ON THE THERMAL AND HYDRODYNAMIC CHARACTERISTICS OF LIQUID-LIQUID TAYLOR FLOWS


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Supervisors
Dr. Vanessa Egan and Dr. Jeff Punch
Stokes Institute
Department of Mechanical, Aeronautical and Biomedical Engineering
University of Limerick

Submitted to the University of Limerick, April 2014
Declaration
The substance of this thesis is the original work of the author, and due reference and ac-
knowledgement has been made, where necessary, to the work of others. No part of this
thesis has been submitted in canditure for any degree.

________________________________________
Marc Mac Giolla Eain (Candidate)

________________________________________
Dr. Vanessa Egan (Supervisor)

________________________________________
Dr. Jeff Punch (Supervisor)

This thesis was defended on the 27th of May, 2014.
Chairman Dr. Jeremy Robinson University of Limerick
External Examiner Prof. Anthony Robinson Trinity College Dublin
Internal Examiner Prof. Harry Van den Akker University of Limerick
Abstract

Two phase liquid-liquid flows offer significant heat and mass transfer enhancements over single phase flows and, as a result, have found use in numerous emerging technologies employing microfluidics. Such technologies include lab-on-chip devices for chemical and biological diagnostics, and biosensors. Liquid-liquid flows have also shown potential for use in high-heat flux removal systems. Although these flows are found in numerous applications, little is known about the complex fluid mechanics that govern them. Consequently, there is a need for a greater knowledge base to serve as a foundation for future system design and characterisation. This thesis presents a fundamental investigation of the hydrodynamic and thermal characteristics of liquid-liquid slug or Taylor flows confined to minichannel geometries.

There were three principal aspects to this thesis, which encompassed the measurement of film thickness, pressure drop and heat transfer in liquid-liquid Taylor flows. Experiments were carried out using a number of different carrier fluids – while maintaining water as the dispersed phase throughout. Dimensionless slug length, Capillary and Reynolds numbers were varied over several orders of magnitude.

High speed imaging was used in conjunction with microscopy to measure the mean slug velocity and liquid film thickness. Images of the dispersed slugs revealed that the thickness of the liquid film was not constant along the length of the slug. However, above a threshold dispersed slug length, a region of constant film thickness existed. The thickness of the film was found to be heavily dependent on the Capillary number. Analysis of the experimental data revealed that it fell into two distinct flow regimes: a visco-capillary regime and a visco-inertial regime. A modified Taylor’s Law is proposed for flows in the visco-capillary regime, while a novel correlation – based on the Capillary and Weber numbers – is put forward for flows in the visco-inertial regime.

The pressure drop induced by the liquid-liquid flow regimes was measured using a differential pressure transducer, and the results were compared to the most referenced correlations in the literature. Comparisons highlighted a lack of robustness in the liquid-liquid pressure drop correlations. Interpretation of the data using liquid-gas Taylor flow correlations unearthed a threshold viscosity ratio, above which liquid-gas correlations may be used to model the flow. Below this threshold, a modification to an existing correlation is proposed, where the interfacial pressure drop is normalised by the volumetric channel fraction occupied by the carrier phase.

A heat transfer facility was designed and commissioned to subject the flow to a constant wall heat flux boundary condition. Local temperature measurements were acquired using a high resolution infrared thermography system. Slug length and film thickness were found to have a significant effect on the local heat transfer rates, with enhancements up to 600% over conventional Poiseuille flow noted. Nusselt number oscillations were observed in the lower Capillary number flows. However, these oscillations damped out as the Capillary number, and hence film thickness, increased. Based on the characteristics identified, a novel correlation is proposed to model the flow in the thermal entrance and fully developed regions.

The findings of this thesis are of fundamental and practical relevance for the design of systems and devices incorporating liquid-liquid Taylor flow regimes.
Acknowledgements

I would like to thank my supervisors, Dr. Vanessa Egan and Dr. Jeff Punch, for providing me with the opportunity to pursue this research and for all their encouragement and guidance over the past three years. I would also like to thank them for allowing me the time and space to work in my own way, to say that it required some monumental patience on both your parts would be an understatement.

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Thanks to my fellow postgrads within Stokes for creating a fantastic working environment and ensuring there were plenty of distractions over the last three years! Pitch and putt anyone? Few quiet pints tonight? To name and thank everyone that assisted me over the course of this work would require a chapter all to itself and I’m sure I would leave someone out, so thank you to everyone who has helped me over the past three years. However, not to mention the following people would be an injustice to the help they gave me. Thanks to Dr. Bubbles for answering my endless questions, no matter how stupid they were! Thanks to Conor Mac for the great discussions on all things liquid-liquid and GAA related. Thanks to Seamus and Ellen for putting up with me, feeding me and everything in-between. Someday in the very very very distant future I will thank Nick Jeffers and Ger Kelly for convincing me to do a PhD.

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1 Variation in transducer output over time for a dodecane/water Taylor flow regime.
## Nomenclature

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$\nu$  Kinematic Viscosity  \( \text{m}^2/\text{s} \)

$\Phi$  Two Phase Multiplier  -

$X$  Martinelli Parameter  -

**Subscripts**

A  Apparent

act  Actual

B  Bubble

BM  Bulk Mean

C  Continuous Oil Phase

c  Curvature

D  Dispersed Water Phase

Dev  Developed

Ent  Entrance

exp-total  Total Experimental

$f$  Film

G  Gas

h  Hydrodynamic

in  Inlet

Int  Interfacial

L  Liquid

m  Mean

Nat  Natural

out  Output

Plug  Plug Flow

Pois  Poiseuille Flow

r  Local Radial Position
S  Slug Flow
s  Surface
st  Stored
T  Total
TP  Two Phase
U  Unit
w  Wall
x  Local Axial Position
Chapter 1

Introduction

Over the past two decades, the potential financial and analytical benefits attributed to the scaling down of numerous large scale processes and devices has led to significant research in the area of microfluidics. Fluidics is a broad field that encompasses physics, chemistry, biology and engineering, while microfluidics involves studying the behaviour of fluids at the microscale. At present, there is considerable interest in the development of micro-electro mechanical systems (MEMS) which, in the most general form, are miniaturised mechanical and electro-mechanical devices and structures, that can vary from as little as 20 μm up to 1 mm in scale. MEMS have applications in a number of different fields including chemical and biological diagnostics, communications and sensing. The motivations in transferring existing technologies to the small scale are numerous, and include portability of devices and substantial savings in cost, for both the consumer and the manufacturer. Over the last decade, multiphase flow regimes have become a prominent feature within devices that incorporate microfluidics. As the name suggests, a multiphase flow regime consists of multiple phases – solid, liquid or gas – flowing together. This thesis presents a fundamental investigation of the thermal and hydrodynamic characteristics of liquid-liquid multiphase flow regimes confined to minichannel geometries. Liquid-liquid flows have potential use in numerous application areas, however, the focus of the work in this thesis is for use in:

1. Chemical and biological diagnostics - where chemical or biological samples are separated into distinct fluidic samples, and are separated from one another by an organic
carrier phase. Although significant attention has been focused on the outputs of the systems and devices that incorporate these flow regimes, little is known about the complex fluid dynamics that govern the flow within them; and

2. High heat flux removal. Thermal management has emerged as a critical component in the performance and reliability of numerous electronic systems and devices. System level heat fluxes are approaching the limits of conventional forced air cooling, and there is a need to develop alternative cooling techniques.

The remainder of this chapter is divided into five sections. Section 1.1 presents and defines the different types of multiphase flow regimes. Sections 1.2 and 1.3 overview the potential use of liquid-liquid flows in diagnostic, 1.2, and electronics cooling, 1.3, applications. The primary objectives of this thesis are stated in section 1.4, and the chapter concludes with an overview of the remainder of this thesis in section 1.5.

## 1.1 Multiphase Flows

A multiphase flow regime is generated when two immiscible phases flow together in a channel at a variety of flow rates. The generated flow regimes are the result of a balance between interfacial, viscous, inertial and gravitational forces, and depend on a number of parameters including flow rate, fluid properties, channel diameter and surface roughness, Tsaoulidis et al. (2013). The most commonly observed flow regimes, presented in Fig. 1.1, include: bubbly, droplet, slug, annular, stratified and wavy. Much of the early research into two phase flows was on developing an understanding, and prediction of the formation of the different flow regimes. As a result, authors have developed flow maps, which act as a predictive tool that graphically illustrate the transitions between the different flow regimes.
a) Bubbly flow

Figure 1.1: Images of typical flow regimes encountered in liquid-liquid flows, Jovanovic et al. (2012).

b) Droplet flow

c) Slug flow

d) Annular flow

e) Stratified flow

f) Wavy flow

Fig. 1.2 presents a flow map developed by Jovanovic et al. (2012) which identifies the dependencies of the anticipated flow regimes on the Weber numbers of the constituent liquid phases. (The Weber number is a non-dimensional number that presents the balance between the inertial and interfacial forces of the flow). Flow maps are a useful tool, however they are not universal between geometries and scales. Consequently, the flow patterns seen at the mini and microscale are different from those observed at the macroscale. At the mini/microscale, the dominance of the surface forces over gravitational forces results in slug and droplet flow regimes being the most commonly encountered.
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Figure 1.2: A flow map put forward by Jovanovic et al. (2012) for a circular microcapillary with an internal diameter of 250 μm.

Liquid-liquid slug flow or, as they are more commonly known, Taylor flow regimes, moving in horizontal channels are the focus of this thesis. A liquid-liquid Taylor flow regime, presented in Fig. 1.3, consists of a continuous carrier phase that is segmented into slugs by an immiscible dispersed phase, which also forms slugs. This segmentation of the liquid phases into separate fluidic packages has a significant effect on the local and global fluid dynamics, resulting in enhanced radial transport of heat and mass, Janes et al. (2010), compared to single phase flows. Consequently, Taylor flow regimes have found use in numerous application areas including chemical and biological diagnostics and have also shown potential for cooling of high heat flux electronics, Walsh et al. (2010). The use of Taylor flows in these application areas will be discussed in the following sections.

Figure 1.3: Image of a liquid-liquid Taylor flow regime in a minichannel geometry.
1.2 Biological Diagnostics

In 1990 the US Department of Energy and National Institute of Health, in conjunction with an international consortium of geneticists from the UK, France, Australia and Japan, announced plans to map the human genome. The human genome is the complete set of genetic information of a human, and it is encoded in a person’s DNA. Decoding the sequence of DNA provides information regarding how organisms survive and the genetic factors in disease. The project was declared completed in April 2003 and a sequenced map of the human genome was published. The success of the project has had a major impact on clinical diagnostics, with the discovery of over 1,800 disease genes and more than 350 biotechnology based products in clinical trials. These products have the potential to impact many areas of life, from providing cost effective health screening to modifying crops in order to make them resistant to certain pests and diseases. However, significant advances are required in the fields of engineering, molecular biology and chemistry before this potential can be realised.

The area of small scale, cost effective clinical diagnostics has attracted noticeable attention in recent years. This new approach to processing large numbers of samples in serial format makes use of two phase flows, or more specifically liquid-liquid two phase flows. This methodology - where chemical or biological samples are separated into discrete separate streams of droplets or slugs by a segmenting fluid - has garnered much attention in the literature, with numerous publications from authors such as Dong et al. (2001), Lo (2009), Yung et al. (2009) and Henkel et al. (2004), assessing its potential. The benefits of this approach over the single phase continuous flow methodology are numerous and include: reduced sample size, reduced consumption of expensive reagents, elimination of cross contamination between samples, reusability of devices, enhanced mixing within samples, and reduction in processing times, to name but a few. The potential of this methodology is clear, for example if one sample is added every second, using a single channel, over 80,000 samples could be processed in a day. However, many engineering and technological issues – such as flow generation and stability, pressure drop, heat transfer characteristics and data processing – need to be overcome before the full potential of this methodology can be realised, Jensen and Lee (2004).
Although numerous publications exist where liquid-liquid flows are used to perform chemical and biological diagnostics, little research work has been completed examining the fundamental governing characteristics of these flow regimes. For instance, separating these samples from the channel walls is a very thin film of the segmenting fluid, which protects the samples from any asperities within the channels and aids in the prevention of cross contamination between samples. Numerous works have been undertaken to measure and characterise the thickness of this film in liquid-gas flows, which have yielded correlations to predict its thickness. These correlations are used in the design of microfluidic devices that employ liquid-liquid flows. However, little is known about the thickness of the film in liquid-liquid flows and the applicability of these correlations to liquid-liquid flows as there are only a handful of publications examining this topic. Similarly, there are numerous correlations to predict the pressure drop in a liquid-gas flows, while the topic has received significantly less attention in liquid-liquid flows. To this end, work is required to develop a thorough understanding of the governing flow parameters in liquid-liquid flows and to determine if they differ from those in liquid-gas flow regimes. This work will aid greatly in the design of future technologies that employ liquid-liquid Taylor flow regimes.

1.3 Thermal Management

Contemporary electronic devices and systems permeate virtually every aspect of our lives. These devices and systems can generate high level heat fluxes at the component level, which is the result of two main trends: the escalation of power dissipation from components, and the reduction in the available surface area from which to transfer heat. Today’s microprocessors can generate heat fluxes of order 100 W/cm², while in high end microprocessors this value can be closer to 300 W/cm², Asthana et al. (2011). Consequently, a significant body of research exists that is focused solely on the development of high heat flux removal technologies. The primary objective of these technologies is to dissipate these high heat loads while maintaining components within safe operating temperature limits. For decades this has been accomplished using forced air convection cooling, the benefits of which include: simplicity, low cost, ease of maintenance and high reliability, Egan et al. (2009) and
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Walsh et al. (2008). However, escalating power dissipation levels necessitate higher air flow rates and hence, larger and more powerful fans, thereby exacerbating the problems associated with profile, noise and vibration control. Hence, conventional forced air convection cooling techniques are no longer sufficient, thus driving the need for alternative cooling modes to the fore. At present, liquid cooling is seen as the prevailing alternative. Liquid cooling yields significantly enhanced heat transfer coefficients compared to air based counterparts, thus allowing further advances in processor performance and miniaturisation. Examples of liquid cooling include: cold plates (Jeffers et al. (2007)), impinging jets (Jeffers et al. (2009)), spray cooling (Fabbri and Dhir (2005)), microgap cooling (Rahim et al. (2012)) and microchannels (Morini (2004)), with microchannels seen as the pre-eminent solution.

The use of microchannels for high power density cooling has been widely studied since the pioneering work of Tuckerman and Pease (1981). Theoretically, a microchannel flow possesses high surface-area to volume ratios, thereby generating high heat transfer coefficients. However, a result of the reduced channel dimensions is a laminar flow regime, Squires and Quake (2005), hence Nusselt numbers do not exceed those of equivalent macroscale systems. Accordingly, microchannel research is now focused towards methods of enhancing heat transfer rates above macroscale systems. Potential enhancement methods include the addition of: specifically engineered nano-particles to enhance the thermal conductivity of the fluid (Anoop et al. (2009)), micro-encapsulated phase change materials (Zeng et al. (2009)), vortex promoters to the channel surface (Icoz and Jaluria (2006)) and multiphase flows (Janes et al. (2010)), with multiphase flows thought to offer the best potential for increased heat transfer rates. Thermally, multiphase flows have been studied from both boiling and non-boiling perspectives. In a boiling multiphase flow regime, the surface temperature exceeds the saturation temperature of the coolant. This results in a change in phase within the heat exchanger, which generates very high local heat transfer coefficients. However, boiling heat transfer can be difficult to control and quantify; and it requires sealed vessels for containment. Nonetheless, a considerable body of research exists in this area, Harirchian and Garimella (2011), Liu and Garimella (2007) and Thome (2004). Alternatively, the use of non-boiling multiphase flows to remove high heat loads.
has received considerably less attention in the literature.

Some of the earliest works examining the heat transfer rates in non-boiling two phase slug flows were those of Prothero and Burton (1961), who noted that segmenting a continuous stream of liquid with gaseous bubbles was almost twice as effective in transferring heat as single phase flow. Oliver and Wright (1964) reported enhancements up to 2.5 times above that of single phase flows, however, more recent studies by Howard et al. (2011), Walsh et al. (2010) and Majumder et al. (2013) have reported heat transfer rates up to six times that of single phase flows. In the majority of these non-boiling Taylor flow studies, a continuous liquid stream is segmented by gaseous bubbles. However, the thermophysical properties of gases render them less attractive than liquids as coolants. Their comparatively lower density, \((\rho \approx 1.2 \text{ kg/m}^3)\), and specific heat capacities, \((c_p \approx 1 \text{ kJ/kg K})\), curtail their effectiveness in heat removal by convection. Water is a common liquid coolant and possess thermophysical properties much more conducive to cooling by convection, with higher densities, \((\rho \approx 900 \text{ kg/m}^3)\) and specific heat capacities, \((c_p \approx 4 \text{ kJ/kg K})\), providing a greater capacity to transfer heat. Therefore, replacing the gaseous phase with an immiscible liquid phase should result in improved heat transfer rates over liquid-gas Taylor flows. Asthana et al. (2011) examined the potential of liquid-liquid Taylor flows for the removal of high heat loads, reporting a four fold increase in the Nusselt number over single phase flows. Although the authors report a number of interesting findings, the work was very limited and was designed to assess the potential of liquid-liquid Taylor flows in removing high heat loads. Consequently, a number of questions were left unanswered, such as the effects of slug length and carrier phase variations on the thermal behaviour of the flow. As a result, work is required to determine the thermal characteristics of liquid-liquid Taylor flows.

1.4 Objectives

This thesis presents an investigation of the thermal and hydrodynamic characteristics of a liquid-liquid Taylor flow regime confined to a minichannel geometry. Two immiscible liquids, an organic carrier phase and water, are pumped into a circular channel to create a continuous stream of mono-dispersed water slugs in an oil phase. The effects of slug length
and continuous oil phase variations on the thermal and hydrodynamic characteristics are analysed. The objectives of this thesis are to:

- Measure the thickness of the film that separates the dispersed slugs from the capillary walls and assess the influence of slug length and carrier phase thermophysical properties upon its magnitude.

- Validate existing film thickness correlations from the literature and, if necessary, develop new correlations to predict the thickness of the film.

- Measure the pressure drop induced by a liquid-liquid Taylor flow regime and determine the accuracy of a number of analytical and semi-empirical pressure drop models, establishing, if necessary, modifications for liquid-liquid Taylor flows.

- Complete a thorough thermal characterisation of liquid-liquid Taylor flows subject to a constant wall heat flux boundary condition, assessing the influence of slug length, film thickness and Prandtl number variations upon the heat transfer rates attainable.

- Develop a novel model to predict the thermal behaviour of liquid-liquid Taylor flows over a wide range of parameters.

The conclusions from these objectives will contribute greatly to the knowledge base pertaining to liquid-liquid Taylor flows.

1.5 Thesis Structure

The remainder of this thesis is divided into six chapters. Chapter 2 presents the theory relevant to two phase internal channel flows. The experimental facilities and measurement techniques used in this thesis are detailed in Chapter 3. Chapters 4 and 5 address the hydrodynamics of liquid-liquid Taylor flows, with the thickness of the liquid film examined in Chapter 4 and the associated pressure drop analysed in Chapter 5. Included in both chapters are detailed assessments of a series of analytical and semi empirical correlations from the literature. Chapter 6 presents a thermal characterisation of liquid-liquid Taylor flows subject to a constant wall heat flux boundary condition. Finally, Chapter 7 details
the primary conclusions from this thesis and a number of recommendations to further this work.
Chapter 2

Theory

This chapter introduces a number of theoretical concepts relevant to the study of single and two phase flows and is separated into three main sections. The fluid dynamics that result in the enhanced heat transfer rates generated by Taylor flows are quite complex and require a comprehensive understanding of the fundamental thermal and hydrodynamic characteristics of single phase flows. These theories are detailed in section 2.1.

The addition of a second immiscible fluid stream creates a mono-dispersed flow structure of aqueous slugs dispersed in a continuous carrier phase. Consequently, there are a number of flow features unique to Taylor flows regimes. These features, and their resultant effects on the hydrodynamic and thermal boundary layers, are presented in section 2.2.

The nature of the flow is the result of viscous, inertial and interfacial force interactions, where the relative magnitudes of these forces control the dynamics of the flow. Dimensionless numbers provide a means of quantifying the magnitudes of these forces relative to each other and, presented in the final part of this chapter, are the relevant dimensionless numbers used in the thermal and hydrodynamic characterisation of Taylor slug flows.

2.1 Single Phase Flow in Capillaries

The principles of single phase flow in circular capillaries are detailed, from a hydrodynamic and thermal perspective, in this section. These principles provide an insight
into the flow conditions encountered within the capillaries used during experimentation in this thesis. The analysis begins by focusing on the formation of the hydrodynamic boundary layer, and is followed by a brief description of the relevant modes of heat transfer and thermal boundary layer development.

2.1.1 Fluid Dynamics

The flow regimes in microfluidic technologies, especially those used in clinical diagnostics, are laminar in nature. A laminar flow regime is characterised by fluid streams that flow parallel to each other and mix only through convective-diffusive mass transport. Steady laminar flow in a circular capillary is governed by Hagen-Poiseuille Law, which defines the velocity, \( u(r) \), at any given point by:

\[
    u(r) = -\frac{1}{4\mu} \left( \frac{dp}{dx} \right) \left( R^2 - r^2 \right)
\]

(2.1)

where \( \mu \) represents the dynamic viscosity, \( dp/dx \) is the streamwise pressure gradient, \( x \) is the co-ordinate system in the direction of the flow, \( R \) is the radius of the capillary and \( r \) is the radial distance from the centre of the capillary, Massey (1998). As the velocity varies over the cross-section of the capillary, it is necessary when examining internal flows to define the mean velocity, \( U_m \), where:

\[
    U_m = -\left( \frac{R^2}{8\mu} \right) \left( \frac{dP}{dx} \right)
\]

(2.2)

It is evident from Eq. 2.1 that the maximum velocity occurs at the centre of the capillary, where \( r = 0 \). Therefore the maximum velocity can be expressed as:

\[
    u_{max} = -\left( \frac{dP}{dx} \right) \left( \frac{R^2}{4\mu} \right)
\]

(2.3)

The expressions presented in Eqs. 2.1, 2.2 and 2.3 provide means of solving for the velocity at any point in the capillary. However, the precision syringe pumps that deliver the liquid phases used in this thesis infuse at defined flow rates, not pressures. Consequently, Eqs. 2.1, 2.2 and 2.3 can be rewritten in terms of flow rate, \( Q \), rather than pressure:

\[
    u = \frac{2Q}{\pi R^2} \left( 1 - \frac{r^2}{R^2} \right)
\]

(2.4)
\[ U_m = \frac{Q}{\pi R^2} \]  

and:

\[ u_{max} = \frac{2Q}{\pi R^2} \]

where:

\[ Q = -\left(\frac{dp}{dx}\right) \frac{\pi R^4}{8\mu} \]

When considering external flow, for example flow over a flat plate, it is only necessary to consider whether the flow is laminar or turbulent. For an internal flow, however, the existence of entrance and fully developed regions must also be considered. For example, for laminar flow in a circular capillary of radius \( R \), the fluid enters with a uniform velocity \( u \) and is shown here schematically in Fig. 2.1.

As soon as the fluid makes contact with the walls of the capillary, viscous forces begin to influence the flow, and a hydrodynamic boundary layer begins to form as the flow moves downstream. The boundary layer development occurs at the expense of the inviscid flow region and concludes with the merging of the boundary layers at the capillary centreline. Viscous effects now extend over the entire capillary cross-section, and the velocity profile is now invariant to changes in \( x \). The flow is now said to be fully developed, and the distance travelled from the entrance to reach this state is known as the hydrodynamic entrance length, which is defined as the distance from the inlet to where the maximum velocity is within 1% of the final theoretical value, Massey (1998). For a laminar flow regime, the hydrodynamic entrance length is given by:

![Figure 2.1: Hydrodynamic boundary layer development in laminar circular tube flow.](image-url)
\[
\frac{x_h}{D} = 0.057 \left( Re_D \right) \tag{2.8}
\]

where \( x_h \) refers to the distance downstream from the entrance, \( D \) is the inner diameter of the capillary and \( Re_D \) is the Reynolds number of the flow based on the inner diameter of the capillary. Within the entrance region, the flow is still developing and, as a result, has a flatter velocity profile. However, once the flow transitions to a fully developed state, the velocity profile becomes parabolic in nature.

### 2.1.2 Heat Transfer

The term heat transfer denotes the energy transfer that occurs as a result of a temperature difference between media or within a medium. This energy transfer is in the form of heat and may be transferred by three different modes: conduction, convection and radiation. For this thesis, however, forced internal convection is the mode of interest, where the mechanisms responsible for the transfer of heat are: diffusion and advection.

As stated in section 2.1.1, for internal flows, as soon as the fluid makes contact with the capillary walls, a hydrodynamic boundary layer begins to form. Similarly, if the walls of the capillary are heated, a thermal boundary layer begins to form, shown here in Fig. 2.2.

![Figure 2.2: Thermal boundary layer development in a heated circular capillary.](image)

The fluid enters the capillary at a uniform velocity, \( u \), and temperature, \( T(r,0) \), that is less than the surface temperature, \( T_s \). Convective heat transfer occurs, and a thermal boundary layer begins to develop. If the capillary surface condition is fixed, by imposing either
a uniform temperature, (constant $T_s$), or constant wall heat flux, ($q''$), a thermally fully developed state is eventually reached. The shape of the fully developed temperature profile, $T(r,x)$, differs depending on the thermal boundary condition imposed upon the flow. There are closed form equations that can be solved to determine $T(r,x)$ for either thermal boundary condition. For the work presented in this thesis, see Chapter 6, the flow is subject to a constant wall heat flux boundary condition. Accordingly, the fully developed temperature profile, $T(r,x)$, can be expressed as:

$$T(r,x) = T_s(x) - \frac{2U_m R^2}{\alpha} \left( \frac{dT_{BM}}{dx} \right) \left( \frac{3}{16} + \frac{1}{16} \left( \frac{r}{R} \right)^4 - \frac{1}{4} \left( \frac{r}{R} \right)^2 \right)$$  (2.9)

where $U_m$, $\alpha$ and $dT_{BM}/dx$ refer to the mean flow velocity, thermal diffusivity and axial variation of the bulk mean temperature, $T_{BM}$. The full derivation of Eq. 2.9 can be found in numerous texts, including those of Incropera et al. (2007), Holman (2002) and Bejan (1993). Using Eqs. 2.1 and 2.9, it is possible to solve for the mean temperature distribution in the axial direction of the flow. If the flow is fully developed, both hydrodynamically and thermally, this yields a constant heat transfer coefficient, which, when presented in terms of the dimensionless heat transfer rate, the Nusselt number, equates to $Nu = 4.36$.

Similar to the hydrodynamic boundary layer, prior to reaching a fully developed state, the thermal boundary layer is still forming. The formation of the boundary layer takes place in the thermal entrance region and, for laminar flow, the thermal entry length may be calculated by:

$$\left( \frac{x_T}{D} \right) = 0.05 Re_D Pr$$  (2.10)

where $x_T$ refers to the distance downstream from the thermal entrance and $Pr$ is the Prandtl number of the fluid, Holman (2002). Comparing Eqs. 2.8 and 2.10, it is evident that if $Pr > 1$, the hydrodynamic boundary layer develops more rapidly than the thermal boundary layer, while the inverse is true if $Pr < 1$. The work presented in Chapter 6 examines the thermal boundary layer development in liquid-liquid Taylor slug flows, in which $Pr$ of the different liquids lies in the range 5.7 - 265.4. Consequently, $x_h$ will always be less than $x_T$ and the hydrodynamic boundary layer, in both liquid phases, will develop
at a faster rate than the thermal boundary layer.

The introduction of a second immiscible fluid stream to the flow results in a number of different flow regimes. The following section introduces a number of flow features unique to Taylor flow regimes, and documents the impact of these features on the hydrodynamic and thermal boundary layers.

### 2.2 Two Phase Taylor Flows

A segmented flow regime consists of a series of slugs dispersed at regular intervals, suspended in a continuous carrier phase, as shown in Fig. 2.3. Two aspects unique to slug flow regimes result in significant changes to the hydrodynamic and thermal boundary layers presented in Figs. 2.1 and 2.2. These are: the thin liquid film that encapsulates the dispersed slugs; and the hemispherical caps at the front and rear of the dispersed slugs.

![Segmented flow structure consisting of slugs dispersed at regular intervals in a continuous carrier phase.](image)

Figure 2.3: Segmented flow structure consisting of slugs dispersed at regular intervals in a continuous carrier phase.

The liquid film that encapsulates the dispersed slugs acts as a barrier between the slugs and the capillary walls, thus reducing the area through which the dispersed phase flows. Consequently, the velocity profile of flow within the dispersed phase differs from that in the carrier phase. Using a method similar to that of Abiev (2008), Howard and Walsh (2013) solved for the velocity profiles in the dispersed phase and the liquid film by taking a momentum balance at the interface between the two phases. The authors made the following assumptions:
• The flow is steady, incompressible and axisymmetric;
• There is a no slip condition at the capillary wall;
• The velocity in both phases is equal at the interface;
• The shear stress at the interface between the phases is equal; and
• The slug is sufficiently long to ensure a hydrodynamically fully developed state at its centre.

Using these assumptions Howard and Walsh (2013) developed the following expressions:

\[
U_D(r) = \left( \frac{2U_m}{1 + \frac{R_D^4}{R^4} (\frac{\mu_C}{\mu_D} - 1)} \right) \left( \frac{R_D^2 \frac{\mu_C}{\mu_D}}{R^2} \right) \left( 1 - \frac{r^2}{R^2} \right) + 1 - \frac{R_D^2}{R^2} \quad (2.11)
\]

for \( R_D > r > 0 \)

\[
U_f(r) = \left( \frac{2U_m}{1 + \frac{R_D^4}{R^4} (\frac{\mu_C}{\mu_D} - 1)} \right) \left( 1 - \frac{r^2}{R^2} \right) \quad (2.12)
\]

for \( R > r > R_D \)

where \( U_m, R_D, R, \mu_C, \) and \( \mu_D \), refer to the mean two phase velocity, dispersed slug radius, capillary radius, carrier and dispersed phase viscosities respectively. Fig. 2.4 presents a comparison of the velocity profiles in the liquid film, carrier and dispersed phases, calculated using the expressions presented in Eqs. 2.12, 2.4 and 2.11 respectively. These equations show that the velocity profiles within the respective phases are parabolic in nature and that the relative magnitudes are dependent on the ratio of slug to capillary radius and the carrier to dispersed phase viscosity ratio. Generally, in liquid-liquid slug flows, the ratio of slug to capillary radius varies between 0.7 - 0.97, Grimes et al. (2007). Therefore, the viscosity ratio between the phases has a greater influence on the velocity profile within the dispersed phase. Hence, the larger the viscosity ratio, the greater the difference between the dispersed and carrier phase velocity profiles. In a liquid-gas flow regime, for instance, a large viscosity difference exists between the phases due to the almost negligible viscosity of the gaseous phase. This disparity in viscosities can range from 50, for a water-air
flow, up to 1000, for an Ethylene Glycerol-air flow, and generates a velocity profile more equivalent to a bullet in shape within the dispersed phase.

![Velocity profile in the liquid film](image)

**Figure 2.4:** Velocity profiles within the dispersed slug, liquid film and carrier phase, calculated for a liquid-liquid flow with a liquid film to capillary radius ratio, \((h/R)\), of 0.07 and a carrier to dispersed phase viscosity ratio of 4.

The viscosity ratio also has an effect on the velocity in the liquid film. Assuming that the shear stress at the interface between the phases is equal, Eq. 2.13, then the fluid in the liquid film in a liquid-gas flow regime can be assumed stationary.

\[
\mu_f \frac{\partial U_f}{\partial r} \bigg|_{r=R_D} = \mu_D \frac{\partial U_D}{\partial r} \bigg|_{r=R_D} 
\]  

(2.13)

In a liquid-liquid flow regime, however, the dispersed phase viscosity is non-negligible. Consequently, the viscosity ratio between the phases is considerably less, which results in an appreciable velocity within the film; this is highlighted in Fig. 2.4. The velocity profiles presented in Fig. 2.4 are those expected at the centre of the dispersed and carrier slugs, provided they are of sufficient length to be in a hydrodynamically fully developed state.
However, due to the presence of the interfaces between the respective phases, the velocity profiles near the front and rear caps are much flatter and are similar in shape to those in the entrance region in Fig. 2.1. This is the result of the axial flow velocity in these regions approximating the mean flow velocity, i.e. the mean slug velocity in the dispersed phase and the mean two phase velocity in the carrier phase, both of which are highlighted in Fig. 2.4.

When viewed from a moving wall reference frame the velocity profiles presented in Fig. 2.4 take on a completely different appearance. Fig. 2.5 presents a schematic illustration of the velocity profiles within the respective phases relative to the mean translation velocity, and highlights the presence of internal circulations within the flow. These circulations have been shown, through micro Particle Image Velocimetry (μPIV), by authors such as Meinhart et al. (1999), Miessner et al. (2008), Malsch et al. (2008) and King et al. (2007), to be toroidal in shape and axisymmetric with respect to the centre axis of the capillary.

![Moving walls](image)

**Figure 2.5:** Typical flow fields present in the dispersed and continuous phases when viewed from a moving wall reference frame.

As the fluid within the slug moves towards the front cap, it is blocked by the interface and moves radially outward towards the wall of the capillary. At the wall, frictional effects force the liquid to move towards the rear interface, where the flow is then forced towards the centre of the slug. Consequently, the presence of the front and rear interfaces induce a recirculation of the flow. A similar phenomenon occurs within the carrier phase, generating a similar recirculation of the flow. The relative strength of these recirculation vortices is dependent on the thickness of the film that surrounds the dispersed slugs, Howard and Walsh (2013). In the dispersed slugs, the effect is at a maximum when the film is at its
thickest and diminishes with reducing film thickness, King et al. (2007); while the inverse is true in the continuous phase, which is the result of slug velocity exceeding the mean flow velocity. This trend continues until, according to Taylor (1960), the slug velocity approaches the maximum flow velocity, i.e. twice the mean slug velocity, and the flow in the continuous phase bypasses the dispersed slug.

The presence of these toroidal vortices results in significant changes to the development of the thermal boundary layer which, according to Che et al. (2013), occurs in three stages:

1. **Thermal boundary layer development.** In this first stage, the liquid in the continuous phase comes in contact with the heated wall, a thin liquid layer in the immediate vicinity of the wall rapidly increases in temperature, and a thermal boundary layer begins to form. The thermal boundary layer is thinner at the front and thicker at the rear of the slug due to the relative flow fields, shown schematically in Fig. 2.5.

2. **Advection of fluid within the slug.** The presence of the front interface forces cooler fluid towards the heated wall, while at the rear interface warmer fluid is transported from the wall toward the capillary centreline, resulting in a redevelopment of the thermal boundary layer.

3. **Fully developed flow.** In the final stage, a constant temperature difference exists between the slug and the capillary wall, culminating in a thermally fully developed flow.

The thermal boundary layer develops in the same manner in the dispersed slug and the continuous phase. This modified thermal boundary layer, which is the product of the modified flow field, results in significant heat transfer enhancements over single phase flows and is a significant motivation behind the work in this thesis. The following section introduces the dimensionless groups implemented during the course of this thesis. These groups have physical interpretations that relate to the hydrodynamic and thermal boundary layers presented in the preceding sections.
2.3 Dimensionless Groups

Within any fluidic system, be it mini, micro or macro, there are a number of interacting forces that control the dynamics of the flow. Dimensionless numbers provide a means of assessing the relative magnitudes of these interacting forces. In single phase flow, the dynamics are controlled by no more than two or three different contributions. However, the addition of a second immiscible phase introduces a number of additional parameters that influence the fluidic balance within the system. For the most part, two phase flows may be characterised, both hydrodynamically and thermally, with the following parameters: Reynolds, Capillary, Weber, Bond, Nusselt, Prandtl and inverse Graetz numbers. In addition to these classical dimensionless groups, there are a series of groups used in the thermal, Janes et al. (2010) and Walsh et al. (2010), and hydrodynamic, Kreutzer et al. (2005) and Warnier et al. (2010), characterisation of Taylor flows.

2.3.1 Dimensionless Hydrodynamic Groups

This sub-section presents a brief outline of the dimensionless groups used to hydrodynamically characterise Taylor flows.

Reynolds Number:

The Reynolds number is defined as:

\[ Re = \frac{\varphi U_m D}{\mu} \]  \hspace{1cm} (2.14)

where \( \rho \), \( U_m \), \( D \) and \( \mu \) refer to the density, mean two phase velocity, inner capillary diameter and viscosity respectively. The Reynolds number defines the ratio of inertial and viscous forces and quantifies the relative importance of these forces. The Reynolds number is most often used in the characterisation of flow regimes as either laminar or turbulent and can also be used as a measure of non-dimensional flow velocity.

Capillary Number:

The Capillary number is defined as:
\[ Ca = \frac{\mu U_m}{\sigma} \]  

(2.15)

where \( \sigma \) is the interfacial tension that exists between the phases. The Capillary number is a measure of the relative importance of the viscous and capillary forces. In the case of two phase Taylor flows, it may be thought of as a measure of the scaled axial viscous drag force and the capillary or wetting force.

**Weber Number:**

The Weber number is the product of the Reynolds and Capillary numbers and represents the balance between the inertial and interfacial forces. It is defined as:

\[ We = ReCa = \frac{\sigma U_m^2 D}{\rho} \]  

(2.16)

**Bond Number:**

The Bond number represents the ratio of the gravitational or buoyancy forces and the capillary forces and may be represented by the following equation:

\[ Bo = gD^2 (\Delta \rho) \frac{1}{\sigma} \]  

(2.17)

where \( g \) and \( \Delta \rho \) refer to the gravitational effects and density difference between the respective phases. In Taylor flows where the Bond numbers are greater than unity, gravitational effects will cause the slugs to deviate from their streamlines and, potentially, come into contact with the walls of the capillary. In chemical and biological diagnostic applications this is undesirable as it leads to cross-contamination effects.

### 2.3.2 Dimensionless Thermal Groups

The following dimensionless groups have been used in the thermal characterisation of Taylor slug flows: Nusselt, Prandtl and inverse Graetz numbers.
Nusselt Number:

In heat transfer at a boundary surface with a fluid, the Nusselt number expresses the ratio of convection to conduction heat transfer normal to the surface and may be viewed as the dimensionless heat transfer rate and expressed as:

\[ \text{Nu} = \frac{hD}{k} \]  

(2.18)

where \( h \) and \( k \) refer to the convective heat transfer coefficient and thermal conductivity of the fluid respectively.

Prandtl Number:

The Prandtl number represents a measure of the relative momentum and energy transport by diffusion in the hydrodynamic and thermal boundary layers respectively, and may be expressed mathematically as:

\[ Pr = \frac{\mu c_p}{k} \]  

(2.19)

where \( c_p \) is the specific heat capacity of the fluid. These diffusion rates determine the relative thickness of the thermal and hydrodynamic boundary layers, hence the Prandtl number provides a link between the velocity and temperature fields.

Inverse Graetz Number:

The inverse Graetz number, \( x^* \), provides a measure of dimensionless position in a channel and is defined as:

\[ x^* = \frac{x}{DRePr} \]  

(2.20)

2.3.3 Dimensionless Taylor Flow Groups

There are, within Taylor flows, a series of dimensionless groups that have been identified as critical to the thermal and hydrodynamic characterisation of the flow. The groups are:
the void fraction of the flow, \( \varepsilon \), and the dimensionless slug length, \( L^* \).

**Void Fraction:**

The void fraction of the flow presents a ratio of volumetric flow rates within the system and is defined as:

\[
\varepsilon = \frac{Q_D}{Q_C + Q_D} \tag{2.21}
\]

where \( Q \) is the volumetric flow rate and the subscripts D and C refer to the dispersed and continuous phases respectively. This parameter is more commonly encountered in liquid-gas Taylor flows, Walsh et al. (2010) and Horvath et al. (1973). In these types of flow regimes, the continuous phase is the only contributor to the heat transfer. However, only a percentage of the continuous phase is in contact with the heat transfer surface due to the presence of gaseous bubbles in the flow. Consequently, the void fraction is used to normalise the heat transfer data.

**Slug Length:**

In both liquid-liquid and liquid-gas Taylor flow studies, slug length has been identified as a key parameter in characterising the enhanced heat and mass transfer rates generated, Oliver and Wright (1964), Muzychka and Yovanovich (2004) and Ghaini et al. (2010). The dimensionless slug length is defined as:

\[
L^* = \frac{L}{D} \tag{2.22}
\]

where the dispersed slug length is measured from the nose to the tail of the slug and includes the hemispherical caps, and the continuous slug length is measured from the tail of one slug to the nose of the following slug, as illustrated in Fig. 2.6.
Aqueous slugs suspended in a continuous carrier phase

Figure 2.6: Schematic illustration of slug unit cell, included in the figure is a graphical definition of slug length.

The ranges in dimensionless groups over which the liquid-liquid Taylor flows were analysed, both hydrodynamically and thermally, will be presented in Chapter 3.

2.4 Closure

This chapter presented a number of fundamental concepts related to the development of the hydrodynamic and thermal boundary layers in both single and two phase flows. These concepts play a key role in understanding the relevant experimental results presented in the subsequent results chapters. A summary of the relevant dimensionless numbers required to characterise the flow both hydrodynamically and thermally was presented. Within the literature, there are a series of analytical and empirical expressions that can be used for characterising the liquid film thickness, pressure drop and heat transfer rates of Taylor flow regimes. These expressions will be presented and discussed later with the relevant experimental results in Chapters 4, 5 and 6.
Chapter 3

Experimentation

This chapter provides detailed descriptions of the experimental facilities and measurement techniques used in this thesis. The top level objective of this thesis was to research the fundamental thermal and hydrodynamic characteristics of liquid-liquid Taylor flow regimes confined to a minichannel geometry. In order to achieve this objective, three different experimental facilities were designed and constructed that allowed the flow to be studied non-invasively over a wide range of non-dimensional parameters.

Firstly, however, the measurement of a number of important thermophysical properties fundamental to both the thermal and hydrodynamic characterisation was completed, the results of which are presented in section 3.1. Section 3.2 presents a summary of the techniques used in this thesis to generate the liquid-liquid Taylor flows. Detailed descriptions of the three experimental test facilities are presented in sections 3.3, 3.4 and 3.5. The first and second test facilities were designed to examine the principal hydrodynamics, namely the film thickness, section 3.3, and pressure drop, section 3.4, of liquid-liquid Taylor flows. The third experimental facility, presented in section 3.5, was designed to enable a thermal characterisation of the flow when subjected to a constant wall heat flux boundary condition. The chapter concludes with an uncertainty analysis in section 3.6. Uncertainty values are presented for both the primary measurands and derived variables considered in this thesis.
3.1 Fluid Property Measurements

The flow physics related to liquid-liquid slug flows can, for the most part, be characterised by the non-dimensional Reynolds, Capillary, Nusselt and Prandtl numbers. In this regard, the fluid properties of interest are the: density, viscosity, interfacial tension, thermal conductivity and specific heat capacity of the liquid phases. Experiments were carried out using a selection of different carrier liquids, while water was used as the dispersed phase throughout. In total, four oil/water combinations were examined, thus allowing the effects of a wide range of thermophysical properties on film thickness, pressure drop and heat transfer to be investigated. The four carrier phases were:

- Pd5, which is a silicone oil;
- Dodecane, which is a silicone oil;
- AR20, which is a silicone oil; and
- FC40, which is a fluorinert fluid.

and were supplied by Momentive and Sigma Aldrich Ltd. The relevant thermophysical properties of the different liquid media used over the course of this thesis are presented in Table 3.1. The data presented in Table 3.1 was measured at room temperature (~25°C) and atmospheric pressure. Specific heat capacities and refractive index values were provided by the respective suppliers. Provided below is a brief summation of the different techniques that were used to measure the respective properties.

Density

A 10 mL sample of each liquid was pipetted into a cuvette sample holder. The cuvette was weighed, to within four decimal places, before and after the addition of the sample using a Kern ALS I20-4N digital scales. The density of the sample was determined from the resulting difference in cuvette weights.
Table 3.1: Relevant thermophysical properties, at room temperature and atmospheric pressure, of the different liquids used during this thesis.

<table>
<thead>
<tr>
<th>Medium</th>
<th>Density (kg/m³)</th>
<th>Viscosity (kg/m s)</th>
<th>Interfacial Tension (mN/m)</th>
<th>Thermal Conductivity (W/m K)</th>
<th>Specific Heat Capacity (kJ/kg K)</th>
<th>Refractive Index (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>997.1</td>
<td>0.0009</td>
<td>-</td>
<td>0.6</td>
<td>4.1</td>
<td>1.33</td>
</tr>
<tr>
<td>Pd5</td>
<td>911.8</td>
<td>0.0036</td>
<td>39.7</td>
<td>0.1</td>
<td>1.7</td>
<td>1.44</td>
</tr>
<tr>
<td>Dodecane</td>
<td>754.3</td>
<td>0.0014</td>
<td>52.0</td>
<td>0.1</td>
<td>2.2</td>
<td>1.42</td>
</tr>
<tr>
<td>FC40</td>
<td>1854.1</td>
<td>0.0039</td>
<td>51.1</td>
<td>0.1</td>
<td>1.1</td>
<td>1.29</td>
</tr>
<tr>
<td>AR20</td>
<td>1142.2</td>
<td>0.0209</td>
<td>30.2</td>
<td>0.1</td>
<td>1.7</td>
<td>1.44</td>
</tr>
</tbody>
</table>

**Viscosity**

The viscosities of the liquids were measured using a Brookfield LVDV-II+ rotary viscometer with an ultra low adapter water jacket. For each measurement, the cylinder was filled with 16 mL of fluid. The spindle was lowered into the cylinder and set to rotate at a rotational speed that would provide a torque resistance of between 10% - 100% of full scale. The instrument measured the torque at a particular rpm and from this inferred the viscosity of the fluid. The water jacket was connected to a Lauda Ecoline Staredition RE104 thermostatic bath and allowed the variation in viscosity with temperature to be accurately measured.

**Interfacial Tension**

The interfacial tension between the oil phases and water was measured using a KSV CAM 2000 apparatus, which utilised the pendant drop technique, Saad et al. (2011). This method involved the suspension of a small droplet of the denser liquid, with a volume of between 5 - 15 μL, from the tip of an FEP (fluorinated ethylene propylene) Teflon capillary, inner diameter of 406 μm, into a borosilicate cell, 40 mm x 40 mm x 60 mm. The balance between gravitational and interfacial forces causes the droplet to assume a pendant shape. A simple optical analysis was completed, where a series of curves were fitted to the shape of the droplet and the Young-Laplace equation was solved to provide a value for the interfacial tension.
**Thermal Conductivity**

A KD2 Pro thermal analysis device was used in conjunction with a Lauda Ecoline Staredition RE104 thermostatic bath to measure the thermal conductivities of the different liquids. The KD2 Pro thermal analysis device uses a single needle probe, which acts as an infinite line heat source with a constant heat output and zero mass, and housed within the probe are a heater and a temperature sensor. The probe works by passing a current through the heater and monitoring the temperature of the surrounding fluid over time. The device uses the temperature profile to solve a series of analytical equations, based on transient line heat source methods, to calculate the resultant thermal conductivity.

### 3.2 Slug Flow Generation

When immiscible fluid streams are brought into contact at the inlet of a channel, the generated flow regimes depend upon the channel geometry, flow rates and instabilities at the liquid-liquid interface. At the microscale, however, the dominance of surface forces over gravitational forces results in segmented flow regimes being the most commonly encountered. These segmented flow regimes, which are more commonly referred to as Taylor or slug flows, can be generated in a number of different ways. Comprehensive reviews, by authors such as Christopher and Anna (2007), Gunther and Jensen (2006) and Seemann et al. (2012), of the different generation techniques and their associated fluid dynamics, can be found in the literature. Provided below is a brief summation of these techniques, followed by a detailed description of the devices, and their associated fluid dynamics, used to form Taylor flow regimes during the course of this thesis.

The methods used in the formation of Taylor flows can be either active or passive. Active techniques rely upon an external force to control slug size and frequency. Typically, an actuation mechanism utilises a flexible membrane or micro-valve that is deflected using a piezoelectric transducer. This is referred to as the “drop-on-demand” technique, a well known example of which is inkjet printing, where a droplet of ink is forced out of a microfabricated nozzle by the deflection of a membrane in contact with the reservoir of ink. However, most methods used in the formation of liquid slugs are passive in nature. These
methods take advantage of the flow field to deform the interface and promote the natural growth of interfacial instabilities, thus obviating local external actuations. Passive methods may be grouped into three categories, where the formation of the slugs is characterised by the nature of the flow near the slug break-off point. These categories are: breakup in co-flowing streams; breakup in cross-flowing streams; and breakup in elongated or stretching flows. Schematic illustrations of the different categories are presented in Fig. 3.1.

Figure 3.1: Schematic illustration of the three different geometries used in the passive generation of liquid-liquid plug/slug flows. (a) co-flowing streams, (b) cross-flowing streams and (c) elongated or stretched flow in a flow focused geometry.

Two alternative methods were used during the course of this thesis to generate a segmented slug flow regime: segmenters, which are an example of slug formation due to break up in an elongated flow in a flow focused geometry, Fig. 3.1 (c) and T-junctions, which are an example of slug formation due to break up in cross flowing streams, Fig. 3.1 (b). Segmenters, Fig. 3.2, work by periodically creating and rupturing a liquid bridge between two opposing capillary tips and have been used previously by Curran et al. (2005)
to dispense sub-micro litre volumes of reagents to be used as microfluidic chemical reactors in the PCR process. The dispersed phase forms a droplet at the tip of the inlet, Fig. 3.2 (a), such that, when sufficiently large, it forms a stable liquid bridge between the opposing capillaries, Fig. 3.2 (b). Fluid is then drained from the liquid bridge, Fig. 3.2 (c), until it reaches the minimum volume stability limit and ruptures, dispensing an aqueous slug, Fig. 3.2 (d). The cycle is then repeated, creating a chain of aqueous slugs punctuated by oil. Segmenters require the reservoir to be filled with a density matched oil to ensure that a buoyancy free environment exists for axisymmetric liquid bridging. Consequently, in this thesis only the Pd5/water slugs flows were generated through segmentation. For all other oil/water combinations T-junctions were used.

![Figure 3.2: Schematic depicting the mode of operation of a segmenter.](image)

T-junctions offer a simple, cheap and reliable method for generating segmented flow regimes and, as a result, have been used in numerous studies, Howard et al. (2011); Tice et al. (2003); Walsh et al. (2009) and Hestroni et al. (2009). An example of a T-junction is depicted in Fig. 3.3, which shows the immiscible liquid phases flowing through two circular channels that are normal to one another. An array of different T-junctions were used during experimentation, where variations in the internal diameter of the T-junctions, presented as $D_{out}$ in Fig. 3.3, allowed for changes in the dispersed and continuous slug lengths.
Depending on the ratio of the channel diameters, $x = \frac{D_{in}}{D_{out}}$, the liquid slugs form by either one of two different mechanisms. For cases where the outer channel diameter is greater than the inner channel diameter, $x < 1$, the aqueous slugs are emitted before they can block the channel and their formation is entirely due to the viscous stresses overcoming the interfacial forces. For cases where $x \geq 1$, the discontinuous phase protrudes across the channel and restricts the flow of the continuous phase. This reduction in space through which the continuous phase can flow results in an increase in dynamic pressure upstream of the slug. This causes the interface to neck and pinch off into an aqueous slug, shown here in Fig. 3.3. This cycle is then repeated, resulting in the formation of a stable train of aqueous slugs dispersed in a continuous carrier phase.

![Figure 3.3: Example of slug formation with a T-junction.](image)

### 3.3 Film Thickness Measurements

The objective of this experimentation was to measure the thickness of the liquid film that separates the aqueous slugs from the channel wall. A custom measurement facility was built and commissioned that allowed a direct and non-invasive measurement of the film. The non-invasive optical technique that was used to capture images of the flow, the experimental test procedure and necessary post processing steps that were required to extrapolate the film thickness data are detailed in this section. The resulting film thickness measurements are presented in Chapter 4.
3.3.1 Film Thickness Test Facility

Fig. 3.4 is a schematic of the experimental apparatus that was used to measure the magnitude of the liquid film. Two precision Harvard PhD 2000 programmable syringe pumps were used to set the volumetric flow rates. Separate syringe pumps allowed the relative flow rates, and hence slug lengths, to be varied. The liquid phases were delivered from 100 mL capacity Hamilton 1100TLL gas tight glass syringes. Glass syringes were used as they were found to deliver steadier flows than comparable plastic syringes. The flows then merged inside either a segmenter or T-junction to generate the slug flow regime.

![Schematic of experimental facility](image)

Figure 3.4: Schematic of experimental facility used to measure the magnitude of the liquid film separating the aqueous slugs from the channel wall.

Once generated, the aqueous slugs proceeded to the main capillary which was horizontal in orientation and consisted of transparent Upchurch Scientific ® FEP Teflon tubing. The tubing was approximately 1m in length and had an internal diameter of 1.59 mm. The flow was imaged at a single location 0.75 m upstream from the entrance of the main capillary; this was to ensure the full hydrodynamic development of the flow prior to imaging. The region that was imaged was immersed in a transparent water bath. The refractive index
of water is 1.33, Meinhart et al. (1999), while the refractive index of the tubing, according to the manufacturer, is 1.338. This almost perfect refractive index matching resulted in excellent visualisation of the flow, shown here in Fig. 3.5.

![Figure 3.5: Refractive index matching between capillary and water allowing excellent flow visualisation.](image)

However, it can be seen in Fig. 3.5 that there is a dark space along the capillary wall. The carrier phase that separates the dispersed slugs from the capillary wall has a different refractive index to that of the capillary and water. Consequently, the light rays are refracted within the capillary and distort the image seen in the microscope. A series of images of the capillary cross-section incorporating a needle with a distinct profile were analysed to assess the influence on this dark region on the measurement of the film. These images are presented in Appendix B, and show that the dark region does not result in a distortion of the near wall region. Consequently, they do not need to be considered in the analysis.

Images of the flow were captured using an IDT X-Stream XS-4 CMOS high-speed camera mounted on a Zeiss Axioskop microscope. The microscope had a built in light source that was used to illuminate the flow. Images were recorded using 1.5x and 20x lenses at a frequency of 5000 Hz (1 image every 200 $\mu$s), and an exposure of 122 $\mu$s. Images captured using the 20x lens were analysed in MatLab (version 2010a), where a custom code extracted measurements of liquid film thickness along the length of the aqueous slugs. The images captured using the 1.5x lens were used to provide a measure of the aqueous and
oil slug lengths. These images were also used to determine the aqueous slug velocity. For any specific test, the distance traveled by a slug over a series of images could be measured, and, by knowing the rate at which the images were recorded, the slug velocity could be calculated.

### 3.3.2 Film Thickness Measurement Procedure

The following procedural steps were followed for each test:

1. The experimental apparatus was arranged as shown in Fig. 3.4.

2. The syringe pumps were set to the desired volumetric flow rates, which ranged from 0.5 - 11.5 ml/min.

3. The system was initially primed with the carrier phase. This was done to remove all air from the system and ensure the carrier phase would preferentially wet the capillary wall.

4. The light source, high-speed camera, microscope and syringe pump delivering the aqueous phase were powered on.

5. Once a stable flow regime had been created, the microscope objective was set to 1.5x and the light source intensity and microscope focus were adjusted until a clear image of the flow was achieved. Images of the flow were recorded at a rate 5000 Hz for a period of 5 seconds, resulting in 25,000 images of the flow.

6. The microscope objective was then changed to 20x and the previous step was repeated.

7. Keeping the total flow rate constant, the flow rate ratio between the oil and water phases was adjusted, resulting in changes to the oil and aqueous slug lengths. The previous two steps were then repeated. In total, five different oil/water flow rate ratios were examined for a constant total volumetric flow rate.
8. The total volumetric flow rate was then adjusted and the previous three steps were repeated. This procedure was completed for each of the four different oil/water combinations examined.

As the oil phase preferentially wets the walls of the tubing and leaves a residual film, it was necessary to change the tubing with each change in carrier phase. Presented in Table 3.2 is the range in non-dimensional parameters over which the flow was analysed. It was stated in section 2.3.1 that in Taylor flows where the Bond numbers are greater than unity, gravitational effects can cause the slugs to deviate from their streamlines and, potentially, come into contact with the capillary walls. The ranges in $Bo$ presented in Table 3.2 are less than unity, hence gravitational effects could be assumed negligible and as a result the flow was assumed axisymmetric and imaged at a single location. This was also confirmed experimentally by comparing images of different orientations of the capillary.

Table 3.2: Range of non-dimensional parameters over which the variations in film thickness were studied.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_D^*$</td>
<td>0.87</td>
<td>8.11</td>
</tr>
<tr>
<td>$L_C^*$</td>
<td>0.72</td>
<td>14.14</td>
</tr>
<tr>
<td>$Ca$</td>
<td>0.002</td>
<td>0.119</td>
</tr>
<tr>
<td>$Re$</td>
<td>3.68</td>
<td>100.96</td>
</tr>
<tr>
<td>$We$</td>
<td>0.047</td>
<td>1.671</td>
</tr>
<tr>
<td>$Bo$</td>
<td>0.053</td>
<td>0.414</td>
</tr>
</tbody>
</table>

The measurement technique presented in section 3.1.1 provided a straightforward and non-invasive measurement of the magnitude of the film. However, a series of post-processing steps are required, due to the refraction of light through the liquid phases, to determine the true liquid film thickness. The necessary post-processing steps are documented in the following sub-section.
3.3.3 Optical Corrections

As light rays pass through the capillary wall they are scattered, due to the changes in refractive index, presented in Table 3.1. These effects were corrected using a series of trigonometric operations coupled with the geometrical optics of light refraction. Fig. 3.6 graphically illustrates the effects of refractive index changes on the light rays.

![Diagram showing optical corrections](image)

Figure 3.6: Schematic illustration of the technique used to correct for refractive index mismatches.

As the tubing and the surrounding water have almost identical refractive indices, it was assumed that the refraction of the light through the tubing could be neglected and the position of the capillary wall seen in the images was the true position of the capillary wall. However, depending on the refractive index of the carrier phase, the visible film would appear to be larger or smaller than the actual film. A measure of the apparent film thickness was taken and from this the apparent aqueous slug radius, $r_A$, was inferred. This allowed the angle $\alpha$ to be determined and from this the angle $\theta_1$, the angle the light ray makes with the inner wall of the capillary and an axis normal to the surface.

\[
\cos(\alpha) = \frac{r_A}{R} \tag{3.1}
\]

\[
\theta_1 = 90 - \alpha \tag{3.2}
\]
Use of Snell’s Law allowed the refracted angle, $\theta_2$, to be solved:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (3.3)$$

Where $n_1$ and $n_2$ are the refractive indices of the tubing and the carrier phase respectively. The difference between the apparent and actual film thickness can then be solved using a series of simple trigonometric relations. The length $L$ is calculated using Pythagoras’s theorem and the angle $\xi$, is simply the difference between the angles $\theta_1$ and $\theta_2$.

$$L = \sqrt{R^2 - r_A^2} \quad (3.4)$$

$$\delta = (L) \tan (\theta_1 - \theta_2) \quad (3.5)$$

The actual aqueous slug radius, $r_{act}$, can then be solved and, from this, the actual film thickness can be determined.

$$r_{act} = r_A - \delta \quad (3.6)$$

The magnitude of the refraction index induced errors varied from 4.3-77.9 $\mu$m, or relative to the capillary radius, $\delta/R$, 5.4-9.8% for the Pd5, Dodecane and AR20 based flows. These carrier phases have refractive indices that are greater than that of the tubing, see Table 3.1. Consequently, the film appears thinner than the actual film. In the FC40 based flows the opposite is true, as the refractive index of FC40 is less than that of the tubing and as a result the film appears larger than the actual film, by 23-31 $\mu$m, or relative to the capillary radius, $\delta/R$, 2.8-3.9%.

### 3.4 Pressure Drop Measurements

The addition of a second immiscible liquid phase to create a segmented liquid-liquid slug flow regime results in an increase in pressure drop relative to the single phase flow case. The objectives of this series of experiments were to measure the pressure drop associated
with liquid-liquid slug flows, and to examine the effects of specific parameters on pressure drop; aqueous and carrier slug lengths, and carrier phase thermophysical properties. The results are presented and discussed in Chapter 5. The experimental apparatus and test procedure are detailed in this section.

3.4.1 Pressure Drop Test Facility

Fig. 3.7 is a schematic illustration of the experimental facility that was used to measure pressure drop. The methods used to generate the liquid-liquid slug flow regimes have been documented previously in sections 3.2 and 3.3.

Figure 3.7: Schematic of the experimental facility used to measure the pressure drop associated with the liquid-liquid slug flows.

Once generated, the flow proceeded to the test section where the pressure drop was measured in hard walled FEP Teflon capillaries which were horizontal in orientation and circular in cross-section. The capillary, supplied by Upchurch Scientific®, had an internal
diameter of 1.59 mm and was transparent in nature, thus allowing the flow to be visualised to ensure that a stable and consistent chain of aqueous slugs punctuated by oil passed between the pressure tappings. A CCD camera was used to record images of the flow, such as the one shown in Fig. 3.8, which were then analysed in MatLab (version 2010a) to extract aqueous and oil slug length data. The maximum variation in aqueous and oil slug length was found to be less than 10%.

Figure 3.8: An image of the liquid-liquid Taylor slug flow regime acquired during pressure drop measurements. Aqueous and oil slug lengths were determined by analysing images, such as this, in Matlab using the scale (1 mm divisions) at the bottom.

The distance between the pressure tappings varied between 0.6 m and 2.0 m, depending on the pressure range of the oil/water combination examined. The pressure drop itself was measured using a Druck PDCR 4170 differential pressure transducer, with a manufacturer’s accuracy of 0.1% of full scale deflection, and a range of 35 kPa. The ± 10V output from the transducer was measured using a Duck digital process indicator (DPI) 280 which connected to Labview via a DAQ card. A custom Labview program allowed precise recordings of the transducer output throughout each test, hence allowing steady state measurements to be identified and recorded. The range of the transducer allowed for the range in pressure drops associated with the liquids of differing viscosities. Repeatability trials demonstrated a maximum variation of 3% between tests. The pressure transducer was calibrated over the full 35 kPa range using single phase flow theory for fully developed laminar flow in a circular capillary, presented here in Eq. 3.7.

\[
\Delta P = \frac{8 \mu LQ}{\pi R^4}
\]  

(3.7)
The liquids were collected in a reservoir at the exit of the capillary where a k-type thermocouple was used to measure their temperature. Temperatures were not controlled, rather monitored to ensure that any fluctuations in fluidic properties, particularly viscosity, were accounted for.

### 3.4.2 Pressure Drop Test Procedure

All pressure drop measurements were recorded using the following procedural steps:

1. The experimental apparatus was arranged as shown in Fig. 3.7.

2. The separate syringe pumps were set to the required volumetric flow rates, ranging from 0.5 - 11.5 ml/min.

3. The system was primed with the carrier phase and then inspected to ensure no air bubbles were trapped anywhere within the fluidic network, particularly at the connections with the pressure transducer.

4. Once the system was primed, the data collection software and syringe pumps were activated. The output was monitored using a custom Labview code. Data was collected at a rate of 10Hz or 0.1s. The duration of the test varied from 2 - 20 minutes depending on the overall flow rate through the system. The code allowed the user to identify when steady state had been achieved. Results were averaged within this steady state region.

5. Once a stable chain of aqueous slugs punctuated by oil was established through the main capillary, a series of 10 - 20 images of the flow were recorded.

6. The test duration varied between 5 and 20 minutes depending on the relative flow rates of the different liquid phases.

7. The relative oil and aqueous flow rates were then adjusted and steps 2 - 6 were repeated.

This procedure was repeated for the desired range in volumetric flow rates for the different oil/water combinations used as part of this study. To eliminate cross-contamination effects,
a different capillary was used for each oil/water combination. Table 3.3 presents the range in non-dimensional parameters tested as part of this study.

Table 3.3: Range of non-dimensional variables over which the pressure drop was measured.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>L_D^*</td>
<td>1.05</td>
<td>11.01</td>
</tr>
<tr>
<td>L_C^*</td>
<td>0.71</td>
<td>14.14</td>
</tr>
<tr>
<td>Re</td>
<td>1.82</td>
<td>98.63</td>
</tr>
<tr>
<td>Ca</td>
<td>4.5 x 10^{-4}</td>
<td>0.06</td>
</tr>
</tbody>
</table>

3.5 Local Heat Transfer Measurements

The objective of this experimentation was to investigate the thermal characteristics of non-boiling, liquid-liquid Taylor flow regimes. The experimental facility was designed and constructed such that the flow was subjected to a constant wall heat flux boundary condition, which is a boundary condition commonly encountered in heat exchange devices. The details of the core components of the thermal facility, including the commissioning of the facility, are presented in section 3.5.1. Details of the Infrared (IR) thermography system used to acquire local temperature measurements and the steps required to extrapolate the relevant temperature data are presented in sections 3.5.2 and 3.5.3. System calibration and experimental procedures are then outlined and the section is concluded. Results of the thermal characterisation are presented in Chapter 6.

3.5.1 Local Heat Transfer Measurement Facility

A schematic of the experimental facility used to complete the thermal analysis of the flow is presented in Fig. 3.9. The devices and modes of slug generation have been documented previously in sections 3.2 and 3.3 respectively. Once generated the liquid slug flows passed into a 1.5 mm ID transparent hydrophobic tubing. The transparent nature of the tubing allowed visualisation of the flow to ensure that a stable and consistent slug train existed and that no coalescence of aqueous slugs had occurred. A series of 10 - 15 images, such as that in Fig. 3.10, were recorded and analysed using a custom code developed in MatLab.
(version 2010a) to extract the aqueous and oil slug lengths. The maximum variation in slug length was found to be less than 10%.

![Figure 3.9: Schematic illustration of the local heat transfer test facility.](image)

Figure 3.9: Schematic illustration of the local heat transfer test facility.

![Figure 3.10: Sample image of liquid-liquid slug flow regime captured prior to entering the main test section.](image)

Figure 3.10: Sample image of liquid-liquid slug flow regime captured prior to entering the main test section.

The flow then proceeded to the heated test section, which consisted of 1m length of 1.5 mm internal bore stainless steel tubing. However, only a short length, 0.3 m, was heated during experimentation. Prior to reaching the heated test section, the flow passed through
a 0.4 m unheated entry length in order to ensure the full hydrodynamic development of the flow. To minimise axial conduction losses and the temperature gradient through the wall, the tubing used had a wall thickness of 0.25 mm. A constant wall heat flux boundary condition was achieved by Joule heating using a TTi TSX1820P high current DC power supply. Electrical connections to the tube were made using custom-made knife edged copper ring contacts, shown here in Fig. 3.11, which allowed the thermal entrance point to be clearly identified. Additionally, the use of copper as the material for electrical connections ensured minimal internal heat generation, and their sharp edged connection minimised conductive heat loss.

![Copper ring contacts](image)

**Figure 3.11:** Copper ring contacts that were used to make electrical connection to the capillary.

To ensure the flow was, at all times, subject to a hydrophobic boundary condition, the inside of the stainless steel tubing was coated with a thin layer of polyurethane. Stainless steel is naturally hydrophilic and, consequently, the attractive forces between the water molecules in the dispersed slugs and the tube wall can result in the breakdown of the thin liquid film of the carrier phase. This breakdown of the liquid film would result in a wetting of the tube wall and subsequent disruption of the well-ordered slug flow. The thickness of the coating was measured using a scanning electron microscope (SEM) and images of the tubing, presented in Fig 3.12 showed it varied from 5 - 15 μm in thickness. To ensure that the coating had no adverse affects on the flow, a series of images of the slugs were captured, in a transparent 1.5mm ID capillary, after the heated test section. A comparison
CHAPTER 3 Experimentation

of the oil and aqueous slug lengths, pre- and post-stainless steel tubing, was completed and the maximum variation in slug lengths, between pre- and post-stainless steel tubing was found to be less than 10%.

![Stainless steel tube wall Polyurethane coating applied to the wall of the stainless steel tube 10 μm](image)

Figure 3.12: SEM images of the stainless steel tubing: a) without polyurethane coating and b) with polyurethane coating.

3.5.2 Infrared System

External surface temperatures, $T_W$, of the heated test section were obtained using a high resolution IR thermography system. This consisted of a FLIR systems ThermaCam Merlin series IR camera and ThermaCAM Researcher Pro 2.8 software. Due to resolution issues the field-of-view of the IR camera extended over 110 mm of the heated test section during experimentation. All local temperature measurements were obtained within this region. For each test, 320 x 256 pixel images were recorded at a frequency of 1 Hz for a period of up to 10 minutes, depending on the flow rates. Steady state was typically achieved after a period of 1 minute, however, extended recording times were necessary due to the unsteady nature of the heat transfer processes under examination resulting from the periodic passing of aqueous slugs. The heated test section and IR camera lens were placed in a custom made enclosure to shield them from their environment, thereby minimising the effects of ambient convection currents within the laboratory and long wavelength radiation from high temperature surrounding bodies such as light fixtures or monitors.
3.5.3 Data Reduction

The IR images were analysed using a custom code developed in MatLab (version 2010a). This code extracted the transient temperature profiles along the tube surface. Fig. 3.13 illustrates the main steps of the code.

Figure 3.13: Images summarising the main steps in the Matlab code that was used to extrapolate the transient temperature profile of the capillary wall. (a) Image used for scaling, (b) Region of interest for analysis selected, (c) Contour map of transient surface temperatures and (d) Output profiles of upper, mean and lower surface temperatures.

Fig. 3.13 (a) shows an image of the test section prior to heating and incorporates a ruler to scale the images. A region of interest is selected, Fig. 3.13 (b), and is highlighted by the dashed line. Once selected, the code uses an edge detection algorithm to identify the heated tube edges and, subsequently identify the centreline of the tube. Surface temperatures are extracted along this line at each pixel, resulting in a temperature measurement approximately every 300 μm. This step is repeated for each image, resulting in a temperature contour map, presented in Fig. 3.13 (c). The centreline temperatures are sequentially placed together and represented on the y-axis, which is representative of the time domain through the recording frequency, while plotted on the x-axis is the distance along the tube.
from the inlet. The contour colour map represents the tube wall temperatures in °C, with the magnitude indicated by the legend shown at the bottom of Fig. 3.13 (c). The code allows the user to input the range of images from which the average is calculated and the output is the mean temperature along the surface of the tube, Fig. 3.13 (d). The upper and lower limit lines represent three standard deviations of the periodically fluctuating temperature along the length of the tube. These fluctuations do not represent uncertainty, rather they highlight the unsteady nature of the heat transfer processes associated with such a flow regime.

3.5.4 System Calibration

Prior to testing, it was necessary to calibrate both the IR thermography system and the experimental facility. The details of the necessary calibration steps are as follows:

Emissivity

The use of IR thermography required the external surface of the tubing to be sprayed matt black to enhance its emissivity. The emissivity of the matt black coating was calculated prior to testing by comparing IR thermography measurements with measurements from four k-type thermocouples that were mounted on the surface of the tube, just outside the field of view of the camera. To reduce the uncertainty in the thermocouple temperature measurements, the thermocouples were calibrated prior to testing using a Lauda E100 RE104 thermostatic bath over a temperature range of 10ºC - 70ºC. This reduced the uncertainty in the temperature measurements from the quoted tolerance of ±2.2ºC to ±0.1ºC, as indicated by ASTM (1993).

Water was pumped through the channel at three different temperatures, where the temperature range spanned the range of experimental values. The flow rate was sufficiently high to ensure that the temperature drop, due to natural convection losses, between the thermocouples was no greater than 0.1K. IR and thermocouple measurements were then compared and the test section emissivity was found to be 0.96, which would be typical of a matt black surface. This procedure, to determine the test surface emissivity, was repeated for each stainless steel tube used during the course of experimentation.
**Electrical Resistance**

The experimental facility, presented in Fig. 3.9, was calibrated over five different flow rates, spanning several orders of magnitude of inverse Graetz number, using single phase flow theory. Comparing experimental tube wall temperature measurements with theoretical wall temperatures allowed the electrical resistance of the stainless steel tubing and the heat fluxes to be determined. The theoretical wall temperatures were calculated using the following expressions:

\[
Nu_x = \frac{hD}{k} = \frac{q''D}{k(T_w - T_{BM})} \quad (3.8)
\]

Rearranging this expression in terms of the wall temperature:

\[
T_w = \frac{q''D}{Nu_xk} + T_{BM} \quad (3.9)
\]

where \(q''\) is the supplied heat flux and \(T_{BM}\) is the bulk mean temperature of the flow. The local Nusselt number, \(Nu_x\), was calculated using a piecewise analytical expression based on the Graetz solution, Graetz (1883) and Graetz (1885). The expression presented in Eq. 3.10 was developed by Muzychka and Yovanovich (2004) and defines the variation in local Nusselt number with dimensionless position when the flow is subject to a constant wall heat flux boundary condition and the fluid Prandtl number is greater than unity.

\[
Nu_x = \left( \frac{1.302}{x^{1/3}} \right)^5 + (4.36)^5 \quad (3.10)
\]

Piecewise solutions are separated into two distinct regions. In the first region, an expression is defined for the entrance region where the flow is still developing thermally while, in the second region, the flow is fully developed thermally and the Nusselt number is constant.

As a constant wall heat flux is applied to the flow, the local bulk mean temperature increases linearly from the inlet temperature, \(T_{in}\). The value of \(T_{in}\) is extrapolated from the IR images immediately upstream from the thermal entrance point and is highlighted in Fig. 3.13 (b). The bulk mean temperature was calculated using an energy balance between the
heat added and the enthalpy of the fluid such that:

\[ T_{BM} = \frac{q'' \pi D x}{mc_p} + T_{in} \]  \hspace{1cm} (3.11)

Fig. 3.14 presents a comparison of the experimental data points with those calculated theoretically, using Eqs. 3.9 and 3.10. The electrical resistance of the stainless steel tube was determined to be 0.14 \( \Omega \). Using Eq. 3.12, where \( \rho, L \) and \( A \) refer to the electrical resistivity, length and cross sectional area of the tubing, the resistance was found to be 0.13 \( \Omega \). The agreement between the calculated resistances validates the technique that was used to determine the tube resistance and heat fluxes.

\[ R = \frac{\rho L}{A} = \frac{(75 \times 10^{-8})(0.3)}{1.76 \times 10^{-6}} = 0.13 \Omega \]  \hspace{1cm} (3.12)

Figure 3.14: Comparison of experimental and theoretical single phase wall temperature profiles, allowing verification of experimental setup and heat flux calibration. The flow has an \( Re = 175.6 \) and is subject to a constant wall heat flux of approximately 5,000 W/m\(^2\).
3.5.5 Thermal Measurement Procedure

The following procedural steps were taken during the thermal characterisation of the flow:

1. The experimental apparatus was arranged per the schematic in Fig. 3.9.

2. The system was initially primed with the carrier phase to remove all air bubbles from the fluidic network.

3. The syringe pumps were set to the desired volumetric flow rates and were powered on, producing a steady stream of aqueous slugs dispersed in the carrier oil phase.

4. Once steady flow had been established, the IR camera recorded a series of images of the test section prior to heating.

5. The desired current was set on the power supply and it was activated.

6. Simultaneous to the IR measurements, a series of 10 - 20 images of the flow were recorded at locations both pre- and post- the stainless steel tubing using CCD cameras.

7. Once the test was completed, the power supply, IR camera and syringe pumps were powered down. The system was allowed to cool down and return to room temperature.

8. Steps 3 - 7 were repeated for a range of oil/water ratios over the desired range of volumetric flow rates.

The above procedure was carried out for the three different oil/water combinations, resulting in the range in non-dimensional parameters presented in Table 3.4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re</td>
<td>2.31</td>
<td>92.12</td>
</tr>
<tr>
<td>Pr</td>
<td>23.6</td>
<td>265.4</td>
</tr>
<tr>
<td>Ca</td>
<td>0.001</td>
<td>0.119</td>
</tr>
<tr>
<td>$x$</td>
<td>8.11x10^{-5}</td>
<td>9.17x10^{-2}</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.04</td>
<td>0.75</td>
</tr>
<tr>
<td>$L_{C^*}$</td>
<td>0.23</td>
<td>22.86</td>
</tr>
<tr>
<td>$L_{D^*}$</td>
<td>1.02</td>
<td>8.17</td>
</tr>
</tbody>
</table>
3.6 Uncertainty Analysis

The following section introduces the uncertainties associated with the experimental measurement techniques, presented in sections 3.3, 3.4 and 3.5, and the uncertainties in the resulting measurements. The uncertainties in the data presented in Chapters 4, 5 and 6 of this thesis were calculated using the method of Kline and McClintock (1953). This method determines the uncertainty in a given variable above and below the actual calculated value, where the uncertainty of the independent variables is known. The calculated result, \( f \), is given as a function of several independent measured variables such that:

\[
f = f(x_1, x_2, x_3, \ldots, x_n)
\]  

The uncertainty of the calculated value, \( w_f \), is calculated from:

\[
w_f = \left( \frac{\delta f}{\delta x_1} w_1 \right)^2 + \left( \frac{\delta f}{\delta x_2} w_2 \right)^2 + \ldots + \left( \frac{\delta f}{\delta x_n} w_n \right)^2 \right]^{1/2}
\]

3.6.1 Primary Uncertainties

The following primary uncertainties were common to the majority of tests conducted in this thesis and were minimised as follows:

- **Temperature Measurement**: A Lauda E100 calibration bath was used for the calibration of thermocouples. A Pt 100 temperature probe was used to measure the actual temperature and for control. All thermocouples used in this thesis were calibrated over 8 data points between 10°C and 70°C, resulting in each thermocouple having an uncertainty of \( \pm 0.1°C \).

- **Flow Rate**: The flow rates were set by the Harvard PHD 2000 programmable syringe pumps. The pumps were capable of delivering flow at a rate of between 0.0001 \( \mu \text{L}/\text{min} \) and 220.82 ml/min. The syringe pumps were calibrated by Harvard systems to generate a flow within 0.35% of the target value.

- **Measurement Length**: Measurement length uncertainty was minimised by using a digital vernier callipers. This resulted in an uncertainty in measurement of 0.1 mm -
this uncertainty value includes the error associated with the operation of the vernier callipers. Where measurements of length were extrapolated from images, the images were resolved to within 1 pixel.

- Pressure: Uncertainty in pressure drop measurements was minimised by calibrating the transducer over its full 35 kPa range and conducting repeatability trials for each test.

The following sub-sections detail the measurement uncertainties associated with the: film thickness, pressure drop and heat transfer measurements.

### 3.6.2 Film Thickness Measurement Uncertainty

An optical technique was used to acquire a series of images of the film that separates the aqueous slugs from the capillary wall. The images were analysed in Matlab (version 2010a) and could be resolved to the nearest pixel. The resolution obtained was 44.38 μm and 2.68 μm per pixel for the 1.5x and 20x lenses respectively. Table 3.5 details the uncertainties in the primary measurements: flow rate, carrier phase thermophysical properties and distance, and in the derived variables: film thickness and Capillary number.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Variable</td>
<td></td>
</tr>
<tr>
<td>Flow Rate</td>
<td>0.35%</td>
</tr>
<tr>
<td>Thermophysical Properties</td>
<td>5%</td>
</tr>
<tr>
<td>Pixels</td>
<td>± 1</td>
</tr>
<tr>
<td>Derived Variables</td>
<td></td>
</tr>
<tr>
<td>h (film thickness)</td>
<td>± 8%</td>
</tr>
<tr>
<td>Ca</td>
<td>± 6%</td>
</tr>
</tbody>
</table>

### 3.6.3 Pressure Drop Measurement Uncertainty

Table 3.6 presents the uncertainties in the technique that was used to experimentally measure pressure drop.
Table 3.6: Experimental uncertainties for pressure drop measurement facility.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Variable</td>
<td></td>
</tr>
<tr>
<td>Flow Rate</td>
<td>0.35%</td>
</tr>
<tr>
<td>Thermophysical Properties</td>
<td>5%</td>
</tr>
<tr>
<td>Pixels</td>
<td>± 1</td>
</tr>
<tr>
<td>Pressure</td>
<td>± 35 Pa</td>
</tr>
<tr>
<td>Derived Variable</td>
<td></td>
</tr>
<tr>
<td>Re</td>
<td>± 10%</td>
</tr>
<tr>
<td>Ca</td>
<td>± 6%</td>
</tr>
<tr>
<td>( f/Re )</td>
<td>± 11%</td>
</tr>
</tbody>
</table>

3.6.4 Local Heat Transfer Measurement Uncertainty

This section details the relevant uncertainties in the local heat transfer measurements and is split into two sections: uncertainties in measurements caused by natural convection, radiation and axial conduction; and the subsequent uncertainty analysis.

a) Uncertainties caused by natural convection, radiation and axial conduction

The system was analysed using an energy balance, presented by Stafford et al. (2009), to quantify the effects of secondary heat transfer mechanisms, such as natural convection, radiation and axial conduction, on the overall heat transfer performance of the system. The governing equations in this analysis are: the Joule heating equation and the energy balance for an element of the tube, as shown in Fig. 3.15. The energy balance in the system is given by:

\[
\dot{E}_{in} + \dot{E}_{generated} - \dot{E}_{out} = \dot{E}_{st} \tag{3.15}
\]

where \( \dot{E}_{in} \) is the energy input into the control volume, \( \dot{E}_{generated} \) is the Joule heating input, \( \dot{E}_{out} \) is the energy dissipated from the control volume and \( \dot{E}_{st} \) is the energy stored in the control volume. A number of assumptions have been made to simplify the energy balance of the control volume. These assumptions are as follows:

- The problem is assumed to be axisymmetric.
• There is a negligible temperature gradient radially through the tube wall and the layer of black paint.

• The Fourier number, the ratio of heat conduction to the rate of thermal energy stored, was found to $33.3 \times 10^3$, therefore the storage term $E_{st} = 0$.

![Diagram](image)

Figure 3.15: Energy balance within an element of the heated tube.

The energy balance, presented in Eq. 3.15, expands as:

$$ q_{in} - q_{\text{Forced-convection}} - q_{\text{natural-convection}} - q_{\text{radiation}} + q_{\text{conduction}} = 0 \quad (3.16) $$

The different heat transfer mechanisms within the experimental facility were quantified using the following expressions:

$$ q_{\text{conduction}} = kA \frac{\Delta T}{\Delta x} \quad (3.17) $$
\[ q_{\text{natural-convection}} = h_{\text{nat}} A (T_x - T_\infty) \quad (3.18) \]

where \( h_{\text{nat}} \) is the natural convection heat transfer coefficient, \( T_x \) is the local surface temperature and \( T_\infty \) is the ambient air temperature. The natural convection heat transfer coefficient was calculated using a correlation, presented in Eq. 3.19, developed by Churchill and Chu (1975) for a horizontal cylinder subject to a constant wall heat flux boundary condition.

\[
h_{\text{nat}} = \frac{0.6 + 0.387 \left( \frac{\text{Ra}_D}{D} \right)^{1/6}}{1 + \left( \frac{0.559}{\text{Pr}} \right)^{9/16}}^{8/27} \quad (3.19)
\]

where \( \text{Ra}_D \) is the Rayleigh number based on the diameter of the tube, and is the product of the Grashof and Prandtl numbers:

\[
\text{Ra}_D = \text{Gr}_D \text{Pr} = \frac{g \beta (T_x - T_\infty) D^3}{\nu \alpha} \quad (3.20)
\]

where \( g \), \( \beta \), \( \nu \), and \( \alpha \) refer to gravity, the coefficient of thermal expansion, kinematic viscosity and thermal diffusivity respectively. The heat radiated from the tube surface to the surrounding enclosure, illustrated in Fig. 3.9, was calculated using the following equation:

\[
q_{\text{radiation}} = \varepsilon \sigma A \left( T_x^4 - T_\infty^4 \right) \quad (3.21)
\]

where \( \sigma \) is the Stefan-Boltzmann constant and \( \varepsilon \) is the emissivity of the matt black tube surface, and it is assumed that heat is being radiated from a small body in a large enclosure, Holman (2002). An order of magnitude analysis of the different heat losses within the system was completed, the results of which are presented in Fig. 3.16. The analysis was conducted at the lowest flow rates and over the maximum range in temperatures, observed, as, under these conditions, the losses to the environment, through natural convection and thermal radiation, would be greatest. Possible sources of heat loss in the system were: natural convection and radiation to the environment; and axial conduction along the length of the tubing. It can be seen in Fig. 3.16 that natural convection and radiation are
the greatest source of loss in the system. The magnitude of these losses increase with axial
distance, as the temperature difference between the heated wall and surroundings increases,
thereby creating a larger potential for natural convection and radiation. Losses due to ax-
ial conduction are at a maximum at the thermal entrance point and decrease as the axial
distance increases, however, the magnitude is negligible. Total heat losses in the system,
due to these secondary heat transfer mechanisms, were found to be less than 3% and could
therefore be neglected.

![Figure 3.16: Secondary losses in the local heat transfer measurement facility plotted as a
function of axial distance.](image)

**b) Uncertainty analysis of experimental data**

The uncertainties in the local heat transfer measurement facility are presented in Table 3.7.
The primary uncertainties in local temperature measurements were minimised by calibrat-
ing both the thermocouples and IR camera, as discussed previously in section 3.5.4 of this
chapter.
Table 3.7: Uncertainty analysis of the local heat transfer measurement facility.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Rate</td>
<td>0.35%</td>
</tr>
<tr>
<td>Pixel</td>
<td>± 1</td>
</tr>
<tr>
<td>Temperature</td>
<td>± 0.1°C</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Derived Variable</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>3.39% - 8.43%</td>
</tr>
<tr>
<td>Nu_x</td>
<td>4.44% - 10.13%</td>
</tr>
</tbody>
</table>

Such uncertainties were at a maximum in the early entrance region and quickly diminished towards their lower values as the flows developed.

3.7 Closure

This chapter presented the experimental facilities and measurement techniques used over the course of this thesis. The experimental facilities allowed the flow to be studied non-invasively over a wide range of non-dimensional parameters. The measurement techniques employed allowed a fundamental investigation of the hydrodynamic, film thickness and pressure drop, and thermal characteristics of liquid-liquid Taylor flows confined to minichannel geometries. The results of the hydrodynamic analysis are presented in Chapters 4 and 5 respectively, and the results of the thermal analysis are presented and discussed in Chapter 6.
Chapter 4

Film Thickness Measurements

This chapter provides a detailed discussion of the hydrodynamic performance of liquid-liquid Taylor flows. In a Taylor flow regime, the dispersed slug is separated from the capillary wall by a thin film of the carrier phase. In chemical and biological diagnostic applications, this thin film protects the chemical or biological samples from any asperities within the channels and aids in the prevention of cross contamination between samples. Hence, film thickness and its prediction are an important design criterion in such applications. The objective of the investigation presented in this chapter was to measure the thickness of this liquid film and to evaluate the validity of a series of analytical and semi-empirical expressions from the literature. Experimental measurements were inferred from images captured using the facility presented in section 3.3.

The chapter consists of three sections. Firstly, in section 4.1, the formation of the liquid film is discussed and a series of images are presented that document the influence of specific parameters on the morphology of the slug and, hence, the film thickness. In section 4.2, the experimental measurements are compared with the predictive models from the literature, and the applicability of these models to liquid-liquid Taylor flows is assessed. The final part of this chapter introduces a number of alternative expressions developed to estimate the thickness of the film. These expressions have been developed specifically for use with liquid-liquid Taylor flow regimes and account for the interactions that occur within the flow.
4.1 Film Formation and Variation

This section presents the mechanisms responsible for the formation of the liquid film and investigates the influence of flow velocity and aqueous slug length on slug morphology and the resulting liquid film thickness.

4.1.1 Liquid Film Formation

The study of a long gas bubble flowing in a capillary is a classical problem in fluid mechanics. Traditionally, a gas bubble was incorporated into the flow to act as a tracer, thus allowing the velocity of the liquid phase to be determined. However, Fairbrother and Stubbs (1935) noted that a thin film of liquid was deposited on the wall of the capillary when the liquid phase was displaced by the gaseous bubble. Examining the motion of a gas bubble in a liquid filled capillary, Bretherton (1960) divided the bubble structure into several regions; this has been adapted for liquid-liquid flows and is shown here schematically in Fig. 4.1.

The form of the slug is the result of the competing interfacial and confinement effects.

For a slug at rest, there are no viscous stresses and the front and rear menisci assume shapes of constant mean curvature. Assuming the slugs are at rest, hence axisymmetric, the resultant cap shape is that of a hemisphere. The shape of these caps is maintained by a balance between the uniform pressure within the slug and the interfacial tension between
the immiscible phases. Moving along the length of the slug, these caps pass through transition regions and then proceed to a region where a uniform film of fluid exists between the slug and the capillary wall. As the slug begins to move, viscous effects at the solid-liquid interface result in the formation of the liquid film at the nose of the slug. The thickness of this film, however, depends on the: inertial, viscous and interfacial forces. The slug morphology presented in Fig. 4.1 is that typically encountered in slugs whose length is several times the capillary diameter and flowing at low flow velocities. The slug profile, however, can vary greatly with both mean flow velocity and dispersed slug length. The effects of these features on slug morphology and, hence, film thickness are discussed in the following sub-sections.

4.1.2 Capillary Number Effects on Slug Morphology

Fig. 4.2 presents a series of images of each of the oil/water combinations examined in this thesis and depicts the changes in aqueous slug morphology with flow velocity. Images of the flow were captured using a 10x lens using the experimental facility presented in section 3.3.1. Each image in Fig. 4.2 is a compilation of a series of images stitched together. Due to the low Bond numbers of the flow, see Table 3.2 in section 3.3.2, gravitational effects are negligible. Hence, the flow was assumed to be axisymmetric so only part of the capillary cross-section was imaged. The $Ca$ was calculated based on the mean flow velocity, carrier phase viscosity and interfacial tension between the liquid phases. The $Ca$ ranges from 0.002 - 0.12. Although it was possible to achieve $Ca$ outside of this range, these results were not considered as, at higher $Ca$, a breakdown occurred in the well-ordered Taylor flow regimes and this is not the focus of this investigation. At lower $Ca$, moreover, there was no difference in slug morphology from that at $Ca = 0.002$. Included in Fig. 4.2 is the range in Ca for each liquid-liquid combination examined.

At the minimum $Ca$, 0.002, the transition regions that separate the front and rear caps from the flat film region are non-existent and the aqueous slug nose merges almost directly with the flat film region. At this low $Ca$, the low viscous effects and; uniform pressure within the aqueous slug and the interfacial tension are sufficient to maintain the hemispherical cap shapes, which are the same at the front and rear of the slugs. The hemispherical cap
shapes are the result of interfacial and confinement effects. Interfacial tension seeks to form a slug of with the minimums surface area, a sphere. However, due to the confinement imposed on the sphere by the walls of the capillary, a slug forms. Increases in viscous effects within the capillary result in appreciable changes in slug morphology, Fig. 4.2 (b) - (d), \( Ca > 0.003 \). The front cap becomes more prolate in shape, resulting in a smaller slug nose and a longer transition region between the nose and the flat film region. At the tail, the cap is flatter and more oblate in form, resulting in an aqueous slug profile more akin to a bullet in shape. These changes in aqueous slug morphology, due to the increased viscous effects on the slugs are particularly striking in the AR20/water flow, \( Ca 0.03 - 0.12 \), and similar changes in slug morphology with flow velocity have been documented by Giavedoni and Saita (1999) and Taha and Cui (2004).

![Figure 4.2: Images depicting the changes in aqueous slug morphology with Ca, from 0.002 - 0.12.](image-url)
4.1.3 Dispersed Slug Length Effects on Slug Morphology

Numerous authors, including Han and Shikazono (2009), Olbricht and Kung (1992) and Taha and Cui (2004), have presented slug profiles similar to that described by Bretherton (1960) in his seminal paper. In these studies, the dispersed slugs were of sufficient length to allow the caps to transition into a flat film region. A series of images are presented in Fig. 4.3 that document the development of the aqueous slug profile and the influence of aqueous slug length on the formation of the flat film region. The images presented in Fig. 4.3 are of an FC40/water flow, captured using a 20x lens, and at a constant Capillary number, \( Ca \), of 0.0072. As \( Ca \) is constant, the changes in aqueous slug profile, and hence film thickness, are purely due to changes in aqueous slug length rather than a combined effect with other parameters, namely viscous, inertial and interfacial forces.

![Image](image_url)

Figure 4.3: Experimental images of the FC40/water flow, flowing at \( Ca = 0.0072 \), highlighting the variation in slug profile with length at \( L_D^* \) of (a) 1.15, (b) 1.81 and (c) 2.76.

For shorter aqueous slug lengths, Fig. 4.3 (a), \( L_D^* = 1.15 \), no region of uniform film
thickness exists. The transition from the front cap continues directly to the rear cap and the profile varies along the entire length of the aqueous slug. However, as the aqueous slug length increases, Fig. 4.3 (b), a change in aqueous slug morphology is observed. The transition region at the front of the slug increases in length, while at the tail a wave disturbance is seen prior to the rear cap. This change in slug profile at the tail has also been noted by Goldsmith and Mason (1963) and Olbricht and Kung (1992), and it has been described by Bretherton (1960) as the transition from the body of the slug to the rear cap. Fig. 4.3 (b) shows that the front and rear transition regions intersect, however, no flat film region exists along the slug. A further increase in aqueous slug length, as shown in Fig. 4.3 (c), results in an aqueous slug profile similar to that described by Bretherton (1960), illustrated in Fig. 4.1, where a region of constant film thickness exists.

Similar images to those presented in Fig. 4.3 were recorded for all carrier/water combinations examined, revealing similar trends in aqueous slug morphology for changes in aqueous slug length. Analysis of the experimental data, over the ranges in $Ca$ and $Re$ examined in this thesis, has resulted in the development of the following thresholds for changes in aqueous slug profile with changes in aqueous slug length. For $L_D^* \leq 1.25$ and $1.25 < L_D^* < 1.8$, the aqueous slug profile varies from nose to tail, however, for the latter limits, distinct transition regions are present at the front and rear of the aqueous slugs. Once $L_D^* \geq 1.9$, a flat region forms along the length of the aqueous slug, and a film of uniform thickness separates this flat region from the capillary wall.

Although the slug profile varies with length, the effects of this variation on film thickness are minimal. Fig. 4.4 is a non-dimensional plot of film thickness against aqueous slug length. It can be seen in Fig. 4.4 that for an $L_D^* < 2$, the magnitude of the film varies with slug length. Although there are deviations in film thickness between the different data sets, data for a wide range in $Ca$ are included in the plot. However, at a constant $Ca$, the variation in film thickness above $L_D^* > 1.9$ are minimal, as highlighted in the Pd5/water data. For $L_D^* < 1.9$, the profile of the slug is still developing and no flat film region has formed along the length of the slug. As a result, the magnitude of the film is measured within the transition region between the hemispherical caps, as shown in Fig. 4.3 (a) and (b). Consequently, it is proposed that the location of measurement, resulting
from the non-existence of a flat film region, is the cause of this variation in film thickness with aqueous slug length. However, for $L_{D}^* \geq 1.9$, a flat film region forms, and, similar to observations presented by Howard and Walsh (2013), Irandoust and Andersson (1989) and White and Beardmore (1962), variations in film thickness with aqueous slug length are minimal. Moreover, it is believed that these variations fall within experimental uncertainty, hence any effects of aqueous slug length on film thickness cannot be clearly identified.

Figure 4.4: Non-dimensional plot of film thickness, $h/R$, against aqueous slug length, $L_{D}/D$.

### 4.2 Film Thickness Models

This section introduces the analytical and semi-empirical film thickness models that are most pertinent to this thesis. The predictions of these models are plotted against the experimental liquid-liquid data, obtained using the experimental facility presented in section 3.3.1, and a comparative analysis is completed that assesses the validity of these models over the ranges in non-dimensional parameters considered in this thesis.
CHAPTER 4  Film Thickness Measurements

4.2.1 Correlations

Numerous researchers have completed numerical, (Araujo et al. (2012), Heil (2001) and Taha and Cui (2004)), and experimental, (Grimes et al. (2007), Irandoust and Andersson (1989) and Jovanovic et al. (2010)), works examining the deposition of a liquid film on a capillary wall. Some of these works have resulted in the development of models that can be used to estimate the magnitude of the film. The details of the models most pertinent to this study are presented in Table 4.1.

Table 4.1: Film thickness correlations from the literature.

<table>
<thead>
<tr>
<th>Author</th>
<th>Correlation</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fairbrother and Stubbs (1935)</td>
<td>( \frac{h}{R} = 0.5Ca^{1/2} )</td>
<td>( Ca &lt; 0.014 ) Liquid-gas flow Horizontal flow Single slug</td>
</tr>
<tr>
<td>Bretherton (1960)</td>
<td>( \frac{h}{R} = 1.34Ca^{2/3} )</td>
<td>( 10^{-3} &lt; Ca &lt; 10^{-2} ) Liquid-gas flow Horizontal flow Single slug</td>
</tr>
<tr>
<td>Aussillous and Quere (2000)</td>
<td>( \frac{h}{R} = \frac{1.34Ca^{2/3}}{1+1.34(2.5Ca^{2/3})} )</td>
<td>( 10^{-3} &lt; Ca &lt; 1.4 ) Liquid-gas flow Horizontal flow Single slug</td>
</tr>
<tr>
<td>Han and Shikazono (2009)</td>
<td>( \frac{h}{R} = \frac{1.34Ca^{2/3}}{1+3.13Ca^{2/3}+0.54Ca^{0.672}Re^{0.589}-0.352W_e^{0.629}} )</td>
<td>( Ca &lt; 0.40 ) Liquid-gas flow Horizontal flow Multiple slugs</td>
</tr>
</tbody>
</table>

Examining the deposition of the liquid film on a capillary wall, Fairbrother and Stubbs (1935) noted that, at low bubble speeds, the thickness of the film was proportional to \( Ca^{1/2} \). Taylor (1960) experimentally measured the mean liquid film thickness by measuring the difference between the bubble and mean flow velocity and confirmed the validity of the Fairbrother and Stubbs (1935) model up to \( Ca = 0.1 \). Using an analytical approach, Bretherton (1960) assumed a creeping flow within the liquid film and used lubrication theory to calculate the magnitude of the film. The resulting expression is presented in Table 4.1.

Aussillous and Quere (2000) developed an empirical correlation using the data of
Taylor (1960) and, similar to Bretherton (1960) and Fairbrother and Stubbs (1935), found that the normalised film thickness was dependent on a single parameter, $Ca$. This correlation became known as Taylor’s Law, where the coefficient 1.34 was derived by Bretherton (1960) and the coefficient 2.5 is empirical. It is stated to be valid up to $Ca = 1.4$. The authors noted that at higher flow velocities, the magnitude of the film was underpredicted by Taylor’s Law, and further increases in velocity resulted in a reduction in the rate of film growth until the magnitude of the film reached an asymptotic limit. This limit in film thickness was attributed to the confinement effects of the capillary, and it has also been observed in experimental works of Cox (1964) and Howard and Walsh (2013). Aussillous and Quere (2000) noted that these changes in film thickness, above and below Taylor’s Law, were dependent on the magnitude of the $Ca$ relative to the ratio of $Ca/Re$.

In the majority of experimental works, $Ca$ is varied by increasing the viscosity of the liquid phase and maintaining a low flow velocity, for ease of measurement. Consequently, the bulk of the experimental works in the literature have collected data from flows at low $Re$, where inertial effects are negligible. Addressing this deficit, Han and Shikazono (2009) measured the magnitude of the film, using an interferometer and laser focus displacement meter, in high $Re$ flows, $4.6 < Re < 2000$, where the inertial forces eclipse viscous forces. The authors found that at low flow velocities, inertial forces could be neglected and that $Ca$ alone could be used to determine the thickness of the film. However, increases in $Ca$ and $Re$ resulted in a reduction in film thickness to a minimum value before subsequently increasing again. Similar observations were made by Heil (2001) and Kreutzer et al. (2005), and have been attributed to increased inertial effects. To account for inertial effects, Han and Shikazono (2009) developed a universal expression, presented in Table 4.1, to predict the magnitude of the film over a wide range in $Ca$ and $Re$.

It is worth noting that the models presented in Table 4.1 were developed for liquid-gas flows, and, although numerous applications employ liquid-liquid Taylor flows, little is known about the thickness of the film in such a flow regime. Systems and devices that incorporate liquid-liquid Taylor flows, such as those developed by Taheny (2010) and Dalton (2012) to perform biological diagnostic processes, are designed using the models presented in Table 4.1 to estimate the magnitude of the film. The following sub-section examines the
validity of these expressions when applied to liquid-liquid Taylor flow regimes over a wide range of flow conditions.

4.2.2 Experimental Data

Fig. 4.5 is a plot of the non-dimensional film thickness, $h/R$, for each of the carrier oil/water combinations examined in this thesis against Capillary number, $Ca$. Included in the plot is an enlarged view of the data in the lower $Ca$ range, $Ca < 0.01$. Experimental data points are compared with data points evaluated using the models listed in Table 4.1.

Section 4.1.2 revealed that the aqueous slug profile varied with length. This can make the choice of measurement location difficult. Some authors report measurements based on the thickness at the flat section midway between the front and rear caps, Han and Shikazono (2009) and Jovanovic et al. (2010). However, this is only possible when aqueous slug lengths are greater than the previously defined threshold of $L_D^* \geq 1.86$. Other authors have overcome this variation in film thickness with aqueous slug length by using indirect measurement techniques, Grimes et al. (2007), Irandoust and Andersson (1989) and Kashid et al. (2005), that give an average of the film thickness along the length of the aqueous slug. Where possible, the experimental data presented in this section was measured at the flat region. However, for slugs where $L_D^* < 1.86$, the measurement was taken towards the rear of the slug. This location was chosen as, similar to cases where the flat film region was present, the aqueous slug was at its broadest, and hence the film was at its thinnest.

Overall, the model of Bretherton provides the best approximation of the experimental data, to within $\pm 25\%$, over the $Ca$ range examined, $0.001 \leq Ca \leq 0.07$. This is particularly evident at $Ca \geq 0.01$, where the other models greatly underpredict the magnitude of the film. The data in Fig. 4.5 show a strong dependence of film thickness on $Ca$. There is, however, a question over the definition of the $Ca$. The Capillary numbers presented in Fig. 4.5 were calculated based on the carrier phase viscosity, the mean two phase velocity and the interfacial tension between the phases. It was stated at the beginning of this chapter that the magnitude of the film depended on a number of parameters, including the velocity at the interface between the phases, i.e. the mean dispersed slug velocity, $U_D$. 
Figure 4.5: Non-dimensional film thickness, $h/R$, plotted against $Ca$, where the $Ca$ is calculated based on the carrier viscosity, mean two phase velocity and interfacial tension between the liquid phases. Experimental data points are compared to the most referenced film thickness models from literature.

At low $Ca$, this is approximately equal to the mean two phase velocity, however as
CHAPTER 4 Film Thickness Measurements

*Ca* increases, the film increases in thickness and, as a result, the dispersed slug velocity increases. This change in slug velocity with film thickness is shown in Fig. 4.6. Mean dispersed slug velocities were calculated using the images captured with the 1.25x lens, from which corresponding *Ca* were calculated. This redefined *Ca*, unlike the *Ca* based on the mean two phase velocity, provides a truer representation of the forces influencing the magnitude of the film at the point of measurement.

![Graph](image)

Figure 4.6: Ratio of experimental slug velocity to mean two phase velocity, $U_D/U$, plotted against dimensionless film thickness, $h/R$, depicting the change in slug velocity with film thickness.

Fig. 4.7 is a plot of the non-dimensional film thickness against the redefined *Ca*. Included in Fig. 4.7 is an enlarged view of the data in the lower *Ca* range, $Ca \leq 0.01$. Similar to Fig. 4.5, at $Ca < 0.012$, the experimental data is best approximated by the model of Bretherton. Although developed for low *Ca* flows, $Ca < 10^{-2}$, the model assumes creeping flow, $Re \approx 1$. However, the experimental data in this region is populated by fluids with $Re$ in the range 15 - 100.96, hence the influence of inertia on the magnitude of the film should be greater. Consequently, the model of Han and Shikazono (2009) should provide a better approximation of the experimental data.
Figure 4.7: Non-dimensional film thickness, $h/R$, plotted against $Ca$ based on the mean slug velocity. Included in the plot for comparative purposes are the most referenced film thickness correlations from literature.

In a study measuring the film thickness in liquid-gas flows, Howard and Walsh (2013)
found that the expression of Han and Shikazono (2009) provided excellent agreement with their experimental data up to $Re = 112.7$. Nonetheless, as can be seen in the enlarged view in Fig. 4.7, this expression underpredicts the magnitude of the film in a liquid-liquid Taylor flow regime, by 4.5 - 41.3%. As the $Ca$ increases, $Ca > 0.012$, the experimental data deviates from the model of Bretherton and begins to follow a similar trend to Taylor’s Law. However, this model still underpredicts the magnitude of the film, by 5 - 19.1%. Although there are a number of differences between liquid-liquid and liquid-gas flows, the primary disparity between the flow regimes is the viscosity difference that exists between the two phases. In a liquid-gas flow regime, the viscosity of the discontinuous gaseous phase can be assumed negligible compared to that of the carrier phase. Consequently, the shear forces at the interface can also be assumed negligible and, hence, no flow occurs in the liquid film, as discussed in section 2.2. However, in a liquid-liquid flow regime, the reduced viscosity difference induces significant shear forces at the deformable interface between the phases. This results in a non-negligible flow in the liquid film that surrounds the aqueous slugs, as highlighted in Fig. 2.4 in section 2.2. Hence, it is proposed that the shear induced flow in the liquid film, is the cause of the thicker film between the capillary walls and the deformable interface in liquid-liquid flows compared to those encountered in liquid-gas flows.

The data presented in Figs. 4.5 and 4.7 show that the models, for the most part, underestimate the magnitude of the film, by 3.4 - 46.5%, and highlights the need for an alternative approach to estimate the magnitude of the film in liquid-liquid Taylor flows.

### 4.3 New Film Thickness Models

This section introduces a number of new expressions developed specifically for liquid-liquid Taylor flows. The section begins by identifying the appropriate groupings for the experimental data, based on critical Capillary numbers and, following this, details of the development of the new expressions are presented.
4.3.1 Capillary Number Groupings

In a study measuring the magnitude of the liquid film in liquid-gas flows, Aussillous and Quere (2000) identified three distinct flow regimes that influenced the deposition of the liquid film. These flow regimes are:

1. **Visco-capillary flow regime:** In a visco-capillary regime, the normalised film thickness is dependent on a single parameter, $Ca$, and can be estimated using Taylor’s Law. Flows in a visco-capillary regime are characterised by low deposition velocities and significant viscous effects.

2. **Visco-inertial flow regime:** Increases in deposition velocity result in a transition to the visco-inertial regime. Aussillous and Quere (2000) noted that this transition occurred at a critical Capillary number, $Ca^*$, presented in Eq. 4.1, which is based on the carrier phase viscosity and density, capillary radius and interfacial tension. Flows in a visco-capillary regime are subject to increased inertial and reduced viscous effects, and it has been observed by Aussillous and Quere (2000) and Han and Shikazono (2009) that the magnitude of the film in this regime is thicker than that predicted by Taylor’s Law.

3. **Confinement dominated flow regime:** Further increases in deposition velocity result in a flow where the growth of the film is restricted by geometrical effects. Transition to this flow regime is proposed to occur at $Ca^{**}$, presented in Eq. 4.2, provided $Ca/Re < 1$.

\[
Ca^* = \left( \frac{Ca}{Re} \right)^{3/4} = \left( \frac{\mu^2}{\rho R \sigma} \right)^{3/4} \quad (4.1)
\]

\[
Ca^{**} = \left( \frac{Ca}{Re} \right)^{1/2} = \left( \frac{\mu^2}{\rho R \sigma} \right)^{1/2} \quad (4.2)
\]

Table 4.2 presents the $Ca$, $Ca^*$ and $Ca^{**}$ ranges for the different liquid-liquid combinations examined in this thesis. It can be seen in Table 4.2 that the the AR20/water data is in the visco-capillary regime, $Ca < Ca^*$, while the remainder of the experimental data lies in the visco-inertial regime, $Ca^* < Ca < Ca^{**}$. 

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Table 4.2: $Re$, $Ca$, $Ca^*$ and $Ca^{**}$ ranges for the current study, $Re$ and $Ca$ are calculated using the mean slug velocity.

<table>
<thead>
<tr>
<th>Oil/Water Combination</th>
<th>$Re$</th>
<th>$Ca$</th>
<th>$Ca^*$</th>
<th>$Ca^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pd5/Water</td>
<td>14.46 - 47.53</td>
<td>0.003 - 0.011</td>
<td>0.003</td>
<td>0.021</td>
</tr>
<tr>
<td>Dodecane/Water</td>
<td>61.95 - 95.86</td>
<td>0.002 - 0.003</td>
<td>0.007</td>
<td>0.008</td>
</tr>
<tr>
<td>FC40/Water</td>
<td>31.02 - 100.96</td>
<td>0.002 - 0.007</td>
<td>0.001</td>
<td>0.120</td>
</tr>
<tr>
<td>AR20/Water</td>
<td>3.68 - 15.13</td>
<td>0.029 - 0.119</td>
<td>0.126</td>
<td>0.355</td>
</tr>
</tbody>
</table>

4.3.2 Visco-Capillary Regime

The experimental AR20/water data lies in the visco-capillary regime, thus the influence of inertial forces on the magnitude of the film should be negligible, $Re < 15$, and $Ca$ alone should be sufficient to model the experimental data. It was seen in Fig. 4.7 that the AR20/water measurements followed a similar trend to Taylor’s Law, however, the model underpredicted the magnitude of the film, particularly as the $Ca$ increased, $Ca > 0.04$. The empirical constant in this correlation, presented in Table 4.1, is stated as 2.5, however for the data presented in this thesis on liquid-liquid flows, a value of 1.6 was found, through observation, to give better agreement with the experimental data, within $\pm$ 10%. A modified Taylor’s Law for liquid-liquid flows in the visco-capillary regime is presented in Eq. 4.3 and, for completeness, it is plotted in Fig. 4.8 against the experimental AR20/water data.

$$\frac{h}{R} = \frac{1.34Ca^{2/3}}{1 + 1.34(1.6Ca^{2/3})}$$ (4.3)
Figure 4.8: Non-dimensional film thickness, $h/R$, of AR20/water flows plotted against $Ca$, based on the mean slug velocity. Included in the plot is a modified Taylor’s Law for liquid-liquid slug flows.

### 4.3.3 Visco-Inertial Regime

The majority of the experimental data lies within the visco-inertial regime, where $Ca^* < Ca < Ca^{**}$. Similar to Aussillous and Quere (2000), Han and Shikazono (2009) and Howard and Walsh (2013), the thickness of the film in this regime, shown in the enlarged view of Fig. 4.7, is underpredicted by Taylor’s Law. Moreover, it is also underpredicted by the model of Han and Shikazono (2009). Consequently, the following approach was used to model the thickness of the film.

The experimental data exhibits the following trends: $Re >> 1$, $Ca << 1$ and $We << 1$. These non-dimensional groups represent the forces that exert the greatest influence on the magnitude of the film. Relatively high Reynolds, $Re > 15$, and low Capillary, $Ca < 0.01$, numbers indicate that the inertial and interfacial forces are the predominant influences on film thickness. The non-dimensional Weber number, which presents the relationship between these forces, ranges from 0.047 - 0.697 which implies that the interfacial forces should have the most substantial influence on the thickness of the film. To determine which
forces, or combination of forces, have the greatest influence on the magnitude of the film, a regression analysis of the experimental data was completed using the curve fitting toolbox in Matlab (version 2010a).

Regression analysis is a statistical tool that can be used to identify the form of a relationship that exists between the dependent, film thickness, and independent variables, namely the dimensionless slug lengths, Capillary, Reynolds and Weber numbers. The regression analysis determined that the $Ca$ and $We$ were the non-dimensional groups that best scaled the experimental data. The form of this relationship, presented in Eq. 4.4, was determined by the scaling technique used which, due to the low residual levels, $R^2 = 0.93$, identified it as the most pertinent means of representing the data. The form of Eq. 4.4 presents a slight variation on those of Bretherton (1960), Fairbrother and Stubbs (1935) and Aussillous and Quere (2000), where, in these models, inertial effects on film thickness are assumed negligible, and presents a much simpler expression than that of Han and Shikazono (2009).

\[
\frac{h}{R} = 0.72(Ca)^{0.47} 0.08(We)^{0.13}
\] (4.4)

\[
\frac{h}{R} = \frac{\varphi^{0.13} D^{0.13} \mu^{0.47} U_D^{0.73}}{\sigma^{0.60}}
\] (4.5)

Eq. 4.5 presents a breakdown of Eq. 4.4 into its primary constituents, thus providing an insight into which physical mechanisms have the greatest influence on the magnitude of the film. Similar to the expression developed by Han and Shikazono (2009), the mean dispersed slug velocity and interfacial forces exert the greatest influence on film thickness, $U_D^{0.73}/\sigma^{0.60}$. Eq. 4.4 provided a good collapse of the experimental data and further analysis of the scaled data identified that a power law trend, of the form presented in Eq. 4.6, accurately represented the scaled data.

\[
\frac{h}{R} = 0.35\left((Ca)^{0.47} (We)^{0.13}\right)^{0.75} = 0.35(Ca)^{0.35} (We)^{0.10}
\] (4.6)

Eq. 4.6 incorporates both $Ca$ and $We$. Although $We = ReCa$, and the inertial and viscous forces affect film thickness, it is their interaction with the interface, i.e. their balance...
with interfacial tension, and not one another that affects the thickness of the film. Consequently, the incorporation of \( We \), and not \( Re \) in Eq. 4.6, makes more physical sense to the problem under analysis. Similar to the expressions proposed by Bretherton (1960), Fairbrother and Stubbs (1935) and Aussillous and Quere (2000), Eq. 4.6 illustrates that film thickness is highly dependent on \( Ca \). Analysis of the \( Ca \) and \( Ca^* \) data, presented in Table 4.2, shows that the experimental data has, in some cases, just entered into the visco-inertial regime. Consequently, the small weighting of the Weber number, \( We^{0.10} \), compared to the Capillary number, \( Ca^{0.35} \), is unsurprising and highlights the dependence of film thickness on \( Ca \) over the \( Ca, 0.002 - 0.119 \), \( We, 0.047 - 0.697 \), and \( Re, 14.46 - 100.96 \), ranges examined in this thesis.

A comparison of the experimental data with the expression presented in Eq. 4.6 and the model of Han and Shikazono (2009), the most appropriate model for flows in a visco-inertial regime, is presented in Fig. 4.9.

Figure 4.9: Non-dimensional film thickness, \( h/R \), plotted against \( Ca \) based on the mean slug velocity. Experimental results are compared with the correlation of Han and Shikazono (2009) and the expression presented in Eq. 4.6.
Although the expression in Eq. 4.6 was developed using a small number of data points, the ranges in non-dimensional parameters reflect those encountered in most applications that employ liquid-liquid Taylor flow regimes. Eq. 4.6 also provides a better approximation of the experimental data than the models extrapolated from the literature, within ± 15%.

### 4.4 Closure

This chapter focused on the thin liquid film that separates the dispersed aqueous slugs from the capillary wall in a liquid-liquid Taylor flow regime. The objectives were to obtain a non-invasive direct measure of its thickness, and to examine the influence of aqueous slug length and the thermophysical properties of the carrier phase. Images of the flow revealed that the morphology of the aqueous slugs changed with changes in aqueous slug length. A series of thresholds, based on dimensionless aqueous slug length, were identified, above which a region of constant film thickness exists that separates the aqueous slugs from the capillary wall.

A comparative analysis between the experimental data and the classical models from the literature was completed. The experimental data showed reasonable agreement with the Bretherton model when the Capillary number was calculated based on the mean two phase flow velocity. However, significant differences were observed when the Capillary number was redefined to account for the mean velocity at the liquid interface, i.e. the mean dispersed slug velocity and, overall, the existing models were found to underpredict the thickness of the film.

Analysis of the experimental data revealed that the data fell into two distinct flow regimes: a visco-capillary regime and a visco-inertial regime. A modified Taylor expression was presented to estimate the thickness of the film for flows in the visco-capillary regime, to within ± 10%. A new model was proposed, based on the Capillary and Weber numbers, for flows in the visco-inertial regime, which predicted the data to within ± 15%.

Chapter 5 presents a further discussion on the hydrodynamics of liquid-liquid Taylor flows by examining the pressure drop induced by such a flow regime.
Chapter 5

Pressure Drop Measurements

This chapter presents pressure drop measurements induced by liquid-liquid Taylor flows confined to minichannel geometries. Pressure drop is a key design parameter in any engineering application, particularly when trying to minimise pumping power requirements and overall system costs. The purpose of the investigation in this chapter is to determine the accuracy of number of analytical and semi-empirical pressure drop models, and to establish, if necessary, improvements for liquid-liquid Taylor flows. Experimental time-averaged pressure drop measurements were recorded using the experimental facility and test procedure outlined in section 3.4. The ranges in non-dimensional slug lengths, 0.71-14.14, and Capillary number, $4.5 \times 10^{-4} - 0.06$, are expected to reflect those encountered in most micro/macro fluidic systems that incorporate liquid-liquid Taylor flows.

Sections 5.1 and 5.2 compare the experimental data to the homogeneous and separated flow models. These approaches represent the most fundamental, yet widely applied means of modelling the flow. There are a number of models in the literature developed specifically for Taylor flow regimes. These models are presented and discussed in section 5.3 and a comparative analysis between the experimental data and the data predicted using the models is presented.
5.1 Homogeneous Flow Models

The homogeneous modelling approach is the most basic and straightforward means of analysing the pressure drop associated with a two phase flow regime. This modelling technique assumes that the two phase flow behaves as a pseudo single phase fluid with average or mixture defined thermophysical properties. The resultant pressure drop per unit length, $\Delta P/L$, is calculated using expressions from single phase flow theory, presented in Eq. 5.1:

$$\left(\frac{\Delta P}{L}\right)_{TP} = \frac{16}{Re_m} \left(\frac{1}{2} \rho_m U_m^2\right) \frac{4}{D}$$

where the subscript $TP$ refers to the two phase pressure drop and $Re_m$, $\rho_m$ and $U_m$ are the mixture Reynolds number, density and velocity. As the homogeneous model is based upon pressure drop theory for single phase flows, a number of mixture models have also been developed for the thermophysical properties of the flow. The most referenced and pertinent of these models will be presented in the following sub-sections.

5.1.1 Effective Density Models

The mean density of the flow can be calculated using the volume fraction of the dispersed phase, $\varepsilon$, and is presented in Eq. 5.2.

$$\rho_m = (1 - \varepsilon) \rho_C + \varepsilon \rho_D$$

where the subscripts $C$ and $D$ denote the continuous and dispersed phases; and the volume fraction of the dispersed phase is given by:

$$\varepsilon = \frac{Q_D}{Q_C + Q_D}$$

The mean density can also be written in terms of the mass fraction of the dispersed phase:

$$\rho_m = \left(\frac{1-x}{\rho_C} + \frac{x}{\rho_D}\right)^{-1}$$
where:

$$x = \frac{\rho_D Q_D}{\rho_C Q_C + \rho_D Q_D} \quad (5.5)$$

### 5.1.2 Effective Viscosity Models

The effective viscosity of the flow has received far more attention in the literature than the effective density. The models vary depending on the particular flow species, liquid-liquid, liquid-gas or solid-liquid, employed. In liquid-gas flows, the most commonly encountered viscosity models are those of Cicchitti et al. (1960) and Dukler et al. (1964), which are presented in Eqs. 5.6 and 5.7. Although developed for liquid-gas flows, Salim et al. (2008) found that their experimental liquid-liquid pressure drop data correlated well with the homogeneous model when used in conjunction with these viscosity models.

$$\mu_m = (1 - x)\mu_C + x\mu_D \quad (5.6)$$

$$\mu_m = \rho_m \left(1 - x\right) \left(\frac{\mu_C}{\rho_C} + x\frac{\mu_D}{\rho_D}\right) \quad (5.7)$$

In a review of existing viscosity models, Awad and Muzychka (2008) proposed several new models based on analogous thermal conductivity models for two phase flow systems. The authors found that for flows in micro and macrochannels, the Maxwell based models provided improvements in homogeneous flow modelling. The most applicable of these models is the Maxwell-Eucken I model, which was developed for flows where the viscosity of the continuous phase is greater than that of the dispersed phase and takes the following form:

$$\mu_m = \mu_C \left(\frac{2\mu_C + \mu_D - 2(\mu_C - \mu_D)x}{2\mu_C + \mu_D + (\mu_C - \mu_D)x}\right) \quad (5.8)$$

The following sub-section assess the effectiveness of the homogeneous modelling technique when used in conjunction with the aforementioned density and viscosity models.
5.1.3 Homogeneous Model Performance

Fig. 5.1 is a dimensioned plot of pressure drop against the total volumetric flow rate, $Q_T$, and presents a series of measurements for an FC40/water flow recorded using the experimental facility and procedure presented in section 3.4. The total volumetric flow rate varies from 2.5 - 13 ml/min and the pressure drop varies from 3 - 16 kPa. Included in the plot are data points calculated using a homogeneous model in conjunction with the effective density and viscosity models presented in Eqs. 5.4, 5.6, 5.7 and 5.8. By varying the mass fraction, due to a variation in relative flow rates, multiple data points are generated at a particular overall flow rate. It is evident in Fig. 5.1 that a homogeneous model greatly underpredicts the magnitude of the pressure drop, by approximately 1.8 kPa, or 60%, at 2.5 ml/min up to 8 kPa, or 60%, at 13 ml/min.

![Figure 5.1: Plot of pressure drop, $\Delta P$, against the total volumetric flow rate, $Q_T$, for an FC40/water flow. Included in the plot for comparative purposes are data points calculated using the homogeneous model.](image)

Comparisons between the other oil/water combinations examined in this thesis and
the homogeneous model revealed the same trend; where the pressure drop was under-
predicted. For clarity, these data points have not been included in Fig. 5.1, however, Table
5.1 presents a summary of the differences between all the experimental and model data.

Table 5.1: Summary of the minimum and maximum differences between the experimental
pressure drop data and data points calculated using the homogeneous model.

<table>
<thead>
<tr>
<th>Liquid-Liquid Combination</th>
<th>Cicchitti et al. (1960) kPa (%)</th>
<th>Dukler et al. (1964) kPa (%)</th>
<th>Maxwell-Euken I kPa (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pd5/Water</td>
<td>1.0 - 6.4 (15.6 - 78.4)</td>
<td>0.9 - 6.3 (15.3 - 78.3)</td>
<td>1.1 - 6.8 (16.5 - 79.6)</td>
</tr>
<tr>
<td>Dodecane/Water</td>
<td>1.4 - 10.7 (24.7 - 82.3)</td>
<td>1.4 - 10.6 (24.7 - 82.2)</td>
<td>1.4 - 10.7 (24.7 - 82.3)</td>
</tr>
<tr>
<td>FC40/Water</td>
<td>1.8 - 11.2 (60.0 - 70.0)</td>
<td>1.9 - 11.9 (63.2 - 74.4)</td>
<td>1.8 - 11.5 (60.0 - 71.8)</td>
</tr>
<tr>
<td>AR20/Water</td>
<td>1.0 - 3.3 (19.2 - 40.6)</td>
<td>1.1 - 3.7 (21.2 - 45.5)</td>
<td>1.2 - 4.5 (23.2 - 53.5)</td>
</tr>
</tbody>
</table>

A number of alternative expressions were presented in sections 5.1.1 and 5.1.2 to
calculate the effective density and viscosity of the flow. However, the data presented in
Fig. 5.1 and Table 5.1 shows little difference between the pressure drops predicted by the
homogeneous model using the different density and viscosity models. Consequently the
choice of effective density and viscosity model makes little difference as, ultimately, the
homogeneous model underpredicts the pressure drop.

The homogeneous model assumes that the two phases mix together and a single
fluid flows in the capillary. Examining the pressure drop induced by liquid-liquid Taylor
flows flowing in microchannels, Jovanovic et al. (2010) noted that the interface between
the phases accounted for between 60 - 80% of the total pressure drop, depending on the
flow rate. By varying the relative flow rates and hence the slug lengths, it was possible
to vary the number of interfaces present in the channel. For example, in Fig. 5.1 at 12
ml/min, the relative oil and water flow rates were adjusted from 9/3 to 5/7, which resulted
in an approximate increase in pressure drop of 4 kPa. As the homogeneous model assumes
no interface exists between the phases, the poor performance of the model is unsurprising,
and has also been reported by Sun and Mishima (2009). Consequently, it can be concluded
that homogeneous flow modelling is not an accurate means of calculating the pressure drop
generated by liquid-liquid Taylor flows.
5.2 Separated Flow Modelling

Unlike the homogeneous flow modelling approach, presented in section 5.1, separated flow models account for the differing thermophysical properties and flow velocities of the constituent phases. This approach considers the contributions of both phases separately and predicts the resultant pressure drop to be the sum of their single phase contributions. The most widely used models in the literature include those of Lockhart and Martinelli (1949) and Turner-Wallis, see Wallis (1969). The following sub-sections will introduce these models and, subsequently, present a comparison between the models and the experimental data.

5.2.1 Separated Flow Models

This section presents a general overview of the Lockhart-Martinelli and Turner-Wallis models, which are, in fact, related.

a) Lockhart-Martinelli Model

Lockhart and Martinelli (1949) proposed a correlation scheme for liquid-gas flows on the premise that the static pressure drop for the two phases, flowing simultaneously, are equal at any point along the channel, such that:

\[
\frac{\Delta P}{\Delta L}_{TP} = \Phi_L^2 \left( \frac{\Delta P}{\Delta L} \right)_L = \Phi_G^2 \left( \frac{\Delta P}{\Delta L} \right)_G
\]  
(5.9)

where:

\[
\Phi_G = \sqrt{\frac{\left( \frac{\Delta P/L}{\Delta P/L} \right)_{TP}}{\left( \frac{\Delta P/L}{\Delta P/L} \right)_G}}
\]  
(5.10)

\[
\Phi_L = \sqrt{\frac{\left( \frac{\Delta P/L}{\Delta P/L} \right)_{TP}}{\left( \frac{\Delta P/L}{\Delta P/L} \right)_L}}
\]  
(5.11)

The subscripts \( G \) and \( L \) refer to the liquid and gas phases, and \( \Phi \) is the two phase multiplier which defines the ratio of the two phase flow pressure gradient to a reference single phase pressure gradient. The authors also introduce a parameter, \( X \), known as the
Martinelli parameter, presented in Eq. 5.12. The Martinelli parameter is a reference scale that defines the extent to which the two phase pressure drop is dominated by either the liquid or gas phase.

\[ X = \frac{\Phi_G}{\Phi_L} = \sqrt{\frac{(\Delta P/L)_L}{(\Delta P/L)_G}} \]  

(5.12)

The model proposed by Lockhart and Martinelli (1949) is actually an empirically based set of curves and tabulated data sets developed for liquid-gas flows. Chisholm and Laird (1958) completed a semi-empirical analysis of the tabulated data sets and proposed a series of simple fits for the data. These equations, presented in Eqs. 5.13 and 5.14, have been widely accepted in the literature, Muzychka and Awad (2010) and Kawahara et al. (2002), and are known as the Lockhart and Martinelli model.

\[ \Phi_L^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \]  

(5.13)

\[ \Phi_G^2 = 1 + CX + X^2 \]  

(5.14)

where the value of C depends on whether each phase is laminar or turbulent, and is presented in Table 5.2.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Gas</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminar</td>
<td>Laminar</td>
<td>5</td>
</tr>
<tr>
<td>Turbulent</td>
<td>Laminar</td>
<td>10</td>
</tr>
<tr>
<td>Laminar</td>
<td>Turbulent</td>
<td>15</td>
</tr>
<tr>
<td>Turbulent</td>
<td>Turbulent</td>
<td>20</td>
</tr>
</tbody>
</table>

b) Turner-Wallis Model

Two decades after Lockhart and Martinelli presented their pioneering approach to two phase flow modelling, Wallis (1969) put forward a semi-empirical model based on the equations of Lockhart and Martinelli (1949). The authors proposed that the two phase flow could be analysed as two single phase streams flowing parallel to one another in separate
channels, each with a cross-sectional area equal to its actual flow area. The model assumes that the pressure gradient across the parallel single phase streams equates to that in the two phase flow, and that the cross-sectional areas of the two smaller channels is equal to that of the actual channel carrying the two phase flow. The authors proposed that the following generalised equation could be used for any two phase flow regime:

\[
\left( \frac{1}{\Phi_L^2} \right)^p + \left( \frac{1}{\Phi_G^2} \right)^p = 1
\]  

(5.15)

Given the relationship of Eq. 5.12, it is possible to rearrange Eq. 5.15 to obtain the following one parameter models:

\[
\Phi_L^2 = \left( 1 + \left( \frac{1}{X^2} \right)^p \right)
\]  

(5.16)

\[
\Phi_G^2 = \left( 1 + \left( X^2 \right)^p \right)
\]  

(5.17)

where the value of \( p \) has been found using the separate cylinders approach to equal \( p = 2 \) for laminar-laminar flow, \( 2.375 < p < 2.5 \) for turbulent-turbulent flows based on the friction factor, and \( 2.5 < p < 3.5 \) for turbulent-turbulent flows calculated on a mixing length basis, Wallis (1969).

The following sub-section compares these models with the experimentally measured liquid-liquid data. To allow a comparison with the experimental data, the models were modified by replacing the gas phase with the less viscous liquid phase. For all liquid-liquid combinations examined in this thesis, this corresponded to water.

### 5.2.2 Separated Flow Model Performance

Figs. 5.2 and 5.3 present comparisons between the experimentally measured liquid-liquid pressure drop data and data points calculated using the Lockhart-Martinelli and Turner-Wallis models. To highlight the accuracy of these models, lines representing ±20% deviations from parity are included in the plots. The predicted pressure drop was calculated using Eq. 5.9 and \( \Phi \) was calculated using the expressions presented in Eqs. 5.13 and 5.16.
Figure 5.2: Comparison between the measured pressure drop data and the data calculated using the Lockhart-Martinelli model.

Figure 5.3: Comparison between the measured pressure drop data and the data calculated using the Turner-Wallis model.
Examining the data in Figs. 5.2 and 5.3, it can be seen that there is significant scatter in the Pd5/water, Dodecane/water and FC40/water data, with the models, for the most part, underpredicting the magnitude of the two phase pressure drop. Differences between the experimental and predicted pressure drops range from 4.9 - 72.3% using the Lockhart-Martinelli model and 7.7 - 82.8% using the Turner-Wallis model. However, the models provide a reasonable approximation of the AR20/water data, with the majority of the data falling within the ± 20% bands. It is unsurprising that the models perform similarly, as the Turner-Wallis model is based on the tabulated data sets and empirical curves developed by Lockhart and Martinelli (1949).

Unlike the homogeneous flow models, presented in section 5.1, Lockhart and Martinelli (1949) attempted to account for the additional pressure drop associated with the interface between the two phases by incorporating the empirical parameter $\Phi$. The model, presented in Eq. 5.9, was developed to provide a solution for most types of two phase flow regimes, such as; bubbly, mist, or stratified, presented in Fig. 1.1. Consequently, the models presented in section 5.2.1 have garnered much popularity and use, Saisorn and Wongwises (2008), Lee and Lee (2001), Kawahara et al. (2002) and Chen et al. (2007). However, these models do not distinguish between the unique flow physics associated with each flow regime. In general, the resulting interfacial pressure drop predictions are an order of magnitude estimate, consequently, the total two phase pressure drop is also an order of magnitude estimate, as can be seen in Figs. 5.2 and 5.3. For applications such as chemical or biological diagnostics, where precise control of the flow is required, this would not be an acceptable approximation. Thus, separated flow modelling is not an appropriate technique for modelling the pressure drop in liquid-liquid Taylor flows.

### 5.3 Taylor Flow Models

The introduction of a second immiscible phase into a laminar flow regime significantly alters the hydrodynamic characteristics of the flow. Consequently, a new set of fluid dynamics equations are required as the addition of a second phase introduces non-linearities into the otherwise linear Navier-Stokes laws, thus making it difficult to make *a priori* design
calculations. The previously discussed homogeneous and separated flow models, sections 5.1 and 5.2 respectively, fail to account for any of the unique hydrodynamic characteristics associated with liquid-liquid Taylor flows. However, there does exist within the literature, a number of models developed specifically for Taylor flow regimes. These models are presented and discussed in two separate sub-sections that compare the experimental liquid-liquid data with the most pertinent and referenced liquid-liquid and liquid-gas pressure drop models from the literature.

5.3.1 Liquid-Liquid Taylor Flow Models

There are two categories of pressure drop models for liquid-liquid Taylor flows.

1. The first assumes that the dispersed phase makes an appreciable contribution to the total pressure drop.

2. The second assumes that the thin liquid film that encapsulates the aqueous slugs contributes significantly to the total pressure drop.

a) Dispersed Phase Effects

In this category, the total pressure drop consists of three components:

1. the frictional pressure drop of the carrier phase;
2. the frictional pressure drop of the dispersed phase; and
3. the pressure drop due to the interface between the phases.

Two models fall into this category and have been developed by Kashid and Agar (2007) and Jovanovic et al. (2010). In the model developed by Kashid and Agar (2007), presented in Eq. 5.18, the flow is modelled as a series of slug unit cells, each consisting of a dispersed and continuous slug pair, as illustrated in Fig. 5.4. This approach assumes fully developed Hagen-Poiseuille flow, and that no liquid film separates the aqueous slugs from the capillary wall, Fig. 5.4 a). The interfacial pressure is calculated using the Young-Laplace equation, presented in the final term of Eq. 5.18.
\[ \Delta P_{TP} = \frac{L}{L_u} \left( \frac{8\mu_C U (1 - \varepsilon) L}{R^2} + \frac{8\mu_D U \varepsilon L}{R^2} \right) + \frac{2L - L_u}{L_u} \left( \frac{2\sigma}{R_c \cos \theta} \right) \] (5.18)

where \( L, L_u, \varepsilon, \sigma, \theta \) and \( R_c \) refer to the length of the capillary, the slug unit length, dispersed phase fraction length, \( L_D/L_u \), interfacial tension, contact angle and curvature of the interface respectively.

![Aqueous slugs suspended in a continuous carrier phase](image)

Figure 5.4: Slug unit cell, consisting of a single continuous carrier slug and dispersed slug, a) shows an aqueous slug wetting the capillary wall and b) an aqueous slug completely encapsulated by the carrier phase.

The model developed by Jovanovic et al. (2010), presented in Eq. 5.19, assumes that a stagnant liquid film separates the aqueous slugs from the capillary wall, Fig 5.4 b). Consequently, the aqueous slugs travel at a higher velocity than the mean two-phase velocity. This results in a modified Hagen-Poiseuille expression for the pressure drop experienced by the dispersed phase. Interfacial effects are accounted for using Bretherton’s theoretical expression, presented in the final term in Eq. 5.19.

\[ \Delta P_{TP} = \frac{8\mu_C U (1 - \varepsilon) L}{R^2} + \frac{8\mu_D U \varepsilon L}{(R - h)^2} + \frac{L}{L_u} \frac{7.16 (3Ca)^{2/3} \sigma}{D} \] (5.19)
where \( h \) is the thickness of the film that separates the aqueous slugs from the capillary wall. Fig. 5.5 is a plot of pressure drop against the total volumetric flow rate for an FC40/water flow. The flow rate and pressure drop range from 2.5 - 13 ml/min and 3 - 16 kPa respectively. Included in the plot are data points calculated using the expressions presented in Eqs. 5.18 and 5.19.

![Figure 5.5: Plot of \( \Delta P \) against \( Q_T \) for an FC40/water flow. Experimental measurements are compared to data points calculated using the models of Kashid and Agar (2007) and Jovanovic et al. (2010).](image)

It is evident from Fig. 5.5 that the model of Kashid and Agar (2007), Eq. 5.18, greatly overestimates the total pressure drop in the capillary, by approximately 10.1 kPa, (333.6%), at the lower flow rates up to 14.5 kPa, (103.4%), at the higher flow rates. However, the model of Jovanovic et al. (2010), Eq. 5.19, underpredicts the total pressure drop, by 1-5 kPa, (28.8 - 61.4%), at the lower flow rates and up to 6 kPa, (41.4%), at the higher flow rates. Similar results were observed in the other liquid-liquid combinations examined in this thesis and the results are summarised in Table 5.3.
Table 5.3: Summary of the differences between the experimental liquid-liquid data and the liquid-liquid Taylor flow pressure drop models.

<table>
<thead>
<tr>
<th>Liquid-Liquid Combination</th>
<th>Liquid-Liquid Taylor Flow Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Kashid and Agar Eq. 5.18 kPa (%)</td>
</tr>
<tr>
<td>Pd5/Water</td>
<td>8.7 - 20.6 (154.9 - 417.1)</td>
</tr>
<tr>
<td>Dodecane/Water</td>
<td>6.7 - 17.3 (79.4 - 518.1)</td>
</tr>
<tr>
<td>FC40/Water</td>
<td>8.3 - 15.0 (68.6 - 365.3)</td>
</tr>
<tr>
<td>AR20/Water</td>
<td>8.6 - 17.8 (76.1 - 420.8)</td>
</tr>
</tbody>
</table>

The model of Kashid and Agar (2007) greatly overpredicts the magnitude of the pressure drop, in some cases the model data is double the magnitude of the experimental data. This is due to the method used to calculate the interfacial pressure drop. The model assumes no liquid film separates the aqueous slugs from the capillary wall, as shown in Fig. 5.4 a). Consequently, the interfacial pressure drop is calculated at a constant contact angle and the contributions of the front and rear menisci are summed together. However, it has been shown by numerous investigators; Han and Shikazono (2009), Howard and Walsh (2013), Olbricht and Kung (1992) and Taha and Cui (2004); and in Chapter 4 of this thesis, that a thin liquid film separates the aqueous slugs from the capillary wall, Fig. 5.4 b). Therefore, the contact angles are substantially different from the dry wall case and the receding and advancing contact angles can only be assumed equal at very low velocities. Furthermore, due to the presence of the film, the rear cap of the aqueous slug assumes a profile that is inverse to that of the front cap; hence the front cap has a positive contribution and the rear cap a negative contribution to the pressure drop. Accordingly, the contributions should be subtracted from one another rather than summed.

In the model developed by Jovanovic et al. (2010), Eq. 5.19, the theoretical solution of Bretherton is used to calculate the interfacial pressure drop. This expression underestimates the interfacial contribution as it assumes that there are negligible inertial effects, and that the film thickness is small compared to the capillary radius, \( h/R < 10^{-2} \). However, it has been shown in section 4.2.2 and in the experimental works of Grimes et al. (2007), Aussillous and Quere (2000) and Tsaoulidis et al. (2013) that this assumption is invalid and that the theoretical solution of Bretherton breaks down at \( Ca < 10^{-4} \) and \( Ca > 10^{-1} \), and in cases where inertia is no longer negligible.
b) Liquid Film Effects

The second category of pressure drop models developed for liquid-liquid Taylor flows assumes the thin liquid film that encapsulates the dispersed slugs generates a significant pressure drop. As in the previous category, the models have been developed by Kashid and Agar (2007) and Jovanovic et al. (2010). In a liquid-liquid flow regime, depending on the viscosity difference that exists between the phases, interfacial shear can be significant. This results in non-negligible flow in the film and was highlighted in Fig. 2.4 in section 2.2. Both models assume that the pressure drop induced by the flow in the liquid film makes a significant contribution to the total pressure drop.

The second model developed by Kashid and Agar (2007), presented in Eq. 5.20, assumes that the pressure drop induced by the film is the dominant pressure drop in a slug unit. Hence, the pressure drops associated with the interface and the dispersed phase can be completely neglected. This approach is based on the work of Charles et al. (1961), who modelled the pressure drop of solid capsules flowing through a capillary, and relates the pressure drop along the film to the single phase carrier pressure drop along the length of the capillary. To avoid confusion with the other pressure drop model developed by Kashid and Agar (2007), presented in the previous sub-section, the model presented in Eq. 5.20 will be referred to henceforth as the capsule model.

$$
\Delta P_T = \left( \frac{\varepsilon}{1 - C^4} \right) \Delta P_C
$$

(5.20)

where $\varepsilon$ is the dispersed phase fraction and $C$ is a coefficient calculated using:

$$
C = \frac{R - h}{R}
$$

(5.21)

The model of Jovanovic et al. (2010), Eq. 5.22, assumes that the pressure drop caused by the non-negligible flow in the liquid film is greater than that of the dispersed phase. Consequently, the expression presented in Eq. 5.19 can be rewritten in the following form and is referred to as the moving film model:
\[ \Delta P_{TP} = \frac{8\mu_U U (1 - \varepsilon)L}{R^2} + \frac{4U_D L\varepsilon}{(R^2 - (R-h)^2)} + \frac{L}{L_u} \frac{7.16 (3Ca)^{2/3}}{\sigma D} \]  

(5.22)

where the middle term in Eq. 5.22 is the pressure drop induced by the liquid film.

Fig. 5.6 is a plot of the same experimental FC40/water measurements as presented in Fig. 5.5 and includes data points calculated using the capsule and moving film pressure drop models presented in Eqs. 5.20 and 5.22.

Figure 5.6: Plot of the total volumetric flow rate against pressure drop for an FC40/water flow. The plot contains data points measured experimentally, using the experimental facility presented in Fig. 3.7, and data points calculated using the capsule, Eq. 5.20, and moving film, Eq. 5.22, models.

Unlike the data presented in Fig. 5.5, both models now underpredict the magnitude of the pressure drop, from 1 - 5 kPa, or 28.9 - 61.6%, at the lower flow rates to between 5 - 8 kPa, or 17.9 - 52.1%, at the higher flow rates. Similar results were observed in the other liquid-liquid combinations examined in this thesis, as summarised in Table 5.4.
Table 5.4: Summary of the differences between the experimental data and data points calculated using the models presented in Eqs. 5.20 and 5.22.

<table>
<thead>
<tr>
<th>Liquid-Liquid Combination</th>
<th>Liquid-Liquid Taylor Flow Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Capsule Model Eq. 5.20 kPa (%)</td>
<td>Moving Film Model Eq. 5.22 kPa (%)</td>
</tr>
<tr>
<td>Pd5/Water</td>
<td>0.1 - 3.6 (2.1 - 48.9)</td>
<td>0.2 - 3.2 (3.5 - 60.9)</td>
</tr>
<tr>
<td>Dodecane/Water</td>
<td>0.5 - 9.1 (18.2 - 74.7)</td>
<td>5.4 - 25.8 (85.5 - 306.7)</td>
</tr>
<tr>
<td>FC40/Water</td>
<td>0.2 - 8.3 (5.3 - 54.9)</td>
<td>0.8 - 6.7 (17.9 - 61.6)</td>
</tr>
<tr>
<td>AR20/Water</td>
<td>0.3 - 2.4 (2.5 - 28.7)</td>
<td>0.3 - 2.6 (2.5 - 30.9)</td>
</tr>
</tbody>
</table>

Both models developed by Jovanovic et al. (2010) provide similar approximations of the total pressure drop. This result is unsurprising, as both models assume that the interface and carrier phase make the greatest contributions to the total pressure drop, between 90% - 95%, and both models use the same expressions to calculate these contributions. Similar findings were reported by Jovanovic et al. (2010), and it was concluded by the authors that the velocity of the flow in the liquid film had a negligible influence on the total pressure drop in the channel. However, the model underestimates the total pressure drop due to the method used to calculate the interfacial pressure drop, as discussed in the previous sub-section. Alternatively, the capsule model put forward by Kashid and Agar (2007) in Eq. 5.20, provides a better approximation than the model presented in Eq. 5.18. Nonetheless, the capsule model still provides poor agreement with the experimental data, underestimating the magnitude of the pressure drop by 0.1 - 6.7 kPa, or 17.9 - 61.6%. The capsule model, based on that of Charles et al. (1961), assumes that the aqueous slugs are representative of solid capsules. Thus implying that there is no circulation of flow within the aqueous slugs, such as that shown in Fig. 2.5 in section 2.2, and a constant cap shape exists. This is certainly not the case for liquid-liquid Taylor flows, where the circulation of flow within the aqueous slugs has been documented in numerous μ-PIV studies, such as those by; Miessner et al. (2008), Malsch et al. (2008) and Meinhart et al. (1999), and the changes in cap shape with flow velocity have been presented in Fig. 4.2 in section 4.1.2 of this thesis. The capsule model is simplistic, ignoring key contributors to the pressure drop, and, ultimately underpredicts the magnitude of the pressure drop.

In summary, the liquid-liquid Taylor flow models presented in this section provide poor approximations of the experimentally measured pressure drop data. The major short
falling of these models lies in the methods used to calculate the interfacial pressure drop. The expressions used are very limited in terms of their applicability, to cases where there are negligible inertial effects or the dispersed slug wets the capillary wall. As a result, these models are not suitable for use in the design of most systems that incorporate liquid-liquid Taylor flows. Numerical modelling of liquid-gas Taylor flows by Ratulowski and Chung (1989), Heil (2001) and Fujioka and Grotberg (2005) has shown that the flow structure around the gaseous bubbles is influenced by inertia, and this influence of inertia on the interfacial pressure drop has been confirmed experimentally by Kreutzer et al. (2005), Walsh et al. (2009) and Warnier et al. (2010). Consequently, the interfacial pressure drop in a liquid-liquid Taylor flow regime should also be expected to be influenced by inertia. Thus, liquid-gas Taylor flow models should provide some direction in modelling the pressure drop in liquid-liquid Taylor flows. The following sub-section presents a series of liquid-gas Taylor flow models and examines the methods used to model the interfacial pressure drop.

5.3.2 Liquid-Gas Taylor Flow Models

The introduction of a second immiscible phase into a channel, to create a segmented flow regime, results in an increase in pressure drop relative to the single phase case. In a liquid-gas flow regime, the frictional losses in the gaseous phase are considered negligible when compared to those in the liquid phase and, as a result, are not taken into account. Consequently, liquid-gas pressure drop models consist of two components: the frictional pressure drop due to the liquid phase and the interfacial pressure drop, and this can be expressed as:

\[
\Delta P_{TP} = \Delta P_C + \Delta P_{Int}
\]  

(5.23)

Similar to the homogeneous, separated and liquid-liquid Taylor flow models presented in the preceding sections, the flow in the carrier phase is assumed to be fully developed Hagen-Poiseuille flow, given by:

\[
\frac{\Delta P_C}{L} = \frac{16}{Re} \left( \frac{1}{2} \rho U^2 \right) \frac{4}{D} = \frac{4(fRe)_C}{2D^2} \frac{\mu U}{2D^2}
\]

(5.24)
where the product of the friction factor, \( f \), and the Reynolds number, \( Re \), is equal to a constant:

\[(fRe)_C = 16\]  

(5.25)

As a result, Eq. 5.23 may be rewritten as:

\[fRe_{TP} = 16 + fRe_{Int}\]  

(5.26)

Focusing on the interfacial component, \( \Delta P_{Int} \), a number of expressions have been developed to model the interfacial pressure drop, with that of Bretherton (1960), presented in section 5.3.1, being the most referenced in the literature. This expression is a theoretical solution derived for the pressure drop caused by a single gas bubble in a Taylor flow regime. The expression was designed to be all-encompassing, accounting for changes in curvature due to the presence of the liquid film and the Laplace pressure term.

Kreutzer et al. (2005) examined the contribution of the interface both numerically and experimentally, and determined that the interfacial pressure drop was a function of the dimensionless carrier slug length, \( L_C^* \), Capillary, \( Ca \), and Reynolds, \( Re \), numbers. The authors developed the expression presented in Eq. 5.27 to account for the pressure drop caused by the presence of the interface:

\[fRe_{Int} = \alpha \left( \frac{Re}{Ca} \right)^{\beta} \]  

(5.27)

where \( \alpha \) and \( \beta \) are coefficients whose values are presented in Table 5.5. This expression implies that the interfacial pressure drop is dependent on the physical properties of the continuous phase and the capillary diameter, but is independent of velocity. Walsh et al. (2009) completed a rigorous experimental analysis of the problem and came to the same conclusion, that the interfacial pressure drop was a function of \( L_C^* \), \( Ca \) and \( Re \). However, examining the problem over a much greater range, \( 1.58 \leq Re \leq 1024 \) and \( 0.002 \leq Ca \leq 0.2 \), and incorporating the data of Kreutzer et al. (2005), Walsh et al. (2009) found that a value of \( a = 1.92 \) provided a much better collapse of the data than those put forward by Kreutzer et al. (2005).
Table 5.5: Coefficient values used in Eq. 5.27 by Kreutzer et al. (2005) and Walsh et al. (2009).

<table>
<thead>
<tr>
<th>Model</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kreutzer et al. (2005) (numerical)</td>
<td>1.12</td>
<td>0.33</td>
</tr>
<tr>
<td>Kreutzer et al. (2005) (experimental)</td>
<td>2.72</td>
<td>0.33</td>
</tr>
<tr>
<td>Walsh et al. (2009) (experimental)</td>
<td>1.92</td>
<td>0.33</td>
</tr>
</tbody>
</table>

More recently, Warnier et al. (2010) examined the contribution of the bubble to the total pressure drop in a Taylor flow regime. Similar to the work of Aussillous and Quere (2000), where the authors modified the expression of Bretherton (1960) and incorporated the data of Taylor (1960) to develop Taylor’s Law, Warnier et al. (2010) reworked Bretherton correlation to incorporate Taylor’s Law to account for the presence of the film and its effects on pressure drop. The resultant expression, presented in Eq. 5.28, can be arranged in terms of $fRe$ to allow for comparison with Eq. 5.27.

$$fRe_{Int} = \left( \frac{7.16 \left( \frac{3^{2/3}}{32} \right)}{L_C^{5/2}} \right) \left( \frac{A}{A_B} \right) \frac{1}{Ca^{1/3} + 3.34Ca}$$

(5.28)

where $A$ and $A_B$ refer to the capillary and bubble cross-sectional areas and $Re$ and $Ca$ are based on the characteristic velocities $U_m$ and $U_D$ respectively. The total pressure drop induced by the two phase Taylor flow regime can be calculated by:

$$\Delta P_{TP} = \frac{4\mu_c U L (fRe)_{TP}}{2D^2}$$

(5.29)

where $fRe_{TP}$ can be calculated using Eq. 5.25 with either Eq. 5.27 or Eq. 5.28. In numerous microfluidic devices that incorporate liquid-liquid Taylor flows, such as those presented by Sugiura et al. (2004), Cramer et al. (2004) and Zheng and Ismagilov (2005), the viscosity difference between the phases can range from 0.3 up to 100. Hence, the viscosity difference between the phases can be comparable to those encountered in a liquid-gas flow regime. Consequently, provided a significant viscosity difference exists between the phases, liquid-gas pressure drop models should provide an accurate approximation of the pressure drop induced by liquid-liquid Taylor flows.
5.3.3 Liquid-Gas Theory Applied to Liquid-Liquid Taylor Flows

The viscosity ratios, viscosity of the carrier phase relative to the dispersed phase, of the different liquid-liquid combinations examined in this thesis are presented in Table 5.6 and are quite modest, 1.6 - 24, compared to those encountered in most liquid-gas studies, 54 - 1105, Walsh et al. (2009), Oliver and Young-Hoon (1968), Saisorn and Wongwises (2008) and Irandoust and Andersson (1989). Nonetheless, in a previous study by Salim et al. (2008), where the carrier phase was approximately 30 times more viscous than the dispersed phase, the authors found good agreement between their experimental data and data modelled using liquid-gas models.

Table 5.6: Viscosity differences between the different liquid-liquid combinations examined in this thesis.

<table>
<thead>
<tr>
<th>Liquid-Liquid Combination</th>
<th>Viscosity Ratio $(\mu_C/\mu_D)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pd5/Water</td>
<td>4.1</td>
</tr>
<tr>
<td>Dodecane/Water</td>
<td>1.6</td>
</tr>
<tr>
<td>FC40/Water</td>
<td>4.5</td>
</tr>
<tr>
<td>AR20/Water</td>
<td>24</td>
</tr>
</tbody>
</table>

Fig. 5.7 presents a direct comparison of the experimental friction factor-Reynolds number products, $(f_{\text{exp total Re}})_{TP}$, and those calculated using the models of Walsh et al. (2009), Fig. 5.7 a), and Warnier et al. (2010), Fig. 5.7 b), $(f_{\text{Re}})_{TP \text{ Theory}}$. The experimental two phase friction factor was calculated using the following expression:

$$f_{\text{exp total}} = \left(\frac{\Delta P}{L}\right) \left(\frac{\rho_C U^2}{\frac{4}{3}}\right) \left(\frac{\rho_D}{\frac{4}{3}}\right) + \left(\frac{\rho_C U^2}{\frac{4}{3}}\right) \left(\frac{\rho_D}{\frac{4}{3}}\right)$$  (5.30)

To highlight the accuracy of the Walsh and Warnier models, lines representing ± 20% deviations from parity are included in the plots. Starting with the Walsh model, Fig. 5.7 a), there is considerable scatter in the Dodecane/water data, with the majority of the data points falling outside of the ± 20% bandwidth. As the viscosity ratio increases to 4.1, the model begins to overpredict the Pd5/water data. A further increase in the viscosity ratio to 4.5 results in an improved model performance.
Figure 5.7: Experimental pressure drop presented as $(f_{\text{exp total Re}})_{TP}$, as described by Eq. 5.30, plotted against $(f_{Re})_{TP \text{ Theory}}$. The latter is calculated using Eq. 5.26, where the interfacial component is calculated using models developed by: a) Walsh et al. (2009), Eq. 5.27, and b) Warnier et al. (2010), Eq. 5.28.
This improvement in model performance continues as the viscosity ratio increases, with all of the AR20/water data falling inside the ± 20% bandwidth. This highlights the applicability of this model for high viscosity ratio flows. Given the range in Ca of the AR20/water flows, 0.014 ≤ Ca ≤ 0.052, falls perfectly within that of the Walsh model, 0.002 ≤ Ca ≤ 0.2, this result is unsurprising. Taking a closer look at the Walsh model, it consists of a linear superposition of the single phase Poiseuille flow limit and a semi empirically derived interfacial flow limit and was developed using an addition of asymptotes approach. The transition from a Poiseuille to interfacially dominated flow occurs when:

\[ 16 \sim \frac{\alpha}{L_c^* \left( Ca \right)^{0.33}} \left( \frac{Re}{Ca} \right) \]

Poiseuille Flow \sim Interfacial Flow

\[ L_c^* \left( \frac{Ca}{Re} \right)^{0.33} \sim \left( \frac{1.92}{16} \right) \sim 0.12 \]

Thus, when \( L_c^* (Ca/Re)^{0.33} \ll 0.12 \), the pressure drop in the flow is dominated by the interface. While the opposite is true when \( L_c^* (Ca/Re)^{0.33} \gg 0.12 \), the pressure drop in the flow is characteristically Poiseuille flow and the interface has a negligible influence. Experimental fRe values are plotted against the Walsh model in Fig. 5.8 and it can be seen that the experimental data is scattered about the Walsh model. At \( L_c^* (Ca/Re)^{0.33} > 0.12 \), the flows are subject to reduced interfacial effects and the slugs are of sufficient length for the pressure drop in the capillary to be characteristically Poiseuille. Due to its viscous nature, the data in this region is almost exclusively populated by the AR20/water flows. Similar to Fig. 5.7 a), excellent agreement is observed between the experimental data and the model. Thus highlighting the applicability of the Walsh model to flows where a significant viscosity difference, \( \mu_L/\mu_D > 4.5 \), exists between the liquid phases in a liquid-liquid Taylor flow regime.

Very short slug lengths demonstrate the most significant deviation from the model as they have the greatest interfacial contribution, as described by \( L_c^* (Ca/Re)^{0.33} < 0.12 \). A noteworthy feature from Fig. 5.8 is that the data in the region \( L_c^* (Ca/Re)^{0.33} < 0.12 \), is
populated by the flows where the viscosity difference between the flows is quite low, with a maximum of 4.5. Although developed to model flows with a large interfacial contribution, Kreutzer et al. (2005) stated that Eq. 5.27 was only applicable for Re of order 100, while the interfacial contribution calculated using the Bretherton correlation should only be used for Re of order 1. The Pd5/water, Dodecane/water and FC40/water flows, have Re between these bounds of 1 and 100, Re ranges from 14.4 - 98.6. Consequently, the poor performance of the Walsh model at these intermediate Re values, as highlighted in Figs. 5.7 a) and 5.8, is unsurprising.

Figure 5.8: Experimental pressure drop, presented as $f \cdot Re$, plotted against $L'(Ca/Re)^{0.33}$, where Ca and Re are calculated based on the carrier phase thermophysical properties. Data is compared with the model presented in Eq. 5.27 using the coefficient $\alpha$ put forward by Walsh et al. (2009).

A similar observation was made by Warnier et al. (2010), who highlighted this dependence on Re at intermediate values. The authors developed a correlation, presented in Eq. 5.28, to model the pressure drop in Taylor flows at these intermediate Re values.
Fig. 5.7 b) presents the experimental data, ($f_{\text{exp total Re}}$)$_{TP}$, plotted against data points calculated using the model of Warnier et al. (2010), ($f_{Re}$)$_{TP}$ Theory. Similar to the Walsh model, Fig. 5.7 a), there is excellent agreement between the Warnier model, Fig. 5.7 b), and the AR20/water data. The Warnier model provides an improved approximation of the Pd5/water, Dodecane/water, and FC40/water data and exhibits better agreement with changes in $Re$ compared to the Walsh model, Fig. 5.7 a). The model incorporates the works of Aussillous and Quere (2000) and accounts for the increased dispersed slug velocity, $U_D$, due to the presence of the liquid film, when determining the interfacial pressure drop. The dispersed slug velocity, $U_D$, was inferred from calculations of the film thickness using Eqs. 4.3 and 4.6. However, a number of the Pd5/water, Dodecane/water and FC40/water data points still reside outside of the ± 20% bandwidths. A modified form of the interfacial component of the Warnier model, Eq. 5.28, is presented in Eq. 5.31. The interfacial component is normalised by the volume fraction of the capillary occupied by the carrier phase, or $1 - \varepsilon$, where $\varepsilon$ is the volume fraction of the dispersed phase and can be calculated using Eq. 5.3.

$$fRe_{Int} = \left( \frac{8.16(3^{2/3})}{32} \right) L_C^{e-1} \left( \frac{A}{A_B} \right) \frac{1}{C a^{1/3} + 3.34C a} (1 - \varepsilon)^{1/3} \right) \tag{5.31}$$

The normalisation of the interfacial frictional component implies that the changes in frictional pressure drop induced by the varying volumetric presence of the dispersed slugs is negated by the volumetric channel fraction occupied by the carrier phase, $1 - \varepsilon$. In the original paper Warnier et al. (2010) noted that in most engineering applications of Taylor flow regimes the bubble velocity is not measured. Taking a mass balance, the authors showed that the term $(L/L_C)(A/A_B) = f_B/U_L$, where $f_B$ is the bubble frequency and $U_L$ is the carrier phase velocity. Imposing the limits $\varepsilon = 0$ and $\varepsilon = 1$ on Eq. 5.31 results in the interfacial pressure component tending towards zero as the bubble frequency is zero for both cases. Thus the resulting frictional pressure drop in the channel is that of the single phase flow. The empirical curvature parameter in the original Warnier model, Eq. 5.28, is stated as 7.16 and is based on the works of Bretherton (1960). Jovanovic et al. (2010) found that the curvature parameter was dependent on We. However, at $We > 0.2$, the curvature
parameter became a constant, the value of which increased with channel diameter. For the present work, a value of $We > 0.2$ and $8.16$ was found to give a better approximation of the experimental data. Fig. 5.9 evaluates the performance of the modified Warnier model, where plotted on the abscissa are the theoretical $f/Re$ values and plotted on the ordinate are the experimental results. The outlying data from Fig. 5.7 b) now resides within the $\pm 20\%$ bandwidths. The Warnier model was developed for water-air flows and the following ranges in dimensionless parameters: $41 < Re < 159$, $2.3 \times 10^{-3} < Ca < 8.8 \times 10^{-3}$, $3 < L_C^* < 23.95$ and $1.67 < L_D^* < 14.25$. The agreement between the experimental data and the modified model greatly extends the applicability of this model to: $1.82 < Re < 159$, $4.5 \times 10^{-4} < Ca < 0.06$, $0.71 < L_C^* < 23.95$ and $1.05 < L_D^* < 14.25$ and liquid-liquid Taylor flows.

This model assumes that the total pressure drop in the capillary consists of two components: the frictional pressure drop of the continuous phase and the pressure drop due to
the interface between the phases. Although there is a third phase, the dispersed phase, that induces a pressure drop, the agreement between the experimental data and the modified Warnier model, presented in Fig. 5.9, implies that it is negligible compared to the other two components and does not need to be considered in the analysis.

5.4 Closure

This chapter focused on the pressure drop in liquid-liquid Taylor flow regimes. A comprehensive experimental programme was completed, where the pressure drop was measured over dimensionless slug, Capillary and Reynolds numbers that spanned several orders of magnitude.

Comparisons between the experimental data and the homogeneous and separated flow models showed poor agreement. These models are non-phenomenological, and fail to account for interfacial tension effects and the hydrodynamic details of the flow regime, such as the spatial liquid-liquid distribution and the velocity distribution within the respective phases.

A series of models, developed specifically for use with liquid-liquid Taylor flows, were then presented. These models were developed by Kashid and Agar (2007) and Jovanovic et al. (2010) and were found to give poor approximations of the experimental data. In the case of Jovanovic et al. (2010), this was attributed to the method used to determine the interfacial component, where it was assumed that the flow was not subject to any inertial effects, \( Re \approx 1 \). Whereas the works of Kashid and Agar (2007) are limited to very specific flow conditions, where the dispersed phase wets the walls of the capillary and the flow within the dispersed phase behaves in a similar manner to that in a solid plug.

The liquid-gas Taylor flow pressure drop models of Kreutzer et al. (2005), Walsh et al. (2009) and Warnier et al. (2010) were used for some guidance in modelling the pressure drop in liquid-liquid Taylor flows. The former two authors identified that the interfacial pressure drop was dependent on the dimensionless slug length, \( L_C^* \), and \( Ca/Re \). However, a comparative analysis between the experimental data and data points calculated using this model unearthed significant disparities between the data sets in the Pd5/water,
Dodecane/water and FC40/water flows. However, the model was found to predict the pressure drop in the AR20/water data to within ± 20%. Hence, provided a significant viscosity difference exists between the phases, as in the case of AR20/water, 23, the model of Walsh et al. (2009) can be used to calculate the pressure drop in a liquid-liquid Taylor flow regime.

The model proposed by Warnier et al. (2010) was developed to account for changes in pressure drop with changes in $Re$, a limitation of the previous models. The model was found to give an improved approximation of the experimental data over that of Walsh et al. (2009). However, a significant quantity of data was still outside the ± 20% bandwidths. A modified Warnier model was presented here, where the interfacial pressure drop was normalised by the volumetric channel fraction occupied by the carrier phase, $1 - \varepsilon$. This resulted in an improved model performance where all the experimental data fell within ± 20% of the model data.

Chapter 6 presents a thermal characterisation of liquid-liquid Taylor flows subject to a constant wall heat flux boundary condition. Similar to the hydrodynamic characterisation completed in Chapters 4 and 5, the influence of slug length and carrier phase thermophysical properties on the flow will be analysed.
Chapter 6

Heat Transfer Measurements

This chapter addresses the thermal behaviour of liquid-liquid Taylor flows. The advent of microchannel technology has led to the development of numerous liquid cooled devices, such as cold plates and compact heat exchangers. The addition of a Taylor flow regime to such devices would represent a novel cooling solution. The aim of this experimentation was to thermally characterise liquid-liquid Taylor flows when subject to a constant wall heat flux boundary condition. Results are drawn from time-averaged temperature profiles generated over the thermal entrance and thermally developed regions of the experimental test section, presented in section 3.5.

The chapter begins by outlining a series of expressions that are used as benchmarks to gauge the magnitude of enhancement offered by liquid-liquid Taylor flows, and a series of heat transfer correlations relevant to Taylor flow regimes. Section 5.2 presents the experimental results, first in a dimensioned format, highlighting the augmentation in heat transfer. The results are then generalised by presenting them dimensionlessly. In section 5.3, the effectiveness of liquid-gas Taylor flow models, when applied to liquid-liquid flows, is assessed. The final part of this chapter outlines the development of a correlation to model the thermal behaviour of liquid-liquid Taylor flows subject to a constant wall heat flux boundary condition.
6.1 Analytical Models

This section consists of two sub-sections that highlight the relevant analytical expressions used in thermally characterising both single and two phase flows in both the thermally developing and fully developed regions of internal channel flows. The specific problem under investigation examines the thermal boundary layer development in a laminar liquid-liquid Taylor flow regime subject to a constant heat flux boundary condition. Dimensionless heat transfer rates for internal flows are characterised by the local Nusselt numbers, $Nu_x$, and can be calculated using Eq. 3.8 from section 3.5.4. The local heat transfer rates are dependent on position: the inverse Graetz number, $x^*$, as defined in Eq. 2.20 in section 2.3.2, provides a measure of the dimensionless position in the channel downstream of the thermal entrance point.

6.1.1 Single Phase Flows

For fully developed single phase laminar flow, the velocity profile at any cross-section in a circular capillary can be solved using Eq. 2.1 from section 2.1.1. Consequently, the energy conservation equation can be solved to provide an exact solution of the temperature field, and is presented in Eq. 2.9 in section 2.1.2. A solution of this nature was initially reported by Graetz (1883) and Graetz (1885), but the Graetz-Leveque equation was later developed as a simplified piecewise analytical expression. This expression, presented in Eq. 6.1, defines the dimensionless heat transfer rates as a function of the inverse Graetz number and was first reported by Shah and London (1978):

$$Nu_x = \begin{cases} 
1.302x^{*-1/3} & \text{if } x^* \leq 0.0015 \\
4.364 + 8.68(10^3 x^*)^{-0.506} \exp(-41x^*) & \text{if } x^* \geq 0.0015 
\end{cases}$$

(6.1)

As Eq. 6.1 shows, solutions of this nature are separated into two distinct regions where, in the first region an expression is defined for the thermal entrance region, while in the second region, the flow is fully developed thermally and the Nusselt number is a constant. Using the addition of asymptotic limits approach, Muzychka and Yovanovich (2004) combined such expressions and arrived at Eq. 3.10, presented in section 3.5.4. This
expression defines the variation in $Nu_x$ with $x^*$ for flows subject to a constant heat flux boundary condition when the fluid Prandtl number, $Pr$, is greater than unity.

Also of interest in this study is the theoretical solution obtained as the $Pr$ approaches zero. Such a solution is known as the plug flow limit, Bejan (1993), and occurs when either an inviscid fluid or solid rod passes through a channel. Hence, the fluid flows as a solid plug with a uniform velocity profile across the channel cross-section. By way of analytical considerations, Bejan (1993) showed that the fully developed $Nu_x$ increases to 7.96 for a constant heat flux boundary condition. Muzychka et al. (2009) developed an analytical solution, presented in Eq. 6.2, that describes the variation in $Nu_x$ with $x^*$ in terms of asymptotic limits:

$$Nu_{x,\text{Plug}} = \left( \frac{0.886}{x^{1/2}} \right)^2 + (7.96)^2 \right)^{1/2} \quad (6.2)$$

The models presented in Eqs. 3.10 and 6.2 will be used as indicators to gauge the level of enhancement offered by the liquid-liquid Taylor flows, as illustrated in Fig. 6.1.

Figure 6.1: Plot of local Nusselt number, $Nu_x$ as a function of the inverse Graetz number, $x^*$. Plot includes theoretical plug and Poiseuille flow limits that will act as indicators to gauge the level of enhancement offered by liquid-liquid Taylor flows.
6.1.2 Two Phase Flow Correlations

Although a number of experimental and numerical works have been undertaken examining the heat transfer enhancements associated with Taylor flows – Prothero and Burton (1961), Verntas et al. (1978), Oliver and Young-Hoon (1968), Lim et al. (2013) and Che et al. (2013) – few have proposed correlations to model their thermal behaviour. This sub-section outlines the correlations most pertinent to this thesis. It should be noted that, within the literature, there are no correlations pertaining to liquid-liquid Taylor flows. Consequently, it is necessary to turn towards liquid-gas Taylor flow models for some guidance in characterising the thermal behaviour of liquid-liquid flows. Experimental and numerical works by authors such as Kreutzer et al. (2001), Horvath et al. (1973), Thome (2003), Hughmark (1965) and Narayanan and Lakehal (2008) have resulted in correlations to model the heat transfer rates in liquid-gas Taylor flows subject to an isothermal boundary condition. Thermal management applications typically subject flows to a constant wall heat flux boundary condition. However, relatively little work has been done on two phase flows in this area, with only of the work of Walsh et al. (2010) resulting in a correlation to model their behaviour, though it was developed for a water-air flow regime only.

Walsh et al. (2010) examined the heat transfer rates attainable from liquid-gas Taylor flows when subject to a constant wall heat flux boundary condition. The authors derived separate expressions for the developing and fully developed asymptotic limits and combined these expressions, in a similar manner to Muzychka and Yovanovich (2004) and Muzychka et al. (2009), using the blending approach of Churchill and Usagi (1972). Walsh et al. (2010) proposed that in the entrance region $N_{ux}$ varied between the limits of Poiseuille and plug flow, presented in Fig. 6.1, depending on the carrier slug length. The following expression was proposed to define $N_{ux}$ within the entrance region:

$$N_{ux,(s,Ent)} = N_{ux,(Pois,Ent)} + \frac{1}{L_c} \left( N_{ux,(Plug,Ent)} - N_{ux,(Pois,Ent)} \right)$$  \hspace{1cm} (6.3)

where $N_{ux,(s,Ent)}$, $N_{ux,(Pois,Ent)}$ and $N_{ux,(Plug,Ent)}$ refer to the local Nusselt numbers in the entrance region in slug, Poiseuille and plug flows respectively. Through experimentation, Walsh et al. (2010) found that the enhancements in the fully developed $Nu$ were
inversely dependent on slug length. Consequently, for a slug of infinite length, the fully
developed $Nu$ should approach that of Poiseuille flow. The authors proposed the following
expression for the fully developed region:

$$Nu_{x(s,Dev)} = Nu_{x(Pois,Dev)} + 25(L_C^*)^{-1/2}$$  \hspace{1cm} (6.4)

where the level of enhancement offered decreases with increasing slug length. The
authors combined the developing, Eq. 6.3, and developed, Eq. 6.4, asymptotes to create
the following expression:

$$Nu_{x(s)} = \left(\left(Nu_{x(s,Ent)}\right)^{10} + \left(Nu_{x(s,Dev)}\right)^{10}\right)^{1/10}$$  \hspace{1cm} (6.5)

The authors noted that the transition from the developing to the fully developed states
was quite abrupt and a value of $n = 10$ best represented this. The validity of this model,
when applied to liquid-liquid flows, will be assessed in section 6.3. The following section
addresses the experimental heat transfer data, and highlights a number of unique features
present within the flow.

6.2 Experimental Data

This section presents and analyses the results of the thermal investigation of liquid-liquid
Taylor flows. The flows were subject to a constant wall heat flux boundary condition using
the experimental test facility presented in section 3.5.1. The primary results of this study are drawn from time-averaged wall temperature profiles generated over the thermally
developing and developed regions of the experimental test section. Fig. 6.2 presents a dimensioned plot of the tube wall temperature profile, $T_w - T_{BM}$, versus the distance from the entrance of the heated section, $x$. Included in the plot are experimental results for a Dodecane/water combination where the dimensionless carrier, $L_C^*$, and dispersed, $L_D^*$, slug lengths vary from 0.69 - 6.26 and 1.66 - 3.75 respectively. Theoretical single phase tube wall temperatures are plotted as a reference, and were calculated using Eqs. 3.9 and 3.10 from section 3.5.4. The plot highlights two key features:
1. The addition of a second immiscible liquid phase to create a segmented Taylor flow regime results in a marked reduction in the time-averaged tube wall temperature profile compared to that offered by an equivalent single phase flow regime; and

2. The effect of slug length on the time-averaged tube wall temperature profile. This effect is particularly striking in the entrance region and is visible throughout the entire test length, where reductions in $L_{C}^*$ and increases in $L_{D}^*$ result in appreciable reductions in the tube wall temperature.

Figure 6.2: Dimensioned plot of the time-averaged wall temperature profile as a function of the distance from the thermal entrance highlighting the influence of Taylor flows on the local surface temperatures. The flow has a constant $Re$ and input heat flux of 46.1 and 5,000 W/m² respectively.

Taking a closer look at the Dodecane/water temperature profiles, and specifically at $L_{C}^* = 6.26$, initially the temperature difference increases with axial distance, $x$, and peaks at 0.01 m downstream from the thermal entrance point. After this point, the temperature difference oscillates towards a constant value, at approximately 0.042 m. Howard et al. (2011) stated that these oscillations were a transient effect due to the alternating slug flow structure. The constant temperature difference is maintained for a period before it begins to decrease again with increasing $x$. The temperature profiles of $L_{C}^* = 0.69$ and $L_{C}^* = 1.49$
differ from that of $L_C^* = 6.26$ as the flows tend towards a constant temperature difference, rather than oscillate towards such a condition. However, similar to the case of $L_C^* = 6.26$, after maintaining a constant temperature difference for a period, the temperature difference also begins to diminish with increasing $x$. A noteworthy feature of Fig. 6.2 is that these regions of constant temperature difference occur closer to the thermal entrance point as $L_C^*$ decreases and $L_D^*$ increases. The most appropriate way of analysing and presenting data of this nature is in dimensionless form, as shown in Fig. 6.3. The experimental temperature differences plotted in Fig. 6.2 are now replotted in the form of $Nu_x$, calculated using Eq. 3.8 from section 3.5.4, while the axial distance from the thermal entrance point is replotted as the inverse Graetz number, $x^*$. 

![Image of graph showing dimensionless plot of $Nu_x$ as a function of $x^*$ for a Dodecane/water Taylor flow regime. The data is plotted at a constant $Re$ of 46.1 and is subject to a constant wall heat flux of approximately 5,000 W/m². Included in the plot are the theoretical Poiseuille and plug flow limits.]

Figure 6.3: Dimensionless plot of $Nu_x$ as a function of $x^*$ for a Dodecane/water Taylor flow regime. The data is plotted at a constant $Re$ of 46.1 and is subject to a constant wall heat flux of approximately 5,000 W/m². Included in the plot are the theoretical Poiseuille and plug flow limits.

The dimensionless position was calculated based on the inlet $Re$ and the mean two phase velocity, $U_m$, and the local Nusselt numbers were calculated using the thermophysical properties of the carrier phase, as this is the phase that wets the heated tube walls. A notable feature of Fig. 6.3 is that, irrespective of $L^*$, the presence of a Taylor flow regime results in substantial enhancements in $Nu_x$ over Poiseuille flow in both the entrance and
fully developed regions. The improved performance in the entrance region would be of particular importance in applications where the heat exchanger length is of the order of the thermal entrance length. Enhancements up to 600% over conventional Poiseuille flow were observed in the fully developed region. Similar enhancements over Poiseuille flow were reported by Walsh et al. (2010) and Howard et al. (2011) for liquid-gas flows, however only after the results were normalised by the percentage contact area of the cooling fluid. When presented as a direct comparison however, the authors documented enhancements up to 200%. Similar enhancements over single phase flow were reported by Mehta and Khandekar (2014), Betz and Attinger (2010), Narayanan and Lakehal (2008) and Prothero and Burton (1961). Although the Nu provides a dimensionless measure of the heat transfer rates, for thermal management applications it is often more pertinent to compare data in terms of heat transfer coefficients. A fully developed single phase flow will generate a heat transfer coefficient, \( h \), of approximately 200 W/m\(^2\) K. The Taylor flow regimes presented in Fig. 6.3 generate heat transfer coefficients of the order 1400 - 1800 W/m\(^2\) K, hence providing a much more effective means of removing heat by convection.

Fig. 6.3 also highlights the effects of slug length on \( \text{Nu} \). The dimensionless slug lengths, \( L_{C^*} \) and \( L_{D^*} \), vary from 0.69 - 6.26 and 1.66 - 3.75. It is evident from Fig. 6.3 that reductions in \( L_{C^*} \) and increases in \( L_{D^*} \) result in improved thermal performance, with the fully developed \( \text{Nu}_x \) increasing from approximately 15 to 22, and similar observations were documented by Horvath et al. (1973) and Oliver and Young-Hoon (1968). Consequently, shorter carrier slugs and longer water slugs should be preferred in order to provide the greatest reduction in tube wall temperature. It is also worth noting that slugs have a shorter thermal entrance length compared to continuous flow. This feature, which was also reported by Asthana et al. (2011), Mehta and Khandekar (2014) and Majumder et al. (2013), has been attributed to the increased mixing within the fluid segments resulting in the flow reaching thermal equilibrium faster. Consequently, the flow reaches a fully developed state earlier in the channel.

In section 2.2 of Chapter 2, the three stages of the thermal boundary layer development in Taylor flows, according to Che et al. (2013), were outlined. These three stages are highlighted in Fig. 6.3 and are compared to \( L_{C^*} = 6.26 \). In stage 1, the thermal boundary
layer is initially thin, resulting in a small temperature difference between the wall and the flow, $T_w - T_{BM}$, and a large $Nu_x$. However, as the boundary layer develops, $Nu_x$ decreases and $T_w - T_{BM}$ increases, (see Fig. 6.2), with increasing $x^*$. At stage 2, due to the presence of the hemispherical caps, fresh fluid is advected within the slugs resulting in a renewal of the thermal boundary layer. Thus, $T_w - T_{BM}$ decreases and $Nu_x$ increases. This recirculation of flow within the slugs, presented in Fig. 2.5 in section 2.3, has been shown by Muzychka et al. (2011) to be the primary mechanism responsible for the enhanced heat transfer rates seen in Taylor flows. This increase in $Nu_x$ occurs until the heated fluid comes into contact with the heated tube wall again and corresponds with the peak in $Nu_x$. According to Walsh et al. (2010), this peak should correspond to one circulation length of the slug, which equates to twice the slug length plus the tube diameter. Circulation lengths for both the carrier and dispersed phases are highlighted by pink and blue circles in Fig. 6.3, with the peak in $Nu_x$ corresponding approximately to one circulation of the carrier phase for $L_C^* = 6.26$. Following this, a similar pattern in $Nu_x$ occurs in the downstream direction, however its magnitude is significantly damped and a constant temperature difference, $T_w - T_{BM}$, is approached. In this final stage, stage 3, a constant temperature difference exists between the wall and the flow stream and the flow is said to be fully developed thermally, resulting in a constant $Nu_x$.

It is also evident from Fig. 6.3 that the $Nu_x$ profiles for Dodecane/water flows at $L_C^* = 0.69$ and $L_C^* = 1.49$ do not follow the same trend as that of $L_C^* = 6.26$. Similar $Nu_x$ profiles were observed in the works of Howard et al. (2011) and Walsh et al. (2010) for flows where $L_C^* \approx 1$. In the thermal entrance region, these flows have heat transfer characteristics similar to that of plug flow. Howard et al. (2011) postulated that the enhancements in $Nu_x$ above the plug flow limit were due to a form of impingement at the leading edge of the slug where contact is made between the fluid and the wall, illustrated in Fig. 6.4. Due to the small volume of fluid in the slug, the flows quickly approach a fully developed state and no peak in $Nu_x$ is observed in the data. It is thought that this constant $Nu_x$ is a result of the continuous phase reaching a thermally fully developed state. However, similar to the case where $L_C^* = 6.26$, after this thermally fully developed state is reached, $Nu_x$ begins to increase with increasing $x^*$. 
Dispersed slug

Figure 6.4: Schematic illustration of the postulated impingement at the leading edge of the carrier slug.

It is postulated that these increases in \( Nu_x \), after steady state is achieved, are the result of dispersed phase effects. In a liquid-gas flow regime, the dispersed gaseous phase makes a negligible contribution thermally to the overall flow. In a liquid-liquid flow regime, however, the dispersed phase can make a significant contribution thermally, depending on its thermophysical properties. Water was used as the dispersed phase in all experiments in this thesis and has a density, \( \rho \), and specific heat capacity, \( c_p \), of approximately 920 kg/m\(^3\) and 4.2 kJ/kg K respectively, thus it has a much larger thermal capacitance compared to air, \( \rho \approx 1.2 \text{ kg/m}^3 \) and \( c_p \approx 1 \text{ kJ/kg K} \). Separating the dispersed slugs from the heated tube wall is a thin film of the carrier phase. Due to its low thermal conductivity, \( k \approx 0.1 \text{ W/m K} \) see Table 3.1, the carrier phase acts as an insulator or a thermal barrier between the dispersed slugs and the heated walls, thus limiting the role the dispersed phase plays in the removal of heat. The liquid film thickness was calculated using Eq. 4.6 from section 4.3.3 and found to equal 15.7 \( \mu \text{m} \) or, relative to the tube radius, 2.1\%. The non-negligible flow within the liquid film results in heat being convected away from the tube walls. However, the velocity in both phases is equal at the liquid-liquid interface. Consequently, heat is conducted into the dispersed slugs and due to the recirculation of flow within the dispersed slug – similar to that in the continuous phase – this heat is continuously removed from the film resulting in the reduction \( T_w - T_{BM} \) seen in Fig. 6.2 and the increase in \( Nu_x \) seen in Fig. 6.3. As in the carrier phase, this would continue until the dispersed slug reaches thermal equilibrium. However, extensive computational modelling would be required to verify this, which is beyond the scope of this thesis.

Examining potential film effects further, Fig. 6.5 is a plot \( Nu_x \) as a function of \( x^* \) for Dodecane/water flowing at an \( Re \) of 92.1 and \( Ca \) of 0.003. Thus a thicker liquid film separates the dispersed slugs from the heated tube walls. The film thickness for this flow was calculated to be 32 \( \mu \text{m} \) or 4.2\% relative to the radius, which is twice that of the film.
present for the data presented in Fig. 6.3. It can be clearly seen in Fig. 6.5 that the different Dodecane/water Taylor flows reach a constant $N_u_x$ and, similar to Fig. 6.3, do not maintain this constant $N_u_x$ with increasing $x^*$. To highlight the increase in $N_u_x$, fully developed asymptotes are included in the plot. However, unlike the data plotted in Fig. 6.3, the disparity between the fully developed asymptotes and the experimental data plotted in Fig. 6.5 is greatly diminished. As the film separating the dispersed slugs from the heated tube walls in Fig. 6.5 is approximately twice as thick as that in Fig. 6.3 it provides a greater thermal barrier between the dispersed slugs and the heated tube wall, thus restricting the involvement of the dispersed phase in the removal of heat.

![Graph](image)

**Figure 6.5:** Dimensionless plot of $N_u_x$ as a function of $x^*$ for a Dodecane/water Taylor flow regime flowing at an $Re = 92.1$ and subject to a constant wall heat flux of approximately 5,000 W/m$^2$. The plot highlights the effects of liquid film thickness, $h$, on $N_u_x$, where relative to the tube radius, the film occupies 4.2% of the tube.

This trend continues as film thickness is increased further, Figs. 6.6 and 6.7 present dimensionless plots of $N_u_x$ as a function of $x^*$ for Pd5/water and AR20/water Taylor flows respectively. It was seen in Figs. 4.5 and 4.7 in section 4.2.2 that the more viscous Pd5 and AR20 deposited a thicker film on the walls of the capillary than Dodecane. Liquid film thicknesses were calculated using Eqs. 4.3 and 4.6 and found to equal 53 μm, or 7%
relative to the tube radius, for the Pd5/water flow and 150 μm, or 20% relative to the tube radius, for the AR20/water flow. It is clear in both Figs. 6.6 and 6.7 that these flows do deviate from the fully developed asymptotes, however, the deviations, with increasing $x^*$, are practically negligible, particularly in the AR20/water results presented in Fig. 6.7.

Figure 6.6: Dimensionless plot of $N_u_x$ as a function of $x^*$ for a Pd5/water Taylor flow regime flowing at an $Re = 42.9$. The plot highlights the effects of liquid film thickness on $N_u_x$, where, relative to the tube radius the film occupies approximately 7% of the tube.

An interesting feature from Figs. 6.3 and 6.5 is that for carrier slugs of approximately equal length, $L_C^* \approx 0.65$, an increase in film thickness of 16.3 μm, or 2.1%, corresponds to a reduction in $N_u_x$ from approximately 23 to 19. In an exploratory investigation, Talimi et al. (2012) examined the influence of film thickness on heat transfer in liquid-gas Taylor flows. The authors noted that increases in film thickness were accompanied by an increase in the wall temperature. This reduction in was attributed to the increase in flow bypass through the liquid film, which results in a reduction of the recirculation of flow in the continuous phase. Consequently, the flow is more akin to Poiseuille flow and $N_u_x$ decreases. However, the authors noted that this effect diminished as the flow moved downstream and reached a fully developed state. Similarly, as the film increases in thickness to approximately 7% of the channel, Fig. 6.6, the oscillations seen in the $N_u_x$ profiles, Figs. 6.3 and 6.5, are
dampened out. Similar observations were documented by Howard et al. (2011) and were attributed to the increase in flow bypass through the liquid film.

It is evident that the liquid film plays a significant role in the ability of liquid-liquid Taylor flows to remove heat. In flows where the film occupies more than 4% of the channel, the film acts as a thermal barrier between the heated tube walls and the dispersed slugs, thus limiting the role of the dispersed slugs in the removal of heat. However, below 4%, the dispersed slugs are more actively involved in the transfer of heat. Increases in film thickness result in a reduction in $Nu_x$ and also dampen out the oscillations observed in the $Nu_x$ profiles.

![Figure 6.7: Dimensionless plot of $Nu_x$ as a function of $x^*$ for an AR20/water Taylor flow regime at an $Re = 8.7$. The plot highlights the effects of liquid film thickness on $Nu_x$, where, relative to the tube radius the film occupies approximately 20% of the tube.](image)

6.3 Taylor Flow Correlations

This section assesses the validity of the Walsh et al. (2010) model, presented in Eq. 6.5, when applied to liquid-liquid Taylor flows. Fig. 6.8 is a plot of $Nu_x$ as a function of $x^*$ for a Dodecane/water Taylor flow regime flowing at an $Re$ of 30.7. The dimensionless carrier
and dispersed slug lengths vary from 0.79 - 9.03 and 1.69 - 4.07 respectively. It is evident from Fig. 6.8 that the model provides a poor approximation of the liquid-liquid data, particularly at the lower $L_{C^*}$ values. Starting at the entrance region, the model transitions to the developed region while the experimental data is still developing, indicating that the entrance region correlation, Eq. 6.3, is inapplicable to liquid-liquid flows. Fully developed experimental $Nu_x$ values are up to 40% lower than those predicted by the model, with the greatest difference observed in the shortest carrier slug result. Similar results were observed in the other liquid-liquid combinations examined in this thesis, as summarised in Table 6.1.

![Diagram](image)

Figure 6.8: Dimensionless plot of $Nu_x$ as a function of $x^*$ for a Dodecane/water Taylor flow regime flowing at an $Re = 30.7$. The plot highlights the accuracy of the Walsh et al. (2010) model, Eq. 6.5, when applied to liquid-liquid Taylor flows.

<table>
<thead>
<tr>
<th>Liquid-Liquid Combination</th>
<th>Difference between Experimental and Model Data (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pd5/Water</td>
<td>2.9 - 37.6</td>
</tr>
<tr>
<td>Dodecane/Water</td>
<td>4.8 - 88.3</td>
</tr>
<tr>
<td>AR20/Water</td>
<td>160.5 - 384.1</td>
</tr>
</tbody>
</table>

Table 6.1: Summary of the differences between the fully developed experimental $Nu_x$ values and those calculated using the Walsh et al. (2010) model.

The model of Walsh et al. (2010) was developed using a water-air Taylor flow regime,
where only one phase plays an appreciable role in heat removal. It was shown in section 6.2 that in a liquid-liquid Taylor flow regime, both phases are actively involved in cooling. Consequently, the thermal behaviour of liquid-liquid Taylor flows incorporate factors which are not accounted for in this model. This highlights a requirement for further data reduction parameters to allow for accurate modelling of liquid-liquid Taylor flows; these will be presented in the following section.

### 6.4 Thermal Characterisation of Liquid-Liquid Taylor Flows

This section presents a novel approach to modelling the thermal behaviour of liquid-liquid Taylor flows when subject to a constant wall heat flux boundary condition. By isolating specific experimental parameters, such as slug length, it allows for a direct comparison in $Nun$ for the different liquid-liquid combinations. Fig. 6.9 is a dimensionless plot of $Nun$ as a function of $x^*$ for a constant $L_C^* ≈ 1.4$.

![Figure 6.9: Dimensionless plot of Nun as a function of x* for a constant L_C^* ≈ 1.4. Plot contains experimental data for the different liquid-liquid combinations examined in this thesis over the following range in Re, 3.2 - 46.1, ε, 0.42 - 0.7, and Pr, 23.6 - 265.4.](image)
The experimental data presented in Fig. 6.9 has the following ranges in $Re$, 3.2 - 46.1, void fraction, $\varepsilon$, 0.42 - 0.7, and Prandtl numbers, $Pr$, 23.6 - 265.4. Included in the plot, for comparative purposes, are the theoretical Poiseuille and plug flow limits. It is evident from Fig. 6.9 that the differences in $Nu_x$ in the fully developed region between the different liquid-liquid combinations are minimal, 13.5 - 18.5. However, significant differences are observed in the transition from thermally developing to thermally fully developed. Few works have examined the effects of varying carrier fluids on the thermal performance of Taylor flows. Of these, Urbant et al. (2008) varied the carrier phase to validate a single phase numerical model, and Hughmark (1965) examined the effects in turbulent liquid-gas flows, which is beyond the scope of this thesis. Varying the carrier phase, Howard et al. (2011) noted that variations in Prandtl number, $Pr$, resulted in notable changes to the transition from thermally developing to fully developed. However, once the data had been normalised by the correct void fraction, excellent agreement was observed in the fully developed region between the different liquid-gas flows. The authors make no specific comment in relation to this agreement. In a single phase flow regime, irrespective of the fluid, hence $Pr$, flowing in the capillary, a constant $Nu$ is obtained once the flow reaches a thermally fully developed state. In a two-phase flow regime, a similar phenomenon should occur; the flow should tend towards a constant $Nu$ when both phases are fully developed thermally. Consequently, any variations in the fully developed $Nu_x$ should be the result of changes in slug length, as highlighted in section 6.2. Additionally, a change in $Pr$ between the fluids is accounted for in the inverse Graetz number, $x^*$, see Eq. 2.20 in section 2.3.2.

Fig. 6.10 examines the influence of carrier slug length on the fully developed $Nu_x$. Plotted on the ordinate is the enhancement in $Nu$ over Poiseuille flow, $Nu_{x(S,Dev)} - Nu_{x(Pois,Dev)}$, while plotted on the abscissa is the dimensionless carrier slug length, $L_{C^*}$. Experiments were carried out over a wide range in $L_{C^*}$, presented in Table 3.4, in order to obtain an accurate approximation of the dependence of $Nu_x$ on $L_{C^*}$. In flows where $L_{C^*} \geq 10$, the flow had yet to reach its fully developed limit within the experimental test section. Consequently, values of $Nu_{x(S,Dev)}$ were chosen at a distance equal to one circulation length from the thermal entrance point. This location was chosen as, according to Walsh et al. (2010), the flow reaches its steady state $Nu$ at one circulation length and oscillates about
this value thereafter. Both Howard et al. (2011) and Talimi et al. (2012) found that the film that separates the dispersed slugs from the wall has an effect on $Nu_x$. However, Talimi et al. (2012) stated that this effect dissipated as the flow moved downstream and reached a fully developed state. A similar observation was made in section 6.2, where it was also postulated that in flows where the film occupied a small fraction of the channel, $h/R < 4\%$, the superior thermal properties of the dispersed water phase resulted in an additional temperature reduction at the wall. However, the $Nu_x$ values plotted in Fig. 6.10 are the steady state $Nu_x$ achieved prior to the dispersed phase effects being felt at the wall, as highlighted in Fig. 6.3.

![Figure 6.10: Plot of the increase in fully developed $Nu_x$ above the theoretical single phase limit as a function of dimensionless carrier slug length, $L_c^*$.](image)

The plot highlights the effects of $L_c^*$ on the enhancement in $Nu_{x(S,Dev)}$ over Poiseuille flow, particularly at values of $L_c^* \leq 3$. Included in the plot is a best fit line of the data, which follows a power law trend. The fully developed asymptotic limit for a liquid-liquid Taylor flow regime is given by Eq. 6.6, where the experimental data points presented in Fig. 6.10 fall within $\pm 20\%$ of the correlation. The disagreement between the experimental data and the correlation is most prominent in carrier slugs approaching the channel diameter, $L_c^* \approx$
1.

\[ \textit{Nu}_{x(S,Dev)} = 4.36 + 13.7(L_C^*)^{-0.23} \tag{6.6} \]

Focusing on the thermal entrance region, Walsh et al. (2010) found that \textit{Nu}_x varied between the Poiseuille and plug flow limits. Similar to the fully developed state, \textit{Nu}_x was found to vary inversely with \( L_C^* \) in the thermal entrance region. Experimental results in Figs. 6.3 - 6.7 show that the \textit{Nu}_x varies between the Poiseuille and plug flow limits as \( L_C^* \) varies. Similar to the correlation proposed by Walsh et al. (2010) in section 6.1.2, this behaviour was found to be best correlated using a weighted mean approach between these two limits and is presented in Eq. 6.7. Slugs of infinite length should fall on the Poiseuille flow limit, while slugs of order the channel diameter should fall on the plug flow limit. Local \textit{Nu} will vary linearly between these limits at intermediate carrier slug lengths.

\[ \textit{Nu}_{x(S,Ent)} = ((\textit{Nu}_{x(Plug,Ent)} - \textit{Nu}_{x(Pois,Ent)})(L_C^*)^{-0.4}) + \textit{Nu}_{x(Pois,Ent)} \tag{6.7} \]

These expressions, for the thermal entrance and fully developed regions, represent the asymptotic limits of the flow. Using the blending approach of Churchill and Usagi (1972), these limits are combined in Eq. 6.8 to form a new correlation to model the thermal behaviour of liquid-liquid Taylor flows subject to a constant wall heat flux boundary condition:

\[ \textit{Nu}_{x(S)} = \left( \left( \textit{Nu}_{x(S,Ent)} \right)^4 + \left( \textit{Nu}_{x(S,Dev)} \right)^4 \right)^{\frac{1}{4}} \tag{6.8} \]

The accuracy of this correlation is graphically illustrated in Fig. 6.11, where experimental measurements are compared with the predictions of \textit{Nu}_x from Eq. 6.8 over a wide range of experimental parameters. Dimensionless carrier slug length, 1.05 - 14.38, \( Re \), 4.92 - 92.12, and \( Pr \), 23.6 - 265.4, span several orders of magnitude for three different liquid-liquid combinations. Included in the plot are circles that indicate one full circulation of the carrier phase. It can be seen in Fig. 6.11 that the model provides excellent agreement with the experimental data in the thermal entrance region and the short and intermediate slug lengths, \( L_C^* < 5 \). At \( L_C^* = 14.38 \) there are significant deviations in the early entrance...
region, ± 30%, however, this deviation diminishes as $x^*$ and the flow begins to transition towards fully developed. Similarly, the correlation provides an excellent approximation of the transition from developing to fully developed in the short and intermediate slug lengths. There is a renewal of the thermal boundary layer in the Dodecane/water flow, described previously in section 2.2 and 6.3, which the correlation does not account for. Nonetheless, the correlation still approximates the $Nu_x$ to within ± 10%. In the fully developed region, the correlation predicts the experimental $Nu_x$ to within ± 10%. The correlation is also in accordance with the steady state values proposed to occur at one circulation length of the carrier phase, with correlation predictions within ± 10% of the experimental measurements. Overall, the correlation presented in Eq. 6.8 provides a novel means of predicting the $Nu_x$ in liquid-liquid Taylor flows subject to a constant wall heat flux boundary condition.

Figure 6.11: Dimensionless plot of $Nu_x$ as a function of $x^*$ over the following range in dimensionless parameters; $L_C^*$: 1.05 - 14.38, $Re$: 4.92 - 92.12 and $Pr$: 23.6 - 265.4. The plot highlights the accuracy of the proposed correlation in Eq. 6.8 at predicting the experimental data in the entrance, transition and fully developed regions. Included in the plot are indicators marking one circulation length.
6.5 Closure

This chapter focused on the local heat transfer characteristics of liquid-liquid Taylor flows subject to a constant wall heat flux boundary condition. The objectives of this study were to identify and understand the primary heat transfer mechanisms in such a flow regime and to examine the effects of varying slug length and Prandtl number on the local heat transfer rates. High spatial resolution local wall temperature measurements were obtained using an IR thermography system, allowing the flow to be characterised in the entrance and fully developed regions.

Reductions in the tube wall temperature were observed throughout the experimental test section, with Nusselt number enhancements up 600% noted in the fully developed region. Slug length was found to have a significant effect on heat transfer rates attainable, with a reduction in carrier slug length and an increase in the dispersed slug length resulting in increases in Nusselt number. The thickness of the film surrounding the dispersed slugs was also found to have an appreciable effect on the Nusselt number. The film, due to its low thermal conductivity, \( k \approx 0.1 \text{ W/m K} \), acted as a thermal barrier in flows where the liquid film occupied more than 4% of the channel. However, in flows where the film occupied less than 4% of the channel, the dispersed aqueous slugs had a greater thermal contribution and further increases in Nusselt number were observed. Oscillations were observed in the Nusselt number profiles in the lower Capillary number flows. These oscillations were the result of the circulation of fluid within the carrier slugs. However, these oscillations were dampened out as the Capillary number, and hence film thickness, increased.

Thermally fully developed Nusselt numbers were found to be inversely proportional to the carrier slug length and a correlation was proposed to model the flow in this region. Similarly in the thermal entrance region an expression was proposed, using a weighted mean between the Poiseuille and plug flow limits, to model the flow. Using the blending approach of Churchill and Usagi (1972), a correlation was proposed, consisting of the thermal entrance and fully developed region expressions, to model the thermal behaviour of liquid-liquid Taylor flows.
Chapter 7

Conclusions and Recommendations

This chapter presents the main conclusions drawn from the research presented in this thesis. The primary objective was to investigate the fundamental thermal and hydrodynamic characteristics of liquid-liquid Taylor flows confined to minichannel geometries. An in-depth experimental programme was completed that allowed the flow to be studied non-invasively over a wide range of parameters. In total, four different liquid-liquid combinations were examined as part of this thesis. This chapter outlines the conclusions from this research in section 7.1, and recommendations for further investigation are presented in section 7.2.

7.1 Conclusions

This sections consists of three sub-sections that present the main findings of this thesis. The first two sub-sections discuss the findings of the hydrodynamic studies, namely the film thickness and pressure drop associated with liquid-liquid Taylor flows. The final sub-section addresses the thermal behaviour of liquid-liquid Taylor flows when subject to a constant wall heat flux boundary condition.

7.1.1 Film Thickness Study

The thickness of the thin liquid film that separates the dispersed slugs from the capillary wall was measured as part of this study. Optical microscopy was used to acquire images of the flow in order to determine the thickness of the film. The principal aims of this study
were to examine the effects of dispersed slug length and carrier phase thermophysical properties upon the thickness of the film. Although a number of correlations are presented in the literature, none of these pertain to liquid-liquid flows. The accuracy of these correlations in predicting film thicknesses in liquid-liquid flows was also investigated. The following conclusions have been established:

- Images of the flow revealed variations in film thickness with dispersed slug length. Below a threshold value of slug length, $L_D^* \leq 1.25$, the slug profile and hence film thickness varied along the entire length of the slug. As the dispersed slug length increased, $L_D^* \geq 1.25$, distinct transition regions emerged between the front and rear slug caps. Above an upper threshold of $L_D^* \geq 1.86$, a uniform film of fluid was found to exist between the slugs and the capillary wall.

- The experimental data showed a strong dependence on Capillary number and the correlation of Bretherton provided the best agreement with the experimental data when the Capillary number was calculated based on the mean two phase velocity, within $\pm 25\%$. However, none of the correlations provided accurate estimates of the film thickness when the Capillary number was redefined to account for the velocity at the interface.

- New expressions are presented to calculate film thickness in liquid-liquid flows in the visco-capillary and visco-inertial regimes. The thickness of the film in the visco-capillary flow regime was found to be solely dependent on the Capillary number, and a modified Taylor's Law is presented that estimates the thickness of the film to within $\pm 10\%$. Once the flow transitioned into the visco-inertial regime, the film thickness was no longer solely dependent on Capillary number, consequently, a correlation that accounts for the increased inertial effects was developed. This equation incorporates the Weber number and estimated the film thickness to within $\pm 15\%$.

7.1.2 Pressure Drop Study

The pressure drop induced by liquid-liquid Taylor flows was measured as part of this study. The purpose of this study was to determine the accuracy of a number of analytical and
semi-empirical pressure drop correlations and to establish, if necessary, improved relations for liquid-liquid Taylor flows. Experimental measurements were conducted over ranges in dimensionless slug, Capillary and Reynolds numbers that spanned several orders of magnitude. The main conclusions from the analysis are:

- The homogeneous and separated flow models showed poor agreement with the experimental data and should not be used to model the pressure drop in Taylor flows. These models fail to account for the interfacial effects or any of the hydrodynamic details of a Taylor flow regime.

- The pressure drop models proposed by Jovanovic et al. (2010) and Kashid and Agar (2005) were developed specifically for use with liquid-liquid Taylor flows. However, these models showed poor agreement with the experimental data, and this was attributed to the assumptions used in their composition.

- The liquid-gas Taylor flow model proposed by Kreutzer et al. (2005) and modified by Walsh et al. (2009) demonstrated good agreement with the experimental data, within $\pm 20\%$, provided a significant viscosity difference existed between the liquid phases, $\mu_C/\mu_D \approx 23$.

- Results showed that the dispersed slug velocity, $U_D$, needs to be accounted for in flows with high interfacial contributions. This dependence is captured by the Warnier et al. (2010) model, however significant scatter was observed in the experimental data. A modification to the Warnier et al. (2010) model is proposed, where the interfacial component is normalised by the volume fraction of the capillary occupied by the carrier phase. The agreement between the experimental data and the model, within $\pm 20\%$, implies that the pressure drop induced by the dispersed phase can be neglected from the analysis.

### 7.1.3 Heat Transfer Study

An objective of this aspect of this thesis was to characterise the thermal behaviour of liquid-liquid Taylor flows subject to a constant wall heat flux boundary condition. The effects of
slug length and carrier phase thermophysical properties on the flow were documented. This section summarises the main findings from the thermal characterisation:

- The addition of a liquid-liquid Taylor flow regime resulted in heat transfer enhancements throughout the experimental test section, with enhancements of up to 600% over single phase flows observed. These enhancements were attributed to the recirculation of flow within the slugs.

- Results showed that the enhancements were dependent on slug length, with reductions in carrier slug length and increases in dispersed slug length inducing enhancements in Nusselt number.

- Film thickness was found to have an appreciable effect on the heat transfer rates attainable. In flows where the film occupied a significant part of the channel, $h/R > 4\%$, the film was found to act as a thermal barrier between the heated tube wall and the dispersed slugs. However, in flows where $h/R < 4\%$, the dispersed slugs were found to make a greater contribution to the removal of heat and further enhancements in $Nu_x$ were observed.

- A novel correlation is presented to model the thermal behaviour of liquid-liquid Taylor flows. The correlation consists two expressions, for the thermal entrance and fully developed regions, blended together. Excellent agreement was observed between the model and the experimental data over the range $L_{C^*} \leq 5$. However, deviations up to $\pm 30\%$ were observed in the early thermal entrance region for slug lengths in the range $L_{C^*} > 5$. These deviations dissipate as the flow moves downstream and transitions into fully developed, where the correlation predicts the fully developed $Nu_x$ to within $\pm 10\%$.

### 7.2 Recommendations

This section outlines a number of recommendations for the continuation of this work. These recommendations include improvements to the experimentation and a number of
lines of investigation that would further advance the understanding of liquid-liquid Taylor flows.

- In all of the experimental works in this thesis, the viscosity of the carrier phase was greater than that of the dispersed phase, $\mu_C > \mu_D$. Consequently, the pressure drop induced by the carrier phase was always greater than that of the dispersed phase. Experimentation is required to examine the pressure drop in liquid-liquid Taylor flows where the viscosity of the dispersed phase is greater than that of the carrier phase, for example in a benzene/water or AR20/AS100 flow.

- Likewise the effects of $\mu_D > \mu_C$ on film thickness and heat transfer characteristics are unknown, and experimentation in this area, particularly a $\mu$-PIV study, would add significantly to the knowledge base of liquid-liquid Taylor flows.

- Experimentation in this thesis highlighted that at a sufficiently high viscosity ratio, $\mu_C/\mu_D \approx 23$, liquid-gas Taylor flow pressure drop models could be used to accurately calculate the pressure drop in liquid-liquid Taylor flows. It was also noted that at $\mu_C/\mu_D \leq 4.5$, these models were not applicable to liquid-liquid flows. A hydrodynamic characterisation over the range $4.5 \leq \mu_C/\mu_D \leq 23$ would determine at which viscosity ratio liquid-gas Taylor flow pressure drop models could be accurately used.

- The field of view of the IR camera was limited by resolution issues. Mounting the camera on a stage and using a traverse system would allow for measurements over the entire length of the heated test section. This would provide measurements that extend further into the thermally fully developed region and would provide a greater insight into the liquid film and dispersed slug effects on the local Nusselt numbers. However, as the data is presented on a log scale, the tube length would have to be increased to at least 1m in order to gain a proper insight.

- It was postulated in section 6.2 that in flows where the film occupied greater than 4% of the channel, the film acted as a thermal barrier, restricting the effects of the dispersed water slugs in the transfer of heat. Computational modelling of the flow would give a better insight into the role the film plays in the heat transfer process.
Bibliography


Appendix A

Published Work

Journals


Conference Papers


Appendix B

Capillary Wall

This appendix contains a series of images that highlight the presence and negligible effect of a dark region at the wall of the capillary. Fig. 1 a) is an image of a needle in a capillary filled with water. There is excellent visualisation of the flow due to the refractive index matching between the capillary and water. There is a small dark region along the capillary wall, however its magnitude is negligible. Fig. 1 b) shows a needle positioned in a capillary filled with Pd5 silicone oil. The distinct profile of the needle is clearly visible in Fig. 1 b). If the dark region was a distortion of the near wall region, this would be a dead space and anything in the near wall region would be distorted. For example, if a ruler was moved into this region, a truncated form of the ruler would be visible in the image. However, as Fig. 1 c) shows as the needle moves into the dark region no distortion of the needle profile occurs and no truncated form of the needle is visible in the image. Fig. 1 d) shows the needle passing through the wall of the capillary. As in Fig. 1 c) there is no distortion of the needle profile as it passes through the dark region and into the wall of the capillary. Thus, the dark region does not influence the measurement of the liquid film.
Figure 1: Variation in transducer output over time for a dodecane/water Taylor flow regime.