An Investigation into the Effects of Introducing Algebra Using a Function-Based Approach

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For the award of Masters of Science Degree

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Submitted to the University of Limerick, October 2012
Abstract

Ireland is currently witnessing a major overhaul of its mathematics syllabus for second level education. This syllabus is known as ‘Project Maths’ and came about as a results of concerns relating to the mathematics performance of students in Ireland in international comparative studies such as the PISA (Program for International Student Assessment) tests (Close & Oldham 2005; Cosgrove, Shiel, Sofroniou, Zastrutzki & Shortt 2005; Perkins, Moran, Cosgrove and Shiel 2010; Oldham 2002, 2006).

The author found inspiration for this research when she identified concerns in her own classroom. These concerns were two-fold; firstly the author found that first year students began secondary school with a poor attitude towards mathematics and secondly, the author found that first year students had a lot of difficulty grasping and retaining basic algebraic concepts. The author followed an action research approach to implementing an intervention in her classroom aimed at overcoming these problems. In the first phase of this research, the author carried out a comprehensive review of literature on affect pertaining to mathematics education and on the teaching and learning of algebra. As a result of this review, the author decided to use a function-based approach to teaching algebra as a means of improving students understanding of basic algebra. A collaborative peer learning environment was chosen as the main pedagogical tool for improving attitude towards mathematics. The second phase of this research saw the development and implementation of an intervention in the author’s classroom during which fourth year students tutored first year students. Quantitative and qualitative data was gathered during this phase. The third phase comprised of an analysis of data, presentation of results and discussion of findings.

The data gathered produced a lot of contradicting and conflicting results. There was no significant change (p > 0.05) in either first or fourth year students’ overall attitude towards mathematics. Results indicate, however, that there was a significant increase (p < 0.05) in first year students understanding of basic algebra. This result may have significance for the Project Maths Development Team as they advocate a function-based approach to the teaching and learning of algebra in the new syllabuses. Due to the limited amount of students involved in the study, however, the author does not believe that these results can be generalised in a national or global sense.
Declaration

This thesis is presented in fulfilment of the requirements for the degree of Masters of Science. It is entirely my own work and has not been submitted to any other University or higher education institution, or for any other academic award in this University. Where use has been made of the work of other people it has been fully acknowledged and fully referenced.

Signature: __________________________

Nicola Sterritt

September, 2012
Dedication

This thesis is dedicated to my Mum, Dad and sister, Sharon who have always encouraged and supported me.
Acknowledgements

I would like to thank my supervisors Prof. John O’Donoghue and Dr. Miriam Liston for their constant support and guidance. I would like to thank them for giving me this opportunity.

I would like to thank everyone in the NCE-MSTL for being so kind and helpful over the past two years.

I would also like to thank my friends, especially those that I live with, for their continued encouragement and patience!

I would like to thank all the students that took part in this research.
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Chapter 1: Introduction

1.1 Introduction

“Neither attitude nor achievement is dependent on each other; rather they interact with each other in complex and unpredictable ways”.

(McLeod, 1992, p. 582)

The issue of affect on mathematics learning and teaching has been well documented by many researchers internationally (Liston, 2008, McLeod, 1992, DeBellis & Goldin, 1997, Ma & Kishor, 1997, Martino & Zan, 2001, Hannula, 2002, 2004, Goracke, 2009). These issues have played a valuable role informing directions in mathematics education worldwide. In light of such research, Ireland is currently rolling out a major overhaul of its mathematics syllabus known as Project Maths. This new syllabus aims to simultaneously “develop the mathematical knowledge, skills and understanding needed for continuing education, for life and for work” and “foster a positive attitude to mathematics in the learner” (NCCA, 2012, p.6).

This research seeks to investigate the ‘complex’ and ‘unpredictable’ ways in which an intervention might increase students’ attitude towards mathematics and improve students’ understanding of basic algebra. This intervention focuses on one aspect of the new Irish Mathematics syllabus which pertains to the teaching and learning of algebra. It reports the methods employed to introduce algebra to first year students through a function-based approach, as advocated by the Project Maths team. The intervention utilises collaborative peer teaching as the main pedagogical approach for improving attitude towards mathematics.

This chapter serves to describe the background against which mathematics education research is being undertaken in Ireland and provide a justification for this research study. The purpose of the research as well as its scope and significance are outlined. The research methodology and theoretical perspectives underpinning the intervention are then discussed, before an explanation of the Irish second level system and associated terms are given. Finally, this chapter concludes with a brief overview of each chapter within this thesis.
1.2 Background and Significance of the Study

1.2.1 Background of the Study

The results of two very large scale international studies, the Trends in International Mathematics and Science Studies (TIMSS) and the Program for International Student Assessment (PISA), provide reason to review and reform current educational practices pertaining to the teaching and learning of mathematics in schools in Ireland. PISA, a project sponsored by the Organisation for Economic Co-operation and Development (OECD), assesses the knowledge and skills of 15-year-olds in reading, mathematics and science. In the latest PISA round of testing (2009), Ireland achieved a mean score of 487.1 on the combined mathematics scale, which is significantly below the OECD average of 495.7. Ireland’s mean mathematics score dropped 16 points, from 502.8 to 487.1. From figure 1.1, it can be noted that most of this decline (14 of the 16 points) occurred between 2006 and 2009. Just one other country, the Czech Republic, experienced a greater decline (24 points). Ireland’s rank dropped from 20th to 26th among countries that participated in both cycles. Applying a 95% confidence interval which takes into account measurement and sampling error, it can be said that Ireland’s rank is between 22nd and 29th among OECD countries and between 28th and 35th among all countries (Shiel, Moran, Cosgrove & Perkins, 2010).

Figure 1.1: Distribution of proficiency levels on combined mathematical literacy in Ireland, 2003 and 2009

These international comparative studies have fuelled the view that there has been a decline in the mathematics performance of students in Ireland; however, a number of researchers attributed the poor performance of students’ PISA tests to the types of questions asked in the PISA mathematics assessment. Comparisons of the PISA test items and the content of the
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Junior Certificate mathematics curriculum and examination indicate that there is a relatively low level of familiarity with the content of some of the PISA questions, and that this is more marked at Ordinary and Foundation levels (Close, 2006). The PISA questions were viewed as not reflecting national curricula and rather testing what are believed to be the knowledge and skills needed for life after school.

Inevitably, concerns pertaining to the results of these studies have led to questions about what is happening in Irish mathematics classrooms and how students can be better prepared for the types of questions they are faced with (Conway & Sloane, 2005; Lyons, Lynch, Close, Sheerin & Boland 2003; NCCA, 2005; Oldham, 2006). Influenced by national as well as international research and perhaps media pressure the NCCA (National Council for Curriculum and Assessment) has developed and is currently implementing a new mathematics syllabus at second level, known as Project Maths (as previously mentioned in the introduction to this chapter). This syllabus aims “to provide for an enhanced student learning experience and greater levels of achievement for all” (Project Maths Development Team, 2012, http://www.projectmaths.ie/overview/). The initiative will also focus on developing students’ problem-solving skills due to a much greater emphasis being placed on student understanding of mathematical concepts, and an increased use of contexts and applications that will enable students to relate mathematics to everyday experience.

1.2.2 Significance of the study

Ernest (2000, p.44) describes the need for mathematics in our everyday lives as “functional numeracy” – the basic mathematical skills needed to function in society. Basic mathematical skills such as counting, measurement, and geometry provide people with an understanding that is necessary to survive. Not only is it essential to personal life, however, it is also seen as a key ingredient in developing a “knowledge economy”, which provides for economic success.

The Irish Business and Employers’ Confederation (IBEC, 2007), claim that mathematics is fundamentally important to the educational and economic well-being of this country. They have expressed dissatisfaction at the levels of proficiency of graduates from mathematical courses. This reflects the longstanding international disquiet that students are entering mathematics intensive courses with fewer of the basic mathematical skills essential for course success (Hourigan & O’Donoghue, 2007). In an Irish context, the body of research on this phenomenon is limited but it can be noted that Mathematics Departments within Irish Third
Level Institutions (e.g. Cork Institute of Technology, University of Cork, and University of Limerick (UL)) have expressed dissatisfaction with the mathematical ability of entrants since the mid-1980’s (Hourigan & O’Donoghue, 2005). Hourigan & O’Donoghue (2005, p.462) identify that one area of concern relates to algebra where students exhibit “little competence in algebraic manipulation/simplification”.

The author identifies two key areas in Irish students' performance, attitudes and understanding, in which improvement could be achieved. This research questions whether a function-based approach to teaching algebra through a collaborative peer teaching methodology will improve student understanding of basic algebra and increase students’ attitude towards mathematics. The findings have significance as the function-based approach is advocated by the Project Maths syllabus and active learning is recommended as a tool for delivery (Project maths development team, 2012). As research of this nature has not occurred in an Irish context, results should indicate the potential of using such approaches.

This investigation has potential benefits for practitioners. The new Project Maths syllabus in which Strand 4 - “Algebra” has been introduced in the 2011/12 school year sees investigating patterns and problem solving play a larger role within the syllabus. The student handbook and teacher guidelines developed will provide teachers with materials, which are in line with the Project Maths syllabus.

**1.3 Rationale for Research**

As discussed, Irish students had difficulty interpreting the questions asked on the PISA tests. An example of this can be related to algebra; the PISA test questions explicitly referred to patterns, relationships, functions, graphs and statistics and although all of these occur in the Junior Cycle curriculum (Department of Education and Science/NCCA, 2000) in practice, the emphasis is on formal and technical skills rather than on applications involving ideas of change (Oldham, 2006). This is of interest because this research arose due to the personal experiences of the author who repeatedly encountered problems teaching algebra to first year students in a meaningful and worthwhile manner. Students were able to learn the steps and processes (formal and technical skills) needed to deal with basic algebra but were unable to explain why they were performing these steps (applications involving ideas of change).
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There was also evidence in the author’s classroom that first year students portrayed a poor attitude towards and dislike of the topic. For these reasons, the author decided to put in place an intervention that would see a movement away from a traditional “follow the steps” method of teaching algebra in order to investigate if attitudes towards mathematics and understanding of basic algebra could be enhanced.

This type of investigation has scope beyond the author’s own classroom due to the new Project Maths syllabus. This new syllabus in which Strand 4 - “Algebra” has been introduced in the 2011/12 school year, sees a function-based approach playing a much larger role.

1.4 Purpose of the research

This research aims to investigate issues currently facing the mathematics education system in Ireland in relation to attitudes towards mathematics and introductory algebra. The principle objectives of this project are:

- To investigate by means of a literature review both international and Irish research on the ways in which algebra is taught in post-primary schools.
- To investigate by means of a literature review both international and Irish research on students’ attitude towards mathematics.
- To implement an intervention that will see the introduction of algebra through a function-based approach in a collaborative peer environment.
- To gather quantitative and qualitative data, through the use of questionnaires and focus groups, on students’ attitudes toward mathematics pre and post intervention.
- To gather quantitative and qualitative data, through the use of diagnostic tests, to ascertain student’s levels of understanding in basic algebra pre and post intervention.

1.5 Research Questions

The following research questions were derived and helped guide each phase of the research. The first two questions guided the author throughout the literature review and development and implementation of the intervention. Questions 3-7 guided the author in the discussion of results and research findings.
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1. What are the issues contributing to and theoretical perspectives underlying effective mathematics teaching which can stimulate and improve students’ attitude towards mathematics at Junior Cycle level?

2. What are the issues contributing to and theoretical perspectives underlying effective mathematics teaching which can improve students’ understanding of basic algebra at Junior Cycle level?

3. Does the introduction of algebra using a function-based approach improve first year students’ understanding of basic algebra?

4. Does the introduction of algebra using a collaborative peer teaching approach improve first and transition year students’ attitudes towards mathematics?

5. Does the introduction of algebra using a function-based approach improve first and fourth year students’ attitudes towards mathematics and in turn improve understanding?

6. Does the use of manipulatives improve students’ attitude towards mathematics and aid students in their understanding of basic algebra?

1.6 Theoretical Perspectives

This intervention has two main concerns. The first concern relates to attitude towards mathematics and the second concern relates to students’ understanding of basic algebra.

Figure 1.2, which is discussed briefly in section 4.5 and in more depth in section 5.3, outlines the theoretical framework for this project. DeBellis & Goldin (1997) provide the overarching theory in relation to the affective domain. They categorise affect into four main concerns, one of which was chosen to be the main focus of this investigation; attitude. Fennema & Sherman (1976) provided the research instrument, the revised attitude questionnaire, which allowed the author to quantitatively measure the students’ attitude pre and post intervention. Rubin and Hebert (1998) suggest collaborative peer teaching as a method for improving attitude towards mathematics. The author utilised this approach under the guidance of Billson and Tiberius (1991), Forsyth and McMillan (1991) and Svinicki (1991).
Bernarz, Kieran and Lee (1996) outline a variety of approaches for teaching algebra. One of these approaches, the function-based approach, is advocated by the Project Maths Development Team and so was chosen as an appropriate medium through which to introduce first year students to algebra. Lesh & Zawojeski (2007) suggest an approach to problem solving that can be used to improve understanding within a function-based approach to teaching algebra. Lesh et al. (2003), Yerushalmy and Chazan (2003) Ferrara, Pratt & Robutti (2006) and Haimes (1996) guide the author in the development of the intervention and its’ resources, the student handbook and teacher guidelines.

Figure 1.2 Theoretical framework

1.7 Research Methodology

This project was carried out in different stages so as to fulfill all its objectives successfully. An intervention at the heart of an action research methodology is the primary research tool used in this investigation.

Phase one incorporates a comprehensive review of the literature. An investigation into general issues in mathematics education, as well as a more detailed analysis of the teaching of algebra will be carried out. This work will provide a basis for the design of the theoretical
Chapter 1

framework employed. It will also provide the author with the knowledge to design and implement an intervention as well as an outline of the methodological approaches used.

Phase 2 is known as the ‘action’ phase and is cyclical in nature. The first cycle of the action phase involves developing the student handbook, getting critical feedback and amending the handbook. The second cycle sees transition year students being taught introductory algebra through a function-based approach using the student handbook developed in cycle one. It also involves getting feedback from the transition year students and amending the handbook further. The third part of this phase involves tutors being selected from the transition year class to act as peer tutors within first year classes. A collaborative peer learning environment is established for all classes in which algebra is introduced to first year students. Within this phase, all first year students complete pre and post diagnostic tests and the Fennema-Sherman questionnaire. Transition year students complete a post diagnostic test and questionnaire after tutoring first year students. Groups of students from first year and fourth year are then interviewed in the form of semi-structured focus groups on completion of the intervention (see section 3.74). Data is gathered and evaluations and conclusions are drawn from the action phase to inform the final phase, thesis writing.

The final phase ‘thesis writing’ centres on the write up of the thesis. It involves both quantitative and qualitative analysis. The combination of methods provides the author with a deep insight into the impact that the intervention has on participant attitudes towards mathematics and understanding of basic algebra. Accordingly, conclusions and recommendations from the investigation are drawn and reported in Chapter 6 - Discussion of Key Findings. The final write up of the thesis then takes place followed by evaluations, comments and proof reading.

1.8 Limitations of this study

The author recognises that this study is focused on just one first year class in one school in the Mid-West region of Ireland and that it is oriented to the Western world where the development of the various scales and theory in relation to affect in mathematics education were developed e.g. Australia and the US. Further details of the limitations of this study are discussed in section 4.9 as they follow on from a detailed account of the methodology employed in this study.
1.9 Education in Ireland – System and Terms

In order to clarify terms that may be unique to Ireland, this section aims to provide a broad outline of the Irish education system with reference to terms that may be repeated throughout this document.

In Ireland, children generally begin schooling at either 4 or 5 years of age. They complete 8 years in primary school prior to entering post-primary education (often referred to as secondary school or second level). This is compulsory for all children and is referred to as elementary school in North America. The traditional progression of Irish students is that they enter second level aged 12 or 13. The typical second level education is divided into two cycles. The Junior Cycle is a three year cycle of study which ends in a terminal examination known as the Junior Certificate or Junior Cert. This year alone, 60,000 students completed these examinations (Stack, 2012). Students complete between 11 and 13 exams in different subjects depending on the school they attend. The examinations are independently created, supervised and corrected by a state body, the State Examinations Commission. The students then continue to Senior Cycle.

Senior Cycle can be a two or three year programme. The three year programme consists of 4th year, also known as Transition Year. Transition Year was introduced in 1994 and aims to promote maturity:

- Maturity in studies by making students more self-directed learners through the development of general, technical and academic skills,
- Maturity in relation to work and careers by developing work-related skills,
- Personal maturity by providing opportunities to develop communication skills, self-confidence and a sense of responsibility,
- Social maturity by developing greater ‘people’ skills and more awareness of the world outside school,
- Maturity that will help the student make a more informed choice of subject for their Leaving Certificate studies.

(Second Level Support Service (SLSS), 2012, http://ty.slss.ie/)
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It is available in the majority of schools in Ireland as an optional year for students. In some schools it is compulsory, while a minority of schools do not offer it at all.

The three year programme continues from 4th year to 5th and 6th year (5th and 6th year make up the two year programme). Senior cycle concludes with the Leaving Certificate, popularly known as the Leaving Cert. This year saw 57,000 Irish Students complete these examinations (Stack, 2012). Entry to third level is based on performance in these examinations. A points system based on the students’ best six subjects determines whether they meet the necessary requirements for entry into the third level courses they wish to study.

Mathematics can be taken at three levels in Junior Certificate and Leaving Certificate. The highest level is referred to as Higher (or Honours), a lower level is referred to as Ordinary (or Pass) and the lowest level that can be taken is called Foundation and is only available in three core subjects, English, Irish and Mathematics.

Having completed the Leaving Certificate, the majority of Irish students move on to what is known as third level, higher or tertiary education. The third-level education sector in Ireland consists of universities, institutes of technology, and colleges of education - collectively known as higher education institutions or HEIs.

1.10 Structure of the thesis

Chapter 2 reviews general issues related to mathematics education today. The chapter begins investigating problems facing mathematics education today including failure to appreciate mathematics in today’s society, levels of attainment in mathematics and poor attitude towards mathematics. A review of teaching and learning theories will then take place in both the broader sense and in relation to mathematics education. The chapter concludes with a more in-depth look at the affective domain and its standing in the literature.

Chapter 3 looks specifically at teaching algebra. The chapter begins by defining the term ‘algebra’ and exploring its development throughout history. It reviews algebra education through an international perspective and identifies problems associated with the teaching and learning of algebra. The main section of this chapter reviews the pedagogical approaches to the teaching and learning of algebra and focuses on research pertaining to what is known as the ‘function-based approach’ to teaching algebra.
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**Introduction**

Chapter 4 outlines the methodology used in this research study. It explains action research methodology as well as outlining the research design and the instruments used to collect both the quantitative and qualitative data. The theoretical framework underpinning the research is illustrated and discussed briefly. The importance of validity, reliability and triangulation are examined and the ethical considerations involved in the study are outlined. Limitations of the study are also discussed.

Chapter 5 outlines the development and implementation of the intervention used in this project. It outlines the aims of the intervention and gives a deep insight into the theoretical perspectives underlying the development of the intervention design, student handbook and teacher guidelines.

Chapter 6 will present the key findings emerging from the data analysis. The method of data analysis employed and the school and student profiles are outlined. The chapter then provides a comprehensive statistical analysis of the quantitative data (questionnaires and diagnostic tests) using the software package SPSS (Version 18 for Windows). Reliability of the scales used in the study are examined and discussed along with the relevant preliminary and inferential statistical results. This is followed by a grounded theory approach to analysing the qualitative data which involved focus groups. A summary of findings is presented to conclude the chapter.

Chapter 7 provides an in-depth discussion of findings in light of relevant national and international literature discussed in Chapters 2 and 3. It is guided by the research questions outlined at the beginning of the thesis.

Chapter 8 begins by drawing conclusions from chapter 7 and making recommendations based on these conclusions and the research undertaken. The contribution made by the research is also outlined and the limitations of the study explained. Recommendations for future research are highlighted and the thesis concludes with final remarks.

1.11 Conclusion

This introductory chapter was designed to set out the nature and the scope of the thesis. It began with a description of the background and significance of the research and sought to justify the need for such research in Ireland. This chapter then outlined the overall purpose of the research and the research questions that are the driving force behind the project. The theoretical framework used within the research is outlined and discussed briefly. In addition, the research
methodology is described for each phase of the research. To conclude, terms that may be unfamiliar to an international audience are explained and this is followed by an overview of the chapters. As mentioned, chapter 2 proceeds to set out in greater detail, the background for the research by investigating problems facing mathematics education today and issues associated with the affective domain and its standing in the literature.
Chapter 2: Issues in Mathematics Education today

2.1 Introduction

The purpose of this chapter is to explore and discuss a broad range of general issues related to mathematics education. It is guided by the first research question, ‘What are the issues contributing to and theoretical perspectives underlying effective mathematics teaching which can stimulate and improve students’ attitude towards mathematics at Junior Cycle level?’

Firstly, the importance of mathematics education will be discussed, and a short synopsis of the history of mathematics from an Irish perspective will be outlined. This research is concerned with students’ attitude towards mathematics and understanding of basic algebra. For this reason, problems facing mathematics education today, including failure to appreciate mathematics in today’s society, levels of attainment in mathematics, poor attitude towards mathematics and recent changes in the uptake of mathematics will all be discussed. A review of teaching and learning theories will then be developed in both the broader sense and in relation to mathematics education.

The affective domain is then introduced in more detail and the main affective components that are recognised in the literature as influencing learning are highlighted. A more in-depth analysis of collaborative peer teaching and its effects in a mathematics classroom will then take place.

2.2 Mathematics Education – An Irish Perspective

The Established Leaving Certificate was introduced in Ireland in 1924. The development of the current syllabuses, however, can be traced back to the sixties (NCCA, 2011). The sixties was the era of major reform in mathematics education worldwide. According to the NCCA (2011) this reform was characterised by an emphasis on structure and rigour. In Ireland, teachers attended seminars given by mathematicians; the early years of the Irish Mathematics Teachers’ Association were enlivened by discussions of this new material. These first changes occurred in the Leaving Certificate syllabuses in 1964 and were then introduced to Junior Certificate in 1966. These syllabuses were available at two levels; higher and lower but the higher was only available to boys (NCCA, 2011). They introduced topics such as sets and statistics that are now a core element of mathematics syllabuses today.
Throughout the seventies, the number of students taking the Intermediate Certificate increased dramatically and it was discovered that the new syllabuses were not suitable for all students within this larger cohort. A package of three syllabuses, A, B and C, was introduced in 1987, for examination in 1990 (NCCA, 2011).

In the year 2000, a revised Junior Certificate was introduced that witnessed small changes to content. Unlike previous reforms, however, these revised changes brought about the first dedicated maths support service with full time seconded mathematics teachers. This service provided in-career development for teachers of mathematics. It focussed on syllabus content as well as the types of teaching methodology (i.e. active methodologies) that might ‘facilitate achievement of the aims and objectives of the revised syllabus’ (NCCA, 2002, p.6).

‘Guidelines for teachers’ were developed advocating and providing practical resources for the use of active learning methodologies in the classroom. This full time service lasted from 2000-2008 after which a new syllabus "Project Maths" was established. This syllabus is currently being rolled out and is discussed in more detail in section 2.2.2.

The topics covered by the Higher and Ordinary levels of this revised syllabus were wide-ranging. These topics included algebra, trigonometry, geometry, sequences and series, functions and calculus and discrete mathematics and statistics. Ordinary level also covered elements of arithmetic. These courses allowed students to choose one topic from four to study as their ‘option question’. These topics for Higher level included further probability and statistics, further calculus and series, groups and further geometry. Topics at Ordinary level included further geometry, plane vectors, further sequences and series and linear programming.

In the year 2000, a revised Junior Certificate was introduced that witnessed small changes to content. Unlike previous reforms, however, these revised changes brought about the first dedicated maths support service with full time seconded maths teachers. This service provided in-career development for teachers of mathematics. It focussed on syllabus content as well as the types of teaching methodology (i.e. active methodologies) that might ‘facilitate achievement of the aims and objectives of the revised syllabus’ (NCCA, 2002, p.6).

‘Guidelines for teachers’ were developed advocating and providing practical resources for the use of active learning methodologies in the classroom. This full time service lasted from 2000-2008 after which a Project Maths team was established. Currently a new syllabus, Project Maths, is being rolled out and is discussed in more detail in section 2.2.2.
Chapter 2  
Review of Literature

2.2.1 Problems associated with Mathematics Education in Ireland

Historically, students in Ireland have not performed well in international assessments of mathematics achievement, though primary level students have done somewhat better than their post-primary counterparts (OECD, 2000). In an analysis of the TIMMS (1995) data by the OECD, students in fourth class at primary school in Ireland achieved a mean score that was above the OECD country average, whereas students in Second Year (Grade 8) at post-primary achieved a mean score that was not significantly different (OECD, 2000). Ireland’s participation in the mathematics component of TIMSS in 2011 will provide further evidence relating to the performance of primary-level students in Ireland.

By the late 20th century researchers began to identify problems with the syllabuses that might have attributed to the poor performance in international assessments. O’Donoghue (1999) and Hourigan & O’Donoghue (2007) alleged that the curriculum was overcrowded and this resulted in teachers focusing on recall and routine procedures which in turn led students to relying heavily on rote memory and special purpose algorithms. With the demands of the ‘information age’, mathematics education was no longer preparing students for the mathematics they would encounter outside of school. The publication of The Review of Mathematics in Post Primary Education – A Discussion Paper (NCCA, 2005) resulted in proposals for a reformed curriculum. This new curriculum is known as Project Maths.

2.2.2 Project Maths

In an effort to promote international standards and principles outlined by the OECD, a new initiative in mathematics education in Ireland called ‘Project Maths’ is currently being led by the NCCA. According to the Project Maths Implementation Support Group (2010), Project Maths endeavours to teach mathematics in a way that promotes real understanding. It aims to bring about improved standards in mathematics literacy through reforming the curriculum. Project Maths strives to develop students’ mathematical knowledge, skills and understanding in an effort to prepare them for continuing their education or for the challenges they will face in the work place (NCCA, 2009). An emphasis on problem solving is a key component of this initiative. It is envisaged that “students will develop skills in analysing, interpreting and presenting mathematical information; in logical reasoning and argument, and in applying their mathematical knowledge and skills to solve familiar and unfamiliar problems” (NCCA, 2010, p. 14).
In order to begin the process of improving these standards, curricula have aimed to provide better continuity with primary school mathematics through developing a bridging framework (see appendix K) that links the various strands of mathematics in the primary school to topics in the Junior Certificate mathematics syllabuses. This has led to five distinct strands being developed:

- Strand 1: Statistics and Probability
- Strand 2: Geometry and Trigonometry
- Strand 3: Number
- Strand 4: Algebra
- Strand 5: Functions

These strands were phased into 24 pilot schools in September 2008. The Leaving Certificate students of 2012 were the first cohort to be examined on the entire Project Maths curriculum. The Junior Certificate students of 2013, in these pilot schools, will be the first to be examined on the entire Junior Certificate curriculum. Figure 2.1 displays the process by which all schools will be introduced to Project Maths. By 2015 both Junior Certificate and Leaving Certificate examinations will be completely based on the Project Maths syllabus.

*Figure 2.1 Project Maths timeline*


Unlike previous reforms in mathematics education, Project Maths places a much greater emphasis on the methods by which this curriculum is to be taught. According to the syllabus the focus should be on the learner understanding the concepts involved, building from the
concrete to the abstract and from the informal to the formal. Project Maths hopes that learners’ experiences contribute to the development of problem-solving skills. Also, particular importance is placed on promoting learners’ confidence in them-selves (that they can ‘do’ mathematics) and in the subject (that mathematics makes sense). Through the use of meaningful contexts, opportunities are presented for the learner to achieve success (Junior Certificate Syllabus, for examination in 2013).

In order to gain a deeper understanding of the issues associated with the teaching and learning of mathematics in the context of Project Maths, the author will now review teaching and learning theories in light of the literature.

2.3 Teaching and Learning Theories

This section of the literature review aims to describe a variety of approaches to teaching and learning supported in the research. It will investigate constructivism, problem solving, Lesh’s Translation model, Realistic Math’s Education (RME) and active learning methodologies in light of the literature.

2.3.1 Constructivism

The constructivist approach to education has been one of the buzz terms for many years. A variety of names have been used to describe this approach including discovery learning (Anthony, 1973; Bruner, 1961), problem-based learning (PBL; Barrows & Tamblyn, 1980; Schmidt, 1983), inquiry learning (Papert, 1980; Rutherford, 1964), experiential learning (Boud, Keogh, & Walker, 1985; Kolb & Fry, 1975) and constructivist learning (Jonassen, 1991; Steffe & Gale, 1995). It appears to be a popular approach through which educators believe that students learn best in an unguided or minimally guided environment. It is believed that students must discover or construct essential information for themselves (e.g., Bruner, 1961; Papert, 1980; Steffe & Gale, 1995 cited in Kirschner, Swellar and Clarke, 2006). On the other side of this argument are those advocating that beginner learners should be provided with direct instructional guidance and should not be left to discover procedures for themselves (e.g. Cronbach & Snow, 1977; Klahr & Nigam, 2004; Mayer, 2004; Shulman & Keisler, 1966; Sweller, 2003).
Chapter 2

Review of Literature

Noddings (1990), Ball and Cohen (1999) and Chazan (1999) observe that the constructivist approach is closely associated with the dubious assertion that “telling is bad” because it deprives students of the opportunity to construct knowledge for themselves (Cobb, 2007, p. 5). Kirschner, Swellar & Clarke’s (2006) research in this field has led them to conclude that there is no body of research to support the technique (constructivism). They argue that even for students with considerable prior knowledge, strong guidance while learning is most often found to be equally effective as unguided and that, in fact, unguided is normally less effective and can even have negative results. Their findings support Mayer’s (2004, p.18) recommendation that we “move educational reform efforts from the fuzzy and unproductive world of ideology—which sometimes hides under the various banners of constructivism—to the sharp and productive world of theory-based research on how people learn”.

2.3.2 Problem Solving

Polya’s (1957) work on how to solve problems influenced an initial debate on the complex mathematical topic of problem solving. It has been given extensive consideration in mathematics curricula throughout the world particularly in the last thirty years (English et al., 2008). In the 1970’s the idea of problem solving as a necessary educational tool in mathematics instruction began to take momentum. In 1976, at the third International Congress on Mathematical Education in Germany, problem solving was one of the themes addressed (Allevato & Onuchic, 2008). It was then, as predicted in the NCTM (Krulik, 1980, p. xiv), a dominant theme throughout the 1980’s and set the scene for reform in mathematics education in Ireland as discussed in section 2.2. Since this time, problem solving has become inherent in Irish mathematics curricula. The Project Maths syllabus places a large emphasis on teaching students through a problem solving approach.

This discussion thus far has outlined the inclusion of problem solving in research and school curricula. It is important that we now look towards a definition in order to gain an understanding of what is meant by the term ‘problem solving’. Lesh & Zawojeski (2007, p. 782) define problem solving as:

“the process of interpreting a situation mathematically, which usually involves several iterative cycles of expressing, testing and revising mathematical interpretations—and of sorting out, integrating, modifying, revising, refining clusters of mathematical concepts from various topics within and beyond mathematics.”
This definition views problem solving as iterative cycles of understanding the givens and goals of a problem (Lesh & Zawojeski, 2007). It assumes that in response to a complex situation, problem solvers engage in “mathematical thinking” in order to solve the problem. A variety of approaches have been developed to categorise problems used in school mathematics. Diaz (1999) describes problems as having four characteristics which include real, realistic, fantasy and purely mathematical. Orton (2004) echoes these when he categorises problems into routine problems, novel problems, word problems and real life applications. English et al. (2008) suggest an approach that is used by students to solve problems:

- Initial learning of concepts and procedures
- Practice on story problem
- Exposure to a range of strategies e.g. drawing diagrams, using tables etc.
- Applying the above competencies to solve novel or non-routine problems.

(English et al., 2008, p.1)

English et al. (2008) argue that research in this field stagnated for much of the nineties. Lesh & Zawojeski (2007) believe, however, that current research on mathematical problem solving may be migrating to other areas of research, under different names. Their explanation for this is that there has been a shift in viewing problem solving as a “thing” toward viewing it as related to a “large category of things” that need to be learned (Lesh & Zawojeski, 2007). Three categories are suggested: situated cognition, communities of practice and representational fluency. These are seen as essential skills for mathematics education because in an information age, in knowledge economies, in learning organisations, in global societies, people need to be able to describe and explain the systems within which they operate Lesh & Zawojeski (2007). These skills break problem solving into three clearly defined categories that can inform class instruction:

- **Situated cognition** refers to learning and problem solving in context (e.g. Greeno, 1998; Greeno & the Middle-School Mathematics Through Application Project Group, 1997). The research from situated cognition consistently demonstrates that nearly all people that are engaged in problem solving in their local contexts are able to develop
mathematical concepts and conceptual tools for problem situations that are powerful and reusable (Lesh & Zawojeski, 2007).

- **Communities of practice** acknowledge that while knowledge is contextually situated (as in situated cognition) it is also socially situated. Vygotsky’s (1978) notion of zone of proximal development has been used in many modern theories of learning based on social perspectives. Vygotsky’s (1978) ideas have been adapted to consider how student-to-student interactions facilitate learning. Yackel & Cobb (1996) suggest that a role for the teacher in mathematical problem solving is to ensure that the learning community develops productive socio-mathematical norms in order to optimise learning via student-to-student interactions. Lesh & Zawojeski (2007) suggest that instead of solely depending on the teacher-student interaction to prompt learning, the main learning is thought to occur through student interaction, as the teacher or a tutor orchestrates the experience and ensures that the intended learning takes place.

- **Representational Fluency** refers to students’ capabilities to solve problems that appear in multirepresentational forms and require a solution to be developed that involves a variety of media for the purposes of explanation, simplification, justification or prediction (Lesh, 1987). Newly developed types of expressive media emphasise the importance of graphics-orientated, dynamic, and creative representations (e.g. tables, graphs and equations) (Kaput, 1991). Representational fluency is vital for professional uses of mathematics, where the problem solver often is in a team of diverse specialists communicating remote sites, and where resources and expertise tend to be distributed in different geographic locations (Lesh & Zawojeski, 2007).

These skills outline some very important pedagogical approaches that should be incorporated into mathematics classes. They stress the need for problems to be contextual in nature so that students can relate the problem to life outside of school. They require students to move fluidly between words, graphs, diagrams, tables and to be proficient in ICT. They also require students to become more engaged in group work, to learn how to problem solve in a social context. These three skills are essential if students are to become proficient enough to enter the working world in this age of information technology.
2.3.3 Lesh’s Translation Model

As highlighted in section 2.3.2, representational fluency is essential to students of this era. This fluency is an important component of student’s ability to model situations in mathematics contexts and it is an important part of understanding many topics ranging from early number concepts to rational number concepts to concepts in algebra, geometry, probability, statistics and calculus (Cramer, 2003). The Lesh Translation Model (2003)(see figure 2.2), highlights the importance of students’ abilities to represent mathematical ideas in multiple ways including manipulatives, real life situations, pictures, verbal symbols and written symbols (Lesh et al., 1979). He suggests that students should be able to move between the various methods of representations fluently.

Figure 2.2 Lesh’s Translation Model

![Lesh's Translation Model](image)

(Lesh et al., 2003)

Before identifying the underlying principles of the Lesh Translation Model, it is useful to gain some understanding of how this model is grounded in earlier theories developed by Piaget (1960) and Dienes (1960). Piaget’s stages of intellectual development still influence research on the impact of concrete embodiments of the mathematical systems on the development of underlying concepts at different ages (Post, 1988). The use of these concrete materials to teach mathematics is well founded in mathematics education research and is discussed further in Chapter 3.

The Lesh model relates to Dienes (1960) work in so far as they both promote active student learning and instructional sequences for utilizing concrete representations. Dienes (1960) proposes that these instructional sequences move through three stages:
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1. The play stage where learners have experience with the concept in informal ways.
2. The structured activity stage where activities are designed to lead students to understand a concept.
3. The becoming aware stage is characterised by the explicit examination of the relevant mathematical concept.


Dienes (1960) also suggests that conceptual learning is maximised when children are exposed to a concept through a variety of physical contexts or embodiments. This idea supports the use of several manipulative materials for any given mathematical idea to be learned. An example offered by Cramer (2003) is one in which students use base ten blocks, chip trading and unifix cubes to study place value. Dienes (1960) proposes that what is common among the different materials is the mathematical abstraction that students are to construct.

The Lesh model (2003) emphasises interactions with and among representations. Arrows that connect different modes depict translations whilst the internal arrows depict translations within modes. A translation requires reinterpretations of an idea from one representation into another representation. This model has been used in a wide variety of research studies. For example, The National Science Foundation supported the development of the Rational Number Project (RNP) fraction curriculum through using the Lesh translation model (Cramer, 2003). This was a large scale study with fourth and fifth graders and its success showed the value of the Lesh Translation model in developing curricula for fraction understanding over traditional curricula. Mathematics courses developed with this model have also been successfully used with elementary and middle school Minneapolis teachers (Cramer, 2003). They were so popular that the districts new Urban Systemic Initiative aimed at improving mathematics instruction decided to use courses developed using the Lesh model as a foundation for its teacher enhancement activities.

More examples of where the Lesh model appears in the literature will be discussed in Chapter 3 where applications to a function-based approach to teaching algebra will be outlined.
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Use of Manipulatives in Mathematics Instruction

The Lesh translation model encourages the inclusion of manipulatives in mathematics education. As attested to by frequent references in mathematics standards, mathematics methods textbooks, in-service course offerings, professional journals and commercial resource catalogues, the use of manipulatives is well-situated in the mainstream of mathematics instruction also (Moyer, 2001). Many researchers have found that the use of manipulatives is necessary for supporting students’ understanding. Zoltan Dienes’s work in 1969 convinced researchers that the use of various representations of a concept, or ‘multiple embodiments,’ was imperative for aiding in this understanding. Furthermore, Piaget (1952) suggested that children need many experiences with concrete materials and drawings for learning to occur as they do not have the mental maturity to grasp abstract mathematical concepts presented in words or symbols alone. Skemp’s (1987) theories supported the belief that students’ early experiences and interactions with physical objects formed the basis for later learning at the abstract level.

Manipulatives have both visual and tactile appeal and can be manipulated by learners through hands-on experiences. The effective use of these manipulatives for mathematics instruction is more complicated than it might appear, however. “Although kinaesthetic experience can enhance perception and thinking, understanding does not travel through the fingertips and up the arm” (Ball, 1992, p. 47). Manipulatives are externally generated manufacturers’ representations of mathematical ideas. It cannot be assumed that teachers and students can automatically attach meaning to the manipulatives (Meira, 1998). Leinebach (1996) found that manipulatives may serve as tools for teachers to translate abstractions into a form that enables learners to relate new knowledge to existing knowledge. Students sometimes learn to use manipulatives in a rote manner, with little or no learning of the mathematical concepts behind the procedures (Hiebert & Wearne, 1992) and the inability to link their actions with manipulatives to abstract symbols (Thompson & Thompson, 1990). These manipulatives, however, are often regarded as fun and used as a reward for students but it is important to understand that in order for them to be effective learning tools, students must connect their own internal representations of ideas with the external representations or manipulatives (Leinebach, 1996). The importance of teachers having to guide students to translate between representations in the form of mathematical objects, actions and abstract concepts so that students can see the relationship between their knowledge and new knowledge cannot be stressed enough according to Moyer (2001). Research has also shown that for children to use
concrete representations effectively without increased demands on their processing capacity, they must know the materials well enough to use them automatically (Boulton-Lewis, 1998).

Many researchers have investigated the effects of incorporating manipulatives into mathematics instruction. This research has focused on the effects of manipulative use on students of different ability levels (Prigge, 1978; Threadgill-Sowder & Juilfs, 1980), on the frequency of verbal interactions (Stigler and Baranes, 1988) and on students’ attitudes towards mathematics (Sowell, 1989). The findings of much of this research have shown that students who use manipulatives outperform students who do not (Driscoll, 1983; Greabell, 1978; Raphael & Wahlstrom, 1989; Sowell, 1989; Suydam, 1986). Some studies show that the teachers’ experience in using the manipulatives impacts on student achievement levels (Sowell, 1989; Raphael & Wahlstrom, 1989). According to Moyer (2001), there is a lack of observations and interviews of teachers using manipulatives in the literature with much of the documented research on the use of manipulatives being carried out by meta-analysis and research reviews.

Another aspect of Lesh’s translation model is that of situating the mathematics that students are learning into a real life context. Research on Realistic Mathematics Education provides an insight in to the how and why one might include this theory in their classroom.

**Realistic Mathematics Education**

Realistic Mathematics Education (RME) arose as a response to ‘modern’ mathematics and the worldwide reform of mathematics teaching and learning (Oldham, 2002). Over the past 30 years, researchers at the Freudenthal Institute in The Netherlands have developed a mathematics curriculum and a theory of pedagogy that is based on RME. Despite, its initial formation in the 1970’s, it only became a dominant theme at the 8th International Congress on Mathematics Education (ICME 8) in 1996, an entire three decades since its foundation (Oldham, 2002).

The present form of RME has been mostly determined by Freudenthal’s (1977) view on mathematics. He felt mathematics must be connected to reality, stay close to children’s experience and be relevant to society, in order to be of human value (Van den Heuvel-Panhuizen, 2000). The word ‘realistic’, refers not just to the connection with the real-world, but also refers to problem situations which can be imagined by students. For the problems to
be presented to the students this means that the context can be real-world but this is not always necessary. RME curriculum uses "imaginable" contexts to help students to develop mathematically, with a strong emphasis on students "making sense" of the subject (Hough & Gough, 2007).

This model is characterised by several features:

1. The use of realistic (real to the learner) contexts as sources from which to develop mathematics and as situations in which the problems to be solved are presented.

2. The use of models to develop mathematical concepts, thinking and skills. Models evolve from ‘models of’ to ‘models for’ (Gravemeijer, 1994) where the student moves from the concrete informal level of doing mathematics to the more abstract and formal level. This process is known as progressive formalisation.

3. The use of guided reinvention where students can experience a similar process compared to the process by which mathematics was invented.

4. The use of students’ own productions and own constructions to demonstrate understanding and to reflect on the learning process. This means that students are asked to create problems themselves.

5. The use of various instructional modes (individual, group work, pairs with and without technology); together with interaction (i.e. through discussion) is the key to explicate the learning and teaching.

6. The use of intertwined learning strands. Mathematics is seen as one subject. At primary and secondary school there are no separate course on algebra, geometry and calculus, but the topics are integrated in one course named mathematics.

(Wijers & van Reeuwijk, 2004, p. 80)

Later on, Treffers (1978, 1987) explicitly formulated the idea of two types of mathematization in an educational context; he distinguished ‘horizontal’ and ‘vertical’ mathematization. Freudenthal (1991, p.71) stated that “horizontal mathematization involves going from the world of life into the world of symbols, while vertical mathematization means
moving within the world of symbols.” He adds that the difference between these two types is not always clear cut.

An adapted method of assessment was gradually developed in The Netherlands in the wake of the development of RME. Developers of assessment in the Netherlands believe that not only must assessment lead to good education, but it must simultaneously improve learning by giving the students feedback on their learning processes. De Lange (1993, p. 179) articulated this latter point explicitly:

“The first and main purpose of testing is to improve learning”

Assessment in RME is principally sought through assessment problems. These problems are viewed as situations requiring a solution, which can be reached through organization, schematization and processing of data. An essential aspect in RME assessment is the reflection on the mathematical activities that can enable a rise in level; for example, the discovery of a certain short cut that can then be applied to new problems (Van den Heuvel-Panhuizen, 1996).

The Lesh Translation model (2003) gives the author an excellent basis for teaching algebra. The author believes that every aspect of the model; pictures, spoken and written symbols, manipulative models and real life situations are of importance for effective teaching. The author will now investigate active learning as an approach to employing the Lesh translation model within a classroom setting.

2.3.4 Active Learning

It is difficult to define the term ‘active learning’ as it appears to have different meanings in different contexts. Chickering & Gamson (1987) suggest that active learning involves students doing more than just listening. They believe students must read, write, discuss, or be engaged in solving problems and most importantly be actively engaged in such higher-order thinking tasks as analysis, synthesis, and evaluation. For many years, the idea of learners getting involved in their learning rather than passively receiving information from an instructor has been viewed as the essence of education. Recent empirical studies have found that student-centred methods are superior to teacher dominated methods. McKeachie, Pintrich, Lin and Smith (cited in Sorcinelli, 1991, p. 17) attribute this superiority to students
developing the ability to “apply concepts, problem solve, lead, improve attitude and motivation and be involved in group membership”.

Some theories of active learning expand our view beyond the individual student and focus on the social context of learning. Approaches to learning that promote social constructivism, or learning within a social context, and which feature active group constructions of knowledge (Jaworski, 1994) provide an ideal environment for some learners. Studies on active learning suggest that methods such as student-centred discussions and cooperative-learning groups develop committed and positive relationships among class members. Sorcinelli (1991) found that students engaged in learning in a social context felt more responsible for preparing and coming to class, for paying attention during class, and for taking active responsibility for their own learning. Under the umbrella of social constructivism, Rubin & Hebert (1998) believe that collaborative peer learning and peer teaching are particularly encouraging. The next section will look at these methodologies in more detail.

**Collaborative Peer Teaching**

Mckeachie *et al.* (1991) maintain that one of the most effective methods of teaching is “students teaching other students” (cited in Sorcinelli, 1991, p. 17). This collaborative peer teaching approach has many advantages for students including intellectual and social development according to Gerlach (1994). The benefits pertain not only to the peer teachers but to the students that are being taught as well. There is evidence in the research that students learn more in a cooperative classroom atmosphere and that they are more willing to take responsibility for their educational experiences than they would in learning situations where they are subordinate to the teacher (Rubin & Hebert, 1998). Another advantage of such an approach lies in the motivation students develop within a collaborative peer environment and the interest that it sparks (Forsyth & McMillan, 1991).

Rubin and Hebert (1998) suggest that three theoretical perspectives underpin collaborative peer teaching (see figure 2.3).
Social Context

This context relates to the conductivity of the classroom environment for learning. Dialogue is viewed as a very important element of this context with interaction and cooperation being key principles.

Motivational Theory

Motivational theory requires students taking responsibility for their learning as it looks at how learning is initiated and sustained (Forsyth & McMillan, 1991).

Cognitive Approach

This approach focuses on strategies of information processing – how can the student make the learning more meaningful? Svinicki (1991) suggests that students must organise information, make their own connections with the information and then apply it to new contexts.

(1995) organised for groups of 4-5 students to take responsibility for selecting, reading and planning activities for a play for their peers. This approach was deemed very successful and so Rubin & Hebert (1998) wrote into the requirements of 3 humanities courses that groups of students must teach a class at some point in the semester.

These peer taught classes met both praise and criticism. Peer teachers found that patterns that were habitual and unconscious could be evaluated and changed. They became more conscious of their own intellectual and interpersonal skills and expressed pride in their accomplishments and the new skills they developed as peer teachers (Rubin & Hebert, 1998). Rubin & Hebert (1998) also found that 75% of students believed that teaching was harder than expected due to the preparation and effort needed. They also had difficulties managing their peers – keeping their attention, getting responses and keeping them happy. Students’ responses to the collaborative peer teaching suggest that they were more relaxed and comfortable in these lessons and that participation increased due to the lack of an authority figure (Rubin & Hebert, 1998).

Another aspect of this approach is its ability to alter students’ attitude towards mathematics and teaching. Rubin & Hebert (1998) found that as a result of the difficulties the peer teachers encountered, they appeared to have a lot more empathy when they returned to being students. Rubin & Hebert (1998) suggest that this new awareness meant that students became active partners in the teaching-learning process sooner than they would have otherwise. Also, Bandura (1997) has argued that collaborative decision making (the exchange of ideas) raises the sense of self-efficacy in students.

Although both students and peer teachers gave the experience a positive evaluation and felt it should be used again, some problems arose. Firstly, the evaluations tended to focus on social aspects, i.e. Sasha presented well because she tried a few jokes, as opposed to the learning that occurred - did they feel they understood and learnt more material in these classes? Peer teachers also became very frustrated when classmates were unresponsive and unhelpful. Rubin & Hebert (1998) make suggestions to improve the collaborative peer teaching experience and believe that with these improvements, it could be used very successfully in other educational domains, such as mathematics. Tobias (1995) echoes this when he suggests that teachers should make greater use of collaborative small groups to work on problems, establish a non-threatening learning environment, explore different ways of approaching
mathematical problems, and focus on conceptual understanding. In conclusion, collaborative peer teaching is a flexible method that can be profitable in a variety of disciplines.

This chapter has so far highlighted research that pertains to teaching and learning theories. It has sought to investigate theories such as constructivism, problem solving, Lesh’s translation model, Realistic Mathematics Education and active learning methodologies in light of the literature in order to establish appropriate methods to be used in this project. Findings reveal that there are positives related to all theories discussed. In light of this, the author had a difficult decision to make when selecting a theoretical framework and so looked to further her research in the areas of teaching for understanding and the affective domain.

2.4 Teaching for Understanding

Mathematics educators such as Colburn, Brownell & Van Engen wrote about the ideal of teaching for understanding long before the reform movement of the 1960’s (Carpenter & Lehrer, 1999). An ever-growing body of research and knowledge about the teaching and learning of mathematics is forming new directions for classroom instruction so that students learn with understanding. Kilpatrick, Swafford & Findell (2001) emphasized that mathematics is a tool to understand and interpret the real world. They argued that those who can understand and apply mathematics have significantly enhanced opportunities to achieve success in continuing education and in daily life.

2.4.1 Perspectives on teaching for understanding

A number of mathematical researchers have written comprehensively on this topic. Skemp (1978), for example, identifies two types of understanding: instrumental understanding and relational understanding. Hiebert (1986) had a similar view but referred to the different types of understanding as procedural and conceptual knowledge. Instrumental understanding/procedural knowledge is described as ‘rules without reasons’, i.e. students know the procedures for performing calculations. This type of instruction has dominated classrooms for much of the last century. Skemp (1978) suggests that the reason for this is due to the easier and immediate comprehension of material. Also, due to less knowledge being required, learners can calculate answers more quickly with less mathematical errors, which in turn leads to immediate rewards for correct answers (Davidenko, 2000).
Relational understanding/conceptual knowledge, however, is defined by Skemp (1978, p.9) as “knowing both what to do and why”. The most important feature of learning with understanding is that this type of learning is generative. Carpenter & Lehrer (1999, p.19) believe that if students acquire knowledge with understanding, they can “apply that knowledge to learn new topics & solve unfamiliar problems”. Irish mathematics students have been identified as having difficulties with this type of transfer of knowledge (Lyons et al. 2003; O’Donoghue, 2002 cited in NCCA, 2005). If students do not fully understand a topic they perceive it as being an individual skill. If this occurs, whatever knowledge they acquire is of little value to them outside of school. Relational understanding is long-term as opposed to immediate and context specific (Ni Riordáin, 2008) as it requires students to gain a more in-depth and meaningful grasp of material. Through understanding in this way, students are enabled to apply their knowledge to tasks and problems that they may not have seen before.

Carpenter and Hiebert (1992) explain that the degree of students’ understanding is determined by the strength and number of connections formed. The more extensive the network of relationships or connections created, the greater the degree of student understanding. Teaching for understanding should therefore involve a pedagogical approach, which promotes the construction of connections rather than memorisation of procedures.

2.4.2 Promoting understanding in the classroom

Carpenter & Lehrer (1999) believe that there are five forms of mental activity from which mathematical understanding emerges. They believe that classrooms must provide students with opportunities to engage in these mental activities in order to learn with understanding. The five forms are outlined below:

- **Construct relationships** - students must relate new ideas or processes to ones they already understand. If students do not do this they will learn in two separate ways, one way for school and one way for outside of school.

- **Extend & apply mathematical knowledge** - simply connecting knowledge is not enough; students must create rich, integrated knowledge structures. If students see a number of critical relationships among concepts and processes, they are less likely to forget and more likely to recognise when the knowledge can be related to new situations.
• Reflect about their own maths experiences - this involves a conscious examination of one’s own actions and thoughts. If students can do this they stand a better chance of acquiring the ability to consciously examine the connections between their own knowledge and conditions of a problem situation.

• Articulate what they know - an ability to communicate what they know verbally, in writing or through pictures, diagrams, graphs or models. It can be thought of as a public form of reflection.

• Make mathematical knowledge their own - students need to adopt a stance that knowledge is evolving and provisional and that in order to understand it they cannot perceive knowledge as simply something else that someone has told them, they must make a personal connection with it.

(Carpenter & Lehrer, 1999, p. 21-23)

In order to provide these opportunities Carpenter & Lehrer (1999) suggest three dimensions of instruction that must be considered. The first dimension relates to the tasks or activities that students engage in and problems that they solve. Students must be engaged in these tasks for the purposes of understanding and not simply for the purpose of completing the task. The second dimension relates to the tools that represent mathematical ideas and problem situations. These tools can be pencil, paper, manipulatives, ICT etc. Carpenter & Lehrer (1999) stress the importance of students linking critical steps in procedures to abstract symbols in order to gain meaning from them. The final dimension is normative practice, which refers to the standards regulating mathematical activity, agreed by the student and teacher. These practices form the basis for the way the first two dimensions are used for learning. They govern the “nature of the arguments that students and teachers use to justify mathematical conjectures and conclusions” (Carpenter & Lehrer, 1999, p. 25).

Teaching for understanding has many benefits. Research that focuses on school mathematics instruction repeatedly demonstrates that memorisation of facts or procedures without understanding often results in fragile learning (Ambrose, Clement, Philipp & Chauvot, 2003; Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti & Perlwitz, 1991; Fennema & Romberg, 1999; Hiebert & Wearne, 1993; Schoenfeld, 1988; Zohar & Dori, 2003). These studies emphasise the importance of sense making, of conceptual understanding and many
researchers believe that such an alliance could be very powerful (Carpenter & Lehrer, 1999; Hiebert & Carpenter, 1992; Resnick & Ford, 1981; Schoenfeld, 1988).

The powerful nature of teaching for understanding lies in its ability to help students solve new and unseen problems. O’Donoghue (2010) explains that this skill is highly valued in today’s workforce as it is impossible to predict technological advances. The Project Maths curriculum also acknowledges the strengths of this approach:

“Much greater emphasis will be placed on student understanding of mathematical concepts, with increased use of contexts and applications that will enable students to relate mathematics to everyday experience”


This is not, however, the first attempt at bringing these areas into second level mathematics in Ireland (Oldham & Close, 2009; NCCA, 2005). Other approaches such as Problem Solving and RME have a lot of support amongst researchers and studies have shown many benefits of these approaches. School practice in Ireland is undoubtedly characterised by a heavy preference towards behaviourism and other out-dated learning theories (Lyons et al., NCCA, 2005) but, with the influence of Project Maths this may well be on its way to changing for the benefit of students, teachers and Irish society as a whole. This chapter will now examine, the affective domain and its standing in both national and international research.

2.5 The Affective Domain

There is extensive research that highlights the importance of addressing affective issues during any course of study (McLeod, 1992, DeBellis & Goldin, 1997, Ma & Kishor, 1997, Martino & Zan, 2001, Hannula 2002, 2004, Goracke, 2009, Roswal, Mims, Evans, Smith, Young, Burch, Croce, Horvat & Block, 2010). This importance is highlighted by theorists such as Atkin & Helms (1993) who suggest that affective components are as important as the content itself. Affect is difficult to define. Fleckenstein (1991) suggests that it is an integral, if not the initiating, part of all knowledge construction. She acknowledges Vygotsky’s (1962, p. 150) thoughts that point out, "a true and full understanding of another's thought is possible only when we understand its affective-volitional basis".
Using affect as an umbrella term can be seen as a solution to the difficulties encountered when defining affect. This umbrella term includes emotions, feelings, moods, preferences, beliefs, attitudes, motivations, and evaluations. McLeod (1992) suggests classifying these into three concepts; beliefs, attitudes and emotions (see figure 2.4). This classification characterises the nature of each concept on an affective continuum and assigns different levels of stability and intensity to them.

Figure 2.4 McLeod’s (1992) classification of the concepts of the affective domain.

![Emotions, Attitude, Beliefs]

(Mcleod, 1992 cited in Hannula 2006, p. 213)

More recently, DeBellis & Goldin (1997) suggest four facets of affective states: emotional states, attitudes, beliefs, and values/morals/ethics. They suggest that the fourth facet, however, cannot be ordered on a single stability/intensity dimension. Research in relation to these classifications has taken two broad directions according to Zan et al. (2006). One direction aimed to critique and revise McLeod’s basic concepts whilst the other aimed to break new ground. Using DeBellis & Goldins (1997) four facets it is possible to briefly outline how research has progressed under each.

‘Attitude’ has received a lot of focus in research. A comparison of different definitions has taken place amongst researchers, its significance in mathematics education and suggestions of how to integrate quantitative and qualitative methodologies have all been referred to by authors such as Ruffell et al. (1998), Di Martino & Zan (2001) and Hannula (2002). Research on ‘beliefs’ has continued to examine mathematics-related beliefs of teachers and students (e.g. Leder et al., 2002) but has broadened to include a focus on ‘self-efficacy’ beliefs (e.g. Philippou & Christou, 2002). ‘Emotion’ has received less attention in the research according to Zan et al. (2006). They argue that repeated experience of emotion may
be seen as the basis for more stable attitude and beliefs and so may be the most fundamental concept of affect. Further to this, emotions are seen to involve physiological reactions that affect cognitive processing in several ways. They found that they bias attention and memory and activate action tendencies (Zan et al., 2006). Moreover, Evans (2000), Hannula (2002), DeBellis & Goldin (2006) have discovered that emotions are seen to be functional, with a key role in human coping and adaptation. ‘Value’ has, according to Zan et al. (2006) been the least popular topic of research under the four facets outlined by DeBellis & Goldin (1997). Bishop (2001) has sought to investigate what research on this topic can offer mathematics education.

The second broad direction that research in this field has taken is that of breaking new ground. Constructs such as motivation, mood and interest are among this ‘new ground’ and have been investigated by authors such as Hannula (2006) and Mendick (2002).

For the purposes of this project, the author will use this section of the literature review to explore one of the terms common to both McLeod (1992) and DeBellis & Goldin (1997) research - ‘attitude’ in relation to mathematics. It will begin by defining what is meant by attitude in relation to mathematics education. It will then give a brief outline of findings from the research in relation to attitude and mathematics education. Because this research project is concerned with a group of first year students, this section will continue by briefly examining issues relating to attitude and the transition from primary to secondary school.

### 2.5.1 Defining Attitude

Attitude can be defined in many different ways according to the research. Fishbein & Ajzen (1975, p. 1) suggest that a definition for attitude is “characterised by an embarrassing degree of ambiguity and confusion”. They believe that this may be due to it being a widely used concept with attribution of dispositions, liking and behavioural intentions all coming under the general label of “attitude”. Over twenty years later Martino & Zan (2001) as well as Ruffell, Mason & Allen (1998) agree with Fishbein & Ajzen (1975) when they point out that attitude is an ambiguous construct that is often used without proper definition.

Hannula (2002) suggests that it needs to be developed theoretically. He refers to an everyday notion of attitude being “someone’s basic liking or disliking of a familiar target” (Hannula, 2002, p.1). Aiken (1996, p. 168) gives a more comprehensive definition when he postulates
that attitude “consists of cognitive (knowledge of intellect), affect (emotion and motivation) and performance (behaviour or action) components”.

Although both Hannula and Aiken offer definitions for attitude and although slightly contradictory to the recent calls for a new, less ambiguous definition, the author believes that the definition given by Neale (1969, p. 632) for attitude towards mathematics is most appropriate for the purposes of this project:

“Attitude to mathematics is an aggregated measure of a liking or disliking of mathematics, a tendency to engage in or avoid mathematical activities, a belief that one is good or bad at mathematics and a belief that mathematics is useful or useless”.

Due to its simplistic language and general ease of understanding, the author is referring to the above definition by Neale (1969), when speaking of attitude throughout this research.

2.5.2 Attitude towards Mathematics

Research on attitude has been motivated by the belief that “something called ‘attitude’ plays a crucial role in learning mathematics” (Neale, 1969). Studies in this field have shown that, for example, girls tend to have more negative attitudes towards mathematics than boys (Frost et al., 1994, Leder, 1995), and that attitudes tend to become more negative as students move from elementary to secondary school (McLeod, 1994). Haladyna et al. (1983) discovered that the general attitude of the class is related to the quality of the teaching as well as to the social-psychological climate of the class.

Reform movements in mathematics education (Australian Education Council 1991; Cockroft Report 1982; NCTM, 1989) have sought to improve students’ attitude towards mathematics in order to improve the learning of mathematics. Interestingly, Ma & Kishor (1997) carried out a meta-analysis of 113 classical studies and their analysis of the correlation between attitude and achievement produced no educationally statistically significant results. Although the correlations were weak in the overall sample, they were stronger throughout grades 7-12. Farooq & Shah (2008), however, suggest that the relationship that exists between a positive attitude towards mathematics and performance in mathematics is emphasised repeatedly throughout the mathematics literature. McLeod (1992) and OECD (2004) also support this view. It is recognised that the exact nature of this relationship is complex (McLeod, 1992). Another
study by PISA (2003) asked students about four different factors in an effort to examine their influence on performance. These were motivation, self-related beliefs, emotional response (anxiety) and learning strategies. The analysis of the data showed that there was a strong relationship between these factors and the students’ performance. The most significant finding, according to the OECD (2004, p. 148), was that “students who are less anxious perform better regardless of other characteristics” and “that anxiety and interest and enjoyment of mathematics are closely interrelated”. They also discovered that in terms of students’ perception of the usefulness of mathematics, “most students believe that success in mathematics will help them in their future work and study” (OECD, 2004, p.121).

McLeod (1992, p. 582) suggests a reason for the lack of consensus on whether there is a direct relationship between attitude and achievement; “research suggests that neither attitude nor achievement is dependent on each other; rather they interact with each other in complex and unpredictable ways”. Meta-analysis studies on this topic allow the reader an insight into the general narrative of what the literature is saying. Richardson & Suinn (1972) developed the Mathematics Anxiety Rating Scale (MARS) and found results that suggest high levels of mathematics anxiety appear to interfere with achievement in mathematics. A meta-analysis study undertaken by Hembree (1990) based on 151 studies, confirmed that mathematics anxiety is related to poor performance in mathematics. Another interesting finding by Middleton & Toluk (1999) suggests that attitude can affect engagement in an activity. This is a significant finding as it demonstrates that attitude towards mathematics can be linked to the most advanced student’s achievement.

There appears to be an acceptance of underachievement in mathematics of students who believe that some people have a natural flair for mathematics and others do not (NCCA 2005, National Research Council 1989). Students inherently feel comfortable in a didactic setting where routine questions and presentation mode are key and the Cockroft Report (1982) suggests that students informally push teachers in that direction. These initial discomforts appear to affect student achievement but Nicksons (2000) suggests that positive attitudes to mathematics can flourish when students recover from these initial discomforts. Studies by Carpenter et al. (1998), Cobb et al. (1991), and Verschaffel & De Corte (1997) support this as unhappiness was cited in the beginning phases of projects.
2.5.3 Improving Attitude towards Mathematics

A variety of approaches have been used across the literature as a means of improving attitude. A number of these have been discussed throughout this literature review and will continue to be discussed in the next chapter in relation to the teaching of algebra. A short synopsis of interesting studies aimed at improving attitude will now be discussed.

Use of Manipulatives to Improve Attitude towards Mathematics

Research by Sowell (1989) involved the results of 60 studies being combined to determine the effectiveness of mathematics instruction with manipulative materials. Students ranged from kindergarten through to postsecondary. These studies indicate that manipulatives can be effective in improving attitude towards mathematics provided teachers were knowledgeable about their use (Sowell, 1989). She also found that long-term use of manipulatives was more effective than short-term use in improving attitude. Similarly, Goracke (2009) used an action research approach to investigate the use of manipulatives and its impact on student attitude and understanding. She discovered that student attitude toward mathematics improved when greater manipulative use was infused into the lessons. She found that students felt more confident that they understood the material, which translated into a better attitude regarding mathematics class.

Collaborative Peer Teaching to Improve Attitude towards Mathematics

Another interesting approach to improving attitude towards mathematics is that of collaborative peer tutoring. Forsyth & McMillan (1991) found that an environment involving collaborative peer learning develops students’ motivation and interest in mathematics. More recent research by Boaler (1997a,b, 1998) and Ridlon (1999) confirms this as it suggests that collaborative approaches can promote positive attitudes among students.

As recently as 2010, Roswal, Mims, Evans, Smith, Young, Burch, Croce, Horvat & Block (2010) studied the effects of a collaborative peer tutor teaching programme on the self-concept and school-based attitudes of seventh-grade students at a large urban junior high school. The study consisted of the 282 students and many of these had been previously identified to be at risk by traditional school identification strategies. The Piers-Harris Self-Concept Scale was used to measure self-concept in subjects and the Demos D (Dropout)
Scale was used to measure student tendency to drop out of school. Data was collected immediately before the programme began and immediately after the programme reached completion. A post hoc analysis revealed that students in the collaborative peer tutor teaching programme demonstrated significant improvement in dropout scores compared with students in both the traditional class using group learning activities and the traditional class using individual learning activities (Roswal et al., 2010).

Furthermore, a study was carried out by Townsend & Wilton (2003) to assess whether traditional pre-test post-test procedures also indicate positive changes in mathematics attitude during a programme of cooperative learning. Their research involved 141 undergraduate students enrolled in a 12-week statistics and research design component of a course in educational psychology. Multivariate procedures, pre-test, post-test and measures of mathematics self-concept and anxiety were examined in conjunction with a cooperative learning approach to teaching. The results showed statistically significant positive changes for both mathematics self-concept and mathematics anxiety. The results of these studies indicate that a collaborative peer tutor teaching program can be effective in eliciting improvements in attitudes toward school and mathematics.

Use of Games to Improve Attitude towards Mathematics

Another approach for improving the negative effects of attitude and motivation (Druckman, 1995) supports the inclusion of simulation games in mathematics education. A review by Van Sickle (1986) looked at 42 studies of simulation games and found an effect size of .16 for attitude toward subject matter in favour of gaming versus other forms in half of the studies. Other research has also shown that attitude toward content can be influenced by using games (Barak, Engle, Katzir, & Fisher, 1987; Malouf, 1988; Pascale, 1974). According to the NCTM, (as cited in Bright, Harvey & Wheeler, 1985), mathematics games may be the best way to ensure that students get the basic mathematics skills they need to be successful in life, while at the same time promoting an enjoyment of mathematics and motivation to learn mathematics. Heyman (1982, p.4) agrees: "Simulation games contribute to young people's learning in the affective domain", and the incorporation of game-like features and simulation elements can promote motivation to learn (Brophy, 1987). Although few studies have examined the effect of combining agents and games and none have examined their effect on attitude toward mathematics, Van Eck (2006) put in place a study designed to determine the effect of contextual pedagogical advisement (CPA) and competition on attitude toward
mathematics in a computer-based simulation game. A total of 123 seventh and eighth grade students were randomly assigned to one of five conditions formed by crossing the two independent variables and adding a control group. Results indicate that contextual pedagogical advisement can result in lower anxiety toward mathematics scores, especially under competitive conditions.

2.5.4 Measurement of Affect and Attitude

Early attempts to measure aspects of affectivity relied heavily on self-report paper and pencil measures, including Thurstone, Likert, Guttman and Osgood’s semantic differential scales (Leder, 2006). Very few of these measurement devices have lasted the test of time. Thurstone scales have lost their popularity in recent years due to their now recognised doubtful validity. Guttman scales have been used sparingly due to their difficulty to construct (Leder, 2006). Osgood’s semantic differential scales rely heavily on responses to lists of bipolar adjectives covering evaluative, potency and activity aspects. These scales enable measurement congruent with cognitive, affective and behavioural components of attitudes and beliefs captured in definitions such as Aiken’s cited above (p. 35). Likert scales such as those devised by Fennema and Sherman (1976) have proved to be the most popular amongst the literature. They consist of a series of statements about the attitude object or activity of interest and represent a common approach to the measurement of affect with a particular emphasis on attitude (Leder, 2006). Fennema-Sherman (1976) revised their scale and produced the Fennema Sherman mathematics attitude scale [MAS]. It was first used to explore the link between gender differences in mathematics achievement, spatial visualisation and affective factors. It has since spawned much of the research in relation to this topic and has been identified as among the most frequently cited publications in mainstream journals of educational psychology (Wallberg & Haertel, 1992).

Over two decades ago Cockcroft (1982, p. 61) wrote about the importance of engaging students affectively as well as cognitively in his influential Mathematics counts:

“It is to be expected that most teachers will attach considerable importance to the development of good attitudes among the students whom they teach...Attitudes are derived from teachers’ attitudes...and to an extent from parents’ attitudes...Attitude to mathematics is correlated...with the peer group’s attitude”.
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Research in the same vein has continued steadily throughout the past few decades with much of it utilising Fennema and Sherman’s work. As discussed, many of these studies have sought to successfully identify the links that exist between attitude and achievement. Findings from the recent Programme for International Student Assessment (PISA) concluded that “self-related beliefs and emotional factors are linked to the adoption of effective learning strategies and thus can help students become lifelong learners” (OECD, 2004, p.12).

The author aims to examine the effects of an intervention on student’s attitude in this study. A revised Fennema-Sherman questionnaire will be used for this analysis and will look at four aspects of attitude that link closely with Neale’s (1969) definition for attitude:

- **Confidence**: “a liking or disliking of mathematics”
- **Effective motivation**: “a tendency to engage in or avoid mathematical activities”
- **Anxiety towards mathematics**: “a belief that one is good or bad at mathematics”
- **Usefulness**: “a belief that mathematics is useful or useless”.

(Fennema-Sherman, 1976; Neale, 1969)

2.5.5 Attitudes towards Mathematics during Transition from Primary to Secondary Education

International studies suggest that the transition to middle level schools is associated with a decline in motivation and performance for a number of children (Eccles & Midgley, 1989). Research has found that one’s beliefs about mathematics can determine how one chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on (Schoenfeld, 1985, p.45). In early 2002, as part of the review of the Junior Cycle, the NCCA commissioned the Educational Policy Research Centre of the Economic and Social Research Institute (ERSI) to engage in a longitudinal study of students’ experiences of curriculum in the first three years of their post-primary schooling. The first report focused on first year students and the role of transition. This report made it clear that there was a lack of continuum between primary and post-primary schooling and this was having an impact on the success or otherwise of students’ successful transition. It also noted that there was a discontinuity in teaching approaches between primary schools, where the
focus of teaching is ‘child as active learner’, and post primary schools where the emphasis is on instruction rather than participation. Other interesting findings noted that mathematics was the second least favourite subject in first year and that the curriculum discontinuity caused significant disruption to academic progress (NCCA, 2006). This research contributed in a major overhaul of the mathematics syllabuses in Ireland for both Junior and Leaving Certificate Mathematics’ known as ‘Project Maths’. In order to counteract the discrepancies in curriculum continuity and lack of focus on students as active learners, one of Project Math’s key objectives is to build on and extend students’ experience of mathematics from primary school. In order to achieve this, a bridging framework has been developed (see appendix K) which aims to link the various strands of mathematics in the primary school to topics in the Junior Certificate mathematics syllabuses. Its roll out will involve changes to what students learn in mathematics, how they learn it and how they will be assessed. With Project Maths only entering its first roll out phase in 2010 there is a clear opportunity for research in the area of attitude and transition from primary to secondary education in Ireland. A recent document published by the Department of Education and Skills (DES, 2012), called ‘Literacy and Numeracy for Learning and Life’ aims to improve literacy and numeracy among children and young people from 2011 until 2020. One of the key areas within the strategy is to get the content of the curriculum for literacy and numeracy “right at primary and post primary levels by making sure that the curriculum is clear about what to expect and what we expect students to learn” (DES, 2011, p.5). Another target of this strategy is that of improving attitude towards mathematics. The strategy plans to raise public awareness of the importance of oral and written language in all its forms, create greater awareness of and attitude towards mathematics among the public and in general promote better attitude towards maths among children and young people (DES, 2012).

2.6 Conclusion

This chapter began with a brief review of mathematics and mathematics education in an Irish context. The author carried out a detailed exploration of the key domains on which the design framework is to be based:

- Teaching and learning theories
- Teaching for understanding
- The affective domain.
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Through this exploration, a deeper understanding of effective teaching and learning has undoubtedly been gained. A comprehensive review of teaching and learning theories such as constructivism, problem solving, Lesh’s Translation model, Realistic Mathematics Education and active learning highlighted the benefits of teaching using these models. A review of literature pertaining to teaching for understanding helped the author to gain a perspective on methods for teaching mathematics that leads to lifelong learning. A comprehensive analysis of attitude towards mathematics deepened the author’s understanding of this topic and highlighted methods that may be employed to improve attitude towards mathematics.

This analysis allowed the author to choose an appropriate means through which first years could be taught for the duration of the intervention. In light of the literature, The Lesh Translation Model (2003) combined with a collaborative peer teaching approach were deemed appropriate models for fostering and improving attitudes towards mathematics. Teaching for understanding was thought to be an excellent model to help first year students improve their understanding of basic algebra but the author felt a more in-depth investigation of this topic was required to proceed. The next chapter, ‘teaching algebra’, outlines a basic history of algebra, algebra in Irish education and approaches to teaching algebra and aims to provide the author with a more comprehensive understanding of the most suitable methods to employ for teaching first year algebra.
Chapter 3: Teaching Algebra

3.1 Introduction

This chapter is guided by the second research question outlined in section 1.5, ‘What are the issues contributing to and theoretical perspectives underlying effective mathematics teaching which can improve students’ understanding of basic algebra at Junior Cycle level?’ This chapter will provide a brief history of algebra and algebra education at both national and international level. It will discuss the importance of algebra, difficulties associated with algebra and the theoretical perspectives underlying the various approaches to teaching algebra. The chapter will conclude by giving a more detailed outline of the approach selected for this intervention; the function-based approach.

Before beginning, it is important that we have some understanding of what the term ‘algebra’ means. Colin Maclaurin wrote, in his 1748 algebra text, “Algebra is a general method of computation by certain signs and symbols which have been contrived for this purpose, and found convenient. It is called a universal arithmetic, and proceeds by operations and rules similar to those in common arithmetic, founded upon the same principles” (Maclaurin, 1756, p. 1). Leonhard Euler, in his own algebra text of 1770 wrote, “Algebra has been defined as the science which teaches how to determine unknown quantities by means of those that are known” (Euler, 1984, p. 186). That is, in the 18th century, algebra dealt with determining unknowns by using signs and symbols and certain well-defined methods of manipulation of these. More recently, Lee (1996) presented the question ‘What is algebra?’ to a cohort of mathematicians, teachers, students and mathematics education researchers. Themes that emerged from her interviews were:

- Algebra is a school subject
- Algebra is generalised arithmetic
- Algebra is a tool
- Algebra is a language
- Algebra is a culture
- Algebra is a way of thinking
- Algebra is an activity

(Kieran, 2004, p.22)
Two of these themes are particularly interesting as they appear more widely in the literature. These are algebra as a language and algebra as generalised arithmetic. Natural language is seen as too inept for communicating abstract and complex ideas (Kellett, 1998). Algebra takes on the role as the language by which these ideas can be conveyed. The view of algebra as generalised arithmetic is widely debated in mathematics education circles. It is argued that simple equations can be solved using arithmetic means but as ideas become more abstract and complex, the significance of algebra becomes more apparent (MacGregor, 2004). It is at this point that students must make the transition from arithmetic to algebraic thinking. MacGregor (2004) believes that once this transition is complete, students will have the opportunity to engage with conceptual ideas and to experience the pleasure and satisfaction of using a powerful symbol system to support logical thinking.

3.2 A Brief History of Algebra

In many history texts, algebra is considered to have three stages in its historical development: the rhetorical stage, the syncopated stage, and the symbolic stage.

- The rhetorical stage refers to when all statements and arguments are made in words and sentences.

- The syncopated stage is where abbreviations are used when dealing with algebraic expressions.

- The symbolic stage refers to when there is total symbolization – all numbers, operations, relationships are expressed through a set of easily recognized symbols, and manipulations on the symbols take place according to well-understood rules.

(Katz, 2004, p.1)

According to Katz (2004) there are four conceptual stages that have happened alongside these three stages of expressing algebraic ideas. The conceptual stages are the geometric stage, where most of the concepts of algebra are geometric; the static equation-solving stage, where the goal is to find numbers satisfying certain relationships; the dynamic function stage, where motion seems to be an underlying idea; and finally the abstract stage, where structure is the goal. Naturally, neither these stages nor the earlier three are disjoint from one another; there
is always some overlap. The first set of stages (Katz, 2004) is well known and discussed in detail by Luis Puig in the recent *ICMI Study on Algebra* (Puig, 2004) and so will be discussed while outlining a brief history of algebra.

The Old Stone Age, the Palaeolithic age, can be attributed with the first conceptions of number and form (Struik, 1987). The history of algebra, however, began in ancient Egypt and Babylon. It originated as a practical science in order to facilitate computation of the calendar, administration of the harvest, organisation of public works and collection of taxes. Its’ initial emphasis was on practical arithmetic and mensuration, where people used geometry as the underlying rationale to solve problems (the rhetorical stage). The Egyptians of this period (1750 B.C.) developed algorithms and procedures to solve simple linear equations \((ax = b)\) that eventually began to replace geometry. The Babylonians, however, were in full possession of the technique of handling quadratic equations \((ax^2 + bx = c)\) at about the same time (Struik, 1987). Diophantus (3rd century AD) was an Alexandrian Greek mathematician and was sometimes referred to as “the father of algebra”. He continued the traditions of Egypt and Babylon through a series of books called *Arithmetica*. These books dealt with solving algebraic equations and were at a much higher level than any previous algebra (Katz, 2004).

The word algebra comes from the Arabic language, “al-jabr” which means “restoration”. ‘Al-jabr’ was the title of one of the most famous algebraic textbooks of all time, written by the Arab mathematician Mohammad ibn Musa al-Khwarizmi, who was also, referred to as “the father of algebra”. It was written in Baghdad around 825 (Al-Khwarizmi, 1831) and established algebra as a mathematical discipline that is independent of geometry and arithmetic. Algebra has now moved decisively from the original geometric stage to the static equation-solving stage. Al-Khwarizmi wanted to solve equations (Katz, 2004). Al-Kharizmi’s work plays a vital role in the history of mathematics as it is one of the main sources through which Indian numerals and Arabic algebra came to Western Europe (Struik, 1987).

Islamic algebra was transmitted to Europe in the twelfth and thirteenth centuries. There were several routes that Al-Khwarizmi’s algebra took into Europe, including the work of Leonardo of Pisa (Fibonacci), and Abraham bar Hiyya (in Spain, 1136), as well as the direct translations made by Robert of Chester and Gerard of Cremona (Struik, 1987).

By the sixteenth century, Greeks and Orientals had failed to discover solutions to the third degree equation. Cardan and Ferrari discovered methods for finding solutions to such cases but were unable to do anything with the so called “irreducible case” in which three solutions
appeared as the sum or difference of what we now call complex numbers. Bombelli, whose algebra appeared in 1572, introduced the consistent theory of imaginary complex numbers which was not fully accepted until the nineteenth century (Struik, 1987). Rene Descartes (c. 16th century) made a significant contribution to mathematics when he helped to introduce symbols for the unknown and for algebraic powers and operations. His book, Géométrie, was published as “an application of his general method of rationalistic unification, in this case the unification of algebra and geometry” (Struik, 1987, p. 96). He discovered analytic geometry, which reduces solutions of geometric problems to algebraic ones (Katz, 2004).

At this point, algebra was still just about finding solutions to equations. With a new notation coming into place in the seventeenth century, however, a great change in point of view was also taking place in algebra itself. Mathematicians started asking questions other than “find the solution to that problem expressed as an equation” (Katz, 2004, p. 194). This was the beginning of a very rapid change from the Islamic rhetorical algebra through the syncopated stage into the modern symbolic stage.

By the eighteenth century, algebra had entered its modern phase moving from solving polynomial equations to studying structure of abstract mathematical systems. Gauss made the most leading discovery of this era when he proved that every polynomial equation has at least one root in the complex plane. Mathematicians of this era, began to worry and question whether their algebraic manipulations were correct. They began to realise that as long as you had some form of axiom system in place, then you could be assured that your calculations gave correct results. It was when Hamilton discovered quaternions that mathematicians realised that there could be other sets of axioms that could produce very interesting results. These algebraic groups and quaternions, which share some of the properties of number systems but also depart from them in important ways, lead the way for mathematicians in the 19th century (Katz, 2004). Augustin Cauchy (French), Arthur Cayley (British) and Neils Abel (Norweigan), Sophus Lie (Norweigan) and William Rowan Hamilton (Irish) made important contributions to the development of algebra over the past two hundred years.

By the beginning of the twentieth century, algebra had become less about finding solutions to equations and more about looking for common structures in many diverse mathematical objects, with the object being defined by sets of axioms (Katz, 2004). Abstract algebra has continued to develop in recent times. Important new results have been discovered, and the
subject has found applications in all branches of mathematics and in many of the sciences as well.

3.3 An International Perspective: Algebra and Education

Algebra has grown, through the later stages of history, into a powerful tool for describing and using mathematical systems. Traditionally students’ introduction to school algebra was concerned with using and operating on literal systems (Royal Society/JMC Working Group, 1997). The emphasis was on repeatedly practising algebra by working through a multitude of exercises. These transformational activities (e.g. describing numerical patterns algebraically) involved presenting students with symbolic code as a means of generalising from arithmetic where ‘letters stand for numbers’. Katz (2004) believes that teaching algebra in this way usually degenerates into a sequence of algebraic skills that are unrelated to each other and can only be used in recognisable mathematical situations. According to Kieran (2003), studies of the late 1970’s and 1980’s show that an exclusively skills-based approach to the teaching of algebra did not lead to skilled performance amongst students (Carry, Lewis & Bernard, 1980), nor did it help them to interpret adequately the various ways in which letters are used in algebra (Kuchemann, 1981; Matz, 1982) or the structural features of algebraic expressions (Davis 1975). This research suggests that this approach has not been pedagogically successful.

The reform movement, in the 1960’s, introduced changes to the ways in which algebra was taught in Britain and across Europe. This movement reflected the changing role of algebra as a language for representing structures. Students’ first introduction to algebra became more likely to be in the context of set theory. There was also an emphasis put on functions, mappings and the language of sets during this reformation stage. Research accumulated throughout the 1970s and 1980s showing that students were not interpreting literal symbols in ways which were appropriate to algebra (Booth, 1984; Kieran, 1989; Kuchemann, 1981). This greatly influenced a shift away from the use of literal symbols in algebra education (Sutherland, 1990).

School algebra has never been the same and has not developed in the same ways in all countries. Since the mid-1980s, the content of school algebra has been experiencing a tug of war between traditional and reformist views. Currently, content varies not only from country to country but also within countries (Lester, 2004). According to the document ‘Teaching and
Learning Algebra pre-19’ changes during the 1990’s in the USA were heavily influenced by changes in curriculum in the Netherlands and the UK (Royal Society/JMC Working Group, 1996). France and Germany, however, appeared to develop their curriculum, but in a different manner. The changes in the US, England and Wales focused on a problem solving approach. It was noted that algebraic word problems which are problem solving in nature were part of the traditional course but that they had all but disappeared from textbooks in England and Wales. Section 3.4 highlights the development of the function-based approach in an international context.

3.4 The Difficulties associated with the Teaching and Learning of Algebra

Many issues and problems have arisen in relation to the teaching and learning of algebra. A plethora of research highlights the difficulties that teenage students have encountered and still encounter with algebra:

a) Students believe that the equals sign only represents a unidirectional operator that produces an output on the right side from the input on the left (Booth, L., 1984; Kieran, 1981; Vergnaud, 1985; Vergnaud, Cortes, A. & Favre-Artigue, P., 1988b),

b) Students focus on finding particular answers (Booth 1984),

c) Students do not recognise the commutative and distributive properties (Boulton-Lewis, 2001; MacGregor, 1996),

d) Students do not use mathematical symbols to express relationships among quantities (Bednarz, 2001; Bednarz & Janvier, 1996; Vergnaud, 1985; Wagner 1981),

e) Students do not comprehend the use of letters as generalised numbers or as variables (Booth, L., 1984; Kuuchemann, 1981; Vergnaud, 1985),

f) Students have great difficulty operating on unknowns.

g) Students fail to understand that equivalent transformations on both sides of an equation do not alter its truth value (Bednarz, 2001; Bednarz & Janvier, 1996; Filoy & Rojano, 1989; Kieran, 1989; Steinberg, Sleeman, & Ktorza, 1990).

(Lester, 2004, p.670)
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Researchers and teachers have become frustrated with this wide range of issues associated with the teaching and learning of algebra. Kaput (2000, p. 2) argues that “Algebra has been experienced as an unpleasant, even alienating event - mostly about manipulating symbols that don't stand for anything”.

Evidence of this confusion can also be found in Irish classrooms (Prendergast, 2011). An Irish study carried out by McConway (2006) identified algebra as an area of difficulty for students. Similarly, Chief Examiners for Junior and Leaving Certificate examinations report that questions that require the use of algebra are low scoring and unpopular. They conclude that attention must be focused on improving students’ proficiency in this topic (Prendergast, 2011).

A number of explanations for such student difficulty are highlighted in the literature. Herscovics and Linchevski (1994) feel that the problem may be linked to the pace at which the topic is covered and also the formal approach often used in its instruction. Teachers appear to be unaware of the cognitive difficulties that students encounter when learning algebra. Grouws (1992) and Lyons et al. (2003) suggest that teachers see no alternative to teaching mathematics other than the traditional, teacher dominated method of following sections through a textbook. This forces students to memorise procedures that they do not fully understand and leads to students’ inability to relate the mathematics they learn to their everyday lives (Carpenter, Lindquist, Matthews & Silver, 1983).

3.5 Pedagogical approaches to the Teaching and Learning of Algebra

In response to the problems associated with the way in which algebra is taught in schools, teachers and researchers changed their perspectives regarding the core elements of school algebra. This change can be tracked back to the 1960’s (Kieran, 2007). These new approaches were often influenced by the advances in computing technology that were evident at this time. This next section seeks to outline alternative pedagogical approaches for teachers.

3.5.1 Kieran’s (2004) Model for Conceptualising Algebra

Many researchers have sought to identify new approaches for the teaching of algebra. Carolyn Kieran (2004) is one such researcher. She developed a model in which she identified three important activities of school algebra.
The model Kieran (1996) developed identifies three important components of school algebra namely generational, transformational and global / meta level activities. Kieran (1996) suggests that teachers must place equal emphasis on each particular activity when teaching the domain. Each activity will now be explained in more detail.

**Generational Activities**

Generational activities involve the forming of expressions and equations that are objects of algebra (Kieran, 1994). Kieran (2004, p. 23) outlines typical cases which include (i) Equations containing an unknown which represent number problem situations (ii) Expressions of generality from geometric patterns or numerical sequences and (iii) A numerical relationship leading to an algebraic expression. The focus of generational activities according to Kieran (2004, p.24) is the “representation (and interpretation) of situations, properties, patterns and relations”. She highlights that much of the research supporting technology that is aimed at giving meaning to algebraic objects, finds that it is the functional approach “with its multiple representations that has to be capitalised on” (Kieran, 2004, p. 24).

**Transformational Activities**

The second type of algebraic activities is known as transformational activities. These can often be referred to as rule-based activities (Kieran, 2004). Such activities include for example; “collecting like terms, factorising, expanding, substitution, manipulating and simplifying algebraic expressions, working with inverse operations and solving equations” (Kieran, 2004, p. 24). Transformational activities are concerned with changing the form of an expression or equation in order to maintain equivalence (Kieran, 2004).
Global/ Meta Level Activities

Global/Meta Level Activities involve activities that can be engaged in without the use of algebra at all, for example, problem solving, modelling, noticing structure, studying change etc. (Kieran, 2004). This level of activity is important because it gives purpose to the activities. Students are provided with contexts which encourage them to seek reasons for why something works (Kieran, 2004).

Kieran’s model recognises the need to place an equal emphasis on each particular type of algebraic activity. Such a position is far removed from the practical, transformational based approach to algebra that has dominated Irish classroom up to now, as well as its successor in other countries that emphasises almost exclusively sense making generational activity (Kieran, 2004).

Kieran’s Model (2004): Implications for Teaching

Kieran’s model aims to provide a balance between algebraic activities. The model acknowledges that an equal emphasis be placed on each particular type of activity (Prendergast, 2011). Sutherland (1997) supports this view and emphasises the importance of traditional exposition and practice alongside more opportunities for practical work, problem solving, investigations and discussion in order to provide purpose to the mathematical activities that students are engaged in (Prendergast, 2011). To summarise it is important that traditional methods and active learning methodologies be taught together rather than in opposition to each other. The challenge for teachers is to find ways of teaching that create classroom environments which allow students to learn with understanding and generate a genuine interest in the topic.

3.5.2 Bednarz, Kieran and Lee’s (2004) Model for conceptualising algebra

Bednarz, Kieran & Lee (Kieran, 2004) suggest a number of different ways in which the teaching of algebra could be approached. These categories include:

- The generalisation approach

This approach is generally confined to finding a formula for patterns (Stacey & MacGregor, 2001).
• **The problem solving approach**

The problem solving approach aims to introduce students to the concept of moving from arithmetic to algebra, in terms of symbolism as well as reasoning. Traditional word problems are often used to introduce this idea and these often focus on solving equations and viewing letters as unknowns.

• **The modelling approach**

‘Mathematical narratives’ which are constructed in describing events and situations are used to introduce algebra through a modelling approach (Nemirovsky, 1996).

• **The function-based approach**

This approach views functions as fundamental mathematical objects (Sutherland, 2004). It requires students to develop an understanding of tables of values, graphs and the concept of the variable.

(Kieran, 2004, p.73 cited in Prendergast, 2011, p. 73/74)

3.5.3 **Rationale for a function-based approach to teaching algebra**

A brief look at the appearance of the term ‘function’ throughout a history of mathematics highlights some interesting developments. The following is edited from the website: Earliest Known Uses of Mathematical Words (2012).

*The word* function *first appears in a Latin manuscript “Methodus tangentium inversa, seu de fuctionibus” written by Gottfried Wilhelm Leibniz (1646-1716) in 1673. From the beginning of his manuscript, however, Leibniz demonstrated that he already possessed the idea of function, a term he denominates relatio. In . . . 1694, . . . Leibniz used the word function almost in its technical sense, defining function as “a part of a straight line which is cut off by straight lines drawn solely by means of a fixed point, and of a point in the curve which is given together with its degree of curvature.” Function is found in English in 1779 in Chambers’ Cyclopedia: “The term function is used in algebra, for an analytical expression any way compounded of a variable quantity, and of numbers, or constant quantities”*

This extract highlights how the function was developed through geometry, but also took much more use before its analytical sense was fully developed. It followed the pattern of algebra development throughout history, from a rhetorical stage, through a syncopated stage and onto a symbolic stage.

It has long been recognised that students have been ‘given’ a formula and told to use it in certain situations (Hovis et al., 2003). A movement towards a function-based approach became more elaborated in integrated curricula throughout the nineties when it was recognised that mathematics curricula should require students to make connections among topics rather than just using formula. During this reformation stage, in many countries, generalisation activity (e.g. describing numerical patterns algebraically) had all but replaced transformational activity (Kieran, 2007). A project developed and implemented in South Nottingham, England, known as the “Journey into Maths” (Bell et al., 1978, 1979) incorporated an approach to the teaching of algebra in which students were encouraged to discover concepts by manipulating concrete materials, generalising pattern and sequences and describing relationships in words and symbols as well as drawing graphs. This project, along with numerous documents such as Mathematics Counts (Cockcroft, 1982) led to both curriculum and pedagogical changes in the teaching of mathematics in England. The reform envisaged a change of focus from a “generalised arithmetic” approach to one in which functions provide the medium through which the concept of variable is developed (Haimes, 1996). American research, notably, “A nation at Risk” (Gardner, 1983) and the Standards documents of the National Council for Teachers of Mathematics (NCTM, 1989, 1991, 1995) led to reform in the teaching of algebra in North America and at about the same time the national mathematics curriculum in Australia was developed to incorporate focuses on patterns, sequences and functions. (Western Australia, Ministry of Education, 1990 cited in Haimes, 1996) Japan, British Columbia, England, Australia, North America and Singapore are examples of educational jurisdictions which emphasise this type of approach in the early stages of teaching algebra (Stacey et al., 2004, p. 335).

3.5.4 Teaching and Learning of Algebra – an Irish Perspective

In Ireland, research (Grouws, 1992; Lyons et al. 2003) has suggested that there has been an over reliance on traditional methods when teaching algebra. Transformational (rule and procedure) based activities have dominated lessons. Algebra was a paper and pencil activity involving the following of rules and procedures. A minimalist approach to algebraic sense
making took place. Each day of instruction was text-book led and focused on a particular type of manipulation. The textbook started by introducing the concept of a variable, followed by the notion of algebraic expressions and then equations (Kieran, 1992). This structure fitted this approach to a curriculum that considered algebra as a series of skills to be mastered (Chazan, 1996). Success in the subject was determined by the ability to memorise procedures by rote, nothing else (Bracey, 1992). The need for technicians (e.g. engineering and construction) to be able to communicate with co-workers, in both written and oral presentations drives home the need for a movement away from these traditional methods. Hovis et al. (2003, p.4) describe it as “the need for technicians to be able to understand and use mathematics with formulas, graphs and tables and communicate it”. For this reason, the new Irish curriculum, ‘Project Maths’ has followed the lead of the USA and advocates a function-based approach for teaching algebra.

**Algebra in the context of Project Maths**

The first year of the Junior Certificate curriculum is known as the ‘common introductory course’. A bridging framework was developed by the Project Maths team that links all strands in primary school mathematics to those of secondary mathematics (see Appendix K). For the purposes of this project the focus is on the introduction of algebra under the Project Maths syllabus.

The aims of the primary school curriculum for algebra are:

- Explore, perceive, use and appreciate patterns and relationships in numbers,
- Identify positive and negative integers on the number line,
- Understand the concept of variable and substitute values for variables in simple formulae, expressions and equations,
- Translate verbal problems into algebraic expressions,
- Acquire an understanding of properties and rules concerning algebraic expressions,
- Solve simple linear equations,
- Use acquired concepts, skills and processes in problem solving.

These aims are fulfilled through a staggered approach to introducing algebra at primary level. Infant classes are introduced to pattern recognition and extending patterns. First and second class explore patterns and using patterns. Third and fourth class develop the knowledge on
number patterns, sequences, and number sentences. The final two classes of the primary school cycle, fifth and sixth class, work with directed numbers, rules and properties, variables and simple equations.

It is evident that a pattern based approach is heavily emphasised throughout the primary school curriculum. A continuation of this approach is now emphasised through the aims of the algebra strand within the Junior Certificate Project Maths curricula. These aims are outlined below:

- Introduce ratio and proportional reasoning as the backbone of the study of algebra,
- Emphasise relationship based algebra,
- Connect graphical and symbolic representations of algebraic concepts,
- Use real life problems as a vehicles to motivate the use of algebra thinking,
- Use appropriate graphing techniques (graphing calculators, computer software) throughout the strand activities.

(NCCA, 2012, p. 26)

We can see from these aims that students should build on their proficiency to move between equations, tables and graphs and become more adept at solving real world problems. For the purposes of this research, the function-based approach was chosen as the medium through which algebra would be introduced as it is most closely in line with the aims of the Project Maths syllabus. The next section will examine the function-based approach in light of the literature.

3.6 The Function-based approach to teaching algebra

This section will look at a rationale for choosing the function-based approach for the introduction of algebra in this research project. It will examine the ways in which the function-based approach can be used in a school context, the strengths and weaknesses of such an approach as well as resources required for teaching.

3.6.1 What is the function-based approach to teaching algebra?

Societies solved problems of algebra long before algebraic notation (Harper, 1987). The function-based approach to algebra envisages that students may be able to work with
variables and the rules of arithmetic and learn to use algebraic notation and techniques
themselves (Lester & Ferrin-Mundy, 2004) in the same way that it was discovered
throughout history. Traditional approaches to the teaching of introductory concepts in algebra
have focused on developing the concept of the variable by having students interpret algebraic
expressions and statements as generalisations of numerical ones (Haimes, 1996). These
traditional approaches enforce the use of rules and formulas from the beginning of secondary
school. A function-based approach, however, assumes the function to be a central concept
around which the school algebra curriculum can be organised. Schwartz (1999) proposes that
it be the fundamental object of algebra instruction (Schwartz & Yerushalmy, 1992a,b,c: see
also, Chazan 1993; Chazan & Yerushalmy, 2003; Yerushalmy & Chazan, 2002). Pegg &
Redden (1990) outline why this approach is so attractive - they suggest that it points to an
opportunity for students to see algebraic notation arising as a natural and useful consequence
of expressing generality. Brown & Coles (1999, p. 23) reinforce this notion when they talk
about helping students to develop a “need for algebra” so that instead of looking for contexts
that might be real for the students, one must expect students to develop the need for algebra
through asking and answering their own questions.

Functions can be thought of in a number of ways; tables of values, a graph of a relationship
and as a rule which is usually expressed in algebraic symbols. A function approach to algebra
focuses on these representations through developing understanding and developing the idea
of a variable (Sutherland, 2004). School textbooks often introduce functions in such a way
that procedures must be followed and pencils and graph paper must be used. It is hoped that a
new approach will see three levels of functional thinking come into classrooms:

1. Numerical level
2. Functional reasoning level
3. Dynamic Functional reasoning level.

These levels would see students move from a systematic search for a solution to a rich
understanding of the concept of function (Ferrara, Pratt, Bobutti 2006). Tabach and
Friedlander (cited in Kieran & Yerushalmy, 2004) use their own words to describe the
importance of these three levels for students: ample numerical tables, a need to use general
expressions to express data in these tables and the possibility to obtain a wide variety of
corresponding graphs.
This approach aims to deepen students’ understanding about the ‘inherent complexities of algebra’ (Yerushalmy, 2000) through offering more opportunities for students to ask questions. It may not be a simpler approach but it hopes to facilitate the teacher when promoting inquiry in algebra lessons (Yerushalmy, 2000). It hopes to achieve this through promoting more than just a single solution which, in turn, hopes to support a long problem solving process. The chapter will continue by illustrating how researchers have approached using the function-based approach for mathematics instruction.

3.6.2 Approaches to teaching through a function-based approach

Within this approach it is expected that at primary school level children will investigate patterns and sequences and make generalisations about them in everyday language (MacGregor & Stacey, 1992). For example in infant years students could be asked to complete the pattern of colours:

*Figure 3.2 A basic pattern for primary school students*

![Pattern](image)

(NCCA, 2008)

As students progress they may be asked to make more complex observations about patterns. For example a pattern of matches:

*Figure 3.3 A more complex pattern that uses matchsticks*

![Pattern](image)

(MacGregor & Stacey, 1992, p. 362)

This pattern could be described by children as “You start with four for the first square and then add on three for each extra square” (MacGregor & Stacey, 1992).

In post-primary school this new approach must reflect inquiry methods through which students take responsibility when dealing with new problems rather than rehearsing known procedures (Heid, 1996). The match stick problem above illustrates nicely the way in which
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this pattern can be used in a post-primary school setting. One student may see the pattern as “four matches for the first square and then three for each extra square” and should then, at this stage, be enabled to write the rule algebraically: \( N = 4 + 3 \times (s-1) \), where \( N \) is the number of matches required for \( s \) squares. Another student may have a different perspective and see the pattern as “starting with one match, add on three for each square”. They would then be expected to write \( N = 1+3s \), where \( N \) is the number of matches required for \( s \) squares. According to (MacGregor & Stacey, 1992), a comparison of these two correct expressions establishes some aspects of the distributive property.

3.6.3 Representing functions

According to Yerushalmy (2000), functions should be represented in different forms; words, numbers, graphs and symbols. This approach calls for students to express generalisations mathematically using algebraic symbolism i.e. to conceive letters as variables instead of unknowns, to interpret expressions as rules for functions and to use the Cartesian coordinate system as a space to display the results of calculations as well as consider a variety of meanings of the equals sign (Chazan & Yerushalmy, 2003).

As discussed functions should, according to Yerushalmy (2000) be represented in different forms; words, numbers, graphs and symbols. Another important representation, however is mentioned and illustrated by Ferrara, Pratt & Robutti (2006) and this is a tabular representation (see figure 3.4). A combination of these five representations may act as a good guide to teaching through this function-based approach. They can be applied to most problems and could act as a guide for students to help them in the problem solving phase.

*Figure 3.4 Representing functions using symbols, graphs and tables*

(Ferrara, Pratt & Robutti, 2006, p. 247)
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Lesh et al. (2003) suggests a model for these representations. As discussed in Chapter 2, section 2.3.4, the Lesh translation model (see figure 3.5) highlights the importance of students’ abilities to represent mathematical ideas in multiple ways including manipulatives, real life situations, pictures, verbal symbols and written symbols.

Figure 3.5 Lesh’s Translation Model

![Lesh’s Translation Model](image)

(Lesh et al., 2003)

Chazan (2003) examined a course aimed at introducing functions and proportionality to teachers (see figure 3.6). The activities within the course were developed using the Lesh translation model. The model suggests that teachers need to experience mathematical ideas through multiple representations in order to gain a deep understanding of them. The model recommends that teachers should be able to translate from one representation to another fluidly. These representations (see figure 3.5) include contexts, manipulatives, pictures, verbal symbols and written symbols.

Figure 3.6 Mathematics content for a functions and proportionality course

<table>
<thead>
<tr>
<th>Functions and Proportionality:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Looking for Patterns and Regularities in Number</td>
</tr>
</tbody>
</table>

Represent concrete and real-world examples of functions using tables, graphs, verbal rules and algebraic expressions: Linear, Quadratic and Exponential

- Look for patterns among different representations
- Make connections among different representations based on patterns observed.

Explore Linear Functions in detail:

- Multiple representations
- Connections among representations
- Slope and y-intercept
  - Multiple representations
  - Changes in context and rule affect graph

Proportionality as a Special Linear Function:

- Multiple representations; math characteristics
- Examples and non examples
- Mathematical contexts

(Cramer cited in Lesh and Doerr, 2003, p.457)
The first step in the process was the development of activities using concrete models such as geoboards, pattern blocks, folding paper and string. The next stage involved teachers collecting data based on these concrete materials and using their own language (with the help of leading questions) to describe the patterns and functional relationships. This encourages a translation from manipulative to symbolic to informal language. The teachers were then expected to translate this informal language into written symbols (function rule), followed by a graphical representation (pictorial mode). The course continued in this manner and after three lessons teachers interacted with linear, quadratic and cubic functions using concrete models, tables, graphs and written symbols.

Reflection on such a course highlighted the way in which this model can help teachers determine the types of activities to supplement a curriculum so they can effectively meet students’ instructional needs. A study carried out by Brenner et al. (1997) introduced a pre-algebra course in which students (11-13 year olds) learned about functions in a unit that emphasized (a) representing problems in multiple formats, (b) anchoring learning in a meaningful thematic context, and (c) problem-solving processes in cooperative groups. Results of this study indicate that students that took part in the unit were more successful in representing and solving a function word problem and were better at problem representation tasks such as translating word problems into tables and graphs than students in a control group. Also, Cramer & Bezuk (1991) suggests that this model can also be an effective tool for developing assessment. They suggest that assessment tasks can be constructed around translations within and between modes of representation. They believe that this leads to teachers assessing understanding beyond procedural skill.

To summarise, Yerushalmy and Chazan (2003) has outlined the benefits of using the Lesh Translation Model for curriculum development, for classroom curriculum decisions and for assessment.

3.6.4 Progressing through the Function-based Approach to teaching Algebra

Haimes (1996) suggests that these types of patterns and number sequences be further developed into the following progressive stages: Linear, quadratic, exponential, reciprocal and periodic relationships. Yerushalmy and Chazan’s (2003) investigation of a course that
introduced mathematics content for a functions and proportionality course teachers (see figure 3.6) reciprocates Haime’s suggested progressive stages.

A function-based approach links tables, graphs and algebraic expressions and equations and supports a goal towards understanding. This type of approach allows students to begin to explore these tasks before even being taught an algorithm. For example, students may be given a linear expression in the form $ax + b$ and asked to factorise it. Through using the function-based approach, students can use their ability to approximate the $x$ intercept of a graph to make a first attempt at a solution to the task. As seen in Yerushalmy (1991) and Yerushalmy and Gafni (1992) students may use software to create graphs which will compare two expressions. Chazan also suggests that elements from the function-based approach which asks students to use graphical and tabular feedback to correct mistakes in factorisation can help explain the nature of the mistake to the student. It helps students to identify patterns in expressions. Also, through guiding students to build functions out of functions (i.e. linear functions multiplied together to form a quadratic), students become better equipped to tackle a variety of problems such as the relationship between areas of rectangular figures and lengths of their sides (Chazan, 1999). Students may also come to notice that there are quadratic functions that cannot be created by the multiplication of two linear functions with rational coefficients. This in turn leads them to the understanding that solving a quadratic equation is equivalent to finding the $x$ intercept of the graph of the related quadratic function and that if the $x$ intercepts do not exist, then there are no solutions over the real numbers (Chazan, 1999). An example of a class that encouraged this function-based approach can be seen in figure 3.7 below. It highlights the way in which the function-based approach can help students to discover meaning in the mathematics that they engage in and explore.

**Figure 3.7 Example of a function-based approach in a class**

For example, one year, I had a student who had an interesting (and wrong) conjecture about the relationship between the vertex and $y$ intercepts of a parabola. He claimed that the distance between the $x$ intercepts of a parabola and between the vertex and the $x$-axis had to be the same.
In exploring his conjecture, we came to realize that to find the x intercepts of a parabola one moves from the vertex \((h, k)\) the square root of the quantity \(k/a\) to the right and left (equivalent of solving an equation in the form of trinomial square minus a constant is equal to zero).

In other words, we had written a new quadratic formula. For functions in the form, \(a(x-h)^2 + k\), to find their x-intercepts, evaluate:

\[ h \pm \sqrt{\frac{k}{a}}. \]

This way of thinking about the quadratic formula suggests to me that the formula describes how the x intercepts of a parabola depend on the coefficients of the vertex form of the function’s expression. The x intercept depends on the location of the vertex and the steepness of the opening of the parabola. In terms of the standard quadratic formula, the remaining step is to connect the \(ax^2+bx+c\) form of the function’s expression with its vertex form.

Thus, instead of thinking of the quadratic formula as a “formula” from the theory of equations for determining the two numbers which “solve” a quadratic equation, the quadratic formula became a function on the coefficients of the quadratic function in the standard form. This function on quadratic functions returns the values of the x intercepts of the input quadratic function. With the graphical representation of functions in mind, we can see this “function” as making connections between the vertex point of a quadratic function and its x intercepts. The outputs of this “function” can be used to rewrite the original function in a factored form which highlights its x intercepts.

(Chazan, 1999, p. 138)
3.6.5 Strengths and Weaknesses associated with the Function-based Approach to Teaching Algebra

MacGregor & Stacey (1992) investigated the effects of introducing a function-based approach to algebra to a group of students. The items below (see figure 3.8) were given to 512 students in 26 classes from yr 7 to yr 10 (11-15 year olds) across seven schools in Melbourne, Australia. Students were given sufficient time to complete the questions.

**Figure 3.8 Function-based questions given to 512 students**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>(i) When x is 2, what is y?</th>
<th>(ii) When x is 6, what is y?</th>
<th>(iii) When x is 20, what is y?</th>
<th>(iv) Use algebra symbols to write the rule connecting x and y.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>..</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>7</td>
<td>..</td>
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<tr>
<td>8</td>
<td>..</td>
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<tr>
<td>..</td>
<td>..</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>First number</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second number</td>
<td>-1</td>
<td>9</td>
<td>19</td>
<td>29</td>
<td>39</td>
<td>..</td>
</tr>
</tbody>
</table>

(i) Work out the missing number.
(ii) Explain how you worked it out.
(iii) Use algebra symbols to write a rule for working out the second number. In your rule, use $F$ to represent the first number and $S$ to represent the second number.

**Figure 1:** Test items

(MacGregor & Stacey, 1992, p.364)

Results from item A revealed that the form of many responses lacked an understanding of the meaning and use of algebraic notation. Examples of results from item B (see figure 3.9), suggest that students have a lot of difficulty constructing formulas from tables. Arzarello, (1991) suggests that students’ way of thinking are locked in to arithmetic concepts which caused them to search for ways to predict the next number in a table from the value of its predecessor. Exposure to these types of questions may help students develop the ability to focus on the relationship between successive values of the dependent and independent
variable rather than on the differences between successive values of the dependent variable (MacGregor & Stacey, 1992).

**Figure 3.9 Sample responses to questions in figure 3.8**

<table>
<thead>
<tr>
<th>Category</th>
<th>Explanation</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pattern involving 9</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A pattern of 9 being the last number. [Yr 8]</td>
<td>S+F</td>
<td></td>
</tr>
<tr>
<td>Every number that has 9 in it. [Yr 9]</td>
<td>F+10S</td>
<td></td>
</tr>
<tr>
<td>Every 10th number has 9 added on. [Yr 10]</td>
<td>S=10+9F</td>
<td></td>
</tr>
<tr>
<td><strong>Add ten</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The second the number is counting by ten. [Yr 7]</td>
<td>Fx10-1=S</td>
<td></td>
</tr>
<tr>
<td>Bottom line is going up by tens. [Yr 8]</td>
<td>F+1=S+10</td>
<td></td>
</tr>
<tr>
<td><strong>Describe two sequences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I added 10 to each number by the increasing 1 number. [Yr 7]</td>
<td>F+10=S</td>
<td></td>
</tr>
<tr>
<td>First number progressed by 1, second number progressed by 10. [Yr 8]</td>
<td>F+1=F10</td>
<td></td>
</tr>
<tr>
<td>When the first number increases by 1, the second number increases by 10. [Yr 9]</td>
<td>no attempt</td>
<td></td>
</tr>
<tr>
<td>Each first number is added on ten in the second. [Yr 10]</td>
<td>no attempt</td>
<td></td>
</tr>
<tr>
<td><strong>Relate two variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtract 1 from the first number multiplied by ten. [Yr 7]</td>
<td>Fx10-1=S</td>
<td></td>
</tr>
<tr>
<td>10 times the top column and take 1. [Yr 8]</td>
<td>S=10F-1</td>
<td></td>
</tr>
<tr>
<td>Multiply the first by 10, subtract 1. [Yr 9]</td>
<td>Fx10-1=S</td>
<td></td>
</tr>
<tr>
<td>Subtracting one from the first number where it is multiplied by 10. [Yr 10]</td>
<td>no attempt</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 3:** Students’ explanations and rules for finding the missing number in item B

(MacGregor & Stacey, 1992, p. 368)

Yerushalmy (2000) attempted to analyse students’ constructions of a function-based problem solving method in introductory algebra. One of the questions (illustrated in figure 3.10) is used to see how students would implement their knowledge of the function concept and software after a year of familiarity.
Figure 3.10 Problem solving within a function-based approach—an example

The Parking problem story

Two parking lots are located one next to the other. In one lot one pays 2.7 IS for an hour and a proportional price for fractions of the hour. The second lot charges 2 NIS an hour and an additional 6 NIS charge for entrance. Half an hour parking costs 1 NIS. Describe the two parking conditions in a way that would allow the customer to choose the best offer.

Problem equation: \(2.7x = 6 + 2x\)

(Yerushalmy, 2000, p.131)

The solutions (see figure 3.11) produced by the lesser able students show that they used the tabular approach as well as the graphical approach to attempt to solve this problem.

Figure 3.11 Sample solutions to the problem in figure 3.10

(Yerushalmy, 2000, p. 132)

High achievers on the other hand used symbolic representation through formulating a symbolic comparison model and immediately asking to use the software (Yerushalmy, 2000).
In conclusion, the vision of how algebra is learned has been widened over the decades as it has moved from a letter symbolic symbol-manipulation view to one that encompasses multiple representations, use of technology and problems solving in real life contexts (Kieran, 2007).

3.7 Resources for Algebra Education

As a result of the wide availability of the internet in schools and at home and due to the innovations in technology, an approach for teaching and learning mathematics using manipulatives and computers is emerging. These will be discussed in the following section.

3.7.1 The use of Manipulatives for the Teaching and Learning of Algebra

With considerable research supporting the use of manipulative materials (Chapter 2) and widely available teacher professional development workshops focusing on their use, mathematics manipulatives are becoming more common in Irish classrooms. A number of studies have investigated the use of manipulatives for the teaching and learning of algebra. Algebra tiles are manipulatives which allow abstract variables and constants to be given a physical representation. They consist of small squares, large squares and rectangles which represent different variables or constants. Rather than learning via their logical/mathematical intelligence, these tiles allow students to learn via their visual and tactile intelligence (McConway, 2006 cited in Prendergast, 2011). Developing algebraic concepts from a geometric perspective where non abstract thinkers can see sequences develop from a concrete level, through a pictorial level and onto an abstract level is believed to be of benefit to students (Leitze and Kitt, 2000). McConway (2006) identifies them as an invaluable source for weaker students.

McClung (1998) carried out a study on the use of manipulatives and their effect on student achievement in a high school algebra 1 class. The results were analysed using a two-sample t-test and at the .05 level of significance and it was discovered that there was a significant difference between a group that used manipulatives and a control group. When the mean scores were compared, it was discovered that the mean score of Group A (control group) was higher than that of Group B (manipulatives group) which would indicate that the students taught using the traditional method of lecture, homework, and in-class worksheets
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outperformed the students taught using the manipulatives. The author looked to Piaget for help in explaining the results. Piaget describes the concrete operational stage as the basis for use of manipulatives. This stage begins at about seven and carried on until about 12. The characteristics of this stage are “logic of classes and relations, understanding of numbers and thinking bound to the concrete” (Philips & Soltis, 1995, p.43). The students in this study were between the 15-17 years of age and so the author believed that they had moved into a formal operational stage (12-adulthood). The formal operational stage refers to students having complete generality of thought and an ability to deal with the hypothetical.

Another study was carried out by Leinebach and Raymond (1996) in an urban setting with 120 high-school students. She engaged in a two year collaborative action research study on the effects of a ‘hands-on’ approach to learning algebra. She taught algebra in three sections, first using the textbook, then letting students engage with manipulatives and finally just using the textbook again. She found that individual scores were higher during the manipulative phase than the original textbook phase but that they weakened in the third phase. She cited three reasons for why scores may have weakened. Firstly, she considered whether students were able to work out the algebra without the manipulatives in the third phase having got used to them in the second phase. Secondly, she wondered whether students were unable to retain what they had learnt in the manipulatives phase and lastly she reflected that the third phase may have involved more difficult material than the previous two phases.

A plethora of research supports the use of manipulatives for mathematics. Hinzman, (1997) found that students are significantly more positive when learning with manipulatives. It is, however, important to bear in mind that there is evidence that not all research indicates a positive outcome in relation to the use of manipulatives in algebra classes.

3.7.2 The use of ICT in the Teaching and Learning of Algebra

As mentioned, the transformational based approach to teaching algebra which has dominated Irish classrooms was common in the U.K. and various other countries up until about the mid 1960’s (Kieran, 2004). Research discovered that when solving problems, students emerging from arithmetic found it difficult to represent the situation (Kieran, 2004). These difficulties led to a belief that more time needed to be spent creating meaning and sense for the objects that were being manipulated. Research that turned to teaching experiments to try out new approaches related to generational activities led the way for computing technology in the
learning and teaching of algebra (Kieran, 2004). Transformational activities were all but forgotten and it became accepted that if students could solve algebra type problems with spread sheets and other such tools, there appeared to be little need to learn algebraic transformations (Prendergast, 2011). This search for meaning led to a situation in the early 1990s where students were not engaged in enough symbol manipulation (Sutherland, 1997). Kieran (2004) points out that problem solving by whatever means had caused this lack of symbol manipulation taking place and had all but replaced algebra. In the 1990s, it was hoped that if students understood how to solve a problem, algebraic techniques would come naturally (Prendergast, 2011). A study carried out by Artique in France in the mid 1990’s on the use of DERIVE\(^1\) found that the techniques did not occur naturally (Kieran, 2004). Researchers found that the teachers were placing a much greater emphasise on the conceptual elements of algebra and neglecting the role of the procedural work in algebra learning (Prendergast, 2011). This emphasis on conceptual work was producing neither a definite enhancement of students’ conceptual understanding nor a clear understanding of the procedural aspects, “easier calculation did not automatically enhance students reflections and understanding” (Lagrange, 2003 as cited in Kieran, 2004, p. 28). A more structured approach was needed for the use of technology in the teaching and learning of algebra.

The use of multiple representations and the ability to translate among representational models has been shown to be an important factor in students’ abilities to model and understand mathematical constructs (Cifarelli, 1998; Fennell & Rowan, 2001; Goldin & Shteingold, 2001; Kamii, Kirkland, & Lewis, 2001; Lamon, 2001; Perry & Atkins, 2002). In a summary of over 100 research studies, Marzano (1998; see also Marzano, Gaddy, & Dean, 2000) found that one instructional technique that demonstrated a consistent positive impact on student achievement was the use of graphic/non-linguistic formats to explore and practice new knowledge (Su et al., 2005, cited in Marzano, 1998).

### 3.7.3 Technology and the Function-based Approach to Teaching Algebra

According to Heid (1996), ‘functions’ must play an important role in school mathematics in order to envisage algebra in a technological world. As the vision of school algebra has widened considerably over the decades - moving from a letter symbolic and symbol manipulation view to one that ‘encompasses multiple representations, realistic problem settings and the use of technological tools - so too has the vision of how algebra is learned’

---

1 DERIVE is a computer algebra system
In order for our students to be ready for a technological world it is envisaged that technology will play an important role in promoting effective teaching of new and traditional topics i.e. functions (Fey and Good, 1985). Moyer (2002) suggests that technological tools be known as virtual manipulatives and that they can be broken into two categories: static and dynamic visual representations of concrete material (Spicer and Stratford, 2001). Static representations are essentially pictures and are not what Moyer calls true manipulatives. Dynamic visual representations are, however, like pictures but can be manipulated as if it were a three dimensional object. Students have access to many graphing tools, spread-sheets of ever increasing sophistication and interactive websites. The software programs, ‘Geogebra’ (free) and ‘Microsoft Excel’ are currently being promoted by the Project Maths team for use in Irish classrooms. Both Geogebra and Excel allow the student to use static and dynamic manipulatives. Dorward and Heal (1999) found that virtual manipulatives foster as much engagement as physical manipulatives.

Technology can be used in a variety of ways for introducing the function-based approach. It can be used to develop tables, graphs and also to build patterns. An example of the use of technology for the development of tables and graphs can be seen in figure 3.12. It demonstrates a problem that is presented to students that they must solve.

**Figure 3.12 Use of technology in an algebra class**

The Bonus problem story

A factory owner wishes to give a special bonus for the holidays. His account suggests two methods for the special payment. Method A: The January payment will include additional 450 IS per person. Method B: In the January payment each employee will receive an additional amount equal to 1/5 of his regular monthly salary. Find a way to present the cost of both methods in order to help the owner to choose the better bonus strategy.

The problem equation: 

\[ x + 450 = x + \frac{x}{5} \]

Background

9th grade

Current work in class consists of linear and quadratic motion problems of various types; usually more complicated than the interview problem. Most word problems solved in class cannot be solved by simply reading the solution from the graph and therefore graphs are mostly used as sketches of situated models. Graphing software is being used less frequently, and more attention is given to equivalency of expressions and equations throughout symbolic manipulations.

(Yerushalmy, 2000, p. 136)
The solutions which can be seen in figure 3.13 demonstrate that there is a variety of ways that students approach such a problem. Students sketched graphs using pencil and paper, used symbolic representation or used graphing software to solve this problem.

Figure 3.13 Sample solutions to the problem in figure 3.12

An example of the use of technology for building shape patterns can be seen in figure 3.14.

Figure 3.14 Growing shapes pattern created using virtual pattern blocks

(Yerushalmy, 2000, p. 137)

(Moyer, 2005, p. 441)
These examples show the powerful nature of technology in the teaching and learning of algebra. The NCTM (2000) also acknowledge that by utilising the graphing and symbol manipulation of modern day computers, students are enabled to think differently, not just faster.

3.8 Conclusion

This chapter has provided an overview of some of the educational research regarding algebra. Algebra has been a pivotal component in mathematics for many thousands of years. It developed from a rhetorical stage, when all statements and arguments are made in words and sentences through a syncopated stage, where abbreviations are used when dealing with algebraic expressions and is now in a symbolic stage where there is total symbolisation.

Problems associated with the teaching and learning of algebra have been highlighted. These include student’s difficulty operating on unknowns, student’s belief that the equals sign is unidirectional (Booth, L., 1984; Kieran, 1981) and student’s inability to recognise the commutative and distributive properties (Boulton-Lewis, 2001; MacGregor, 1996). According to Herscovics and Linchevski, (1994), Grouws (1992) and Lyons et al. (2003) the pace of lessons, teacher dominated classrooms and the inherent difficulties students possess in relation to algebra can contribute to these problems.

Approaches to the teaching and learning of algebra have developed in response to these issues. Four approaches; the generalisation approach, the problem solving approach, the modelling approach and the function-based approach are outlined by Bednarz, Kieran and Lee (1996). Kieran (2002) then developed a model for conceptualising algebraic activity. This model identifies three important components; generational activities, transformational activities and global/Meta activities. The objective of this model is to find a balance between these algebraic activities.

This chapter continued by outlining the Project Maths approach to teaching algebra. It highlighted that there is an emphasis on patterns and functions within the new Project Maths syllabus for Junior Certificate. An overview of the function-based approach to teaching was then given. This section discussed the importance of using a function-based approach for aiding students in making connections among topics. It includes a rationale for using this approach in this research project and gave a synopsis of methods for using this approach in the classroom. Resources were seen as an important aspect of introducing algebra using the
function-based approach. The positive influence of the use of manipulatives and ICT were discussed in light of this approach.

In conclusion, there are many approaches to teaching algebra and many problems associated with its instruction. This research project aims to investigate the effects of one of these approaches - the function-based approach - on students’ attitudes towards mathematics and understanding of basic algebra. A detailed description of the intervention booklet that was used to introduce algebra using the function-based approach is outlined in chapter 5.
Chapter 4: Methodology

4.1 Introduction

The purpose of this chapter is to discuss and design a suitable methodology for the collection and analysis of data that will address the research questions outlined in the introductory chapter. Careful consideration must be given towards the decision of which research tools to employ and ultimately which methodology is deemed appropriate for this research. The author was faced with many possible choices. Poulson & Wallace (2003) point out that it is important to justify your choices of methodology in this type of research and so this chapter aims to discuss a methodological approach and give a lengthy justification for the decisions made.

4.2 Research Aims and Objectives

The aim of this research is to investigate the effects of introducing algebra using a function-based approach. The benefits of introducing algebra in this way were discussed in detail in Chapter 3.

The principle objectives of this research project are:

- To investigate by means of a literature review both international and Irish research on students’ attitudes towards mathematics.
- To investigate by means of a literature review both international and Irish research on the ways in which algebra is taught in secondary schools.
- To implement an intervention that will see the introduction of algebra through a function-based approach in a collaborative peer environment.
- To gather quantitative data, through the use of a questionnaire on students’ attitudes toward mathematics.
- To gather quantitative data to ascertain student’s levels of understanding in basic algebra.
- To gather qualitative data in the form of focus groups as a means of triangulating analysis within the research.
4.3 Research Questions

The following research questions were derived and helped guide each phase of the research.

1. What are the issues contributing to and theoretical perspectives underlying effective mathematics teaching which can stimulate and improve students’ attitude towards mathematics at Junior Cycle level?

2. What are the issues contributing to and theoretical perspectives underlying effective mathematics teaching which can improve students’ understanding of basic algebra at Junior Cycle level?

3. Does the introduction of algebra using a function-based approach improve first year students’ understanding of basic algebra?

4. Does the introduction of algebra using a collaborative peer teaching approach improve first and transition year students’ attitudes towards mathematics?

5. Does the introduction of algebra using a function-based approach improve first year students’ attitudes towards mathematics and in turn improve understanding?

6. Does the use of manipulatives improve students’ attitude towards mathematics and aid students in their understanding of basic algebra?

4.4 Theoretical Perspectives

A review of literature highlighted key issues pertaining to this research project. These theoretical considerations and the methodological approaches underpinning the author’s research are discussed briefly in this section. They have been expanded extensively in Chapter 2 and 3. DeBellis and Goldin (1997) highlighted four facets of the affect domain: beliefs, attitude, emotions, values/moral/ethics. The author chose to focus on one of these facets; attitude because it was of most interest to her. Fennema-Sherman (1976) provided the instrument for the measurement of four subscales of attitude; confidence, usefulness, effective motivation and anxiety. Boaler (1997a, b, 1998) and Ridlon (1999) suggest that collaborative approaches can promote positive attitudes among students (e.g. Boaler, 1997a, b, 1998; Ridlon, 1999) and so the author chose this as the dominant teaching and learning
approach for this project. Rubin and Hebert (1998) provided the concept for collaborative peer teaching that would be used in this project. Svinicki (1991), Forsyth and McMillan (1991) and Billson and Tibertus (1991) support Rubin and Hebert’s (1998) collaborative peer teaching approach by providing a cognitive approach, motivational theory and social context for the author to work from.

The author was also concerned with improving students’ understanding of basic algebra. Bednarz, Kieran and Lee (1996) developed a variety of approaches to teaching algebra. The function-based approach was one of these and it views functions as fundamental mathematical objects (Sutherland, 2004) around which a curriculum can be built. Research by Yerushalmy (2000), Ferrara, Pratt and Robutti (2006) and Haimes (1996) informed the author of approaches that could be incorporated into the student handbook to help students’ develop their understanding of basic algebra. Lesh & Zawojeski (2007) provide a way in which problem solving can be incorporated within the function-based approach. Lesh’s (2003) Translation Model also informed the author during the development of the student handbook. This model helps students to move fluidly between representations (see Chapter 5). The following diagram (figure 4.1) illustrates clearly the overall theoretical framework behind this research study:

**Figure 4.1 Theoretical framework**
There are a number of theoretical approaches that underpin this study and there is a need for a range of methodological instruments to truly interpret students’ attitudes and understanding. It is the intention of the researcher to deepen the understanding of the phenomenon in question through the systematic process of collecting information and analysing data. This will be discussed in further detail throughout this chapter.

4.5 Research Methodologies

The author recognised that a standard research design and a valid research methodology had to be chosen. Research methodologies are standardised models which provide the researcher with an outline of how they should approach their study and take many differing forms (Mertens, 1998). This study used an educational intervention within an action research model.

4.5.1 Action Research

According to Elliott (1991) action research emerged as an aspect of the school based curriculum reforms in innovatory schools during the 1960s and is becoming an increasingly recognised field of educational research (Cardelle-Elawar, 1993; Clift, Veal, Johnson and Holland, 1988; Miller and Pine 1990 cited in Raymond 2000). There are many definitions of action research. Miller and Pine (1990) define it as an on-going process of systematic study in which teachers examine their own teaching and students’ learning through “descriptive reporting, purposeful conversation, collegial sharing and critical reflection for the purpose of improving classroom practice” (p. 57). Elliott (1991) suggests that action research “integrates teaching and teaching development, curriculum development and evaluation, research and philosophical reflection, into a unified conception of a reflective education practice” (p. 54).

The fundamental aim of action research, according to Elliott (1991), can be thought of as a means to improve practice rather than to produce knowledge. It is guided by a collection of interrelated ideas about the nature of education, knowledge, learning, curriculum and teaching. It provides a medium for teachers to find practical solutions to problems they have identified in their classrooms (Raymond, 2000).
What are the goals of action research?

Raymond (2000) identified six goals of collaborative action research which can be seen in table 4.1. The first goal recognises that collaborative research helps to bridge the much talked about gap between researcher and teacher. It provides an opportunity for those with common interests from two different settings to come together and share ideas and achieve common goals.

Table 4.1 Goals of Collaborative Active Research

<table>
<thead>
<tr>
<th>Goals of Collaborative Action Research</th>
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</thead>
<tbody>
<tr>
<td>To bridge the gap between universities and schools</td>
</tr>
<tr>
<td>To provide opportunities for teacher enhancement</td>
</tr>
<tr>
<td>To stimulate classroom reform</td>
</tr>
<tr>
<td>To enhance the professional status of the teacher</td>
</tr>
<tr>
<td>To improve teaching and learning</td>
</tr>
<tr>
<td>To generate theory and knowledge</td>
</tr>
</tbody>
</table>

(Raymond, 2000, p. 285)

Through action research, teachers have the opportunity to enhance their teaching and improve classroom practices, as mentioned in the second goal and fifth goals respectfully. Teachers must become more reflective about the ways in which they are teaching and are in turn more likely to make changes that lead to improvements in their classroom. Elliott (1991) suggests that there are two approaches through which teachers can develop their practices. These are:

1. **Reflection initiates action.** The teacher undertakes research and on this basis changes some aspect of his or her teaching. The development of understanding precedes the decision to change teaching strategies.

2. **Action initiates reflection.** The teacher changes some aspect of teaching in response to a practical problem. This change is monitored as to its effectiveness in resolving the problem. Through the evaluation the teacher’s initial understanding of the problem is modified and changed. The decision to adopt a change strategy therefore precedes the development of understanding.
The type of reflective pedagogy is an important step in the development of curriculum as it helps to validate both the pedagogical processes, their contexts and content and can therefore stimulate classroom reform (Elliot, 1991).

Another goal cited by Raymond (1996) is the professional development of the teacher. Simons (1978) points out that teachers involved in ‘insider research/evaluation’ activities see themselves first and foremost as classroom teachers but Miller and Pine (1990) reinforce the notion that these teachers are setting an example for excellence in ‘reflective practice’ and are enhancing the professional status of teaching.

**Approaches to action research**

Action Research can be broken into its various parts as follows:

1. Identifying and clarifying the general idea;
2. Reconnaissance;
3. Constructing the general plan;
4. Techniques and methods for gathering evidence.

Action research projects are cyclical or spiral in nature (Lewin, 1948; Elliot 1991; Riding, Fowell and Levy, 1995). A problem is identified, research in the relevant literature is carried out and some form of intervention is decided upon. This intervention or action is then trialled and evaluated. From this evaluation, problems with the intervention are identified, carried out and the intervention is refined. The process continues in a cyclical or spiral pattern, slowly circling upwards until it reaches a point where the researcher is content that he has produced an action that has resolved, satisfactorily, the problem initially identified (Lewin, 1948; Elliot 1991; Riding, *et al.* 1995). This action is then implemented and conclusions are drawn which inform the write up of the thesis.

A more detailed layout of this approach can be seen in figure 4.2.
4.5.2 Educational Interventions

A teaching intervention was designed and implemented as part of this study to allow the author to investigate students’ attitudes towards mathematics and understanding of basic algebra, while remaining consistent with the theoretical framework employed in this research study. This section examines the definition of the term ‘intervention’ from a variety of perspectives and backgrounds, in particular educational interventions. This enables the author to identify the general approaches to evaluating an educational intervention and consequently facilitate the evaluation strategy employed.
Defining Interventions

In order to gain an understanding of the term ‘intervention’, the author sought a variety of definitions. According to the Oxford Concise Dictionary (2012), an intervention is ‘the action or process of intervening’. Teaching expertise (2010) define it as:

“Any change or programme of change which is implemented in the classroom, it could be formal or informal and might be initiated by the individual teacher, the school or another organisation from outside.”


Essentially, an intervention means imposing a change in an already on-going relationship with the primary goal of improving it. Interventions are widely used in educational and healthcare issues, where an intervention is aimed at a positive outcome rather than a perceived negative one that would ordinarily occur (Wilkes & Bligh, 1999). In order for a positive outcome to occur a decisive action needs to take place. This research study involves the utilisation of social constructivism as a vehicle for improving attitude towards mathematics and the introduction of the function-based approach as a means of improving understanding of basic algebra.

Evaluation of Interventions

Wilkes and Bligh (1999, p.1269) acknowledge educational evaluations as the: ‘systematic appraisal of the quality of teaching and learning’. They attempt to improve education from two perspectives: formative evaluation and summative evaluation. Formative evaluation identifies areas where teaching can be improved, while summative evaluation judges the effectiveness of the teaching (Wilkes & Bligh, 1999).

Shapiro (1987) outlines four key parameters by which intervention research can be evaluated. These are:

- **Treatment effectiveness**: The effectiveness of an intervention is typically a quantitative measure of the change observed as a result of the strategy.
• **Treatment integrity:** Treatment integrity refers to the extent to which specified scheme is actually the manner prescribed in the intervention documentation.

• **Social Validity:** “Social validity refers to the evaluation of the intervention by the clients or consumers” (Shapiro, 1987, p. 293).

• **Treatment acceptability:** “Judgements by laypersons, clients and others on whether treatment procedures are appropriate, fair, reasonable for the problem or client” (Kazdin, 1981, p. 494).

Furthermore Edwards (1999) proposes a four stage cyclical process when evaluating an educational intervention (see figure 4.3).

*Figure 4.3 Evaluation Cycle*

> Consideration of these evaluation techniques were acknowledged by the author and incorporated into the research design which is outlined below.
4.5.3 Research Design

Action research was the primary research methodology used in this project. An intervention was used within this action research in order to fulfill the project’s objectives successfully. This intervention involved the development of an intervention booklet that introduced algebra using a function-based approach. This intervention booklet was then taught by transition year tutors to first year students over twelve classes.

This action research project underwent a number of cycles in order for it to be completed. It adopted the revised model of action research outlined by Zuber-Skerritt and Perry (2002) (see figure 4.2). The student handbook went through the action research cycle twice before the author was satisfied with the student handbook. It is important to note that the literature review continued through these cycles. Each time the author planned or revised the plan, the literature was examined and the review in chapters 2 and 3 was expanded upon. The methods of data collection are described in more detail later in this chapter, while the data collected is presented in chapter 6. An outline of the project design can be seen in figure 4.4 and explanations of each phase are given below.
Chapter 4  
Methodology

Figure 4.4 Research design

1. Identify the problem
2. Plan the theoretical framework/inter-
vention
3. Seek critical advice
4. Reflect and amend

Phase 1: Thesis Research

5. Plan methodology
6. Action Research/Phase2
7. Evaluation/Conclusion from field work

Phase 2: Action

1. Plan theoretical framework/inter-
vention
2. Observe
3. Reflect
3. Reflect and amend

Phase 3: Thesis Writing

1. Planning of final draft
2. Writing of final draft
3. Evaluating, seeking comments, revising and proof reading
4. Reflections and conclusions on research

Future Research

1. Introduce student handbook to 4th years
2. Amend student handbook
3. Implementation of intervention with first year students
Phase 1: Thesis Research

This phase incorporated a comprehensive review of the literature. An investigation into general issues in mathematics education, as well as a more detailed analysis of the teaching of algebra was carried out. Issues associated with attitude towards mathematics and the uses of collaborative peer teaching were investigated. This work provided a basis for the design of the theoretical framework employed. It also provided the author with the knowledge to design and implement an intervention (Chapter 4) as well as an outline of the methodological approaches used. By using multiple sources of data in Phase 1, validity is ensured.

Phase 2: Action/Fieldwork

The ‘action’ phase followed. The first cycle of the action phase involved developing the intervention booklet, getting critical feedback and amending the handbook. The second cycle saw elements of the intervention take place in a transition year class. Ethical approval was sought from the headmistress of the school and all potential participants. Transition year students were first introduced to algebra using the function-based approach. They completed a pre diagnostic test and Fennema-Sherman questionnaire. Teaching the transition years prior to the first year had a dual purpose. It taught students the material that they in turn were going to be teaching to first year students. It also informed the researcher of any changes, additions or omissions that needed to be made to the student handbook.

The third part of this phase involved tutors being selected from the transition year class (see section 5.3) to act as peer tutors within first year classes. A collaborative peer learning environment was established for all classes in which algebra was introduced using the function-based approach. Within this phase, all first year students completed a pre and post diagnostic tests and Fennema-Sherman questionnaire. Transition year students completed a post diagnostic test and questionnaire after tutoring first year students. Groups of students were then interviewed in the form of semi-structured focus groups on completion of the intervention (see section 4.7.5). Data was gathered and evaluations and conclusions were drawn from the action phase to inform the final phase, thesis writing.
Phase 3: Thesis Writing

The final phase centred on the write up of the thesis. It witnessed both quantitative and qualitative analysis (see section 4.7, description of methods employed). The combination of methods used provided the author with a deep insight into the impact that the intervention had on participant attitudes towards mathematics and understanding of basic algebra. Accordingly, conclusions and recommendations from the investigation are suggested and reported in Chapter 6 - Discussion of Key Findings. The final write up of the thesis then took place followed by evaluations, comments and proof reading.

4.5.4 Limitations of Action Research

Action research, though it struggles to produce quantitative data that can be generalised for the whole population, can indisputably produce perfectly valid qualitative data. There are many significant benefits to action research but it does have limitations. Action research as a methodology does lack scientific rigour. The very characteristics which make it unique - it is situational and specific - ensure that there is little or no control over the sample and other independent variables. This makes findings hard to generalise, though as action research projects become extensive in a specific area this becomes less of an issue (Cohen and Manion, 1994).

These difficulties have led to repeated discussions on how to assess the validity and quality of action research (Hodgkinson, 1957; Foster 1999; Hammersly, 2004). McMahon and Jefford (2009) believe that by following the three criteria, which they refer to as “Elliott’s context-related criteria” (Elliott, 2007), action research can be judged on its quality and validity. The question of validity and reliability will be discussed in more detail later in this chapter.

4.5.5 Evaluating Action Research

Cohen and Manion (1994, p. 186) describe action research as “a small scale intervention in the functioning of the real world and a close examination of the effects of such intervention”. Punch (2009) clarifies this view when he talks about how action research brings together the action (the doing) and the researching (inquiry). Evaluation is an essential part of action research and according to the literature must be carried out through critical trialling or through the need for triangulation (Stenhouse, 1975; Elliott 1991; McKernan, 1996). Essentially triangulation demands mixed-method procedures; evaluation from multiple
perspectives. These multiple perspectives serve to reinforce the research’s validity and reliability (Jick, 1979). In this study this was achieved through the use of a modified Fennema-Sherman attitude scale to examine the students’ perspective as well as a diagnostic test which produced quantitative data. Focus groups provided qualitative data on the intervention ensuring this mixed method approach was achieved.

### 4.5.6 Chronology of Research

As the author is a full time teacher of mathematics, this research took place over a two year period and was completed on a part-time basis. Below, an outline of the chronology of research is displayed in figure 4.5.

**Figure 4.5 Chronology of Research**

<table>
<thead>
<tr>
<th>September 2010 - June 2011</th>
<th>Phase 1: Review of Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 2011 - September 2011</td>
<td>Phase 1; Plan theoretical framework</td>
</tr>
<tr>
<td>October 2011</td>
<td>Phase 2; Cycle 2 Ethical approval sought</td>
</tr>
<tr>
<td>October/November 2011</td>
<td>Phase 2, cycle 2 continued; Redesign of Intervention</td>
</tr>
<tr>
<td>November/December 2011</td>
<td>Phase 2, cycle 3</td>
</tr>
<tr>
<td>January 2012</td>
<td>First and Transition year participate in focus group interviews.</td>
</tr>
<tr>
<td>February - June 2012</td>
<td>Analysis of data</td>
</tr>
<tr>
<td>June - September 2012</td>
<td>Conclusions and Recommendations</td>
</tr>
</tbody>
</table>
4.6 Research Paradigms

Denzin & Lincoln (2000, p.33) refer to paradigms as a framework for interpretation led by “a set of beliefs and feelings about the world and how it should be understood and studied”. Five research paradigms are identified by Lincoln & Guba (2000) but two of these are used regularly in educational research; positivism and interpretivism.

4.6.1 Quantitative Research

Flick (2011) describes these paradigms as umbrella terms for a number of approaches, methods and theoretical backgrounds on each side. The positivist paradigm, otherwise known as the quantitative paradigm is interested in causalities and data collection is designed in a standardized way (Flick, 2011). Quantitative research requires observations and measurements to be made in an objective manner so that they can be repeated by other researchers (Tuli, 2011). Investigators that adopt this quantitative approach to the social world must treat the world of natural phenomena as being hard, real and external to the individual and they will choose from highly standardised instruments such as surveys and questionnaires (Cohen et al, 2007). This methodology has dominated social science for much of the 20th century but over the past decades, researchers have expressed dissatisfaction with this method as a means of conducting research and generating knowledge.

4.6.2 Qualitative Research

The interpretist or qualitative paradigm has risen in popularity since researchers expressed dissatisfaction with quantitative methods. It is described as an understanding of human conduct and permits the researcher to observe actions and humankind from the viewpoint of a person involved in the study (Bryman & Bell, 2003). Interpretist purists believe that reality is subjective, multiple and socially constructed by its participants (Krauss, 2005; Bryman, 1984; Lincoln & Guba, 2000; Guba and Lincoln, 1994; Amare, 2004 cited in Tuli, 2011). Investigators adopting this approach view the world as a much softer, personal and humanly created place and will select recently developed and emerging techniques such as interviews, focus groups and participation observation for example (Cohen et al., 2007). Qualitative approaches possess a number of very distinguishable features:
• People are deliberate and creative in their actions; they act intentionally and make meanings in and through their activities;
• People actively construct their social world—they are not the ‘cultural dopes’ or passive dolls of positivism (Garkinkel, 1967; Becker, 1970);
• Situations are fluid and changing rather than fixed and static; events and behaviour evolve over time and are richly affected by context—they are ‘situated activities’;
• Event and individuals are unique and largely non-generalizable;
• A view that the social world should be studied in its natural state, without the intervention of, or manipulation by, the researcher (Hammersly and Atkinson 1983). Fidelity to the phenomena being studied is fundamental;
• People interpret events, contexts and situations, and act on the bases of those events;
• There are multiple interpretations of and perspectives on, single events and situations;
• Reality is multi-layered and complex;
• Many events are not reducible to simplistic interpretation; hence ‘thick descriptors’ (Geertz 1973b) are essential rather than reductionism;
• We need to examine situations through the eyes of participants rather than the researcher.

(Chen, 2007, p. 21)

4.6.3 Quantitative vs. Qualitative Research

According to Tuli (2011), both positivist and interpretive researchers believe that human behaviours may be patterned and regular. The difference, however, lies in how they view these patterns. Quantitative researchers believe that human behaviour is as a result of cause and effect, whereas qualitative researchers see this behaviour as an evolution of meaning systems generated as humans socially interact (Neuman & Kreuger, 2003). Ions (1977) believes that quantification is a form of depersonalisation that becomes an end to itself:

“a branch of mathematics rather than a humane study seeking to explore and elucidate the gritty circumstances of the human condition”.

(Ions, 1977 cited in Cohen et al., 2007 p. 17)
This is echoed by many critics of the positivist approach that believe that it is the mathematization of concepts about nature and that it is a form of alienation from our true selves (Horkheimer, 1972, Roszak, 1970, 1972).

While there are a larger number of critics of the positivist approach, the interpretive approach is not without its opponents. Some believe that naturalistic researchers have gone too far in abandoning scientific procedures of verification (Cohen et al, 2007). Bernstein (1974), for example, believes that subjective reports may be incomplete and misleading. Another concern is that of the power of a researcher to impose their ideas beliefs on the subjects being researched. It is believed that circumstances may arise that lead to inequalities in power being imposed on unequal participants. Easterby-Smith et al. (1991) provide us with a very clear outline of the strengths and weaknesses of Quantitative and Qualitative Research as seen below in table 4.2.

Table 4.2 Strengths and Weaknesses of Quantitative and Qualitative Research

<table>
<thead>
<tr>
<th>Theme</th>
<th>Strengths</th>
<th>Weaknesses</th>
</tr>
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<tbody>
<tr>
<td>Positivist (quantitative paradigm)</td>
<td>They can provide wide coverage of the range of situations</td>
<td>The methods used tend to be rather inflexible and artificial</td>
</tr>
<tr>
<td></td>
<td>They can be fast and economical</td>
<td>They are not very effective in understanding processes or the significance that people attach to actions</td>
</tr>
<tr>
<td></td>
<td>Where statistics are aggregated from large samples, they may be of considerable relevance to policy decisions</td>
<td>They are not very helpful in generating theories</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Because they focus on what is, or what has been recently, they make it hard for policy makers to infer what changes and actions should take place in the future</td>
</tr>
<tr>
<td>Phenomenological (qualitative paradigm)</td>
<td>Data-gathering methods seen more as natural than artificial</td>
<td>Data collection can be tedious and require more resources</td>
</tr>
<tr>
<td></td>
<td>Ability to look at change processes over time</td>
<td>Analysis and interpretation of data may be more difficult</td>
</tr>
<tr>
<td></td>
<td>Ability to understand people’s meaning</td>
<td>Harder to control the pace, progress and end-points of research process</td>
</tr>
<tr>
<td></td>
<td>Ability to adjust to new issues and ideas as they emerge</td>
<td>Policy makers may give low credibility to results from qualitative approach</td>
</tr>
<tr>
<td></td>
<td>Contribute to theory generation</td>
<td></td>
</tr>
</tbody>
</table>

(Amarantunga et al., 2002, p. 20)
4.6.4 Mixed Method Approach

The dominant research paradigm discussed above has resulted in two distinct research cultures developing. One of these cultures professes superiority due to its collection of ‘deep, rich, observational data’ whilst the other argues that its virtues of “hard, generalizable data” are of more worth to research (Sieber, 1973, p.1335).

Mixed method research is becoming recognised as the third major research paradigm. In their article ‘Towards a definition of mixed method research’, Johnson, Onwuegbuzie & Turner (2007), state their belief that the primary philosophy of mixed method research is that of pragmatism. They see it is an approach to knowledge that ‘attempts to consider multiple viewpoints, perspectives, positions and standpoints’ (Johnson, Onwuegbuzie & Turner, 2007, p. 113). Burke, Johnson & Onwuegbuzie (2005), hope that the two dominant research paradigms will no longer be in competition with each other and that this third major paradigm will offer a chance for harmony. The goal of mixed method is not to replace but rather support qualitative and quantitative research through drawing on the strengths of each and minimising the weaknesses of both in a single research study (Johnson & Onwuegbuzie, 2005). The use of multiple methods reflects an attempt to secure an in depth understanding of the phenomenon in question and allows for broader and better results (Denzin & Lincoln, 1994). It may also overcome the biases inherent in any single method (Creswell, 2003). Bryman (2006) developed a scheme that identifies justifications for combining quantitative and qualitative research:

1. **Triangulation**: “convergence, corroboration, correspondence or results from different methods. In coding triangulation, the emphasis was placed on seeking corroboration between quantitative and qualitative data”.

2. **Complementarity**: “seeks elaboration, enhancement, illustration, clarification of the results from one method with the results from another”.

3. **Development**: “seeks to use the results from one method to help develop or inform the other method, where development is broadly construed to include sampling and implementation, as well as measurement decisions”.

4. **Initiation**: “seeks the discovery of paradox and contradiction, new perspectives of [sic] frameworks, the recasting of questions or results from one method with questions or results from the other method”.
5. **Expansion**: “seeks to extend the breadth and range of enquiry by using different methods for different inquiry components”.

(Greene *et al.*, 1989, p. 259)

### 4.7 Description of Methods Employed

According to Cohen *et al.* (2007), before an investigator can decide on which method to use, assumptions of an ontological nature must be considered. The investigator must be concerned with the essence of the social phenomena being investigated. Are the phenomena external to the individuals, ‘out there’ in the world or is it created by one’s own mind? Having considered this, assumptions of an epistemological kind come into play according to Burrell and Morgan (1979 cited in Cohen *et al.*, 2007). These assumptions include the nature and form of the research and how it can be acquired and communicated to other human beings. If there is a view that the knowledge is “hard, objective and tangible” then the researcher must acquire an observer role and use methods of a positivist nature. If, however, the knowledge is seen as “personal, subjective and unique”, the researcher must assume allegiance with the interpretivism scientist (Cohen *et al.*, 2007, p. 7).

Having completed a review of the various options, the author found that the research was not definitively “hard and objective” or just “personal and subjective”, it was a mixture of both. It was decided that the mixed method approach was most closely in line with the goal of the author’s study, to find a solution to the problem experienced in the classroom. The author employed a number of data collection instruments during the study. A modified Fennema-Sherman Attitude Scale (1976) and a diagnostic test designed to test for understanding were given to students as a means of providing the author with quantitative data. Focus groups were identified as the most appropriate way to gather qualitative data for the project. These data collection instruments produced a mixture of data, providing the author with rounded and important insights as well as enhanced validity and reliability for the study by providing triangulation (Bryman, 1988).

#### 4.7.1 The Study Sample

The author teaches in a co-educational day and boarding school in an urban area in the Munster region of circa 500 students. All the students who participated in this study attended this school and were taught by the author. Though this raises issues of validity or ability to generalise the
findings onto the wider population, in action research studies the researcher’s primary goal is to solve the problem within the context or situation it arose (Cohen and Manion, 1992), which is the author’s classroom.

As illustrated earlier in the description of this action research study, transition year students were the first group taught using the intervention booklet. They were a Higher level class of 28 students, 13 of which got an A at Junior Certificate Higher Level. Five of these thirteen students (see section 5.3) became tutors for first year classes in the next stage of the intervention. The first year class was a mixed ability class made up of 22 students, 14 female and 8 male. This will be discussed in more detail in Chapter 5, intervention design and implementation.

4.7.2 The Questionnaire

A modified version of the Fennema-Sherman (1976) attitude scale was chosen as the principle quantitative tool for this research. These attitude scales have been described as the “most influential” (McLeod, 1994, p.639) and “popular” (Tapia and Marsh 2004, p.1) scales developed in measuring the affective domain and have been used in hundreds of studies.

The Fennema-Sherman attitude questionnaire (1976, Appendix E) is designed to test for any changes in students’ attitude towards mathematics. It is broken down into four subscales; confidence, anxiety, perception of usefulness and effective motivation. There are twelve statements in each scale, 6 negative statements and 6 positive statements. The 48 questions were mixed in order in the questionnaire to avoid patterns emerging. Each respondent was asked to indicate their level of agreement or disagreement with each item. For the positive items, 1 = strongly disagree, 2 = disagree, 3 = not sure, 4 = agree, 5 = strongly agree. For negative worded statements the order was reversed ( 1 = strongly agree, 2 = agree, 3 = not sure, 4 = disagree and 5 = strongly disagree.) These scales mean that the higher the score the more positive the attitude. The highest possible score is 60 (12 x 5) for each of the subscales. This means that a score of 60 indicates the lowest possible anxiety level but the highest possible confidence, perception of usefulness and effective motivation level. The results of each of the four subscales are arranged in tabular form. Mean scores and standard deviations are displayed for each of the twelve questions within each subscale. Below, mean scores and standard deviations are given for each statement in the Confidence Subscale.
4.7.3 The Diagnostic Test

The diagnostic tests (Appendix F) were designed by the author in order to evaluate students understanding of material taught throughout the intervention. The same diagnostic test was used for both fourth and first year students as neither had previously been introduced to algebra using a function-based approach. The author’s experience and a review of the literature guided her in the development of the diagnostic tests. They were made up of three sections. The first section began with basic shape patterns and continued with more difficult number patterns that students had to complete. The second section required students to use a function-based approach involving pattern recognition, tables and graphs in order to solve a problem. The third section required students to solve basic algebraic equations. Questions were informed by Yerushalmy (2002) and Keating, Mulvany, Murphy and O’Loughlin (2010). They were reviewed by the author’s supervisor before being distributed to transition year tutors and first year students pre and post intervention.

4.7.4 Data Analysis (Quantitative data)

The data resulting from the pre and post intervention questionnaires (see Appendix E) was analysed using the statistical software package SPSS (Version 18 for Windows). Questions were coded for later analysis. All questions were given a unique code number and responses were entered into SPSS (Version 18 for Windows) using these codes. Missing data were also coded so as to ensure that no question in particular was answered with a significantly lower frequency than other questions.

Each of the four subscales of the modified Fennema Sherman questionnaire: Confidence, Effective motivation, Usefulness and Anxiety were analysed followed by the diagnostic tests. Both the questionnaires and diagnostic tests were analysed using the SPSS (version 18) computer package. The reliability of each of the scales was analysed using Cronbach’s Alpha scores. While the scales used in the questionnaire are those tested in the literature, a confirmatory factor analysis, for this population/sample, was carried out on all items of each scale and is discussed in this chapter. Descriptive statistics reveal the mean and standard deviation for all items. Further, more in-depth analysis took place in the form of inferential statistics. Tests for normality highlighted that all data was normally distributed (p > 0.05), therefore paired sample t-tests were used to test for significance on each of the four subscales and the diagnostic tests. Pearson’s correlation coefficient was used to check for correlations
between overall attitudes and diagnostic test results. A significance level of 5% was used for all statistical tests.

4.7.5 Focus Groups

Semi structured focus groups were selected as a vehicle for collecting qualitative data. This form of qualitative research enables participant’s to ‘bounce’ ideas off one another and as a result of this sharing of views they often produce data and insights that may not have been attained in a one – to – one interview. It is only recently that there has been significant growth in popularity of this type of research tool (Morgan, 1996). Another benefit of focus groups is what Stewart & Shamdasani (1990) labelled the ‘snowballing effect’. In essence, this means that a remark made by one member of the group can very easily be picked up on by another and can in turn trigger a number of relevant thoughts, comments and ideas. Furthermore, this group synergy also leads to participants comparing their experiences and opinions with others in the group and this collaboration provides the researcher with further valuable insights into the thought processes and opinions of participants (Morgan & Kreuger, 1993).

The author agrees with Cohen et al. (2007) who suggested that a role for focus groups lies in their usefulness for triangulation with other forms of research such as questionnaires, interviews etc. The author used focus groups to ensure a mixed method approach was employed in this research to ensure validity and reliability.

The focus groups used in this study incorporated some elements of the funnel approach proposed by Morgan (1996). This approach offers a compromise between very structured and unstructured focus groups as the interview starts off by letting participants discuss their views on the topic in general before funnelling their thoughts to specific, pre-determined questions. The first question the author used in this study for focus groups was “Do any of you like maths?” This was used to generate general discussion on the topic of mathematics before more structured, open ended questions were used. These open ended questions varied slightly from group to group but generally remained the same to ensure interview standardisation. This helped the participants to avoid drifting into generalities, a major problem for focus groups according to Merton et al. (1990).
Open ended questions

Open ended questions (Appendix I) were used throughout the focus groups in this study. Research has shown that there are many benefits to using this type of questioning in semi structured focus groups and interviews. Frankfort-Nachmias & Nachmias (1996) believe that open ended questions are beneficial and worthwhile because they do not compel the respondent to adhere to preconceived answers. As a result, the researcher gets more honest and personal responses which in turn gives a truer assessment of what the individual believes. The open ended questions as well as some probing questions were prepared in advance of the first focus group and were used throughout all focus group sessions. These questions were used to determine students’ views on their attitude towards mathematics, their opinion of the function-based approach to teaching algebra and the effects of transition years tutoring first year students.

Details of the Research Instrument (semi-structured focus groups)

The TAP paradigm was selected for developing the questions to be used in the focus groups. This paradigm, which was developed by Foddy (1993), requires the author to be aware of three different factors when designing and developing the open ended questions:

1. Topic: The topic should be properly defined so that each respondent clearly understands what is being talked about.
2. Applicability: Questions should be applicable to each respondent. That is, respondents should not be asked to give information they do not have.
3. Perspective: The perspective that respondents, when answering the questions, should be asked in such a way that each respondent gives the same kind of answer.

In essence, the topic should be properly defined so that each respondent clearly understands what is being referred to, questions should be applicable to each respondent and have a specified perspective. By using this well established model the author will obtain a plentiful supply of relevant qualitative data from the focus groups while simultaneously avoiding bias and ensuring reliability and validity (see sections 4.8.1 and 4.8.2 below). Each section of the focus group and the objective of the section are discussed in the table below.
Table 4.3 Main topics to be introduced during focus groups

<table>
<thead>
<tr>
<th>Topic</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attitude toward mathematics</td>
<td>To gain an insight into students’ opinions on their own attitude towards mathematics. To ascertain whether students enjoy mathematics, see its relevance, are motivated to complete mathematical tasks or feel some level anxiety in relation to mathematics.</td>
</tr>
<tr>
<td>Transition years tutoring first years</td>
<td>To establish strengths and weaknesses of collaborative peer teaching</td>
</tr>
<tr>
<td>Function-based approach to teaching algebra</td>
<td>To determine students’ opinions on the intervention and the use of the function-based approach as a method for introducing algebra.</td>
</tr>
</tbody>
</table>

4.7.6 Data Analysis (Qualitative data)

The computer software package NVivo was used to analyse the data obtained from the focus groups. NVivo is a form of computer assisted analysis of qualitative data (CAQDA) and research suggests that it has numerous benefits including allowing the author to store, fragment and organise large amounts of qualitative data in order to identify patterns and common responses. According to Glaser & Strauss (1967) a constant comparative approach allows us to extract theories that are grounded in the data collected. Therefore, the author employed a constant comparative strategy when using NVivo:

1. Delineate Categories
2. Categorise the data
3. Code the data
4. Connect the categories and extract some theory or meaning

(Boeije, 2002)

An example of how the data was gathered using NVivo can be seen in appendix J.
Research Sample Employed for Focus groups

Focus groups were selected as a direct result of the groups that students were placed in for the duration of the intervention (see Appendix H). These groups were mixed ability and mixed gender. There were five groups in total, four first year groups and one transition year group. Two of the first year groups had five students and two had six students. Five transition year tutors were selected from the transition year class on the basis of their junior certificate results, pre diagnostic test results and willingness to commit to the intervention. These tutors were the participants in the transition year focus group. Four of the tutors were assigned a group each whilst the fifth acted as a helper for all groups.

4.8 Validity and Reliability

The author had to give due consideration to many different methods being employed in this research project. Issues included the need to validate the research and ensure the research and data was reliable so that the project can be deemed worthwhile. Attention must also be drawn to the issue of ethics and triangulation, all of which will be discussed in the following section.

4.8.1 Validity

Validity is an essential component of effective research. In quantitative research validity is ensured through selecting appropriate instruments, careful sampling and suitable statistical analysis of data (Cohen et al., 2000). Qualitative research is more difficult to validate. It relies on the objectivity of the researcher and the honesty, depth and scope of the data achieved (Cohen et al., 2000). The validity and reliability of research methods employed will determine the significance of the data collected and analysed and thus influence the significance of the conclusions obtained from the data (Leedy & Ormrod, 2001).

Internal Validity

Internal validity refers to the need to describe the phenomena being researched accurately. In other words, internal validity means that there must be evidence within the data that a particular event or issue can be explained (Cohen et al., 2007). Issues of validity can also be addressed using different instruments for data collection. According to Carr (2010),
triangulation is a powerful way of demonstrating concurrent validity, particularly in qualitative research. Descriptive and inferential statistics were used throughout the analysis process of the research project to ensure appropriate levels of validity were evident.

**External Validity**

External validity refers to the degree to which “the results can be generalised to the wider population, cases or situations” (Cohen et al, 2007, p. 136). This research project took place in one school type in a city in the Munster region. Due to the composition of this sample, the option to generalise is very limited.

**Content Validity**

Content validity refers to the extent to which a measurement instrument is deemed to be comprehensive, fair and that it measures what it is supposed to measure (Maguire, 2005). Validity was ensured through the author’s comprehensive review of literature prior to the use of the Fennema-Sherman questionnaire and focus group sessions.

**4.8.2 Reliability in Quantitative Research**

The reliability of a measurement in quantitative research is the extent to which it yields consistent results when the characteristic being measured hasn’t changed (Leedy and Ormond, 2001). Reliability makes assumptions that ‘instrumentation, data and findings should be controllable, predictable, consistent and replicable’ (Cohen et al, 2007, p.148). In other words, it suggests that there should be a high correlation between results if the same research problem were to be carried out using the same instruments but with a different sample.

Guided by Cooper and Schindler’s (2001) outline of ways to improve reliability in this paradigm, the author implemented the following to ensure reliability:

- Conditions under which data was measured and collected were standardised and controlled.
- Minimising any external sources of variation.
- Excluding extreme responses from the data analysis (outliers in SPSS).

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• The time between the pre and post questionnaires and diagnostic tests was not long enough to allow situational factors to change and not too short so as to allow students remember the questionnaires and tests.

Also, the use of the internationally respected Fennema-Sherman (1976) attitude scale, allows the author to claim reliability. This attitude scale has been described as the ‘most influential’ (McLeod 1994, p.639) and “popular” (Tapia and Marsh 2004, p.1) scales developed in measuring the affective domain and has been used in hundreds of studies. The degree of anonymity enjoyed as well as the 100% response rate, ensures a high degree of reliability with the data collected from both groups.

4.8.3 Reliability in Qualitative Research

Reliability of quantitative research differs from that of qualitative research. The uniqueness of situations that qualitative research strives to comprehend leads one to believe that difficulties arise when attempting to replicate qualitative data. Denzin and Lincoln (1994) suggest that this reliability can be addressed in several ways:

• Stability of observations - Would the same observations be made if a researcher observed at a different time or in a different location.
• Parallel forms - Would the researcher have paid the same attention to an observation if they had paid more attention to different phenomena prior to the subsequent observation.
• Inter-rater reliability - Would a different observer with the same theoretical framework and observed the situation in the same way.

In order to ensure reliability, the author took the above observations into consideration. Denzin (1970a) suggests that triangulation addresses issues of reliability. The author used three sources of data, both quantitative and qualitative to support relevant findings and ensure reliability.

4.8.4 Triangulation

Triangulation in research refers to the use of multiple techniques for gathering and/or handling data within a single study. It enhances the reliability and validity of a research
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project. A reason for this is provided by Cohen et al. (2000) who propose that relying on one method of observation is disadvantageous due to the fact that it may distort the researcher’s picture of the investigation. They (Cohen et al., 2007) believe that the more the methods contrast with each other, the greater the researcher’s confidence about its findings. Triangulation is an attempt to map out the richness and complexity of human behaviour by studying it from more than one viewpoint (Cohen et al., 2007).

Denzin (1970b) acknowledges the multi method approach as a type of triangulation but provides us with a more extended view that incorporates 5 other types:

- *Time triangulation* - this type attempts to take into consideration the factors of change and process by utilising cross-sectional and longitudinal designs;
- *Space Triangulation* - this type attempts to overcome the parochialism of studies conducted in the same country or within the same sub culture by making use of cross cultural techniques;
- *Combined levels of triangulation* - this type uses more than one level of analysis from the three principal levels used in the social sciences; individual level, interactive level (groups) and levels of all collectives (organisational, cultural and societal);
- *Theoretical triangulation* - this type draws upon alternative or competing theories in preference to utilizing one viewpoint only;
- *Investigator triangulation* - this type engages more than one observer, data are discovered independently by more than one investigator (Silverman, 1993);
- *Methodological triangulation* - this type uses either the same method on different occasions, or different methods on the same object of study.

(Chen et al., 2007, p.142)

4.8.5 Researcher Distance

It is expected that the author’s experience of full time teaching, as well as her own second and third level education, will result in her bringing her own biases, assumptions and expectations to the project. A number of conscious decisions made by the author, including the use of a mixed method approach and the choice of a number of different methods of inquiry, as well as the acknowledgement of her biases and expectations allowed the author to
establish researcher distance and objectivity. In turn this researcher distance will help contribute to the validity, reliability and overall acceptance of the researcher’s findings.

4.8.6 Ethical Issues

Cohen et al. (2000) stress the importance of taking into account the effects of research on participants, and to act in such a way as to preserve their human dignity. As a result, the author identified any ethical issues at the beginning of this project and ethical guidelines, as outlined by the University of Limerick Research Ethics Committee (ULREC) were adhered to throughout the design and implementation phases of this study. In order to adhere to these ethical guidelines the author had to ensure the following:

- A consent form had to be signed by the school principal;
- Subject information sheets clearly outlined the purposes of the research and what was required of them;
- Consent forms were signed by all participants and their parent/s prior to the intervention commencing;
- Subjects were aware that participation in the study was voluntary and that they could withdraw from the project at any stage;
- Code numbers were allocated to participants to ensure anonymity;
- Any data collected was used for research purposes only and all data stored according to ULREC Regulations.

The author received ethical approval from ULREC for this study in August 2011. Prior to implementation of the intervention the author sought consent from the headmistress of the school. The author then issued a principal information sheet and consent form for both first and transition year students (Appendices A & B) that participated in the study. The primary objective of the information sheets was to provide participants with the necessary information regarding the intervention, whilst also outlining the requirements of each participant. Participation was voluntary and participants were aware that they could withdraw from the study at any time.

At all times the researcher followed university guidelines in obtaining student data. Data analysis of the student questionnaire was carried out using SPSS (version 18 for Windows).
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All entries into SPSS were coded. Analysis of the focus groups was carried out using NVivo. Confidentiality was ensured throughout the interview and analysis process through the use of coding. The information sheets explained that information about students would not be identified in the final report and all data would be stored safely with access only to the investigators as specified by university guidelines for such data.

4.9 Limitations of the Study

There were a number of limitations encountered during the study. These are outlined below. Firstly, all research took place in the author’s workplace raising questions about the contextual nature of the research. Triangulation methods were used in order to limit researcher bias. Secondly, this project was set up in one school with one first year class and one transition year class. Due to timetabling restraints and limitations put in place by the maths department in the school, there was no opportunity to have an experimental group and a control group. Time constraints meant that the intervention could not be repeated with another group. This project required transition years to tutor first year students. Due to the nature of transition year, these students were not available for the complete duration of the study. They were available to tutor for 7 out of the 12 designated classes.

Although there are clearly defined limitations in this research study the author feels that due to the mixed method approach used, these limitations will not have a significant impact on the outcome or results of the study.

4.10 Conclusion

This chapter outlined the research objectives and questions that informed the researcher on many occasions throughout this research. It also provides an outline of the theoretical framework that guided the author’s research decisions and explains the methodologies employed in the study. Action research was the natural methodology to use in this inquiry. By identifying a problem within her classroom, the author had already begun the process of action research. Whilst the author realises that there are many methodologies, she acknowledges that the action research model provides guidance for those teachers who wish to engage in research where the aim, ‘is to solve the immediate and day-today problems of practitioners’ (McKernan, 1996,
A breakdown of each phase of the research was provided in addition to the overall research design.

After ample research, a mixed method approach towards the analysis of data was decided upon in order to ensure both the validity and reliability of the study. The findings reached following these methods of evaluation will be discussed in a later chapter (Chapter 6) but first the intervention, its format and content needs to be given further thought.
Chapter 5: Intervention Design and Implementation

5.1 Introduction

This chapter aims to outline the design and development of the intervention in light of the literature review in chapters 2 and 3. Interventions are widely used in educational contexts, where an intervention is aimed at a positive outcome rather than a perceived negative one that would ordinarily occur (Wilkes & Bligh, 1999). This chapter will give the reader a clear picture of the goals of the intervention as well as the development and implementation of the intervention. A detailed analysis of the structure and content of the intervention booklet used for the delivery of the intervention will take place.

5.2 Aims of Intervention

The purpose of this intervention is to introduce algebra to both first year and transition year students using a function-based approach within a collaborative peer environment. The intervention aims to:

- Improve first and transition year students’ understanding of variables and the need for algebra in society,
- Increase first and transition year students’ attitude and confidence in and towards mathematics,
- Introduce algebra to first year students in a way that blends effortlessly with the approach advocated in primary schools,
- Introduce algebra to first year students in a collaborative peer environment in order to improve students’ attitude toward mathematics.

5.3 Selection of Participants

Two classes were used for the implementation of this intervention. One was a large transition year class of 28 students. These students were between the ages of 15 and 16. The class consisted of 15 female and 13 male students (see Appendix H). All Irish students in this class completed the Higher level Junior Certificate in June 2011. There were 4 Spanish students and 1 German student within the class who had not completed the Junior Certificate and were
new to the class. Due to the transition year maths classes being timetabled together, the author was able to choose the level of the class that she would teach for the school year 2011/2012. The author chose this class because they were high achievers and enthusiastic about the mathematics. As part of the intervention (discussed in section 4.5.3), transition year students were selected to act as tutors in first year classes. The author took a number of factors into consideration when choosing these tutors. These included:

- Performance in Junior certificate/international alternative;
- Results of the diagnostic tests (see appendix F) given to students pre intervention;
- Ability to present in a clear and confident manner in front of a group (students had to complete a presentation on a mathematician to their peers in September, 2011);
- Ability to work well in groups (observed throughout the completion of the student handbook (see appendix L) in cycle 2 of the action phase (see figure 4.1));
- Interest and willingness/ability to commit to the project;
- Ability to explain concepts within the function-based approach.

Having analysed students under the above criteria, 9 were deemed appropriate for selection as tutors. The next step involved distinguishing the students that would be available to come into first year mathematics classes. By coming into first year mathematics classes, it meant that transition year students would miss some of their own timetabled classes. In order to establish availability, timetables were collated and teachers were approached. Having completed this process, 7 of the 9 students identified were available to become transition year tutors with two being unavailable due to commitments in other classes. These 7 tutors then had a meeting with the author to discuss what would be expected of them throughout the classes. Students were given the opportunity to ask questions in relation to the material they would be teaching and in relation to teaching skills that they would require. Having discussed their role, two of the tutors expressed an unwillingness to commit to the project and so the author was left with 5 tutors for the intervention.

The other class used in the intervention was a first year class. This was chosen as a direct result of the author’s timetable. The class was made up of 22 students, 14 female and 8 male, between the ages of 12 and 13. Students were from a wide range of backgrounds and had very mixed mathematical ability.
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5.4 Development and Implementation of the Intervention

Once a review of the literature was complete and the overall intervention was designed, it was deemed appropriate that two main resources be developed in order to facilitate implementation. A ‘handbook’ (see appendix L) was developed for students as a means of guiding them through a function-based approach to algebra. This handbook will be discussed in more detail in section 5.5. The teacher guidelines (see appendix M) were developed as a direct result of the student handbook and are discussed in section 5.7.

The development of these documents occurred in Phase 2 (see figure 5.1) of this project which was known as the action phase (see section 4.6.3). This action phase witnessed two cycles occurring prior to the introduction of algebra using the function-based approach with first year students.

**Figure 5.1 Cycle 2: Action Phase**

The student handbook was designed in the first cycle of the action phase and amended after receiving critical advice. It was then used to introduce a function-based approach to algebra to transition year students. The second cycle informed the author of other improvements that could be made and so the handbook was amended again. The third part of the action phase witnessed the student handbook being used in first year classes. Details of the content and format of the student handbook will now be discussed in light of the literature.
5.5 Content and Format of the Student Handbook

A review of the literature informed the design and content of the student handbook. A number of learning outcomes were identified in relation to this intervention (see figure 5.2). The author believed that through completing the handbook students should be able to:

- Use words, tables, graphs, diagrams and manipulatives to represent a repeating pattern situation,
- Generalise and explain patterns and relationships in words and numbers,
- Write arithmetic expressions for particular terms in a sequence,
- Use tables, diagrams and graphs as tools for representing and analysing patterns and relations,
- Develop and use their own generalising strategies and ideas and consider those of others,
- Present and interpret solutions, explaining and justifying methods, inferences and reasoning.

Ten lessons were devised to lead students through an introductory course in algebra which used a function-based approach.

5.5.1 Lesson 1:

The NCCA commissioned the ERSI to engage in a longitudinal study of students’ experiences of curriculum in the first three years of their post-primary schooling. The first report focused on first year students and the role of transition. This report made it clear that there was a lack of continuum between primary and post-primary schooling and this was having an impact on the success or otherwise of students’ successful transition. For this reason, the handbook begins with a lesson that discusses and investigates basic patterns that students should be familiar with from primary school (see appendix K, bridging document for algebra from primary to secondary school within project maths). The main aim for lesson one is to explore patterns in the world around us and to revise how to identify and continue basic patterns using pictures and words.
Central Outcome

Students will be enabled to use mathematical terminology and notation, algebraic symbols, diagrams, text and tables to communicate mathematical ideas.

Key Ideas

Students will be able to:

- Use letters to represent patterns
- Translate between words and algebraic symbols and between algebraic symbols and words

Overview of Learning Outcomes

- Using letters to represent numbers
- Model algebraic expressions such as a+1, 2a, 2a +1 using manipulatives, tables, and graphs.
- Substitute a given number for a letter
- Link algebra with generalised arithmetic
- Generate a variety of equivalent expressions that represent a problem
- Answers questions applied to real-life situations

Answers questions applied to real-life situations

Generate a variety of equivalent expressions that represent a problem

Model algebraic expressions such as a+1, 2a, 2a +1 using manipulatives, tables, and graphs.

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- Substitute a given number for a letter
- Link algebra with generalised arithmetic
- Generate a variety of equivalent expressions that represent a problem
- Answers questions applied to real-life situations

Answers questions applied to real-life situations

Generate a variety of equivalent expressions that represent a problem

Model algebraic expressions such as a+1, 2a, 2a +1 using manipulatives, tables, and graphs.
The lesson begins with students being asked to describe, in words, a definition of the term ‘pattern’. They are asked to do this in order to trigger any previous knowledge that they may have gained in primary school. They are then asked to think about, write and draw where they might see patterns in the world around them. Transition year tutors guide group discussions, after which students are asked to share the group’s thoughts with the class. The author felt that this introduction was important as she agreed with Freudenthal’s (1977) view that mathematics must be connected to reality, stay close to children’s experience and be relevant to society, in order for it to be of human value (Van den Heuvel-Panhuizen, 2000). A short video (Cristóbal Vila, 2010, http://www.youtube.com/watch?v=kkGeOWYOFoA) relating patterns to nature is then shown to the class. Dale’s (1946) Cone of Experience (see figure 5.3) highlights that as the students are increasingly challenged in a manner that is active, practical and multi-sensory their understanding of the topic they are studying increases.

**Figure 5.3 Dale’s cone of experience**

![Dale’s Cone of Experience](image)

After watching the video, students are asked to use manipulatives (unifix blocks) to create patterns in an effort to have students ‘simulate’ or ‘model the real experience’ (see figure 5.3) that they will have just witnessed on the video.
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The next section of the first lesson is a group activity called ‘Guess my rule!’ This involved one student, Paul, for the purposes of this example, thinking of a rule that can be applied to a number. For example, Paul might think of “times the number by 3”. Other students in the group must then give Paul a number to which he applies the rule and gives them a response. For example, Sarah, might say ‘4’ and Paul’s response would be ‘12’. The first student to figure out Paul’s rule wins and takes over Paul’s role. This activity was chosen because it is fun and it encourages group work and thinking about the concept of ‘a rule’. McKeachie, Pintrich, Lin and Smith (1986) found that student-centred methods, such as group work, are superior to teacher dominated methods. They attribute this superiority to students developing the ability to “apply concepts, problem solve, lead, improve attitude and motivation and be involved in group membership” (p. 17).

The final section of this lesson involves completing worksheets (adapted in part from Yerushalmy & Gilead, 1999) at the end of class and then for homework. The first worksheet is pictorial in nature, the next two involve number patterns and revisit the idea of rules applying to patterns whilst the third involves students decoding a message by identifying patterns. These worksheets act as a bridge from the primary school curriculum to the secondary curriculum.

5.5.2 Lesson 2

The second lesson focuses on helping students to describe patterns in a multitude of ways. The Lesh Translation Model (1979, see figure 5.4) highlights the importance of students’ abilities to represent mathematical ideas in different ways including manipulatives, real life situations, pictures, verbal symbols and written symbols (Lesh et al., 2003).

There is a distinct overlap between the representations used in Lesh’s translation model and the function-based approach. Yerushalmy (2000) and Ferrara, Pratt and Robutti (2006) suggest that functions be represented in different forms: words, numbers, graphs and symbols (Yerushalmy, 2000) and tables (Ferrara, Pratt and Robutti, 2006).
The author chose to combine these and created a pictorial illustration of the variety of ways in which a pattern can be represented; pictures, words, tables, graphs and symbols (see figure 5.5). The author felt that Lesh’s ‘real life situations’ or ‘manipulative models’ were of great value to students and utilised them throughout the intervention. She felt, however, that in all cases, manipulatives and real life situations might not be applicable for solving problems and so did not include them on the diagram. This overlap of representations leads to a deep theoretically grounded approach to teaching algebra.

**Figure 5.5 Representing patterns**
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This diagram is placed in the top right hand corner of every page of the student handbook. The representation that is the focus of the lesson is highlighted so that students have a clear understanding of the method by which they are being asked to represent the patterns in that particular lesson. Lesson one focuses on using words to describe patterns and includes a worksheet that requires students to use pictures to continue patterns. Figure 5.6 displays the diagram used for the second lesson.

Figure 5.6 Representing patterns in lesson 2

![Diagram showing representation methods]

The first four exercises in lesson 2, ask students to represent patterns using manipulatives. The author chose to have students interacting with manipulatives such as unifix cubes as a result of research by Goracke (2009). She used an action research approach to investigate the use of manipulatives and its impact on student attitude and understanding. She discovered that student attitude toward mathematics improved when greater manipulative use was infused into the lessons. She found that students felt more confident that they understood the material, which translated into a better attitude regarding math class.

After using manipulatives, students are asked to draw diagrams and/or complete tables in relation to a pattern and are then asked to describe the pattern using words. As the exercises progress, the term ‘rule’ is included in the questions that ask students to describe the pattern. This rule must then be used to answer follow up questions. This concept is similar to that of the ‘Guess my rule’ game in the first lesson. An example of this is displayed in figure 5.7.
Figure 5.7 Exercise 2.2 that requires students to use pictures, words and tables to represent a pattern.

### Exercise 2.2

#### Ice Cream Cones

1. Draw 4 cones.

2. How many blocks are needed for 4 cones? ______

3. Complete the table using the information that you see above:

<table>
<thead>
<tr>
<th>No. of cones</th>
<th>No. of blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

4. Write down, in words a rule that describes the pattern in the table.

5. How many blocks will 10 cones need? __________

6. How many blocks will 100 cones need? __________

The next activity requires students to work in pairs to draw their own repeating patterns similar to the exercises they had just completed. They are then asked to complete a table using the pattern and as a challenge can be asked to create questions about the pattern. The author includes pair and group work within lessons in order to establish a cooperative learning environment. Rubin and Hiebert (1998) found that students learn more in a cooperative classroom atmosphere and that they are more willing to take responsibility for
their educational experiences than they would in learning situations where they are subordinate to the teacher.

The final task in lesson two requires students to study the multiplication tables, a concept that should be very familiar to them. This reflects research pertaining to Realistic Maths Education that advocates relating the mathematics to a context familiar to the students (Wijers & Van Reeuwijk, 2004, p. 80). The multiplication tables should be very familiar to the students and so the aim of this activity is to have students identify patterns that they may not have thought about or discovered previously. The author believes that this activity requires students to engage the five concepts advocated by Carpenter & Lehrer’s (1999) for improving students levels of (see 2.4.2):

- Construct relationships
- Extend & apply mathematical knowledge
- Reflect about their own maths experiences
- Articulate what they know
- Make mathematical knowledge their own

5.5.3 Lesson 3

The main aim of the third lesson is to introduce students to the Cartesian plane and drawing of graphs. The first three activities relate directly to introducing and developing students ability to work with a Cartesian plane in a fun and interactive manner.

The first activity uses a constructivist approach of guided discovery. Prasad (2011, p.32) states that in guided discovery, the teacher “leads a class along the right path, rejecting incorrect attempts, asking leading questions, and introducing key ideas as necessary.” It asks students to think about how they might describe the position of a fly on the board and requires students to discover the need for a coordinate diagram. According to Wijers & Van Reeuwijk, 2004, p. 80) important aspects of a Realistic Mathematics Education include the use of ‘realistic (real to the learner) contexts, the movement from a concrete to abstract level of thinking and the use of guided reinvention where students experience a similar process compared to the process by which mathematics was invented.’ This exercise can be said to fulfill all three aspects advocated by Wijers & Reeuwijk (2004).
position of something can be seen as a concept that is very real to the learner. The question of how to describe the position leads students from a concrete to an abstract level of thinking as it asks students to develop the idea of an axis and coordinates. Finally, this short activity leads students through an experience similar to Descartes who first developed the Cartesian plane which fulfils the third aspect of guided reinvention. Also, research suggests that the inclusion of historical data can stimulate interest amongst students (Mikk, 2000).

The next activity called ‘Star Wars’ has the same concept of the game ‘Battleship’. Students must draw a coordinate plane on a graph board and colour in three ‘stars’. Their partner must then attempt to ‘shoot down’ their stars by naming coordinates. The third activity requires students to match letters on a Cartesian plane to coordinates in order to break a code. Students are then asked to create their own code for a partner to crack. Students view this activity as a game. Research has shown that attitude toward content can be influenced by using games (Barak, Engle, Katzir, & Fisher, 1987; Malouf, 1988; Pascale, 1974). According to the NCTM, (as cited in Bright, Harvey, & Wheeler, 1985), mathematics games may be the best way to ensure that students get the basic mathematics skills they need to be successful in life, while at the same time promoting an enjoyment of mathematics and motivation to learn mathematics.

Having familiarised themselves with the Cartesian plane, students are then asked to work together to complete activities in class or for homework, that are very similar to those in the second lesson. Students must represent patterns using tables and words but this time they must use the pattern to draw graphs. They will be asked to recognise that the graphs developed are linear in nature. This activity advocates working in pairs which creates a community of practice and use representational fluency. Communities of practice and representational fluency are two the three problem solving skills advocated by Lesh and Zawojeski (2003) and are important in an information age, in knowledge economies, in learning organisations and in global societies where people need to be able to work together to describe and explain the systems within which they operate.

5.5.4 Lesson 4

The fourth lesson aims to introduce the concept of the variable. This is the final representation advocated by Yerushalmy (2000) and Ferrara, Pratt and Robutti (2006) that has not yet been explored. Exercises from lesson 3 are repeated at the beginning of this lesson
in order to create continuity of thought. In this lesson, however, students are asked to represent the patterns with letters.

Exercise 4.2, provides a question within a realistic setting for students (see figure 5.8). Representing problems in real life contexts is advocated by Lesh et al. (2003). Exercise 4.2 and 4.3 begin to introduce concepts of proportionality and require student to interpret graphs rather than just representing patterns using graphs.

Exercise 4.3 requires the use of manipulatives again, which McConway (2006) identifies as an invaluable source for weaker students. Another important aspect to note, however, was highlighted in a study by McClung (1998). She discovered that the age (and therefore stage of development, concrete/formal) of the student plays an important role in the success or otherwise of manipulative use in the classroom. Students in this project were between the ages of 12 and 13 which means that they were either at a concrete operational stage or just progressing to a formal operational stage. Those still in the concrete operational stage were more likely to benefit from the use of manipulatives than those at a formal stage who have complete generality of thought and ability to deal with the hypothetical (Philips & Soltis, 1998).

5.5.5 Lesson 5

This lesson aims to challenge students to solve problems using the multitude of representations introduced in previous lessons. The first activity, the caterpillar problem, was adapted from the Project Maths guidelines to the function-based approach to teaching algebra. The second activity was created by the author in response to a similar problem she had experienced and requires students to provide an answer to a problem in relation to seating arrangements in a restaurant. It was included to provide a real life context within the lesson and was adopted from Math Wire (2011). The third activity, the string activity, was adopted from work by Sobel and Maletsky (1975) that aimed to introduce algebra using patterns.
Figure 5.8 Exercise 4.2 requires a pattern from a realistic context to be represented using tables, graphs and symbols.

**Exercise 4.2**

The Clarion hotel in Limerick has five rooms on each floor.

Complete the table below to represent this information:

<table>
<thead>
<tr>
<th>Number of Floors (F)</th>
<th>Total Number of Rooms (R)</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Write down, in words, a pattern that emerges from the table:

_________________________________________________________________________
_________________________________________________________________________
_________________________________________________________________________

2. Represent the pattern using algebraic symbols.

_________________________________________________________________________

3. How many rooms in total would be in a building which has 12 floors? Describe how you got your answer.

_________________________________________________________________________
_________________________________________________________________________
_________________________________________________________________________

4. If you double the number of floors what happens to the total number of rooms in the building?

_________________________________________________________________________
_________________________________________________________________________
_________________________________________________________________________
5. If you treble the number of floors what happens to the total number of rooms in the building?

_________________________________________________________________________
_________________________________________________________________________
_________________________________________________________________________

6. Are the variables proportional to each other?

_________________________________________________________________________

7. The graph of this pattern is shown below. Write down three things you notice about the graph.

The important concept of proportionality is introduced in this lesson (Chazan, 2003). This can be confusing for students and may take time for them to get to grips with. Students will be made aware that graphs that go through the origin are proportional and those that do not go through the origin are non-proportional. The difficulty lies in establishing the position of the 0 point in a pattern. For example, in the string activity, it is easily established that if you cut the string ‘0’ times that you will have ‘1’ piece of string. This means that the graph will go through the coordinate (0,1) and therefore be non-proportional. It is more difficult, however, to identify for example what happens the ‘0’ case in the caterpillar problem.
If you look at the pattern of the stripes in figure 5.9, one can establish that the yellow occurs in the 1st, 4th and 7th position, the black occurs in the 2nd, 5th and 8th position and the green occurs in the 3rd and 6th position. When investigating the pattern of each colour, how do you establish what happens at 0? We will look at the case of the green pattern (see figure 5.10).

*Figure 5.10 Investigating the pattern of green blocks in the ‘caterpillar problem’*

<table>
<thead>
<tr>
<th>Green</th>
<th>Block</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>0</td>
<td>?</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>15</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>18</td>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>3n</td>
<td>n</td>
<td>3n</td>
</tr>
</tbody>
</table>

In order to establish what happens at 0 we must work backwards using the pattern. If we do this we can see that 9 goes to 6, 6 goes to 3 and therefore 3 must go to 0. We have established that the first coordinate that we can now plot is (0,0) which means this graph is proportional. This concept can be difficult for students to get to grips with and must be emphasised clearly.

This lesson also involves the use of manipulatives as both a tool to help engage students in the lesson and as a fun aspect. The first problem requires the students to use unifix cubes while the third problem requires the use of string and scissors. Up until now, completed graphs have been displayed using Geogebra. This lesson involves tutors showing students how to create a graph using Geogebra. Each group will be required to go to the interactive...
whiteboard at different stages throughout the lesson and display one of their solutions on a graph using Geogebra. This will prepare students for a project that they will be required to complete later on in the intervention.

By the end of lesson 5 students should be aware that they can use pictures, words, tables, graphs and symbols to help to solve problems. They should be comfortable plotting coordinates and representing patterns using graphs and should recognise that graphs thus far have been linear in nature and are either proportional or non-proportional. They should also be aware that manipulatives can be a valuable tool in helping them to identify and represent patterns and that these questions can occur in a real context.

5.5.6 Lesson 6

Haimes (2006) provided the author with a structure through which the handbook progressed. He suggests that patterns and number sequences be developed into the following progressive stages: linear, quadratic, exponential, reciprocal and periodic relationships. For this reason, the focus of lesson 6 moves from that of problems that are linear in nature to those that are quadratic in nature. The first activity, the building shapes worksheet was adapted from the NSW Department of Education (2012). It requires students to work their way through an active approach to discovering the difference between variables that are added or subtracted from numbers (example, $2 + c$) to variables that are multiplied by numbers ($2c$). This is a challenging task and so tutors are required to go through a number of examples and students are encouraged to work in pairs. The next activity, hexagon dragons which was adapted from the website Mathwire.com (2011), revisits the use of Geogebra in order to establish the level of students’ understanding of how to use the program.

After completing the first two activities, students work through three exercises (see example in figure 5.11). The first two introduce the concept of squaring a variable to represent a pattern, i.e. the idea of a quadratic function whilst the third revisits the concept of a linear functions. This was done in order to emphasise the difference between the two types of functions and allow for comparisons to be drawn. The ‘Squares’ exercise was adapted from Mathwire.com, the swimming pool problem was adapted from Stacey et al. (2004, p. 336).
Figure 5.11 Exercise 6.3; ‘Squares’ activity for introducing the concept of a quadratic function.

6.3 Squares!

1. Above you can see the second and third step in a pattern. Use unifix cubes to create the first, third and fifth steps in this pattern. Draw the pattern below.

2. Complete the table and draw a graph to represent this information:

<table>
<thead>
<tr>
<th>Length of side</th>
<th>Area</th>
<th>Change</th>
<th>Change of Changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Write down a rule that emerges from the diagrams above.

4. Use this rule to ascertain:
   - How many cubes would be needed for a square of length 11.

   - What is the length of a square whose area is 169.
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At this point, it is important that the idea of examining the rate of change within patterns is introduced (Nayland College of Mathematics, 2011). If we look at a linear pattern (see figure 5.12), we can see that the rates of change (in red below) are constant.

*Figure 5.12 Solution to the string activity, part one; a linear pattern.*

<table>
<thead>
<tr>
<th># Cuts (c)</th>
<th># Pieces</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>c</td>
<td>2c + 1</td>
</tr>
</tbody>
</table>

If, however, we look at a quadratic pattern, the first rates of change (see red in figure 5.13) are not constant. We must look at the change of the rates of change (see green in figure 5.13)! If these are constant, we know the pattern is quadratic in nature.

*Figure 5.13 Solution to squares activity; a quadratic Pattern*

<table>
<thead>
<tr>
<th>Diagram</th>
<th># unifix blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1 +1 +2</td>
</tr>
<tr>
<td>2</td>
<td>4 +3</td>
</tr>
<tr>
<td>3</td>
<td>9 +5 +2</td>
</tr>
<tr>
<td>4</td>
<td>16 +7 +2</td>
</tr>
<tr>
<td>5</td>
<td>25 +9 +2</td>
</tr>
<tr>
<td>d</td>
<td>d^2</td>
</tr>
</tbody>
</table>
Geogebra is used to display the graphs (see figure 5.14) of the functions and from this students are asked to identify and discuss the differences between linear and quadratic graphs and functions.

*Figure 5.14 Graph of the function $a = t^2$*

5.5.7 Lesson 7

The vision of school algebra has widened considerably over the decades-moving from a letter symbolic and symbol manipulation view to one that ‘encompasses multiple representations, realistic problem settings and the use of technological tools’ (Lester, 2007, p. 747). Fey & Good (1985) envisaged over twenty five years ago that technology would play an important role in promoting effective teaching of new and traditional topics, i.e. functions, to students that need to be ready for a technological world.

In light of this, the aim of this lesson is to develop student’s ability to use IT to help them to represent and solve a problem. The software programs used in this project, ‘Geogebra’ and ‘Microsoft excel’ are currently being promoted by the Project Maths team for use in Irish classrooms. Both Geogebra and Excel allow the student to use static and dynamic manipulatives. Moyer (2002) suggests that static representations are essentially pictures and are not what Moyer calls *true manipulatives*. Dynamic visual representations are, however, like pictures but can be manipulated as if it were a three dimensional object. Dorward and Heal (1999) found that virtual manipulatives foster as much engagement as physical manipulatives.
Within this lesson groups will work together with their tutor to complete the questions relating to a staircase problem through using both static and dynamic manipulatives. The tutor will first demonstrate how to create a table using excel. Students will then create their own tables for the area and perimeter of the staircase. The tutor will then demonstrate how to translate the tables into graphs in Geogebra after which the students will generate their own graphs. At this point, students will work together to write equations to represent the area and perimeter of the staircase. They will then be shown how to compile all their information into a word document.

5.5.8 Lesson 8

Lesson 8 aims to give students the freedom to complete a project using Microsoft excel, Microsoft word and Geogebra. Students are presented with a problem (see figure 5.15) and asked to answer ten questions in relation to the problem. These questions include creating tables and graphs, representing information using variables and making recommendations in light of the problem. They must compile all information into a word document and must present this document to the author. The tutors and teachers act as true facilitators within this lesson, helping students who encounter difficulty.

Figure Exercise 5.15; The car park problem

John goes by car every work day to the centre of Dublin, where his office is located. Nearby there are two car parks. The first demands €4 to enter and €2 per hour. The second demands €3 per hour. John does not have a regular timetable. His choice about where he parks his car depends on how many hours he will stay at his office.

(Adapted from Yerushalumy, 2000, p.131)

Websites requiring students to work with virtual manipulatives in a simulated game context are listed at the end of this lesson. These are made available for students that might finish the project quickly and they are also a resource that can be used for homework. Brophy (1987, p.4) found that "Simulation games contribute to young people's learning in the affective
domain” and that the incorporation of game-like features and simulation elements can promote motivation to learn.

5.5.9 Lesson 9

The entirety of this lesson involves a game called ‘equation bingo’. At this point students have gone through a process in which they are required to use letters, to solve problems relating to patterns. The equations they are presented with here are not linked to patterns but they should now recognise the need for algebra to solve problems and should understand that behind every algebraic equation, there is a pattern of some description. The aim of this lesson is to help students to solve algebraic equations. The game will take place as follows:

- Each group is given a pack of equation cards (see figure 5.16) and each student is given a bingo card with numbers and whiteboard and marker;
- An equation card is turned over within the group;
- The tutor and the group discuss methods that would be appropriate for solving such a problem. Students are encouraged to use tables, graphs, words and symbols where appropriate to help them. When equations become too difficult the students should realise a need for some sort of process to solve the problems. At this point, the idea of transposing is introduced;
- Once a solution is reached, students with the answer on their bingo cards are allowed to cross it off;
- As the game progresses, the tutors give more responsibility to the students to solve the problems.

This group work approach encourages an active methodology through which social constructivism is envisaged.
5.5.10 Lesson 10

Having completed the project and the equation bingo game, students are then presented with algebraic word problems. Students are encouraged to work together to use pictures, tables, graphs, words and symbols where appropriate to help them to solve the problems.

Students then participate in a recap of the overall intervention. This recap involves tutors asking students to identify the most important concepts they learnt throughout the completion of the student handbook. A post diagnostic test is issued in the next lesson to gauge the level of understanding that students gained in relation to basic algebra.

5.6 Content and Format of the Teacher Guidelines

The teacher guidelines were developed as a step by step guide for teachers for the implementation of this intervention. They developed as a result of the student handbook. They include:

- An Introduction
- Background of the intervention
Aims of the resource and learning outcomes
Organisation of lessons
Teacher outcomes
Overview of lessons
Resources required
Detailed overview of each lesson

The detailed overview of each lesson provides the teacher/tutor with the main focus of each lesson as well as a brief description of each lesson. It also outlines resources required, key terms and concepts within the lesson and a copy of the power-point presentation that accompanies the lesson. This power point presentation contains solutions to the problems within each lesson.

5.7 Problems Encountered with the Implementation of the Intervention

The student handbook was designed to take ten classes but in reality took fourteen. Whilst many positives emerged from these classes, a number of issues arose in relation to the implementation of the intervention:

- Lack of consistency. When the 5th tutor became available, another tutor was unavailable and so for this reason, the 4 groups remained for the duration of the intervention. This meant however that there was a lack of consistency for one of the groups as their tutor changed;

- If all 5 tutors were available, the author assigned one as a ‘floater’. This meant that they acted as extra support for tutors, groups or individual students. When this occurred it was difficult to assign which tutor should go with which group. Should the original tutor go back with their group or the new tutor remain? In the end, the new tutor remained;

- Lack of consistency due to the nature of transition year, student tutors were unavailable for 5 of the classes. This is because they were involved in a transition year play and also due to excursions outside of the school. When tutors were unavailable, students remained in their groups and worked through the student handbooks. The
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author moved around the groups helping with any difficulties. At times, lessons became teacher led, to facilitate the introduction of a new topic or concept to the class;

- Different groups moved at different paces and so the power point presentation had to be changed regularly to facilitate this. In the end, the author printed large slides of the power point as an alternative for tutors to use;

- Lack of examples for students. Tutors used the activities in the handbooks as examples for students to follow. Some first year students, however, moved at a much faster pace to others and so looked for examples within the handbook that they could follow so that they could progress;

- Difficulty in the ICT suite – tutors found introducing first years to Microsoft Excel and Geogebra very challenging. Some first year students were extremely proficient having watched the tutor/author use these programs in other lessons. Others, however, had a lot of difficult and took a lot of time to progress through the project. For this reason and because of the lack of time available in the ICT suite, some students did not complete the project.

5.9 Conclusion

This chapter has sought to outline the development and implementation of the intervention used in this project. It has outlined the aims of the intervention as well as how participants were selected and grouped within fourth and first year classes. Most importantly it gave a deep insight into the theoretical perspectives underlying the development of the student handbook and teacher guidelines. Problems associated with the implementation of the intervention were also alluded to. Chapter 6 will now present the key findings that emerge from both the quantitative and qualitative data.
Chapter 6: Research Findings

6.1 Introduction

This chapter will present the key findings emerging from the data analysis. The author used a combination of qualitative and quantitative research in order to gain different perspectives and allow conclusions to be drawn from the data. The quantitative approach saw pre and post questionnaires and diagnostic tests being distributed and collated. The quantitative approach included a series of focus groups with both first and transition year students. This chapter includes a description of the evaluation procedures and an examination of the findings in light of this mixed method approach. Other emerging themes will also be analysed and discussed and a summary of the findings will be outlined. Chapter 7 will discuss the findings and draw conclusions in light of the literary review conducted in chapter 2 and chapter 3.

6.2 Data Analysis - methodology

The data was collected between October 2011 and January 2012. This chapter will proceed to analyse the quantitative data, including the questionnaires and diagnostic tests followed by the qualitative data and the focus groups.

Figure 6.1 outlines the order through which the quantitative data will be analysed within this chapter. The questionnaire was analysed first, followed by the diagnostic tests results and then correlations between overall attitude scores and diagnostic test results. Each of the four subscales of the modified Fennema Sherman questionnaire: Confidence, Effective motivation, Usefulness and Anxiety were analysed followed by an examination of the overall attitude scores. The reliability of each of the scales was analysed using Cronbach’s Alpha scores. Within each section of analysis, first year scores were examined followed by transition year scores. Descriptive statistics revealed the mean and standard deviation for all items. Further, more in-depth analysis took place in the form of inferential statistics. Tests for normality highlighted that all data was normally distributed (p > 0.05), therefore paired sample t-tests were used to test for significance on each of the four subscales at pre and post intervention stage. A significance level of 5% was used for all statistical tests. Pearson’s correlation was used to see if a significant relationship exists between firstly, pre diagnostic test results and pre overall attitude and secondly post diagnostic test results and post overall attitude for both first and transition year data. Quantitative data was analysed using the SPSS (version 18) computer package.
Following an analysis of the quantitative data, the author continues by analyzing the qualitative data (focus groups) collected. The computer package NVivo (version 10) was used to perform this analysis. This chapter presents the main or emerging issues/themes that emerge from the data.

A discussion of findings will be presented in Chapter 7.
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6.3 School and students’ profiles

The research sample was taken from one second level school in an urban area in the Munster region. The author has taught in this school on a full time basis for four years. It is a co-educational, boarding and day school. First year students are between 12 and 13 years old and come from a wide range of backgrounds. They are in classes of mixed ability. All students in the transition year class used for this study have completed the Higher level Junior Certificate course. All students that acted as tutors received an ‘A’ at this level. In terms of gender, 64% of first year students are female and 36% are male, while the transition year tutors are all female. Male students within the transition year intervention class were unwilling to commit to tutoring first year classes, hence the dominance of female tutors.

Table 6.1 Students’ profiles

<table>
<thead>
<tr>
<th>Year</th>
<th>Number</th>
<th>Gender</th>
<th>Age</th>
<th>Previous Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Year</td>
<td>22</td>
<td>Male : 8</td>
<td>12 yrs : 5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Female : 14</td>
<td>13 yrs : 17</td>
<td></td>
</tr>
<tr>
<td>Transition year</td>
<td>5</td>
<td>Male : 0</td>
<td>15 yrs : 3</td>
<td>Junior Certificate ‘A’ grade: 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Female : 5</td>
<td>16 yrs : 2</td>
<td></td>
</tr>
</tbody>
</table>

6.3.1 Response rate

All guardians and students signed consent forms and agreed to take part in the study. Students completed the questionnaire/diagnostic tests during timetabled mathematics classes in school. The high level of consent and timetabled classes contributed to the response rate being 100% for the student questionnaires and diagnostic tests.

6.3.2 Attendance Rate

In general, the attendance of first years was high for the duration of the intervention but due to school activities and illness a number of students missed a number of classes. Four students missed 4 classes, three missed 3 classes and eight missed 1 class out of the twelve that it took to complete the intervention. Due to the nature of transition year, tutors were unavailable for 5 of the designated first year classes.
6.4 Quantitative Analysis (Questionnaire & Diagnostic tests)

Students completed a pre and post intervention attitudes questionnaire (Appendix E). They also completed a pre and post diagnostic test in order to assess their understanding of the topics taught (Appendix F). The data was collected from September-January 2011/2012 and was analysed using SPSS (version 18).

6.4.1 Reliability of Scales

The reliability of the scales was verified using a Cronbach Alpha Score for both pre and post intervention questionnaire’s. The value was calculated and is given in Table 6.1 below.

<table>
<thead>
<tr>
<th>Reliability Analysis</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence</td>
<td>.931</td>
<td>.914</td>
</tr>
<tr>
<td>Usefulness</td>
<td>.879</td>
<td>.926</td>
</tr>
<tr>
<td>Anxiety</td>
<td>.924</td>
<td>.885</td>
</tr>
<tr>
<td>Motivation</td>
<td>.871</td>
<td>.814</td>
</tr>
</tbody>
</table>

The Cronbach alpha value indicated very good reliability (> 0.8) for all subscales (confidence, effective motivation, anxiety and usefulness), therefore all data produced by the scales can be considered to be of a reliable quality.

6.4.2 The Questionnaire

The Fennema-Sherman attitude questionnaire (1976), (Appendix C) produced valid quantitative data allowing tests of significance to be carried out. This questionnaire is designed to test for any changes in students’ attitude towards mathematics. It is broken down into four subscales; confidence, anxiety, perception of usefulness and effective motivation. There were twelve statements in each scale, 6 negative statements and 6 positive statements. These 48 questions were randomly assigned to the overall questionnaire to avoid patterns emerging. Each respondent was asked to indicate their level of agreement or disagreement with each item. For the positive items, 1 = strongly disagree, 2 = disagree, 3 = not sure, 4 = agree, 5 = strongly agree. For negative worded statements the order was reversed (1 = strongly agree, 2 = agree, 3 = not sure, 4 = disagree and 5 = strongly disagree.) These scales mean that the higher the score the more positive the attitude. The highest possible score is 60 (12 x 5) for each of the subscales. This means that a score of 60 indicates the lowest possible anxiety level but the highest possible confidence, perception of usefulness and effective motivation level. The results of each of the four subscales are arranged in tabular form. Mean
scores and standard deviations are displayed for each of the twelve questions within each subscale. Below, mean scores and standard deviations are given for each statement in the confidence subscale.

**Attitude Subscale 1: Confidence**

*Table 6.3 Attitude subscale – personal confidence of first year students in subject matter*

<table>
<thead>
<tr>
<th>No.</th>
<th>Statement</th>
<th>N</th>
<th>Pre-Test Mean(SD)</th>
<th>Post-Test Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(1)</td>
<td>I am sure that I can learn maths</td>
<td>22</td>
<td>4.5(.74)</td>
<td>4.5(.74)</td>
</tr>
<tr>
<td>2(5)</td>
<td>I don’t think I could do advanced maths.</td>
<td>22</td>
<td>3.3(.98)</td>
<td>3.6(1.18)</td>
</tr>
<tr>
<td>3(9)</td>
<td>Maths is hard for me.</td>
<td>22</td>
<td>3.5(1.22)</td>
<td>3.6(1.05)</td>
</tr>
<tr>
<td>4(11)</td>
<td>I am sure of myself when I do maths.</td>
<td>22</td>
<td>3.7(.94)</td>
<td>3.5(1.01)</td>
</tr>
<tr>
<td>5(17)</td>
<td>I’m not the type to do well in maths</td>
<td>22</td>
<td>3.4(1.14)</td>
<td>3.5(1.1)</td>
</tr>
<tr>
<td>6(24)</td>
<td>Maths has been my worst subject</td>
<td>22</td>
<td>3.7(1.35)</td>
<td>3.8(1.34)</td>
</tr>
<tr>
<td>7(25)</td>
<td>I think I could handle more difficult maths</td>
<td>22</td>
<td>3.1(1.12)</td>
<td>3.2(1.81)</td>
</tr>
<tr>
<td>8(31)</td>
<td>Most subjects I can handle OK, but I just can’t do a good job with maths</td>
<td>22</td>
<td>3.8(.89)</td>
<td>3.6(1.22)</td>
</tr>
<tr>
<td>9(32)</td>
<td>I can get good grades in maths</td>
<td>22</td>
<td>3.5(1.01)</td>
<td>3.8(1.01)</td>
</tr>
<tr>
<td>10(37)</td>
<td>I know I can do well in maths</td>
<td>22</td>
<td>3.9(.92)</td>
<td>3.9(1.13)</td>
</tr>
<tr>
<td>11(41)</td>
<td>I am sure I could do advanced work in maths</td>
<td>22</td>
<td>3.4(.902)</td>
<td>3.6(0.91)</td>
</tr>
<tr>
<td>12(44)</td>
<td>I’m no good in maths</td>
<td>22</td>
<td>3.8(1.04)</td>
<td>3.8(1.07)</td>
</tr>
<tr>
<td>Total</td>
<td>Personal Confidence in Subject Matter</td>
<td>22</td>
<td>43.6(9.49)</td>
<td>44.1(8.75)</td>
</tr>
</tbody>
</table>

The minimum possible score for each statement is 0 and the maximum possible score is 5. The maximum total score is 60. The statement “I am sure that I can learn maths” had the highest mean (4.5) both pre and post intervention. This statement also had the lowest standard deviation (.74) which shows that there was the least amount of variance within this statement. The data also indicates that students believe they can do well in maths pre and post intervention (mean = 3.9, SD = 1.13) and get good grades (mean = 3.8, SD = 1.01) post intervention. Contradictory to this, however, with a mean of 3.8 and standard deviation of 1.07, the statement “I’m no good in maths” indicates that some students feel they cannot do well in maths.

The overall mean score increased from 43.6 to 44.1 and the standard deviation dropped from 9.49 to 8.75 for first year students. A paired sample t-test was carried out to test if the overall mean score on the pre intervention questionnaire was statistically different from that of the
post intervention questionnaire (\(H_0 = \) there is no difference between overall pre and post intervention confidence scores, \(H_a = \) there is a difference between overall pre and post intervention confidence scores). The test was not statistically significant (\(p > 0.05\)) so we therefore, accept the null hypothesis and can confirm that there is no difference between the means for the pre intervention questionnaire and the post intervention questionnaire.

Figure 6.2 Overall total scores for confidence from pre to post intervention for first year students

The boxplots above illustrate that the spread of data decreased from pre to post intervention. They also demonstrate that the median decreased over the course of the intervention from 47 to 46.

Table 6.4  Attitude subscale – personal confidence of transition year students in subject matter

<table>
<thead>
<tr>
<th>No.</th>
<th>Statement</th>
<th>N</th>
<th>Pre-Test Mean(SD)</th>
<th>Post-Test Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(1)</td>
<td>I am sure that I can learn maths</td>
<td>5</td>
<td>4.6(.89)</td>
<td>4.6(.89)</td>
</tr>
<tr>
<td>2(5)</td>
<td>I don't think I could do advanced maths.</td>
<td>5</td>
<td>4.2(.84)</td>
<td>3.4(.89)</td>
</tr>
<tr>
<td>3(9)</td>
<td>Maths is hard for me.</td>
<td>5</td>
<td>3.2(1.48)</td>
<td>3.4(1.34)</td>
</tr>
<tr>
<td>4(11)</td>
<td>I am sure of myself when I do maths.</td>
<td>5</td>
<td>4(.71)</td>
<td>3.2(1.1)</td>
</tr>
<tr>
<td>5(17)</td>
<td>I'm not the type to do well in maths</td>
<td>5</td>
<td>3.2(1.48)</td>
<td>3.8(1.1)</td>
</tr>
<tr>
<td>6(24)</td>
<td>Maths has been my worst subject</td>
<td>5</td>
<td>4.7(.58)</td>
<td>4.4(.89)</td>
</tr>
<tr>
<td>7(25)</td>
<td>I think I could handle more difficult maths</td>
<td>5</td>
<td>3.6(1.14)</td>
<td>3.4(1.14)</td>
</tr>
<tr>
<td>8(31)</td>
<td>Most subjects I can handle OK, but I just can't do a good job with maths</td>
<td>5</td>
<td>3.8(1.64)</td>
<td>4(1.22)</td>
</tr>
<tr>
<td>9(32)</td>
<td>I can get good grades in maths</td>
<td>5</td>
<td>4.4(1.34)</td>
<td>4.4(.89)</td>
</tr>
<tr>
<td>10(37)</td>
<td>I know I can do well in maths</td>
<td>5</td>
<td>4.6(.89)</td>
<td>4.6(.89)</td>
</tr>
<tr>
<td>11(41)</td>
<td>I am sure I could do advanced work in maths</td>
<td>5</td>
<td>3.6(1.14)</td>
<td>3.4(.89)</td>
</tr>
<tr>
<td>12(44)</td>
<td>I'm no good in maths</td>
<td>5</td>
<td>3.8(1.3)</td>
<td>4.2(.84)</td>
</tr>
<tr>
<td>Total</td>
<td>Personal Confidence in Subject Matter</td>
<td>5</td>
<td>47.6(10.43)</td>
<td>46.8(9.88)</td>
</tr>
</tbody>
</table>
reasonably high level of confidence pre intervention with the overall mean being 47.6 out of a possible 60. Responses to the statements “I am sure that I can learn maths” and “I know I can do well in maths” both had high mean scores of 4.6 and low standard deviations of .89 pre intervention. Interestingly the responses to these statements remained the same post intervention. The lowest mean scores pre intervention were for the statements “maths is hard for me” and “I’m just not the type to do well in math” and these scores both improved to 3.4 and 3.8 respectively.

Unlike the first years, the overall confidence of transition year students decreased post intervention from 47.6 to 46.8. The standard deviation also dropped from 10.43 to 9.88 which must be taken into consideration. A paired sample t-test was carried out to test if the overall mean score on the pre intervention questionnaire was statistically different from that of the post intervention questionnaire (Ho = there is no difference between overall pre and post intervention confidence scores, Ha = there is a difference between overall pre and post intervention confidence scores). The test was not statistically significant (p > 0.05) so we therefore, accept the null hypothesis and can confirm that there is no difference between the means for the pre intervention questionnaire and the post intervention questionnaire.

**Attitude Subscale 2: Motivation of the students**

The mean score and standard deviation for each statement in this subcategory are displayed in table 6.5 over leaf.
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Table 6.5  
Attitude Subscale –Effective Motivation of First Year students towards Mathematics

<table>
<thead>
<tr>
<th>No.</th>
<th>Statement</th>
<th>N</th>
<th>Pre-Test Mean(SD)</th>
<th>Post-Test Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (2)</td>
<td>Mathematics is enjoyable and stimulating to me</td>
<td>22</td>
<td>3.6(.91)</td>
<td>3.4(.85)</td>
</tr>
<tr>
<td>2 (8)</td>
<td>I don’t understand how some people can spend so much time on maths and seem to enjoy it.</td>
<td>22</td>
<td>3.2(1.13)</td>
<td>3(1.31)</td>
</tr>
<tr>
<td>3 (14)</td>
<td>I am challenged by maths problems I can’t understand immediately.</td>
<td>22</td>
<td>3.5(1.1)</td>
<td>3.7(1.13)</td>
</tr>
<tr>
<td>4 (18)</td>
<td>The challenge of maths problems does not appeal to me.</td>
<td>22</td>
<td>3.3(.98)</td>
<td>3.1(.99)</td>
</tr>
<tr>
<td>5 (19)</td>
<td>I would rather have someone give me the solution for a difficult maths problem than have to work it out myself.</td>
<td>22</td>
<td>3.6(1.18)</td>
<td>3.3(1.08)</td>
</tr>
<tr>
<td>6 (20)</td>
<td>I like maths puzzles.</td>
<td>22</td>
<td>3.5(1.01)</td>
<td>3.6(1.14)</td>
</tr>
<tr>
<td>7 (28)</td>
<td>I do as little work in maths as possible.</td>
<td>22</td>
<td>4.2(1.61)</td>
<td>3.8(1.81)</td>
</tr>
<tr>
<td>8 (35)</td>
<td>When a maths problem arises that I can’t immediately solve, I stick to it until I have the solution.</td>
<td>22</td>
<td>3.4(1.1)</td>
<td>3.1(1.16)</td>
</tr>
<tr>
<td>9 (36)</td>
<td>When a question is left unanswered in maths class, I continue to think about it afterwards.</td>
<td>22</td>
<td>3.1(1.28)</td>
<td>3.3(1.29)</td>
</tr>
<tr>
<td>10 (40)</td>
<td>Once I start working on a maths puzzle I find it hard to stop</td>
<td>22</td>
<td>2.8(.97)</td>
<td>2.7(1.08)</td>
</tr>
<tr>
<td>11 (43)</td>
<td>Figuring out mathematical problems does not appeal to me.</td>
<td>22</td>
<td>3.6(.5)</td>
<td>3.1(1.06)</td>
</tr>
<tr>
<td>12 (45)</td>
<td>Maths puzzles are boring</td>
<td>22</td>
<td>3.4(1.1)</td>
<td>3(1.21)</td>
</tr>
<tr>
<td>Total</td>
<td>Effective Motivation of the students</td>
<td>22</td>
<td>41.1(7.12)</td>
<td>39(6.28)</td>
</tr>
</tbody>
</table>

Results here illustrate that pre intervention students were slightly more effectively motivated with the mean score dropping from 41.1 to 39. The highest mean score of 4.2 (SD = .61) was for the statement “I do as little work in maths as possible” pre intervention. This dropped to a mean of 3.8 post intervention (SD = .81). The lowest mean score of 2.8 (SD = .97) was for the statement “Once I start working on a maths puzzle I find it hard to stop” and dropped further to 2.7 (SD = 1.08) post intervention. Response rates for first years dropped in relation to how enjoyable and stimulating they found mathematics and how challenging they found maths problems that they could not understand straight away. These results do indicate however, that first year students liked maths puzzles more and that figuring out maths problems became more appealing over the course of the intervention.

The overall mean score decreased from 41.1 to 39 and the standard deviation dropped from 7.12 to 6.28. A paired sample t-test was carried out to test if the overall mean score on the pre intervention questionnaire was statistically different from that of the post intervention questionnaire (Ho = there is no difference between overall pre and post intervention effective
motivation scores, $H_a = \text{there is a difference between overall pre and post effective motivation scores}$). The test was not statistically significant ($p > 0.05$) so we therefore, accept the null hypothesis and can confirm that there is no difference between the means for the pre intervention questionnaire and the post intervention questionnaire.

**Figure 6.3 Overall total scores for effective motivation of first year students pre and post intervention**

The box plots above (figure 6.3) illustrate that the spread of data was greater pre intervention in comparison to post intervention. They also indicate that the median score was higher pre intervention to post intervention for first year students.
Levels of motivation dropped for both first and transition year students post intervention. The first years overall mean score fell from 41.1 to 39. Transition year students began with a very high level of motivation towards mathematics (47.4 out of a total of 60) and this dropped to 47.2. The mean scores for all statements are above three or four both pre and post intervention indicating that they agree or strongly agree with the statements presented. Pre intervention, a very high mean of 4.8 and low standard deviation of .45 illustrates that students strongly agreed with the statement “I am challenged by maths problems I can’t understand immediately”. The mean for this statement dropped to 4.2 with the standard deviation remaining the same.

The main attributes for the small drop in the level of effective motivation appears to be transition years response to the statements ‘the challenge of maths problems does not appeal to me’, ‘once I start working on a maths puzzle I find it hard to stop’ and ‘maths puzzles are boring’. Transition year appear to dislike puzzles and problems solving more post...
intervention in comparison to pre intervention. They do, however, find mathematics more stimulating and enjoyable according to the above results.

A paired sample t-test was carried out to test if the overall mean score on the pre intervention questionnaire was statistically different from that of the post intervention questionnaire (Ho = there is no difference between overall pre and post intervention effective motivation scores, Ha = there is a difference between overall pre and post effective motivation scores). The overall mean score decreased from 47.4 to 47.2 and the standard deviation dropped from 10.9 to 10.64. The test was not statistically significant (p > 0.05) so we therefore, accept the null hypothesis and can confirm that there is no difference between the means for the pre intervention questionnaire and the post intervention questionnaire.

**Figure 6.4 Overall total scores of effective motivation of transition year students pre intervention**

![Figure 6.4](histogram_pre_intervention)

**Figure 6.5 Overall total scores of effective motivation of transition year students post intervention**

![Figure 6.5](histogram_post_intervention)
histograms above display the overall total scores for the transition year students at pre and post intervention stage. The normal distribution of the data can also be noted from these histograms.

**Attitude Subscale 3: Anxiety felt towards mathematics**

The mean score and standard deviation for each statement in this subcategory are displayed in table 6.7.

**Table 6.7  ** *Attitude subscale –anxiety felt towards mathematics by first year students.*

<table>
<thead>
<tr>
<th>No.</th>
<th>Statement</th>
<th>N</th>
<th>Pre-Test Mean(SD)</th>
<th>Post-Test Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (3)</td>
<td>Maths doesn’t scare me at all.</td>
<td>22</td>
<td>4.1(1.06)</td>
<td>4.1(.77)</td>
</tr>
<tr>
<td>2 (7)</td>
<td>Mathematics usually makes me feel uncomfortable and nervous.</td>
<td>22</td>
<td>3.7(1.16)</td>
<td>3.8(1.07)</td>
</tr>
<tr>
<td>3 (13)</td>
<td>It wouldn’t bother me at all to take more maths courses.</td>
<td>22</td>
<td>3.5(.86)</td>
<td>3.6(.96)</td>
</tr>
<tr>
<td>4 (15)</td>
<td>Mathematics makes me feel uncomfortable, restless, irritable and impatient.</td>
<td>22</td>
<td>3.8(1.01)</td>
<td>3.7(1.04)</td>
</tr>
<tr>
<td>5 (21)</td>
<td>I haven’t usually worried about being able to solve maths problems.</td>
<td>22</td>
<td>3.3(.98)</td>
<td>3.2(.99)</td>
</tr>
<tr>
<td>6 (23)</td>
<td>I get a sinking feeling when I think of doing maths problems.</td>
<td>22</td>
<td>3.8(.81)</td>
<td>3.6(1.18)</td>
</tr>
<tr>
<td>7 (26)</td>
<td>My mind goes blank and I am unable to think clearly when working mathematics.</td>
<td>22</td>
<td>3.5(1.37)</td>
<td>3.4(1.18)</td>
</tr>
<tr>
<td>8 (30)</td>
<td>A maths test would scare me.</td>
<td>22</td>
<td>3.1(.99)</td>
<td>3.3(1.42)</td>
</tr>
<tr>
<td>9 (34)</td>
<td>I almost never have got nervous during a maths exam.</td>
<td>22</td>
<td>2.9(1.11)</td>
<td>2.7(1.35)</td>
</tr>
<tr>
<td>10 (39)</td>
<td>Mathematics makes me feel uneasy and confused.</td>
<td>22</td>
<td>3.6(.95)</td>
<td>3.4(.91)</td>
</tr>
<tr>
<td>11 (47)</td>
<td>I usually have been at ease during maths tests.</td>
<td>22</td>
<td>3.1(1.23)</td>
<td>3(1.21)</td>
</tr>
<tr>
<td>12 (48)</td>
<td>I usually have been at ease in maths classes.</td>
<td>22</td>
<td>3.5(1.18)</td>
<td>3.7(1.09)</td>
</tr>
<tr>
<td>Total</td>
<td>Anxiety felt towards mathematics</td>
<td>22</td>
<td>41.9(9.56)</td>
<td>41.3(8.48)</td>
</tr>
</tbody>
</table>

It should be, again, noted that the higher the score the less anxious the student in this questionnaire. From table 6.7, we can see that the levels of first year anxiety were relatively low before the intervention began (41.9 out of 60) and rose by a small amount to 41.3. This means that students were slightly more anxious about mathematics post intervention. Interesting findings emerge from this data in relation to the statement “Maths doesn’t scare me at all” which had a mean of 4.1 for both pre and post intervention but for which the standard deviation fell from 1.06 to .77. Both pre and post scores indicate that the average
first year student agrees that mathematics doesn’t scare them at all and that they have usually been at ease in maths classes. The lowest mean score, 2.9 (SD = 1.11), is in relation to the statement “I almost never have got nervous during a maths exam” and this dropped to 2.7 (SD = 1.35) post intervention.

A paired sample t-test was carried out to test if the overall mean score on the pre intervention questionnaire was statistically different from that of the post intervention questionnaire (Ho = there is no difference between overall pre and post intervention anxiety towards mathematics scores, Ha = there is a difference between overall pre and post anxiety towards mathematics scores). The overall mean score decreased from 41.9 to 41.3 and the standard deviation dropped from 9.56 to 8.48. The test was not statistically significant (p > 0.05) so we therefore, accept the null hypothesis and can confirm that there is no difference between the means for the pre intervention questionnaire and the post intervention questionnaire.

The histograms below display the overall total scores of first years for this third subscale—anxiety towards mathematics. The first histogram (figure 6.6) gives the results from the pre intervention questionnaire and the second histogram (figure 6.7) gives the results for the post intervention questionnaire.

**Figure 6.6 Overall total score of anxiety felt towards mathematics for first year students pre intervention**

![Histogram](image.png)
Figure 6.7 Overall total score of anxiety felt towards mathematics for first year students post intervention

The mean score and standard deviation for each statement for transition year students in this subcategory are displayed in table 6.8.

Table 6.8  Attitude subscale –anxiety felt towards mathematics by transition year students

<table>
<thead>
<tr>
<th>No.</th>
<th>Statement</th>
<th>N</th>
<th>Pre-Test Mean(SD)</th>
<th>Post-Test Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (3)</td>
<td>Maths doesn’t scare me at all.</td>
<td>5</td>
<td>3.6(.55)</td>
<td>3.2(1.3)</td>
</tr>
<tr>
<td>2 (7)</td>
<td>Mathematics usually makes me feel uncomfortable and nervous.</td>
<td>5</td>
<td>3.8(1.3)</td>
<td>3.8(1.3)</td>
</tr>
<tr>
<td>3 (13)</td>
<td>It wouldn’t bother me at all to take more maths courses.</td>
<td>5</td>
<td>3.8(1.3)</td>
<td>3.8(1.3)</td>
</tr>
<tr>
<td>4 (15)</td>
<td>Mathematics me feel uncomfortable, restless, irritable and impatient.</td>
<td>5</td>
<td>4(1.22)</td>
<td>4(1.22)</td>
</tr>
<tr>
<td>5 (21)</td>
<td>I haven’t usually worried about being able to solve maths problems.</td>
<td>5</td>
<td>2.8(1.3)</td>
<td>3.2(1.3)</td>
</tr>
<tr>
<td>6 (23)</td>
<td>I get a sinking feeling when I think of doing maths problems.</td>
<td>5</td>
<td>3.8(.84)</td>
<td>3.8(1.1)</td>
</tr>
<tr>
<td>7 (26)</td>
<td>My mind goes blank and I am unable to think clearly when working mathematics.</td>
<td>5</td>
<td>3.4(1.14)</td>
<td>4.2(1.3)</td>
</tr>
<tr>
<td>8 (30)</td>
<td>A maths test would scare me.</td>
<td>5</td>
<td>2.6(1.14)</td>
<td>3(.71)</td>
</tr>
<tr>
<td>9 (34)</td>
<td>I almost never have got nervous during a maths exam.</td>
<td>5</td>
<td>4.2(.84)</td>
<td>3(1)</td>
</tr>
<tr>
<td>10 (39)</td>
<td>Mathematics makes me feel uneasy and confused.</td>
<td>5</td>
<td>3.2(1.3)</td>
<td>3.4(.89)</td>
</tr>
<tr>
<td>11 (47)</td>
<td>I usually have been at ease during maths tests.</td>
<td>5</td>
<td>4.2(1.3)</td>
<td>3(1)</td>
</tr>
<tr>
<td>12 (48)</td>
<td>I usually have been at ease in maths classes.</td>
<td>5</td>
<td>4(.71)</td>
<td>4.2(1.3)</td>
</tr>
<tr>
<td>Total</td>
<td>Anxiety felt towards mathematics</td>
<td>5</td>
<td>43.4(9.94)</td>
<td>42.6(11.41)</td>
</tr>
</tbody>
</table>
Transition year results are similar to the first year results in that students appear to have become slightly more anxious about mathematics post intervention (43.4 to 42.6; a .8 difference in scores). Four responses to questions remained the same pre and post intervention but statements with regard to anxiety in relation to mathematics tests appear to be the cause of the increase in anxiety for students. Pre intervention, the mean for the statement “I almost never have got never during a maths exam” was 4.2 (SD = .84) and then it dropped to 3 (SD = 1). This indicates that students became more nervous about mathematics tests post intervention.

Although anxiety towards mathematics was heightened over the course of the intervention, a paired sample t-test was carried out to test if the overall mean score on the pre intervention questionnaire was statistically different from that of the post intervention questionnaire (Ho = there is no difference between overall scores for students anxiety towards mathematics scores, Ha = there is no difference between overall scores for students anxiety towards mathematics). The overall mean score decreased from 43.4 to 42.6 and the standard deviation dropped from 9.94 to 11.41. The test was not statistically significant (p > 0.05) so we therefore, accept the null hypothesis and can confirm that there is no difference between the means for the pre intervention questionnaire and the post intervention questionnaire.

The histograms below display the overall total scores of transition years for this third subscale-anxiety towards mathematics. The first histogram (figure 6.8) gives the results from the pre intervention questionnaire and the second histogram (figure 6.9) gives the results for the post intervention questionnaire.

*Figure 6.8 Overall total scores for anxiety towards mathematics of transition year student’s pre intervention*
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Figure 6.9 Overall total scores for anxiety towards mathematics of transition year student’s post intervention

Attitude Subscale 4: Students’ perspective of the usefulness of mathematics

The mean score and standard deviation for each statement with respect to students’ perspective of the usefulness of mathematics is given below.

Table 6.9  Attitude subscale – first year students’ perspective of the usefulness of mathematics

<table>
<thead>
<tr>
<th>No.</th>
<th>Statement</th>
<th>N</th>
<th>Pre-Test Mean(SD)</th>
<th>Post-Test Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (4)</td>
<td>Knowing mathematics will help me earn a living.</td>
<td>22</td>
<td>4.5(.96)</td>
<td>4.2(1.07)</td>
</tr>
<tr>
<td>2 (6)</td>
<td>Maths will not be important to me in my life’s work</td>
<td>22</td>
<td>4.1(1.21)</td>
<td>3.3(1.58)</td>
</tr>
<tr>
<td>3 (10)</td>
<td>I’ll need mathematics for my future work.</td>
<td>22</td>
<td>4(1.15)</td>
<td>3.8(1.18)</td>
</tr>
<tr>
<td>4 (12)</td>
<td>I don’t expect to use much maths when I get out of school</td>
<td>22</td>
<td>4(.87)</td>
<td>4(1.13)</td>
</tr>
<tr>
<td>5 (16)</td>
<td>Maths is a worthwhile, necessary subject</td>
<td>22</td>
<td>4.3(.72)</td>
<td>4(1.11)</td>
</tr>
<tr>
<td>6 (22)</td>
<td>Taking maths is a waste of time</td>
<td>22</td>
<td>4.6(6)</td>
<td>4.5(1.86)</td>
</tr>
<tr>
<td>7 (27)</td>
<td>I will use mathematics in many ways as an adult.</td>
<td>22</td>
<td>4.1(1.13)</td>
<td>4.1(1.02)</td>
</tr>
<tr>
<td>8 (29)</td>
<td>I see mathematics as something I won’t use very often when I get out of school</td>
<td>22</td>
<td>3.8(1.05)</td>
<td>4(1.98)</td>
</tr>
<tr>
<td>9 (33)</td>
<td>I’ll need a good understanding of maths for my future work</td>
<td>22</td>
<td>4(.87)</td>
<td>4(1.98)</td>
</tr>
<tr>
<td>10 (38)</td>
<td>Doing well in maths is not important for my future</td>
<td>22</td>
<td>4(1.17)</td>
<td>3.9(1.92)</td>
</tr>
<tr>
<td>11 (42)</td>
<td>Maths is not important for my life</td>
<td>22</td>
<td>3.9(1.27)</td>
<td>3.9(1.11)</td>
</tr>
<tr>
<td>12 (46)</td>
<td>I study maths because I know how useful it is</td>
<td>22</td>
<td>4.1(0.97)</td>
<td>3.8(0.91)</td>
</tr>
<tr>
<td>Total</td>
<td>Perception of the usefulness of mathematics</td>
<td>22</td>
<td>49.2(8.3)</td>
<td>47.5(9.69)</td>
</tr>
</tbody>
</table>
The above table displays the first years’ mean scores for their perspective of the usefulness of mathematics. Results indicate that students either agree or strongly agree with all the statements within this scale as all, but one, of the results are above 3.5 (“maths will not be important to me in my life’s work” received a 3.3 post intervention). The highest mean, 4.6 pre intervention and 4.5 post intervention, suggest that students strongly disagree that taking mathematics is a waste of their time. Another result of note is that of the mean 4.5, pre intervention, for the statement “knowing maths will help me to earn a living”. This mean dropped to 4.2 but the standard deviation rose from .96 to 1.07. Before the intervention they strongly believed that maths would help them earn a living and whilst they still agree that this is the case, the overall mean scores relating to statements of this nature fell.

The overall mean score fell from pre to post intervention (49.2 to 47.5), whilst the overall standard deviation rose (8.3 to 9.69). A paired sample t-test was carried out to test if the overall mean score on the pre intervention questionnaire was statistically different from that of the post intervention questionnaire (Ho = there is no difference between the overall scores for student’ perspective on the usefulness of mathematics, Ha = there is a difference between the overall scores for student’ perspective on the usefulness of mathematics). The test was not statistically significant (p > 0.05) so we can accept the null hypothesis and can confirm that there is no difference between the means for the pre intervention questionnaire and the post intervention questionnaire.

**Figure 6.10 Overall scores for first year student’s perspective of the usefulness of mathematics from pre to post intervention**
The box plots above illustrate that the spread of data increased but the median of the data decreased. The pre intervention boxplot displays an interesting outlier. Student 6 received an overall score of 26 out of a maximum of 60 for her pre intervention perspective of the usefulness of mathematics. All other scores for the pre intervention ranged from 37 to 58.

Table 6.10  Attitude subscale – Transition year students’ perspective of the usefulness of mathematics

<table>
<thead>
<tr>
<th>No.</th>
<th>Statement</th>
<th>N</th>
<th>Pre-Test Mean(SD)</th>
<th>Post-Test Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Knowing mathematics will help me earn a living.</td>
<td>5</td>
<td>5(0)</td>
<td>5(0)</td>
</tr>
<tr>
<td>2</td>
<td>Maths will not be important to me in my life’s work</td>
<td>5</td>
<td>4.2(.84)</td>
<td>4.2(.84)</td>
</tr>
<tr>
<td>3</td>
<td>I’ll need mathematics for my future work.</td>
<td>5</td>
<td>4.2(.84)</td>
<td>4.2(.84)</td>
</tr>
<tr>
<td>4</td>
<td>I don’t expect to use much maths when I get out of school</td>
<td>5</td>
<td>3.8(1.3)</td>
<td>4(1)</td>
</tr>
<tr>
<td>5</td>
<td>Maths is a worthwhile, necessary subject</td>
<td>5</td>
<td>4(1.73)</td>
<td>4.6(.55)</td>
</tr>
<tr>
<td>6</td>
<td>Taking maths is a waste of time</td>
<td>5</td>
<td>4.6(.55)</td>
<td>4.4(.55)</td>
</tr>
<tr>
<td>7</td>
<td>I will use mathematics in many ways as an adult.</td>
<td>5</td>
<td>4.4(.89)</td>
<td>4.4(.89)</td>
</tr>
<tr>
<td>8</td>
<td>I see mathematics as something I won’t use very often when I get out of school</td>
<td>5</td>
<td>4.4(.55)</td>
<td>4.2(.84)</td>
</tr>
<tr>
<td>9</td>
<td>I’ll need a good understanding of maths for my future work</td>
<td>5</td>
<td>3.6(1.67)</td>
<td>4(1)</td>
</tr>
<tr>
<td>10</td>
<td>Doing well in maths is not important for my future</td>
<td>5</td>
<td>4.4(.55)</td>
<td>4(1.41)</td>
</tr>
<tr>
<td>11</td>
<td>Maths is not important for my life</td>
<td>5</td>
<td>4.6(.55)</td>
<td>4.4(.89)</td>
</tr>
<tr>
<td>12</td>
<td>I study maths because I know how useful it is</td>
<td>5</td>
<td>4.4(.55)</td>
<td>5(0)</td>
</tr>
<tr>
<td>Total</td>
<td>Perception of the usefulness of mathematics</td>
<td>5</td>
<td>51.6(5.32)</td>
<td>52.4(7.27)</td>
</tr>
</tbody>
</table>

Through analysing the data in table 6.8 it can be noted that transition year students are of an even stronger opinion that mathematics is useful having completed the intervention (51.6 to 52.4; a difference of .8). At pre-intervention stage, all five transition year students strongly agree that knowing mathematics will help them to earn a living (mean = 5). Post intervention, they also strongly agree (mean = 5) that they know how useful it is.

A paired sample t-test was carried out to test if the overall mean score on the pre intervention questionnaire was statistically different from that of the post intervention questionnaire (Ho = there is no difference between the overall scores for student’ perspective on the usefulness of mathematics, Ha = there is a difference between the overall scores for student’ perspective on the usefulness of mathematics). The overall mean score increased from 51.6 to 52.4 over the
course of the intervention. The standard deviation also increased from 5.32 to 7.27. The test was not statistically significant (p > 0.05) so we can accept the null hypothesis and can confirm that there is no difference between the means for the pre intervention questionnaire and the post intervention questionnaire.

*Figure 6.11 Overall scores of transition year students’ perspective of the usefulness of mathematics from pre to post intervention*

The boxplots above indicate that both the median and range of scores increased from pre to post intervention. Both scales begin above 40 illustrating that all results are very high for this subscale.

**Combined Subscale: First year students’ overall attitude to mathematics pre and post intervention**

Below is a summary of overall scores for each of the four sub categories included on the questionnaire; confidence, effective motivation, anxiety towards mathematics and perspective on the usefulness of mathematics. Mean and standard deviations are displayed in table 6.11. The overall attitude score of first year students pre-intervention was 175.8 out of a possible 240. This score dropped to 171.9. A paired sample t-test was carried out to test if the overall mean score on the pre intervention questionnaire was statistically different from that of the post intervention questionnaire (Ho = there is no difference between the overall scores for attitude towards mathematics, Ha = there is a difference between the overall scores for attitude towards mathematics). The test was not statistically significant (p > 0.05) so we can
accept the null hypothesis and can confirm that there is no difference between the means for the pre intervention questionnaire and the post intervention questionnaire.

Table 6.11 Attitude towards Mathematics – Summary of first year subscales

<table>
<thead>
<tr>
<th>Subscale</th>
<th>Pre</th>
<th>Post</th>
<th>t-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence</td>
<td>43.6(9.49)</td>
<td>44.1(8.75)</td>
<td>P &gt; 0.05</td>
</tr>
<tr>
<td>Effectance Motivation</td>
<td>41.1(7.12)</td>
<td>39(6.28)</td>
<td>P &gt; 0.05</td>
</tr>
<tr>
<td>Anxiety</td>
<td>41.9(9.56)</td>
<td>41.3(8.48)</td>
<td>P &gt; 0.05</td>
</tr>
<tr>
<td>Usefulness</td>
<td>49.2(8.3)</td>
<td>47.5(9.69)</td>
<td>P &gt; 0.05</td>
</tr>
<tr>
<td>Overall Attitude</td>
<td>175.8(29.63)</td>
<td>171.9(27.35)</td>
<td>P &gt; 0.05</td>
</tr>
</tbody>
</table>

The histograms below display the distribution of scores that students achieved for their overall attitude towards mathematics.

Figure 6.12 First year students’ overall attitude towards mathematics pre and post intervention.

Combined Subscale: Transition year students’ overall attitude to mathematics pre and post intervention

Overall scores for transition year students for each of the four sub categories from the questionnaire are displayed in table 6.12. Similar to the first year results, the overall attitude of transition years dropped from 190 to 189. A paired sample t-test was carried out to test if the overall mean score on the pre intervention questionnaire was statistically different from that of the post intervention questionnaire (Ho = there is no difference between the overall scores for attitude towards mathematics, Ha = there is a difference between the overall scores for attitude towards mathematics). The test was not statistically significant (p > 0.05) so we can accept the null hypothesis and can confirm that there is no difference between the means for the pre intervention questionnaire and the post intervention questionnaire.
Factors accounting for the drop in transition years’ overall attitude towards mathematics will be discussed in the next chapter.

Table 6.12 Attitude towards Mathematics – Summary for Transition year subscales

<table>
<thead>
<tr>
<th>Subscale</th>
<th>Pre</th>
<th>Post</th>
<th>t-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence</td>
<td>47.6(10.43)</td>
<td>46.8(9.88)</td>
<td>P &gt; 0.05</td>
</tr>
<tr>
<td>Effectance Motivation</td>
<td>47.4(10.9)</td>
<td>52.4(7.27)</td>
<td>P &gt; 0.05</td>
</tr>
<tr>
<td>Anxiety</td>
<td>43.4(9.94)</td>
<td>42.6(11.41)</td>
<td>P &gt; 0.05</td>
</tr>
<tr>
<td>Usefulness</td>
<td>51.6(5.32)</td>
<td>47.2(10.64)</td>
<td>P &gt; 0.05</td>
</tr>
<tr>
<td>Overall Attitude</td>
<td>190(32.54)</td>
<td>189(37.40)</td>
<td>P &gt; 0.05</td>
</tr>
</tbody>
</table>

The histograms below display the distribution of scores that students achieved for their overall attitude towards mathematics. It is clear, from these graphs, that overall attitude towards mathematics fell for transition year students over the course of the intervention.

Figure 6.13 Transition year students’ overall attitude towards mathematics pre and post intervention.

6.4.3 Pre and Post Diagnostic Test

A diagnostic test (Appendix F) was distributed before the intervention to the fourth year classes and also before any algebra was introduced to the students in the first class. A follow up diagnostic test was distributed at the end of the intervention to both fourth and first year students. The tests were designed to assess students’ levels of understanding of basic algebra. The response rate for the diagnostic tests was very high. One first year student (S1) and one transition year student (S24) were absent for the pre diagnostic tests.
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Pre and post intervention diagnostic test results for first year students

The author analysed the results of first year students pre and post intervention diagnostic test results. Means and standard deviations were calculated for both the pre and post results (see Appendix G). The author noted that nearly all test scores improved with only two first year students remaining consistent from pre to post intervention (students 7 and 18). Reasons accounting for students remaining consistent may be due to the following: student 7 missed four of the twelve intervention classes, student 18 has extreme difficulty with mathematics, in general. The post test results show that 15 of the 22 first year students received a grade C or higher which demonstrates that they had a reasonable level of understanding. The mean increased from 54% pre intervention to 70.14% post intervention, whilst the standard deviation increased by .08.

Having analysed the pre and post diagnostic tests, the author carried out a paired sample t-test to test if the overall mean score on the pre intervention diagnostic test was statistically different from that of the post intervention diagnostic test for first year students (Ho = there is no difference between students understanding of algebra from pre to post intervention, Ha = there is a difference between students understanding of algebra from pre to post intervention). The test was statistically significant (p < 0.05) so we can reject the null hypothesis and can confirm that there is a difference between the means for the pre intervention diagnostic test and the post intervention diagnostic test. The diagram below (figure 6.14) displays the first year results from the pre and post intervention diagnostic tests. It should be noted that pre result for student one is zero because the student was absent.

Figure 6.14 Pre and post diagnostic test results for first year students
Pre and post diagnostic test results for transition year students

The author analysed the results of transition year students’ pre and post intervention diagnostic test results. Means and standard deviations were calculated for both the pre and post results (see Appendix G).

All transition year students received an A (>85%) in all test scores. One particularly talented student received 100% in both pre and post diagnostic tests. Having analysed the pre and post diagnostic tests, the author carried out a paired sample t-test to test if the overall mean score on the pre intervention diagnostic test was statistically different from that of the post intervention diagnostic test for first year students (Ho = there is no difference between students understanding of algebra from pre to post intervention, Ha = there is a difference between students understanding of algebra from pre to post intervention). The test was not statistically significant (p > 0.05) so we can accept the null hypothesis and can confirm that there is no difference between the means for the pre intervention diagnostic test and the post intervention diagnostic test. The diagram below displays the first year results from the pre and post intervention diagnostic tests.

Figure 6.15 displays the pre and post diagnostic test results for transition year students. The graph confirms that the results of the transition year diagnostic tests remained reasonably consistent from pre to post intervention.

**Figure 6.15 Pre and post diagnostic test results for transition year students**

6.4.4 Correlation between diagnostic test results and overall attitude

As discussed earlier, the literature predicts a relationship between students’ ability in mathematics and students attitude towards mathematics (see section 2.5.2). The author wished to test if the same was true for this sample.
Pearson’s correlation was used to see if a significant relationship exists between firstly, pre diagnostic test results and pre overall attitude and secondly post diagnostic test results and post overall attitude for both first and transition year data.

**Correlation between diagnostic test and overall attitude for first year students**

The relationship between pre diagnostic test results and pre overall attitude for first years is not a statistically significant one (p < 0.05). It is however a positive (r = .3) one. The same is true for post diagnostic test results and post overall attitude as there is no significance (p < 0.05) but again the relationship is positive where r = .4. While it is a somewhat weak relationship it is still a positive one suggesting that those with high diagnostic tests results also have a high overall attitude towards mathematics. This is evident from the scatter plots below (figure 6.16 and 6.17).

*Figure 6.16 Correlation between pre diagnostic test results and pre overall attitude for first year students*
Correlation between diagnostic test and overall attitude for transition year students

The relationship between pre diagnostic test results and pre overall attitude for transition years is not a statistically significant one (p < 0.05). It is negative relationship (r = -.196). The negative nature of the pre intervention relationships indicates that students with high diagnostic results have lower overall attitude test scores. This is evident from the scatter plots below (see figure 6.18). The relationship between post diagnostic test results and post overall attitude is not a significant one either. The transition years all received 100% on their post diagnostic tests which meant that their scores were taken as a constant. This led to an inconclusive statistical analysis (see figure 6.19).

Figure 6.18 Correlation between pre diagnostic test results and pre overall attitude for transition year students
6.5 Qualitative Analysis (Focus Groups)

A series of focus groups took place after the intervention was implemented. As described by Silverman (2005), this approach allowed the author to start off with some data and then extract theories from this data. The aim of the focus groups is to provide the author with an insight into the students’ own attitudes and beliefs about the introduction of algebra and the use of transition year tutors in first year classes.

6.5.1 Focus group profiles

A profile of the students involved in the focus group can be seen in Appendix H and transcribed interviews are provided in Appendix I. The focus groups were chosen as a direct result of the grouping that students were placed in for the duration of the intervention (described in Chapter 3). There were four first year groups with 5-6 students in each. Three of the four first year groups and the transition year group took part in the focus group. The author was unable to complete the fourth first year group due to constraints put on the author’s timetable. Table 6.13 summarises the information for each focus group. A more detailed analysis of this information can be seen in Appendix H.
Table 6.13 Summary of focus group information

<table>
<thead>
<tr>
<th>Focus Group</th>
<th>Year</th>
<th>Age range</th>
<th>No.</th>
<th>Gender</th>
<th>Range of post diagnostic results</th>
<th>Range of post overall attitude score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1st</td>
<td>12-13</td>
<td>5</td>
<td>3 Female 2 Male</td>
<td>54% - 95%</td>
<td>138 - 200</td>
</tr>
<tr>
<td>2</td>
<td>1st</td>
<td>12-13</td>
<td>5</td>
<td>2 Female 3 Male</td>
<td>25% - 95%</td>
<td>129 - 231</td>
</tr>
<tr>
<td>3</td>
<td>1st</td>
<td>12-13</td>
<td>6</td>
<td>5 Female 1 Male</td>
<td>40% - 95%</td>
<td>141 - 197</td>
</tr>
<tr>
<td>4</td>
<td>4th</td>
<td>15-16</td>
<td>5</td>
<td>5 Female</td>
<td>100%</td>
<td>130 - 220</td>
</tr>
</tbody>
</table>

6.5.2 Research Instrument (qualitative study)

Four semi-structured focus groups were conducted between January and February 2012, the term following the intervention and collection of quantitative data. These took place during first year SPHE (Social, Personal and Health Education) classes as the author was free from her own timetable and so could conduct the focus groups. Focus groups occurred at weekly intervals as SPHE classes only occur once a week. Each focus group was constrained to 40 minutes as this is the length of an SPHE class.

Computer assisted qualitative data analysis (CAQDA) was incorporated through the use of the software package NVivo, a tool used to aid the researcher in analysis of qualitative data. The focus groups were guided by the modified Fennema-Sherman questionnaire and its subscales: Confidence, Usefulness, Effective Motivation and Anxiety. They were also guided by the research questions discussed in the introduction and methodology chapter. Table 6.14 outlines the topics to be introduced during the focus groups and the objective of each topic.
Table 6.14 Main topics to be introduced during focus groups

<table>
<thead>
<tr>
<th>Topic</th>
<th>Objective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attitude toward mathematics</td>
<td>To gain an insight into students’ opinions on their own attitude towards mathematics. To ascertain whether students enjoy mathematics, see its relevance, are motivated to complete mathematical tasks or feel some level anxiety in relation to mathematics.</td>
</tr>
<tr>
<td>Transition years tutoring first years</td>
<td>To establish strengths and weaknesses of collaborative peer teaching</td>
</tr>
<tr>
<td>Function-based approach to teaching algebra</td>
<td>To determine student’s opinion on the intervention and the use of the function-based approach as a method for introducing algebra.</td>
</tr>
</tbody>
</table>

6.5.3 Data Analysis (qualitative study)

The author conducted four stages within NVivo as follows:

- The focus groups were led and informed by the topics and objectives outlined in table 6.14.
- All responses were recorded, saved as a word document and imported into NVivo. Interviewees were coded using numbers (S1 = student 1).
- Following a careful analysis a list of nodes emerged from the data.
- Once the nodes had been developed and finalized the author then analyzed each of the nodes, identified trends both between and across nodes and extracted meaning from the data.

Within these restructured nodes, there were key issues that presented themselves (see table 6.15 below).
Table 6.15 Emerging Themes

<table>
<thead>
<tr>
<th>Emerging Themes</th>
</tr>
</thead>
<tbody>
<tr>
<td>- The effects transition year tutors have in a first year maths class.</td>
</tr>
<tr>
<td>- The effects tutoring first year students have on transition year students.</td>
</tr>
<tr>
<td>- The use of manipulatives for introducing algebra.</td>
</tr>
<tr>
<td>- Anxiety felt amongst students in relation to mathematics.</td>
</tr>
<tr>
<td>- Students thoughts on the usefulness of mathematics.</td>
</tr>
<tr>
<td>- The use of a function-based approach for introducing algebra to first year students.</td>
</tr>
</tbody>
</table>

The key issues that emerged as the most dominating themes from the focus group analysis relate to the introduction of algebra, the use of manipulatives, transition year tutors and the four subscales of the Fennema-Sherman questionnaire: Confidence, Usefulness, Anxiety and Effective Motivation. These themes will now be discussed in further detail.

6.5.4 The effects transition year tutors have on a first year maths class

Students were asked to comment on the experience of having transition year tutors in class. The majority of students noted that they found it to be a positive experience. An illustration of this can be seen when a first year comments:

*S6: Am oh, ya ya it was helpful having the transition years like coming around helping you, like being in groups just made it easier because you could ask more questions because there was like more teachers to one, a smaller group.*

This student recognises that the transition years were available to help them and that the group setting encouraged students to ask questions. Other students reiterated this point:

*S13: I thought it was brilliant that there was like someone assigned to a small group, whereas, when there is just one teacher teaching a class and you don't understand something, she's to try to help everyone, whereas one person with a small group means they can tell them what to do and how to do it.*
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S5: I like having them in cause they are helpful and they’re like if you don’t understand something you can ask them and they don’t give homework.

Interestingly, four students identified the student’s youth and inexperience of teaching as a bonus. Two such students commented:

S9: Well, they kind of, like they’ve been through this more recently than the teacher so they know what you’re going through and can help you and understand a bit more than the teacher.

S21: Ya they’re a lot more, sort of easy going, like Loni said, they’ve been through it more recently so understand and they’re a lot more fun than just the normal classes.

Thirteen of the sixteen first year responses indicate that they were comfortable and confident in group settings led by a transition year tutor. They appear to enjoy associating with transition year students and deemed the experience as positive. The other three students made reference to how they didn’t mind when the class was led by the teacher and that it ‘worked for them’ (S6). They also referred to the pace of lessons that were tutor led:

S2: I thought that it was a bit slower paced

S20: I suppose, they’re not as experienced as teachers so they might take a bit longer to explain something to people who can’t get it as fast.

6.5.5 The effects tutoring first year students have on transition year students.

Transition years had a slightly different angle on this aspect of the intervention. Firstly, they believed that the first year students worried about asking questions and that they were too shy to ask if they were genuinely unable to complete a task.
S27: I think they were kind of nervous, they wouldn’t ask questions. I had one in five that would ask questions and if you asked if they were ok they would say ya and then they would copy what you would say.

S27: Like if I was in first year I would have been embarrassed by the transition years, like you don’t want to look stupid in-front of them.

Secondly, it was, in fact, the transition year tutors that were very nervous in classes. They were introduced to algebra using traditional methods in first year and despite being introduced to algebra using the function-based approach at the beginning of transition year all tutors were not confident in their ability to teach algebra using this method.

S26: I enjoyed it, I’m not going to lie, I got kind of scared coming in because I thought it was pressure cause I didn’t know what I was doing so they were looking at me like ‘but you’re here to help us’

S23: Like you’re just trying to make it make sense yourself sometimes.

S27: I think it was just harder because we were trying to teach them a whole different way and I use really strange logic if I’m trying to figure something out, it’ll work but it won’t be the proper way but with this everything had to be structured.

S23: Like we had the rule and we just learned the rule but they had to figure out why and it’s hard to teach them why when I wasn’t sure of it myself at times.

Finally, two of the transition years discussed the merits of being a tutor. They discuss how teaching the material forced them to clarify the underlying concepts of algebra. Interestingly student 23’s overall attitude pre intervention was 217 and rose to 220 post intervention, whilst students 24’s pre intervention attitude was 217 and fell to 209 post intervention. These are very high scores as the highest overall attitude score possible is 240.

S23: It was fun but it kind of reminded you of the basics, one or two little things that you hadn’t remembered so it helped a bit with my maths.

S24: It kind of brought the algebra to life like how it could be used in every day, made it a bit more realistic.
Both first and transition year students identified the strengths and weaknesses associated with transition year students tutoring first year students. The weaknesses included the lack of clarity transition year students experienced when teaching this topic to first year students as well as the perceived shyness of first year students. The strengths, however, outweigh the weaknesses. These include the first years’ view that there were more opportunities to ask questions and that the responses they got were tailored to their needs and level. They also include the realisation by the transition years that were actually learning whilst tutoring.

6.5.7 Students’ thoughts on the usefulness of maths

Another node that emerged from the qualitative data was that of the discrepancies between students and their thoughts on the use of mathematics outside of school. Overall, thirteen of the first year students made reference to the need for some knowledge of mathematics in a real life context:

S10: Yes because when you leave school whatever job you do you’re going to need maths no matter what it is.

S2: It’s in your daily life, like if you’re going shopping, how much you need to spend or save to buy something.

S18: You have to use it for shopping and working and everything.

S9: Cause, we kind of, we use maths for everything really, like if we go out of school, most of us like even if they don’t maths like if they wanted to set up a bakery, they still need to be able to do maths, like set up accounts for that so its important.

S21: I’d say that practically every job would need maths because like even a shop keeper would need to count up the money, the change and even for us going into to town with our friends or something and we had like a 20 and we bought something we would need to know how much we would get back.

When asked about the need for algebra outside of school there was a wide range of responses. Five of the first year students commented that they were unsure of the value of algebra outside of school. These students post diagnostic test results ranged from 25% to 75% and overall attitude scores ranged from 129 to 173.
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S13: I think it is important in some ways but in others it’s not because like algebra, you’d never really use that in your everyday job whether as you would use multiplication, division, addition and subtraction.

S6: I kind of, I think that some jobs might not need as much maths as others and like I agree with Emily that you don’t really need algebra but you need multiplication, division, addition and subtraction and all that kind of stuff.

S2: Maths is a subject that you have to do for your life to help you along, algebra is a part of maths but I don’t see it as something that you actually need unless its part of your job say.

Five students (whose post test scores ranged from 60% to 95%) were able to identify, to some extent, a use for algebra outside of the school context. These students all had a high score for their post intervention overall attitude (173 to 231). Student 20 scored 231 out of a possible 240 on his post intervention score and was one of the only students able to give a concrete example of how algebra may be used outside of school (see below). It is also interesting to note that student 20 scored the highest on the post-test intervention scores.

S20: Cause if you have a job and you get in wages you might work per hour you could put in a letter instead of the number so then you could find out how much you earn per day by multiplying this and that, putting in the money you earn each day where the x was or the y was.

Also, the five students that scored the lowest on the diagnostic test were in the top ten lowest scores for overall attitude. These students were unable to find a use for algebra outside of a school setting.

All transition year students recognised the importance of mathematics in their futures:

S25: Ya I dunno, you need it for loads of stuff, like in college and everything, like it’d be more important than like Geography for getting into College and stuff. I suppose if you do honours it keeps your options open.

In conclusion, 81% of first year students responses recognised the need for basic mathematics in their future lives, be it as part of the process of progressing through school, college or on into their working lives. 69 % of first year students spoke about having a lot of difficulty associating the need for algebra with anything outside of a school context. 31% of first year
students were able to put algebra into some sort of context outside of school. 100% of the transition year students see mathematics as a way of keeping their career options open.

6.5.8 Anxiety felt amongst students in relation to mathematics

It appears, from the data, that there is a lack of anxiety amongst first year students in mathematics classes. Students did not comment or appear to want to speak about the topic. Only two of the sixteen first year students acknowledged some sort level of anxiety in relation to mathematics.

*Interviewer: Do you ever feel scared or anxious about doing maths?*

*Group: No response, heads shaking*

*S7: I don’t get nervous just annoyed when I can’t do a question*

*S15: I get nervous in tests*

*S17: head shakes*

*S22: A little bit sometimes until I get it I’m a bit scared that I won’t get it*

*S7: I’m not very good at maths. I don’t really get nervous, just have to get on with it, like I usually can’t do it*

The lack of response may be due to the environment created by the teacher and tutors throughout the intervention. This environment was a group based, tutor led environment in which students were encouraged to ask questions of each other, the tutor and teacher. The author believes, however, that there may be an element of students not wanting to admit to being anxious in front of their peers or in front of their class teacher. This belief has merit due to the fact that the author has spoken, on numerous occasions, with parents about this topic. Parents have openly talked about the nervous nature of their son/daughter before doing mathematics homework and before mathematics tests.

Transition year students take a different stance to the first years and talk openly and honestly about their anxiety in relation to mathematics. All 5 transition year students made some reference to being anxious at some stage during mathematics classes.
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S26: I find it really difficult in maths when there is a question that everyone else gets and I don’t. I really don’t like that cause then I’m like, am I missing something huge or do I really not get it and then I’m trying to get it and I really can’t and its really annoying.

S26: Sometimes if its ages and I haven’t got something but I wouldn’t say in every class.

S23: Sometimes when you didn’t get something, you couldn’t figure out why you didn’t get it.

The transition year tutors, as a group, are very ambitious and are driven young individuals. They place a big emphasis on doing well in school and see mathematics as a key element in their road to success. For this reason, these students appear to become anxious in classes as they are afraid that they will not be able to successfully complete the course.

6.5.9 Use of manipulatives for introducing algebra

Students had to engage with a lot of manipulatives throughout the intervention in order to successfully answer questions. Over 50% of first year students were very quick to identify manipulatives used:

S13: Ya we used the whiteboards and markers.

S21: Lollipop sticks

S9: Matches

S6: String

Responses were mixed with regard to their use. Four of the sixteen first year students noted that they had fun using the manipulatives. Five made reference to the manipulatives acting as a visual aid in helping them to complete questions:

S7: Ya you could but it was more fun to use them and I couldn’t picture some of the stuff unless I did it with the cubes.

S9: Am you can kind of see what’s happening, whereas when it’s explained in the booklet you’re kind of like ya but when you’re doing it with blocks you can see what’s happening, it makes it easier to work it out in your head.
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Two transition year students agreed that they found that the use of blocks, string, matchsticks etc. helped them to internalise what they were doing within questions. Two transition year students did not make reference to manipulatives within the focus group.

S23: But you actually see it

S26: You could see it and not think it’s just in your head, you had it in front of you!

One first year (who scored 91% on the pre diagnostic test and 95% on the post diagnostic test) and one transition year (who scored 95% on the pre diagnostic test and 100% on the post diagnostic test) did not see the merit in using concrete objects and found them to be fun but unnecessary:

S17: No not really, like the questions were easy, you could just draw the stuff out instead of using the things.

S27: I didn’t really think they were necessary like you were putting the diagram in your book. I think drawing would be faster.

Even though there was not a huge response rate in relation to the use of manipulatives, it appears from the data that more able students find the use of manipulatives unnecessary.

6.5.10 The use of a function-based approach for introducing algebra to first year students

Having introduced algebra using the function-based approach: students were subsequently taken through the traditional method of introducing algebra that was displayed in their textbook. This was a requirement of the school so that students would stay in line with their peers in other classes. Some of the first year focus groups took place after they had been taken through the algebra in their textbook. This led to some interesting responses when students were asked to discuss which way they would introduce algebra to first year students. Ten of the first year students appeared to enjoy using the handbook and found that it helped them to grasp a basic understanding of algebra.

S13: Ya we looked at patterns like in everyday life, whereas the book just kind of states out the questions.
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S9: Well I thought it was good to start off using it but once we had finished it I felt I was ready to move onto the stuff in the book.

S13: I think it was better to start off with the booklets because it showed us what algebra kind of was whereas the book just kind of laid it out and said do this sum now whereas the booklet had like patterns and showed us the algebra in nature and stuff.

S9: Ya like it’s been explained in more detail. I can kind of I feel I understand it more, like the letters are replacing something else.

The above students believe that the booklet gave them time to get to grips with the concept of algebra as it forced them to think about how they were answering questions. Four first year students, however, did not like this method. They found it difficult to grasp the types of questions being asked in the handbooks and preferred the structured, repetitive nature of the textbook:

S22: I didn’t really get where the letters were coming from like they were just there but it was easy cause you just copied the example.

S6: I actually preferred the book. Just the sums, I didn’t really like them, the algebra in the booklets. I just preferred when they gave you the sums and you had to work it out rather than making the stuff.

S2: I prefer the textbook, it does start off a bit hard but if you’re taught it the way you like it you learn it.

The responses above are very divided. One student made particular reference to the booklet being confusing whilst another made particular reference to how the textbook was confusing. Two students spoke about the benefits of both methods being introduced to first years. Three students opted for the textbook approach whilst eight students said they would prefer that the function-based approach be used to introduce algebra.

Transition year students were similar to the first year in that their opinions differed:

S24: It kind of brought the algebra to life like how it could be used in every day, made it a bit more realistic.

S27: I don’t know it kind of confused me more.
S26: I didn’t like it at all.

Two transition year tutors found that the function-based approach helped them to put algebra into a real life context. Two transition year tutors found the approach confusing. Three transition year tutors believed that the best approach would be to use the function-based approach followed by the textbook approach. When asked, if they had to choose between approaches, four responded that they would prefer the textbook. This may be due to the anxiety they felt whilst teaching the function-based approach to first years (see 6.5.5).

A small number of students (three first years and two transition years) recognised that the student handbook used a problem solving approach in contrast to the textbooks that just ‘state out the question’ (S13, Appendix I). Some students recognised that the problem solving approach helped them to have a better understanding of algebra, whereas others found it frustrating and difficult.

S26: Ya like sometimes the way the questions were phrased, I didn’t understand what they were asking me, like in the maths book, it would just say the question but with these you had to figure out the question first of all.

S25: Ya you’d be worrying about how to phrase answers without getting the actual maths answer. I thought the graphs were helpful though.

To summarise, students’ responses indicate that a dual approach to teaching algebra would be most beneficial. They believe that introducing algebra using a pattern based approach followed by lessons using a more traditional approach would be the best way of introducing algebra to first years.

6.6 Summary of Findings

At the beginning of this chapter the methodology used to analyse the intervention was outlined.

After examining the data the key findings to emerge are:

1. There was no significant difference \((p > 0.05)\) in students’ responses to the 48 statements presented to them pre and post intervention.
2. There was a statistically significant difference ($p < 0.05$) in first year students’ pre to post intervention diagnostic test results. This may suggest that a function-based approach to teaching algebra aided students’ understanding of basic algebra.

3. The data is inconclusive and often contradictory as to whether there is a link between the use of a function-based approach taught by transition year tutors and an improvement in students’ general attitude towards mathematics.

4. A strong theme that emerged from the data is that there is evidence that both parties benefit when transition year students tutor first year students and that a dual approach involving both a pattern based and traditional method of introducing algebra may help learning.

Each of these findings will be elaborated on in light of the literature review in chapter 2 in the following chapter “Discussion of Findings”. Overall conclusions will be drawn and discussed in chapter 8.
Chapter 7: Discussion of research findings

7.1 Introduction

This chapter will discuss the key findings that emerged from the analysis of the data. Findings from both quantitative and qualitative aspects of this research (as presented in Chapter 6) are compared and contrasted to that in the literature. The research questions outlined in section 1.5, guide the layout of this chapter.

7.2 Key Findings for the Research Questions

The first two research questions guided the review of literature that took place in Chapter 2 and 3.

1. What are the issues contributing to and theoretical perspectives underlying effective mathematics teaching which can stimulate and improve students’ attitude towards mathematics at Junior Cycle level?

2. What are the issues contributing to and theoretical perspectives underlying effective mathematics teaching which can improve students’ understanding of basic algebra at Junior Cycle level?

The final four research questions will now be discussed in detail with reference to the review of literature in chapters 2 & 3. These research questions are:

3. Does the introduction of algebra using a function-based approach improve first year students’ understanding of basic algebra?

4. Does the introduction of algebra using a collaborative peer teaching approach improve first and transition year students’ attitudes towards mathematics?
5. Does the introduction of algebra using a function-based approach improve first and fourth year students’ attitudes towards mathematics and in turn improve understanding?

6. Does the use of manipulatives improve students’ attitude towards mathematics and aid students in their understanding of basic algebra?

7.2.1 Key Findings for Research Question 3

‘Does the introduction of algebra using a function-based approach improve first year student’s understanding of basic algebra?’

As discussed, success in mathematics was, for a long time, determined by the ability to memorise procedures by rote (Bracey, 1992). With long-standing international and national disquiet that students are entering mathematics intensive courses with fewer of the basic mathematical skills essential for course success (Hourigan & O’Donoghue, 2005), a movement away from ‘memorising procedures by rote’ was deemed appropriate by the Project Maths movement in Ireland. This movement advocates the introduction of algebra through a function-based approach.

This intervention aimed to introduce algebra in this way using a student handbook developed by the author. In order to determine whether students’ understanding of the material improved, they had to complete a pre and post diagnostic test (Appendix E, discussed in chapter 4). Results (section 6.5.1) indicate that the transition year students, all of whom received an A in the pre and post diagnostic test, had a very good grasp of the concepts associated with the function-based approach to algebra. Results of the first year tests indicate that there was a significant improvement (p < 0.05) in their test scores. The post test results show that 15 of the 22 first year students received a grade C or higher which demonstrates that they had a reasonable level of understanding. Focus group analysis reflects these results to some degree with 10 students explaining how the handbooks ‘showed them what algebra was’ (see appendix I) and how it helped them to understand basic algebra.

These results do not reflect research by MacGregor & Stacey (1992) who investigated the effects of introducing a function-based approach to a group of students between the ages of 7 to 10. They found that students lacked an understanding of the meaning and use of algebraic notation. They also found that the students in their study had a lot of difficulty constructing
formulas from tables. The author found that whilst some students experienced difficulty, in
general, they displayed a good level of understanding of algebraic notation and ability to
construct formulas from tables. Reasons for the difference in findings may be attributed to the
age difference of the students in the studies. The first year students in this study were 12/13
years of age and transition year students were 15/16 years of age and were therefore reaching
a formal level of understanding in comparison to those in the study by MacGregor & Stacey
(1992) that were still at a concrete level of understanding.

Another interesting aspect that relates to transition year students’ understanding of algebra is
outlined by Thorpe and Wood (2000). They define cross-age tutoring (referred to as
collaborative peer teaching in this research) as “a form of cooperative learning in which an
older student, often one who can benefit from additional reinforcement, is paired with a
younger student who may or may not be in need of remediation” (p. 239). The author
acknowledges that the transition year tutors in this intervention were high achievers but that
they could still benefit from additional reinforcement of the concept of the function-based
approach which was new to them. One transition year student highlighted this in the focus
group:

S23: It was fun but it kind of reminded you of the basics, one or two little things that you
hadn’t remembered so it helped a bit with my maths.

7.2.2 Key Findings for Research Question 4

‘Does the introduction of algebra using a collaborative peer teaching approach improve
first and transition year students’ attitudes towards mathematics?’

Evidence suggests that collaborative approaches can promote positive attitudes amongst
students (e.g. Boaler, 1997a, b, 1998; Ridlon, 1999). This intervention sought to utilise
collaborative peer teaching as a means of improving first year attitude towards algebra and
more generally towards mathematics.

An analysis of the responses to the Fennema-Sherman questionnaire were categorised into
four subscales; confidence, anxiety, perception of usefulness and effective motivation using
SPSS (version 18). Overall mean scores indicate that first year students attitudes dropped
from 175.8 to 171.9 These results were not statistically significant (see section 6.11) but they
do indicate that attitude towards mathematics moved in a negative direction. The drop in first
year students’ overall attitude score is in stark contrast to the findings of Forsyth and McMillan (1991). They found that an environment involving collaborative peer learning develops students’ motivation and interest in mathematics. More recent research by Boaler (1997a,b, 1998) and Ridlon (1999) confirms this as it suggests that collaborative approaches can promote positive attitudes among students. McLeod (1994) offers a reason for the drop in first year students’ attitude; he found that attitudes tend to become more negative as students move from elementary to secondary school. These findings are also in line with those of Fennema and Sherman (1979) who found that transition from elementary to high school had a negative effect on students’ attitude towards mathematics. They found that the value of mathematics for students who move from teachers they perceive to be more supportive to teachers they perceive to be less supportive will decline. In this study, students were taught by peer tutors for the majority of the time and so the teacher’s role may have appeared less supportive to first year students which may have led to the decline in students’ perceptions of the usefulness of mathematics and in turn overall attitude towards mathematics.

The transition year overall attitude scores dropped from 190 to 189 post intervention. This was not a statistically significant result (see section 6.12) but was similar to the first year’s result in that it moved in a negative direction. These transition year overall mean scores, however, are very high for attitude when you consider that the highest score is 240. The very slight drop in their overall mean scores may be attributed to the rise in anxiety of students throughout this intervention. This anxiety is acknowledged in the results of the questionnaires, where transition year students overall mean score for ‘anxiety felt towards mathematics’ (see section 6.7) dropped from 43.4 to 42.6 indicating a rise in anxiety. Focus group analysis highlighted anxiety of students as a theme. Responses to a question in the focus groups which asked students about their anxiety levels in relation to mathematics generated some interesting data. One transition year student acknowledged that she got anxious when she ‘didn’t get something and couldn’t figure out why’. Another stated; ‘I’m not going to lie, I got kind of scared coming in’. These results are not in line with those of Townsend et al. (1998) whose study used multivariate procedures, pre-test, post-test and measures of mathematics self-concept and anxiety to examine whether positive changes in mathematics attitude occur during a programme of cooperative learning. The results which were found by Townsend et al. (1998) showed statistically significant positive changes for both mathematics self-concept and mathematics anxiety. A number of reasons may be attributed to this. Firstly, the study carried out by Townsend et al. (1998) was part of a twelve week programme which was in contrast to this intervention which only lasted just over two
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weeks. Secondly, the study by Townsend et al. (1998) involved undergraduate students, unlike this study which involved 15/16 year olds. The difference in age may have led the peer tutors to be less confident and more anxious in a cooperative learning environment.

Interestingly, research suggests that students inherently feel more comfortable in a didactic setting where routine questions and presentation mode are key (Cockroft Report, 1982). This study involved a classroom intervention that did not encourage a didactic approach. The increase in anxiety scores of both first and transition year students (see table 6.7 and 6.8) show that students may have felt more uncomfortable than usual in a classroom setting that did not encourage a didactic approach. Findings from the focus group support this as two of the transition year students acknowledge the difficulties first year students had with the cooperative learning environment; ‘I think they were kind of nervous, they wouldn’t ask questions’ (see section 6.5.6). However, Nickson’s (2000) research suggests that positive attitudes to mathematics can flourish when students recover from these initial discomforts. Also, studies by Carpenter et al. (1998), Cobb et al. (1991) and Verschaffel and De Corte (1997) support this finding as unhappiness was cited in just the beginning phases of their respective projects. The author (N. Sterritt) feels that the intervention she carried out for this research project was very short at just 14 classes and so students may not have had time to recover from these ‘initial discomforts’ (Nickson, 2000). This is also reflected in work by Hannula (2002) whose findings indicate that attitudes can change in a very short space of time but it is critical to note that a short space of time in his study was deemed to be half a year. To summarise, the short length of the intervention may have contributed to a drop in overall scores for attitude.

7.2.3 Key Findings for Research Question 5

‘Does the introduction of algebra using a function-based approach improve first and fourth year students’ attitudes towards mathematics and in turn improve understanding?’

This research question is particularly challenging to answer due to the complex nature of the relationship that exists between attitudes and understanding of mathematics (McLeod, 1992). In this study, results indicate that attitude declined (but not significantly) for both first and transition year students. Understanding, however, increased significantly (P < 0.05) for first year students with the mean increasing from 54% pre intervention to 70.14% post
intervention, whilst the standard deviation increased by .08. The transition year students’ results were not statistically significant due to the fact that all received an A (>85%) in both pre and post intervention test scores.

Research in the literature both supports and contrasts with such a finding. For example, according to Beaton et al. (1996) the TIMSS data reveal that 8th grade students with more positive attitudes had higher average mathematics achievement. Seggers and Boekaerts (1993) argue that learners’ beliefs about their capacities exert a strong influence on task performance. Another such study, conducted by the OECD (2004, p. 148), found that “students who are less anxious perform better regardless of other characteristics”. The results of the subscale relating to anxiety show that first year student’s levels of anxiety rose from 41.9 to 41.3 but not significantly. With regard to this subscale, the higher the score the less anxious the student. With the overall score being 60, first year students were not overly anxious in class. This is backed up by the findings of the focus groups. Students did not comment or appear to want to speak about the topic when it was brought up in the focus group interviews. Only two of the sixteen first year students that took part in the focus groups acknowledged some sort of level of anxiety in relation to mathematics (see section 6.5.7). This display of lack of anxiety may have helped students perform better regardless of other characteristics (OECD, 2004). These findings are backed up by Richardson and Suinn (1972) who developed the Mathematics Anxiety Rating Scale (MARS). They found results that suggest that high levels of mathematics anxiety interfere with achievement in mathematics. A meta-analysis study undertaken by Hembree (1990) based on 151 studies, confirmed this.

As discussed, the quantitative data reported a drop in overall attitude scores in this study but it can be noted that first year students’ scores under the subscale ‘confidence’, increased. This was not a statistically significant increase (see section 6.4.2). Focus group interviews also identify aspects within the intervention to which students had a positive attitude. Thirteen of the sixteen first year students that took part in the focus groups had a very positive attitude towards the use of peer tutors (see section 6.5.4). A reason as to why understanding increased in this study is offered by Middleton and Toluk (1999) and relates to these positive aspects of students’ attitude. They suggest that attitude can affect engagement in an activity. Both quantitative and qualitative data show that some level of positivity in attitude was evident in classes. This positive attitude increased students’ levels of activity in class which in turn helped to increase students’ levels of understanding of the topic. Zimmerman & Schunk, 2004 (p.323) contradict this view as they found that “instructional efforts that lead to positive
learning outcomes do not always produce sustained motivation and conversely, instructional efforts to boost motivation of students without simultaneously improving their learning processes or competencies do not always produce sustained achievement”.

Transition year results also indicate a decline in attitude but an increase in level of understanding. Svinicki (1991) suggests that students must organise information, make their own connections with the information and then apply it to new contexts in order to understand it. Within this intervention, transition year students were required to introduce first year students to algebra using the function-based approach. In order to teach this effectively, students understood that they had to have a deep understanding of this approach. They had to ‘organise’ the information and make their own ‘connections’ with the function-based approach in order to teach it effectively in a context which was very unfamiliar to them. These connections are highlighted in comments made by transition years in the focus group interviews (see section 6.5.6).

Although achievement in academic performance can be attributed to a complex and dynamic interaction between cognitive, affective and motivational variables (Volet, 1997), the findings of this study are similar to those of Papanastasiou (2000) which indicate that further research should be initiated to examine the influence of attitudes and beliefs on the mathematics outcome.

7.2.4 Key Findings for Research Question 6

‘Does the use of manipulatives improve students’ attitude towards mathematics and aid students in their understanding of basic algebra?’

Research by Sowell (1989) involved the results of 60 studies being combined to determine the effectiveness of mathematics instruction with manipulative materials. Students ranged from kindergarten through to post-secondary. These studies indicate that manipulatives can be effective in improving attitude towards mathematics provided teachers were knowledgeable about their use (Sowell, 1989).

First year students, according to focus group responses, indicate a positive attitude towards manipulatives and claim that they aided in their understanding of the topic (see section 6.5.8).
Chapter 7

Discussion

The overall findings of this project in relation to attitude, however, are in contrast to those of Goracke (2009). She used an action research approach to investigate the use of manipulatives and its impact on student attitude and understanding. She discovered that student attitude toward mathematics improved when greater manipulative use was infused into the lessons. This project indicated a decline in students’ overall attitude score when greater manipulative use was infused in lessons. She also found that students felt more confident that they understood the material, which translated into a better attitude regarding mathematics class. Interestingly, first year students overall mean scores in relation to the subscale confidence increased but not significantly from 43.6 to 44.1 and as previously discussed, understanding of basic algebra increased significantly. It may be possible that the greater use of manipulatives contributed to this but unlike Goracke’s (2009) study, this did not in turn increase overall attitude.

Two transition year students agreed that they found that the use of blocks, string, matchsticks etc. helped them to gain a deeper understanding of the concepts within a function-based approach (see section 6.5.8). One aspect that the author feels may have contributed to the decrease in attitude scores in both first and transition year scores, however, is that of manipulative use. As Sowell (1989) identifies, it is crucial that teachers are knowledgeable about how and why manipulatives are being used in lessons. Responses from the focus groups in relation to the manipulatives indicate that two of the five transition year students found them beneficial whilst two other students found them unnecessary. These responses indicate that some of these transition year students may be still transitioning from the concrete stage to the formal operational stage whilst others are at a formal operational stage and are able to move from a concrete to abstract levels of thought (Philips & Soltis, 1995).

Another finding by Sowell (1989) offers a reason as to why the use of manipulatives may not have had an impact of students’ attitude towards mathematics. Sowell (1989) found that long-term use of manipulatives was more effective than short-term use in improving attitude. As discussed, this intervention lasted for just over two weeks and cannot be deemed to be ‘long term’ as Sowell (1989) recommends for improving attitude.

7.3 Other Emerging Themes

Another theme that arose from the analysis of data was that of students’ beliefs about the usefulness of mathematics in their future education, vocation and other activities (Fennema & Sherman, 1979). All scores recorded for first year students and transition year students were
above 47 out a maximum of 60 which is very high. This means that both first year and transition year students see mathematics as something that is relevant to them and their lives. It must be noted, however, that first year scores decreased (but not significantly) from pre intervention to post intervention whilst transition year scores increased from pre intervention to post intervention (see tables 6.9 and 6.10). Focus group analysis offers some explanation for the drop in first year students’ results (transition year results will be analysed later in this section). Thirteen of the sixteen first years interviewed acknowledged that they needed mathematics in life to some degree. Only five of these students, however, were able to identify a use of algebra outside of school (this may have been the case pre intervention also!). Interestingly these five students had high scores for overall attitude (173 to 231) and high scores for post diagnostic tests (60%-95%). The student that scored the highest in overall attitude (231 out of 240) also scored the highest on the diagnostic test (95%) and was the only student that could give a very clear example of how algebra could be used outside of a school context. Also, the five students that scored the lowest on the diagnostic test were in the top ten lowest scores for overall attitude.

Through analysing the data the author suggests that the students’ lack of ability to relate algebra to the world outside the classroom may have caused the decline in first year students’ scores for perceptions of usefulness. This is a worrying finding because according to Fennema & Sherman (1979), if the value of mathematics decreases for many low-achieving students when they move from elementary to the junior high school environment, they may be especially likely to give up trying to achieve in mathematics. Problems with algebra in Irish classrooms have been long recognised. McConway (2006) identified algebra as an area of difficulty for students in his study based in Ireland. Similarly, Chief Examiners’ Reports for Junior and Leaving Certificate examinations conclude that questions that require the use of algebra are low scoring and unpopular. We will now look at transition year students’ perceptions of the usefulness of mathematics.

All transition year students were able to identify uses for and recognise the importance of mathematics outside of the school setting. In 1981, Fennema et al. studied the effects of an intervention programme on high school students’ intent to enrol in optional mathematics classes in high school and college. One of the focuses of the study was to stress the usefulness of mathematics outside of school and as it turned out, this focus played a large role in motivating students to take part in extra classes (Kloosterman & Stage, 1992). Transition year students’ perceptions of the usefulness of mathematics may be both an input and an
outcome of the intervention. The students may have chosen to take part because of this perception and, from the data, it can be noted that their perception of usefulness increased as a result of the intervention.

7.4 Conclusion

This chapter has sought to identify and discuss the key findings of this action research project. The research questions outlined in section 1.5 were used to guide this chapter. Chapter 8 will draw conclusions from the findings identified in this chapter. It will also describe the contribution that this research has made to existing knowledge in this field of mathematics education and its national and international significance will be evaluated.
Chapter 8: Conclusions, Contributions, Recommendations and Further Research

8.1 Introduction

The main aim of this chapter is to collate and present the main findings of this research. A summary of the thesis as well as the main conclusions drawn from the research questions are detailed at the beginning of this chapter. Following this, the contributions this research has made to the field of mathematics education are discussed. Finally, the author’s recommendations and possible directions for future research in this area are considered. The chapter concludes with final comments by the author.

8.2 Summary and Conclusions

Having identified a problem in her classroom the author designed and implemented an action research project whose overall aim was to improve the teaching and learning of algebra in the classroom. The research was focused on the development of an intervention for the teaching of algebra to a mixed ability first year class. A function-based approach to teaching algebra was used within the intervention. Transition year students acted as tutors for first year students so that a collaborative peer teaching environment was created. Through implementing such a classroom intervention, the author aimed to improve students’ attitude towards mathematics and develop students’ understanding of basic algebra.

An action research methodology saw the investigation move through a cyclical process. The first cycle involved the identification of a problem (Chapter 1) and a literature review in relation to the problem. The first chapter of the literature review (Chapter 2) focused on international and national research pertaining to current issues facing mathematics education. It also gave an overview of the role of affect and a more detailed analysis of the role of attitude in mathematics education. The author found that there was a plethora of research in this field but that it was limited in an Irish context. The second chapter of the literature review (Chapter 3) began by investigating research relating to the teaching of algebra. It concluded with an exploration of the function-based approach to teaching algebra. The author found a limited amount of research in this field internationally and could not find any
research based here in Ireland, even though the Project Maths syllabus now advocates this approach to teaching algebra.

Cycle 2, otherwise known as the action cycle, saw the development and implementation of the intervention (chapter 5). The Intervention required two documents to be produced, the ‘student handbook’ and the ‘teacher guidelines’, which were informed by the literature review in Chapters 2 and 3. These documents went through two cycles of amendments before being used to teach first year students. The implementation of the intervention witnessed both quantitative and qualitative data being gathered. A collaborative peer learning environment was established during the intervention as the main pedagogical tool. This saw transition year students coming in to class to tutor first year students. The intervention was deemed as successful in achieving the teachers overarching aim of improving the teaching and learning of algebra in the classroom.

The third stage in this cyclical process witnessed the write up of the thesis. An evaluation of the intervention took place followed by a presentation of the findings (chapter 6). Findings from the quantitative (questionnaires and diagnostic tests) were examined using the computer software program, SPSS version 18. Findings from the qualitative data (focus groups) were examined using the computer software program NVivo 20. The write up continued with a discussion of the key findings that emerged from the analysis of the quantitative and qualitative data (chapter 7). This chapter will now draw significant overall conclusions from the findings identified in Chapter 7 that relate to the research questions (see section 1.5) outlined at the beginning of this thesis.

8.3 Significant Overall Conclusions

The author will now present the significant overall conclusions relating to the research questions outlined in section 1.5. These conclusions are drawn from the literature review in Chapter 2 and 3 as well as the discussion of findings in Chapter 7 and are outlined below:

There is an abundance of research both nationally and internationally on issues pertaining to the teaching and learning of mathematics. There is also a plethora of research pertaining to the affective domain and mathematics education and more specifically, attitude in light of mathematics education. There is a distinct lack of research both
internationally and nationally, however, on collaborative peer teaching where it is the sole basis of an intervention. Whilst the function-based approach is the dominating theme of research in algebra in countries such as the US, it is more difficult to identify literature outside of this pertaining to this topic. The author was unable to find any research study in Ireland that focused on this as an approach for teaching algebra.

There was a drop in overall attitude scores of both first and transition year students post intervention. Reasons for this were contradictory and conflicted much of the literature. Possible explanations are explained in brief below:

- Attitudes tend to become more negative as students move from elementary to secondary school (McLeod, 1994).
- Levels in transition year students’ anxiety levels rose, possibly because they were too young and therefore not confident enough to act as peer tutors (Townsend & Wilton, 2010).

There was a significant rise in first year students’ understanding of basic algebra whilst transition year students either improved or received the same result pre and post intervention. Again, reasons for this were contradictory to the literature. Possible explanations are given below:

- First year students were approaching or were at a formal operational stage of development and so were able to grasp abstract concepts such as the use of variables to solve problems (Philips & Soltis, 1995).
- Transition year students benefitted from the extra reinforcement of concepts that occurred as a result of peer tutoring (Thorpe and Wood, 2000).

Findings relating to the complex nature of whether attitude affects achievement or vice versa were multifaceted:

- First year students’ scores in relation to anxiety found that anxiety levels rose throughout the intervention (though not significantly). Focus group analysis presented an opposite finding, that first years were not anxious in class. The OECD (2004, p. 148) found that “students who are less anxious perform better
regardless of other characteristics”. First year students’ understanding increased and so it may be possible that the focus group analysis was a truer reflection.

- Whilst overall attitude scores dropped, first year students’ scores in relation to confidence rose (but not significantly). Also, focus group analysis highlighted positive attitude towards peer tutors. It may be possible that these positive elements of attitude increased first students’ engagement in activities and hence increased students’ levels of understanding (Middleton and Toluk, 1999).

- Whilst transition year students’ scores in attitude decreased, their understanding of the topic increased. As highlighted earlier, a rise in anxiety may have led to the decrease in attitude scores but this anxiety may have been due to the way in which transition year students had to organise information for themselves, make their own connections with and apply these connections to new contexts. Svinicki (1991) cites this as a necessary process for understanding.

- Findings relating to the use of manipulatives are again contradictory and difficult to summarise. Findings from the focus groups strengthen research that links manipulative use to better understanding. Whilst overall scores in attitude dropped for first year, focus group responses indicate that this was not related to the use of manipulatives. This is different for transition years, however. Focus group responses indicate that the use of manipulatives may have attributed to a drop in attitude scores as transition years felt they were unnecessary. Again, these findings may link to the fact that transition year students should all be at a formal operational stage of development and therefore should be able to move from concrete to abstract thought processes (Philips & Soltis, 1995). First year students are approaching or have just entered this formal operational stage and so may find that manipulatives are a good way of bridging the gap between the concrete and abstract thought processes. In addition, transition year students may not have experienced the use of manipulatives in mathematics until now, unlike first year students who may be more open to the use of manipulatives due their experiences of using them in primary school.

A final theme emerged from the analysis of data. It relates to the students’ beliefs about the usefulness of mathematics in their future education, vocation and other activities. Both first and transition year students appear to see mathematics as useful outside of school. First year scores for perception of usefulness of mathematics, however, dropped post intervention. The author attributed this drop to the large majority (69%) of first year
students who were unable to relate algebra to life outside of the classroom. The author was interested to note that the students that could relate algebra to a real life context, were the students that scored very highly on the overall attitude scores and highly on the post diagnostic test results.

8.4 Thesis Contribution

This project has made contributions to research in mathematics education on a number of levels. These levels include international and third level research in mathematics education. They also include contributions to reform that are currently taking place at a national level and contributions to teachers of mathematics who are the implementers of this reform movement.

Contributions made internationally and to third level pertain to both the topics that were investigated and the gaps in research that the author has discovered whilst completing her thesis. She has discovered that there is a gap in the literature relating to collaborative peer teaching. Research that involves an intervention solely investigating collaborative peer teaching in education is minimal both in an international and Irish context. The author also found that whilst there was not an abundance of literature on the best methods of implementing a function-based approach or research involving the function-based approach internationally, there was no literature on this approach at a national level.

This research took place during a disruptive time in mathematics education in Ireland. The research began in 2010 during which time the first phase (strands 1 and 2) of the Project Maths syllabus was being rolled out in schools nationwide. This research is of significance to the Project Maths team as the new mathematics syllabus encourages the use of such an approach to teaching algebra (NCCA, 2011). On a national level, this research may strengthen the argument for encouraging such an approach to be taken in Irish classrooms as it was found to increase students’ understanding of basic algebra significantly. This research also found that the inclusion of active learning methodologies and use of manipulatives in classrooms may contribute to aiding students’ understanding of basic algebra and may improve first year students’ confidence but not overall attitude in relation to mathematics. This is also relevant at a national level as these approaches are advocated by the Project Maths syllabus. A note of caution is offered by Sarason (1993) in relation to research involving interventions etc. and its contribution. He claims that there is an abundance of
literature that has involved interventions and other research methodologies and that these have advocated many changes in how classrooms function. He notes that many of these changes have taken place in the form of reform and that the widespread failure of these reforms to take hold has been well detailed (Sarason, 1993).

At a local level, the author believes that this study is directly relevant to second level mathematics teachers. This is because the student handbook and teacher guidelines offer a resource for teachers nationwide who wish to introduce algebra using this approach. Teachers may not choose to use it as a replacement for the textbook or guidelines outlined by the Project Maths team but it can offer some interesting exercises that may contribute to a teacher’s instruction. Boaler (1997, 2002) found that students who worked predominantly through textbooks found it difficult to apply their mathematics in different situations. The author in her role as a teacher deems that this research project will very much contribute to her own classroom experience. Firstly, it has informed her of many different approaches to teaching mathematics and more specifically algebra. It has also led to the development of resources that she will use in the future.

8.5 Limitations of the Study

Limitations to this study have been discussed on numerous occasions (see section 4.9, 7.2 and 8.2). The main limitations included:

- The first limitation is the size of the student sample; 22 first year students and 5 transition year students were investigated in this research. The author believes that this sample size is too small and that the students were too local (i.e. the author had taught the students prior to the investigation);

- The second limitation concerns the context in which the research took place. It was carried out in one particular school with one particular teacher over a very short time period; 14 classes (about three weeks) meaning unique factors could be at play when analysing the data produced by the students. This raises concerns over the external validity or the ability to claim that the results can be generalised to a wider population. Also a three week intervention does not provide enough time for students initial discomfort at changes in pedagogical approaches to be overcome. This means that the data gathered may not be a reliable guide as to long-term effect of these approaches on attitudes and understanding.
Chapter 8

Conclusion

Although there are clearly defined limitations in this research study the author feels that due to the mixed method approach used, these limitations will not have a significant impact on the outcome or results of the study.

8.6 Recommendations

The author wishes to make a number of recommendations based on the findings and conclusions reached during this research:

1. The adoption of a function-based approach to teaching algebra as a method for introducing algebra to first year students. This is in-line with the aims of the Project Maths syllabus (NCCA, 2010). She recommends that this introduction be complemented with a traditional approach to teaching algebra to help students gain a better understanding of algebra;

2. A collaborative peer teaching environment be established in and between classes. The author recommends that tutors are given defined topics to teach over short periods of time. A review of ten studies by Maida (2012) showed that results from each study suggest that peer tutoring has a positive effect on adolescents;

3. Problem solving, as an active learning methodology, including group work should be included within lessons. Communities of practice (i.e. group work) are one of three problem solving strategies recommended by Lesh and Zawojeski (2003);

4. The inclusion of hands on and virtual manipulatives in classes, particularly for younger and less able students. The author recommends that the teacher/tutor be very familiar with the manipulatives and how it is best used in the classroom to aid students in their understanding. The author also recommends that students are given time to become familiar with the manipulatives so that they too understand how they work and how they can aid in understanding;

5. A continuation of the Project Maths examinations which give students the scope to solve problems in a variety of ways. Representational fluency is one of the three problem solving skills advocated by Lesh and Zawojeski (2003) who believe that it is important in an information age, in knowledge economies, in learning organisations and in global societies where people need to be able to work together to describe and
explain the systems within which they operate. Also, O'Donoghue and Maguire (2005, p.438) highlight that the “Irish education system has been very successful in developing a well-educated and highly skilled workforce but the knowledge economies of the future need lifelong learners who are competent, capable, adaptable, flexible and innovative – an area where governments admit that national education systems are not yet sufficiently successful (e.g. OECD, 2004b)”;

6. The author’s final recommendation is that mathematics teachers are actively encouraged to undertake action research studies. If real change is to occur teachers need to be made feel more part of reform movement. Battista (1999) clarifies this by explaining that once teachers fully understand and believe in the reform movement, they will lead the way in implementing it.

8.7 Further Research

On completion of this action research, the author believes that teacher educators and government bodies have a duty to investigate issues raised in this thesis such as the effects of collaborative peer teaching and the use of the function-based approach for introducing algebra. She believes that action needs to be taken in the following ways:

- More research needs to take place on the effectiveness of peer tutoring in secondary schools. According to Maida (2012) many of the empirical studies see peer tutoring being implemented in conjunction with another form of instruction. As multiple forms of instruction can affect results, peer tutoring cannot be given sole credit for changing students’ attitude or understanding of mathematics (Maida, 2012) and so peer tutoring needs to be the main focus of a research investigation.

- The author believes that further research is needed on the use of a function-based approach for introducing algebra. A quantitative analysis involving a much larger sample of first year students should take place.

8.8 Final Comments

This intervention has provided the author and reader alike with some insights into the implementation of an intervention in an Irish classroom. As Cohen and Mannion (1994) argue, it is the view of the practitioner/researcher that is of greatest importance in action
research projects. This study was initiated after the identification of a specific problem within the author’s mathematics classroom, which the intervention succeeded in solving to the satisfaction of the practitioner/researcher. The author believes that the success of this research is three-fold. Firstly, both the first and transition year students involved in the study were exposed to a new and exciting pedagogical approach within the classroom. Secondly, they were exposed to the importance and significance of patterns in the world around us that they can refer to in future years of study. Finally, the author believes that she gained invaluable experience carrying out an action research project of this nature. The author felt that she developed a deeper understanding of issues pertaining to mathematics education and her own teaching. With the introduction of the Project Maths syllabus representing a significant change in how mathematics is taught in Ireland (NCCA, 2010), the author feels motivated to continue researching and is excited to implement new pedagogical approaches in the classroom.
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Appendices


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Appendices


APPENDICES
Appendices are on a CD attached to the back of this thesis.