using addition we know that we can only add like terms to like terms and that when adding fractions ‘like fractions’ are those with the same denominator. So what happens when we need to add fractions that do not have the same denominator, such as \( \frac{1}{2} + \frac{1}{3} \).

Fraction blocks can help us to see what is happening when we need to add these fractions. Many of you will have used fraction blocks in primary school. Here is an example of what a fraction block looks like.

<table>
<thead>
<tr>
<th></th>
<th>One</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>1/6</td>
<td>1/6</td>
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<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>1/6</td>
<td>1/6</td>
</tr>
<tr>
<td>1/6</td>
<td>1/6</td>
</tr>
</tbody>
</table>

These blocks help us to visualise equivalent fractions.

**Objectives**

You will be able to:

- demonstrate where we use fractions in the real world.
- add fractions with different denominators.
- use fraction addition to solve problems.
We have already seen that when adding fractions the denominators must be the same. We want to add \(\frac{1}{2}\) and \(\frac{1}{3}\). So let's look at these units from our fraction block separately. There are no common block sizes between halves and thirds so we look down further on the block to see if we can find a connection. Now we can see that \(\frac{1}{2}\) is equal to \(\frac{3}{6}\) and that \(\frac{1}{3}\) is equal to \(\frac{2}{6}\).

<table>
<thead>
<tr>
<th></th>
<th>1/2</th>
<th></th>
<th>1/2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/4</td>
<td></td>
<td>1/4</td>
</tr>
<tr>
<td></td>
<td>1/6</td>
<td></td>
<td>1/6</td>
</tr>
<tr>
<td></td>
<td>1/6</td>
<td></td>
<td>1/6</td>
</tr>
<tr>
<td></td>
<td>1/6</td>
<td></td>
<td>1/6</td>
</tr>
</tbody>
</table>

Therefore:

\[
\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}
\]

\[
\frac{3}{6} + \frac{2}{6} = \frac{5}{6}
\]

**Activity 3.1:**

Equivalent fractions are those which represent the exact same amount. Pattern blocks help to explore the idea of equivalent fractions. Using the cut-out page that is attached, discover the relationship between these shapes.
Fraction Addition

- How many green blocks are equivalent to a blue block, red block, black block, yellow block or pink block?
- How many red blocks are equivalent to a yellow block or a pink block?
- How many blue blocks are equivalent to a black block, yellow block or a pink block?
- How many black blocks are equivalent to a pink block?

Now use the the double hexagon (pink).
Cover the pink double hexagon with a red block and a yellow block.
What fraction of the whole does it cover?
Can we replace the yellow block with any other blocks without changing the fraction of the whole that we have covered?

Example 3.1:
The home economics teacher hands out the recipe for Wednesday’s cooking class every Monday. This Monday the recipe reads as follows:

<table>
<thead>
<tr>
<th>Ingredients:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{11}{6} ) cups wholemeal flour</td>
<td></td>
</tr>
<tr>
<td>4 teaspoons baking powder</td>
<td></td>
</tr>
<tr>
<td>( \frac{9}{5} ) cups plain white flour</td>
<td></td>
</tr>
<tr>
<td>1 teaspoon salt</td>
<td></td>
</tr>
<tr>
<td>6 tablespoons butter</td>
<td></td>
</tr>
<tr>
<td>2 tablespoons sultanas</td>
<td></td>
</tr>
<tr>
<td>1 tablespoon sugar</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{2} ) pint milk</td>
<td></td>
</tr>
<tr>
<td>1 egg, beaten</td>
<td></td>
</tr>
</tbody>
</table>

How much flour do I need in total for my recipe?
Fraction Addition

Wholemeal Flour required = $\frac{11}{6}$
Plain Flour required = $\frac{9}{5}$
Total Flour required = $\frac{11}{6} + \frac{9}{5}$

Now in order to add these fractions we need to create equivalent fractions - which have the same denominator. As we can see our fraction block does not have every fraction which we require. Therefore we must consider another approach to finding equivalent fractions.

Firstly we must decide what denominator the equivalent fractions should have. We can do this by finding the Lowest Common Denominator (LCD).

For a quick revision of lowest common multiple why not check out the following websites:

www.helpwithfractions.com
www.aaamath.com

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48 ..... 
Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45 ..... 

Demominator of first fraction = 6
Demominator of second fraction = 5
$\text{LCM} = 30$
$\Rightarrow \text{LCD} = 30$

Lets look at $\frac{11}{6}$ first. We know that the $\text{LCD} = 30$, therefore we are looking for a fraction equivalent to $\frac{11}{6}$ whose denominator is 30;

\[ \frac{11}{6} = \frac{?}{30} \]

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(6 × 5 = 30), so therefore to change the denominator from 6 to 30 we have multiplied 6 by 5. If we change the denominator then we must make the same change to the numerator otherwise we will have changed the relationship between the numerator and the denominator (changed the proportion of the fraction).

\[
\frac{11}{6} \times 5 = \frac{55}{30}
\]

Then we do the same for the second fraction \( \frac{9}{5} \)

\[
\frac{9}{5} \times \frac{6}{6} = \frac{54}{30}
\]

Now I have two fractions with the same denominator and so we can begin addition:

\[
\frac{55}{30} + \frac{54}{30} = \frac{55 + 54}{30} = \frac{109}{30}
\]

Now I know that the total flour needed is \( \frac{109}{30} \) cups.

Question: Can you rewrite this as a mixed fraction?
Reminder
Fraction Addition:
Fractions can only be added if they are ‘like fractions’, that is they must have the same denominator. Therefore when you need to add fractions that have different denominators we rewrite them using equivalent fractions. For example, $\frac{2}{5} + \frac{1}{3}$ is the equivalent of adding $\frac{6}{15} + \frac{5}{15}$.

Exercise 3.1:
Q. 1.
Shade in one quarter of the following shapes:

Q. 2.
To illustrate the fact that $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent fractions Mrs. Gomez drew the picture shown here. Explain how this picture would suggest that $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent fractions.
Q. 3.
There are eight equal slices of pizza. Justin ate five-eighths of the pizza for dinner. He ate \( \frac{1}{4} \) of the pizza for a bedtime snack. How much of the pizza has Justin eaten?

Q. 4.
Conor had €12. He spent two-thirds of his money on a cinema ticket and another one-quarter of his money on popcorn. What fraction of his money did he spend altogether?

Q. 5.
The cost of a ticket to see Liverpool play Manchester United at Anfield is €100. For his birthday, Harry’s Auntie gave him money, which amounted to \( \frac{5}{8} \) of the price of a ticket. Harry then used some of his savings to pay for the rest of the ticket. How much did Harry take out of his savings?

Q. 6.
The capacity of Anfield is 45,362. At this Liverpool match \( \frac{2}{5} \) of the seats were occupied by Manchester United fans. \( \frac{1}{7} \) of the seats were empty. What fraction of the seats were occupied by Liverpool fans?

Q. 7.
Miss Lee had her students draw pictures of their homes. \( \frac{5}{10} \) of the students used coloured pencils, \( \frac{1}{8} \) used crayons, and the rest of the class used paint. What fraction of the class did not use paint to draw the pictures?
Q. 8.
Kate has a bag of marbles. The bag only has yellow, green, blue and red marbles. In the bag:
\[ \frac{1}{3} \text{ of them are green,} \]
\[ \frac{3}{4} \text{ of them are blue} \]
\[ \frac{1}{10} \text{ of them are red,} \]
30 of them are yellow.

How many marbles does Kate have?

How many marbles of each colour does Kate have?

Q. 9.
Solve the problem using at least two different methods. Record each of your methods in some detail (separate into steps as much as possible).
My fabric softener (Comfort) comes in a 750ml bottle. The bottle says that it lasts for 21 loads of washing. How many milliliters (mls) of liquid should be used per wash? Give your answer as a fraction (or mixed number.)

Suppose I realised that 20mls was enough fabric softener for every wash, how many loads of washing will the bottle last for?

**Did you know:**
A commonly used symbol for fractions is the “division symbol,” or \( \div \). This symbol is called an obelus. It is used by many people when working with calculators to indicate division and/or fractions. Fractions are now commonly used in the real world in reading the time, shoe-sizes, recipes, carpentry, clothing manufacture, and multiple other places. A study carried out by Education Week (2007) found that \( \frac{27}{35} \) employers require their workers to at least have a good knowledge base in basic math and fractions.
Equivalent fractions represent the same amount of a whole object.

Fractions can only be added if they have the same denominator.

Use equivalent fractions to add fractions that do not have the same denominator.

When faced with a problem don’t be afraid to experiment with different approaches.

Remember, there is more than one way to do everything so if unsure discuss your solutions and methods/approaches.
Appendix F

Fraction Concept Map
Appendix G

Model Chapter - Teacher Booklet
The diagram on the next page outlines the framework for teaching fractions. The following must be applied throughout your teaching:

- **Discussion** - regularly open up discussion as this highlights any misconceptions which students might have and also helps them to learn from each other.

- **Benchmark** - set achievable (yet challenging) goals for your class to encourage motivation.

- **Make links** between what they know and new knowledge, help them to build the existing structures that they have in place (partial examples are extremely useful).

- **Help students** to embed their knowledge by reinforcing ideas and concepts this can be done effectively with discussion and linking old and new concepts.

- **Have pupils read and write** their mathematics, Mathematics is much more than the computation of numbers, test their concepts by asking for written explanations of certain terms (links nicely with discussion)
Fractions:

Note:
Additional handout/worksheets have been included at the back of this booklet and can be used at your discretion.

Regularly look back to the fractions framework and ensure that you are considering and applying all areas of concern.
Introduction

In primary school you will have come across at least four different sets of numbers:

- Whole Numbers, e.g 1, 2, 3, 4 ...
- Ordinary Fractions such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$,...
- Decimals such 0.10, 0.5, 0.25 ...
- Percentages such as 50%, 10%, 75%...

In this section you will look at the fraction number system which will help you to learn about fraction addition. Fractions are an important part of everyday life. Most people use fractions on a daily basis and you too will have met fractions in your life. For example: sharing out sweets (quarter of a bag of jellies), time (quarter past 2), distance (running half 1 kilometer) etc.
The aim of this section is to learn how to add fractions.

**What you already know**

In primary school you will have studied addition, subtraction, multiplication and division of whole numbers. From 1st to 6th class you will have looked at the following:

*(1st and 2nd class)*
What is a Fraction?
Introduction to Halves.

*(3rd and 4th class)*
Introduction to Equivalent Fractions.
Compare and Order Fractions.

*(5th and 6th class)*
Addition and Subtraction of Fractions.
Finding the Lowest Common Multiple.

A fraction is a number which is written as one number above another, separated by a fraction bar.

Here we can see that someone has eaten half the pizza.

\[
\frac{1}{2}\text{ of the pizza:}
\]
Before starting this unit let us do some quick revision:

Here are some fractions that you may have met before.

<table>
<thead>
<tr>
<th>A disk divided into 2 equal pieces:</th>
<th>A disk divided into 3 equal pieces:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each piece is ( \frac{1}{2} )</td>
<td>Each piece is ( \frac{1}{3} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A disk divided into 4 equal pieces:</th>
<th>A disk divided into 5 equal pieces:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each piece is ( \frac{1}{4} )</td>
<td>Each piece is ( \frac{1}{5} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A disk divided into 6 equal pieces:</th>
<th>A disk divided into 7 equal pieces:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each piece is ( \frac{1}{6} )</td>
<td>Each piece is ( \frac{1}{7} )</td>
</tr>
</tbody>
</table>

**Revision Question 1.**
(a) What fraction of the people shown below are have blue hair?
(b) What fraction of the people are wearing a necklace?

---

A fraction is just a number so like all numbers, fractions can be added, subtracted, divided and multiplied.

---

(a) If a pizza is divided into eight equal slices. What fraction does each slice represent of the total pizza?

(b) If it is divided into two unequal sized parts, is each part equal to \( \frac{5}{7} \)?
Unit 1: Proper, Improper & Mixed Fractions

In this Unit we will look at three types of fractions: Proper Fractions, Improper Fractions and Mixed Fractions.

Before beginning this section let’s remind ourselves of the special words that you need to know:

**Vocabulary List:**

- **Numerator:** A numerator is the number above the ‘—’ (fraction bar) in a fraction.
- **Denominator:** A denominator is the number below the ‘—’ (fraction bar) in a fraction.

1.1 Proper Fractions

Consider the strip shown below. It is divided up into five equal segments. One of these segments is shaded in red. One in five can be written as \(\frac{1}{5}\). Therefore \(\frac{1}{5}\) of the strip is red.

**Did you know:**
Our modern method of writing fractions was first used by Hindu mathematicians in the 12th Century. While they were the first people to write fractions as one number above another they did not use the line (fraction bar) in between. It wasn’t until the 17th century that this method reached Europe.
Proper fractions are those whose numerator is less than the denominator. For example fraction $\frac{3}{7}$ is a proper fraction, the numerator is 3 and the denominator is 7. The fraction $\frac{3}{7}$ tells us that the ‘whole’ is divided into 7 equal parts and the fraction $\frac{3}{7}$ represents three of these parts.

**Exercise 1.1**

**Q. 1.**

What fraction of the following diagrams are shaded?

(a) ![Diagram](image1)

(b) ![Diagram](image2)

(c) ![Diagram](image3)

(d) What fraction of the suns rays are shaded?

![Sun](image4)

**Q. 2.**

(a) Draw a whole object and shade $\frac{1}{3}$ of it.

(b) Draw a whole object and shade $\frac{3}{8}$ of it.

(c) Draw a whole object and shade $\frac{4}{7}$ of it.

**Reminder:**

The term proper fraction means part of a whole.

**Did you know:**

The fraction bar (also called the vinculum) seemed to cause a few problems when it came to printing or typing because it took up more than one line. This led to the introduction of the diagonal fraction bar. The earliest known usage of the diagonal fraction bar is in 1718, when Thomas Twining listed quantities of tea using the diagonal fraction bar (e.g. 1/4 pound green tea).
1.2 Improper Fractions

An Improper fraction represents more than a whole object (more than one). \( \frac{8}{3} \) is an example of an improper fraction. Improper fractions are those whose numerator is greater than the denominator.

Consider the strip shown. One full strip is divided into four equal segments. Here we have one full strip (\( \frac{1}{4} \)) shaded plus one extra segment (\( \frac{1}{4} \)). This means that in total we have five shaded segments.

\[ \begin{array}{cccccc}
\text{\includegraphics{strip.png}}
\end{array} \]

We know that the denominator tells how many pieces the whole is divided into (here one whole strip has 4 segments), and the numerator tells how many of these pieces we are talking about (we have 5 segments in total). This gives the fraction \( \frac{5}{4} \).

**Exercise 1.2**

**Q. 1.**

(a) Draw a diagram to represent \( \frac{4}{3} \).

(b) Draw a diagram to represent \( \frac{5}{2} \).

(c) Draw a diagram to represent \( \frac{7}{6} \).
1.3 Mixed Fractions

A mixed fraction is like an improper fraction because it can be described as being more than one. Mixed fractions are those that combine a whole number plus a proper fraction denominator.

In the fraction $1\frac{3}{4}$, the whole number part is 1 and the proper fraction part is $\frac{3}{4}$. Therefore it is a mixed fraction. The fraction $1\frac{3}{4}$ tells us that we have one whole object and three-quarters of a similar object.

Exercise 1.3
Q.1 What mixed fraction does this diagram represent?

All improper fractions can be written as mixed fractions and all mixed fractions can be written as improper fractions.

For a quick revision of fractions why not check out the following websites:

www.learningplanet.com
www.aaamath.com
www.mathsisfun.com
1.4 Equivalent Fractions

Equivalent fractions are two or more fractions which are written differently but are of equal value. Consider the two strips shown. The first strip has six segments in total with two of these shaded, \( \frac{2}{6} \). The second strip (which is the same size) has three segments with one of these shaded, \( \frac{1}{3} \).

From this diagram we can see that \( \frac{2}{6} \) represents the same amount as \( \frac{1}{3} \), therefore \( \frac{2}{6} \) and \( \frac{1}{3} \) are equivalent fractions. We can also say the \( \frac{1}{3} \) is a reduced fraction. That is, \( \frac{2}{6} \) can be simplified (or reduced) to give the fraction \( \frac{1}{3} \).

Exercise 1.4
Q.1

Circle every picture that has a shaded part equivalent to the given fraction.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Picture 1</th>
<th>Picture 2</th>
<th>Picture 3</th>
<th>Picture 4</th>
<th>Picture 5</th>
<th>Picture 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} )</td>
<td>![Picture 1]</td>
<td>![Picture 2]</td>
<td>![Picture 3]</td>
<td>![Picture 4]</td>
<td>![Picture 5]</td>
<td>![Picture 6]</td>
</tr>
<tr>
<td>( \frac{2}{3} )</td>
<td>![Picture 1]</td>
<td>![Picture 2]</td>
<td>![Picture 3]</td>
<td>![Picture 4]</td>
<td>![Picture 5]</td>
<td>![Picture 6]</td>
</tr>
<tr>
<td>( \frac{3}{4} )</td>
<td>![Picture 1]</td>
<td>![Picture 2]</td>
<td>![Picture 3]</td>
<td>![Picture 4]</td>
<td>![Picture 5]</td>
<td>![Picture 6]</td>
</tr>
</tbody>
</table>
1.5 Recap of Unit One

Shade or color the portion of each figure to represent the fraction. Then finish the reduced fraction.

\[
\begin{array}{ccc}
\frac{4}{16} &=& \frac{1}{4} \\
\frac{6}{9} &=& \frac{2}{3} \\
\frac{5}{10} &=& \frac{1}{2} \\
\frac{2}{4} &=& \frac{1}{2} \\
\frac{10}{12} &=& \frac{5}{6} \\
\frac{2}{10} &=& \frac{1}{5} \\
\frac{10}{16} &=& \frac{5}{8} \\
\frac{4}{8} &=& \frac{1}{2} \\
\frac{2}{8} &=& \frac{1}{4} \\
\frac{8}{12} &=& \frac{2}{3} \\
\frac{10}{12} &=& \frac{5}{6} \\
\frac{8}{12} &=& \frac{2}{3} \\
\frac{2}{14} &=& \frac{1}{7} \\
\frac{2}{16} &=& \frac{1}{8} \\
\frac{12}{16} &=& \frac{3}{4}
\end{array}
\]
Fraction Addition

How many robots are there? __________________

What fraction of the robots are made of stars? ______

What fraction of the robots aren't made of stars? _____

What fraction of the robots are made of triangles? _____

What fraction of the robots aren't made of triangles? __

What fraction of the robots are made of rectangles? _____

What fraction of the robots aren't made of rectangles? ___

What fraction of the robots are made of ovals? __________

What fraction of the robots aren't made of ovals? _______
Fraction Addition

(a) How many flowers are in $\frac{2}{3}$ of the set?
(b) How many flowers are in $\frac{5}{6}$ of the set?
(c) How many flowers are in $\frac{7}{9}$ of the set?

Test you vocab:

In your own words write out what each word means:

Numerator:

Denominator:

Proper Fraction:

Improper Fraction:

Mixed Fraction:

Equivalent Fraction:
Unit 2: Adding Fractions with the Same Denominator

When using addition we know that we can only add like terms to like terms i.e. we can only add apples to apples, oranges to oranges etc. So for example, if you have a fruit bowl with 4 apples and 5 oranges and a friend has a fruit bowl with 1 apples and 2 oranges then all together you will have 5 apples and 7 oranges.

With fractions we consider those with the same denominators as ‘like’ fractions. For example all the quarters are like fractions, \(\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \ldots\). This means that we can only add quarters to other quarters.

This rule for number addition, *we can only add like terms to like*, is also the standard rule for adding fractions.

Let’s look at an example:

\[
\begin{array}{c|c|c|c}
\hline
& & \mathbf{+} & \\
\hline
\mathbf{3} & \frac{1}{5} & \frac{1}{5} & = \\
\hline
\end{array}
\]

This gives us the following:

\[
\frac{3}{5} + \frac{1}{5} = ?
\]
Therefore when we are adding fractions we are looking to find out how many pieces of the whole we end up, so we add the numerators. Using the diagrams can help us to see this.

\[
\frac{3}{5} + \frac{1}{5} = \frac{3 + 1}{5} = \frac{4}{5}
\]

Example 2.1:

Your younger sister buys a bag of gummy bears and she empties out the bag to count them. There are 72 sweets in the bag. As she counts them she puts the ones she has counted to her left-hand side and without her noticing you first take 5 sweets and then when you get a little braver you take 16.

What fraction of the bag did you first take?

Total number of sweets = 72
Number of sweets you took = 5
Total fraction of sweets you took = \(\frac{5}{72}\)

What fraction of the bag did you take the second time?

Total fraction of sweets you took = \(\frac{16}{72}\)

What fraction of sweets did you take in total?

\[
\frac{5}{72} + \frac{16}{72}
\]
Fraction Addition

These fractions are ‘like fractions’ because the denominators are the same. Therefore to add these fractions we just add the numerators.

\[
\frac{5}{72} + \frac{16}{72} = \frac{5 + 16}{72} = \frac{21}{72}
\]

Practice Exercises 2.1:

Q. 1.
Add the following:
(a) \(\frac{5}{17} + \frac{3}{17}\)  
(b) \(\frac{5}{11} + \frac{3}{11}\)
(c) \(\frac{2}{9} + \frac{5}{9}\)  
(d) \(\frac{1}{14} + \frac{8}{14}\)

Q. 2.

Shade one half of the figure to the right.  
How many sixths did you shade? _____

\[
\text{One half} = \frac{1}{6}
\]

Shade one third of the figure to the right.  
How many sixths did you shade? _____

\[
\text{One sixth} = \frac{1}{6}
\]

Shade one half plus one sixth of the figure to the right.  
How many sixths did you shade? _____

\[
\frac{1}{2} + \frac{1}{6} = \frac{2}{6} = \frac{2}{3}
\]

Did you know:
Fractions are also needed in the world of sport. Statisticians use fractions to create statistics and averages for games and players.
Exercises 2.1:

Q. 1.
Groupwork
What fraction is your group of the entire class?
What fraction of your group wear glasses?
What fraction of your group has a red school bag?
What fraction of the class wear glasses?
What fraction of the class have red school bags?
What fraction of your group have nokia mobile phones?
What fraction of your group support Irish rugby?
What fraction of your group likes shopping?
What fraction of your group have pets?
What fraction of your group play sport?
What fraction of your group have no brothers or sisters?

Q. 2.
A large bar of Cadbury Dairymilk has 20 pieces. If my sister ate 8 squares, and I ate 6 squares, what fraction of the bar is left over?

Q. 3.
During P.E. class Shane is asked to divide the class into teams. The teacher tells him to give \( \frac{2}{9} \) of the class red bibs, \( \frac{3}{9} \) of the class blue bib and the rest of the class were to get green bibs. What fraction of the class were given green bibs?

Reminder
Fractions can only be added if they have the same denominator.
For example:
\[ \frac{2}{5} + \frac{1}{5} = \frac{3}{5} \]
Exercises 2.1:

Q. 4.
You have €400 in savings. You spend €80 on a new Irish rugby jersey, you spend €150 buying a new mobile phone, and €70 on a Wii Fit game. What fraction of your savings did you spend?

€ 80 € 150 € 70

Q. 5.
Make a path through each matrix so that the sum of the fractions is equal to the answer. Use only horizontal and vertical paths. You may not have to use all the fractions. Don’t use a fraction more than once.
Example 2.2:

Tracey is training for the under 14’s All-Ireland Athletics competition. As part of her training programme it is important for her to keep track of the distances she has run. Yesterday Tracey ran $2\frac{1}{3}$ km and today she ran $1\frac{2}{3}$ km. How many kilometers did Tracey run in these two days?

Solution:
We need to add $2\frac{1}{3} + 1\frac{1}{3}$.

Step 1:
Add the whole numbers:

$$2 + 1 = 3$$

Step 2:
Add the fractions:

$$\frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$$

Step 3:
Add the answer from Step 1 and Step 2:

$$3 + 1 = 4$$

Tracey ran 4km between yesterday and today.

Practice Exercises 2.2:

Q. 1. Add the following:

(a) $\frac{2}{17} + \frac{6}{17}$
(b) $8\frac{4}{7} + 1\frac{2}{7}$
(c) $4\frac{2}{9} + 7\frac{5}{9}$
(d) $5\frac{1}{14} + 3\frac{8}{14}$

Reminder
Any fraction which has a numerator equal to its denominator is equal to 1.

E.g.

$$\frac{7}{7} = 1$$
$$\frac{5}{5} = 1$$
$$\frac{17}{17} = 1$$

The fraction

$$\frac{6}{5} = \frac{5}{5} + \frac{1}{5}$$

$$= 1\frac{1}{5}$$
Unit 3: Adding Fractions with different Denominators

Vocabulary List:

Equivalent Fraction: Two or more fractions which are of equal value.

LCD - Lowest Common Denominator LCD is the lowest common multiple of the denominators you are working with. For example: the LCD of \( \frac{1}{2} \) and \( \frac{1}{3} \) is the lowest common multiple of 2 and 3 which is 6. LCD = 6.

When we need to add fractions that do not have the same denominator, such as \( \frac{1}{2} + \frac{1}{3} \) we need to:

1. Find the LCD (Lowest Common Denominator)
2. Find equivalent fractions (Use the LCD as the new denominator)
3. Add the equivalent fractions

Fraction blocks can help us to see what is happening. Many of you will have used fraction blocks in primary school. Here is an example of what a fraction block looks like.

We can only add like terms to like terms and when adding fractions ‘like fractions’ are those with the same denominator.
if we look a little closer at the fraction wall we can see connections between halves and sixths and also between thirds and sixths.

We can see that $\frac{1}{2}$ is equal to $\frac{3}{6}$ and that $\frac{1}{3}$ is equal to $\frac{2}{6}$.

Therefore:

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

### Activity 3.1:

Equivalent fractions are those that represent the exact same amount. Pattern blocks help to explore the idea of equivalent fractions. Using the cut-out page that is attached, discover the relationship between these shapes.

- How many green blocks are equivalent to a blue block, red block, black block, yellow block or pink block?
- How many red blocks are equivalent to a yellow block or a pink block?
- How many blue blocks are equivalent to a black block, yellow block or a pink block?
- How many black blocks are equivalent to a pink block?
Example 3.1:
The home economics teacher hands out the recipe for Wednesday’s cooking class every Monday. This Monday the recipe reads as follows:

**Ingredients:**
- \( \frac{11}{6} \) cups wholemeal flour
- 4 teaspoons baking powder
- \( \frac{9}{5} \) cups plain white flour
- 1 teaspoon salt
- 6 tablespoons butter
- 2 tablespoons sultanas
- 1 tablespoon sugar
- \( \frac{1}{2} \) pint milk
- 1 egg, beaten

How much flour do I need in total for my recipe?

\[
\text{Wholemeal Flour required } = \frac{11}{6} \\
\text{Plain Flour required } = \frac{9}{5} \\
\text{Total Flour required } = \frac{11}{6} + \frac{9}{5}
\]

**Step 1:**
Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48 ....
Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45 ....

- Denominator of first fraction = 6
- Denominator of second fraction = 5
- \( \text{LCM } = 30 \)  
  \( \Rightarrow \text{LCD } = 30 \)

1: Find the LCD  
2: Find equivalent fractions  
3: Add the equivalent fractions
**Fraction Addition**

**Step 2:** Let’s look at \( \frac{11}{6} \) first. We know that the LCD = 30, therefore we are looking for a fraction equivalent to \( \frac{11}{6} \) whose denominator is 30;

\[
\frac{11}{6} = \frac{?}{30}
\]

\((6 \times 5 = 30)\), so therefore to change the denominator from 6 to 30 we have multiplied 6 by 5. If we change the denominator then we must make the same change to the numerator otherwise we will have changed the relationship between the numerator and the denominator (changed the proportion of the fraction).

\[
\frac{11}{6} = \frac{?}{30} \\
\Rightarrow \frac{11 \times 5}{6 \times 5} = \frac{55}{30}
\]

Then we do the same for the second fraction \( \frac{9}{5} \);

\[
\frac{9}{5} = \frac{?}{30} \\
\Rightarrow \frac{9 \times 6}{5 \times 6} = \frac{54}{30}
\]

**Step 3:** Now I have two fractions with the same denominator and so we can begin addition:

\[
\frac{55}{30} + \frac{54}{30} = \frac{55 + 54}{30}
\]

\[
= \frac{109}{30}
\]

Therefore the total fraction of flour needed is \( \frac{109}{30} \).

**Question:** Can you rewrite this as a mixed fraction to tell how many cups of flour are needed?
SOLVING WORD PROBLEMS

1. Read the problem carefully.

2. Cross out unnecessary information.

3. Show your work. Don't do it in your head.


5. Re-read your problem and check your answers.

6. Draw a picture that illustrates the problem.

7. Write in your own words how you got your answer.
### Recap:

**Fraction Addition:** Fractions can only be added if they are ‘like fractions’, that is they must have the same denominator. Therefore when you need to add fractions that have different denominators we rewrite them using equivalent fractions. For example, \(\frac{2}{5} + \frac{1}{3}\) is the equivalent of adding \(\frac{6}{15} + \frac{5}{15}\).

### Practice Exercises 3.1:

<table>
<thead>
<tr>
<th>(\frac{1}{6} + \frac{3}{8})</th>
<th>(\frac{1}{2} + \frac{1}{4})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(= \frac{1 \times 4 + 3 \times 3}{6 \times 4})</td>
<td>(= \frac{4 + 9}{24})</td>
</tr>
<tr>
<td>(= \frac{13}{24})</td>
<td>(= \frac{13}{24})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\frac{1}{6} + \frac{3}{5})</th>
<th>(\frac{1}{4} + \frac{5}{8})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(= \frac{1 + 3}{6\times 5})</td>
<td>(= \frac{1 + 5}{4\times 8})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\frac{1}{2} + \frac{5}{6})</th>
<th>(\frac{4}{5} + \frac{1}{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(= \frac{6 + 5}{12})</td>
<td>(= \frac{4 + 5}{10})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\frac{1}{4} + \frac{1}{9})</th>
<th>(\frac{2}{3} + \frac{3}{7})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(= \frac{1 + 1}{12})</td>
<td>(= \frac{6 + 3}{21})</td>
</tr>
</tbody>
</table>

### Exercises 3.1:

**Q. 1.**

To illustrate the fact that \(\frac{2}{8}\) and \(\frac{1}{4}\) are equivalent fractions Mrs. Gomez drew the picture shown on the right. Explain how this picture would suggest that \(\frac{2}{8}\) and \(\frac{1}{4}\) are equivalent fractions.
Exercises 3.1:

Q. 2.
There are eight equal slices of pizza. Justin ate five-eighths of the pizza for dinner. He ate $\frac{1}{4}$ of the pizza for a bedtime snack. How much of the pizza has Justin eaten?

Q. 3.
Conor had €12. He spent two-thirds of his money on a cinema ticket and another one-quarter of his money on popcorn. What fraction of his money did he spend altogether?

Q. 4.
The capacity of Anfield is 45,362. At this Liverpool match $\frac{2}{5}$ of the seats were occupied by Manchester United fans. $\frac{1}{7}$ of the seats were empty. What fraction of the seats were occupied by Liverpool fans?

Q. 5.
An equilateral triangle measures $\frac{1}{7}$ of a metre on one side. What fraction of a metre is the perimeter of this triangle?
Fraction Addition

Example 3.2:

Darren spent 2\(\frac{1}{4}\) hours on his homework on Monday. On Tuesday, he spent 1\(\frac{3}{5}\) hours on his homework. How much time, in hours, did Darren spend doing his homework on Monday and Tuesday?

Step 1:
Add the whole numbers:

\[
2 + 1 = 3
\]

Step 2:
Add the fractions:

\[
\frac{1}{4} + \frac{3}{5} = \frac{5}{20} + \frac{12}{20}
\]

\[
= \frac{5 + 12}{20}
\]

\[
= \frac{17}{20}
\]

Step 3:
Add the answer from Step 1 and Step 2:

\[
3 + \frac{17}{20} = 3\frac{17}{20}
\]

Practice Exercises 3.2:

Q. 1.
Add the following:
(a) \(1\frac{2}{5} + 2\frac{1}{3}\)  
(b) \(2\frac{4}{7} + 3\frac{1}{2}\)  
(e) \(4\frac{1}{6} + 7\frac{3}{5}\)  
(d) \(5\frac{2}{9} + 3\frac{1}{2}\)

The LCD = 20 therefore we must now find the equivalent fractions:

\[
\frac{1}{4} = \frac{1 \times 5}{4 \times 5} = \frac{5}{20}
\]

\[
\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}
\]
Exercises 3.2:

Q. 1.
Miss Lee had her students draw pictures of their homes. \( \frac{5}{16} \) of the students used coloured pencils, \( \frac{1}{8} \) used crayons, and the rest of the class used paint. What fraction of the class did not use paint to draw the pictures?

Q. 2.
Kate has a bag of marbles. The bag only has yellow, green, blue and red marbles. In the bag:
- \( \frac{1}{3} \) of them are green,
- \( \frac{1}{2} \) of them are blue
- \( \frac{1}{10} \) of them are red,
- 30 of them are yellow.

How many marbles does Kate have?
How many marbles of each colour does Kate have?

Q. 3.
Sharon practices two and two-fifths hours of baseball on Sunday and one and four-sevenths hours of baseball after school on Monday. How many total hours of baseball did she practice?

Q. 4.
Sinead is allowed to make no more than three hours worth of phone calls each week. On Monday she rang Kate and they talked for \( \frac{1}{3} \) of an hour. On Wednesday she rang Aoife and they talked for 1\(\frac{1}{5} \) hours and on Thursday she rang her Gran and they talked for \( \frac{1}{2} \) an hour. How many hours did Sinead spend talking (to the people she rang) on the phone?
How many minutes did she spend on her phone calls?
What fraction of an hour has she left for the rest of the week?
## Fraction Addition

**Exercises 3.2:**

Q. 5.

Make a path through each matrix so that the sum of the fractions is equal to the answer. Use only horizontal and vertical trails. You may not have to use all the fractions. Don’t use a fraction more than once.

<table>
<thead>
<tr>
<th>Start →</th>
<th>( \frac{1}{3} )</th>
<th>( \frac{1}{2} )</th>
<th>( \frac{2}{3} )</th>
<th>( \frac{1}{4} )</th>
<th>( \frac{1}{8} )</th>
<th>( \frac{1}{3} )</th>
<th>( \frac{1}{5} )</th>
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<tbody>
<tr>
<td></td>
<td>( \frac{3}{5} )</td>
<td>( \frac{2}{3} )</td>
<td>( \frac{1}{5} )</td>
<td>( \frac{1}{4} )</td>
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</tr>
<tr>
<td><strong>Example</strong></td>
<td>( \frac{1}{2} )</td>
<td>( \frac{2}{3} )</td>
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<td></td>
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<td></td>
<td>( \frac{2}{3} )</td>
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<td></td>
</tr>
<tr>
<td><strong>Answer</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Start → \( \frac{1}{6} \) \( \frac{1}{6} \) \( \frac{1}{6} \)

Start → \( \frac{2}{7} \) \( \frac{1}{7} \) \( \frac{3}{7} \)

Start → \( \frac{3}{2} \) \( \frac{1}{4} \) \( \frac{1}{2} \)

Start → \( \frac{1}{4} \) \( \frac{1}{2} \) \( \frac{3}{4} \)

Start → \( \frac{5}{8} \) \( \frac{1}{4} \) \( \frac{2}{8} \)

Start → \( \frac{5}{9} \) \( \frac{1}{9} \) \( \frac{1}{3} \)

Start → \( \frac{1}{9} \) \( \frac{2}{9} \) \( \frac{1}{3} \)

Start → \( \frac{2}{5} \) \( \frac{4}{5} \) \( 2 \)
Activity 3.1
Fraction Addition

Journal
## Fraction Addition

**Reducing Fractions #2:** Write the fraction of each figure that is dark. Then express the fraction in lowest terms.

<table>
<thead>
<tr>
<th>Fraction of Dark Area in Figure</th>
<th>Fraction written in Lowest Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Fraction 1" /></td>
<td>**( \frac{2}{16} ) \quad \frac{1}{8} )</td>
</tr>
<tr>
<td><img src="image2" alt="Fraction 2" /></td>
<td></td>
</tr>
<tr>
<td><img src="image3" alt="Fraction 3" /></td>
<td></td>
</tr>
<tr>
<td><img src="image4" alt="Fraction 4" /></td>
<td></td>
</tr>
<tr>
<td><img src="image5" alt="Fraction 5" /></td>
<td></td>
</tr>
<tr>
<td><img src="image6" alt="Fraction 6" /></td>
<td></td>
</tr>
</tbody>
</table>


**Fraction Addition**

Shade one half of the figure to the right. How many eighths did you shade? _____

One half \( \frac{1}{8} \)

Shade three eighths of the figure to the right. How many eighths did you shade? _____

Three eighths \( \frac{3}{8} \)

Shade one half plus three eighths of the figure to the right. How many eighths did you shade? _____

\( \frac{1}{2} + \frac{3}{8} = \frac{7}{8} \)

Draw a point on the number line below representing the number one half. Label that point "A."

Draw a point on the number line below representing the number three eighths. Label that point "B."

Draw a point on the number line below representing the sum of one half plus three eighths. Label that point "C."
## Fraction Addition

### Converting Mixed Numbers to Fractions (F)

Write the improper fraction equivalent for each mixed number.

<table>
<thead>
<tr>
<th>Mixed Number</th>
<th>Improper Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \frac{4}{9}$</td>
<td>$3 \frac{1}{9}$</td>
</tr>
<tr>
<td>$7 \frac{1}{2}$</td>
<td>$3 \frac{5}{10}$</td>
</tr>
<tr>
<td>$1 \frac{1}{5}$</td>
<td>$7 \frac{6}{9}$</td>
</tr>
<tr>
<td>$2 \frac{1}{2}$</td>
<td>$5 \frac{1}{9}$</td>
</tr>
<tr>
<td>$8 \frac{1}{5}$</td>
<td>$9 \frac{1}{10}$</td>
</tr>
<tr>
<td>$7 \frac{5}{7}$</td>
<td>$2 \frac{2}{10}$</td>
</tr>
<tr>
<td>$9 \frac{3}{4}$</td>
<td>$2 \frac{1}{2}$</td>
</tr>
<tr>
<td>$2 \frac{2}{5}$</td>
<td>$9 \frac{2}{9}$</td>
</tr>
<tr>
<td>$5 \frac{2}{3}$</td>
<td>$8 \frac{1}{3}$</td>
</tr>
<tr>
<td>$2 \frac{1}{2}$</td>
<td>$8 \frac{5}{7}$</td>
</tr>
</tbody>
</table>
Identifying Fractions

Word Problems #1:

Draw pictures to help solve these word problems.

1. Danny has 4 green jellybeans and 3 yellow jellybeans. What fraction of his jellybeans are green?

   Answer

2. Emma had 6 pink flowers and 1 red flower. What fraction of her flowers are red?

   Answer

3. Nick had 8 red apples and 3 green apples. What fraction of his apples are red?

   Answer

4. Sandy had 5 red balloons and 3 blue balloons. What fraction of her balloons are blue?

   Answer
Fraction Word Problems #8:  
For each problem, draw a picture to help solve the problem.

Name: ____________________________

June picked 14 flowers, and gave one half of them to her friend. How many did she have left?

| __________________________________________________________________________ |
| __________________________________________________________________________ |

Seth checked out 5 library books and read three fifths of them. How many does he have left to read?

| __________________________________________________________________________ |
| __________________________________________________________________________ |

Tabitha had 12 dollars in her bank account, but withdrew one sixth of it to buy a toy. How much money does she have left?

| __________________________________________________________________________ |
| __________________________________________________________________________ |
Fraction Addition

Make a path through each matrix so that the sum of the fractions is equal to the answer. Use only horizontal and vertical trails. You may not have to use all the fractions. Don't use a fraction more than once.

Start: \( \frac{1}{5} \quad \frac{1}{2} \quad \frac{2}{3} \)  
End: \( \frac{12}{3} \)  
Example: \( \frac{2}{3} \)  
Answer:

Start: \( \frac{4}{2} \quad \frac{1}{8} \quad \frac{1}{3} \)  
End: \( \frac{5}{8} \)  
Start: \( \frac{2}{7} \quad \frac{1}{7} \quad \frac{3}{7} \)  
End: \( \frac{5}{7} \)  
Start: \( \frac{2}{8} \quad \frac{1}{8} \quad \frac{1}{8} \)  
End: \( \frac{1}{4} \)  

G Model Chapter - Teacher Booklet
### Adding Fractions #D3:

Add the fractions and put the answers in lowest terms.

<table>
<thead>
<tr>
<th>Fraction 1</th>
<th>Fraction 2</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{8} + \frac{7}{10} = \frac{3 \times 5}{8 \times 5} + \frac{7 \times 4}{10 \times 4} = \frac{15 + 28}{40} = \frac{43}{40} )</td>
<td>( \frac{5}{6} + \frac{3}{8} = )</td>
<td></td>
</tr>
<tr>
<td>= 1 ( \frac{31}{40} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{5}{7} + \frac{3}{14} = )</td>
<td>( \frac{1}{4} + \frac{5}{12} = )</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{6} + \frac{3}{9} = )</td>
<td>( \frac{5}{12} + \frac{7}{8} = )</td>
<td></td>
</tr>
<tr>
<td>( \frac{2}{3} + \frac{5}{9} = )</td>
<td>( \frac{1}{8} + \frac{3}{10} = )</td>
<td></td>
</tr>
</tbody>
</table>
Fraction Addition

\[ \frac{1}{2} + \frac{1}{4} + \frac{1}{4} \]

\[ -\frac{1}{4} - \frac{1}{2} - \frac{1}{2} \]

\[ -0 + \frac{1}{4} \]

\[ \frac{2}{3} - \frac{1}{3} + \frac{2}{3} \]

\[ +\frac{1}{3} + \frac{4}{3} - \frac{1}{3} \]

\[ +\frac{2}{3} - 1 \]

G-XLIII
Add Mixed Numbers With Like Denominators (G)

Add the whole numbers. Reduce the fraction. The whole number stays the same. Add the fractions.

\[
\begin{align*}
3 \frac{1}{8} + 5 \frac{5}{8} &= \\
1 \frac{9}{12} + 6 \frac{1}{12} &= \\
8 \frac{2}{8} + 1 \frac{2}{8} &= \\
1 \frac{1}{6} + 6 \frac{3}{6} &= \\
2 \frac{1}{10} + 9 \frac{3}{10} &= \\
4 \frac{3}{6} + 4 \frac{1}{6} &= \\
3 \frac{1}{12} + 3 \frac{3}{12} &=
\end{align*}
\]

Free Math Worksheets at http://www.math-drills.com
## Fraction Addition

### Add Fractions With Like Denominators (E)

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Fraction</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{7} + \frac{2}{7} = )</td>
<td>( \frac{1}{10} + \frac{6}{10} = \frac{7}{10} )</td>
<td>( \frac{6}{8} + \frac{1}{8} = )</td>
</tr>
<tr>
<td>( \frac{1}{12} + \frac{10}{12} = )</td>
<td>( \frac{2}{10} + \frac{3}{10} = \frac{1}{5} + \frac{1}{5} = )</td>
<td></td>
</tr>
<tr>
<td>( \frac{3}{6} + \frac{2}{6} = )</td>
<td>( \frac{1}{10} + \frac{2}{10} = \frac{1}{6} + \frac{4}{6} = )</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{5} + \frac{1}{5} = )</td>
<td>( \frac{4}{7} + \frac{2}{7} = \frac{3}{12} + \frac{4}{12} = )</td>
<td></td>
</tr>
<tr>
<td>( \frac{2}{4} + \frac{1}{4} = )</td>
<td>( \frac{1}{5} + \frac{1}{5} = \frac{1}{3} + \frac{1}{3} = )</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{3} + \frac{1}{3} = )</td>
<td>( \frac{3}{8} + \frac{4}{8} = \frac{1}{7} + \frac{1}{7} = )</td>
<td></td>
</tr>
<tr>
<td>( \frac{2}{6} + \frac{3}{6} = )</td>
<td>( \frac{3}{8} + \frac{2}{8} = \frac{1}{7} + \frac{5}{7} = )</td>
<td></td>
</tr>
<tr>
<td>( \frac{2}{4} + \frac{1}{4} = )</td>
<td>( \frac{4}{9} + \frac{3}{9} = \frac{7}{12} + \frac{4}{12} = )</td>
<td></td>
</tr>
</tbody>
</table>

Free Math Worksheets at http://www.math-drills.com
Add Fractions With Like Denominators (G)

Add the numerators. Keep the same denominator.

\[
\frac{4}{8} + \frac{2}{8} = \frac{3}{12} + \frac{5}{12} = \]

\[
\frac{2}{12} + \frac{6}{12} = \frac{3}{10} + \frac{5}{10} = \]

\[
\frac{3}{8} + \frac{1}{8} = \frac{3}{9} + \frac{3}{9} = \]

\[
\frac{7}{10} + \frac{1}{10} = \frac{1}{8} + \frac{1}{8} = \]

\[
\frac{2}{6} + \frac{2}{6} = \frac{1}{9} + \frac{5}{9} = \]

\[
\frac{1}{6} + \frac{2}{6} = \frac{5}{12} + \frac{4}{12} = \]

\[
\frac{3}{8} + \frac{3}{8} = \frac{5}{9} + \frac{1}{9} = \]

After you add the fractions, you must reduce the answer. Divide the numerator and denominator by the greatest common factor.

Free Math Worksheets at http://www.math-drills.com
**Fraction Addition**

**Add Fractions With Like Denominators (E)**

1. \( \frac{7}{10} + \frac{5}{10} = \frac{12}{10} = 1 \frac{2}{10} = 1 \frac{2}{5} \)

2. \( \frac{9}{10} + \frac{3}{10} = \frac{4}{6} + \frac{5}{6} = \frac{9}{6} = 1 \frac{3}{6} = 1 \frac{1}{2} \)

3. \( \frac{10}{12} + \frac{4}{12} = \frac{3}{6} + \frac{5}{6} = \frac{8}{6} = 1 \frac{2}{6} = 1 \frac{1}{3} \)

4. \( \frac{6}{10} + \frac{8}{10} = \frac{10}{12} + \frac{5}{12} = \frac{15}{12} = 1 \frac{3}{12} = 1 \frac{1}{4} \)

5. \( \frac{5}{12} + \frac{10}{12} = \frac{8}{9} + \frac{4}{9} = \frac{12}{9} = 1 \frac{3}{9} = 1 \frac{1}{3} \)

6. \( \frac{9}{10} + \frac{7}{10} = \frac{11}{12} + \frac{3}{12} = \frac{14}{12} = 1 \frac{2}{12} = 1 \frac{1}{6} \)

7. \( \frac{4}{9} + \frac{8}{9} = \frac{11}{12} + \frac{3}{12} = \frac{14}{12} = 1 \frac{2}{12} = 1 \frac{1}{6} \)

8. \( \frac{3}{4} + \frac{3}{4} = \frac{4}{6} + \frac{5}{6} = \frac{9}{6} = 1 \frac{3}{6} = 1 \frac{1}{2} \)

Free Math Worksheets at http://www.math-drills.com
Add Fractions (G)

Find equivalent fractions using the least common denominator (LCD).

\[
\frac{6}{9} + \frac{1}{2} = \frac{12}{18} + \frac{9}{18} = \frac{21}{18} = 1 \frac{3}{18} = 1 \frac{1}{6}
\]

LCD: 18

Add.

Change to a mixed number if necessary.

Add.

Reduce the fraction if necessary.

\[
\frac{10}{11} + \frac{5}{9} =
\]

\[
\frac{6}{10} + \frac{1}{5} =
\]

\[
\frac{3}{9} + \frac{5}{9} =
\]

\[
\frac{1}{2} + \frac{9}{10} =
\]

\[
\frac{7}{9} + \frac{2}{7} =
\]

\[
\frac{7}{10} + \frac{4}{11} =
\]

\[
\frac{3}{6} + \frac{6}{11} =
\]

\[
\frac{10}{11} + \frac{1}{4} =
\]

\[
\frac{5}{12} + \frac{1}{2} =
\]

Free Math Worksheets at http://www.math-drills.com
**Fraction Addition**

**Add Fractions (H)**

Find equivalent fractions using the least common denominator (LCD).

\[
\frac{10}{15} + \frac{50}{19} = \frac{190}{285} + \frac{750}{285} = \frac{940}{285} = 3 \frac{85}{285} = 3 \frac{17}{57}
\]

\[\text{LCD: } 285\]

Change to a mixed number if necessary.

Add.

Reduce the fraction if necessary.

\[
\begin{align*}
\frac{39}{7} + \frac{16}{13} & = \\
\frac{21}{12} + \frac{32}{20} & = \\
\frac{41}{16} + \frac{29}{4} & = \\
\frac{13}{17} + \frac{26}{2} & = \\
\frac{16}{5} + \frac{20}{7} & = \\
\frac{40}{16} + \frac{9}{17} & = \\
\frac{35}{12} + \frac{30}{20} & = \\
\frac{33}{10} + \frac{5}{2} & = \\
\frac{21}{4} + \frac{28}{4} & = 
\end{align*}
\]

Free Math Worksheets at http://www.math-drills.com
Adding Mixed Numbers (8)

1) \( \frac{1}{5} + 1 \frac{6}{7} \)  
2) \( \frac{3}{5} + 1 \frac{3}{4} \)

3) \( \frac{4}{7} + 1 \frac{1}{6} \)  
4) \( \frac{3}{5} + 4 \frac{1}{2} \)

5) \( 2 \frac{1}{4} + 2 \frac{7}{8} \)  
6) \( 2 \frac{1}{2} + 1 \frac{6}{7} \)

7) \( 7 + 1 \frac{2}{5} \)  
8) \( 3 \frac{7}{8} + 2 \frac{2}{5} \)

9) \( 1 \frac{1}{2} + 2 \frac{4}{5} \)  
10) \( 2 \frac{1}{5} + 3 \frac{3}{4} \)

Free Math Worksheets at http://www.math-drills.com
Adding Mixed Numbers (6)

1) \( \frac{3}{7} + \frac{20}{21} \)

2) \( \frac{1}{20} + \frac{1}{21} \)

3) \( \frac{5}{19} + \frac{5}{22} \)

4) \( \frac{7}{8} + \frac{11}{14} \)

5) \( \frac{8}{11} + \frac{7}{11} \)

6) \( \frac{1}{15} + \frac{21}{22} \)

7) \( \frac{1}{17} + \frac{4}{5} \)

8) \( \frac{2}{5} + \frac{11}{14} \)

9) \( \frac{8}{9} + 2\frac{1}{2} \)

10) \( \frac{1}{5} + 3\frac{1}{2} \)

Free Math Worksheets at http://www.math-drills.com
Show your work.

1. At my birthday party, the girls ate \( \frac{3}{2} \) pizzas and the boys ate \( \frac{7}{2} \) pizzas. How many pizzas were eaten at my party?

2. I bought \( \frac{2}{2} \) gallons of paint but I only used \( \frac{2}{4} \) gallons of the paint. How much paint do I have left?

3. My recipe calls for \( \frac{2}{3} \) cups of white flour and \( 2 \frac{1}{5} \) cups of whole wheat flour. How much flour do I need in total for my recipe?

4. My dog is \( 5 \frac{1}{2} \) years old. My cat is \( 4 \frac{1}{2} \) years younger than my dog. How old is my cat?

5. During the pie eating contest my dad ate \( 5 \frac{4}{4} \) pies and my mom ate \( 2 \frac{1}{4} \) pies. How many pies did they eat altogether?

6. I need to drink \( 8 \frac{2}{4} \) cups of water and \( 2 \frac{1}{5} \) cups of milk every day. How much fluid do I have to drink?
Appendix H

RMARS - Mathematics Anxiety Inventory
RMARS - Revised Mathematics Anxiety Rating Scale

General Directions:
The purpose of this questionnaire is to measure your attitude towards mathematics and mathematics related activities. By using the scale provided, answer how you FEEL TODAY regarding the items on this questionnaire.

Name: ____________________________ Year/Class: __________
School: ____________________________ Gender: M____ F____

Please read the following statements and indicate by circling one answer. It is vital that you answer each question honestly.

1. Watching a teacher work on an algebraic equation on the blackboard.
   1 Not anxious  2 A little anxious  3 Moderately anxious  4 Pretty Anxious  5 Very Anxious

2. Buying a textbook.
   1 Not anxious  2 A little anxious  3 Moderately anxious  4 Pretty Anxious  5 Very Anxious

3. Being given a homework assignment of many difficult problems which are due for tomorrow’s class.
   1 Not anxious  2 A little anxious  3 Moderately anxious  4 Pretty Anxious  5 Very Anxious

4. Thinking about tomorrow’s mathematics test.
   1 Not anxious  2 A little anxious  3 Moderately anxious  4 Pretty Anxious  5 Very Anxious

5. Solving a fraction addition problem.
   1 Not anxious  2 A little anxious  3 Moderately anxious  4 Pretty Anxious  5 Very Anxious
6. Reading and interpreting graphs and charts.


7. Signing up for an after school mathematics study group.


8. Listening to another student explain a mathematics formula.


9. Walking into a mathematics class.


10. Looking through the pages of my mathematics textbook.


11. Starting a new chapter in a mathematics textbook.

12. Walking onto school grounds and thinking about mathematics class.

1  Not anxious
2  A little anxious
3  Moderately anxious
4  Pretty Anxious
5  Very Anxious

13. Picking up a mathematics textbook to begin homework.

1  Not anxious
2  A little anxious
3  Moderately anxious
4  Pretty Anxious
5  Very Anxious


1  Not anxious
2  A little anxious
3  Moderately anxious
4  Pretty Anxious
5  Very Anxious

15. Reading the word “statistics”.

1  Not anxious
2  A little anxious
3  Moderately anxious
4  Pretty Anxious
5  Very Anxious

16. Working on an abstract mathematical problem such as: “If \( x = \) outstanding bills and \( y = \) total income, calculate how much you have left for recreational expenditures”.

1  Not anxious
2  A little anxious
3  Moderately anxious
4  Pretty Anxious
5  Very Anxious

17. Reading a formula in science.

1  Not anxious
2  A little anxious
3  Moderately anxious
4  Pretty Anxious
5  Very Anxious

18. Taking a summer exam in a mathematics.

1  Not anxious
2  A little anxious
3  Moderately anxious
4  Pretty Anxious
5  Very Anxious

H-IV
19. Getting ready to study for a mathematics test.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not anxious</td>
<td>A little anxious</td>
<td>Moderately anxious</td>
<td>Pretty Anxious</td>
<td>Very Anxious</td>
</tr>
</tbody>
</table>

20. Being given a class test in mathematics.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not anxious</td>
<td>A little anxious</td>
<td>Moderately anxious</td>
<td>Pretty Anxious</td>
<td>Very Anxious</td>
</tr>
</tbody>
</table>

21. Waiting to get a mathematics test returned on which you expect to do well.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not anxious</td>
<td>A little anxious</td>
<td>Moderately anxious</td>
<td>Pretty Anxious</td>
<td>Very Anxious</td>
</tr>
</tbody>
</table>

22. Listening to a lecture on mathematics.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not anxious</td>
<td>A little anxious</td>
<td>Moderately anxious</td>
<td>Pretty Anxious</td>
<td>Very Anxious</td>
</tr>
</tbody>
</table>

23. Having to do fraction addition without a calculator.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not anxious</td>
<td>A little anxious</td>
<td>Moderately anxious</td>
<td>Pretty Anxious</td>
<td>Very Anxious</td>
</tr>
</tbody>
</table>

24. Being told how to interpret fraction addition.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not anxious</td>
<td>A little anxious</td>
<td>Moderately anxious</td>
<td>Pretty Anxious</td>
<td>Very Anxious</td>
</tr>
</tbody>
</table>
Appendix I

Expert Panel Evaluation
My name is Lisa O’Keeffe and I am a postgraduate researcher at the University of Limerick, working with Professor John O’Donoghue. For completion of my doctoral research I am undertaking a study aimed at improving the teaching and learning of mathematics. My study focuses on an analysis of the current Junior Cycle mathematical textbooks, with the aim of identifying the key features which can enhance student learning.

For this study I plan to implement and test a sample selection of an ideal textbook and will initially run a pilot study. The pilot study also requires a number of independent individuals to examine the booklet and based on their opinions, complete a questionnaire which is aimed at obtaining feedback from their experiences of the booklet. All information will be confidential and used only for the purpose intended. Please be advised that these booklets have been peer reviewed.

The questionnaires can be returned to me electronically at lisa.okeeffe@ul.ie or be posted to:
Lisa O’Keeffe
Department of Mathematics & Statistics
University of Limerick
Co. Limerick
If you have any further questions or queries, do not hesitate to contact me.
Yours Sincerely,

Lisa O’Keeffe
University of Limerick
Telephone: 086-1214924
Email: lisa.okeeffe@ul.ie
Expert Panel Evaluation

Please circle the relevant response

1. The objectives of the lesson are realistic and reasonable.

   1 2 3 4 5
   Strongly Disagree        Strongly Agree

2. The layout of the unit is appropriate for student learning.

   1 2 3 4 5
   Strongly Disagree        Strongly Agree

3. The content is relevant.

   1 2 3 4 5
   Strongly Disagree        Strongly Agree

4. The content layout is conducive to student learning.

   1 2 3 4 5
   Strongly Disagree        Strongly Agree

5. The presence of problems within the unit is suitable and effective.

   1 2 3 4 5
   Strongly Disagree        Strongly Agree

6. The colours used are suitable and attractive.

   1 2 3 4 5
   Strongly Disagree        Strongly Agree
7. All diagrams present in the unit have an obvious purpose.

1 2 3 4 5
Strongly Disagree Strongly Agree

8. More references to prior knowledge is required.

1 2 3 4 5
Strongly Disagree Strongly Agree

9. More historical references are required.

1 2 3 4 5
Strongly Disagree Strongly Agree

10. What changes/recommendations would you make to this textbook unit to make it more effective in its role in student learning.

__________________________________________________________________________
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Appendix J

Student Feedback and Evaluation
Parent/Guardian Consent Form

Please read the following information sheet, if you see no reason to prevent your child for completing the questionnaire please indicate so by signing at the end of this letter.

Dear Parent/Guardian,

My name is Lisa O’Keeffe and I am a postgraduate researcher at the University of Limerick, working with Professor John O’Donoghue. For completion of my doctoral research I am undertaking a study aimed at improving the teaching and learning of mathematics. My study focuses on an analysis of the current mathematical textbooks with the aim of identifying the key features which can enhance student learning.

For this study I plan to implement and test a sample selection of an ideal textbook which has been peer reviewed and pilot tested. There are nine mathematics teachers involved in my study, your child’s mathematics teacher is one of these teachers. This study requires six teachers to use the textbook unit provided to teach the topic of fraction addition and three others to act as a control group. Prior to teaching with this booklet all teachers involved will ask each student to complete a questionnaire. This questionnaire aims to identify what your child already knows about the topic of fraction addition and their attitude about mathematics in general. Similar questionnaires will be redistributed following the fraction addition unit. All information will be confidential and used only for the purpose intended.

If you agree to allow your child to complete this questionnaire then please indicate so by signing below, no child will be allowed to complete the questionnaire without consent.

Yours Sincerely,

Lisa O’Keeffe
University of Limerick
Telephone: 061-234788
Email: lisa.okeeffe@ul.ie

I, __________________________ the parent of __________________________, agree to allow my child complete the above mentioned questionnaire.

.[Signing]

J-11
Pupil Opinion

Please circle the relevant response

1. The content of this booklet is challenging.
   1 2 3 4 5
   Strongly Disagree Strongly Agree

2. The layout of the booklet is easy to read.
   1 2 3 4 5
   Strongly Disagree Strongly Agree

3. The problems used are relevant to my life.
   1 2 3 4 5
   Strongly Disagree Strongly Agree

4. The examples used are easy to understand.
   1 2 3 4 5
   Strongly Disagree Strongly Agree

5. The problems provided are interesting.
   1 2 3 4 5
   Strongly Disagree Strongly Agree

6. The references to fractions in the workplace are interesting.
   1 2 3 4 5
   Strongly Disagree Strongly Agree
Please indicate the most relevant answer to you by ticking either a, b, c, or d. Only tick one answer for each question. If you feel you have more to say about any question then add it to the comments section in Q.14

7. The introduction unit (first two pages):
   (a) is pointless
   (b) helps to explain what I will be studying
   (c) outlines what the teacher will be expecting from me

8. The blue box used to tick off the objectives:
   (a) is pointless
   (b) helps explain what I will be studying
   (c) outlines what the teacher will be expecting from me
   (d) lets me get more involved in working with the booklet.

9. The pink extra information (Did you know) boxes:
   (a) are pointless
   (b) give interesting notes
   (c) help me to see the purpose of fractions
   (d) provide extra, helpful information.

10. The yellow vocabulary boxes:
    (a) are pointless
    (b) help me to remember definitions
    (c) help me to use the correct vocabulary
    (d) are not used often enough.

11. The violet internet link boxes:
    (a) are pointless
    (b) do not give enough information
    (c) provide a good source of extra help
    (d) let me get more involved in working with the booklet.
Rate the following by circling the most appropriate number:

12. The real life problems used in this booklet are:

1 2 3 4 5
Really Interesting Boring

13. The real life problems used in this booklet are:

1 2 3 4 5
More interesting than my usual textbook Far worse than my usual textbook

Give as much information as you can to the following questions:

14. If you have some difficulty with your homework, is your current mathematics textbook useful in helping you find the answer?

______________________________

15. Can you explain why the textbook is or is not useful in helping you with challenging questions?

______________________________

______________________________

16. What do you think is difficult about learning fraction addition?

______________________________

______________________________

______________________________

17. Can you explain what you think the term equivalent fractions means, in your own words?

______________________________

______________________________

______________________________
18. Can you explain (in your own words) what you have to do in order to add fractions with different denominators?

________________________________________________________________________

________________________________________________________________________

19. What, in your opinion, should be included in every textbook to help students learn fractions?

________________________________________________________________________

________________________________________________________________________

20. What changes/recommendations would you make to this booklet to make it more effective in helping students to learn?

________________________________________________________________________

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Appendix K

Two-Tier Diagnostic Test Instrument
Calculators are not permitted.

Name: ___________________________ Year/Class: __________
School: ___________________________ Gender: M _____ F _____

Q.1
What is 6 in this fraction \( \frac{6}{11} \)?
(a) Numerator
(b) Denominator
(c) Quotient
(d) Sum

The reason for my answer is:
(1) The numerator tells what type of fraction it is
(2) The denominator tells what type of fraction it is
(3) The numerator tells how much of the fraction it is
(4) The sum tells how much of the fraction it is

Q.2
What are \( \frac{1}{2} \) and \( \frac{1}{3} \)?
(a) mixed numbers
(b) equivalent fractions
(c) reduced fractions
(d) exponential fractions

The reason for my answer is:
(1) They have the same numerator
(2) Their common denominator is 6
(3) They can be simplified down
(4) They can’t be simplified down

Q.3
Which is a mixed number?
(a) \( \frac{81}{100} \)
(b) \( 3 \frac{1}{2} \)
(c) 58
(d) 8.3

The reason for my answer is:
(1) It has mixed numbers
(2) It contains numbers and decimals
(3) It contains numbers and fractions
(4) It is a fraction
Q.4
Which group is equivalent fractions?
(a) \(\frac{1}{2}\) and \(\frac{6}{12}\)
(b) \(\frac{3}{3}\)
(c) 58
(d) 8.3

The reason for my answer is:
(1) They have the same denominators
(2) They contain the same numbers
(3) They are equal in value
(4) They are both fractions

Q.5
What is the simplest expression for \(\frac{8}{12}\)
(a) \(\frac{1}{3}\)
(b) \(\frac{1}{6}\)
(c) \(\frac{3}{4}\)
(d) \(\frac{2}{3}\)

The reason for my answer is:
(1) It is simplified by dividing by 2
(2) It is simplified by subtraction
(3) It is simplified by 3
(4) It is simplified by 4

Q.6
What is the lowest common denominator for \(\frac{1}{3}, \frac{1}{5}, \frac{1}{8}\)
(a) 80
(b) 120
(c) 70
(d) 230

The reason for my answer is:
(1) This is the lowest common divisor of 3, 5, and 8
(2) This is the lowest common multiple of 3, 5, and 8
(3) This is the answer when you have \(3 \times 5 \times 8\)
(4) The lowest common denominator is the same as HCF
Q.7
If $7 = \frac{p}{3}$, what is the value of $p$?
(a) 10
(b) 4
(c) 21
(d) 12

The reason for my answer is:
(1) $P$ tells me how many fractions I am looking for
(2) $P$ tells me how much of the fraction I am looking for
(3) $P$ can be found by adding $7 + 3$
(4) $P$ can be found subtracting three from seven and finding the LCM

Q.8
What is the smallest common denominator of $\frac{7}{8}$ and $\frac{5}{6}$?
(a) 12
(b) 16
(c) 24
(d) 8

The reason for my answer is:
(1) This is the lowest common multiple of 7, 8, 5, and 6
(2) This is the lowest common multiple of 8, and 6
(3) This is the lowest common multiple of 7, and 5
(4) This is the common multiple of 8, and 6 which can be divided by 7 and 5

Q.9
Convert 30 into a fraction which has 5 as the denominator
(a) $\frac{35}{5}$
(b) $\frac{30}{5}$
(c) $\frac{30}{5}$
(d) $\frac{25}{5}$

The reason for my answer is:
(1) 30 is the numerator and 5 is the denominator
(2) There are 150 fifths in 30
(3) There are 25 fifths in 30
(4) $30 + 5 = 35$
Q.10
Which is an improper fraction
(a) $\frac{1}{4}$
(b) $\frac{5}{8}$
(c) $\frac{100}{101}$
(d) $\frac{3}{6}$

The reason for my answer is:
(1) The numerator is larger than the denominator
(2) The denominator is larger than the numerator
(3) It is a reduced fraction
(4) It represents a large fraction

Q.11
Find the answer: $\frac{7}{8} + \frac{5}{8} + \frac{3}{8} + \frac{1}{8}$
(a) 3
(b) 2
(c) $\frac{15}{8}$
(d) $\frac{16}{12}$

The reason for my answer is:
(1) You must find the LCD by multiply the denominators
(2) You can only add fractions which are less than one
(3) You can add fractions once they have the same numerator
(4) You can only add like fractions

Q.12
Find the answer: $\frac{1}{2} + \frac{5}{8} + \frac{3}{4} + \frac{1}{8}$
(a) $1\frac{3}{8}$
(b) 2
(c) $\frac{10}{13}$
(d) $\frac{10}{27}$

The reason for my answer is:
(1) You must find the LCD by multiply the denominators
(2) You can only add fractions which are less than one
(3) You can add fractions once they have the same numerator
(4) You can only add like fractions
Q.13
A family of 6 shares 4 ham and cheese pizzas and 3 cheese pizzas. What fraction of a pizza does each family member get in total if each gets an even share?
(a) \( \frac{5}{6} \)
(b) \( \frac{6}{7} \)
(c) \( 1\frac{1}{6} \)
(d) 1\( \frac{1}{6} \)

The reason for my answer is:
(1) They can each have a pizza to themselves
(2) The common denominator is 7
(3) 7 pizza’s for 6 people is the same as having one pizza each and dividing the other into sevenths
(4) 7 pizza’s for 6 people is the same as having one pizza each and dividing the other into sixths

Q.14
Your mother buys a 750ml (milliliter) bottle of fabric softener, on the bottle it says that it lasts for 25 loads of washing. What fraction of the bottle should be used per wash to get 25 loads?
(a) 30 ml
(b) \( \frac{1}{6} \)
(c) \( \frac{1}{30} \)
(d) \( \frac{1}{4} \)

The reason for my answer is:
(1) 25ml is \( \frac{1}{4} \) of a litre
(2) 30ml is \( \frac{25}{750} \)
(3) 30ml \( \frac{1}{25} \)
(4) 25 loads requires 30ml each time
Q.15
Your mother has noticed that 15ml is enough fabric softener per load of washing. If she only uses 15ml the bottle of fabric softener (from Q.14) would last longer. How many washes would the bottle now last for?
(a) 30
(b) 50
(c) 40
(d) 375

Please explain your answer in detail:
(1) 30ml - 15ml gives 15ml which is half of the original amount of fabric softner so will last twice as long
(2) 30ml - 15ml gives 15ml, therefore 25 loads of washing + 15 gives 40
(3) 25 washes × 15ml gives 375 loads of washing
(4) none of the above

Q.16
Which fraction does the letter x represent on the diagram

\[ \frac{1}{2} \quad \text{x} \quad \frac{15}{16} \]

(a) \( \frac{9}{16} \)
(b) \( \frac{3}{4} \)
(c) \( \frac{5}{8} \)
(d) \( \frac{11}{16} \)

The reason for my answer is:
(1) \( \frac{9}{16} \) is the next step on the number line from \( \frac{1}{2} \)
(2) \( \frac{3}{4} \) is the next step on the number line from \( \frac{5}{8} \)
(3) \( \frac{5}{8} \) is the same as \( \frac{10}{16} \)
(4) None of the above, my reason is __________________________

Before returning this sheet please double check that you have answered every element of each question. If you feel the reason for your answer is not found in the available list feel free to write in your reason however, you must not leave any part unanswered.
Appendix L

Framework for Structure Analysis
## Framework for Structure Analysis

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Appendix M

Distribution of Problems within each Textbook
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Appendix P

School Principal Consent Form
Dear Principal,

My name is Lisa O’Keeffe and I am a postgraduate researcher at the University of Limerick, working with Professor John O’Donoghue. For completion of my doctoral research I am undertaking a study aimed at improving the teaching and learning of mathematics. My study focuses on an analysis of the current Junior Cycle mathematical textbooks, with the aim of identifying the key features which can enhance student learning.

For this study I plan to implement and test a sample selection of an “model” textbook which has been peer reviewed and pilot tested. Having already spoken with you and having emailed on all associated information, three mathematics teacher in your school have agreed to take part in my study. The study requires two teachers to teach fraction addition, using the booklet provided and the other to act as a control group. Prior to teaching with the booklet the students involved will complete a questionnaire aimed at obtaining feedback about their attitude to mathematics and current knowledge about the topic, similar questionnaires will be redistributed following the study. The teachers involved will be asked to track, in journals, their experiences of teaching the topic. All information will be confidential and used only for the purpose intended.

I would really appreciate if could sign this form, giving your consent for my study to run in your school. This form can be returned to the teacher involved who will include it with the returning questionnaires. If you have any further questions or queries, do not hesitate to contact me.

Yours Sincerely,

Lisa O’Keeffe
University of Limerick
Telephone: 061-234788
Email: lisa.okeeffe@ul.ie

I, ___________________________ the Principal of ___________________________ , agree to allow my school to take part in the above mentioned study.
Appendix Q

Interview Transcripts
Interview Questions:

1. What does the word fraction mean?
2. What’s the connection between numbers and fractions?
3. Can every number be written as a fraction?
4. What vocab do you use specifically in fractions?
5. Can fractions be drawn on a number line?
6. Is it possible to add, subtract, divide and multiply fractions the same way you do for numbers?
7. Are fractions useful?
8. Where do you use fractions in your daily live?
9. How do you find a fraction of a quantity?
10. When comparing fractions what must you look at?
11. What is the easiest way to represent fractions with diagrams?
12. What do you think is the most important thing you need to know about fractions?
13. Have you any suggestions about the way fractions should be taught?
Interview Group 1:

Q.1. What does the word fraction mean?
1, 2, 3: ... piece of a whole number (all three respond together)

Q.3. Can every number be written as a fraction?
... Yes (2 respond)
Any number that can’t be?
Any number that you have to ignore or have to leave out,
No No,
Or you can’t use?
No

Q.4. Am what is the vocab that you use when working with fractions?
1: Ah like the ah rect. the bars and stuff?
The vocabulary, so the words that you use?
1: Denominator
1 & 2: Numerator
3: common denominator
Anything else?
Am...
That’s it or anything else?
2: No

Q.6. Okay, am can you add, subtract, divide and multiply fractions the same way you do with numbers??
1: Yes
2: Yes
3: Yes

Q.7. Am are they useful? Fractions?
1, 3: Yes
2: Sometimes
How would you say they are useful? What would you find them useful for?
1: coz. Dividing teams, if there was 25 people, divide them into five groups
3: and cards
They are the main ones?

Q.8. Where do you use fractions in your daily life?
1: am doing teams, sweets, marbles all that dividing them into equal groups
2: An orange dividing into segments
Yeah, anything else?
3: No
Is there anywhere do you think you would actually use the process of adding fractions in your own life?
1: am if am …Don’t think so
You might not have done it but is there anywhere where you think you could or if you could encounter the situation where you might need to
1: When you’d have to add fractions am I don’t think so to be honest
2: No
Q.10. If you are comparing fractions what must you look at?
1: am finding the common denominator (all three respond together)

Q.11. Okay... am what is the easiest way to represent fractions using a diagram?
1: using a diagram, like ah the segments, like putting them into... like like getting a rectangle and if it was two thirds am put two lines and you’d have three segments and colour in two of them
2: am
Does your book draw them any other way or does Mr. Cronin draw them any other way?
2: No
1: there would be, sometimes there would be a pie chart out into fractions, a circle
Okay so a circle would be another way, which way is easier to understand?
2: circle I think,
3: the rectangle
1: I think the bars
Everybody has their own way of thinking about it, what is easier about the circle?
2: I just find it easier

Q.13. Last question, have you any suggestions about how you would improve the way fractions is taught.
1: am... like not make it so complicated really like just to kinda simplify it down a bit.
What's complicated about it?
1: am like it’s the way sometimes there explained in maths books like… the way they are am ...
2: they give examples
1: not really the examples but like the way its said, if it could just be simplifed like Into words you would use?
1: yeah
Anything else?
Am...
Is there anything about the type of questions that you use or that you come across or that you’re expected to do them?
1: like am the way they are done?
The way you are asked to do them, like if had to put together a group, lets say you had to go and teach fourth class for twenty minutes and you had to do fractions with them would you choose to do like am practice questions where you have like one half plus a quarter and do ten of those or would you give them word problems where they have to figure out the question and then try and figure out what they have to add so it wasn’t given directly to them?
3: practice
2: word problems
1: they would probably be the best but the easier would probably be just like the half plus the quarter.
2: get used to the first ones used to the easy ones
3: yeah
1: read the problem first and then do the fractions
And in your normal book, that's action maths isn't it is there a lot of word problems
or is it mostly practice questions?
2: Yeah
1: There's a fair bit like but in the revision chapters now it's just all a, b, c, d like.
Which would you find it easier to work off, do you learn more sorry. Do you learn
more from the practice questions which just gives you the fractions to be added or
would you learn more from the...
1: The fractions just adding coz then you have to figure them out in your head and then
if you don't get them right then you learn how to make them right.
So you learn more from the practice questions than the word problems?
1, 2, and 3: Yeah
Okay I think that's it really does anyone have anything else they want to add?
**Interview Group 2:**

Q.1. So the first one is just what do you think the word fraction means? You can’t be wrong it’s just your opinion.
1: It’s like a part of something like, a certain amount of it
Yeah, any other way of thinking about it?
2: Like its broken up in pieces
Anything else?
No okay.

Q.2. So what’s the connection between numbers and fractions?
3: Am when it’s broken up to pieces am there would be a certain number of them and you’d pick a certain number and put it over the whole amount
Yeah, anything else?
2: Yeah like if you had one big thing and halved it up you would have two halves
Yeah, anything to add?

Q.3. Can every single number be written as a fraction?
All: Yeah
Is there any number that you can’t write as a fraction?
No

Q.5. What are the words that you use specifically when you are working with fractions?
1: Common denominators
Yeah, anything else
3: Not really no
Just common denominators
2: Fraction bar
Fractions bar yeah, okay

Q.5. Can all fractions be drawn on a number line?
3: Yes
Yeah, anybody think not?
2:1: No

Q.6. Can you add, subtract, divide and multiply all fractions the same way you would for normal numbers?
1, 2, Yeah
So there is nothing different? Nothing very obvious? Okay

Q.7. Are fractions useful?
3: Yeah they are useful in real life
What way would you use them?
3: Am … if you were building a house you’d want a certain number of bricks and a fraction of them would be used for one part of the house and another fraction for another.
Yeah perfect. Any other places where you might think fractions could be useful?
1: Maybe in shop like for say like half price or something.
Yeah working out those, anything else?
2: In buildings like doing carpentry and stuff
Interview Transcripts

So you would have to make sure your pieces fit together you’d have to work out fractions of inches, is it inches or which ever they would use

Q.8. Is there anywhere that you would use fractions in your life, outside of the classroom?

1: not really
3: am if you were going shopping with your mother and were buying things a certain fraction of them would be vegetables and a certain fraction of them would be dairy products
Yeah or you would even have to get a certain amount of vegetables,
1: can’t think of any
Is there anywhere where you think you could use fractions but you don’t, anything in real life that you know if someone wanted to give you a real life example what would be a good example of how someone your age could use fractions if they wanted to.
2: in a game of soccer like they might have half of the possession
Yeah that’s a good way to have it, anything else?
3, 1: No
So you would say people your age don’t use fractions that much would you think? Or would you say that they do?
1: not really
Could you use them more than you do?
1: yeah

Q.9. How do you find a fraction of something, just your own way of thinking about how you’d find a fraction of something?

2: divide it up and
Yeah so how would you, you’d divide it up into pieces first. How would you, if I asked you to find a fraction of a whole object?
3: I’d divide it up and find out what the whole was and then multiply it out
Okay
1: am I’d divide up and them multiply by whatever, like if I wanted three fifths I’d divide by 5 and multiply by 3.

Q.10. And when your comparing fractions what is the most important thing that you look at when comparing fractions?

1: common denominator

Q.11. Okay am, what is the easiest way to represent fractions as a diagram most people usually have there own way of thinking about it so if I asked you to draw a diagram of two thirds what kinds of diagram would you draw?

3: am I’d a block and divide it into three pieces
So like a square block or?
3: like a rectangle one
So kinda like a strip?
1: like in our maths copy there’s squares and you could use three of them and then divide
So you’d use squares
2: you could use that yeah
Is there any other thing you could use?
No? Okay

Q-VII
Q.12. Am what do you think is the most important thing for you to know how to add subtract multiply and divide fractions. What’s the most important thing that you need to know about fractions?
3: how to find the common denominator
2: yeah like if you have two quarters and three sixths you’d have to round it up to like twelfths
Okay and specifically for adding fractions then?
3: am when adding fractions it’s important to know the common denominator for sums with fractions its important to know how to get it
So you think the common denominator is the most important thing for fractions in general
All: yeah
Okay

Q.13. And have you any suggestions about how fractions should be taught? When you start learning fractions in fourth class?
Any suggestions about how teachers could put fractions to you in such a way that it would be easier to understand?
1: no
3: no
Any changes you would make to action maths the book that you are using?
2: no it’s good
1: no
Okay and the last question is just the types of questions that you do when you are working with fractions
okay so we called them practice questions the last time you know where you just get like a half plus a third and you might get a block with like ten of those to do or alternatively you can get word problems where you have to figure out what fractions you have to work with and then figure out what you have to do with the fractions and then do the routine procedure. Which one of these do you do most often?
3: the block one
The word problem or the practice ones?
3: the practice questions
And which do you find most helpful to you when you want to learn?
3: when I was learning fractions I found the practice ones easier Easier, and would you say now that you know how to do fractions which do you think you actually learn more from the practice one or the word problems
3: the word ones are easier now
2: word ones
The word ones are easier now? And do you think they are more beneficial to you to than the practice ones or would the practice ones?
3: the practice ones are generally simple
Okay, so you all feel the same? That you get more out of the word problems or do you get more from doing the practice problems?
Yeah
Both of them?
2: the word ones
Sorry I couldn’t hear you, so you think; ye all think that the word ones are better?
All: yeah
*Okay, I think that’s it does anyone want to add anything?*
All: no
Interview Transcripts

Interview Group 3:

Q.1. What do you think the word fraction means? Anybody?
1: like it’s a part of something
Part of something yeah, any other way of thinking about it?
1: a segment
Okay will I move on?

Q.2. What do you think is the connection between numbers and fractions? You can’t be wrong its just your opinion.
2: a fraction is half of is something of a number
Of a number, yeah so you’re working on a piece of a number anything else?
No? Okay

Q.3. Can every single number be written as a fraction?
12: yes
What do you think?
3, 4: yes
So there is no number that you can not write?
Shake heads
Okay
Q.5. What words or vocab do you use, what was that?
2: zero, coz like you can’t get a fraction of zero
You can’t get a fraction of zero? Yeah very good, you’re the first person actually that said that one.

Q.4. What vocab or words do you use only when you’re working with fractions, so words that are specific to fractions? Is there any words?
2: like a quarter of something
A quarter yeah
4: half
A quarter is specific to fractions
4: a half
Half yeah, all of those. Is there any other words?
3: whole number
Whole number yeah
1: mixed fractions yeah
Mixed fractions
2: improper fractions
Yeah, any more? Getting way more out of ye!

Q.5. That’s it we’re out of them, okay am can every single fraction be drawn on a number line?
1: yeah?
Everybody?
2, 3, 4 ah yeah? Yip unhhm
Is there any fraction that you don’t think can be drawn on a number line?
2: well ten tenths or something just be written as far as ten
Ten tenths? Would be?

Q-X
2: or eleven tenths
Eleven tenths
Yeah could you expand your number line to fit it or?
2: yeah
Okay so every single number be it a fraction or a whole number, everything you think can be graphed or can be drawn on the number line?
1: yeah

Q.6. Do you add subtract divide and multiply fractions in the exact same way that you would add, subtract, divide and multiply normal numbers.
No
Okay what’s different?
2: in the division you turn one over and then you multiply
Okay so technically you can’t do division of fractions
2: no
Anything else that’s different?
2: am when you multiplication its not as simple or no no no in the plus, adding you kinda have to find the common factor of the,
1: denominator
2: common denominator and then you go like am and then you go through
So its not as easy as just adding
1: u can’t just minus them straight away
You have to figure them out first, yeah.

Q.7. Are fractions useful?
3: yeah
4: yes
How would you find them useful?
4: am because they involve everything in your life,
3: yeah
4: like there used everyday
Do you think there useful?
3: yeah
Okay, where would you use fractions? Where could anybody use fractions outside of the classroom, just a general for anybody now?
3: if you were cutting a cake
Yeah
4: if you were diving sweets
Okay so dividing sweets and food is a big one, anything else? Anybody in the whole world not just someone that’s?
1: if you were taking prices off clothes in a sale you’d use fractions
2: getting a discount
Anything else?
2: money?
So like your fifty cent is a fraction of your euro, anything else? Okay.

Q.8. Where do you use fractions in your life? Okay do you use fractions first of all, would you say you actually use fractions outside of the classroom?
1: like sometimes you wouldn’t use them everyday
Where would you use them?
1: like if you were in a shop or anything if you were buying something that’s half price
So you’d actually be processing the fractions
Would you use fractions?
4: yeah
Where would you use fractions?
4: like food like you want to half it for another person
To share?
4: yeah
Anything else?
4: am if you were sharing money
Sharing out money yeah. And would you use fractions in your life?
3: am I dunno I suppose not really
Not really?
Is there anywhere where you think you could use fractions but you don’t? No okay?
Is there anywhere in real life where you think you might actually add two fractions together?
Its just your opinion.
Is there anywhere you think you could, outside the classroom, see yourself adding two fractions?
3: well like if you were splitting food like and had a few different types of food
Yeah
3: and then
Trying to share it out evenly, Yeah
4: if you am there was two people and they put their money together it would make like a bigger fraction then
Yeah so if they were saving for something in particular
2: or if were like wanted to buy like more than one thing in a shop
1: ud have to split up your money
You’d have to figure out someway of putting your finances together to afford it.
Anything else?
Okay

Q.9. How do you find a fraction of a quantity?
1: am like if looking for an eighth of something you’d divide it by eight an multiply by whatever
Is there any other way of thinking about it
2, 3, 4, no
Okay so that’s how ye would all go about it?

Q.10. Am if you’re comparing fractions what’s the most important piece that you look at?
2: am you must like find the common denominator its like, if you were comparing two quarters or a half you find like, if you put wrote them out and put like two goes diagonal and goes to each
1: there the same thing but they look different
Anything else to add?
3, 4: no
Q.11. Am what is the easiest way to represent fractions as a diagram, people usually have there own ways of drawing fractions so if I asked you to draw two thirds or something on a diagram what kind of a diagram would you use?
1: am two thirds, like two over three
So if I asked you to draw it as a picture to show me instead of to tell me what it was?
1: am you could say that there’s three things like say there’s three apples and two are coloured but one isn’t so that means it would be two thirds
So you would use objects, you use objects to show it. Anybody use anything different to draw fractions?
2: maybe pie charts?
You could use it kinda like just a circle and split it up yeah, anything else?
2: or you could do like a fraction wall or something.
Yeah a fraction wall that’s another good way of doing it. SO a fraction wall uses rectangles doesn’t it, it splits them up into strips. Anything else?
2: no
1: no

Q.12. Okay, only two questions left.
What do you think, and this is your own personal opinion, is the most important thing you need to know about fractions, just fractions in general.
1: changing fractions to am percentages
Okay
1: like you’d see that they’re actually the same thing if you break them down
And what do you think is the most important thing?
2: like that and changing them to decimals (whispered help from 1)
Yeah?
3: How to add fractions
Yeah, so what’s the most important thing about adding fractions?
3: that you can join things
4: am like mixed numbers like you can change them,
You can work between yeah
And what’s the most important thing then if you were to just think about adding fractions, just the process of adding fractions what’s the most important thing that you need to know in order to be able to add fractions?
1: common denominators
2: be able to add
Be able to add yeah that’s a good one you’re not going to get far without that.

Q.13. And the last one then have you any suggestions about how fractions are taught and how it could be improved a little bit, what would make it easier for you?
4: if you bring it down to lowest form when you get a big number
So if we just think of your maths book which is Action Maths 6, so if you think that inside of that book there are two types of questions that you usually get and one block I’d call practice questions where you just get like a half plus a third where you get the fractions given to you and you are asked to add them or subtract them. However the other ones then would be like where you’re given a word problem and you have to figure out what the question is asking you. Which do you think is the easiest of those questions?
1: when you actually get the numbers
When you get the practice questions so is it?
1: you don’t have to work it out
2: I like the ones with the story
You prefer the story ones? Yeah okay so if you, for yourself when you’re doing those
type of questions which do you think you learn more doing? DO you learn more doing
the practice questions when you’re given the numbers of do you learn more when
you’re given the word problems?
1, 2: the word problems
1: you have to work a bit harder
Do you all agree with that or?
3, 4, and 2: yeah
So you all think the word problems, that you get more from them, you learn more from
them but which do you do more often?
4: the other ones
1, 3; yeah, the ones your given
And in your maths book is more practice problems or more word problems
3, 4: practice problems
1: amm
2: be like am, there might be like 1 a, b, c and all that and then 2 a,b and all that up to
5 and then they would four questions would be word problems
So you’d probably have thirty or more practice problems and then four word
problems?
All: yeah
And if you were changing that textbook would you leave it the same or would you
change it in anyway?
3: I’d put in more of the word problems
2: instead of like a,b,c,d,e, I’d just put a,b coz like you’d probably get the point after
the second one.

Yeah fair enough, is there anything that anybody would like to add about adding
fractions, or learning fractions or fractions in general, any improvements that you
think you can make?
No okay.
Appendix R

Academic Publications and Presentations
R.1 Publications


R.2 Conference Presentations


Academic Publications and Presentations