Improving Mathematics Teaching at Second Level through the Design of a Model of Teacher Knowledge and an Intervention Aimed at Developing Teachers’ Knowledge

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Abstract

The importance of mathematics is recognised and acknowledged worldwide and its importance as a school subject has been confirmed universally. A primary concern in Ireland among educators, policy makers and the Department of Education and Skills is the finding that many students complete their second level studies with a poor grasp of mathematics and are not prepared for the mathematics they will face at third level or in the workplace. Researchers now accept that the supply of well qualified second level students in mathematics for higher education or the workplace is crucially dependent upon the quality of teaching they receive especially in the formative years. Teachers’ levels of knowledge helps determine the quality of mathematics teaching and underpin much of what is done in the mathematics classroom.

Research has shown that numerous attributes of effective teaching are affected by, inter alia, a mathematics teacher’s knowledge base. Furthermore an extensive knowledge base on the part of teachers will allow them to teach for understanding and foster an appreciation of mathematics among their students while research also suggests that this knowledge can result in increased uptake and attainment levels in mathematics. However, despite such findings, research indicates that at the moment teachers simply do not have a sufficient knowledge base to carry out their duties effectively and as a result the teaching and learning of mathematics is being detrimentally affected in Ireland and elsewhere worldwide.

Such considerations and findings led the author to investigate the issue of the knowledge required for mathematics teaching. The research analysed the special relationship between the teaching and learning of mathematics and teachers subject knowledge. The author then focussed on defining a model of the knowledge base required for teaching and examined the types of knowledge required in order to teach mathematics effectively. The author then developed and supported, and validated this model through a proof of concept approach involving an action research Continuous Professional Development (CPD) initiative. This enabled the author to see if her model is a ‘fit for purpose’ vehicle for improving of knowledge among teachers and in turn improving the teaching and learning of mathematics in Ireland.
Declaration

This thesis is presented in fulfilment of the requirements for the degree of Doctorate of Philosophy. It is entirely my own work and has not been submitted to any other university or higher education institution, or any other academic award in this University. Where use has been made of the work of other people it has been fully acknowledged and fully referenced.

Signature: _______________________

Niamh O’Meara

December, 2010
Dedication

This dissertation is dedicated to my Mam and Dad. Thanks for the unwavering support you have given me and for the years of selfless devotion, love and sacrifice. I couldn’t have done this without you. Thanks for everything!
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1. Introduction

1.1 Introduction

“Mathematics is an integral part of a general education. It can enhance our understanding of our world and the quality of our participation in society. It is valuable to people individually and collectively, providing important tools which can be used at the personal, civic and vocational levels”

(Australian Education Council, 1991: 13)

The importance of mathematics is recognised and acknowledged worldwide. In recent years researchers and international organisations have acknowledged the extensive uses of mathematics in today’s society and hence its importance as a school subject has been confirmed universally (London Mathematical Society, 1995; Smith, 2004, U.S. National Mathematics Advisory Panel, 2008). While Ireland always placed a high premium on mathematics education it has also witnessed a surge in interest in mathematics education in recent times and all stakeholders now recognise the critical nature of this subject in all walks of life.

The National Council for Curriculum and Assessment [NCCA] (2006) underline the importance of mathematics in developing problem solving skills which we rely on a daily basis. Also the fact that mathematics provides us with important foundations
for other subjects such as science and technology is well understood and the relationship between mathematics and economic development is acknowledged.

“Mathematical knowledge and skills are held in high esteem in Ireland and are seen as having a significant role to play in the development of the knowledge society to which Ireland aspires.” (NCCA, 2006: 1)

Many other government bodies such as the Task Force on Physical Sciences (2001), Expert Group on Future Skills Needs [EGFSN] (2008) and the Innovation Task Force (2010) highlight the crucial role mathematics education now plays in the wider economy. Furthermore Brabazon (2010) found that a strong focus on mathematical literacy is required in Ireland today in order for us to become the innovation island of tomorrow. This means that mathematics is now seen as a crucial subject in Ireland at primary, secondary and tertiary level and the emphasis on research and curriculum development in the field in recent years duly reflects this.

Reports such as the PISA Report (2003) and TIMSS Report (2007) have provided large scale insights into the mathematical standards of adolescents and have provided valuable information on the future direction of mathematics education in numerous countries. In Ireland, recent reports published by the NCCA (2005), Gill & O'Donoghue (2006), Ni Riordain & Hannigan (2009) and Engineers Ireland (2010) have also provided those involved in the field of mathematics education with insightful findings on a range of issues. Such issues include problems regarding the transition from secondary to tertiary mathematics, the current uptake of higher level mathematics in Ireland, teaching approaches adopted in Irish classrooms and the prevalence of out-of-field teaching in Ireland.

A primary concern in Ireland among educators, policy makers and the Department of Education and Skills is the finding that many students complete their second level studies with a poor grasp of mathematics and are not prepared for the mathematics which many will face at third level (Chief Examiner’s Reports, 2000, 2005; O’Donoghue, 2002; Gill et al., 2010). However this problem is not only confined to Ireland; international findings have suggested that similar problems exist worldwide. For example Smith (2004) and the LMS (1995: 5) found that a large proportion of students are under prepared for third level mathematics on completion of their second level studies:
“The problem is more serious; it is not just the case that some students are less well – prepared, but that many ‘high attaining’ students are seriously lacking in the fundamental notions of the subject”

The supply of well qualified second level students in mathematics for higher education and the work place is crucially dependent upon the quality of the teaching they receive especially in the formative years (Ni Riordan & Hannigan, 2009). However, despite the need for well qualified and knowledgeable teachers, research suggests that currently some teachers have an insufficient knowledge base to carry out their duties effectively and as a result mathematics teaching and learning is less than optimal at primary and secondary level, for example, in the U.S. and Ireland (Ball et al., 2005; Hourigan & O’Donoghue, 2007).

1.2 Background and Context to the Research

Many problems still exist in this area despite the acceptance of the importance of mathematics education and the investigations that have been undertaken both in Ireland and abroad in recent years. According to the Chief Examiners Report (2005) students understanding of mathematics in their final year of schooling in Ireland is below the expected standards both at Higher and Ordinary level (These terms are explained in section 1.10). Levels of procedural understanding, that is, the knowledge of rules and procedures, were reported to be adequate among Senior Cycle students but conceptual understanding, that is, their understanding of mathematical concepts, operations, relationships and applications was highlighted as a matter of concern. Ireland’s performance in mathematics has also been compared to that of students in other OECD countries with the general consensus being that Irish students are currently underperforming in comparison to their international counterparts (NCCA, 2005; OECD\(^1\), 2007; FORFAS\(^2\), 2008). Such findings suggest that at present mathematics education in Ireland is facing numerous challenges, including overemphasis on skills and procedural knowledge and insufficient emphasis on problem solving, understanding and concepts.

\(^1\) Organisation for Economic Co-operation and Development

\(^2\) FORFAS is Ireland’s policy advisory board for enterprise, trade, science, technology and innovation.
These issues are matters of serious concern for the Irish Government in their efforts to promote future economic development based on a Smart Economy. Thus mathematics education is currently receiving priority in educational and economic policy terms from the Irish government and a major national effort is underway at second level to revitalise mathematics education, namely Project Maths.

The inadequate nature of mathematics teaching and its negative effects especially at secondary school level have been acknowledged widely in Ireland in education and government circles. Several recent reports have detailed the shortcomings of mathematics teaching at secondary level as a lack of emphasis:

- on teaching for understanding
- on problem solving and applications
- on conceptual development.

The didactical nature and exam orientation of mathematics teaching is widely recognised in Irish education circles at this level (NCCA, 2006; Innovation Task Force, 2010; Project Maths Implementation Support Committee, 2010). These issues are underlined by the NCCA in their report which describes the main emphasis of the new major national initiative in secondary school mathematics called Project Maths and are detailed as follows:

- To provide a bridging framework from the revised primary curriculum into second level.
- To place a greater emphasis on the understanding of mathematical concepts and the application of mathematical skills and knowledge.
- To contribute to the development of higher order thinking skills including logical reasoning and problem solving
- To encourage greater uptake of higher level mathematics
- To provide a solid foundation which prepares students for careers in science, technology, engineering, business or humanities.

(Project Maths Implementation Support Group, 2010: 13)

Further new research on the composition of the mathematics teaching force at second level shows that approximately 48 per cent of the teachers of mathematics are out-of-
field teachers (Ni Riordain & Hannigan, 2009). The latter study points to a serious structural problem in the mathematics teaching profession at secondary level in relation to teachers’ subject knowledge in mathematics and their deployment across the system.

The author’s intention in this study is to improve the mathematics education experience for second level students and in no way does she seek to blame mathematics teachers for a state of affairs that is not of their own making. The author recognises current teachers as key stakeholders in the future of mathematics education in Ireland. She is also aware of the context and the environmental constraints on mathematics teachers in Irish secondary schools particularly as regards parental expectations, societal pressures, exam success and other metrics. However the emergence of teaching as a profession is a relatively new phenomenon in Ireland and this impacts on teaching across the board but especially on the teaching of mathematics and science. Prior to the establishment of the Teaching Council in 2006 an undergraduate degree and a Higher Diploma in Education was sufficient to qualify as a secondary school teacher as is the case in many other countries. The nature of this qualification led to the issue of out – of – field teaching as discussed by Ní Riordain & Hannigan (2009). Despite the large number of out – of – field teachers practising as mathematics teachers, the performance of mathematics teachers was successful and effective by the standards of the day. Because they saw themselves as being successful teachers they began to conform to the sociological idea ‘habitus’ proposed by Bourdieu (1980) (Wedge, 2011). This stance adopted by teachers means that due to perceived past success they saw no need to change their teaching habits. However in recent years mathematics has evolved together with our understanding of teaching and learning in mathematics. Firstly there has been a significant change in emphasis on mathematics and science whereby these subjects are now seen as instruments of economic policy. Consequently, this has led to a better understanding of the important role of school mathematics and science and it is now more critical than ever to ensure students graduate from secondary school with a good foundation in these two subjects. This can be facilitated through revised curricula and effective teaching. Secondly the introduction of Project Maths has called on mathematics teachers to engage in new teaching practices and to teach new content. These are practices and ideas which
teachers may not have encountered before, are ill prepared for and, due to perceived past success, do not believe necessary. As a result it is important that initiatives, such as this research project, are undertaken to guide teachers through the new curriculum and to help them develop the wider knowledge base and qualifications necessary to teach mathematics at second level through CPD in order to help the teaching profession evolve in line with the demands of mathematics education today.

1.3 Research Intent

The author’s qualification as a secondary school mathematics teacher as well as her research interests that began at Final Year Project stage as part of her undergraduate study kindled a strong desire to work to improve the quality of mathematics teaching at secondary level. Various reports point to a lack of emphasis on understanding, problem solving and applications and conceptual development and place the onus on the mathematics teachers for this state of affairs. This led the author to focus initially on improving teaching quality through improving teachers’ ability to deal with applications. The author brought these motivations and intentions with her when she joined the research team at the National Centre for Excellence in Maths and Science Teaching and Learning (NCE-MSTL).

The mission of the National Centre is to address issues of national priority in mathematics and science teaching. To this end the centre is concentrating efforts on supporting the national effort in Project Maths in a way that addresses core issues and adds value. The centre has identified inappropriate teaching as a systemic issue that could undermine the national effort (Ní Riordain & Hannigan, 2009). This points to the need to address the related issue of CPD for out-of-field teachers of mathematics.

1.4 The Problem

In light of priorities in mathematics education in Ireland, as reflected in various reports cited earlier, and her role as member of a research team at the University of Limerick, the author was motivated to engage in this area particularly in issues
related to improving the quality of mathematics teaching at second level. After an initial focus on the teaching of applications the author widened her study to related factors such as teacher knowledge(s) for effective mathematics teaching. While there was ongoing development work in this area of applications undertaken by the Project Maths Development Team there was no specific research in the knowledge requirement for such teaching and the wider issue of mathematics knowledge for teaching.

Further, the author was aware of the structural issues raised by Ni Riordain & Hannigan (2009) regarding the qualifications of teachers of mathematics in Irish secondary schools. Together with the Minister for Education and Skills figures, based on the Teaching Council’s data, which estimate that 35% of teachers of mathematics do not have mathematics in their degree (O’Keeffe, 2010), this points to a significant challenge related to subject matter knowledge and other teacher knowledge in the current cohort of teachers of mathematics. This motivated the author to move her research in the direction of teacher knowledges and Continuous Professional Development (CPD) as a way of addressing problems in the Irish system.

In Ireland however, there is a paucity of research in the area of teacher knowledge. While there are studies on teacher knowledge for primary school teachers (Delaney et. al., 2008; Hourigan & O’Donoghue, 2009) the author is aware of no specific research in mathematics teacher knowledge for secondary school mathematics teachers conducted in Ireland. It appears that most of the international studies also relate to primary rather than secondary education, for example Ball et al., (2005) and Rowland et al. (2007).

Teachers have been shown to affect students’ attitudes towards mathematics (Grouws & Cramer, 1989; Klinger, 2005; Beswick, 2006); their perceptions and expectations of the subject (Osborne et al., 1997) and their overall achievement in the subject (An et al., 2004; US National Mathematics Advisory Panel, 2008). Such research suggests that teachers influence students’ cognitive and affective development in the subject and students’ entire mathematical experience could be
improved if they were taught by effective teachers. In addition to this, a recent report conducted by Ní Riordain & Hannigan (2009: 4) stated that teacher quality is “…one of the most important factors affecting student learning”. In essence second level mathematics teachers play a defining role in preparing students for the many mathematical tasks and problems they will face in their everyday lives, at third level and in the workplace while also helping to foster positive attitudes and interest in the subject. Consequently, the challenge is to improve the standards of teaching in Ireland (NCCA, 2005). Only when greater numbers of more effective teachers are in place will we see meaningful and positive change in mathematics education.

The author’s research ideas and the needs of the centre converged and evolved to encompass the wider issues related to mathematics teacher knowledge. The author is convinced effective mathematics teaching is a key to the successful implementation of Project Maths and has taken up the challenge by addressing the issue of teacher knowledge. After due consideration the author focussed on:

- improving teacher knowledge
- dealing with serving teachers through CPD.

Internationally, over the last number of years research has repeatedly pointed to the need for teachers to have a deep understanding of the mathematical content which they teach in order to improve their teaching while at the same time enabling them to develop competency in other types of knowledge which they may also require for effective teaching including pedagogical knowledge and transformational knowledge (Carpenter et al., 1996; Cuoco, 2003; An et al., 2004). These ‘packages of knowledge’ have already been discussed at length, in an international context, by Shulman (1986), Ernest (1989), Fenema & Franke (1992), Rowland (2007) and Ball et al. (2008) but the author feels that these packages must be modified and adopted for use in an Irish setting.
1.5 Research Study

1.5.1 Purpose of the Study

The main purpose of this study is to improve the quality of teaching in Irish senior cycle mathematics classes. In order to achieve this, the project aims to identify and quantify the package of knowledge required by mathematics teachers in Irish secondary schools and improve their command of these knowledges by various means.

1.5.2 Objectives of the Study

The principal objectives of this project are:

1. To establish and clarify key issues in mathematics education internationally and in an Irish context with specific attention afforded to the areas of teacher knowledge and Continuous Professional Development.

2. To investigate the models of mathematics teacher knowledge proposed by researchers at both primary and secondary level with a view to possible use or adaptation in the Irish context.

3. To develop a model of teacher knowledge which has a strong focus on applications and that meets the needs of mathematics education in Ireland.

4. To develop, support, corroborate and validate the author’s model using various approaches including a ‘Proof of Concept’ approach.

5. To further investigate the model through a CPD intervention.

1.5.3 Research Design and Methodology

This is a mixed method study organised in three phases identified as Phases 1, 2 and 3. Phase 3 is further sub-divided into sub-phases 3A, 3B and 3C. An overview of the research phases involved in this doctoral study is presented in Figure 1.1, overleaf.
Improving Mathematics Teaching at Second Level Using an Intervention Aimed at Developing Teachers’ Subject Knowledge

Began with

Phase 1

Consisted of

A review of the literature on general issues in mathematics education

An investigation into the area of teacher subject knowledge

A comprehensive review of issues relating to the current provision of CPD

An analysis of the packages of knowledge required for mathematics teaching as proposed by researchers in the

Informed

Phase 2

Consisted of

Was developed, supported, corroborated and validated by

Phase 3

The design of the “Ladder of Knowledge” – a model of the knowledge required by Irish mathematics teachers

Phase 3A

Required the author to

Conduct focus groups in order to pilot the questionnaire/test to be used in phase 3B and to obtain teachers views on the knowledge required for mathematics teaching.

Phase 3B

Required the author to

Design and distribute a questionnaire to ascertain levels of knowledge among a sample number of teachers and to obtain their views on the importance of different knowledge domains for the purpose of mathematics teaching

Phase 3C

Required the author to

Design, implement and evaluate an innovative CPD intervention based on the Ladder of Knowledge

Figure 1.1: Research Design
Chapter 1 Introduction

Research Methodology

Phase 1 and 2 were the principal phases of this doctoral study while Phase 3 served to develop, support, corroborate and validate the previous phases. In Phase 1 the author carried out an extensive literature review. In Phase 2 the author designed the prototype model of teacher knowledge based on the literature review and the analysis of the five theoretical frameworks. The focus groups used in Phase 3A served to give feedback on the author’s model and to pilot the questionnaire used in the subsequent phase. Finally, in Phase 3B, the author administered the questionnaire and in Phase 3C she designed, implemented and evaluated the CPD intervention.

The methodology and methods chosen must best reflect and cater for the issues in the research context. In recent years the merits of both qualitative and quantitative research have been widely debated (Leeroy & Ormrod, 2005). The author combined two research paradigms, positivist (quantitative) and interpretive (qualitative) to pursue her research objectives (Leeroy & Ormrod, 2005) because of the advantages offered by this approach. The author chose to use a mixed method approach combining both qualitative and quantitative strategies. By making use of both quantitative and qualitative methods in this manner the author is subscribing to the idea of triangulation (Cohen et al., 2000). Furthermore, this multi method approach allows for valid and reliable research, as Cohen et al (2000: 112) found that one can only be confident that the data produced is unbiased and not simply ‘artefacts of one specific model’ when different methods give way to similar results.

The different methods and methodologies employed for the purpose of this research are detailed in Chapter 4.

1.5.4 Research Questions

The research questions associated with each of the phases are as follows:
Chapter 1
Introduction

Phase 1
- What are the main issues affecting mathematics education internationally and in Ireland?
- What role can teachers’ knowledge base play in improving the quality of teaching?
- Do teachers internationally have a knowledge base sufficient for the purpose of teaching and are third level courses adequately preparing prospective teachers for the task of teaching?
- What constitutes effective CPD and what are the characteristics common to CPD initiatives that have proven successful internationally with specific reference to mathematics teachers?
- What is the current state of CPD in Ireland and what problems, if any are being encountered?

Phase 2
- What models of teacher knowledge have been put forward by previous researchers and what common links exist between these?
- What issues need to be addressed in the design of a model of mathematics teacher knowledge for secondary mathematics teachers?
- How can the author adapt and develop a model of mathematics teacher knowledge that is fit for purpose in Irish secondary schools?

Phase 3
This phase is used to develop, support and corroborate issues relating to the author’s model and is divided into three sub – phases, 3A, 3B and 3C.

Phase 3A (Focus Groups)
- How do teachers perceive the author’s model of knowledge required for teaching?
- How do teachers perceive the adequacy of their current levels of knowledge in the domains outlined in the authors’ model?
- Which knowledge domains do teachers believe to be most critical for mathematics teaching?
Chapter 1  
Introduction

- Do teachers feel competent in all elements of the authors’ model of teacher knowledge?
- What changes do teachers feel could be made to the questionnaire which will be used in Phase 3B?

**Phase 3B (Questionnaire/Test)**

- How do teachers rate the types of knowledge domains outlined on the authors model?
- What are the current levels of knowledge in the model’s domains among this group of mathematics teachers
- What aspects of the model do these teachers find most problematic?
- Is there a difference between these teachers’ procedural understanding and their relational understanding?
- Are the knowledge bases of those teachers involved in this small scale study significantly more extensive than that expected of their students or is it limited to material covered in the textbook and on State Examinations?

**Phase 3C (Intervention)**

Design Questions:
- What knowledge domains must this CPD intervention focus on in order to be effective?
- How can these knowledge domains be integrated in an effective manner?

Implementation Questions
- What delivery method should be used when implementing this CPD intervention?
- What are the relative merits of different approaches to CPD delivery?

Evaluation Questions
- How effective was the CPD initiative?
- Did the teachers find the CPD intervention acceptable?
- What recommendations would teachers make for future work in this area?
1.6 Scope and Significance of the Research

Due to its perceived importance in society and the fact that it often acts as one of a number of gateways to third level education, mathematics is essentially compulsory for all students up to Leaving Certificate level\(^3\). As a result, the majority of young people in Ireland, over 96\%, are exposed to mathematics teaching up until they leave Second Level education (Chief Examiners Report, 2005). In order to ensure an annual supply of well qualified students of mathematics graduating from second level schools it is essential that these teachers be of a very high quality. Many recent reports investigating mathematics education in Ireland have recommended that a strong focus be placed on improving the standard of teaching and the knowledge of mathematics teachers both at primary and secondary level (NCCA, 2006; EGFSN, 2008; Innovation Ireland, 2010). However, despite major reports advocating the need to improve teachers’ levels of knowledge very little research has been undertaken to devise strategies for this and no action has yet been taken in this regard. As a result there is a significant gap in the research into mathematics education in an Irish context and it is this gap which the author has chosen to address in her research.

The problem of insufficient knowledge among teachers is one which research has shown to be extremely significant for the future of mathematics education. For several decades researchers have discussed the importance of a teacher’s knowledge base and the significant role which it can play in classroom proceedings and mathematical attainment. For example Conant (1963: 93) claimed that

“If a teacher is largely ignorant or uninformed he can do much harm”

while Fennema & Franke (1992: 148) found that:

“…one of the most widely offered explanations of why students do not learn mathematics is the inadequacy of their teachers’ knowledge of mathematics”

Further research carried out by the US National Mathematics Advisory Panel (2008) found there to be a positive correlation between the mathematical knowledge of mathematics teachers and students’ achievements. As a result of such findings it is accepted that a teacher’s knowledge base is of paramount importance and without an

\(^3\) It is not compulsory to take mathematics as part of the LCVP course (See Chapter 3) but it is highly recommended. As a result in 2005 only 4\% of all Leaving Certificate students did not take mathematics at this level (Chief Examiners Report, 2005)
extensive knowledge base it is unlikely that effective mathematics teaching will be possible. Figure 1.2, below, summarises the significance of this project for Irish education.

**Figure 1.2: Significance of the Project**

- Leads to Poor knowledge base among teachers
- Leads to Inadequate and ineffective teaching
- Leads to Poor understanding among students
- Leads to Poor uptake of higher level mathematics and poor performance in mathematics

Due to the impact that a teacher's knowledge can have on their ability to teach effectively, this project has potential to lead to improvements in mathematics education at secondary level in Ireland precisely in those priority areas of attainment, uptake, interest and preparedness, as Figure 1.3 overleaf highlights.
It is anticipated that this project will help improve the current number of students opting for higher level mathematics at Senior Cycle level. Currently, these numbers are low; in 2010 16.04% of students who sat mathematics chose to sit the higher level paper (www.examinations.ie) but the NCCA state that this figure needs to be closer to 30% (NCCA, 2005). Although bodies such as Engineers Ireland (2010) have said that this target may be too ambitious in the short term the author is convinced that if students are exposed to more knowledgeable and more effective teachers then such targets may be reached sooner than people have anticipated.

Similarly, research has also pointed to a problem in Ireland in relation to the underpreparedness of students on entry into third level. A recent study carried out by Gill et al. (2010) in the University of Limerick found that the number of students entering third level, unprepared for the mathematics course they will face, is increasing and of the 2008 cohort 46.2% of students, an increase of 16.2% from 2003, will struggle with third level mathematics. Again the author’s work will contribute to improving the quality of teaching that such students are exposed to at second level resulting in
students graduating from second level who possess a better understanding of mathematics.

Finally the author chose to address this problem through Continuous Professional Development (CPD) and this too was a conscious and significant decision on her part. It would be imprudent for anyone to think that prospective teachers would be provided with all the knowledge, resources or ideas that they need for the duration of their teaching career during the 3 – 5 years spent in third level institutions. Just as athletes must spend numerous hours in the gym improving and refining their skills so too must teachers engage in regular CPD in order to maintain high standards of teaching (Barton, 2008). Furthermore, Resnick (2005) presented evidence to show that CPD which focuses on developing a teacher’s mathematical knowledge base can have a significantly positive knock on effect on student learning. This need for, and importance of, CPD was one of the main reasons why the author chose to address the problem in this manner. In addition to this, figures show that there are significantly more teachers in service than teachers entering service in Ireland at secondary level. Therefore by seeking to address the problem through CPD this project will reach a larger number of teachers earlier.

### 1.7 Theoretical Framework

In the past, many researchers have found the area of teacher knowledge to be under – researched (Shulman, 1986; Askew & Williams, 1995). For example Shulman (1986) found that most of the research in mathematics education focussed on questioning, direct instruction and timing issues as opposed to teacher knowledge. However, recent years have seen a surge of interest in the area of teacher knowledge and many frameworks of teacher knowledge have been identified internationally. The complex nature of a teachers’ knowledge base resulted in the author combining five different models of knowledge which would act as a theoretical framework that would support the author’s research. These models are listed overleaf:
1. Shulman (1986); A Perspective on Teacher’s Knowledge
2. Ernest (1989); A Model of Mathematics Teacher’s Knowledge, Beliefs and Attitudes
3. Fennema & Franke (1992); Components of Teacher Knowledge
4. Rowland (2007); The Knowledge Quartet
5. Ball et al. (2008); The Egg Framework.

Each of these frameworks offers a valuable insight into the knowledge domains required for effective teaching and provide the author with a foundation on which her own model can be built. Each framework is discussed in detail in Chapter 6 but Table 1.1, below, briefly outlines the main features in each of the theoretical frameworks employed in this project.

<table>
<thead>
<tr>
<th></th>
<th>Mathematics Specific</th>
<th>Strong Focus on Content Knowledge</th>
<th>Discusses the interrelated nature of teachers knowledge base</th>
<th>Specifically includes a knowledge of applications</th>
<th>Number of Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shulman (1986)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>3</td>
</tr>
<tr>
<td>Ernest (1989)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>6</td>
</tr>
<tr>
<td>Fennema &amp; Franke (1992)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>4</td>
</tr>
<tr>
<td>Rowland (2007)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 1.1: Features of the Theoretical Frameworks on which the Research is Based
1.8 The Proof of Concept Approach

The main purpose of this study is to improve the quality of teaching in Irish senior cycle mathematics classes. The author has concentrated on developing a knowledge construct for teacher knowledge, namely the “Ladder of Knowledge”, incorporating applications, among other elements, as a solution to problems in effective mathematics teaching. Having analysed the models in the literature and constructed a model which the author felt would serve her purposes it was important that such a model be tested with teachers. Due to the small number of teachers willing to participate in the third and final phase of the project the author adopted a Proof of Concept approach. Similar to a manufacturer designing a new product the author wanted to ensure that the consumer (i.e. teachers) would accept and engage with the model which she designed. Proof of concept has been employed for many years now in the fields of medicine, engineering and product design. Literature in this area offers a number of definitions of this approach, for example:

“…proof of concept can be viewed as studies that demonstrate the clinical relevance of a novel mechanism to treat a disease”  
(Vose, 2001: 6)

“A proof of concept can refer to a partial solution that involves a relatively small number of users acting in business roles to establish whether the system [product] satisfies some aspects of the requirements”  
(www.primatex.ca/blog/definitions/)

For the purpose of this project the proof of concept approach is defined as:

“…the use of evidence which demonstrates that a model or innovative approach is viable, feasible and capable of solving or diminishing a particular problem”  
(Ferry et al, 2010: 2)

This “Proof of Concept” approach led the author to validate the model using a case study approach and all results are therefore indicative of the thoughts and opinions of Irish second level mathematics teachers in this regard.
1.9 Limitations of the Study

The author recognises that there are limitations to this study. One such limitation is the cycle of under preparedness (Furinghetti, 2000) can be counteracted from two different angles – during pre – service undergraduate education and through CPD. However, the author has decided to focus on CPD. The author believes it would be impossible due to time constraints to develop initiatives from both perspectives and, as the author has already discussed, CPD is preferred as there are more in service teachers than pre service. This approach will, therefore, benefit the majority while at the same time it is anticipated that if the levels of knowledge among practicing teachers can be improved then those entering the profession may learn from their senior colleagues.

Another problem, which is discussed in greater detail in Chapter 4, is the poor response rate obtained during phase 3B of the project. This poor response rate resulted in the author having a small pool of subjects and this means that the findings in this phase must be treated as indicative of Irish teachers views.

Finally, the author used a convenience sample of students to get an indication of second level students’ perspective on the Calculus Tool Kit. The convenience sample consisted of a group of students attending the MACSI Mathematics Summer School which was run in the University of Limerick from June 21st – June 25th 2010. The response received from this distinct, and possibly elite, group of students are not representative of the entire student population. However, findings from this convenient sample do add to our overall understanding of the effectiveness and appropriateness of the Calculus Toll Kit.

1.10 Terms Introduced throughout this Study

*Junior Cycle (Lower Secondary School)*: Post primary education in Ireland is divided into two cycles. The first of these two stages is Junior Cycle. Students generally enter Junior Cycle between the ages of 12 and 13. It is a three year programme that builds on the knowledge and skills developed by students during their primary school experience while at the same time preparing them for the transition to Senior Cycle.
Senior Cycle (Upper Secondary School): Senior Cycle is the second phase of post primary education in Ireland. Students tend to enter this cycle between the age of 15 and 16. The cycle includes an optional transition year in the first year of Senior Cycle and so this cycle can be either of two or three year’s duration. The main role of Senior Cycle education in Ireland is to prepare students for adult life and the tasks and challenges they will face at third level or in the workplace.

State Examinations (Junior Certificate and Leaving Certificate): Both Junior and Senior Cycle culminate with a state examination in each of the topics studied by students. For the Junior Certificate students generally study between ten and twelve subjects of the twenty eight subjects that are on offer (www.citizensinformation.ie). Students are provided with results in the form of grades and the percent range which each grade represents at Junior Certificate level is outlined in Table 1.2.

<table>
<thead>
<tr>
<th>Grade</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>No Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>% Range</td>
<td>85 - 100</td>
<td>70 - 84</td>
<td>55 - 69</td>
<td>40 - 54</td>
<td>25 - 39</td>
<td>10 - 24</td>
<td>0 - 9</td>
</tr>
</tbody>
</table>

Table 1.2: Percentage range for Junior Certificate Grades

The Leaving Certificate requires students to study a minimum of five subjects (but the majority of students study seven) from a possible thirty four. Again students receive their results as a grade but this time each grade is divided into two or three sub grades. The percent range which each grade represents at Leaving Certificate Level is outlined in Table 1.3, overleaf.
<table>
<thead>
<tr>
<th>Grade</th>
<th>% Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>90 - 100</td>
</tr>
<tr>
<td>A2</td>
<td>85 - 89</td>
</tr>
<tr>
<td>B1</td>
<td>80 – 84</td>
</tr>
<tr>
<td>B2</td>
<td>75 – 79</td>
</tr>
<tr>
<td>B3</td>
<td>70 – 74</td>
</tr>
<tr>
<td>C1</td>
<td>65 – 69</td>
</tr>
<tr>
<td>C2</td>
<td>60 – 64</td>
</tr>
<tr>
<td>C3</td>
<td>55 – 59</td>
</tr>
<tr>
<td>D1</td>
<td>50 – 54</td>
</tr>
<tr>
<td>D2</td>
<td>45 – 59</td>
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<tr>
<td>D3</td>
<td>40 – 44</td>
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<td>E</td>
<td>25 - 39</td>
</tr>
<tr>
<td>F</td>
<td>10 – 24</td>
</tr>
<tr>
<td>No Grade</td>
<td>0 – 9</td>
</tr>
</tbody>
</table>

Table 1.3: Percentage range for Leaving Certificate Grades

*Higher Level, Ordinary Level and Foundation Level:* In Ireland mathematics at both Junior and Senior Cycle is offered at three levels; higher, ordinary and foundation. All three levels of mathematics at Junior Cycle level cover a wide range of topics. The higher level course (also known as honours) covers topics at a deeper level and requires students to have a more thorough understanding of the mathematical concepts involved than that expected of ordinary level students. Foundation is the lowest of the three levels and mathematics is only one of three
subjects to offer this level at Junior Cycle while mathematics and Irish are the only subjects which offer it at Senior Cycle.

*Leaving Certificate Points System (CAO)*: The Central Applications Office (CAO) has been delegated the responsibility of processing most applications made for first year undergraduate programmes in all third level institutes in Ireland. Students coming towards the end of Senior Cycle education who wish to apply for places in undergraduate courses do so through the CAO rather than applying directly to the university or institute. Once students meet the entry requirements for the course and there are places available on the course of their choice they will be offered a position. However, on many occasions there are more applicants for courses than there are places. In such instances the CAO place eligible applicants in an order of merit and places are offered to students with the highest CAO points. Points are awarded to students based on their performance in the Leaving Certificate exam. Table 1.4, overleaf, outlines the points awarded for each individual grade at both higher and ordinary level. A students total points are calculated by adding the points attained in their top six subjects (Maximum points = 600)
Chapter 1

### Introduction

#### Table 1.4: Guide to Leaving Certificate Points System

<table>
<thead>
<tr>
<th>Higher Level</th>
<th>Ordinary Level</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Grade</strong></td>
<td><strong>Points</strong></td>
</tr>
<tr>
<td>A1</td>
<td>100</td>
</tr>
<tr>
<td>A2</td>
<td>90</td>
</tr>
<tr>
<td>B1</td>
<td>85</td>
</tr>
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<td>B2</td>
<td>80</td>
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<td>B3</td>
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<td>C3</td>
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<td>D1</td>
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<tr>
<td>D2</td>
<td>50</td>
</tr>
<tr>
<td>D3</td>
<td>45</td>
</tr>
</tbody>
</table>

| Project Maths: | Project Maths involves the introduction of a revised mathematics syllabi for both Junior and Senior Cycle. The main aim of Project Maths is to enhance levels of mathematical understanding among students and it will focus on developing students’ problem solving skills. It seeks to raise awareness of the relevance and applicability of mathematics so that second level students will begin to see its importance in their everyday life. It is hoped that the revised syllabi will lead to higher levels of achievement in mathematics in Ireland and an increase in the number taking higher level mathematics at both Junior and Senior Cycle. Project Maths was piloted in 24 schools starting in September 2008 and the first two strands (Statistics & Geometry and Trigonometry) will be introduced nationally in 2010. |
1.11 Outline of Chapters

In this chapter the background to the study was presented and the author discussed its significance, the research questions which guided the study and the three phase approach adopted during this study. An outline of the remaining chapters in the thesis follows.

Chapter 2: is a review of the current literature on the problems facing mathematics education from both an international and Irish perspective. The review encompasses the ramifications of the previously discussed problems and analyses possible causes and remedies. This chapter concludes with the author identifying teacher content knowledge as the root cause of a number of the identified problems and looks at the impact that this can have on the standard of mathematics teaching and learning.

Chapter 3: explores the role of teachers’ subject knowledge and Continuous Professional Development in changing practices in mathematics classrooms. The chapter identifies the different types of knowledge required for mathematics teaching, the current levels of knowledge of mathematics teachers and problems with teacher training courses. Issues in relation to the importance of CPD for improving teachers’ levels of knowledge as well as what constitutes effective CPD are also examined.

Chapter 4: provides the reader with an overview of the research methodology used in the process of addressing the phenomenon of mathematics teachers’ subject knowledge. It details the research design, methodologies and research tools employed in this research study. Data collection methods are considered and research instruments are discussed. The relevant issues of sampling, ethics, reliability, validity and triangulation in relation to the development and administration of the various methods used in the three phases are explained as appropriate and the data analysis process is discussed.
**Chapter 5:** presents the five theoretical frameworks used to frame this research study. An analysis of each of the five models is presented and the need for a new model of teacher knowledge is discussed. Finally a new model of teacher knowledge suitable for Irish mathematics teachers is offered and this model highlights the knowledge domains necessary to teach second level mathematics in Ireland. A detailed discussion of the model follows where the author discusses the different knowledge domains and justifies the inclusion of each.

**Chapter 6:** provides the reader with findings from the qualitative and quantitative study. Initially the qualitative and quantitative findings are analysed in isolation before comparisons between the two are identified. The chapter details teachers opinions of the model discussed in Chapter 5, their perceptions of their own levels of knowledge as well as the current levels of knowledge displayed by teachers in this study. The impact of such findings on students is also discussed.

**Chapter 7:** focuses explicitly on the design, implementation and evaluation of an innovative CPD initiative. It provides details on the three critical stages of an educational intervention and examines teacher’s and student’s reactions to the different elements of the intervention.

**Chapter 8:** concludes the thesis with a re-examination of the findings of the research, a summary of the local, national and international contributions’ of the study. Recommendations and suggestions for future work are also offered.
2. General Issues in Mathematics Education

2.1 Introduction

The purpose of this chapter is to identify problems that are associated with mathematics education at second level both in Ireland and abroad. Research has shown that these problems have had numerous knock on effects at third level while at the same time they have contributed to economic problems in the affected countries. Thus it is important to also look at these effects and to provide insights into the ramifications of these problems. Finally it is necessary, once the problems and the implications have been identified and analysed, to uncover some of the potential causes of these problems and to look at what is being done to rectify these issues as well as ways in which this doctoral thesis can help remedy some of the problems that we currently face in mathematics education today.
Chapter 2 General Issues in Mathematics Education

2.2 The History of Mathematics Education-An Irish Perspective

The Established Leaving Certificate was introduced in Ireland in 1924. In 1992, fundamental changes were made to the Leaving Certificate mathematics syllabus but the style of the syllabus did not change from that introduced in the 1960’s when mathematical reform was occurring worldwide (Gjone, 2003; NCCA, 2005). The mathematical topics covered by the higher and ordinary levels of this syllabus were wide-ranging. These topics included algebra, trigonometry, geometry, sequences and series, functions and calculus and discrete mathematics and statistics while the ordinary level course also covered some elements of arithmetic. Both these courses also allowed students/teachers to choose one topic from four to study as their ‘option question’. These topics for higher level included further probability and statistics, further calculus and series, groups and further geometry while the topics in this section on the ordinary level course included further geometry, plane vectors, further sequences and series and linear programming. By the late 20th Century researchers began to identify problems with this syllabus and many concluded that the overcrowded curriculum resulted in teachers focussing on recall and routine procedures and in turn students were relying heavily on rote memory and special purpose algorithms (O’Donoghue, 1999; Hourigan & O’Donoghue, 2007). Mathematics education in Ireland was no longer serving its intended purpose as it was failing to prepare students for the mathematics they would encounter outside of school. Change was required and the publication of The Review of Mathematics in Post Primary Education – A Discussion Paper (2005) and The Review of Post Primary Mathematics (2006) resulted in proposals for a reformed curriculum, namely Project Maths.

2.2.1 Project Maths

According to the Project Maths Implementation Support Group (2010), Project Maths endeavours to teach mathematics in a way that will promote real understanding. The NCCA (2009) continue by saying that under this new programme Leaving Certificate mathematics will strive to develop students’ mathematical knowledge, skills and understanding in an effort to prepare them for continuing their education or for the challenges they will face in the work place.
They also see Project Maths as a vehicle that can promote the beauty and power of mathematics in students’ everyday lives. The main objectives of Project Maths as outlined by the Project Maths Implementation Support Group (2010) are to:

- place a greater emphasis on the understanding of mathematical concepts and the application of mathematical skills and knowledge;
- contribute to the development of higher order thinking skills, including logical reasoning and problem solving;
- provide a solid foundation which prepares students for careers in science, technology, engineering, business or humanities;
- increase the use of context and applications so as to enable students to relate mathematics to everyday experiences.

In essence, the initiative aims to strike a balance between developing an understanding of mathematical theory and developing an ability to apply mathematics to practical situations among students. In summary, the main differences between the old Leaving Certificate mathematics syllabus and the new Project Maths syllabus are summarised by Engineers Ireland (2010: 48) and these differences are outlined in Figure 2.1 overleaf.
The initiative commenced in September 2008 in 24 pilot schools and was rolled out nationally in September 2010. In June 2010 students in the 24 pilot schools completed the first State Examination of this new syllabus. Under this new syllabus a number of strands that appeared in the old syllabus have been amalgamated while others have been omitted completely. As a result there are now five main strands to be studied at both Junior and Senior Cycle. These are:

- Strand 1: Statistics and Probability
- Strand 2: Geometry
- Strand 3: Number
- Strand 4: Algebra
- Strand 5: Functions.

The timeline for implementation of these strands, at both Junior and Senior Cycle is highlighted in Figure 2.2 overleaf:
It is anticipated that this undertaking by the NCCA will change the face of mathematics education in Ireland in the future and will lead to a more mathematically proficient workforce in Ireland. However it is worth noting that there is no guarantee that Project Maths will succeed. It can only succeed if mathematics teachers and other stakeholders in mathematics education work to ensure that it is successfully implemented then students, teachers, schools and the economy will experience the anticipated benefits.

Such reform will prove challenging for teachers. For example this initiative will require teachers to extend their knowledge base and alter their teaching strategies in order to achieve the aims and objectives outlined previously. There is also an issue with the timetable for introduction, the acceptance of a teaching for understanding approach and new types of assessment which makes it difficult for teachers to embrace. While the CPD provision is exceptional in the Irish context it would be seen as relatively ordinary in an international context and this in turn is likely to lead to problems in the implementation phase. It is essential, therefore, that teachers are
well supported during the early years of this initiative through Continuous Professional Development (Project Maths Implementation Support Group, 2010). Only then will teachers be able to overcome the challenges which Project Maths presents and furthermore only then will the true benefits of Project Maths, such as an appreciation of the subject of mathematics and an improved understanding of mathematics among students, be experienced.

2.3 Problems Facing Mathematics Education Today

Despite advancements, such as Project Maths, in mathematics education around the globe those involved in the field of mathematics education recognise that there are many problems that still must be addressed. Many of the problems discussed in the upcoming sub-sections have been in existence for many years yet they still continue to affect the standard of mathematics teaching and learning today.

2.3.1 Failure to Appreciate Mathematics in Today’s Society

“*There is no branch of mathematics, however abstract, which may not someday be applied to phenomena of the real world*”

(Glaister & Glaister 2000: 1)

This quote highlights the importance of mathematics in today’s society. In the changing environment in which we live, mathematics is becoming more important everyday. The director of ICT\(^4\) Ireland, Kathryn Raleigh (2006), reiterates this sentiment when she states that mathematics is currently more important than ever before in relation to students’ success outside of a school setting. Researchers also highlight how a wide range of everyday tasks require the use of mathematics to some degree, for example managing/exchanging money, saving money, stock control, increasing task efficiency, reading maps as well as time management (London Mathematical Society, 1995; Alsina, 2002). Mathematics, therefore, has become central to almost everything we do in life while at the same time it has the ability to change the world in which we live for generations to come. In terms of the

\(^4\) Information Communication Technology (the overall ICT policy development arm of IBEC)
importance of mathematics beyond the school years, Ball (2001) states that despite
the increasing importance of technology in society, high levels of mathematics will
continue to be required in the work place. Smith (2004) also found mathematics
receives more recognition in the work place and he fears that this will result in
school – leavers, who struggle with numeracy, finding it more difficult to secure
employment and they may even face ‘social exclusion’. Finally, according to
Brabazon (2010) a strong focus on mathematical literacy is required in Ireland today
in order for us to become the ‘Innovation Island’ of tomorrow.

In addition to the importance of mathematics in the workplace, mathematics also
feeds into a wide range of other subjects that students will study during their school
or college years. Research points to subjects such as science, engineering, social
sciences, biology, business and economics that rely heavily on a good knowledge
and understanding of some basic mathematical concepts (London Mathematical
Hence, mathematics will be called upon to succeed in a number of school subjects,
further emphasising the crucial role it plays in society today and in helping young
people live successful and fulfilling lives. Niss (2006) goes as far as saying that
mathematics can in fact empower students to enter society as competent, active and
critical members of the community.

Despite the importance of mathematics, research carried out in Northern Ireland by
Eaton & Bell (2006) found that students did understand the usefulness of
mathematics to some extent but they failed to see the importance or relevance of
some of the more complex aspects of the mathematics curriculum, such as
Pythagoras’ Theorem, which they studied in post primary school. Similarly other
researchers found students believed that mathematics was important due to the value
that was placed on it by society, particularly employers, but at the same time these
students failed to see where they might need or use many of the concepts learnt in
school (Frid & White, 1995, Smith, 2004).

However, the findings of Glaister & Glaister (2004) highlight that teachers are
unaware of many of the possible applications of mathematics in society while
Engineers Ireland (2010: 37) found that as a result of this lack of knowledge,
students are only being prepared to pass mathematics examinations and are not
“...given an understanding of mathematics or of how to apply mathematical concepts”. Such findings indicate that neither teachers nor students understand or appreciate the impact that mathematics can have on so many different aspects of their lives. This lack of awareness on the part of teachers and the subsequent failure to share such information with students could leave many impressionable teens bewildered and uninterested in a mathematics course that they fail to see the importance of. As a result Metje (2007) notes that educators now have a responsibility to make mathematics more relevant and applicable to students everyday life and they must show them how our ever-growing and competitive global environment relies heavily on mathematics. By fulfilling this responsibility teachers may help students to appreciate mathematics and as they begin to see the relevance and usefulness of the subject, students’ attitudes towards the subject may in turn improve. At the moment, however, the poor attitudes of students towards mathematics is another problem facing mathematics education.

2.3.2 Poor Attitudes towards Mathematics

“...there is considerable evidence to support the idea that learners’ attitudes, beliefs and feelings about mathematics and their confidence (or lack of it) in their own mathematical abilities have an effect on their learning”

(Cohen, 2003: 92)

In his recent work, Cohen found that attitudes and the entire affective domain can play a very influential role in the learning of mathematics. In addition to this, Philippou & Christou (1998) acknowledged that attitudes can be vital in determining the teaching methods or practices adopted by teachers. Thus, the affective domain plays a part in both the teaching and learning of mathematics in today’s education system and so this section is devoted to gaining a deeper understanding of the attitudes of students and teachers, how they are developed and how they affect classroom proceedings and learning. But firstly what do we mean when we talk about attitudes? Attitudes are defined as a complex mental state which are dependent on beliefs, feelings and dispositions to act in a certain way (www.wordnet.princeton.edu/perl/webwn). Attitudes are complex issues and so
further investigation is needed on the part of all those involved in the education system to fully understand them and the role they play.

In recent years many studies including those carried out by Frid & White (1995) and Eaton & Bell (2006) have found very negative attitudes exist among students towards mathematics. Some words used to describe mathematics in these studies include dull and boring as well as complaints about an ‘overcrowded’ curriculum. Ball (2001: 18) emphasises that such negative attitudes towards mathematics are both “widespread and socially acceptable” while research in Ireland points to the prevalence of negative attitudes towards mathematics and discusses how these have led to people now finding it acceptable not to be good at the subject (NCCA, 2005). According to Smith (2004) there also exists a widespread belief that mathematics demands much more work than other GCSE subjects and such views also contribute to negative attitudes among parents and students. These findings were also echoed by researchers in Ireland. Both English et al. (1991) and the NCCA (2005) found that many students hold negative attitudes towards mathematics as they believe it to be a difficult subject only suitable for academically able students. These findings are a cause for concern for education systems as it provides schools, teachers and pupils with little hope of ever progressing or succeeding in mathematics.

Also Philippou & Christou (1998) argue that students entering primary school usually do so with a positive attitude towards mathematics and it is during their time in school that these attitudes begin to deteriorate. This is worrying as it means that school mathematics, for many students, is having a detrimental effect on their learning, development and general feelings towards the subject. Over the past number of years researchers have identified a critical factor that often contributes to the development of negative attitudes on the part of students, namely levels of attainment.

The Link between Attainment and Attitudes
Metje (2007) discusses how achievement to date, especially failure, can help determine an adolescent’s attitude towards a subject. She acknowledges that if students get caught up in the failure cycle (See Figure 2.3 overleaf) then some of the main results will be low confidence in the subject, a fear of the subject and an overall
bad attitude, and in extreme cases an avoidance of the subject. On the other hand Frid & White (1995) note that the success cycle, Figure 2.4, below, contributes to the development of interest in mathematics on the part of students and as a result a more positive attitude towards the subject.

Figure 2.3: Failure Cycle in Mathematics (Metje, 2007)

Figure 2.4: Success Cycle in Mathematics (Frid & White, 1995)
Figures 2.3 and 2.4 highlight what Metje (2003) and Frid & White (1995) refer to when they propose that past achievement/attainment can have an effect on students’ attitudes towards mathematics. These diagrams also emphasise the effect attitudes can have on students’ achievement. It has been debated for many years now whether poor attitudes lead to poor results or whether it is poor results that lead to poor attitudes. Due to the diagrams’ cyclical nature, researchers have concluded that neither one can be identified as the cause or effect but what is important to take from both the success and failure cycles is that attitudes can be affected by achievement levels while future success in the subject could also be dependent on these attitudes. As a result, teachers need to be aware of the existence of the failure and success cycles and where possible try to break the failure cycle and guide students through the success cycle in order to address the issue of negative attitudes among students (Metje, 2007).

Attainment levels can significantly contribute to the negative attitudes toward mathematics which students currently possess. However this is not the only contributing factor and the next factor which the author will analyse is often one of the most critical while at the same time it is the one most often overlooked. It is the impact of a teacher’s attitudes on his/her students’ attitudes.

**The Impact of Teachers’ Attitudes on Those of their Students**

The quality of teaching to which students are exposed impacts upon their learning. This section looks specifically at teachers’ attitudes and the influential role that they can play in relation to the development of students’ attitudes. Research has found that teachers’ attitudes help to determine students’ attitudes. According to Phillipou & Christou (1998: 190):

> “Teachers’ beliefs about mathematics and mathematics teaching play a significant role in shaping their instructional practice and consequently influence their pupils’ attitudes, interests and achievements”

Thus the attitudes held by teachers are crucial to the development of students’ attitudes and this in itself has presented many problems. One of the first issues is the belief that success at mathematics depends on innate ability. According to the NCCA (2005) many teachers in Irish classrooms currently believe that mathematics skills
cannot be developed or improved through schooling and instead mathematics requires an innate ability. Lyons et al (2003) also argue that teachers are entering classrooms worldwide believing that ‘innate ability’ has a role to play in the study of mathematics. If teachers entering a classroom firmly believe that natural ability is one of the key attributes required to succeed at mathematics then they are immediately neglecting a proportion of their class who may be struggling, but with some guidance could succeed in the subject. In 1999, the Californian Department of Education recognised this problem and realised that such myths could become even more widespread among teachers unless attempts were made to eradicate them. As a result in the mathematical standards which they devised they state in their introduction that aptitude in mathematics is not an innate characteristic and instead it is accomplished through perseverance, effort and practice on a students’ part and through careful and effective teaching on the part of teachers. It can only now be hoped that more statements of this nature are made in the coming years and that teachers, and in turn students, will develop more positive attitudes towards the subject than currently exist.

Research has also found that teachers can affect the attitudes of their students through their own teaching. If teachers fail to use clear explanations and a pace with which students are comfortable then this could also affect students’ attitudes (Lyons et al., 2003). Specifically, Lyons et al. (2003) found students could develop negative attitudes when the teacher moved too fast without satisfactory explanations. As a result teachers must ensure that they are capable of explaining mathematical content in a way that students can understand and relate to.

Finally, Philippou & Christou (1998) paint a bleak picture for the future of mathematics in relation to teachers’ attitude. There is evidence in their work to show that internationally teachers are entering this profession with little or no motivation and a negative attitude towards the subject, an issue which they envisage will persist in the future. Boero et al. (1996) also highlight this trend as they found only a small proportion of teachers enter the profession with a positive attitude towards mathematics while the vast majority dislike both the profession of teaching and the subject of mathematics! This will have a knock on effect on young people as a teacher with a poor attitude towards the subject will not inspire students to appreciate the subject of mathematics and so these teachers are influencing students and
potential future teachers to develop negative attitudes towards mathematics. This in turn can become a vicious cycle and one that will be difficult to break in the future (Phillipou & Christou, 1998). Also as the author outlined previously if teachers’ negative attitudes do lead to negative attitudes on the part of their students then this in turn could detrimentally affect their levels of attainment in mathematics. This is an issue which is already a cause for concern, as the author now highlights.

### 2.3.3 Current Levels of Attainment in Mathematics

This section looks at the issue of mathematical attainment both from an international and an Irish perspective.

**Current Levels of Attainment in Mathematics Internationally**

Levels of achievement in mathematics are a way of looking at a country’s success rate in mathematics education. In the British Isles, the current levels of attainment are worrying. The 1994 SOEID Assessment of Achievement Programme [AAP], as quoted in Macnab (2000), thoroughly investigated this issue in a Scottish setting and they reported that the mathematical results of students aged between nine and fourteen had been in decline since 1988. More recently the 2003 TIMSS Report also identified this trend in 4th grade students when they found that the average mathematics scale score in Scotland fell from 493 in 1995 to 491 in 2003. Overall these reports combined emphasise that mathematical achievement in Scotland has been in decline since 1988.

In an English context the TIMSS Report is more positive as it states that England’s average score is up 47 points over the same eight year period while the OECD (2010) shows that England’s mean score in mathematics is not statistically different from the OECD average. The LMS (1995) also found a significant improvement in relation to grades achieved. For example 10% of A - level students received a grade A in 1986 while this figure rose to 25% by 1994. Small increases have been reported since 1994 and by 2010 27.61% of students studying A-Level mathematics received an A grade ([www.furthermaths.org.uk](http://www.furthermaths.org.uk)). At face value these findings appear to be encouraging for mathematics educators in England but the LMS (1995) acknowledge
that improvements such as these can be attributed to a devaluation of grades rather than any improvement in the teaching and learning of mathematics. Smith (2004) also argues that, despite these findings, mathematics achievement is still extremely low when compared to other core subjects. His findings are detailed in Table 2.1, below.

<table>
<thead>
<tr>
<th>Subject</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>71.8</td>
<td>75.6</td>
<td>76.3</td>
</tr>
<tr>
<td>English</td>
<td>94.1</td>
<td>93.7</td>
<td>94.2</td>
</tr>
<tr>
<td>History</td>
<td>93.5</td>
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<td>89.6</td>
<td>90.1</td>
</tr>
<tr>
<td>Physics</td>
<td>86.3</td>
<td>83.7</td>
<td>82.5</td>
</tr>
</tbody>
</table>

Table 2.1: AS Overall Percentage Pass Rates for 17 year olds in England (Smith, 2004:60)

Table 2.1 shows the percentage of students that passed each of the five subjects in the given years. It highlights the fact that although attainment rates in mathematics appear to be improving currently there is a larger percentage of students succeeding in other subjects, such as English and Physics. This problem needs to be addressed in order to dispel the notion that mathematics is one of the more difficult subjects and so allow more positive attitudes to emerge and an eagerness on the part of students to engage in the subject.

Internationally, away from the British Isles, the 2003 TIMSS Report shows that levels of attainment fluctuate from country to country. Hong Kong and Latvia have seen the greatest improvements in mathematics scores both for 4th grade students and 8th grade students between 1995 and 2003. On the other hand the Nordic countries of Sweden and Norway as well as Bulgaria have witnessed the severest decline in mathematical performances with the latter falling from an average score of 527 in 1995 to 476 in 2003. Reasons including the time allocated to mathematics at second level and the duration of teacher training courses have been cited as reasons for these fluctuating results and both these issues will be analysed at a later stage. These
negative findings have numerous repercussions for the society and the economy in the affected countries as well as on third level institutions.

**Current Levels of Attainment in Mathematics in Ireland**

“...poor performance in Maths can exclude students from almost all third level courses in universities and institutes of technology [in Ireland]”

(Donnelly, 2007)

This quote, from an article in the Irish Independent, underlines the importance of achieving good grades in Senior Cycle mathematics. In Ireland, entry requirements for the majority of colleges require students to obtain at least a grade D in ordinary level mathematics\(^5\) (Healy, 2003). This highlights just how important Senior Cycle mathematics is and the important role good grades can play in deciding the future career choices of many students. At present, levels of attainment for both higher and ordinary level appear to be improving but when we investigate these issues further it is apparent that many issues still exist in this area of mathematics education.

**Recent Changes in Attainment in Higher Level Mathematics**

The NCCA discussion paper (2005) presents a positive picture of the current levels of achievement in higher level Senior Cycle mathematics education. Despite a drop in the number of students choosing the higher level paper, which the author will look at in section 2.4, Table 2.2, overleaf, highlights an increase in the overall percentage of students obtaining A and B grades while a drop has also been witnessed in the number of students receiving a grade D or below in the period 2000 - 2010.

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\(^5\) Many colleges require students to obtain at least a C3 grade on the ordinary level paper while some courses with high mathematical content require students to obtain a C3 grade in higher level mathematics.
Table 2.2: Percentage of Candidates Achieving Various Grades in Higher Level Leaving Certificate Mathematics 2000 - 2010

Table 2.2 highlights how the number of students receiving an honour (Grade A/B/C) has risen from 74.2% in 2000 to 77.7% in 2010. Furthermore the percentage of students who passed the higher level Leaving Certificate mathematics (obtained a D3 or higher) rose from 94.9% in 2000 to 96.3% in 2010.

Overall the current levels of attainment at higher level are encouraging and if this trend continues it may encourage more students to study mathematics at this level in Senior Cycle as they will begin to understand that mathematics is not only for an elite group of advanced students. However the situation does not appear as positive for students opting for the ordinary level course.

Recent Changes in Attainment in Ordinary Level Mathematics

The Chief Examiner’s Report (2001) initially presents a worrying scenario in relation to levels of achievement in ordinary level mathematics. Their findings indicate a drop in the percentage of students receiving ‘high grades’ (i.e. A, B, C) from 67.5% in 1999 to 62.1% in 2001. In addition to this, the percentage of those receiving a grade D or below rose from 32.4% in 1999 to 37.8% in 2001. Since the turn of the century, however, this trend appears to have improved, as highlighted in table 2.3, overleaf:
Table 2.3 shows that between 2000 and 2010 there was a small increase of 2.8% in the number of students receiving a ‘high grade’ on the ordinary level paper while there has been a corresponding decline of 2.8% in the number of students receiving a grade D or lower during this period. However, we must note that the period 2004 – 2008 witnessed a decline of 1.8% in the number of candidates receiving these high grades (DES, 2008) and it was only in 2010 that an increase was recorded. Those involved in mathematics education in Ireland must now strive to ensure that this trend remains positive and the number of students attaining high grades continues to rise.

As with the situation at higher level, this decade has witnessed an improvement in relation to attainment levels at ordinary level. This is commendable and credit needs to be attributed to those involved in the teaching and learning of mathematics at Senior Cycle level but people also need to be made aware that many problems still exist in relation to levels of achievement.

**Problems in Relation to Attainment Levels in Ireland**

Despite the improvements in attainment levels witnessed in recent years many problems still exist in this regard. One of the main problems in this area is the fact that when compared to other countries Ireland appears to be lagging behind in terms of achievement. According to the Irish Mathematical Society (2006:17 - 18)
“...higher performing Irish students do less well than their counterparts in countries which record comparable overall levels of achievement in international Mathematics tests.”

This stresses that Ireland’s top mathematics students are failing to reach the same levels of mathematical competency as students of the same age in different countries. Furthermore, a recent report published by the OECD (2010) found that Junior Cycle students in Ireland performed significantly lower than the OECD average in mathematics. The mean mathematics score for Ireland was 487 while the average OECD score was 496. However, Senior Cycle mathematics education in Ireland has not been investigated in reports such as TIMSS or PISA and so it is impossible, at this point, to compare levels of achievement in Ireland at Senior Cycle level with those in other countries. Nevertheless, according to the Chief Examiner’s Report (2001) international studies of achievement have found that younger students in Ireland (i.e. Junior Cycle students) perform better, when compared with international students, on routine skills and procedures but they appear to fare worse on questions that require a deeper understanding of mathematical concepts and ideas. The report argues that the situation at Leaving Certificate is the same. It highlights how students lack sound conceptual understanding and cannot apply mathematics in contexts which, although familiar, are not precisely like the examples they have rehearsed in class. This shows that despite research displaying signs of improvement in the situation, as measured by grades, students’ understanding still falls far behind their international counterparts.

In addition to this, as is the case in the UK (Smith, 2004), when achievement levels of different subjects on the Irish curriculum are compared mathematics is again found wanting. According to Donohue (2007), as cited in Donnelly (2007), 12.5% of the entire student cohort obtained an honour in the 2007 higher mathematics paper compared with 44% who obtained an honour in the higher English paper that year. Similarly, in the Chief Examiner’s Report on Ordinary Level Geography (2002) it was found that in 2002 73.3% of students sitting the Ordinary Level paper obtained an honour and this is compared with 62.6% in mathematics. In addition to this, Figure 2.5, overleaf, compares the percentage of students who got an honour in the three core Leaving Certificate subjects, namely English, Irish and Maths, in 2010. From this it is evident that students performed better in the higher level mathematics
exam than in the higher level English exam in 2010 but they did not do as well in Irish. Also at ordinary level a significantly lower percentage of students attained an honour (Grade A – C) in mathematics when compared with the results in the ordinary level English and Irish exam results.

![Figure 2.5: Comparing Levels of Attainment across the Three Core Subjects](image)

**Figure 2.5: Comparing Levels of Attainment across the Three Core Subjects**

Although mathematics results appear to be improving, problems still exist when we compare mathematical results in Ireland with those in other countries as well as when we compare achievement levels in mathematics with other subjects on the curriculum.

Another problem in this regard relates to the high number of students who fail to achieve a grade D at ordinary level. Table 2.3 showed that this figure was in decline between 2000 and 2010 but despite this approximately 3,700 students of the 2010 cohort failed to attain a grade higher than D. This figure in conjunction with the 5,977 students who sat the foundation level paper that year meant that in 2010, approximately 9,677 students were, as a result of their performance in Leaving Certificate mathematics, prohibited from entering the majority of Universities and Institutes of Technologies in Ireland. This meant that 17.8% of students last year
were unlikely to progress to third level education solely as a result of their performance in mathematics. Similarly statistics from the DES (2008) show that in 2008, 4,404 students failed to attain a D3 and when this figure is combined with the number of students who sat the foundation level paper that year (5,803) it meant that approximately 20.4% of students would fail to gain entry to most third level institutes due to their mathematics results.

As is the case internationally, there appears to be many issues relating to the levels of attainment of Irish students and like other countries these problems will have onward effects at third level and in the workplace. Another problem which will affect the standard of mathematics among students graduating from second level is the greater number taking ordinary level mathematics. This is the next problem to be analysed in this section.

2.3.4 Recent Changes in the Uptake of Mathematics in Second Level Education

This section will look at the uptake of mathematics both internationally and in Ireland.

Changes in the Uptake of Mathematics in Second Level Education Internationally

“\textit{The report recognised that the number of students continuing with mathematical education post GCSE level is declining.}”

(Metje, 2007: 146)

In a society where it is necessary to call upon mathematics on a daily basis, trends such as those being discussed by Metje (2007) are worrying. Much investigation has been carried out in this area and other researchers have come to a wide range of conclusions. For example, over a decade ago the LMS (1995) found that numbers studying mathematics at A-Level in England and Wales had been in decline since 1965 while numbers taking mathematics at all levels had also dropped between 1985 and 1995. More recently, Smith (2004) found that numbers in mathematics continued to decline in England, Wales and Northern Ireland up until 2004 while a decrease was also witnessed in subjects that had high mathematical content (e.g. physics). Table 2.4, overleaf, which is taken from the work of Smith (2004), along
with the findings of the LMS (1995), highlights that the decline in the take up of mathematics had occurred over many years.

<table>
<thead>
<tr>
<th>Year</th>
<th>No of A – Level Entries (for Mathematics)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>68,853</td>
</tr>
<tr>
<td>2001</td>
<td>65,891</td>
</tr>
<tr>
<td>2002</td>
<td>53,940</td>
</tr>
</tbody>
</table>

Table 2.4: Total A Level Entries for Mathematics (all UK, all ages)

This decline of 21% over a five year period was, according to Smith (2004), a direct result of the perception that mathematics is one of the most difficult and time consuming subjects and requires an innate ability (Cuoco, 2003). However, recent years have seen significant increases in the uptake of mathematics at A Level and in 2010, 77,001 students completed their A-Levels in mathematics in the U.K. (LMS, 2010).

At first glance, Australia presents a promising picture but across the globe there appears to be differing trends in the uptake of mathematics. According to a report published in ‘The Bulletin’ in 2002 enrolments in mathematics courses in senior years in Australia has remained steady over the last decade. However, when researchers examined these figures further they found that the majority of the 88% taking mathematics were opting for the easy mathematics course while there have been decreases in the numbers enrolling for advanced courses. In this report, Bagnall (2002) uses the example of New South Wales where numbers taking advanced mathematics courses fell from 15,273 in 1991 to 9,547 in 2000.

Two countries that do show improvement are France and the US. Research has indicated that between 1965 and 1995 numbers enrolling for higher level mathematics at upper secondary level in France ‘tripled’ (LMS, 1995: 14) while many US states have also, in recent years, reported increases in the number of students choosing mathematics at second level. For example the Department of Education in California reported in 2003 that there was a significant increase in the number of students studying standards based mathematics while there was a 43%
increase in the number of students completing a three year college preparatory mathematics program.

Increased levels of participation and attainment as reported in England, France and the US are worth striving for. If achieved they will have a knock on effect on the numbers taking mathematics at third level (including those choosing courses in mathematics education which are in serious decline (Smith, 2004)) while at the same time the current levels of attainment could be improved in line with such increases.

**Uptake of Mathematics in Ireland**
Mathematics is a virtually compulsory subject for the Leaving Certificate in Ireland. Research shows that the number of students studying mathematics in Ireland in upper second level education is above the international average (NCCA, 2005). For example in 2009, 95.7% of students sat the mathematics exam, the highest participation in any subject on offer at Leaving Certificate level (Project Maths Implementation Support Group, 2010). One major problem still exists in relation to the uptake of mathematics despite the positive finding outlined by the Project Maths Implementation Support Group (2010).

**Higher Level vs. Ordinary Level**
An issue that has arisen in recent years, and one which has also been found to be problematic internationally, is the small number of students who opt for higher level mathematics at senior level (Bagnall, 2002). Table 2.5, overleaf, which is from the NCCA (2005) discussion paper, shows the number of students studying mathematics at each of the three different levels between 2002 and 2004 while Figures 2.6 and 2.7, show more recent trends in the uptake of higher and ordinary level mathematics.
### Chapter 2  General Issues in Mathematics Education

<table>
<thead>
<tr>
<th>Year</th>
<th>Total No. of Examination Candidates</th>
<th>Maths: Foundation Level</th>
<th>Maths: Ordinary Level</th>
<th>Maths: Higher Level</th>
<th>Total Maths</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>55,496</td>
<td>5,296</td>
<td>38,932</td>
<td>9,430</td>
<td>53,658</td>
</tr>
<tr>
<td>2003</td>
<td>56,237</td>
<td>5,702</td>
<td>39,101</td>
<td>9,453</td>
<td>54,256</td>
</tr>
<tr>
<td>2004</td>
<td>55,254</td>
<td>5,832</td>
<td>37,796</td>
<td>9,429</td>
<td>53,057</td>
</tr>
</tbody>
</table>

Table 2.5: Examination Candidates taking Mathematics at Each Level (2002 – 2004)  
(NCCA, 2005)

![Figure 2.6: Take Up in Higher Level mathematics 2005 - 2010](image)

Figure 2.6: Take Up in Higher Level mathematics 2005 - 2010
These figures and table highlight one of the problems that mathematics education is currently experiencing in Ireland. In 2002 only 17.5% of students opted for the higher level paper and although these figures rose slightly in 2004 to 17.7% there have been further decreases in recent years, and in 2010 only 16.04% of students who sat Leaving Certificate mathematics opted for the higher level paper. In contrast the percentage of students choosing to sit the ordinary level mathematics paper has risen from 70.48% in 2005 to 72.49% in 2010. Such findings highlight how, over the last number of years, despite large numbers studying mathematics in Ireland, the number opting to study it in its most advanced form is critically low. In addition to these figures, it is also worrying to note that in 2009, 79 schools (10.63%) in Ireland had no student sitting the higher level exam paper an increase from 58 schools (7.81%) in 2008 (Project Maths Implementation Support Group, 2010)

The former Minister for Education, Batt O’Keeffe, aimed to increase this figure from 16% to 30% in the coming years (Engineers Ireland, 2010) and it is anticipated that these increases will be facilitated through the ‘Project Math’ initiative and the introduction of a bonus point system for higher level mathematics. However, even if
this goal is achieved a problem still exists for mathematics education in Ireland when
the numbers taking higher level are compared with similar statistics for other
subjects. For example the NCCA (2005) found that in 2004, 60% of students sitting
English for the Leaving Certificate chose the higher level course while in 2010 this
figure rose to 64.09%.

Similar to the findings of Smith (2004) in a British context, research in Ireland
acknowledges that low numbers choosing higher level mathematics in this country
may be attributed to poor attitudes and a lack of interest among students and
teachers’ attitudes and approach to teaching mathematics or the perception of
mathematics as a difficult subject only suitable for academically talented students
who are capable of handling the ‘overcrowded curriculum’ (English et al, 1991;
NCCA, 2005).

2.3.5 Procedural Approach to Teaching

Researchers have long held the view that the procedural approach favoured by the
majority of mathematics teachers in many countries has a negative impact on student
learning (Wearne & Hibbert 1988; Watson 2004; Cockroft, 1982). For example
Wearne and Hibbert (1988) found that primary school pupils who had been taught by
teachers using this procedural approach found it more difficult to extract any real
meaning from mathematics than those who had been taught using a conceptual or
‘teaching for understanding’ approach. This highlights the negative impact of such
an approach and yet common practice in Irish schools today (English et al. 1991;
Hourigan & O’Donoghue, 2007). In addition to this, the work of de Corte et al
(1996) demonstrates that this procedural approach leads to poor understanding
among pupils. Their research notes that poor levels of understanding, which in itself
is a significant problem, are a direct result of the way in which students are taught in
an inflexible and procedural manner as opposed to a conceptual, flexible approach
which is advocated for secondary schools today.

Similarly in Ireland research has found that the procedural or top down approach to
teaching is leading to poor understanding among students.
“Students’ poor levels of mathematical understanding are typified by concerns about schools’ focus on procedural, routine, inflexible, abstract, and inert knowledge rather than fostering students’ capacity in conceptual problem focused, practical and flexible use of mathematical knowledge.”

(Conway and Sloane, 2005:16)

At the moment the problem discussed by Conway and Sloane is affecting the standard of mathematics teaching and learning in Ireland. Researchers such as Oldham (2001) discuss a number of reasons for this problem in Ireland. One of the main reasons is the strong emphasis that schools in our society place on the Junior and Leaving Certificate examinations. Hourigan & O’Donoghue (2007) acknowledge that the pressure on schools and teachers in particular to deliver good exam results is impacting negatively on the overall aim of providing students with a high quality mathematics education. This idea of ‘teaching towards the exam’ is resulting in didactic lessons with no student involvement in the form of group work, whole class discussion or reflection and students are beginning to believe that mathematics is simply about learning a technique or procedure or remembering a lot of formulae (English et al. 1991; NCCA, 2005; Hourigan & O’Donoghue, 2007). As a result many students are engaging in rote learning and when they learn procedures by rote many of them lose sight of the meaning or reasoning behind the mathematical concepts hence leading to this current problem. It is anticipated that the introduction of the new Project Maths curriculum will lead to a greater emphasis being placed on conceptual understanding and the application of mathematical concepts but in order for this vision to become a reality we must ensure that teachers are prepared and motivated to teach and deliver the course appropriately. However, researchers have found that this is not always the case and often mathematics teachers are not well motivated to teach mathematics at second level.

2.3.6 Unmotivated Teachers

Another problem facing mathematics educators in today’s society is the recruitment of unmotivated teachers to the profession. Boero et al. (1996: 1098) found that, in a large number of countries, teachers enter the profession of teaching for reasons which are far from a genuine vocation to teach mathematics. This is a difficult scenario that we are faced with as it means teachers are entering classes with
negative attitudes which negatively impact on their students’ attitudes, performance and most importantly appreciation of the subject (Thompson, 1992; LMS, 1995; Gorard et al. 2001; Lyons et al. 2003). These concerns are also raised by Phillipou and Christou (1998) as their research emphasised that in the majority of countries unsuitable candidates are being accepted into the profession of teaching and, as a result, teacher training colleges worldwide currently number too many unenthusiastic students among their numbers. Similarly the LMS Report (1995) report that in England and Wales many trainee teachers in their final year of study, hold extremely negative attitudes towards mathematics as well as their own ability and yet they will soon graduate and be regarded as ‘well – qualified’ teachers. Phillipou and Christou (1998) identify the risk that these potential teachers entering classrooms poses for students in terms of their attitudes and experiences but even more significant is the fact that they do not foresee a change in this situation in the near future. As a result students for generations to come, according to their research, will be exposed to ideas of mathematics being a ‘dead discipline’ with little use outside of a school setting (Phillipou & Christou, 1998).

2.3.7 Overcrowded Curriculum

The final two problems to be addressed are problems that are beyond the control of second level teachers and schools. Firstly, mathematics educators are currently faced with an overcrowded syllabus which is difficult to teach. This ‘overcrowded curriculum’ and the need to prepare students for external examinations prevents even the most motivated of teachers from introducing new teaching approaches that have been found to facilitate better understanding and learning (Eaton & Bell, 2006). Eaton & Bell (2006) drew up a number of trial lessons which were designed to enhance the teaching and learning of mathematics but despite the teachers involved being willing to implement them, they reported that the need to prepare students for internal and external examinations had prevented them from doing so. Similarly, English et al (1991), found that the overcrowded curriculum in Ireland was another major obstacle that teachers had to overcome while Engineers Ireland (2010) report that the overcrowded curriculum is contributing to low numbers opting for higher level mathematics in Senior Cycle. These researchers note that the aesthetic and
pleasurable aspects of mathematics are being lost as a result of such curricula and in turn this encourages and to some extent promotes negative attitudes and disillusionment among students. Another problem previously analysed in this section, the procedural approach to teaching, also stems from the overcrowded curriculum and the idea of ‘teaching towards the exam’.

2.3.8 Time Allocated to Mathematics at Senior Cycle Level

The final problem to be looked at in this section is specific to the Irish context and one which the author referred to when looking at different attainment levels internationally. It concerns the time allocated to mathematics at second level. As with many other aspects of mathematics education in Ireland the time allocated to mathematics at Senior Cycle level varies from school to school but the average number of classes per week is between four and five, each of approximately forty minutes duration. In total, it is estimated that Irish students in Senior Cycle are exposed to approximately three hours of mathematics teaching per week. Figure 2.8, overleaf, highlights that this figure is lower than the number of mathematics teaching hours in other countries:
By comparing the weekly instructional time in this manner, Ireland appears to only fall slightly behind countries such as Russia and Japan, both of whom have performed above average in international mathematical assessment, e.g. TIMSS (2003). On the surface this appears to be a positive finding for Ireland but when the issue is investigated further problems do arise. According to the OECD (2004) Ireland has 33.1 instructional weeks per year and this is compared with 38.6 for Canada, 39.7 for Germany, 38.9 for Japan, 35.0 for Russia and 36.0 for the United States. Table 2.6, overleaf, shows the effect this has on the total amount of time allocated to mathematics in these countries in any given year.

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Figure 2.8\(^6\): Average Hours Spent Per Week on Mathematics Learning in At – School Settings

(http://nces.ed.gov/surveys/international/IntlIndicators/pdf/2007006_timespent.pdf)

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\(^6\) To date, there is very little information in relation to the number of hours, if any, that Irish students spend in remedial or enrichment classes and so Figure 3.4 will be used solely to compare the instructional time for mathematics between countries.
<table>
<thead>
<tr>
<th>Country</th>
<th>Instructional Time Per Week (Hours)</th>
<th>Instructional Weeks Per Year</th>
<th>Total Instructional Time Per Year (Hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ireland</td>
<td>3</td>
<td>33.1(^7)</td>
<td>99.3</td>
</tr>
<tr>
<td>Canada</td>
<td>3.7</td>
<td>38.6</td>
<td>142.82</td>
</tr>
<tr>
<td>Germany</td>
<td>3</td>
<td>39.7</td>
<td>119.1</td>
</tr>
<tr>
<td>Japan</td>
<td>3.6</td>
<td>38.9</td>
<td>140.04</td>
</tr>
<tr>
<td>Russian Federation</td>
<td>3.5</td>
<td>35</td>
<td>122.5</td>
</tr>
<tr>
<td>United States</td>
<td>3.7</td>
<td>36</td>
<td>133.2</td>
</tr>
</tbody>
</table>

Table 2.6: Average Instructional Time Afforded to Mathematics on an Annual Basis

This table highlights that, despite initial findings in relation to instructional time per week, Ireland is currently falling behind many countries when the total number of hours per year are analysed. Research has also shown that Singapore, who were the highest achievers in TIMSS (2003) and were ranked second in the 2009 PISA Report (OECD, 2010), also allocate significantly more time than Ireland to mathematics. According to Ruddock (1998) students in Singapore spend about 5.5 hours per week engaged in mathematics lessons and their school year consists of 280 days – approximately 40 weeks (Year Round Schools, 2001). As a result, it is estimated that Singapore schools dedicate 220 hours per school year to mathematics – over twice what Ireland currently allocates. As with Singapore, Japan also did extremely well in international assessments, placed among the top three countries in TIMSS (2003) and among the top ten in PISA (2009) and this would suggest a strong correlation between time afforded to mathematics in schools and student performance in the subject. The Irish authorities need to examine this issue carefully if they wish to improve performance.

\(^7\) This figure does not take into account classes missed during mock examinations or oral/aural examinations which are all compulsory for different subjects at Leaving Certificate level.
2.3.9 Summarising the Problems

In this section the author explored a wide range of problems that exist in mathematics education worldwide and more specifically in Ireland. These problems ranged from poor attitudes towards mathematics to the low uptake of mathematics; from a failure to appreciate the relevance of mathematics to unmotivated teachers teaching mathematics. Due to the presence of these problems a number of students, but not all students, across the world and in Ireland, have had negative experiences of mathematics during their time at second level and consequently there is a strong possibility that these students do not appreciate and may even dislike or detest the subject. However this is not the only effect these issues have. Due to the nature of the problems identified the onward effects in Higher Education are much more extensive. These issues are the focus of section 2.4.

2.4 Ramifications of the Problems Affecting Mathematics Education

The list of problems discussed by the author in section 2.3 have numerous knock on effects. These effects will be felt universally and will affect mathematics education and the image of mathematics education for years to come. One of the most pressing issues which results in part from the aforementioned problems relates to the transition from second to third level mathematics education.

2.4.1 The Transition from School Mathematics to Third Level Mathematics

For many year researchers both in Ireland and abroad have investigated problems relating to the transition from second to third level education and many of these problems arise directly from issues discussed in section 2.3. Many of the investigations carried out to date focus specifically on mathematics and the author will now look into some of the findings in this regard.

“The problem is more serious; it is not just the case that some students are less well – prepared, but that many ‘high attaining’ students are seriously lacking in fundamental notions of the subject” (LMS, 1995: 5)
A large proportion of students are under prepared for third level mathematics on completion of their second level studies. The prevalence of a procedural approach to teaching as well as the low numbers choosing to study mathematics in its highest form is resulting in students graduating from secondary school ill prepared for third level courses. Even those who attain high grades at second level struggle with further study in the field. However as Metje (2007) points out in England a GCSE grade C in mathematics (and a C3 on the ordinary level paper in Ireland) is still accepted as an entry requirement for many third level courses with high mathematical content. This problem appears to have intensified in recent years.

Research carried out in the University of York and Coventry University in the U.K., as cited by the Engineering Council (2000), showed how levels of preparedness among students entering these institutions were in serious decline. Table 2.7, below, and Figure 2.9, overleaf, show the results that first year students attained in diagnostic tests carried out during the first week of college in Coventry University and the University of York respectively. Such tests test students on the basic mathematical skills that they are required to use in their mathematical modules and identify any problematic areas.

<table>
<thead>
<tr>
<th>Grade</th>
<th>1998</th>
<th>1997</th>
<th>1991</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Score (max 50)</td>
<td>Sample Size</td>
<td>Score (max 50)</td>
</tr>
<tr>
<td>B</td>
<td>36.8</td>
<td>27</td>
<td>36.4</td>
</tr>
<tr>
<td>C</td>
<td>32.1</td>
<td>52</td>
<td>34.0</td>
</tr>
<tr>
<td>D</td>
<td>30.6</td>
<td>58</td>
<td>30.3</td>
</tr>
<tr>
<td>E</td>
<td>28.2</td>
<td>46</td>
<td>30.3</td>
</tr>
<tr>
<td>N</td>
<td>25.7</td>
<td>22</td>
<td>27.8</td>
</tr>
<tr>
<td>U</td>
<td>22.8</td>
<td>9</td>
<td>23.8</td>
</tr>
</tbody>
</table>

Table 2.7: Diagnostic Test Scores by A Level Grades from Coventry University (Engineering Council, 2000: 9)
Both Table 2.7 and Figure 2.9, above, highlight the decline in performance in these diagnostic tests and show how a problem now exists in terms of the mathematical knowledge that students have acquired by the time they leave school and enter Higher Education Institutions (HEIs). These findings highlight that an increasing number of students are struggling with basic mathematical concepts which are the foundation of many tertiary mathematical modules while the problem of the devaluation of grades as discussed by Smith (2004) is also evident. It is not surprising then that a gap exists between school and college mathematics (LMS, 1995; Glaister & Glaister 2000; Smith, 2004). Research shows that factors other than entry requirements, as discussed by Metje (2007), also contribute to the formation of this ‘gap’. For example, Glaister & Glaister (2000) found that the didactic nature of the subject and the lack of student involvement at second level fail to prepare students for the project work and oral assignments that they will encounter at third level. Furthermore, Smith (2004) argues that the lack of interaction that currently takes place between schools and colleges also allows for the development of this gap and hinders the formation of any possible links between the two types of institutions.
This is another problem that Ireland has failed to avoid. According to Hourigan & O’Donoghue (2007) mathematics departments in third level institutions around Ireland are dissatisfied with the mathematical ability of entrants since the mid 1980’s. Furthermore the NCCA (2005) notes that the level of mathematical expertise among students entering third level institutions is inadequate and they claim that it fails to match expectations. Research carried out in an Irish context has pointed to ‘mathematical under-preparedness’ as one of the main problems facing students entering further or higher education (Morgan et al. 2001; O’Donoghue, 2002; Moore 2004). According to the Irish Mathematical Society (2006), despite students satisfying the entry requirements for mathematics, many enter third level institutes across the country with insufficient mathematical skills and knowledge and this is placing them at an immediate disadvantage. The work of O’Donoghue (2002) supports this finding as he too found many students entering third level education lack basic arithmetic and algebraic skills as well as a basic understanding of trigonometry, calculus and complex numbers. These are all elements of the Leaving Certificate syllabi and yet some students seem to have a very poor understanding once they reach tertiary education. This would indicate that the Leaving Certificate course is failing to fulfil its aims as a number of students are progressing from it without the knowledge required for the most basic mathematical modules in third level education. Furthermore, this problem has worsened in Ireland in recent years. A diagnostic test is carried out every year in the University of Limerick (UL) in order to identify the ‘at risk’ students and in recent years this has served to highlight what appears to be the serious decline in the mathematical standards of students entering third level. In 1998 28.2% of students failed this diagnostic test, in 2005 35.44% failed while by 2008 this figure had risen to 36.78% (Gill et. al, 2010). This trend must be curtailed immediately in order to prevent the gap between second and third level widening and in order for this to be achieved work is needed both at second and third level.

In addition to this, another concern is that some students entering college currently appear unable to apply mathematics when it is presented in a form that they may not have encountered before in secondary school (O’Donoghue, 2002). the work of Conway & Sloane (2005) found that the dominance of procedural teaching at primary and secondary level, as discussed by English et al (1991) and Hourigan &
O’Donoghue (2007), results in post primary graduates lacking these basic skills and leads to them being unable to apply basic concepts to real life situations. This statement is echoed in the findings of Hourigan & O’Donoghue (2007:473) as they found:

“\textit{The limitations of a quick – fix, teacher led didactic approach, which has the terminal exam as a primary focus, means that the ‘typical’ pre-tertiary mathematics experience fails to provide pupils with the necessary foundations for everyday life, not to mention tertiary level mathematics courses}”

Other reasons for the mathematical under preparedness of students were proposed by O’Donoghue (2002) and include the devaluation of grades, the dilution of the Leaving Certificate course and different expectations from schools and colleges. These causes are all in line with international findings put forward by LMS (1995), Glaister & Glaister (2004) and Smith (2004). Such findings highlight how a gap has opened up in Ireland between second and third level mathematics and the impact of such a gap can have many detrimental effects (Morgan et al, 2002; Gill & O’Donoghue, 2006).

There is evidence in the work of Morgan et al. (2001) to suggest that poor levels of competency in mathematics is one of the leading factors contributing to drop out from courses in Institutes of Technology in Ireland. Their research found that many students enter courses without having acquired sufficient mathematical knowledge in school (despite having the necessary entry requirements) and as a result struggle with many modules, leading to their drop out or failure. O’Donoghue (2002) also discusses how this ‘under-preparedness’ results in students failing to complete courses both in Universities and Institutes of Technology while Gill and O’Donoghue (2006) found that mathematical under-preparedness is leading to problems in relation to retention in technology, engineering, science and business courses. O’Donoghue (2002) notes that the problem of retention due to under-preparedness is having a detrimental effect on both departments within colleges and colleges themselves as they are losing students and hence funding as a result of the problem of under preparedness. There is also evidence in his work to suggest that this under preparedness can have other negative implications for students such as hindering their progress in their major discipline as many disciplines require students
to have a good grasp of basic mathematical concepts, and weakening the overall value of their degree. Overall, the gap that currently exists between school mathematics and college mathematics, which stems from problems facing mathematics education at second level, is having numerous detrimental effects both on students, the higher level institutes and the Irish economy.

2.4.2 The Impact of these Problems on the Economy

According to the EGFSN (2008) the current problems attributed to mathematics education, and more specifically the problem of low levels of attainment in mathematics, are of serious concern to employers in Ireland. Ireland is currently experiencing a severe economic crisis yet this group report that if mathematical performance were to improve then many problems in the economy could be resolved and the number of people on the Live Register could be reduced. Furthermore Donnelly (2010) found that problems in relation to uptake and attainment are costing our government approximately 8 billion a year. According to her research findings, until Ireland reaches Finnish standards (top performing country in PISA studies) in mathematics and science education then mathematics and science will continue to cost our economy. Finally Flynn (2010) states that if Ireland transforms itself into a centre of innovation and enterprise then this will lead to the creation of 120,000 jobs. However, in order for this to occur he argues that a concentrated effort is needed to improve the standard of mathematics and science teaching and learning. Overall, the problems affecting mathematics education are having a serious effect on the already struggling Irish economy but improvements in this regard could help resolve issues of unemployment as well as help the Irish government save money. Similar findings were reported in England in the past. It is anticipated that if the problems discussed in section 2.3 continue to exist then, the knock on effect will involve Britain being unable to keep abreast of international developments in science, engineering and technology (LMS, 1995; Smith, 2004). As a result the LMS (1995: 6) warned that Britain will become even more dependent on foreign nations for “…inventions, specialists and products” and the entire economy will suffer.

The poor image of mathematics education portrayed by the author in section 2.3 is having serious repercussions at third level while at the same time detrimentally
Chapter 2  General Issues in Mathematics Education

affecting the economy. Due to the seriousness of these problems and the subsequent knock on effects it is essential that the root of these problems is identified as soon as possible and strategies are put in place to rectify the situation.

2.5 Potential Causes and Possible Remedies

A number of factors have led to these problems evolving over the past number of years and in this section the author will outline the three that have had the most significant impact. Steps have been taken to overcome the first two while the third and final cause will be addressed in this doctoral thesis.

Firstly the overcrowded curriculum, which the author described in section 2.3.5, contributes to a wide range of problems which mathematics education must overcome. In addition to being a problem in its own right, the overcrowded curriculum has been identified as a cause of other problems including poor uptake of and attainment in mathematics, the prevalence of a procedural approach to mathematics teaching and poor attitudes towards mathematics (Hourigan & O’Donoghue, 2007; Engineers Ireland, 2010). However, the national effort in Project Maths is seeking to overcome this problem. As outlined in section 2.2, the number of strands to be studied at higher level for Leaving Certificate has been reduced from seven to five while ordinary level students also only have to study five strands as opposed to the eight which they previously were required to study for the Leaving Certificate. In addition to this a number of sub topics, for example matrix algebra, have also been removed hence further reducing the amount of material to be studied.

A second cause that has been identified is the time afforded to mathematics at Senior Cycle. As with the overcrowded curriculum the author has already identified the time allocated to mathematics as a problem in its own right but it again contributes to numerous other problems that have been discussed. Brumbaugh & Rock (2006) found that time constraints are preventing teachers from incorporating innovative teaching strategies in their mathematics classes. Furthermore evidence put forward in their research shows that the lack of time afforded to mathematics is leading to mathematics teachers promoting rote learning rather than understanding. The issue of time is also leading to a strong focus on the terminal exam during mathematics
classes and the aesthetic and pleasurable side of mathematics are being overlooked as a result hence leading to negative attitudes and low numbers taking higher level (Hourigan & O’Donoghue, 2007). It is expected that Project Maths will again contribute, somewhat, to the resolution of this problem but further action is also necessary. As the author previously stated, it is essential for those in power to realise the correlation between time spent at mathematics and achievement levels in the subject and that action is taken to bring Ireland in line with countries such as Singapore and Japan with regards the time spent on mathematics during second level.

The final factor which will contribute to the resolution of the aforementioned problems is effective teaching. For many years now researchers have investigated the issue of effective teaching and have identified many factors which they believe contribute to such effective teaching. In order to resolve the problems currently facing mathematics education it is critical that teachers demonstrate these characteristics in the classrooms. These characteristics include the use of clear and concise explanations that students can easily identify with (Ball, 2001; Manallung, 2005), effective lesson design and delivery methods (Million, 1987), the use of a variety of teaching methods and approaches (Lieberman, 1995), the ability to involve students in the learning process (Rogers, 2002; Watson, 2004) and the effective use of questioning (Ball, 2001).

However the characteristic which many have found to underpin all those previously discussed and which has been identified as the most important attribute of an effective teacher is an extensive knowledge base in mathematics. In order to teach effectively it is important for a teacher to have an extensive knowledge base as the following Chinese proverb suggests:

“If you want to give the students one cup of water, you (the teacher) should have one bucket of water of your own”

(An et al., 2004: 146)

This statement suggests that in order to be able to develop students’ knowledge of mathematics, to improve their understanding and interest in the subject and hence to improve uptake and attainment levels teachers must have a much deeper knowledge of the material. Furthermore, research has shown that without an extensive knowledge base in the subject teachers will struggle to demonstrate many of the
other characteristics associated with effective teaching. For example research previously conducted has found that teachers who have a broad knowledge base are more capable of involving students in the learning process than those teachers who’s knowledge is deemed weak (Fennema & Franke, 1992; Irwin & Britt, 1999). Similarly, Ernest (1989) acknowledges that the standard of explanations provided by teachers in class can be enhanced by a teacher’s level of knowledge while Carlsen (1991) found that a strong knowledge base on the part of teachers can improve the standard of questioning in the mathematics classroom. Therefore, just as the author found the quality of teaching to be at the core of many of the problems associated with mathematics education, research has allowed her to ascertain that in order to improve the standard of teaching it is critical that the focus is initially on positively influencing teachers’ levels of knowledge.

Overall research has shown there is strong correlation between teachers’ levels of knowledge and their ability to teach effectively. The impact of such effective teaching is immense and can significantly help overcome the problems facing mathematics education today. For example:

- Researchers such as Cobb (1988), Niss (1996), NCTM (2000), Metje (2007) propose an extensive list of aims for mathematics education in general. In order for teachers to achieve these goals they must have an extensive knowledge base. For example, one of the primary goals of mathematics education, according to Comiti & Ball (1996), is the fostering of an appreciation of the relevance and applicability of mathematics. However, without a knowledge of the applications of mathematics, teachers will not be able to achieve this goal. Similarly, the aims of the new Project Maths syllabus, which strive to increase uptake and attainment in mathematics rely on a deep understanding of applications and connections on the part of teachers.

- Research has found that teachers and in particular the quality of teaching has a positive effect on students attitudes towards the subject (Phillipou & Christou, 1998; Stigler & Hiebert, 2004). However, the quality of teaching has, in turn, been found to be underlined by a teachers level of knowledge (Ernest, 1989; Carlsen, 1991; Irwin & Britt, 1999) and hence by improving
levels of knowledge and in turn teacher effectiveness the problem of negative attitudes towards mathematics may be overcome.

✓ Teachers need to develop a deeper knowledge of pedagogical principles and teaching approaches in order to move away from the didactic approach to mathematics teaching. In doing so teachers can better help prepare students for third level mathematics (Glaister & Glaister, 2000).

✓ Teachers need to develop an understanding of the relevance and importance of mathematics and share this knowledge with students in order to improve their interest in the subject (Conway & Sloane, 2005). The problem of poor uptake of higher level mathematics has been partly attributed to a lack of interest in the subject on the part of students. As a result increasing levels of interest in the subject, through improved teacher knowledge, may help alleviate the problem of low numbers opting for higher level mathematics.

✓ Improved levels of knowledge on the part of teachers, when shared with students in an effective manner (this also requires improved knowledge), will undoubtedly lead to improved levels of knowledge on the part of students.

✓ Finally, improved knowledge on the part of teachers will lead to more innovative mathematics classes and a shift from the procedural approach to mathematics that is common place today. De Corte (1996) reports that this shift will have a knock on effect on student understanding and will allow for conceptual understanding to take place.

2.6 Conclusion

In conclusion, this chapter initially painted a poor image of mathematics education in Ireland however there now appears to be some hope for the future. As highlighted by the author, it is anticipated and hoped that the national effort in Project Maths will, in the future, lead to a number of the issues being resolved while improved levels of knowledge among mathematics teachers can significantly help alleviate many of the other problems discussed in the early stages of this chapter. Such issues include student disinterest in mathematics, poor uptake of higher level mathematics, procedural approach to teaching and poor conceptual understanding among students.
Now that teacher knowledge has been identified as a possible remedy to these problems, it is critical that action is taken immediately in this regard in order to curtail the problems currently affecting mathematics education. The first step in this procedure is to look at the knowledge required for mathematics teaching, the current levels of knowledge of mathematics teachers and strategies for improving teacher knowledge. The author will address each of these issues in the following chapter before narrowing her focus to the Irish context by identifying and improving the package of knowledge required for teaching.
3. Changing Practice in Mathematics

Classrooms: The Role of Teachers’ Subject Knowledge and CPD

3.1 Introduction

The research findings discussed in Chapter 2 highlight the critical nature of a mathematics teacher’s knowledge base. According to Askew & Williams (1995:42)

“…many aspects of mathematics teaching are under researched”

One such area of mathematics is the knowledge base of mathematics teachers and the impact of such knowledge on student learning (Shulman, 1986; Stigler & Hiebert, 2004). Shulman (1986) found that research to date has focused on questioning, direct instruction and timing issues as opposed to teacher knowledge and he cites subject matter knowledge as the missing paradigm in previous research in mathematics education. Stigler & Hiebert (2004) also recommend that future research in this field should shift from superficial issues and, instead, concentrate on
issues that have a more significant impact on the teaching and learning of mathematics.

In the early stages of this chapter different components of a teachers’ package of knowledge are analysed. The type of knowledge that is discussed and analysed during the early part of this chapter is neither static nor monolithic (Fennema & Franke, 1992). That is to say that the knowledge base of teachers is forever changing and integrates a number of different strands. It is these strands or types of knowledge that are the focus of section 3.2. The current state of teacher training and the contribution it makes to the development of a teacher’s knowledge base is the focus of section 3.3 while strategies for improving subject matter knowledge are investigated in section 3.5 as research has acknowledged that small improvements such as this can have an extremely positive effect on classroom proceedings and student learning (Stigler & Hiebert, 2004).

During the second half of the chapter, the focus is on one particular strategy that could be used to improve teachers’ levels of knowledge, namely Continuous Professional Development (CPD).

“The report [ACME Report] recognises and the Inquiry accepts that it is not possible for ITT\(^8\) to provide future teachers of mathematics with all they should know about the subject they will teach, how pupils learn it or how to teach it effectively. There is a need therefore for mathematics specific CPD…”

(Smith, 2004: 109)

Smith (2004) highlights just one of the reasons why there exists a critical need for CPD among mathematics teachers. In addition to this, the Inquiry carried out by Smith (2004) argues that the ever changing nature of school mathematics, as witnessed in Ireland in recent years, is another reason why CPD is essential for both newly qualified and experienced teachers. CPD cannot help teaching and learning if it occurs in isolation (Bradshaw, 1997) but it is one of the most important strategies that will enable teachers to enhance their knowledge and overall teaching ability throughout their professional careers. During the latter stages of this chapter the author looks in detail at a number of different aspects of CPD in general including different types of CPD that are available, the way in which CPD initiatives and strategies can affect teacher effectiveness and, more specifically, teachers’

\(^8\) Initial Teacher Training
knowledge of mathematics. The author investigates what constitutes effective CPD, the importance of CPD in relation to mathematics teachers’ subject matter knowledge and the benefits and problems associated with CPD. Finally effective CPD initiatives currently in place worldwide are analysed. These two issues are investigated in greater detail by the author and a package of knowledge required by teachers in Ireland is proposed (Chapter 5) as well as effective strategies for improving current levels of knowledge among Irish teachers (Chapter 7).

3.2 Different Types of Knowledge

“To be a teacher requires extensive and highly organised bodies of knowledge”

(Shulman, 1985:447)

This quote highlights that in order for a teacher to be effective then he/she needs a combination of different types of knowledge. Ernest (1989) elaborated on this when he drew up a list of knowledge domains that he believed were critical for the teaching of mathematics. This list included:

- Knowledge of mathematics
- Knowledge of other subject matter
- Knowledge of teaching mathematics
- Knowledge of the students taught.

It is now accepted that a knowledge of mathematics alone does not guarantee good teaching and instead it is imperative that teachers develop the entire package of knowledge required for teaching (Barton, 2008). In addition to the list compiled by Ernest (1989) many other authors have discussed at length the types of knowledge that a teachers ‘package of knowledge’ must contain and although they each offer a different package the core of each model remains the same (Shulman, 1986; Ernest, 1989; Fennema & Franke, 1992; Rowland, 2007; Ball et al., 2008).

The packages of knowledge proposed by these researchers reinforce the proposition that being a good mathematician is no longer sufficient for being a good teacher of mathematics (Comiti & Ball, 1996; Blanco, 2003). Researchers now accept that ‘…there is more to “knowing the subject” than meets the eye’ (Ball et al., 2005: 20).
Numerous models of teacher knowledge have been put forward by researchers in recent years and these are analysed in detail in Chapter 5, prior to the author presenting her own model. However, first it is important to develop an understanding of the different components included in these models as well as identifying and analysing a component that appears to have been overlooked in the 5 models that will serve as theoretical frameworks for this doctoral study.

### 3.2.1 Content Knowledge

Ball et al. (2005) state that content knowledge includes the common knowledge of mathematics that the majority of adults have access to as well as specialised knowledge for the purpose of teaching. It is the knowledge of mathematics that people begin to develop in early childhood while prospective teachers continue to learn it throughout their third level education and even through their own teaching experience (Ernest, 1989). For example data collection and statistics is one of the first areas that a child encounters in mathematics in primary school and they continue to develop their knowledge and ability in this area throughout their primary and secondary education. Then at third level, prospective mathematics teachers are required to study modules in statistics while it is also a dominant feature of many pedagogy modules. Finally whilst teaching, teachers continue to learn more about this strand of mathematics and through research, CPD and collaboration with colleagues they also continue to develop and use new teaching approaches for statistics. Hence, statistics, as with every other strand of mathematics, is introduced to all children at an early age and teachers of mathematics continue to develop their knowledge until the end of their professional career. Rowland (2007) also identified this content knowledge as one of the many ‘teacher knowledges’ in his ‘Knowledge Quartet’. Although he identifies it as foundation knowledge he still referred to the need for teachers to have explicit subject matter knowledge which would include knowledge of theories, procedures, terminology and purposes of mathematics.

Content knowledge, in tandem with pedagogical knowledge (Section 3.2.2), trains teachers on how to offer clear and simple explanations that students can understand as well as how to introduce topics using language and symbols that students can relate to. In Chapter 2 clarity of explanations and the effective use of language and
symbols were both identified as characteristics of effective teaching and as both are seen to be affected by a teacher’s level of content knowledge, such knowledge is shown to contribute greatly to effective teaching. Fennema & Franke (1992) carried out extensive work in this area too. This work acknowledges that a thorough knowledge of mathematics is required in order for teachers to comprehensively plan their teaching in a way that ensures student learning. Similarly Ball (1988) found a knowledge of mathematics is critical if we wish to help someone else learn it.

3.2.2 Pedagogical Knowledge

“Pedagogy concerns the actions teachers take, the intentions behind the actions, and how these relate to the progress of the learners within a mathematical environment”  
(Watson, 2004: 363)

In essence, pedagogical knowledge is the practical knowledge that teachers require to teach mathematics. It refers to the transfer of a teacher’s subject matter knowledge into representations, explanations, analogies, illustrations, examples and demonstrations comprehensible for their students (Shulman, 1986). Magnusson et al. (1999) found that this type of knowledge encompasses teachers’ knowledge of how to best help students understand mathematical content. According to Ernest (1989) pedagogical knowledge incorporates a deep understanding of students’ approaches and ideas as well as the areas where they tend to struggle. This type of knowledge also features in two of the four sections in the ‘Knowledge Quartet’: Transformations and Connections (Rowland, 2007). In this model Rowland places an emphasis on the choice of examples and representations as well as the anticipation of problems that students may encounter.

Prior to the 1980s pedagogical knowledge was omitted from the bulk of research carried out in the field of mathematics education. It wasn’t until Shulman made his AERA presidential address in 1985 that this knowledge domain was identified as critical for the purpose of teaching (Ball et al., 2008). In recent years the importance of this type of knowledge has been further acknowledged. Fennema & Franke (1992), who carried out extensive work in relation to teacher knowledge, note that pedagogical knowledge is critical in order for teachers to effectively represent mathematical ideas in a way that students can understand. Similarly Smith et al.
(2004) found that this type of knowledge was required in order to facilitate learning for all students despite ability or levels of motivation, which vary in each class. Despite these important findings some teachers demonstrate inadequate levels of pedagogical knowledge (Ball, 1990; Orton, 1988). For example Orton (1988) and the Project Maths Implementation Support Group (2010) found that teachers tend to rely heavily on procedures and simple symbolic representations rather than try to enhance learning by including real-life representations and explanations that students could relate to in their lessons. Consequently, poor pedagogical knowledge results in teachers being unable to expose mathematics as a relevant subject and hence students fail to see its usefulness. This highlights once again how inadequate knowledge contributes to the problems discussed in Chapter 2.

Finally, teacher knowledge is not monolithic and this is evidenced by the strong relationship that exists between pedagogical knowledge and content knowledge. Despite these two domains of teacher knowledge often being investigated and viewed in isolation there is a strong connection between the two (Shulman, 1986). Even (1993) argues that pedagogical knowledge is influenced and often determined by content knowledge while Wu (2005: 2-6) argues this point further:

“...sound pedagogical decisions can only be based on sound content knowledge... I do not believe a teacher can have pedagogical content knowledge without a firm command of content knowledge”

It must be recognised that although both content knowledge and pedagogical knowledge are critical to teachers’ overall repertoire of knowledge, educators must first try to develop and allow prospective teachers to become competent in content knowledge before concerning themselves with pedagogical knowledge. Hence, one of the biggest challenges for those designing teacher education programmes is to produce teachers who have sound mathematical knowledge as well as being pedagogically skilled (Comiti & Ball, 1996)

3.2.3 Knowledge of Students

Knowledge of students is another area of teachers’ knowledge that has strong links with those previously discussed. Knowledge of students requires teachers to have an
in-depth knowledge of the thought processes of students as well as the way in which they acquire knowledge and develop positive attitudes, both towards the subject and about themselves (Fennema & Franke, 1992). There is an overlap between this knowledge and pedagogical knowledge, and this again highlights the interrelated nature of the different types of knowledge needed by teachers.

There are currently differing views on the relevance and value of this type of knowledge. Due to its extremely broad scope, Putnam and Liendhart (1986) failed to see the need for this type of knowledge in the ‘package of knowledge’ they believed necessary for teaching. They alleged that expert teachers involved in their study never displayed this type of knowledge and no detrimental outcomes were witnessed as a result. In addition to this Rowland (2007) makes no reference to knowledge of students’ ways of thinking in his ‘Knowledge Quartet’ suggesting that he too places little emphasis or value on this aspect of teacher knowledge. On the other hand, the findings of a number of other studies indicate that this type of knowledge can yield positive results in terms of classroom proceedings (Carpenter et al, 1989; Fennema et al, 1989; Ball et al., 2008). Additionally Fennema & Franke (1992) found that such knowledge can make a substantial contribution to teachers’ instructional decision making and this in turn can lead to an improved learning experience for students. On balance, despite this ongoing debate and the need for further investigation in this area, teachers need to be knowledgeable in this area in order to enhance their own teaching as well as to further develop other types of knowledge (i.e. pedagogical knowledge).

3.2.4 Curricular Knowledge & Knowledge of Other Subjects

The final two domains of teacher knowledge that appear frequently in models of teacher knowledge are a knowledge of the curriculum and knowledge of other subjects and their link to mathematics. The mathematics curriculum, as with any other curriculum, provides teachers with a wide range of topics, programs and materials that they can employ in their classroom in order to augment learning. Again Shulman was one of the first to propose that this domain be included in a
teachers’ repertoire of knowledge and the true importance of this type of knowledge is described by Shulman (1986:10) when he states:

“We expect the mature physician to understand the full range of treatments available to ameliorate a given disorder, as well as the range of alternatives for particular circumstances…Similarly, we ought to expect that the mature teacher possess such understandings about the curricular alternatives available for instruction”

In the modern era physicians around the world are held in very high regard and their profession is truly valued. In order for teachers to gain such status and recognition they must demonstrate an in depth knowledge of the curriculum just as physicians are required to have a comprehensive knowledge of different illnesses, symptoms and treatments.

As discussed in Chapter 2, many connections exist between mathematics and other school subjects. Ernest (1989) notes that it is necessary for teachers to have some knowledge of other subjects and, more importantly, the links between these subjects and mathematics. There is evidence in his work to show that such knowledge is valuable for helping teachers highlight the purpose and applications of mathematics as well providing them with the opportunity to offer students an incentive to study the subject. Moreover Fennema & Franke (1992) found that a large number of people working in the field of mathematics education believed that this type of knowledge would allow teachers to gain a deeper insight into the general pedagogical principles of other subjects which they could then effectively apply to their own teaching in the mathematics classroom.

It is widely accepted that each of the different types of knowledge discussed in this section play an important role in the effective teaching of mathematics. The author has discussed the close relationship that exists between the multiple knowledges while the work of Peterson (1988) further emphasises this point. She argues that strong knowledge in one area is worthless without proficiency in the other areas. Despite a number of types of knowledge being referred to by researchers in the past one fundamental knowledge domain has been omitted in previous models.
3.2.5 Knowledge of Applications

In Chapter 2 the author looked in detail at the importance of mathematics in the world around us and spoke of the need to expose students to the applicable and relevant side of mathematics. In order to do this, teachers themselves must understand the importance of mathematics and hence must develop a knowledge of applications and mathematical modelling. A knowledge of applications refers to an understanding of “any representational relations whatsoever between the real world and mathematics” (Blum & Niss, 1991:40).

A key area of focus for the new Project Maths curriculum in Ireland is mathematical applications while Blum et al. (2002) found that, in recent years, many initiatives focussing on improving the teaching of mathematical applications have been undertaken worldwide. Furthermore Burkhardt (2006) found that mathematical applications and modelling are now common features in the mathematics curricula in countries around the globe. Therefore, it is now crucial that teachers develop a knowledge of this aspect of mathematics in order to share such information with students and researchers must be aware of this knowledge domain when creating future models of teacher knowledge. As with the other knowledge domains discussed to date this domain is informed by other areas of knowledge while also serving to enhance teachers’ levels of knowledge in other domains. Burkhardt (2006) found that developing an understanding of applications improves teachers’ levels of pedagogical knowledge as the teaching of applications requires a wide range of teaching strategies. Therefore when learning about applications teachers will also develop an understanding of such teaching strategies. As a result, teachers need to develop this type of knowledge in conjunction with those previously discussed rather than in isolation.

Due to the reform of the mathematics curriculum in Ireland it is now more important than ever for teachers to acquire a knowledge of mathematical applications. Without such knowledge teachers will find it impossible to deliver the new curriculum in an effective manner. Hence it is also of paramount importance that a new model of teacher knowledge is designed for Irish teachers so as to allow for the effective implementation and delivery of Project Maths.
In conclusion numerous different types of knowledge are required in order for a teacher to be able to teach effectively. The majority of these have been proposed by researchers in the past but another critical domain now required, due to recent developments in mathematics education, is the knowledge of applications. It is essential that teachers understand the importance of each of these domains, including a knowledge of applications, and appreciate how each can contribute to their classroom practices and teacher effectiveness. This understanding and appreciation can be facilitated through CPD or through initial teacher training. The latter will now be analysed in order to determine if teacher training is providing teachers with the knowledge required for teaching and in turn adequately preparing prospective teachers for the profession they wish to enter.

3.3 Teacher Training

“Learning to teach effectively requires that teachers have sustained opportunities to learn mathematics, about students, and about ways to help students learn particular mathematical ideas...before they begin teaching”
(Ball, 2001: 21)

Initial teacher training for second level teachers plays a significant role in helping teachers develop the different types of knowledge discussed in section 3.2. Comiti & Ball (1996) and Boero et al. (1996) note that such training should involve the expansion of mathematical knowledge, know how, approaches, thoughts and habits among prospective teachers of mathematics. The aim of this section is to analyse the current state of teacher training in colleges/universities (with particular focus on their attempts to develop teachers’ subject matter knowledge) as well as the role of teaching practice during this process.

3.3.1 Current State of Teacher Training

Comiti & Ball (1996) carried out extensive research into teacher training internationally. Their study found very differing approaches to teacher education worldwide. Table 3.1, overleaf, summarises their findings in relation to initial teacher training from a number of different countries:
<table>
<thead>
<tr>
<th>Country/Region</th>
<th>Approach to Initial Teacher Training</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scandinavia &amp; Holland</strong></td>
<td>- Teacher education is incorporated into departments of education within universities.</td>
</tr>
<tr>
<td></td>
<td>- It is common practice for a teacher of second level mathematics to be qualified at masters degree level.</td>
</tr>
<tr>
<td><strong>Italy</strong></td>
<td>- The courses in place for teacher training have remained relatively unchanged since 1923.</td>
</tr>
<tr>
<td></td>
<td>- Secondary teachers are required to have a university degree and it is at a college’s discretion whether or not students receive any pedagogical training.</td>
</tr>
<tr>
<td><strong>Germany</strong></td>
<td>- Since the 1970s initial teacher training has undergone many reforms.</td>
</tr>
<tr>
<td></td>
<td>- Teacher training involves two distinct elements: academic studies and practical experience (at Haupt &amp; Fach seminars)</td>
</tr>
<tr>
<td></td>
<td>- The primary goal of teacher training is to develop prospective teachers’ subject matter knowledge</td>
</tr>
<tr>
<td><strong>England &amp; Wales</strong></td>
<td>- Teacher training courses are largely dictated by the British government under the Teacher Training Agency but individual universities still control the content of a lecture and assessment.</td>
</tr>
<tr>
<td></td>
<td>- In recent years teacher training has moved from the apprenticeship approach to a focus on competencies.</td>
</tr>
</tbody>
</table>

Table 3.1: Approaches to Initial Teacher Training Worldwide

This overview of the work of Comiti & Ball (1996) displays the many different approaches that are in place for teacher education programmes across the globe. In Ireland each third level institute designs its own teacher training courses. However in order to be registered as a teacher of mathematics with the Teaching Council an individual must have a recognised teaching qualification and a degree in which mathematics features at least 30% of the time over a minimum of three years (Project Maths Implementation Support Group, 2010). Many other aspects of teacher training

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9 The Teaching Council is the professional body for teaching in Ireland.
also vary between different countries/regions and such differences often lead to problems in teacher training.

**Contact Hours**

One aspect of teacher training that differs between countries and even between individual institutions is the number of hours allocated to developing prospective teachers’ subject matter knowledge. Many studies have analysed teacher training from this quantitative perspective. For example Malaty (2004) found teacher training programmes in Finland yield extremely successful results and one reason for such success is the dedication of between 700 and 1200 teaching hours to the study of mathematics during teacher training programmes. The allocation of such time to developing teachers’ subject knowledge will have an effect on the standards of teaching and learning and it is no surprise, therefore, that international mathematical assessment reports have shown a strong performance from Finland. For example the TIMSS Report (1999) found the average score of Finnish students to be 33 points above the international average while the OECD Report (2010) found that in 2009 Finland’s mean mathematical score was again significantly higher than the OECD average. Not all countries/institutions are willing to invest such time in developing the knowledge of trainee teachers. In Genoa University, Boero et al (1996) found only 144 teaching hours were dedicated to education in mathematics while Comiti & Ball (1996) found that the I.U.F.M programme in place at Grenoble offers 392 hours of subject matter knowledge in first year and circa 70 hours in year two. The latter two studies indicate that many universities are failing to provide their students with enough time or modules in which they can develop their mathematical knowledge for the purpose of teaching and it is therefore worthwhile noting that both France and Italy were below the international average in the TIMSS Report of 1995 and 1999 respectively and in the PISA Report of 2009. The findings of the TIMSS and PISA Reports for these three countries suggest a correlation exists between the number of hours allocated to developing teacher knowledge during the tertiary education of prospective teachers and the mathematical performance of students at second level. It is reasonable to conclude that prospective teachers will continue to enter classrooms without the mathematical proficiency required of them until models like those in place in Finland are introduced globally.
Significance of Mathematical Modules
Another challenge facing teacher training programmes, according to Cuoco (2003), is the generic nature of many of the modules which prospective teachers must study. Ball et al. (2005) note that teachers require professional training to develop different, and in many cases, superior mathematical knowledge than that required by those in an arts programme or even those entering professions with high mathematical content. In an attempt to help teachers develop this superior subject matter knowledge, universities/colleges of education place prospective teachers into modules with future engineers, scientists or even astronomers. As a result prospective teachers are subjected to broad modules aimed at a much wider clientele (Cuoco, 2003). The mathematical content of such modules does not provide teachers with the relevant knowledge needed in second level classrooms and instead other, more relevant, modules are being omitted. Even (1993) argued that teacher training courses may not require more modules but instead different modules to allow for teachers to develop a more appropriate and comprehensive knowledge of mathematics. Change such as that advocated by Even (1993) is needed in teacher training in order to better prepare potential teachers in the subject matter they will be required to teach.

Communication and Collaboration
One final problem considered here is the lack of communication and co-operation between internal college departments. An example of this is evident between departments of mathematics and departments of education. According to Cuoco (2003), both these departments are currently involved in a power struggle to obtain control of mathematics education and the repercussions of this struggle are being felt by students. Instead of combining forces and enhancing the third level education of students these departments are working in isolation and students are given a number of mathematics courses and a number of education courses separately. Consequently they enter schools with a large number of educational theories and a substantial knowledge base but with no idea how to integrate the two.
In the main the challenges facing the majority of initial teacher training courses relate to the insufficient focus on mathematical knowledge. For example, Blanco (2003) found prospective mathematics teachers have inadequate levels of mathematical knowledge when they first enter the workforce and are ill prepared to teach mathematics due to shortcomings at third level.

In section 3.4 the true extent of the effects of these problems are shown when the current levels of knowledge among mathematics teachers are analysed. Due to these problems and their effects Brown & Borko (1992) and Ball (1990) concluded that knowledge for mathematics teaching needs to become the focal point of teacher training in order to enhance teachers’ understanding of mathematical concepts as well as to improve teaching and learning in secondary schools. The upcoming section includes an analysis of another aspect of teacher training; teaching practice, in the hope that it will provide a more comprehensive view of the initial teacher training practices currently in place.

3.3.2 Teaching Practice

“Student teaching, or the clinical field experience, is a component of virtually every pre-service teacher preparation program and is commonly considered to be an essential – if not the essential – element of these programs”

(Brown & Borko, 1992: 210)

Teaching practice is a valuable element of initial teacher training programmes and researchers have acknowledged this for many years. For example, Ernest (1989) found teaching practice to be a critical aspect of teacher training as it provided students with the best opportunity to develop the wide knowledge base required for their future profession. He accepted that practical experience was one of the best ways for teachers to develop their pedagogical knowledge, curricular knowledge as well as a much deeper knowledge of mathematics while Fennema & Franke (1992) also found that teachers learn mathematics indirectly when teaching it. They argue that some elements of a teachers’ knowledge base evolve through the art of teaching. In essence, teaching is a process within which new knowledge is created. However, a prospective teachers’ knowledge base will only improve if this aspect of teacher
training is designed and implemented correctly. Research shows that this is currently not the case.

The biggest problem in relation to the practical element of teacher training programmes is identical to the main problem being experienced in other elements of teacher training and that is the lack of focus on developing mathematical knowledge. Strong & Baron (2004) found that feedback given to trainee/novice teachers in California focussed, almost exclusively, on teaching and student matters with little or no focus on a teacher’s knowledge base. This problem was also explored by Brown et al (1999) in an English context and they too found that issues relating to classroom management, equipment availability, space and pedagogy took precedence over subject knowledge during the evaluation of a student’s performance in teaching practice by both tutors and co – operating teachers. Finally the National Numeracy Strategy which commenced in Britain in 1996 lists a number of items that tutors need to evaluate in order to comprehensively assess a trainee teacher. This list is comprehensive and includes aspects of teaching such as well structured lessons, good use of questioning, the use of demonstration, employing a suitable pace for a lesson and the inclusion of oral work. Despite the detailed nature of this checklist mathematical knowledge was never referred to hence indicating that they too place little emphasis on this facet of teaching during placement.

This seems to be the case in an Irish context too with little emphasis being placed on subject matter knowledge during the evaluation of students on teaching practice. The evaluation form used for teaching practice in UL is the same for all teaching degrees and therefore it mainly deals with generic issues such as classroom management, lesson coherence, teacher professionalism and planning. Only one of the eleven equally weighted sections on the assessment form refers to a teacher’s knowledge base hence highlighting the lack of attention and importance afforded to it.

The research presented in this section suggests that initial teacher training currently faces numerous problems such as the use of inappropriate mathematical modules that student teachers must complete and the lack of co – operation between different departments. However, the most pressing issue is the failure of the majority of institutions to place sufficient emphasis on a teacher’s package of knowledge during students’ academic studies as well as their practical experience. Carter et al. (1993)
found that very little change in knowledge resulted from teacher training programmes and these findings add credibility to the authors decision to tackle the issue of inadequate knowledge among teachers through CPD.

This problem has repercussions on the teaching and learning of mathematics as many teachers are entering classrooms without sufficient knowledge of the material they have to teach. Comiti & Ball (1996) acknowledge that this problem is resulting in teachers’ knowledge of mathematics being based on memories from their own time in school rather than a deeper understanding which they should have acquired during their time in third level. Such findings indicate that this is a problem that needs to be addressed immediately. This will not be an easy task as research has shown that teaching traditions are stronger than teacher education, so a change is needed in people’s thinking as well as in teacher training programmes in order for significant improvements in teachers’ subject matter knowledge to occur. At the moment however few improvements are evident in the levels of mathematical knowledge among teachers and the current low levels knowledge is a concern for all those involved in the education system.

### 3.4 Current Levels of Subject Matter Knowledge among Teachers

“…many teachers simply do not know enough mathematics”

(Post et al., 1988:210)

Post et al. reached this conclusion over twenty years ago yet there appears to be little improvement in this area in recent years. In this section the author discusses the current level of subject matter knowledge among teachers globally while special attention is afforded to the United States where teachers’ level of content knowledge has, in recent years, been identified as a major challenge facing mathematics education.

### 3.4.1 The Problem in the United States

In the wake of their under performance in the second and third TIMSS researchers in the United States began to investigate levels of subject matter knowledge among
mathematics teachers. The results were not encouraging. Schmidt et al. (1997) found US teachers to be weaker in subject matter knowledge than teachers in a number of other countries. Further research found that the U.S. public regularly lamented the weak mathematical knowledge of many mathematics teachers at both primary and secondary level (Ingersoll, 1999; Ball, 2001). Ma (1999), building on the work of Ball (1988), also carried out extensive comparative studies between China and America. She asked teachers in both America and China a number of mathematical questions (e.g. Compute $\frac{13}{4} \div \frac{1}{2}$ and then make up a story/problem which models this computation.). There was a significant difference in the quality of answers received from the two countries and Ma (1999) concluded that Chinese educators had a much deeper understanding of mathematical concepts than their American counterparts. It is worth noting, that China performed extremely well in the TIMSS reports that gave America such cause for concern.

Researchers in the U.S. have tried to explain these worrying findings and have identified two main causes. Firstly the prevalence of unqualified teachers in American secondary schools is contributing to these worrying statistics. Figure 3.1 and 3.2, overleaf, from the work of Ingersoll (1999:29) highlight the extent of this problem, especially within the subject of mathematics:
Figure 3.1: Percentage of Public Secondary School (grades 7 – 12) Teachers in Each Field without a Major or Minor in that Field

Figure 3.2: Percentage of Public High School (grades 9 – 12) Teachers in Each Field without a Major or Minor in that Field
These figures highlight how a large proportion of mathematics teachers in the U.S. are teaching without any qualifications in mathematics. When we see that at one time 34% of mathematics teachers in American secondary schools were unqualified to teach mathematics we cannot be surprised with findings that suggest that the levels of subject matter knowledge are exceptionally low in the U.S. More recently the work of Cuoco (2003) found that little has changed since 1994 as teachers of science, computer science or even history continue to be employed as mathematics teachers at second level, without any formal training. As a result there still exists a large number of teachers relying solely on the mathematics they acquired during their own primary and second level education. This problem is not restricted to the United States however and in section 3.4.2 and 3.4.3 it is evident that many other countries including Australia, England, Wales and Ireland have also encountered this problem.

Another factor contributing to this problem in the U.S., according to research, is the teacher training programmes currently in place. As discussed in section 3.3, teacher training programmes are poor and failing many of their students. One of the knock on effects of this problem is insufficient levels of knowledge among teachers. Ball et al (2005:14) further elaborated on this issue when they stated:

“Equally unsurprising is that many U.S. teachers lack sound mathematical understanding and skill. This is to be expected because most teachers – like most other adults in [America] – are graduates of the very system that we seek to improve. Their own opportunities to learn mathematics have been uneven and often inadequate…”

Overall it is clear that the majority of researchers that investigated levels of subject matter in the U.S. reached similar conclusions (Schmidt, 1997; Ma, 1999, Cuoco, 2003). They all agree that this is a serious challenge facing mathematics educators and one that needs to be addressed in the near future. However this problem is not confined to America and many other regions will also need to address this issue.

3.4.2 Levels of Subject Matter Globally

Insufficient levels of content knowledge, which have proven to be problematic in the U.S., have also been investigated in other geographical regions. For example Bagnall
(2002) found a similar problem to exist in Australia. Her work points to a serious shortage of mathematics teachers with high levels of subject matter knowledge involved in teaching at second level. This has led to a scenario similar to that discussed in the U.S. whereby a large percentage of teachers (40%) are recruited to teach mathematics without any qualifications in the subject area. Again, when such statistics are revealed it is unsurprising that Australia also performed poorly in the 2003 TIMSS report where it fell well behind Japan, China, England and even the United States.

The report written by Smith et al. (2004) also indicates that many teachers of mathematics do not have the required subject matter knowledge needed for teaching in England, Wales and Northern Ireland. Their report found that 30% of teachers relied solely on the mathematical knowledge they acquired prior to their A-levels as they have no other relevant qualifications. In addition to this, approximately 25% of teachers who are qualified to teach mathematics are currently employed to complete other tasks. These teachers with the required qualifications are likely to have a greater level of subject matter knowledge than those with no qualification in the area of mathematics teaching and so it is critical that they are reassigned to the mathematics classroom.

Finally research carried out in Spain by Blanco (2003) and in Taiwan by Huang (2002) highlights that the problem of deficient subject matter knowledge among teachers is a worldwide one. Both these researchers found, in their respective countries, that teachers lacked a deep understanding of mathematics and this was having a negative impact on the students in their classes.

Overall the analysis carried out to date has found that the level of subject matter knowledge among teachers is proving problematic worldwide. Similar findings have also emerged from the research that has been carried out in Ireland in this regard.

3.4.3 Levels of Subject Matter among Irish Teachers

In Ireland no studies have explicitly sought to quantify Irish mathematics teachers’ levels of mathematical knowledge. However in recent years numerous reports have referred to the pervasiveness of out-of-field teaching in Ireland. Firstly in 2007 the
Royal Irish Academy (RIA) found that there were numerous unqualified teachers without any element of the ‘package of knowledge’ required for teaching mathematics, teaching in Irish secondary schools. However it was not until a report by Ní Riordáin and Hannigan in 2009 that the true extent of this problem was revealed. One of the key findings of this report was that almost half of the mathematics teachers surveyed (48%) did not have a mathematics teaching qualification and hence were deemed to be out-of-field teachers. The majority of these teachers were deployed in Junior Cycle classes as the following table shows:

<table>
<thead>
<tr>
<th>Teaching Qual. in Maths</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Fourth</th>
<th>Fifth</th>
<th>Sixth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes (n = 168)</td>
<td>85</td>
<td>100</td>
<td>134</td>
<td>95</td>
<td>133</td>
<td>131</td>
</tr>
<tr>
<td></td>
<td>(51%)</td>
<td>(60%)</td>
<td>(80%)</td>
<td>(57%)</td>
<td>(79%)</td>
<td>(78%)</td>
</tr>
<tr>
<td>No (n = 156)</td>
<td>81</td>
<td>94</td>
<td>79</td>
<td>18</td>
<td>45</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>(52%)</td>
<td>(60%)</td>
<td>(51%)</td>
<td>(12%)</td>
<td>(29%)</td>
<td>(24%)</td>
</tr>
</tbody>
</table>

Table 3.2: Numbers teaching in each year by whether or not they had a teaching qualification in mathematics (% of total in teaching qualification category)  
(Ní Riordain & Hannigan, 2009: 15)

In addition to this Ní Riordáin & Hannigan (2009) found that these out of field teachers were also regularly used in ordinary level classes both at Junior and Senior Cycle. This means that these teachers, who rely primarily on the mathematical knowledge that they acquired in primary and secondary education, were teaching some of the most vulnerable students in our secondary schools (Smith, 2004).

Overall the problem of out – of - field teaching is prevalent in Ireland and although no study has been carried out to analyse current levels of knowledge among Irish mathematics teachers this finding alone would suggest insufficient knowledge among a large number of mathematics teachers. In the latter stages of this thesis the author’s own model of teacher knowledge is used to evaluate the current levels of knowledge of a small sample of mathematics teachers in Ireland and this will provide the readership with a greater insight into the nature of this problem in Ireland.
3.5 The Need and Means for Enhancing Subject Matter Knowledge

According to Resnick (2005), effective teachers are the cornerstone on which good schools are built. Therefore, improving levels of knowledge among teachers is one of the most important investments of time and money that those involved in the field of mathematics education can make. After an analysis of the current levels of subject matter knowledge among teachers, the current state of teacher training courses and the impact and importance of a teacher’s knowledge base (Chapter 2), it is apparent that both teaching and learning is suffering as a result of teachers’ mathematical knowledge. Initiatives must be undertaken to remedy this problem in the near future. Once such initiatives are implemented research has shown that teaching performance will improve (Mannullang, 2005). The challenge of improving levels of knowledge among mathematics teachers can be addressed in two different ways, namely through improvements at third level in pre service teacher education programmes or through Continuous Professional Development.

3.5.1 Possible Initiatives to help Develop Teachers’ Subject Matter Knowledge

The bulk of research points to the need to change the assessment procedures currently in place in third level institutes in an effort to improve the levels of subject matter knowledge among graduates. Researchers, such as Kennedy (1998), note that an important strategy for enhancing subject matter knowledge among prospective teachers is to change the assessment criteria hence forcing potential teachers to develop a proficient knowledge base prior to entering the classroom. She condemns the constant assessment of recitational knowledge at third level and instead calls for assessment to test more than a prospective teacher’s ability to recall facts and rules. A similar argument was put forward by Ball (1990) and Hill et al. (2005). They found that assessment must require third level students to demonstrate a deep understanding of mathematical knowledge that will in turn enhance their own explanations, demonstrations and representations in the classroom. Furthermore Madaus & Mehrens (1990) argue that assessment, if it is to be of any benefit, needs to measure potential teachers’ competency in the material they will be required to teach. Until such assessment procedures are put in place potential teachers will
continue to graduate with insufficient knowledge for teaching thus making any future initiatives even more difficult to implement.

However, as suggested by Shulman (1986), although such changes in assessment are important they will serve little purpose if implemented in isolation. Other schemes for enhancing the knowledge base of mathematics teachers also need to be looked at by third level institutes. Additional ideas that have been proposed by researchers include more time allocated to teaching practice as well as Teaching Councils insisting that mathematics teachers must specialise in the subject of mathematics during their time in college. Research has shown that many aspects of teachers’ knowledge evolve in the classroom and many argue that through the act of teaching much subject matter knowledge is constructed (Turner & Rowland, 2008; Fennema & Franke, 1992). As a result of such findings it is essential that more time be assigned to teaching practice during teacher training programmes. Sykes (1990) advocates increased teaching practice or apprentice like arrangements in schools as he found such initiatives to be paramount in helping teachers develop subject matter knowledge during their time as an undergraduate.

The work of Cockroft (1982) established that those who trained with mathematics as their first or specialist subject constituted good teachers. There is evidence in his work to suggest that those who specialised in the study of mathematics had a much better grasp of the underlying concepts and in turn proved to be more effective teachers. Hence, another possible way of improving subject knowledge among teachers is to ensure that only those who major in mathematics are entitled to teach mathematics at the highest level (Smith, 2004; Ball et al 2005). Smith (2004: 6) suggests consideration be given to the idea of awarding different certifications to mathematics teachers and such certifications would then determine the level to which teachers were allowed to teach:

“The Inquiry recommends that consideration be given to the introduction of new mathematics teacher certification schemes which award certification to teach mathematics only up to certain specified levels, e.g. Key Stage 3”

In conclusion, change is needed in order to improve the current state of teacher training and initiatives such as those proposed in this section will allow for teachers to enter classrooms with improved levels of mathematical knowledge. This in turn
will provide them with more opportunities to perform the act of teaching effectively. However graduation from college does not signal the end of their development of the knowledge required for mathematics teaching. Instead there are a number of ways that they can continue enhancing their mathematical knowledge throughout their careers thus enhancing their overall ability to teach effectively and such strategies are the focus of the remainder of this chapter.

3.5.2 Improving Levels of Knowledge through CPD

To date this chapter has looked at issues relating to the quality of teacher training programmes and the prevalence of out – of – field teachers. This research allowed the author to conclude that a large number of teachers in Irish schools have not yet had the opportunity to develop the knowledge required to teach mathematics effectively. CPD is one approach that will allow them to develop these knowledge domains in the short term. There are many more in-service teachers than there are pre-service and the majority of teachers who will be teaching in Irish schools over the next decade or two have had a poor third level experience and many are not even qualified in the field of mathematics. CPD for such teachers is essential and research has highlighted the pivotal role it can play in developing teachers’ subject matter knowledge (Watson, 2008; Smith, 2004; Wilson & Berne, 1999). However prior to designing a CPD intervention, it is necessary to investigate the purpose of CPD, the benefits which it has to offer and how effective interventions can be developed.

3.6 Purpose of CPD

Despite the changes that have occurred in mathematics education in terms of initial teacher training, the curriculum and teacher development, the purpose of CPD has remained stable and consistent. Research has pointed to three main purposes for CPD – change, growth and development (Griffin, 1983; Guskey, 1986; Basinger 2003).

CPD, despite its format, has always been viewed as an initiative that would help bring about change. According to Griffin (1983) and Guskey (1986) this change
involves a change in teaching practice, a change in attitudes and beliefs among teachers, a change in the learning outcomes of students and a change in teachers’ levels of knowledge. Chapter 2 has already pointed to the urgent need for change in attitudes among teachers of mathematics and improved learning outcomes for students in mathematics. Furthermore both Chapter 2 and the early stages of this chapter have advocated the need for improved levels of mathematical knowledge among teachers thus indicating the critical need for CPD in mathematics education. However the pressing question which remains is in what order should this change occur and how can CPD be designed in order to successfully fulfil this role?

Guskey (1986) believes that CPD results in change in a sequential manner. Figure 3.3, below, gives a clear indication of the way in which Guskey (1986:7) believes change occurs and CPD facilitators that wish to succeed in bringing about change should bear this in mind during the design process.

![Figure 3.3: A Model of the Process of Teacher Change](image)

In terms of mathematics education this type of change would mean mathematics teachers first attending a CPD program before implementing the proposed changes or initiatives in their own classroom. Once teachers witness change in their
mathematical understanding or even improved mathematical appreciation among their students a change in their beliefs and attitudes should follow. According to Guskey (1986) this model means that only practices which a teacher sees to yield a positive change in student behaviour are retained while all others are discarded. One obstacle facing the fulfilment of this role of CPD is the fact that many people, including teachers, fear change (Lortie, 1975). According to Lortie’s sociological study the risk of failure that often accompanies change makes teachers extremely reluctant to engage in any process that advocates drastic change. In addition to this the sociological idea of ‘habitus’, as discussed in Chapter 1, may lead teachers to view change as unnecessary because of perceived past success. Programme designers and facilitators need to be aware of these issues prior to the development or commencement of any CPD program. However if the issues of how change occurs and why people might be wary of or resist such change are taken into consideration then CPD programs should be seen to yield worthwhile change in both teaching and learning.

Teacher growth and development is widely viewed as another of the main purposes of CPD. For almost three decades now, researchers such as Fullan (2001) have been discussing how CPD contributes to staff development while at the same time helping teachers to ‘grow on the job’. In more recent years the work of Bassinger (2003) has again reinforced this idea that one of the primary functions of CPD is to allow for teachers to develop their knowledge and skills throughout their professional career. She argues that teachers need to keep abreast of the ever changing curricula, such as Project Maths, and engaging in CPD initiatives, designed with the intention of yielding growth and development among teachers, is the best way for teachers to successfully do this. Guskey (1986) found that this is why so many teachers are attracted to CPD programs. He acknowledges that some teachers are required to attend such programs as a result of government or school regulations; however, there is evidence in his work to suggest that the vast majority of teachers attend such courses in order to grow and develop in a professional capacity.

Overall the research carried out in this area suggests that change, growth and development are the main purposes that CPD courses serve. It is for these purposes and the desire to improve their teaching that teachers are willing to engage in CPD programs throughout their career (Berman & McLaughlin, 1978). It is crucial that
everyone involved in designing and implementing CPD programs are aware of the purposes that these programs must fulfil in order to attract more teachers to them. In recent years there have been calls for CPD to extend its focus beyond these three aspects .and there is now a consensus among researchers that future CPD initiatives should also focus on developing teachers’ subject matter knowledge (Resnick, 2005).

### 3.7 The Importance of CPD in Relation to Subject Matter Knowledge

Inadequate knowledge among mathematics teachers is a serious problem in Ireland and abroad. However CPD has proven to be a very efficient and effective way to tackle this problem. Finucane (2004) found teacher subject knowledge and teaching competence to be two of the areas most affected by CPD. Resnick (2005) states that policy makers need to ensure that professional development programs continue to have a strong focus on subject matter knowledge. Similarly Ball (2001) found that in order to develop mathematics education and allow for progression in the teaching and learning of mathematics in the future then CPD must concentrate on helping teachers to develop their subject matter knowledge. In recent years research in this area displays contradicting positions in relation to the current emphasis on subject matter knowledge during CPD initiatives. Finucane (2004) asked Irish subjects involved in her study to identify what they believed to be the focus of in service courses which they attended and from Table 3.3, below, it is evident that content knowledge and curriculum change were most commonly the focus of in service courses.

<table>
<thead>
<tr>
<th>Course Topics</th>
<th>Predominantly (%)</th>
<th>Often (%)</th>
<th>Never (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject Matter Oriented</td>
<td>74.5</td>
<td>20</td>
<td>5.5</td>
</tr>
<tr>
<td>Curriculum/ Syllabus Changes</td>
<td>69.6</td>
<td>14.3</td>
<td>16.1</td>
</tr>
<tr>
<td>New Teaching Methodologies</td>
<td>26.8</td>
<td>48.8</td>
<td>24.4</td>
</tr>
<tr>
<td>Whole School Planning</td>
<td>38</td>
<td>18</td>
<td>44</td>
</tr>
<tr>
<td>Classroom Management</td>
<td>5.3</td>
<td>13.2</td>
<td>81.6</td>
</tr>
<tr>
<td>Personal Development</td>
<td>10.5</td>
<td>21.1</td>
<td>68.4</td>
</tr>
</tbody>
</table>

Table 3.3: In – Service Course Content
This table shows that the majority of teachers involved in Finucane’s study believed subject matter knowledge was the focal point of most CPD courses while only 5.5% believed that it had never been the focus of a CPD initiative which they had attended. On the other hand, however, findings from the report of Smith (2004) show that not enough attention is being paid to subject matter knowledge as part of CPD initiatives. His research found that teachers, in Northern Ireland especially, believed that CPD needed to have a stronger focus on knowledge of mathematics and should be designed to help teachers develop mathematical proficiency. Such findings were also supported by the ACME Report (2002) as they found that there was an urgent need for CPD to help teachers develop their mathematical knowledge regardless of the qualification or grade they received in their initial teacher training programme. These three studies underline that there are contrasting views in relation to the amount of time currently being allocated to subject matter knowledge during CPD courses. However one aspect that all researchers agree on is that mathematical knowledge needs to be held in high regard by the developers and implementers of these courses and initiatives and the following sub section highlights the reasons why.

3.7.1. The Positive Effect of Knowledge Focussed CPD on Subject Matter Knowledge

CPD which focuses on developing a teacher’s knowledge base can have a significantly positive knock on effect on student learning (Resnick, 2005). Resnick (2005) found that student levels of attainment can be affected by such CPD and he notes that CPD which is rooted in subject matter knowledge can in fact have a strong influence on students mathematical achievement. This finding is supported by Ball (2001). She established that in countries, such as Japan and China, where students have performed above average in international assessments such as TIMMS, CPD initiatives concentrate mainly on mathematical content and subject matter knowledge. Therefore, CPD which focuses on subject matter knowledge allows for a positive change in student achievement while the work of Resnick (2005) demonstrates that the opposite is also true. That is, CPD initiatives which fail to focus on subject matter knowledge will fail to bring about change in either teaching
practices or student learning. Hence content knowledge needs to be at the centre of CPD initiatives in order for CPD to fulfil one of its most basic purposes – change.

Watson (2004) also found that CPD which concentrates primarily on developing subject matter knowledge can yield many benefits for both teachers and learners. The following list highlights the advantages that teachers can gain by studying mathematics during their professional career through such CPD initiatives, according to Watson (2004):

- Be able to use their own knowledge directly
- Be aware of new mathematical connections
- Be aware how to study mathematics
- Gain new insights into the teaching of mathematics
- Be aware of approaches to mathematics that are less procedural based.

This list highlights the importance of focussing on mathematical knowledge during CPD courses. For example, the author has already discussed the problem of mathematics teaching being too procedural but subject matter knowledge focussed CPD appears to offer a viable solution. In addition to this, Watson’s (2004) findings show that the benefits that knowledge focussed CPD has to offer will have a positive effect on student learning. If, for example, teachers become aware of new mathematical connections through CPD they will be able to bring this knowledge with them to the classroom thus helping students appreciate the relevance and links that exist in the world of mathematics and hence making it easier for them to understand interrelated concepts and ideas.

Overall, CPD initiatives which focus on improving teacher’ levels of knowledge often yield the best results. As a result it is important that future CPD initiatives, including the author’s CPD initiative, concentrate on developing the knowledge of mathematics teachers. Another way in which the effectiveness of a CPD intervention can be improved is through the method of delivery chosen for the CPD initiative. It is necessary therefore to analyse different methods of delivery.
3.8 Different Types of CPD

In any study that involves the design of a CPD intervention it is important for the researcher to develop a thorough understanding of the different forms of CPD that have been employed in the past. Once the researcher has developed this knowledge he/she will then be better positioned to choose the type of CPD intervention which will allow him/her to achieve the targets which have been set out for the intervention. The author looked at a number of different types of CPD interventions that have been proposed and researched in the past.

3.8.1 Traditional CPD

Traditionally CPD involves a large group of teachers being exposed to the thoughts, knowledge and ideas of an expert speaker on a particular theme or topic (Finucane, 2004). This type of CPD takes the form of a one day in-service course. However recent studies have shown that the depth of teacher change is directly influenced by the length of time they are involved in any CPD and one day courses will not allow for significant change in teachers’ levels of knowledge. (Shields et al., 1998). Furthermore traditional CPD tends to be extremely didactic in nature with very little input from attending teachers. There is little opportunity for participants to practice the ideas or innovations being presented in a real life situation and individual needs are not catered for with this approach (Fullan, 2001). As a result when teachers adopt the role of learner they are exposed to an approach to teaching that many parties involved in the field of mathematics education are trying to divert teachers away from (Lieberman, 1995). For this reason this method of CPD has been criticised for many years now (Corey, 1957; Howey & Joyce, 1983). Despite such criticism this traditional approach is the most prevalent type of CPD which teachers experience (Lyons et al, 2003). For example, 9 out of 10 teachers in their Irish study revealed that this was the only form of CPD they had ever experienced. However it is not the only option available and it is essential that other options are considered in order to improve the standard and variety of CPD available to all teachers.

Other options that have been proposed by a number of researchers include an increase in systematic, reflective practice among teachers, the formation of
mentoring systems in schools as well as the establishment of communities of practice in different geographical regions.

### 3.8.2 Reflective Practice

The use of reflection is an alternative approach to CPD that has been advocated to help practicing teachers develop their subject matter knowledge as well as other aspects of effective teaching. Kyriacou (1998) found that the majority of teachers reflect on their own practice in an informal but instinctive manner and such analysis enables teachers to improve their own knowledge base (Hill & Ball, 2004). Britt et al. (1993) also see the value of reflective practice as their work shows this approach to be central in helping teachers change and develop their own teaching as well as their attitudes and beliefs about teaching. Furthermore Turner & Rowland (2008) found reflection on one’s own teaching to contribute immensely to the development of a teacher’s mathematical knowledge as well as their understanding of the mathematics which they are teaching. However this practice, if it is to further enhance knowledge and skills among all teachers, needs to be more systematic and carried out by every practicing teacher on a regular basis. Its success, according to Eraut (1995), depends on teachers being provided with high levels of support from school management while much research has also shown that this reflection can be improved when carried out with the support of colleagues (Jaworski, 2001; Lerman, 2001). This, therefore, gives substance to another possible CPD initiative – the establishment of a mentoring system within schools or communities.

### 3.8.3 Mentoring Systems and Communities of Practice

Collaboration with others presents teachers with the opportunity to enhance their own subject matter knowledge:

> “Experience of doing mathematics...with others, in an environment that encourages listening, questioning and pedagogic reflection (which may be the teacher’s own classroom), develops and deepens mathematical knowledge both in and for teaching.”

Watson (2008: 1)
Her findings show that the advice and support that teachers receive from their colleagues as well as from the wider communities of practice enable them to significantly improve their knowledge of the material they are required to teach. There is also evidence in the work of Ling & Mackenzie (2001) to support this argument. They found that the establishment of communities of practice is a form of CPD that can have a positive and lasting impact on the teaching and learning of a number of school subjects including mathematics. According to Finucane (2004) this form of CPD involves the collaboration of teachers in a way that will allow for meaningful and purposeful professional development and often requires a facilitator who can offer a more detached perspective.

Watson (2008) also states that all such communities of practice should involve mentors/facilitators who offer support to teachers while they are developing their skills and mathematical knowledge. This idea of mentoring has already proved successful in England and Northern Ireland following its introduction in 1998 and its success has resulted in financial investment from industry and commerce (Smith, 2004). The way this mentoring system works in the British Isles is that highly skilled teachers must first pass a rigorous assessment in order to become an Advanced Skills Teacher (AST) and once they attain this role they continue to teach but also dedicate some of their time to working with teachers in their locality in order to help raise teaching and learning standards. Conway & Sloane (2005) found such mentoring systems to be one of the best ways to allow for a change in teachers’ knowledge base with minimal disturbance to teaching and learning. They acknowledge that the idea of ‘coaching’ is still new in an educational setting but argue that in order for every type of teacher to be able to enhance their teaching ability, initiatives need to move away from the “one size fits all model” (such as the traditional approach to CPD) and begin to look into individually helping teachers through such mentoring/coaching initiatives.

3.8.4 Possible Other Forms of CPD

The three forms of CPD discussed above have been analysed in great detail in recent years and the latter two are now being promoted in order to revolutionise the current state of CPD. However many more under researched options are also available. A
Chapter 3 Changing Practice in Mathematics Classrooms

list of possible options from the work of Finucane (2004), Smith (2004) and Hyland and Hannafin (1996) is presented below:

♦ Workshop discussion with colleagues
♦ Training of trainer’s model
♦ Model Mixing
♦ Action Research Model
♦ Demonstration lessons by Leading mathematics teachers
♦ Conferences/working seminars run by professional subject associations
♦ Conference attendance.

These are all viable options for CPD programs but more research on their effectiveness and feasibility is required before they can be promoted with conviction nationally and internationally.

Numerous forms of CPD can be implemented in schools and the author investigates and employs a variety of these ideas during her own fieldwork, as discussed in Chapter 7. Over the past two sections the author has described the decisions that are necessary in order to ensure an effective CPD intervention. For example choices relating to the focus of the CPD intervention as well as the mode of delivery, contribute significantly to the development of an effective CPD intervention programme. The benefits of such effective CPD initiatives are outlined in section 3.9.

3.9 Benefits of CPD

Previous sections in this thesis have highlighted how teachers can enhance their knowledge base substantially as a result of engaging in effective CPD initiatives while their students’ levels of achievement have also been seen to improve. However these are not the only advantages associated with teachers engaging in lifelong learning initiatives. The work of Desimone et al (2002) and Borko (2004) shows that CPD has a wide range of benefits to offer for both teachers and students but to date this area is under researched in mathematics education. Despite this, research has already brought some of these benefits to our attention including the change in attitudes that is associated with engagement in CPD.
In Chapter 2 the author documented the findings of the LMS (1995) and Phillipou & Christou (1998) in relation to the poor attitudes that currently exist among teachers. In Chapter 2 the serious nature of this problem and the impact it has on student learning and students’ attitudes towards mathematics was analysed. However CPD can play a positive role in improving teachers’ attitudes towards mathematics. The study of Guskey (1986) found that teachers who engaged in CPD programs which yielded positive results reported having a more positive attitude towards teaching after such courses. Flecknoe (2000) also discovered this to be the case. His work showed that after engaging in CPD courses, teachers can be seen to demonstrate more constructive attitudes towards mathematics as well as the art of teaching. Furthermore he found that if some teachers in a school were willing to participate in CPD courses then often times the attitudes of the entire teaching community in that school is seen to improve (Flecknoe, 2000). This means that even the attitudes of teachers who did not engage in the CPD programme can be enhanced by observing the change in the attitude and behaviour of their colleagues after they participate in such courses. Overall research has highlighted the significant and positive impact that CPD can have on teachers’ attitude and this alone is a sufficient reason for encouraging teachers to get involved in such programs.

However it is not the only factor that may encourage teachers to engage in such courses. Just as poor attitudes among teachers can lead to poor attitudes among students (Phillipou & Christou, 1998) improved attitudes among teachers can lead to improved attitudes among students. Flecknoe (2000) found that after teachers engaged in CPD and introduced new initiatives and teaching approaches in the classroom their students showed a desire to continue learning and stay involved in the education system. Borko (2004) also reported that attitudes changed and confidence increased among students of teachers who engaged in CPD. Therefore CPD affects not only the attitudes, confidence and beliefs of teachers who engage in it but can also help improve the attitudes of the students they teach. Hence, CPD has been shown to help remedy another critical problem that is currently widespread in mathematics education.

Another issue that the author discussed in Chapter 2 was the lack of motivation of teachers entering the profession (Boero et al., 1996). Again this problem can be counteracted through engagement in CPD. In recent years researchers have found
teachers’ levels of motivation and commitment to be enhanced through their involvement in CPD initiatives. CPD plays a role in helping to motivate teachers. The work of Smith (2004) demonstrates how successful CPD programmes can lead to a more motivated, eager and enthusiastic teaching force in mathematics. As with many other professions, teachers need to face new challenges and develop new skills regularly in order to keep them motivated and eager to do the job they do. The work of Finucane (2004) also supports this idea of CPD resulting in increased motivation among participants while her findings also show that participation in CPD courses demonstrates a commitment to teaching that, along with motivation, has often been lacking among newly recruited teachers in recent years.

The benefits of engaging in CPD are plentiful. Researchers have shown that CPD can lead to increased levels of motivation and commitment among teachers, more positive attitudes for both students and teachers and a greater knowledge base for teachers. Researchers also highlight how CPD can help inform teachers of the ever increasing number of applications for mathematics in our everyday lives and helps them to better comprehend student thinking (Smith, 2004; Borko, 2004) hence further contributing to a teachers’ package of knowledge. Such benefits further justify the author’s decision to tackle the problem of teacher knowledge in Ireland through CPD.

Despite this positive picture of CPD, programmes have yet to be perfected and many problems still exist. It is only when such problems are identified and addressed that teachers and all those involved in the education system will begin to see and experience all the possible benefits of CPD.

### 3.10 Problems with CPD

In spite of the array of benefits that we have seen associated with CPD researchers such as Borko (2004) have found that CPD is still being neglected and overlooked by those involved in the education system. There is a consensus worldwide that more time needs to be allocated to the development and improvement of CPD if it is to continue yielding worthwhile benefits (Macnab, 2000). There are many problems still associated with CPD. Such problems include the low numbers of teachers
Chapter 3  Changing Practice in Mathematics Classrooms

attending these programmes, the structure and timing of many of these courses, the contradictory nature of CPD and the numerous needs that CPD is failing to meet.

3.10.1 Contradictory Nature of CPD

A problem that has been identified by many researchers in mathematics education is the prevalence of didactic teaching in the mathematics classroom (Rogers, 2002; Irish Mathematics Society, 2001; Hourigan & O’Donoghue, 2007). The identification of this problem has led those in the field of mathematics education to encourage teachers to change from this style of teaching and instead try to adopt an approach that allows for student involvement and interaction in a lesson. This approach has been commended by researchers such as Lieberman (1995) but it is very difficult for teachers to develop such an approach when they themselves continue to be exposed to didactic methods of teaching during their own learning experiences. According to Finucane (2004), the top down approach is currently dominating the majority of CPD courses with most having little or no input from teachers. She notes that teachers at these courses are treated as ‘passive receivers’ of information and they are not given the opportunity to test or question any of the material ‘handed down’ to them. Lyons et al. (2003) also found that CPD courses favour the didactic approach to teaching. Their investigation showed that ninety percent of teachers who engaged in CPD were taught in a traditional, didactic manner. The author has previously discussed Watson’s (2004) ideas in relation to creating communities of learners in the classroom in order to enhance the learning that occurs. Despite being encouraged to implement such strategies in their classrooms teachers have very little exposure to such communities of learners in their own professional development (Borko, 2004). Such findings highlight that although people involved in education are advocating a shift from didactic teaching, their actions during CPD courses are promoting the opposite.

This problem with CPD is making it extremely difficult for teachers to bring ideas and initiatives that they learn at these courses into the classroom in an innovative manner. As a result the courses are not proving as beneficial as they could be. As previously suggested, one method for counteracting this problem would be for those
involved with the design and implementation of CPD programmes to become less dependent on the traditional style of CPD and instead test new methods such as mentoring systems for establishing communities of practice. Only then, when teachers themselves are immersed in the learning process and given the opportunity to test new ideas and initiatives in a controlled and supporting environment, will they begin to understand and appreciate alternative approaches to teaching that they can use in their classroom.

3.10.2 Numbers Attending CPD Programmes

Another problem that CPD currently faces is the small number of teachers that attend the programmes. Wilson & Berne (1999) highlight how only a select number of teachers are truly motivated to engage in CPD while Boero et al (1996) found that many teachers fail to participate in CPD of any kind. The reasons for these small numbers are yet to be conclusively identified but the risk of failure and the unwillingness on the part of teachers to openly admit they require assistance have been cited as possible contributing factors. The work of Boero et al (1996) also shows that not only do teachers fail to enrol for professional development programmes but they also do not read journals or reviews written for teachers of mathematics, they do not attend meetings related to their subject and some do not even subscribe to associations for mathematics teachers. As a result many qualified mathematics teachers in schools worldwide have had no experience of mathematics since their own time in tertiary education and may be totally unaware of advancements that have taken place in the field. This problem also means that many teachers’ subject matter knowledge is limited and maybe even outdated while many others may be blissfully unaware of a large number of the applications of mathematics as well as having very little knowledge of new, effective approaches to teaching. In Ireland the problem has not yet been researched thoroughly but some of the findings that do exist are cause for concern. For example in the 2001/2002 academic year Finucane (2004) found that only 67% of teachers involved in her study attended in-service courses. However the findings of Finucane (2004) give no further information in relation to other forms of professional development that
teachers may have engaged in (e.g. reading journals) and so the author is unsure of the true extent of this problem in Ireland.

As with the previous problem discussed this issue is having a detrimental effect on the development of teaching and learning internationally. A solution, according to Smith (2004), may be to make CPD a condition of service. He argues that the contracts of all teachers should include a clause that requires them to attend CPD programmes throughout their term of employment and in this way it would be guaranteed that every teacher, including those unwilling to ask for help and those who fear failure, was in some way developing their own knowledge and skills throughout their career. Furthermore over three decades ago Berman & McLaughlin (1978) found that extrinsic rewards such as monetary incentives would not entice teachers to engage in CPD and instead they had to be intrinsically motivated. However as time has progressed so too have many people’s beliefs. In a world driven by financial gain the opposite is now thought to be true. Darling-Hammond & McLaughlin (1995) found that career advancement and monetary rewards were two of the three factors which attracted teachers to CPD. As a result the provision of pay increases or job promotion is considered by many as another viable solution to this problem.

3.10.3 The Structure and Time Allocated for CPD

The final two problems to be looked at in relation to CPD are the structure and timing of CPD initiatives. These issues have already been touched upon by the author in section 3.10.1. In relation to the structure of CPD the ‘one size fits all’ model is not having the desired effect. Flecknoe (2000) found that this approach to professional development is only working in a minority of schools while no gains are being experienced by the majority of students, teachers and schools. It is now time for those involved in the development of CPD programmes to acknowledge the fact that no two school settings are identical, no two learning experiences are alike and no two teachers approach lessons in the same manner. As a result CPD cannot be implemented homogeneously (Finucane, 2004). CPD must now begin to focus on individual needs so that everyone who devotes time and effort to CPD can experience the benefits that it has to offer.
“...if you really want to alter teaching practice, you need more than a two hour workshop” (Kennedy, 1999: 2)

The final problem to be looked at in relation to CPD is the amount of time spent engaging in it. The time currently allocated to CPD is insufficient (Kennedy, 1999). This problem needs to be addressed if we are to witness significant changes in teachers’ levels of knowledge as a result of CPD. Irwin et al (1999) and Cuoco (2003) have both condemned the one day in-service course that currently constitute professional development in a number of countries worldwide. Furthermore Cuoco (2003) found that attempts to allocate more time to CPD by designing a set of these one day seminars over a number of weeks is also proving ineffective. Instead CPD initiatives must be allocated more time on consecutive days/weeks depending on its purpose and only then will it result in worthwhile teacher development (Irwin & Britt, 1999)

An abundance of research has been carried out into general issues in professional development in recent years. Although CPD has much to offer both students and teachers when structured and conducted correctly its current state leaves a lot to be desired. Many problems still exist in this area of teacher development and until such problems are overcome, CPD will continue to result in fragmented learning and development for teachers (Wilson & Berne, 1999). The author aims to avoid many of the shortcomings outlined in this section by analysing programmes that have already proved successful and looking at the policies in place worldwide for professional development. In this way the author will understand ways to counteract the problems facing CPD. This in turn will enable her to gain a deeper understanding of what constitutes effective CPD and how effective CPD initiatives have managed to overcome the age-old problems associated with CPD.

The following sections in this chapter involve a comprehensive analysis of CPD practices and policies globally to identify effective professional development initiatives in different geographical regions. This review will give the author a sound basis for devising an effective, informed and grounded intervention for use in the latter stages of this project.
3.11 CPD Policies and Practices Worldwide

In order to gain a deeper understanding of what constitutes effective CPD policies, practices in New Zealand, the United States, England, Wales and Northern Ireland are analysed. Each sub section is devoted to an investigation of one effective CPD initiative that is in operation in each of these regions and the different factors that have contributed to its success are discussed.

3.11.1 CPD in New Zealand

New Zealand is a country similar in size and population to Ireland and boasts strong gains in mathematics education in recent years. Prior to the turn of the century, researchers reported that the lack of a sustained CPD programme was preventing any real improvement in teaching in New Zealand and the levels of achievement in mathematics was becoming a major concern (Higgins et al. 2003). This led to researchers advocating a more sustainable professional development programme in order to fully fulfil its role (Thomas 1999). As a result of such research, there has been a greater emphasis on professional development in this region in recent years. According to the Ministry of Education [MOE] (2006), CPD is now highly valued in New Zealand as there now exists a strong belief that the strengthening of effective teaching will result in the enhancement of student achievement. They have shown effective CPD to be grounded in research and involves:

“…provision of advice, materials, specialist interventions and tools that reflect evidence – based curriculum pedagogical and assessment knowledge”

(MOE Annual Report, 2006: 9)

Money, which according to Garet et al. (2001) often restricts CPD initiatives, is also now being invested by the government in professional development in New Zealand. For example the budget published by the MOE (2006) shows substantial funds being allocated to CPD for the year ending 30/6/05 ($94,437,000) and this increased the following year to $100,711,000. These increased funds have contributed to a greater
focus on CPD and a number of different strategies have been investigated and implemented. Literacy and numeracy projects are now common practice in New Zealand and are yielding positive results while much attention has also been afforded to supporting the teachers of Maori and Pasifika students. These students have in turn showed significant advancements as a result of such initiatives (Education Gazette, 2005).

This greater appreciation for CPD in New Zealand has also occurred in line with a shift from the traditional approach to CPD to a more innovative, updated approach. Authorities in New Zealand have begun to promote communities of learners as they too see the benefits that researchers such as Watson (2008) and Ling & McKenzie (2001) have been promoting. Furthermore, and most likely as a result of the grounded approach that the MOE have adopted, initiatives are now in place to encourage teachers to engage in CPD. In line with the findings of Darling – Hammond & McLaughlin (1995), the MOE (2006) found that improved career pathways, will help increase the number of teachers attending such programmes and in turn the effectiveness of teachers in classrooms around the country.

This analysis of professional development in New Zealand highlights how in recent years the authorities responsible for CPD in this region are promoting a highly professional, evidence based approach to CPD. Many of the policies they have in place are up to date, effective and valued. This approach has also paved the way for many effective initiatives in this region including the CAS Pilot Project, Te Kotahitanga, the Literacy Development Project and the Numeracy Development Project (NDP). The author feels that the NDP initiative has much to commend it in the context of her own research.

Numeracy Development Projects (NDP)
This series of initiatives designed to improve mathematics teaching and learning in primary, intermediate and secondary schools around New Zealand commenced in 2000. There are five different projects currently ongoing namely:

♦ Early Numeracy Project (For years 0 – 3)
♦ Advanced Numeracy Project (For years 4 – 6)
As is the trend with CPD in New Zealand, this series of initiatives was only undertaken after extensive research was carried out which helped inform the designers and implementers of the programme of the most effective route to take (MOE, 2006). The overall aim behind the NDPs is to improve the professional capabilities of teachers and in turn to improve student performance (http://www.tki.org.nz/r/literacy_numeracy/num_projects_e.php). The main characteristics of these five projects are very similar with only slight variations between each and so only one project will be looked at in detail.

The Secondary Numeracy Project (SNP) has only been in operation since 2005 and has been reviewed for the 2005 and 2006 academic years. In its inaugural year 320 mathematics teachers across 43 schools participated while this number increased to 556 across 80 schools in 2006 (Harvey & Higgins, 2007). One of the most noteworthy aspects of this project was the ‘in – school facilitation’ used. This project involves one member of a mathematics department in any given school receiving training and support and then returning to his/her school setting and acting as a facilitator. Some researchers have shown that in order for CPD to be effective it must be conducted ‘off site’ away from the classroom (Finucane, 2004). However evaluations of the SNP highlight that conducting the programme ‘on site’ in this manner allows for teachers to see the ideas or strategies being implemented in their own classrooms on a regular basis. According to Bolster (1983) and Crandall (1983) change will only occur in teaching practice after teachers have had the chance to implement and observe new practices and so this is an extremely beneficial aspect of this initiative.

Support, both during and after initiatives, has proven to be another critical feature of effective CPD (Guskey, 1986; Abdal – Haqq, 1995; ACME 2002). Again this in school facilitation approach allows for such support. Evaluations of the SNP carried out in 2006 show that the in - school facilitator offers teachers support on a continuous basis and this enhances the programme’s overall credibility. Also the fact
that the facilitator will remain in the school once the programme is complete means that post initiative support is also readily available.

Finally, those involved with the design and implementation of this initiative in New Zealand opt for more alternative forms of professional development and this project is no different. This is another aspect of the programme that contributes to its effectiveness. Garet et al (2001) found that alternative forms of CPD yield more positive outcomes than traditional CPD. Professional development has also been found to be more effective when it involved groups of teachers from the same school or faculty (Newmann & Associates 1996). According to Harvey & Higgins (2007) the SNP often included up to 16 teachers from the one school while small schools involved in the project formed clusters to increase the number of teachers working in any one group. This allowed for the development of communities of practice as well as whole school participation in the project while at the same time promoting collegial support and as research has shown these factors have a positive impact on the project outcomes.

A number of the characteristics of the SNP have already been shown to yield positive outcomes and so it is unsurprising that benefits have been witnessed and experienced in both the teaching and learning of mathematics in New Zealand. The Education Gazette (2008) notes that students involved in this project in 2006 made progress on all topics while they also found it to have been a significant impact on teachers’ knowledge and practice. Meanwhile Harvey & Higgins (2006) also found that 43% of teachers felt that the SNP had been the cause for positive change in their teaching practice. In addition to this all teachers and facilitators in this review agreed that their knowledge of mathematics was enhanced as a result of this while they felt the initiative had an even more significant impact on their knowledge of teaching mathematics and their knowledge of how students learned mathematics. Finally, there were also significant improvements in students’ performance (Tagg & Thomas, 2007). The percentage of students in the top two stages\(^\text{10}\) of all three domains (multiplication, addition and proportions) increased by an average of 20% while average gains of 19% were also recorded in their performance on fractions, place

\(^{10}\)The stages are outlined in the Number Framework as set out in the New Zealand Curriculum 2006
value and basic facts. On the other hand the percentage of students in stages five or below in the first three domains fell by an average of 22.33%.

Overall the SNP is a very effective and beneficial professional development initiative and other initiatives could easily be modelled on such an approach. It adopted a grounded approach and as a result it demonstrates many of the features that researchers have shown to constitute effective CPD. This has enabled it to yield a wide range of worthwhile benefits for students, teachers and facilitators. Although this project is ongoing and people have acknowledged that it will take much more time and commitment from all those involved (Harvey & Higgins, 2006) it is a project from which much can be learned.

3.11.2 CPD in the United States

As was the case in New Zealand, in recent years there is increased attention on professional development for teachers in the United States. Researchers in the United States have now come to accept the fact that:

“Professionals begin their preparation in the university but do not arrive in the workplace ready to practice. They continue their preparation on the job” (Wise, 2007: 59)

One of the main committees responsible for professional development in mathematics teaching is the Professional Development Services Committee [PDSC], which is a subcommittee of the National Council of Teachers of Mathematics. Established in 2006 this committee is responsible for designing, implementing, reviewing and evaluating professional development initiatives for mathematics teachers in the United States. One of the principal aims of the PDSC is to provide coherent professional development for both teachers and principals throughout their careers (www.nctm.org/about/committees/index.aspx?commID=78). According to this committee, it is critical that CPD be both coherent and ongoing while researchers in this area have, for many years, argued that CPD needs to occur throughout ones career and in a coherent fashion in order for it to be successful (Feiman – Nemser, 1983; Garet et al, 2001). The No Child Left Behind Act [NCLB] (2001) also recognises the need for ongoing CPD right through one’s career and they
have shown that CPD of this nature can result in improved academic achievement among students in the United States.

As in New Zealand, the government in the United States have also heeded the research carried out by Garet et al. (2001) among others, and recognised the importance of funding in order to ensure the successful design and implementation of CPD programmes. As a result, according to the NCLB Act (2001), funds of up to $100,000,000 are readily available for professional development programmes that will enhance the teaching and learning of science and mathematics. Research carried out in recent years has found that CPD needs to be interspersed over time as this allows for more coherent and constructive programmes as well allowing for experimentation for teachers and the provision of support (Kennedy, 1999; Garet et al., 2001). The government in the United States has also realised this and they are no longer advocating one day courses for professional development but instead are seeking to fund initiatives that take place over time. Examples of such initiatives are summer workshops or institutes and professional development schools (NCLB, 2001; Wise, 2007; Messina, 2010). Both these initiatives occur over a period of time and do not yield immediate results. However it is hoped that in the future they will prove more beneficial as teachers receive better training/knowledge, more opportunities to experiment and more support than would be possible in a course that lasted one or two days.

Finally, two more aspects of CPD that are being promoted in the United States are the use of mentoring systems and a stronger focus on subject matter knowledge during CPD initiatives. Many of those involved in the area of professional development in the United States have begun promoting CPD that incorporates some form of mentoring system (NCLB Act, 2001; Wise, 2007). In addition to this, the same researchers have been advocating the formation of a much stronger alliance between second level schools and third level institutes and both this mentoring and collaboration has been shown in research to contribute greatly to effective CPD (Little, 1988; Finucane, 2004). Legislation in the United States has also forced CPD to concentrate more specifically on developing teachers’ content knowledge particularly in the areas of mathematics and science. As the author has already discussed funding is readily available in the United States to support the design and implementation of CPD initiatives, however when such funds are being allocated...
priority is given to those initiatives which focus on enhancing subject matter knowledge of mathematics and science teachers (NCLB Act, 2001). By prioritising the development of content knowledge in this manner, the government of the United States is aiming to ensure that well funded CPD initiatives are as effective as possible (Garet et. al, 2001).

In conclusion, as was the case in New Zealand, recent years have witnessed renewed interest in CPD in the United States. The research carried out in this area is being acknowledged by all those involved in teacher professional development and, as a result, initiatives grounded in this research are becoming more effective and worthwhile. A variety of different initiatives are now in operation in the United States and the majority appear to be following the guidelines set out by the variety of government bodies, laws and research that exists in this area. Such initiatives include the Eisenhower Professional Development Programme and the STAAR Project. The latter will now be analysed in greater detail.

**STAAR Professional Development Initiative**

The STAAR Professional Development Programme is a three tiered programme designed to enhance teaching and students’ algebraic reasoning through teacher professional development. Pedagogical content knowledge is the primary focus of this project. The project was implemented in two states – Colorado and Wisconsin – and it has been evaluated annually since 2003. One of the most significant aspects of this particular project was the time that was spent working with developing teachers’ knowledge and ability. Alibali et al. (2006) found that the project requires teachers to participate in a summer institute that runs over 5 days and then monthly professional development courses throughout a full academic year. In total teachers are required to attend workshops/institutes for approximately 54 hours in any given year and this extensive amount of time is what is required in order for CPD to yield effective results (Abdal – Haqq, 1995). The work of Garet et al (2001) demonstrates that prolonged initiatives also tend to promote coherence and this appears to be the case with the STAAR project. Connections existed between the different workshops in the STAAR initiative. The work that teachers did and the knowledge they acquired in each workshop was brought forward to the following one. Table 3.4, overleaf,
shows the goals, activities, and the knowledge developed in the first three workshops of this initiative and from this table it is evident that coherence and continuity were core features of this project.

<table>
<thead>
<tr>
<th>Workshop</th>
<th>Goals</th>
<th>Activities</th>
<th>Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Develop content knowledge to teach the PSC(^{11}) problem</td>
<td>Solve problem, debrief strategies. Plan lesson</td>
<td>Specialised content Pedagogical content</td>
</tr>
<tr>
<td>2</td>
<td>Analyse teacher’s role</td>
<td>Analyse video, focussing on the teacher’s role</td>
<td>PCK (content &amp; teaching)</td>
</tr>
<tr>
<td>3</td>
<td>Analyse student thinking</td>
<td>Analyse video, focussing on student thinking</td>
<td>PCK (content &amp; students)</td>
</tr>
</tbody>
</table>

**Table 3.4: Goals, Activities, and Knowledge Foregrounded in Each PSC Workshop**

The table above highlights the coherent nature of this project as it shows how each workshop session builds on the material covered in the previous session. For example in the first session content knowledge is developed and strategies for teaching particular problems are looked at while the following session involves teachers analysing how they actually put what they learned into practice and the teaching approaches they adopted for different problems.

In addition to highlighting the coherent nature of the initiative, Table 3.4 also serves to emphasise two more critical features of this project – the provision of feedback and the allocation of time for experimentation. A report written by ACME (2002) in the U.K. states that all CPD programmes should:

“…allow opportunities to relate theory to practice in the classroom”

(ACME, 2002: vii)

Such recommendations were acknowledged in the design of the STAAR initiative as the workshops involve a theoretical element first and then the opportunity for teachers to test what they have learned in a real life classroom setting. In addition to

\(^{11}\) PSC = Problem Solving Cycle
this teachers are provided with feedback on how they implemented their new skills/knowledge and the author has already pointed to research that found such feedback to be of paramount importance for effective CPD (Guskey, 1986).

The final aspect of this initiative which contributes to its effectiveness, is its focus on student learning. One of the stated aims of this initiative is to develop students’ understanding of algebraic concepts as well as their ability to solve algebraic problems. A third of the entire project is dedicated to achieving this (Alibali et al, 2006). This commitment to enhancing the mathematical experience, knowledge and ability of students again contributes to the project’s effectiveness as many researchers have found that one of the prominent characteristics of an effective CPD initiative is its focus on student learning as well as teacher development (Abdal – Haqq, 1995)

The STAAR initiative demonstrates many of the qualities of an effective CPD programme such as the allocation of extensive time periods, its coherent and connected nature, the provision of feedback, the provision of time and support for experimentation and its strong focus on student development. These qualities contribute to a range of benefits that have been associated with this initiative. According to Alibali et al (2006) this project has resulted in significant improvement in teachers’ content and pedagogical knowledge while tests carried out before and after participation in this programme suggest teachers gain a much deeper understanding of student reasoning as a result of involvement in this project. The favourable responses of teachers towards many aspects of this project such as the more thorough examination of student work and the use of self reflection and video analysis also highlight how many of the ideas presented in this initiative will continue to be used in the classroom hence making way for a more positive mathematical experience for students.

In conclusion, as with the SNP initiative in New Zealand the STAAR project has proved successful in the United States. As a result of the various benefits which STAAR yields many of its core characteristics, which have also been shown in literature to be effective, should feature in any new CPD initiative that is to be designed and implemented in the future.
3.11.3 CPD in England, Wales & Northern Ireland

CPD in both New Zealand and the United States has gathered momentum since the turn of the 21st century and both regions now have CPD policies and structures in place that are in line with research findings on this issue. Similar developments have not yet happened in England, Wales and Northern Ireland. It was not until 2006 that people in this region began to understand the urgent need for improved CPD and action was initiated; first with the publication of the ACME report and followed soon after with the establishment of the National Centre for Excellence in the Teaching of Mathematics (NCETM) and the Smith Report (2004). The current state of CPD in this region is not ideal and many of the problems long associated with CPD still exist. For example the problem of attendance, which was highlighted by Wilson & Berne (1999) and Boero et al (1996), continues to be an issue in England, Wales and Northern Ireland as the NCETM (2007) found that a large percentage of teachers still fail to engage in CPD courses or interventions.

Therefore, there is a serious issue in relation to mathematics teachers attending CPD courses but there is another finding from the report which indicates that there is a willingness on the part of teachers to attend CPD courses but there are very few opportunities for them to do so. Despite the fact that by March 2007 the NCETM (2007) had a directory containing over 600 learning opportunities for teachers they felt many of these initiatives were generic in nature and dealt with general issues such as behaviour and classroom management rather than focussing specifically on the teaching and learning of mathematics. As a result of the generic nature of the majority of courses there is very little focus on subject matter knowledge during any of the CPD initiatives currently in place and this in turn is having a detrimental effect on CPD effectiveness (NCETM, 2007). Consequently, teachers do not believe such initiatives to be worthwhile and the existence of these problems means that the problem of attendance remains.

Despite the numerous problems associated with CPD in England, Wales and Northern Ireland the future appears to hold some hope. Research is ongoing in this region in order to devise the best possible initiatives for the future and many of the recommendations from ACME (2002) and NCETM (2007) are being implemented. Any future initiative will be grounded in research and this can only serve to enhance
the effectiveness of such initiatives. For example researchers have found that teacher input is crucial when designing suitable CPD programs as their needs have to be attended to in order for any programme to be effective (Guskey, 1986; Kennedy, 1999). ACME (2002) advocate surveying teachers prior to the design of any initiative in order to understand, more fully, their needs. Another recommendation made by ACME (2002) is for any future scheme to provide support to participants subsequent to its completion, a recommendation in line with international research on CPD (Garet et al., 2001; Finucane, 2004). Similarly NCETM (2007) have made recommendations based on findings from international researchers. They advise that in order to develop logical and effective CPD it is necessary to incorporate a collaborative approach. This call for collaborative CPD as well as other recommendations including the provision of support during and after CPD and the allocation of substantial time periods to CPD initiatives is all supported by research in this field. If such recommendations are implemented then future programmes in this region will prove effective and worthwhile.

Overall despite the fact that CPD standards in England, Wales and Northern Ireland currently fall behind those set in New Zealand and the United States, the grounded approach that has been adopted in recent years offers hope for the future. Already some initiatives have begun based on the recommendations of Smith (2004), NCETM (2007) and ACME (2002) and one such initiative is The Mathematics Development Programme for Teachers.

**The Mathematics Development Programme for Teachers**
The Mathematics Development Programme for Teachers [MDPT] is a professional development initiative for non-specialist mathematics teachers, teachers who trained in mathematics a long time ago or simply those who feel they need to refresh or update their mathematical knowledge. The first noticeable feature of the MDPT programme is the timeframe in which it takes place. As with the NDP in New Zealand and the STAAR Project in the United States, this initiative has shifted from the traditional idea of a one day programme and the author has already analysed the positive research findings on this matter (Abdal – Haqq, 1995; Irwin & Britt, 1999). The MDPT now involves a substantial 30 day programme in one of the UK’s leading
universities and 10 days follow – up in schools. Therefore the contact time for this project is extensive and, as predicted by Garet et al. (2001), the allocation of longer time periods allows for follow up support to be incorporated into the CPD initiative. As the author has already discussed at length, such support has been found to be another critical aspect of effective CPD (Guskey, 1986; Abdal – Haqq, 1995; ACME 2002).

Course leaders play an important role in determining the effectiveness of any CPD initiative (Guskey, 1986; Wilson & Berne, 1999; Borko, 2004). Such findings were acknowledged when the MDPT programme was being designed and implemented as the course outline specifically states that the initiative is to be delivered by ‘skilled practitioners on a flexible basis’ (http://www.wlv.ac.uk/Default.aspx?page=12615). According to Borko (2004) facilitators can contribute greatly to the success of any professional development initiative but in order to do this they must have a good knowledge of content and participants and must carry out their duties in a flexible manner. As a result of such findings, the careful selection of course leaders for this initiative will enhance the overall effectiveness of the project.

The structure of this programme is another commendable feature. As discussed previously the programme involves participants being based in college for 30 days and in their own school for 10 days. Until now there has been much debate on whether on site CPD is more effective than off site CPD and vice versa. The evaluations of the SNP in New Zealand indicate that on site CPD can be extremely effective as it allows for experimentation and support in a setting familiar to participants. On the other hand Finucane (2004) found that teachers preferred to engage in CPD off site. With such uncertainty surrounding the most suitable location for CPD programmes the MDPT has obviously tried to cater for both preferences by structuring their project in such a way that allows for a mixture of both on site and off site CPD. This is likely to have a positive effect on the outcomes of this programme in reports published in the future.

Finally the mixture of teaching approaches used by facilitators results in this project again moving from the traditional didactic approach to a more modern approach. Garet et al (2001) state that teachers must be provided with access to alternative teaching methods in order for any programme to be considered worthwhile and this
is the case with the MDPT. According to the course prospectus this programme involves taught sessions, group work and e – learning meaning that teachers are exposed to all these teaching approaches during their time in this initiative. There is also an element of experiential and discovery learning as well as peer teaching involved in this programme and these are other approaches that teachers can take from the initiative and introduce in their own classroom.

In conclusion, despite the mixed state of CPD in England, Wales and Northern Ireland this is one project from which much can be learned. It is grounded in relevant literature and many of its features, such as its time span and choice of location, have already been shown to contribute to successful CPD elsewhere. Because no report has yet been published for this initiative it is hard to identify the exact benefits which such features yield for this project but once such a report is published the benefits of this well grounded project will be available for all to see.

3.11.4 Summary of Findings on CPD Globally

To summarise Table 3.5, overleaf, highlights the effective aspects of CPD initiatives in each of the three regions analysed in section 3.11. These findings are used to inform the initiative used in this project to help teachers develop their subject matter knowledge in Ireland.
Table 3.5: Findings on International CPD

<table>
<thead>
<tr>
<th>Country/Region</th>
<th>Effective Aspects of CPD Initiatives</th>
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<td>New Zealand</td>
<td>- Grounded approach needs to be adopted.</td>
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<td>- Provision of support is critical</td>
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<td>- On site facilitation allows teachers to witness the implementation of ideas thus encouraging change</td>
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<td>- Alternative approaches to CPD are effective</td>
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<td>United States</td>
<td>- Extensive period of time is required in order to witness worthwhile change and improvement</td>
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<td>- Coherence and connectedness of the different stages is critical</td>
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<td>- Provision of feedback is very important</td>
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<td>- Experimentation is needed in order for teachers to witness the theory in practice and see the benefits it can yield</td>
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<td>- CPD needs to focus on students as well as teachers</td>
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<td>England, Wales &amp; Northern Ireland</td>
<td>- Extensive time frame which allows for the incorporation of support</td>
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<td>- Carefully selected skilful and flexible course leaders</td>
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<td>- A mixture of on and off site facilitation allows an initiative to cater for all needs/desires</td>
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<td>- A mixture of approaches used when delivering the course content</td>
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3.12 Conclusion

In conclusion this chapter addressed two issues that are central to this doctoral thesis, namely teachers’ subject knowledge and Continuous Professional Development. For many years research has pointed to the critical importance of teachers’ levels of mathematical knowledge and researchers have spoken of how a concentrated effort is needed in order to improve the current levels of knowledge being displayed by mathematics teachers around the globe. In this chapter the author has shown how the issue of insufficient knowledge can be addressed at third level or through CPD.
Many simple initiatives that are available to third level institutes, which will allow them to enhance the tertiary education experienced by all prospective teachers and in turn lead to more knowledgeable teachers entering the workforce were proposed for consideration. Initiatives such as a change in assessment procedures and increased time for teaching practice have been advocated for many years now. Until such initiatives are implemented worldwide their impact will not be truly felt and the level of subject matter knowledge among teachers entering the workforce will remain a contentious problem for mathematics education.

In recent years, mathematics educators have come to accept that CPD plays a central role in improving the teaching and learning of mathematics. Research has shown that the more time teachers are willing to dedicate to CPD the greater the change that will occur in their practice (Resnick, 2005). In addition to this change, this chapter has investigated many of the other benefits associated with CPD such as improved subject matter knowledge, improved attitudes and improved motivation on the part of both students and teachers. However despite the array of benefits associated with effective CPD, many problems relating to CPD still exist including the prevalence of traditional, didactic CPD, the contradictory nature of CPD and the poor structure of CPD. These problems are preventing CPD from being as effective and worthwhile as possible and need to be overcome. Finally the author has looked at CPD initiatives that have succeeded in overcoming these problems in recent years such as the NDP, STAAR and the MDPT. Any future initiatives, including the authors’ intervention, should look at these as exemplars of good practice. Only then will teachers be provided with the best possible opportunities to continue their education throughout their own career and to improve their current levels of knowledge.
4. Methodology

4.1 Introduction

The purpose of this chapter is to discuss the research methodology and design employed by the author for this study. From the outset the author was faced with numerous possible methodological approaches, procedures and instruments. These methodological choices as well as a justification for the decisions made in relation to the selection of methods are discussed in this chapter. Literature has repeatedly pointed to the need to justify all methodological choices when conducting a study of this nature (Poulson & Wallace, 2003). All decisions and choices made by the author were done with the final outcome in mind, that is improving the quality of mathematics teaching in Ireland. The methods discussed in this chapter refer to the procedures and instruments employed in order to gather the necessary information; while the methodology is the analysis of these procedures (Cohen & Mannion, 1992).

All methodological decisions made by the author were made on the basis of a ‘fitness for purpose’ criterion. (Cohen et al., 2004: 104). The research started as a qualitative study and one of the critical outcomes of this was the design of a model
of teacher knowledge which is discussed in Chapter 5. In order to ‘test’ this model among teachers the author then employed qualitative and quantitative methods. The research was, at all times, guided by and supported by an in-depth literature review. However before this literature review was conducted it was essential that the research problem was first identified. The research conducted is unique in Ireland and therefore the methodology and methods chosen must best reflect and cater for issues in the Irish context.

4.2 Research Problem

The initial focus for this doctoral thesis was on the teaching of mathematical applications in Irish secondary schools. However this research problem was quickly widened in order to allow the author to study related factors such as teacher knowledge(s) for effective mathematics teaching. Ireland is currently witnessing fundamental change in the mathematics curriculum at secondary level with the new curriculum having a strong focus on applications (Project Maths Implementation Support Group, 2010). However no specific research into the knowledge requirement for teaching such content has been carried out nor has any investigation been conducted in Ireland into the broader issues of the knowledge required for secondary mathematics teaching.

Researchers have identified many characteristics that contribute to high-quality teaching and the majority of researchers agree that an extensive knowledge base is one of the fundamental attributes required in order to teach effectively (Ernest, 1989; Carlsen, 1991; Fennema & Franke, 1992; Irwin & Britt, 1999). Therefore in order to improve the standard of mathematics teaching it is necessary to identify requisite knowledge types and to improve the levels of knowledge among practising teachers in each of these domains. Only when such developments occur will meaningful change transpire in the standard of mathematics teaching and learning.
4.3 Research Objectives and Questions

This section outlines the research objectives which guided this study as well as the questions which the author sought to answer over the course of this doctoral thesis.

4.3.1 Research Objectives

The principal objectives of this research project are:

1. To investigate, establish and clarify key issues in mathematics education internationally and in an Irish context with specific attention afforded to the areas of teacher knowledge and CPD.
2. To investigate the models of mathematics teacher knowledge proposed by researchers at both primary and secondary level with a view to use in the Irish context.
3. To develop a model of teacher knowledge which has a strong focus on applications and that meets the needs of mathematics education in Ireland.
4. To develop, support, corroborate and validate the author’s model using various approaches including a ‘Proof of Concept’ approach.
5. To further investigate the model through a CPD intervention.

4.3.2 Research Questions

The research questions which guided this doctoral study were determined by the different phases of the study, as discussed in Chapter 1. Table 4.1, overleaf, outlines the research questions which the author wishes to address:
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<th>Phase</th>
<th>Questions</th>
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| Phase 1 | 1. What are the main issues affecting mathematics education internationally and in Ireland?  
2. What role does teachers’ knowledge play in improving the quality of teaching?  
3. Do teachers internationally have a knowledge base fit for purpose e.g. secondary mathematics teaching, and are third level courses adequately preparing prospective teachers for the task of teaching?  
4. What constitutes effective CPD and what are the characteristics common to CPD initiatives that have proven successful internationally with specific reference to mathematics teachers?  
5. What is the current state of CPD in Ireland and what problems, if any, are being encountered? |
| Phase 2 | 1. What models of teacher knowledge have been put forward by previous researchers and what common links exist between these?  
2. What issues need to be addressed in the design of a model of mathematics teacher knowledge for secondary mathematics teachers?  
3. How can the author adapt and develop a model of mathematics teacher knowledge that is fit for purpose in Irish secondary schools? |
| Phase 3 3A | 1. How do teachers perceive the author’s model of the knowledge required for teaching?  
2. How do teachers perceive the adequacy of their current levels of knowledge in the domains outlined in the author’s model of knowledge?  
3. Which knowledge domains do teachers believe to be most critical for mathematics teaching?  
4. Do the teachers feel competent in all elements of the author’s model of teacher knowledge?  
5. What changes do teachers feel could be made to the questionnaire which will be used in phase 3B? |
## Phase 3 ctd.

1. How do teachers rate the types of knowledge domains outlined on the author’s model?
2. What are the current levels of knowledge in the model’s domains among this group of mathematics teachers?
3. What aspects of the author’s model do these teachers find most problematic?
4. Is there a difference between these teachers’ procedural understanding and their relational understanding?
5. Are the knowledge bases of those teachers involved in this small scale study significantly more extensive that that expected of their students or is it limited to material covered in the textbook and on State Examinations?

## 3C

1. What knowledge domains must this CPD intervention focus on in order to be effective?
2. How can these knowledge domains be integrated in an effective manner?
3. What delivery method should be used when implementing this CPD intervention?
4. What are the relative merits of different approaches to CPD delivery?
5. How effective was the CPD intervention?
6. Did the teachers find the CPD intervention acceptable?
7. Did students enjoy being exposed to the material included in the intervention pack?
8. What recommendations would teachers make for future work in this area?

| Table 4.1: Research Questions | 1. How do teachers rate the types of knowledge domains outlined on the author’s model? | 2. What are the current levels of knowledge in the model’s domains among this group of mathematics teachers? | 3. What aspects of the author’s model do these teachers find most problematic? | 4. Is there a difference between these teachers’ procedural understanding and their relational understanding? | 5. Are the knowledge bases of those teachers involved in this small scale study significantly more extensive that that expected of their students or is it limited to material covered in the textbook and on State Examinations? | 1. What knowledge domains must this CPD intervention focus on in order to be effective? | 2. How can these knowledge domains be integrated in an effective manner? | 3. What delivery method should be used when implementing this CPD intervention? | 4. What are the relative merits of different approaches to CPD delivery? | 5. How effective was the CPD intervention? | 6. Did the teachers find the CPD intervention acceptable? | 7. Did students enjoy being exposed to the material included in the intervention pack? | 8. What recommendations would teachers make for future work in this area? |
Chapter 4  Methodology

In order to achieve the objectives and to address the large number of research questions outlined in this section a combination of desk and field research was conducted by the author. However the author had to first identify the theoretical frameworks which would underpin this study. These frameworks are discussed in section 4.4.

4.4 Theoretical Frameworks

This section examines the theoretical considerations which underpin this research study. The theoretical frameworks incorporate or cite the main features of a given theory and in this project provide scaffolding for the author’s model. A variety of models of teacher knowledge have been proposed by researchers and the models analysed for the purpose of this study include a purposeful selection of these models. For example the author chose:

1. Mathematics specific and non–mathematics specific models.
3. Models with a number of knowledge domains ranging from three to six.

The author, like many researchers of teacher knowledge, began by analysing the generic framework of teacher knowledge proposed in the work of Shulman (1986) where he identified three types of knowledge domains critical for teaching. The author then narrowed her focus to concentrate on mathematics specific models and four such models were analysed. Figure 4.1, overleaf, outlines the five main frameworks analysed by the author for the purpose of this study and identifies the important characteristics of each.
Figure 4.1 highlights the theoretical models used to underpin this. These frameworks are discussed and analysed in detail in Chapter 5.

4.5 Chronology of the Research

Figure 4.2, overleaf, gives an overview and timeline for this research study.
### Figure 4.2 Chronology of Research

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<td>8</td>
<td>Writing Tasks</td>
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4.6 Research Design

This section details the research design that was implemented through the use of a wide range of methodologies.

4.6.1 Research Paradigms

According to Cohen, Mannion & Morrison (2000) research paradigms allow us to interpret social reality and provide us with a basis for understanding this reality while Denzin & Lincoln (2000: 33) refer to paradigms as a framework for interpretation led by “a set of beliefs and feelings about the world and how it should be understood and studied.” Burrell & Morgan (1979) submit that being situated in a particular paradigm is simply to view the world in a particular way. Lincoln & Guba (2000) outline five different research paradigms, namely:

- Positivism
- Interpretivism
- Critical Post-Modernism
- Constructivism
- Participatory.

However only two of the five outlined by Lincoln & Guba (2000) are used regularly in educational research. These are positivism and interpretivism and both are employed in this research study in a mixed methods approach.

The interpretivist or qualitative paradigm is broadly defined as any form of research that does not rely on statistical procedures or other forms of quantification to produce findings (Strauss & Corbin, 1994). A more detailed definition is offered by Denzin & Lincoln (2000: 3):

“…a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world…into a series of representations, including field notes, interviews, conversations, photographs, recordings and memos to the self. At this level, qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers
Due to the nature of qualitative research many researchers also refer to it as the subjectivist paradigm as it is seen as a paradigm which is highly subjective i.e. influenced by individual bias and relies heavily on the researchers’ interpretation of events or comments. Essentially the interpretist paradigm involves the researcher developing an understanding of a situation or phenomenon from the subject’s viewpoint (Lincoln and Guba, 2000). For the purpose of this study this paradigm is employed in order to gain an insight into teachers’ perspectives on the knowledge domains which are critical for the task of teaching and their opinions in relation to their own competency in these domains.

The second paradigm which is used regularly in educational research is the positivist paradigm also known as the quantitative paradigm. The quantitative paradigm seeks to describe, predict and/or control phenomena of interest through the use of numerical data (Leedy and Ormond, 2005). Quantitative purists believe that by following rational methods of enquiry the researcher, using this paradigm, can validly and reliably find regularities, identify relationships and discover the causes of a particular phenomenon. In this study this paradigm will be used to quantify the levels of knowledge of a sample number of teachers and to position them on the Ladder of Knowledge.

One of the fundamental, and possibly most notable, differences between the positivist and the interpretist paradigm is that the positivist paradigm is essentially objectivist, and it is possible for the researcher to remain detached from the study and uninvolved with it (Al Zeera, 2001) while the interpretist paradigm is highly subjective. However other differences also exist between the two and these differences are looked at in greater detail below.

**Positivist vs. Interpretist**

Researchers have debated the merits of both these paradigms but no one paradigm has yet been identified as superior to the other. Researchers in favour of the interpretist paradigm claim that this approach yields more in-depth findings and allows researchers to get a more detailed picture of social life through observational
data. On the other hand those in favour of the positivist paradigm argue that only when human behaviour can be expressed numerically can it be accurately measured (Jones, 2007). Many differences between the two paradigms have also been identified over the past number of years and some of these differences, which were put forward in the work of Leedy and Ormond (2005) and Krauss (2005) are outlined below:

- When using qualitative methods, reality is constructed by the study’s subjects and as a result reality may be constructed differently by different individuals. Alternatively, if a quantitative approach is adopted, reality is independent from participants (Krauss, 2005).
- The positivist paradigm leads to the focus of the study being on general trends while the inquiry focus is on the study of multiple social realities when using the interpretivist paradigm (Leedy and Ormond, 2005).
- The positivist paradigm may involve hypothesis testing and allows for variables of interest to be expressed as a numerical scale. On the other hand the interpretivist paradigm involves identifying patterns that emerge from qualitative data by thoroughly analysing the data (Leedy and Ormond, 2005).
- The qualitative paradigm involves the study of individual cases and requires the researcher to make verbal descriptions of the observations. Conversely, the quantitative paradigm relies on the use of mathematics to represent and analyse features of interest (Krauss, 2005).

Despite the differences between these two paradigms the benefits of both make it very difficult for one paradigm to be chosen at the expense of the other. So for the purpose of this research study the author chose to use both paradigms in her research and in doing so adopted a mixed method approach.

**Mixed Methods Approach**
The unique benefits of both qualitative and quantitative methods have already been highlighted and it is believed that combining these methods through a mixed method approach allows for researchers to gain different perspectives. A mixed method approach is one that combines or associates both these forms of research (Creswell, 2009). According to researchers a mixed method approach is advantageous as it
allows researchers to consider different voices and perspectives, answer both exploratory and firmatory research questions and investigate multifaceted issues (Deacon et al., 1998; Tashakkori & Teddle, 2003). The author uses a mixed method approach as opposed to a multi method approach as the qualitative and quantitative methods are used for the same purpose, which is to pilot the model of teacher knowledge (Cook & Reichardt, 1979). This mixed method ensures the research findings are triangulated. In addition to this Creswell (2009) found a project to be strengthened when qualitative and quantitative approaches are used in tandem.

The three phases of this study, outlined in Chapter 1, required different research methods and Table 4.2, below, highlights how a mixed method approach was employed at different phases of the project.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Qualitative</th>
<th>Quantitative</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Phase 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Literature Review</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>Investigation into the area of teacher subject knowledge</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Review of CPD</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td><strong>Phase 2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analysis of models of mathematics teacher knowledge</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>Design of the Ladder of Knowledge</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td><strong>Phase 3</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Phase 3A: Focus Groups</td>
<td>✓</td>
<td>x</td>
</tr>
<tr>
<td>Phase 3B: Questionnaire/Test</td>
<td>x</td>
<td>✓</td>
</tr>
<tr>
<td>Phase 3C: Design, implementation and evaluation of CPD intervention</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 4.2: Mixed Method Approach Used in this Research Study
4.6.2 Research Framework

Having analysed the five theoretical models the author made an informed judgement on the research framework which she would use for the purpose of this project. The research framework adopted is similar to that proposed by Creswell (2003: 5). His framework for design details how three different elements of inquiry merge to form different research approaches and in turn these approaches are translated into design processes of research. This framework permitted the author to recognise gaps in the research, identify her research problem, decide on the research approach that is required and design research tools accordingly. These three elements allowed the author to identify the approach to inquiry and this approach is detailed in Figure 4.2 below:

<table>
<thead>
<tr>
<th>Elements of Inquiry</th>
<th>Approaches to Research</th>
<th>Design Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>4. Data Analysis</td>
</tr>
<tr>
<td></td>
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<td>5. Write Up</td>
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<td></td>
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<td>6. Validation</td>
</tr>
</tbody>
</table>

Figure 4.3: Knowledge Claims, Strategies of Inquiry and Methods Leading to Approaches and the Design Process

Mixed method approaches must be of a sequential nature in order to adhere to this research design (Creswell, 2009). This study began with a qualitative investigation into the phenomenon of teacher knowledge and this qualitative analysis then led to the design of a model of teacher knowledge. Subsequently the model was prototyped among teachers through qualitative and quantitative means before all qualitative and quantitative findings informed the CPD initiative. This CPD intervention was evaluated through quantitative methods.
4.6.3 Multi Phase Approach to the Research Study

Figure 4.4/1.1\textsuperscript{12}, overleaf, clearly highlights the multi phase approach adopted by the author for the purpose of this research study. In total there were three main phases required to complete this project described as follows.

\textsuperscript{12} Same figure as that which appeared in Chapter 1
Improving Mathematics Teaching at Second Level Using an Intervention Aimed at Developing Teachers’ Subject Knowledge

Began with Phase 1
Consisted of:
- A review of the literature on general issues in mathematics education
- An investigation into the area of teacher subject knowledge
- A comprehensive review of issues relating to the current provision of CPD
- An analysis of the packages of knowledge required for mathematics teaching as proposed by researchers in the past

Informed Phase 2
Consisted of:
The design of the “Ladder of Knowledge” – a model of the knowledge required by Irish mathematics teachers

Was developed, supported, corroborated and validated by

Phase 3

Phase 3A
Required the author to
Conduct focus groups in order to pilot the questionnaire/test to be used in phase 4 and to obtain teachers views on the knowledge required for mathematics teaching.

Phase 3B
Required the author to
Design and distribute a questionnaire to ascertain levels of knowledge among a sample number of teachers and to obtain their views on the importance of different knowledge domains for the purpose of mathematics teaching.

Phase 3C
Required the author to
Design, implement and evaluate an innovative CPD intervention based on the Ladder of Knowledge.

Figure 4.4/1.1: Research Design
Chapter 4

Methodology

**Phase 1**

Phase 1 incorporates a review of the literature on the general issues which are affecting mathematics education both from an international perspective and in Ireland. It identifies problems that are affecting mathematics teaching and learning internationally and assesses whether these problems are (a) occurring in Ireland and (b) impacting on interest, uptake and attainment levels in mathematics in Ireland. In addition to this review, this phase also requires the author to investigate the idea of mathematics teachers’ knowledge. It involves an analysis of the current levels of knowledge demonstrated by teachers worldwide as well as issues relating to the knowledge required for mathematics teaching, the quality of teacher training programmes and the importance of subject matter knowledge for the purpose of mathematics teaching. Finally Phase 1 also investigates the issue of CPD. The author identifies the purpose of CPD, discusses different types of CPD, analyses the importance of CPD in relation to mathematics teacher subject knowledge and evaluates CPD initiatives that have been conducted worldwide. This phase allows for the development of a detailed methodology and allows for informed methodological decisions to be made in Phase 2 and Phase 3 of the project.

**Phase 2**

This phase involves two key elements fundamental to the research, namely the review of theoretical frameworks and the development of the model of teacher knowledge. This phase first requires a review of the literature which relates to the package of knowledge required for mathematics teachers. As previously outlined the author analyses five different models in this phase and identifies common features across all five. These models then frame and inform the remainder of the research study. The latter stage of this phase involves the design of a new model of teacher knowledge. This model is informed by the literature review and investigation carried out during Phase 1 and the analysis of models of teacher knowledge which is conducted in the earlier stage of this phase. As a result the model designed by the author is grounded in research and this grounding ensures that the model is fit for purpose in an Irish context. This phase is informed and to an extent driven by national priorities related to the reform of secondary school mathematics in Irish schools through the initiative called Project Maths. The author has identified a
pressing need for foundational subject knowledge (mathematics) in Irish secondary school mathematics teachers and an urgent need for CPD interventions.

Phase 2A

The model designed during the main stage of phase 2 then needed to be piloted among practising Senior Cycle teachers. The topic selected for the trial was calculus as:

“Calculus is one of the milestones of human thought. Every well educated person should be acquainted with the basic ideas of the subject. In today’s technological worlds, in which more and more ideas are being quantified, knowledge of calculus has become essential to a broader cross section of the population”

(Blank & Krantz, 2006: vii)

Blank & Krantz (2006) emphasise the fact that calculus is one of the most important topics that students will study in Senior Cycle mathematics (upper secondary school). In addition to calculus playing a critically important role in the world around us, a wide range of disciplines depend heavily on a thorough understanding of calculus. As a result, for many students, calculus is their entry point into third level courses in mathematics, science, business and engineering (Sabella & Redish, 1998). Furthermore Adams (2003) found that by developing an understanding of calculus students will in turn develop useful skills and acquire means to analyse problems in a wide range of fields of interest.

Despite the evident importance of calculus, research has shown students perform badly in this topic. Internationally (Orton, 1986: 660) found that:

“...in the case of differentiation and integration there never was much understanding of the underlying concepts”

Orton (1986) is not the only researcher to reach such conclusions. For example Gleason & Hallet (1992: 1) found that calculus was mainly taught as a set of rules and procedures and requires students to engage in ‘mindless algebra practice’ while Feffer (2010) suggested that in the United States there exists high failure rates in
calculus at both high school and third level. This is having a knock on effect on the number of students entering and graduating from science and engineering courses.

Irish research has noted similar problems in this regard. The most recent Chief Examiner’s Report (2005) found that a high percentage of students, 86% at ordinary level and 91% at higher level, chose to complete the calculus questions on the Leaving Certificate paper. The results were adequate. On the ordinary level paper the average mark out of 50 in the three calculus questions was 29. This is compared to an average mark of 32 out of 50 in the 3 algebra questions, 34 out of 50 on the two arithmetic questions and 33 out of 50 on the two discrete mathematics and statistics questions. Similarly at higher level the average mark for the four calculus questions (35 out of 50) was less than the average mark in the 3 geometry questions and the two trigonometry questions and only equivalent to the average mark attained by students in the three algebra questions. In addition to these statistics the comments and recommendations provided by the Chief Examiner show that these students hold fundamental misconceptions and misunderstandings in relation to the topic of calculus. For example his comments suggest that:

- Students can apply the rules of differentiation but cannot apply their knowledge to real life contexts or to unseen and unfamiliar questions.
- Students cannot identify any links between differentiation and the slope of a line.
- Students encounter significant problems when they are required to manipulate formulae.
- Students fail to extract clear meaning and conclusions from their work.
- Students cannot apply their knowledge of calculus in real life situations and therefore do not understand how to use calculus to solve problems, abstract and generalise.

These comments highlight a lack of understanding on the part of students and demonstrate how rote learning and procedural approaches to teaching dominate during the teaching and learning of calculus in Irish second level schools (Hourigan & O’Donoghue, 2007).
In addition to this, as discussed in Chapter 2, each year in UL students studying service mathematics modules are required to take a diagnostic test in their first mathematics class to help identify at-risk students. 8 topics including algebra, trigonometry, geometry, co-ordinate geometry, arithmetic, complex numbers, modelling and calculus are tested in this examination. When deciding on the topic to focus on for the purpose of this study the author also analysed the diagnostic test results of the 2009 cohort. In total 505 students, who had come directly from Irish secondary schools\textsuperscript{13}, sat this exam in 2009 and of this cohort 180 had studied mathematics at higher level while the remaining 325 studied ordinary level mathematics for Leaving Certificate. The author analysed the results in each topic and again problems appeared to arise with calculus, among higher level students as Table 4.3, below, outlines.

<table>
<thead>
<tr>
<th>Topic</th>
<th>No. of Questions</th>
<th>Average Percentage</th>
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<tbody>
<tr>
<td>Calculus</td>
<td>5</td>
<td>59.2</td>
</tr>
<tr>
<td>Algebra</td>
<td>6</td>
<td>79</td>
</tr>
<tr>
<td>Geometry</td>
<td>4</td>
<td>85.9</td>
</tr>
<tr>
<td>Modelling</td>
<td>1</td>
<td>83.8</td>
</tr>
<tr>
<td>Trigonometry</td>
<td>3</td>
<td>63.3</td>
</tr>
<tr>
<td>Arithmetic</td>
<td>13</td>
<td>69.4</td>
</tr>
<tr>
<td>Co-ordinate Geometry</td>
<td>4</td>
<td>59.6</td>
</tr>
<tr>
<td>Complex Numbers</td>
<td>2</td>
<td>44.72</td>
</tr>
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</table>

Table 4.3: Analysis of Higher Level Students Performance in Different Topics in the 2009 UL Diagnostic Test.

Table 4.3 shows that the set of calculus questions was one of the poorest answered among higher level students with the average result being below the average result achieved in 6 of the 7 other topics tested.

When analysing the results of ordinary level students the author omitted questions on trigonometric differentiation, exponential differentiation and integration topics which the majority of ordinary level students will not have encountered at second level. As

\textsuperscript{13} I.e. Mature or non-standard students were not included in this analysis.
a result ordinary levels student performance in one simple differentiation question was analysed. When asked to differentiate $5x + 3$, 16% of ordinary level students submitted an incorrect answer. Again this highlights gaps in some students knowledge of calculus on graduation from second level and indicates that a number of students struggle with the very basic concept of calculus.

Overall these findings from the diagnostic test results in 2009 and those of the Chief Examiner’s Report (2005), coupled with international findings, show that secondary level students struggle with calculus. Despite its importance in future study and professions, students appear to have a poor grasp of many fundamental calculus concepts and they are unable to apply such procedures to unfamiliar or real life problems.

Consequently, as part of her research study the author investigated teachers’ levels of knowledge in the topic of calculus and to see if their knowledge was adequate for teaching this topic. The project also requires the author to design an intervention based on this topic that may help improve teachers’ understanding of the applications, relevance, historical origins and the background of many calculus concepts. If the knowledge of the sample group of teachers could be improved in these critical areas it would be hoped that they would then be able to incorporate more effective teaching strategies, for example better explanations, and make much of their newly acquired knowledge and resources available to their students when teaching the topic of calculus.

Once a topic was selected the author could then design the necessary resources and tools that would help her to pilot the Ladder of Knowledge in Phase 3.

**Phase 3**

As outlined in Chapter 1, phase 3 is subsidiary to phase 2 and serves to develop, support, corroborate and validate the model which was designed in phase 2. It involves three subsidiary phases, namely phase 3A, phase 3B and phase 3C.
Phase 3A

This phase serves a dual purpose. Firstly it provides, through the use of focus groups, an insight into the thoughts and attitudes of a sample number of practising mathematics teachers in relation to the knowledge required for teaching. This phase analyses how appropriate teachers believe the domains on the “Ladder of Knowledge” to be and how adequate they believe their knowledge to be in each of the domains. In addition to prototyping the model this phase involves piloting the questionnaire/test which is used in phase 3B.

Phase 3B

With phase 3A complete the author refined and distributed the amended research tool during phase 3B. This phase involves analysing the current levels of knowledge of a sample of teachers (a different sample from that used in phase 3A). The questionnaire evaluated teacher’s levels of knowledge in each of the domains outlined on the “Ladder of Knowledge”. The subsequent quantitative analysis provides an insight into any gaps in these teachers’ knowledge bases and informed the researcher about the content that needed to be included in the CPD intervention which was designed, implemented and evaluated in phase 3C.

Phase 3C

The final sub-phase of phase 3 requires the author to design, implement and evaluate a CPD intervention. This CPD intervention was informed by the literature review from phase 1, the author’s model of teacher knowledge which was designed during phase 2 and the qualitative and quantitative analysis carried out in the previous sub-phases of phase 3. As a result this final sub-phase brings together every other phase conducted by the author as part of this research study and serves to further corroborate the model of teacher knowledge.

Overall this sequential, multi-phase approach will enable the author to develop a model that, although similar to those previously proposed, will include unique aspects that will cater for the changing demands and obstacles facing mathematics teachers in Ireland. In addition to this, the approach adopted by the author allows for
this newly designed model to be tested among practising teachers using a ‘Proof of Concept’ approach and provide insights about whether this new model may also be capable of acting as a vehicle for improving levels of knowledge among mathematics teachers.

4.7 Phase 3A (Qualitative Study – Focus Groups)

Once the ‘Ladder of Knowledge’ was designed it was then piloted among a sample of teachers. The first phase in this piloting procedure was the qualitative study. The qualitative study follows a grounded theory approach. According to Creswell (2009: 13) this grounded theory approach is:

“...a strategy of inquiry in which the researcher derives a general, abstract theory of a process, action or interaction grounded in the views of participants”

Silverman (2005) describes how this approach allows researchers to start off with some data and, from this data theories can be extracted. It typically includes interviews, surveys, documents and observations and its most recognisable characteristic is that the data collected provides the author with an insight into the perspectives and voices of the people being studied (Strauss & Corbin, 1994).

The author used focus groups to collect data and generate insights. These semi structured focus groups provided the author with an insight into the thoughts, opinions and perspectives of second level mathematics teachers. As is explained in section 4.7.2, this data was analysed by the author and categorised and re-categorised as certain themes of interest emerged. These themes were then matched to previous research carried out in the area of teacher subject knowledge (Post et. al, 1998; Ball, 2001; Smith et al., 2004) and were also compared to the quantitative results obtained in phase 3B.

4.7.1 Semi Structured Focus Groups

Semi structured focus groups were selected as a vehicle to prototype the ‘Ladder of Knowledge’ for many reasons. This form of qualitative research enables participants to ‘bounce’ ideas off one another and as a result of this sharing of views they often
produce data and insights that may not have been attained in a one–to–one interview. For many years focus groups have been a popular research tool in market research (Hayes & Tatham, 1989). It is only in recent years that their popularity has seen significant growth in the social sciences (Morgan, 1996). Focus groups can serve a number of purposes and are often seen as effective research tools both when used in isolation and when combined with other research methods (Morgan, 1997). One of the most commonly discussed benefits of focus groups is what Stewart & Shamdasani (1990) labelled the ‘snowballing’ effect. In essence, this means that a remark made by one member of the group can very easily be picked up on by another and can in turn trigger a number of relevant thoughts, comments and ideas. Furthermore this group synergy also leads to participants comparing their experiences and opinions with others in the group and this collaboration provides the researcher with further insights into the thought processes and opinions of participants (Morgan & Kreuger, 1993).

Focus groups are reliant on interactions among the group on topics or questions that are supplied by the researcher who typically plays the role of moderator (Morgan, 1997). According to Morgan (1996) interview standardisation and researcher involvement help determine the nature of focus groups. The focus groups used in this study incorporated some elements of the funnel approach proposed by Morgan (1996). This approach offers a compromise between very structured and unstructured focus groups. The interview starts off by letting participants discuss their views on the topic in general before funnelling their thoughts to specific, pre determined questions. This funnel approach allows the structure of the focus groups to vary from group to group but the general open ended questions remain the same (Appendix A) thus making way for interview standardisation and this approach prevented participants drifting into generalities, a major problem for focus groups according to Merton et al (1990).

Open Ended Questions
Open ended questions were used throughout the focus groups in this study. Research has shown that there are many benefits to using this type of questioning in semi-structured focus groups or interviews. Frankfort-Nachmias & Nachmias (1996)
found that open ended questions are beneficial and worthwhile as they do not compel
the respondent to adhere to preconceived answers. Once the intent of the question is
understood subjects are free to express their opinions in their own words. Therefore
the researcher gets a more honest and personal response through the use of this type
of questioning and in turn is able to obtain a truer assessment of what the subjects
believe. Other advantages of this type of questioning are that the questions are
flexible, they encourage freedom of expression and they allow the researcher to test
the limits of respondent’s knowledge (Cohen & Mannion, 1994).

Prior to conducting the focus groups the author spent time deliberating and
researching ways to prepare and conduct effective focus groups. The open ended
questions, as well as some probing questions, were prepared in advance of the first
focus group and these general questions remained the same throughout all focus
groups. These open questions were designed so as to allow the author to gain an
insight into teachers’ thoughts and attitudes in relation to the knowledge required for
teaching as well as their perceptions regarding the adequacy of their current levels of
knowledge. Finally the open ended questions used in the focus groups allowed the
author to obtain teachers’ views on the suitability and practicability of the research
instrument used in phase 3B.

4.7.2 Details of the Research Instrument

After carrying out a review of the literature relating to focus groups and open ended
questions, the TAP paradigm was selected for developing the questions to be used in
the focus group. This paradigm, which was developed by Foddy (1993), requires the
author to be aware of three different factors when designing and developing the open
ended questions. These factors are detailed in figure 4.5 overleaf:
Figure 4.5: The TAP Paradigm

In essence the TAP paradigm advises that each topic/question introduced in the focus group should be explicitly defined in order to ensure that each subject understands the question and that the questions are applicable and have a clear perspective. By using this well established model the author obtained a plentiful supply of relevant qualitative data from the focus groups while simultaneously avoiding bias and ensuring reliability and validity (See Section 4.12).

Each topic introduced during the focus groups and the objective of each topic is outlined in table 4.4 overleaf:

<table>
<thead>
<tr>
<th>Topic</th>
<th>Applicability</th>
<th>Perspective</th>
</tr>
</thead>
<tbody>
<tr>
<td>The topic must be properly defined so that each respondent clearly understands what is being talked about.</td>
<td>The applicability of the questions to each respondent should be established. Respondents should not be asked to give information which they are not expected to have.</td>
<td>The perspective that respondents should adopt when answering the questions should be specified so that each respondent gives the same kind of answer</td>
</tr>
</tbody>
</table>
# Chapter 4

## Methodology

<table>
<thead>
<tr>
<th><strong>Topic</strong></th>
<th><strong>Objective</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Importance of Different Types of Knowledge</td>
<td>To gain an insight into teachers’ opinions on the importance they attribute to each of the different knowledge domains included in the author’s model and to see if they believe there is a need for teachers to develop all these types of knowledge.</td>
</tr>
<tr>
<td>Knowledge of Applications and Connections (i.e. the relevance of mathematics)</td>
<td>To ascertain the importance which teachers attribute to these knowledge domains, to see if these teachers disclose this type of information to students in their class and to assess how teachers perceive their own levels of competence in these domains.</td>
</tr>
<tr>
<td>Knowledge of History of Mathematics</td>
<td>To analyse teachers’ understanding of the historical origins of mathematics, to assess the importance they attribute to this knowledge domain and to investigate if their students are ever exposed to information relating to the history of mathematics during their classes.</td>
</tr>
<tr>
<td>Knowledge of School Maths &amp; Transformational Knowledge</td>
<td>To examine the importance which teachers attribute to the knowledge required to provide students with clear explanations and analogies, to investigate if teachers recognise the connections between school mathematics, real life and other subjects and to assess their perceived levels of knowledge in both of these domains</td>
</tr>
<tr>
<td>Pedagogical Knowledge</td>
<td>To determine the teachers’ perceived level of competency in this domain and to gain an insight into teachers’ thoughts regarding the importance of this knowledge domain.</td>
</tr>
<tr>
<td>Piloting the Research Instrument</td>
<td>To obtain teachers’ opinions on the suitability of the questionnaire and to ascertain if there are any gaps in the questionnaire or if they feel some material could be omitted from the questionnaire.</td>
</tr>
</tbody>
</table>

**Table 4.4: The main topics introduced during focus groups**
4.8 Data Collection (Focus Groups)

Prior to data being collected for Phase 3A the author had to identify her sample. The strategies used for selecting this sample are outlined in section 4.8.1.

4.8.1 Research Sample Employed for the Focus Groups

At first the author used convenience sampling to select the locations of the schools that would be contacted for this phase of the project. According to Cohen & Mannion (1992) this type of sampling involves the researcher choosing subjects/institutions based on their location and their proximity to where the study is being conducted. When drawing up a list of possible schools to contact the author selected 6 locations within a 100 km radius of her institution and another location near her home. This was due to professional and time commitments. The list of counties selected using this convenient sampling strategy were Cork, Clare, Limerick, Galway, Kerry, Tipperary and Dublin. In total this meant that the sample from which the individual schools could be picked consisted of 426 out of a total of 743 schools in Ireland. Once these counties had been identified a list of all schools in these counties was drawn up and the schools to be contacted were selected through simple random sampling. This strategy, according to Frankfort-Nachmias & Nachmias (1996: 186), is one which ensures that each member of the population, i.e. each school in these 7, counties had ‘an equal and unknown non-zero probability of being selected’. Initially two schools from each county (14 schools in total) were randomly selected and a letter was sent to the principal of each of these schools (Appendix B). The author wished to conduct four focus groups and, within two weeks, four of the fourteen schools contacted had replied to say they would be interested in partaking in the project. An overview of the research sample for the qualitative study is provided in Table 4.5, overleaf.
Region | Number of Subjects | Male Subjects | Female Subjects
---|---|---|---
Cork | 3 | 1 | 2
Galway | 4 | 1 | 3
Tipperary | 3 | 3 | 0
Kerry | 3 | 1 | 2

Table 4.5 Research Sample for Phase 3A by region and gender

This table highlights the small but geographical and gender representative sample. The sample was also representative in terms of age and teaching experience with the former ranging from 22 – 61 years while the latter ranged from 3 – 39 years.

Once the sample had been identified the author made contact either through email or through a telephone call with the lead teacher in each school. During this contact the teachers were provided with further information regarding the nature of the project while the researcher also outlined in greater detail what was expected of them during this phase. They were informed that their anonymity would be ensured and that they could withdraw from the project at any stage. Once all this information was provided to teachers a date and time was arranged when the author would travel to their school to conduct the focus group.

Finally with the sample in place and a number of teachers willing to participate the author travelled to the four different regions and conducted the four focus groups. Each of these focus groups were conducted at a time that suited all volunteering teachers and in order to accommodate volunteers the focus groups were conducted in the convenience of their own school. All focus groups were recorded, with subjects’ permission, using an IC recorder and once each focus group was complete the recording was transferred to the voice editing PC software Voice Editor. Using this software the author was able to transcribe each of the focus groups. Research shows that the transcribing of focus groups/interviews is essential in order to ensure that no valuable data is lost, distorted or overlooked (Cohen et al., 2000). Once the data had been transcribed into word documents the author was then able to analyse the data obtained from all four focus groups and from this, necessary conclusions could be drawn.
4.8.2 Data Analysis

The computer software package NVivo was used to analyse the data obtained from the focus groups. NVivo is a form of computer assisted analysis of qualitative data (CADQA) and research suggests that it has numerous benefits including allowing the author to store, fragment and organise large amounts of qualitative data in a bid to identify patterns and common responses. When using NVivo the author also employed a constant comparative strategy. According to Glaser & Strauss (1967) this constant comparative approach allows us to extract theories that are grounded in the data collected. The work of Boeije (2002) states that this strategy requires the author to conduct four key stages namely:

1. Delineate categories
2. Categorise the data
3. Code the data
4. Connect the categories and extract some theory or meaning.

The author conducted these four stages within NVivo as follows:

- The focus groups were led and informed by the topics and objectives outlined in Table 4.4
- All responses were recorded, saved as a word document and imported into NVivo.
- Following careful analysis a list of nodes emerged from the data. In total three main nodes were identified. Theses were Poor Levels of Knowledge, Problems with Mathematics Teaching and The Contrast between Ordinary Level Teachers and Higher Level Teachers. All interviews were first coded in NVivo under these nodes
- Once the nodes had been developed and finalised the author then analysed each of the nodes, identified trends both between and across nodes and extracted meaning from the data.
4.9 Phase 3B (Pilot Study – Questionnaire)

Phase 3B helped the author to further corroborate the model developed in Phase 2. The first step undertaken by the author was the development of the research instrument.

4.9.1 Development of the Research Instrument

At the start of Phase 3A the author designed a questionnaire (Appendix C) that contained five sections. The first section required the teacher to provide the author with some background information, sought their opinion on the importance of the knowledge domains included in the author’s model and investigated their perceptions of their own levels of competence in each of the knowledge domains. The findings from this section could then be compared with the opinions expressed by teachers during Phase 3A (Focus Groups) of the study. The subsequent four sections all sought to test teachers’ competency in each of the domains outlined on the author’s model. Table 4.6, below, highlights the focus of each section and demonstrates how each component in the author’s model was tested in this questionnaire/test.

<table>
<thead>
<tr>
<th>Section</th>
<th>Knowledge Domains Addressed</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Background Information</td>
</tr>
<tr>
<td></td>
<td>Opinions on the importance of different knowledge domains</td>
</tr>
<tr>
<td></td>
<td>Self perceptions in relation to competency</td>
</tr>
<tr>
<td>B</td>
<td>Procedural Content Knowledge</td>
</tr>
<tr>
<td></td>
<td>Relational Content Knowledge</td>
</tr>
<tr>
<td></td>
<td>Knowledge of Applications</td>
</tr>
<tr>
<td>C</td>
<td>Knowledge of Connections</td>
</tr>
<tr>
<td></td>
<td>Knowledge of School Maths</td>
</tr>
<tr>
<td></td>
<td>Knowledge of the History of Maths</td>
</tr>
<tr>
<td>D</td>
<td>Pedagogical Knowledge</td>
</tr>
<tr>
<td></td>
<td>Transformational Knowledge</td>
</tr>
</tbody>
</table>

Table 4.6: Knowledge Domains Assessed in Each Section of the Questionnaire
This questionnaire was designed with the intention of gaining an insight into where these particular teachers were positioned on the author’s Ladder of Knowledge (See Chapter 5) and to analyse how effectively the Ladder of Knowledge could act as a tool by which teachers’ levels of knowledge could be measured. The questionnaire is based on the author’s model of teacher knowledge and hence it is supported by the models put forward by Shulman (1987), Ernest (1989), Fennema & Franke (1992), Rowland (2007) and Ball et al. (2008). Each domain assessed in the questionnaire, with the exception of a knowledge of applications, has been identified as a knowledge domain critical for teaching, while the need for a knowledge of applications will be discussed in Chapter 5. Overall this questionnaire, although original and unique, is very much grounded in theory and this will help produce valid and reliable results (See Section 4.12)

4.9.2 Piloting the Questionnaire

According to Rattray & Jones (2007) it is critical that a new research instrument is piloted during the development phase. They believe that such piloting will allow the researcher to identify areas that lack clarity, are inappropriate or that discriminate between respondents. As this was a newly designed questionnaire it was essential that the author piloted it among a similar sample to those who would be completing it during the pilot study. The author used the teachers in the focus groups to pilot the questionnaire and hence the focus groups served a dual purpose (as mentioned previously). During the focus groups teachers were asked about the content of the questionnaire, its suitability, the length of the questionnaire and the wording of the questionnaire. As with all other feedback obtained during the focus group all issues regarding the questionnaire were recorded and analysed using NVivo. The feedback received from teachers was then considered and the necessary changes were made to the original questionnaire. Overall by piloting the questionnaire in this manner and amending the research instrument on the advice of teachers the author was ensuring validity and reliability in the pilot study.
4.9.3 Final Research Instrument

A copy of the research instrument used in this phase is attached (Appendix D). Although based on an original concept, the author’s model of mathematics teachers’ subject knowledge, the author was able to incorporate the Likert Scale in Section A of the questionnaire. This is a well researched scale that is used to obtain the personal opinions of subjects in a questionnaire. It is an ordered one dimensional scale from which respondents must choose an option provided that best matches their own stance or viewpoint on a particular issue. The subsequent sections of the final research instrument are fully based on the author’s model and will, as outlined above, assess teachers’ levels of competency in each of the domains on the Ladder of Knowledge.

4.9.4 Limitations of the Questionnaire

A serious issue in relation to the questionnaire relates to the uncontrolled test environment. Initially the author intended to conduct the questionnaires over a one hour period whereby she would sit with teachers while they completed the questionnaire. However when this was proposed during the focus groups teachers felt that their peers would be more willing to participate if they were given the questionnaire to do in their own time. This recommendation was heeded by the author in an attempt to ensure that more teachers were comfortable with the task and in the hope of increasing the response rate. In an attempt to ensure that teachers did the questionnaire independently and without assistance from resources available on the internet/in books the author requested that participating teachers sign a form stating that they completed the questionnaire without receiving assistance from a third party or from extra resources (Appendix H). All teachers signed this form and therefore despite the test not being carried out in a controlled environment it is hoped that the work they submitted was done independently.
4.10 Data Collection (Questionnaire/Test)

Once the changes recommended by teachers in the piloting stage had been implemented, the author sought to recruit subjects who would be willing to participate in this phase of the study. As with the focus groups the target audience was Senior Cycle mathematics teachers and initially it was the author’s intention to recruit fifty teachers for this phase.

4.10.1 Research Sample Employed for the Questionnaire

The sampling methods chosen by the author for this phase of the project are similar to those employed by the author in Phase 3A. As was the case when selecting subjects for the focus groups the researcher again chose to use convenient sampling in order to identify a list of schools. The same counties as those identified for the focus groups were used in this phase and as a result the author had 412 schools to choose from\(^{14}\). Once the author had identified the list of possible schools she then employed random sampling methods to select the fifty schools she would contact initially.

When the fifty schools had been selected the researcher sent letters and consent forms to the principal (Appendices E and F). The consent forms gave principals the opportunity to accept or decline the invitation to be involved in the study on behalf of the Senior Cycle mathematics teachers in their school. Many of these consent forms were not returned within two weeks and so the researcher followed them up with phone calls to the principal. At the end of this process only 26 teachers were willing to participate in the project and so it was necessary for the researcher to contact a further fifty schools. Again letters and consent forms were sent to all principals and follow up phone calls were made if necessary. At the end of this cycle a further 18 teachers declared an interest in participating. Due to time constraints, the author agreed to conduct the pilot study with these 44 teachers as opposed to the 50 teachers she originally sought. The breakdown of teachers from different counties is detailed in table 4.7, overleaf:

\(^{14}\) Only 412 schools were available for selection as schools who were contacted in Phase 3A were not contacted again in Phase 3B.
### Table 4.7: Geographical breakdown of teachers who agreed to participate in the study

<table>
<thead>
<tr>
<th>County (Province)</th>
<th>Number Teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limerick</td>
<td>16</td>
</tr>
<tr>
<td>Dublin</td>
<td>8</td>
</tr>
<tr>
<td>Galway</td>
<td>8</td>
</tr>
<tr>
<td>Clare</td>
<td>2</td>
</tr>
<tr>
<td>Cork</td>
<td>5</td>
</tr>
<tr>
<td>Tipperary</td>
<td>5</td>
</tr>
</tbody>
</table>

### 4.10.2 Conducting the Pilot Study

The 44 teachers who expressed an interest in participating in this phase of the study were then contacted by the researcher. The researcher sent all teachers an information sheet detailing exactly what was expected of them in this phase of the study (Appendix G). They were also informed that their anonymity would be ensured and that they could withdraw from the project at any stage. A suitable time for the researcher to call out to the school was confirmed at this point.

The researcher called out to each individual school and spent time with each teacher explaining the questionnaire, its purpose and how teachers should go about completing it i.e. how long should be spent on it etc. Teachers were also asked to sign a form stating that they completed the questionnaire independently and without the help of a third party or additional resources (Appendix H). Finally during this meeting teachers were also given a form in which they could express an interest in participating in the final phase of the project, Phase 3C (Appendix I). Teachers were asked to complete the questionnaire within the school week and the author returned to the school the following week to collect all completed questionnaires. On a number of occasions when the author returned to the schools some teachers expressed a desire to withdraw from the study and in total only half the questionnaires distributed were completed. Table 4.8 overleaf provides a gender and geographical breakdown of the teachers who completed this phase of the study.
Table 4.8: Geographical and Gender Breakdown of Teachers who Returned Questionnaires

<table>
<thead>
<tr>
<th>County</th>
<th>Male Teachers</th>
<th>Female Teachers</th>
<th>ID Withheld</th>
<th>Total Number of Teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limerick (Munster)</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>Dublin (Leinster)</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Galway (Connaught)</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>Clare (Munster)</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Cork (Munster)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Tipperary (Munster)</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>School Information &amp; ID Withheld</td>
<td>N/A</td>
<td>N/A</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Once the questionnaires had been completed and collected the author was able to analyse the data obtained during this pilot study.

4.10.3 Data Analysis

The data collected from the questionnaires was analysed using the statistical software package SPSS (Version 16 for Windows). All subjects were assigned a number and each question was then uniquely coded. Any missing data was also coded in order to ensure that no question in particular was answered with significantly lower frequency than other questions. When the data was entered the author first carried out frequency checks to ensure that all the data was entered correctly and to detect, at this early stage, any coding errors that may have occurred.
Chapter 4                                                                       Methodology

The majority of the data was analysed mainly using small counts and frequency tables. This information then allowed the author to represent the data graphically through the use of bar charts, histograms, pie charts etc. Due to the aim of this phase of the research, the ‘Proof of Concept’ approach adopted by the author and the small sample employed this analysis yielded no inferential statistics and instead the questionnaire simply sought to see where this particular group of teachers were positioned on the Ladder of Knowledge prior to them engaging in Phase 3C – the intervention.

4.11 Phase 3C (Intervention)

The final sub phase of phase 3 involved the design, implementation and evaluation of an intervention which was again based on the author’s model of mathematics teacher knowledge. An intervention can be defined as:

“An activity, process, event or system that is designed to correct a problem, or change a situation and improve performance”

[http://www.reproline.jhu.edu/english/6read/6gloss/glossps.htm]

As the definition suggests the author designed an intervention to correct, or at least improve, the current problem of poor knowledge among mathematics teachers. A second aim of this intervention was to see whether the author’s model could act as a vehicle to improve the levels of knowledge among practising mathematics teachers. The intervention designed for the purpose of this project was entitled The Calculus Tool Kit. This intervention ran over a ten day period and participating teachers were required to engage in the activity in their own free time during this period. Further details relating to the design, implementation and evaluation of this tool kit are provided in the forthcoming pages.

4.11.1 Design and Development of Intervention

In order to design an effective intervention it is crucial for the author to first gather sufficient information about the problem under investigation as well as the participants and their needs. The desk research conducted in phase 1 and phase 2 and
the qualitative and quantitative analysis carried out to date in phase 3 help ensure that the author is well informed in each of these areas and is in a position to design an intervention that addresses the challenge in question, improving the knowledge of teachers involved in this study. A detailed explanation of this stage of the intervention is be provided in section 7.2.

4.11.2 Implementation of the Intervention

The next stage in the intervention process is the implementation phase. This is the process whereby the intervention is conducted in a supportive environment. It requires mutual trust, respect and co-operation between all parties. The two critical decisions to be made during this phase were:

1. What teachers were to be involved in the intervention?
2. What method of delivery was to be employed for this intervention?

Research Sample Employed in Phase 3C

When completing the questionnaire in Phase 3B teachers who were interested in participating in the intervention were asked to complete a form relating to the intervention. Of the 22 teachers who completed Phase 3B ten teachers expressed an interest in participating in Phase 3C. The ten teachers were teaching in four different schools in four different counties, namely Galway, Clare, Tipperary and Dublin. When the intervention was designed the author contacted each of these ten teachers and explained what they were required to do in Phase 3C. As with all other phases teachers involved in this phase were again informed that there anonymity would be ensured and that they could withdraw from the project at any stage.

Method of Delivery

Once the intervention was designed and the subjects identified the author then had to decide how to deliver the intervention. Prior to deciding this, the author investigated other forms of CPD and looked at the benefits and drawbacks of each. This detailed
analysis is discussed in section 7.3. Following this analysis the author opted for an innovative and unique method of delivery for the purpose of this CPD intervention. This innovative method incorporated many elements of other approaches to CPD but also eliminated some of drawbacks such as the didactical nature of traditional CPD. Again a detailed account of the delivery method chosen for Phase 3C is provided in Chapter 7.

4.11.3 Evaluation of the Intervention

The evaluation phase of an intervention is, according to Regan (2005), the most important and so the author gave much consideration to this phase. In order to evaluate an intervention four different aspects of the intervention must be considered. These are intervention effectiveness, intervention integrity, intervention acceptability and social validity (Shapiro, 1987). These aspects are explained in greater detail in figure 4.6 overleaf:
The author sought to evaluate each element through the use of a questionnaire (Appendix J). The questionnaire was distributed to all ten teachers on completion of the intervention. This questionnaire was divided into two sections. The first section incorporated the Likert Scale and allowed the author to get an insight into subjects opinions in relation to the intervention and how they perceived it.
The second section allowed participants to express their opinions in more detail through the use of open ended questions. Eight of the ten teachers involved in the final phase of the project returned the evaluation form and the data was again analysed using SPSS (Version 16 for Windows). The findings that emerged from this evaluation are outlined in Chapter 7.

In order to further triangulate the data relating to the intervention and in turn the Ladder of Knowledge the author shared some components of the *Calculus Tool Kit* with a convenient sample of second level students. The components were used by the author during a MACSI Mathematics Summer School held in the University of Limerick in 2010. There were fifty three students involved in one session that the author delivered during the Summer School and forty six in a second session. Their thoughts and opinions were also obtained in order to see their views on different elements of the intervention. Again the findings of this evaluation will be outlined in section 7.4.5.

### 4.12 Issues within the Research

When deciding on the different methods to be employed in this project many issues arose to which the author had to give due consideration. Such issues included the need for ethics, the problem of researcher distance and the need for validity and reliability in order for the project to be deemed worthwhile. The author will now look at each of these issues in more detail and outline how these issues were accounted for in this study.

#### 4.12.1 Ethics

Within a discipline such as education working and interacting with human subjects is common practice and as a result ethical implications must be looked at closely (Leedy & Ormrod, 2005). As a result the author identified any ethical issues at the beginning of this project and ethical guidelines, as outlined by the UL Research Ethics Committee (ULREC), were adhered to throughout the design and
implementation of the studies involved in this research. In order to adhere to these ethical guidelines the author had to ensure the following:

- Subjects were aware that participation in the study is strictly voluntary.
- Subjects were aware that participants can withdraw from the project at any stage.
- Principals and participating teachers were provided with information sheets which clearly explained the purpose of the research and what was required of them over the course of the study.
- Consent forms were signed by all participants prior to the fieldwork commencing.
- Code numbers/pseudonyms were allocated to all participants throughout the study to guarantee their anonymity.
- Any data collected was used for research purposes and all data will be stored according to UL Ethic’s Regulations.

The author obtained ethical approval from ULREC for this study in March 2008.

4.12.2 Researcher Distance

As discussed in Chapter 1 the author’s brief experiences of teaching mathematics, her third level education and her experience of school mathematics at second level contributed to her decision to pursue a research project in the field of mathematics education. However as a result of this experience it is expected that the author will bring her own biases, assumptions and expectations to the project. A number of conscious decisions made by the author, including the use of a mixed method approach and the choice of a number of different tools of inquiry, as well as the acknowledgement of her biases and expectations allowed the author to establish researcher distance and objectivity. In turn this researcher distance will help contribute to the validity, reliability and overall acceptance of the researcher’s findings.
4.12.3 Validity

“If a piece of research is invalid then it is worthless”

(Cohen et al., 2000: 105)

In order to extract any useful information from a research project the measurement tools used must be shown to be both valid and reliable. The validity of a tool of inquiry is defined as the extent to which it measures what the researcher wants it to measure (Leedy & Ormrod, 2005). Validity can be improved upon by selecting an appropriate research methodology in tandem with suitable research instruments. For the purpose of this research project the author used two different instruments (focus groups and questionnaires) in order to ascertain teachers opinions about the types of knowledge required for teaching, their views on their own levels of competence and their knowledge in a number of different domains. During the quantitative pilot study the author strove to ensure validity through the careful selection of a sample and appropriate analysis of the data. In the qualitative phase of stage 3, it was not as easy to ensure validity but the author aimed to do so through the depth, richness and scope of the data collected (Cohen et al., 2000). Furthermore the researcher distance discussed in section 4.12.2 will help eliminate bias while collecting qualitative data during the interviews and observations.

Research has shown there to be a number of different types of validity (Frankfort-Nachmias & Nachmias, 1996; Creswell, 2009) but the two most critical to the integrity of this project are:

- Internal Validity
- Content Validity.

**Internal Validity**

This is the extent to which results of the data analysis support the relationships and conclusions drawn from the data. This can be addressed by employing a number of different methods of inquiry, as was the case in this project, and through using a constant comparative method during the analysis phase.
Content Validity

This type of validity refers to the extent to which a measurement instrument e.g. the questionnaire/test is comprehensive and fair and measures what it is supposed to measure (Maguire, 2005). Content validity was ensured through two means. Firstly the author’s extensive literature review prior to the development of the questionnaire, the focus group questions and the intervention and secondly by designing all research tools based on the Ladder of Knowledge.

4.12.4 Reliability

Reliability refers to the consistency with which a measurement instrument can yield similar results when the phenomenon considered is not changing. It suggests that were the same research project to be carried out using the same instruments but with a different sample then there would be a high correlation between results. Within this research project the author sought reliability through the use of a mixed method approach. Furthermore all participants, in the focus groups, the pilot study (Phase 3A and 3B) and the intervention evaluation (Phase 3C) engaged in/completed the same research tool. The reliability of both the qualitative and quantitative study was also improved upon through the rigorous approach to data collection, analysis and write up that was adopted by the author; and by triangulation.

4.12.5 Triangulation

Triangulation enhances reliability and validity. In order to increase confidence in one’s findings it is important to avoid using only one source for data collection. The use of a number of sources, as is the case in this project is referred to as triangulation. According to Cresswell (1994: 174):

“The concept of triangulation was based on the assumption that any bias inherent in particular data sources, investigator and method would be neutralised when used in conjunction with other sources, investigators and methods”

Researchers such as Cohen & Mannion (1992) and Denzin & Lincoln (1994) discuss a number of different types of triangulation but the three employed for the purpose of
this study are methodological triangulation, theory triangulation and data triangulation

Methodological triangulation refers to the use of a number of different methods of inquiry in any given project. The use of a mixed method approach ensures that methodological triangulation is a feature of this doctoral study.

Theory triangulation involves the use of a number of different theories when examining data and drawing conclusions. The extensive literature review and the use of five different theoretical frameworks in the development of her own model allowed the author to incorporate theory triangulation during the research.

Data triangulation is the collection of data using a number of different strategies at a number of different times. This type of triangulation was incorporated through the use of a number of different tools of inquiry throughout the different phases of the project.

4.13 Limitations of the Study

The author recognises that there are some limitations with this study. These limitations are outlined below:

- Firstly the author accepts that the task of improving teacher knowledge could be addressed in two different ways, either though CPD or at third level. However it was decided to solely focus on one approach, that being CPD. Although such an approach does not cater for prospective teachers currently studying at third level the author felt that CPD would cater for a larger number of teachers and also help alleviate the problem of out of field teaching which is a serious issue in Ireland today (Ni Riordain & Hannigan, 2009)

- Another limitation which the author encountered was the poor response rate during phase 3B of the study. This poor response rate resulted in the author having a small pool of subjects and this means that the findings in this phase must only be treated as indicative of Irish teachers’ views.
Finally when evaluating the intervention from a student’s perspective, in order to ensure data triangulation, a convenient sample was chosen. The convenience sample consisted of a group of students attending the MACSI Mathematics Summer School which was run in the University of Limerick from June 21<sup>st</sup> – June 25<sup>th</sup> 2010. The response received from this distinct, and possibly elite, group of students are not representative of the entire student population.

Although there are three clearly defined limitations in this research study the author feels that due to the mixed method approach used and the richness of the data obtained that these limitations will not have a significant impact on the outcomes or results of the study.

### 4.14 Conclusion

This chapter has identified the author’s stance in relation to the theoretical considerations that guided the research process and explains her methodological decisions. A breakdown of each phase of the research was provided in addition to the overall research design. The research aims and questions were outlined as well as a description of the three-stage mixed method approach adopted during this research study. In this chapter the issues of sampling, ethics, reliability, validity and triangulation in relation to the development and administration of the various methods used in the three phases are discussed. Finally the research aims and questions detailed in this chapter set up the issues that will be addressed in the chapters following on to this one.
5. The Ladder of Knowledge

5.1 Introduction

“No one questions the idea that what a teacher knows is one of the most important influences on what is done in classrooms and ultimately on what students learn.” (Fennema & Franke, 1992: 147)

Chapters 2 and 3 show how a mathematics teacher’s knowledge base has an important role to play in improving the standard of mathematics teaching and learning worldwide. Thus it is essential that those involved in mathematics education understand what types of knowledge a teacher must develop in order to teach effectively. Furthermore, people involved in teacher training must strive to ensure teachers are proficient in each of the domains either through teacher training or Continuous Professional Development. This chapter is devoted to identifying a ‘package of knowledge’ that will meet the demands placed on teachers as a result of Project Maths and help overcome the obstacles that Irish teachers now face due to this recent curriculum reform. This model will also serve to highlight the intricate
nature of a teacher’s knowledge base and will help alleviate the problems which stem from inadequate levels of knowledge among Irish teachers. It is first necessary, however, to discuss models that have been put forward by researchers in the past and analyse ways in which these models can be adapted and improved upon to meet the needs of Irish mathematics teachers.

5.2 Models of Teacher Knowledge

Five ‘packages of knowledge’ have been identified and selected to act as theoretical frameworks for this doctoral thesis. A summary of these models was provided in Chapter 1 (Table 1.1). The models to be analysed in this chapter are:

- Shulman’s Package of Knowledge [for teaching in general] (1986)
- Fennema & Franke’s Package of Knowledge [for mathematics teaching] (1992)
- Rowland’s Knowledge Quartet [for mathematics teaching] (2007)

5.2.1 Shulman’s Package of Knowledge

The mid-1980’s witnessed a major breakthrough in mathematics education and more specifically in the area of teacher knowledge. During this era the idea of teachers requiring a number of different knowledge types was first introduced in the work of Shulman (1986). He proposes three different knowledge domains that secondary school teachers need to develop in order to be capable of teaching effectively. His model is not specific to the subject of mathematics and instead is designed, based on observations of mathematics, science, English literature and history teachers. The three domains which Shulman and his colleagues found to be most critical for effective teaching are:

- Subject Matter Content Knowledge
- Pedagogical Content Knowledge
- Curricular Knowledge.
In section 3.2 these knowledge types were defined but it is now necessary to analyse the importance attributed to each by Shulman (1986). The first thing to note is his constant referral to content knowledge. This term appears in two of the three knowledge domains he found to be important thus suggesting that he places a great deal of emphasis on this knowledge type. Other aspects of his work also confirm that subject matter knowledge takes precedence over all other knowledge types in his research. For example he argues that a teacher’s main concern should be developing their content knowledge in order to be, at the very least, on a par with a layperson who majored in the same subject (in our case mathematics). Only when this is achieved should they focus on pedagogical knowledge. This finding was also supported by Wu (2004: 2 - 7) as he states:

“…sound pedagogical decisions can only be based on sound content knowledge…without such a mastery [of content knowledge], good pedagogy is impossible”

Shulman (1986) continues by discussing pedagogical knowledge. He acknowledges that there are two dimensions to this type of knowledge; the pedagogical knowledge of teaching and pedagogical content knowledge. It is the latter that his work investigates, again highlighting the importance of content knowledge for teaching specific subjects. When Shulman (1986) discusses pedagogical content knowledge he constantly refers to content knowledge and the link that exists between the two. He notes that pedagogical content knowledge is simply an extension of subject matter knowledge. There is evidence in his work to show that content knowledge feeds into pedagogical knowledge and it is only when one has mastered content knowledge can he/she become concerned with identifying and using the appropriate representations, analogies, demonstrations and examples that lead to effective pedagogy.

Although Shulman (1986) found that pedagogical content knowledge plays a secondary role to subject matter content knowledge he is still concerned with the lack of emphasis placed on it by teacher educators. Despite it not being the most critical of knowledges, Shulman (1986) argues that time still needs to be dedicated to developing this knowledge during teacher training and throughout one’s professional career as it still plays an important role in teachers’ overall knowledge base. He also notes that the strong links between pedagogical knowledge and content knowledge
mean that by developing pedagogical knowledge, one is, at the same time, improving their content knowledge.

Shulman’s concerns in relation to the time afforded to pedagogical knowledge resurface when he discusses the final element of his model:

“If we are regularly remiss in not teaching pedagogical knowledge to our students in teacher education programs, we are even more delinquent with respect to the third category…”

(Shulman, 1986: 10)

Shulman views the curriculum as a tool capable of enhancing pedagogy by providing teachers with alternative approaches, instruction methods and texts. As with pedagogical knowledge this knowledge base has little use without profound subject matter knowledge but once this has been attained the third knowledge in Shulman’s package can serve to enhance the teaching of mathematics as well as helping teachers to develop another type of knowledge – pedagogical content knowledge.

From the outset Shulman (1986) states that this model of teacher knowledge is a blueprint and in no way a finished article. By 1987 he and his colleagues had elaborated on the model and extended pedagogical content knowledge to include general pedagogical knowledge and knowledge of learners and their characteristics. Knowledge of the curriculum was also extended to include knowledge of educational contexts and knowledge of educational ends (Shulman, 1987). However these extensions acted as placeholders and the three knowledge domains outlined in the original model remained the main focus of his work (Ball et al., 2008); hence the original model was selected for analysis in this section.

Overall Shulman’s work indicates that subject matter knowledge is the most important of the three but he also acknowledges that the remaining two cannot be forgotten. Despite the importance attributed to content knowledge in his work he found a pressing issue in education to be the lack of time and attention afforded to pedagogical content knowledge and curricular knowledge which also contribute to one’s ability to teach effectively.

This generic model of teacher knowledge was the foundation on which many other models were built and a number of these models will now be analysed.
5.2.2 Ernest’s Model of Teacher Knowledge

Ernest (1989) was another researcher in the field who investigated the types of knowledge which second level teachers require for teaching. He too developed a model of the knowledges and beliefs which he found to be essential for teaching. Unlike Shulman’s, his is mathematics specific as opposed to a model intended for secondary teachers in general. His ‘package of knowledge’ is more detailed than that devised by Shulman three years previously and includes more components. The following list outlines the types of knowledge which Ernest (1989: 15) deems most important:

- Knowledge of Mathematics
- Knowledge of Other Subject Matter
- Knowledge of Teaching Mathematics
  - Mathematics Pedagogy
  - Mathematics Curriculum
- Classroom Organisation and Management for Mathematics Teaching
- Knowledge of the Context of Teaching Mathematics
  - The School Context
  - The Students Taught
- Knowledge of Education
  - Educational Psychology
  - Education
  - Mathematics Education.

This model appears very complex with many different elements yet it is interesting to note that once again content knowledge appears at the top of the list. Like Shulman (1986), Ernest (1989: 16) found that

“...this knowledge provides an essential foundation for the teaching of mathematics”

Furthermore, despite the complexity of the model put forward by Ernest (1989), there is evidence in his work to support the fact that nothing can be achieved without teachers first developing a substantial knowledge of mathematics. It is this knowledge type which he found to underpin everything else required for effective teaching including good explanations, demonstrations and curriculum choices. It is
only when this foundation is established that people can begin to concern themselves with other domains of teacher knowledge. In addition to a deep understanding of mathematical content, it necessary for teachers to develop knowledge of other subjects which mathematics may feed into such as science, engineering, business and economics (Ernest, 1989; LMS, 1995; Smith, 2004, EGFSN, 2010). Ernest’s (1989) work shows that this knowledge will contribute to a teacher’s overall ability to teach mathematics as it provides them with information on the applications and usefulness of mathematics. This information will motivate and raise interest in the subject among students and teachers. His work, along with that of Fennema & Franke (1992), also highlights how this element of his ‘package of knowledge’ enables teachers to gain a deeper insight into the pedagogical principles of other subjects which may be useful for the teaching of mathematics. These pedagogical principles form the foundation of the next element of the ‘package of knowledge’, knowledge of teaching mathematics, and this again emphasises the link between the various elements of this model of teacher knowledge.

In the ‘package of knowledge’ proposed by Shulman (1986) he discusses pedagogical knowledge and curricular knowledge as two separate entities. Ernest (1989) also found these two knowledge types to be important for teachers but he combines both under the heading ‘knowledge of teaching mathematics’. After content knowledge, Ernest (1989) shows this to be the next critical element of a teacher’s ‘package of knowledge’. In accordance with the findings of Shulman (1986), Ernest argues that this strand is required once teachers have developed sufficient content knowledge. This type of knowledge is broad but it is critical in order to create a connection between what teachers know and how they present this knowledge to students (Ernest, 1989). The pedagogical knowledge domain involves knowledge of representations, student thinking and common errors among other things while curricular knowledge in this package refers to a knowledge of the physical tools which can be useful for teaching, such as textbooks, computer programmes and assessment methods. According to Elbaz (1983), this type of knowledge is the most practical of all types of knowledge and the work of Ernest (1989) supports this finding. For example, Ernest (1989) often refers to this element of his model as ‘practical knowledge’. Due to its practical nature Ernest (1989) states that this type of knowledge is developed through practical teaching experience.
Therefore it is generally after teachers complete their studies at third level that they increase their knowledge in this area.

According to Ernest (1989) the next type of knowledge, knowledge of classroom organisation and management for teaching, is also developed in this experiential manner. Knowledge of organisation for teaching is again a very practical type of knowledge that teachers require. As with pedagogical knowledge, there is a strong connection between this knowledge type and a teacher’s ability to plan for any given lesson. Lieberman (1995) states that children learn best through active involvement and in order to facilitate this type of learning Ernest (1989) notes that teachers must develop knowledge of organisation for teaching mathematics. It is this knowledge that enables them to become aware of effective methods for organising group work and for maintaining order and attention in the classroom. However Ernest (1989) states that it is only when the previously discussed types of knowledge have been acquired that teachers should begin to concern themselves with this knowledge type.

The final two types of knowledge which Ernest (1989) includes in his model do not receive as much attention as those previously discussed. Ernest (1989) acknowledges that prior to the commencement of his own research these knowledge types received very little recognition but he states that it would be incorrect for his model to ignore them completely. Instead he found that in order for teachers to be capable of teaching effectively they must at some stage develop a knowledge of students (both as individuals and as members of a wider group) and a knowledge of the school environment in which they teach. In addition they must acquire knowledge of educational concepts, theories and empirical results. According to Stones (1983) meaningful instruction relies heavily on a good knowledge of education while a report written by Clark & Peterson (1986) recognises the importance of a knowledge of the context for teaching hence justifying the inclusion of these final two domains.

Overall the detailed model which Ernest (1989) proposes contains many of the components discussed by Shulman (1986). Content knowledge is again the most important type of knowledge in this model and no other knowledge can be acquired before teachers have a strong foundation in this area. In addition to the similarities there are also a number of differences evident between these two models, most notably the number of knowledge domains that Ernest (1989) proposes in
comparison to Shulman (1986). Despite the detailed nature of the model Ernest (1989) puts forward strong arguments for the inclusion of each type of knowledge. Although the latter few are not dealt with in great detail it is important, according to the work of Ernest, that they are recognised as important elements of a teacher’s package of knowledge.

5.2.3 Fennema & Franke’s Package of Knowledge

Although they adopt an approach similar to that of the author, whereby they look at the different types of knowledge required for teaching in isolation first, Fennema & Franke (1992) remain aware of the integrated nature of a teacher’s knowledge base and highlight this during the latter stages of their work.

“It [teacher knowledge] is a large integrated functioning system with each part difficult to isolate”

(Fennema & Franke, 1992: 148)

Due to this, the knowledge domains which they found important are represented in a Venn diagram in Figure 5.1, below:

![Venn diagram of Fennema and Franke’s Model of Teacher Knowledge](image)

**Figure 5.1: Fennema and Franke’s Model of Teacher Knowledge**

As with both previous researchers Fennema & Franke (1992) again attributed most importance to content knowledge. From the outset they discuss the importance of
this component of a teacher’s knowledge package. Their work found that poor content knowledge results in inadequate learning among students and they support the finding of Ball (1988) that in order for teachers to help students learn mathematics they themselves need a strong foundation in the subject. Fennema and Franke (1992) accept the fact that in the past the majority of investigations carried out have found little or no link between student achievement and teachers’ levels of subject matter knowledge (Schools Mathematics Study Group, 1972; Eisenberg, 1977). Nonetheless, Fennema & Franke (1992) refuse to dismiss the importance of subject matter knowledge. Instead they note that the research carried out by Eisenberg (1977) and the Schools Mathematics Study Group [SMSG] (1972), which sought to identify a relationship between teacher knowledge and student learning, measured content knowledge solely by the number of mathematical modules teachers had studied during their third level education. They failed to take into consideration the knowledge base they may have acquired prior to this time, outside of college hours or during the act of teaching. As a result the measure of content knowledge does not appear to be entirely accurate and this therefore results in any findings being questionable. In addition to this Fennema & Franke (1992) also found that by looking at content knowledge in isolation, as the SMSG (1972) and Eisenberg (1977) did in their studies, it is impossible to measure the impact which it can have on classroom proceedings. Fennema & Franke (1992) argue that although content knowledge may not be considered to have a direct impact on student learning the impact which it has on other forms of knowledge such as pedagogical knowledge and knowledge of learning means that it will affect students’ learning and contribute to a teacher’s ability to teach effectively.

The next component of a teacher’s ‘package of knowledge’ which Fennema & Franke (1992) discuss is knowledge about students’ learning. With diversity and mixed ability becoming more prevalent in classrooms around the globe this type of knowledge has begun to play a much more significant role in the teaching of mathematics. According to Fennema & Franke (1992), a knowledge about students, learning and thought processes enables teachers to adapt instruction for different needs while it can also help them with individual instruction and in the identification of student errors. As with all other types of knowledge in this model, Fennema & Franke (1992) note that knowledge of student learning will have little impact when
analysed in isolation (Putnam & Leinhardt, 1986). Instead consideration must be
given to the fact that it also helps teachers develop other important types of
knowledge such as pedagogical knowledge as it helps them make informed
instructional decisions. Furthermore Fennema & Franke (1992) concur with the work
of Carpenter et al. (1989) when they found that knowledge of student learning can
only be developed when teachers have a strong knowledge of mathematical content.
When knowledge of student learning is analysed in this integrated manner the impact
it has on classroom learning becomes clear (Carpenter et al, 1989). Finally, Fennema
& Franke (1992) accept that much more investigation is needed in order to determine
how much knowledge of student learning teachers need in order to carry out their
professional duties but even at this early stage of investigation there is evidence in
their work to suggest that this type of knowledge is another critical component of
any ‘package of knowledge’.

The third type of knowledge discussed by Fennema & Franke (1992) is knowledge
of mathematical representations. This involves teachers being able to transform their
own mathematical knowledge into representations and analogies which students can
easily understand. According to Fennema & Franke (1992: 153) it is ‘not totally
disparate from content [knowledge]...’ yet it plays a crucial role in distinguishing a
mathematics teacher from a mathematician. It also plays an important role in
facilitating learning for understanding as opposed to rote learning which has proved
problematic in recent years (NCCA, 2005). In spite of the importance of this
knowledge domain, Fennema & Franke (1992) cite two projects in which teachers
were found to have a poor grasp of this type of knowledge. Orton (1988) and Ball
(1990) found in their two studies that teachers were unable to transform their
knowledge about the division of fractions into appropriate representations for
students. This presents a challenge for mathematics education as teachers who are
considered to be proficient in the most important type of knowledge, content
knowledge, are unable to pass this knowledge onto students in a comprehensible
manner. To overcome this problem, teachers need to develop all the types of
knowledge previously discussed by Fennema & Franke (1992). They need to become
more competent not only in subject matter knowledge but also in the real life
applications of such knowledge and be able to know enough about students’ learning
and thought processes to present mathematical ideas in a way which they can understand.

The final component of Fennema & Franke’s model is pedagogical knowledge. Many may argue that knowledge of mathematical representations falls under this heading but Fennema & Franke (1992) viewed them as two separate knowledge types which are strongly interlinked. They refer to pedagogical knowledge as teachers’ general knowledge of teaching and it is affected by each individual teacher’s beliefs and knowledge. It relates to the decisions teachers make during instruction as well as planning prior to the commencement of any lesson and post lesson analysis. The three other types of knowledge which Fennema & Franke (1992) look at all inform this knowledge type and pedagogical knowledge is seen to bring the entire ‘package’ together. As with all previous types of knowledge in this model, this final domain can have a considerable influence on student learning.

Overall the package of knowledge put forward by Fennema & Franke (1992) incorporates many of the same types of knowledge that Shulman (1986) and Ernest (1989) include in their models. Throughout their work Fennema & Franke (1992) highlight the integrated nature of a teacher’s knowledge base and it is essential that future researchers take this into consideration when designing new or updated models.

5.2.4 Rowland’s Knowledge Quartet

The next model to be analysed is one which was devised by Tim Rowland and his colleagues at Cambridge University in 2005. Unlike that of Ernest (1989) and Fennema & Franke (1992) this model outlines the knowledge necessary for primary school mathematics teachers. Rowland et al. (2005) labelled the package of knowledge which he designed as the ‘Knowledge Quartet’ and the components are outlined in Figure 5.2, overleaf.
All researchers thus far have attributed huge importance to subject matter knowledge and Rowland (2007) continues this trend. His entire ‘quartet’ is based on the premise that content knowledge is fundamental for teaching and it is this knowledge that allows for and instigates the development of other knowledge types. According to Rowland et al. (2005) and Rowland (2007) the foundation component relates to teachers’ mathematical knowledge and beliefs. The name, *foundation*, highlights how mathematical content knowledge creates a basis from which all other knowledges can be developed. It is only when one has become proficient in this area can they concern themselves with the other knowledges required for teaching (Rowland et al., 2005). However, the researchers also argue that at this early stage teachers should begin to develop some knowledge in relation to the teaching and learning of mathematics. However the bulk of this pedagogical knowledge is the focus of the second component of the quartet, a component which Rowland (2007) describes as being ‘knowledge – in – action’.

Transformational knowledge is what researchers would have traditionally called pedagogical knowledge. Despite not being considered the most important knowledge Rowland et al (2005) accept that it is this component that distinguishes a mathematics teacher’s knowledge from that of a mathematician. Rowland et al
(2005) concur with the findings of Shulman (1987: 15) who stated that the knowledge base for teaching is distinguished by

“...the capacity of a teacher to transform the content knowledge he possesses into forms that are pedagogically powerful”

The third element of Rowland’s’ quartet relates to teachers’ knowledge of students and their learning. Rowland (2007) discusses the need for teachers to arrange/sequence topics in an order comprehensible and coherent for students. It also involves having sufficient knowledge of mathematics to make connections between topics or a knowledge of other subjects in order to make connections between these subjects and mathematics. Due to the nature of this element it is clear that this type of knowledge draws on/connects the other knowledges previously discussed in this quartet again highlighting the integrated nature of a teacher’s package of knowledge. Rowland (2007) also cites the work of Brown et al. (1999) in order to emphasise the importance of this component in the ‘knowledge quartet’. According to their investigation five out of six teachers that they considered to be effective demonstrated this type of knowledge during their teaching thus demonstrating the role that this component can play in the effective teaching of mathematics.

The final component of the quartet, contingency, relates to a teacher having the knowledge and flexibility to deal with the unexpected. As the name contingency suggests this knowledge is one which may only be called upon in rare situations and it involves the teacher being aware of how to respond to unexpected questions or answers from students as well as having the awareness to know when and the flexibility to know how to alter an intended lesson plan due to unforeseen circumstances. As with the previous component of the quartet this element draws on the knowledge types which have been discussed previously. Without a strong content knowledge (foundation) teachers will be unable to deal with any variance in a given lesson while pedagogical knowledge (transformation) will enable teachers to alter lesson plans with relative ease and an awareness of the applications of mathematics (connection) will make it easier for teachers to cope with uninterested students who question the relevance of any topic.
In conclusion, the work of Rowland (2007) highlights a number of different types of knowledge which teachers require in order to teach effectively. Many aspects of this model, for example the integrated nature of a teacher’s knowledge base or the critical need for subject matter knowledge, coincide with the findings of previous researchers and such elements will also be evident in the last model to be analysed.

5.2.5 Ball et al.’s Package of Knowledge

The final model to be analysed is again one which was designed for primary school mathematics teachers. This model was based solely on the first model put forward in 1986 by Shulman and sought to elaborate or extend the components which he saw fit to include. The ‘egg’ framework of teacher knowledge which Ball et al. propose is outlined in Figure 5.3, below.

![Figure 5.3: Ball et al.'s Model of Teacher Knowledge](image-url)
The first domain in this model is again content knowledge. Ball et al. (2008) split this domain into two parts namely common content knowledge (CCK) and horizon knowledge. CCK is the knowledge necessary to carry out simple calculations and to solve mathematical problems while horizon knowledge refers to knowing how mathematical topics are related and how one topic serves to inform another. Horizon knowledge is similar to the knowledge of connections which was proposed in Rowland’s ‘Knowledge Quartet’ and is necessary in order for teachers to show students the interrelated and coherent nature of mathematics. According to Ball et al. (2008), this knowledge domain is not specific to mathematics teachers and instead, the majority of people in society should possess such knowledge.

Once teachers are satisfied with their levels of common content knowledge they must, according to this model, develop this knowledge further and acquire specialised content knowledge (SCK) for mathematics teaching. Similar to the other models analysed, Ball et al. (2008) acknowledge that teachers must develop a type of knowledge that is unique to the profession of teaching and that helps differentiate a teachers’ knowledge base from that of a mathematician or layperson. They propose SCK to serve this purpose. Ball et al. (2008: 401) justify including this knowledge domain when they state:

“The mathematical demands of teaching require specialised mathematical knowledge, needed by teachers and not needed by others. Accountants have to calculate and reconcile numbers and engineers have to mathematically model properties of materials, but neither group needs to explain why, when you multiply by ten you ‘add a zero.’”

Therefore this is the knowledge type that is not commonly needed for purposes other than teaching but is critical for the teaching profession. Ball et al. (2008: 400) elaborate on this when they provide a list of teacher tasks which this knowledge domain serves to enhance:

- Responding to students “why” questions
- Finding an example to make a specific mathematical point
- Recognising what is involved in using a particular representation
- Linking representations to underlying ideas and to other representations
- Connecting a topic being taught to topics from prior or future years
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- Explaining mathematical goals and purposes to parents
- Appraising and adapting the mathematical content of textbooks
- Modifying tasks to be either easier or harder
- Evaluating the plausibility of students’ claims (often quickly)
- Giving or evaluating mathematical explanations
- Choosing and developing useable definitions
- Using mathematical notation and language and critiquing its use
- Asking productive mathematical questions
- Selecting representations for particular purposes
- Inspecting equivalencies.

The comprehensive nature of this list highlights how SCK is a fundamental element of a teacher’s package of knowledge and it is the defining feature of Ball et al.’s model. They found that if teachers do not develop this knowledge domain they will be unable to carry out tasks which they are required to routinely do. Furthermore they have shown this knowledge domain as one which bridge the gap between content knowledge and pedagogical knowledge, which is the focus of the latter half of their model.

The third knowledge domain in this model is a knowledge of content and students (KCS). According to Ball et al. (2008), this knowledge domain combines knowing about students and knowing about mathematics and again it a type of knowledge that helps to connect the two main knowledge domains put forward in Shulman’s model, namely content knowledge and pedagogical knowledge. The work of Ball et al. (2008) demonstrates how the demands placed on teachers results in them needing to develop this knowledge which is at the intersection of content and students. This knowledge type allows teachers to understand students’ thought processes, to anticipate what they will find difficult or confusing and to identify where misconceptions may occur. It also requires them to understand the curriculum and use mathematical examples and anecdotes that they predict will arouse interest among their students.

The next domain which Ball et al. include is a knowledge of content and curriculum.
“Because teaching involves showing students how to solve problems, answering students’ questions, and checking students’ work, it demands an understanding of the content of the school curriculum”

(Ball et al., 2008: 395)

This element of a teacher’s knowledge base combines teachers’ content knowledge with a knowledge of the curriculum. In essence it requires teachers to match their mathematical knowledge with the mathematics that appears on the curriculum they are required to teach. Ball et al. (2008) found that the additional content knowledge which teachers possess can then be used to enhance levels of understanding and develop an appreciation for the subject of mathematics in the classroom. Furthermore when content knowledge and curricular knowledge are combined in this way it will allow for teachers to understand why a particular topic is central to a discipline and why others do not receive the same attention hence helping them when selecting topics to teach and the order in which to teach them (Shulman, 1986).

The final domain in the model which Ball et al. propose is knowledge of content and teaching. It is this domain that allows a teacher to combine what they know about teaching with what they know about mathematics (i.e. combining pedagogical and content knowledge). This type of knowledge will enable teachers to sequence particular content in a way that will allow for student understanding and appreciation. Again it is similar to a component included in Rowland’s model viz. connection knowledge. According to Ball et al. (2008) this knowledge is also critical as it allows teachers to evaluate the significance and worth of representations which could assist in their teaching of specific areas. Finally this knowledge domain assists teachers when they are required to make instructional decisions. For example Ball et al. (2008) note that this type of knowledge can help teachers identify when extra clarification is necessary, when to ask questions and when to proceed to a new task and in turn this type of knowledge can again contribute significantly to their ability to teach effectively.

Overall Ball et al.’s model was built on the work put forward by Shulman (1986) and by elaborating on the domains proposed in his model they have designed a new model which is both comprehensive and if developed by teachers at either primary or secondary level will serve to enhance the standard of mathematics teaching. Many of
the knowledge types discussed in this model, although assigned different names, are similar to those we have encountered before and it is these similarities between the five models, as well as a number of differences that the author will now analyse.

5.2.6 Analysis of the Theoretical Frameworks

The review of the five different ‘packages of knowledge’ carried out to date, has allowed the author to identify common trends and discrepancies between each of the models. Such trends will serve to highlight important features required for the author’s ‘package of knowledge’ and will also help in identifying the most critical types of knowledge required by all mathematics teachers.

Firstly the above analysis shows that all types of knowledge required by teachers are integrated. Fennema & Franke (1992) state this at the beginning of their work and every other researcher also implies it in their findings. For example Rowland et al. (2005) highlight how one component in their model allows for the development of another while Ernest (1989) emphasises how content knowledge must be acquired before knowledge of teaching mathematics can be developed while a knowledge of organisation for teaching is supported by a combination of the previous three types of knowledge in his package. Furthermore Ball et al. (2008) states that “It is not always easy to discern where one of our categories divides from the next...” and instead they argue that each knowledge domain must be viewed as an element of an intertwined knowledge base where each knowledge domain has strong connections with the next. Shulman (1986) also states that a teacher developing content knowledge alone is ‘useless’ unless he/she is capable of combining it with a knowledge of pedagogical principles or curricular knowledge. Due to this integrated nature, it is accepted that analysing any type of knowledge in isolation is not worthwhile. Instead of viewing the different knowledge types as separate entities, as has been done in the past (Eisenberg, 1977), researchers must now begin to view the different types of knowledge required by teachers as an integrated package with each knowledge domain affected or enhanced by a different knowledge(s). Only when everyone accepts that a mathematics teachers’ knowledge is integrated will the true effect of a teacher’s knowledge base on classroom teaching and learning be realised.
The second notable feature across all five packages is the perceived importance of subject matter knowledge. In Chapters 2 and 3 the author discussed the importance of subject matter knowledge for teaching and the above analysis further underlines this point. Each of the five models discussed above prioritise content knowledge as the basic knowledge required by all teachers from the outset. Ernest (1989: 16 - 17) summarises the importance which all other researchers attribute to this knowledge domain in the following quote:

“Whatever means of instruction are adopted the teacher needs a substantial knowledge base in the subject in order to plan for instruction and to understand and guide the learners’ responses. The teacher's knowledge of mathematics will underpin the teacher’s explanations, demonstrations, diagnosis of misconceptions, acceptance of children’s own methods, curriculum decisions (such as emphasising central concepts), and so on. Thus knowledge of mathematics provides a foundation for the teacher’s pedagogical knowledge and skills for teaching mathematics.

This statement highlights the importance of content knowledge in each of the models discussed. As Ernest (1989) states it has a profound impact on pedagogical knowledge but by highlighting the other aspects of teaching which it underpins he also shows its impacts upon a teacher’s knowledge of students, curricular knowledge and transformation knowledge. All these types of knowledge are critical elements in the models proposed by Shulman (1986), Fennema & Franke (1992), Rowland (2007) and Ball et al. (2008). As a result content knowledge is the one type of knowledge which no model can afford to omit. Although researchers such as SMSG (1972) and Eisenberg (1977) deemed this knowledge to have very little impact on student learning when we see the effect it has on a teachers overall repertoire of knowledge, its impact and importance is unquestionable. Hence this knowledge domain must act as the base on which all future models are built.

Another feature that was common across the five ‘packages of knowledge’ was the need for a specific knowledge that distinguishes mathematics teachers from mathematicians. Three of the four researchers who developed models specifically for mathematics teachers refer to the critical need for this type of knowledge. Rowland et al. (2005) label it as ‘transformation knowledge’ while Fennema & Franke (1992) include it as a core component of a teacher’s knowledge of mathematical representations. Finally Ball et al. (2008) label this elusive knowledge domain as
'specialised content knowledge’. Shulman’s model was one designed to meet the needs of all second level teachers and so no domain exclusive to mathematics teaching was included in his model. However the year after designing his model Shulman (1987) spoke of the need for teachers to develop a unique type of knowledge that would allow them to transform their own content knowledge into representations that students could understand and hence he refers to a knowledge that is unique to the profession of teaching. Such findings show that despite the broad and complex nature of a teacher’s knowledge base, as highlighted in these models, ‘transformation knowledge’ or a knowledge specific for mathematic teachers, is the one knowledge which truly separates a teacher’s knowledge base from that of a mathematician. Its inclusion, therefore, is critical in any package of knowledge required for teaching and it will also provide the profession of teaching with added credibility as it highlights that teachers possess something unique from those invovled in different professions.

Finally one aspect that differed across the four packages was the complexity of each model. As can be seen in the anaysis, Shulman (1986), Fennema & Franke (1992) and Rowland (2007) all present a concise package of knoweldge with no more than four types of knowledge discussed in each of their individual models while Ball et al.’s model is based on two knowledge domains but these domains were then divided into 6 sub-domains. Ernest (1989), on the other hand, proposes a model which contains six different types of knowledge and many of these also contained sub headings which Ernest (1989) deemed important. Following the analysis of each of the different models the author accepts that the more concise models are more beneficial and easier to work with during CPD initiatives. There is evidence in the work of Lortie (1975) to show that teachers often fear new developments and as a result a complex model such as that proposed by Ernest (1989) may prove too overwhelming for teachers. As a result any CPD initiative that attempts to develop such a package of knowledge among teachers may meet with resistance before it even commences. On the other hand, however, concise models such as the other four proposed in this chapter manage to incorporate all the important aspects without appearing too convoluted and hence teachers may be more open to such models.
In conclusion the work to date highlights how some aspects of the models discussed are critical for any package of knowledge which the author designs. The inclusion of subject matter knowledge and a knowledge that distinguishes mathematics teachers from mathematics graduates is crucial while every package of knowledge needs to demonstrate the interrelated nature of a teacher’s knowledge base. Furthermore any model should be concise so as not to appear off-putting to teachers who are wary of change. These findings will be used by the author when designing her own model but first it is necessary to discuss why there is a need for a new model of teacher knowledge in Ireland.

5.3 The Need for a New Model

This section is devoted to explaining why the author chose to develop a new model of teacher knowledge and use this to act as a vehicle that would help improve mathematics teachers’ knowledge as opposed to using one of the models previously designed and tested. One of the main reasons for this decision was the fact that all the models analysed thus far have been designed to meet the needs of teachers in countries such as the U.S. and the U.K. No model has ever been designed specifically for Irish teachers. In recent years a lot of time and resources have been invested in mathematics curriculum reform in Ireland and in order to ensure that such efforts are not in vain it is important for mathematics teachers to be adequately prepared to teach the new syllabus effectively. Ensuring that teachers understand the different types of knowledge necessary to teach the new material as well as helping them develop such knowledge will be key to ensuring this. Project Maths aims to secure a change in teaching approaches and practices and according to Putnam et al. (1992) such change will also require a change in a teacher’s knowledge base. Project Maths, therefore, requires teachers’ to extend and deepen their knowledge and so it is now more critical than ever before for those involved in mathematics education in Ireland to understand the types of knowledge necessary in order to teach the subject of mathematics effectively. Although the models put forward previously are comprehensive and would serve to enhance the standard of mathematics teaching and learning in Ireland the author submits that two essential components, which will allow teachers to deliver Project Maths effectively, have been overlooked in previous
models. These two categories are a knowledge of applications and a knowledge of schools maths.

As discussed in Chapter 2, a central focus of the new Irish curriculum is the relevance and applications of mathematics. Therefore the lack of focus on a knowledge of applications in previous models is of concern to the author. In addition to this mathematical applications are becoming central to mathematics curricula around the globe (Burkhardt, 2006). However in order to teach mathematical applications teachers must first have an extensive knowledge of such applications and so it is no longer viable to omit this knowledge domain from models of mathematics teachers’ knowledge. This knowledge of applications requires teachers to develop an understanding of the uses of mathematics in the real world and how this subject can be used to solve numerous problems facing people outside of a school setting (Blum & Niss, 1991). Despite the importance of this knowledge domain no model of teacher knowledge studied saw fit to include this component. Due to the lack of emphasis on a knowledge of applications many teachers are failing to show students the importance and relevance of mathematics. For example Doerr (2007) found that the reason for the limited use of mathematical applications and mathematical modelling at second level is a lack of knowledge of applications on the part of teachers. Consequently, until teachers and teacher educators in Ireland recognise the importance of a knowledge of applications and in turn improve their proficiency in this regard they will find it impossible to deliver a curriculum which has such a strong focus on applications and modelling. Therefore, it is critical that a model of teacher knowledge be designed so as to incorporate this aspect of teacher knowledge and alert people to its importance and the role it can play in improving mathematics teaching and learning.

Knowledge of school mathematics is another knowledge domain that was overlooked in previous models. Although Ball et al. (2008) alerted the readership to the need for teachers to understand the mathematics that is on the curriculum they are required to teach, they did not elaborate on or discuss this knowledge domain in detail. Hence it is necessary for a new model to be designed that will incorporate this knowledge domain and highlight its true importance. Ball et al. (2008) discuss how it has been suggested over the years that mathematics teachers need to know the
mathematics that appears in the curriculum plus the additional mathematics knowledge which they acquire at third level. The extent of the ‘additional knowledge’ required for mathematics teaching has never been explicitly defined. Analysis of third level courses from around the globe shows that teachers are expected to develop a knowledge base greater than that needed at second level. For example the Open University in the U.K. offers prospective teachers the following modules:

1. Using Mathematics
2. Analysing Data
3. Exploring Mathematics
4. Mathematics Methods and Modelling
5. Pure Mathematics
6. Developing Mathematical Thinking at Key Stage 3
7. Teaching Mathematical Thinking at Key Stage 3
8. Developing Algebraic Thinking
9. Developing Statistical Thinking
10. Developing Geometrical Thinking
11. Applications of Probability
12. Complex Analysis
13. Computer Algebra, Chaos and Simulations
14. Electromagnetism
15. Graphs, Network and Design
16. Groups and Geometry
17. Linear Statistical Modelling
18. Mathematical Methods and Fluid Mechanics
19. Number Theory and Mathematical Logic
20. Optimisation
21. The Quantum World
22. Topology
23. Waves, Diffusion and Variational Principles.

All modules form 1 – 5 are compulsory while students must choose two modules from those numbered 6 – 10 and then choose one of the remaining 13 modules. This
list shows how teachers in this institution are expected to develop a substantial knowledge base beyond the material they are expected to cover at second level. Similarly in the U.S. prospective high school teachers must study modules in the following areas:

- Algebra and Number Theory
- Geometry and Trigonometry
- Data Analysis, Statistics and Probability
- Discrete Mathematics and Computer Science

(Conference Board of the Mathematical Sciences, 2001)

Finally in U.L., where this project is based, prospective mathematics teachers must study the following modules:

- 3 Algebra modules
- 2 Calculus modules
- 1 Analysis module
- 1 Statistics module
- 1 Differential Equations module
- 1 History of Mathematics module
- 1 Group Theory module
- 1 ICT and Mathematics module
- 2 Pedagogy modules

This analysis of teacher training programmes in Ireland, the U.S. and the U.K. highlights that although teachers study a number of modules at third level, sometimes these modules do not specifically prepare prospective teachers for the task of teaching. For example a pressing issue is the fact that although the modules listed above will prove beneficial to teachers when they enter the workforce, no teacher trainee course analysed provides students with the opportunity to develop an
understanding of the mathematics that they will be required to teach. Borko et al (1992) found that the teacher in their case study had not acquired such knowledge and this, in turn, was detrimentally affecting her teaching. They found that despite Ms. Daniels studying calculus for two years and completing courses in mathematical proof, modern algebra and computer science she was unable to explain why, when dividing fractions, the invert and multiply algorithm works. This indicates that although a teacher studied the necessary modules at third level and developed the ‘additional knowledge’ required, she still was unsure of some of the fundamental concepts that appear on a primary level mathematics curriculum. In order to rectify this problem it is essential that researchers begin to recognise the importance of prospective and practising teachers developing a knowledge of school maths and in turn including this component in future models of teacher knowledge. Furthermore the Conference Board of Mathematical Sciences (2001) recommends that teacher training colleges include a module related to the fundamental ideas of school mathematics. Such a module would serve to inform teachers of the basic mathematical skills required at second level, connect their third level modules with the material they will teach at second level and “develop careful reasoning and ‘common sense’ in analyzing conceptual relationships” (Conference Board of Mathematical Sciences, 2001: 7). However only when this knowledge domain is included in models of teacher knowledge will those involved in initial teacher training and CPD accept the importance of this knowledge domain and heed the advice of bodies such as the Conference Board of Mathematical Sciences (2001).

As a result of these two shortcomings, as well as the need for a model to address the needs of Irish teachers the author felt it was necessary to design a new model of teacher knowledge which would (a) meet the needs of Irish teachers teaching in a new and challenging climate and (b) provide teachers with more information on the types of knowledge required to teach effectively. The model which the author proposes to serve this purpose is ‘The Ladder of Knowledge’.

5.4 The Ladder of Knowledge

Based on investigations carried out into packages of knowledge proposed by researchers in the past the author designed a model which she felt best suited the
needs of Irish teachers and one which would help, in some way, resolve the problems currently facing mathematics education in Ireland. The model which she designed has been labelled “The Ladder of Knowledge” and is illustrated in Figure 5.4 below.

![Figure 5.4: The Ladder of Knowledge](image)

The ladder analogy chosen by the author highlights the three types of knowledge which teachers must develop in order to have a sufficient knowledge base for teaching. It also demonstrates the importance she attributes to each. Subject matter knowledge occupies the bottom rung due to the associated importance of this knowledge type. The author has previously highlighted the importance that researchers such as Carlsen (1991), Boero et al (1996) and Smith (2004) have attributed to content knowledge and as a result no teacher can even attempt to mount the ladder without a deep understanding of content knowledge. Only when teachers have mastered this type of knowledge can they become concerned with reaching the second rung.

“...it does not make good sense...to make teachers believe that they can make a full scale assault on pedagogical content knowledge without first acquiring a strong content knowledge. If we ask them to run before they can walk, they will surely fall flat on their faces”

(Wu, 2005: 2)
Teachers must next develop pedagogical knowledge in order to be able to transfer the knowledge of mathematics, which they have acquired, to students in an effective manner. They must begin to present mathematics in a different, more appealing way by highlighting its relevance and applications as well as demonstrating understanding of the links between different mathematical topics and the relevance of the mathematics that appears on the second level curriculum. Rowland (2007) found this domain to be necessary in order to distinguish mathematics teachers from mathematicians and the author agrees with him to an extent. However it is this type of knowledge that makes way for another knowledge type and it is the latter which truly distinguishes teachers of mathematics from mathematicians.

According to the National Centre for Excellence in the Teaching of Mathematics [NCETM] (2007) the main characteristics of effective teaching are the ability to connect different mathematical strands as well as mathematical ideas within the same strand, the willingness and capacity to teach certain topics in a number of different ways in order to ensure understanding on the part of students, and the capability to allow for whole class discussion and questioning during mathematic lessons. The final rung on the ladder, knowledge for effective teaching, enables teachers to demonstrate such characteristics. Once teachers have developed knowledge for effective teaching they will be able to combine their knowledge of mathematical content and their pedagogical knowledge. They will no longer view these two important knowledge domains as separate entities. This will allow them to present mathematics to second level students in an informative and interesting manner. This knowledge for effective teaching will also enable teachers to move away from the didactical, top down approach to teaching and to present mathematics as a relevant school subject with connections both across the school curriculum and in the outside world. Essentially it will bring together all previous knowledge domains and in doing so will bridge the gap between content and pedagogy and ensure a well rounded knowledge base for all mathematics teachers.

The knowledge acquired on the first rung provides teachers with a knowledge of mathematical concepts while ensuring they are also aware of the connections within mathematics, the connections between mathematics and other subjects, the relevance and applicability of mathematics and the historical origins of mathematics. It also
allows teachers to become experienced and confident enough in mathematical content to be capable of teaching in it a number of different ways. Competency in pedagogical knowledge, the second rung, will enable teachers to use a number of different teaching approaches as well as allowing them to ask effective questions and involve students in the learning process at appropriate times. In essence, it is clear that competency in the first two types of knowledge make it possible for teachers to reach the final rung and in turn develop a type of knowledge unique to the profession of teaching. Subsequently this will lead to more knowledgeable and effective teachers in our society.

Overall the three types of knowledge proposed by the author, if developed by teachers would help to resolve many of the issues that mathematics education is currently facing in Ireland. For example a proficient knowledge base will help resolve the issue of poor understanding among teachers and students while reaching the final rung and developing knowledge for effective teaching will help create a more attractive image of school mathematics. This in turn will help second level students overcome their negative feelings towards mathematics and encourage them to opt for higher level mathematics. However many questions still remain in relation to this model including how do teachers develop the three different knowledge types and how can one progress from one rung to the next. The extended version of the ladder, which the author looks at in depth in the following section, will answer these questions.

5.4.1 The Extended Ladder of Knowledge

This section outlines the structure and domains of mathematical knowledge needed for teaching according to the author’s model. Each of the three rungs on the Ladder of Knowledge proposed by the author are complex types of knowledge and so in an attempt to clarify what is required to reach each rung, Figure 5.5 overleaf shows an extended version of the ladder with the associated knowledge domains needed to proceed from one rung to the next.
The extended version of the author’s ladder highlights what is required to reach the summit of the Ladder of Knowledge. Unlike models previously proposed, the ‘Ladder of Knowledge’ indicates the knowledge needed in order to progress from one rung to the next and helps teachers to bridge the gap between the important knowledge domains. In order to initially get on the ladder it is important that teachers develop a strong knowledge of mathematical content. Once on the ladder they must then develop and expand on this knowledge in order to reach the second rung. This expansion, which enables them to then develop pedagogical knowledge, first involves developing a collateral knowledge of applications and connections, i.e. mathematics which is relevant to students’ lives both inside and outside of the school setting. This domain is critical in order to help students appreciate the importance and relevance of school mathematics and is a domain which is unique to the ‘Ladder of Knowledge’. By developing a knowledge of applications teachers will be able to highlight the numerous everyday problems which can be solved using mathematics as well as the various professions which rely heavily on mathematics. Additionally a knowledge of connections will enable the teachers to show how mathematics feeds into many areas of a young person’s life including the other subjects they study at secondary school and the mathematics they will be faced with once they graduate.
from second level. The final element required before progressing to the second rung of the ladder is again related to the knowledge of connections but this time refers to connections between mathematics and historical milestones in mathematics. This knowledge, when shared with students, will again help arouse interest in the subject, develop their confidence and highlight the importance of mathematics (Siu & Siu, 1979). Furthermore Jahnke et al. (2002) state that this type of knowledge will allow teachers to plan more innovative classes and will help students see mathematics in a more positive light, thus again helping alleviate some of the problems discussed in Chapter 2.

If teachers become competent in these three areas they will then have sufficient knowledge to progress from subject matter knowledge to pedagogical knowledge. Strong pedagogical knowledge will allow teachers to use alternative approaches to teaching as well as demonstrate many of the characteristics of effective teaching (Fennema & Franke, 1992; Smith, 2004). However in order to teach effectively they must again develop this knowledge type further.

The first step in achieving this involves transforming the knowledge they have already acquired. As discussed by researchers such as Rowland et al. (2005) and Fennema and Franke (2002) it is essential that teachers know how to transform the knowledge they have acquired into material comprehensible to students. Teachers must combine their knowledge of general mathematical concepts and applications as well as their pedagogical knowledge and convert it into representations, explanations and analogies that students will understand and appreciate. In essence teachers must use an amalgam of all the knowledge domains discussed to date in order to transfer their knowledge to students in a logical manner. If this area is developed sufficiently teachers will not only help students to develop a better understanding of mathematics but they will also help students develop a better attitude than those which they are currently displaying. Furthermore the pedagogical knowledge which teachers have developed will inform them that questioning is essential in order to ensure student learning while content knowledge will make them aware of the material on which they must question students. Therefore by combining both and transforming these two types of knowledge teachers should then be able to ask the correct type of question at the correct time in order to yield the best results, hence contributing to
their ability to teach effectively. Finally this transformation knowledge incorporates the knowledge of students as in order to transform one’s knowledge into something students understand you must first have an in-depth knowledge of your students.

The last step required to reach the final rung is a knowledge of school mathematics. In section 5.3 the author put forward a strong argument for the inclusion of this knowledge domain and showed how this knowledge domain can also contribute to a teacher’s ability to teach effectively. There is a distinct difference between the mathematics encountered at third level and that which will be required for teaching at second level. Therefore it is critical for teachers to become familiar with what school mathematics entails as well as the relevance and applications of such content, hence linking this component with previous domains on the ladder. As discussed previously there is an onus on third level institutions to change the content of some modules in order to help trainee teachers develop this element of their knowledge base however it can also be done through continuous professional development initiatives.

In conclusion teachers must connect all the different knowledge types they have acquired to date during this final phase and combine them in order to demonstrate sufficient knowledge for teaching. They must link what they know about mathematics to what they know about the effective teaching of mathematics and in doing so they will reach the summit of the ladder of knowledge and develop a profound knowledge base suitable for teaching mathematics.

5.4.2 ‘Folding Back’: Another Important Feature of the Ladder of Knowledge

Another notable feature of the Ladder of Knowledge is that it adheres to the ‘folding back’ principle proposed by Pirie & Kieran (1991). When discussing growth in mathematical understanding they define folding back as:

“A person functioning at an outer level of understanding when challenged may invoke or fold back to inner, perhaps more specific local or intuitive understandings. This returned to inner level activity is not the same as the original activity at that level. It is now stimulated and guided by outer level knowing...This folding back allows for the reconstruction and elaboration of inner level understanding to support and lead to new outer level understanding”

(Pirie & Kieran, 1991: 172)
This principle applies to the Ladder of Knowledge. Although teachers may have progressed up the ladder they can, at any stage, return to the lower rungs in order to enhance or update their mathematical knowledge or understanding. It is critical that teachers continually revisit the bottom rung of the ladder and strengthen their mathematical understanding so they can reach even higher levels at the top of the ladder. By revisiting the lower rungs teachers will not only ensure that they have a thorough understanding of modern mathematics but they can also use their newly acquired and reconstructed knowledge to enhance their understanding in domains that appear higher on the ladder. Furthermore, as Pirie & Kieran (1994) discuss, this idea of folding back also allows the ladder to act as a non-unidirectional model. Although it is essential for teachers to first develop content knowledge and then pedagogical knowledge, when they encounter a problem that they cannot solve they can revisit any domain on the Ladder of Knowledge to help them develop the knowledge or understanding that will allow them to solve this problem and in turn improve their teaching. By adhering to this idea of folding back the author is ensuring that the ladder is as essential a tool for the expert teacher who needs to refresh their knowledge in certain domains as it is to the novice or out-of-field teachers.

5.5 Quantum of Knowledge

In order for prospective teachers to develop the different knowledge domains outlined on the author’s model it is necessary to understand the nature of the knowledge which they need to develop at third level. In section 5.2 the author outlined the current modules which trainee mathematics teachers must study in the U.S., the U.K. and UL. Although these modules may provide prospective teachers with the necessary content knowledge as well as the requisite pedagogical knowledge it is essential that trainee teachers also study some module which will allow them link these two critical areas of teachers’ knowledge base. This idea is reinforced in Figure 5.6, overleaf.
Figure 5.6, which is based on the teacher trainee programme available at Harvard University, Massachusetts, clearly highlights that in order for prospective teachers to develop one of the unique aspects of the author’s model, namely a knowledge of school mathematics and in turn develop the knowledge required for effective teaching, it is necessary that future third level programmes include a capstone module that will link their mathematics knowledge to their pedagogical knowledge. It is this module which will provide students with a deeper understanding of the mathematics they are required to teach at second level. Furthermore this module will inform prospective teachers of how the ‘additional knowledge’ which they acquire during third level can inform their teaching and enhance their levels of knowledge on topics that appear on the syllabus.

The Teaching Council currently recommends that Irish mathematics teachers study mathematics modules worth 54 ECTS credits as well as additional pedagogical modules and practical experience (Teaching Practice). However the author submits that in order to allow for teachers to develop the required knowledge students must
study more than the minimum requirement. As a result the author proposes the following programme structure for prospective mathematics teachers:

- ✓ 10 mathematics content modules (60 ECTS)
  
  3 Algebra & Discrete Mathematics modules
  
  2 Calculus modules
  
  1 Differential Equations module
  
  1 Statistics module
  
  1 History of Mathematics module
  
  1 Analysis module
  
  1 Modern Applications of Mathematics module.

- ✓ 1 capstone module (6 ECTS)

- ✓ 2 mathematics pedagogy modules [including ICT and Mathematics](12 ECTS)

- ✓ Teaching Practice (18 ECTS)

This would result in prospective mathematics teachers requiring 96 ECTS credits before graduating from third level and entering the teaching profession. The author believes that only when such targets are reached will third level courses fulfil their duties and ensure that teachers are adequately prepared to teach mathematics on graduation.

In addition to identifying the quantum of knowledge which prospective teachers must develop at third level The Ladder of Knowledge can also offer future direction for mathematics specific CPD as highlighted in Chapter 7.
5.6 Conclusion

This chapter provides the readership with a detailed analysis of the different types of knowledge required to teach mathematics effectively. Having analysed five different models of teacher knowledge the author was able to identify critical aspects of a teacher’s knowledge base and this analysis in turn served to inform her own model of teacher knowledge – *The Ladder of Knowledge*. This model of teacher knowledge encompasses many of the features included in other models but also has two unique elements viz. knowledge of applications and knowledge of school mathematics. It is these two elements that helps ensure that this model is unique and suitable for Irish mathematics teachers. The author’s model emphasises the importance the author attributes to content knowledge and her belief that nothing can be achieved without a solid foundation in this area. The model also highlights the types of knowledge which the author deems critical in order for teachers to develop a knowledge that will allow for effective teaching.

Despite the unique nature of this model the author accepts that more work is needed in order to develop and refine this model further. The author agrees with Ball et al. (2008) when they state that models of teacher knowledge need to be constantly revisited and revised in order to keep abreast of developments in mathematics and mathematics education. For example future work in this area may involve rethinking the ladder analogy and instead presenting the knowledge domains on a spiral structure to further emphasise the non – unidirectional nature of the model. Furthermore some knowledge domains may need to be revised in line with future changes in the curriculum while it may also be worthwhile combining some of the domains on the extended version of the Ladder of Knowledge in order to avoid the model being viewed as a complex structure. In addition to this despite the importance of this model, for assisting teachers’ master sufficient knowledge for teaching, much work is still required in order to help teachers, through continuous professional development, reach the summit of the ladder and in turn improve the teaching and learning standards in Ireland. Over the next two chapters the author will explain how she developed, supported, corroborated and validated the model and demonstrate how this model can in fact be used as a vehicle for improving teachers’ levels of knowledge.
6. Levels of Knowledge among Teachers

participating in this Study: Findings

from the Focus Groups and Pilot Study

6.1 Introduction

This chapter is devoted to an analysis of the levels of knowledge among the teachers involved in this study as referred to on the Ladder of Knowledge. In order to assess the levels of knowledge of participating teachers at this early stage the author conducted a series of focus group meetings first before administering a questionnaire to 22 teachers as a pilot study. These methods enabled the author to gain an insight into common trends relating to the types of knowledge domains in which teachers are excelling and those in which they are struggling. The responses also allowed the author to analyse teachers’ views of their own level of knowledge and to see if their own perceptions match their performance. This chapter takes an in – depth look at these trends and looks at the implications of these for students while Chapter 7
explains how these findings were used to inform the CPD intervention and the changes in teacher knowledge as a result of this intervention.

### 6.2 Focus Groups

In Chapter 4 the author outlined the design and administration of the calculus questionnaire. This investigative tool was piloted with focus groups with the primary aim of ensuring the suitability of the instrument for the targeted audience. As discussed in Chapter 4 one of the purposes of these focus groups was to help the researcher gain feedback on the original questionnaire, before distributing it to teachers in the pilot study. The focus groups also allowed the researcher to gain a deeper insight into teachers’ own attitudes and beliefs about the knowledge required for teaching and how they rate themselves in each of the knowledge domains. The feedback received from teachers is used to inform the latter stages of this project, which involves the design, implementation and evaluation of a CPD intervention.

For this phase of the research four focus groups were used by the researcher in four different geographical regions, namely, Galway (4 teachers), Cork (3 teachers), Kerry (3 teachers) and Tipperary (3 teachers). A total of thirteen teachers were involved, six male and seven female. All teachers were fully qualified mathematics teachers who are currently teaching Leaving Certificate mathematics at either higher or ordinary level. Their teaching experience ranged from three to thirty nine years. For the purpose of analysis each teacher is assigned a pseudonym in order to protect their identity and ensure confidentiality. The alias assigned to each teacher remains consistent throughout the project.

The findings outlined in this chapter facilitated the author in determining the beliefs, attitudes and perceptions of participating teachers regarding the knowledge required for mathematics teaching. These findings, coupled with additional information on the current levels of knowledge of participating teachers, will then serve to inform the third phase of the project – the CPD initiative. In the process of unearthing these attitudes, beliefs and perceptions the author discovered numerous problems that exist, the majority of which are not of a teachers own making . Many of these problems have been discussed by researchers in an international context but the
author will now look at problems from an Irish perspective, such as the need to improve mathematics teachers’ levels of knowledge; negative attitudes towards ‘weaker’ students; the overdependence on the textbook; the examination driven nature of mathematics classes, and perceived failings of the education system. However, the author will begin by analysing one of the positive findings to come from the focus groups – the acceptance of the importance of a number of different types of knowledge for the purpose of mathematics teaching.

6.2.1 The Acceptance of the Importance of Different Types of Knowledge

For a number of years researchers have discussed the different types of knowledge that are essential for mathematics teaching, for example content knowledge, knowledge of connections, knowledge of students and pedagogical knowledge (Shulman, 1986; Ernest, 1989; Fennema & Franke, 1992; Rowland 2004). Despite this realisation recent research studies carried out in the field of mathematics education, have shown how a number of mathematics teachers currently do not have the knowledge required for mathematics teaching (Blanco, 2003; Cuoco, 2003; Ball et al., 2005). In spite of the need to improve levels of knowledge among mathematics teachers it is encouraging to see that there is a widespread belief among participating teachers that it is important to develop a number of different types of knowledge in order to teach mathematics effectively. As discussed previously the questionnaire sought to investigate teachers’ levels of knowledge in a number of different domains including instrumental content knowledge, relational content knowledge, knowledge of connections, knowledge of applications and pedagogical knowledge. When questioned about the relevance of these types of knowledge eight of the thirteen teachers interviewed agreed, at some point, that these were essential for teaching. For example when asked if it is important for teachers to understand how mathematics is applicable to students everyday life, Eilis replied:

Eilis: *Ah extremely important, like for teachers to be able to relate it…ah if you can relate it to their, em, how they can use it in everyday life they will straight away, like, it will up their interest…If we could relate it to real life and be able to say that to the students it would make learning a whole pile easier you know.*
When asked about other areas of knowledge that the author sought to assess the responses were consistent in the four different schools:

Gary: *I think it’s laudable, the aim is laudable to make, em, calculus and differentiation relevant to other subjects, cross curriculum, I agree with that…*

Andrew: *Well yeah the practical side of it I would agree with, at least that gives them some reason why they’re learning it…I think a lot of them have that question well why, why do we need to learn it, at least if you have a practical problem it gives them some reason why they need to learn it…*

In addition to the acceptance of the importance of these knowledge types there is also a realisation that the aims and objectives of mathematics education are evolving and so too must a teacher’s knowledge base. Due to advancements in technology these teachers now accept the fact that students no longer need to simply recall or remember facts and figures as there are machines capable of carrying out such tasks. Instead mathematics education needs to focus, among other things, on highlighting the applicability and relevance of the subject. As a result, teachers, such as Gary and James, have now become conscious of the fact that it is this appreciation and understanding which they must strive to instil in their students.

Gary: *You know the old poem about the headmaster, how one, one small head could carry all he knew. That’s not relevant anymore, that skill with anybody, not just a headmaster because you have data banks now so you don’t need people carrying all the information in their heads, it’s more how to use the information…*

James: *Or to be able to work their, their old heads better we’ll say, themselves work it out there and not be depending on formulas and all this sort of thing.*

These extracts highlight the importance which these teachers attribute to the different types of knowledge. They now realise that mathematics teaching and learning can be improved by highlighting the relevance of certain topics to students. The impact that mathematics can have on students’ lives has been shown in the research carried out by Glaister & Glaister (2001) and Smith (2004) and it is encouraging to see that
teachers now accept this importance. Other knowledge domains such as cross curricular knowledge, historical knowledge and pedagogical knowledge are also accepted as critical for mathematics teaching as can be seen from Gary’s response to the question “Is it important to be aware of and use a wide range of resources in the classroom other than the text book?”

Gary: Yes, in this day and age, definitely, yes…the visual impact, it’s a lot more interesting for the kid

Similar responses were received from the majority of teachers involved when asked how they felt about the other types of knowledge. These responses reflect the importance that these teachers attribute to the different types of knowledge required for teaching. This acceptance may well be the first step needed to improve the levels of knowledge among practicing teachers. They need to first accept that mathematics teaching requires an extensive knowledge base before they agree to allocate time to improving their own knowledge base.

Despite the positive image depicted by the author thus far, there are still a number of issues to be overcome before the standard of mathematics teaching can improve. One such issue relates to teachers perceived levels of knowledge. Despite the acceptance of the importance of a wide range of knowledge domains many participants in this phase of the study have yet to develop these types of knowledge in order to experience an improvement in their teaching and in the learning of their students.

6.2.2 Levels of Knowledge among Mathematics Teachers

The previous section indicates that teachers in this phase of the study have begun to accept the importance of a number of different types of knowledge for mathematics teaching. However their responses show recognition of a gap between the knowledge they believe they need and that which they currently possess.

James: I’d like to be able to do all that stuff in actual fact [highlight the relevance of calculus and involve students in a calculus class]…but I can’t
This response from James was not unique. Although other teachers may not have explicitly stated it, their responses indicate that they do not feel competent in a number of domains included on the Ladder of Knowledge. When discussing their own levels of knowledge the majority of these qualified teachers only felt competent in instrumental content knowledge.

In Chapter 3 the author highlighted how research has shown that mathematical content knowledge underpins all other knowledge types. For example, Hill et al. (2004) found that subject matter knowledge is central to effective mathematics teaching at primary school level while Boero et al. (1996) argue that only the person who knows mathematics is capable of teaching the subject at second level. The author also reviewed research carried out which highlighted that teachers worldwide do not have the required content knowledge that will facilitate a high standard of teaching (Schmidt, 1997; Ma, 1999; Blanco, 2003). These findings were also reflected in teachers’ responses throughout the focus groups. Jason provided us with an example of this poor understanding of relational mathematical content knowledge. On the original questionnaire (Appendix C) question B3 required teachers to integrate a function in order to determine the initial velocity needed for Michael Jordan to jump a height of 4.5 feet. On seeing the word velocity Jason instantly said:

Jason: That is more, that’s more an ordinary level question really

This statement highlights the need for improved content knowledge among mathematics teachers as Jason failed to realise that it was integration (which is not a topic on the ordinary level course) which was needed and not differentiation. By eliciting responses such as these it became clear that teachers in this phase of the study did not fully understand some of the fundamental concepts about calculus. Hence it is not surprising that students are failing to apply the basic instrumental knowledge to applied questions in examinations:

“Candidates handled basic differentiation well. However only the high achievers were able to apply their skills…” (Chief Examiner’s Report, 2001: 9)

Due to the fact that researchers have found that subject matter knowledge underpins all other types of teacher knowledge required for teaching (Hill et. al, 2004) the author was unsurprised to find that many of the teachers involved in this study also
believed their knowledge and the knowledge of Senior Cycle teachers needed to be improved upon. The following extracts from the focus group interviews highlight teachers’ beliefs about their current levels of knowledge in the remaining four areas:

Anne: *Em like this is a famous thing about Irish teachers they never know who invented anything or, *em* the history of it. I know they do in other countries…*

Jason: *I’d never connect differentiation [to real life] to be honest. You can connect co-ordinate geometry to some sort of job or career, you could connect linear programming, something like that now at ordinary level, but I’d never even think of connecting differentiation to something to be honest. There’s little practical side to it.***

Eimear: *Yeah because they do ask you, they do ask you oh when am I ever going to use this again…and you literally have to go [say] the only place you will see this is on your Leaving Cert exam paper and you’re not going to see it again.*

James: *But we’ll say, I always found that the disadvantage of maths with, I taught economics as well, at least with economics everything is relevant and you can see it all around you. The maths you’re just nearly talking to yourself and they say to you sure I’ll never, I won’t need any of that when I leave school…in fairness to them they are probably right.*

Shane: *As regards the applying it there to different ones [subjects] I wouldn’t put anything down there.*

Eimear: *They’re, like as in what resources do you use is a good question but I’ll almost guarantee ninety per cent of the teachers will say the blackboard, the whiteboard, the overhead projector*

This array of quotes highlights that there is a need to improve mathematics teachers’ knowledge base and this is a serious challenge facing mathematics education. Inadequate knowledge on the part of teachers has been shown to result in students failing to reach the standards outlined in the national syllabi and as a result efforts must be made to improve the levels of knowledge among mathematics teachers generally.
Finally, another problem in relation to the knowledge base of teachers involved in this study is the way in which it is limited to material covered in the text books. Many teachers appear to have little additional knowledge to that expected of students. For example when looking at participating teachers’ relational content knowledge a number of teachers were unable to attempt the questions on the questionnaire without being provided with a diagram or some additional hints, as would be expected on a Leaving Certificate examination or in a Leaving Certificate textbook.

Anne: *Just on B2 would have been helped by a diagram. I started drawing a diagram out and I went to your solutions then sure you had a different thing in mind altogether so, and like you know nineteen times out of twenty they get a diagram in, em, an exam, so like you know we nearly expect the same thing.*

Maeve: *Yeah it’s just a wonder, the questions are very different, they’d be very different to what we’re used to working with from the textbooks and exam papers and now I wonder would people spend a long time trying to figure them out…*

Research carried out by Hiebert et al. (2003) found that in countries such as the Czech Republic, the Netherlands and Australia between 91 and 100 per cent of teachers relied heavily on the textbook in their mathematics classrooms. This is now the case with Irish teachers as they too rely heavily on textbooks that promote a drill and practice technique (NCCA, 2006). This study indicates that it is not only teachers’ content knowledge that is limited to the material in the textbook but their knowledge in the other domains is also influenced by this resource. For example when discussing the application of calculus some teachers, such as Shane, Jason and Eimear, were only able to suggest those which were discussed in textbooks nationwide. In relation to differentiation Jason felt he could connect it to speed and velocity and turning points while Shane felt the only way in which integration could be connected to real life would be in relation to area and volume. These examples are a combination of those provided in all Leaving Certificate textbooks and represent a narrow coverage of applications. The author submits that additional examples, from
a wider selection of areas, are needed to motivate students and to arouse interest in the topic of mathematics.

Overall the responses from participants in this phase of the study indicate that a concentrated effort must be made to improve the knowledge base of mathematics teachers. Future reform must focus on helping teachers develop the different knowledge domains that appear on the Ladder of Knowledge. Once their own levels of knowledge improve teachers will then be in a better position to offer students extra information and insights into calculus and elaborate on the information that is provided in Senior Cycle textbooks. However inadequate levels of knowledge is only one of a number of issues we have to contend with in the field of mathematics education in Ireland.

6.2.3 Additional Problems with Irish Mathematics Teaching in Schools

In the previous section the author highlighted problems in relation to levels of knowledge among mathematics teachers and the reliance on the textbook. However the responses from the focus groups indicate other problems also exist. In Chapter 2 the author analysed a number of challenges facing mathematics education. The focus groups show that issues including the examination - driven nature of mathematics classes, the procedural approach to mathematics teaching and the poor attitudes of teachers are also affecting the standard of teaching and learning among subjects in this study and their students.

*Examination Driven Mathematics Classes*

Gerry: *See everything comes back to it [the exam] doesn’t it?*

When conducting the focus groups it became evident to the author that everything a teacher does in a Senior Cycle Irish mathematics classroom is concerned with the examination which students must face in their final year of formal education. Due to the importance attributed to the Leaving Certificate examination teachers are unwilling to elaborate on anything that will not appear on the final assessment. For
example when teachers were asked if they feel it is worthwhile to discuss applications of calculus with students Jackie gave the following response:

*Jackie: Will they be asked it in the exam? Like I'd be very much exam paper.*

Similarly James said

*James: You're just constantly focussing on the teaching for the exam. All the time you're teaching the ordinary, well I would be anyway when teaching the pass course [ordinary level], I'm always conscious of what the exam paper is.*

In 2006, Eaton & Bell found that the need to prepare students for external examinations was preventing even the most motivated of teachers from introducing new approaches that may facilitate better understanding and learning in Northern Ireland. The same plight appears to be affecting teaching in the Republic of Ireland. In a case study carried out by Hourigan & O’Donoghue (2007) they found that the Leaving Certificate examination was the core focus of every mathematics class they observed and this State Examination was the only motivation offered by teachers to students to encourage them to engage in mathematics. The results of this study would suggest similar attitudes and behaviours among the teachers involved in this project. As discussed previously teachers in this study believe that students’ learning and levels of interest in mathematics can be improved by highlighting the relevance, applicability and importance of different topics in mathematics but despite this perceived importance they feel that the examination is preventing them from introducing such ideas during mathematics classes.

*Eilis: Like trying to bring this in [how mathematics connects to real life] you could spend a whole class bringing in, em, doing these examples, you know and I know yeah realistically we should but when it comes down to it you have to teach to an exam*

The attitude among these teachers is that only material which is guaranteed to appear on the Leaving Certificate will be covered in detail in class as time does not allow for any other information to be divulged to students. Again such findings were also reflected in the work of Hourigan & O’Donoghue (2005) when they found that it was
common practice for teachers to omit certain topics that tend not to be examined in the Leaving Certificate. Similarly teachers in this study are of the opinion that until theoretical questions relating to the relevance and applicability of mathematics appear on the external examination paper then these important aspects of mathematics will not be dealt with in the classroom:

Maeve: ...what I’d be hoping for is that on the Project Maths that the emphasis would be put on this [the relevance and applicability of mathematics] ...now I don’t mean in terms of the curriculum, but that the exam will reflect it so that we would have to spend more time because they will be asked...a question like this list three ways in which differentiation is used in every day life.

This response from Maeve highlights how the examination is dictating the content being covered in class and restricting the teaching methodologies being employed. As a result, until changes in the examination, as advocated by Hourigan & O’Donoghue (2007), occur these teachers will fail to expose students to the relevant and enjoyable nature of mathematics. This in turn will have a detrimental effect on student learning. According to the Chief Examiner’s Report (2005) this is currently leading to students at ordinary level being competent in the routine application of procedures but failing to deal with problems which require higher order thinking while at higher level they report that candidates cannot engage with problems that are not of a routine or well rehearsed nature. This strong focus on the Leaving Certificate examination, among the participants in this study, is also leading to problems in relation to the teaching approaches adopted by these teachers and is contributing to the prevalence of a procedural approach to teaching in the mathematics classroom.

**Procedural Approach to Mathematics Teaching**

Almost twenty years ago English et al. (1991) found that a procedural approach to mathematics teaching was seen as common practice in schools in Ireland. Over the last two decades this finding does not appear to have changed (Lyons et al., 2003; Oldham, 2006). Teachers’ responses in the focus groups suggest that they still favour teaching mathematics as a set of procedures or as a routine that can be followed
every time. This approach means that every mathematics class is identical with only the topic changing. Classes involve the teacher explaining the steps, providing students with examples and then getting students to practice the procedure by completing a number of very similar examples with little or no focus on applications. The following conversation between Jason, Eilis and Maeve highlights this:

Maeve: *In practice at the moment I’d say we’re just trying to drill the different techniques into them.*

Jason: *Drill the steps in.*

Eilis: *It’s the steps yeah, it’s not even, it’s not even trying to relate it to real life, it’s like this is your formula, write it out, fill in your values, you know?*

A similar conversation was held between Eimear and Jackie:

Jackie: *…it’ll just be learn the method.*

Eimear: *It’s all about learning, practice, practice, practice and get it down.*

Jackie: *And copy the previous example.*

It is evident from these exchanges that these teachers, despite what they may have said about the importance of highlighting the relevance or applicability of mathematics, still teach mathematics as a set of rules to be followed every time. This situation might be due to time constraints and the examination driven nature of mathematics in Ireland. It is not surprising then that the TIMSS Report (1995) claimed that over 60% of Irish second year students were exposed to teachers who felt it unnecessary to disclose the uses of mathematics in the real world. Furthermore teacher’s responses in this study indicate that they view teaching as a one way process and the ‘mug and jug’ approach is the teaching approach favoured by these teachers. That is, they see students as empty vessels that they can literally fill with
knowledge that they deem appropriate. This approach, according to researchers, leads to passive modes of learning and develops instrumental rather than relational understanding among students (Wood et al., 1991). Furthermore this small scale study indicates that even questions which aim to analyse students’ ability to apply their knowledge are being taught in this manner:

Jason: …say with B3 there that kind of type of question [applicable question] if that comes up in question seven part C, the first thing you drill into them is if you write down the function, you differentiate it, you differentiate it again because straight away you get seven out of twenty…so straight away you’re just drilling into them you just do your step, you don’t even link the practical part really.

This quote, combined with the conversations outlined above highlight practising teachers’ ideas about the best and most effective ways to teach mathematics at Senior Cycle. However the views expressed by teachers in this study are not supported by research in this area. For the past number of years researchers such as de Corte et al. (1996) and Watson (2004) have found that this procedural approach to teaching is having a negative impact on student learning. Consequently, this notion that effective teaching involves drilling procedures into students must change in order for any real change in the standard of mathematics teaching and learning in Ireland to occur.

The Prevalence of Negative Attitudes among Teachers
Teachers in this study also demonstrated negative attitudes on a number of occasions throughout the focus group meetings. These poor attitudes appeared more prevalent among teachers involved with ordinary level classes. Research has found that teachers’ attitudes and expectations help to determine students’ attitudes (Phillipou & Christou, 1998; Wilkins & Ma, 2003). Therefore it is a concern that some of the students in ordinary level classes, who already tend to have low self esteem, a poor sense of self worth and negative attitudes towards mathematics (Liston, 2008), are being hampered further by being exposed to these negative attitudes on the part of their teachers. The following responses from teachers highlight their belief that ordinary level students should be shown little more than the procedures required to complete basic mathematical skills and do not feel it necessary to develop their
interest in the subject by highlighting the usefulness and importance of different mathematical topics.

David: …when you’re trying to teach a pass student calculus you’re just trying to get them to understand it so most of this stuff here would be absolutely pointless to ordinary level students…you make it as simple as possible and if they ask what it is for it’s to get into college.

James: …as regards ordinary level when I’m doing differentiation with ordinary level I just do the basics. You’d have pupils in there that wouldn’t have; you know they’d be so weak you might barely be able to get them through the Leaving Certificate…you know I’d move away from kind of explaining all this they wouldn’t know what would be going on.

Shane: …for a very weak student talking about something like that [the applicability of mathematics] it’s the very same as Paul Daniel’s [magician] stuff…I think it’s crazy in fairness.

Eimear: So your motto, your aim for them [ordinary level students] would be to get as much into them…you’re not going to tell them where it applies or anything you’re just going to go [say] you want 40% so you can pass your Leaving Cert.

These responses show how teachers in this study have very low expectations for ordinary level students and as a result feel it unnecessary for them to be informed about the importance or relevance of the subject. Many of the teachers believe it to be acceptable not to provide such students with basic explanations. In order to encourage ordinary level students to do well and excel in mathematics it is essential that these attitudes change. Otherwise there is a risk that ordinary level students will become disillusioned and unmotivated with the subject.

Another attitude that is common among these teachers is that the topic of calculus is not hugely important for many students in ordinary level classes. Despite previously acknowledging the importance of the topic and the numerous applications it has in every day life, teachers such as Gerry and Shane feel that ordinary level students do not need to study it at all at second level.
Gerry: …the people James is talking about, the weak student, I don't think they need to do calculus at all.

Shane: It was too abstract for them, calculus, you know what I mean?

A number of teachers involved in phase one of this study do not believe calculus to be relevant to students in ordinary level classes. However it is interesting to note that in 2008 over half the cohort of students (56%) who entered service mathematics modules in U.L. sat the ordinary level paper in this year. These two modules, Technological Mathematics and Science Mathematics, have a strong focus on calculus. Therefore it was critical that the 381 students who opted for the ordinary level had a basic understanding of the concepts and the applicability of the topic prior to entering third level. The only way that this can be achieved is through effective teaching. Contrary to the views of teachers such as Gerry and Shane, these figures suggest that it is critical that students at both higher and ordinary level are exposed to calculus at second level in order to prevent the problem worsening in relation to the transition from second to third level, as discussed by the NCCA in 2005.

Finally, research has found that students’ past experiences in the mathematics classroom affect their attitudes (Ponte et al, 1994) and this too appears to be the case with teachers in this study. The focus groups highlighted that teachers’ previous experiences had resulted in them developing negative attitudes and they also developed a resistance to trying out new approaches or to incorporating different procedures in their classes.

Shane: They go to town on you. Some of them that would be weak …they're only waiting for the chance to get in with a few other lads and start messing...

Eimear: …even with the whiteboards here now like if I brought that into boys…if it was my fifth years they’d be hitting each other on top of the head with it.
Gerry: …they [those advocating group work/project work] don’t allow for the Tommy in the corner who’s a disruptive little f***** and it can ruin the class.

These teachers all appear to be very wary of disruptive and weak students in the class and as a result of the negative experiences they have encountered due to the behaviour of these students in the past they appear unwilling to try any new approaches for fear of such students misbehaving. As a result, despite accepting the need for extra resources to be used and the importance of student involvement in the classroom, many of these teachers’ attitudes towards students, which stem from negative past experiences, result in them being unwilling to incorporate such ideas into their class.

Overall the attitudes demonstrated by teachers throughout the focus groups present those involved in mathematics education with a number of challenges. As pointed out previously these negative attitudes are resulting in similar pessimistic views of mathematics among students and this can prove detrimental. For example Middleton & Toluk (1999) found that poor attitudes among students detrimentally affect their willingness to engage with the subject of mathematics while in 2003 Papanastasiou & Bottiger found a link between positive attitudes and high attainment. In addition to this, the project’s teachers’ low expectations and the little faith that they appear to have in students will also result in students developing a perception that mathematics is a subject beyond them.

In conclusion the responses obtained from teachers throughout the focus groups indicate that improving mathematics teachers’ knowledge base will help overcome many of the problems facing mathematics education. However it is not the only challenge which those involved in mathematics education face. Numerous other problems in the field also exist. The number of problems outlined in this section exposes a worrying image of mathematics education and paints a bleak image for the future of mathematics teaching and learning. These problems will continue to affect the state of mathematics education if they are not addressed but before we begin to search for solutions there is one more issue that must be considered – ‘The Age Issue’
6.2.5 ‘The Age Issue’

In recent years researchers have found that teachers resist change (Richardson, 1998). However the author found, in her research, this resistance to be more common among older teachers or teachers who have been involved in the system for a number of years. Younger teachers appeared to embrace many of the ideas proposed by the author. From the responses it is clear that these teachers accept that change is needed in order to improve the teaching and learning of mathematics and are therefore willing to engage in such change.

Eimear: 
*Younger teachers are probably more open to change…*

On the other hand, however, all teachers appear to accept the fact that older teachers resist change. Over the years they have developed a system with which they are happy and are unwilling to divert from their set ways. They do not view the changes being advocated by the author as essential. Instead perceived past success has led them to believe that there is no need for change and again this indicates that they are conforming to Bourdieu’s idea of habitus. i.e. they do not see any need to change as the results they have been achieving to date have been deemed satisfactory.

Gerry: 
*We’re set in our ways. You know it’s harder to change once you have a system that works kind of, kind of well…It’s easier for younger teachers.*

James: 
*When you’re at it for nearly thirty years it’s very hard to [change]…*

Jackie: 
*You’ll find the older ones [teachers] , there’s a method, this works for me, this is how I do it and they’re not going to be open to change…a lot of teachers are going [saying] this is how I get the grades and they are not open to any kind of change.*

Shane: 
*And another thing that teachers have concerns, like myself we’ll say, this new course [Project Maths] now that, will it make teaching harder in a sense that we’ll say as of now you know it’s fairly straight forward in the class and they do their own bit of work and everything else but if you introduce this stuff they’re all intermingling and moving around and it’s going to make it very difficult…*
These responses highlight how older teachers in the study resist change. Many of these teachers had previously accepted the importance of the different knowledge domains yet such findings indicate that they are unwilling or do not see the need to do much to develop such knowledge or practices based on these knowledges. Moritimo (1973) found that teachers often resist change as they feel threatened by it and wish to cling onto their old ways. The past three decades have witnessed little change in this regard. Teachers such as Shane appear to fear change and as Moritimo (1973) suggested feel threatened by new developments, such as Project Mathematics which may force them to change their ways.

Overall, if much needed change is to occur in mathematics teaching and learning then all teachers must be open to it in order for it to result in improved standards. Teachers who may be involved in the system for a number of years must be encouraged to engage in initiatives that will help them to improve many aspects of their teaching and to enable them to evolve in line with the evolution of mathematics education. Only then will true reform happen in mathematics classes.

6.2.6 Perceived Problems Beyond the Control of Teachers

In addition to the problems which stem from teachers’ levels of knowledge, as outlined above, teachers involved in the first phase of this project also complained about problems outside of their control. As with the other problems discussed these too have been found to be problematic around the world and include an overcrowded curriculum, the problem of limited time, the curriculum and examination structure and issues with teacher training (Oldham, 2001; Cuoco, 2003; Eaton & Bell 2006) . As a result of such problems teachers, such as Jackie, would like to develop the different types of knowledge being advocated by researchers but believe that issues beyond their control often prevent them

Jackie: …it would be great but there is a lot of things out of our control.

Firstly the author looks at the problem of the over crowded curriculum and consequent timing issues. Over the past number of years researchers such as English et al. (1991) and Eaton & Bell (2006) found that the overcrowded mathematics
curriculum is proving problematic for teachers in Ireland and Northern Ireland. These researchers argue that congested curricula result in teachers being unable to find the time to inform students of the relevance of mathematics and as a result are unable to present mathematics as an enjoyable and worthwhile subject. In turn this led to disillusionment among students as they simply saw mathematics as another subject that they were forced to do during their school years and not something that they would use in the future. Such findings were also reflected in the focus groups as the teachers involved felt that the Senior Cycle curriculum in Ireland gave them little opportunity to engage students in lessons in a manner that would allow them to develop an appreciation for the subject.

Eilis: ...you’re just so restricted to get the course covered...

Maeve: But it’s a case of like you’re kind of just trying to get everything else covered. It’s like oh ... I don’t have time to be [explaining real life connections and applications] you know?

Due to the amount of material on the curriculum teachers involved in this study are finding it extremely difficult to even get the course completed let alone engage in any other activities, such as project work, that may enhance understanding and appreciation. The packed curriculum is also restricting the time these teachers can allocate to arousing interest among the students. Although many accept the importance of developing and maintaining interest and enthusiasm among students a large majority cite time as one of the main reasons for not attending to it:

Eilis: See I think you know when you’re teaching Senior Cycle you’re so restricted with you’re time, especially at higher level, you don’t have the time to, you just have to basically to try and get the kids to understand it and like get them working on it

Shane: The way things are at the moment Gary you’d hardly get time to do that, with the honours Leaving Certificate anyway.
Maeve: *At the moment no, we don’t have the time but em, that is what we should be doing and I think everyone would agree to that, that we should be showing how maths is part of everything we do.*

As a result, although these teachers do acknowledge the importance of the ideas being proposed by the researcher they feel the time simply is not there to allow them to adopt such approaches and to fully engage with the material.

Another issue that participating teachers appear to have with the curriculum relates to its content and the structure of the Leaving Certificate examination. As discussed in Section 6.2.1, teachers are beginning to accept that they need an extensive knowledge base in order to be able to teach effectively yet they believe that this is not reflected in the current Leaving Certificate programme. Many of the teachers, who felt it essential that the applications of mathematics be discussed with students, state that until such information is included in the curriculum it will never be accepted by everyone involved. At the moment teachers feel that the lack of focus on applications on the current Leaving Certificate curriculum is making it extremely difficult to concentrate on this aspect of mathematics in class:

Anne: *Well you see there isn’t an awful lot of applications of calculus to real life in the syllabus…but yeah I know yeah they’re very interesting questions.*

Maeve: *…definitely at higher level, there’d be a lot more emphasis on em, differentiating types of functions as opposed to, and integrating, as opposed to applying it you know?*

James: *What’s happening there now you see, even your stuff there now, say questions kind of based on what you’re giving there now would be much more relevant to a Leaving Cert exam than what they get at the moment. You know at least they can see some practical stuff in that.*

If these teachers are willing to accept the importance of developing these different types of knowledge, then it is important that their efforts are rewarded. One way of doing that would be to ensure that the value and importance of applications and
connections is reflected in future curricula and examinations. Although there is an onus on teachers to develop these types of knowledge independently it would be seen as a much more worthwhile venture if they saw those in authority also promoting the validity and worthiness of their efforts. This is the case with Project Mathematics and offers hope for the future.

In addition to the perceived problem of a poorly structured curriculum, teachers in the focus groups also believe the examination which is designed to assess the Leaving Certificate course is flawed. They feel that the current structure of the exam is proving detrimental to the teaching and learning of mathematics. Practising teachers believe that both teachers and students can be successful in the examination without having an in-depth knowledge of the important concepts.

Andrew: And you see the [exam] system is designed in such a way that you can get through the net without knowing certain topics...you can definitely get through and get an A and get 100% without knowing certain areas.

This is a cause for concern, as it means lazy or unmotivated teachers, who researchers argue are common in many schools (Boero et al, 1996; Phillipou & Christou, 1998), will fail to expose students to a number of important mathematical concepts. Instead they are aware that they will succeed without an understanding of these. As a result the system is failing a number of students by failing to provide students with a well-rounded fundamental understanding of mathematics and an appreciation for the subject (NCCA, 2006)

The final area where participants in this project believe problems beyond their control exist is in teacher training courses at third level. In Chapter 3 the author discussed the significance of the mathematics modules in teacher training courses and there appears to be a general consensus among researchers that many of the modules to which trainee teachers are exposed are irrelevant and inadequate. They believe that teacher training courses do not necessarily require additional modules but instead teachers need to study different modules (Even, 1993; Cuoco, 2003). Such findings are also echoed by teachers in this study. The practising teachers involved believe that the material which they studied at third level failed to help them to develop knowledge in the seven areas analysed in the authors’ questionnaire.
Teachers explicitly stated that their third level education did not facilitate their understanding of the history of mathematics, the relevant applications of mathematics or the cross curricular links between other subjects and mathematics. Instead they felt that the modules which they engaged in simply covered irrelevant concepts that they may never use in their day – to – day teaching.

Eilis: *Do you know I was taught a lot of stuff at University that I will never ever again use”*

Jason: *You’d never be told about it [the applicability of mathematics] in college really to be honest...You’re not trained. There’s not, they don’t talk about it.*

Shane: *There was nothing like that [promotion of group work and project work and a shift from the procedural approach during lectures] now in my time. You just, all that happened was you did your lecture there, your man came in and he wrote down something on the board and he just kept going and going and you wrote it down.*

These findings indicate that many teacher training courses are currently not providing trainee teachers with the knowledge that researchers such as Fennema & Franke (1996) and Rowland (2007) have advocated in the past. Although the author accepts that it is important for teachers to engage in CPD in order to develop the knowledge proposed in the Ladder of Knowledge, third level institutes must also accept some responsibility and strive to ensure that teacher training courses serve to enhance the knowledge required for teachers among all prospective mathematics teachers.

Overall these problems relating to the curriculum, the examination and teacher training programmes are having a negative effect on the standards of teaching and learning. It is clear that teachers face many obstacles when attempting to carry out their professional duties and the climate in which they currently teach is a very difficult one. These problems which are not of their own making must be dealt with by external bodies such as Department of Education and Skills, the National Council for Curriculum and Assessment and in turn the State Examinations Commission and Third Level Institutions. These bodies must act to help alleviate the numerous
challenges facing mathematics education in Ireland. It would be hoped that the new Project Mathematics course will be the first of a number of key steps taken by these bodies to help rectify some of the problems which will in turn lead to improved mathematics teaching and learning in this country.

6.2.7 Summary

In conclusion there were seven main themes emerging from the focus groups, namely:

- The need to improve levels of knowledge among mathematics teachers
- The examination driven nature of Senior Cycle mathematics
- The prevalence of a procedural approach to mathematics teaching
- Negative attitudes on the part of teachers
- A resistance to change especially among older teachers
- The acknowledgement of problems with the education system
- The acceptance of the importance of an extensive knowledge base for teaching.

Six of these themes depict a limited image of mathematics education in the four schools involved in this phase of the study and it is clear that work is needed to improve mathematics teaching and learning. Despite teachers in this phase of the study acknowledging the fact that they need to develop a number of different types of knowledge it is evident that strategies are needed to bridge the gap between what teachers believe they should know and what they actually know. Furthermore following on from the focus groups, it is clear to the author that the teachers involved are currently required to teach in a very difficult climate and problems beyond their control also need to be addressed. Only then when all parties work together and strive to improve standards at all levels will true change happen. However, for now, the author will analyse whether the claims made by teachers in this phase are supported by responses submitted by teachers during the pilot study. If such findings
are supported this will have repercussions for students and these implications are reported before a combination of findings from the pilot study and the focus groups are used to help the researcher design an intervention that will assist teachers in developing the necessary knowledge required for effective teaching.

6.3 The Pilot Study

Once the original questionnaire designed by the author was amended, in line with teachers’ recommendations from the focus groups, it was distributed to 44 teachers as a pilot study. 22 of these were returned to the researcher. This pilot study served a number of purposes. Primarily it served to validate the author’s model. Secondly it allowed the author to get further insight into the levels of knowledge of the teachers involved in this study and offered support to many of the claims made by subjects during the focus groups. Finally, the pilot study allowed for the author to gather information on the baseline knowledge of teachers prior to them engaging in the CPD intervention. As a result the pilot study may also be used to assess the areas in which teachers felt the CPD intervention helped them improve and can validate teachers opinions on their levels of knowledge before and after participating in the study.

For the purpose of the pilot study, schools were selected at random in the Munster, Connaught and Dublin region. 22 teachers were involved in the pilot phase; 7 males, 13 female and 2 IDs withheld. These teachers taught in ten different schools across six different counties (from a total of 26 possible counties). 8 of the 22 (36%) subjects involved in the pilot study held concurrent teacher education degrees with mathematics while a further 5 teachers (23%) had a BA/BSc. with mathematics primary degree. Another 2 out of 22 teachers (9%) of the sample would have done a significant amount of mathematics in their degree but would not be specialist mathematics teachers while 3 teachers (14%) were out of field teachers. Finally 4 out of the 22 subjects involved (18%) withheld this information. All subjects are currently teaching Leaving Certificate mathematics at either higher or ordinary level and teaching experience ranged from two years to thirty four years.
6.3.1 Common Findings from the Pilot Study and the Focus Groups

**Teachers Perceptions on their Levels of Knowledge**
Throughout the focus groups teachers constantly referred to their low levels of knowledge and teachers such as James, Anne, Jason and Eimear admitted to not understanding the applications, connections and history of calculus or the resources that could be used in a calculus class. The pilot study yielded similar results. Content knowledge and transformational knowledge were the only knowledge domains of the seven set out on the ‘Ladder of Knowledge’ in which the majority of teachers felt competent in. 14 of the 22 teachers surveys teachers strongly agreed or agreed with the statement *I have a strong content knowledge of calculus* while 15 teachers strongly agreed or agreed with the statement *I find it easy to get my ideas about calculus across to students*. On the other hand, however, when questioned about their perceived knowledge in the other domains on the Ladder of Knowledge only 5 teachers (23%) believed they were aware of a wide number of applications, 3 teachers (14%) agreed or strongly agreed with the statement *I know how calculus links to students everyday lives* while 15 of the teachers (68%) stated that they did not have a good understanding of the historical origins of calculus. Table 6.2, overleaf, categorises teachers’ perceptions of their levels of knowledge in each of the knowledge domains outlined on the Ladder of Knowledge as either strong, unsure\(^{15}\), weak.

\(^{15}\) This was the percentage of teachers who were undecided on any given statement.
Table 6.1: Teachers Perceptions of their Levels of Knowledge

<table>
<thead>
<tr>
<th>Domain</th>
<th>Strong</th>
<th>Unsure</th>
<th>Weak</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Subject Matter Knowledge</strong></td>
<td>14 Teachers</td>
<td>4 Teachers</td>
<td>4 Teachers</td>
<td>22 Teachers</td>
</tr>
<tr>
<td></td>
<td>64%</td>
<td>18%</td>
<td>18%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>Knowledge of Real Life Applications</strong></td>
<td>5 Teachers</td>
<td>8 Teachers</td>
<td>9 Teachers</td>
<td>22 teachers</td>
</tr>
<tr>
<td></td>
<td>23%</td>
<td>36%</td>
<td>41%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>Knowledge of Connections</strong></td>
<td>3 Teachers</td>
<td>10 Teachers</td>
<td>9 Teachers</td>
<td>22 Teachers</td>
</tr>
<tr>
<td></td>
<td>14%</td>
<td>45%</td>
<td>41%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>Historical Knowledge</strong></td>
<td>4 Teachers</td>
<td>3 Teachers</td>
<td>15 Teachers</td>
<td>22 Teachers</td>
</tr>
<tr>
<td></td>
<td>18%</td>
<td>14%</td>
<td>68%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>Pedagogical Knowledge</strong> (2 questions)16</td>
<td>7 Teachers</td>
<td>5 Teachers</td>
<td>10 Teachers</td>
<td>22 Teachers</td>
</tr>
<tr>
<td></td>
<td>34%</td>
<td>21%</td>
<td>45%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>Transforming Knowledge</strong></td>
<td>15 Teachers</td>
<td>5 Teachers</td>
<td>2 Teachers</td>
<td>22 Teachers</td>
</tr>
<tr>
<td></td>
<td>68%</td>
<td>23%</td>
<td>9%</td>
<td>100%</td>
</tr>
<tr>
<td><strong>Knowledge of School Maths</strong></td>
<td>9 Teachers</td>
<td>8 Teachers</td>
<td>5 Teachers</td>
<td>22 Teachers</td>
</tr>
<tr>
<td></td>
<td>41%</td>
<td>36%</td>
<td>23%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 6.1 highlights teachers’ beliefs in relation to their own levels of knowledge. Teachers in the pilot study do not feel competent in the majority of domains outlined in the author’s Ladder of Knowledge, hence it can be assumed that they do not feel that they have the knowledge required to teach effectively.

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16 In cases where two questions help to establish teachers perceptions of their levels of knowledge in a given domain the percentage is calculated for each individual question and the mean result is then taken.
Levels of Knowledge Displayed by Teachers in relation to Knowledge of Applications

In the focus groups, as well as stating their beliefs that their knowledge was inadequate, teachers such as James also demonstrated poor understanding of mathematical applications. Such findings were also evident in the pilot study. In Section B of the questionnaire teachers were required to answer a number of calculus questions (See Appendix D). The first four questions examined teachers procedural knowledge, that is their knowledge of the rules and procedures that are frequently encountered in calculus classrooms while the last 3 questions investigated their levels of relational understanding i.e. their ability to apply such procedures to real life problems. When analysing the questionnaires the author marked each question using the same scale. Each question in Section B received one of five ratings. It was either marked fully correct, attempt made with slight error/numerical slip, attempt made but with fundamental error/numerous numerical slips, attempt made but with no correct work to note or no attempt made. In order to compare the standard of answers the author first analysed each question individually before combining the four procedural questions and calculating the average percentage of teachers in each of the aforementioned categories. The exact same procedure was carried out when analysing the final three applicable questions and the results from this analysis is highlighted in Figure 6.1, below.

![Figure 6.1: Procedural Knowledge vs. Applicable Knowledge](image)
Teachers in this phase struggled with questions which assessed their relational understanding. On average 56% of participants were able to correctly answer questions using rules and procedures that appear on the Leaving Certificate syllabus with a further 16% only making a simple numerical slip. In contrast only an average of 12% was able to correctly apply such procedures to solve everyday problems while only a further 6% were close to getting it fully correct (one numerical slip). Furthermore, on average, less than 4 teachers in this study were unable to attempt a question that required them to demonstrate procedural knowledge compared with over a 10 teachers who could not attempt a question requiring them to apply their knowledge while a further 5 teachers attempted the questions but none of the work submitted was correct.

The remaining question in Section B (Question B2) when compared with the applicable questions mentioned previously alerted the author to another issue relating to teachers levels of knowledge. Again this issue was first exposed during the focus groups and relates to the correlation between a teachers knowledge base and the material that is covered in Leaving Certificate textbooks.

**Levels of Knowledge and Leaving Certificate Textbooks**

The author has already shown the answers submitted by teachers for the applied questions in Section B (questions B3, B4 and B5). However another applied question which has yet to be analysed is question B2. This was again an applied question but this time it was similar to questions that teachers would encounter in a text book or on past Leaving Certificate examinations. There were three elements to the question and it was interesting to note that 16 of the 22 teachers surveyed got at least two of the three questions fully correct. Only 2 teachers failed to attempt any of the three questions. When we then compare this with the other three application questions it is interesting to note that only 2 teachers were able to correctly answer two or more of the questions on applications (requiring simple calculus) that they may not have seen before. The number of teachers who failed to attempt any of the three questions (7 teachers) was also considerably higher than the same figure for question B2. Overall such findings, coupled with responses from teachers in the focus group, would suggest that teachers in this study, although some may believe they know
applications of calculus, such applications are limited to the material they have covered either in text books or on past exam papers. Hence, they are unsure of how to apply calculus to numerous other, more relevant, problems that they and their students often encounter in their daily lives.

Knowledge of applications was not the only type of knowledge which was limited to the textbook in both the focus groups and the pilot study. As discussed previously teachers in the focus groups were only able to connect calculus to areas such as speed and velocity, turning points or area and volume. Similarly the majority of responses attained from the pilot study showed that teachers were unsure of any further connections between calculus and the real world. In section C (question C1 and C2 in Appendix D) teachers were asked to list four areas where differentiation and integration may be used outside of school. 10 of the 22 teachers (45%) were able to connect differentiation to 3 or more areas while 3 teachers (14%) were able to do likewise for integration. Such findings would suggest that efforts need to be made to help these teachers improve in this domain. When the answers submitted by teachers were analysed further, further challenges arose. Figures 6.2, below, and 6.3, overleaf, highlight the most common links between calculus and everyday life, as given by teachers in this pilot study.

Figure 6.2: Most Common Links to Differentiation
When these links are compared with those outlined in Leaving Certificate textbooks there are very few discrepancies between the two. All the connections provided by teachers, with the exception of ‘Business Problems’, appear in all the books available in Ireland for Senior Cycle mathematics. This again highlights the idea that teachers’ knowledge in the area of connections is limited to what is contained inside the resource they rely on most, the textbook (Hourigan & O’Donoghue, 2007). These examples provided by teachers in the two phases of this study represent a narrow coverage of the connections which calculus has with the real world and highlights gaps in the knowledge base of these teachers.

**Knowledge of Historical Origins**

During the analysis of focus group feedback the author noted a response from Anne whereby she claimed that a famous thing about Irish teachers was that they never knew who invented anything or the history behind any topic that they were teaching. The findings from the pilot study corroborate this view. In section C and D teachers were asked questions in relation to the historical origins of calculus (Question C4 and D4 in Appendix D). In question C4 teachers were asked to name one or more mathematician/scientist credited with developing ideas about calculus and to discuss briefly the contributions they made. Table 6.2, overleaf, highlights the standard of answers received from teachers in the pilot study for this question.
In addition to this, Question D4 required teachers to give a brief explanation of the history of calculus as they would to students. 16 teachers in the pilot study were unable to give an explanation while a further 4 only gave a very brief and uninformative explanation from which students would gain little. Such findings from the pilot study support the claim made by Anne during the focus group and highlight that the majority of teachers in this study are not aware of the historical origins of calculus. As a result, in addition to helping teachers improve their understanding of mathematical applications and connections, efforts must also be made to help teachers improve their level of knowledge in the final domain in the lower level of the ‘Ladder of Knowledge’ (Historical Knowledge). Otherwise teachers will not have the required content knowledge to move onto the second phase of the author’s model. If teachers do not acquire the types of knowledge outlined in the lower level of the ladder they will face many obstacles and challenges when trying to teach mathematics effectively.
**Procedural Approach to Learning**

Another issue identified by the author in the analysis of her focus group responses was in relation to the procedural approach to teaching which a lot of teachers appeared to adopt. From conversations held with teachers in different focus groups it was noticeable that the main aim for many teachers was to ‘drill’ knowledge into students, to show them the rules and procedures to follow and then to get them to practice continuously. The responses received during the pilot study suggested that this was a problem among these twenty two teachers too. Figure 6.4, below, highlights teachers’ responses to the question *Do you think it is possible to allow for student involvement in your calculus classes.*

![Figure 6.4: Is Student Involvement Possible in the Mathematics Classroom](image)

Figure 6.4 shows that teachers in the pilot study are of a similar opinion as teachers in the focus groups. The majority do not believe it possible to involve students in the classroom and instead, for numerous reasons including an overcrowded curriculum and time constraints, as well as their own poor levels of knowledge in this regard, they see mathematics teaching and learning as a process whereby they must share the information and expect students to learn it off, without fully involving them in classroom procedures. In addition to this even when teachers attempted to give examples of activities that may allow for student involvement the three suggestions provided were:
1. Questioning

2. Group discussion

3. By adopting a practical approach.

Although it is encouraging to see teachers attempting to involve students in a classroom this involvement is limited and the tasks that they are alluding to are neither rich nor of huge interest for adolescents.

In addition to poor student involvement many of the responses received for different questions also support the supposition that a procedural approach is the one favoured by teachers in this study and teachers do not have the knowledge to incorporate alternative teaching approaches. For example when asked to explain the importance of calculus for other topics on the Leaving Certificate Mathematics syllabus (Question D3) Teacher 2 wrote:

Teacher 2: *Teach it as a separate topic on syllabus with a question on exam paper.*

Similarly Teacher 99 responded to question D4 by writing:

Teacher 99: *I don’t give any history to the students I teach, as coverage of curriculum basics is my aim”*

Finally when asked to explain what is actually being done during the process of differentiation, Teacher 16 gave the following explanation as her answer:

Teacher 16: *Follow the rule of differentiation:* \[
\frac{d}{dx}(x^n) = nx^{n-1}, \quad x \in \mathbb{N},
\]
Such responses to these probing questions further emphasise the ‘mug and jug’ approach that a large number of teachers opt for. Teachers in this study view mathematics as a set of rules to be followed and believe that there is little need for students to be exposed to other aspects such as mathematical applications, connections or historical information. Such findings are also supported by the TIMSS Report (1995) which found that almost 70% of second and third year students in Ireland were exposed to teachers who felt it was very important for students to remember formulae and procedures in order to succeed in mathematics. Of the 45 countries involved in this study only one country, Kuwait, reported a higher percentage of students exposed to such teachers. In contrast, only 20% of students were taught by teachers who felt that it was important to promote an understanding of the usefulness of mathematics, the lowest percentage among the 45 countries in the study. Due to the findings discussed to date as part of this research project as well as those presented in the TIMSS Report (1995), it is unsurprising that the TIMSS report went on to find that 69% of students believed that in order to succeed in mathematics it was necessary to solely memorise the text book or class notes.

Use of Resources
The procedural approach to teaching discussed in the previous section is exacerbated in its effects by teacher’s understanding of possible resources that could be used in the mathematics classroom. During one of the focus groups Andrew, Eilis and Jackie referred, on numerous occasions, to the resources available to them. They stated that they did not have access to relevant resources and felt that the only resource available to them was the marker and board. A similar opinion was expressed by Eimear when she predicted that 90% of teachers in the pilot study would only cite using resources such as the blackboard and the overhead projector in their classroom. Eimear’s prediction was relatively accurate as Figure 6.5 shows:
Figure 6.5 portrays an image of the resources used most frequently by teachers in the pilot study. Traditional resources such as the textbook, the blackboard and the overhead projector are still the most popular among these practising Senior Cycle teachers. It is encouraging to see some of the teachers also being open to the idea of using technology in the classroom and the need for concrete examples in order to develop student understanding. However these two resources are not the most popular in this study and in fact were only referred to by a limited number of teachers. As a result there is a need for teachers to become aware of alternative resources that can be used to aid and improve their teaching and that can assist in overcoming the problem of procedural teaching. Such resources can also be used in both Higher and Ordinary Level classes and so may help improve the standard of teaching in both. This improvement across the two levels, not just at Higher Level, is necessary as the following section shows.
Focus Groups and Pilot Study Findings

Higher Level Teachers vs. Ordinary Level Teachers
Another matter arising from the focus groups related to the difference in attitudes among teachers depending on the level they taught. Across all focus groups conducted many of the teachers involved with ordinary level classes seemed to disregard the need to highlight the relevance or applicability of the subject. Instead they firmly believed that at ordinary level mathematics should be taught as a set of rules and procedures without any explanations or background information needed. Although teachers attitudes were not explicitly examined during the pilot study it is interesting to note the discrepancies in the responses submitted by higher level teachers and ordinary level teachers. In the focus groups teachers did not believe it to be important to highlight the connectivity of mathematics to students and so it is interesting to note that ordinary level teachers in the pilot study do not appear to have an in-depth knowledge of connections. Tables 6.3, below, and 6.4, overleaf, compare the number of connections provided by both higher and ordinary level teachers for both differentiation and integration.

<table>
<thead>
<tr>
<th>Current Teaching Level</th>
<th>Number of Connections Provided</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Higher</td>
<td>2</td>
</tr>
<tr>
<td>Ordinary</td>
<td>0</td>
</tr>
<tr>
<td>Both(^{17})</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.3: Comparison of Higher and Ordinary Level Teachers Ability to Connect Differentiation to Real Life

\(^{17}\)This refers to teachers who teach mathematics at both Higher and Ordinary level.
Table 6.4: Comparison of Higher and Ordinary Level Teachers Ability to Connect Integration to Real Life

<table>
<thead>
<tr>
<th>Current Teaching Level</th>
<th>Number of Connections Provided</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Higher</td>
<td>1</td>
</tr>
<tr>
<td>Ordinary</td>
<td>4</td>
</tr>
<tr>
<td>Both</td>
<td>2</td>
</tr>
</tbody>
</table>

At first glance this shows that two Higher Level teachers cannot connect differentiation to any real life problems while further investigation highlights more pressing concerns. These tables show that higher level teachers are able to connect calculus to a wider range of areas than ordinary level teachers. Three quarters of higher level teachers were able to provide two or more real life applications of differentiation compared with only a half of ordinary level teachers. Only one ordinary level teacher was able to suggest more than one link between integration and the outside world. Although integration is not a topic on the ordinary level course it would still be expected that mathematics teacher at either level would be able to relate integration, which is essentially anti differentiation, to real life issues, especially when 50% of the teacher’s currently teaching ordinary level in this study have taught the higher level course within the last 10 years.

As mentioned previously many teachers in the focus groups, including David, believed that if ordinary level students wanted to know the need for calculus it would be sufficient to tell them that it was a requirement for entry into college and nothing more. Responses from teachers in the pilot study suggest that they are also unable to connect mathematics to other school subjects. Figure 6.6, overleaf, presents a graphical representation of the standard of answers received when teachers in this study were asked to identify ways in which they could link calculus to six school subjects, namely science, business, engineering, technology, physical education and geography (Question C3).
Chapter 6                      Focus Groups and Pilot Study Findings

Figure 6.6: Comparison of Higher and Ordinary Level Teachers Ability to Connect Calculus to Other School Subjects

Figure 6.6 shows how higher level teachers again performed better when asked to connect calculus to other school subjects. A half of Higher Level teachers in the pilot study were able to link calculus to five or six of the subjects listed but no ordinary level could do the same (Only one ordinary level teacher could link calculus to five subjects). Instead 40% of ordinary level teachers could link calculus to one subject or less. In addition to this, teachers in this pilot study also appear inadequately prepared to inform students of the importance of calculus for other topics on the Leaving Certificate mathematics syllabus (Question D3 in Appendix D). 7 out of 10 ordinary level teachers were unable to explain where calculus may be used in other mathematical topics while only one ordinary level teacher was able to give a detailed explanation. In comparison only one higher level teacher was unable to provide some explanation and 3 out of 8 provided a detailed explanation that would help students understand the usefulness of mathematics for other mathematical topics.

Finally, the focus group findings indicated that teachers such as James saw no need to thoroughly explain concepts to ordinary level students during the process of teaching. Similar findings emerged during the pilot study. There was a significant difference between the standard of answers submitted by ordinary level teachers and those submitted by higher level teachers when asked to explain the idea of
differentiation and to give an overview of the historical origins of calculus. Table 6.5, below, highlights the standard of answers obtained from both higher and ordinary level teachers when asked to explain what is being done during the process of differentiation while Table 6.6, below, outlines the standards from the same cohort of teachers when they were asked to explain briefly the history of calculus as they would to students.

<table>
<thead>
<tr>
<th>Current Teaching Level</th>
<th>Standard of Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unable to Explain</td>
</tr>
<tr>
<td></td>
<td>Examination given</td>
</tr>
<tr>
<td></td>
<td>with reference to</td>
</tr>
<tr>
<td></td>
<td>slope and/or rate</td>
</tr>
<tr>
<td></td>
<td>of change of two</td>
</tr>
<tr>
<td></td>
<td>variables</td>
</tr>
<tr>
<td>Higher</td>
<td>3</td>
</tr>
<tr>
<td>Ordinary</td>
<td>4</td>
</tr>
<tr>
<td>Both</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6.5: Comparison of Standard of Responses from Higher and Ordinary Level Teachers for Question D5

<table>
<thead>
<tr>
<th>Current Teaching Level</th>
<th>Standard of Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Explanation</td>
</tr>
<tr>
<td></td>
<td>Given</td>
</tr>
<tr>
<td></td>
<td>Brief and</td>
</tr>
<tr>
<td></td>
<td>Uninformative</td>
</tr>
<tr>
<td></td>
<td>Explanation Given</td>
</tr>
<tr>
<td></td>
<td>Detailed</td>
</tr>
<tr>
<td></td>
<td>Explanation Given</td>
</tr>
<tr>
<td>Higher</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Ordinary</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Both</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6.6: Comparison of Standard of Responses from Higher and Ordinary Level Teachers for Question D4

Again these tables highlight concerns in relation to ordinary level teachers and echo the findings that have been discussed to date.
6.3.2 Summary of Pilot Study Findings

To conclude, it is now evident that work is needed to improve the levels of knowledge of teachers involved in this study so as to help them progress on the Ladder of Knowledge. The pilot study supports the findings from the focus groups whereby teachers do not appear to have the knowledge required for effective teaching as outlined in the Ladder of Knowledge. Each of the domains outlined by the author in the Ladder of Knowledge was included due to its perceived importance in the Irish context yet when the knowledge of a number of Irish teachers was analysed it is clear that teachers must engage in initiatives to help them develop these knowledge domains. Teachers’ level of subject knowledge and related knowledges goes some of the way towards explaining why Irish students fail to match the mathematical standard of students of the same age in different countries (Irish Mathematical Society, 2006; NCCA, 2006).

Furthermore both the pilot study and the focus groups suggest there are discrepancies between the levels of knowledge of qualified ordinary level teachers and that of qualified higher level teachers. When this is coupled with the finding of NiRiordain & Hannigan (2009), who found out-of-field teachers are primarily assigned to ordinary level classes this presents further concerns. For example the majority of Senior Cycle students, 84% in 2009, study mathematics at ordinary level and such findings therefore suggest that the majority of students are being placed at a disadvantage due to their teachers’ level of knowledge or third level qualification.

Overall the levels of knowledge demonstrated by all teachers at different stages of this pilot study is a concern for everyone involved in mathematics education and the author is in no doubt that until efforts are made to improve teachers’ subject knowledge, problems in mathematics education will continue to prevail and the implications will be felt by students nationwide. The exact nature of these repercussions will be discussed by the author in section 6.4.

6.4 Implications for Students

The most critical finding from both the focus groups and the pilot study was the low levels of knowledge in the majority of domains outlined on the Ladder of
Knowledge that participating teachers either admitted to or demonstrated. This in itself has numerous repercussions. Firstly research has already indicted that in order to be a teacher you must develop an extensive knowledge base (Shulman, 1985). Knowledge of mathematics alone is no longer seen as a guarantee for good teaching and instead teachers must develop a combination of different types of knowledge in order to teach effectively (Barton, 2008). However the teachers in this study have not yet had the opportunity to develop this extensive knowledge base. Research has found that low levels of knowledge among mathematics teachers results in them doing much harm in the classroom (Conant, 1963) while Skemp (1976) found that poor knowledge on the part of teachers will lead to them failing to help their students develop a conceptual understanding of mathematics. More recently Ball et al. (2001) found that low levels of knowledge affect teacher’s ability to select appropriate resources for use in the classroom, their capacity to respond to students work effectively and their ability to assign effective and motivating homework assignments. Such problems arising from low levels of knowledge among teachers will have a negative impact on the mathematical experiences of their students and their attitudes, beliefs and knowledge of mathematics will also be negatively affected.

A second problem in relation to the levels of knowledge displayed by teachers in this study related to their positioning on the Ladder of Knowledge. As outlined in Chapter 5 the author submits that competency in each of the domains on the Ladder is necessary in order for teachers to develop the knowledge required for effective teaching. By admitting and demonstrating inadequate levels of knowledge in the majority of domains included in this model it can only be assumed that based on this package teachers do not believe they have the knowledge to teach effectively. This again will have serious repercussions for second level students. An abundance of research has been carried out in the area of effective teaching and the findings of Even (1993), Brown et al., (2001) and Ball (2001) have found that effective teaching can lead to improvements in levels of attainment among students. However, the opposite has also been found to be true and it is believed that if students are not exposed to effective teachers during their formative years then this can have a lasting, negative impact on their mathematical ability which is then hard to reverse at a later date (London Mathematical Society, 1995). As a result of such research, the
findings discussed by the author present those involved in the field of mathematics education with numerous challenges and it is essential that efforts are made to help teachers improve their knowledge base so as to ensure a positive mathematical experience for second level students. Without the knowledge to teach effectively it is believed that teachers will actually have a detrimental impact on their students and only when teachers develop the knowledge that will allow them to teach effectively will they be able to help these students develop a deeper understanding of mathematics and its usefulness and in turn contribute to their cognitive and affective development (Eaton & Bell, 2006).

Another notable finding from both the focus groups and the pilot study was that teachers appear confident in their ability to reproduce formulae and procedures but many of the same teachers are then unwilling to attempt to apply the knowledge they are so competent in to real life situations. If these teachers are unable to apply mathematical procedures to real life problems then it is unlikely that their students will be informed of the real life applications of many mathematical concepts. It would be expected, therefore, that many Irish students would, like so many other students referred to in international research, fail to see the importance or relevance of mathematics in the world around them (Frid & White, 1995; Smith, 2004; Eaton & Bell, 2006). As the author has already suggested this could result in these impressionable teens becoming disillusioned and uninterested in the subject of mathematics and there will be little hope of reaching the target of 30% of the cohort, outlined by the Engineers Ireland (2010), sitting the higher level mathematics paper at Senior Cycle. Furthermore, if teachers are unable to apply their procedural knowledge to real life situations it is impossible for them to instruct their students in this regard. Hence without a deep understanding of mathematical applications, Irish students will continue to struggle in international assessments such as PISA (2003) when their relational understanding is compared to that of international students and the problem of inadequate performance in international tests will persist. These students will also find it difficult to adapt to the new Project Maths curriculum which aims to deepen student understanding and increase the use of mathematical contexts and applications (www.projectmaths.ie). Consequently until teachers develop a thorough understanding of mathematical applications, attainment rates in State Examinations will continue to be less than satisfactory and the poor standard of
mathematics teaching and learning will continue to cost our government money, resources and highly qualified personnel in the coming years (Donnelly, 2010).

In the past very little research was carried out in the area of teacher dependency on the textbook (Freeman & Porter, 1989). However in recent years much research has suggested that the textbook is the main resource employed by teachers internationally. The TIMSS Report (2003) found that on average 65% of students were taught by teachers who used the textbook as a primary resource. Similarly, in Namibia, Katonyala (1999) found the textbook to be the most commonly used instructional tool while Tholey (1994), as cited in Polaki (2006: 2) believes that

“...textbook dependency is a reality to be found in many countries”.

The findings of both the focus groups and the pilot studies suggest that this is the reality in many Irish classrooms too. The textbook was cited as the most popular resource by the majority of teachers involved in this study and this dependency on the textbook could lead these teachers to ignore some of the aims and objectives outlined in the national syllabus as their lessons are entirely dictated by the textbook (Katonyala, 1999). This in turn will lead to teachers in this study failing to allow for the holistic development of their students as Irish textbooks rarely refer to the affective or psychomotor domains. Furthermore the author has already indicated how the knowledge of teachers in this study is limited to the textbook. According to Polaki (2006) and Katonyala (1999) the content contained in the majority of international textbooks is mainly of a procedural or computational nature while O’Keeffe & O’Donoghue (2010) found that in Ireland the textbook afford more attention to procedure as opposed to understanding. Therefore if teachers’ knowledge is limited to the textbook it can be assumed that their knowledge too is confined to mathematical procedures and concepts.

Another issue identified in the findings outlined earlier relates to the procedural approach to teaching. Research carried out by English et al. (1991) suggests that a procedural approach to teaching is prevalent in the majority of Irish schools and this approach is favoured by teachers in this study. As teachers do not have sufficient knowledge of applications, connections and historical origins, the ladder analogy suggests that they do not have the pedagogical knowledge that will allow for an innovative approach to mathematics teaching. This results in the sole aim of
mathematics teaching among these teachers being course coverage and exam preparation with little focus on arousing student interest or increasing enthusiasm for the subject of mathematics. Such an approach does not allow teachers to highlight the relevance or applicability of the subject and has already shown to have a negative effect on student learning and attitudes. For example Wearne & Hiebert (1988) found that the procedural approach to teaching was resulting in students failing to extract any true meaning from mathematics while Corte et al. (1996) believe this approach is leading to poor levels of understanding among students. Therefore, the implications of this approach are serious and when combined with the other problems arising from other issues identified in this study it would appear that levels of knowledge of participating teachers are negatively impacting on the mathematical understanding and attainment of their students.

This procedural approach to teaching is also leading to another problem namely the limited number of resources being used by teachers. The majority of teachers in this study are uncomfortable with employing resources other than those traditionally used in the mathematics classroom i.e. the textbook, the blackboard and the overhead projector. According to Kyriacou (1998) there are a wide range of resources available for use by mathematics teachers including computer packages, worksheets and simulation materials. Furthermore Ball et al. (2001) found that millions of dollars had been invested in curriculum materials in recent years in the United States resulting in the development of a wide range of resources that can, if used correctly, assist mathematics teaching and learning. However similar investment is yet to be witnessed in Ireland and although money is being invested in the new Project Maths curriculum and associated resources Engineers Ireland (2010) still believe that further monetary investment is required in order to develop resource materials. Due to these perceived problems in Ireland and the lack of investment in the area it is not surprising that teachers in this study firstly were unsure of the resources available and secondly appear unwilling to incorporate such resources into their classroom. Without the availability and use of such resources and if teachers are left to their own devices it is likely that their students will be engaged in monotonous and repetitive mathematics classes which again will lead to disinterest and disillusionment among these students.
The final issue which the author chose examine and one which again impacts heavily on students, relates to the differing attitudes and levels of knowledge of higher level and ordinary level teachers. Research carried out by Liston (2008) as part of her doctoral study found that students in ordinary level classes tend to have lower self esteem and more negative feelings towards mathematics than their peers in higher level classes. The results unearthed in this study show that teachers of ordinary level classes do not feel it necessary to expose their ‘weaker’ students, as they labelled them, to the relevant applications and uses of mathematics or to offer thorough explanations of key concepts or the historical origins of the topics they teach. In turn, these students will not develop a thorough understanding of the uses of mathematics and will simply see it as another subject they must endure during their time in secondary school (Liston, 2008). In addition to this Leaving Certificate results from 2009\textsuperscript{19} show that only 3.2\% of the cohort who sat the higher level paper failed compared with 11.33\% of ordinary level students. Again such problems will not be assisted by the findings of this study. Throughout both phases of this study the author found that ordinary level teachers did not feel the need to provide ordinary level students with in-depth explanations of the concepts being covered and actually displayed lower levels of knowledge than those teachers involved with higher level classes. This is to be expected as recent findings in a report published by Ni Riordain & Hanigan (2009) found that out-of-field teachers, who will undoubtedly have a poorer understanding of mathematics, are more likely to be assigned to ordinary level classes than qualified mathematics teachers. As a result of such findings ordinary level students appear to be exposed to teachers with low levels of knowledge and as already outlined in this section this will have a negative impact on students levels of attainment. Overall discrepancies exist between the attitudes and levels of knowledge of ordinary and higher level teachers and this will impact on students and lead to the most vulnerable students having a more negative mathematical experience and encounter more difficulties at Senior Cycle

Overall the implications of a select number of problems have been analysed by the author in this section. However many of the problems which the author did not refer to here including the examination driven nature of mathematics classes, the negative

attitudes of teachers and the poor knowledge of historical origins demonstrated by teachers, which were discussed in section 6.2 and 6.3, will result in students suffering many of the same problems that are caused by the issues that were selected for analysis. These implications include:

- A negative experience of mathematics
- Failure to expose students to the relevance and applicability of mathematics
- The persistence of problems in relation to uptake and achievement among students in mathematics despite the curriculum reform being introduced
- Disinterest and disillusionment in mathematics among students
- Failure to allow for the holistic development of students
- The prevalence of boring and repetitive mathematics classes
- Poor understanding of mathematics, its usefulness, its relevance, its links to other subjects and its origins among teachers and in turn students.

6.5 Conclusion

In conclusion the findings from both the pilot study and the focus group which the author carried out suggest that teachers must make an effort to improve their levels of subject knowledge. Teachers are poorly positioned on the Ladder of Knowledge, the package of knowledge which the author feels teachers must strive to develop in order to be able to teach effectively. Many of the findings from the focus group were supported by the pilot study and all teachers at some stage in this study showed inadequate levels of knowledge in one or more of the domains outlined on the Ladder. Due to the repercussions of this issue for students, which the author outlined in section 6.5, it is clear that mathematics teachers’ knowledge is something that needs to be addressed in order to avoid them having a negative and possibly harmful impact on their students in the future. As a result of such a realisation the author felt it necessary to develop a small scale intervention which aimed to further substantiate the author’s model, help teachers overcome their shortcomings and in turn improve the standard of mathematics teaching and learning among a select number of teachers that have been involved in this study to date. The design, implementation and evaluation of this intervention will be the focus of Chapters 7.
7. An Intervention Aimed at Improving Levels of Knowledge among Senior Cycle Teachers

7.1 Introduction

"You do not need to be sick to get better" (Barton, 2008)

This quote highlights how CPD interventions are not only needed to overcome the issues raised by the author in Chapter 6 but also to help teachers who are highly qualified to continue to develop their mathematical knowledge and teaching skills. Having thoroughly researched effective CPD and following on from the investigation into the levels of knowledge among participating teachers in each of the domains outlined in the Ladder of Knowledge, the author was now in a position to design a unique and effective intervention aimed at helping to improve teachers' level of knowledge. According to The Oxford English Dictionary (1989) an intervention is
The action of intervening, ‘stepping in’ or interfering in any affair, so as to affect its course or issue”

Another definition which the author found for the term intervention was:

“An activity, process, event or system that is designed to correct a problem, or change a situation and improve performance”

(http://www.reproline.jhu.edu/english/6read/6gloss/glossps.htm)

In essence an intervention is a course of action designed to positively affect an outcome which otherwise may yield undesirable results.

With the term intervention defined the author determined that for the purpose of this study the main aim of the intervention was to positively affect or improve mathematics teachers’ levels of knowledge in the domains outlined on the Ladder of Knowledge. Research has shown that this would then have a positive influence on the standard of mathematics teaching and learning in the classrooms of the teachers involved. In Chapter 4 the author highlighted how students performed poorly in the topic of calculus and how procedural approaches to teaching were favoured when teaching this topic. As a result phase 3A and phase 3B focussed on calculus and as the intervention was designed based on the findings from these two phases it too focussed on the topic of calculus.

As discussed in Chapter 4 there are three key stages in any educational intervention, namely the design and development phase, the implementation phase and the evaluation phase. Over the course of this chapter the author will discuss each of these different phases in detail as well as offering recommendations for future interventions with similar aims and the future direction for her own intervention.

7.2 The Design and Development Phase

In order to design and develop an effective intervention it is crucial that the author first gathers sufficient information about the problem under investigation as well as the participants and their needs. The desk research allowed the author to develop a better understanding of the phenomenon of mathematics teacher subject knowledge and effective CPD. The field work and particularly the focus groups conducted
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during phase 3A provided the author with an insight into the needs of teachers and the material that they would like to see covered in this intervention. According to ACME (2002) it is essential that teachers are consulted prior to the design of any effective CPD intervention in order to fully understand their needs and so the focus groups also played a critical role in the design phase of the intervention. The focus groups allowed teachers to discuss the areas that they believed future CPD initiatives should focus on and elaborate on the material they feel should be included future intervention. Therefore as a result of the desk research and the field work carried out the author was confident of her ground when she moved to design an intervention that will address the issue of improving mathematics teachers’ subject knowledge.

7.2.1 Goals of the Intervention

Prior to designing this intervention the author first needed to clarify the goals of the intervention and identify what she wished to achieve from the CPD initiative. The primary aim of the intervention, as discussed previously, was to reduce the gap between what teachers currently know and what the author believes they ought to know in order to teach effectively. Despite this being the primary aim, the intervention also sought to address the following issues:

- The intervention seeks to provide teachers with resources that can be easily adapted and introduced in the mathematics classroom. The resources provided will be innovative and informative and as a result they will help these teachers develop and plan interesting, educational and creative mathematics lessons.
- The intervention aims to provide teachers with sample problems (and solutions) that are more relevant to student’s everyday life than those offered by the textbooks currently available at Senior Cycle. These examples can be used to encourage creative thinking among students, to prepare both students and teachers for the calculus strand of Project Maths or simply to highlight the numerous uses of mathematics in the world around us.
- The intervention will endeavour to highlight how simple strategies will allow for worthwhile student involvement in the classroom.
• Finally the intervention intends to provide teachers with anecdotes, stories and mathematical questions and activities that they can use to arouse interest in mathematics among Senior Cycle students.

Therefore in addition to the overall aim of improving teacher subject knowledge this intervention also aims to address many of the other issues which, for a long time now, have affected mathematics education in Ireland. For example if the intervention accomplishes its aims it is anticipated that issues, including the prevalence of negative attitudes towards mathematics in Ireland as discussed by Eaton & Bell (2006) and the over dependence on the text book among Irish teachers as reported by TIMSS (1995), will be overcome for those involved in this study.

7.2.2 Type of Intervention

Very little literature, relevant to the topics in this study, is available on interventions previously designed to overcome or address issues in the field of education. However Sandoval (1993) found that any interventions that have been employed in an education setting, the majority of which have sought to address behavioural issues among students, have required researchers to obtain data and such data will then inform and direct the intervention. In order to obtain such data the author first analysed different forms of CPD available to her (Chapter 3), then investigated teachers needs and concerns in phases 3A and 3B (Chapter 6) before investigating the different types of educational interventions which could be used.

There are two types of instructional intervention, behavioural interventions and process – based interventions, and it is the latter type which was used to address the issue of improving teachers’ levels of knowledge. This type of intervention has a strong focus on the cognitive processes of participants and requires an evaluation to be carried out first in order to determine a starting point for the intervention. This type of intervention also focuses on providing all subjects with a developmentally appropriate educational experience (Sandoval, 1993). This type of intervention clearly fits in with the aims and objectives of this project and is the one that the author deemed most suitable.
7.2.3 Topic Chosen for Intervention

The area of interest for both the focus groups and the questionnaire was calculus. The reasons for initially choosing the topic of calculus were outlined in Chapter 4. As the focus group and pilot study helped to determine teachers’ knowledge in the area of calculus the author decided that this too should be the focus of the intervention. The reasons for this are simple. Firstly when participating in the focus groups or the pilot study volunteering teachers were forced to think about their current levels of knowledge of mathematics and more specifically of calculus. By continuing to focus on calculus during the intervention the author was ensuring that prior to the intervention evaluation, participating teachers were aware of their prior knowledge of calculus and were able to make an informed and reliable judgement on the effectiveness of the intervention and the impact it had on their own levels of mathematical knowledge. Secondly, the feedback that the author received from teachers after they engaged in either phase of this study was that they would be eager to develop their knowledge of mathematics and in particular calculus. As a result it would have been unfair if the author had changed the topic of focus for the purpose of the intervention. Instead by focussing on the same topic it ensured that teachers who were keen to develop and enhance their own knowledge base were provided with resources and information that allowed them to do so.

7.2.4 Knowledge Domains to Be Addressed During Intervention

The data analysed from the focus groups and the pilot study in Chapter 6 indicated that teachers needed to improve their knowledge in most domains included on the Ladder of Knowledge. As a result the intervention sought to address every domain outlined in the authors’ model with the exception of procedural subject matter knowledge, an area where the majority of teachers demonstrated competency in both the focus groups and the pilot study. That is to say that the intervention will aim to address the following knowledge domains:

- Relational Content Knowledge i.e. Knowledge of applications
- Knowledge of Connections
- Historical Knowledge
Each area may not necessarily be addressed individually but the intervention designed endeavoured to provide teachers with relevant information about each of the domains outlined. As a result the intervention also highlighted the interrelated nature of the model designed by the author and encouraged teachers to share their knowledge with students in this manner. Finally as well as seeking to provide the teacher with information about each of the aforementioned domains the intervention also provided teachers with resources that will help improve their own levels of mathematical knowledge in each of the six areas outlined and in turn may improve that of their students.

7.2.5 Resources Developed for Intervention

In order to help improve teachers’ levels of historical knowledge the author first designed a resource book with a section dedicated to historical anecdotes and short stories. This section provided teachers with information on the two men credited with developing the main ideas about calculus, Isaac Newton and Gottfried Leibniz. It also detailed the historical meaning of the word ‘calculus’ as well as information regarding the development of the Fundamental Theorem of Calculus and the Great Debate which followed the publication of work by both Leibniz and Newton. Finally the author also provided teachers with a short story highlighting how an Irishman, George Berkeley, played a pivotal role in the development of calculus. Overall these short stories, anecdotes and information could be used by teachers to help develop their own knowledge about the history of calculus while they could also be introduced in many mathematics classes to highlight the importance of mathematics (particularly calculus), both in the past and in modern society, and to arouse interest among students.

The resource book designed for the CPD intervention also focussed on another knowledge domain that appears on the Ladder of Knowledge, namely pedagogical knowledge. The resource book provided teachers with a number of different
resources that could be used when teaching the topic of calculus. Firstly the researcher introduced teachers to the computer package Geogebra. Despite research carried out by Jonasson et al. (1999) which found that that the use of technology in the classroom can help students construct knowledge only 4 out of 22 teachers in the pilot study said that they used computers in the classroom. The resource book described how teachers or students could create applets in Geogebra, which is a free, dynamic mathematics software package. The teachers were provided with step by step guidelines that will help them to create Geogebra applets relevant to the topic of calculus. The researcher also outlines specific higher order questions that could be used to encourage students to think more about the concepts being taught using the applets. This will ensure that even if the students aren’t involved in the construction of the applets, due to time constraints, they will still be able to play a worthwhile role in the class and will benefit from the use of such applets in the mathematics classroom. In addition to providing teachers with guidelines for constructing a number of applets relevant to calculus the author also included a section on relevant websites and one of the websites provided would offer teachers more applets that covered different topics and instructions on how to construct such applets.

The other websites included in the resource book also offer teachers alternative resources such as interactive tutorials or games that could easily be used in the classroom and through the use of such websites the teachers will again be able to incorporate technology in their classroom for the benefit of students.

A final aspect of the resource book that focussed on pedagogical knowledge was the ‘Games, Quizzes and Competitions’ chapter. This chapter outlined four games, namely Calculus Countdown, Calculus Snap, Who Wants to be a Millionaire, and Calculus Bingo. Teachers in a study carried out by Borko et al. (1992) stated that games such as those included in this intervention helped develop an enjoyment and appreciation of the subject among students. Furthermore, as with Geogebra, these games again offer teachers alternative resources that they can use in order to ensure classes are unique and innovative, something which does not currently appear to be the case in Ireland. The games allow students to fully engage in mathematics lessons while also helping them to revise or learn about the basics of calculus, the applications of calculus, the importance of calculus in the world around us and those responsible for developing calculus. All items needed to play these games were
included in the resource pack and once provided with examples teachers could easily create some additional resources or alter those offered in the resource pack to meet the needs of a specific class.

Finally the author also included a PowerPoint presentation outlining the advantages of using alternative approaches to teaching. The benefits of group projects, technology, games and competitions were outlined in the hope of convincing teachers that by becoming aware of alternative pedagogical approaches they will help make mathematics more appealing to students, arouse their interest in the subject and in turn have a positive effect on their levels of attainment. It is hoped that this PowerPoint presentation, combined with the resource book, may help change the views of teachers, such as Jason who was involved in this study, and lead them to see that there is more involved in mathematics than simply drilling in a procedure in preparation for an exam.

Another PowerPoint presentation created by the author for the purpose of this intervention contains questions which students may have. The presentation entitled “Definitions and Explanations: Their Questions Answered” outlines 8 different questions which students may have when studying the topic of calculus. Questions range from “What is calculus” to “Why do turning points occur at the first derivative equal to zero?” The primary aim of this section is to address the need for teachers to improve their ability to transform the knowledge they hold into representations, explanations and examples that students can understand. The detailed analysis carried out in Chapter 6 highlighted how some teachers felt it was sufficient to explain differentiation as a procedure that required us to multiply by the power and reduce the power by one (Power Rule). However, according to Carpenter & Lehrer (2009) simply informing students of a procedure will not allow them to fully understand new concepts and without such understanding they claim that we cannot expect students to be able to apply these concepts to new topics or unfamiliar problems that they may encounter in mathematics. As a result of such research this component of the intervention was designed to help teachers explain different calculus concepts in a more detailed manner. It aims to help teachers transform their knowledge and present it to students in a way that shows the relevance of different concepts, the applicability of such concepts and how previously acquired knowledge informs such explanations. It is only when students are exposed to such mental
activities will they truly understand the material to which they are being introduced (Carpenter & Lerner, 2009).

This PowerPoint presentation also helped intervention teachers develop a knowledge of school mathematics and the questions that students may have at this level. Even (1993) pointed to the need for mathematics modules for trainee teachers to become more relevant while Cuoco (2003) found that the current mathematics modules to which prospective teachers are exposed are too generic to help them develop a better understanding of third level mathematics. Results from the pilot study showed that less than 50% of participating teachers felt confident in their knowledge of school mathematics despite the fact that the majority of teachers surveyed had five years or more experience. Therefore as well as helping teachers transform their knowledge this PowerPoint presentation also seeks to help teachers to understand the issues they will face in second level mathematics classrooms.

In addition to this, another PowerPoint presentation that sought to improve teachers’ knowledge of school mathematics was entitled “Calculus and its Applications”. At third level prospective teachers are often taught different topics in isolation. For example in the National University of Ireland Galway (NUIG), prospective mathematics teachers are required to study modules in algebra, analysis, geometry, probability, calculus, statistics, discrete mathematics and mathematics modelling (http://www.nuigalway.ie/education/ite/ba_mathsed/ba_mathsed.html). When teachers are exposed to these topics in isolation, as is the case in the majority of Irish universities, it is unlikely that they will develop a thorough understanding of the links between these different topics. This is bound to have a detrimental effect on their levels of knowledge. This was evident in the pilot study when less than a quarter of teachers (23%) were able to explain the importance of calculus for other topics in mathematics. Therefore the author felt it was critical for the intervention to outline the links between different topics on the Senior Cycle mathematics syllabus. In order to do this the author produced a presentation that helped teachers develop in three different knowledge domains, namely knowledge of school mathematics, knowledge of applications and knowledge of connections. The latter two will be discussed in greater detail at a later stage. In order to help teachers develop a better understanding of school mathematics the author was able to use this section to
highlight how different topics in mathematics could be used together in order to solve real life problems. Teachers were provided with different applications and each problem required at least two other topics in mathematics in order to solve it. From ten real life problems this strategy enabled the researcher to link calculus to eight other topics that feature regularly on the senior cycle syllabus. Figure 7.1, below, outlines the topics that the author linked calculus to and an understanding of these topics when combined with that of calculus will allow teachers and in turn students to solve the real life problems outlined in this presentation.

Figure 7.1: Linking Calculus to Other Mathematical Topics

The questions designed for this presentation clearly showed teachers the numerous links that exist between different topics on the Senior Cycle mathematics curriculum and it is expected that the identification of such links will help to deepen teachers’ knowledge of school mathematics.
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As mentioned previously the aforementioned presentation also helped to develop teachers’ knowledge of applications and connections. The face of mathematics education in Ireland is changing, and now more than ever, teachers need to impart a strong knowledge of connections to students. Only then will they succeed in the new Project Mathematics curriculum and will the standard of mathematics understanding and attainment among Senior Cycle students improve. Due to the importance of these two domains there was a strong focus on them throughout this intervention. Firstly the PowerPoint presentation entitled ‘Calculus and Its Applications’ aimed to provide teachers with a pool of questions that clearly highlighted how calculus could be used to solve real life problems. The problems ranged from solving aviation problems to using mathematics for solving a crime. The set of questions was designed to highlight the applications at both Higher and Ordinary level. Basic differentiation, which appears on both the Higher and Ordinary level courses was required for five of the questions, implicit differentiation which is unique to the Higher Level course was required for 2 different questions while integration, again a topic that is only studied at Higher Level was the focus of three different questions. The mix of questions was designed to ensure that teachers, despite the level they teach, could benefit from this element of the intervention. This PowerPoint presentation also allowed the author to help teachers to understand the use of mathematics in the workforce. In total the author showed how calculus can be linked to twenty seven different professions in this presentation. These professions ranged from pilots to sport scientists to economists and highlighted the numerous ways in which mathematics can be used in the workforce today.

Finally this presentation also gave the author the opportunity to highlight the numerous connections between mathematics and other school topics. When focussing on the topic of calculus alone the author was able to highlight ten connections between mathematics and other school subjects that students can study at Senior Cycle. Figure 7.2, overleaf, highlights the connections which the author was able to make between mathematics and other school subjects in this presentation.

20 These applications were also included in a booklet that was a key component of the intervention. This meant that it was easy for teachers to photocopy sample problems for students and also meant that the pool of questions designed by the author were available in electronic and physical form.
Another component of the intervention that sought to improve teachers’ knowledge of mathematical connection was the fourth and final PowerPoint presentation entitled “Calculus at Work and at Third Level”. This presentation was necessary in order to reinforce the idea that there are numerous connections between mathematics and the real world and students will depend on this subject for many years to come. The presentation aimed to help teachers answer the question that so many students have when studying mathematics: “When will I ever use this [mathematics] again?” It is also hoped that by providing teachers with this information answers such as that supplied by Eimear in the focus group: “...you literally have to go [say] the only place you will see this is on your Leaving Cert exam paper and you’re not going to see it again” will be avoided. The first part of the presentation outlines seventeen different professions that rely heavily on mathematics and more specifically calculus in the work place. As well as naming the professions it also outlines tasks that these
professionals are required to complete on a daily basis that will involve them having a good understanding of mathematics. The second half of the presentation looked at the importance of mathematics at third level. According to Sabella & Redish (1998) calculus is the entry point to college mathematics for a large number of college entrants. As a result, and in order to reduce the gap between third level and second level education in Ireland which Hourigan & O’Donoghue (2007) allude to, the author felt it was important for teachers to begin to understand the importance of mathematics at third level and the links between it and various college courses. This element of the intervention looked at five different third level institutes in Ireland namely University of Limerick (UL), University College Dublin (UCD), National University of Ireland, Galway (NUIG), Limerick Institute of Technology (LIT) and Dublin City University (DCU). The author analysed courses in each college individually before outlining the percentage of courses in each institute that required students to study calculus as well as the number of schools/faculties in the colleges that offered courses with a large calculus content. Finally the author looked at some of these courses individually and looked at some of the jobs students could aim for once they complete these courses. Therefore, this part of the presentation not only allowed teachers to see the importance of calculus at third level but also showed that even if a students’ chosen profession may not rely on mathematics on a daily basis in order to get the necessary qualifications for such jobs they would need to have a good understanding of mathematics.

The final element of this intervention also endeavoured to improve teachers’ knowledge of applications and connections. The DVD which the author created gave substance to all the claims made in the previously discussed presentations. The DVD contains interviews which the researcher conducted with five individuals in different professions. The interviewees were Dr. Gerald Fleming (Meteorologist), Prof. Tom Cosgrove (Civil Engineer & Consulting Engineer on Thomond Park), Dr. Declan Phillips (Soil Mechanics and Geotechnical Engineer), Mr. Oisin Doyle (Product Designer) and Professor Stephen Ressler (Civil & Mechanical Engineer in the West Point Military Academy). Each professional interviewed spoke about the importance of mathematics in his career, highlighted what they would not be able to achieve without mathematics, provided examples of real life problems that they have encountered that required calculus and in general provided the audience with an
insight into the need for mathematically proficient students in the workforce. As well as supporting the claims made by the researcher in all the other resources that have been discussed, this DVD also provided teachers with first hand evidence of the relevance, applicability and connectivity of mathematics and was another innovative resource that could be used to highlight the same to disinterested or unmotivated second level students.

In summary the following is a list of items included in the *Calculus Tool Kit*:

- 1 CD containing 4 PowerPoint presentations.
- 1 A4 book entitled ‘Calculus Resource Pack’
- 1 A5 book entitled ‘Calculus and It’s Applications’
- 1 DVD
- 1 ‘Who Wants to Be a Millionaire’ Game
- 1 pack of Calculus Playing Cards
- 1 evaluation form (to be returned).

The researcher is confident that the resources designed for the purpose of this intervention will improve teachers levels of knowledge in the necessary domains outlined on the Ladder of Knowledge. The next issue which she then had to address was how to deliver such content. This issue is the focus of section 7.3.

### 7.3 Implementation of Intervention

With the necessary resources in place and having decided on the type of intervention, the topic, key areas to be addressed and the overall goals of this intervention the author then had to decide how best to deliver this intervention. There were a wide range of options available to the author. In Chapter 3 the author discussed numerous different forms of CPD including:

- Traditional CPD
- CPD through reflective practice
- Mentoring Systems and Communities of Practice
- Action Research Model
- Conferences or working seminars
• Training of trainers model.

Each of these delivery methods was given due consideration and the advantages and disadvantages of each were carefully analysed. This analysis led the author to conclude that a new and unique approach was needed for the purpose of this project. This approach combines the benefits of previously analysed models but has a number of unique aspects which help the author to overcome the shortcomings of previously established models of CPD.

7.3.1 The Chosen Approach

The unique approach to CPD adopted by the author demonstrated many of the characteristics of the models discussed in Chapter 3 but the author also included a new and distinctive element. The numerous resources that were designed were combined into six different components which then made up the ‘Calculus Tool Kit’ (Appendix K). For example all four PowerPoint presentations discussed in section 7.2.5 were placed on a disc and a further presentation was included at the start to introduce each PowerPoint and to outline what each presentation aims to achieve. Resources needed for the games, which the author also outlined in section 7.2.5, were also included. For example a pack of cards that could be used for Calculus Snap was included in the resource kit as well as a disc containing an interactive version of Calculus Who Wants to Be a Millionaire. The A4 resource book and the DVD made up the final two components of the resource pack.

With the Calculus Tool Kit in place the author then contacted teachers who declared an interest in participating in the intervention after engaging in either the pilot study or the focus groups. In order to incorporate some element of mentoring and communities of practice the author sought to deliver the intervention to a number of teachers in each school. At the time of contact, ten teachers were willing to engage in the intervention and all but one of these teachers taught with another interested party. As a result the author travelled to four schools in four different counties in Ireland. One school had four teachers participating in the intervention, another school had three while a third had two teachers and only one school had one participating teacher. Although the author ensured that every element of the tool kit was self
explanatory she still felt it necessary to organise a 40 minute induction session with interested teachers in each school. Every teacher in the school that was interested in participating in the intervention engaged in these sessions. Although these sessions were informal they still provided teachers with the support that they needed prior to participating in the intervention. These sessions gave the author the opportunity to show teachers each component of the tool kit, to highlight what the intervention sought to achieve and to encourage teachers to work together when using the tool kit to enhance their own levels of knowledge. The informal forty minute session also gave teachers the opportunity to raise any issues they may have initially and to ask about anything they were unsure of. Once the forty minute session was completed the author left the teachers with the tool kit but remained in contact with each participant through emails and telephone calls over the next ten days. This was one of the unique aspects of this intervention and it allowed teachers to work through the intervention at their own pace. Furthermore it meant that there was constant support available to teachers throughout the ten days they spent engaged in the intervention and studying the components of the tool kit, both in the form of their colleagues and the course administrator. By ensuring that the facilitator is constantly on hand to offer support while teachers strive to develop their own knowledge will, according to Watson (2008), enhance the standard of the intervention and yield more positive results. Furthermore by putting in place a mentoring system such as this the author helped to ensure that there is minimal disturbance to the teaching and learning that was still ongoing in the classroom (Conway & Sloane, 2005). In addition to this even though the teachers were required to use technology both to view many elements of the tool kit and to engage in correspondence with the author the intervention itself was not overly dependent on it and so even teachers who did not engage with ICT on a regular basis could participate. Finally the administrator (i.e. the author) encouraged teachers throughout the intervention to keep a written record of their thoughts, reflections and feelings towards the intervention in order to allow them to complete an intervention evaluation once the ten day intervention was completed. This ensured that the teachers were constantly engaging in reflective practice throughout the intervention. Another benefit of this approach is that this unique intervention has universal appeal, that is, it will be beneficial for both the expert and the novice teacher. By leaving teachers to work through the intervention at their own
pace the author is ensuring that even the highly qualified teacher with an extensive knowledge base can benefit from participating in this type of intervention. For example a teacher who has an extensive knowledge of mathematical content knowledge, mathematical applications and transformational knowledge can use this intervention to enhance his/her knowledge of the historical origins of calculus. On the other hand an out–of–field teacher who has previously developed procedural content knowledge can use this intervention to gradually develop the other types of knowledge outlined on the author’s model, hence highlighting the suitability of this type of CPD for all mathematics teachers.

Overall this method of delivery was unique and does not appear to have been done elsewhere. The author combined many aspects of other approaches that have been put forward in the past. For example the author ensured that the teachers engaged in reflective practice, were provided with the necessary support both from their peers and the facilitator and were exposed to alternative approaches to instruction through the use of technology and group work. Some of the drawbacks of the alternative approaches such as the over reliance on ICT, the time consuming and at times disruptive nature of CPD and the short time frame within which CPD was conducted were avoided. As a result, the literature relating to CPD would suggest that an intervention combining the advantages of numerous alternative approaches to CPD as well as avoiding the disadvantages associated with such approaches and those that affect traditional CPD would be effective and yield the desired results. However this cannot be assumed true until the intervention has been evaluated.

7.4 Intervention Evaluation

The evaluation phase of an intervention is very important and so the author gave much consideration to this phase. As discussed in Chapter 4 in order to evaluate an intervention four different aspects of the intervention must be considered. These are intervention effectiveness, intervention integrity, intervention acceptability and social validity (Shapiro, 1987). The author sought to evaluate each element through the use of a questionnaire (Appendix D) that was distributed to all ten teachers on completion of the intervention. Eight of the ten teachers involved in the final phase of the project returned the evaluation form and the results of this evaluation are
outlined in the forthcoming pages. In addition to this some of the elements of the Calculus Tool Kit were used by the author during a Mathematics Summer School held in the University of Limerick. There were fifty three students involved in one session that the author delivered during the Summer School and forty six in a second session. Their thoughts and opinions were also obtained in order to see their views on different elements of the intervention. The findings of this evaluation will be outlined in section 7.4.5.

7.4.1 Intervention Effectiveness

According to Shapiro (1987) intervention effectiveness requires us to evaluate if and when change came about as a result of the intervention. It relates to the amount of change, the immediacy of change and the strength of change that occurs. The responses from teachers suggest that change did occur as 7 of the 8 teachers who responded believed that the intervention improved their knowledge of Senior Cycle calculus while the eighth teacher was unsure. In addition to this every teacher who responded disagreed or strongly disagreed with the statement “I do not believe the Calculus Tool Kit will improve my teaching of calculus”. Such responses indicate that this small group of teachers believe that this intervention has had a positive impact on their teaching and hence has resulted in positive change. Furthermore when these teachers were asked to evaluate their levels of knowledge before and after partaking in the intervention it was again noted that the intervention helped enhance teacher’s level of knowledge in a number of the targeted domains. Prior to the intervention 4 of the 8 teachers felt their subject knowledge was very good while the remaining 4 teachers claimed their knowledge to be adequate. However once these teachers had participated in the intervention 2 teachers felt their subject knowledge was now excellent while the remaining 6 believed their knowledge in this domain to be very good. These teachers believe that the intervention has had a positive impact on their level of mathematical knowledge. Six of the eight teachers who responded felt that the intervention had led to an improvement in their level of subject knowledge while the remaining two saw no positive or negative change. Furthermore, after participating in the intervention not one teacher involved in the study felt that their content knowledge was adequate, poor or very poor. When
questioned about the other domains on the Ladder of Knowledge similar responses were attained. For example all the teachers involved believed that the intervention helped improve their knowledge of applications. Prior to the intervention commencing 6 of the teachers who responded felt their knowledge of applications was poor or very poor but 5 of these 6 teachers believed that after the intervention they had a very good understanding of mathematical applications while the sixth teacher believed that their knowledge of applications was now excellent. Again this highlights these teachers’ beliefs that the intervention resulted in positive changes in their levels of knowledge. Table 7.1, below, highlights the change in teachers perceived levels of knowledge pre- and post- intervention.

<table>
<thead>
<tr>
<th>Knowledge Domain</th>
<th>Rated Excellent</th>
<th>Rated Very Good</th>
<th>Rated Adequate</th>
<th>Rated Poor</th>
<th>Rated Very Poor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre</td>
<td>Post</td>
<td>Pre</td>
<td>Post</td>
<td>Pre</td>
</tr>
<tr>
<td>Knowledge of Connections</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Historical Knowledge</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Pedagogical Knowledge</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Knowledge of Transformations &amp; School Mathematics</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 7.1: Teacher’s Perceived Levels of Knowledge Pre and Post Intervention

This table highlights the levels of improvement that teachers who responded to the survey felt resulted from their involvement in the intervention. It is encouraging to see that only one teacher felt their knowledge in any of the four domains remained poor after the intervention and instead the majority felt that the intervention had enabled them to develop an excellent or very good knowledge base in each of the domains. Furthermore when teachers were asked whether the intervention met its
aim of enhancing teachers levels of knowledge every teacher felt it had and the following responses clearly highlight the change teachers felt occurred as a result of participation in the intervention:

**Teacher FG1:** “…overall it improved my knowledge of calculus. I am now more effective at applying my knowledge of calculus”

**Teacher FG2:** “…It has helped improve my knowledge of calculus. Now I have additional resources for teaching calculus”

Such responses combined with the figures put forward previously highlight the positive effect that this ten day intervention had on the eight teachers’ perceived levels of knowledge.

Positive change was reported by all teachers who participated in this phase of the study and this change occurred almost immediately as all evaluation forms were returned within a week of the intervention reaching a conclusion. Overall, therefore, the evaluation of intervention effectiveness would suggest that the intervention was indeed effective and achieved the desired aims. The author then proceeded to evaluate the intervention under the second element of Shapiro’s (1987) evaluation module, namely intervention integrity.

### 7.4.2 Intervention Integrity

According to Shapiro (1987) intervention integrity relates to the appropriateness of the intervention, the importance of the outcome and the significance of the intervention goals. The author has already addressed the latter two issues in her previous work. In Chapter 6 she outlined the critical need for the levels of knowledge of participating teachers to be improved and the impact that this may possibly have on student learning. Therefore improved teacher knowledge, which was the main change reported by teachers, is critical and the importance of the outcome therefore cannot be denied. Similarly the goals of the intervention, which were outlined in section 7.2.1, were grounded in research and were developed after
the author gained a good understanding of the needs and current levels of knowledge of teachers involved in the study primarily through the focus groups and also through the pilot study. By designing the intervention goals in this manner the author ensured that these goals were significant and if achieved would result in meaningful and much needed change.

Therefore, in order to ensure intervention integrity, the author only needed to concentrate on evaluating the appropriateness of the intervention. A number of statements were put to teachers in this regard and this provided the author with an insight into teachers’ opinions on the suitability of the intervention. Again the results were positive as figure 7.3, below, indicates

![Figure 7.3: Teachers Views on the Appropriateness of the Intervention](image)

Figure 7.3: Teachers Views on the Appropriateness of the Intervention

Although teachers were asked to state whether they strongly agreed, agreed, were unsure, disagreed or strongly disagreed with a wide range of statements it is encouraging to see that every teacher in the study either agreed or strongly agreed with these particular statements. Such findings would suggest that teachers believed this intervention to be appropriate and they felt that it met their needs. They are of the opinion that the content that they were provided with was suitable and will help improve the content that they deliver and the manner in which they deliver it during
future mathematics lessons. In addition to this when teachers were questioned about
the usefulness of different components in the Calculus Tool Kit the responses were
again positive and once more they suggested that teachers found the intervention to
be appropriate. For example when questioned about the usefulness of the short
stories and anecdotes provided, all teachers in the study stated that they found them
beneficial and fully intend to make use of these resources during their mathematics
classes. Similarly all eight teachers who returned their evaluation forms said that
they are determined to use the games provided with Senior Cycle students. Again
these are encouraging findings as they suggest that these teachers are now willing to
accept the need for change in their teaching strategies and are embracing the idea of
introducing students to the history of mathematics as well as allowing students to
play a more active role in the mathematics classroom.

Finally when teachers were asked “What, if any, aspects of the Calculus Tool Kit did
you find most beneficial” the responses indicate that the resources and content
included in the Tool Kit were relevant and met the teachers needs. The following are
a selection of responses received from teachers:

Teacher FG1: “The CD ‘Calculus: Its Relevance, Applications, Resources and
Pedagogy’. I have found the resources to be a great aid in my teaching of calculus”

Teacher FG2: “All formats of the kit are helpful – the PowerPoint presentations are
good at getting the main points across. The booklets give a record of the material
and are useful for getting teachers/students to think about problems in advance of
the PowerPoint presentations. ‘Calculus Snap’ is a novel and engaging way for
students both to connect with calculus and to learn it. I look forward to using it in
class.”

Teacher 19: “The CD’s. I feel the students would enjoy watching them –
particularly the interviews with the meteorologists and engineers…I felt it was all
very relevant and useful”

Furthermore when asked what areas of the Calculus Tool Kit could be improved
upon five of the eight teachers who responded felt that no improvements or
alterations were necessary.
Such responses reiterate teachers’ opinions that the intervention was useful and relevant and as a result this was a case study that they were happy to engage in. Overall the responses received from teachers indicate that they believe this intervention to be pertinent and appropriate. As a result the author is now confident that the integrity of the intervention has been guaranteed.

7.4.3 Intervention Acceptability

The next area of concern for the author was the acceptability of the intervention. Again from the work of Shapiro (1987) the author was aware that in order to ensure intervention acceptability the following issues must be considered:

- Time and Cost of the Intervention
- Method of Delivery
- Effectiveness and Integrity
- Possible Side Effects
- Understanding of Intervention
- Is it replicable and transportable.

**Time and Cost**

As the author discussed previously the time allocated to an intervention helps determine its success. As a result a lot of time was invested in this intervention both by the author and the participants. The development of the *Calculus Tool Kit* took over two months and was, at all times, informed by the responses received in both the focus groups and the pilot study as well as the desk research that the author engaged in. When the *Calculus Tool Kit* was distributed to teachers they then were given ten days to study the material and engage with the resources. Throughout this lengthy period of time the participants were in constant contact with the author in order to keep her abreast of the progress they were making and the problems, if any, that they were encountering. Overall the time dedicated to this intervention was substantial and contributed to the positive evaluation of the intervention that the author has discussed to date.
The cost of this intervention was relatively low. Due to the low numbers participating in the intervention only one facilitator was required to visit all schools involved. This reduced the cost of delivering the intervention substantially. Greatest cost was incurred in the design of the *Calculus Tool Kit*. The cost included the printing and binding of the two books as well as the material required for games outlined in the resource book. However these expenses were not excessive and as a result this intervention required very little financial investment.

**Method of Delivery**

This CPD intervention was delivered in an innovative manner and one which teachers may not have been used to. Despite this the results of the evaluation suggest that these teachers were happy with the delivery mode employed for the purpose of this project. Figure 7.4 highlights the responses of teachers when they were questioned about the delivery of the intervention.

![Figure 7.4: Teachers Responses to the Statement “The CPD intervention was not delivered in an effective manner”](image)

Again teachers’ responses in relation to the method of delivery are encouraging. Only one teacher was unsure if the method of delivery was effective while the majority of teachers strongly believed that the way in which the intervention was delivered was effective. These responses suggest that teachers participating in this study were happy with this novel approach to CPD.
Integrity and Effectiveness

Both intervention effectiveness and integrity have been analysed in the previous two sub sections. All responses submitted by teachers have suggested that this intervention is effective in nature and demonstrates integrity.

Possible Side Effects

One of the researchers’ early concerns was that the intervention would impinge on teachers daily duties. However by leaving them to work on the intervention in their own time over a long period the author helped alleviate the risk of this occurring and such a side effect never actually materialised.

Understanding of Intervention

Teachers’ understanding of the intervention was another area of concern for the author prior to the implementation of the intervention. The author was concerned that teachers understanding may be affected due to the fact that she was not delivering the content to them directly. However these fears were allayed when the author analysed teachers’ evaluation of the intervention. The first question that was put to teachers in the evaluation was “The content contained in the ‘Calculus Tool Kit’ was easy to understand”. All teachers said that they found the content easy to understand and 7 of the teachers involved strongly agreed with this statement. In addition to this teachers were also asked to respond to the following statement “The content in the ‘Calculus Tool Kit’ was not easy to follow”. As with the majority of other questions included on the evaluation form teachers were asked to state whether they strongly agreed, agreed, were unsure, disagreed or strongly disagreed with the statement. The responses from teachers are highlighted in Figure 7.5 overleaf.
Figure 7.5: Did teachers find the material in the ‘Tool Kit’ difficult to follow?

Again this finding is encouraging as not one teacher claimed to have any difficulty following the intervention independently. Such findings would again suggest that teachers understood the intervention despite the author not being present to deliver all the content. Finally in relation to understanding it was also encouraging to note that teachers felt that the material and knowledge that they gained during the intervention actually helped develop their own understanding. All teachers that were involved in this phase of the study claimed that the ‘Calculus Tool Kit’ enhanced their understanding while the responses from teachers in section 3 of the evaluation suggest that they see the Calculus Tool Kit as an intervention that will help both their own understanding and that of their students:

Teacher 13: “It [The Calculus Tool Kit] helps your own personal knowledge and understanding and also provides you with resources in the classroom. I think it would be a brilliant idea to design tool kits for other topics”

Teacher 15: “I think it will improve students understanding, interest and motivation. I think it would be worthwhile in designing tool kits for as many topics as possible”
Is it replicable and transportable?

This intervention could be easily replicated and adapted in order to develop teachers’ knowledge of different topics in mathematics. In addition to this teachers could replicate many of the resources provided during this intervention to meet the needs of specific classes or to develop similar resources for other classes they will teach. In relation to the transportability of the intervention the author believes that an intervention such as this would be very easy to carry out in a larger number of schools. Although it may require more than one facilitator to implement the intervention with a larger number of teachers, these facilitators would only be required for the induction meeting with teachers and then teachers could remain in contact with the main facilitator/intervention co-ordinator for the remainder of the project. As a result the author foresees no insurmountable problems in relation to the replicable and transportable nature of this intervention.

Overall, responses from teachers indicate that the intervention addresses each of the areas which define intervention acceptability and as a result this is another domain in Shapiro’s model which the authors’ intervention satisfies.

7.4.4 Social Validity

The final area to be assessed before reaching a conclusion on the worthiness and success of the intervention is its social validity. Again Shapiro (1987) outlined a number of different issues that the intervention must address in order to ensure that it is socially valid. These issues include:

- The immediacy of change
- The effort in implementation
- The degree of change
- The theoretical orientation
- The intervention facilitator.
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Immediacy and Degree of Change

The author has previously discussed these two issues and concluded that the degree of change was significant and such change would allow the intervention to achieve its stated aims. According to Pearson Assessment Support (2010) a strong intervention will lead to immediate change. As the author previously noted all change was reported within seven days of the conclusion of the intervention which suggests that the change in teachers’ perceived levels of knowledge occurred almost immediately.

The Effort in Implementation

Again this was an area that could not be assessed directly on the evaluation form but the author feels that the effort dedicated to the intervention both from participants and the author herself was substantial. As previously discussed the design of the intervention took a lot of time and effort on the part of the author while engagement in the intervention required teachers making a conscious effort to improve their levels of knowledge over the course of a ten day period. Participating teachers and the facilitator were also required to make an effort to keep in regular contact in order to ensure that the intervention was fully understood by teachers and to make certain that teachers had no issue with the intervention throughout its duration. Overall, in order for this intervention to work as well as it appears to have requires an extensive effort from teachers and the facilitator alike.

The Theoretical Orientation

Every aspect of this intervention was grounded in theory. The Ladder of Knowledge, which played a central role in the design of the intervention and the purpose of the intervention, was only devised after the benefits and shortcomings of previous models of teacher knowledge were analysed. Similarly the author only made choices regarding the type of intervention, the delivery of the intervention and the duration of the intervention following thorough investigation in each of these areas. For example, as outlined earlier in this chapter, the author considered many possible
CPD approaches and only when each one was methodically analysed was the innovative approach chosen. This approach demonstrated many aspects of other CPD interventions which had been proven to contribute to the overall success of the intervention and such theoretical orientation allowed the author to be confident about the effectiveness of the intervention prior to its implementation. Finally the proposed goals of the intervention were also grounded in previous research in mathematics education and the field research carried out by the author previously. As a result a mixture of theory and analysed data informed the intervention goals and as a result of such grounding the goals were considered worthwhile and achievable.

The Intervention Facilitator

According to the website [www.exforsys.com](http://www.exforsys.com) “A failed or poorly carried out activity may be partially blamed to an ineffective facilitator”. As a result of such findings it was critical that the author, acting as facilitator, demonstrated the qualities needed for effective facilitation of an intervention. According to Clarke (2005) these qualities include:

- Recognising the strengths and abilities of individual participants.
- Ensuring all participants feel comfortable about sharing their hopes and concerns
- Supporting the group and encouraging the implementation of new ideas
- Leading by example through attitudes, approach and actions.

The author has already noted how teachers responded positively to the delivery method and appeared happy to engage with the material independently while also maintaining contact with the facilitator. In addition to this the author feels that when acting as facilitator she also demonstrated many of the qualities proposed by Clark (2005). For example the author’s thorough analysis of responses received in both the focus groups and the pilot study provided her with an excellent insight into the strengths and weakness of each participant and enabled her to deliver the content accordingly. Similarly the author constantly encouraged teachers, both during the induction session and in all correspondence thereafter, to maintain open lines of communication both with their colleagues who were participating in the intervention
and with the facilitator. This provided all participants with an opportunity to voice their opinions and concerns to a number of interested personnel. Thirdly the author designed the *Calculus Tool Kit* in a way that ensured teachers were provided with all the resources that may be required to implement some of the proposed ideas in their own classrooms. She also encouraged teachers to implement such strategies and/or activities in their own classroom and questioned their intent to do so in the evaluation form (See Section 7.4.2). Finally the author felt that she led by example throughout this study. As well as promoting improved knowledge among teachers in order to improve the standard of mathematics teaching and learning the author also provided the teachers with a means by which to improve their levels of knowledge. Similarly the intervention sought to promote more student centred classes which advocate the relevance of mathematics and a shift from the didactic lessons that are currently in place. By departing from the traditional approach to CPD the author ensured that these beliefs were also reflected in the delivery of the intervention and the intervention was no longer promoting one thing and then exposing teachers to something completely different. Overall although the effectiveness of the facilitator was not explicitly evaluated in the evaluation form, the satisfaction demonstrated by participants in relation to the delivery of the CPD as well as the author’s judgement of her own performance as facilitator, in line with the qualities outlined by Clarke (2005), suggest that again the intervention facilitator had a positive impact on the intervention.

In conclusion, teachers’ evaluation of the intervention using the model of evaluation proposed by Shapiro (1987) would suggest that this intervention was effective, worthwhile and resulted in meaningful change. Some of the concluding comments on the evaluations forms support this point

**Teacher 17:** *Excellent resource – the initial handout (questionnaire) proves how unprepared teachers may be in teaching aspects of maths and as a recently qualified teacher, I feel more efforts need to be made at third level to prepare them but especially for teachers to partake in projects such as this one highlight the importance of Continuous Professional Development.*

**Teacher 14:** *[Intervention was] very well put together, the presentations were excellent as well as the resources suggested – look forward to using them all.*
Teacher 13: A copy of this [intervention] should be sent to Project Maths before they implement the new calculus strand.

These quotes highlight how these particular teachers viewed the intervention as an effective form of CPD and a course from which they gained a lot. Furthermore these teachers were specifically asked to reflect on the majority of areas outlined in the evaluation model and the responses received were positive in every regard hence leading the author to believe that the intervention was (a) effective, (b) demonstrated integrity, (c) acceptable to the targeted audience and (d) socially valid. In addition the responses indicate that the aims outlined for the intervention at the outset were also met and, as the author discussed when outlining these aims, the improvements that resulted from the intervention will lead to improved standards of teaching in the classroom. However the author also wanted to see if such an intervention, if adapted and employed for use in the classroom, could improve students’ interest in the subject and levels of mathematical achievement and section 7.4.5 seeks to address this issue.

7.4.5 Student Evaluation

Due to time constraints and the nature of this project, the author was unable to determine if there was a change in the attitudes, interest and attainment levels among the students of participating teachers. Instead in order to ascertain students’ opinion of the material included in the Calculus Tool Kit the author introduced three components of the Tool Kit to students during a Mathematics Summer School run by the Mathematics Applications Consortium for Science and Industry (MACSI) in the University of Limerick from June 21st – June 25th 2010. These students were all in Senior Cycle and ranged in age from 16 – 18 years. The author accepts that this group is not representative of the entire student population and so the resource pack will simply be piloted among these students in order to gain an insight into their own personal views on the usefulness and importance of using the information in the resource pack in mathematics classes. The three components of the Calculus Tool Kit to which students were exposed were the DVD combined with some slides from
the PowerPoint presentation entitled “Calculus at Work and at Third Level” and an adaptation of the Who Wants to be a Millionaire21 game. The sessions were each 45 minutes in duration and this time slot is only slightly longer than the average length of time for a single class in Irish secondary schools, suggesting that much the same content covered in these sessions could be covered in a typical mathematics class. Students’ opinions on these three components were obtained in two different questionnaires (Appendices L & M) and responses were analysed by the author. The results of this analysis are outlined below.

**Students’ Appreciation of Mathematics**

Despite the profound importance of mathematics (Australian Education Council, 1991; Glaister & Glaister, 2000), research has found that students do not appreciate its many uses in everyday life (Frid & White 1994; Smith, 2004). As a result of such findings the author felt it was necessary to see if the content delivered to teachers in this intervention, when disclosed to students, would increase their levels of appreciation and help them to understand the relevant nature of mathematics. The first statement put to students after the Mathematics and Careers session was “I feel this course helped me understand the importance of mathematics”. As with the teacher evaluation forms, students were required to say whether they strongly agreed, agreed, were unsure, disagreed or strongly disagreed with each statement and Figure 7.6 overleaf shows the responses received.

21 The adapted version of Who Wants to Be a Millionaire involved a change in format so that it ran more like another TV game show namely The Weakest Link. This adaptation allowed for greater involvement among the fifty students participating in the session.
Figure 7.6: Do students feel the course improved their understanding of mathematics.

It is encouraging to see the majority of students agreeing or strongly agreeing with this statement. These responses suggest that the majority of students involved in the Summer School felt that exposure to clips from the interviews with professionals and the content outlined in the presentation heightened their understanding of the usefulness of mathematics in the world around them. This finding was even more encouraging when the author saw that over half of students involved in the session displayed a lack of knowledge of the uses of mathematics in everyday life prior to seeing the DVD and the PowerPoint presentation. In addition to this students also developed an appreciation of the relevance of mathematics as a result of this session. For example 83% of students involved felt that the DVD and presentation enabled them to see the relevance of mathematics outside of the classroom while only 4% of students felt that it did not improve their understanding of the relevance of mathematics. Combined with the previous findings such responses show that exposing students to the DVD and the information contained in the PowerPoint presentation can enhance their appreciation of the importance and relevance of school mathematics.
The second session involved students engaging in a modified version of ‘Who Wants to be a Millionaire’. Again the author sought to evaluate students’ appreciation levels after participation in this component. Responses received to the statement “I believe games such as this would help arouse students’ interest in the subject of mathematics” are outlined in Figure 7.7 below.

Figure 7.7: Students’ opinions in relation to the use of games to arouse interest in mathematics.

Again the responses, when analysed, were extremely positive and suggested that by incorporating games, such as those included in the Calculus Tool Kit, teachers can enhance students level of interest in the subject and in turn their willingness to engage in different activities in the mathematics classroom. Furthermore exactly 50% of students involved in the study felt that games such as Who Wants to be a Millionaire provided them with an insight into the uses of mathematics both in the past and in the world we live in today. An additional 24% of students were unsure about this while only 26% disagreed with it. This also suggests that for a large number of students in this sample such games can again alter their view of mathematics and lead them to see the critical need for it outside of the classroom.

Overall such findings suggest that if teachers were to implement some of the strategies put forward during the intervention then this would lead to students
gaining a better insight into the usefulness of mathematics and they would begin to appreciate the relevant and enjoyable nature of this important subject.

**Students’ Level of Understanding in Mathematics**

Another area of concern for people involved in mathematics education is students’ level of mathematical understanding. As the author has already pointed out, international research has indicated that in recent years students have demonstrated insufficient levels of understanding in mathematics. For example De Corte et al (1996) found that the prevalence of a procedural approach to teaching is leading to students failing to develop a solid understanding of the mathematics they encounter while the Chief Examiner’s Report (2001) in Ireland claimed that Irish students fare poorly on questions that require them to demonstrate a deep understanding of mathematical concepts and ideas. Therefore the author felt it was critical to see if students felt that participation in these sessions improved their levels of understanding. Again a number of statements were put to students in this regard. Firstly the author investigated whether or not students felt the DVD and the PowerPoint presentation helped develop their understanding of how simple mathematics can be used to solve real life problems. In total 85% of students involved felt that exposure to the DVD and presentation improved their understanding of mathematical applications while only 6% felt that these components failed to improve their understanding. This would suggest that short clips from the DVD as well as exposure to some of the content in the PowerPoint presentation can truly enhance students understanding of mathematics and help to further develop their appreciation of the subject. Students were also asked if they found the DVD and presentation informative and helpful and again the majority of responses were positive. 57% of students claimed that they found these resources to be informative and helped them gain a deeper understanding of mathematics. Although a further 21% of those involved felt that the session did little to contribute to their understanding it is still encouraging that the majority of students believe that exposure to short DVD clips and the presentation helped them develop a better understanding of mathematics and the application of concepts which they learn in school.
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In relation to the second session (Who Wants to Be a Millionaire) the following statement was one of the first to be put to students “I feel games like this could help improve my understanding of mathematics” and students’ responses are highlighted in Figure 7.8 below:

![Figure 7.8](image)

**Figure 7.8: Students views on the effectiveness of mathematics games for the purpose of improving understanding.**

Figure 7.10 again portrays the positive attitudes towards mathematical games expressed by the majority of students. As with all other statements put to this sample group, the majority again appear to find the different components of the Calculus Tool Kit helpful and beneficial to their understanding and appear willing to engage with such content. In addition to this 59% of students reported that they acquired new knowledge while participating in the game and in contrast only 11% of students involved felt that the games did nothing to improve their knowledge base. Overall the opinions expressed by students after both sessions suggest that these two components of the Calculus Tool Kit contributed to increased understanding on the part of students. Although not all students felt their learning had improved it was encouraging for the author to see that not many students disagreed or strongly disagreed with the statements put to them in this regard and instead the majority...
either felt that it had a positive effect on their understanding or else they were unsure of the type of impact it had, if any.

**Students Enjoyment of Mathematics**

Research has found that at present negative attitudes exist among students towards mathematics (Eaton & Bell, 2006). Both Eaton & Bell (2006) and Frid and White (1995) reported that words such as dull and boring were often used by students to describe mathematics classes. Furthermore Hannula (2002) argues that attitudes deteriorated and in fact became more negative during students time in secondary school. These findings again present researchers in the field of mathematics education with concerns for the future of mathematics education and the author, therefore, sought to see if this intervention could contribute, even in a small way, to solving this problem. After watching clips from the DVD and seeing the content in the PowerPoint presentation students were asked if they would like to see more material of this nature in their mathematics classroom. 68% of students involved claimed that they would like to encounter such material in class and this would suggest that the majority of students in this part of the study enjoyed the material and appreciated its relevance. Less than 6% of students would not like to see this material in their second level mathematics class and again such findings are encouraging and indicate that the majority of the 53 students involved in this session enjoyed it and would like to see it repeated. Similar opinions were expressed following the second session. 89% of students reported enjoying participating in the mathematics game while only 9% felt that it was not an enjoyable experience. Clearly, therefore, the introduction of games and other alternative teaching approaches can help arouse interest in and enthusiasm for the subject and can lead students to seeing mathematics as a subject that is anything but dull and boring. Furthermore over three fifths of students involved said that if games such as these were introduced in their mathematics class then they would be more willing to get involved in mathematics lessons. This again suggests that students enjoy activities such as this and with issues concerning student involvement in the mathematics class prevalent in Ireland today games such as these may offer viable solutions to this as well as other problems facing mathematics education.
The Need for Change in Mathematics Classrooms

Finally the author noticed that students’ responses indicated that they wanted to see change in the content and structure of their mathematics classes. The author has already discussed in detail how procedural and didactic classes are common place in Irish schools and these approaches are leading to students developing a limited understanding and appreciation of mathematics (Conway & Sloane, 2005). As a result alternative approaches, such as those to which these students were introduced, are required and the students in this study now also appear to be calling for these changes in mathematics education. 60% of students would like to get more information on the importance of mathematics in the workplace during mathematics classes while only 11% felt it was unnecessary to get any more information on this topic. As a result it is evident that the majority of these adolescents want to be taught about the relevance of mathematics and its numerous uses outside of the classroom.

As outlined in the responses received from teachers in the focus group and the pilot studies teachers themselves do not know enough about this topic to even consider introducing it in their classroom. However, it is hoped that this will soon change and once teachers become competent in areas such as mathematical connections then classes like this may be common practice in second level schools in Ireland. In addition to this, 79% of students in this sample group would also like to see mathematical games become a feature of their mathematics classes whenever possible. The exact same figure felt that the inclusion of such games in mathematics classes would allow for more worthwhile student involvement and this was something that these students longed for in their mathematics classes. These statistics indicate that students as well as researchers are aware of the need for change. They, like researchers, would appreciate a shift from the didactic and monotonous classes that they regularly encounter and would instead value alternative approaches to teaching that enables them to see the usefulness of mathematics and allows for meaningful participation in the class on their part.

In conclusion the findings outlined by the author in this section indicate that this intervention has the potential to lead to improved understanding among Senior Cycle students. It is evident that by adapting and using many of the resources provided in the resource pack teachers can engage students in the class and help develop their mathematical ability. The responses of students in this sample also suggests that if
teachers were to develop the different types of knowledge outlined on the Ladder of Knowledge and share this newly acquired knowledge with students then improvements in students attitudes towards and appreciation of the subject would follow.

Overall the responses from teachers and students alike indicate high levels of satisfaction with the intervention and suggest that such an intervention may well lead to the desired improvements in mathematics teaching and learning. However the author accepts that the intervention was not perfect and acknowledges that it may need to be altered in the future to ensure that it caters for all teachers. For example the main criticism from teachers who completed the evaluation form was that the intervention did not focus enough on the ordinary level course. The author may need to look at this issue when designing future initiatives of this kind in order to ensure that the content can be adapted for both higher and ordinary level classes. Alternatively, in order to overcome this problem the author could design two different packs with one aimed at teachers of the higher level course and a second aimed at ordinary level teachers. However such concerns were only voiced by 2 of the 8 teachers who participated in this case study and so, for the majority of teachers and students we can conclude that participation in this intervention was a beneficial, worthwhile and positive experience.

7.5 Conclusion

This chapter presented the analysis of the CPD intervention that was designed, implemented and evaluated for the purpose of this project. The innovative CPD intervention which sought to improve teachers’ subject knowledge proved to be effective for these teachers and the teacher evaluation indicates that this initiative succeeded in achieving its aim. In addition to this the student evaluation suggests that when the information contained in this CPD intervention is shared with second level students it increases their appreciation of, interest in and understanding of mathematics. These findings, the possible future direction and recommendation for the future provision of CPD in Ireland of this project will be discussed in more detail in the next chapter.
8. Thesis Contributions, Recommendations & Future Work

8.1 Introduction

In this chapter, the author collates the main findings of this research. A summary of the thesis as well as the main conclusions drawn from the research questions are detailed at the beginning of the chapter and following this the contributions that this doctoral thesis has made to the field of mathematics education are discussed. Finally, the author’s recommendations and possible directions for future research in this area are considered. The chapter commences with a summary of the overall research study.

8.2 Summary and Conclusions

The issue at the core of this doctoral thesis was improving the quality of mathematics teaching at second level. In order to improve the quality of mathematics teaching the author focussed on mathematics teachers’ knowledge and identified the package of
knowledge required for second level mathematics teaching in Ireland. In Ireland, to date, very little research has been carried out in the area of teacher knowledge and the author could find no studies specifically concerned with identifying the types of knowledge required by Irish second level mathematics teachers or the current levels of knowledge of these mathematics teachers. This paucity of research combined with the introduction of Project Maths (which presents teachers with new challenges and obstacles) and the statistics emerging from a report compiled by Ní Riordáin and Hannigan (2009) motivated the author to investigate this issue further. This led to the creation of a theoretical construct for describing and promoting teachers’ mathematics knowledge called the *Ladder of Knowledge*. Furthermore it led to the design of an intervention which could help both out-of-field and qualified mathematics teachers improve their levels of knowledge. This is the first investigation of this kind in Ireland and the findings and insights in this area are significant.

From the outset the author recognised the importance of teacher subject knowledge and she sought to investigate the effects that this characteristic of effective teaching was having on mathematics education in general. She concluded that numerous problems identified worldwide also existed in Ireland and many of these problems were caused, to a significant degree, by the levels of knowledge demonstrated by mathematics teachers. Such problems included:

- A lack of appreciation of mathematics among students and teachers
- The prevalence of negative attitudes towards mathematics
- Poor levels of mathematical attainment
- Poor uptake of mathematics
- Dominance of a procedural approach to teaching
- The presence of unmotivated teachers in secondary schools.

Further research led the author to see that these problems were affecting students’ transition from second to third level education as well as hindering efforts to produce a knowledgeable workforce in Ireland. These realisations led the author to focus on teacher subject knowledge for the remainder of the doctoral study. In addition to this, findings presented in a report compiled by Ní Riordáin and Hannigan (2009) highlighted the prevalence of out-of-field teaching and as a result the author realised
that it was more critical to address the issue of teacher knowledge among practising teachers, rather than pre-service. This literature review and the conclusions reached by this stage informed the next two stages of this doctoral thesis.

Having established the importance of a teacher’s knowledge base, the author then analysed the package of knowledge required for teaching. Following an analysis of the wider literature and models put forward by Shulman (1986), Ernest (1989), Fennema and Franke (1992), Rowland et al. (2005) and Ball et al. (2008) the author concluded that mathematics teaching requires more than just a knowledge of mathematics. Teachers need to be made aware of the different types of knowledge which they must develop for the purpose of mathematics teaching. The analysis of the models proposed by the aforementioned researchers also led the author to see a gap in the research and to identify two knowledge domains which had been largely overlooked in previous models viz. knowledge of applications and knowledge of school mathematics. These knowledge domains were found to be critical factors in an Irish context and were required by teachers if they were to deliver the new Project Maths curriculum effectively. As a result the author recognised the need for a new model of teacher knowledge which incorporated these two domains and this led to the development of the Ladder of Knowledge. The Ladder of Knowledge offers Irish teachers and teacher educators a model of teacher knowledge which will enhance the standard of mathematics teaching and learning in Ireland. This model incorporates many of the features of previously designed models but also exhibits two unique aspects which the author believes to be critical for any model designed for the Irish context.

For the remainder of this project the author sought to develop, support, validate and corroborate this model using a Proof of Concept approach through qualitative and quantitative means and through the use of a CPD intervention. This would inform the author if the Ladder of Knowledge could be used as a bench mark for measuring levels of knowledge and whether it could also act as a vehicle for enhancing levels of knowledge among a sample of Irish teachers. A number of conclusions were drawn from this final stage of the research as outlined below:

- Firstly when the model of teacher knowledge was first designed the author concluded that changes were needed at third level in order to help
prospective teachers develop the knowledge required for mathematics teaching in Ireland. Until such changes in teacher education programmes and the pre service training of mathematics teachers are made, prospective teachers will continue to graduate from third level without the knowledge required to teach mathematics.

- The qualitative and quantitative study led the author to conclude that initiatives were needed to help the teachers involved in this study develop the different types of knowledge outlined on the Ladder with the exception of procedural content knowledge. Although teachers recognised the importance of the different knowledge domains the large majority of those involved in this study admitted to poor levels of understanding and needed help if they were to climb the Ladder of Knowledge. Furthermore the qualitative and quantitative analysis highlighted that mathematics teachers’ knowledge was limited to textbooks and as a result their knowledge of applications, connections, historical origins and pedagogy was detrimentally affected.

- The author then highlighted how levels of teacher knowledge were having implications on student learning. Essentially the author found that teachers’ levels of mathematical knowledge can impact upon students experience of mathematics at second level. For example the author found that low levels of knowledge will affect teachers’ overall ability to teach effectively as without developing the knowledge required for effective teaching, teachers will not be able to teach mathematics for understanding, involve students in the learning process, respond to students’ questions and queries or highlight the importance and relevance of mathematics.

- The findings in relation to teachers’ levels of knowledge and the consequences of this problem led the author to conclude that action was necessary in order to improve the levels of knowledge of the teachers involved in this study. An innovative CPD intervention was designed for this purpose. The evaluations of this intervention indicate that this new approach to CPD in Ireland was effective and well received. The evaluation showed how teachers found the intervention informative, interesting, coherent, well delivered and above all they felt it improved their knowledge in the areas outlined on the Ladder of Knowledge. As a result the author concluded that
such a CPD initiative, although new, yielded the desired results and is a model on which future interventions could be designed.

Finally this three phase study yielded many conclusions of note. Most importantly, the author was able to conclude that the Ladder of Knowledge is a model which can effectively identify and improve levels of knowledge among Irish mathematics teachers. However, it is now evident that teachers in Ireland must concentrate on developing a number of different types of knowledge, including a knowledge of applications and a knowledge of school mathematics, in order to enable them to carry out their duties effectively. This research project suggests that such knowledge must be developed both at third level and through CPD while it also offers suggestions on how teacher educators can help teachers develop such knowledge. Only when such conclusions are realised nationally and suggestions acted upon will we witness meaningful change in mathematics education in Ireland.

8.3 Contributions

The author is optimistic that this project has made significant contributions to the issue of mathematics teachers’ knowledge in Ireland. This research is of critical importance to those who wish to ensure that teachers have the knowledge required to teach mathematics effectively. The research is therefore relevant to people involved in mathematics education both at second and third level and also those responsible for developing teacher trainee programmes and CPD initiatives for mathematics teachers. The author has already pointed to the large body of research internationally on this issue however it is critical for every country to undertake their own research at a local level in order to address the specific needs of teachers in that context. Therefore the study is significant at local, national and international levels.

At a local level the study is directly relevant to mathematics educators in the University of Limerick and those who will design mathematics specific CPD in the National Centre for Excellence in Mathematics and Science Teaching and Learning. The author’s Ladder of Knowledge provides information relating to the different types of knowledge required by Irish mathematics teachers and hence informs educators of the topics that should be central to mathematics teacher trainee courses
or mathematics specific CPD initiatives. In addition to this the author, in Chapter 5, described an additional module which would contribute to the development of the knowledge domains outlined on the *Ladder of Knowledge*. In doing so she not only provided teacher educators with a model of teacher knowledge but also with a blueprint for developing the requisite knowledge among pre-service teachers.

In addition to advising third level educators of the modules needed to develop this model among prospective teachers, the author also designed and implemented an innovative and effective CPD initiative that allowed practising teachers to develop the knowledge required for mathematics teaching. This CPD intervention was evidence based, informed by practising teachers and different to any that had previously been conducted in Ireland. It incorporated many of the advantageous features of established CPD modules while at the same time included some new elements, such as the delivery method, that helped overcome the shortcomings of other approaches. As discussed in Chapter 7 the CPD intervention designed by the author in this study proved beneficial for teachers and students and as a result this intervention could act as a model when designing future subject matter focussed CPD in Ireland.

This study also provides national contributions. Prior to the author undertaking this doctoral thesis there was no empirical evidence relating to the current levels of knowledge of mathematics teachers in Ireland. The author’s work took the first steps towards quantifying the levels of knowledge of Irish mathematics teachers nationally through the design of a model of teacher knowledge and the development of a research instrument which could test teachers’ knowledge in each of the domains. The *Ladder of Knowledge* can be used as a benchmark to measure teachers’ knowledge and by designing future assessments based on the author’s research instrument (Phase 3B) it would be possible to gain an insight into the levels of knowledge of a larger sample of Irish mathematics teacher.

Finally the international significance of this project again relates to the *Ladder of Knowledge*. Prior to this doctoral thesis, the most recent model of teacher knowledge was designed by Ball et al. (2008). When discussing their model Ball et al. (2008) spoke of the need for their categories to be refined and updated in line with changes in mathematics curricula and teaching approaches. In recent years mathematics
curricula around the world have come to recognise the importance of applications (Burkhardt, 2006). Therefore by refining the models put forward by previous researchers and designing an updated model with a strong focus on mathematical applications the author is ensuring that this model gains attention in the mathematics education community.

8.4 Recommendations

The author wishes to make a number of recommendations based on the findings from her study:

- Much research has been done internationally on the issue of teacher knowledge (Shulman, 1986; Ernest, 1989; Fennema & Franke, 1992; Rowland et al., 2005; Ball et al., 2008). Teacher educators must now recognise that mathematics teaching requires an extensive knowledge base. As well as developing knowledge of mathematics, prospective teachers must develop knowledge of applications, knowledge of school mathematics, pedagogical knowledge etc. In order to facilitate trainee teachers, the author recommends a revision of teacher trainee courses and advocates that all such courses should include a capstone module to generate knowledge of school mathematics and to bridge the gap between knowledge of mathematics and knowledge of teaching.

- The author strongly recommends that efforts be made to reform Continuous Professional Development in Ireland. Traditional CPD has proven to be ineffective as researchers found that the time allocated to such CPD is insufficient. According to Shields et al. (1998) the extent of teacher change is affected by the time spent engaged in CPD and one day courses are not enough. As a result the author believes that it is essential that future CPD initiatives run over a longer period of time and with the use of more innovative resources and teaching strategies.

- Smith (2004) recommended that CPD should be a condition of service for all mathematics teachers in the U.K. and the author would like to echo this sentiment. The author recommends that the government consider, in the context of the Teaching Council, making CPD compulsory for all
mathematics teachers. In doing so they will be ensuring that teachers continue to develop their knowledge and skills throughout their professional career and all teachers will be kept abreast of developments in the field of mathematics education.

- The author believes that it is necessary to establish a national network that will promote and alert teachers of initiatives they can partake in or resources that they can use to enhance their levels of knowledge. Similar to the directory developed by the NCETM (2007) Irish educators need to establish such a network in order to ensure that teachers are aware of professional development opportunities that are available to them. It is anticipated that this network will help increase the number of teachers engaging in CPD in Ireland.

- Finally prior to this research study there was a paucity of research in Ireland in relation to mathematics teachers’ subject knowledge. This study is simply a starting point and the author recommends that much more time, effort and money is invested in this area of research in the future in order to ensure that Irish teachers have the knowledge required to teach mathematics effectively and to improve mathematical uptake and attainment in Ireland.

8.5 Future Work

On completion of this doctoral study the author wishes to continue to investigate the effectiveness of the CPD intervention and the Ladder of Knowledge as a vehicle for improving teacher knowledge. The future work which will further support and corroborate the Ladder of Knowledge and the CPD intervention, both of which were central to this study, include:

- Re testing the teachers who participated in the intervention to see if their levels of knowledge have improved in the domains outlined on the Ladder of Knowledge as a result of this CPD intervention

- Analysing whether the material included in the Calculus Tool Kit, when shared with students, enhances their interest in and attitude towards mathematics as well as their levels of attainment. The author envisages this being done through the use of control and test groups,
Chapter 8 Contributions, Recommendations & Future Work

- The author would like to introduce trainee teachers in the University of Limerick to the Calculus Tool Kit and develop further tool kits with these students for different topics. Again through the use of control and test groups the author will seek to analyse the impact that this intervention can have on perspective teachers’ levels of knowledge prior to them entering the workforce. Funding for this future work has been received from the National Academy for Integration of Research, Teaching and Learning (NAIRTL – Application Number S59)

The author believes that teacher educators and government bodies have a duty to investigate issues raised in this thesis such as pre-service teacher education and CPD. She submits that action needs to be taken in the following two ways:

- Firstly the author believes that a quantitative analysis, similar to the pilot study carried out as part of this research study needs to be conducted with a national sample. This will provide teachers, school principals, parents, teacher educators, third level institutes and government bodies with an insight into the current levels of knowledge among mathematics teachers nationally and allow these interested parties address any issues that may arise from such an investigation.

- Finally this research study focussed on improving teacher knowledge through the medium of CPD. Although the author provided information on the modules required to help teachers develop the domains outlined on Ladder of Knowledge at third level there is much scope for further research in this area. Future work at third level should be aimed at improving teacher trainee courses through the revision of modules and the introduction of the capstone module as discussed in Chapter 5.

8.6 Final Comment

This thesis reports on the first investigation into the knowledge required for mathematics teaching in Ireland. This project was necessary in order to complement the national effort in Project Maths and to ensure teachers and teacher educators were aware of the knowledge required to deliver this course. The main feature of this
project, The Ladder of Knowledge, will serve this purpose. However the task now lies in further developing this model and ensuring that it is seen as a model which can help identify and improve levels of knowledge among mathematics teachers in Ireland.
APPENDIX A
Guide Questions for Focus Groups

- How would you rate your own knowledge in each of the domains outlined on the *Ladder of Knowledge*?
- Would you consider the types of knowledge analysed in section B (knowledge of applications) to be important for teaching?
- Do you think it is important to allocate time to explaining the connections and the relevance of topics to students in class?
- In relation to the history of mathematics, is it important firstly for teachers to understand this and secondly to explain it to students?
- Is it important for teachers to develop the type of pedagogical knowledge discussed in Section D?
- How important do you think it is for teachers to understand how mathematics is applicable to students’ everyday life?
- Do you believe teachers have a duty to explain the relevance and applicability of mathematics to students?
- Do you feel you have a good understanding of the relevance of mathematics and the connections between mathematics and other subjects?
- Do you feel it is important to allocate time to explaining applications to students?
- How often would you discuss the historical origins of mathematics with your students?
- Do you believe students need to understand the history of the mathematics they study?
- How would you rate your own ability to explain mathematical concepts to students?
- What are your initial views on the questionnaire?
- Do you think the questions in section B are relevant?
- How would you rate the difficulty of the questions in section B?
- Is there any question(s) you feel I should omit from section B?
- Is there anything you think I have failed to include?
- What do you make of the questions in section C?
• Do you think they would be too difficult?
• Are there any changes I could make?
• What is your opinion on the questions put forward in section D?
• How would you rate the level of difficulty of the questions in this section?
• Do you believe this section to be relevant?
• Any additions I could make?
• Do you feel I could afford to leave anything out?
• Overall what do you think of the length of the questionnaire?
• How would you feel if you were required to complete this questionnaire?
Dear Principal,

I am a doctoral candidate at the University of Limerick pursuing a PhD in Mathematics Education under the supervision of Prof. John O’Donoghue, Director of the National Centre for Excellence in Mathematics and Science Teaching and Learning. My research is entitled ‘Improving Mathematics Teaching at Second Level Using Interventions Aimed at Improving Teachers’ Subject Knowledge’.

In recent years researchers have uncovered evidence to support the fact that a cycle exists whereby poor mathematics teaching at second level can contribute to unmotivated and mathematically under prepared students entering higher education. Many of these students then graduate, as teachers of mathematics, returning to the second level classroom with the same low levels of interest and poor understanding of the subject that they first entered university with, thus continuing the vicious cycle. As a result there are problems to be addressed within mathematics education at both second and third level.

In order to overcome a number of issues currently affecting mathematics education in Ireland adversely I feel it is critical that this cycle is broken. This project aims to do this by first identifying the package of knowledge required for effective mathematics teaching and then designing an intervention that will help senior cycle mathematics teachers develop and improve these domains of knowledge. In this way, the various problems that researchers have found stem from insufficient levels of mathematical and pedagogical knowledge, such as overly didactic lessons and an over reliance on the textbook, will be overcome and improvements will be witnessed in the teaching and learning of mathematics in Ireland. Consequently, and most importantly, we will witness the cycle of mathematical under preparedness being broken.

The project will involve three different phases namely: Focus Groups, Completion of a Calculus Questionnaire by practicing senior cycle mathematics teachers and an Intervention. Calculus was chosen as the focus for this project as it is an area/topic that has proven popular among students at Leaving Certificate Level yet one where understanding and performance is sub standard (Chief Examiners Report, 2005)
Participation in this study is voluntary, and I am aware that this is undoubtedly a busy time of the year for you and teachers in your school but I would greatly appreciate your assistance with this work. I am currently looking for participants to complete the first phase of the project so I would be looking for teachers to engage in a focus group. This would require me meeting with Senior Cycle Mathematics Teachers for one hour during which time they will discuss the model of teacher knowledge which I have designed and analyse a questionnaire which I have designed for the second phase of the study. All teachers who participate in the focus group will then have the option of participating in the intervention which I will design based on the information obtained in the focus groups and the subsequent questionnaire and which will seek to improve levels of knowledge among senior cycle mathematics teachers.

I have enclosed an information sheet for teachers involved in the study too and I would greatly appreciate it if you could distribute this to senior cycle mathematics teachers in your school. Finally I would ask you to contact me either by phone or by email (details are given below) to let me know if teachers are/are not interested in this study. If you agree to participate I will then make contact with you to arrange a suitable time for all teachers.

The information gathered will be used for research purposes only and anonymity will be maintained. If at any time the school wishes to withdraw from this research it may do so.

I look forward to hearing from you.

Kind Regards,

__________________________

Niamh O’Meara

September 2010.

Email: Niamh.omeara@ul.ie John.ODonoghue@ul.ie

Phone: (061) 234788 (061) 202481

Mobile: 087 9335573

If you have concerns about this study and wish to contact someone independent, you may contact

The Chairman of the University of Limerick Research Ethics Committee

c/o Vice President Academic and Registrar's Office

University of Limerick

Limerick

Tel: (061) 202022
Calculus Questionnaire

Investigator: Niamh O’Meara

Supervisor: Professor John O’Donoghue
## Section A

### A1

<table>
<thead>
<tr>
<th>Name:</th>
<th>No of Years Teaching Experience:</th>
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<thead>
<tr>
<th>Qualification:</th>
<th>Level you are currently teaching to Leaving Certificate? (Tick One)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>□ Higher □ Ordinary □ Both</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Institute from which you received your degree</th>
<th>Highest level you have taught to a Leaving Certificate class and when was this?</th>
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</table>

### A2.

Rate these statements using the following scale:

1 – Agree Strongly  2 – Agree  3 – Not Sure  4 – Disagree  5 – Disagree Strongly

1. I have a strong content knowledge of calculus
2. I am aware of a wide range of applications for calculus
3. I know how calculus links to students’ every day lives
4. I have a strong knowledge of the origins of calculus
5. I find it easy to get my ideas about calculus across to students
6. I use a wide range of resources when teaching any aspect of calculus
7. I am aware of how calculus links in to other topics in mathematics
8. I can relate to students when teaching calculus

### A3.

For mathematics in general rate these types of knowledge in order of importance (1 – Most Important; 5 – Least Important)

- Content Knowledge
- Knowledge of Applications
- Knowledge of Connections
- Pedagogical Knowledge
- Knowledge of Student Learning

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A4.

For the topic of calculus rate these types of knowledge in order of importance (1 – Most Important; 5 – Least Important)

Content Knowledge
Knowledge of Applications
Knowledge of Connections
Pedagogical Knowledge
Knowledge of Student Learning
Section B

B1. (i) Differentiate the following:

\[ y = 3x^2 - x + \frac{3}{x} \]
\[ f(x) = \frac{2x^2 - 5x - 3}{x - 3} \]

B1. (ii) Integrate the following:

\[ \int 2x \sin(3x - 2) \, dx \]
\[ \int_{0}^{1} 4x(3x^2 - 1)^3 \, dx \]

B2. An offshore oil well is located in an area 5km from the closest onshore point in Rosslare (A) on a straight shoreline. Oil is to be pumped from the well to another area in Rosslare (B) that is 8 km from A by piping it on a straight line under water from the well to some shore point (P) between A and B and then onto B via pipeline along the shoreline. The cost of laying pipe is €1,000,000 /km under water and €500,000/km over land. Wexford Co. Council is looking to minimize the cost of the pipe laying, where should they locate point P in order to do this? What would the cost be?
B3. It has been reported that ex-basketball star Michael Jordan had a vertical leap of 4.5 feet. Ignoring air resistance and taking gravity to be 32ft/sec what would be the initial velocity required to jump this high?

B4. The new manager of IKEA Dublin has been given €1,500 to develop one more display area in the Dublin store. His idea is to build a 600 metre rectangular enclosure in the stores sheltered car park. Three sides of the enclosure will be built of redwood fencing at a cost of €14 per metre while the fourth side will be built of cement blocks at a cost of €28 per metre. Before he reports his idea to the head office he wants to make sure it is a feasible project. Find the dimensions of the enclosure that will minimise cost and with these dimensions will he have sufficient money to fund his plan?

B5. The rate of change of income produced by a vending machine located at Dublin Airport is given by \( f(t) = 5000e^{-0.04t} \) where \( t \) is time in years since its installation. It costs €20,000 to have the machine in place for five years (maintenance included). Will the company make a profit in this 5 year period?
Section C

C1. Can you name 4 instances where you think differentiation could be used outside of the classroom in a way that students would relate to?

C2. Can you name 4 instances where you think integration could be used outside of the classroom in a way that students would relate to?

C3. Can you identify any ways you could link calculus to the following Leaving Certificate subjects?

<table>
<thead>
<tr>
<th>Leaving Certificate Subject</th>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Science</td>
<td></td>
</tr>
<tr>
<td>Business</td>
<td></td>
</tr>
<tr>
<td>Engineering</td>
<td></td>
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<tr>
<td>Technology</td>
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<tr>
<td>Physical Education</td>
<td></td>
</tr>
<tr>
<td>Geography</td>
<td></td>
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</tbody>
</table>
C4. Can you name one or more of the mathematicians/scientists credited with developing ideas about calculus which we are familiar with today and briefly discuss the contributions they made?
Section D

D1. What resources would you use when teaching the topic of calculus? How would you propose to use such resources?

D2. Do you think it is possible to allow for student involvement in your calculus classes? If yes, how?

D3. Explain as you would to students the importance of calculus for other topics on the Leaving Certificate Mathematics syllabus.
D4. Explain briefly the history of calculus as you would to students. (Refer to contributions it may have made to items they may take for granted in today’s world)

D5. Explain what is actually being done during the process of differentiation.

Sincere Thanks for Taking the Time to Complete the Survey
APPENDIX D
Calculus Questionnaire

Investigator: Niamh O’Meara

Supervisor: Professor John O’Donoghue
### A1

<table>
<thead>
<tr>
<th>Name:</th>
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<table>
<thead>
<tr>
<th>Subjects which you teach with mathematics</th>
<th>Number of teaching hours you have for each subject</th>
</tr>
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</tr>
</tbody>
</table>

### A2.

Rate these statements using the following scale:

1 – Agree Strongly  2 – Agree  3 – Not Sure  4 – Disagree  5 – Disagree Strongly

<table>
<thead>
<tr>
<th>Statement</th>
<th>Rating</th>
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<tbody>
<tr>
<td>I have a strong content knowledge of calculus</td>
<td>1  2  3  4  5</td>
</tr>
<tr>
<td>I am aware of a wide range of applications for calculus</td>
<td>1  2  3  4  5</td>
</tr>
<tr>
<td>I know how calculus links to students’ every day lives</td>
<td>1  2  3  4  5</td>
</tr>
<tr>
<td>I am well informed about the new Project Maths syllabus</td>
<td>1  2  3  4  5</td>
</tr>
<tr>
<td>I have a good knowledge of the origins of calculus</td>
<td>1  2  3  4  5</td>
</tr>
<tr>
<td>I find it easy to get my ideas about calculus across to students</td>
<td>1  2  3  4  5</td>
</tr>
<tr>
<td>I use a wide range of resources when teaching any aspect of calculus</td>
<td>1  2  3  4  5</td>
</tr>
<tr>
<td>I am aware of how calculus links in to other topics in mathematics</td>
<td>1  2  3  4  5</td>
</tr>
<tr>
<td>I can relate to students when teaching calculus</td>
<td>1  2  3  4  5</td>
</tr>
<tr>
<td>I am very apprehensive about the new Project Maths initiative</td>
<td>1  2  3  4  5</td>
</tr>
</tbody>
</table>
A3.
For mathematics in general rate these types of knowledge in order of importance (1 – Most Important; 5 – Least Important)

Content Knowledge  ___
Knowledge of Applications  ___
Knowledge of Connections  ___
Pedagogical Knowledge  ___
Knowledge of How Students Learn  ___

A4.
For the topic of calculus rate these types of knowledge in order of importance (1 – Most Important; 5 – Least Important)

Content Knowledge  ___
Knowledge of Applications  ___
Knowledge of Connections  ___
Pedagogical Knowledge  ___
Knowledge of Student Learning  ___
Section B

B1. (i) Differentiate the following:

\[ y = 3x^2 - x + \frac{3}{x} : \]
\[ f(x) = \frac{2x^2 - 5x - 3}{x - 3} : \]

B1. (ii) Integrate the following:

\[ \int 2x \sin(3x - 2) \, dx : \]
\[ \int_0^1 4x(3x^2 - 1)^3 \, dx : \]

B2. A parachutist jumps out of an aeroplane. The distance \( s \) (in metres), through which he falls in \( t \) seconds is given by:

\[ s(t) = 10t - \frac{6t}{t + 1} \]

Find: (i) The velocity of the parachutist after 1 second has elapsed and when 3 seconds have elapsed

(ii) The acceleration of the parachutist at these times (\( t = 1 \), \( t = 3 \))

(iii) As time continues to elapse (i.e. approaches \( \infty \)) what speed is the parachutist approaching?
B2. An offshore oil well is located in an area 5km from the closest onshore point in Rosslare (A) on a straight shoreline. Oil is to be pumped from the well to another area in Rosslare (B) that is 8 km from A by piping it on a straight line under water from the well to some shore point (P) between A and B and then onto B via pipeline along the shoreline. The cost of laying pipe is €1,000,000/km under water and €500,000/km over land. Wexford Co. Council is looking to minimize the cost of the pipe laying, where should they locate point P in order to do this? What would the cost be?

B4. The new manager of IKEA Dublin has been given €1,500 to develop one more display area in the Dublin store. His idea is to build a 600 metre² rectangular enclosure in the stores sheltered car park. Three sides of the enclosure will be built of redwood fencing at a cost of €14 per metre while the fourth side will be built of cement blocks at a cost of €28 per metre. Before he reports his idea to the head office he wants to make sure it is a feasible project. Find the dimensions of the enclosure that will minimise cost and with these dimensions will he have sufficient money to fund his plan?
The rate of change of income produced by a vending machine located at Dublin Airport is given by $f(t) = 5000e^{-0.04t}$ where $t$ is time in years since its installation. It costs €20,000 to have the machine in place for five years (maintenance included). Will the company make a profit in this 5 year period?
Section C

C1. Can you name 4 instances where you think differentiation could be used outside of the classroom in a way that students would relate to?


C2. Can you name 4 instances where you think integration could be used outside of the classroom in a way that students would relate to?


C3. Can you identify any ways you could link calculus to the following Leaving Certificate subjects?

<table>
<thead>
<tr>
<th>Leaving Certificate Subject</th>
<th>Connections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Science</td>
<td></td>
</tr>
<tr>
<td>Business</td>
<td></td>
</tr>
<tr>
<td>Engineering</td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td></td>
</tr>
<tr>
<td>Physical Education</td>
<td></td>
</tr>
<tr>
<td>Geography</td>
<td></td>
</tr>
</tbody>
</table>
C4. Can you name one or more of the mathematicians/scientists credited with developing ideas about calculus which we are familiar with today and briefly discuss the contributions they made?
Section D

D1. What resources do you use when teaching the topic of calculus? How do you use such resources?

D2. Do you think it is possible to allow for student involvement in your calculus classes? If yes, how?

D3. Explain as you would to students the importance of calculus for other topics on the Leaving Certificate Mathematics syllabus.
D4. Explain briefly the history of calculus as you would to students. (Refer to contributions it may have made to items they may take for granted in today’s world)

D5. Explain what is actually being done during the process of differentiation.

D6. Is there a common approach used for mathematics teaching at senior cycle in your school? If not are you aware of the approaches used by other senior cycle mathematics teachers?

Sincere Thanks for Taking the Time to Complete the Survey
APPENDIX E
Principal’s Information Sheet

Dear Principal,

I am a doctoral candidate at the University of Limerick pursuing a PhD in Mathematics Education under the supervision of Prof. John O’Donoghue, Director of the National Centre for Excellence in Mathematics and Science Teaching and Learning. My research is entitled ‘Improving Mathematics Teaching at Second Level Using Interventions Aimed at Improving Teachers’ Subject Knowledge’.

In recent years researchers have uncovered evidence to support the fact that a cycle exists whereby poor mathematics teaching at second level can contribute to unmotivated and mathematically under prepared students entering higher education. Many of these students then graduate, as teachers of mathematics, returning to the second level classroom with the same low levels of interest and poor understanding of the subject that they first entered university with, thus continuing the vicious cycle. As a result there are problems to be addressed within mathematics education at both second and third level.

In order to overcome a number of issues currently affecting mathematics education in Ireland adversely I feel it is critical that this cycle is broken. This project aims to do this by first identifying the package of knowledge required for effective mathematics teaching and then designing an intervention that will help senior cycle mathematics teachers develop and improve these domains of knowledge. In this way, the various problems that researchers have found stem from insufficient levels of mathematical and pedagogical knowledge, such as overly didactic lessons and an over reliance on the textbook, will be overcome and improvements will be witnessed in the teaching and learning of mathematics in Ireland. Consequently, and most importantly, we will witness the cycle of mathematical under preparedness being broken.

The project will involve three different phases namely: Focus Groups, Completion of a Calculus Questionnaire by practicing senior cycle mathematics teachers and an Intervention. Calculus was chosen as the focus for this project as it is an area/topic that has proven popular among students at Leaving Certificate Level yet one where understanding and performance is sub standard (Chief Examiners Report, 2005)
Participation in this study is voluntary, and I am aware that this is undoubtedly a busy time of the year for you and teachers in your school but I would greatly appreciate your assistance with this work. I am currently looking for participants to complete the second phase of the project so I would be looking for teachers to complete the calculus questionnaire which I have designed and piloted. This would require me meeting with Senior Cycle Mathematics Teachers for fifteen minutes to explain the process and leave the questionnaire with them to be returned and collected at a later date. This should take no longer than 50 minutes in total. All teachers who complete the questionnaire will then have the option of participating in the intervention which I will design based on the information obtained in the questionnaire and which will seek to improve levels of knowledge among senior cycle mathematics teachers.

I have enclosed an information sheet for teachers involved in the study too and I would greatly appreciate it if you could distribute this to senior cycle mathematics teachers in your school. Finally I would ask you to contact me either by phone or by email (details are given below) to let me know if teachers are/are not interested in this study. If you agree to participate I will then make contact with you to arrange a suitable time for all teachers.

The information gathered will be used for research purposes only and anonymity will be maintained. If at any time the school wishes to withdraw from this research it may do so.

I look forward to hearing from you.

Kind Regards,

_______________
Niamh O’Meara
November 2010.

Email: Niamh.omeara@ul.ie John.ODonoghue@ul.ie
Phone: (061) 234788 (061) 202481
Mobile : 087 9335573

If you have concerns about this study and wish to contact someone independent, you may contact

The Chairman of the University of Limerick Research Ethics Committee
c/o Vice President Academic and Registrar's Office
University of Limerick
Limerick
Tel: (061) 202022
Senior Cycle mathematics teachers in ____________________________

(Name of School)

have been informed about this project and have agreed/disagreed to participate in your study.

Number of teachers willing to participate: ________
APPENDIX G
Dear Teacher,

I am a doctoral candidate at the University of Limerick pursuing a PhD in Mathematics Education under the supervision of Prof. John O’Donoghue, Director of the National Centre for Excellence in Mathematics and Science Teaching and Learning. My research is entitled ‘Improving Mathematics Teaching at Second Level Using Interventions Aimed at Improving Teachers’ Subject Knowledge’.

In recent years researchers have uncovered evidence to support the fact that a cycle exists whereby poor mathematics teaching at second level can contribute to unmotivated and mathematically under prepared students entering higher education (Furinghetti, 2000; NCCA, 2006). If this occurs many of these students then graduate, as teachers of mathematics, returning to the second level classroom with the same low levels of interest and poor understanding of the subject that they first entered university with, thus continuing the vicious cycle. As a result there are problems to be addressed within mathematics education at both second and third level.

In order to overcome a number of issues currently affecting mathematics education in Ireland adversely I feel it is critical that this cycle is broken. This project aims to do this by first identifying the package of knowledge required for effective mathematics teaching and then designing an intervention that will help teachers develop and improve these domains of knowledge. In this way, the various problems that researchers have found stem from insufficient levels of mathematical and pedagogical knowledge, such as overly didactic lessons and an over reliance on the textbook, will be overcome and improvements will be witnessed in the teaching and learning of mathematics in Ireland. Consequently, and most importantly, we will witness the cycle of mathematical under preparedness being broken.

The project will involve three different phases namely: Focus Groups, Completion of a Calculus Questionnaire and an Intervention. Calculus was chosen as the focus for this project as it is an area/topic that has proven popular among students at Leaving Certificate Level yet one where understanding and performance is sub standard (Chief Examiners Report, 2005)

Participation in this study is voluntary. I am aware that this is a busy time of the year for you but I would greatly appreciate your assistance with this work. The second phase, for which I am seeking your involvement, requires you to complete the calculus questionnaire. I will meet with you for fifteen minutes before you are required to do this so you have the opportunity to ask any questions and clarify any issues you may have and I will then leave the questionnaire with you to be returned/collected at a later date. The questionnaire seeks to analyse the areas which the researcher needs to concentrate on during the third and final phase of this project.

Please note that teacher knowledge is just one of the issues in mathematics education in Ireland and is just one aspect of my research. As a second level mathematics teacher myself it is not my intention to offend or attribute blame to anyone.

Finally the information gathered will be used for research purposes only and anonymity will be maintained. If at any time you wish to withdraw from this research you may do so.

Kind Regards,

_____________________
Niamh O’Meara

November 2010

Email: Niamh.Omeara@ul.ie John.ODonoghue@ul.ie
Phase 2: Questionnaire

I, ______________________________, have completed this questionnaire of my own accord and without assistance from a third party or from any resource such as a textbook or the internet.

Thank You for your Time
APPENDIX I
Phase 3: Intervention

I would/would not (circle one) like to be involved in the intervention phase of this project which will involve me receiving a resource pack that could be used for the purpose of teaching calculus and briefly analysing the usefulness and effectiveness of this pack.

If you would like to participate in Phase 3 please complete the following questions:

Name:_________________________________________________

Contact Number 1 (School):______________________________

Contact Number 2 (Home/Mobile):_________________________

Email Address:__________________________________________

Signature:_____________________________________________
Section A: Rating the Intervention

For each of the questions below please tick (√) one of the columns

<table>
<thead>
<tr>
<th>Statement</th>
<th>Strongly Agree</th>
<th>Agree</th>
<th>Unsure</th>
<th>Disagree</th>
<th>Strongly Disagree</th>
</tr>
</thead>
<tbody>
<tr>
<td>The content contained in the ‘Calculus Tool Kit’ was easy to understand.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The CPD intervention was <em>not</em> delivered in an effective manner.</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>The material provided will be useful for future classes.</td>
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</tr>
<tr>
<td>I feel the material provided will improve my own levels of calculus knowledge.</td>
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</tr>
<tr>
<td>The content in the ‘Calculus Tool Kit’ was <em>not</em> easy to follow.</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>The intervention enhanced my understanding of calculus.</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>The applications provided are ones which I could use in my own class.</td>
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<td></td>
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</tr>
<tr>
<td>I was aware of the majority of the applications outlined before the intervention</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I did <em>not</em> realise prior to the intervention that calculus was so important in the workplace.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Through this intervention I’ve become aware of the importance of calculus at third level</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The short stories &amp; anecdotes provided have improved my understanding of the foundations of calculus.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I intend to introduce students to the anecdotes and humour provided in the tool kit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I intend to try to include some of the games in my own class.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The explanations and visual aids provided will prove extremely helpful in future classes.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I believe my knowledge of Senior Cycle calculus has improved</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I do <em>not</em> believe that the ‘Calculus Tool Kit’ will improve my teaching of calculus.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Section B: The Ladder of Knowledge

For each question in this section please rate your answers from 1 – 5

<table>
<thead>
<tr>
<th>1 – Excellent</th>
<th>2 – Very Good</th>
<th>3 – Adequate</th>
<th>4 – Poor</th>
<th>5 – Very Poor</th>
</tr>
</thead>
</table>

✓ Prior to engaging in this project how would you have rated your calculus subject knowledge

✓ After receiving the ‘Calculus Tool Kit’ how would you rate your calculus subject knowledge

✓ Prior to engaging in this project how would you have rated your knowledge of the applications of calculus

✓ After receiving the ‘Calculus Tool Kit’ how would you rate your knowledge of the applications of calculus

✓ Prior to engaging in this project how would you have rated your ability to connect calculus to students everyday life and possible future career choices

✓ After receiving the ‘Calculus Tool Kit’ how would you rate your ability to connect calculus to students everyday life and possible future career choices

✓ Prior to engaging in this project how would you have rated your knowledge of the history of calculus

✓ After receiving the ‘Calculus Tool Kit’ how would you rate your knowledge of the history of calculus

✓ Prior to engaging in this project how would you have rated your calculus pedagogical knowledge

✓ After receiving the ‘Calculus Tool Kit’ how would you rate your calculus pedagogical knowledge

✓ Prior to engaging in this project how would you have rated your ability to effectively explain difficult concepts in calculus

✓ After receiving the ‘Calculus Tool Kit’ how would you rate your ability to effectively explain difficult concepts in calculus
Section C: Your Thoughts

Did you think the ‘Calculus Tool Kit’ met the aims outlined in the introductory letter?

_________________________________________________________________________________
_________________________________________________________________________________
_________________________________________________________________________________
_________________________________________________________________________________

What, if any, aspects of the ‘Calculus Tool Kit’ did you find most beneficial?

_________________________________________________________________________________
_________________________________________________________________________________
_________________________________________________________________________________
_________________________________________________________________________________

What, if any, aspects of the ‘Calculus Tool Kit’ did you feel could be improved?

_________________________________________________________________________________
_________________________________________________________________________________
_________________________________________________________________________________
_________________________________________________________________________________

Do you feel that the ‘Calculus Tool Kit’ will benefit your teaching in anyway? If so, do you think it would be worthwhile designing Tool Kits for other topics?

_________________________________________________________________________________
_________________________________________________________________________________
_________________________________________________________________________________
_________________________________________________________________________________

Additional Comments

_________________________________________________________________________________
_________________________________________________________________________________
_________________________________________________________________________________

Thank you for your time in completing this evaluation
APPENDIX K
APPENDIX L
**Mathematics and Careers**

**Course Evaluation**

State whether you strongly agree, agree, are unsure, disagree or strongly disagree with the following statements (Please circle one number for each statement):

1: Strongly Agree  2: Agree  3: Unsure  4: Disagree  5: Strongly Disagree

<table>
<thead>
<tr>
<th>Statement</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>I feel this course helped me to understand the importance of mathematics.</td>
<td></td>
</tr>
<tr>
<td>I now can see how relevant mathematics is outside of the classroom.</td>
<td></td>
</tr>
<tr>
<td>I would like to see material like this covered in my school mathematics class.</td>
<td></td>
</tr>
<tr>
<td>This type of material would help improve my interest in the subject of mathematics.</td>
<td></td>
</tr>
<tr>
<td>I found the DVD really informative and helpful.</td>
<td></td>
</tr>
<tr>
<td>Before this session I didn’t realise that there were so many uses for mathematics in the workplace.</td>
<td></td>
</tr>
<tr>
<td>The application provided showed me how simple mathematics can be used to solve real life problems.</td>
<td></td>
</tr>
<tr>
<td>I would like to get more information on this topic.</td>
<td></td>
</tr>
</tbody>
</table>

**Additional Comments**

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
APPENDIX M
Mathematics Games: The Weakest Link

Session Evaluation

State whether you strongly agree, agree, are unsure, disagree or strongly disagree with the following statements (Please circle one number for each statement):

1: Strongly Agree    2: Agree     3: Unsure    4: Disagree    5: Strongly Disagree

<table>
<thead>
<tr>
<th>Statement</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>I enjoyed participating in the Weakest Link game.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I feel games like this could help improve my understanding of mathematics.</td>
<td></td>
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</tr>
<tr>
<td>I would like to see games like this included in my school maths class whenever possible.</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>I felt I learned something from the different questions asked during this session.</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>I believe games such as this would help arouse students’ interest in the subject of mathematics.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Games such as this would make me more willing to engage in mathematics classes in school</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>This game helped me to see some of the uses of mathematics and provided me with information on the mathematical historians</td>
<td></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Games such as these would allow me to become more involved in the mathematics class.</td>
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<td></td>
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<td></td>
</tr>
</tbody>
</table>

Additional Comments
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
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- [Link](http://www.projectmaths.ie) Date Accessed: 14th June 2010