Partnership Formation with Age-Dependent Preferences

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Abstract

We analyze a model of partnership formation in which players' preferences are based on the age of a prospective partner. There are two classes of individuals, called for convenience here male and female. Males and females are fertile for the same length of time, normalized to one unit. A male enters the mating pool at age 0 and meets prospective partners according to a Poisson process. At equilibrium, he accepts a female if the utility from mating exceeds the expected utility from future search, which depends on the acceptance strategies of all males and females and the corresponding steady-state distribution of age in the pool of unmated individuals. Females face an analogous problem. Mating pairs are only formed by mutual consent and individuals leave the pool of unmated individuals on finding a mating partner or reaching the age of 1. A policy iteration algorithm is used to determine the equilibrium acceptance strategies and the corresponding steady-state distribution of the age of individuals in the mating pool. Two examples are presented.

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1. Introduction and Main Results

The age of a prospective partner is a crucial criterion in mate choice (see Dugatkin and Godin [7], Gil-Burman *et al.* [9], Ritchie [20] and Watt *et al.* [23]). Many models of mate choice are based on common preferences. According to such preferences, individuals prefer attractive partners and each individual of a given sex agrees on the attractiveness of a member of the opposite sex. Some work has been carried out on models in which preferences are homotypic, i.e. individuals prefer partners who are similar (e.g. in character) to themselves in some way. In such models the attractiveness (or character, as appropriate) of an individual is assumed to be fixed. By definition, the age of an individual must change over time. Very little theoretical work has been carried out on such problems.

One could think of such a problem in terms of introducing an innovative product onto the market. One partner could be the inventor who holds a patent. The other partner could be a businessman who would market the new invention. The older the patent the less valuable the innovation. Also, the scientist might gain more from forming a partnership with a young partner than with an older partner. In job search problems, an employer might prefer a young applicant, while applicants might prefer long established employers.

This paper presents a model of a game in which mate choice is based on the age of a prospective partner. In any period, the number of males reaching maturity equals the number of females reaching maturity, i.e. the incoming sex ratio is 1. Individuals enter the pool of unmated individuals at age zero (one's 'age' is how long one has been in the pool) and remain there until they are either mated, or become infertile (at age 1). Hence, the steady-state distribution of the age of those still searching depends on the strategy profile adopted. A strategy profile can be interpreted as a description of the strategies used within a population. We look for an equilibrium symmetric with respect to sex, i.e. both males and females follow the same strategy profile. The corresponding probability that an individual does not find a partner before age x, $a(x)$, is called the age profile.

While in the pool, searchers meet potential partners according to a Poisson process with rate λ and the prospective partner's age is chosen at random from the age profile. If acceptance is mutual, they form a mating pair and obtain a common utility based on their 'ages at marriage'. A prospective partner should be accepted if and only if the utility obtained from mating is at least the expected utility obtained from future search.

Under the 'simple fertility model', the common utility obtained when a male of age x pairs with a female of age y, $u(x, y)$, is the length of time for which both remain fertile, i.e. $u(x, y) = 1 - \max\{x, y\}$. Hence, a male of age x prefers younger females to older females in the age range $[x, 1)$, but does not differentiate between two females of ages below x. Biologically, the utility function is a measure of the expected number of offspring a couple produce. More generally, it measures the expected output of a partnership.

A male's strategy specifies which females are acceptable at each age x. Under the simple fertility model, intuitively a male of age x should accept a female of age below some threshold, say $g(x)$. A strategy profile given by two such rules (one for each sex) is called a threshold profile. As we are looking for a symmetric equilibrium, assume that all searchers use the same threshold rule. Hence, the numbers of males and females in the mating pool are equal, i.e. the operational sex ratio is one. A strategy profile is an equilibrium profile if searchers accept exactly those prospective partners who give them at least the same utility as the expected utility from future search. This is analogous to the concept of subgame perfectness in extensive form games.

The outcomes of an interaction under a symmetric threshold profile are illustrated in

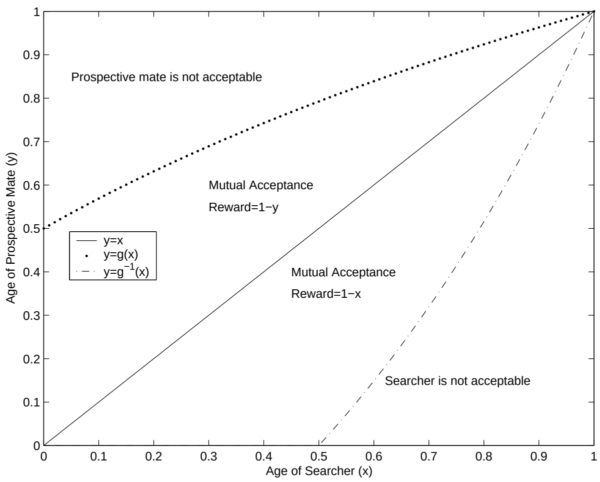

Figure 1: Illustration of the decisions made at equilibrium for the simple fertility model. A searcher of age x will accept a prospective mate of age y if and only if $y \leq g(x)$. However, the prospective mate will accept the searcher if and only if $x \leq g^{-1}(y)$. Hence, mutual acceptance only occurs when the point (x, y) lies between these two curves.

Figure 1. This paper uses policy iteration to determine a symmetric equilibrium profile and corresponding age profile. Figure 2 illustrates the equilibrium strategy and age profiles when the interaction rate is $\lambda = 10$ under the simple fertility model. From the threshold profile, a male of age 0.3 will accept females of age $\leq g(0.3) \approx 0.6$ and is accepted by females of age $\geq g^{-1}(0.3) \approx 0.1$. Hence, an individual of age 0.3 will pair with prospective partners of age in $[g^{-1}(0.3), g(0.3)] \approx [0.1, 0.6]$. From the age profile, it can be seen that around 40% of fertile adults are still searching for a partner at age 0.1. A simpler, discrete time model with integer ages is discussed in a concurrent paper by the authors [4].

There is an extensive literature on mathematical models of mate search, going back to the work of Janetos [11]. He assumed that only females are choosy and the value of a male to a female is from a distribution known to the females. There is a fixed cost for observing each prospective mate, but no limit on the number of males a female can

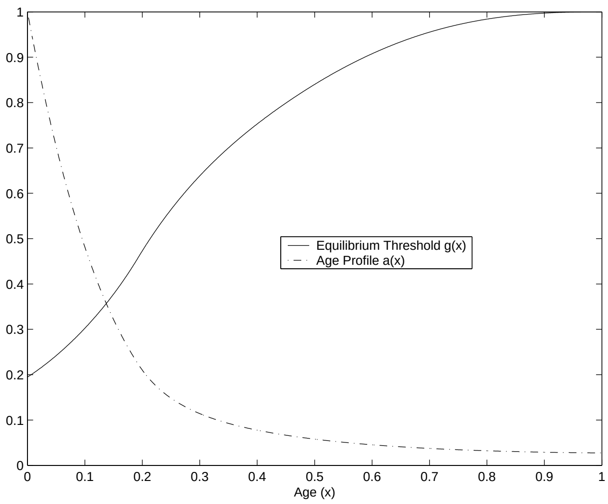

Figure 2: Estimate of the equilibrium threshold rule and age profile for the simple fertility model: $\lambda = 10$. Using the equilibrium threshold, an individual of age x will accept a prospective partner if and only if the prospective partner's age is $\leq g(x)$. The age profile, $a(x)$, gives the proportion of individuals of age x who have not yet found a mate.

observe. Real [18] developed a more rigorous mathematical framework for such models.

When both sexes are choosy, such problems are game theoretic. Parker [15] presents a model in which both sexes prefer mates of high value. He concludes that assortative mating occurs with individuals divided into classes. Class i males pair with class i females and there may be one class of males or females who do not mate.

In the mathematics and economics literature such problems are formulated as marriage problems or job search problems. McNamara and Collins [14] consider a job search game in which job seekers observe a sequence of job offers and, correspondingly, employers observe a sequence of candidates. Both groups have a fixed cost of observing a candidate or employer, as appropriate. Their conclusions are similar to those of Parker [15]. Real [19] developed these ideas within the framework of mate choice problems. For similar problems in the economics literature see e.g. Shimer and Smith [21] and Smith [22].

Under the above models, the distribution of the value of prospective partners is in a steady state. There may be a mating season and as it progresses the distribution of the value of available partners changes. Collins and McNamara [6] formulate such a model as a one-sided job search problem with continuous time. Ramsey [16] considers such a problem with discrete time. Johnstone [12] presents numerical results for a discrete time, two-sided mate choice problem. Alpern and Reyniers [3] use a more analytic approach to similar problems. Such models are further analyzed in Alpern and Kantrantzi [1] and Mazalov and Falko [13]. Alpern and Reyniers [2] consider a model where individuals have homotypic preferences. Eriksson *et al.* [8] describe a model where individuals optimize the expected rank of their partner. Burdett and Coles [5] present a model where the outflow resulting from partnership formation is balanced by job seekers and employers entering the job market. Their derivation of the steady-state distribution of attractiveness is similar to the approach used here to find the steady-state distribution of age.

Section 2 outlines the model. The form of a symmetric equilibrium is considered in Section 3. Section 4 describes the simple fertility model and a numerical procedure to derive a symmetric equilibrium profile and the corresponding age profile. A model where 'middled-aged' adults reproduce faster than others is considered in Section 5. Section 6 considers the joint distribution of the age of partners. Numerical results are presented in Section 7. Section 8 gives a brief conclusion and directions for future research.

2. The Model

Assume that equal numbers of males and females of age 0 enter the population of searchers per unit time. Hence, the incoming sex ratio equals one. Both males and females are fertile for one unit of time. Hence, males and females have a continuum of types (i.e. ages) x and y in the interval $[0, 1]$.

The common utility of a male of age x and a female of age y from pairing is denoted by $u(x, y)$. We consider so called fertility models in which partners obtain utility at some rate, $\gamma(x, y)$, dependent on their ages as long as they both remain fertile. This rate may be interpreted as the expected reproduction rate. Hence, this utility is the expected number of offspring. By assumption γ is non-negative and bounded. The utility obtained by both partners when a male of age x mates with a female of age y is

$$u(x, y) = \int_0^{\min(1-x,1-y)} \gamma(x + t, y + t) \; dt. \tag{1}$$

Under the simple fertility model, this reproduction rate is constant. Without loss of generality, we may assume that $\gamma(x, y) = 1$. From Equation (1), it follows that

$$u(x, y) = \min(1 - x, 1 - y) = 1 - \max(x, y),$$

which is simply the length of time that the male and female are simultaneously fertile.

Note that if the utilities are defined by a fertility model and $\gamma(x, y) = \gamma(y, x)$, then the utility function is symmetric, i.e. $u(x, y) = u(y, x)$.

Definition 1. *A game of the form considered is called symmetric (with respect to sex) if:*

1. *The incoming sex ratio is equal to one.*

2. *Males and females are fertile for the same length of time and meet prospective partners at the same rate, λ.*

3. *The utility function is symmetric.*

A strategy defines the set of acceptable prospective partners at each age x, $0 \leq x \leq 1$. Since we are looking for a symmetric (with respect to sex) pure equilibrium, it suffices to assume that all females use the pure strategy π_f and all males follow the pure strategy

Table 1: Summary of the notation used.

Symbol	**Description**
λ	interaction rate
x	age of male
y	age of female
$u(x, y)$	common utility from a male of age x mating with a female of age y
$\gamma(x, y)$	reproduction rate
π	strategy profile
$a_\pi(x)$	age profile under π, proportion of individuals of age x still searching
$\overline{a}_\pi$	proportion of fertile adults in mating pool
$\hat{a}_\pi(x)$	density function of age under π
$a(x), \hat{a}(x)$	age profile and density at equilibrium
$v_\pi(x)$	optimal utility of individual searcher under π
$g_\pi(x)$	optimal threshold rule under π
$g(x)$	equilibrium threshold rule

π_m. Suppose that the population follow a strategy profile $\pi = [\pi_f, \pi_m]$. A strategy profile is symmetric (with respect to sex) when $\pi_f = \pi_m$. Under a symmetric strategy profile, the operational sex ratio is 1 and the steady-state age distribution is independent of sex.

Henceforth, it will be assumed that π is a symmetric strategy profile. Let $a_\pi(x)$ denote the steady-state proportion of individuals of age x who are still searching for a mate. By definition $a_\pi(0) = 1$. Also, a_π is a non-increasing function. The function a_π will be called the age profile. In general, $a_\pi(1) > 0$, i.e. some individuals will never mate.

The steady-state proportion of individuals that have not yet mated, $\overline{a}_\pi$, is given by

$\overline{a}_\pi = \int_0^1 a_\pi(x)dx$. Hence, the steady-state probability density function of the age of unmated individuals in such a population is given by $\hat{a}_\pi(x)$, where $\hat{a}_\pi(x) = a_\pi(x)/\overline{a}_\pi$.

Let $v_\pi(x)$ be the optimal expected utility of an individual searching at age x when the population follows π. We adopt the **equilibrium criterion** used in McNamara and Collins [14], namely: a prospective partner is accepted if and only if the utility from such a partnership is at least the optimal expected utility from future search. Denote such an equilibrium profile by π^*. The value function corresponding to π^* is $v(x) \equiv v_{\pi^*}(x)$, $0 \leq x \leq 1$. The corresponding age profile, proportion of individuals in the mating pool and age density function are denoted a, $\overline{a}$ and $\hat{a}$, respectively. Table 1 gives a summary of the notation used.

3. Form of a Symmetric Equilibrium in Symmetric Games

This section presents several results regarding the form of a symmetric equilibrium. These will be useful in developing an algorithm for deriving such an equilibrium.

Theorem 1. *The value function $v(x)$ is non-increasing in x.*

PROOF. An individual of age x obtains an expected utility of $v(x+\delta)$, where $\delta > 0$, by rejecting any partner when between the ages x and $x+\delta$ and then using the optimal response to π^*. □

Theorem 2. *At a symmetric equilibrium of a symmetric game, if the younger of two prospective partners accepts the other, then acceptance is mutual.*

PROOF. Suppose the ages of the prospective partners are x and y, where $x < y$. If the younger of the two accepts the older one, then the equilibrium condition states that $v(x) \leq u(x, y)$. From the symmetry of the equilibrium and Theorem 1, $v(y) \leq v(x) \leq u(x, y)$. It follows that the older individual should accept the younger one. □

Theorem 3. *Consider a symmetric game in which $u(x, y)$ depends purely on the maximum of the two arguments, i.e. $u(x, y) = u_2(\max\{x, y\})$. In addition, assume that u_2 is non-increasing and continuous in its argument. The optimal response of a searcher to the strategy profile π is a threshold strategy of the form: at age x accept a prospective partner of age y if and only if $y \leq g_\pi(x)$. Any individual should always accept a prospective partner of the same or lower age, i.e. $g_\pi(x) \geq x$. The threshold function g_π is non-decreasing.*

Note 1: This theorem holds for the simple fertility model, since $u_2(x) = 1 - x$.

Note 2: If the conditions of this theorem hold, then the equilibrium strategy profile is a threshold profile of the form: an individual of age x should accept a prospective partner of age $\leq g(x)$, where g is a non-decreasing function and $g(x) \geq x$.

PROOF. By pairing with a prospective partner of age $\leq x$, an individual of age x obtains a utility of $u_2(x)$. The utility obtained by mating at a later age is $\leq u_2(x)$. Hence, at equilibrium an individual must accept a prospective partner of the same or lower age. An individual of age x should accept a prospective partner of age y, where $y > x$, if and only if $u_2(y) \geq v_\pi(x)$. Since $u_2(y)$ is non-increasing in y, an individual of age x should accept a prospective partner of age below some threshold $g_\pi(x)$, where $g_\pi(x)$ is the largest value of w in $[0, 1]$ such that $u_2(w) = v_\pi(x)$. Since both u_2 and v_π are non-increasing in their arguments, g_π must be non-decreasing in its argument. □

Note that when the equilibrium strategy profile is described by a threshold profile g, then we also use g to denote the strategy profile.

4. Example 1: The Simple Fertility Model

From the analysis in Section 3, the utility obtained from accepting the oldest acceptable prospective partner must be equal to the expected utility from future search. Hence, for the simple fertility model $v(x) = 1 - g(x)$. Since an individual becomes infertile at age 1, we obtain the boundary condition $v(1) = 0$, i.e. $g(1) = 1$. Consider a searcher of age x. Prospective partners are found at rate λ. Hence, the probability of encountering a prospective partner in a small interval of time of length δ is $\lambda\delta$. We consider two cases:

1. $\mathbf{x < g(0)}$. The searcher is acceptable to prospective partners of any age y. The prospective partner is acceptable if $y \leq g(x)$. The probability of mutual acceptance is

$$\frac{\int_0^{g(x)} a(y)dy}{\overline{a}}.$$

It follows that the probability that a searcher mates between age x and age $x + \delta$ is

$$\frac{\lambda\delta}{\overline{a}} \int_0^{g(x)} a(y)dy + O(\delta^2).$$

Hence,

$$\begin{aligned} a(x+\delta) &= a(x)\left[1 - \frac{\lambda\delta}{\overline{a}} \int_0^{g(x)} a(y)dy\right] + O(\delta^2) \\ \frac{a(x+\delta) - a(x)}{\delta} &= -\frac{\lambda a(x)}{\overline{a}} \int_0^{g(x)} a(y)dy + O(\delta). \end{aligned}$$

Letting $\delta \to 0$, we obtain

$$a'(x) = -\frac{\lambda a(x)}{\overline{a}} \int_0^{g(x)} a(y)dy. \tag{2}$$

2. $\mathbf{x \geq g(0)}$. The searcher must be acceptable to the prospective partner, i.e. $x \leq g(y)$. Since g is non-decreasing, it follows that $g^{-1}(x) \leq y$, where $g^{-1}(x)$ is the age of the youngest prospective partner who accepts a searcher of age x. Note that $g^{-1}(x) \leq x$. Also, g^{-1} is a generalisation of the inverse function of g. If g is strictly increasing, then g^{-1} is the inverse function of g. Thus acceptance is mutual if $g^{-1}(x) \leq y \leq g(x)$. The probability that a searcher mates between age x and age $x + \delta$ is

$$\frac{\lambda\delta}{\overline{a}} \int_{g^{-1}(x)}^{g(x)} a(y)dy.$$

Calculations analogous to the ones made in Point 1 lead to

$$a'(x) = -\frac{\lambda a(x)}{\overline{a}} \int_{g^{-1}(x)}^{g(x)} a(y)dy. \tag{3}$$

For $x < g(0)$, dividing both sides of Equation (2) by $a(x)$ and differentiating with respect to x, we obtain the following second order differential equation:

$$\frac{a'(x)}{a(x)} = -\frac{\lambda}{\overline{a}} \int_{0}^{g(x)} a(y)dy \Rightarrow \frac{a(x)a''(x) - [a'(x)]^2}{[a(x)]^2} = -\frac{\lambda a(g(x))g'(x)}{\overline{a}}. \tag{4}$$

Equation (4) is very difficult to solve directly, even numerically, due to the presence of the composite function $a \circ g$. We have the boundary condition $a(0) = 1$, but in order to use a difference equation to estimate $a(\delta)$ for small δ, we need to know $a(g(0))$. However, there is no obvious boundary condition for $g(0)$. Hence, we define a policy iteration algorithm to estimate the threshold rule and age profile corresponding to a symmetric equilibrium.

4.1. A Policy Iteration Algorithm

One might think that a policy iteration algorithm of the following form would work:

1. Choose an arbitrary symmetric strategy profile g_1.

2. Given g_i, derive the age profile a_i and then determine the optimal response g_{i+1}. Repeat until some convergence criterion has been met.

We might try to calculate the age profile a_1 numerically using Equation (2). Although we have the boundary condition $a_1(0) = 1$, in order to calculate $a_1'(0)$, we need to know $a_1(g(0))$. Thus we propose a different iterative procedure, described below.

Suppose we begin by positing an initial strategy profile g_1 (where g_1 is a non-decreasing function) and any non-increasing age profile a_1. Given g_1 and a_1, we can determine an individual's expected utility from search at age x and hence find the optimal threshold

$g_2(x)$. This calculation will be outlined in Subsection 4.2. We denote this calculation by $g_2 = H_1(g_1, a_1)$.

We then compute the function $a_2(x)$ defining the probability that an individual using g_2 is still searching for a mate at age x when the age profile is a_1 and the strategy profile is g_1. Note that, in general, a_{i+1} is not the age profile of the population when they all use the strategy profile g_{i+1}. We denote this computation (outlined in Subsection 4.2) by $a_2 = H_2(g_1, a_1)$.

Denote $(g_2, a_2) = H(g_1, a_1) = (H_1(g_1, a_1), H_2(g_1, a_1))$. In general, we can define

$$(g_{i+1}, a_{i+1}) = H(g_i, a_i) = (H_1(g_i, a_i), H_2(g_i, a_i)). \tag{5}$$

Theorems 4 and 5 (the proofs are available in an online appendix) state that the iteration scheme based on Equation (5) is well defined. Theorem 6 states that if this procedure converges, it converges to a symmetrical equilibrium profile and the corresponding age profile. In Section 7, we show numerically that the scheme converges for a suitable initial pair (g_1, a_1).

Note: This iteration procedure is adapted to the derivation of symmetric equilibria, but can be adapted to asymmetric equilibria. This will be considered in the conclusion.

Theorem 4. *Consider the simple fertility model. There is a unique best response to any pair of strategy profile, $g_i(x)$, and age profile, $a_i(x)$, such that $g_i(x) \geq x$, $g_i(x)$ is non-decreasing in x and $a_i(x)$ is non-increasing in x. This best response, $g_{i+1}(x)$, is also non-decreasing in x and satisfies $g_{i+1}(x) \geq x$.*

Theorem 5. *The age profile a_{i+1} corresponding to the best response g_{i+1} to the strategy profile g_i and age profile a_i is uniquely defined. Also, $a_{i+1}(x)$, is also non-increasing in x.*

Theorem 6. *Suppose that for some initial strategy-age profile pair (g_1, a_1), the iterates $(g_{i+1}, a_{i+1}) = H(g_i, a_i)$ converge to a limit (g, a). Then*

- *the symmetric strategy profile given by g is an equilibrium profile and*
- *the age profile is given by a.*

PROOF. (of Theorem 6) In the limit we have $(g, a) = H(g, a)$. It follows from the definition of H_1 that g is the best response function of an individual when the strategy profile is given by g and the age profile is a. From the definition of H_2, the probability that an individual using g is still searching at age x is given by $a(x)$. This individual is using the same strategy as the rest of the population, thus $a(x)$ is simply the proportion of individuals of age x who are still in the mating pool, i.e. a is the age profile. □

4.2. Derivation of the Best Response

We derive the best response of a searcher, g_{i+1}, when the population use g_i and the age profile is a_i. The optimal expected utility of a searcher of age x is $1-g_{i+1}(x)$. Let the next encounter with a prospective partner occur at age W. The density function of W is $p(w|W>x)=\lambda e^{-\lambda(w-x)}$. Note that at $w=1$ there is a probability mass equal to the probability that a searcher of age x does not meet another prospective partner. In this case, the utility is 0. Let the age of the prospective partner be y. To simplify the notation, assume that for $x \leq g_i(0)$, $g_i^{-1}(x)=0$, otherwise the inverse function is defined as normal. Denote $\overline{a}_i = \int_0^1 a_i(x)dx$. The pairing is mutually acceptable if $y \in [g_i^{-1}(w), g_{i+1}(w)]$. If $y \in [g_i^{-1}(w), w]$, then the searcher has a utility of $1-w$. If $y \in (w, g_{i+1}(w)]$, then the searcher has a utility of $1-y$. Otherwise, the expected utility is $1-g_{i+1}(w)$. Conditioning on the age of the prospective partner and taking the expected value, it follows that

$$\begin{aligned} 1 - g_{i+1}(x) &= \frac{\lambda e^{\lambda x}}{\overline{a}_i} \int_x^1 e^{-\lambda w} \Bigg[[1 - g_{i+1}(w)] \int_0^{g_i^{-1}(w)} a_i(y)dy + \int_{g_i^{-1}(w)}^{w} (1-w)a_i(y)dy + \\ &\quad + \int_w^{g_{i+1}(w)} (1-y)a_i(y)dy + [1 - g_{i+1}(w)] \int_{g_{i+1}(w)}^1 a_i(y)dy \Bigg] dw. \end{aligned}$$

Multiplying by $-e^{-\lambda x}$ and differentiating with respect to x, we obtain

$$
\begin{aligned}
e^{-\lambda x}[g'_{i+1}(x)+\lambda\{1-g_{i+1}(x)\}] \;=\; & \frac{\lambda e^{-\lambda x}}{\overline{a}_i}\left[[1-g_{i+1}(x)]\int_0^{g_i^{-1}(x)} a_i(y)dy + \int_{g_i^{-1}(x)}^{x}(1-x)a_i(y)dy + \right.\\
& \left. + \int_x^{g_{i+1}(x)}(1-y)a_i(y)dy + [1-g_{i+1}(x)]\int_{g_{i+1}(x)}^1 a_i(y)dy\right]
\end{aligned}
$$

Multiplying by $e^{\lambda x}$ and using the fact that $\hat{a}_i(x) = a_i(x)/\overline{a}_i$ is a density function,

$$
\begin{aligned}
g'_{i+1}(x) \;=\; & \frac{\lambda}{\overline{a}_i}\left[\int_{g_i^{-1}(x)}^{x}(1-x)a_i(y)dy + \int_x^{g_{i+1}(x)}(1-y)a_i(y)dy + \right.\\
& \left. -[1-g_{i+1}(x)]\left(\int_0^1 a_i(y)dy - \int_0^{g_i^{-1}(x)} a_i(y)dy - \int_{g_{i+1}(x)}^1 a_i(y)dy\right)\right].
\end{aligned}
$$

Finally, using the additivity properties of integration,

$$
\begin{aligned}
g'_{i+1}(x) &= \frac{\lambda}{\overline{a}_i}\left[\int_{g_i^{-1}(x)}^{x}(1-x)a_i(y)dy + \int_x^{g_{i+1}(x)}(1-y)a_i(y)dy - \int_{g_i^{-1}(x)}^{g_{i+1}(x)}[1-g_{i+1}(x)]a_i(y)dy\right]\\
&= \frac{\lambda}{\overline{a}_i}\left\{[g_{i+1}(x)-x]\int_{g_i^{-1}(x)}^{x} a_i(y)dy + \int_x^{g_{i+1}(x)}[g_{i+1}(x)-y]a_i(y)dy\right\}. \qquad (6)
\end{aligned}
$$

Equation (6) can be solved numerically, using the boundary condition $g_{i+1}(1) = 1$ and estimating $g_{i+1}(x)$ sequentially at $x = 1-h, 1-2h, \ldots, 0$.

Now we calculate a_{i+1} given the optimal response g_{i+1}, current strategy profile g_i and age profile a_i. Analogous to the derivation of Equation (3), we obtain

$$
a'_{i+1}(x) = -\frac{\lambda a_{i+1}(x)}{\overline{a}_i}\int_{g_i^{-1}(x)}^{g_{i+1}(x)} a_i(y)dy. \qquad (7)
$$

Equation (7) can be solved numerically, using the boundary condition $a_i(0) = 1$ and estimating $a_i(x)$ sequentially at $x = 0, h, 2h, \ldots, 1$.

5. Example 2

Consider a fertility model where $\gamma(x,y) = 3(x - x^2 + y - y^2)$, thus the payoff rate is maximized when both partners are of age 0.5. From Equation (1), we obtain

$$u(x,y) = \begin{cases} u_1(x,y) = (1-x)(1-3y^2+x+3yx-2x^2), & y < x \\ u_2(x,y) = (1-y)(1-3x^2+y+3yx-2y^2), & y \geq x \end{cases}. \tag{8}$$

For $y \geq x$, differentiating Equation (8) with respect to y, we obtain

$$\frac{\partial u(x,y)}{\partial y} = 3x - 6y - 6yx + 6y^2 + 3x^2. \tag{9}$$

This derivative is negative whenever y lies between the roots of the following equation (treated as an equation for y):

$$2y^2 - 2y(1+x) + x(1+x) = 0. \tag{10}$$

The smaller root of Equation (10) is $y_0(x)$, where

$$y_0(x) = \frac{1 + x - \sqrt{(1-x)(1+x)}}{2} \leq x.$$

In an analogous manner, it can be shown that the larger root of this equation is greater than 1. Hence, for fixed x, $u(x,y)$ is decreasing in y when $x < y < 1$.

For $0 \leq y \leq x$ it can be shown that for fixed x, $u(x,y)$ has a unique extreme point, a maximum when $y = x/2$. Hence, $\forall \pi$, $v_\pi(x) < u(x, x/2) = 1 - 9x^2/4 + 5x^3/4 \equiv h_1(x)$.

Since $u(x,y)$ is not monotonic in y for a fixed x, it is unclear whether the equilibrium strategy should be a threshold strategy (it might be optimal for an individual to accept only individuals of intermediate age). The maximum possible utility of 1 is obtained when two individuals of age 0 mate (as in the simple fertility model considered in Section 4).

Define $h_2(x) = u(x,x) = 1 - 3x^2 + 2x^3$. The function $h_2(x)$ is decreasing for all $x \in (0,1)$, $h_2(0) = 1$ and $h_2(1) = 0$. Note that $h_2(x) \leq h_1(x)$ for $x \in [0,1]$.

Result 1. *Suppose that for a given x, $v_\pi(x) \leq h_2(x)$. The optimal response of a searcher of age x is to accept a prospective partner if and only if $x \leq g_\pi(x)$, where $g_\pi(x)$ is the value of y which a) lies in $[x,1]$ and b) satisfies the following equation*

$$h_3(x,y) \equiv 2y^3 - 3y^2(1+x) + 3yx(1+x) + 1 - 3x^2 - v_\pi(x) = 0. \tag{11}$$

PROOF. Suppose the prospective partner is of age y, where $y \leq x$. From the above analysis, the utility obtained by an individual of age x has a unique local extremum (a maximum) at $y = x/2$. It follows that the least preferred non-older partners must be either of age 0 or age x. We have $u(x,0) = u(x,x) = h_2(x)$. It follows that if $v_\pi(x) \leq h_2(x)$, then a searcher of age x should accept any prospective partner of age $\leq x$.

A searcher of age x should accept a prospective partner of age y, where $y > x$, if and only if $v_\pi(x) \leq u(x,y)$, i.e. $h_3(x,y) \geq 0$. From Equations (9) and (11), $\frac{\partial h_3(x,y)}{\partial y} = \frac{\partial u(x,y)}{\partial y}$. From the analysis of the function u, $h_3(x,y)$ is decreasing in y for $y \in (x,1)$. Also,

$$h_3(x,x) = 1 - 3x^2 + 2x^3 - v_\pi(x) \geq 0$$

and $h_3(x,1) = -v_\pi(x) < 0$. Thus there exists exactly one value of y, say y_0, in $[x,1)$ which satisfies $h_3(x,y) = 0$. Also, for $y \leq y_0$, $h_3(x,y) \geq 0$ and for $y > y_0$, $h_3(x,y) < 0$. □

Corollary 1. *If $v(x) \leq h_2(x)$, $\forall x \in [0,1]$, then the equilibrium strategy profile is given by a threshold profile.*

We will derive an iterative procedure of the same form as given in Section 4 for estimating the value function under the assumption that the equilibrium is given by a threshold profile. In Section 7, after some numerical results have been presented, we then give an argument that the the equilibrium strategy profile is always of such a form.

Suppose that at stage i of the iteration procedure the strategy profile and age profile are g_i and a_i, respectively. As before, under this threshold strategy, the youngest prospective partner who accepts a searcher of age x is defined to be of age $g_i^{-1}(x)$.

We now calculate the function describing the optimal expected utility of an individual searcher, v_{i+1}. From this function, we then derive the optimal strategy of this searcher (given by the threshold profile g_{i+1}). In order to calculate the optimal expected future utility of a searcher at age x, as before we consider the age of the searcher when the next prospective partner is found, w, and condition on the age of this prospective partner, y.

1. If $y < g_i^{-1}(w)$, then the searcher is rejected and has future expected utility $v_{i+1}(w)$.

2. If $y \in [g_i^{-1}(w), w]$, then acceptance is mutual and the utility of the searcher is $u_1(w, y)$.

3. If $y \in (w, g_{i+1}(w)]$, then acceptance is mutual and the utility of the searcher is $u_2(w, y)$.

4. If $y > g_{i+1}(w)$, then the searcher rejects the prospective partner and has future expected utility $v_{i+1}(w)$.

It follows that

$$v_{i+1}(x) = \int_x^1 \frac{\lambda e^{-\lambda(w-x)}}{\overline{a}_i} \left[\int_0^{g_i^{-1}(w)} v_{i+1}(w) a_i(y) dy + \int_{g_i^{-1}(w)}^{w} u_1(w, y) a_i(y) dy + \right.$$
$$\left. + \int_w^{g_{i+1}(w)} u_2(w, y) a_i(y) dy + \int_{g_{i+1}(w)}^1 v_{i+1}(w) a_i(y) dy \right] dw, \quad (12)$$

where $\overline{a}_i = \int_0^1 a_i(x) dx$. After dividing (12) by $e^{\lambda x}$ and differentiating with respect to x, calculations analogous to those in Section 4.2 lead to

$$v'_{i+1}(x) = \frac{\lambda}{\overline{a}_i} \left\{ \int_{g_i^{-1}(x)}^{x} [v_{i+1}(x) - u_1(x, y)] a_i(y) dy + \int_x^{g_{i+1}(x)} [v_{i+1}(x) - u_2(x, y)] a_i(y) dy \right\}. \quad (13)$$

We estimate $v_{i+1}(x)$ recursively at $x = 1-\delta, 1-2\delta, \ldots, 0$ using $v_{i+1}(1) = 0$. First calculate the thresholds $g_{i+1}(x+\delta), g_{i+1}(x+2\delta), \ldots, g_{i+1}(1)$. Since we have previously estimated $v_{i+1}(x+\delta), v_{i+1}(x+2\delta), \ldots, v_{i+1}(1)$, we can estimate $g_{i+1}(w)$ for $w > x$ by solving $h_3(w, g_{i+1}(w)) = 0$. From the above analysis, if $v_{i+1}(w) \leq h_2(w)$, then there is exactly one solution, $g_{i+1}(w)$, of this equation such that $w \leq g_{i+1}(w) \leq 1$. If $g_{i+1}(w) > h_2(w)$, then set $g_{i+1}(w) = w$. Hence, using the conditions $v_{i+1}(1) = 0, g_{i+1}(1) = 1$, we can numerically estimate $v_{i+1}(1–\delta)$ and use this to numerically calculate $g_{i+1}(1–\delta)$. Working iteratively, we can numerically estimate $v_{i+1}(1-2\delta), g_{i+1}(1-2\delta), v_{i+1}(1-3\delta), g_{i+1}(1-3\delta), \ldots, v_{i+1}(0), g_{i+1}(0)$.

Once the procedure has converged, we check the condition $v(x) \leq h_2(x)$, $\forall x \in [0,1]$, where $v(x)$ is the estimate of the value function. If this condition is satisfied, (as in the numerical examples considered in Section 7) then the equilibrium is a threshold rule.

The question of when the equilibrium is a threshold rule will be addressed in the conclusion and in future work.

6. Joint Distribution of the Age of Partners

The value of $a(t)$ can be interpreted as the probability that the age of an individual on pairing is at least t. Let T denote the age of an individual on pairing and $d(t)$ be the density function of this age. It is assumed that if an individual does not find a mate then $T = 1$, i.e. the distribution of T has a point mass at 1 of weight $a(1)$. We have

$$a(t) = 1 - P(T < t) \Rightarrow d(t) = -a'(t). \tag{14}$$

The numerical procedure for calculating $a(t)$ involves first deriving $a'(t)$, thus the estimation of $d(t)$ does not involve any additional computation.

The density function of T given that mating occurs is denoted $\tilde{d}(t)$. We have

$$\tilde{d}(t) = \frac{d(t)}{P(T < 1)} = \frac{d(t)}{1 - a(1)}. \tag{15}$$

Let $d_2(x,y)$ be the joint density of the ages of partners on pairing (given that pairing occurs) at equilibrium. Here, x and y denote the male's and female's ages, respectively. The conditional density of the female's age on pairing given the male's age is denoted $d(y|x)$. The age of the female is chosen from the age profile under the sole condition that she is mutually acceptable to the male [i.e. aged between $g^{-1}(x)$ and $g(x)$]. Hence,

$$d(y|x) = \begin{cases} \frac{a(y)}{\int_{g^{-1}(x)}^{g(x)} a(y)dy}, & g^{-1}(x) \leq y \leq g(x) \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

The joint distribution $d_2(x,y)$ can then be derived using $d_2(x,y) = \tilde{d}(x)d(y|x)$. The correlation between the ages of partners on pairing, $\rho_{X,Y}$ can then be calculated using

$$\rho_{X,Y} = \frac{E(XY) - E(X)E(Y)}{\sqrt{Var(X)Var(Y)}} = \frac{E(XY) - E(X)^2}{Var(X)},$$

where $E(XY)$ is the expected value of the product of a pair's ages on pairing (calculated with respect to the joint density function d_2). $E(X)$ and $Var(X)$ are the expected value and the variance of the age of an individual on pairing conditioned on an individual finding a mate (i.e. calculated with respect to the density function $\tilde{d}$).

7. Numerical Results

It is assumed that $g_1(x) = a_1(x) = 1$ for $x \in [0,1]$. Hence, initially individuals accept any prospective mate, but no-one leaves the mating pool, i.e. the age profile does not have to correspond to the strategy profile. A MATLAB programme was written to estimate the equilibrium rule and age profiles at $0, h, 2h, \ldots, 1$ based on the appropriate difference equations, using double precision and Simpson's rule to calculate integrals. The inverse to the threshold rule was estimated at the same points by linear interpolation. Comparison of various step sizes suggested that a step size of $h = 10^{-4}$ allowed estimation of the threshold and age profile to 3 decimal places for $\lambda \leq 50$. Estimation of the joint distribution of the ages of partners was less accurate for larger values of λ in this range and so numerical results are given for the expected utility from search, the proportion of adults searching for a mate, the proportion of searchers eventually finding a mate and the correlation between the ages of partners on mating for $\lambda \leq 20$ in Tables 2 and 3. The expected utilities are higher in Example 2, since under this model individuals have a low potential reproduction rate when they are very young (i.e. likely to be searching). As expected, the correlation between the ages of partners is increasing in the interaction rate. Note that the proportion of individuals not mating is greater in Example 2 than in Example 1. This may be due to the fact that in Example 2 older adults are less fertile and thus less attractive as mates. At low interaction rates, the proportion of adults searching for a mate at equilibrium is also greater in Example 2. This relation is reversed at high interaction rates. This could be due to the fact that in Example 2 young adults are initially not very fertile, which means that they are happy to find a prospective partner who is reasonably young (the exact age is not important) and are likely to find one due to the high interaction rate. In Example 1, the exact age of a prospective partner is more important.

The equilibrium profiles are illustrated in Figures 2 (in the Introduction) and 3. For higher values of λ a smaller step size or more advanced procedure is necessary, since the maximum value of the second derivative of the threshold rule is increasing in λ. For the values of λ considered, convergence was quite rapid. The iteration procedure for Example 1 is illustrated in Figure 4. The rule obtained after five iterations is very similar to the equilibrium rule. After 20 iterations the procedure had converged to four decimal places and the corresponding rule is thus a good approximation of the equilibrium rule.

The rules evolved are similar in both examples. The thresholds in Example 2 initially rise more slowly (young adults have a low reproductive rate and do not lose much by not

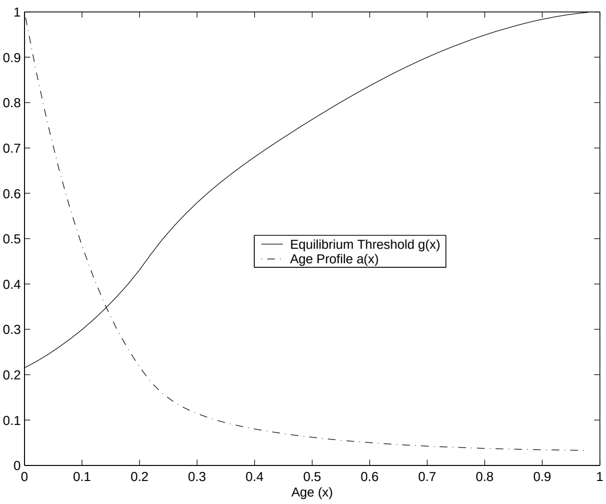

Figure 3: Estimate of the equilibrium threshold rule and age profile for Example 2: $\lambda = 10$. An individual of age x will accept a prospective partner if and only if the prospective partner's age is $\leq g(x)$. The age profile $a(x)$ gives the proportion of individuals of age x still searching for a mate.

mating early) and rise more rapidly as searchers approach their reproductive peak.

Figure 5 illustrates the distribution of age on pairing (given that pairing occurs) for Example 1 when $\lambda = 10$. There is a kink at age $g(0) \approx 0.1946$. Those of age $< g(0)$ are acceptable to all prospective partners, while those of age $> g(0)$ are unacceptable to sufficiently young ones. Thus the rate at which older searchers find a partner drops.

Table 4 illustrates the joint distribution of the ages of partners on mating, (x, y), for Example 1 when $\lambda = 10$. This highlights the correlation between the ages of partners. To make the results more interpretable, the distribution is discretized.

Note that the rules evolved for Example 2 satisfy $v(x) \leq h_2(x)$, i.e. the equilibrium profile is a threshold profile. Since a searcher must obtain a utility of less than $u(0,0) = h_2(0) = 1$, $v(0) < h_2(0)$. Thus from the continuity of $v(x)$ and $h_2(x)$, for some $x_0 > 0$, $v(x) \leq h_2(x)$ for $x \leq x_0$. Assume that for $x < x_0$, $g(x)$ is increasing in x, $v(x_0) = h_2(x_0)$ and $v(x) > h_2(x)$ for x in some interval $(x_0, x_0 + \epsilon)$. From the form of the equilibrium,

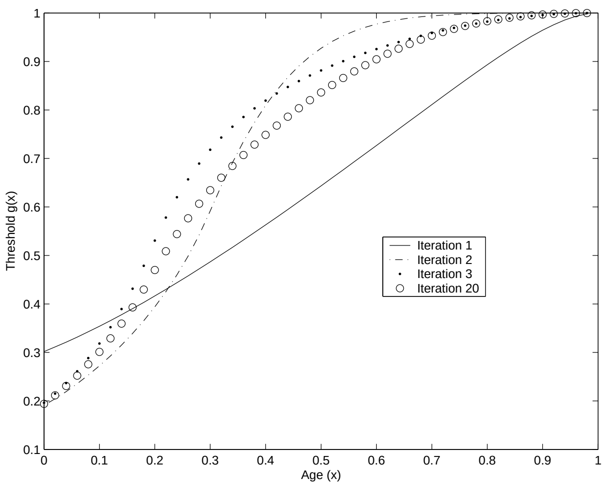

Figure 4: Convergence of the threshold profiles for Example 1: $\lambda = 10$. A searcher accepts a prospective partner if the prospective partner's age is $\leq g(x)$. The iterative procedure converges to 4 decimal places after 20 iterations.

Table 2: Estimates of parameters of the threshold rule and age profile for Example 1 - prop. searching: proportion of all fertile adults who are searching for a partner - prop. not mating: proportion of individuals who never find a mate - corr. of ages: correlation between the ages of partners at the time of mating

λ	Expected Utility	Prop. searching	Prop. not mating	Corr. of ages
1	0.2628	0.6666	0.4453	0.2629
2	0.4264	0.4968	0.2509	0.4172
5	0.6645	0.2755	0.0809	0.6178
10	0.8054	0.1548	0.0267	0.7178
20	0.8954	0.0813	0.0077	0.7846

Table 3: Estimates of parameters of the threshold rule and age profile for Example 2 - prop. searching: proportion of all fertile adults who are searching for a partner - prop. not mating: proportion of individuals who never find a mate - corr. of ages: correlation between the ages of partners at the time of mating

λ	Expected Utility	Prop. searching	Prop. not mating	Corr. of ages
1	0.2774	0.6946	0.5022	0.3921
2	0.4577	0.5289	0.3082	0.5144
5	0.7305	0.2937	0.1026	0.6527
10	0.8816	0.1584	0.0329	0.7174
20	0.9581	0.0777	0.0088	0.7502

Table 4: Joint distribution of ages of male and female on mating, x and y respectively, for Example 1, $\lambda = 10$. The values given are percentages. The table shows that pairs tend to form at an early age and the ages of partners on mating are highly correlated.

$y \in (0.9, 1]$	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04	0.05	0.05
$y \in (0.8, 0.9]$	0.00	0.00	0.00	0.00	0.01	0.07	0.08	0.07	0.06	0.05
$y \in (0.7, 0.8]$	0.00	0.00	0.00	0.02	0.13	0.11	0.09	0.08	0.07	0.04
$y \in (0.6, 0.7]$	0.00	0.00	0.01	0.21	0.18	0.14	0.11	0.09	0.08	0.02
$y \in (0.5, 0.6]$	0.00	0.00	0.26	0.32	0.22	0.17	0.14	0.11	0.07	0.00
$y \in (0.4, 0.5]$	0.00	0.14	0.66	0.42	0.30	0.22	0.18	0.13	0.01	0.00
$y \in (0.3, 0.4]$	0.00	1.26	0.96	0.58	0.42	0.32	0.21	0.02	0.00	0.00
$y \in (0.2, 0.3]$	3.06	3.38	1.56	0.96	0.66	0.26	0.01	0.00	0.00	0.00
$y \in (0.1, 0.2]$	15.82	7.31	3.38	1.26	0.14	0.00	0.00	0.00	0.00	0.00
$y \in [0.0, 0.1]$	34.33	15.82	3.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
x	[0.0,0.1]	(0.1,0.2]	(0.2,0.3]	(0.3,0.4]	(0.4,0.5]	(0.5,0.6]	(0.6,0.7]	(0.7,0.8]	(0.8,0.9]	(0.9,1.0]

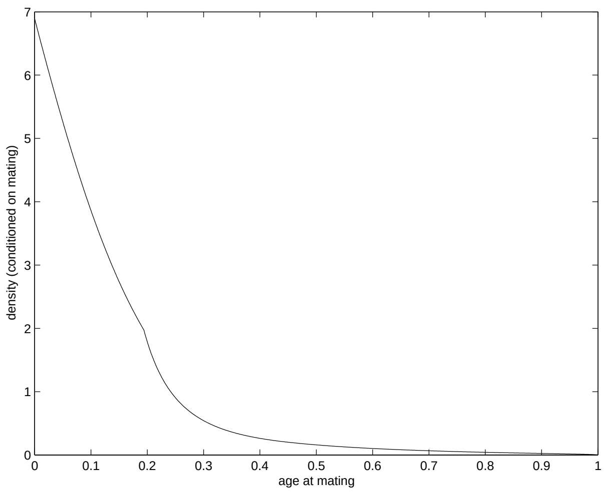

Figure 5: Density function of age on mating (conditioned on mating): $\lambda = 10$. Individuals tend to mate at a relatively early age. The kink in the density function occurs at $x = g(0)$, i.e. the age at which searchers start to become unacceptable to the youngest prospective partners.

$g(x_0) = x_0$. An individual of age between x_0 and $x_0 + \epsilon$ does not accept a prospective partner of the same age or older, but would not be accepted by one of age $\leq x_0$. Hence, $v'(x) = 0$ for $x_0 \leq x \leq x_0 + \epsilon$. Thus for $x \in [x_0, 1]$, $v(x) = 1 - 3x_0^2 + 2x_0^3 > h_2(x)$. However, this is a contradiction, since $v(1) = 0$.

Hence, if $g'(x) \geq 0$ whenever $v(x) < h_2(x)$, then the equilibrium profile is a threshold profile. From the numerical results, the monotonicity of $g(x)$ seems reasonable. However, a proof of this would be technical. Hence, we state the following conjecture:

Conjecture 1. *The equilibrium strategy profile for the model considered in Example 2 is given by a threshold profile.*

8. Conclusion

This article has considered a model of mutual choice based on the ages of prospective partners. Male and female adults enter the mating pool at equal rates and are fertile for one unit of time. Each observes prospective partners as a Poisson process until he/she finds a mutually acceptable partner. On finding a partner, an individual leaves the mating pool. Hence, the distribution of the age of a searcher of a given sex (the age profile) depends on the rules used within the population. When a prospective partner is found, his/her age is chosen at random from the age distribution in the appropriate sex.

Under a fertility model, the utility from mating is the number of offspring. A pair produces offspring at a positive rate depending on their ages, until one individual becomes infertile. Hence, when a male of age x mates with a female of age y both obtain the same utility, denoted $u(x, y)$. A game of this type is called symmetric if $u(x, y) = u(y, x)$.

We derive symmetric equilibria of such games using a policy iteration algorithm. If the utility from mating depends purely on the older partner's age, then the equilibrium profile is of the form: at age x accept a prospective partner if and only if his/her age is $\leq g(x)$, where $g(x) \geq x$ and $g'(x) \geq 0$, $\forall x \in [0, 1]$. One case is the **simple fertility model**, where a pair produces offspring at a constant rate until either becomes infertile.

Another example is considered in which the rate at which pairs produce offspring is greatest when both are 'middle-aged'. It is shown that as long as the value function, $v(x)$, is bounded from above by the utility obtained when two individuals of age x mate, then the equilibrium strategy profile is given by a threshold profile. The numerical results obtained indicate that the equilibrium strategy profile is indeed a threshold profile.

Future work should consider the conditions under which an equilibrium profile is a threshold rule. For example, suppose that the reproduction rate is $\gamma(x, y) = 1$ when both x and y are in $[0.5 - \delta, 0.5 + \delta]$ and $\gamma(x, y) = \epsilon$ otherwise, where δ and ϵ are appropriately small. An individual of age close to $0.5 - \delta$ should only accept a prospective partner of a similar age, i.e. the equilibrium rule is not a threshold rule. However, it seems that the equilibrium rule will be a threshold rule for a wide range of "sensible" γ.

It would be of interest to look at asymmetric games, i.e. games in which adult males and females enter the mating pool at different rates, males and females are fertile for different lengths of time and/or the utility function $u(x, y)$ is asymmetric. Consider the simple fertility model and suppose that the incoming sex ratio is male biased (i.e. in any given interval of time more males mature than females). If males and females are fertile for the same length of time, then intuitively there will be more males than females searching for a partner. Hence, males will be less choosy than females.

The algorithm presented here needs to be adapted to find the equilibria of such problems. One can initiate such a procedure by choosing a strategy and age profile for males. The optimal female response to this strategy and then the corresponding female age profile can be found by deriving and solving the appropriate differential equations (see Sections 4 and 5). After that, one would calculate the best male response to the female strategy and age profile. This procedure is repeated until both the male and female strategies have converged. Such an algorithm could also be used to look for asymmetric equilibria of symmetric games. The recent work of Ramsey [17] considers such problems. The iterative procedure always converged to a symmetric equilibrium. However, no proof that any equilibrium of a symmetric game is a symmetric equilibrium has yet been found.

The assumption that all individuals of a given sex are fertile for a fixed length of time is also unrealistic. One could look at problems in which the rate at which individuals become infertile (or the mortality rate) depends on age. If the mortality rate is increasing, then under the simple fertility model we expect the equilibrium profile to be a threshold rule, since the length of time for which any individual can be expected to breed will be decreasing in their age. However, suppose the mortality rate is lowest for middle-aged individuals. In this case, it may well be that middle-aged individuals will accept middle-aged prospective partners, but reject young prospective partners.

Also, the model considered here assumes that singles meet at the same rate, regardless of the proportion of adults in the mating pool. This could be thought of as a 'singles bar' model. At the opposite end of such a spectrum, assume that the rate at which singles meet is proportional to the fraction of adults in the mating pool. Such a model can be thought of as one in which the populations of singles and paired individuals mix freely.

Another natural extension of the model would be to allow an individual to remate when his/her partner becomes infertile. Thus when a young female meets an older male, she must weigh up the advantages of forming a pair with such a male against the costs of having to return to the mating pool when being a less attractive partner. It is possible that in some scenarios random mating might be an equilibrium strategy (e.g. for the simple fertility model, as older partners reproduce at the same rate as younger partners).

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