An Investigation into the Integration of Mathematics and Science at Junior Cycle in Irish Post Primary Schools.

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For the award of Doctor of Philosophy.

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Submitted to the University of Limerick, November 2012.
Abstract

Within the mathematics education community there have been calls for a greater range of assessment practices as well as a more holistic approach to learning as research has shown that the current approach to instruction that has generally been adopted is producing students who struggle to solve problems and display large gaps in their Mathematical knowledge and understanding. One of the main necessities, according to research, is the need for mathematics to be placed in context and, thus, linked with other subjects. As such, a range of international education groups (NCTM, NRC, SSMA, Curriculum Corporation) have lent their support to the drive to integrate mathematics with other subject areas, especially science, within second level education.

Attempts at integrating mathematics and science have been made but no definitive, widely adopted teaching model has been developed to date. Research suggests that hands-on, practical, pupil-centred, authentic activities should form a central element when designing an effective model for the integration of mathematics and science. The ‘Authentic Instruction’ model, developed by Fred Newmann and his associates in the early 1990’s, provides the basis for a model for the integration of mathematics and science as it is integrative in its very nature, and there is considerable empirical evidence backing up its merits. The author has taken the key elements of ‘Authentic Instruction’ and modified them to produce a new model entitled ‘Authentic Integration’ which caters for the specific needs of integration of mathematics and science. This model requires that each lesson be based around a rich task which relates to the real world and ensures that hands-on group work, inquiry and discussion are central to the lesson.

This teaching model was tested through an intervention which was carried out in four Irish post-primary schools. Six lessons which integrated mathematics and science were created for 2nd year pupils using the Authentic Integration model, three of these lessons were implemented in each school. Analysis of the intervention was completed using teacher interviews, assessment of pupil work,
pupil focus groups, and teacher questionnaires. It was found that the approach employed positively affected pupil understanding; integration of mathematics and science can be incorporated into regular tuition in Irish post primary schools; and the teachers that completed the intervention displayed a very positive attitude towards the approach, intimating that they would continue to implement the practice in their classrooms.

Furthermore, testing of this model led to the creation of explicit design principles for the integration of mathematics and science which will guide mathematics and science teachers in the development of their own lessons to integrate the subjects.
Declaration

This thesis is presented in fulfilment of the requirements for the degree of Doctorate of Philosophy. It is entirely my own work and has not been submitted to any other University or higher education institution, or for any other academic award in this University. Where use has been made of the work of other people it has been fully acknowledged and fully referenced.

Signature: ______________________________

Páraic Treacy

November, 2012
Dedication

A hero is an ordinary individual who finds the strength to persevere and endure in spite of overwhelming obstacles – Christopher Reeves

To my parents, Pat and Margaret, for all that they have endured and overcome so that I may succeed.
Acknowledgements

To my supervisor Professor John O’Donoghue, I count myself extremely fortunate to have spent the last three years under your tutelage. Your instruction and guidance throughout this process is greatly appreciated.

To Dr. George McClelland for his input as co-supervisor throughout this process.

To my family, for all your encouragement and support.

To Maria, for being there. You’re weird but I like you.

To Dr. Olivia Fitzmaurice and Dr. Jenny Johnstone, I’m so grateful to both of you for providing direction and advice at various stages of my study.

To all the gang: Mark, Brian, Fiona, Ricky, Lisa, Gráinne, Niamh, Miriam, Máire, and Paddy – thanks for the help and, more importantly, the laughs.

To the teachers and pupils who took part in the intervention, thanks for your time and effort.

To the National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL), the Department of Mathematics and Statistics in UL, and Kilkenny County Council, thank you for your valuable support, both financial and otherwise.
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Chapter 1

Introduction
1.1 Introduction

One would have to have completely forgotten the history of science so as to not remember that the desire to know nature has had the most constant and the happiest influence on the development of mathematics.

Henri Poincaré

Integrating elements of mathematics and science has long been an issue of discussion amongst academics, with the practice enjoying waves of popularity over a number of decades (Furinghetti and Somaglia 1998, Hirst 1974). Education groups such as the National Council of Teachers of Mathematics (NCTM), the National Research Council (NRC), the Curriculum Corporation (Australia) and the School Science and Mathematics Association (SSMA) have endorsed the integration of Mathematics and Science, while this practice has also been strongly supported by a number of academics (Berlin and White 1992, Furner and Kumar 2007, Daniels et al. 2005, Miller and Davison 1999, Pang and Good 2000).

Brain research would appear to support the integration of mathematics and science as findings reflect positively on the practice of thematic teaching (Cohen 1995) while the Carnegie Council on Adolescent Development (1989) found that integration of content areas aids students’ ability to think critically. It has also been determined that there are significant similarities between the NCTM’s (2009) key mathematical process standards and the NRC’s (1996) 5 E’s – the instructional stages and processes through which students learn science – hence indicating that the characteristics of each subject are such that tuition within both can be carried out simultaneously through integrated instruction, enabling enhanced learning (Bossé et al. 2010).

McBride and Silverman (1991, pp. 286-287) listed the following as the main reasons for integrating mathematics and science:

2
1. Science and mathematics are closely related systems of thought and are naturally correlated in the physical world.

2. Science can provide students with concrete examples of abstract mathematical ideas that can improve learning of mathematics concepts.

3. Mathematics can enable students to achieve deeper understanding of Science concepts by providing ways to quantify relationships.

4. Science activities illustrating mathematics concepts can provide relevancy and motivation for learning mathematics.

These benefits, along with some of the issues prominent in mathematics education (to be discussed later), and the research backing the integration of mathematics and science, sparked the author’s interest in carrying out an investigation into this facet of education. As such, in this thesis, the author explores some of the issues pertaining to mathematics education, both in Ireland and internationally, and how the adoption of an integrative approach aids in solving these issues.

The major challenge of this research was to develop a model and, subsequently, lessons which would successfully integrate mathematics and science in a manner which was theoretically sound and attractive to the practitioners, i.e. the teachers, as well as being effective in providing a positive learning experience for the pupils involved. The process of taking on this challenge, and the findings and conclusions which were drawn from the experience are recorded and analysed throughout this thesis.

1.2 Background to the Research

The author, with his background being firmly rooted in mathematics education, has approached this study from a mathematics perspective rather than a science perspective, thus this has had a great influence on his motivations for conducting
this investigation. Such motivations can be traced to the present issues that are regularly flagged in mathematics education.

The current state of Irish mathematics education has come in for some criticism for the performance of its pupils and the way in which mathematics is taught. In 2003 and 2006, the OECD (Organisation for Economic Co-operation and Development) implemented their Programme for International Student Assessment (PISA) which assesses 15 year olds in OECD member countries in the areas of mathematics, reading literacy, science and problem solving. Mathematics is assessed in a holistic manner, gauging the students’ ability to analyse, reason and communicate effectively within a range of problems that require the use of quantitative, spatial, probabilistic or other mathematical concepts (OECD 2007). The results of the 2003 and 2006 assessments showed that Irish students are performing at a distinctly average level when compared to other OECD countries as they finished 17th of 29 and 16th of 30 in those respective years, scoring very close to the average mark on both occasions (OECD 2004, OECD 2007).

The manner in which mathematics has been taught in Ireland is at the root of such mediocre performances as it has been described as being overly didactic, with the drill and practice approach forming the central focus of each lesson while there is little or no importance placed on the explanation of concepts, and few opportunities to use the mathematics learned in applied problem contexts or everyday situations familiar to students (Lyons et al. 2003, Childs 2006). This results in many students, upon entry into third level education, lacking the ability to apply their mathematical knowledge, except in the most basic or familiar contexts (NCCA 2005b, O'Donoghue 2004).

This approach to mathematics education can be traced back to the adoption, in 1964, of a curriculum based on the ‘New Maths’ movement which had originated in the U.S. (Oldham 1991). ‘New Maths’ (also referred to as ‘Modern Maths’), was a movement which radically altered the manner in which mathematics was
taught and, also, adjusted the focus of the subject (Schoenfeld 2004a). Through this movement, the U.S. mathematics education system began to incorporate new aspects of the subject such as set theory, modular arithmetic, and symbolic logic – an obvious shift towards more ‘abstract’ forms of mathematics (Schoenfeld 2004a). Structure, proof, generalization, and abstraction were at the core of this reform (Jones and Coxford Jr 1970).

The mathematics curriculum in Irish post primary schools was radically changed to adopt the principles of New Maths (Oldham 2001). It maintained these principles even though, internationally, the New Maths movement was being rejected with the development of Realistic Mathematics Education in the Netherlands (Van den Heuvel-Panhuizen 2000) and the release of the NCTM’s ‘An Agenda for Action’ in 1980 (Schoenfeld 2004a). Later, the development of Standards by the NCTM in 1989 led to a period of debate and change within the American mathematics education which became known as the ‘Math Wars’ (Klein 2007).

Meanwhile, in Ireland, revision of the Leaving Certificate syllabus did not come until the 1990’s: new ordinary and higher level syllabi were introduced in 1992 and first examined in 1994 (Conway and Sloane 2006). These syllabi, which had characteristics that linked closely to previous syllabi, have only recently been replaced with the adoption of Project Maths. Even with the new syllabus, it may take time for the traditional approach to mathematics teaching, which is based in the principles of New Maths, to change as, according to Oldham (2001), the ‘New Maths’ curricular culture, with its focus on abstraction as its core principle, had, at that time, dominated Irish post-primary mathematics teaching for the past forty years.

The focus on abstract forms of mathematics inherent in the New Maths movement may be behind Boaler’s (1994) attribution of some of the blame for the shortcomings of mathematics education such as the frustration and disillusionment that pupils feel as they fail to see the meaningfulness behind the
tasks they are completing during mathematics tuition. Numerous academics and
groups have called for a greater emphasis on a more holistic approach which
places mathematics in context and links it to the real world (William 1992,
of the recommendations put forward by the National Council of Teachers of
Mathematics (2009, p.3) in the U.S. is to apply mathematics to “describe and
predict events in almost all academic disciplines”. In other words, integrate
mathematics with other subjects. science is the logical partner of mathematics for
such integration (McBride and Silverman 1991), thus the author explored and
developed this area of research through this study.

1.3 Scope and Significance of the Research

The separate subject approach to education has been claimed to have a deadening
effect on pupils’ experiences as they fail to see the links between subjects, instead
believing them to be isolated blocks of knowledge with little or no links and, as
such, the curriculum can only increase its relevance by linking the subjects
(Beane 2009, Jacobs 1989).

The manner in which mathematics and science are integrated varies from
coordinating a school’s mathematics program with its science program (NRC
1996) to integrating material from both subjects in such a way that “it becomes
indistinguishable as to whether it is mathematics or science” (Berlin and White
1992, p.341). However, no widely-adopted, specific teaching model for
integrating mathematics and science has been developed to date thus this practice
has no definitive direction in which to develop and evolve but rather a cluster of
disconnected approaches which endorse various teaching methods.

Having said that, there are certain criteria which various studies recommend when
integrating mathematics and science, i.e. the need for the content to be
contextually based and taught in an authentic manner with plenty of hands-on

Such an approach would aid in dealing with issues which have affected the quality of students entering higher education e.g. their inability to solve problems. Galbraith and Haines (2000) discovered, when testing 423 beginning engineering and mathematics undergraduates, that students develop and depend on translation algorithms that work for textbook problems, thus they struggle when asked to analyse and interpret mathematical information. Application of contextually-based, hands-on group work, with plenty of inquiry and discussion through the integration of mathematics and science within secondary education (and higher education) would aid in addressing this issue (Frykholm and Glasson 2005, Daniels et al. 2005, Furner and Kumar 2007, National Research Council 1996).

This pedagogical approach offers the starting point for the development of a teaching model that integrates mathematics and science in a manner which ensures significant learning takes place. After extensive exploration of a wide range of teaching strategies and models, the author concluded that the Authentic Instruction model, developed by Newmann and his associates in the 1990’s, would provide a good basis from which to develop a model for integrating mathematics and science. This was due to the fact that this model is integrative in its very nature and embodies many of the characteristics of effective integration recommended by academics in the field. Thus, it inspired the creation of the Authentic Integration model which the author developed specifically for the integration of mathematics and science.

1.4 Research Problems and Aims of the Research

The overall challenge of this study is to investigate the manner in which mathematics and science can be integrated at Junior Cycle in Irish Post Primary schools. This takes into account issues such as how the subjects can be effectively linked; what structure lessons of this nature would assume; whether
teachers can capably undertake such lessons; and the educational effects of these lessons. Careful consideration of these issues resulted in the following primary aims of the study:

1. To review the literature in relation to mathematics education in Ireland and internationally; integration of subjects; integration of mathematics with other subjects, especially science; and methods of teaching and learning which will aid in developing a means for integrating mathematics and science in a school setting.

2. To develop a model for integrating mathematics and science in post primary schools based on sound principles derived from research which can be effectively applied to the creation and implementation of lessons of that nature.

3. To create lessons based on this model and implement them in a number of Irish Post Primary schools to investigate as to whether they can be incorporated into the current school structure and what effect, if any, they have on pupil learning.

4. To gather data, both quantitative and qualitative, through questionnaires, semi-structured interviews, and focus groups to determine the impact of the lessons designed and implemented to integrate mathematics and science while also gaining a greater understanding of where the lessons were successful and areas which need to be improved.

5. To develop design principles that can aid teachers in their efforts to create lessons which integrate mathematics and science.
1.5 Research Questions

Every aspect of this study was guided by specific research questions. Each research question offered the opportunity to explore various elements of the phenomena inherent in this investigation:

- How can mathematics and science be effectively integrated in the classroom? Is there a model which can be adopted or adapted to fit the needs of such an undertaking?

- Is the education system, in its current form, flexible enough to incorporate such a radical change, taking into consideration resources, teacher knowledge within their non-specialist subject, and timetabling, among other issues?

- Is integrating mathematics and science more effective than teaching the subjects separately?

- Do teachers value the process of integrating mathematics and science? Will they want to continue to integrate mathematics and science on a regular basis?

- Can the author pinpoint explicit design principles to aid teachers in creating lessons which integrate mathematics and science?

1.6 Research Objectives

To answer the research questions outlined above, certain objectives need to be achieved. These objectives are listed as follows:

- Identify the areas of mathematics and science which are best suited for integrative activities.
• Develop a working model for integrating the subjects i.e. a blueprint for lesson plans of this nature.

• Create lessons/activities of this nature and experiment with them; receive feedback and adjust these lessons accordingly.

• Investigate how such lessons can be incorporated into the traditional secondary school timetable and implement them in Junior Cycle mathematics and science classes.

• Gauge teacher and pupil reaction and attitude to integration and the methods used to employ it through questionnaires, focus groups, interviews, and assessment of pupil work.

• Determine whether the Irish Education system at Junior Cycle level is flexible enough to incorporate integrative modules/subjects and assessments.

• Carefully develop and critically assess design principles specific to the creation of lessons which integrate mathematics and science.

• Outline further research which is required in this area of study.

1.7 Research Methodology

The general approach to research adopted in this study was largely influenced by investigations linked to Authentic Instruction (Newmann et al. 2007) as that particular model provided the primary basis for the theoretical framework applied in this investigation. The model designed by the author, Authentic Integration, evolved from Authentic Instruction thus the means for testing the new model followed a similar pattern to the means for testing the original.
A mixed methods approach was adopted, whereby quantitative and qualitative research techniques were applied. Such an option allows for broader and better results than depending solely on collection of qualitative or quantitative data (Denzin and Lincoln 1998). The tools of research which were applied were semi-structured interviews with the teachers; focus groups with the pupils; questionnaires for both the teachers and the pupils; and assessment of pupil work using a rubric. Each of these tools, with the exception of the pupil questionnaire, was designed by the author through reference to various literature and similar studies which will be discussed further in chapter 3.

The overall research method was underpinned by the Educational Design Research model. Through this model, a problem is identified and analysed; solutions to the problem are developed, tested, and refined; and the success of any solutions implemented is assessed (Nieveen et al. 2006). Continuous reflection is an important element of every stage of Educational Design Research as the researcher constantly looks for ways to improve their analysis, solutions, and quality of assessment throughout their investigation (Plomp 2009). This model is reflected in the work carried out by the author as he:

- researched the problems identified through research of mathematics education, both in Ireland and internationally;
- formed a solution to some of these problems by integrating mathematics and science through a model adapted from Authentic Instruction;
- tested this solution by applying lessons based on the new model in Irish schools;
- assessed the success of its implementation through research tools previously mentioned.
- created a set of design principles which can be implemented when creating lessons which integrate mathematics and science.

The final achievement noted above – the creation of a set of design principles – came about through a key aspect of the research methodology referred to as ‘Proof
Proof of concept, derived from engineering, is defined as the implementation of a model of a design which tests whether a certain concept will actually work as it is theoretically proposed (Dym et al. 2009). Authentic Integration was implemented to ascertain whether it would work as expected. From this process, the set of design principles for the creation of lessons which integrate mathematics and science was finalised.

Reflection was a key part of the process as the author was always looking for ways to improve the model developed and the lessons that resulted from this model, as well as the resultant overall design principles. Such reflection was also vital in each of the six stages of research which defined this investigation:

1. Literature Review.
2. Design of a Mathematics-Science integration model.
5. Implementation of the Intervention.

Each of these stages will be discussed in detail later (see section 3.5). Figures 1.1 and 1.2 (below) give a basic outline of what occurred during the study and how each stage of the study paved the way for the next.
Figure 1.1: The six stages of the study – Stages 1-3.

Stage 1
Literature Review

- Conclusion: Mathematics needs to be placed in context.
- Integration with Science would achieve this.
- Research Teaching Models that suit integration.
- Result: Authentic Instruction best suited.

Stage 2
Creation of Authentic Integration

- Adapt Authentic Instruction to suit specific needs of Mathematics and Science integration.
- Result: Authentic Integration model

Stage 3
Creation of Resource Pack based on Authentic Integration

- Create lessons based on the Authentic Integration model
- Research teacher knowledges for integration. Conclusion: teachers need training and resources.
- Result: Create resources for lessons and to aid with understanding. Create a plan for training teachers.
Stage 4  
Development of the intervention

- Research methods used in similar studies e.g. Newmann et al. (2007)
- Develop tools for gathering data e.g. rubric, questionnaires, semi-structured interviews.

Stage 5  
Implementation of the intervention

- Train teachers, provide resources, and outline what is to take place during intervention.
- Teachers complete 3 of the 6 lessons with the pupils.
- Collect data i.e. interviews, focus groups, pupil work, and questionnaires.

Stage 6  
Evaluation of the Intervention

- Analyse data collected, collate findings, and draw conclusions from these findings.
- Create Design Principles for lessons integrating Mathematics and Science.
- Complete thesis.

Figure 1.2: The six stages of the study – Stages 4-6.
The final stage of the study, the evaluation of the intervention, requires some further elaboration as elements of this stage guided many aspects of the previous stages. To assess the quality of the Authentic Integration model, the author employed Schoenfeld’s (2000) criteria for evaluating models and theories in mathematics education. This evaluation took into account criteria such as the descriptive and explanatory power of the model as well as scope, rigour, replicability, and falsifiability amongst other factors (Schoenfeld 2000). In a similar manner, the overall intervention process was evaluated through Shapiro’s (1987) four parameters for evaluating an educational intervention. These parameters were:

- Treatment effectiveness,
- Treatment Integrity,
- Social validity,
- Treatment acceptability.

The intervention carried out was guided by the specifics of these parameters (outlined in section 3.5.6) to ensure that it met the required standards so that the results obtained and conclusions drawn could be fully endorsed.

1.8 Limitations of this Research

Inherent in the research carried out are a number of limitations. These limitations include the following:

- The lessons pertaining to this intervention carried out by the teachers involved were not observed to ensure that they were being taught using the characteristics of the Authentic Integration model. This could have led to differences in how the lessons were taught between the four schools involved.
• Interviews and focus groups were conducted by the main researcher in this intervention, i.e. the author. This could lead to bias in the answers provided by the subjects involved to the questions posed. Every attempt was made to avoid this by asking for all subjects involved to reply with honesty.

Further information pertaining to these limitations, and how each limitation was accommodated as much as possible, is outlined in the methodology chapter of this thesis (section 3.10).

1.9 Overview of the Thesis

Each chapter of this thesis contributes significantly to the overall investigation carried out by the author. The noteworthy features of these seven chapters are outlined as follows:

*Chapter 1*, the introduction, outlines the background to the study along with its scope and significance, the research questions and research objectives which framed the study, the manner in which the study was carried out, and the specialist terms which are used throughout the subsequent chapters.

*Chapter 2* is a review of the literature which offers a clear insight into mathematics education, both in Ireland and internationally, while also outlining the research linked to the integration of mathematics and science and the findings which were vital to the approach developed by the author when integrating these subjects. Key elements of this chapter were the issues of concern which have been regularly identified within the mathematics education system, both from an Irish perspective (Lyons et al. 2003, Childs 2006) and from an international perspective (Boaler 1994, William 1992, National Council of Teachers of Mathematics 2009); as well as analysis of the integration of mathematics and science (Jacobs 1989, Frykholm and Glasson 2005, Berlin and White 1994).
While the work of Newmann et al. (1996, 1998, 2007) informed the development of Authentic Integration which was the focus of Chapter 4.

Chapter 3 provides a detailed outline of the methods used to carry out this research. Each element of the design and implementation of this investigation is analysed and justified to ensure findings and conclusions drawn in later chapters are valid and reliable.

Chapter 4 is based around the development of the Authentic Integration model and the issues which guided this development. The chapter initially explores the knowledges required for teachers to educate their pupils effectively. This work is largely based on the research carried out by Shulman (1986), Fennema and Franke (1992), and Rowland et al. (2005). As such, chapter 4 offers an insight into what knowledges a teacher requires to effectively integrate subjects, thus informing the process of implementing lessons of this nature in Irish post primary schools. The chapter also describes in detail the design of the Authentic Integration model, how it evolved from Newmann et al.’s (2007) Authentic Instruction model, and provides a clear justification for each characteristic of the model. There is also an in-depth analysis of the design of the lesson guides created for this intervention and how it was inspired by similar projects, especially the New Basics venture adopted by Education Queensland (2001).

Chapter 5 contains the resource pack created to aid the teachers in participating in the intervention connected with this study. This pack comprises the six lesson guides developed using the Authentic Integration model; resources pertaining to all the mathematics and science topics which were contained within each lesson; clear explanation of what was expected of the teachers; details regarding how the intervention would be carried out; and a detailed description of the Authentic Integration model.
Chapter 6 presents the findings gathered through collection of data from interviews, focus groups, questionnaires, and analysis of pupil work. Discussion on the implications of these findings is also included in this chapter.

Chapter 7 is a presentation of the conclusions drawn from this study and recommendations for future research which were influenced by completion of this investigation.

A number of appendices, which will be referred to throughout the proceeding chapters, will be located at the end of the thesis along with a comprehensive bibliography of the literature which aided this research.
Chapter 2

Literature Review
The following chapter discusses literature pertaining to mathematics education in an Irish and an international context, both past and present. Similarly, teaching models; studies based on integrating subjects; as well as the specific focus of this study – integrating mathematics and science – are each discussed in detail. Progress is made from identifying the initial problems facing mathematics education to solutions suggested by academics and educational groups, followed by how and why integrating mathematics and science can be part of the solution to the previously identified problems and how it can be deployed in schools.

2.1 Introduction

“We do need to find ways to connect mathematics with the broader culture. We need to find ways to break down the barriers between mathematics and art, music, humanities and the social sciences, as well as, of course, science and technology. We need to find entry points into the preoccupation and aspirations of children in ways that respect the integrity of their interests rather than patronizing and inevitably disappointing them.”

(Noss and Hoyles 2000)

The musings of Noss and Hoyles (2000) above indicate the current direction that mathematics education needs to take to reinvent the curriculum in such a way as to integrate the subject more meaningfully into the everyday undertakings of pupils’ lives. Breaking down the barriers between mathematics and other disciplines can be achieved through cross curricular activities which are high in meaningful intellectual work.

The notion of integrating mathematics with other subjects, especially science, in an educational context has been discussed quite frequently by the likes of Berlin and White (1992, 1994, 1998, 1999, 2010), Furner and Kumar (2007), Frykholm and Glasson (2005), and Pang and Good (2000) to name but a few. The following chapter will discuss such literature but it will also provide the background as to
how research by a number of groups and academics into mathematics education led to the conclusion that integrating mathematics with science would aid in solving some of the problems currently facing mathematics education, both in an Irish and an international context. Furthermore, approaches to integrating mathematics and science which have been implemented will be discussed and analysed along with a model which provides a blueprint for integrating these subjects in secondary school classrooms.

As such, this chapter provides a comprehensive review of the literature pertaining to the central elements of this investigation while also offering an insight into how the author progressed from examining the problems inherent in mathematics education to applying integration of mathematics and science to solve some of these problems to adopting a model which would allow him to successfully implement lessons of an integrative nature in Irish secondary schools.

The following pages will discuss the history of mathematics education, nationally and internationally; the problems that are present in the current curriculum; the recommendations offered by various academics and groups as to how to tackle these problems; and support for the adoption of a cross curricular approach using, to a large extent, the Authentic Instruction framework developed by Newmann and associates as a guide would be a viable path to take.

2.2 The Development of Mathematics Education

The manner in which mathematics education developed both nationally and internationally in the last century has had a profound effect on the field today. The major events, starting with the ‘New Maths’ movement of the 1950’s and 1960’s, which shaped modern-day mathematics education offer a vital insight into how and why certain practices and approaches to teaching the subject are prominent within the classroom.
2.2.1 The ‘New Maths’ Movement of the 1950’s and 1960’s

In the 1950’s and 1960’s great effort was put into the development of mathematics education in the US to aid the country in its attempts to win the ‘space race’, a term used to describe the competition between the USSR and the USA to be the first nation to land on the Moon. Schoenfeld (2004a) pinpointed the Russian launch of the satellite ‘Sputnik’ in 1957 as the catalyst for change within American Mathematics Education. He intimated that after this event, the American scientific community were spurred into action and, with the support of the National Science Foundation (NSF), began overhauling the mathematics and science curricula. ‘New Maths’, as the movement became known (also referred to as ‘Modern Maths’), began to incorporate new aspects of the subject such as set theory, modular arithmetic, and symbolic logic – an obvious shift towards more ‘abstract’ forms of mathematics (Schoenfeld 2004a). Structure, proof, generalization, and abstraction were at the core of this reform (Jones and Coxford Jr 1970).

Beberman of the University of Illinois was one of the leading academics in the implementation of this new curriculum. He emphasised the need for students to develop understanding through discovery learning while also building a precise mathematics vocabulary (Lagemann 2002) although this was not adhered to within the majority of schools where behaviourist approaches were largely adopted (Rappaport (1966), cited in Woodward (2004)). Rappaport (1966) suggested that due to this approach, rote learning and memorization became central to mathematics education practices within the average classroom. Schoenfeld (2004a) believes that the failure to properly induct teachers into the new curriculum caused them to either shy away from it or to ‘bastardize’ it. While parents felt disenfranchised and disconnected from the ‘New Maths’ curriculum as they did not feel competent enough to aid their children, and called for a change as the movement ultimately failed.

This experience taught academics and educators an important lesson as regards the need to involve teachers from the start when attempting to develop and
implement a curriculum model which will illicit meaningful change within the subject, an issue to be discussed later in this study.

Woodward (2004) agreed that the ‘New Maths’ movement failed due to the absence of comprehensive broad-based professional development for teachers and also cited the use of abstract mathematics at an elementary level as a major contributory factor to the downfall of the venture. In America, the 1970’s saw the education system go ‘back-to-basics’ i.e. concentrate on the 3 R’s of reading, writing and arithmetic. Hence the mathematics syllabus resembled that of the pre ‘New maths’ era i.e. a return of focus to the topics arithmetic, geometry, algebra and trigonometry; although greater attention was placed on skills and procedures – a lasting residue from the ‘New Maths’ movement (Conway and Sloane 2006). The reaction from the wider mathematics education community was to embark on a reshaping of mathematics education through alternative curriculum models such as Realistic Maths Education (RME).

### 2.2.2 Realistic Mathematics Education (RME)

The Dutch mathematics education system managed to avoid becoming part of the ‘New Maths’ movement due to the establishment of the Wiskobas project in 1968 which acted as a precursor for Realistic Mathematics Education (RME) (Van den Heuvel-Panhuizen 2000). In 1970, the Freudenthal Institute in the Netherlands began to develop the curriculum model RME which, at the time of writing, continues in its evolution. This model uses “Imaginable contexts” in which mathematics is placed so that students can make sense of what they are learning (Hough and Gough 2007). Freudenthal “felt mathematics must be connected to reality, stay close to children’s experience and be relevant to society, in order to be of human value. Instead of seeing mathematics as a subject to be transmitted, Freudenthal stressed the idea of mathematics as a human activity” (Van den Heuvel-Panhuizen 2000, p.3). In essence, RME aims to place the mathematics that the pupils are learning in a familiar or an imaginable context so that it
becomes something which they can identify as being real and useful as well as aiding them in making sense of the material.

Key features of Realistic Mathematics Education include:

- The use of informal strategies
- Informal to formal progression
- Pupils making sense of the maths
- The use of models
- Process-led rather than content-led objectives

(Hough and Gough 2007, p.35)

Within RME the student both develops and implements mathematical skills and knowledge within real life situations and context problems. At first, they develop methods and strategies which are closely linked to the context they are working within, and then they adjust this model so that it can be applied to similar contexts i.e. solving other but related problems (Van den Heuvel-Panhuizen 2000). Eventually the student will incorporate more formal mathematics into the model, in other words they will move from informal to formal strategies (Van den Heuvel-Panhuizen 2000). The methods implemented within RME allow the pupil to use their own basic methods and language to find a solution while gently guiding them towards more formal mathematical language and procedures as they develop a model for solving problems in various contexts (Hough and Gough 2007).

As the principles of RME have been central to the mathematics curriculum in the Netherlands, it is not out of order to suggest that this model has been a major factor in the consistently prolific performance of the Dutch students within the assessments carried out by The Programme for International Student Assessment (PISA). In 2009, the Netherlands ranked 11th (out of 65) overall in the PISA survey with a mean score (525), performing significantly above the average, while
Ireland’s performance, 32\textsuperscript{nd} with a score of 487, was considered below average (Gill et al. 2010).

### 2.2.3 International Change – Math Wars

The reaction to the ‘New Maths’ movement in the US was the publication of a series of documents by the National Council of Teachers of Mathematics (NCTM), three of which were considered to be especially influential: ‘An Agenda for Action’ (1980), ‘Curriculum and Evaluation Standards for School Mathematics’ (1989) (commonly referred to as Standards), and ‘Principles and Standards for School Mathematics’ (2000) (Klein 2007). The ‘Maths Wars’ of the 1990’s can largely be traced back to these texts and how they influenced Mathematics education policy as they induced a period of heated debate amongst the wider mathematics education community (Schoenfeld 2004a).

Schoenfeld (2004a) identified two distinct sides within the debate: extreme reformists and extreme traditionalists, although elements of each side had varying reasons for their stance – some reformists had a vision of pure discovery learning, others were firm believers of the 1989 Standards, and some were just provoked by the traditionalists. Traditionalists cited the move away from core values of mathematics as a reason for their stance whilst others felt that the reform curricula would weaken mathematical achievement.

‘An Agenda for Action’ (1980) advocated a move away from the ‘New Maths’ curriculum as it called for problem solving to become the focus of school mathematics, while also pushing for the widespread use of calculators, and the de-emphasis of calculus (Klein 2007). ‘Curriculum and Evaluation Standards for School Mathematics’ (1989) built on these recommendations by endorsing student-centred, discovery learning through real world problems. Mathematical skills and principles were to be learned through these problems rather than knowledge transfer, the drive for widespread introduction of calculators intensified, and mathematics for its own sake was not encouraged (Klein 2007).
Social justice and the needs of business were often referred to as reasons for the adjustments (Schoenfeld 2004a).

In the 1990’s virtually all American states adopted the principles of the 1989 Standards in their mathematics curriculum design and implementation (Klein 2007). Levels of acceptance of the new curricula varied between states e.g. by 1997 California had rejected the 1989 Standards principles in favour of new standards which relied more on the traditional ‘Modern Maths’ (Klein 2007). Debate on the merits of various educational principles continued and, currently, the issue is yet to be fully resolved.

2.2.4 Ireland and the ‘New Maths’ Movement

With the adoption (and eventual rejection) of the ‘New Maths’ movement as the international backdrop, the Irish education system decided to join the movement and, in 1964, effected radical changes in the post-primary mathematics syllabus. A curriculum based on the principles of New Maths was rolled out. This program, in keeping with the ‘New Maths’ movement, was more abstract in nature and over the next twelve years was tweaked with a succession of updates (Oldham 1991). As previously outlined, the movement was rejected in the US as the NCTM’s ‘An Agenda for Action’ in 1980 and their development of Standards later in 1989 led to the ‘Math Wars’ (Klein 2007).

While, internationally, change was occurring, in Ireland ‘New Maths’ remained the status quo with no real change to the Junior Cycle Curriculum until 1987 and even at that, there was only a reduction of material rather than a change in the overall type of content (Oldham 1991). Revision of the Leaving Certificate syllabus did not come until the 1990’s: the ordinary and higher level syllabi were introduced in 1992 and first examined in 1994 (Conway and Sloane 2006). These syllabi, which have characteristics that still link closely to previous syllabi, have been retained since but the on-going implementation of ‘Project Maths’ aims to move the focus of the mathematics syllabus away from abstract concepts. The
effects of radical curriculum change in the 1960’s are still evident in current practice – at the turn of the century, Oldham (2001) stated that the ‘New Maths’ curricular culture, with its focus on abstraction as its core principle, had dominated Irish post-primary mathematics teaching for the previous forty years. With little change up until very recently, it could be contended that this is still the case. The introduction of Project Maths intends to alter this culture.

2.2.5 Project Maths

Project Maths is the first major overhaul of the Irish mathematics curriculum since the adoption of the ‘New Maths’ model in the 1960’s. This curriculum initiative is currently being implemented nationally with the first set of associated examinations completed in June 2012 (NCCA 2008). The National Council for Curriculum and Assessment (NCCA) (2008, p.2) have developed this new curriculum to:

- provide greater coherence and progression between the mathematics experiences of students in primary school and in the post-primary junior cycle, and between junior cycle and senior cycle mathematics
- make mathematics more relevant to the lives and experiences of students and provide an appropriate mathematics education to meet the needs of all learners
- give greater emphasis to the understanding of mathematical concepts and the application of mathematical knowledge and skill
- encourage more students to study higher level mathematics; the intended targets are 60% at Junior Certificate and 30% at Leaving Certificate
- contribute to the development of higher order skills, including logical reasoning and problem solving
• ensure that there is closer alignment between how mathematics is taught and learned and how it is assessed

• engender an appreciation of the value of a good mathematics education to the present and future lives of students.

The above aims for Project Maths demonstrate that meaningful change, with a view to addressing the problems diagnosed within Irish mathematics education (to be discussed later), is taking place. These aims would also suggest that the curriculum is edging closer to a constructivist approach, which is more in line with current international trends (NCCA 2005b).

The new syllabus aims to ensure that the fashion in which mathematics is taught at post-primary level will fall in line with the type of instruction students experienced at primary level. This will be achieved through a more investigative approach to mathematics and the development of a bridging framework which will link the various strands within primary level to those studied during Junior Cycle (NCCA 2008).

2.2.6 Conclusion

Overall, a review of the history of curriculum change shows the influences which shaped the current attitude to mathematics: major change occurred in line with the ‘New Maths’ movement in the 1960’s (Oldham 1991) but there was very little deviation from the principles adopted at that time (i.e. a focus on more abstract forms of mathematics with little emphasis on real world uses) until the introduction of the new Junior and Senior Cycle syllabi in 1992 (Conway and Sloane 2006). Even with the introduction of these syllabi, the dependence on abstract forms of mathematics remained with only a recognition of the need for the promotion of the use of applications (NCCA 2005b). Oldham (2001) highlights this lack of change and innovation within mathematics education by suggesting that “whereas the official Irish curricular philosophy of the 1960s was
avant garde, that of the 1990s appeared rather dated” (p.270). With the adoption of Project Maths, the Irish education system is moving closer to the mathematics principles advocated internationally but it is questionable whether a change in policy will ensure a change in general attitude amongst the wider mathematics education community. This brief history of Irish mathematics education offers an insight into the current attitudes and practices that have had an effect on the performance of Irish pupils, of which there has been some criticisms which need to be analysed and addressed.

2.3 Criticisms of Current Post Primary Mathematics Education

2.3.1 Irish Students’ Performance

Mathematics has always commanded a position of great importance both nationally and internationally and, as such, all aspects of the subject – especially the results obtained by students in state examinations - are scrutinized on a regular basis to ascertain the success or, indeed, shortcomings of the subject. The most significant and commonly used indicator of the performance of Irish students within mathematics is the OECD (Organisation for Economic Co-operation and Development) Programme for International Student Assessment (PISA). This programme assesses 15 year olds in OECD member countries in the areas of mathematics, reading literacy, science and problem solving. Within mathematics, this group assesses students in a holistic manner, gauging their ability to analyse, reason and communicate effectively within a range of problems that require the use of quantitative, spatial, probabilistic or other mathematical concepts (OECD 2007).

In 2003 PISA made mathematics the major focus of their investigation whilst the other three domains were given a lower emphasis. Ireland was ranked 17th of 29 OECD countries, 20th of 40 participating countries, and achieved a mean score that was only slightly above the average for OECD countries (OECD 2004). There was little change in the most recent PISA survey in 2006 – Ireland ranked
16th of 30 OECD countries; 22nd of 57 participating countries; and, once again, Ireland achieved a score that was very close to the average for OECD countries (Average = 500; Ireland scored 501). By 2009, Ireland’s performance was rated as ‘below average’ as they ranked 32nd of the 65 participating countries (Gill et al. 2010). In contrast, the levels achieved by Ireland within reading and science were well above OECD country averages (OECD 2007). Thus, it is not a stretch to suggest that Irish students lack intelligence but rather that the mathematics education structure needs to be altered so that Irish students can reach or at least come close to their potential in this subject in the same manner as they do in the areas of reading and science.

Ireland places high value on education in the pursuit of creating an ‘Enterprise Economy’ as commonly referred to by Taoiseach Enda Kenny – “This Government will build a new Enterprise Economy. A place where we value, support and encourage the role of the entrepreneur….The Enterprise Economy will require a new generation of creative people, with innovative ideas to capture new markets” (Department of Taoiseach 2011). Thus the country cannot afford to implement a curriculum which produces students of average mathematical ability. If Ireland continues to do so then industry will begin to falter as economic rivals stride ahead on the back of a superior workforce. Such a scenario is detrimental to the country’s ambitions thus the problems within the mathematics curriculum and pedagogy need to be identified and dealt with.

2.3.2 The Approach to Instruction

The main cause for concern according to research is the nature of teaching within the school system: it has been termed as didactic, using drill and practice of similar mathematics problems as the central focus of each lesson with little or no importance placed on the explanation of concepts, and few opportunities for using the mathematics learned in applied problem contexts or everyday situations familiar to students (Lyons et al. 2003). Such pedagogy practices inevitably affect student development whereby they are unable to transfer mathematics they have learned to contextual problems which are unfamiliar (Kerslake 1986, Lave 1988,
Mitchelmore & White (1995) described concepts learned with no link to the contexts in which they occur as ‘abstract-apart’. They observed that students can only use abstract-apart concepts in the context(s) in which they were taught, thus students find it difficult to apply their learning to varying situations. Similarly the NCCA (2005b) reported that there is concern regarding students transitioning from second level to third level education in so far as many seem incapable of applying basic mathematics skills or understanding the required concepts needed to progress. Furthermore, their report suggests that students appear unable to apply the mathematical knowledge they have unless they are asked to do so in the most basic or familiar contexts.

The approach to instruction and the shortcomings of Irish mathematics students are linked – the way in which the students are taught socialises them into a certain way of approaching mathematics which affects their ability to transfer the skills and knowledge they acquire in different contexts. In essence, students are being told how to approach certain problems which will ‘come up in the test’ with little or no explanation as regards the reason(s) behind these approaches or how these methods could be applied to different contexts and/or situations (Mitchelmore and White 1995). Thus, outside of school situations, this information is of little or no use to students (Lyons et al. 2003).

Childs (2006) highlights clearly the problems with the mechanics of the current approach to science and mathematics pedagogy within Ireland by indicating that current practices in this area are not in line with what has been learned from recent research within the field. He argues this point of view by specifying that most teachers tend to teach in the same manner in which they themselves were taught i.e. concentrating on the content and structure of the subject rather than focussing on the various elements involved in the learning process which pertain to the students.

Childs (2006) suggests that more emphasis on the actual process of learning is required so that students develop a greater understanding of exactly what they are
encountering. But there are mitigating factors which would obstruct such an approach; these barriers are outlined by the NCCA (2005a, p.11) in their investigation into Assessment for Learning (AfL) in which they suggest that ‘the system’ places pressure on teachers to conform to certain expectations including

- The need to use grades as a record of progress;
- The limited time available to cover the content on the course; and
- The anxiety attached to attempts to incorporate new or novel approaches.

They found that these restrictions combine to slow down or even completely stall any innovation in teaching and learning practices within the classroom.

From these findings it would seem that the emphasis on high stakes summative assessment places pressure on teachers to adopt the didactic, drill and practice approach with emphasis on exam style questions thereby limiting any application of learned mathematics to unfamiliar contexts, creativity or independent thinking as observed by William (1992, p.3):

“I have visited a lot of mathematics classrooms, and it seemed to me that in most of them, it was as if there were a kind of check-in desk just outside the classroom door labelled 'common sense', and as the pupils filed into the classroom, they left their common sense at the check-in desk saying 'Well we won't be needing this in here.'”

What William (1992) is trying to convey is that students see mathematics as something which is neither logical nor applicable to their lives outside the classroom. The use of ‘common sense’ should be imperative within mathematics problems but, according to Boaler (1994), the approach being used within the classroom is fostering such an attitude leading to widespread frustration and disillusionment as students fail to see the ‘meaningfulness’ behind the tasks they are completing. She indicates that this leads to students adopting the practice of ‘context-conditioning’ whereby they approach every problem by attempting to
recall the correct method or procedure, giving no thought to any contextual aspect because “students know that approaching a task holistically and exploring the real world and mathematical variables would, in a mathematics question, result in failure.” (p.555)

Boaler (1994) and William (1992) are suggesting that students are being socialised or conditioned into a certain way of approaching mathematics i.e. practice the test questions ad nauseum to learn the procedure, identify the questions in the test which require this procedure and apply it in the exact same fashion as was practiced. Creative thinking or the use of common sense is generally not required.

So literature claims that the current approach to teaching mathematics requires adjustments if progress is to be made. More emphasis needs to be placed on the process of learning rather than on the summative assessments (Lyons et al. 2003, William 1992, Mitchelmore and White 1995). Such an approach is emphasised at primary level (NCCA 2007) so it would seem that change is required at Junior Cycle first to offer a smooth transition to second level as well as a platform for such a curriculum reform at senior cycle.

The Economic and Social Research Institute (ESRI) longitudinal study of post-primary level students carried out in 2003 suggests that students have difficulty transitioning from primary to secondary school due to the rigid nature of disconnected, subject-based curricula that are characterised by rote learning and summative assessment – an approach which is in direct contrast to the more holistic education they have experienced at primary level (NCCA 2007). While this is clear evidence that change is required, the most damning indictment of current practices, both nationally and internationally, comes from O’Donoghue (2004). He reported on the under-preparedness, in relation to mathematics, of students entering universities in Ireland, UK, Australia and the US by drawing on the findings of several sources to offer an insight into the characteristics associated with this problem:
Students lack numerate skills that are required to cope with every day life not to mention studying university mathematics.

Elementary Algebra is the centrepiece of student deficiencies.

Students demonstrate large gaps in their knowledge. Diluted syllabus results in fragmented understanding.

Substantial evidence that students are unable to solve problems. Students develop translation algorithms that work for textbook problems.

Inadequate conceptual knowledge.

Judgements and interpretations made by students often lack validity or explanation in terms of mathematical reasoning.

The fourth and sixth points above indicate that students are deficient when it comes to using mathematics to solve authentic problems or to interpret areas in which mathematics is present – yet more evidence which would suggest that a move away from the didactic, drill and practice approach (highlighted earlier), which leads to the practice of ‘context-conditioning’ within students, needs to be seriously considered. Improvement in the observed fragmented, inadequate conceptual knowledge has to be addressed also within such a pedagogical change if the mathematical characteristics of students entering third level are to be enhanced. These problems have been recognised by various elements of Irish society, not least the business sector.

2.3.3 Needs of Irish Industry

The development of improved pedagogical methods is an obvious issue within the Irish business sector as their success depends greatly on the quality of graduates produced by the Irish Education system. In fact, the OECD (2010) highlighted a
direct correlation between student performance and GDP growth: they found that small improvements in mathematical skills of a country’s work force can have a significant impact on their economic performance.

The Irish Business and Employers Confederation (IBEC) offer recommendations for progress within educational practice on a regular basis, recently (2011) suggesting that teachers must be given the opportunity to teach mathematics in a way that promotes deeper thinking and greater innovation so that students can develop real understanding and engage in practical problem solving which is relevant to everyday life challenges and tasks (IBEC 2011).

IBEC (2008) further highlights what they believe is required with one of their overall recommendations in which they call for an increased variety within methods of assessment through the “development of a new Junior Cert Programme which addresses the multiple intelligences of the child, uses multiple methods of assessment, has the potential for local interpretation and customisation by teachers” (IBEC 2008, p.2)

The Expert Group on Future Skills Needs (EGFSN) outlined similar reform demands in 2008, calling for the relevance of mathematical concepts to be displayed and deeper understanding of said concepts to be gained by displaying them through disciplines such as science, engineering, business studies, and social sciences. The EGFSN suggest that a more interactive teaching approach would be best to demonstrate the application of these concepts in scenarios that students can relate to, starting at Primary education and continuing into Second Level (Finfacts 2008). The National Council of Teachers of Mathematics (NCTM) echoed these sentiments in 2009:

“[S]tudents should recognize and apply mathematics in contexts outside mathematics. Students need experiences applying mathematics concepts and representations to describe and predict events in almost all
academic disciplines, as well as in the workplace as we develop a fully informed citizenry.”

NCTM (2009, p.3)

When analysed carefully, it is noticeable that the above recommendations have similar themes running through them:

- The need for alternative methods of assessment.
- Implementing a more holistic curriculum which combines aspects of the cognitive, affective and psychomotor domains as well as an overlap between subjects.
- Establishment of a flexible curriculum which will be of use to students outside of the classroom and within other disciplines.

The reaction from teachers to the need for change is positive as described in the NCCA’s (2005b) report in which they outlined that teachers placed priority on increasing the amount of practical work, placing more focus on specific aspects of the curriculum and escalating the use of mathematical language. This is quite an important element for meaningful change as the adjustments required within the teaching and learning of the subject would be impossible if the primary carriers of change are not fully convinced of its merits.

2.3.4 Conclusion

The distinctly average performance of Irish students indicates that there are problems within the approach to delivering the subject. Childs’ (2006) criticism of the didactic methods commonly implemented indicates that this teaching method is at the root of the problem. Meaningful change needs to occur as regards how the subject is delivered to students if the requisite mathematics skills, sought by employers and universities, and relevant to real life scenarios, are to be developed and retained. The general consensus, outlined previously, within the academic community calls for a greater range of assessment practices as well as a
more holistic approach to learning which encompasses the cognitive, psychomotor, and affective domains. Allied to these changes is the need to integrate mathematics with other subject areas. Hence change should involve all these recommendations and combine them in a manner which serves the needs of the student as they progress through their developmental stages and offer a product which caters to the needs of universities and the business community.

2.4 Integrating Mathematics and Science

2.4.1 Introduction

The integration of subjects in a school setting has become an issue of note amongst the education research community in recent times. Amongst the most obvious and most often discussed subjects to be linked in this sense are mathematics and science. Berlin (1999) noted that research in this area had increased greatly in the 90’s, a trend which has continued into the 00’s. The momentum for such a shift in education practice can be attributed to the decision of organisations such as the National Council of Teachers of Mathematics (NCTM), the National Research Council (NRC), the Curriculum Corporation (Australia) and the School Science and Mathematics Association (SSMA) to position the issue of integrating mathematics and science (and, in some cases, other subjects also) centrally in their plans for instigating change within education.

Table 2.1: Education groups that have indicated their support for the integration of Mathematics and other subjects.

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<tr>
<th>Group</th>
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<tbody>
<tr>
<td>National Council of Teachers of Mathematics</td>
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<tr>
<td>National Research Council</td>
<td>USA</td>
</tr>
<tr>
<td>The Curriculum Corporation</td>
<td>Australia</td>
</tr>
<tr>
<td>School Science and Mathematics Association</td>
<td>USA</td>
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</table>
A prime example of support for integration is the ‘Connection’ standard adopted by the NCTM (2009, p.3) in the US which states that students should have the ability to “recognize and apply mathematics in contexts outside of mathematics”. The Science Education Program Standards (NRC 1996, p.214) goes a step further in suggesting that “The science program should be coordinated with the mathematics program to enhance student use and understanding of mathematics in the study of science and to improve student understanding of mathematics.” Berlin & White have written extensively on this topic and suggest that the barriers between the school subjects of mathematics and science, in their current guise, should be broken down and the integrated material taught in such a way “that it becomes indistinguishable as to whether it is mathematics or science” (Berlin and White 1992, p.341).

Evidence of the increase in support for integration can be identified in its inclusion in teacher training. Berlin and White (2010) reported on a Masters of Education degree established in a large university in the Midwestern region of the U.S. which focusses primarily on teacher preparation for the integration of mathematics, science, and Technology for grades 7-12. This indicates the extent to which integration of these subjects in second level schools is gaining ground in education circles as a worthwhile endeavour.

Central to the argument for the adoption of an integrative approach is the belief that the separate-subject approach, which has long been dominant, has a deadening effect on the experiences of students as they are unable to make connections or develop a deeper understanding of new information derived from disconnected disciplines (Beane 2009). Jacobs (1989) agrees with this point of view, as he found that students see subjects as separate blocks of knowledge with little or no links. He attributed this to the practice of dividing the school day into timed sections of instruction of separate subjects and stated that the curriculum can only become more relevant when connections between subjects are made.
Conversely, it has been argued that it is vital to maintain the structure of disciplines such as mathematics and science so that subject-specific problems and challenges can be encountered to allow students to develop and improve specialised skills and knowledge (Gardner 2004, Leonardo 2004, Schoenfeld 2004b).

Brain research adds weight to the argument in favour of integration as findings support the practice of thematic teaching (Cohen 1995) while the Carnegie Council on Adolescent Development (1989) found that integration of content areas aids students’ ability to think critically. It has also been determined that there are significant similarities between the NCTM’s (2009) key mathematical process standards and the NRC’s (1996) 5 E’s – the instructional stages and processes through which students learn science – hence indicating that the characteristics of each subject are such that tuition within each can be carried out simultaneously through integrated instruction, enabling enhanced learning (Bossé et al. 2010).

2.4.2 Defining Integration and the Integration Continuum

The above indicates that there is support from vital elements of the education community but it also shows that there are varying degrees of ‘integration’ i.e. the NCTM suggest making basic connections; the NRC feel the mathematics and science programs should be co-ordinated whilst Berlin & White (1992) recommend complete assimilation of the disciplines – each of these suggestions can be placed on varying positions of the integration continuum. This continuum ranges from simply mentioning, in passing, particular areas or topics in which mathematics and science are linked to interdisciplinary teaching to fully integrated instruction.

Interdisciplinary instruction includes the assumption, according to Frykholm & Glasson (2005), that the subjects will not be completely merged but will maintain their boundaries through investigation of common contexts that are beneficial to
the learning of both science and mathematics. They believe that for this to be successful, teachers must have both the content and relevant pedagogical knowledge to educate students in both disciplines successfully. This approach suggests that both subjects will be taught in tandem but the disciplines will not be assimilated completely i.e. a task may include both mathematical and scientific work but they will be carried out separately to each other.

A step above ‘Interdisciplinary’ instruction is ‘Integrated’ teaching which Frykholm & Glasson (2005, p.130) believe implies that “science and mathematics can be blended seamlessly so that it is difficult to tell where the mathematics stops and the science begins”. This approach would suggest that mathematics and science should cease to be subjects on their own (at least in the context of secondary school) and assimilate to form a new subject which combines both, something akin to applied mathematics but on a larger scale and incorporating more material.

Various authors define ‘integration’ differently and use different terms (e.g. transdisciplinary, connected, multidisciplinary, blended, co-ordinated). Such is the range of terms and definitions that in 1991, a group of 60 scientists, mathematicians, science and mathematics educators, teachers, curriculum developers, educational technologists and psychologists gathered at a conference organised by the National Science Foundation (NSF) to define the practice of integrating mathematics and science (Berlin and White 1992). After three days, no consensus was reached although one group did come close with a proposed working definition:

“Integration infuses mathematical methods in science and scientific methods into mathematics such that it becomes indistinguishable as to whether it is mathematics or science”

(Berlin and White 1992, p.341)
But this definition was not fully endorsed by all as some had reservations about the propensity for such a definition to cause people to lose important philosophical, methodological, and historical differences between the subjects.

The issue as regards a working definition or working definitions of the practice of integration is that it can mean different things to different researchers and educators; thus establishing its meaning in the context of this author’s research becomes an important undertaking. Frykholm & Glasson’s (2005) attempts to form some sort of working definitions are fine at a basic level but they do not fully outline the range of integration techniques which are available to educators.

Hence, a more specific and transparent continuum would allow for the creation of a better defined approach to integration – this is provided by Jacobs’ (1989) Ten “Design options for an Integrated Curriculum” (see Fig. 2.1). The ten design options for an integrated curriculum lay out sequentially the levels of integration which can be achieved within and/or across disciplines and learners. Each option is assigned to one of three broad types of integration; hence this continuum provides a basic set of terms i.e. integration within single disciplines, across several disciplines, and within and across learners. The ten design options then provide in-depth specific definitions of any attempt at integration of subjects. Thus this continuum can be used when describing, broadly or specifically, any type of integration.
Figure 2.1: Jacobs’ (1989) Ten “Design options for an Integrated Curriculum”
Jacobs (1989) gives an insight above into the characteristics of each design option and how they might be implemented but some elaboration is required as well as how, or even if, these options can be utilized to aid the teaching of science and mathematics in some sort of integrative manner.

The ‘Within Single Disciplines’ type of integration is in the same mould as traditional teaching practices of today where subjects are taught in separate time blocks with slight variations between Cellular, Connected, and Nested options. More relevant to this study are the options contained within the ‘Across Several Disciplines’ type of integration. Beginning with the Sequenced option: teachers of different subjects (e.g. mathematics and science) plan the sequence of their lessons to correspond to each other’s lessons. In this design, students are expected to make their own connections across the different disciplines. The Shared approach brings together certain disciplines that are related to one another in a formal unit to study a theme or issue; for example, the theme of velocity within science and mathematics.

This design is expanded by the next option, Webbed, which purposefully utilizes a range of disciplines to examine a general theme or issue. Disciplines such as Language arts, mathematics, science, social studies, music, and physical education, amongst others, are all brought together in support of each other to establish an integrated curriculum. Each discipline deals with an aspect of the chosen theme hence the central theme is explored from various angles.

The ‘Integrated’ option is similar to the ‘Webbed’ option but rather than studying a central theme this approach calls on students to utilize their knowledge and skills from various and diverse disciplines to solve a problem or complete a task (Jacobs 1989). Hence all aspects of student learning are assimilated to achieve a central aim rather than being assigned to separate subjects with no relation between skills and knowledge learned being required to complete a set task/problem.
The Threaded option is not as relevant as previously mentioned options as regards this study, the reason being that it focuses on the development of a general skill within various subjects rather than the assimilation of content (Jacobs 1989). In other words, change would need to occur in all school subjects to make the development of particular skills the central aim rather than using the act of combining two subjects (mathematics and science) to develop skills which are inherent in both.

As stated, this study is concerned with the combination of two subjects, science and mathematics, hence the ‘Across several disciplines’ section is the one which is most relevant, ‘within and across learners’ can and may be touched on, but, considering the starting point within the current Irish education system, employing the ‘Immersed’ and ‘Networked’ design options seems highly unlikely. The ‘Sequenced’ and ‘Shared’ options will be central in this study while the ‘Webbed’ option will be striven for. The ‘Integrated’ option is an approach which would take radical change within the current education system hence the chances of applying it are quite remote but it remains an interesting reference point when considering possible change in the longer term i.e. when progress has been made in implementing a program for integrated learning.

2.4.3 Barriers to the Implementation of Integration

Pang and Good (2000) believe that integrating mathematics and science is one of the most daunting challenges that an educator can encounter. This belief is echoed by Huntley (1998) as he outlined impediments to integration such as:

- The need to coordinate students;
- The increased time and effort required to implement this change; and
- The lack of instructional models and curricular materials.
Watanabe & Huntley (1998) found similar hurdles when attempting to infuse integration into regular tuition. They found that teachers had positive attitudes as regards the value of integration but retained reservations as regards the time restrictions present when attempting to imbue integrative practices into an already crowded curriculum. Lehman (1994) also met with a similar set of circumstances whereby the teachers had positive perceptions of integration but these did not carry over into practice as teachers did not feel they had the time to work integrative activities into their lessons.

While the issue of time dedicated to integration is a restrictive factor, one which may be of more concern is that of teacher knowledge within both subjects. Teachers may be open to innovative ideas and dedicate their time to implementing them but inadequate subject matter knowledge can often be the main reason for failure when attempting to implement such a pedagogic approach (Pardhan and Mohammad 2005).

Allied to that, teachers that do not have a foundation of content knowledge within other disciplines can, at best, only make superficial links between disciplines (Pang and Good 2000). Thus, along with the fact that many teachers have displayed weak subject matter knowledge within their own disciplines of mathematics or science (Pardhan and Mohammad 2005, Adams 1998, Ní Riordáin and Hannigan 2009), it is very questionable whether such teachers can develop pupils’ conceptual learning in an integrated setting (Pang and Good 2000).

Frykholm & Glasson (2005) recognised such obstacles and thus suggested that a more realistic approach would be to promote the connections between the two subjects (similar to the suggestion by the NCTM (2009) noted earlier) in an authentic fashion which links to the common experience of the learners. They suggest that most teachers would not have the requisite knowledge to teach both subjects in a fully integrated or “interdisciplinary” setting but should be able to create relevant connections from their own experiences and prerequisite knowledge bases which they can build upon.
This is where Jacobs’ (1989) Design Options can be utilised as there will be a certain option which will suit each situation thus offering educators a defined idea of the type of integration they wish to implement. The following approach suggested by Frykholm & Glasson (2005) would fall under Jacobs’ ‘Shared’ option (1989).

The way in which these ‘connections’ between mathematics and science should be made is something which many researchers in this area agree on. Frykholm & Glasson (2005) are adamant that “any effort to connect science and mathematics with meaning must be situated in authentic contexts” (p.130). Similar beliefs are to be found in the calls for constructivist approaches which support the creation of social constructions and explanations in authentic contexts (Cobb 2000, Cobb and Bowers 1999, Greeno et al. 1996, Putnam and Borko 2000). Furner & Kumar (2007, p.186) also support this view and believe that modern education should combine the subjects in a way which moves away from traditional methods to a more constructivist, student-centred approach suggesting that:

“The separate subject curriculum can be viewed as a jigsaw puzzle without any picture. If done properly, integration of math and science could bring together overlapping concepts and principles in a meaningful way and enrich the learning context.”

There is clear support for integration of the subjects, as outlined above, but does integration of mathematics and science actually work and is there evidence to prove that it does?

2.4.4 Does Integration Work?

There are very few empirical studies in relation to mathematics and science integration, the few that have been conducted largely support the practice of integration. Beane (1995) used traditional measures of school achievement to assess student success and found that students who experience an integrated
curriculum do as well as if not better than students who experience a separate-subject curriculum. Stevenson and Carr (1993) discovered that student interest and achievement benefited from integrated instruction, Greene (1991) reported similar findings. He carried out a study of students in California who participated in year-long thematic units and reported increased student interest and achievement scores on the National Assessment of Educational Progress (NAEP). Similarly, Vars (1991) reported that interdisciplinary programs produced higher standardized achievement scores than did traditional separate subject area courses.

In contrast, Wallace et al. (2001) described a project based around the integration of mathematics, science, and technology in which connection-making, cooperation amongst students, and increased interest and excitement were not evident. These are typically put forward as favourable outcomes when arguing the case for integrating mathematics and science, hence the reason for their failure in this instance is an interesting turn of events. In this project, the students were set the task of making a rocket car powered by compressed air. This required them to develop and implement knowledge from mathematics, science and technology. Concepts from each subject which related to the project (e.g. force, work, power, scale, volume, aerodynamics) were taught during regular tuition to prepare and aid the students as they created a design.

As the project progressed, a lack of learning was apparent in a couple of behaviourally difficult students but, more worrying, was the general lack of motivation amongst the class in general. Wallace et al. (2001) suggested that the students’ lack of experience in this type of learning situation contributed to the poor response observed. It was also suggested that maturity, allied to willingness and readiness to learn, may have been lacking and impinged negatively on the project. As there have been very few actual studies on integration projects carried out, this particular report offers a valuable insight into the pitfalls which can be encountered. In the report, the authors highlighted the fact that in a project carried out later in the year, which was considered to be more successful than the first, the students showed greater motivation and improved learning. It was suggested that
this may have been down to greater familiarity with the style of learning as well as more collaboration between the teachers involved through increased levels of team teaching.

This report wasn’t ideal as there wasn’t much data gathered. The authors depended more on general observations and some interviews with those involved rather than giving the students standardised tests to evaluate progress or detailed questionnaires to fully gauge their experience of the project. Having said that, important information can be gleaned from this study as it shows that integration can have pitfalls and may not offer any significant advantages when compared to typical tuition. It was apparent from the report that connections between subjects have to be highlighted clearly and consistently as some students failed to recognise links between the project they were completing and the material they were studying during regular classes (Wallace et al. 2001).

Frykholm and Glasson (2005) focussed on a rather different aspect of integration to that of Wallace et al. (2001): instead of implementing lessons which integrate mathematics and science, they studied the process of creation of interdisciplinary units connecting these subjects by prospective teachers of mathematics, of which there were 42, and science, of which there were 23. These prospective teachers initially recognised the importance of linking mathematics and science instruction through overlaps in the subjects and, also, through real world events. Although they identified the possibilities for making such connections, the prospective teachers indicated concern regarding their content knowledge. Upon completion of the collaborative integrated units, it was concluded, through analysis of the final data, that the prospective teachers were successful in working together to connect mathematics and science in situated or contextualised units (Frykholm and Glasson 2005).

A similar study conducted by Berlin and White (2010) explored the attitudes and perceptions related to the integration of mathematics, science, and technology amongst preservice teachers completing a M. Ed. which focussed primarily on
teacher preparation for the integration of these subjects. Conclusions drawn from this study were quite similar to those of Frykholm and Glasson (2005) i.e. these preservice teachers valued integration from the outset and throughout but upon completion of the program, they perceived integration to be more difficult than originally thought and identified a number of barriers to its implementation. This resulted in their approach to integration becoming more realistic, practical, and cautious (Berlin and White 2010).

The attempts at integration observed within the studies mentioned vary in their general approach to developing and conducting lessons. As such, a clear pathway is required to guide the development of an integrated curriculum. The Berlin-White Integrated Science and Mathematics Model (BWISM) offers a blueprint for developing a coherent program for the integration of mathematics and science, hence it provides a meaningful starting point in the creation of an integrative model.

2.4.5 BWISM

The Berlin-White Integrated Science and Mathematics Model (BWISM) was developed by Donna F. Berlin and Arthur L. White in 1994. It is an interpretive theory or framework that characterizes the integration of science and mathematics. It is not a specific model that is to be adhered to but something to be interpreted and adjusted by researchers and educators to suit the context and the philosophy and practices of the individual (Berlin and White 1994). It is a broad template which has been designed to guide researchers and educators in the characterisation of current resources and the development of new material to aid with the integration of mathematics and science. Michelsen (2006, p.274) champions the merits of the model, suggesting that it “offers a typology to describe and understand the complex nature of integration of mathematics and science from both a content and a pedagogical position”. In essence, it places order on and also describes the various aspects and characteristics of an integrative approach to the teaching of mathematics and science.
The flexibility of this model offers a gateway to meaningful implementation of cross-curricular activities as educators can interpret and adjust this model in whatever way they see fit. There are six general aspects to this model:

- Ways of Learning
- Ways of Knowing
- Content Knowledge
- Process and Thinking Skills
- Attitudes and Perceptions
- Teaching Strategies

(Berlin and White 1994, p.59)

Berlin & White (1994) elaborated on these general aspects:

*Ways of Learning* refers to how students experience science and mathematics, suggesting that students must be actively involved in the process of learning.

*Ways of Knowing:* Knowledge within mathematics and science is usually interpreted through induction and deduction. Induction refers to the study of information to form an idea of what is occurring and translating this into a rule or definition. Applying this rule or definition in a new context would act as deduction in this scenario.

*Content Knowledge:* Many areas within mathematics and science overlap and hence can be easily taught in tandem. Ideas and/or themes such as models, scale, and symmetry can incorporate elements of both subjects so that the material is learned in such a way that it is easily linked and can be experienced in authentic settings.

*Process and Thinking Skills:* There is greater scope for developing processes and ways of thinking such as investigation, exploration, experimentation and problem solving when the subjects are combined in the form of projects or investigations.
**Attitudes and Perceptions:** Similar attitudes and approaches within mathematics and science can coexist in an integrated setting. These include a desire for knowledge; basing decisions on evidence and data; relying on logical reasoning; and working with others to solve problems or challenges.

**Teaching Strategies:** Teachers should make use of problem solving and inquiry based learning methods to deliver the broad range of content whilst providing opportunities to use laboratory tools and technology. It is also suggested that assessment should be embedded within instruction (something akin to assessment for learning).

BWISM, in the form of the six general aspects outlined above, is more of a guide in the creation of models for the effective integration of science and mathematics than a model itself. It is a check list of sorts to ensure that all meaningful bases are covered. In this sense it is quite useful as a tool for analysis or as a reference as it can indicate where a model falls short i.e. what aspect outlined in BWISM is weak within the model or requires further development.

A more precise model than that which can be taken from BWISM is required to offer a distinct route to the effective integration of science and mathematics; such defined models are very few and far between (Sharkawy et al. 2009) hence one either has to be created or adjusted to suit the Irish education system. There are certain characteristics, which authors on the topic seem to agree, are required for any such model.

### 2.4.6 Working towards an Integrative Model

One element which is advocated by most is the need for the content to be contextually based or taught in an authentic manner (Frykholm and Glasson 2005, Daniels et al. 2005, Furner and Kumar 2007, National Research Council 1996, National Council of Teachers of Mathematics 2009, NRC 1996). Putnam & Borko (2000) suggest that “How a person learns a particular set of knowledge and skills, and the situation in which a person learns, become a fundamental part of
what is learned” (p. 4), a point of view which current researchers seem to share and thus the suggestion that contextual learning is best in this instance can be linked to the need for students to improve their ability to apply the knowledge they have to scenarios which are similar to that which they will face in future jobs or everyday tasks.

Hands-on cooperative group work involving plenty of discussion and inquiry comes highly recommended when designing integrative lessons for mathematics and science (Furner and Kumar 2007, Daniels et al. 2005, Miller and Davison 1999). The research carried out by these authors would suggest that the students should be the central focus, combining their knowledge and skills to complete tasks, with the teacher playing the role of facilitator within a problem-based learning atmosphere.

This type of approach was put into action by Judson & Sawada (2000) in a study, involving eighth grade pupils, which used science inquiry-oriented activities with technology that generated data (Calculator Based Labs, graphing calculators and probeware) to integrate mathematics, specifically statistical concepts and techniques. This hands-on, cooperative group work was found to be very beneficial in relation to the pupils’ mathematics development as 75% of the students in the experimental group scored an A or B in a mathematics statistics unit test while only 35% of the control group achieved a similar mark (Judson and Sawada 2000). This was the only study that focused on the integration of these subjects with respect to content, so similar empirical evidence is not available to support the results obtained. Also, it could be argued that the use of technology may have been the catalyst for the marked improvement when compared to the control group rather than the act of integrating the subjects.

In general, as discussed above, experts on the topic of integration more or less agree that student-focused, hands-on, cooperative group work involving high levels of discussion and reflection are necessary to develop a deeper understanding of the subjects within an authentic context. The use of technology
in this scenario would also be beneficial (Daniels et al. 2005, Judson and Sawada 2000) but considering the distinct possibility of there being limited resources within most schools, this should be classed as an option rather than an essential requirement.

Improving or adjusting factors such as professional development, scheduling of classes, and acquisition of materials would assist in the implementation of an integrative model for science and mathematics (West et al. 2006). This is due to the fact that there is a great need for teachers to improve their knowledge within both disciplines so as to improve their delivery of each (Frykholm and Glasson 2005). Teachers should engage in “active learning opportunities in which authentic contexts provide fertile ground for understanding mathematics and science connections” (Frykholm and Glasson 2005, p.138).

In a recent review of mathematics in Irish post primary schools, teachers indicated that, within the mathematics curriculum, completing practical work with their pupils was their “greatest success” (NCCA 2005b, p.3). There was also an awareness among teachers themselves of the need to integrate mathematics with other areas of the curriculum. The results of Ransom’s (1998, p.1) research highlight the gains to be made from the fusion of Mathematics and other subject areas in a hands-on, cooperative, student-centred fashion:

- Mathematics teachers saw how they could exploit real data from other subjects to enrich different areas of the mathematics curriculum.
- Working collaboratively allowed teachers to expand their knowledge of their own and other subjects.
- The use of real data and instantaneous feedback gave pupils a greater understanding of the diagrams and formulas involved in mathematics and science.
- The use of graphing calculators allowed the pupils to work with greater independence.
• Planning cross-curricular groups between mathematics and other subjects allowed the interchange of ideas.
• Students became more interested in learning mathematics and science.

This research indicates that participation in such tasks has positive effects on the teaching community, student knowledge, student motivation and collaborative work amongst students. It also identifies the fact that students are given greater opportunities to work independently rather than taking all their direction from the teacher.

2.4.7 Conclusion

The evidence above suggests that hands-on, practical, student-centred, authentic activities are best when developing an effective integrative model of mathematics and science. Jacobs’ (1989) ten Design options for an Integrated Curriculum offer an effective method for defining aspects of this model whilst BWISM can be used to ensure that all vital aspects of integration are recognised and catered for within any model. What is required also is a framework to ensure that any tasks, problems or activities used in any attempt to integrate mathematics and science have the requisite standards for intellectual quality; otherwise these activities will not provide meaningful learning. Such a framework is Newmann’s ‘Authentic Instruction’.

2.5 Authentic Instruction

Authentic Instruction is a model which is integrative in its very nature and embodies many of the characteristics of effective integration recommended by academics in the field. This model provided key insight and direction throughout this study. Such an important role requires this model to be described and analysed with great depth.
2.5.1 What is Authentic Instruction?

The Authentic Instruction model is rooted in constructivism and relies on formative assessment in the same style as Assessment for Learning. Central to this approach is the setting of meaningful, engaging, intellectual tasks which replicate the challenges pupils will deal with in the real world (Ladwig et al. 2007). Particular focus is placed on problem solving and requires pupils to attain and use deep understanding and relevant skills to complete these real world problems or ‘Rich tasks’ as they are commonly referred to within the literature (King et al. 2009). This model, developed chiefly by Newmann, has three underlying guiding principles: Construction of Knowledge; Disciplined Enquiry; and Value Beyond School, each of which will be discussed in detail later.

Authentic Instruction relies on tasks which encourage students to synthesize knowledge and skills from various disciplines. The cross curricular nature of this model is especially suited to mathematics as putting mathematics into various contexts aids genuine understanding of many of its strands (McBride and Silverman 1991). King et al. (2009) argue that if true mastery is to be obtained then students must experience tasks which require “original application of knowledge and skills, rather than just routine use of facts and procedures” (p.3). They offer an example for the ‘real world’, designing a bridge, which identifies the complex tasks which working people face on a regular basis, tasks which they attempt to replicate in a school setting through Authentic Instruction. To design a bridge, an engineer must use his knowledge from a range of domains including mathematics, physics, engineering, architecture and local knowledge of the area. The engineer must synthesize all this knowledge to create a unique product that suits the specific needs of the task.

This type of example is used as the blueprint for tasks adapted for a school setting: the use of prior knowledge from various disciplines; analysis of the needs of the situation which has been presented, organising and interpreting all this
information so that a plan of action can be created and implemented to achieve the goal(s) of the task.

Even though this model takes its cues from situations outside of school, it does not suggest that all aspects of its use should conform exactly to these situations. Newmann et al. (1996) confirm this, calling on educators “not to insist that schoolwork should imitate all work outside of school but to consider examples of authentic intellectual accomplishment outside of school to help define standards of intellectual quality for schooling” (p.282).

2.5.2 The Three Defining Characteristics of Authentic Instruction

There are three underlying, guiding principles within Authentic Instruction: Construction of Knowledge; Disciplined Enquiry; and Value Beyond School (Newmann et al. 2007). A more in-depth look at each will give a clearer picture of the aims of the model and how it is delivered.

2.5.2.1 Construction of Knowledge

This aspect of Authentic Instruction involves the student “organizing, interpreting, evaluating, or synthesizing prior knowledge to solve new problems” (King et al. 2009, p.4). This construction of knowledge can be in the form of individual research and analysis and/or can be derived from interacting with peers in a group setting. Students should take advantage of all outlets of information open to them so that they can achieve a clearer picture of the task or problem they have been presented with in order to develop a solution. It is important to note that tasks within Authentic Instruction cannot be completed using set routines which have been taught to the students (King et al. 2009); in other words they do not attempt to remember a similar situation and follow the exact same steps to achieve a solution thus constructing knowledge will always be central to any problem faced within an Authentic Instruction setting.
2.5.2.2 Disciplined Inquiry

The inclusion of Disciplined Inquiry is largely to ensure high standards are maintained throughout student work in assigned tasks. It would be easy for a student to produce a large volume of work which is not of much value or did not require any higher order thinking so Disciplined Inquiry ensures that any approach to tasks set will have shape and direction. King et al. (2009) break disciplined inquiry into three sub categories:

1. **Prior Knowledge Base**: Students must have a firm base of knowledge and skills to work from e.g. facts, concepts, theories, algorithms. This base will aid them in various facets of their inquiry.

2. **In-depth understanding**: Students must have an intricate understanding of the relevant knowledge with which they can gain a deeper understanding of specific problems and tasks through intellectual work.

3. **Elaborated Communication**: This aspect calls for the use of the symbols and language used within various disciplines i.e. the vernacular of the discipline. “The tools they (accomplished adults) use – verbal, symbolic, graphic, and visual – provide qualifications, nuances, elaborations, details, and analogies woven into extended narratives, explanations, justifications, and dialogue.” (King et al. 2009, pp.4-5). These tools and details, which are often unique to certain disciplines, need to be adopted and used as required.

In general, disciplined inquiry ensures that students have the requisite approach, knowledge and tools to take on the task or problem set, so that meaningful learning takes place rather than having the students attain only superficial awareness (King et al. 2009).
2.5.2.3 Value Beyond School

This aspect of Authentic Instruction offers the clearest dissimilarity between it and regular educational practice whereby it ensures that each task will have utilitarian, aesthetic, or personal value compared to assessments such as spelling quizzes, laboratory exercises or final examinations which usually lack any significance beyond tracking the documentation of success in school (King et al. 2009).

King et al. (2009) believe that most of the activities and assessments within a school are designed to suit the system rather than provide any authentic learning opportunities i.e. the manner in which ‘achievement’ is measured in a school setting is not necessarily considered worthwhile outside the classroom.

2.5.3 Including the Three Characteristics at All Times

The developers of Authentic Instruction believe that it is of the upmost importance to include each of the three aspects (Construction of Knowledge, Disciplined Inquiry, and Value Beyond School) when assessing whether the performance of a task is of the requisite standard as regards intellectual quality and authenticity. For example, an interesting mathematics problem may require multifaceted construction of knowledge and disciplined inquiry to solve but if it has no value beyond determining the competence of the learner in order to pass an assessment then its authenticity is lacking (Newmann et al. 1996).

So Newmann and his associates believe that any educational task suffers if it does not satisfy all three criteria, an issue which not all commentators on Authentic Integration agree on. The aspect which has caused most argument amongst authors on this subject is ‘Value Beyond School’. Splitter (2009), for example, believes that this aspect is not as vitally important as the other two characteristics, claiming that the requirement of applying the material to real life is sometimes not necessary as it wouldn’t improve the significance or the quality of the lesson.
Splitter (2009) goes on to suggest that the interpretation of ‘Value Beyond School’ (i.e. imitating what is performed within various industries and disciplines) should not be taken literally. He suggests that pedagogic methods must correspond to how tasks should be performed rather than the approaches commonly used in the ‘real world’ i.e. an idealized version of what should be done rather than what is done. He suggests that the authenticity of a task should not be diminished just because it is not strictly a real world task (Splitter 2009).

This suggests that replicating what occurs in the real world is not always the best option and that educators should strive to instil practices and habits of greater value which produce superior results than the approaches commonly seen in the ‘real world’ which may be flawed in certain aspects.

It is clear that Splitter’s (2009) main problem with Authentic Instruction is the interpretation and, in certain situations, the inclusion of ‘Value Beyond School’ as a required element in each educational assignment. Newmann is insistent on all three criteria being present but does offer a little leeway by claiming that the three criteria are to be strived for but not all tasks would be able to meet all the criteria all the time. A radical movement away from all traditional forms of education is not required but rather an adjustment of focus so that authentic achievement is the target to be aimed for (Newmann et al. 1996).

The suggestion that authentic achievement should be the guide to achieving meaningful learning suggests that there are plenty of positives when Authentic Instruction is adopted.

2.5.4 Advantages of the Implementation of Authentic Instruction Model

Many of the problems with mathematics education which were previously outlined (section 2.2) can be counteracted when Authentic Instruction is central to the pedagogic approach (Newmann et al. 2007). Newmann et al. (1996) believe that calls for education reform can be attributed to the general belief that the type of mastery achieved by pupils is largely ‘trivial’ or ‘contrived’, citing the need for
more meaningful intellectual accomplishment which can be achieved by means of authentic intellectual work through Authentic Instruction.

They also suggest that “learning will be more adaptive or powerful when students can connect new information to their own experiences” (Newmann et al. 1996, p.286). It is this aspect of Authentic Instruction which is the most appealing: students will learn something worthwhile in a way which offers them greater control and ownership thus increasing motivation and, in many cases, enjoyment which can only be a positive outcome.

Completing tasks through the application of ‘Authentic Intellectual Work’ (AIW) is commonly referred to in literature relating to Authentic Instruction as being a goal for pupils to strive for (Newmann et al. 2007). AIW is in evidence when students develop in-depth understanding rather than superficial awareness by addressing the complex intellectual challenges of work, civic participation, and managing personal affairs in the contemporary world (King et al. 2009). There are three general reasons offered by King et al. (2009) as to why schools should promote authentic intellectual work:

1. **Better preparation for intellectual demands of the workplace, citizenship, and personal affairs**: they believe that the experience built up from solving problems and tackling tasks will aid students greatly in how they approach similar challenges in various contexts. The ability to correlate information from different sources, reasoning skills and putting a plan of action in place will be valuable in these contexts. This would aid in addressing the pressing worry alluded to by O’Donoghue (2004) (cited in section 2.2.2) that students are unable to solve problems and just rely on translation algorithms that allow them to solve textbook questions.

2. **Increased opportunities for student engagement in learning**: central to the problems in modern education is that students are just given information which they must memorise and reproduce in exams, a process which is
often boring to students. These limitations within mathematics pedagogy were also observed by Childs (2006), Williams (1992) and Boaler (1994) (See section 2.3.2). Within Authentic Intellectual work students are given the opportunity to develop knowledge and skills through investigation and developing ideas which they can express to peers, with many teachers reporting that students find the authentic intellectual work they encounter to be more interesting and meaningful than the typical drill and practice tasks that are more widely used (Newmann et al. 2007).

3. **Intellectual mission strengthens professional community**: The criteria and specific standards within Authentic Instruction ensure that there is plenty of teacher dialogue and cooperative planning across all subjects. Educators can work in tandem to achieve common goals within generic intellectual activities which encompass various disciplines. School mission statements can become more meaningful as the teaching community can work together to achieve them rather than taking on individual parts of the statement and attempting to achieve them in isolation.

### 2.5.5 Criticisms of an ‘Authentic’ approach

The advantages of Authentic Instruction have been discussed and analysed but to achieve a clear insight into this pedagogical approach, the pitfalls need to be investigated and assessed accordingly. Newmann et al. (1996) warn of the authentic assessment “trap,” implying that student participation in authentic activities “can become an end in itself, regardless of the intellectual quality of students’ work” (p. 281). In other words, teachers may view the level of success of the lesson based on how well the group works together to complete a task rather than the quality of learning and level of knowledge attained by each individual in the group.
Testing of the gains in such knowledge and development of learning has been called into question also. Terwilliger (1997) queries how assessment of an authentic nature can be applied on a large scale (e.g. nationally), questioning whether implementation of such assessments in an extensive program would be possible and whether or not it would retain the necessary psychometric quality.

But before assessment can even be considered, the quality of instruction through these means must be reflected upon. As the application of such an authentic approach depends heavily on the commitment and ability of teaching staff to achieve its objectives (Hanley-Maxwell et al. 1999), this could prove to be an issue if implemented within Irish (and/or other) mathematics classrooms as there is quite a large number of under-qualified mathematics teachers employed in Irish schools with 48% of such teachers described as being ‘out-of-field’ i.e. teaching subjects which do not match their training or education (Ni Riordáin and Hannigan 2009), thus they may not have the requisite knowledge to properly apply an authentic approach.

There are areas of Authentic Instruction, and an ‘authentic’ approach in general, which are cause for concern and need to be taken into consideration but the perceived benefits of the adoption of this curriculum model are manifold. Proof that the pros outweigh the cons in this instance are displayed through in-depth research.

2.5.6 Research findings

2.5.6.1 Introduction

These studies had common research aims:

1. To find out if students who experienced higher levels of Authentic Intellectual Work and assessment showed more progress in comparison to those who experienced lower levels of such work and assessment.
2. To determine what factors inside and outside of the school setting helped and hindered the implementation of Authentic Intellectual Work and assessment.

2.5.6.2 Measuring Teacher and Student Authentic Intellectual Performance

Student performance was measured in two general ways to aid in achieving the first research aim:

1. The students assignments were graded according to a rubric based on criteria for Authentic Intellectual Work.
2. Tests which measured basic skills and retention of information studied.

These tests of progress were also valuable within the second research aim i.e. how socio-economic status, race, gender and prior school achievement affected progress and performance when Authentic Intellectual Work (AIW) is implemented. The rubric used to assess the level of Authentic Instruction can be broken down into four standards:

Standard 1: Higher Order Thinking

“Instruction involves students in manipulating information and ideas by synthesizing, generalizing, explaining, hypothesizing, or arriving at conclusions that produce new meaning and understandings for them.”

(Newmann et al. 2007, p.35)
Standard 2: Deep Knowledge

“Instruction addresses central ideas of a topic or discipline with enough thoroughness to explore connections and relationships and to produce relatively complex understandings.”

(Newmann et al. 2007, p.37)

Standard 3: Substantive Conversation

“Students engage in extended conversational exchanges with the teacher and/or their peers about subject matter in a way that builds an improved and share understanding of ideas or topics.”

(Newmann et al. 2007, p.41)

Standard 4: Connections to the World Beyond the Classroom

“Students make connections between substantive knowledge and public problems or personal experiences they are likely to have faced or will face in the future.”

(Newmann et al. 2007, p.44)

Each of these standards was marked on a scale of 1-5, with 5 meaning that the standard was satisfied fully and 1 suggesting that the standard was not present at all within the task or lesson. The scores from each standard were added to give an overall mark (out of 20) which determined the level of authentic instruction i.e. how well the pedagogic approach contributed to creating an atmosphere suited to authentic intellectual work. The four standards above are a general guide for teachers in applying the framework.

There are a further three standards which cover the tasks or assignments given to the students:
Standard 1: Construction of Knowledge

“The assignment asks students to organize and interpret information in addressing a concept, problem, or issue relevant to the discipline.”

(Newmann et al. 2007, p.47)

Standard 2: Elaborated Written Communication

“The assignment asks students to elaborate on their understanding, explanations, or conclusions through extended writing in the relevant discipline.”

(Newmann et al. 2007, p.49)

Standard 3: Connection to Students’ Lives

“The assignment asks students to address a concept, problem, or issue in the relevant discipline that is similar to one that they have encountered or are likely to encounter in their daily lives outside of school.”

(Newmann et al. 2007, p.50)

These three standards are ranked on a scale of 1-3, with 3 signifying that the standard has been fully satisfied and 1 indicating that the standard is barely satisfied, if at all.

The quality of authentic intellectual work is therefore marked out of a total of 29 when scores for each standard are added up. These standards give a clearer picture of what authentic intellectual work is and what areas the authors believe define the framework.
2.5.6.3 Results of the Studies Carried Out

The rubric was used to rate teachers within various studies (Lee and Smith 1995, Newmann et al. 1996, Newmann et al. 1998, Newmann et al. 2001, Avery 1999). Students who were taught by teachers with high scores were compared with students taught by teachers who achieved average and/or low scores when rated using the rubric (see Table 2.2).

Table 2.2: Summary of Research analysed – adapted from Newmann et al. (2007).

<table>
<thead>
<tr>
<th>Study Name and Dates</th>
<th>Number, Type of Schools, classes, Students.</th>
<th>Subjects, Grade Levels</th>
<th>Achievement Measure</th>
<th>Achievement Benefit of Higher vs. Lower Scoring Teachers’ Classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center on Organisation and Restructuring of Schools (CORS) Field Study, 1990-94.</td>
<td>24 elementary, middle, and high schools. 130 Classrooms, 2,100 students. Schools mostly urban, some non-urban.</td>
<td>Mathematics, Social Studies. Grades 4-5, 7-8, 9-10.</td>
<td>AIW Rubrics.</td>
<td>30 percentile points higher than lower scoring.</td>
</tr>
<tr>
<td>Chicago Annenberg Research Project Field Study, 1996-97.</td>
<td>12 Chicago elementary schools. 74 teachers, about 700 students, all urban.</td>
<td>Writing, Mathematics Grades 3, 6, 8.</td>
<td>AIW rubrics.</td>
<td>34-56 percentile points higher than lower scoring.</td>
</tr>
<tr>
<td>Study</td>
<td>Schools/Schools</td>
<td>Subjects</td>
<td>Test</td>
<td>Results</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>-----------------</td>
<td>------------------------</td>
<td>--------------------</td>
<td>------------------------------</td>
</tr>
<tr>
<td>Chicago Annenberg Research Project.</td>
<td>46 Chicago Elementary Schools. 124 Teachers.</td>
<td>Mathematics, writing, reading.</td>
<td>Iowa Test of Basic Skills.</td>
<td>Early gains 40% higher on test score scale.</td>
</tr>
</tbody>
</table>

As expected, the level of proficiency or improvement in students of teachers who achieved higher scores for the promotion of AIW was greater than in students of teachers who achieved average or lower scores for the promotion of AIW. The extent of this difference varied between studies and was sometimes difficult to compare as certain studies used AIW rubrics whilst others depended on standardised tests such as the Iowa Test of Basic Skills (ITBS) and the National Assessment of Educational Progress (NAEP).

Two studies which adopted the AIW rubric for assessing teacher ability and the AIW rubric for assessing student work were Newmann et al. (1996) and Newmann et al. (1998). Newmann et al. (1996) conducted a large study between 1990 and 1994 in 24 elementary, middle, and high schools which were mainly located in urban areas. 2,100 students in 130 classrooms were observed and assignments were collected from each. The subjects focused on were mathematics and social studies in grades 4, 5, 7, 8, 9 and 10.

The results for this survey were grouped together but the authors indicate that differences between results in both subjects were minimal hence it is safe to suggest that, in terms of mathematics, the results are a viable guide to the effect AIW had on the subject. It was found that students of teachers who scored
highest within the AIW instruction rubric performed 30% better in their own AIW assessment (6.8 out of 12) than students of teachers who scored lowest within the AIW instruction rubric (5.4 out of 12). It was also noted that these students performed, on average, about 12% better than students of teachers who scored within the average range on the AIW instruction rubric (6.1).

More significant differences were found by Newmann et al. (1998) in their Chicago Annenberg Research Project. This study was carried out between 1996 and 1997 in 12 Chicago elementary schools in urban areas. 74 teachers and 700 students were involved in the study. Again, assignments were collected and marked using the AIW rubric. 3rd, 6th, and 8th grade mathematics and writing were the focus of the study. 8th grade mathematics is most relevant to this research; it showed a massive 56 percentile point AIW assignment performance difference between students taught by teachers who scored highest in the AIW instruction rubric (82%) when compared to students taught by teachers who scored lowest in the AIW instruction rubric (26%).

Figure 2.2: Results of Newmann et al. (1996) CORS Field Study
Table 2.3: Results of the Chicago Annenberg Research Project

<table>
<thead>
<tr>
<th>Teacher AIW Performance</th>
<th>Average Student AIW Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Quartile</td>
<td>82%</td>
</tr>
<tr>
<td>Bottom Quartile</td>
<td>26%</td>
</tr>
</tbody>
</table>

These studies obviously show that there is a direct correlation between the promotion of Authentic Intellectual work by the teacher through their instruction and the authentic intellectual quality of the work produced by students. The effect of AIW on basic skills and knowledge retention is another important aspect of its value in an educational sense i.e. examining its effect on more conventional academic achievement. Studies carried out by Newmann et al. (2001) and Lee and Smith (1995) offer insights into its performance in this aspect.

Lee and Smith (1995) carried out the ‘National Education Longitudinal Study’ between 1988 and 1992 in 1,000 high schools across the U.S.. 10,000 students from grade 8, 10 and 12 mathematics and science were surveyed, with the National Assessment of Student Progress (NAEP) multiple choice test used as the measure of student performance rather than the AIW rubric used in the previous two studies mentioned. The NAEP only requires recall or simple application of previously learned information. Students were assessed when they were in grades 8, 10, and 12 to gauge how well they improved as they progressed.

The results showed that gains in achievement within mathematics between grades 8 and 10 (similar timeframe and age groups as Junior Cycle) were greatest in students who experienced high levels of Authentic Instruction and got lower as the level of Authentic Instruction decreased. There was an average of a 9 point gain in students whose teacher scored highest in the AIW instruction rubric; a 7 point gain for students whose teacher received an average score in the AIW instruction rubric; and only a 5 point gain for students whose teacher scored lowest in the AIW instruction rubric. The trend is clear: when Authentic Intellectual Work is promoted in the classroom, there is evidence to suggest that
the gains made by students as regards information recall and ability to perform basic skills are likely to be greater than if Authentic Intellectual Work is not present in teacher instruction.

![Figure 2.3: Results of National Education Longitudinal Study](image)

**2.5.6.4 Discussion of Results**

The first set of studies showed that the higher the level of Authentic Intellectual Work promoted by the teacher through instruction, the higher the level of Authentic Intellectual Work produced by students. The second set of studies highlighted the improvements in information recall and execution of basic skills and algorithms when Authentic Intellectual Work is promoted in the classroom. This is particularly relevant to mathematics in its present guise as, rightly or wrongly, high value is placed on the ability to recall information, methods and algorithms, and the ability to apply them correctly.

Even though the results outlined above are very positive, caution must be exercised when attributing plaudits to the framework. Teachers were rated on
their propensity to promote Authentic Intellectual standards in their classroom and given a rank on this basis but it may not be entirely for this reason that their students were successful or unsuccessful in producing better results in standardised testing and in the production of Authentic Intellectual Work. It may be that the teachers who scored in the high bracket were better teachers than those who scored lower as it takes a great deal of preparation, motivation, creativity, class management and attention to detail, amongst other attributes, to implement such tasks, group work activities, assessments and class discussions on a regular basis. It would be much easier to adopt a didactic approach more akin to that which would be termed low in the promotion of Authentic Intellectual Work.

Hence it could be argued that the better teachers, according to the AIW rubric, just have a greater ability to educate the students rather than placing all the credit with the framework. Having said that, the results are too good to ignore and offer solid evidence that promoting Authentic Intellectual Work has a noticeably positive effect on student performance in both Authentic Intellectual Work and traditional knowledge retention and execution of basic skills.

### 2.6 Other Models Considered

#### 2.6.1 Cognitive Apprenticeship

Cognitive Apprenticeship is a model similar to Authentic Instruction which was created around the same time, 1989, by Collins, Brown and Newman. The model is based on a synthesis of apprenticeship and schooling i.e. it takes the characteristics of an apprenticeship and adjusts them to apply them to a classroom context.

The four characteristics of an apprenticeship which they identify are modelling, scaffolding, fading, and coaching (Collins et al. 1991).
• Modelling: the master performs the task, showing the apprenticeship how to complete each stage or sub-skill, explaining his actions as he performs.

• Scaffolding: the master offers help to the apprentice; this ranges from performing most of the task to offering hints and tips on how to complete it.

• Fading: The idea of reducing the amount of help given to the apprentice i.e. giving him/her increasing levels of responsibility

• Coaching: this is the process of supervising the apprentice’s work – interceding regularly to offer hints and tips, adjust the task, create different challenges to aid learning and diagnosing problems they are having.

The Cognitive Apprenticeship curriculum model largely adopts these apprenticeship characteristics but there are some noticeable differences. The main difference, according to Collins et al. (1991), is that both teachers and students must express the thinking behind their actions in a concrete manner as required – this could be in different forms depending on the situation e.g. oral, writing, graphics. This is so there is proof of work being done and it also makes it easier for students to observe and enact the process themselves with the aid of the teacher and peers.

Cognitive Apprenticeship is also intertwined with the core concepts of situatedness and legitimate peripheral participation (Lave and Wenger 1991). Situatedness refers to the cultural, historical, and institutional factors which affect the authentic learning environment that the pupils are immersed in through this process (Dennen 2004). Legitimate peripheral participation, is a process which occurs when learners enter on the periphery and gradually move toward full participation (Dennen 2004). For example, an apprentice may only complete minor elements of the profession at the beginning but would assume greater responsibilities as they progress until they are fully immersed in the profession.
Within apprenticeships, the apprentice can see the finished product thus he/she knows what he/she is working towards. Within the classroom this is not always so clear hence abstract tasks of the curriculum must be placed in contexts which make sense to the student and offer a reason for completing the task (Collins et al. 1991).

Generally within a traditional apprenticeship, some skills within tasks are unique to that discipline i.e. the level of skill transfer is not high. But an important aspect of education is the transfer of skills between tasks hence Collins et al. (1991) believe that a range of tasks need to be presented, varying from basic to complex, containing the target skills as a common theme across these tasks. This will allow the students to apply the skills in various situations so that they can apply them independently when faced with unfamiliar situations which require those skills.

Collins et al. (1991, p.3) outline the general approach required by the teacher:

- Identify the processes of the task and make them visible to students;
- Situate abstract tasks in authentic contexts, so that students understand the relevance of the work; and
- Vary the diversity of situations and articulate the common aspects so that students can transfer what they learn.

Further adjustments to the characteristics of an apprenticeship are the addition of reflection and exploration. These elements complete the synthesis of schooling and traditional apprenticeship. Reflection allows students to self-assess; converse with a peer, teacher, group or whole class on the task or product; or note their experiences within the activity and the skills they acquired or improved upon. Exploration is allowing students to tackle problems themselves, possibly with some scaffolding or hints and tips but generally encouraging them to figure it out for themselves. All these adjustments combine to offer steps to create meaningful tasks which will affect functional learning within the students in a similar fashion to that which is experienced by an apprentice.
The Cognitive Apprenticeship approach is similar to Newmann’s Authentic Instruction framework in many ways, especially in its demand for authenticity in school tasks (Dennen 2004). The main difference between the two models was their performance in a school setting. While Authentic Instruction has been shown to be relatively successful (see 2.5.6.3 and 2.5.6.4), the Cognitive Apprenticeship model did not perform with such distinction when implemented in a study conducted by Hendricks (2001).

Hendricks’ (2001) study aimed to determine whether situated instruction was more likely to result in more usable, transferable knowledge than traditional instruction. A control and treatment group were established; the control group taught using traditional methods, while Cognitive Apprenticeship was applied within the treatment group. Two weeks after instruction, both groups were set a task which would require them to transfer knowledge learned during the instruction phase. It was found that there was no significant difference between the ability of students from the control group and students from the experimental group in their ability to transfer the knowledge learned previously to this new task.

Allied to this mediocre performance, is the proclamation by Dennen (2004) that there is not enough research in Cognitive Apprenticeship, especially in determining as to whether or not instruction based around Cognitive Apprenticeship is preferable to traditional instruction. Since this claim there has been some further investigation within the model e.g. Charney (2007) applied Cognitive Apprenticeship with pre-College students by allowing them to meaningfully engage with scientists in a real working lab. Through this he found that a difference was made to the students’ understanding and beliefs in relation to science. Another example of its application came in a study conducted by Stalmeijer et al. (2009) in which the Cognitive Apprenticeship approach was applied when educating undergraduate medical students. The model was found to be useful in this instance, especially in terms of evaluation, feedback, self-assessment and faculty development (Stalmeijer et al. 2009).
Dennen (2004) suggested that studies based around the application of Cognitive apprenticeship had no focussed program of research which would allow for greater generalizability, this trend continues with the varying applications of the model as displayed by the recent studies of Charney (2007) and Stalmeijer et al. (2009). With this lack of direction in mind, which is in direct contrast to the consistent quality of performance of Authentic Instruction in the second level classroom (as outlined in section 2.5.6.3), the evidence implies that Authentic Instruction is the more proven model within second level education when compared to Cognitive Apprenticeship.

2.6.2 Anchored Instruction

The Anchored Instruction framework was developed by The Cognition and Technology Group at Vanderbilt University, Tennessee. They recognised that within the typical school setting, students learn material in such a way that they can only reproduce it when explicitly asked to rather than remembering to use it in problem solving situations when it is most useful. They called material learned in this manner ‘Inert Knowledge’ (Cognition and Technology Group at Vanderbilt 1990). An example of inert knowledge would be having the know-how to solve a quadratic equation when asked directly but failing to use this information when asked to find the x-axis intercepts when graphing a quadratic. In other words, it is the inability to apply knowledge gained in a different context to which it was learned.

The issue of Inert Knowledge is central to the Anchored Instruction approach – the major aim of this framework is to overcome the Inert Knowledge problem by fostering an environment of sustained exploration in which students and teachers work together to explore the uses of the material they are studying through real life scenarios and from multiple perspectives (Cognition and Technology Group at Vanderbilt 1990). This framework uses open-ended problems which students explore and solve. These problems, usually in the form of a story or a description, contain all the information required to complete them which allows plenty of
opportunity for scaffolding and also caters for scenarios where there is limited
time or limited resources (Oliver 1999). Students are challenged to analyse the
story so that they gather all the relevant data and develop an effective problem
solving approach (Oliver 1999).

Anchored instruction is quite similar to Authentic Pedagogy due to both being
rooted in a constructivist approach. The potential is present for the combination
of mathematics and science within Anchored Instruction as problems which
require knowledge from both areas can be created in the form of a story
containing the relevant data required to tackle the problem. The practice of
supplying a story or a description of a situation for students to interpret, analyse,
and manipulate to solve a problem or complete a task is an approach which could
prove useful in creating lessons through Authentic Instruction.

2.6.3 Constructive Alignment

Within Constructive Alignment the focus is on ensuring that the curriculum and
its intended outcomes, the teaching methods and the assessments employed are
closely linked or, as is suggested in the framework title, aligned. The teacher
creates an environment which allows the students to construct their own learning
by actively engaging with the material of the topic being studied (Biggs 2005).
Central, also, to this framework is the belief that understanding is not something
which is transferred from the teacher to the student but is something that students
must construct themselves; the teacher is there to facilitate this process (Biggs
2005).

John Biggs (2005, p.2), the developer of this pedagogical approach, defined the
framework in four major steps:

1. Defining the intended learning outcomes (ILOs);
2. Choosing teaching/learning activities likely to lead to the ILOs;
3. Assessing students’ actual learning outcomes to see how well they match what was intended;
4. Arriving at a final grade.

In defining ILOs, Biggs (2005) states that these objectives must require the students to demonstrate their understanding rather than regurgitating the information they’ve acquired. In other words, the objective must be for students to develop a level of understanding to such an extent that they can use the knowledge they have gained rather than just being able to state what they know (Biggs 2005).

Biggs (2005) suggests approaches such as interactive group work, peer teaching, independent learning, and work-based learning, amongst others, as being beneficial strategies which foster a positive learning environment allowing students to construct their own understanding. As regards assessment, Ramsden (1992) states that students will study what they believe will come up in the exam rather than what they have covered during class thus Biggs’ (2005) Constructive Alignment calls for assessment to mirror the ILOs set out at the beginning, intimating that the “sought-for qualities of performance” set out in the ILOs must be displayed in the assessment.

2.6.4 Conclusion

The models mentioned above were considered as regards their suitability for integrating mathematics and science but in the end fell short in comparison to Authentic Pedagogy. Each model does offer some key ideas which will inform development of lesson plans and the overall approach to integration e.g. the four steps involved in cognitive apprenticeship (modelling; scaffolding; fading; coaching) would be very useful in defining how to approach the process of being a facilitator of student learning, an aspect that Authentic Pedagogy that Newmann and his associates have not fully explored in their research. The idea of placing the task or challenge in the form of a story, a la Anchored Instruction, is quite
interesting and could offer an effective, alternative outlet to that of simply stating a task or giving basic information. As for Constructive Alignment, the need to mirror intended learning objectives, which call for the learner to construct their own meaning, within assessment is a sound tactic, something which can easily be incorporated into the Authentic Pedagogy approach.

2.7 Conclusion

Considering the needs of mathematics education (outlined section 2.2), the adoption of an integrative model for mathematics and science using the AIW framework would, in theory, satisfy one of the main conclusions that the NCCA (2005b, p.26) came to in their ‘Review of Mathematics in Post-Primary Education:

“Young people need to develop the ability to build connections across knowledge, to identify and explore patterns, to estimate and predict, to interpret and analyse numerical and statistical data, to communicate increasingly complex information, and to apply all of this in their daily lives and work.”

Upon analysing the above statement, the three characteristics of Authentic Instruction (Construction of Knowledge (“identify and explore”; “estimate and predict”); Disciplined Inquiry (“interpret and analyse”); Value Beyond School (“daily lives and work”)) can be clearly identified as well as the need for some form of cross curricular study i.e. “build connections across knowledge”. The NCCA’s (NCCA 2005b) recommendations also echo those of academics cited in this literature review (see section 2.3.2) such as Childs (2006) who suggests that more emphasis needs to be placed on the process of learning, i.e. the construction of knowledge, rather than focussing on the content and structure within mathematics and science, a view backed up by various literature (Lyons et al. 2003, Mitchelmore and White 1995, William 1992).
Boaler (1994) calls for change which would stop the process of ‘context-conditioning’ by offering pupils real world tasks which they can explore in a more holistic manner thus it can be suggested that she endorses disciplined inquiry which fosters the development of skills and knowledge that are useful beyond the classroom. Similarly, the ESRI cite the need for more holistic learning at Junior Cycle (see section 2.3.3), an element central to Authentic Instruction, to bridge the gap from Primary to Post-Primary instruction.

Newmann et al. (2007) claim (and are backed up by various research (see section 2.5.6)) that Authentic Instruction can solve many of the issues which are referred to as being problematic in modern education (2.5.4), many of which are very similar in nature to the characteristics of the ‘Mathematics Problem’ outlined by O’Donoghue (2004) i.e. poor skills; fragmented knowledge; and the inability to apply what is learned to contexts outside of that in which it was learned (see section 2.3.2). Thus it can be claimed that the Authentic Instruction framework has the characteristics to address the needs of modern mathematics education.

Calls for integration of mathematics and science (and/or other subjects) have come from leading mathematics education and general education bodies (National Council of Teachers of Mathematics (NCTM), the National Research Council (NRC), the Curriculum Corporation (Australia) and the School Science and Mathematics Association (SSMA)). Literature on the topic of integration supports doing so using practical, student-centred, authentic activities (see section 2.4.6), an approach supported by the Authentic Instruction framework.

It is due to these findings, outlined in the above paragraphs, that this author concludes that cross curricular activities should be created and implemented within the Irish mathematics curriculum and that the Authentic Instruction framework, due to its outstanding research results (section 2.5.5), will offer a functional blueprint for the creation of such activities, tasks, problems, and experiments of a practical nature.
Chapter 3

Methodology
Designing and implementing a model in an educational setting requires careful analysis and planning to ensure that such a model is backed by quality research and employed in the most efficient manner. This process also requires that such an investigation is assessed using research tools and approaches which are both valid and reliable. This chapter will discuss this requirement and how it was satisfied, as well as providing a detailed description and justification of the methods used and tools applied throughout the investigation. A chronology of the research carried out is provided to offer an insight into the elements of research which underpinned each stage of the investigation. While key considerations such as ethics and the limitations of this study are discussed in detail. To commence this insight into the research methodology of this study, the definition and characteristics of research will be stated and analysed followed by the purpose and objectives of the investigation.

### 3.1 Introduction

Research can be described as the systematic process of gathering, analysing and interpreting information, or data, with a view to improving understanding about a particular phenomenon (Leedy and Ormrod 2010, pp.2-3). Any research typically has eight distinct characteristics:

1. Research originates with a question or problem.
2. Research requires clear articulation of a goal.
3. Research requires a specific plan for proceeding.
4. Research usually divides the principal problem into more manageable subproblems.
5. Research is guided by the specific research problem, question, or hypothesis.
6. Research accepts certain critical assumptions.
7. Research requires the collection and interpretation of data in an attempt to resolve the problem that initiated the research.
8. Research is, by its nature, cyclical or, more exactly, helical.
Each of these characteristics will be analysed and discussed throughout this chapter, beginning with the questions which led to the commencement of this research and the objectives that were established when this endeavour began, leading onto the design of the research, how it was implemented, and the tools employed to analyse the data gathered in a manner that was valid and reliable.

3.2 Research Purpose, Questions, and Objectives

The scope and significance of this investigation (see section 1.3) shaped the purpose of the study which subsequently led to the development of research questions that needed to be considered and answered. The objectives linked to these research questions would aid in achieving the corresponding answers. As such, the research purpose, questions, and objectives shaped the methods applied in this investigation. They are each detailed in this section.

3.2.1 Research Purpose Leading to Research Questions

The purpose of this research is to ascertain whether the subjects of mathematics and science can be integrated successfully within stand-alone lessons at Junior Cycle. The level of success can be determined by answering the following research questions:

- How can mathematics and science be effectively integrated in the classroom? Is there a model which can be adopted or adapted to fit the needs of such an undertaking?

- Is the education system, in its current form, malleable enough to incorporate such a radical change, taking into consideration resources, teacher knowledge within their non-specialist subject, and timetabling, among other issues?
• Is integrating mathematics and science more effective than teaching the subjects separately?

• Do teachers value the process of integrating mathematics and science? Will they want to continue to integrate mathematics and science on a regular basis?

3.2.2 Research Objectives

Investigation into the answers to the research questions outlined above requires that certain objectives be achieved. These objectives are listed as follows:

• Identify the areas of mathematics and science which are best suited for integrative activities.

• Develop a working model for integrating the subjects i.e. a blueprint for lesson plans of this nature.

• Create lessons/activities of this nature and experiment with them; receive feedback and adjust these lessons accordingly.

• Investigate how such lessons can be incorporated into the traditional secondary school timetable and implement them in Junior Cycle mathematics and science classes.

• Gauge teacher and pupil reaction and attitude to integration and the methods used to employ it through questionnaires, focus groups, interviews, and assessment of pupil work.
• Determine whether the Irish Education system at Junior Cycle level is flexible enough to incorporate integrative modules/subjects and assessments.

• Outline further research which is required in this area of study.

3.3 Research Paradigms Leading to the Mixed Method Approach

Analysing the outcomes within this study require the collection of data relative to the research questions and objectives. The author, in order to attain an expansive insight into the effects of the intervention, collected both qualitative and quantitative data (see table 3.1). Discussion of these research paradigms will offer a greater perception of the advantages (and disadvantages) of each, as well as indicating why the combination of the two paradigms provides superior research to an approach which adopts only one particular paradigm.

3.3.1 Research Paradigms – Quantitative and Qualitative

Quantitative data is numeric data which differs in amount or degree; it is placed on a continuum from less to more (Fraenkel and Wallen 2003). To collect this data, inanimate instruments such as scales, tests, surveys, and questionnaires, are commonly used. To apply quantitative research methods, investigation must stress quantification in the accumulation and study of information pertaining to the research at hand (Bryman and Bell 2007). As such, quantitative research lends itself to the breakdown of facts that posit a “social reality as an external, objective reality” (Bryman and Bell 2007) through the prediction, control, description, or confirmation of a given hypothesis according to the precise, numerical data collected (Merriam 2009). A prime example of the application of quantitative research is the teacher questionnaire used within this study which applies the Likert scale to form quantitative results from what would otherwise be qualitative data.
The key strengths of quantitative research are that it is precise; the results are relatively independent of the researcher; and data analysis is relatively less time consuming (Johnson and Onwuegbuzie 2004). While some of the perceived weaknesses of this research approach are that the categories and theories applied by the researcher(s) may not reflect local constituencies’ understandings; and, also, the researcher may focus on testing a particular hypothesis and miss out on phenomena occurring which could be observed by generating a hypothesis instead – this is known as confirmation bias (Johnson and Onwuegbuzie 2004).

Qualitative data, on the other hand, is any data that are not quantitative (Tesch 1990). Qualitative research has the goals of studying all aspects of natural occurring phenomena (Fraenkel and Wallen 2003). As such, understanding, discovery, and meaning, as well as description, form the objectives of this type of investigation, with interviews and the researcher’s observations forming the primary methods of collection of the data involved (Merriam 2009). This would suggest that qualitative research is suited to the investigation of the “quality of relationships, activities, situations, or materials” (Fraenkel and Wallen 2003, p.430). A key element of the author’s research is to gain an in-depth understanding of the successes, failures, opinions, and conclusions of the subjects – teachers and pupils – that experienced this intervention (a process which is explained in detail in section 3.5.4). Collecting qualitative data through interviews and focus groups should aid in achieving these goals as evidenced by the strengths of this type of research.

Some of the strengths of this type of research include its ability to describe complex phenomena in rich detail as they are situated in local contexts; its ability to be responsive to changes that occur during the course of the study (i.e. confirmation bias can be avoided) thus changing the focus of the study; and it can be quite effective when used to explore how and why phenomena occur (Johnson and Onwuegbuzie 2004). Detractors of qualitative research cite the difficulty in testing hypotheses and theories or making quantitative predictions when using qualitative data; the time it takes to collect and analyse the data; and the fact that
results obtained from qualitative research are more susceptible to the researchers’
personal biases (Johnson and Onwuegbuzie 2004).

Table 3.1: Characteristics of Qualitative and Quantitative research (Merriam 2009, p.18)

<table>
<thead>
<tr>
<th>Points of Comparison</th>
<th>Qualitative Research</th>
<th>Quantitative Research</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Focus of Research</strong></td>
<td>Quality</td>
<td>Quantity</td>
</tr>
<tr>
<td><strong>Associated Phrases</strong></td>
<td>Fieldwork, ethnographic, naturalistic, grounded, constructivist.</td>
<td>Experimental, empirical, statistical.</td>
</tr>
<tr>
<td><strong>Goal of Investigation</strong></td>
<td>Understanding, description, discovery, meaning.</td>
<td>Prediction, control, description, confirmation.</td>
</tr>
<tr>
<td><strong>Design Characteristics</strong></td>
<td>Flexible, evolving, developing.</td>
<td>Predetermined, structured.</td>
</tr>
<tr>
<td><strong>Sample</strong></td>
<td>Small, non-random, purposeful.</td>
<td>Large, random, representative.</td>
</tr>
<tr>
<td><strong>Data Collection</strong></td>
<td>Researcher as primary instrument, interviews, observations</td>
<td>Inanimate instruments (scales, tests, surveys, questionnaires, computers).</td>
</tr>
<tr>
<td><strong>Mode of analysis</strong></td>
<td>Inductive (by researcher)</td>
<td>Deductive (by statistical)</td>
</tr>
<tr>
<td><strong>Findings</strong></td>
<td>Comprehensive, holistic, expansive, richly descriptive</td>
<td>Precise, numerical.</td>
</tr>
</tbody>
</table>
3.3.2 Mixed Methods Approach

The application of multiple methods to research a given area allows for broader and better results (Denzin and Lincoln 1998) thus the author applied research techniques of both the quantitative and qualitative variety. Such an approach can be termed as the application of ‘Mixed Methods’.

![Mixed Methods Approach](image)

Figure 3.1: The Mixed Methods Approach

Qualitative and quantitative research methods can be combined as, even though they appear to be on opposite ends of the same spectrum, quantitative and qualitative research methods have certain similarities:

- Both use empirical observations to address research questions.
- Data is described.
- Explanatory arguments are constructed from this data.
- Outcomes are conjectured.
- Precaution is taken to minimise bias and other sources of validity.

(Sechrest and Sidani 1995)
Johnson and Onwuegbuzie (2004) argue that the goal of the mixed method approach is not to replace qualitative and quantitative techniques but to combine them to capitalize on the strengths of each and minimise the weaknesses which may be inherent in both (see Table 3.2). Such a goal has been attained regularly as the mixed method approach has frequently produced superior research when compared to research which employs quantitative or qualitative techniques only as indicated by table 3.2 below (Johnson and Onwuegbuzie 2004). Such endorsement of this methodology is quite an advance on previous thinking as many believed that qualitative and quantitative methods were incompatible (Sieber 1973). Application of these mixed methods to the author’s research produced results which provided in depth understanding of the experiences of teachers in implementing the Authentic Integration model and the pupils’ experiences in completing the work set through the lessons. While the mixed methods will also ensure that there are precise, numerical findings through collection of quantitative data.

Table 3.2: Strengths and weakness of ‘Mixed Methods’ research. (Johnson and Onwuegbuzie 2004, p.21)

<table>
<thead>
<tr>
<th>Strengths</th>
<th>Weakness</th>
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<tbody>
<tr>
<td>• Words, pictures, and narrative can be used to add meaning to numbers.</td>
<td>• Can be difficult for a single researcher to carry out both qualitative and quantitative research, especially if two or more approaches are expected to be used concurrently; it may require a research team.</td>
</tr>
<tr>
<td>• Numbers can be used to add precision to words, pictures and narrative.</td>
<td>• Researcher has to learn about multiple methods and approaches and understand how to mix them appropriately.</td>
</tr>
<tr>
<td>• Can provide quantitative and qualitative research strengths.</td>
<td></td>
</tr>
<tr>
<td>• Researcher can generate and test a grounded theory.</td>
<td></td>
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<tr>
<td>• Can answer a broader and more complete range of research questions</td>
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</table>
- The specific mixed research designs... have specific strengths and weaknesses that should be considered (e.g. in a two-stage sequential design, the stage 1 results can be used to develop and inform the purpose and design of the stage 2 component).
- A researcher can use the strengths of an additional method to overcome the weaknesses in another method by using both in research study.
- Can provide stronger evidence for a conclusion through convergence and corroboration of findings.
- Can add insights and understanding that might be missed when only a single method is used.
- Can be used to increase the generalizability of the results.
- Qualitative and Quantitative research used together produce more complete knowledge necessary to inform theory and practice.

- Methodological purists contend that one should always work within either a qualitative or a quantitative paradigm.
- More expensive.
- More time consuming.
- Some of the details of mixed research remain to be worked out fully by research methodologists (e.g., problems of paradigm mixing, how to qualitatively analyze quantitative data, how to interpret conflicting results).
3.4 Research Method

3.4.1 Introduction

Educational Design Research was the chosen research method for this investigation into the integration of mathematics and science in Post Primary Schools. Its characteristics and suitability to the author’s study will be outlined, along with alternatives which were considered and, ultimately, rejected.

3.4.2 Educational Design Research

The approach to research which best suited the needs of this endeavour was Educational Design Research. This was mainly due to the fact that the exploration of the integration of mathematics and science within second level education was complex in its nature and provided no clear pathway to solutions to the challenges inherent in its implementation and would require the trial and error design process which is central to Educational Design Research.

Educational Design Research is described as “the systematic study of designing, developing and evaluating educational interventions” (Plomp 2009, p.9). Thus, the study of the design and implementation of an intervention pertaining to the integration of mathematics and science would fall under the umbrella of Educational Design Research.

Application of Educational Design Research, according to Nieveen et al. (2006), largely follows this process:

- Preliminary research: the problem and challenge, as well as the context in which it is placed, are analysed allowing for the development of a conceptual framework which is largely informed by review of the relevant literature.
- **Prototyping stage**: the intervention is designed, formatively evaluated, and revised. This cycle continues until the optimal outcome is reached.

- **Assessment stage (summative evaluation)**: this stage typically assesses transferability and scaling along with small-scale evaluation of how effective the intervention was.

- **Systematic reflection and documentation**: This phase takes place throughout all stages of the research as the intervention is designed, revised and adjusted continually. Although this phase is mainly observed at the end of the research as the entire study is portrayed through retrospective analysis.

The process inherent in design research, rather than the more specific educational design research, has been displayed graphically by a number of authors with Reeves (2006) producing a design (from a technology perspective) which displays the cyclical nature of this type of research:

![Design-Based Research](image)

Figure 3.2: Refinement of Problems, Solutions, Methods, and Design Principles (Reeves 2006)

While Plomp (2009, p.19) displays a simplified version of this process:
Although these designs are not specific to educational design research, they display the cyclical nature of the research whereby problems are analysed, solutions are formed, refined and reflected upon with the cycle repeating itself if necessary. The application of the approach engendered by educational design research will be displayed through the description of the chronology of research (see section 3.5).

3.4.3 Alternative Research Method – Action Research

An alternative research method which was considered in this scenario was Action Research. This model utilises continuous feedback to solve specific problems in one particular school setting (Hopkins 1980). In some ways it suits the author’s research as a problem or, to be more specific, an area in which mathematics education may be improved has been identified, i.e. integrating the subject with science, and attacking this problem using continuous feedback would aid the process. But there are some aspects of Action Research which render it unsuitable to this particular endeavour.

Firstly, the whole rationale of Action Research ensures that the approach to the problem identified and the results of implementing that approach are unique to the specific teacher, pupils, and school involved in the research thus they are not typically generalizable (Green 1999). As this study into the integration of mathematics and science aims to explore the suitability of a model for implementation in a range of schools rather than just one in particular, this characteristic of Action Research forms a significant barrier to its adoption in this scenario. Secondly, Action Research is designed to be used by the curriculum
specialist, e.g. the teacher, rather than the researcher (Hopkins 1980) which, again, is not suitable to the needs of the author’s research as he is not the curriculum specialist within this study. Finally, the essential difference between both is that Action Research is not aimed at generating design principles, while such an aim is central to Educational Design Research (Plomp 2009).

Taking these factors into account, Action Research proved not to be suitable as a research model in this instance, thus it was, after careful consideration, rejected in favour of Educational Design Research.

### 3.4.4 Proof of Concept

A vital element of Educational Design Research is the Assessment Stage where evaluation takes place. At this juncture within this investigation, implementing a large scale field trial of the Authentic Integration model developed proved to be beyond the means of this study due to practical constraints i.e. financial and otherwise. Such a scenario is where proof of concept can be employed to establish important conceptual principles based around the model being tested (Gregg et al. 2001). Proof of concept, derived from engineering, is defined as the implementation of a model of a design which tests whether a certain concept will actually work as it is theoretically proposed (Dym et al. 2009).

For example, an architect or an engineer that designs a large structure would not, due to practical constraints, build the full scale structure to test if it works or not but would instead build a small scale model to verify the design (Horenstein 2010). Such engineering models have the advantage of being able to demonstrate particular behaviours or phenomena which can be used to verify the validity of the theory underpinning the model (Dym et al. 2009). A further example of the application of proof of concept comes from market surveys of new products whereby feedback from members of the target market indicates whether or not the new product will be well received (Dym et al. 2009).
In a similar manner, it may not always be possible for teaching models developed for implementation in an educational setting to be tested on a large scale, as is the case in the author’s study. As such, achieving proof of concept would become the target so as to generate important conceptual principles which can be further tested, if desired, in a full field trial (Plomp 2009, Gregg et al. 2001). Thus, micro evaluation of the final prototype of the teaching model, Authentic Integration, would provide the proof of concept (Plomp 2009). Micro evaluation, within this study, took the form of testing Authentic Integration in four Irish post primary schools. This proof of concept approach to testing this model was incorporated into the overall Educational Design research process. It became an integral part of the assessment stage and, as a result, the reflection and documentation stage (see section 3.4.2) where design principles pertaining to the implementation of lessons integrating mathematics and science were generated (see section 7.2.5).

3.5 Chronology of Research

An overview of the general process of research will give a greater understanding of how and why the methods described in this chapter were implemented. As such, the following outlines the process from the formation of the initial ideas for integrating mathematics and science, to the creation of a specific approach, followed by application and evaluation. The research carried out by the author can be split into six stages:

1. Literature Review.
2. Design of a mathematics-science integration model.
5. Implementation of the Intervention.
The first three stages can be grouped together, as the literature review (Stage 1) provided the theoretical basis for the following two stages. Researching the relevant literature provided the author with a model for implementing integration in the classroom which could be tailored to suit the specific needs of integration of mathematics and science i.e. Authentic Instruction, thus, making ‘Stage 2’, the design of the Authentic Integration model, possible. Subsequently, the model designed in ‘Stage 2’, as well as the knowledge gained through ‘Stage 1’, enabled the author to create lesson plans and resources which formed the bulk of the work completed in ‘Stage 3’ (Creation of a Resource Pack).

These three stages cover the preliminary research and prototyping stage of the educational design research model which the author adopted (see section 3.4). The challenge of integrating mathematics and science in second level education was analysed and informed through the literature review, which thus informed the design of the integration model and resource pack. The model and pack were repeatedly revised and adjusted as they were formatively assessed thus conforming to the prototyping stage as outlined previously (see section 3.4).

![Figure 3.4: Stages 1, 2, & 3 – Preparation for the Intervention.](image)

The final three Stages are also inextricably linked as they form the overall intervention. The assessment stage of the educational design research model was
completed within these three stages while large parts of the systematic reflection and documentation stage (see section 3.4) were also completed at this time.

In ‘Stage 4’ the approach to be taken towards the intervention was devised and developed; it was then implemented in ‘Stage 5’ and, finally, the evaluation of the study was completed in ‘Stage 6’. Each element of the research conducted will be described in detail, indicating time frames, connections between the stages and the manner in which each stage fed into the next.

3.5.1 Stage 1 – Literature Review (Jan 2010 – May 2011)

Upon commencing this research, it was necessary to research the recent history of mathematics education (both nationally and internationally) including:


- The ‘Maths Wars’ of the 1980’s (Klein 2007, Schoenfeld 2004)

- The effect each of these had on the evolution of Irish mathematics education (Oldham 1991, Oldham 2001)

This gave the author an insight into how mathematics education in Ireland developed into its current state. From there, the issues of concern which have been regularly identified within the mathematics education system, both from an Irish perspective (Lyons et al. 2003, Childs 2006) and from an international perspective (Boaler 1994, William 1992, National Council of Teachers of Mathematics 2009) were analysed to determine whether integrating mathematics with other subjects would help to tackle these issues effectively. At this stage it
was noted that various experts and business groups have called for a greater emphasis on a more holistic approach which places mathematics in context and links it to the real world (William 1992, Mitchelmore and White 1995, Lyons et al. 2003, IBEC 2011, Boaler 1994). Similarly, one of the recommendations put forward by the National Council of Teachers of Mathematics (2009) in the U.S. is to apply mathematics to practically all other academic disciplines i.e. integrate Mathematics with other subjects.

As it was concluded that integrating mathematics with other subjects would (theoretically) be a worthwhile endeavour, the author began analysing the literature and research pertaining to this topic (Jacobs 1989, Frykholm and Glasson 2005, Berlin and White 1994) to ascertain how it could be applied in a typical post-primary classroom. Models such as ‘Anchored Instruction’, ‘Cognitive Apprenticeship’, and ‘Constructive Allignment’ were considered as options for forming the basis of a framework for integrating mathematics and science but each was eventually rejected in favour of ‘Authentic Instruction’ (see section 2.6).

The work of Newmann and his Associates (Newmann et al. 2007), through their teaching model ‘Authentic Instruction’, provided the basis for a working model for integrating mathematics and science as it was backed by considerable empirical evidence (Newmann et al. 1998, Newmann et al. 1995a, Newmann et al. 1997) and proved to be the most suitable option for the specific needs of integration of mathematics and science (see section 2.5). As such, ‘Authentic Instruction’ provided the inspiration for the creation of ‘Authentic Integration’ within ‘Stage 2’.
3.5.2 Stage 2 – Design of a Mathematics-Science Integration Model (Nov 2010 – March 2011)

The information gathered from ‘Stage 1’ was utilised to produce a model for integrating mathematics and science which was termed ‘Authentic Integration’. As mentioned previously, this model used ‘Authentic Instruction’ as the blueprint – merging the ideals of that model with the specific needs of integration which were discovered through extensive research in ‘Stage 1’.

In keeping with the systematic reflection which is inherent in all stages of Educational Design research (see section 3.4.2), throughout this stage the Authentic Integration model was analysed and revised through consultation with the author’s supervisors, critical friends, and referral to the appropriate literature. Once the theoretical elements of the model were established, a visual representation of the model – the Authentic Integration triangle – was created. This was designed to add more of a visual element to the subsequent lesson plans rather than depending on text alone to highlight the characteristics of the model.
3.5.3 Stage 3 – Creation of a Resource Pack (Feb 2011 – June 2011)

Once the Authentic Integration model had been established, it was possible to produce lessons and resources of an integrative nature which could be applied in post-primary schools. This was achieved by creating and developing a resource pack for the integration of mathematics and science (see chapter 5). This resource pack contained six lesson guides based on the integration of mathematics and science, with plenty of resources to aid with teacher knowledge and delivery of each lesson.
At this point, a review of further literature based on knowledges for effective teaching was completed to aid the author in understanding what knowledges would be vital in relation to teaching lessons of an integrative nature. It became clear that the teacher must have a relatively significant level of content knowledge in both mathematics and science as well as an appreciation for the benefits of inquiry based instruction. This was catered for within the resource pack by including powerpoint presentations based on every topic explored in each of the six lesson plans. These presentations provided in-depth information about each topic so that the teacher could familiarise themselves with a topic if they were not that comfortable with it, while also acting as a teaching resource if required. Allied to that, the author would aid any of the teachers involved in the project with the development of their content knowledge in any particular area of science and/or mathematics, if required.

As this was a new approach to teaching mathematics and science, the author felt that the corresponding lesson plans should reflect this and also guide teachers in implementing the lessons in the manner intended. This requirement offered the author the challenge of producing a lesson plan design which covers the essentials, displays the information clearly and concisely, and highlights the elements which must be present in the lesson so as to satisfy the requirements of the Authentic Integration model.

One of the most pressing issues in this instance is the fact that these lessons will be applied in a range of different schools to pupils of diverse aptitudes within and towards mathematics and science, and, as such, varying capabilities. Thus a ‘lesson plan’ is not what is required in this scenario but rather, what the author has decided to refer to as, a ‘lesson guide’.

These guides were deliberately designed in such a way as to outline a lesson which would engender all the characteristics of Authentic Integration and outline certain expectations and outcomes which should be achieved within the lesson. But the design also allows for adjustment by the teacher as regards time, level of
difficulty, layout of classroom, discussion topics and length, and the order in which sections of the lesson are completed. The aim is to strike a balance between satisfying the needs of integration (according to research) and allowing teachers to assume ownership of a lesson which they can shape to suit the needs of their pupils. Further details of this process are present in Chapter 4 (see section 4.5).

3.5.4 Stage 4 – Development of the Intervention (March 2011 – Sept. 2011)

Before discussing how the intervention, which was central to this research, was established and implemented, the term ‘intervention’ must be defined to offer a clear insight into what took place. A general interpretation of the term is offered by the World Health Organisation:

An activity or set of activities aimed at modifying a process, course of action or sequence of events, in order to change one or several of their characteristics such as performance or expected outcome.

(World Health Organisation 2001)

This would suggest that an intervention requires changes to be made to a particular process, the effect of which would be analysed by observing the outcome or results of this modified process. Such analysis would be based on quantitative and qualitative data gathered. Further understanding of the term ‘intervention’ can be gained by considering its characterisation in an educational context:

By intervention we mean any change or programme of change which is implemented in the classroom, it could be formal or informal and might be initiated by the individual teacher, the school or another organisation from outside.

( Teaching Expertise 2007)
In this particular scenario, the change which took place within the intervention pertaining to this research was the implementation of a number of lessons which integrated mathematics and science at Junior Cycle. It was initiated by an organisation (the NCE-MSTL within the University of Limerick) outside of the schools involved. As such, this study satisfies the criteria for an intervention – one which required careful planning and consideration as to how it would be deemed a successful intervention.

It should be noted at this point that Junior Cycle was the chosen level as teachers would require content knowledge in both mathematics and science to implement integrative lessons successfully (Frykholm and Glasson 2005) and, as the content level at Junior Cycle is not as advanced as it would be at Senior cycle, the teachers involved in the intervention would have a better chance of mastering the content required within their non-specialist subject, whether that be mathematics or science.

One of the key questions the author considered when developing the intervention was: How would the success of these lessons be measured? As referred to previously in this chapter, a mixed methods approach would provide broader and better results than gathering just qualitative data or just quantitative data (Denzin and Lincoln 1998). This is reflected in the tools used to gather data from this intervention:

- A rubric to grade the quality of work produced by each pupil in each lesson.
- A questionnaire for each teacher.
- A semi-structured interview with each teacher.
- Focus groups with some of the pupils involved.

Data collected through these research tools were used to assess the quality of the pupils’ work within these lessons and to get feedback from the pupils through focus groups. Interviews with the teachers involved determined their opinion on
the strengths and weaknesses of the lessons and the integration process overall. Each of these tools, with the exception of the pupil questionnaire, was designed by the author through reference to various literature and similar studies, a process which will be examined carefully in section 3.5.5.

This approach to gathering data of this nature for this particular intervention was inspired by the work of Newmann et al. (1996). They outlined in detail a rubric applied to grade the quality of work produced by students in relation to the standards set by Authentic Instruction. The rubric applied within the author’s research is based specifically on the rubric designed by Newmann et al. (1996). The process for grading work was also identical as work was collected once completed, analysed and graded according to the rubric. Newmann et al. (2007) describe six studies based around the implementation of Authentic Instruction – four of which applied this rubric to grade pupil performance. This approach is sometimes, but not always, coupled with observation of the lessons (Newmann et al. 2007).

The assessment of pupil work and the questionnaires completed account for the quantitative data gathered. To satisfy the mixed methods approach, qualitative data was gathered through interviews and focus groups as such research tools describe complex phenomena, e.g. the process of implementing the Authentic Integration model, in rich detail (Johnson and Onwuegbuzie 2004).
3.5.5 Stage 5 – Implementation of the Intervention (Oct 2011 – Feb 2012)

3.5.5.1 Research Sample

There are two major sampling strategies – probability/random sampling and non-probability/purposive sampling (Cohen et al. 2007). Purposive sampling was applied in this research as the author needed to test the Authentic Integration model in schools which varied in their characteristics i.e. urban/rural; single-sex/mixed; secondary/vocational/catholic. By using the purposive sampling method, the author was able to recruit schools that varied in the aforementioned characteristics thus it was possible to judge whether the application of the integrative lessons would be affected by the type of school within which it was applied. In contrast, using the random sampling approach would not necessarily
yield such a result. The sample size of pupils was 80 in total and the range of school types are broad. Six schools were recruited for the project but two schools had to drop out, leaving a total of four schools in which the project was completed.

3.5.5.2 How the Intervention was carried out

Prior to the intervention, pupils, parents/guardians, teachers, and principals signed the relevant consent forms after reading a clear and concise description of the project and all that it would entail (see Appendices A and B). Also, before the lessons were taught, each participating pupil completed a questionnaire based on their enjoyment and value of mathematics (see Appendix G). These questionnaires were developed by L.R. Aiken (1974) and are termed the ‘Enjoyment Scale’ (E Scale) and the ‘Value Scale’ (V Scale).

The pupil questionnaires used in this research (see Appendix G) were made up of these two scales. Within each scale, there were 10 (Value Scale) or 11 (Enjoyment Scale) statements which the pupils agreed or disagreed with according to the Likert Scale. Aiken (1974) worded approximately half of the items on each scale in a manner which signified a favourable attitude towards enjoyment or value of mathematics and the remaining statements signified an unfavourable attitude. Favourable statements applied the following values: 0 = Strongly Disagree; 1 = Disagree; 2 = Undecided; 3 = Agree; 4 = Strongly Agree. This scale was reversed on unfavourable statements: 0 = Strongly Agree; 1 = Agree; 2 = Undecided; 3 = Disagree; 4 = Strongly Disagree. Thus a high score would indicate a more favourable attitude towards mathematics and form a quantitative measure of the effect of the intervention on pupil attitudes towards mathematics.

The pupils completed these questionnaires before they experienced the lessons, and they completed the same questionnaires after the lessons were completed i.e. pre- and post-intervention. This was done in order to give the author a picture of
the pupils’ attitudes towards mathematics before and after they experienced these lessons to gauge whether the approach implemented had any effect on their enjoyment and/or valuation of mathematics.

The main aspect of the intervention was the implementation of the lessons. The teacher had to pick three from the six choices of lessons to complete, each pertaining to different topics from science and mathematics. The intention was for these three lessons to be completed one after another i.e. one block of lessons in which mathematics and science are integrated. This design was put in place so that any change in the pupils’ attitudes towards mathematics (as measured by the questionnaire outlined previously) could be largely attributed to the new lessons implemented. It would also ensure that the pupils knew exactly what lessons were integration lessons when they were contributing their views within a focus group. If normal tuition was mixed with these integration lessons then the pupils might have found it difficult to differentiate between the two when asked for their opinions on the integration lessons through the focus groups.

Once these lessons were complete, the author conducted an interview with the teachers involved and asked them to complete a brief questionnaire, while also conducting a focus group with a number of pupils from the class group involved.

3.5.5.3 Semi – Structured Interviews

Qualitative interviews are implemented so as to learn about people’s feelings, thoughts, and experiences (Rubin and Rubin 2011). Typically, interviews are conducted to complement other methods of data collection e.g. to validate other methods; follow up results; or to gain a deeper understanding of the responses put forward by the participants (Cohen et al. 2007). The interviews with teachers were of this nature as there were a set number of questions which were asked in each interview to complement the questionnaire completed, and also to gain a deeper insight into their opinions and observations within the intervention.
Prior to conducting a semi-structured interview, the researcher works out a set of open-ended questions but is free to modify their order, change wording, or leave out particular questions as they may deem them inappropriate in certain scenarios (Robson 2002). It is important to note at this point that the author had such options available to him when conducting teacher interviews but decided against modifying order or wording of the open-ended questions, or leaving any question out as there was no reason to do so.

The aforementioned open-ended questions offer a number of advantages: flexibility; testing respondents’ knowledge; making better informed assessments of what the respondent really believes; encouraging freedom of expression from respondents (Cohen et al. 2007, Foddy and Foddy 1994). As such, the quality of understanding of participating teachers’ opinions and experiences is greatly enhanced within this research when compared to an approach which depends on questionnaire feedback alone. Some examples of the questions asked will give a greater insight into the application of such theory to the design of the interviews:

- Do you think that the pupils learned more or less using these teaching methods than they would if the class was taught using traditional methods? Why?

- What is your opinion of the teaching model employed? How would you improve on it?

- Do you think integration of mathematics and science is beneficial? Why/Why not?

- Do you think you will incorporate similar lessons into your mathematics teaching in the future? Why/Why not?
The rest of the questions are located later in this text (see Appendix F). Their development was an important issue in ensuring the effectiveness and integrity of the intervention (factors which will be discussed later in this chapter), thus the ‘TAP’ paradigm (Foddy and Foddy 1994) was utilised during this development process.

3.5.5.4 Developing Semi – Structured Interview

The process of creating questions for the teacher interviews and questionnaires, as well as pupil focus groups, involved implementing Foddy & Foddy’s (1994) ‘TAP’ paradigm which is an acronym for Topic, Applicability, and Perspective. The primary goal of this paradigm is to ensure that valid, reliable, respondent information is obtained through these research tools. Foddy & Foddy (1994, p.193) outlined each characteristic pertaining to the paradigm:

*Topic*: The topic should be properly defined so that each respondent clearly understands what is being talked about.

*Applicability*: Questions should be applicable to each respondent. That is, respondents should not be asked to give information they do not have.

*Perspective*: The perspective that respondents, when answering the questions, should be asked in such a way that each respondent gives the same kind of answer.

In short, questions should be based around a defined topic, one which the respondents can offer information on in a similar manner to each other. This paradigm is useful when identifying elements of various questions which needed to be adjusted, improved, or discarded.
3.5.6 Stage 6 – Evaluation of the Intervention (Feb 2012 – August 2012)

3.5.6.1 Evaluation of the Teaching model

Central to the evaluation of the intervention employed is the level of success achieved by the teaching model developed (the Authentic Integration model). Schoenfeld (2000) set out a list of criteria for evaluating models and theories in mathematics education which is suited to such an evaluation (See Table 3.3). The employment of these criteria to fully evaluate the Authentic Integration teaching model will be conducted in the discussion chapter (see section 6.8).

Table 3.3: Schoenfeld’s (2000) criteria for evaluating models and theories in mathematics education.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Descriptive Power</td>
<td>Are all the important features of the concept being modelled included?</td>
</tr>
<tr>
<td>Explanatory Power</td>
<td>Providing a clear explanation of ‘how and why’ the various components of the model work.</td>
</tr>
<tr>
<td>Scope</td>
<td>Is the model suitable for just one application or can it be adapted to work in multiple situations?</td>
</tr>
<tr>
<td>Predictive Power</td>
<td>Can the model predict the constraints and likely events to occur through the teaching model, thus allowing for negative elements to be minimised?</td>
</tr>
<tr>
<td>Rigour and Specificity</td>
<td>How well defined are the terms? Would you know one if you saw one? In real life? In the model? How well defined are the relationships among them? And how well do the objects and relations in the model correspond to the things they are supposed to represent?</td>
</tr>
<tr>
<td>Falsifiability</td>
<td>Testing the effect of the model using empirical evidence i.e. putting its ideas on the line.</td>
</tr>
<tr>
<td>Replicability</td>
<td>Will the same thing happen if the circumstances are repeated? Will others, once adequately trained, ‘see’ the same things in the data?</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Multiple sources of evidence (Triangulation)</td>
<td>The more independent sources of data there are, the more robust a finding is likely to be.</td>
</tr>
</tbody>
</table>

This list of criteria was constantly referred to when developing the Authentic Integration model and, also, while describing the model itself (see section 4.4.2) as well as considering the model’s effectiveness during the intervention carried out. It is applied to fully evaluate the Authentic Integration model in the results chapter of this text (see section 6.8)

3.5.6.2 Evaluation of the Intervention Process

It is imperative that the exploration of any aspect of mathematics education meets the required standards to ensure the results obtained and conclusions drawn can be fully endorsed. As such, the critical parameters by which an intervention can be evaluated must be identified and satisfied. Shapiro (1987, p.290) identified four such parameters:

- Treatment effectiveness,
- Treatment Integrity,
- Social validity,
- Treatment acceptability.

These parameters were originally developed by Shapiro (1987) for the purposes of school psychology research but they have been adopted successfully to evaluate several teaching interventions (Regan 2005, Prendergast 2011, Hourigan 2009) Each parameter will be outlined and examined in relation to the research conducted by the author.
Treatment Effectiveness

Treatment Effectiveness deals with the issue of how effective the intervention proved to be. This effect can be determined by the amount of change which takes place; the immediacy of this change; and the strength of the change (Shapiro 1987). Ideally, comparison between an experimental and control group would highlight the effects of the intervention but this is not always possible.

The research carried out by the author was of a particular nature such that comparison with a control group would prove very difficult. Each topic explored through the lessons employed would have to be tested individually by both the control and experimental group to gauge differences in improvement for validity to be assured. As the experimental group would likely encounter three topics in mathematics and three topics in science over the course of about a week and a half through the intervention, a control group would need to do the same through...
regular tuition – a scenario which would just not be feasible in the circumstances of an Irish school. This is due to the fact that these topics would most likely be taught at varying times during the school year, thus requesting that a teacher of a control group totally adjust his/her scheme of work to suit this intervention would very likely prove unsuccessful.

One aspect where the effect of the intervention could be gauged was the change in attitude displayed by the students towards mathematics. Students were given a pre- and post-intervention questionnaire which evaluated their enjoyment and value of mathematics. Any significant changes to the outcome of each would offer an insight into how the intervention affected their attitudes in relation to mathematics thus giving an insight into the treatment effectiveness from one important angle.

Feedback on the effectiveness of the intervention was also gathered through assessment of pupil work (see Appendix E), interviews and questionnaires with the teachers, as well as focus groups with the pupils. This gave a rich and deep insight into how suitable the lessons were; the learning which took place; and the effect the intervention had on interest and attitudes amongst both the pupils and the teachers.

_Treatment Integrity_

When an intervention has the required level of integrity, it ensures that it can be implemented again with replicable results. This is measured by the extent to which the intervention is actually executed in the manner prescribed in the documentation which leads to the need for a comprehensive outline of the intervention program (Shapiro 1987). Such a requirement was satisfied within this research through the creation of a resource pack for each participating teacher which clearly outlined the make-up of the teaching model (Authentic Integration) to be employed, the lessons to be taught, and the assessments to be conducted, as
well as the procedures to be carried out (e.g. consent forms; pre- and post-
intervention questionnaires; time frame involved).

Allied to that, training was provided by the author for each participating teacher
so that they would have the requisite skills and knowledge to implement the
lessons. If the intervention were to be repeated, then there would be clear
guidelines in place for the participating teacher, hence differences would be
minimised from one implementation of the intervention to the next.

Social Validity

Social validity relates to the evaluation of the intervention by the participants
(Shapiro 1987). In this case that would be the students and teachers involved.
Each participating teacher provided a large amount of feedback through
questionnaires and interviews in which they evaluated the effectiveness of the
lessons employed and the suitability of such lessons within the second level
education structure, amongst other aspects. Focus groups were also conducted to
gain feedback from a portion of the pupils involved to allow them to evaluate
what they experienced.

Treatment Acceptability

Treatment acceptability relates to the degree to which the participants accept the
methods employed within the intervention as being suitable with regards to the
problem and the participant (Shapiro 1987). It is defined as “judgements by
laypersons, clients and others on whether treatment procedures are appropriate,
fair and reasonable for the problem and the client” (Kazdin 1981, p.493).
Teachers’ acceptability of interventions can be influenced by factors ranging from
the time required to implement the scheme; the cost; understanding of the
intervention; and the aforementioned treatment effectiveness and integrity
(Reimers et al. 1987).
In designing the intervention, the author accounted for these factors. The time needed to implement the scheme was relatively short: lessons took one to two class periods to complete, thus the required number of lessons (three) that the teacher needed to conduct took a maximum of six class periods which would typically account for all the mathematics lessons within a week and a half. Time taken to plan and develop lessons taught were minimal as the general approach to each lesson had been laid out clearly in the resource pack with plenty of powerpoint presentations to aid the teacher throughout the lesson. There were very few other resources required for each lesson and the author had indicated that he would provide whatever was needed in this respect.

As regards data collection, teachers just needed to hand out a questionnaire before the lessons were implemented and directly after they were implemented. Such questionnaires took about five minutes to complete each time. Teacher interviews and questionnaires at the end of the intervention took approximately fifteen minutes to complete. As such, great care was taken to ensure that the time required from the participants would be short without hampering the quality of the intervention.

There was no additional cost to the teachers or schools involved in the intervention. Materials such as copies in which student work was completed, resource pack, sheets for questionnaires and consent forms, resources, and other materials which a teacher or pupil would not generally use during normal tuition were provided by the researcher. Any unforeseen costs pertaining to the intervention would also have been covered by the researcher had they arisen.

Each participant’s understanding of what was to occur during the intervention was covered in the information sheet they (and their parent/guardian) received before the intervention commenced (see Appendix A). Teachers were provided with more of the specifics of the intervention e.g. manner in which it is taught, number of lessons to be taught, and the amount of time required. They were also given some training by the author in relation to how to implement the given teaching
model (Authentic Integration), as well as familiarising them with material to which they may have been unaccustomed (see section 4.6) e.g. mathematics teachers may not have had a full grasp of certain science concepts while science teachers may not have had a full grasp of certain mathematics concepts.

The measures mentioned above positively affected treatment acceptability, allowing for the successful implementation of the intervention.

3.6 General Tools of Research

The methodology adopted within any particular research represents the general approach taken. This also dictates the tools of research employed. Such tools represent the specific mechanism or strategy applied to collect, interpret, or manipulate data (Cohen et al. 2007). The tools central to this research were the statistics collected and the logic, reasoning, and critical thinking employed when interpreting these statistics.

3.6.1 Statistics

Statistics gathered throughout this research were central to the arguments made in the following chapters. The use of statistics within those arguments served two principal functions – providing information in relation to the data in a descriptive and an inferential manner.

Descriptive statistics summarise the nature of the data that has been collected through the research e.g. describing the average performance of a student in a particular school where the work completed by those students has been graded. Inferential statistics aid the researcher in coming to conclusions relating to the data; e.g. whether differences in the pre- and post-test student questionnaires are large enough to be significant.
These two principal functions of statistics were applied consistently throughout analysis of the data obtained thus helping to condense a large body of data into a form which was easily analysed and interpreted. As such, relationships were established between various elements of research, significance of findings were determined, and conclusions were drawn to offer an insight into the effects of the intervention employed.

Aiding this process was the use of computer programmes such as Nvivo (version 9 for Windows) and SPSS (version 17 for Windows). Nvivo is designed to analyse qualitative data, of which there was a great deal, collected during this research. This programme was used to gain a greater insight into the feedback from interviews and focus groups. The SPSS programme was employed to deal with the organisation and analysis of quantitative data which was mainly generated by questionnaires and the grading of pupil work.

3.6.2 Logic, Reasoning, and Critical Thinking

Information gathered must be interpreted to reach logical conclusions. To do so, a systematic approach to the thought process involved must be adopted. According to research, there are three main cognitive tools which aid in the discovery of knowledge: deductive logic, inductive reasoning, and critical thinking (Leedy and Ormrod 2010).

Deductive logic is based around one or more premises which are self-evident and widely accepted truths. Reasoning proceeds logically from these premises towards the deduction of conclusions that must also be true. In its simplest form there is a major premise which is self-evident and a minor premise which is related to this, followed by a conclusion linking both (Cohen et al. 2007). Within the results and discussion chapter, the author has logically deduced conclusions from the data gathered so as to give a greater insight into the effects of the intervention.
Inductive Reasoning requires that data is collected relating to a particular phenomenon with no preconceived notion about the significance or orientation of this data thus allowing for complete objectivity. From this data, hypotheses are created and tested according to the data allowing for conclusions to be drawn based on the given empirical evidence (Cohen et al. 2007). The author depended greatly on such an approach within this research as he regularly employed evidence from the data gathered to inform conclusions drawn. For instance, teacher attitudes towards the integration of mathematics and science were determined by their answers to the relevant questions in the questionnaire they completed and the interview they gave; the author’s opinion of their attitude had no influence on the conclusion drawn.

The final cognitive tool implemented in the discovery of knowledge was critical thinking. This involves evaluating information or arguments in terms of their accuracy or worth (Beyer 1995). According to Leedy & Ormrod (2010), critical thinking takes on one or more of the following forms:

- **Verbal reasoning** – comprehending and evaluating the persuasive techniques found in oral and written language.

- **Argument Analysis** – Differentiating between reasons that support and fail to support a particular conclusion.

- **Decision making** – Identifying and analysing a range of options and selecting the most suitable one.

These elements were particularly useful when identifying what tools for data collection to employ given the limitations of the study and the research questions which needed to be answered, and also when deciding on what elements of the data collected supported (or contradicted) certain arguments or conclusions.
3.6.3 Constant Comparison

Analysis of the qualitative data produced by conducting semi-structured interviews with the participating teachers was achieved through the application of constant comparison. Tesch (1990, p.96) describes this method and how it contributes to organising the data gathered:

“The method of comparing and contrasting is used for practically all intellectual tasks during analysis: forming categories, establishing the boundaries of the categories, summarizing the content of each category, finding negative evidence, etc. The goal is to discern conceptual similarities, to refine the discriminative power of categories, and to discover patterns.”

Through the constant comparative method, data is organised and understood through explicit coding and analytic procedures. Glasser and Strauss (1967, p.105) described the method succinctly:

- Comparing incidents applicable to each category,
- Integrating categories and their properties,
- Delimiting the theory,
- Writing the theory.

The qualitative data gathered through this study was organised for analysis in text form i.e. recordings of the interviews with teachers were transcribed. In order to become familiar with patterns and noteworthy features the author read and re-read the data and studied it carefully looking for patterns (Hammersley and Atkinson 1983). Aspects of each interview which had related themes were grouped together through the computer program Nvivo. Each category or ‘node’, as it is referred to in Nvivo, was determined after reading and re-reading the data.
The author was able to detect patterns and recurrences, note consistencies, evaluate confirming and disconfirming evidence, and inconsistencies, hence make generalisations (Miles and Huberman 1994). In this manner, the author could make the move from description to deduction.

3.7 Reliability and Validity

Within this research there is a great deal of data collected which would be termed as being qualitative i.e. taking the form of opinions, attitudes, and beliefs through data collected via questionnaires, interviews, and focus groups. As such, great care was taken to ensure that the level of bias present was minimized as much as possible by paying particular attention to the issues of reliability and validity.

3.7.1 Reliability

Reliability is achieved if a particular instrument of data collection ensures consistency of scores or answers from one application of the instrument to another (Fraenkel and Wallen 2003). In other words, the same test under the same conditions with the same subject should achieve the same score. The author strived for reliability of interviews and focus groups by creating a script of the questions to be asked for each. Thus, if any of the subjects were to part take in an interview or focus group again, they would be asked the same general questions and they would give the same or similar answers.

Having said that, there would be small differences in a repeat of a focus group as certain follow up questions were unscripted and depended on the response of the subjects. Such follow-up questions were asked to clarify a subject’s opinion or to get a deeper insight into their views on an issue they had raised. Finally, there was no deviation in the make-up of the questionnaires for both teachers and pupils – a repeat completion of these questionnaires would see the subjects encounter the exact same questions and/or statements.
3.7.2 Validity

The validity of qualitative data can be assessed through the honesty, depth, richness, and scope of the data collected, the participants involved, the level of triangulation, and the objectivity of the researcher (Cohen et al. 2007). Quantitative data can be considered valid through appropriate sampling, suitable instrumentation, and appropriate statistical applications of the data (Cohen et al. 2007). In short, validity “refers to the degree to which evidence supports any inferences a researcher makes based on the data he or she collects using a particular instrument” (Fraenkel and Wallen 2003, p.158). If there are issues with the validity of an instrument of data collection then this lessens the degree to which the data obtained can support conclusions put forward by the author.

When considering the aforementioned honesty, depth, richness, and scope of the data collected within this research, it becomes clear that these criteria have been largely satisfied. As regards honesty and objectivity, the researcher consistently reminded any subject completing a questionnaire or part taking in an interview or focus group to give completely honest answers and not the answers that he/she thinks the researcher desires. The level of objectivity could have been improved if the person collecting the data through questionnaires, focus groups, and interviews had not been the author. Such a scenario would have been desirable but proved impossible as the cost to employ a qualified person to complete the amount of travelling and time spent collecting the data was beyond the resources of this project. To counter this, the author endeavoured to remain as objective as possible in all situations, as described previously.

Depth and richness of the data was achieved through extensive questionnaires, interviews, and focus groups which explored a wide range of issues pertaining to the research while also exploring the work completed by the pupils in class and at home to gain further insight into the effect of the intervention on the pupils’ understanding.
3.8 Triangulation

As mentioned previously, the validity of quantitative data is affected by the level of triangulation applied within the research. Triangulation refers to the application of two or more methods of data collection when researching some aspect of human behaviour (Cohen et al. 2007) and is broken up into six different types.

*Methodological Triangulation* – the use of the same methods of data collection at different times or different methods on the same subject (Cohen et al. 2007) – is the type of triangulation which best describes the approach used within this research. One method of data collection – the student questionnaire – was applied at different times (before the intervention started and upon its completion). This questionnaire was used to analyse the students’ enjoyment and value of mathematics, thus differences in results between the pre- and post-test would aid in identifying any effects the intervention had on the students’ attitudes towards mathematics.

Similarly, different methods of data collection were used on the same subject e.g. each participating teacher completed a questionnaire pertaining to their views on integration and the lessons completed; allied to that, they were interviewed so that they could elaborate on these views, thus giving a richer insight into their experience within the intervention and their attitudes towards the integration of mathematics and science.

3.9 Ethical Considerations

Throughout this process of research, it was necessary to adopt and maintain high ethical standards especially given that the vast majority of the subjects were pupils aged between 13 and 15 years old. These pupils came from four schools. The first school was mixed gender, rural secondary school but with a relatively large
pupil population (circa 600). The teacher from this school was predominantly a science teacher but did teach some mathematics also. The next school was an all-boys, urban school (very close to the centre of a large Irish city) with a pupil population of about 530. The third of these schools was mixed gender with a distinctly Christian ethos, based in a relatively large town with a pupil population of about 600. The final school involved in this project was a Vocational Education Committee (VEC) school which was mixed gender and based in a rural area with a pupil population of about 420.

To apply an intervention in these schools, ethical approval for the research project and all it entailed was sought and gained from the University of Limerick Research Ethics Committee (ULREC) in July 2011.

The primary ethical concerns of mathematics education were taken into account at every stage of the research process, particularly in the case of the design and collection of data pertaining to the qualitative processes. Such concerns included the avoidance of harm and maximisation of benefits; recognition of the effect of the researcher’s presence on the research setting; interpretation of data being as non-biased and accurate as possible; preserving confidentiality and anonymity; and making interviewees aware of how the data will be used (Sowder 1997).

Questionnaires and focus group questions were developed and analysed to ensure that each participant would avoid giving away any personal information or be subject to embarrassment at any point. Allied to that, each student, along with their parent(s) and/or guardian(s), received an information letter detailing what the project would entail and what would be expected of them while also indicating that they could withdraw at any time. Each participant signed a consent form, as did the pupils’ parents/guardians, indicating that they understood what was involved and also indicating their agreement to participate.

Confidentiality was ensured throughout as names were not to be put on any questionnaires, and the names of those involved in the focus groups were not
taken either. The only names known to the author were the teachers’ names – each of whom received a fictitious name (as did their school). Where possible, steps were taken to ensure that any trace of the original source was disguised.

3.10 Limitations of the Study

Certain aspirations for this study proved to be beyond realistic bounds. One such aspiration was to test the improvement displayed by each of the students through tuition which involved the integration of science and mathematics in comparison with similar pupils who experienced regular tuition i.e. to have an experiment group and a control group. For this to be possible, diagnostic tests, which assessed students in all topics inherent in the six lesson plans developed, would need to be carefully created and implemented. As there were twelve different topics, from both mathematics and science, covered by the lessons, this would mean that twelve valid and reliable diagnostic tests must be created – an onerous task, but one that the author was willing to take on.

However, to gain an indication of the improvement in student learning, a control group would also have to study the same topics (topics they probably would have covered previously) and take the same tests – something which proved logistically impossible given that recruiting participants was difficult enough without placing such a request on the participating schools (see section 3.5.6 – treatment effectiveness). As such, the only viable option relating to student work was to collect all work completed during the lessons and homework relating to the lessons, analyse it, and grade it according to the standards set out by the Authentic Integration model through use of a rubric (see Appendix E).

A further limitation to this study was that the lessons were not observed by the researcher to ensure that the Authentic Integration model was being implemented correctly. The reason for this was that some of the participating teachers mentioned that they would not be comfortable with an observer in the classroom. As the degree to which participants accept the methods employed is vital in the
successful implementation of an intervention (see ‘Treatment Acceptability’ in section 3.5.6.2), the author deemed it best to refrain from observing the lessons taught and, instead, ensured that the characteristics of the Authentic Integration model were outlined clearly to the participating teachers on more than one occasion.

3.11 Conclusion

The author adopted a mixed methods approach when gathering data pertaining to the intervention applied in this study so as to combine the strengths of qualitative and quantitative methods while minimising their weaknesses. Questionnaires, focus groups, interviews, and grading of pupil work were the sources of the data collected. Each of these was developed carefully by referring to or applying the work of Aiken (1974), Foddy & Foddy (1994), and Newmann et al. (2007), amongst others. Processes which ensured the methods applied throughout were suitable, valid, and reliable were largely guided by the work of Cohen et al. (2007), Leedy & Ormrod (2010), and Shapiro (1987) to name but three sources. The six stages inherent in this intervention offered a clear picture of how the process was conducted and why certain practices were implemented, while vital elements such as triangulation and ethical considerations were analysed to ensure they were catered for throughout the application of the study.
Chapter 4

Theoretical Perspectives for Integrating Mathematics and Science
4.1 Introduction

The author concluded in Chapter 2 that Authentic Instruction was the model best suited to integrating mathematics and science in an educational setting (see section 2.5). The challenge leading on from that conclusion was to determine how Authentic Instruction could be used to integrate mathematics and science.

This chapter outlines the evolution of the Authentic Instruction model from one which was designed for application in all disciplines to one which is specifically constructed to cater for the specific needs of integration of mathematics and science i.e. the Authentic Integration model. Elements which shaped the evolution of this model such as the knowledges that teachers require to integrate mathematics with other subjects; the specific needs, according to research, of mathematics-science integration; and the characteristics of Authentic Instruction will be outlined throughout with reference to their effect on the final design of the Authentic Integration model. Furthermore, the pros and cons of the model will be discussed thoroughly, as will the readiness of teachers in Ireland to implement lessons of an integrative nature.

This new model will be described in detail, referring to and defining each characteristic. Allied to that, the template designed to display lessons pertaining to the Authentic Integration model will be displayed, described, and justified with reference to the relevant literature. But before such lessons could be applied, the consideration of how ready the teachers were to implement them needed to be scrutinized, thus an investigation into the knowledges required for effective integration of mathematics and science was conducted.
4.2 Knowledges for Effective Integration of Mathematics and Science

4.2.1 Introduction

The types and levels of knowledge required to teach in an integrated setting is an important issue to consider as combining subject matter from two distinct disciplines is not common in 2nd level education (NCCA 2005). Serious consideration needs to be given to questions such as:

- Will the science teacher have a firm enough grasp on mathematics concepts and procedures?
- Will the mathematics teacher have a sufficient level of knowledge with regards to the various laws which guide many areas of science?
- How can any of these perceived gaps be bridged and are they of sufficient importance to cause alarm in relation to the propensity for integration of mathematics and science?

The level and complexity of knowledge held by a teacher affects what is done in classrooms and, as a consequence, also influences what students learn (Fennema and Franke 1992), thus, a closer look at what contributes to such knowledge will give a greater insight into how any perceived pitfalls can be negotiated effectively. Shulman’s (1986) work was the first foray into this area and provided a base for the work which was to follow, hence it is the natural place to start when considering knowledges for effective teaching. Shulman (1986) identified three domains when constructing his model:

- Subject Matter Content Knowledge
- Pedagogical Content Knowledge
- Curricular Knowledge.
He believed that ‘Subject Matter Content Knowledge’ was the most important of the three, claiming that teachers must have a deep understanding of the content in order to teach it effectively (Shulman 1986). This is an important issue when considering integration as, depending on the approach taken, it may require one teacher to educate pupils in mathematics and science simultaneously (see section 2.4.2 and Figure 2.1 for such scenarios). If this occurs then the teacher must, according to Shulman (1986), possess a good depth of knowledge within both subjects. If this depth of knowledge is not present then a solution to this quandary is vital. Such a solution will be discussed later in this chapter.

‘Pedagogical Content Knowledge’ refers to the repertoire of representations of the content that a teacher draws on to aid pupils in comprehending the subject matter. These could be demonstrations, examples, analogies or illustrations which help pupils form a greater understanding of what is being examined (Shulman 1986). In other words, it is the ability of a teacher to draw on various illustrations of the given content to enhance the quality of their instruction so as to ensure a greater depth of understanding amongst the pupils. An example of this could be the use of the example of a scales to explain procedure within equations. With a scales, if a weight is added to one side then a weight of the same magnitude must be added to the other side to maintain balance. It is the same with equations: if a number is added to one side of the equation then the same number must be added to the other side of the equation to ‘maintain balance’. This offers a concrete example which pupils can recognise and refer to, thus improving understanding of the concept.

‘Curricular Knowledge’ refers to knowledge and competency in relation to the range of programmes and materials available to the teacher with regards to a particular subject or topic. It refers to the knowledge of the various ways an educator can teach elements of the curriculum to their pupils and the educator’s recognition of which way is best in given situations (Shulman 1986). In other words, this element refers to the notion that there is more than one way to teach a topic or subject and a teacher should have a certain level of expertise in the
various approaches which could be deployed as well as knowing when best to deploy them.

Schulman’s (1986) work was not subject specific hence it was aimed at encompassing all teaching. It formed the key reference for subsequent attempts at modelling subject specific and non-subject specific knowledges for effective teaching. A quick internet search by this author showed that Schulman’s (1986) work has been cited by over four thousand works of literature which highlights the importance of his findings. As the issue of knowledges for effective teaching began to develop, more authors offered theories in relation to its make-up. Two of these authors, Ernest (1989) and Fennema & Franke (1992) lead the way in defining the knowledge make-up of effective mathematics teachers, leading onto Rowland et al. (2005) and their work on the ‘Knowledge Quartet’.

4.2.2 Knowledges for Effective Mathematics Teaching

Following Shulman’s (1986) ground-breaking work, academics began to apply his theory to specific subjects with Ernest (1989) developing one of the first models of teacher knowledges for effective mathematics teaching. This model was quite detailed, outlining the knowledges, beliefs and attitudes vital for effective mathematics teaching. Similar to Shulman (1986), Ernest (1989) highlighted subject content knowledge, i.e. knowledge of mathematics, as the most important element. When Fennema and Franke (1992) published their model of knowledges for effective mathematics teaching, content knowledge was also identified as the most vital characteristic. This aspect continues to be regarded as being of the utmost importance in present day models of this nature.

Content knowledge has been shown to be negatively related to the use of inquiry-based classroom instruction and to beliefs in the effectiveness of such instruction (Wilkins 2008). Many teachers with strong content knowledge tend to rely on ‘traditional’ methods i.e. focus on rules and procedures (Mewborn 2001). It is, rather, positive attitudes towards the subject that facilitate the adoption of inquiry-
based instruction in the classroom (Wilkins 2008, Karp 1991). These findings show that content knowledge is of great importance for effective mathematics instruction but must be supplemented with positive beliefs in relation to inquiry-based instruction if such an approach is to be adopted.

Returning to the aforementioned models: interestingly, the knowledge characteristic which Ernest (1989) terms as the next most important, ‘Knowledge of Other Subject Matter’, is an endorsement of the assimilation of mathematics with other subjects. Ernest (1989, p.17) claims that knowledge of other subject matter “provides a stock of knowledge of uses and applications of mathematics” which he believes forms an important contribution to the teaching of mathematics. Similarly, Rowland et al. (2005) cited such a characteristic in his ‘Knowledge Quartet’ model – ‘Connection Knowledge’. This aspect deals with the knowledge required to make connections within mathematics i.e. between concepts and/or procedures; and between mathematics and other subjects or disciplines (Rowland et al. 2005).

Other knowledge areas of Ernest’s (1989) model are similar to Shulman’s (1986) as he includes knowledge of pedagogy and the curriculum but he also adds classroom organisation and management, and theoretical knowledge from research within general education and mathematics education. Classroom management is of interest to this author as the intended intervention linked to this research would generally require the teacher to have the ability to organise the pupils and/or the classroom to facilitate active learning in a number of scenarios. Such an ability is mainly acquired through experience (Ernest 1989) hence teachers who regularly employ activities of that nature within their lessons would probably be best placed to reap the benefits of Authentic Integration as such activities are central to this model. This would also inform ratings of how successful such a lesson is and may be in the future as teachers who have little practice in the area of active learning would improve with practice. To put it another way: if a lesson isn’t a success initially this does not mean the content is
greatly lacking, it may be the case that the teacher needs to develop their classroom management skills to maximise the learning which takes place.

4.2.3 The Knowledge Quartet

The ‘Knowledge Quartet’ provides the most recent widely endorsed version of what knowledges it takes to be an effective mathematics teacher. Again, Mathematical knowledge tops the list of most important characteristics; within the quartet it is referred to as ‘Foundation Knowledge’. But there is one important difference in the definition of this aspect compared to previously mentioned models – it not only includes knowledge of mathematics itself but also the beliefs which the teacher holds in relation to mathematics, and it is upon this foundation that the other characteristics of the model are built (Rowland et al. 2005). This is significant due to the observation, discussed earlier, that although content knowledge is vital for effective teaching, beliefs determine whether innovative practices such as active and experiential learning are adopted (Wilkins 2008, Karp 1991).

Thus, if an innovation like the integration of mathematics and science is to be adopted, ‘Foundation Knowledge’ within teachers, which encompasses the desirable beliefs and levels of mathematical knowledge, would be a cornerstone of its implementation. The significance of this to the author is the realisation that an effective intervention which implements an integrative framework would require training for teachers to ensure they have the requisite content knowledge but also, possibly more importantly, it would require teachers to ‘buy into’ the approach being used i.e. hold the belief that experiential and active learning is a worthwhile endeavour.

Next in the Knowledge Quartet is ‘Transformation Knowledge’ which is very similar to Shulman’s (1986) ‘Pedagogical Content Knowledge’ described earlier. In essence this characteristic separates those who know mathematics from those who know how to teach mathematics. This leads into the third element of the
‘Knowledge Quartet’ – ‘Connection Knowledge’ which was alluded to earlier. The ability to make connections to areas within and outside mathematics is of course an essential element of integration of mathematics with other subjects thus it is imperative that teachers pursuing an integrative approach have this characteristic.

Finally, within Rowland et al.’s (2005) ‘Knowledge Quartet’, ‘Contingency Knowledge’ which describes a teacher’s ability to adjust to unexpected situations such as an unforeseen circumstance, or a question which had not been anticipated. It also alludes to a teacher’s recognition of when and how a lesson needs to be adjusted from the original lesson plan if required (Rowland et al. 2005). Once again, this is quite relevant to issues relating to integration as a teacher’s ability to think on their feet is essential in an active or experiential lesson as there is a great element of discovery learning involved which can go down various paths thus calling on the teacher to be able to adjust and react to various scenarios and questions, some of which could (and probably will) be unanticipated.

4.2.4 Conclusion

Previously, the author queried the importance of content knowledge in relation to effective teaching. Analysis of the knowledges required for effective instruction indicates that it is a vital characteristic of successful educators. Content knowledge, characterised in one way or another, was labelled the most important aspect of all the models outlined (Rowland et al. 2005, Fennema and Franke 1992, Ernest 1989). It is of great importance to find ways to counteract any gaps in knowledge within mathematics and/or science amongst teachers prior to attempting an integrative approach. One solution to this would be for teachers of each subject to work in tandem i.e. team teach a lesson. Given the rigid nature of timetabling in Irish schools, this would probably be an unrealistic aim as both teachers would need to be free to work at the same time with one class.
Another solution would be to conduct the mathematics aspect of the intended problem or project during the mathematics lesson and the science aspect during the science lesson i.e. in the manner of Jacobs’ (1989) shared option (see figure 2.1). This may be possible for certain problems or projects but for most lessons it would most likely negatively affect the learning that takes place in the lesson and could greatly reduce the integrative element of the lesson and any positives that come with that.

A third solution, and probably the most practical one, would be to up-skill the teachers in relation to the knowledge gaps they have e.g. improve the mathematics teachers’ science content knowledge in relation to the material which would come up in each lesson they are to teach. In other words, give them the knowledge they require to deal with any questions from the pupils or to guide the pupils through any difficulties they may have.

The other issue which was prominent when researching this area was the beliefs held by the teacher. Although content knowledge is the best indicator of an effective teacher (Shulman 1986), it is a positive attitude towards mathematics which facilitates the adoption of inquiry-based instruction in the classroom (Wilkins 2008, Karp 1991). With this knowledge to hand, it became clear that the need to have teachers ‘buy in’ to the approach of inquiry-based instruction is something which is imperative.

The author ensured that teachers adopted this approach willingly by providing literature which outlined the merits of inquiry-based learning; discussing any reservations they had and developing solutions with them to problems which may have arisen out of such misgivings; and impressed upon them the reality that, as outlined by Ernest (1989), the ability to teach using an inquiry-based approach will only improve with practice. This indicates that if inquiry-based learning doesn’t produce results initially, that does not mean it has failed but rather that with more lessons of that nature, the positive results will begin to show. As teachers develop a range of resources and skills in this area, their ability to teach
using this method increases and enhances the quality of the lesson, benefiting overall learning.

In summation, with this close look at knowledges for effective teaching, it has become clear that some work must be done with teachers prior to an implementation of an integrative framework. Work must be carried out on the level of content knowledge they possess in both mathematics and science, and they must also be gently guided to the realisation that inquiry-based learning can produce very positive results. If this is done then it will provide a basis for integration. Such integration was achieved through the ‘Authentic Integration’ model – the process which led to the creation of this model will be discussed in detail beginning with the basis for the new model – Authentic Instruction.

4.3 Authentic Instruction

Authentic Instruction was analysed in detail in chapter two (see section 2.5), here it will be discussed further and analysed in relation to its suitability for adoption when developing lessons to integrate mathematics and science and how it inspired the creation of the ‘Authentic Integration’ model.

4.3.1 Basis of Authentic Instruction

Authentic Instruction, defined by the characteristics ‘Construction of Knowledge’, ‘Disciplined Enquiry’, and ‘Value Beyond School’, derives much of its theoretical basis from constructivism, which is described through comparison with traditional classrooms by Brooks & Brooks (1993) (see Table 4.1), but there are some notable differences (Newmann et al. 1995a). Firstly, ‘Construction of Knowledge’ bears all the hallmarks of constructivism as it puts the onus on the student to make meaning of the material they are encountering by linking it to prior experience rather than depending on the teacher to dispense the information. But this Authentic Instruction criteria goes a step further than typical constructivist approaches by challenging the student to analyse and interpret this
new information to solve a problem that can’t be solved by information retrieval alone (Newmann et al. 1995a).

In addition to this deep analysis and application, a central characteristic of Authentic Instruction is that the outcome of student work must be of a high intellectual standard. In other words, the work students produce must be worthwhile and significant levels of higher order thinking must have taken place throughout the task completed by the student. This will be achieved if the criteria for ‘Disciplined Inquiry’ are met, a characteristic which is often not found in constructivist approaches (Newmann et al. 1995a)

Such elements of Authentic Instruction render it an attractive option to educators e.g. the New Basics Project applied in Queensland (Education Queensland 2001). This section will outline further reasons as to why the general features which define Authentic Instruction offer the best fit for integrating mathematics and science. Analysis of its characteristics and the research based around the framework will supplement the argument for its use in the piece of writing to follow.
Table 4.1: Traditional Classrooms v Constructivist Classrooms (Brooks and Brooks 1993, p.vii)

<table>
<thead>
<tr>
<th>Traditional Classrooms</th>
<th>Constructivist Classrooms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curriculum is presented part to whole, with emphasis on basic skills</td>
<td>Curriculum is presented whole to part with emphasis on big concepts</td>
</tr>
<tr>
<td>Strict adherence to fixed curriculum is highly valued</td>
<td>Pursuit of student questions is highly valued</td>
</tr>
<tr>
<td>Curricular activities rely heavily on textbooks and workbooks</td>
<td>Curricular activities rely heavily on primary sources of data and manipulative materials</td>
</tr>
<tr>
<td>Students are viewed as &quot;blank slates&quot; onto which information is etched by the teacher</td>
<td>Students are viewed as thinkers with emerging theories about the world (As Howard Gardner puts it &quot;cognitive apprentices&quot;)</td>
</tr>
<tr>
<td>Teachers generally behave in a didactic manner, disseminating information to students. (A sage on the stage)</td>
<td>Teachers generally behave in an interactive manner, mediating the environment for the students (A guide on the side)</td>
</tr>
<tr>
<td>Teachers seek the correct answers to validate student learning</td>
<td>Teachers seek the students’ points of view in order to understand students' present conceptions for use in subsequent lessons (scaffolding)</td>
</tr>
<tr>
<td>Assessment of student learning is viewed as separate from teaching and occurs almost entirely through testing</td>
<td>Assessment of student learning is interwoven with teaching and occurs through teacher observations of students at work and through student exhibitions and portfolios</td>
</tr>
<tr>
<td>Students primarily work alone</td>
<td>Students primarily work in groups</td>
</tr>
</tbody>
</table>
4.3.2 Why Authentic Instruction?

The needs of mathematics education are clear: there must be a reduction in the dependence on didactic methods of teaching; greater focus needs to be placed on real life applications of the subject; and a more holistic approach where the student is central to their own learning (outlined in section 2.2). To educate students in a way which fosters genuine understanding requires, according to Carpenter et al. (2004, p.5), teachers to help students:

(a) connect knowledge they are learning to what they already know,

(b) construct a coherent structure for the knowledge they are acquiring rather than learning a collection of isolated bits of information and disconnected skills,

(c) engage students in inquiry and problem solving, and

(d) take responsibility for validating their ideas and procedures.

They justify these requirements by alluding to the belief that knowledge cannot be attained by listening passively to an authority figure (e.g. a teacher) alone, but rather needs to be internalized by the learner by making connections with pre-existing knowledge and also applying this new information in various scenarios (Carpenter et al. 2004).

The NCTM’s (2009) ‘Guiding Principles for Mathematics Curriculum and Assessment’ cites a similar ideology to that of Carpenter et al. (2004). Understanding of the material which is being learned is, of course, considered essential with proficiency in mathematics defined as “being able to use knowledge flexibly, appropriately applying what is learned in one setting to another” (p.2). To ensure that this happens, the NCTM (2009) recommend that problem solving, reasoning, connections, communication, and conceptual understanding are
centrally placed in the development of student knowledge. These requirements are echoed by a series of mathematics education academics, researchers, and groups (see section 2.3)

Such recommended characteristics can be found in Authentic Instruction. As mentioned previously, Authentic Instruction is defined through Construction of Knowledge, Disciplined Inquiry, and Value Beyond School. These characteristics best satisfy the needs of mathematics education (outlined in section 2.2) as they ensure connections are made with the real world and other education disciplines while ensuring that understanding is developed in a meaningful manner rather than in an aimless style which has little educational value (King et al. 2009).

Construction of Knowledge does not require the student to recall set methods for similar problems but challenges students to develop the knowledge they have by using it in a variety of situations thereby adding to it constantly in a manner which fosters deep understanding rather than superficial awareness (Newmann et al. 1995b). When this requirement (outlined in detail in section 2.5.2.1) is satisfied, students are using all available sources of information which they then synthesize and interpret to form a clearer picture of the resources they have available to them to solve a problem or complete a task assigned to them (Newmann et al. 1995b).

Such a scenario is much more beneficial than the practices which are to be seen in Irish mathematics education currently (see section 2.3) in which students are subjected to learning by ‘drill and practice’ i.e. they are shown an example of how to solve a problem and then given problems of a very similar nature to solve using the same methods (Childs 2006). Such repetitive implementation of an algorithm or approach will not improve understanding to the extent required (Boaler 1994) and, in many instances, just promotes the overuse of rote learning i.e. the student just needs to learn how to do it, understanding of the method used is secondary, if even that (William 1992). The ‘how’ is more important than the ‘why’.
Construction of Knowledge satisfies requirements (b) and (c) outlined by Carpenter et al (2004) and also aids in achieving goals engendered in (a) and (d) outlined above. ‘Disciplined Inquiry’ ensures that students partake in work of genuine value and are challenged to such an extent that higher level thinking is required throughout the task. This aspect of Authentic Instruction is often where other models are lacking, especially in pedagogies where active learning is central – the principles may be sound but often there is no thought given to guaranteeing significant levels of actual learning (Newmann and Wehlage 1993). Disciplined Inquiry combats this pitfall by ensuring high intellectual standards are maintained (Newmann et al. 1995a).

Take Anchored Instruction for example (outlined in section 2.6.2). It has many of the positive characteristics of Authentic Instruction: the students work with the teacher to explore subject matter through real life scenarios which challenges them to implement their existing knowledge and skills as well as developing and improving them (Oliver 1999) but there is no element in the structure of this model to ensure that high standards of intellectual work are produced.

Finally, ‘Value Beyond School’, as a principle, guarantees that the material being learned takes on greater meaning to the students and can be interpreted in an easier manner as it places the topic or topics in a scenario which students can relate to. This also aids in achieving requirement (a) set out by Carpenter et al. (2004) while also engaging the students in the problem in a greater sense. This characteristic also correlates closely with how understanding is developed according to the work of Dewey (1933, p.137):

“To grasp the meaning of a thing, an event or a situation is to see it in its relations to other things: to note how it operates or functions, what consequences follow from it, what causes it, what uses it can be put to.”
This implies that for something to be fully understood it must be placed in context to allow the learner to observe how it can be applied and how it affects related elements.

Results of studies carried out in relation to AIW (outlined in detail in section 2.5.6.3 and in the table above) offer some key information in relation to the effects of implementing Authentic Instruction. Promotion of AIW was shown to effect significant improvements in conventional recall assessments which examine a student’s ability to retain information and execute basic skills and algorithms as well as the expected improvement in ability to produce work of high authentic intellectual quality (See Section 2.5.4). This shows that Authentic Instruction can thrive and aid student improvement significantly in an educational scenario where summative assessment and rote learning is central, or, alternatively, in a scenario where formative assessment, authentic intellectual work and challenging problem solving are the central characteristics.

This flexibility demonstrates that this pedagogical approach can be incorporated successfully into a wide range of existing educational frameworks hence it becomes quite an attractive option considering how diverse the general approaches of various educational institutions tend to be. In an Irish context, currently there is a period of transition from the dependence on abstract forms of mathematics, with little emphasis on real world uses, to the new ‘Project Maths’ syllabus which focuses on applying mathematics to real world scenarios which are relevant to students’ lives (outlined in greater detail in section 2.1). Implementing Authentic Instruction can aid this transition as one of its requirements, ‘Value Beyond School’, fits in well with real world Mathematical applications and any adjustment to assessment approach through Project Maths will not significantly alter the effectiveness of Authentic Instruction as demonstrated in the studies highlighted above.
4.3.3 Is Authentic Instruction Suitable for Integrating Science and Mathematics?

Authentic Instruction was not developed specifically to integrate subjects as there is no mention of such a goal for this particular framework in the works of Newmann and his associates but the examples of Authentic Intellectual Work put forward by these authors commonly combine skills and knowledge from various disciplines. The fact that tasks and assessments are based on the replication of challenges the students will face outside school lends itself to cross-curricular learning as tasks in real life often require knowledge from more than one discipline. As real life scientific tasks commonly require mathematical knowledge to be implemented, and vice versa, the scope for combining these disciplines is quite broad hence it is a logical starting point for linking mathematics to other subjects.

Mathematicians and Scientists also have common approaches to challenges in their fields, e.g. scientific enquiry is a commonly used term which describes the collection of evidence through observation and measurement (Morris and Ernest 1934). Proof is central to mathematics in a similar sense. It is “a demonstration that, given certain axioms, some statement of interest is necessarily true” (WordIQ.com 2010). These processes are similar in the sense that both fields require an ordered approach to proving or disproving hypotheses. The ‘Disciplined Inquiry’ characteristic (described in section 2.5.2.2) of Authentic Instruction mirrors these ordered approaches to investigation as it ensures there is a structure to the methods employed as well as a significant level of higher order thinking. This element of Authentic Instruction can aid in inducting students into thinking like a mathematician or a scientist (or both) which would enhance the students’ development in the disciplines.

Within research, each of the four studies mentioned in section 2.5.6.3 included mathematics as one of the subjects studied. As stated previously, the results of each study were significantly positive as regards the level of learning achieved
through Authentic Instruction, thus it is reasonable to conclude that the framework can be implemented in a mathematics setting.

4.3.4 Review of the Adoption of Authentic Instruction for Integrating Mathematics and Science.

Integrating mathematics and science through applications which replicate tasks from the real world is a challenging task itself. Using the Authentic Instruction framework is currently the best approach as it engenders the positive elements of a constructivist approach through its ‘Construction of Knowledge’ characteristic while ensuring there is a sound structure through the presence of ‘Disciplined Inquiry’ which demands high levels of intellectual thinking throughout. The framework’s ability to effect great improvement in both authentic intellectual work and basic recall of facts and algorithms means that it will thrive in most, if not all, educational scenarios no matter whether summative assessment with rote learning or formative assessment with an holistic approach are the preferred methods.

Theoretically, Authentic Instruction embodies many of the characteristics which are recommended when integrating mathematics and science (as indicated through research outlined above) but there is a need to adjust and develop this framework to meet the specific needs of integration of mathematics and science as, from in depth analysis into publications pertaining to this model (Newmann et al. 1995b, King et al. 2009, Newmann et al. 2007), the Authentic Instruction model was not designed specifically for such a task.

4.4 Evolution towards Authentic Integration

4.4.1 Introduction – The Needs of Mathematics-Science Integration

The key challenge, when considering how to implement an integrative approach to mathematics and science, is the need to develop a working framework which
guides the development and delivery of such an approach. It is clear that there is a great need for a wider range of alternative methods of assessment and an increase in levels of practical work which allows the pupils to experience the application of the knowledge they are developing in the classroom (outlined in section 2.3). Support for integration has been displayed by a wide range of academic organisations (see section 2.4.1) and the methods for implementing lessons of an assimilative nature (see section 2.4.6) include:

- Ensuring the content is contextually based and taught in an authentic manner;
- Guiding hands-on, cooperative group work;
- Facilitating high levels of discussion; and
- Fostering an environment of inquiry and reflection.

These required elements provide evidence to suggest that Authentic Instruction engendered many of the characteristics which suited such an approach to integrating mathematics and science, hence it is the basis for such a framework. As mentioned previously, Authentic Instruction positively affects pupil performance in Authentic Intellectual Work, knowledge retention, and execution of basic skills and algorithms (see section 2.5.4) but this model is not specific to any particular subject and, also, it is not developed explicitly for integration (although it does cater for many of the needs of integration). With this in mind, it became clear that a model must be delivered which retained the key characteristics of Authentic Instruction but also satisfied some other aspects which needed to be included so as to ensure the best possible framework for integrating mathematics and science. Such a model is ‘Authentic Integration’.

4.4.2 Characteristics of the Authentic Integration Model

As Authentic Instruction is based around Construction of Knowledge; Disciplined Inquiry; and Value Beyond School (see section 2.5.2), these characteristics will be
retained within the Authentic Integration model but under different names, with each name change explained and justified. There will also be some addition to or subtraction from their overall meaning of each characteristic, depending on the needs of the model, as transition from a broad model which caters for many subjects (Authentic Instruction) to a model which is more specific (Authentic Integration) requires some adjustment.

The characteristics of the Authentic Integration model, rooted in Newmann et al.’s (1995b) Authentic Instruction, ensure that the content is contextually based, taught in an authentic manner through hands-on, cooperative group work with plenty of inquiry and discussion. To achieve this, there are three base characteristics - Knowledge Development, Synthesis and Application; Focused Inquiry Resulting in Higher-Order Learning; Applicable to Real World Scenarios – which combine to satisfy a fourth characteristic – Rich Tasks. Each of these components of Authentic Integration will be discussed in depth.

The new names prescribed to each characteristic were chosen carefully so that they would link directly to terms which would be commonly used within education literature. For example, “Knowledge Development, Synthesis and Application” is derived from Bloom’s (1956) taxonomy (see Fig. 4.1) which classifies learning objectives. Each term within this characteristic would be familiar to fully trained teachers and describes succinctly the cognitive progression which should take place within these integrative lessons. This term and how it is defined will be discussed initially, followed by the remaining characteristics of Authentic Integration.
4.4.2.1 Knowledge Development, Synthesis and Application

Starting with the first component of Authentic Instruction: ‘Construction of Knowledge’ – a term that did not fully describe what this characteristic involved, hence the change in title. Also, as mentioned previously, the term was changed to form a closer link to Bloom’s (1956) taxonomy to aid in displaying the learning objectives which should be achieved. This element, as it is presented through Authentic Instruction, will remain largely unchanged, with synthesis of information being particularly highlighted as, of course, fusion of information from both subjects is central to the idea of integration.

1. Knowledge Development, Synthesis and Application

Pupils will draw on their previous knowledge (in both Mathematics and Science), assimilate this knowledge and develop it through application to various challenging tasks and scenarios. In other words – they will improve and increase their knowledge in Mathematics and Science (knowledge development) then combine what they have learned in both disciplines (synthesis) so that they may apply it to solve a problem or complete a set task (application). It is vital that pupils identify the
connections between the skills and information they have obtained in both disciplines.

Drawing on knowledge from various areas within mathematics and science is imperative in an integrated setting (Berlin and White 1992). Developing the ability to assimilate this information from different sources to form a solution to a task or a problem is a key skill which will benefit pupils in other areas of education and in real life scenarios (Bossé et al. 2010). Furner & Kumar (2007) believe that integration should combine concepts and principles which overlap in a meaningful manner thus this implies that knowledge from various domains must be accumulated and combined effectively. As such, ‘Knowledge Development, Synthesis and Application’ forms the cornerstone of this integrative model as it breaks down any barriers between mathematics and science. Teachers must tailor their lessons sufficiently to ensure that pupils are exposed to such synthesis of information so that they can develop this skill on a regular basis.

4.4.2.2 Focused Inquiry Resulting in Higher-Order Learning

Disciplined inquiry is a vital aspect of Authentic Instruction as it ensures that the quality of work produced is of a high standard rather than a product of some aimless investigation (see section 2.5.2.2). Thus, it is imperative that such a characteristic be retained. ‘Focused Inquiry Resulting in Higher-Order Learning’ calls on the pupils to demonstrate and apply their knowledge through procedures which are central to mathematics and science. As such, logical reasoning, proof, and/or viable experimentation methods will be present in all tasks carried out with results communicated using the language associated with mathematicians and scientists.

2. Focused Inquiry Resulting in Higher-Order Learning

Pupils will achieve higher-order learning through inquiry into the elements of the topic(s) being studied in a focussed manner. Pupils should engage
in substantial conversation with their teacher and their peers to build a shared understanding. Pupils must display advanced learning through:

- Demonstration of their knowledge of the core elements and concepts being studied using the language and procedures relevant to Mathematics and Science

- Coherent approaches to tasks with presentations of results which suit the data obtained and conclusions drawn.

Within tasks which place the onus on the pupils to assume responsibility for their own learning, the overall objectives of these tasks can get lost and pupils may not reach intended goals (King et al. 2009). For this reason, teachers must demand a high level of quality work with certain goals which must be achieved through the completion of the task. It is imperative that the teacher defines these goals, aids the pupils in striving for them, and ensures that the pupils maintain high standards in relation to the quality of their work (Newmann et al. 1995a). Substantial conversation with the teacher and/or their peers is also imperative as “mathematics deepens and develops through communication” (Silver et al. 1990, p.15).

4.4.2.3 Applicable to Real World Scenarios

One of the key elements of any integrative approach is that it must be contextually based (see section 2.4.6). In other words, the concepts and information relevant to the topics being studied must be placed within a scenario which is real to the pupils. If this is satisfied then pupils will have the opportunity to apply their knowledge in contexts which will be similar to those they will face in everyday tasks and future jobs. Again, this is similar to the Authentic Instruction characteristic of ‘Value Beyond School’. A change of title was required for this characteristic, also, to give a clearer meaning to this element thus the term ‘Applicable to Real World Scenarios’.
This element also takes inspiration from Realistic Mathematics Education (see section 2.2.2). Freudenthal, the creator of RME, “felt mathematics must be connected to reality, stay close to children’s experience and be relevant to society, in order to be of human value” (Van den Heuvel-Panhuizen 2000, p.3). As such, ensuring that the tasks completed and learning attained through these lessons would be useful in their everyday lives became a central component of the model.

3. Applicable to Real World Scenarios

Skills and knowledge obtained, developed and displayed should be useful outside the classroom i.e. in situations which may occur in everyday life. This may pertain to typical domestic tasks, hobbies or tasks to be carried out in further education and/or future jobs.

The manner in which a person develops their knowledge and skills within a discipline becomes a vital aspect of what is actually learned (Putnam and Borko 2000), thus it is important to situate learning in a scenario which is as close as is feasible within a school setting to what a pupil experiences or will experience in real life (National Council of Teachers of Mathematics 2009). It is also imperative that what is learned is of value outside the classroom and not just geared towards success in summative assessments i.e. learning algorithms without understanding the reasons behind them which means they cannot be applied outside of familiar exam style questions (William 1992).

Applying mathematics and science to real world scenarios will also give greater meaning to the topics in the eyes of the pupils as they will recognise that what they are learning is of value and can be used to their advantage outside the classroom (Splitter 2009). Having said that, evidence to the contrary has been discovered as Saeki et al. (2001) noted that, within a project carried out in Japan, mathematics students who experienced real-world problems using real-world data displayed no apparent improvement in attitude. Nevertheless, most research indicates that linking material to real life scenarios will produce positive effects
(Mitchelmore and White 1995, Putnam and Borko 2000, Van den Heuvel-Panhuizen 2000) and will also aid in improving their attitude towards the work being carried out and offer greater motivation to complete the task or problem with due diligence (Boaler 1994).

At this point there has been little deviation away from the Authentic Instruction model characteristics except for title changes and adjusting the content to make it more specific to mathematics and science integration. The major change comes in the addition of a fourth element: ‘Rich Tasks’.

4.4.2.4 Rich Tasks

As stated previously, hands-on cooperative group work involving plenty of discussion and inquiry should be central to any attempt to integrate science and mathematics (see section 2.4.6). As such, this should be a central element of any model for integration of the two subjects. As stated in the description below, pupils will be central to the process thereby giving them responsibility for their own learning and a sense of ownership in relation to what and how they learn. The teacher’s role will be mainly that of a facilitator who will ensure that the previous elements are satisfied, especially the presence of higher-order learning through focussed inquiry.

4. Rich Tasks

Problem based tasks must be of an active or experiential nature with pupils central to the planning, implementation and conclusions drawn. The teacher should act as a facilitator offering hints, tips and advice when necessary as well as ensuring that the work carried out is of a high standard.
Rich tasks are problem based challenges which are transdisciplinary in nature, the outcome of which is of demonstrable and substantive intellectual substance and educational value (Education Queensland 2001). These types of tasks have been adopted by, among other groups, Education Queensland through their ‘New Basics’ Project. Each Rich Task “presents substantive, real problems to solve and engages learners in forms of pragmatic social action that have real value in the world” (Education Queensland 2001, p.5). As such, these tasks offer a fine blueprint for the approach to be taken by this author as the characteristics mentioned match those recommended by research outlined previously (see section 2.4.6).

### 4.4.3 Graphical Representation of Authentic Integration

![Figure 4.2: The Authentic Integration Triangle.](image)

Figure 4.2: The Authentic Integration Triangle.
The Authentic Integration Triangle (Figure 4.2) defines the make-up of this framework:

- The first two elements “Knowledge Development, Synthesis and Application” and “Focused Inquiry Resulting in Higher Order Learning” form the foundation of Authentic Integration. These elements are central to how each lesson will be constructed and delivered as well as providing the basis for applying the relevant knowledge to real world scenarios.

- The characteristic “Applicable to Real World Scenarios”, while not as important as the first two mentioned, is something which will be strived for in all lessons. There are instances where it may not be possible and thus not as prominent as the other characteristics. This links in with the structure of the triangle – it is possible to remove this element and still have a stable structure but it will not be as solid as it would be if this element were present.

- Finally, the previous three characteristics are in place to support the final one which is the implementation of “Rich Tasks”. Satisfying the first three characteristics will ensure that the tasks carried out will be of a high level of quality. The final outcome (the task) is the crucial element but it would be useless without first satisfying the first three elements, especially the first two.

There are advantages in representing the characteristics of Authentic Integration graphically. Firstly, from a learner’s perspective, it has been claimed that visuals increase the level to which the content is comprehended; selectively increase learners’ attention; and positively affect enjoyment (Cronin and Myers 1997). Such outcomes, especially the increase in comprehensibility, would aid teachers in gaining a greater understanding of the Authentic Integration model. Allied to these positive impacts, Baggett (1989) contends that visual representations contain more information and greatly aid recall as visuals can be used to make a relatively
large amount of associative and referential connections with information stored in long term memory.

Thus, with a visual as an aid, understanding and recall of the vital elements of Authentic Instruction should be improved and, as such, improve implementation of the model by the teachers.

4.4.4 Overview of Authentic Integration

The author found that the make-up of Authentic Instruction was not specific enough to the task of integrating mathematics and science hence the evolution towards ‘Authentic Integration’. As shown, the new framework maintains the important elements of Authentic Instruction but it also caters for the specific needs for integrating mathematics and science derived from research. The triangle structure displays clearly the characteristics of the framework and their importance: “Knowledge Development, Synthesis and Application” and “Focused Inquiry Resulting in Higher Order Learning” are of utmost importance thus they form the foundation; “Applicable to Real World Scenarios” supplements and complements the previous two characteristics; while “Rich Tasks” is the overall product of the previous three elements. This framework will produce the best of Authentic Instruction while ensuring that the specific needs of mathematics and science integration are met.

The elements complement each other, for example: regularly, in real world scenarios, information from various sources must be correlated to complete a task or reach a conclusion (Newmann et al. 1996). Also, approaching such a task in a focussed and disciplined manner will generally affect the process positively (Newmann et al. 1995b).

The visual representation of the model makes it much more accessible as teachers can see that there are three elements supporting the major outcome of each lesson i.e. the “Knowledge Development, Synthesis and Application”, “Focused Inquiry
Resulting in Higher Order Learning”, and “Applicable to Real World Scenarios” characteristics are the platform for the overall aim of implementing “Rich Tasks”. It has also been repeatedly proven that carefully constructed illustrations form valuable aids to pieces of text in the learning and knowledge retention process (Carney and Levin 2002) thus such an approach should have a similar impact when aiding teachers in adopting this model.

4.5 Lesson Guide Layout

4.5.1 Introduction

As this is a new approach to teaching mathematics and science, the corresponding lesson plans should reflect this and also guide teachers in implementing the lessons in the manner intended. This requirement offered the author the challenge of producing a lesson plan design which covers the essentials, displays the information clearly and concisely, and highlights the elements which must be present in the lesson so as to satisfy the requirements of the Authentic Integration model.

One of the most pressing issues in this instance is the fact that these lessons will be applied in a range of different schools to pupils of different ages, different stages of development and, as such, varying capabilities. Thus a ‘lesson plan’ is not what is required in this scenario but rather, what the author has decided to refer to as, a ‘lesson guide’.

These guides were deliberately designed in such a way as to outline a lesson which would engender all the characteristics of Authentic Integration and outline certain expectations and outcomes which should be achieved within the lesson. But the design also allows for adjustment by the teacher as regards time, level of difficulty, layout of classroom, discussion topics and length, and the order in which sections of the lesson are completed. The aim is to strike a balance
between satisfying the needs of integration (according to research) and allowing teachers to assume ownership of a lesson which they can mould to suit the needs of their pupils.

4.5.2 Why Not Create Lesson Plans?

Planning a lesson requires information on particular elements, outlined as follows (Reece and Walker 2000, p.28):

1. The title of the lesson.
2. Details of the class (name, size, etc.).
3. The time of the lesson.
4. The expected entry behaviour of the class (expressed in terms of what the students should know or be able to do).
5. The objectives of the lesson (expressed in the same sort of terms as the entry behaviour)

In the scenario pertaining to this research, satisfying elements 2, 3, and 4 are not feasible as the details of the class to be taught and their respective timetables are not known. Element 1 is included in the lesson guides while the fifth element is also present in the cover page of the lesson which will be discussed later.

Reece and Walker (2000, p.29) go on to discuss the rest of a typical lesson plan which they refer to as the ‘body of the plan’, outlining the headings which should typically be included in this part of the plan: time, content, teacher activity, student activity, and aids. Each of these elements needs to be considered in the context of the requirements for these integrated lessons.

The time dedicated to the lesson and each phase of the lesson is omitted from the guides as the total time the teacher has is unknown and such a stance also leaves the teacher with plenty of options as regards how much time they feel needs to be
dedicated to each section, keeping in mind pupil proficiency in the area and other varying elements.

The second heading, content, is included by outlining the task, its evaluation, a brief summary and the targeted practices expected throughout the lesson. Teacher activity and student activity are generally outlined through the description of the task(s) and the targeted practices section but the specifics are largely left to the teacher to determine. Finally, teaching aids are mentioned in task descriptions but again are subject to the teacher’s discretion.

It is clear that the outlines of the lessons do not fully conform to a lesson plan as described by Reece and Walker (2000) but they do provide the information and direction to create a lesson plan which satisfies the Authentic Integration model and suits the developmental level and needs of the pupils according to their teacher. Thus lesson guide, rather than lesson plan, is a suitable title for the outline produced to aid the teachers implementing these educational experiences.

4.5.3 Discussing Each Section of the Lesson Guide

Every aspect of the lesson guide layout will be discussed and justified in this section. Examples of these lesson guides can be viewed in detail in section 5.3.

4.5.3.1 Cover Page

The cover page gives the reader an overview of what is to happen in the lesson as well as immediately outlining the fact that mathematics and science concepts will be assimilated through the lesson. The overview of the lesson was covered using a summary which can be seen at the bottom of Fig. 4.3. This summary explains the task which should be undertaken by the pupils during the lesson. The fact that mathematics and science concepts will be assimilated through the lesson was made clear by placing explanations of the mathematics and science concepts to be
explored in the centre of the page side-by-side, as well as the objectives of each element which should be achieved through implementation of the lesson.

Figure 4.3: Sample lesson guide – cover page.

The ‘Key Information’ panel on the left recaps on the essential material covered in the resource pack, acting as a reminder for the teacher. On the right hand side of the cover page, ‘Targeted Practices for Pupils’ offers an insight into what the teacher should expect from pupils as they complete the set task. It would be beneficial for the teacher to share these expectations with the pupils before the commencement of the task – this would offer the pupils a set of goals to be achieved during the lesson thereby aiding the ‘Focused Inquiry Resulting in Higher Order Learning’ characteristic of the Authentic Integration model.

The ‘Summary’ of the task, the ‘Key Information’ and the ‘Targeted Practices for Pupils’ sections included on the first page were inspired by the approach taken by
Education Queensland (2001) in their adoption of Authentic Instruction through the New Basics Project as alluded to previously (see section 4.4.2.4). The outline of their rich tasks draws upon these headings to describe the task to be completed in a similar manner to how these headings are applied in the lesson guides created for Authentic Integration.

In the Education Queensland (2001) report, ‘Targeted Repertoires of Practice’ are described as typically being repertoires which must be attained by pupils in order to successfully complete the rich task within the lesson. They are usually drawn from subject disciplines or real life endeavours. Such an element contributes to the “Focused Inquiry Resulting in Higher Order Learning” requirement of Authentic Integration as it offers a clear direction for the learner to take while ensuring that such learning is of a relatively high standard. These practices also aim to improve the skills and abilities of the pupils within mathematics and science through adequate application of said practices.

In education, practice plays an important role as it establishes routine which reduces the amount of thinking (or the cognitive load) required when completing a task but such practice can limit pupil progress when they reach a certain level i.e. their performance reaches a plateau (Pegg and Graham 2007). This is very similar to the criticisms of the drill and practice approach commonly applied within mathematics education (see section 2.3.2). For pupils to progress from this point or plateau, they must be challenged to cognitively reorganise their skills, i.e. apply them in a different manner to how they have practiced them previously, through deliberate practice or targeted practice of these skills (Pegg and Graham 2007). So, to achieve greater performance within practices developed previously by the pupils, the ‘Targeted Practices for Pupils’ section of the lesson guides calls on the pupils to advance their performance in these practices by applying them in a more challenging manner than the typical ‘drill and practice’ approach or any such similar method of learning.
Greater detail regarding the task(s) to be carried out and how performance within the task(s) is to be evaluated is present on the second page of each lesson (see Fig. 4.4). These sections offer the blueprint for the lesson, giving plenty of leeway for the teacher to tailor the lesson to suit the needs and abilities of their pupils. This section is modelled on the ‘Task Parameters’ element of the New Basics lessons (Education Queensland 2001) whereby the task is explained while ensuring that it is linked to the ‘Targeted Repertoires of Practice’. A similar approach is adopted within these lesson guides for Authentic Integration, whereby the ‘Targeted Practices for Pupils’ are replicated in the ‘Task Outline’.

The large triangle represents the characteristics of the Authentic Integration model which must be present within each lesson. As such, it provides a clear insight into how the lesson must be delivered thereby giving the teacher guidance when planning the overall implementation of the planned task(s).

Figure 4.4: Sample lesson guide – second page.
The design gives a clear outline of the central element of the lesson (the task), how it satisfies the characteristics of the Authentic Integration model, and the aspects of pupil performance which need to be monitored and evaluated.

4.5.3.3 Final Page – Suggested Questions, Ideas, and Progressions

The final page of each lesson guide (see Fig. 4.5) offers the teacher suggestions for questions to be asked throughout the lesson and topics which may be discussed. The minority of these questions are recall-type questions to evaluate pupil learning as they progress, with the majority of questions listed of on an open-ended nature. Martino and Maher (1999) compiled in-depth research pertaining to the art of questioning in mathematics classrooms, and discovered that asking more open-ended questions aimed at problem solving strategies and conceptual knowledge can aid the development of more sophisticated mathematical knowledge amongst pupils. Thus, this section of the lesson guide should aid in improving pupil knowledge through discussion while also guiding the pupils in their progression through the challenge or task set.

It should be noted at this point that the success of this element of the lesson is largely dependent on the ability of the teacher to facilitate discussion as the art of questioning is something that may take years to develop and requires a high level of understanding of both mathematics and the manner in which pupils learn mathematics (Martino and Maher 1999).

Having a discussion element prominent within each lesson also allows for balance between mathematics and science. For example, if the task implemented requires a lot of mathematics to be applied then the discussion can focus on the science element to redress the balance if needs be and vice versa. Similarly, such discussions greatly aid the ‘Knowledge Development, Synthesis and Application’ characteristic of Authentic Integration, this is due to the fact that such communication helps pupils to interpret and form a deeper understanding of the material (Goos 1995, Lee 2006, Pimm 1987).
Further to that, discussion has been shown to have a positive effect on mathematical achievement (Koichu et al. 2007, D’Ambrosio et al. 1995) although this is not always the case (Shouse 2001) thus great care needs to be taken to ensure that a quality discussion is implemented and that teachers are skilled in the delivery of such.

In addition to the suggested questions element of the third page of the lesson guide, there is a section based on further ideas and progressions which would, if applied, illuminate the lesson to a greater extent and/or offer a greater challenge to those who are quite adept in the topics being studied. This part of the lesson guide is in place to give the teacher suggestions as to how they might fashion the lesson but, again, it leaves leeway for the teacher to adjust the lesson as they see fit. Such options are affected by the amount of time available to the teacher to conduct the lesson – typically lack of time is an issue within lessons of an
integrative nature (Lehman 1994). As such, they may or may not be able to include these extras elements within the lesson but the option is there to do so.

The ‘Ideas and Progressions’ section of the lesson guide bears similarities to the ‘Ideas, hints and comments’ portion of the lessons designed for the New Basics Project (Education Queensland 2001). Within New Basics lessons, this portion offers ideas on where pupils can gather further information, hints on how to present the information they gather and conclusions they draw, and the various perspectives from which a solution to the task could be approached (Education Queensland 2001).

The ‘Ideas and Progressions’ section of the lesson guide was also inspired by research into the ‘Anchored Instruction’ model (see section 2.6.2). Within Anchored Instruction, stories or descriptions are commonly employed to allow the learner to explore the material from a different perspective thus giving a more rounded understanding of the topic(s) being studied (Cognition and Technology Group at Vanderbilt 1990). This led to the elements of the ‘Ideas and Progressions’ section of the lesson guide which provide suggestions for pupils and teachers to work together to explore the uses of the material they are studying through stories, various descriptions, further real life scenarios, and various other multiple perspectives.

Having an ‘Ideas and Progressions’ section also serves to provide further challenge for more gifted pupils who may complete the given task at a quicker rate to the rest of the group and/or class. Diezmann et al. (2004) outlined, with the aid of research, that gifted pupils are capable of, with the appropriate scaffolding, completing complex, open-ended tasks quite quickly and with more sophistication than others thus it is important to ensure that there is the possibility of greater levels of difficulty and/or additional tasks available to the teacher to challenge these pupils further.
While these lesson guides provide the blueprint for putting the Authentic Integration model into action, the question, which was touched on previously and which will now be revisited, must be asked: are the teachers sufficiently prepared to implement these lessons?

4.6 Are the Teachers Equipped to Implement this Model?

The single greatest contributor to student success is teacher effectiveness (Sanders 1999, Wenglinsky 2000), while content knowledge is the best indicator of an effective teacher (Shulman 1986). In general, students learn more from teachers who are skilled, experienced, and know what and how to teach (Darling-Hammond 2000, Goldhaber 2002, Rice 2003). As such, the quality of the teacher and their level of content knowledge within both mathematics and science become important aspects to focus on when considering the adoption of a new approach such as Authentic Integration.

The quality of mathematics teachers in Ireland has been called into question regularly as evinced by a recent report based on a survey of 2,045 mathematics teachers in 258 schools conducted in September 2011. This report indicated that 49 of the 2,045 teachers (2.4%) have no third-level qualification or have done no studies in mathematics, 596 (29.1%) have completed “some studies” in mathematics, while 1,400 (68.5%) teachers are fully qualified to teach the subject (O'Halloran 2011).

More worryingly, an investigation conducted in 2009 stated that, of the mathematics teachers employed at that time in the Irish education system at Post Primary level, 48% were ‘out-of-field’ (Ní Riordáin and Hannigan 2009). The term ‘out-of-field’ is defined as “teachers assigned by school administrators to teach subjects which do not match their training or education” (Ingersoll 2002, p.5). As such, this research indicates that nearly half of all mathematics teachers in Irish secondary schools at that time were not suited to teaching mathematics.
With the knowledge that 31.5% of the mathematics teachers surveyed in one particular study, almost 1 in 3, were under-qualified, and, according to another study, 48% were ‘out-of-field’, it is reasonable to believe that there are a large number of teachers in Ireland that have difficulty teaching mathematics alone, not to mind adding science to their responsibilities within the classroom. This realisation raises serious questions as regards the possibility for widespread adoption of an integrated curriculum in Ireland.

Although this issue of poor teacher quality is a serious dilemma, there is positive news in relation to this problem. Advancements are being made in relation to the issue of teacher qualification as the Irish government will aid the up-skilling of such under-qualified teachers through the Professional Diploma in Mathematics for Teaching (Part-time) over the next 3 years, with approximately 400 enrolling in the course which commenced in Autumn 2012 (University of Limerick 2012). While this development marks a positive step in dealing with the issue of under-qualified mathematics teachers, the current situation is a noticeable barrier in the implementation of lessons which integrate mathematics and science.

A long term solution to this issue would be to implement programs such as the M. Ed., outlined by Berlin and White (2010), which focusses primarily on teacher preparation for the integration of mathematics and science (see section 2.4.4). Such programs would sufficiently prepare pre-service teachers to implement lessons which integrate mathematics and science (Berlin and White 2010). Such a scenario cannot be expected immediately, nor will it affect teachers currently in the field, thus a short term solution to this issue is required.

The short term solution must take into account the content knowledge required for both subjects. There is a great need for teachers to improve such knowledge within both mathematics and science so as to improve their delivery of each in an integrated setting (Frykholm and Glasson 2005) as content knowledge is the most important attribute of an effective teacher (Fennema and Franke 1992, Rowland et al. 2005). A fully integrated setting would be unrealistic but teachers should be
able to make connections between the two subjects (Frykholm and Glasson 2005). Thus, the lessons created through the Authentic Integration model use connections between the subjects at Junior Cycle and are interspersed between regular tuition of the individual subjects to avoid putting pressure on the teachers to adapt to a fully integrated setting. Placing its implementation at Junior Cycle also ensures that the content in either topic is not overly complex and can be mastered with the right support structure.

This issue of content knowledge was previously alluded to, with a solution to this problem was outlined (see section 4.2.4). This solution requires a quality support structure in which work with the teachers involved is conducted to improve their content knowledge in their non-specialist subject, whether that be mathematics or science. It has been demonstrated that improving teachers’ mathematical knowledge through content-focused professional development and preservice programs will enhance their teaching ability and, as a result, positively affect student achievement (Hill et al. 2005). Taking this research into account, training the teachers prior to implementation of the Authentic Integration model is a central element of its promotion. This is to be achieved by spending time with the teacher to guide them through the material to be taught and providing resources which explain clearly all elements of any topic they are unfamiliar with e.g. through a powerpoint presentation (see resource pack in appendix) while also providing guidance on how to conduct the lesson, especially with reference to group work and conducting discussions with the pupils.

4.7 Conclusion

Research pertaining to the integration of mathematics and science calls for certain methods to be adopted when taking on such an endeavour:

- Ensuring the content is contextually based and taught in an authentic manner;
• Guiding hands-on, cooperative group work;
• Facilitating high levels of discussion; and
• Fostering an environment of inquiry and reflection.

The Authentic Instruction model satisfied most of these needs. Allied to that, research of this model proved that it positively affected basic recall and execution of basic skills and algorithms, while also increasing pupil production of quality authentic intellectual work. As such, this model offered a fine starting point when it came to designing lessons of an integrative nature within mathematics and science.

The author took the characteristics of Authentic Instruction to create a model which would suit the specific needs for integrating mathematics and science – Authentic Integration. This model was based around the three defining elements of Authentic Instruction - ‘Construction of Knowledge’, ‘Disciplined Enquiry’, and ‘Value Beyond School’. These elements were renamed and adjusted to suit the needs of integration of mathematics and science and thus became ‘Knowledge Development, Synthesis and Application’; ‘Focused Inquiry Resulting in Higher-Order Learning’; and ‘Applicable to Real World Scenarios’. A fourth characteristic, ‘Rich Tasks’, was added to ensure that each lesson would be centred around a contextually based task which called for hands-on cooperative group work with plenty of discussion and inquiry in order to develop, synthesise, and apply knowledge from both disciplines. As such, the final characteristic was a product of the previous three.

These adjustments to and improvements of the elements of Authentic Instruction created a new model which would be more suited to the integration of mathematics and Science while retaining the key components which made the original model successful within the research carried which tested its perceived merits.
The lesson guides display the implementation of the Authentic Integration model in a way which offers the teachers employing them the flexibility to adjust the lesson according to the needs of their pupils and the particular constraints which they are working under while ensuring that the characteristics of the Authentic Integration model are satisfied. These lesson guides also provide ways for sparking discussion and ways in which the lessons can be extended so that the better abled pupils can be challenged further. As such, teachers can take on the lesson and make it their own while ensuring that the pupils experience a lesson which is centred around a contextually based, hands-on, cooperative group work task with plenty of discussion and inquiry.

The ability of teachers to implement lessons based on the Authentic Integration model is questionable considering recent investigations into the quality of said teachers (Ní Riordáin and Hannigan 2009, O’Halloran 2011). Initial implementation of the model at Junior Cycle is recommended, while quality training of teachers to improve content knowledge pertaining to their non-specialist subject is recommended. Interspersing this type of lesson amongst regular tuition will also allow teachers (and pupils) to adapt to the varying facets of the lesson gradually, allowing for improvement in the vital aspects which may prove challenging initially. These measures should provide a platform for teachers to develop and improve the various skills and knowledges required to implement Authentic Integration lessons successfully.
Chapter 5

Resource Pack
5.1 Introduction

Previously, the manner in which Authentic Integration evolved from Authentic Instruction was discussed along with the central characteristics of the model and the theory that underpinned these characteristics. The next step in the progression of the Authentic Integration model was to create a vehicle for its dissemination i.e. a resource pack. The resource pack produced for this intervention played an important part in informing the teachers of their role in the intervention. It also proved to be a vital resource for lessons, understanding of the teaching model to be implemented, and it also provided vital information regarding the content to be studied through the lessons.

In this chapter, analysis will be carried out on the various facets of the resource pack and the functions performed by each facet. The resource pack itself will also be included to highlight the manner in which the Authentic Integration model can be applied to create lessons, as well as giving a clear insight into the make-up of the intervention carried out.

5.2 Analysis of Each Section of the Resource Pack

The aforementioned resource pack was carefully assembled with each section performing a specific function in the implementation of the intervention pertaining to this study. These functions are outlined in this section, beginning with how the teachers were given a detailed account of what would be expected of them and how the Authentic Integration model would be employed. Aspects of the lesson guides will be discussed as well as the process inherent in their creation, while the resources provided will be described and justified in this chapter. Finally, the resource pack itself will make up the rest of the chapter.
5.2.1 Instructions to Teachers

The first three parts of the resource pack are in place specifically to aid teachers in understanding how the intervention was to be carried out and their role in its implementation. The introduction gives a brief overview of the whole pack, while the chapters entitled “How will this intervention be conducted?” and “How to use this Resource Pack” outline the specific manner in which the intervention was to be carried out and the intricacies of the lesson guides which are outlined later in the pack.

Included in the pack is a detailed explanation of the specific components of the intervention. This was included to ensure that the teachers could achieve a significant understanding of the intervention which was central to the parameter termed “Intervention Acceptability” in Shapiro’s (1987) model for evaluating an educational intervention (see section 3.5.6). Similarly, these chapters also aided “Treatment Integrity”, another of Shapiro’s (1987) parameters (see section 3.5.6), as these specific details regarding how the intervention was to be conducted and how the lesson guides were to be interpreted ensured that each teacher understood their specific roles. Treatment integrity requires that, if the intervention were to be repeated, it would produce replicable results – these sections of the resource pack aimed to satisfy such integrity by ensuring uniformity, to a certain extent, in how the intervention was implemented across the schools.

Furthermore, this part of the resource pack detailed each characteristic of the Authentic Integration model while also explaining the graphic, the Authentic Integration Triangle, which represents the model in each lesson guide. This graphic was vital in the interpretation of how the model was to be implemented as visuals increase the level to which the content is comprehended (Cronin and Myers 1997) thus aiding teachers in gaining a greater understanding of how the lessons were to be conducted.
5.2.2 Lesson Guides

As previously stated (see sections 4.5.1 and 4.5.2), lesson guides were created rather than lesson plans. This was to allow leeway for teachers with regards to the amount of time they could dedicate to the lessons and the varying capabilities of the pupils in each of the schools. Such an approach also enabled the teachers to assume more of an ownership of the lesson so that they could fashion it to suit the needs of their pupils while, at the same time, satisfying the characteristics of Authentic Integration. These lesson guides were designed by the author with certain elements of the guides inspired by the New Basics project, implemented by Education Queensland (2001), and research produced by Pegg and Graham (2007), Reece and Walker (2000), and Martino and Maher (1999) among others.

The cover page in each lesson provides a brief synopsis of the rich task to be completed through the lesson as well as outlining the mathematics topic and the science topic which should be integrated through the task. Key information is listed, as are the practices which pupils should display throughout the lesson e.g. peer learning and advanced discussion of concepts. The second page indicates clearly how each characteristic of Authentic Instruction is satisfied while also outlining how the rich task is to be carried out and how pupil performance in this task can be evaluated. The third (and typically final) page of each lesson guide provides a list of questions which teachers can draw on when conducting meaningful discussions with the pupils before, during, and after completion of the task set. Present in this page also are hints and tips regarding the manner in which the task can be changed, supplemented and/or extended. Further elaboration of the layout and rationale of the design of these lesson guides is presented in Chapter 4 (see section 4.5.3).

The topics which were integrated through these six lesson guides were picked specifically to suit the intervention to be carried out. The intervention was conducted in Irish post primary schools at Junior Cycle with 2nd year pupils in their first term of that school year. The topics that were integrated through the
lessons in this intervention should have been experienced by the pupils previously, preferably in the recent past. Such a requirement ensures that the “Knowledge Development, Synthesis and Application” characteristic of the model is satisfied. This characteristic requires pupils to draw on their previous knowledge (in both mathematics and science), assimilate this knowledge and develop it through application to the rich task set (see section 4.4.2.1). If pupils are to draw on previous knowledge then they must have experienced the topic in mathematics or science class previously so that they actually have knowledge to draw on.

With this in mind, research was conducted regarding when topics are typically taught in mathematics and science at Junior Cycle. Topics taught in late 1st year and, especially, early 2nd year were best suited to the needs of this intervention as the pupils would be experiencing this intervention late into their first term in 2nd year. The author conferred with a number of teachers and education professionals in the areas of both mathematics and science as regards the timing of topics taught to gain an insight into what content would be most suitable to include in the lessons. Of significant value in this endeavour also was a resource entitled “Recommended Sequence for Junior Certificate Science” provided by Discover Sensors (2010) which outlined the typical order of science topics at Junior Cycle, the majority of which conformed to the order suggested by the teachers and professionals consulted during this process. Using this information, six lessons were developed, mainly using the topics best suited to the intervention circumstances. The teachers picked the three lessons from these six that they felt best suited their situations.

The mathematics topics present in the lessons include statistics (bar charts and pie charts); plotting and interpreting graphs; geometry (measuring angles); Ratio and Proportion; Mixed Operations and Percentages; and Distance, Speed, and Time. The science topics selected were Food and Diet; Distance, Speed, and Acceleration; Optics (Angle of Incidence and Angle of Reflection); Gravity; Electricity; Light and Sound. These topics combined to form the six lessons
included in the pack which were created through the Authentic Integration model. Resources for each topic were also included.

5.2.3 Mathematics and Science Resources

A key consideration when developing this pack for the teachers involved in the intervention was that content knowledge is the most important aspect of teacher knowledge (Rowland et al. 2005, Fennema and Franke 1992, Ernest 1989). Research into teacher knowledge (see section 4.2 & 4.6) made it clear to the author that the teachers involved in this study would require sufficient content knowledge in both mathematics and science to successfully implement lessons of an integrative nature. This led to teacher training which was supplemented with powerpoint presentations (contained in the final two chapters of the resource pack in section 5.3) covering every topic applied in each of the six lessons created. These presentations proved a valuable aid in developing the teachers’ content knowledge and also acted as a key resource which could be used in the classroom when teaching these or other lessons. Each element of the resource pack was provided in a printed version and via memory stick so that they could also be viewed and used electronically i.e. through a computer or any such similar device.
5.3 The Resource Pack

Integrated Mathematics and Science Resource Pack

University of Limerick

NCE-MSTL
National Centre for Excellence in Mathematics and Science Teaching and Learning
Contents

1. Introduction

2. How will this intervention be conducted?

3. How to use this Resource Pack

4. Authentic Integration

5. Lesson Guides

6. Mathematics Resources

7. Science Resources
**Introduction**

This resource pack has been carefully developed to aid in the implementation of lessons which integrate Mathematics and Science at Junior Cycle. Inside this pack you will find lesson guides which integrate the subjects through tasks which are designed to be active and pupil-centred in nature. Resources are also provided in the form of powerpoint presentations – these will act as an aid for the teacher if they are unsure of elements of certain topics e.g. a Science teacher may not understand every aspect of geometry and may need to refer to the notes provided for assistance. These presentations can also be used during lessons if desired.

These lesson guides are based on a curriculum model called ‘Authentic Integration’ which has been developed through extensive research into the area of integrating Mathematics and Science. It is based on four characteristics which are explained in detail within this pack. These characteristics will form the foundation for each lesson, giving the teacher a clear pathway for applying best practice when it comes to integrating Mathematics and Science.

Lesson *guides* are provided rather than lesson *plans*. These guides were deliberately designed in such a way as to outline a lesson which would engender all the characteristics of Authentic Integration and outline certain expectations and outcomes which should be achieved within the lesson. But the design also allows for adjustment by the teacher as regards time, level of difficulty, layout of classroom, discussion topics and length, and the order in which sections of the lesson are completed. The aim is to strike a balance between satisfying the needs of integration (according to research) and allowing teachers to assume ownership of a lesson which they can mould to suit the needs of their pupils. Hence the reason they are referred to as guides rather than plans which would be more specific in nature.

Information regarding how this intervention will be implemented and how to use this resource are contained in this resource pack. Páraic Treacy will also be available at any time to answer any questions or deal with any issues if required:

e-mail: paraic.treacy@ul.ie
How will this intervention be conducted?
Plan for Intervention

“An Investigation into the Integration of Mathematics and Science at Junior Cycle in Irish Post-Primary Schools”
By Páraic Treacy

Manner in which it will be taught

- Active or Experiential learning would form the basis of each lesson i.e. through practical work.
- Pupils will brainstorm individually, in groups and/or in a whole class manner, as to what approach they will take.
- Teacher will set a problem, a task or an experiment to be carried out.

Plan will be set out, equipment required gathered and practical work will be completed.

Each pupil will reflect on results of their work, the Mathematics and Science involved, and then present it in a form which best describes the work carried out e.g. poster, presentation, journal, report, speech, etc.

Each pupil will complete a written report (about a page long) as a homework exercise detailing the methods they applied and why they applied them within the task they carried out.

Manner in which it will be taught

- Discussion will form a central part of the lesson as a means of assessment.
- These discussions will be conducted in a whole class, group, or one-to-one fashion.
- Questioning and discussion should typically occur before, during and after tasks are completed.

Who will teach these lessons?

- For the most part, due to timetable issues, one teacher will teach each lesson.
- This may be the Science or the Mathematics teacher.
- The Resource Pack will explain in detail both the Mathematics and Science elements of each lesson.
- Páraic Treacy will also be available to provide training.

General Queries

- Manner in which it will be taught?
- Who will teach it?
- How are the lessons constructed?
- Pupils’ role in each lesson?
- When will it take place and for how long?
- Number of Lessons?
- Equipment needed?
- What type of data will be gathered?
These lessons are designed for pupils to implement the knowledge they have in various contexts thus...

The lessons will be positioned at the end of a block of teaching.

In this situation, topics which have been taught in Mathematics and Science in the previous weeks and months would be integrated within these lessons to improve appreciation and understanding of these topics.

The lessons are based around a central theme which incorporates elements of both Mathematics and Science.

Each lesson will involve a task, a problem or a project to be completed/solved.

Pupils will be given information which they need to organise, interpret and synthesize with their own knowledge so as to apply it in the given context.

As stated previously, the result will be in the form of a poster, presentation, journal, report, speech, etc. – whichever suits the results derived.

It is imperative that both subjects are catered for within the lesson. If the teacher feels the task was overly focussed on the Mathematics element then he/she could use the discussion period of the lesson to redress the balance by focussing on the Science element and vice versa.

The pupils will be central to the lesson. It is up to them to interpret the information, form a plan to complete the task set, carry out the plan and produce the results.

The teacher will be purely facilitating, offering hints and tips as they progress.

Pupils will normally work in groups (3 would be the ideal size).

Each group’s work should usually produce different results i.e. groups shouldn’t just copy each other but come up with their own approach.

The intervention would take about 2 weeks.

This will be discussed with each participating teacher and will vary depending on their circumstances.

When will they teach these lessons?

How are the lessons constructed?

Pupils’ role in each lesson?

When will it take place and for how long?
At least 3 lessons per participating class group. These would be positioned at the end of a block of teaching i.e. at the end of a half term or full term.

Within this block of teaching, the topics to be integrated will have been taught in the normal manner at some point.

Also 1 class may not be enough for each lesson i.e. it may, and probably will in some cases, take 2 timetable slots to complete.

Data gathered would come from questionnaires, interviews and focus groups involving both teachers and pupils. As well as written work produced by the pupils.

The Pupils will complete a questionnaire before the intervention begins and upon it’s completion. It will take no more than 5 minutes to complete on each occasion.

Teachers will be asked to participate in an interview at the end of the intervention. This should take approximately 10 minutes to complete.

Teachers will also be asked to complete a questionnaire at the end of the intervention. This will only take 2 minutes and will not involve any writing of long responses, just circling answers.

Pupil work to be collected will be the work they complete in class during the integrated Maths-Science lessons and the homework assignments linked to these lessons. Each pupil will be provided with a copy for this work.

Focus groups for the pupils will be conducted at the end of the intervention.

This will require one small group of pupils (about 5) from each participating class group to answer questions and discuss topics relevant to the lessons they experienced. This should take 10-15 minutes to complete.

Very little equipment will be needed. This was the intention as labs would not be available to Mathematics teachers and it’s best to keep it simple anyway.

Basic items like a weighing scales, resources to create a poster, food diaries, etc. will be required depending on the lesson and will be supplied to the teachers if they can’t acquire the equipment themselves. An overhead projector (for powerpoint presentations and videos) would be beneficial but not necessary.

A copy will be provided for each pupil to record their work.

For further information or discussion on any issue at any time contact Páraic Treacy:

E-mail: paraic.treacy@ul.ie
How to use this Resource Pack
How to use this Resource Pack

- This resource aids in the integration of Mathematics and Science at Junior Cycle. The aim of this resource is to outline the structure of lessons of this nature and provide the requisite teaching aids to support implementation of these lessons.

- Each lesson has a Mathematics element and a Science element hence there are powerpoint presentations for each element of each lesson to ensure that there are no gaps in knowledge. These can be found in the ‘Mathematics Resources’ and the ‘Science Resources’ chapters in this resource pack.

- These presentations can be used during lessons or as a personal resource for a teacher who is unfamiliar with either the Mathematics or Science element involved in the lesson they wish to teach. How these presentations are used is totally up to the teacher.

- The lesson guides outline the activity to be undertaken in class, the Mathematics and Science objectives, key information and the practices which should be visible in each pupil’s work in class.

- The lesson guides also outline how each element of the ‘Authentic Integration’ model is satisfied. This is usually on the second page of the lesson plan and is inscribed in a triangle.

- ‘Authentic Integration’ is the model being implemented through this intervention; it is a model that has been designed specifically to cater for the integration of Mathematics and Science. The description of ‘Authentic Integration’, which is at the beginning of
this resource pack, should be referred to if the reader is not familiar with the model or its graphic representation, the ‘Authentic Integration Triangle’

➤ The lesson guides leave room for the teacher to adjust them to fit the resources they have, the time available to them, and the needs of their pupils. The main requirement is that all the elements of ‘Authentic Integration’ are satisfied during the lesson. Further information on how to implement these lessons is available in the chapter entitled: ‘How will this intervention be conducted?’

➤ There are also further ideas and progressions which can be implemented if the teacher wishes to expand further on the topic(s) being explored or if they feel they need to offer a greater challenge to some or all the pupils in their class.
Authentic Integration

Rich Tasks

Applicable to Real World Scenarios

Knowledge Development, Synthesis and Application

Focused Inquiry Resulting in Higher Order Learning
Authentic Integration

1. Knowledge Development, Synthesis and Application

Pupils will draw on their previous knowledge (in both Mathematics and Science), assimilate this knowledge and develop it through application to various challenging tasks and scenarios. In other words – they will improve and increase their knowledge in Mathematics and Science (knowledge development) then combine what they have learned in both disciplines (synthesis) so that they may apply it to solve a problem or complete a set task (application). It is vital that pupils identify the connections between the skills and information they have obtained in both disciplines.

2. Focused Inquiry Resulting in Higher-Order Learning

Pupils will achieve higher-order learning through inquiry into the elements of the topic(s) being studied in a focussed manner. Pupils should engage in substantial conversation with their teacher and their peers to build a shared understanding. Pupils must display advanced learning through:

- Demonstration of their knowledge of the core elements and concepts being studied using the language and procedures relevant to Mathematics and Science

- Coherent approaches to tasks with presentations of results which suit the data obtained and conclusions drawn.
3. Applicable to Real World Scenarios

Skills and knowledge obtained, developed and displayed should be useful outside the classroom i.e. in situations which may occur in everyday life. This may pertain to typical domestic tasks, hobbies or tasks to be carried out in further education and/or future jobs.

4. Rich Tasks

Problem based tasks must be of an active or experiential nature with pupils central to the planning, implementation and conclusions drawn. The teacher should act as a facilitator offering hints, tips and advice when necessary as well as ensuring that the work carried out is of a high standard.

Overview

The framework caters for the specific needs of integration derived from research. The triangle structure displays clearly the characteristics of the framework and their importance: “Knowledge Development, Synthesis and Application” and “Focused Inquiry Resulting in Higher Order Learning” are of utmost importance thus they form the foundation; “Applicable to Real World Scenarios” supplements and complements the previous two characteristics; while “Rich Tasks” is the overall product of the previous elements.
Lesson Guides
Nutrition Analysis
**Summary:** Pupils will record their food and drink intake and exercise for a week, deduce the levels of carbohydrates, fats, calories that they consume and compare their intake to the recommended intake using graphs, pie charts and bar charts. Note: take into account potential for embarrassment to pupils during this lesson and adjust accordingly.
Nutrition Analysis

**Task Outline**

- Pupils will keep a diary of the foods and drinks they consume in a week as well as hours of exercise.

- These diaries will be analysed and data on calories consumed/Fat intake/carbohydrate intake/protein intake will be extracted.

- Pupils, in groups of 3 or 4, will analyse one (more than one if desirable) particular aspect and create graphs and charts on the data which they think are relevant to displaying the data.

- Pupils will discuss the shape of the graphs/charts shown and interpret them. Suggestions will be offered pertaining to the improvements required to ensure a healthier lifestyle and the reasons for these suggestions will be discussed also.

**Evaluation**

- Pupils should create neat, accurate graphs and charts with a written explanation of each.

- Pupils will be evaluated on the accuracy and relevance of the graphs as well as interpreting their shape accurately.

- Conclusions drawn must be supported by evidence and include the relevant vocabulary.

- Results may be represented in the form of a poster, a written report, powerpoint presentation, or a verbal report.

**Pupils will actively record, analyse and display their diet using graphs and charts.**

Gaining in-depth knowledge in regards to their diet will aid pupils in making critical decisions to improve their health and overall well-being throughout their lives.

The pupils will use their prior knowledge of constructing graphs and charts to determine the best way(s) to display the data they have gathered thereby developing a deeper knowledge of the uses and construction of various charts and graphs.

The pupils will deepen their knowledge of the construction and uses for graphs and charts as well as the importance of a balanced diet. Knowledge of the functions of certain foods must be displayed within work produced and must be evident in the conclusions they arrive at.
Nutrition Analysis

Questions to be asked / Discussion Topics

- Why is a balanced diet important?
- What are calories for? Why can’t you consume too many of them?
- Are all fats bad? What is fat used for?
- What kind of graphs/charts are there? Why would you use them?
- Is exercise important? Why?
- How many hours of exercise a day/week should you complete?
- What does this graph/chart tell you?
- Can you think of another way of representing this data?
- What changes would you make to your diet to improve your overall health?

Resources:

- http://raisingchildren.net.au/
- http://www.keepkidshealthy.com/nutrition/
- http://www.littlesteps.eu/?/home/

Ideas and Progressions

- Analyse food packaging in greater detail e.g. the percentages for fat, calories, sugars, etc. displayed on the wrappers/boxes. Breakfast cereals are great resources in this instance.

- Discuss the effects additives and preservatives have, and how they can be identified in the ingredients written on the packaging. Some research links them to the development of cancer as well as contributing to obesity. Read more here: http://www.herdaily.com/health/892/the-effects-of-food-additives-and-preservatives.html

- Link to Coursework A in Junior Certificate Science: Mandatory Investigations and experiments such as OB3 and OB5.

- Discuss scenarios where diets must be adjusted to suit the everyday needs of a person. A good example would be Olympic swimmer Michael Phelps’ diet during training and competition. His diet was tailored to his needs and, as such, he had a daily intake of 10,000 calories. Read more here: http://news.bbc.co.uk/2/hi/7562840.stm
Motion Graphs
Motion Graphs

Key Information

The slope of a time-distance curve over a given time indicates the speed of the object over that period of time.

The slope of a time-velocity curve over a given time indicates the acceleration of the object over that period of time.

Measurements:

- Distance – metres (m)
- Time – seconds (s)
- Speed – metres/second (m/s)
- Acceleration – metres/second² (m/s²)

Science Concept:

- Distance, Speed, Acceleration

Objectives:

The pupils will improve their understanding of the relationship between speed, distance and time.

The pupils will practice gathering specific data in an accurate manner.

Vocabulary:

- Displacement, acceleration, data.

Mathematics Concept:

- Graphs - Plotting and interpreting

Objectives:

The pupils will form a greater appreciation for the value of graphs.

The pupils will develop their ability to interpret graphs by applying algorithms to find the slope of the curve.

Vocabulary:

- Slope, curve, graph, axis, plot.

Targeted Practices for Pupils:

- Analyse sprint performance according to the graph and indicate where improvements could be made.

- Explain changes in the graph e.g. change in slope.

- Indicate how these graphs can be used in real life and other applications of such graphs.

- Design an approach which would elicit a more accurate curve.

- Peer Learning: work as a group to ensure each member develops their understanding of the content.

Summary: Pupils will gather data on a sprint performed by another pupil. In their groups, they will compile the data, create a time-distance and/or a time velocity graph which they will analyse for findings to be presented in the form of a poster/verbal presentation/written report.
Motion Graphs

Task Outline
- Groups of 4.
- One designated pupil in each group performs sprint.
- The subject will sprint 30 metres from a standing position.
- He/She will be timed at 10 metre intervals.
- This data will be recorded and used to form a time-distance graph and/or a time-velocity graph in the classroom and presented in the form of a poster/verbal presentation/written report.
- Length and number of time intervals can be adjusted to suit.

Evaluation
Pupils work in their groups to create Time-Distance and/or Time-Velocity graphs of the data gathered.

Pupils will be assessed on their ability to explain the shape of each graph, especially the points where significant change takes place in the direction or slope of the curve.

Pupils will be assessed on their ability to accurately measure the slope of the time-distance curve to determine speed of the pupil at different sections of the sprint.

Pupils will be assessed in relation to the quality of their presentation of their findings.
Motion Graphs

Questions to be asked / Discussion Topics:

- Where have you seen graphs used?
- Are graphs effective? In what way?
- Why did the graph change slope/direction at this point?
- What actually happened at this point in time during the experiment?
- How could you make this graph more accurate?
- How could sports people use graphs to help them?
- Can you estimate the subject’s velocity after x seconds?

Resources:

- [http://www.physicsclassroom.com/class/1dkin/u1l4a.cfm](http://www.physicsclassroom.com/class/1dkin/u1l4a.cfm)
- [http://www.racemath.info/motionandenergy/velocity_time_graph.htm](http://www.racemath.info/motionandenergy/velocity_time_graph.htm)
- [http://www.antonine-education.co.uk/New_items/HFS/Running.htm](http://www.antonine-education.co.uk/New_items/HFS/Running.htm)

Ideas and Progressions

- The 30 m run could involve 10 m walk, 10 m sprint followed by a 10 m jog for one or a number of the groups – this would show up noticeably on the subsequent graph and would indicate to pupils clearly how the slope changes according to speed in a time-distance graph.

- An easy progression would be to create a time-velocity graph based on the same data. In this instance, the slope of the curve would represent the acceleration at a given time.

- A discussion on how the results obtained could affect subsequent training could take place. For instance, if the time taken to reach top speed is too long then the sprinter might work on acceleration or if their speed reduces towards the end then they might practice running slightly longer distances to improve their stamina.
Weight on Different Planets
Weight on Different Planets

Key Information

\[ w = mg \]

Acceleration due to gravity = 9.8 m/s²

Gravity on planets is affected by the mass of the planet and the radius of the planet.

Ratio is the relation between two similar magnitudes with respect to the number of times the first contains the second.

Gravitation, or gravity, is the means by which objects with mass attract one another.

Science Concept:

Forces - Gravity

Objectives:

The pupils will discuss the implications of a change of gravity on their lives.

Pupils will understand how gravity is a force which is constantly acting on them and on the planets of the solar system.

Vocabulary:

Mass, weight, gravity, force, Newtons, acceleration.

Mathematics Concept:

Ratio and Proportion

Objectives:

Pupils will apply their knowledge of ratio to an unfamiliar scenario and use the information obtained to describe the difference between a person’s weight on one planet compared to another.

Vocabulary:

Ratio, proportion, comparison.

Targeted Practices for Pupils:

- Obtaining accurate calculations using correct algorithm and order of operations.
- Determining and discussing the elements which affect the gravity on Earth and other planets.
- Creating ratios of weight on Earth to weight on other planets in the form \( 1 : n \).
- Peer learning: Explain to others orally and in writing.
- Appreciating how mathematics can be used to describe various scenarios.

Summary: Pupils, in groups of 3 or 4, will measure the mass of one of the group, determine their weight and find their weight on other planets and the moon. They will then find the ratio of the pupil’s weight on Earth to their weight on other planets while discussing the factors which affect this.
Weight on Different Planets

Task Outline:

Pupils measure one pupil’s mass in kg using weighing scales and use this info to calculate that pupil’s weight on Earth and on the moon.

Pupils create a ratio of the pupil’s weight on Earth to their weight on the moon.

Pupils progress by finding the pupil’s weight on 3 other planets of their choice and find the ratio of their weight on Earth to their weight on those planets.

Pupils will work together to create ratios in unfamiliar contexts thereby deepening their understanding of the process.

Pupils discuss how gravity affects their lives and how a change in gravity would alter their regular activities.

Creating ratios of a person's weight on Earth to their weight on other planets.

Ratios are commonly applied in statistics e.g. sports.

Basic knowledge of gravity gained is important in helping to understand everyday concepts such as movement of objects e.g. ball flight.

Pupils will routinely apply the given formula to determine the weight of a student on various planets. They will also apply their knowledge in unfamiliar scenario i.e. creating ratios of the pupil's weight on Earth to their weight on other planets.

Evalutation:

Pupils will be assessed on their ability to apply algorithms correctly when determining weight and the ratios of weight on Earth to weight on other planets.

Pupils’ knowledge and intuitiveness should be evaluated through one-to-one, group and/or whole class discussions on the factors which affect gravity e.g. mass of the planet, radius of the planet.

Pupils’ understanding of the effects gravity has should be determined through discussion on the flight of objects and how a reduction or increase in gravity would affect this.
Weight on Different Planets

Questions to be asked / Discussion Topics:

- What is gravity? Gravitation, or gravity, is one of the four fundamental interactions of nature, and is the means by which objects with mass attract one another (Wiki).
- If there was lower/higher level of gravity how would that affect you?
- Would you need to be stronger to do normal things like walking and running?
- How would it affect you in sports?
- Could you hit or kick the ball longer with less gravity?
- Look at the formula for determining the strength of gravity on a planet:
  \[ g = \frac{GM}{r^2} \]
  What factors do you think affect the level of gravity?
- If the radius of a planet was bigger would this increase the strength of gravity?
  Why/why not? Can you prove that?

Sources of Information:

- [http://www.exploratorium.edu/ronh/weight/index.html](http://www.exploratorium.edu/ronh/weight/index.html)
- [http://www.aerospaceweb.org/question/astronomy/q0227.shtml](http://www.aerospaceweb.org/question/astronomy/q0227.shtml)

Ideas and Progressions

- Discuss how gravity impacts on their everyday activities – ask them to outline a scenario where a change in the strength of gravity would affect them positively/negatively.
- Allow them to determine the factors which affect the strength of gravity on a planet. Hint that the formula may provide some clues.
- Compare, with the aid of the pupils, the Earth to other planets with similar levels of gravity (e.g. Venus and Uranus). Mention the mass and radius of these planets and how they affect overall gravity.
- Challenge pupils to come up with other uses for ratio to compare various characteristics.
<table>
<thead>
<tr>
<th>Body</th>
<th>Mass [kg]</th>
<th>Radius [m]</th>
<th>Acceleration due to gravity [m/s²]</th>
<th>g / g-Earth</th>
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<tr>
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<td>$14.07$</td>
<td>1.43</td>
</tr>
</tbody>
</table>
Periscope Lesson
Periscope Lesson

**Key Information**

Light travels in straight lines.

The normal is an imaginary line perpendicular to the surface of an optical medium (e.g. mirror).

In reflection of light, the angle of incidence is the angle between the normal and the incident ray.

The angle of reflection is the angle between the normal and the reflected ray.

When a ray of light bounces off a smooth surface such as a mirror: the angle of incidence equals the angle of reflection.

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**Science Concept:**

Optics – Angle of incidence and reflection

**Objectives:**

The pupils will compound their understanding of the characteristics of lights i.e. the relationship between the angle of incidence and angle of reflection as well as the fact that light travels in straight lines.

**Vocabulary:**

Angle of incidence, angle of reflection, normal, optics.

---

**Mathematics Concept:**

Geometry – measuring angles

**Objectives:**

The pupils will develop an appreciation for the importance of recognizing the mathematical accuracy which must be applied to ensure that the periscope works.

Pupils will learn to compute and measure angles accurately.

**Vocabulary:**

Angles, degrees, opposite angles, complimentary angles.

---

**Targeted Practices for Pupils:**

Explain to others orally and in writing.

Calculate angles accurately.

Justify orally and in writing conclusions reached.

Neat exact drawing of path that light takes as it travels through the periscope.

Appreciate the need to be exact in calculations and positioning of mirrors within the periscope.

Recognise the important role that geometry plays in optics.

---

**Summary:** The pupils, in groups of 3 or 4, will make a simple periscope which they will then examine and determine how and why it works. They will also figure out the path the light travels and calculate the angles of incidence and reflection which occur, explaining how they determined these angles.
Periscope Lesson

Task Outline

- Groups of 3
- Each group gathers required materials and the plan for making a periscope.
- Each group member inspects the periscope.
- Groups work together to determine how the periscope works and explain it using terms such as angle of incidence, angle of reflection, normal, etc.
- Groups figure out the path of the light through the periscope and calculate the angles involved.

Making a periscope and examining how it works.

These laws are applied to fibre optics which are used to connect to the internet.

Pupils will develop their knowledge of the characteristics of light and optics as well as increasing their ability to measure angles and apply knowledge gained previously e.g. perpendicular lines; right angles; equilateral triangles.

Pupils will study the periscope to determine how it works and outline this in detail, both orally and in writing. Pupils will be encouraged to explore the angles involved and how an inaccuracy could affect the reliability of the periscope.

Evaluation

Assess their knowledge by discussing with pupils, in their groups or as a class, how light travels through the periscope and why accuracy of the angles involved is important.

Evaluate pupils based on their understanding when explaining orally and in writing, use of the relevant terms, quality of the periscope made, and accuracy of calculations when determining angles involved.
Questions to be asked / Discussion Topics:

- Why is the mirror at that angle? How does that affect what you would see through the periscope?

- Have you heard of the angle of incidence? What does that mean?

- Have you heard of the angle of reflection? What does that mean?

- What’s the relationship between the angle of incidence and the angle of reflection?

- What’s the angle of incidence and reflection of a beam of light on the mirrors in the periscope? Why is it 45º?

- If you were to change the angle of the mirror, how would that affect how the periscope works?

- Change the angle of the mirror on your periscope… what happened?

Ideas and Progressions

- Change the angle of one of the mirrors and discuss, using drawings of the path of the light, how this affects what is seen through the periscope.

- Discuss other devices which use reflection of light e.g. optical fibres and rearview mirrors in cars.

- Relate to Junior Certificate Science Coursework A: Reference OP38.
Household Electricity Lesson
Household Electricity Lesson

Key Information

Science Concept:
Electricity

Objectives:
The pupils will develop an appreciation for the importance of recognising values such as watts, kilowatt-hours, and voltage, and their effect on electricity consumed and the price of bills.

Vocabulary:
watts, kilowatt-hours, voltage, energy rating.

Mathematics Concept:
Mixed Operations & Percentages

Objectives:
The pupils will develop their ability to compute values using a range of algorithms and operations (addition, subtraction, multiplication, division, finding percentages etc.)

Vocabulary:
Percentages, VAT, levy, fractions

Targeted Practices for Pupils:

- Analysing bills to recognise the procedure involved in computing the final cost.
- Explain why certain appliances use more energy than others.
- Discuss ways of reducing the energy used.
- Sequence the mathematical steps in the correct order and explain why it has to be in this order.
- Peer Learning: ensure each member of the group understands each element of the process.

Summary: Pupils investigate and apply the methods for computing electricity usage and cost by completing calculations for electricity usage in a typical home. The pupils will figure out the cost of running each appliance, add up these amounts to find the total and factor in VAT to compute the final bill.
Household Electricity Lesson

Task Outline:

- Groups of 3 or 4
- Two scenarios of electricity usage in the house (i.e. the number of kWh used by each appliance)
- Pupils work out the final bill, taking into account VAT, standing charges and levys.
- Pupils will outline how they calculated the final bill and will provide recommendations that will reduce the cost of the bill.

Pupils will develop their knowledge of energy ratings of household appliances and their ability to perform basic calculations. They will develop their ability to find the cost of running appliances and work out the total cost of typical electricity bills.

Pupils’ knowledge of which appliances use the most electricity will be deepened along with algorithms for finding percentages of various values.

Calculating Household Electricity Bill

Basic calculations and finding percentages are regular tasks in everyday life that will be enhanced through this work.

Understanding and being able to calculate household bills is important for any independent member of society as it is vital when identifying areas in which money can be saved.

Evaluation:

Pupils will be evaluated based on their ability to identify the correct method to be used in each given aspect of calculating the final cost of the bill.

Pupils will also be evaluated based on the quality of presentation of their findings and the recommendations they make at the end i.e. the imagination and relevance of the approaches suggested.

Assess pupils on their ability to work co-operatively and aid each other in understanding the processes involved.
Household Electricity Lesson

Questions to be asked / Discussion Topics:

- What is electrical energy measured in?
- How do you know what appliances use the most electrical energy?
- How would you calculate the number of kilowatt-hours an appliance uses in a month?
- What is VAT? How is it calculated?
- How did they find that percentage on the bill?
- Do you know of any energy saving methods?

Resources:

- [http://www.esb.ie/esbcustomersupply/business/manage-your-account/understand-your-bill.jsp](http://www.esb.ie/esbcustomersupply/business/manage-your-account/understand-your-bill.jsp)

Ideas and Progressions

- The area of energy conversion could be discussed e.g. the process of creating electricity by burning fossil fuels which is the conversion of chemical energy to electric energy. Another example would be a dam: gravitational potential energy to kinetic energy of moving water (and the blades of a turbine) and ultimately to electric energy through an electric generator.

- Discuss the difference between renewable and non-renewable energy and how each one is harvested for household use. Pros and cons of each could also be discussed.

- If the pupils need to be challenged further then introduce additional elements of a bill such as payment outstanding from the previous bill or how a change in the rate of VAT would affect the bill.
Speed of Light

V's

Speed of Sound
**Summary:** Pupils will be asked to estimate the time difference between when a flash of lightning is seen and when it is heard. They will calculate this difference for storms that are 5 km, 4 km, 3 km, 2 km and 1 km away from the observer. They will then analyse the results to determine how they could estimate the distance a lightning storm is from their current position.
Calculate time difference between when a flash of lightning is seen and when the corresponding clap of thunder is heard.

At a concert or sporting event, people furthest away will see the actions before they hear the sound from the actions. Estimating the distance a storm is from a person’s position.

Pupils will apply their knowledge of calculating speed, distance and time to this task. They will organise the data they have to form a rule of thumb for estimating the distance a lightning storm is from them.

Accurate calculations must be derived. The units of measurement must also be consistent to ensure accurate results.

Task Outline

- Pupils will work together in groups of two or three.
- They will be given the speed of light and the speed of sound and asked to figure out how to calculate how much time it takes to see a flash of lightning and hear a clap of thunder from a given distance.
- They will figure this out for distances of 5 km, 4 km, 3 km, 2 km and 1 km.
- When complete, they will analyse their results to see if there is any pattern and, if so, how can they apply this pattern in the future?

Evaluation

- Pupils must be exact in their calculations.
- Pupils should recognise the need to maintain a consistent use of units e.g. convert all speeds to either m/s or km/s and all distances to metres or km.
- Evaluate their ability to work as a group and explain concepts to fellow group members.
- Pupils should be able to explain what scenarios could be influenced by a difference between the speed of light and the speed of sound.
Speed of Light V’s Speed of Sound

Questions to be asked / Discussion Topics:

• Is there much of a difference between the speed of light and the speed of sound?

• In what scenarios would this have an effect?

• How would you calculate the time taken for a sound to travel if you knew the speed of the sound and the distance it travelled?

• The speed of sound is 340 m/s, what is that in km/h?

• Why is it important to keep the same units for speed, the same units for time and the same units for distance during your calculations?

• How would the difference between when actions are seen and when they are heard affect enjoyment of a sporting event or a concert if you are situated a large distance from the action?

Ideas and Progressions

• Refer to the scene in The Shawshank Redemption where Andy Dufresne is trying to break open a sewage pipe to escape through. He sees the lightning but knows he must wait a little for the corresponding thunder clap which will drown out the sound of him hitting the pipe. Watch here: http://www.youtube.com/watch?v=SheaMMd8H5g

• Discuss how the difference between when actions are seen and when they are heard could affect enjoyment of a sporting event or a concert if you are situated a large distance from the action.

• Discuss the breaking of the sound barrier on October 14, 1947, by Charles E. “Chuck” Yeager in the air and by Richard Noble on 15 October 1997 on the ground. Ask why the speed of light will not be surpassed any time soon.
Mathematics Resources
Basic Percentages

To turn any percentage into a decimal look at the following:

\[ 66\% = \frac{66}{100} = 0.66 \]

To divide by 100 move the point 2 places to the left.

Now change the percentages below to decimals.

(a) \( 87\% = \frac{87}{100} = 0.87 \)
(b) \( 24\% = \frac{24}{100} = 0.24 \)
(c) \( 12\% = \frac{12}{100} = 0.12 \)
(d) \( 4\% = \frac{4}{100} = 0.04 \)

Finding A Percentage Using A Decimal

Q. Calculate 47% of €670

Solution:

47% of €670 = 47% x €670 = 0.47 x €670 = €314.90

Finding A Percentage Using A Decimal

Q. Calculate 7% of 2345 people.

Solution:

7% of 2345 = 7% x 2345 = 0.07 x 2345 = 164.15

164 rounded to the nearest person.

Finding A Percentage Using A Decimal

Q. In a sale 12\% is taken off a ski jacket whose normal price is €145. Find the sale price.

Solution

Cost of ski jacket:

12\% of 145 = €145 - €18.13 = €126.87

12\% x 145 = 0.125 x 145 = €18.125

€18.13 rounded to the nearest cent

Making Calculations Faster

In the last problem 12\% was taken off the price. What percentage of the price did the customer pay?

100 - 12\% = 87\% %

Now calculate 87\% of €145

0.875 x €145 = €126.875

€126.88 rounded to the nearest cent
Percentages

There are six basic types of percentage problems:
- Creating a percentage
- Percentage of
- Percentage change
- Percent more than
- Reverse percentage change
- And one simple ratio:
  - Times more than

1. Creating a Percentage

- In Chicago in 2010 there were approx 1.053 million African Americans, 907,000 whites, 754,000 Hispanics, and 181,000 others. What percentage of Chicagoans were of Hispanic origin?
- Remember: part over whole
- 754,000 / (1,053,000 + 907,000 + 754,000 + 181,000)
- = 0.26 or 26%

2. Percentage Of

- 54% of UL’s student body is female. UL has approx 14,000 students. Approximate how many females attend UL.
- Multiply total by percentage
- 14,000 x 0.54 = 7,560 female students in UL.

3. Percentage Change

- You are examining one thing and how it has changed over time
- Greater Dublin’s population is currently 1.6 million people. It is predicted that the population will grow to 2.1 million people by 2020. By how many percent will it grow?
- Divide difference by original value
  \[
  \frac{(2.1 - 1.6)}{1.6} = 0.3125 = 31.25\%
  \]
3. Percentage Change

- In 2001, 94,007 tons of waste were recycled in Dublin. In 2010, 296,363 tons were recycled. By how many percent did it increase?
- Divide difference by original value
  
  \[
  \frac{(296,363 - 94,007)}{94,007} = 2.15 = 215\%
  \]

4. Percentage More Than

- You are comparing two different examples of something
- The life expectancy in Ireland is 79.5 years while it is 76.0 years in the US. By how many percent is the life expectancy in Ireland higher than in the US?
- Divide difference by latter value stated in problem
  
  \[
  \frac{(79.5 - 76.0)}{76.0} = 0.046 = 4.6\%
  \]

4. Percentage More Than

- The dropout rate of school X is 23.5% while the dropout rate of school Y is 18.2%. By how many percent is the dropout rate of school X higher than the dropout rate of school Y?
- Divide difference by latter value
  
  \[
  \frac{(23.5 - 18.2)}{18.2} = 0.29 = 29\%
  \]

4. Percentage More Than

- Ireland has 3.2 cars per family
- Poland has 1.4 cars per family
- By how many percent is the average cars per family for Ireland greater than Poland?
  
  \[
  \frac{(3.2 - 1.4)}{1.4} = 1.29 = 129\%
  \]

5. Reverse Percentage Change

- According to the official 2007 census, the Polish population of Ireland was 450,000. It rose by 57.89% from 2004 to 2007. What was the Hispanic population in 2004?
- \( \text{old} \times (1 + \% \text{change}) = \text{new} \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( P \times (1 + 0.579) = 450,000 )</td>
<td>( P \times 1.579 = 450,000 )</td>
<td>( P = 450,000 \times 1.579 )</td>
</tr>
<tr>
<td>( P \times 1.579 = 450,000 )</td>
<td>( P = 285,008 )</td>
<td></td>
</tr>
</tbody>
</table>

5. Reverse Percentage Change

- You just spent €46.23 for a new pair of jeans. This was the total price with sales tax. You know that sales tax was 7.5%. What was the price of the jeans before the tax was added?
  
  \[
  P \times (1 + 0.075) = 46.23 \\
  P \times 1.075 = 46.23 \\
  P = 46.23 \div 1.075 = €43.00
  \]
- The price of the jeans before tax was €43
5. Reverse Percentage Change

- The cost of tuition dropped by 14% over the last year at a local boarding school.
- This year tuition costs €3,000.
- What was the cost of tuition last year?

\[ P \times (1 - 0.14) = 3,000 \]
\[ P \times 0.86 = 3,000 \]
\[ P = €3,488 \]

Times More Than

- The life expectancy in Ireland is 79.5 years while it is 76.0 years in the U.S. How many times more is the life expectancy of Ireland higher than the life expectancy in the U.S.?

Divide former by latter

\[ 79.5 \div 76.0 = 1.046 \]

- The dropout rate of school X is 23.5% while the dropout rate of school Y is 18.2%.
- How many times is the dropout rate of school X higher than the dropout rate of school Y?

Divide former by latter

\[ 23.5 \div 18.2 = 1.29 \]
In Mathematics, a straight line is defined as having infinite length and no width.

When a line has end points we say that it has finite length. It is called a line segment. We usually label the end points with capital letters. For example, this line segment has end points A and B. We can call this line ‘line segment AB’.

When two lines meet at a point an angle is formed. An angle is a measure of the rotation of one of the line segments relative to the other. The angle can then be described as $\angle ABC$.

A flat two-dimensional surface is called a plane. Any two straight lines in a plane either intersect once or are parallel.
**Lines in a plane**
- ... or they are parallel.
- **Parallel lines** will never meet. They stay an equal distance apart.
- They are always **equidistant**.

**Perpendicular lines**
- Each angle is 90°. We show this with a small square in each corner.

**Angles**
- Angles are measured in **degrees**.
  - A quarter turn measures 90°. It is called a **right angle**.
  - We label a right angle with a small square.

**Angles**
- Angles are measured in **degrees**.
  - A half turn measures 180°. This is a straight line.

**Angles**
- Angles are measured in **degrees**.
  - A three-quarter turn measures 270°.

**Angles**
- Angles are measured in **degrees**.
  - A full turn measures 360°.
Facts Relating to Angles

Contents:

- Angles between intersecting lines
- Angles on a straight line
- Angles around a point

Vertically opposite angles

When two lines intersect, two pairs of **vertically opposite angles** are formed.

\[ a = c \quad \text{and} \quad b = d \]

Vertically opposite angles are equal.

Angles on a straight line

Angles on a line add up to 180°.

\[ a + b = 180^\circ \]

because there are 180° in a half turn i.e. a straight line.

Angles around a point

Angles around a point add up to 360°.

\[ a + b + c + d = 360^\circ \]

because there are 360° in a full turn.

Complementary angles

When two angles add up to 90° they are called **complementary angles**.

\[ a + b = 90^\circ \]

Angle \( a \) and angle \( b \) are complementary angles.

Supplementary angles

When two angles add up to 180° they are called **supplementary angles**.

\[ a + b = 180^\circ \]

Angle \( a \) and angle \( b \) are supplementary angles.
When a straight line crosses two parallel lines eight angles are formed.

Which angles are equal to each other?

There are four pairs of corresponding angles, or F-angles.

\[ a = e \text{ because Corresponding angles are equal} \]

\[ c = g \text{ because Corresponding angles are equal} \]

There are four pairs of corresponding angles, or F-angles.

\[ b = f \text{ because Corresponding angles are equal} \]

There are two pairs of alternate angles, or Z-angles.

\[ d = f \text{ because Alternate angles are equal} \]
There are two pairs of alternate angles, or Z-angles.

Alternate angles are equal

**Alternate angles**

Calculate the size of angle \( a \).

\[
a = 29^\circ + 46^\circ = 75^\circ
\]

**Calculating angles**

**Angles in a triangle**

The angles in a triangle add up to 180°.

**Calculating angles in a triangle**

We can prove that the sum of the angles in a triangle is 180° by drawing a line parallel to one of the sides through the opposite vertex.

These angles are equal because they are alternate angles. Call this angle \( c \).

\[
a + b + c = 180^\circ
\]

The angles \( a, b \) and \( c \) in the triangle also add up to 180°.

**Angles in a triangle**

Calculate the size of the missing angles in each of the following triangles.

**Calculating angles involving parallel lines**
Calculating angles in a triangle

In an isosceles triangle, two of the sides are equal. We indicate the equal sides by drawing dashes on them. The two angles at the bottom of the equal sides are called base angles. The two base angles are also equal. If we are told one angle in an isosceles triangle we can work out the other two.

Angles in an isosceles triangle

For example, find the sizes of the other two angles.

The two unknown angles are equal so call them both $a$.

We can use the fact that the angles in a triangle add up to $180^\circ$ to write an equation.

$88^\circ + a + a = 180^\circ$
$88^\circ + 2a = 180^\circ$
$2a = 92^\circ$
$a = 46^\circ$

Calculating angles in special triangles

Interior angles in triangles

The angles inside a triangle are called interior angles.

The sum of the interior angles of a triangle is $180^\circ$.

Exterior angles in triangles

When we extend the sides of a polygon outside the shape exterior angles are formed.
Any exterior angle in a triangle is equal to the sum of the two opposite interior angles.

\[ a = b + c \]

We can prove this by constructing a line parallel to this side. These alternate angles are equal. These corresponding angles are equal.

### Calculating angles

Calculate the size of the lettered angles in each of the following triangles.

### Calculating angles

Calculate the size of the lettered angles in this diagram.

Base angles in the isosceles triangle = \((180^\circ - 104^\circ) \div 2\)

\[ = 76^\circ \div 2 \]

\[ = 38^\circ \]

Angle \(a = 180^\circ - 56^\circ - 38^\circ = 86^\circ\)

Angle \(b = 180^\circ - 73^\circ - 38^\circ = 69^\circ\)

### Sum of the interior angles in a quadrilateral

What is the sum of the interior angles in a quadrilateral?

We can work this out by dividing the quadrilateral into two triangles.

\[ a + b + c = 180^\circ \quad \text{and} \quad d + e + f = 180^\circ \]

So,

\[ (a + b + c) + (d + e + f) = 360^\circ \]

The sum of the interior angles in a quadrilateral is 360°.

### Sum of interior angles in a polygon

We already know that the sum of the interior angles in any triangle is 180°.

\[ a + b + c = 180^\circ \]

We have just shown that the sum of the interior angles in any quadrilateral is 360°.

\[ a + b + c + d = 360^\circ \]
In an equilateral triangle, every interior angle measures 60°. Every exterior angle measures 120°. The sum of the interior angles is $3 \times 60° = 180°$. The sum of the exterior angles is $3 \times 120° = 360°$.

In a square, every interior angle measures 90°. Every exterior angle measures 90°. The sum of the interior angles is $4 \times 90° = 360°$. The sum of the exterior angles is $4 \times 90° = 360°$.  

---

**Interior and exterior angles in an equilateral triangle**

In an equilateral triangle, every interior angle measures 60°. Every exterior angle measures 120°. The sum of the interior angles is $3 \times 60° = 180°$. The sum of the exterior angles is $3 \times 120° = 360°$.

**Interior and exterior angles in a square**

In a square, every interior angle measures 90°. Every exterior angle measures 90°. The sum of the interior angles is $4 \times 90° = 360°$. The sum of the exterior angles is $4 \times 90° = 360°$. 
Ratio

- A method to compare the relative quantities of 2 or more items.
- e.g. Squash is made using the ratio

\[
\text{Juice} : \text{Water} = 1 : 5
\]

- For every 1 part of juice, we need 5 parts of water.
- We say Juice to Water in the ratio 1 to 5.

Equivalent Ratio

- If we mix weed-killer using Concentrate and Water in the ratio 1 to 20.
- Using 5 measures of concentrate, how many measures of water do we need?

\[
\text{Concentrate} : \text{Water} \quad 1 : 20
\]

\[
5 : 100
\]

\[
\text{So, } 5 \times 5
\]

Example – If the ratio of red beads to black beads is 3 : 5, how many black beads will I need for 21 red beads?

\[
\text{Red} : \text{Black} = 3 : 5
\]

\[
21 : 35
\]

\[
\text{So, } 5 \times 7
\]
Example – Mixing Sand and Cement in the ratio 1 to 3, how much sand will be needed for 12 spades of cement?

Sand : Cement

So, 1 : 3

Multiply by 4

Therefore, 4 spades of sand will be needed.

Like fractions, cancel to simplest form

When you are given the quantity of each item and asked to convert to a ratio

eg. With 4 red balls and 12 blue balls, what is the ratio of red to blue balls?

Red : Blue

4 : 12

Divide both sides by 4

So, 1 : 3

eg. In a theatre there were 400 boys and 600 girls, what is the ratio of girls to boys?

Girls : Boys

600 : 400

Divide by 100

6 : 4

Divide by 2

3 : 2

These can include decimal numbers

eg. Ratio of girls to boys in the form n:1

Girls : Boys

600 : 400

Divide by 100

6 : 4

Divide by 2

3 : 2

e.g. On a pool table there were 6 red balls and 14 yellow balls, give the ratio of red to yellow balls in the form 1:n

Red : Yellow

6 : 14

Divide by 6

1 : 2.3

So, 14 + 6

Splitting Ratio

Share €36 in the ratio 2:7

1- How many parts in ratio?

2 + 7 = 9

2- What is 1 part worth?

9/36 = €4

3- Multiply up ratio

4- Add together to check

€8 + €28 = €36
### Splitting Ratio
- Split 240 tonnes in the ratio 3:5
- 1- How many parts in ratio? $3 + 5 = 8$
- 2- What is 1 part worth? $\frac{240}{8} = 30$
- 3- Multiply up ratio
- 4- Add together to check $\frac{3}{30} : \frac{5}{30} = \frac{90}{90 + 150} = 240$ tonnes

### Scale
- Uses principles of equivalent ratio and cancelling ratio
- e.g. On a plan using a scale of 1cm to 3m, the length of a room measures 4cm, how long is the room?

#### Plan : Actual
- 1cm : 3m
- 4cm : $12m$

### Scale
- If a road of 3km measures 2cm on a map, what is the scale of the map in its simplest form

#### Map : Actual
- 2cm : 3km
- Make units the same $2cm : 300,000cm$
- $1 : 150,000$

### Scale
- If a boardroom table of 3m measures 15cm on a plan, what is the scale of the plan in its simplest form

#### Plan : Actual
- 15cm : 3m
- Make units the same $15cm : 300cm$
- $1 : 20$

### Direct Proportion
- When 2 or more items increase in relation to each other
- e.g. If a recipe for 6 cakes uses 300g of flour, how much flour is needed to make 24 cakes?
- Uses same method as equivalent ratio

#### 6 cakes = 300g
- Multiply by 4
- 24 cakes = 1200g

### Indirect Proportion
- When one item is increasing in proportion to another item decreasing
- e.g. If it takes 1 dog 6 days to eat a box of dried food, how long will it take 2 dogs?

#### 1 dog = 6 days
- Multiply by 2
- 2 dogs = 3 days
- Divide by 2
Science Resources
Electricity is a form of energy. Electricity is electrons flowing in a current. We call this an electrical current.

Electricity is so useful because it can be easily converted to other types of energy.

Take for example, your television set – in this electrical appliance, electrical energy is converted into light energy and sound energy.

The power needed to produce electricity comes from many different sources e.g. Coal, peat, wind and water.

These sources are used to power turbines and generators. These turbines spin around to cause a build up of electrical charge in a generator which makes an electrical current flow.

Turbines are powered by renewable or non-renewable sources of energy.

Non-renewable sources of energy are called fossil fuels and include coal, oil, gas and peat.

They are burned to heat water for steam powered turbines. Burning fossil fuels releases gases which harm environment.

92% of our electricity comes from burning fossil fuels.

Wind, hydro, solar and biomass are all sources of renewable energy.

The blades in a wind turbine turn a generator to produce electricity.

Renewable sources of energy are by far the better option but provide just 6% of the total installed electrical capacity and 2% of all energy required in Ireland.

Energy efficiency means not wasting energy. It refers to appliances, buildings and even to human behaviour!

If you are only making one cup of tea but your boil a full kettle of water, that’s not very efficient is it? Energy is wasted in heating more water than is needed.

Appliances also waste electricity. They always take in more energy than gets put to good use.
### Energy Efficiency

- For example, a television converts electricity into light and sound energy, which is great, but some also gets converted into heat, which is a waster.

- No energy conversion process is 100% efficient but the more efficient the process, the less energy is wasted.

---

### The Kilowatt-Hour

- The kilowatt-hour (symbolized kWh) is a unit of energy equivalent to one kilowatt (1 kW) of power expended for one hour (1 h) of time.

- \( 1 \text{kW} = 1,000 \text{ Watts} \)

- An energy expenditure of 1 kWh represents 3,600,000 joules (\(3.600 \times 10^6\) J).

---

### Example of Electricity Usage

- An Air Conditioner has a Wattage of 1500W or 1.5kW

- It is used for 200 hours in one particular month.

- How many kilowatt-hours (kWh) are used?

  - Solution: \( 1.5 \times 200 = 300 \text{kWh} \)

---

### Example of Electricity Usage

- 300kWh used by the air conditioner in one month

- How much will that cost if 1kWh costs €0.1260?

  - Solution: \( 300 \times 0.1260 = €37.80 \)

- The air conditioner will cost €37.80 to run that particular month.
Your account number

To ask about this bill
call 1850 372 372
Open Mon - Sat, 9am - 8pm

For emergencies or electricity interruptions
call 1850 372 999
Open 24 hours, 7 days a week
Please have this MPRN number ready

MPRN
10 XXX XXX XXX

Of MC Profile

Date of issue 12 Dec XX
Invoice number 12345678

This bill is for
House Name, House address, Town name

Your electricity bill at a glance
Full details of your account are on the back of this bill

Billing period
12 Oct xx to 10 Dec xx 60 days

Reading type
We read your meter

Bill summary
Your last bill €200.00
Payments €200.00 cr
Balance brought forward €0.00
Charges for this period €161.58
VAT €21.81

Total due €183.39
Pay by Direct debit

Your direct debit is due for collection on XXXXXX. Thank you.

Payment terms are 14 days from date of bill issue or immediately if overdue.
Information on the Fuel Mix and environmental impact is on the back of this bill.

Bank Giro Credit Transfer
Allied Irish Banks plc.
3712 Dame Street. Dublin 2
Giro No. 81900087

Ms Customer Name

Date
Notes/Coins €
Total cash €
Cheques etc €
Total amount € 183.39
## Typical Operating Costs of Electric Household Appliances

<table>
<thead>
<tr>
<th>Appliance</th>
<th>Typical Wattage</th>
<th>Estimated Hours Used Per Month</th>
<th>Estimated Monthly kWh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air Conditioner</td>
<td>1500</td>
<td>200.0</td>
<td>300.0</td>
</tr>
<tr>
<td>Air Conditioner</td>
<td>4500</td>
<td>200.0</td>
<td>900.0</td>
</tr>
<tr>
<td>Auto Engine Heater</td>
<td>600</td>
<td>40.0</td>
<td>24.0</td>
</tr>
<tr>
<td>Battery Charger (Car)</td>
<td>150</td>
<td>15.0</td>
<td>2.3</td>
</tr>
<tr>
<td>Blender</td>
<td>385</td>
<td>2.0</td>
<td>.8</td>
</tr>
<tr>
<td>Bug Zapper</td>
<td>40</td>
<td>300.0</td>
<td>12.0</td>
</tr>
<tr>
<td>CD, Tape, Radio, Receiver System</td>
<td>250</td>
<td>60.0</td>
<td>15.0</td>
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<tr>
<td>Clock</td>
<td>3</td>
<td>730.0</td>
<td>2.2</td>
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<td>4.7</td>
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<td>Computer (With Monitor and Printer)</td>
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<td>1500</td>
<td>8.0</td>
<td>12.0</td>
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<td>1500</td>
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<td>7.2</td>
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<tr>
<td>Dehumidifier</td>
<td>450</td>
<td>360.0</td>
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<tr>
<td>Dishwasher (Dry Cycle)</td>
<td>1200</td>
<td>25.0</td>
<td>30.0</td>
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<tr>
<td>Dishwasher (Wash Cycle)</td>
<td>200</td>
<td>25.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Item</td>
<td>Power (W)</td>
<td>Current (A)</td>
<td>Kilowatt Hours (kWh)</td>
</tr>
<tr>
<td>--------------------------------------------------</td>
<td>-----------</td>
<td>-------------</td>
<td>----------------------</td>
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<tr>
<td>Disposal</td>
<td>420</td>
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<td>80</td>
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<td>Freezer (Automatic Defrost)</td>
<td>440</td>
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<td>350</td>
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<tr>
<td>Fry Pan</td>
<td>1200</td>
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<td>10.0</td>
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<td>Heat Lamp</td>
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<tr>
<td>Heat Tape</td>
<td>180</td>
<td>720.0</td>
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<tr>
<td>Heater (Auto Engine, Winter)</td>
<td>1000</td>
<td>180.0</td>
<td>180.0</td>
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<tr>
<td>Heater (Portable)</td>
<td>1500</td>
<td>40.0</td>
<td>60.0</td>
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<td>Heating System</td>
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<td>288.0</td>
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<td>Humidifier (Winter)</td>
<td>177</td>
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<tr>
<td>Iron</td>
<td>1000</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Jacuzzi (Maintain Temperature, 2 Person)</td>
<td>1500</td>
<td>93.0</td>
<td>139.5</td>
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<tr>
<td>Lighting (Incandescent)</td>
<td>75</td>
<td>100.0</td>
<td>7.5</td>
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<tr>
<td>Lighting (Fluorescent)</td>
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<td>100.0</td>
<td>4.0</td>
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<tr>
<td>Lighting (Compact Fluorescent)</td>
<td>18</td>
<td>100.0</td>
<td>1.8</td>
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<tr>
<td>Item</td>
<td>Power (W)</td>
<td>Current (A)</td>
<td>kVA</td>
</tr>
<tr>
<td>-------------------------------------------</td>
<td>-----------</td>
<td>-------------</td>
<td>-------</td>
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<tr>
<td>Lighting (Outdoor Floor)</td>
<td>120</td>
<td>90.0</td>
<td>10.8</td>
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<tr>
<td>Microwave Oven</td>
<td>1500</td>
<td>11.0</td>
<td>16.5</td>
</tr>
<tr>
<td>Mixer, Hand</td>
<td>100</td>
<td>10.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Motor</td>
<td>1000</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>Power Tools (Circular Saw)</td>
<td>1800</td>
<td>1.0</td>
<td>1.8</td>
</tr>
<tr>
<td>Radio</td>
<td>71</td>
<td>101.0</td>
<td>7.2</td>
</tr>
<tr>
<td>Range (Oven)</td>
<td>2660</td>
<td>8.0</td>
<td>21.3</td>
</tr>
<tr>
<td>Range (Self Cleaning Cycle)</td>
<td>2500</td>
<td>3.0</td>
<td>7.5</td>
</tr>
<tr>
<td>Refrigerator/Freezer (Frostfree)</td>
<td>450</td>
<td>333.0</td>
<td>149.9</td>
</tr>
<tr>
<td>Satellite Dish (Includes Receiver)</td>
<td>360</td>
<td>183.0</td>
<td>65.9</td>
</tr>
<tr>
<td>Television (Color, Solid State)</td>
<td>200</td>
<td>183.0</td>
<td>36.6</td>
</tr>
<tr>
<td>Toaster</td>
<td>1400</td>
<td>3.0</td>
<td>4.2</td>
</tr>
<tr>
<td>Vacuum Cleaner</td>
<td>1560</td>
<td>6.0</td>
<td>9.4</td>
</tr>
<tr>
<td>VCR/DVD</td>
<td>21</td>
<td>12.0</td>
<td>2.5</td>
</tr>
<tr>
<td>Washer</td>
<td>512</td>
<td>17.0</td>
<td>8.7</td>
</tr>
<tr>
<td>Water Heater (Quick Recovery)</td>
<td>4500</td>
<td>89.0</td>
<td>400.5</td>
</tr>
<tr>
<td>Water Pump</td>
<td>460</td>
<td>41.0</td>
<td>18.9</td>
</tr>
</tbody>
</table>
### Heating

<table>
<thead>
<tr>
<th>Watts</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>26,500</td>
<td>Elec. furnace, 2000sf, cold climate</td>
</tr>
<tr>
<td>7941</td>
<td>Elec. furnace, 1000sf, warm climate</td>
</tr>
<tr>
<td>1440</td>
<td>Electric space heater (high)</td>
</tr>
<tr>
<td>900</td>
<td>Electric space heater (medium)</td>
</tr>
<tr>
<td>600</td>
<td>Electric space heater (low)</td>
</tr>
<tr>
<td>750</td>
<td>Gas furnace (for the blower)</td>
</tr>
<tr>
<td>1100</td>
<td>Waterbed heater</td>
</tr>
<tr>
<td>450</td>
<td>Waterbed heater (avg. 10 hrs./day)</td>
</tr>
</tbody>
</table>

### Cooling

<table>
<thead>
<tr>
<th>Watts</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3500</td>
<td>Central Air Conditioner (2.2 tonnes)</td>
</tr>
<tr>
<td>1440</td>
<td>Window unit AC, huge</td>
</tr>
<tr>
<td>900</td>
<td>Window unit AC, medium</td>
</tr>
<tr>
<td>500</td>
<td>Tiny-ass window unit AC</td>
</tr>
<tr>
<td>750</td>
<td>Central AC fan (no cooling)</td>
</tr>
</tbody>
</table>

### More efficient cooling

<table>
<thead>
<tr>
<th>Watts</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>Evaporative cooler</td>
</tr>
<tr>
<td>350</td>
<td>Whole-house fan</td>
</tr>
<tr>
<td>100</td>
<td>Floor or box fan (high speed)</td>
</tr>
<tr>
<td>90</td>
<td>52&quot; ceiling fan (high speed)</td>
</tr>
<tr>
<td>75</td>
<td>48&quot; ceiling fan (high speed)</td>
</tr>
<tr>
<td>55</td>
<td>36&quot; ceiling fan (high speed)</td>
</tr>
<tr>
<td>24</td>
<td>42&quot; ceiling fan (low speed)</td>
</tr>
</tbody>
</table>

### Major appliances

<table>
<thead>
<tr>
<th>Watts</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4400</td>
<td>Clothes dryer (electric)</td>
</tr>
<tr>
<td>3800</td>
<td>Water heater (electric)</td>
</tr>
<tr>
<td>200-700</td>
<td>Refrigerator (compressor)</td>
</tr>
<tr>
<td>57-160</td>
<td>Refrigerator (average)</td>
</tr>
<tr>
<td>3600</td>
<td>Dishwasher (washer heats water)</td>
</tr>
<tr>
<td>2000</td>
<td>Electric oven, 350°F</td>
</tr>
<tr>
<td>1178</td>
<td>Electric oven, self-cleaning mode (takes 4.5 hrs, 5.3 kWh total)</td>
</tr>
<tr>
<td>1200</td>
<td>Dishwasher (dry cycle)</td>
</tr>
<tr>
<td>200</td>
<td>Dishwasher (no water heating or drying)</td>
</tr>
</tbody>
</table>

### Lighting

<table>
<thead>
<tr>
<th>Watts</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>60-watt light bulb (incandescent)</td>
</tr>
<tr>
<td>18</td>
<td>CFL light bulb (60-watt equivalent)</td>
</tr>
<tr>
<td>5</td>
<td>Night light</td>
</tr>
<tr>
<td>0.5</td>
<td>LED night light</td>
</tr>
</tbody>
</table>

### Computers

<table>
<thead>
<tr>
<th>Watts</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>150-340</td>
<td>Desktop Computer &amp; 17&quot; CRT monitor</td>
</tr>
<tr>
<td>1-20</td>
<td>Desktop Computer &amp; Monitor (in sleep mode)</td>
</tr>
<tr>
<td>90</td>
<td>17&quot; CRT monitor</td>
</tr>
<tr>
<td>40</td>
<td>17&quot; LCD monitor</td>
</tr>
<tr>
<td>45</td>
<td>Laptop computer</td>
</tr>
</tbody>
</table>

### Televisions & Videogames

<table>
<thead>
<tr>
<th>Watts</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>191-474</td>
<td>50-56&quot; Plasma television</td>
</tr>
<tr>
<td>210-322</td>
<td>50-56&quot; LCD television</td>
</tr>
<tr>
<td>150-206</td>
<td>50-56&quot; DLP television</td>
</tr>
<tr>
<td>188-464</td>
<td>42&quot; Plasma television</td>
</tr>
<tr>
<td>91-236</td>
<td>42&quot; LCD television</td>
</tr>
<tr>
<td>98-156</td>
<td>32&quot; LCD television</td>
</tr>
<tr>
<td>55-90</td>
<td>19&quot; CRT television</td>
</tr>
<tr>
<td>45</td>
<td>HD cable box (varies by model)</td>
</tr>
<tr>
<td>194</td>
<td>PS3</td>
</tr>
<tr>
<td>185</td>
<td>Xbox 360</td>
</tr>
<tr>
<td>70</td>
<td>Xbox</td>
</tr>
<tr>
<td>30</td>
<td>PS2</td>
</tr>
<tr>
<td>18</td>
<td>Nintendo Wii</td>
</tr>
</tbody>
</table>
### Speed

- Place the following in order of how fast they are travelling (from slowest to fastest) - what information do you need to work out how fast something is travelling?

![Images of SR-71 Blackbird, Lamborghini, Olympic Cyclist, Bullet, Space Shuttle, and Snail]

- SR-71 Blackbird
- Lamborghini
- Olympic Cyclist
- Bullet
- Space Shuttle
- Snail

### Speed

- Speed (and velocity) are measurements of how fast you are going (velocity includes direction) – to work out speed you need to know the distance travelled and the time it took...

- Speed = Distance ÷ Time
- Time = Distance ÷ Speed
- Distance = Speed x Time

### Examples

- Jack ran 100m in 12 seconds. What speed was he traveling at?
  - Speed = 100 ÷ 12 = 8.34 m/s

- Jack then ran 100 m again, but this time it was much more windy, and it took him 15 seconds. What was his new speed, and why was this different?
  - Speed = 100 ÷ 15 = 6.67 m/s (more air resistance)

- My car was going at 50 km/h for 1 hour. How many kilometres did I travel?
  - Distance = 50 x 1 = 50 km

- My car was going at 50 km/h, and I traveled 20 km. How long did this take me?
  - Time = 20 ÷ 50 = 0.4 hours (24 minutes)
Distance-Time graphs are used to show three pieces of information:

- Distance
- Time
- Speed (distance ÷ time)

Complete a rough graph of the following journey (plot the time (seconds) on the x-axis and the distance (meters) on the y-axis (x is across))

I left school to walk to the shop – I walked slowly for 10 minutes, covering a distance of 150 m. At the shop I stopped to talk on the phone for 5 minutes. Having realised I was about to miss my bus I ran for 2 minutes to the bus-stop which was 180 m away...

Key information:

- 10 minute (600 seconds) walk covering 150 m
- 5 minute (300 seconds) stationary covering 0 m
- 2 minute (120 seconds) run covering 180 m

*Total time = 17 minutes (1020 seconds); total distance = 330 m

Work out the speed during the different sections (walking; stationary and running)

Annotate your graph – what does the graph show?

In a distance-time graph the slope = the speed

A flat section is where the object has stopped moving

The steeper the graph the faster the speed

However distance-time graphs can also show acceleration and deceleration:

- Steepening curve = speeding up (slope increase)
- Leveling-off curve = slowing down (slope decrease)

The gradient shows the speed in a distance-time graph: 

(Change in vertical ÷ Change in horizontal)
Annotate the following graph, explaining what is being shown – include the distances covered, the speed (was this constant speed / accelerating / decelerating) and the overall time for the journey...

<table>
<thead>
<tr>
<th>Speed</th>
<th>Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>How to collect data for a distance-time graph using 100 m race:</td>
<td>Acceleration is the rate of change of velocity.</td>
</tr>
<tr>
<td>Measure the distance (100 m) and then every 10 m have someone standing with a stop watch</td>
<td>Acceleration is the change in velocity (ΔV) / time (t)</td>
</tr>
<tr>
<td>Time how long it takes the runner to cover each 10 m, then we can graph and note the speed differences between the start and end of the race...</td>
<td></td>
</tr>
</tbody>
</table>

Velocity-Time Graphs

- Remember, the velocity of an object is its speed in a particular direction (this means that two cars travelling at the same speed, but in opposite directions, have different velocities)
- When an object is moving with a constant velocity, the line on the graph is horizontal
- When an object is moving with a constant acceleration, the line on the graph is straight, but sloped
- The steeper the line, the greater the acceleration of the object

- Acceleration is represented on a velocity-time graph by the gradient of the line (change in velocity / time)
- What is the acceleration - represented by the sloping line?
- Change in velocity from 0m/s to 8m/s = 8m/s
- Time of 4 seconds for change in velocity
- $8 \div 4 = 2 \text{ m/s}^2$
The area under the line on a velocity-time graph represents the distance travelled.

What distance was covered on the above graph?

To find the distance we need to calculate the area of the light blue and dark blue regions.

For rectangle areas use the formula base x height:
- For the light blue rectangle: \( 6 \times 8 = 48 \text{ m} \)
- For the dark blue triangle: \( \frac{1}{2} \times 4 \times 8 = 16 \text{ m} \)

Distance = 48 + 16 = 64 m

Some sample graph questions...

Match A, B, C and D to the following descriptions:
1. Accelerated motion throughout
2. Zero acceleration
3. Accelerated motion, then decelerated motion
4. Deceleration

Which line represents the furthest distance?
Which line represents the least distance?

Match A, B, C and D to the following descriptions:
1. Accelerated motion throughout – A (\( \frac{1}{2} \times 20 \text{ s} \times 8 \text{ m/s} = 80 \text{ m} \))
2. Zero acceleration – C (\( 20 \text{ s} \times 8 \text{ m/s} = 160 \text{ m} \))
3. Accelerated motion, then decelerated motion – D (\( \frac{1}{2} \times 20 \text{ s} \times 6 \text{ m/s} = 60 \text{ m} \))
4. Deceleration – B (\( \frac{1}{2} \times 20 \text{ s} \times 4 \text{ m/s} = 40 \text{ m} \))

Which line represents the furthest distance? – C
Which line represents the least distance? – B

Describe the motion of the cyclist.

Work out the initial acceleration.

Work out the distance travelled by the cyclist in the first 40 seconds.
Describe the motion of the cyclist – accelerates at constant rate for 40 seconds, then decelerates for the next 20 seconds to a standstill

Work out the initial acceleration – $0.2 \text{m/s}^2$ ($8 \text{m/s} ÷ 40 \text{s}$)

Work out the distance travelled by the cyclist in the first 40 seconds – $\frac{1}{2} \times 40 \text{ s} \times 8 \text{ m/s} = 160 \text{ m}$

In a motorcycle test the speed from rest was recorded at intervals

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity (m/s)</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

Plot a velocity-time graph of these results

- What was the initial acceleration?
- How far did it travel in the first 20 seconds?
- How far did it travel in the next 10 seconds

What was the initial acceleration – $-2 \text{m/s}^2$ ($40 \text{ m/s} ÷ 20 \text{ s}$)

How far did it travel in the first 20 seconds – $5 \times 20 \times 40 \text{ m/s} = 400 \text{ m}$

How far did it travel in the next 10 seconds – $40 \times 10 = 400 \text{ m}$
The main function of fibre is to keep the digestive system healthy and functioning properly.

Fibre aids and speeds up the excretion of waste and toxins from the body, preventing them from sitting in the intestine or bowel for too long, which could cause a build-up and lead to several diseases.
Fats

- Nutrient with highest energy content
- Unsaturated – good fat
  - Neurons
- Saturated fats - bad fat
  - cholesterol

Protein

- An organic compound made of amino acids
- 22 total amino acids
- 13 are made in the body
- 9 MUST come from food!!!
- Complete proteins - animal sources have all 9 amino acids
- Incomplete proteins - vegetables, nuts, seeds lack one or two incomplete proteins

Nutritional Advice

- http://www.bbc.co.uk/health/treatments/healthy_living/nutrition/life_adolescence.shtml

Reading Food Labels

- http://kidshealth.org/kid/stay_healthy/food/labels.html

Healthy Eating for Teenagers

- Adolescence is a time of rapid growth, and the primary dietary need is for energy - often reflected in a voracious appetite.
- Ideally, foods in the diet should be rich in energy and nutrients.
- Providing calories in the form of sugary or fatty snacks can mean nutrient intake is compromised, so teenagers should be encouraged to choose a variety of foods from the other basic food groups

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- Providing calories in the form of sugary or fatty snacks can mean nutrient intake is compromised, so teenagers should be encouraged to choose a variety of foods from the other basic food groups.
**Healthy Eating for Teenagers**

- Plenty of starchy carbohydrates - bread, rice, pasta, breakfast cereals, chapattis, couscous and potatoes
- Plenty of fruit and vegetables - at least five portions every day
- Two to three portions of dairy products, such as milk, yoghurt, fromage frais and pasteurised cheeses

**Other important dietary habits to follow during adolescence include:**

- Drink six to eight glasses of fluid a day.
- Eat regular meals, including breakfast, as it can provide essential nutrients and improve concentration in the mornings. Choose a fortified breakfast cereal with semi-skimmed milk and a glass of fruit juice.
- Take regular exercise, which is important for overall fitness and cardiovascular health, as well as bone development.

**Key points**

- Eat regular meals from the main food groups, and minimise intake of high-fat and sugar-rich foods
- Pay particular attention to getting enough iron and calcium in the diet, and eat lean red meat or non-meat iron sources and dairy products every day
- Maintain a healthy weight
- Be physically active
**Light**

**Part 1 – Properties of Light**
- Light travels in **straight** lines:

**Light travels at around 300,000 kilometres per second**
- At this speed it can go around the world 8 times in one second.

**Light Travels a lot faster than Sound**
- Thunder and lightning start at the same time, but we will see the lightning first.
- When a starting pistol is fired we see the smoke first and then hear the bang.

**Light allows us to see objects**
- We see things because they reflect light into our eyes:

**Luminous and Non-Luminous Objects**
- A **luminous** object is one that produces light.
  - e.g. stars, flame, TV screen
- A **non-luminous** object is one that reflects light.
  - e.g. objects like table, chair, book.
Shadows

- Shadows are places where light is “blocked”:

Properties of Light Summary

1) **Light travels in straight lines**
2) **Light travels much faster than sound**
3) **We see things because they reflect light into our eyes**
4) **Shadows are formed when light is blocked by an object**

Part 2 - Reflection

- Reflection from a mirror:

Note: the normal is always perpendicular to the mirror.

Law of Reflection

Angle of incidence = Angle of reflection

In other words, light gets reflected from a surface at the same angle it hits it.

Clear vs. Diffuse Reflection

Smooth, shiny surfaces have a **clear reflection**:

Rough, dull surfaces have a **diffuse reflection**.

**Diffuse reflection** is when light is scattered in different directions.
How to make a periscope

Materials needed:

| Big sheet of Cardboard or thin plastic (something you’re able to cut with scissors) |
| Some scissors |
| Two flat mirrors |
| Sticky tape |

1. Cut out your cardboard into four shapes like the ones below. Note the size of each compared to each other. This is important, that’s why we drew a grid, to make it a little more obvious. (The colours are not important it’s just there to help your know where each bit goes)

2. Tape the shapes together into a tube like that shown below.
3. Next step tape the mirrors onto the ends of the tube as shown.

4. Look though one end with the other end held high. What do you see?

Explain…

1. Why does the periscope work? Mention position of the mirrors; outline the path that light takes as it travels through the periscope by drawing it; and mention terms such as angle of incidence, angle of reflection, normal.

2. What angles are the mirrors at? Why are they at these angles? How would it affect the periscope if the mirrors were not at these angles?
Gravity

Force at Impact
- A falling ball on a paper surface can break through.
  - The ball is exerting a force
- A falling ball on a hard surface rebounds.
  - The surface is exerting a force
- There are forces both ways at contact.

Force is Not Motion
- The ball breaking paper doesn’t stop.
  - The force needed to break through is small.
  - But that small force is also exerted on the ball.
- On a rebound the ground doesn’t move.
  - The force needed to break through is large
  - A force acted in both ways

Third Law: Law of Reaction
For every action there is an equal and opposite reaction.

Forces between two objects act in pairs:

Equal and Opposite
- Newton’s law of reaction also applies to the force of gravity.
  - The Earth pulls the Moon
  - The Moon pulls the Earth
- Newton used this to describe a Law of Gravity.

Universal Gravity
- Newton realized that all objects obey that Law.
  - Other planets
  - Apples
  - People
- The gravitational force is universal.
  - The gravitational constant is $G = 6.67 \times 10^{-11} \text{ m}^3/\text{kg}^2$.
- What is the gravitational force between two students sitting in adjacent seats?
  - Assume the students have a mass of 70 kg each.
  - Assume that they are separated by 1 m.

$$F = \frac{G M m}{r^2}$$

$F = 3.3 \times 10^{-7} \text{ N}$.
The force of gravity on a mass is its weight.

Weight = mass \times \text{gravity}

\[ w = mg \]

Gravity varies over the surface of the Earth.

- The height of the surface varies – so the radius does, too
- The material under the surface is not uniform
- The earth isn’t exactly round
- The tides affect the earth as well as the oceans

One unit of gravitational acceleration used on the Earth is the Galileo.

- 1 gal = 1 \text{cm/s}^2 = 0.001 \text{ m/s}^2 = 0.001 \text{ N/kg}
- \( g \) = 9.81 gal

High areas have a greater distance from the center of the Earth.

An increase of 1 km should decrease \( g \) by 300 mGal.

Type of rock affects \( g \).

As the Earth spins the equator slightly bulges.

- The radius is about 22 km bigger compared to the pole.
- Expect a few Gal difference from equator to pole.

Equator: 9.780 m/s^2

North Pole: 9.832 m/s^2

Earth’s Rotation

- The earth is made of layers of different types of rock.
- These rocks can move due to daily tides.
- Much less than ocean tides
- Period is 22 hours like the ocean

The force of gravity from the Moon is countering some of the force of the Earth.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mass (kg)</th>
<th>Radius (m)</th>
<th>Acceleration due to gravity (m/s^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth</td>
<td>5.98\times10^{24}</td>
<td>6.37\times10^{6}</td>
<td>9.81</td>
</tr>
<tr>
<td>Jupiter</td>
<td>1.90\times10^{27}</td>
<td>7.14\times10^{7}</td>
<td>25.95</td>
</tr>
<tr>
<td>Saturn</td>
<td>5.68\times10^{26}</td>
<td>5.82\times10^{7}</td>
<td>11.08</td>
</tr>
<tr>
<td>Uranus</td>
<td>8.68\times10^{25}</td>
<td>2.33\times10^{7}</td>
<td>10.67</td>
</tr>
<tr>
<td>Neptune</td>
<td>1.03\times10^{26}</td>
<td>2.21\times10^{7}</td>
<td>14.07</td>
</tr>
<tr>
<td>Pluto</td>
<td>1.31\times10^{23}</td>
<td>1.42\times10^{6}</td>
<td>0.42</td>
</tr>
</tbody>
</table>

Gravity of each Planet
The Physics Of Sound

Sound is made when something vibrates

- The vibration disturbs the air around it.
- This makes changes in air pressure.
- These changes in air pressure move through the air as sound waves.

Sound Waves

- The sound waves cause pressure changes against our ear drum sending nerve impulses to our brain.

This is similar to throwing a rock into a pond...

- Air molecules ripple through the air in sound waves like water waves rippling across a pond.

The Three Components of Sound are:

- Pitch (how high or low)
- Loudness (volume)
- Timbre (tone colour)

Pitch

- The vibration patterns of some sounds are repetitive.
- Vibration patterns are also called waveforms.
- Each repetition of a waveform is called a cycle.
- We can hear frequencies between 20 hertz or cycles (vibrations) per second (low pitches) to 20 kilohertz, i.e. 20,000 Hz (high pitches).
When the frequency of a sound doubles we say that the pitch goes up an octave.

We can hear a range of pitches of about ten octaves.

Many animals can make sounds and hear frequencies that are beyond what we can hear.

To create vibrations energy is used.

The greater amount of energy used the louder the sound.

The strength of the changes in air pressure made by the vibrating object determines loudness.

As the distance from the source increases the amount of power is spread over a greater area.

The amount of power per square metre is called the intensity of the sound.

For us to perceive a sound as twice as loud its intensity must be ten times greater.

The perceived intensity level of sound is measured in a logarithmic scale using a unit called the decibel.

The scale begins (0 dB) on the softest sound that a person can hear. This is called the threshold of hearing.

The scale ends at the volume that causes pain (120 dB) and is therefore called the threshold of pain.

From the perspective of the logarithmic scale the threshold of pain is...

1,000,000,000,000 times as great as the threshold of hearing.
The picture above is a wave file of someone singing.

The chart on the left is a representation of different sounds around us and their volume in decibels.

- **Timbre** is the specific property of sound that enables us to determine the difference between a piano and a harp.
- In other words, it is the quality of a musical note or sound or tone that distinguishes different types of sound production, such as voices and musical instruments.

- **Tone Colours**
  - An extremely broad variety of tone colours exist because most sounds that we perceive as pitch actually contain many frequencies.
  - The predominant pitch is called the **fundamental** frequency.

- **Sound from a Vibrating String**
  Although we would perceive a string vibrating as a whole, it actually vibrates in a pattern that at first appears to be erratic producing many different overtone pitches. What results are particular tone colours or timbres of instruments and voices.
Chapter 6

Evaluation of the Intervention
6.1 Introduction

Various aspects of the investigation into the integration of mathematics and science carried out are explored in detail in this section, including how group work was implemented by the teachers; how it was received by the pupils; and the effects it had on learning. Similarly, the issues of aligning mathematics with Science in a school setting; teacher knowledge within both subjects; the challenges faced in applying Authentic Integration; the effects of applying the material studied to real life scenarios; and pupil enjoyment and improvement will be analysed using the research tools outlined previously (see section 3.6) i.e. semi-structured interviews with the teachers involved, focus groups with some of the pupils, questionnaires completed by the teachers, and assessment of pupil work using a rubric specifically designed for this scenario.

The interviews conducted offered rich and in-depth insights into what elements of the model, and integration in general, were successful and which aspects need further work or consideration. Questionnaires completed by teachers highlighted, through quantitative means, the opinions of the teachers expressed already in the interviews. Grading of pupil work and feedback from pupils also served to illuminate the following discussion and inform the conclusions drawn from this research.

The names used to identify teachers throughout this chapter are pseudonyms put in place to protect the anonymity of the participants as promised them through the initial information letter (see Appendix A) and as required for ethical approval (see section 3.9). 24 schools were contacted in relation to this intervention but only 6 indicated a willingness to take part, two of these schools dropped out prior to the commencement of the intervention. Thus, four schools took part with one teacher from each school participating. Of the teachers that were involved in the intervention, John Ryan is predominantly a science teacher but also teaches some mathematics; Martina O’Reilly and Jennifer Collins both teach mathematics only; while Eoin O’Shea is the only teacher in this particular study who regularly
teaches significant amounts of both mathematics and science. Their feedback will provide valuable insights into the themes of the investigation which have been chronicled in this chapter, beginning with the hands-on tasks which were central to each lesson and the group work dynamic which was inherent in each task.

### 6.2 Hands on Tasks and Group Work

Central to the lessons implemented were the tasks completed by the pupils. These tasks were hands-on, group work tasks which encouraged discussion and inquiry. Tasks of this nature are rarely experienced by 2nd level students at Junior Cycle in Ireland, especially within their mathematics lessons as it is not a requirement within their syllabus – only one mention of working in groups is made in the syllabus i.e. as an option when conducting probability experiments (NCCA 2010). The teachers involved in this study also mentioned that group work wasn’t commonly employed in their mathematics lessons, thus it was important to gauge the level of success of these tasks, as well as the group work central to the tasks, in relation to pupil learning, enthusiasm, and interest.

#### 6.2.1 Pupil Involvement, Enjoyment, and Enthusiasm

The advantages of the Authentic Integration tasks were obvious to the teachers who conducted these lessons, both in terms of pupil enjoyment and learning, and in terms of development of their own teaching:

“[T]he group work came very naturally with teaching those particular lessons. Even for me, it was a great outcome to teach the same topic in different ways and to bring a ‘freshness’ or a different approach to it. I actually found it very enjoyable and I think the children definitely did too”.

Eoin O’Shea (science and mathematics teacher)
“(The pupils were) learning a lot more than (if it were) just me standing and telling them what a food pyramid looks like or telling them what they should and shouldn’t be eating. Definitely, when you get them to look up recommended daily allowances for food, they recognise that ‘Oh God, I’m taking in too much or too little’.

John Ryan (science teacher)

One of the key elements of the model applied within these lessons was that pupils would have the opportunity to develop and apply their mathematical and scientific knowledge to everyday scenarios – something which can be seen in this description of Ryan’s experience in exploring the food pyramid and diet with his pupils. Also, it’s clear that Ryan found that allowing the pupils the responsibility to analyse the food pyramid and their own daily intake of certain foods was a more effective approach to learning than applying a didactic style.

It can often be forgotten that teacher motivation and enjoyment is an important issue to consider, as evinced within O’Shea’s statement. His increase in enjoyment and enthusiasm can only be a positive outcome for student learning as suggested by Patrick et al. (2000, p.233) who, within their extensive study, discovered that “when a teacher exhibits greater evidence of enthusiasm, students are more likely to be interested, energetic, curious, and excited about learning”.

Research pertaining to the integration of mathematics and science consistently recommends ensuring that lessons are pupil centred (see section 2.4.6). This became a fundamental element of the Authentic Integration model developed, thus pupil involvement and, as a result, enjoyment within the lessons formed one of the main objectives of each lesson. Interestingly, content of the lessons became something which was discussed outside the classroom with the pupils using what they had learned and applying it to their own everyday lives:

“There were a lot of stories about students eating dinner and saying ‘oh Mam, there’s too much fat in that’ or ‘Mam, there’s not enough
carbohydrates in that’. So, I think that they were really getting the family involved in it at home. The parents could see that the students were getting really interested in the whole Science idea being explored. The kids really enjoyed the task – especially when they got to go home and dictate to Mam and Dad what they should be eating.”

John Ryan (Science teacher)

Ryan’s observations imply that interest and enthusiasm were evident in the pupils not just in the classroom but at home as well while Collins (mathematics teacher) also realised the opportunity to affect and increase pupil interaction and enjoyment through the discussion of real life scenarios pertaining to the material being taught:

“…they were interesting topics, you were able make it really relevant. For instance, the household energy usage: they all brought in a bill and we all talked about what appliances we used every morning and they were amazed and we had a laugh about what types of things we used like hair straighteners and so on. So when it came to assessing it, they were able link it up with something that they enjoyed in class rather than the usual chalk and talk or the teacher mainly talking instead of them interacting in class. So I think that definitely worked because they were way more involved in the lesson.”

Similarly, the other teachers involved also remarked on how the lessons implemented affected the attitudes of their pupils:

“As regards enthusiasm and levels of understanding – they even ask for group work now.”

Eoin O’Shea (science and mathematics teacher)
“It did get some of them (the pupils) talking because some their resource teachers would come up to me asking ‘what are you doing in maths?!’ because the students were asking them all these questions about maths and science. So it’s obviously ignited their interest a bit”

Martina O’Reilly (mathematics teacher)

These positive reports give credence to the notion that active engagement by pupils in the education process enhances memory, learning, and enjoyment. Allied to that, pupils displayed an improvement in enjoyment of mathematics over the course of the intervention as the mean score for the whole class in their Enjoyment scale pre-test was 27.55 (SD = 7.8), while the mean score for the same test taken upon completion of the intervention was 29.58 (SD = 7.9). Although this improvement was not statistically significant, it indicates that the pupils’ enjoyment of mathematics moved in a positive direction following their experience of the integrative lessons implemented.

6.2.2 Pupil Learning and Understanding

Pupil learning was mainly assessed using the rubric to grade pupil work as well as feedback from the teachers and pupils. Evidence of improvement in this aspect is outlined in this section

6.2.2.1 Teachers’ Views of Pupil Learning and Understanding

The rich tasks inherent in each of these integrative lessons bear many similar characteristics to Problem Based Learning (PBL) (Prince 2004). The positive effects of PBL, based on research, include improved retention of information, as well as enhanced critical thinking and problem-solving skills (Prince 2004, Hmelo-Silver 2004).
Two of the teachers, Ryan and Collins, recognised these effects at work through the instances in which the pupils were applying the learning which they were actively engaged in within the classroom to their own everyday lives at home and assessments in a school setting (see section 6.2.1). Such positive proclamations with regards to development of understanding and critical thinking through experiential learning were echoed by O’Shea (science and mathematics teacher):

“’The biggest positive outcome was in terms of understanding. Thinking back to when I taught these topics before, obviously this time I spent more time on it, but I think it was the way in which the pupils engaged with it themselves and the fact that the maths was brought in more naturally or it was there from the start.’”

Getting the pupils active and engaged in the process is central to Authentic Integration through the ‘Focussed Inquiry Resulting in Higher Order Learning’ characteristic of the model. It is an element that teachers had very positive views on, and something which they suggest elicited meaningful learning. The feedback from teachers suggests that understanding has been affected significantly through these lessons:

“I think, as regards short-term, the enthusiasm and the different approach definitely meant that the understanding would be there. I think it would be interesting, and I even said this to the pupils, that if we do a test on these three topics later in the year I would definitely expect that the understanding would be better. I think time will tell.”

Eoin O’Shea (science and mathematics Teacher)

“I definitely think that maths can be something that’s very much didactic and working out of the textbook a lot of the time because, I suppose, that’s relevant and you have exactly what you need there and it’s easy to put up the examples and expect them to work on [through the book]. But,
realistically, when it comes to a test I find that students can’t remember half of it. So I did think that, by using these teaching methods, when it came to the time for assessment, they definitely had a better recall of what we did because there were more concrete examples and they were able link up material between the subjects. As well, they were interesting topics, you were able make it really relevant.”

Jennifer Collins (mathematics teacher)

Collins contends that linking the mathematics and science content to something concrete which they have experienced would enhance recall. Such thinking aligns with Bloom’s (1987) work outlined earlier and can positively affect their learning.

6.2.2.2 Grading Pupil Learning and Understanding

Assessment of pupil learning and understanding was completed through the collection of pupil work which was analysed and marked using a rubric specifically designed for Authentic Integration. This allowed for a more in-depth insight into the level of understanding achieved by the pupils in comparison to a diagnostic test which would only assess basic recall of facts and algorithms.

Newmann et al. (2007) identified six studies into the application of Authentic Instruction in school settings, four of the six studies they described used a rubric for assessment of pupil work that is very similar to the rubric applied for the same purpose in this investigation (see Appendix E). The fact that the rubric was preferred over a diagnostic test in four of the six studies suggests that it offers more worthwhile feedback in this context.

This rubric had three categories: Knowledge Development, Synthesis, and Application; Focused Inquiry Resulting in Higher Order Learning: Concepts; and Focused Inquiry Resulting in Higher Order Learning: Written Communication. Each category was marked on a scale of 1 to 4, thus the highest possible score was
12 and the lowest possible score was 3. The criteria for attaining a score of 1, 2, 3, or 4 in each category are specifically outlined (see Appendix E). For example, a pupil would score a mark of 4 in the “Focused Inquiry Resulting in Higher Order Learning: Concepts” category if they demonstrated exemplary understanding of the concepts that are central to the assignment.

The quality of work produced by the pupils, according to the rubric, was quite high with a mean score of 8.42 out of 12 (SD = 1.82). Pupils were marked on their understanding of concepts; written communication; and knowledge development, synthesis, and application. A histogram of the performance of all pupils involved is outlined below in Fig. 6.1. A total score of 8 was most common (scored by 13 of the 48 pupils assessed) while the majority of pupils (31 of the 48 assessed) scored between 7 and 9 marks out of 12. However, it must be noted at this point that pupils who were absent for the majority of the lessons were not graded as there was too little written work to properly assess their performance. Evidence of understanding of concepts was assessed with the mean score among pupils being 2.9 out of 4 (SD = 0.722) while the pupils’ ability to analyse and apply their knowledge scored 3.0 out of 4 (SD = 0.652).

Assessment of this work was carried out by the author. Reliability of the marks awarded was ensured by comparison with the marking given to the same work by two colleagues of the author, both Doctors of Mathematics Education. The level of agreement was calculated using the following formula:

\[
\frac{\text{Number of Agreements}}{\text{Number of Opportunities for Agreement}} \times 100
\]

Initially, agreement with one colleague was 80% and 75% with the other. Marks in certain cases were discussed with the colleagues involved and adjusted if necessary. As such, marks awarded by both the author and his colleagues were of a very similar nature thus allowing the author to accept with confidence the marks he finally awarded.
While this approach of marking the written work completed by pupils based on the given rubric offers an insight into pupil performance, further research could focus specifically on the effects of integrating a topic from both mathematics and science compared to teaching these two topics separately using diagnostic tests. Such an approach was outside the scope of this research due to a number of factors (outlined in section 3.10) and such an approach would negatively affect, even make impossible, the exploration of other aspects of the process of introducing Authentic Integration lessons to a post primary education scenario. For instance, pupils and teachers would only experience a very narrow range of topics in an integration setting which would limit their experience of the various
applications of Authentic Integration thus greatly reducing their ability to offer meaningful feedback in relation to the model.

Considering the constraints that were present in this research gives rise to the consideration of the barriers encountered during the introduction of lessons of an integrative nature to a post-primary education setting.

6.3 Challenges Faced When Implementing Authentic Integration

The teachers who participated in this study had not previously taught lessons which integrate mathematics and science, thus certain challenges to the successful implementation of such lessons were encountered. Each of these issues is discussed in detail.

6.3.1 Aligning Mathematics and Science Topics

When planning the implementation of these lessons with the schools and teachers involved, one of the main issues which consistently came up was the issue of aligning topics in both mathematics and science. The lessons were developed for 2\textsuperscript{nd} year students, who would ideally have previously experienced (in normal classes) the topics to be explored in these integration lessons so that they could draw on previous knowledge from both subjects, develop this knowledge, and synthesise it. As such, co-ordination between the mathematics and the science departments within each school became important, a practice which wasn’t regularly conducted in each of the schools involved according to Collins:

“If you could come on board with the Science department and teach the [relevant] topics at the same time. I think you’re trying to remind them of something they’ve done ages ago… I think there could be more cross-curricular links if both departments could be on board together and willing to work in cohesion.”
This would draw on one of Jacobs’ (1989) ten “Design options for an Integrated Curriculum” i.e. the “Sequenced” option (see Literature Review Section 2.4.2) in which teachers of different subjects create links by teaching topics which can be interlinked at the same time as each other e.g. an English teacher analyses a text based on an historical event that is being studied at the same time in history lessons.

Connecting with other departments through integration could offer new opportunities to bolster and advance the learning taking place in subject specific lessons and increase the quality of tuition through shared ideas:

“Sometimes we’re (teachers) guilty of being in our own classrooms and saying ‘this is my subject’ and maybe not being aware of [the possibility] that they (the pupils) could’ve being doing gravity the exact same day in Science – I wouldn’t know that. It made me talk to the Science teachers and go: ‘do ye do this?’. Some of the Science teachers asked to see the material I was using so I can see that they might use that again in their classes because I could focus on doing the maths part with the a bit of Science. They could do the opposite and incorporate it (mathematics). So I think that it was good and some of the Science teachers have asked me for the resources so that they might use them.”

Martina O’Reilly (mathematics teacher)

As O’Reilly suggests, being aware of what pupils are studying in other subjects can be a rare occurrence so planning and coordination between subject departments within schools needs to be conducted if effective integration of topics can be applied throughout the school term.
6.3.2 Adjusting to Group Work

The effect of working in a group can have both positive and negative elements which need to be considered when attempting to positively affect the quality and level of learning taking place.

6.3.2.1 Teachers’ and Pupils’ Experiences of Group Work

O’Reilly (mathematics teacher) observed plenty of positive outcomes in relation to the implementation of group work:

“I think one of the main (positive outcomes) is that they learned from each other. They discussed things and if they made mistakes they’d need to figure out how they went wrong and why they went wrong. Sometimes you learn more from making mistakes than getting things right all the time. I think the fact they were in a group made them interact a bit more and question each other’s answers because they always wanted to make sure they were right themselves”.

This statement highlights the ‘Focused Inquiry Resulting in Higher Order Learning’ element of Authentic Integration as the pupils used the group setting to discuss and question their ideas and theories as they worked towards a solution to the problem or task set. But the outcome of such group work was not always positive as suggested by the mixed feelings the pupils of O’Reilly’s class had towards such a setup when asked whether they liked group work:

Pupil 2 (P2), Pupil 3 (P3), Pupil 4 (P4): No

Interviewer: Why don’t you like working in groups?

P4: because you can get distracted.
P2: in the Junior and Leaving Cert you’re by yourself so you should be by yourself doing work to get used to it. Like if you’re doing work…

P1: You won’t have anyone there to help you…

P2: Yeah… There’s some people in the group not doing much and other people that are doing a lot. You just get mixed up with the different abilities.

Interviewer: if you were unsure of how to do a certain question, would you prefer to be in a group or on your own?

P2: on your own ‘cause then you can just ask the teacher.

Interviewer: So you’re not comfortable asking people in your own group?

P2: I am but you’re more sure that you’ll get the right answer if you ask the teacher.

Interviewer: And it’s important to get the right answer is it?

P2: Yeah.

Interviewer: Anyone with any positive views on group work?

P1: Yeah, it’s easier. ‘cause you can ask people instead of always asking the teacher.

P3: It can be easier.

P4: Yeah, at some points.
Interviewer: what are those points?

P3: like, if someone in your group knows then you can just ask him.

It’s interesting to note the attitude towards school work shown by the pupils within this focus group. Even though they were a year and a half away from starting their Junior Certificate examinations, they were adjusting their attitudes and practices in order to be prepared for the nature of these examinations i.e. working on their own to solve problems because they will be on their own when completing the exam. This highlights the importance of the types of assessment applied in dictating the habits and attitudes developed by pupils.

The need to prepare for the Junior Certificate seems to be one reason the pupils have borderline negative feelings towards group work (something which will be discussed in detail later in this chapter), another reason mentioned was the need to get the ‘right’ answer every time and, as such, preferring to consult with the teacher rather than another group member. The pupil that felt this way elaborated on this point later by intimating that group work “gets very messy. Like, if there’s someone who doesn’t like maths and gets the wrong answer and they think they’re right…” This pupil, the author was later informed, is, in the opinion of his teacher, one of the most intelligent in his class, and the tone of this statement suggested that he was a little annoyed when he knew the right way to approach a task and another group member put forward another approach which he believed to be wrong thus slowing down the process of arriving at the right answer. This was also alluded to by O’Reilly when she observed that “One or two of the stronger students in the class didn’t want to be involved in group work or weren’t that great at helping each other out – they wanted teacher input as opposed to group input.”

At this point the thoughts of the teachers with regard to how well pupils work without major teacher input should be considered. The teachers involved in this research, according to their questionnaire responses, had mixed feelings in
relation to this aspect of group work, with two teachers unsure as to how well the pupil groups worked independently of the teacher, one teacher feeling that they did not work well in that manner and one teacher declaring that they did work well without major teacher input, while the other two teachers were unsure. This mixed reaction shows that some pupils may adjust to this type of task, and into group work in general, easily while others may take a little longer to embrace the practice.

It also raises the question as to whether teachers are comfortable in transitioning from the typical teacher-led style of learning to one which is pupil-directed. Dolmans et al. (2001) investigated issues which arose in problem based learning (PBL), an approach which is very similar to that which is central to Authentic Integration. They noticed that when difficulties were encountered within group work that teachers felt the need to interject and impose external standards on the quality of learning taking place within the group rather than put the responsibility for enhanced learning in the hands of the pupils. Dolmans et al. (2001) observed that such an approach led to pupils being coerced into an artificial behaviour which is aimed at impressing the teacher rather than being intrinsically motivated to complete the task set and develop their understanding. As an alternative, they suggest that pupils should be made responsible for their own learning through peer- and self-assessment as well as regular formative assessment e.g. through discussion.

While group work requires the teacher to ‘buy into it’, how group work is introduced should also be considered. O’Shea (science and mathematics teacher) felt that the tasks involved required some level of experimentation in how they were conducted and that “it’s only by going through them first time that you find out you could have given them more structure or less structure when you send them away to do something.”

The thoughts of the pupils and observations of their teachers in relation to group work and the type of tasks set gave the author pause for thought in relation to the
need for pupils to be inducted gradually into this type of approach rather than jumping straight into it. It may be necessary to guide them through the process in the initial lessons of this nature until they are more comfortable with an approach which would probably be quite different to the normal tuition they’d encounter within secondary school. O’Reilly touched on this issue:

“I think if this group method is developed properly, they’d learn more. They had to question everything in their group so when they had their work done themselves, even the guys who wanted to work individually, they had to question everything in their group. They had to compare answers, analyse ‘what you did right’, ‘what you did wrong’. I think they were getting a bigger picture of the aspects of each topic rather than ‘this is the way to do it’.”

O’Reilly mentions that there were pupils that wanted to work individually rather than in a group, something which was mentioned within the focus group script previously. This highlights why group work needs to be eased in and ‘developed properly’, as O’Reilly put it, as it may not be an approach which is regularly applied in a typical Irish Post-Primary school.

The schools involved in this research fit such a description, as group work was something which was not common place within a typical class carried out by these teachers, especially when it came to mathematics. This proved to be somewhat of a barrier that needed to be negotiated when introducing the Authentic Integration lessons which were typically based around group work. It proved to be a challenge to some of the teachers, including O’Reilly:

“I suppose, at the start, getting used to the setup in the room – it can be slightly disruptive. One or two of the stronger students in the class didn’t want to be involved in group work or weren’t that great at helping each other out – they wanted teacher input as opposed to group input. So I think it would take students like that a while to get used to it.”
Collins found this aspect of the lessons challenging also:

“Group work can be difficult to manage, depending on the classroom environment. Some classes might take to it better than others. Sometimes it can be hard to get the students ‘on task’ because they think if they’re in a group they can talk off topic or whatever.”

6.3.2.2 Tackling Initial Difficulties Associated with Group Work

Difficulties with group work in the initial stages are common. Research shows that it typically takes two to three weeks for students to adjust to this type of learning situation if they are not overly familiar with it (Davidson 1990). Obviously this was something which O’Reilly and Collins encountered in the early stages of these lessons. Pupils cannot just be placed in groups and expected to adapt straight away to such a dynamic – it requires time and development of group work skills (Baines et al. 2007). It may have been prudent on the author’s part to have provided greater instruction in relation to aiding pupils in adjusting to the group dynamic. Davidson’s (1990, p.56) set of guidelines for group behaviour would have proved quite useful:

1. Work together in groups of four.
2. Cooperate with other group members.
3. Achieve a group solution for each problem.
4. Make sure that everyone understands the solution before the group goes on.
5. Listen carefully to others and try, whenever possible, to build on their ideas.
6. Share the leadership of the group.
7. Make sure that everyone participates and that no one dominates.
8. Take turns writing problem solutions on the board.
9. Proceed at a pace that is comfortable for your own group.
Davidson (1990) suggests that groups of four work best as they are large enough to allow for the generation of ideas and solutions, and one member being absent would not decimate the group. While four is a small enough number to ensure that every member can actively participate. He also suggests that pair work would provide a useful gateway to group work for those who are not familiar with it. Similarly, responsibility for learning should be gradually transferred from the teacher to the pupils (Baines et al. 2007). Taking these recommendations into account, if the author were to introduce Authentic Integration lessons to another school, he would first analyse the level of group work which the pupils have experienced in the past and, if it is relatively minor, he would recommend introducing pair work first before easing the pupils into group work lessons, developing their skills, and aiding them in adapting to this dynamic. Greater levels of support would be provided in relation to methods for conducting group work in the classroom to ensure that the level of learning is maximised from the outset.

Davidson’s (1990) set of guidelines for group behaviour may need to be cut down to ensure that the pupils don’t have to try and take on too much information in the initial stages of identifying good practice during group work, with points 2, 4, 5, 6, 7, and 9 areas that would be highlighted. O’Shea (mathematics and science teacher) had, on occasion, implemented group work in his Science classes. As such, he had plenty of experience regarding how to conduct these lessons. Similarly, his pupils were familiar with the procedures involved. This experience also allowed him to recognise the positive effects group work can have:

“Once it’s organised properly and they know exactly what they are supposed to have done by the end of it, for a lot of things in Junior Cert Science, time permitting, it is a huge advantage to use it.”

Each of the teachers had a positive view of employing group work within their classroom, O’Shea’s comments being typical of the feedback received. The teachers also observed that the pupils largely enjoyed completing the group tasks
with each teacher agreeing that the pupils appeared to enjoy working together on the tasks, one of whom strongly agreed with the statement. Collins (mathematics teacher) described how this enthusiasm for group work became evident to her:

“…they (the students) really enjoyed it. After completing the lessons, they were really enjoying it and were saying ‘why is it over?’ to the extent where I was having to make my lessons so much more active because it was such a difference to the way I’d normally teach. The students really responded well to it.”

6.3.2.3 Summary of Group Work Experiences

In summation, there were initial problems with the introduction of group work – some teachers had not conducted a great deal of group work previously; some pupils would have favoured working individually; certain pupils preferred teacher help to peer help; and some teachers were quite wary that pupils could easily go ‘off-task’. These problems could have been avoided with a better support system i.e. instruction on how to introduce group work and guidelines for pupils to follow while taking part in group work. Research shows that as both teachers and pupils become more familiar with the group dynamic through experience of cooperative learning, these problems lessen and the potential for learning in groups is realised (Davidson 1990, Baines et al. 2007), Collins recognised this also:

“I think, depending on what way you arrange the groups, the students can gain a lot from doing group tasks. They get to hear different people’s opinions and come up with things together as a group rather than just having individual opinions.”

Jennifer Collins (mathematics teacher)
6.3.3 The Amount of Time Required

When embarking on this research, the author expected a certain level of difficulty, even resistance, when attempting to apply lessons which integrated mathematics and science as to do so would be outside the norm set out by the curriculum for each subject. This could take up time needed to cover the curriculum of mathematics and science, and dedicate this time to something which was relatively untested. As such this was an interesting aspect of this endeavour and was referred to regularly in interviews with teachers hence giving an insight into the barriers that new, innovative practices face within the current education system in Ireland:

“I think the problem that any teacher has in trying to do research like this is it’s outside of the normal curriculum. We have, as a group of 2nd year Science teachers, certain chapters we have to get done by Christmas, so if you deviate from that then it messes up Christmas exams or mid-term exams because you’re not at the same pace as everyone else.”

John Ryan (Science teacher)

So it seems that the pressure placed on teachers to cover the curriculum affects the approach they take to teaching the material at hand thus leaving little room for varying teaching methods. It also discourages the exploration of different topics to the ones being studied in other similar classes within the school as uniformity appears to be of upmost importance. As going outside the norm affects how time is spent in various scenarios, this issue cropped up regularly. The time taken to prepare and implement these lessons was referred to by all four teachers interviewed as a possible barrier to the adoption of such an approach:

“For me the only issue I would see would be the time issue. If I was to really do a good job on the three lessons that we did, it would probably take even more time and I probably would have spent three times as much
time with these three lessons as I would have on these topics before but I think it’s time well spent.”

Eoin O’Shea (science and mathematics teacher)

“…it’d be nice to have a lot of time to do something like that (the Authentic Integration tasks), to spend more time, to explore all the questions that came up. So, I suppose time would be another factor… I would have liked more time to go through it but, apart from that, I liked all the ideas and the lessons.”

Martina O’Reilly (mathematics teacher)

Adopting the approach set out in curriculum model applied within this research, as outlined above, required that more time be spent on the topics than regular tuition would necessitate but, according to O’Shea, this is something which is worth such a sacrifice:

“Time is the thing we keep coming back to and it’s such a pity… but we have a syllabus which you have to keep to in some way. But I would expect the understanding to be massively improved.”

Eoin O’Shea (science and mathematics teacher)

The teachers offered a solution to this problem: spread the lessons throughout the year. This would ensure that pupils can explore the topics in an in-depth manner without impinging on the completion of the curriculum:

“It may be time consuming. It may not be realistic that you’d get to spend so much time connecting up the subjects and teaching them in different ways and using the group tasks every single day in school. I definitely think it’s something that you could integrate throughout the year but I
don’t think it’s realistic to use it in every single class, especially when you’re trying to cover the curriculum.”

Jennifer Collins (mathematics teacher)

Again, the need to cover the curriculum is mentioned – an insight into the summative assessment-driven attitude towards learning which infiltrates the Irish education system, placing the following constraints upon the teachers (NCCA 2005, p.11):

- The need to use grades as a record of progress;
- The limited time available to cover the content on the course; and
- The anxiety attached to attempts to incorporate new or novel approaches.

To accommodate these constraints, using these lessons at various points throughout the year was a common suggestion from teachers rather than implementing a few lessons within a condensed time frame:

“I would love to stretch it out over the entire year, I would love to do your whole pack but (it’s) an awful lot of material to try and get done in a particular time frame.”

John Ryan (Science teacher)

This would suggest that teachers would prefer to pick times in the year where they could use such lessons to vary the teaching styles applied and allow pupils to explore the topics in a different context rather than trying to shoehorn a number of these lessons into a short period of time. This was an important realisation for the author which offers an insight into how these lessons could be incorporated into regular tuition in the future.
6.3.4 Teacher Knowledge

Allied to the need for coordination is the requirement for teachers to have a working knowledge of both subjects, a requirement which was explored in depth previously (see sections 4.2 and 4.6). This can obstruct the application of integration as teachers may be aware of how the subjects connect but may not have the expertise or the confidence to carry through these connections into their teaching:

“[O]bviously I’d be aware of the link [between the subjects], but it took someone to point it out for me to actually explain it perfectly to the students.”

Jennifer Collins (mathematics teacher)

Later in the interview she confirmed this anxiety:

“At the start I was a bit apprehensive about how I’d incorporate it into my teaching, how I’d put the whole lesson across and would I be able achieve what I was hoping to. But as I got into it, and the resources were brilliant that I was provided with, and it was laid out so well that it was easy to follow and deliver the lesson to the students. I really enjoyed it. It also gave me an awakening; it showed me how you can teach a topic in different ways and integrate it with something else.”

Jennifer Collins (mathematics teacher)

The author anticipated that this would be an important element within this research and thus ensured that each teacher would have the requisite level of knowledge within both mathematics and science prior to implementing the lessons by providing plenty of resources as well as training (see section 4.6).
This support system – which included powerpoint files on all the topics, teacher training, availability of the lead researcher for assistance – evidently proved to be quite important in ensuring that teachers were comfortable in implementing the lessons. Collins had some anxiety in relation to her ability to teach lessons which incorporated both mathematics and science (something she divulged to the author at the outset of the research) but made use of the resources and training available which gave her the confidence to conduct the integration lessons.

It proved not to be a stumbling block for the other teachers involved in this project as each of them found the content to be very manageable and none voiced any problems grasping elements that they weren’t previously overly familiar with. In fact, the participants gained great benefit from combining the subjects:

“I think it worked very, very well. From my own point of view, there’s a lot I didn’t know – some of the definitions in science… there’s a lot I didn’t know myself and they’re very maths related. So I think there’s so many areas in science that are related to maths and I think it’ll come through with the project maths type of questions that are coming in”.

Martina O’Reilly (mathematics teacher)

O’Reilly gained a greater insight into elements of science that mathematics could be related to and envisions the benefits that this will provide for her when adapting to the new ‘Project Maths’ syllabus (outlined in section 2.2.5) which is almost completely rolled out in Ireland. This signifies the potential for these types of lessons to be incorporated into the new curriculum as the lessons and the new curriculum have the common ground of making applications of mathematics central to their make-up:

“In particular, I will incorporate it into the Maths lessons. Some people say that’s a criticism of the new Project Maths syllabus – that it’s become more like engineering or too scientific. I would definitely be of the
opinion that that’s not a problem – I think you have to bring Maths into context; you have to make it real for them (the pupils). Even through all this time that I’ve done these classes with the 2nd years, I’ve been thinking of ways to bring it into my 5th year Project Maths class. That’s definitely on my mind for the future.”

Eoin O’Shea (science and mathematics teacher)

6.3.5 Conclusion

In general, teachers found the lessons to be worthwhile and beneficial but worried that they would not have enough time to implement the lessons and teach the curriculum at a typical pace. Having said that, the general consensus was that these lessons could be integrated into their teaching by facilitating lessons of this type at various times throughout the year to supplement their regular teaching. Some of the teachers were both mathematics and science teachers, so content knowledge in both subjects wasn’t a major issue. But for those who did not teach both, the content did not prove to be a problem, quite the opposite in fact as they found it quite beneficial to be exposed to new material which could aid their teaching.

6.4 Applying Material to Real Life Scenarios

One of the central components of Authentic Integration is to make the material ‘Applicable to Real World Scenarios’. The pupils recognised how this made the topics they were studying more relevant:

“A lot of maths… I didn’t really see the point in it – as in having to learn the formulas – I didn’t see the everyday use of them… I didn’t think you’d need it for anything… It (using mathematics in real life scenarios) definitely gives more of a point to maths.”

‘Pupil 2’ (Focus group)
Teachers also recognised the importance of this aspect of the lessons:

“I often hear from the students ‘where would we use this in normal life?’ or ‘why are we studying this topic?’ or ‘Is this just for the Junior Cert? When will I ever use it again?’ So, I suppose by showing its relevance in another subject, that subjects do link up and they’re not an isolated thing that you learn – I think that was beneficial because I was able to tell the students we’re not just learning a topic that has no relevance and they saw it put into use in real life.”

Jennifer Collins (mathematics teacher)

One of the key aims of the new Project Maths syllabus is to make mathematics more relevant to the lives and experiences of students (NCCA 2008). Similarly, this focus on real life applications of mathematics has been and is being adopted internationally (see section 2.2.2 and 2.2.3). As such, Authentic Integration should fit in with the current trends in mathematics education both nationally and internationally. Comments like those from Collins (above) indicate why this approach can pay dividends – pupils see why they are learning a new topic and how it can aid them in real life, offering further motivation to master the new material.

This could also address many of the issues identified by O’Donoghue (2004) regarding inadequate performance of 3rd level students in Ireland, Britain, Australia, and USA (see section 2.3.2) e.g. failure to deal with numerate challenges in everyday life; inability to solve problems; and inadequate conceptual knowledge. O’Shea (science and mathematics teacher) identified such advantages in using integration to apply these subjects, especially mathematics, to real life to aid learning and understanding:

“Sometimes I think pupils are doing their maths in Junior Cert – they’re doing graphs, finding slopes, finding midpoints, etcetera and there’s no
context to them. Whereas when they’re doing it in Science, they might have their own set of data for running 100 metres or something to do with Formula 1 or something they’re actually interested in. Understanding the mathematical concepts of the line would be far more effective when tied in with real data that they’re interested in.”

One of the major points being made here is that, for example: when using graphs in Science, the depth to which the mathematics element is explored would not be as great as it would be in a mathematics lesson whereas in a mathematics lesson the material being studied in relation to graphs may not be put into context in a meaningful way as it may be in science class. But, if mathematics and science were combined then the lesson would have the potential to explore the mathematics element in depth while having a rich context to make the material being learned meaningful. This would ensure that the best characteristics of each lesson would be combined to create a lesson of greater depth and meaning without hindering the level of learning in either subject.

Such an approach would also combat the deadening effect on student experiences of the separate subject approach identified by Beane (2009) while also showing the links between subjects instead of having them viewed, as they are by many pupils, as isolated blocks of knowledge (Jacobs 1989). Ryan (science teacher) also identified with O’Shea’s attitude to putting mathematics in context:

“I definitely intend on continuing on with this pack, especially the electricity lesson that I looked at – bringing the whole electricity bill in, doing the actual maths of finding the cost of the bill, calculating the kilowatts used and stuff like that, because, in terms of Science, you learn what electricity actually is but you don’t necessarily go into the everyday life element of the bill. Bringing the maths into that – the decimals, converting to percentages, etcetera – would be excellent.”
The students were also able to identify how placing the topics in context aided understanding and appreciation of the work they were completing as well as allowing them to apply more mathematics within science:

P4: It (the type of tasks completed) helps you understand Science a bit more.

P1: It’ll help you in later life. Like, if you were calculating your bill and you saw that the ESB made a mistake, you could check it.

P2: we went into more detail in the maths we do in Science than we would in a normal science class. Like in Science you have to learn the formula, then you’ve to do one or two sums but if you do it during maths, you do a lot more of the mathematical side.

One of the pupils (P2) seemed to identify that science lessons often only use mathematics as a means to an end e.g. to solve a problem using a formula, whereas the lessons they experienced go into greater depth when applying the mathematics to the context being explored. They also made the connection between the content they are learning and how it can be useful to them outside the classroom which can only have a positive effect on their appreciation of and motivation to study mathematics and science.

6.5 Overall Views of Integration

So far, the various elements and intricacies of the lessons incorporated into the pupils’ regular tuition have been discussed, but what were the overall conclusions drawn on the application of integration lessons?

“Extremely beneficial. When you’re trying to teach Junior Cert Science, and you’re doing certain lessons, like motions graphs, you realise that
students haven’t done things like the slope formula yet, so they can’t calculate speed from a (time-distance) graph. When you’re doing calculations like percentages or converting grams to kilograms or anything like that, (you notice that) their mathematical ability isn’t at that level yet. So, I think it would be a lot more beneficial to have the two subjects linked together because students will definitely benefit from it.”

John Ryan (science teacher)

Ryan has identified one of the advantages that could be gained from integrating the subjects i.e. ensuring that the pupils are learning elements of one subject that can be applied to the other. He gives the examples of calculating percentages and working out the slope of a graph in mathematics as being topics that should be studied at a similar time to the topics which relate to them in a Science context. The integration of the subjects would ensure that would occur and reinforce the learning taking place in both disciplines. O’Shea (science and mathematics teacher) echoed these sentiments:

“In regard to graphing and working out angles for the angle of incidence and reflection, the fact that the maths went side-by-side with the understanding of the Science topic was a huge thing… From teaching physics, this approach seems to make complete sense to me, where you would bring the mathematical concepts along with the scientific concepts.”

Breaking down the barriers between subjects to show the links between them and aid understanding in both was something the teachers repeatedly referred to as a positive impact made by integration of mathematics and science. Such an advantage has been referred to in research in a similar manner whereby Furner & Kumar (2007, p.186) suggested that “If done properly, integration of math and science could bring together overlapping concepts and principles in a meaningful way and enrich the learning context.” Collins (mathematics teacher) referred to
how this approach enriches learning in the classroom compared to a strictly didactic approach which depends on rote learning:

“The fact that the students have to develop the skills to achieve a particular goal, I think that’s relevant because a lot of it is being taught in such a manner that they are told to put everything down on paper and just learn it off, so I suppose the fact that they understand what they’re doing and why they’re doing it and the different steps involved is important. It also links in with the new ‘Project Maths’ course in that they have to understand the background and why they’re using and synthesising their knowledge while also seeing how it’s relevant in everyday life.

This echoes the suggestions made by Boaler (1994) to focus less on conditioning pupils to answer exam type questions, which only require that they remember the correct procedure that is to be applied, and more on meaningful, real world problems which require common sense. O’Reilly (mathematics teacher) was similarly enthusiastic regarding how the change in teaching approach benefited and will continue to benefit her pupils:

“I think it’s (the new approach) very good for the students. I think one of the problems in the past with maths was the teacher being the all-powerful person at the top… they can learn an awful lot from each other and, also, I think that sometimes they’ll explain certain things in their own language in a way that teachers wouldn’t. They’ll really be able to help each other out because they know how hard it is to grasp things. It’s the way forward.”

The positive views expressed above are reinforced by the positive reaction of the teachers to the process of completing the tasks involved. All the teachers agreed that the tasks were a worthwhile endeavour, three of them strongly agreeing. Much of this, the author suspects, can be attributed to the perceived gains made by pupils in understanding as mentioned previously by teachers (see section 6.2.2).
Further to these advantages, one of the main objectives of post primary tuition is to prepare pupils for 3rd level education. This has implications for the integration of mathematics and Science as many 3rd level institutions combine both subjects in service mathematics modules e.g. Science Maths in the University of Limerick. Collins (mathematics teacher) touched on this connection and identified how integration at post primary level could aid her pupils:

“Yeah, I think it’s definitely beneficial. I think there needs to be such an emphasis put on maths and science at the moment. It’s important that they do see such a link because, when it comes to taking on whatever courses they encounter at university level, maths and science are so closely linked. I think they need to realise that they’re not just isolated subjects but they’re a pairing in their own right. A lot of the things that they learn in each will overlap so excelling in one will also benefit the other. There’s definitely a strong correlation between the two so if they realise that at secondary level then hopefully they’ll see the benefits of it when they get to further education.”

Authentic Integration specifically aims to aid pupils in enhancing their knowledge and skills to be able to tackle everyday tasks and, also, challenges they face in further education or future jobs. Collins recognised this advantage as evinced by the above statement while the other teachers involved also clearly recognised such a benefit as all agreed, three of them strongly, that integrating mathematics and science is a good idea.

The pupils views within the focus group seemed to be somewhat mixed when asked whether they thought working on both mathematics and science in one class would be a good idea:

P4: No.

Interviewer: Why?
P4: Because it gets very confusing.

Interviewer: How does it get confusing?

P4: You're doing a lot of Science then you’re moving straight onto maths and you get lost along the way.

P1: Yeah but there’s a lot of Maths in Science as well, like calculations…
P3: Then you can be good at two subjects

P1: Yeah.

P2: If you’re good at Science and bad at maths or the other way around, it could be better. Like if you learn Science through Maths then it could help you if you're good at maths. If you learn Maths through Science then it could help you if you're good at Science. One might help you understand the other.

One pupil found that the integration of the two subjects confuses matters while another outlined how learning one subject through the other can aid understanding. Such an insight is important as it displays how some pupils are open and willing to take on something new and different while others may be more comfortable sticking with the norm. As such, teachers have to be aware of this and help pupils who aren’t motivated to change practice to become comfortable with integrating two subjects within one lesson.

Further proof of the teachers’ endorsement of the practice of integration of mathematics with other subjects is present in the responses of the teachers regarding their willingness to continue with lessons of this manner. Three of the four teachers strongly agreed that they would integrate mathematics with science and/or other subjects in the future thus indicating that they had a very positive
experience throughout the whole process, O’Shea’s (science and mathematics Teacher) thoughts offering a clear example of this:

“Very glad I got involved. I’m looking forward to using more of the resources with TY’s or other Junior Cert classes in the future. It’s a thing I’m really interested in actually. I think the thing that’s been really lacking in Junior Cert Science and Junior Cert Maths in particular is the lack of understanding of where you apply knowledge that you’ve learned in your maths class to other subjects, in particular Science.”

While Ryan (Science teacher) also fully endorsed the lessons used and the model applied:

“The approach that was taken on – I wouldn’t change it at all. The lessons were excellent – very well laid out, very easy to follow.”

The pupils also identified when integration could be quite useful:

Interviewer: So, do you think it should be separate all the time or bring together the two subjects sometimes?

P1, P2, P3, P5: sometimes would be good.

P2: You could bring them together for some physics classes but for chemistry or biology I don’t think they’d go together with maths. There’s just some parts of Science that Maths doesn’t go into and there’s some that it does.

Interviewer: why is it good then to sometimes bring the subjects together?

P2: ‘cause it can help you understand the subject.
The pupils indicated that integration can aid understanding but also outlined that the subjects can’t be integrated all the time, which, of course, is true as not all the mathematics curriculum links easily with the entire science curriculum in some form. As such, integration can be used at varying times of the school year as an alternative to regular tuition and as a means of enhancing learning:

“It may not be realistic that you’d get to spend so much time connecting up the subjects and teaching them in different ways and using the group tasks every single day in school. I definitely think it’s something that you could integrate throughout the year but I don’t think it’s realistic to use it in every single class, especially when you’re trying to cover the curriculum.”

Jennifer Collins (mathematics teacher)

The suggestions outlined by Collins offer a good starting point for Authentic Integration, whereby it would be incorporated into regular tuition throughout the academic year. By such means, Authentic Integration could act as an alternative teaching method to improve understanding within both mathematics and science until such a time as it could be incorporated into the curriculum and, thus, the assessment of each of the subjects.

6.6 Concluding Remarks on the Data Collected

There were many interesting facets of this research which have been analysed and discussed in depth – how well group work was introduced and received; the effects on understanding; the challenges faced by teachers and the researcher when developing and conducting Authentic Integration lessons; how the application of mathematics and science content to real life scenarios affected attitudes and learning; and the overall opinions of the teachers and pupils in relation to the lessons.
Initially there were minor problems with the introduction of group work to some of the classrooms as some pupils had rarely experienced such a set up. But as the lessons progressed, the teachers noted that the pupils adjusted well with enjoyment and enthusiasm increasing noticeably. This had an impact on understanding which some of the teachers believed improved noticeably (see section 6.2.2). While this was anecdotal, analysis of pupil performance showed that understanding of the material studied was of a relatively high level (see Fig. 6.1).

Innovation in a post-primary school setting has to negotiate the issue of time required to plan for and implement a new approach within a system which can be quite rigid. This proved to be a challenge within this research as ‘time’ and the need to ‘cover the curriculum’ were consistently referred to by the teachers. As such, some of the teachers suggested that the lessons could be implemented at different times throughout the year to supplement regular tuition rather than attempting to apply them in a condensed period of time. This suggestion would seem to be quite logical and will inform further development of these lessons and how they would be implemented.

Teacher knowledge within both mathematics and science was a potential stumbling block for the teachers who specialised in only one of the subjects. This was identified prior to research being carried out and was catered for with a support system which served to aid the teachers quite well as none reported any issues in this respect. Some teachers pointed out that the key challenge in this aspect was to identify how topics in mathematics and science could be merged into one lesson, the lesson guides did this for them thus allowing the teachers to implement the ideas outlined and adjust the tasks to suit the learning outcomes they wished to achieve. Any gaps in knowledge were catered for which meant that the success of this potential problem was one of the main successes of the research completed.
The application of material from mathematics and science to real life scenarios is central to Authentic Integration and proved to have a positive impact on pupil attitudes and understanding as some pupils mentioned how this element of the lesson gave more meaning to the topics they were studying as they could identify how and where they could use it in real life. Teachers endorsed this practice and also recognised how ideas from these lessons could aid them in implementing new material from the Project Maths syllabus currently being introduced nationwide. Similarly, the intervention carried out satisfied the parameters set out to gauge its effectiveness while the model created successfully embodied the vital characteristics set out by Schoenfeld (2000).

Overall, the teachers found the tasks to be worthwhile and had very positive feedback regarding the lessons and tasks they completed with the pupils. Most strongly intimate that they think integrating mathematics and science is a good idea and that they will do so in the future with their pupils. As such, while there are some areas to work on, the lessons were a success.

6.7 Effectiveness of the Intervention

The critical parameters by which an intervention can be evaluated were identified previously in the methodology chapter of this text (see section 3.5.6). Shapiro’s (1987, p.290) work provides the parameters, four in all, by which this intervention is evaluated:

- Treatment effectiveness,
- Treatment Integrity,
- Social validity,
- Treatment acceptability.
Figure 6.2: Shapiro’s (1987) four parameters for evaluating an educational intervention.

6.7.1 Treatment Effectiveness

The first parameter requires evaluation of how effective the intervention proved to be. This is determined by the amount of change which takes place; the immediacy of this change; and the strength of the change (Shapiro 1987). Such transformation is evident in the positive reactions of the teachers to employing integrative lessons through the Authentic Integration model (see sections 6.2, 6.4, 6.5). While the strength of this change is also evident in the attitudes of the teachers towards integration (see section 6.5) and the fact that they intimated that they will continue to implement lessons of this nature in the future.

Furthermore, the fact that these lessons were successfully integrated into the regular tuition of these Junior Cycle pupils in four different post primary schools indicates that such lessons can become a staple part of the Irish mathematics curriculum.
6.7.2 Treatment Integrity

The integrity of this intervention is dependent on whether or not its implementation would garner similar results if it were to be repeated. As such, treatment integrity is measured by the extent to which the intervention is actually executed in the manner prescribed in the documentation which leads to the need for a comprehensive outline of the intervention program (Shapiro 1987).

Integrity in this intervention was achieved through clear instructions to the teachers regarding the implementation of the intervention. These clear instructions were given in the resource pack created and throughout teacher training prior to the implementation of the intervention. Each teacher was made fully aware of what was expected of them in all aspects of the study e.g. how to apply the teaching model (Authentic Integration), the manner in which the lessons were to be taught, and the assessments to be conducted, as well as the procedures to be carried out (e.g. consent forms; pre- and post-intervention questionnaires; time frame involved).

Allied to that, the training provided by the author for each participating teacher ensured that they would have the requisite skills and knowledge to implement the lessons. Thus ensuring that the quality of teaching would be of a consistent and relatively high level throughout the schools involved, hence aiding the integrity of the intervention.

6.7.3 Social Validity

Social validity is referred to as “the evaluation of the intervention” by the participants (Shapiro 1987, p.293). The teachers and the pupils, the participants in this study, rated the intervention at various stages of the interviews, questionnaires, and focus groups completed during the investigation. For example, each teacher was asked their opinion on the tasks completed in class during the intervention (see section 6.5). Each of the four teachers agreed that the
lessons were a worthwhile endeavour, three of them strongly. Each of the teachers had similar views on the value of integrating mathematics and science at the end of the intervention. When some of the pupils were asked for their views on the lessons implemented in the intervention, their reactions were mixed i.e. some positive, some negative (see sections 6.5 and 6.3.2) although all the teachers suggested that the pupils seemed to enjoy working together to complete the tasks set (see section 6.3.2).

6.7.4 Treatment Acceptability

This parameter relates to the degree to which the participants accept the methods employed within the intervention as being suitable with regards to the problem and the participant (Shapiro 1987). In an educational scenario this pertains to the teachers involved and their acceptability is influenced by the time required to implement the scheme; the cost; their understanding of the intervention; and the aforementioned treatment effectiveness and integrity (Reimers et al. 1987).

Time was an issue alluded to by the teachers (see section 6.3.3) but this was more to do with their frustration at not having the flexibility within the structures in their school to allocate more time to lessons of this nature rather than any negative feelings towards the time required to implement the scheme. For example, Ryan (Science Teacher) referred to the pressure within the school to complete a set list of topics in a certain time frame (e.g. a half term or a full term) thus leaving little time for innovation (see section 6.3.3).

When asked in the interviews whether there were any negatives regarding the intervention that the teachers wished to outline, some referred to the aforementioned time issue while there were suggestions that adjusting to group work proved a little disruptive and/or difficult at the outset but no teacher voiced anything above a minor concern (see section 6.3.2).
There was no additional cost to the teachers or schools involved in the intervention. Materials such as copies in which student work was completed, resource pack, sheets for questionnaires and consent forms, resources, and other materials which a teacher or pupil would not generally use during normal tuition, were provided by the researcher.

As regards the general understanding of the intervention, information sheets were given to the pupils, their parents/guardians, the teachers, and the principles of the participating schools. These information sheets (see Appendix A) outlined exactly what was to take place during the intervention along with any vital information pertaining to the intervention. The participating teachers were provided with a resource pack which further described the design of the intervention and their role in it.

These procedures satisfied the need for treatment acceptability, allowing for the successful implementation of the intervention.

**6.8 Evaluation of Authentic Integration**

Below (Table 6.1) is a list of criteria for evaluating models and theories in mathematics education developed by Schoenfeld (2000). This list was a crucial tool in the design of Authentic Integration as it served as a vital method of assessment of the model throughout its development. Each of the criteria listed and described was considered and satisfied in the final design of the model for integrating mathematics and science.
Table 6.1: Schoenfeld’s (2000) criteria for evaluating models and theories in mathematics education.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Descriptive Power</td>
<td>Are all the important features of the concept being modelled included?</td>
</tr>
<tr>
<td>Explanatory Power</td>
<td>Providing a clear explanation of ‘how and why’ the various components of the model work.</td>
</tr>
<tr>
<td>Scope</td>
<td>Is the model suitable for just one application or can it be adapted to work in multiple situations?</td>
</tr>
<tr>
<td>Predictive Power</td>
<td>Can the model predict the constraints and likely events to occur through the teaching model, thus allowing for negative elements to be minimised?</td>
</tr>
<tr>
<td>Rigour and Specificity</td>
<td>How well defined are the terms? Would you know one if you saw one? In real life? In the model? How well defined are the relationships among them? And how well do the objects and relations in the model correspond to the things they are supposed to represent?</td>
</tr>
<tr>
<td>Falsifiability</td>
<td>Testing the effect of the model using empirical evidence i.e. putting its ideas on the line.</td>
</tr>
<tr>
<td>Replicability</td>
<td>Will the same thing happen if the circumstances are repeated? Will others, once adequately trained, ‘see’ the same things in the data?</td>
</tr>
<tr>
<td>Multiple sources of evidence (Triangulation)</td>
<td>The more independent sources of data there are, the more robust a finding is likely to be.</td>
</tr>
</tbody>
</table>
6.8.1 Descriptive Power

*Are all the important features of the concept being modelled included?*

According to research, within lessons which integrate mathematics and science, the content should be contextually based and taught in an authentic manner with plenty of hands-on group work, inquiry and discussion throughout (Frykholm and Glasson 2005, Daniels et al. 2005, Furner and Kumar 2007, NRC 1996). This has been achieved in the Authentic Integration model through the four key characteristics which underpin the model: Knowledge Development, Synthesis and Application; Focused Inquiry Resulting in Higher-Order Learning; Applicable to Real World Scenarios; and Rich Tasks (see section 4.4.2). Each of these characteristics are displayed as being central to the model (see sections 4.4.2.5 and 4.4.3) thus ensuring that all the important features of the concept are being modelled.

6.8.2 Explanatory Power

*Providing a clear explanation of ‘how and why’ the various components of the model work.*

The author goes into great detail defining the characteristics of Authentic Integration and justifies why each element is vital and how it contributes to the overall implementation of the model (see section 4.4.2). ‘How the components of the model work’ is addressed through the description of each characteristic. ‘Why the various components work’ is described in detail with referral to each characteristics’ roots in sound academic research (see section 4.4.2).

6.8.3 Scope

*Is the model suitable for just one application or can it be adapted to work in multiple situations?*
The Authentic Integration model is quite specific, as is required in this scenario. It can only be applied to integrate mathematics and science in an educational setting, but it can be applied to any connection of topics from mathematics and science, thus can be applied through any amount of lessons of this nature. Six such lessons are outlined in the resource pack (see section 5.3). The possibility for adaptation of the model for other scenarios is possible, just as it was possible to adjust Authentic Instruction to create Authentic Integration (see section 4.4) but that would take further investigation and research to realise such a goal.

6.8.4 Predictive Power

Can the model predict the constraints and likely events to occur through the teaching model, thus allowing for negative elements to be minimised?

One of the concerns voiced regarding the type of pupil centred group work which is central to Authentic Instruction and Authentic Integration is that when the onus is put on the pupils to assume responsibility for their own learning, the overall objectives of these tasks can get lost and pupils may not reach intended goals (King et al. 2009). To counteract this, teachers must demand a high level of quality work with certain goals which must be achieved through the completion of the task. This approach to avoiding such a pitfall is built into Authentic Integration through the ‘Focused Inquiry Resulting in Higher-Order Learning’ characteristic (see section 4.4.2.2). As such, the model predicts a possible major constraint and ensures that such a negative element is minimised.

6.8.5 Rigour and Specificity

How well defined are the terms? Would you know one if you saw one? In real life? In the model? How well defined are the relationships among them? And how well do the objects and relations in the model correspond to the things they are supposed to represent?
The terms of the model are clearly outlined, well defined, and theoretically sound as they are based around extensive research by the likes of Bloom (1956), Newmann (1995), Bossé et al. (2010), and Baggett (1989).

Each element of the model should be recognisable in both real life and through the model. The characteristics ‘Knowledge Development, Synthesis and Application’ and ‘Focused Inquiry Resulting in Higher-Order Learning’ can be identified in pupil behaviour and performance throughout the tasks. ‘Applicable to Real World Scenarios’ and ‘Rich Tasks’ are elements which define the tasks or challenges set within the lessons, thus they can be detected within the lesson plan set and the manner in which the lesson is conducted.

The relationships among these four characteristics are very well defined. This is achieved through the Authentic Integration Triangle (see section 4.4.3) which clearly outlines the foundation characteristics - ‘Knowledge Development, Synthesis and Application’ and ‘Focused Inquiry Resulting in Higher-Order Learning’ – as well as the characteristic that supplements this foundation - ‘Applicable to Real World Scenarios’. These three combine to define the final outcome – the rich task.

Finally, the objects and relations in the model correspond clearly to the things they are supposed to represent as each characteristic describes the required design and delivery of the lesson as well as the intended behaviours and quality of performance required.

6.8.6 Falsifiability

*Testing the effect of the model using empirical evidence i.e. putting its ideas on the line.*

The model has been successfully incorporated into four Irish post primary schools at Junior Cycle. It received very positive feedback from the practitioners of the
model i.e. the teachers (see sections 6.2, 6.4, 6.5), and it has been shown to result in a relatively high level of understanding amongst participating pupils (see section 6.2.2). As such, the ideas of the model have been put on the line and succeeded in being integrated into the everyday workings of a number of Irish post primary schools with positive results.

6.8.7 Replicability

*Will the same thing happen if the circumstances are repeated? Will others, once adequately trained, 'see' the same things in the data?*

The manner in which the lessons guides are designed and laid out should satisfy the need for replicability as the targeted practices for pupils, task outline, discussion topics, and ideas and progressions sections of the lesson plans clearly guide the teachers in their implementation of the lesson (see section 4.5.3). Similarly, the fashion in which each characteristic of Authentic Integration must be satisfied is clearly defined in each lesson created (see section 4.5.3.2) thus the lessons in different settings with similar circumstances should be implemented in a very similar manner to each other.

6.8.8 Multiple Sources of Evidence (Triangulation)

*The more independent sources of data there are, the more robust a finding is likely to be.*

As alluded to previously (see section 3.8), multiple sources of evidence were employed when assessing the level of success of the model when it was implemented through the intervention. For example, each participating teacher completed a questionnaire with respect to their views on integration and their experiences within the intervention. To supplement this source of data, they were interviewed so that they could elaborate on these views, thus giving a richer insight into their experience within the intervention and their attitudes towards the integration of mathematics and science.
In a similar manner, multiple sources were used when designing the Authentic Integration model. The work of Newmann et al. (1995, 1997, 2007) was central to the design with inspiration for different aspects of the model design emanating from Furner and Kumar (2007), Education Queensland (2001), Berlin and White (1994, 1998, 2010) to name but a few.

6.9 Summary of the Evaluation of the Intervention

Each of the teachers involved in the study found the implementation of Authentic Integration lessons to be quite a worthwhile endeavour, offering positive feedback on various aspects of the intervention. Such positivity is evinced by the declarations given by each teacher that they will continue to implement lessons of this nature in the future. The Authentic Integration model was also scrutinized theoretically as it satisfied Schoenfeld’s (2000) criteria for evaluating models and theories in mathematics education, while the methods used in testing the model adhered to Shapiro’s (1987) four parameters for evaluating an educational intervention.

This analysis of the intervention paves the way for conclusions to be drawn regarding the integration of mathematics and science, the Authentic Integration model, and how further progress can be made in this field. These conclusions and recommendations will be detailed in the next chapter.
Chapter 7

Conclusions and Recommendations for Further Research
7.1 Introduction

At this juncture, the author has described and discussed the reasons for undertaking this study; the manner in which a teaching model for integrating mathematics and science was created and applied; and the analysis of findings pertaining to the related intervention. Those preceding features of this study will be employed to facilitate the author in drawing conclusions from this investigation as well as outlining recommendations and possible future research pertaining to the Integration of mathematics and science.

7.2 Summary and Conclusions

The research questions outlined in the introduction chapter (see section 1.5) have framed the research which has taken place through this study and, in a similar manner, the answers to these questions inform the conclusions which will be drawn below. Through the process of answering these research questions, a summary of the research and findings was also completed to give a background to the eventual conclusions drawn.

7.2.1 Effectively integrating Mathematics and Science

*How can mathematics and science be effectively integrated? Is there a model which can be adopted or adapted to fit the needs of such an undertaking?*

The needs of mathematics education can be summed up thusly: there must be a reduction in the dependence on didactic methods of teaching; greater focus needs to be placed on real life applications of the subject; and a more holistic approach where the student is central to their own learning is required (outlined in section 2.2). The manner in which mathematics and science should be integrated, according to research, champions similar characteristics i.e. lessons of this nature should be contextually based and taught in an authentic manner with plenty of

This pedagogical approach underpins the type of teaching model which best suits the needs of integration while also addressing many of the issues currently challenging mathematics educators. Integrating mathematics and science in a manner which ensures significant learning takes place required a model which would embody these characteristics. The Authentic Instruction model provides a basis for integrating mathematics and science. This was due to the fact that this model is integrative in its very nature, positively affected learning of pupils throughout extensive studies, and displays many of the characteristics of effective integration recommended by academics in the field (see section 2.5).

Authentic Instruction was not designed specifically to integrate mathematics and science, thus it was adapted to fulfil this need and, as such, evolved into Authentic Integration. The new framework maintains the important elements of Authentic Instruction but it also caters for the specific needs for integrating mathematics and science derived from research. As such, mathematics and science can be effectively integrated by applying the Authentic Integration model. The endorsement of the model by the teachers who applied it (see section 6.5) indicates that the model can be successfully implemented in post primary schools while the manner in which it improved learning and understanding (see section 6.2.2) proves it is a worthwhile endeavour.

7.2.2 Integrating Integration

Is the education system, in its current form, flexible enough to incorporate such a radical change, taking into consideration resources, teacher knowledge within their non-specialist subject, and timetabling, among other issues?

The education system can incorporate the integration of mathematics and science but such an undertaking does require a certain amount of extra effort in particular
areas and consideration of various challenges which need to be discussed in detail beginning with the time required to implement such lessons.

7.2.2.1 Time needed to Integrate

Firstly, the time required to plan and implement these lessons was referred to by all of the teachers involved as being a hurdle they had to overcome (see section 6.3.3). The issue with this wasn’t necessarily the extra effort they had to put in but the fact that they would not complete the mathematics and/or science topics they intended to complete by the end of the term. The general consensus among the teachers as to how to combat this problem would be to intersperse the integration lessons throughout the school year so that the pupils can experience the tasks involved while the teacher can ensure that the targeted topics are completed in each term.

Co-ordinating individual mathematics and science lessons was also considered. This would have been a scenario whereby a task would be designed in which part of it would be completed in science class and another part completed in mathematics class. Such a venture was put to the relevant teachers in the participating schools but they suggested that this would not be possible as coordinating both subjects in this manner would prove nearly impossible due to the varying stages at which they teach topics from both subjects which would link up best. Team teaching, in which the science teacher and mathematics teacher would complete a lesson together was also rejected due to timetable issues i.e. both teachers would not be available to teach one class at the same time. In conclusion, timetabling proved to be a significant barrier but one that was overcome by employing one teacher only to conduct the lessons within either mathematics or science class time.

So it can be concluded that the lessons would be best integrated into typical tuition, in an Irish context, by applying them at various intervals throughout the school year rather than attempting to apply them in succession at a given point
and within either mathematics or science class rather than attempting to coordinate them.

7.2.2.2 Applying Group Work

Group work is another aspect of the Authentic Integration approach which needs to be considered carefully when implementing the model. Pupils had mixed feelings regarding the advantages of group work while teachers similarly differed in opinion when it came to putting the responsibility on the pupils to work in groups without major teacher input although they did indicate that the pupils enjoyed working together in groups (see section 6.3.2). Pupils that haven’t experienced group work too often must be eased into such a process using pair work and then naturally progressing into further group work so that they are fully prepared for the type of interaction that is inherent in Authentic Integration.

7.2.2.3 Teacher Knowledge

Students learn more from teachers who are skilled, experienced, and know what and how to teach (Darling-Hammond 2000, Goldhaber 2002, Rice 2003). A teacher may tick all these boxes when it comes to his/her specialised subject but the adoption of an additional subject within their classroom setting provides a further challenge. This is because their content knowledge, which is the best indicator of an effective teacher (Shulman 1986), within that auxiliary subject might not be of the required standard.

At the commencement of this study, some of the teachers, i.e. those not specialised in both mathematics and science, displayed some anxiety and indicated their trepidation regarding their lack of content knowledge within their non-specialist subject. The allocation of teacher training and the support structure put in place for this investigation allowed them to develop their knowledge within the topics they studied with the pupils through the lessons. It became clear as the
study progressed, and through interviews with the teachers (see section 6.3.4), that such development of knowledge was a vital element in the success of the lessons.

As such, research indicates that a teacher’s content knowledge in the subjects he/she teaches is of upmost importance, this translates to an integrative setting – content knowledge within both mathematics and science must be of a high standard to implement these lessons successfully (see section 4.2.4). As shown in this study, this can be achieved through provision of the relevant resources, a working support structure, and teacher training.

7.2.3 Is Integration of Mathematics and Science more effective?

*Is integrating mathematics and science more effective than teaching the subjects separately?*

There is no clear definitive answer to this question – integrating mathematics and science was deemed a very worthwhile endeavour by the teachers, a practice they will continue to implement (see section 6.5). It also resulted in a good level of understanding amongst the pupils (see section 6.2.2). But feedback from the teachers suggested that, taking into account the current structures inherent in mathematics education, integration of mathematics and science is not an approach which can be applied all the time, instead it should be seen as something which supplements typical tuition:

“I definitely think it’s something that you could integrate throughout the year but I don’t think it’s realistic to use it in every single class, especially when you’re trying to cover the curriculum.”

Jennifer Collins (mathematics teacher)

Taking into account the views of Collins, integrating mathematics and science should not be considered to be in direct opposition to single subject tuition, especially when the widespread prevalence of separate subject curricula is taken
into account. An integrated syllabus may not replace the current single subject approach but lessons of an integrative nature may complement the current system.

This is particularly relevant in an Irish context as the characteristics of Authentic Integration foster an approach which complements the aims of the new post primary mathematics syllabus commonly referred to as ‘Project Maths’ (see section 2.2.5). For instance, Project Maths aims to “make mathematics more relevant to the lives and experiences of students” (NCCA 2008, p.2) – this matches the ‘Applicable to Real World Scenarios’ characteristic which is central to Authentic Integration (see section 4.4.2.3). Allied to that, Project Maths seeks to “give greater emphasis to the understanding of mathematical concepts and the application of mathematical knowledge and skill” (NCCA 2008, p.2) – again this is replicated in the Authentic Integration model through the requirement for ‘Knowledge Development, Synthesis and Application’ as well as the need for pupils to display ‘Focused Inquiry Resulting in Higher-Order Learning’ (see section 4.4.2).

Thus, integration of mathematics and science should, under current circumstances, adopt a niche role whereby it aids learning and understanding within both mathematics and science rather than replacing the separate tuition of the subjects.

**7.2.4 Teachers’ Valuation of Integration of Mathematics and Science**

*Do teachers value the process of integrating mathematics and science? Will they want to continue to integrate mathematics and science on a regular basis?*

Upon completion of the intervention carried out through this study, the teachers involved displayed very strong support for the practice of integrating mathematics and science. All four agreed, through the questionnaire they completed, that such a practice is ‘a good idea’ and that they will integrate mathematics with science and/or other subjects in the future, in each instance three out of the four teachers
strongly agreed. Each of the teachers also highlighted their support for integrating mathematics and science at varying stages in their interviews (see section 6.5).

Thus, in conclusion, the teachers most certainly valued the process of integrating mathematics and science, while the probability of them integrating the subjects on a regular basis is quite high as they strongly agreed they would complete such lessons again in the future.

7.2.5 Design Principles for Integrating Mathematics and Science

The author has put together a number of principles for integrating mathematics and science in an educational setting. These principles emanated from the process of developing and implementing the Authentic Integration model i.e. determining whether it would actually work as proposed (see ‘Proof of Concept’, section 3.4.4).

7.2.5.1 Pupil-Centred, Active, Meaningful, Challenging Tasks

As stated throughout this text, research into the integration of mathematics and science recommends that content should be contextually based and taught in an authentic manner with plenty of hands-on group work, inquiry and discussion throughout (Frykholm and Glasson 2005, Daniels et al. 2005, Furner and Kumar 2007, National Research Council 1996). Such an approach was implemented through this study in the form of the Authentic Integration model. The feedback from the teachers and the assessment of pupil work indicates that pupil understanding benefited from this approach thus it is a central principle emanating from this research.

7.2.5.2 Meaningful Discussion

Discussions should take place throughout the lesson to promote deep thinking; development and explanations of concepts and ideas; and assess pupil learning i.e.
as a means of formative assessment. Such discussions should take place within
groups, between groups, between teachers and pupils, and in a whole class
manner to aid peer learning as well as progress towards viable solutions to the
task(s) set.

7.2.5.3 High Standard Investigation with Specific Purpose

Teachers should set goals and challenge pupils to reach them throughout
completion of the tasks set. Expectations of higher order learning of concepts and
processes present in the topics explored should be set, outlined to the pupils, and
maintained throughout each lesson.

7.2.5.4 Focus on the Group

Pupils should be encouraged to take advantage of their groups to solve problems
or improve learning rather than depending on the teacher as the source of
information or to validate their individual ideas. Whole class discussions should
be employed also as a manner for groups to share ideas.

7.3 Recommendations

The author has, through the findings detailed in this investigation, established a
number of recommendations for mathematics education in general as well as the
integration of mathematics and science. These recommendations aim to improve
practice within these fields.

7.3.1 Reduce Dependence on ‘Drill and Practice’

Mathematics educators need to move away from dependence on the ‘drill and
practice’ approach to teaching the subject. Pupils need to be challenged to apply
the mathematics they learn in different scenarios and contexts so that they develop
a deeper and more rounded understanding of the topics they are exploring rather
than repeating algorithms ad nauseum without understanding what they are doing and why they are doing it.

7.3.2 Greater Implementation of Applications and Real Life Links

Too much of what pupils learn in mathematics, especially at secondary school level, can be termed as ‘abstract-apart’ i.e. concepts are learned with no link to the contexts in which they occur (Mitchelmore and White 1995). The process of challenging pupils to apply mathematics concepts to real world contexts must become common practice in the Irish education system. This, at the very least, will ensure that, once these pupils complete their second level education, they will be able apply what they have learned in the mathematics classroom to challenges they will face in their daily lives. Much of what is studied by these pupils is just learned for the sake of passing examinations without any real understanding of how or where it can be applied (Lyons et al. 2003), this needs to change.

7.3.3 Incorporate Integration into Mathematics Teacher Training

Integrating mathematics with other subjects, such as science, needs to be incorporated into mathematics teacher training courses if it is to become a practice which is commonly adopted within the classroom. Progress is being made in this respect as Berlin and White (2010) reported on a Master’s Degree programme which makes the integration of mathematics and science the central focus of study within the course. As reported previously, teachers require decent levels of content knowledge within both mathematics and science if they are to integrate the subjects effectively (see section 4.2), knowledge which they are not likely to possess (see section 4.6). If this knowledge could be developed, along with greater understanding regarding the techniques for integrating mathematics and science, through third level teaching courses then teachers would be in a much stronger position to integrate the subjects effectively.
7.3.4 Add Practical Applications and Projects to the Mathematics Syllabus

The old cliché: ‘Assessment is the tail that wags the curriculum dog’, is relevant to the current situation in Irish mathematics education whereby there is a great need for more application of mathematics to real life scenarios (Lyons et al. 2003, Boaler 1994). The success of such an addition to the curriculum would most likely be dictated by the manner in which these real life applications would be assessed. If projects which call for applications of mathematics in a real life context were to contribute to a pupils’ final grade at Junior Cycle or Senior Cycle then this would give the practice greater credence and thus encourage teachers to focus on this type of approach to tuition in their classrooms. This could extend to the application of both mathematics and science together through such projects in the future also.

7.4 Contributions

The work carried out through this study has led to a number of contributions to the field of mathematics education and, to a lesser extent, science education. The Authentic Integration model, the lessons produced through the model, the lesson guide design used to outline these lessons, and the manner in which the intervention was conducted will all add to the body of research of mathematics education and, to a greater extent, the integration of mathematics and science, especially from an Irish perspective.

7.4.1 The Authentic Integration Model

The Authentic Integration model was designed using a sound theoretical base, Authentic Instruction, and it succeeds in incorporating the needs of integration of mathematics and science (see section 4.7). As such, it presents a key reference for the planning and implementation of lessons which integrate mathematics and science, thus making a substantial contribution to research in this area.
7.4.2 Design Principles for Integrating Mathematics and Science

The design principles for integrating mathematics and science (outlined in section 7.2.5) offer researchers and educators a clear set of criteria to follow when developing their own lessons which integrate the subjects. These principles will guide future research also as it provides a theoretical basis for advancement in this field.

7.4.3 Authentic Integration Lessons

The lessons created and outlined in Chapter 5 offer vital resources for teachers who wish to integrate mathematics and science. Similarly, the ideas outlined could provide inspiration for further lessons of an integrative nature. The lessons created by the author could also serve as a means for teachers to introduce applications to their mathematics lessons i.e. a reduction in the focus on the science element of the lessons and greater emphasis on the mathematics aspect would allow teachers to use the ideas outlined for applications of mathematics in real life scenarios.

7.4.4 New Lesson Guide Design

The creation of a new teaching model (Authentic Integration) produced a further challenge – designing lesson plans, or lesson guides as they were referred to, which would accurately portray how every aspect of the model should be satisfied through the lessons implemented (see section 4.5). These lesson guides were theoretically sound as their structure was based around the New Basics project, spearheaded by Education Queensland (2001), and the guides also drew from other related research e.g. Pegg and Graham (2007), Reece and Walker (2000), and Martino and Maher (1999). The design provides a new and innovative template for displaying each facet of lessons which Integrate mathematics and science.
7.4.5 Cultivating an Approach to Implementing Lessons of an Integrative Nature in a Single Subject Focussed System

The schools in which the intervention was implemented did not typically integrate subjects. As such, there was no structure in place to achieve integration of subjects such as mathematics and science. The design of the intervention, coupled with what was learned from teacher and pupil feedback, provided an outline of how integrative lessons should be incorporated into schools where single subject focus prevails. As stated previously, these lessons should be implemented at varying stages throughout the school year to provide pupils with the benefits inherent in integrating mathematics and science while allowing the teacher to maintain an acceptable level of progress through the curriculum set (see section 6.3.3).

7.5 Further Research

This investigation into the integration of mathematics and science at Junior Cycle in Irish post primary schools has produced an in-depth insight into the requirements of integration, a model for implementing integration, lessons which successfully integrate topics from both subjects, and a comprehensive intervention which gauged the success of the model. Further research into the area of integrating mathematics and science would include the following:

- Further testing of the Authentic Integration model is required to gain a substantial insight into its effectiveness. This would be achieved by applying it in a greater number of schools and over a longer period of time (e.g. a full school year).

- The knowledges a teacher requires to integrate mathematics and science effectively was discussed in this investigation (see section 4.2). This is an interesting aspect of the study which has the potential for much deeper
investigation. As such, it is an issue which should be considered for future research.

- To implement the Authentic Integration model, six lessons were developed for this purpose and applied through the intervention. More lessons of this nature need to be created and employed so as to include a wider range of topics in any future investigation into the model and, also, to act as further resources for teachers to apply in their regular tuition.

- As outlined in the recommendations located in this chapter (see section 7.3.4), teacher training should incorporate the development of techniques and knowledges pertaining to the integration of mathematics and science. The manner in which this is conducted should be researched thoroughly so that teachers experience the best training possible, thus leading to their delivery of quality lessons which integrate mathematics and science once they enter the teaching profession.
Appendices
Appendix A – Consent Forms
Title of Project:  *An Investigation into the Integration of Mathematics and Science at Junior Cycle in Irish Post-Primary Schools.*

Your child is under **no** obligation to participate in this study. If they agree to participate but, at a later stage, feel the need to withdraw, they are free to do so – it will not affect them in any way.

**Please answer all of the following by ticking the appropriate box:**

- I have read and I understand the accompanying information sheet.  
  - Yes  
  - No

- I understand what the project is about, and what the results will be used for.  
  - Yes  
  - No

- I am fully aware of all the procedures involving my child and of any risks and benefits associated with the study.  
  - Yes  
  - No

- I know that my child’s participation is voluntary and that they can withdraw from the project at any stage without giving any reason.  
  - Yes  
  - No

- I am aware that my child’s contributions will be kept confidential.  
  - Yes  
  - No

**I agree to my child’s participation in the above study.**  

________________________          __________  
Signature of Participant          Date

_______________________          __________  
Signature of Parent/Guardian    Date
Teacher Consent Form

Title of Project: An Investigation into the Integration of Mathematics and Science at Junior Cycle in Irish Post-Primary Schools.

Please answer all of the following by ticking the appropriate box:

I have read the Teacher information sheet and the purpose of the study has been explained to me. □ □

I understand that I may withdraw from the study at any stage and if I do so, any data relating to my participation will be destroyed immediately. □ □

I understand that the data will be stored for the duration of the study with access only by the researcher and the supervisor. I understand that all computer files containing teacher and/or student data and information will be password protected, while all other data relating to participants will be secured in a locked cabinet. □ □

I understand that I may contact the researcher or supervisor if I require any further information about the study. □ □

I understand that a copy of any interview transcript will be made available to me, should I wish to check it for accuracy. □ □

I have read and understand the conditions under which I will participate in this study and give my consent to be a participant. □ □

________________________          __________
Signature of Teacher         Date

________________________          __________
Signature of Researcher       Date
Principal Consent Form

Title of Project: *An Investigation into the Integration of Mathematics and Science at Junior Cycle in Irish Post-Primary Schools.*

Please answer all of the following by ticking the appropriate box:

I have read the Principal information sheet and the purpose of the study has been explained to me.

I understand that I may withdraw the school from the study at any stage and if I do so, any data relating to the schools participation will be destroyed immediately.

I understand that the data will be stored for the duration of the study with access only by the researcher and the supervisor. I understand that all computer files containing teacher and/or student data and information will be password protected, while all other data relating to participants will be secured in a locked cabinet.

I understand that I may contact the researcher or supervisor if I require any further information about the study.

I understand that a copy of any interview transcript will be made available to me, should I wish to check it for accuracy.

I have read and understand the conditions under which I will participate in this study and give my consent, on behalf of the school, to be a participant.

________________________          __________
Signature of Principal        Date

________________________          __________
Signature of Researcher       Date
Pupil Consent Form

Title of Project: An Investigation into the Integration of Mathematics and Science at Junior Cycle in Irish Post-Primary Schools.

You are under **no** obligation to participate in this study. If you agree to participate but, at a later stage, feel the need to withdraw, you are free to do so – it will not affect you in any way.

**Please answer all of the following by ticking the appropriate box:**

I have read and I understand the accompanying information sheet. [ ] Yes [ ] No

I understand what the project is about, and what the results will be used for. [ ] Yes [ ] No

I am fully aware of all the procedures involved and of any risks and benefits associated with the study. [ ] Yes [ ] No

I know that my participation is voluntary and that I can withdraw from the project at any stage without giving any reason. [ ] Yes [ ] No

I am aware that my contributions will be kept confidential. [ ] Yes [ ] No

**I agree to participate in the above study.** [ ] Yes [ ] No

________________________________  __________  __________________________________ _________
Signature of Investigator     Date    Signature of Participant Date

________________________________  __________
Signature of Parent/Guardian      Date
Appendix B – Information Letters
Title: An Investigation into the Integration of Mathematics and Science at Junior Cycle in Irish Post-Primary Schools.

What is the study about?
This study aims to combine mathematics and science through projects and/or tasks to be completed by the pupils during class. It is an attempt to improve the methods used to educate pupils in these subjects by applying a new approach. The effects of this approach on pupil learning and attitude towards the subjects will be gauged using assessment of pupil work (the individual results of which will not be made known to the school), questionnaires, and interviews. The results will be published in a thesis and within academic papers.

What will my child have to do?
Your child will be expected to participate in these tasks/projects and complete a questionnaire at the end of the process. They may also be asked to participate in a focus group (this is where a small group of pupils will be asked about the new approach and will discuss it, putting forward any opinions they may have).

What are the risks?
The risk involved will be minimal: questionnaires will be anonymous and will not contain any unnecessary personal questions to avoid embarrassment as much as possible. The focus group questions will also be closely monitored to avoid any questions that may cause the pupils to divulge any personal information. Questionnaires and focus group questions have received ethical approval from the University of Limerick.
What if I do not want my child to take part?
There is no obligation to take part and your child can be withdrawn by you at any time during the study with no consequences whatsoever.

What happens to the information?
The information retrieved will be dealt with and handled in complete confidence whereby results of the participants as well as their confidentiality are the first priority of the researchers carrying out the study. After the completion of the study, information will be kept electronically on the principal investigator’s password-protected computer and under lock and key at the University of Limerick.

Who else is taking part?
There will be a number of other pupils in the school taking part as well as pupils from various schools in Ireland.

What if something goes wrong?
In the unlikely event that something goes wrong, the testing procedure will immediately cease.

What happens at the end of the study?
At the end of the study the information will be used to present results but the information here will be completely anonymous. All subject detail/information and data will be held by the principal investigator for up to 7 years in a password-protected computer at UL.

What if I have more questions or do not understand something?
If you do not understand any aspect of the study we would urge you to come forward to the researchers and discuss any questions that you might have. It is important that parents and/or guardians of the participants feel completely at ease throughout the study.
What if I change my mind during the study?
Should you feel at any stage that you want your child to discontinue being a participant then you are free to withdraw them so that they take no further part.

Kind regards,
Páraic Treacy (NCE-MSTL, University of Limerick)

E-mail: paraic.treacy@ul.ie       john.odonoghue@ul.ie
Phone:    087 xxxxxxxx            061 xxxxxx

If you have any concerns about this study and wish to contact an independent party, you may contact:

The Chairman of the University of Limerick Research Ethics Committee
c/o Vice President Academic and Registrar’s Office
University of Limerick
Limerick
Tel: 061 202022
Teacher Information Letter

Title: An Investigation into the Integration of Mathematics and Science at Junior Cycle in Irish Post-Primary Schools.

What is the study about?
This study aims to combine mathematics and science through pupil-centred, hands-on projects and/or tasks to be completed by the pupils (mainly in groups) during class. It is an attempt to improve the methods used to educate pupils in these subjects by applying a new approach. The effects of this approach on pupil learning and attitude towards the subjects will be gauged using assessment of the work they’ve produced, questionnaires, and focus groups. The results will be published in a thesis and within academic papers.

What will I have to do?
At the end of a couple of months of normal instruction, you will implement three integration lessons to enhance the learning that has taken place previously. Elements of science and mathematics topics that have been recently studied by the pupils will be integrated within these lessons through challenging tasks of an authentic nature.

How much time will it take?
The study is expected to last for two weeks. It is expected that you will have to dedicate one or two class periods to each lesson, completing three lessons in all i.e. a maximum of 6 class periods. In these lessons you will be expected to complete tasks/projects (that integrate mathematics and science) with the pupils. At the end of the study, a short amount of time will be required to complete pupil
questionnaires (5 minutes to complete) and focus groups (10-15 minutes) as well as teacher questionnaires (2 minutes) and interviews (10 minutes).

**What are the risks?**
The risk involved will be minimal: questionnaires will be anonymous and will not contain any unnecessary personal questions to avoid embarrassment as much as possible. The focus group questions will also be closely monitored to avoid any questions that may cause the pupils to divulge any personal information.

**What if I do not want to take part?**
There is no obligation to take part and you can withdraw at any time during the study with no consequences whatsoever.

**What happens to the information?**
The information retrieved will be dealt with and handled in complete confidence whereby results of the participants as well as their confidentiality are the first priority of the researchers carrying out the study. After the completion of the study, information will be kept electronically on the principal investigator’s password-protected computer and under lock and key at the University of Limerick.

**Who else is taking part?**
There will be a number of other schools in Ireland participating, about 4-6 in total.

**What if something goes wrong?**
In the unlikely event that something goes wrong, the testing procedure will immediately cease.

**What happens at the end of the study?**
At the end of the study the information will be used to present results but the information here will be completely anonymous. All subject detail/information
and data will be held by the principal investigator for up to 7 years in a password-protected computer at UL.

**What if I have more questions or do not understand something?**
If you do not understand any aspect of the study we would urge you to come forward to the researchers and discuss any questions that you might have. It is important that participants feel completely at ease throughout the study.

**What if I change my mind during the study?**
Should you feel at any stage that you want to discontinue being a participant then you are free to withdraw and take no further part.

Kind regards,
Páraic Treacy (NCE-MSTL, University of Limerick)

E-mail:  paraic.treacy@ul.ie  john.odonoghue@ul.ie
Phone:    087 xxxxxxxx      061 xxxxxx

**If you have any concerns about this study and wish to contact an independent party, you may contact:**

The Chairman of the University of Limerick Research Ethics Committee  
c/o Vice President Academic and Registrar’s Office  
University of Limerick  
Limerick  
Tel: 061 202022
Dear Principal,

My name is Páraic Treacy and I am pursuing a PhD in Mathematics Education under the supervision of Professor John O’Donoghue (Director of the NCE-MSTL) at the University of Limerick. My research is based around the integration of Mathematics and Science, entitled: “An Investigation into the Integration of Mathematics and Science at Junior Cycle in Irish Post-Primary Schools”.

To give you an overview of the motivation for this research: within the academic community there have been calls for a greater range of assessment practices as well as a more holistic approach to learning which encompasses the cognitive, psychomotor and affective domains. Allied to these changes, international education groups have lent their support to the drive to integrate mathematics with other subject areas, especially Science, within second level education. Attempts at integrating the subjects have been made but no definitive framework has been developed to date. Evidence suggests that hands-on, practical, student-centred, authentic activities are best when developing an effective integrative model for Mathematics and Science.

My aim is to test this theory by implementing lessons which mix elements of Mathematics and Science through the completion of projects and tasks by 2nd Year pupils. Each lesson will take one or two timetabled classes to complete. Three lessons should be completed by each participating class group – this should take 1-2 weeks to complete. The general approach is as follows: the Mathematics teacher and the Science teacher will complete topics with their pupils as normal; the only change is that, at the end of a couple of months of normal instruction, three integration lessons will be implemented to enhance the learning that has taken place previously. Elements of Science and Mathematics topics that have
been recently studied by the pupils will be integrated within these lessons through challenging tasks of an authentic nature. These lessons will be taught by one teacher only. I will assess the effects of this intervention through questionnaires, focus groups, and assessment of the work produced by the pupils.

Participation in this study is voluntary, and your school’s participation would be greatly appreciated. Lesson plans, materials, resources and any other requirements will be provided for teachers and pupils during this intervention. Teachers will be provided with an extensive resource pack and a memory stick containing all the information included in the resource pack. Also, any information gathered will be carefully kept to ensure complete anonymity. If, at any time, the school wishes to withdraw from this intervention then it may do so. Thank you for considering my proposal, I hope to hear from you in the near future.

Kind regards,
Páraic Treacy
October 2011

E-mail: paraic.treacy@ul.ie john.odonoghue@ul.ie
Phone: 087 xxxxxxx 061 xxxxxx

If you have any concerns about this study and wish to contact an independent party, you may contact:

The Chairman of the Faculty of Science and Engineering Research Ethics Committee
University of Limerick
Limerick
Tel: 061 202217
Appendix C – Teacher Questionnaire
Teacher Questionnaire

Instructions: Please circle the number that corresponds to your opinion.

<table>
<thead>
<tr>
<th>1 = Strongly disagree</th>
<th>2 = Disagree</th>
<th>3 = Not sure</th>
<th>4 = Agree</th>
<th>5 = Strongly agree</th>
</tr>
</thead>
</table>

1. Integrating Mathematics and Science is a good idea

1  2  3  4  5

2. I will integrate Mathematics with Science and/or other subjects in the future

1  2  3  4  5

3. I think that completing the tasks which integrated Mathematics and Science helped improve the pupils’ Mathematics competency

1  2  3  4  5

4. The pupils worked well when given a task to complete without major teacher input

1  2  3  4  5
5. The pupils appeared to enjoy working together to complete the tasks set

1 2 3 4 5

6. Overall, I think the tasks were a worthwhile endeavour

1 2 3 4 5
Appendix D – Pupil Focus Group Script
Pupil Focus Group Script

1. Do you like working in groups during Maths class?

2. Do you like Maths classes where you’re figuring out how to do a question together in groups or do you prefer it when the teacher tells you how to do it?

3. Do you see [insert mathematics topic] as being more relevant since you did [relevant task]?

4. Do you think working on Maths and Science together in one class is a good idea? Why/why not?

5. Do you think there is a connection between Maths and Science?

6. Is there any connection between Maths and other subjects?

7. Did you enjoy the tasks you did in class? Why/why not?

8. Has completing those tasks changed your opinion of maths? How?
Appendix E – Rubric for Assessing Pupil Work
<table>
<thead>
<tr>
<th>Category</th>
<th>Excellent (4)</th>
<th>Good (3)</th>
<th>Adequate (2)</th>
<th>Poor (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge Development, Synthesis, and Application</td>
<td>Analysis and application of knowledge was involved throughout the student’s work</td>
<td>Analysis and application of knowledge was involved in a significant proportion of the student’s work</td>
<td>Analysis and application of knowledge was involved in some portion of the student’s work</td>
<td>Analysis and application of knowledge constituted no part of the student’s work</td>
</tr>
<tr>
<td>Focused Inquiry Resulting in Higher Order Learning: Concepts</td>
<td>The student demonstrates exemplary understanding of the concepts that are central to the assignment</td>
<td>The student demonstrates significant understanding of the concepts that are central to the assignment</td>
<td>The student demonstrates some understanding of the concepts that are central to the assignment</td>
<td>The student demonstrates very little or no understanding of the concepts that are central to the assignment</td>
</tr>
<tr>
<td>Focused Inquiry Resulting in Higher Order</td>
<td>Explanations or arguments</td>
<td>Explanations or arguments are present.</td>
<td>Explanations, arguments, or representations</td>
<td>Explanations, arguments, or representations</td>
</tr>
<tr>
<td>Learning: Written Communication</td>
<td>are clear, convincing, and accurate, with no significant errors</td>
<td>They are reasonably clear and accurate, but less than convincing</td>
<td>are present. However, they may not be finished, may omit a significant part of an argument/explanation, or may contain significant errors</td>
<td>are absent or, if present, are seriously incomplete, inappropriate, or incorrect.</td>
</tr>
</tbody>
</table>
Appendix F – Teacher Interview Script
Teacher Interview Questions

1. Do you think pupil-centred, hands-on, group tasks are beneficial? Why/why not?

2. In your opinion, what were the positive outcomes from completing the tasks?

3. In your opinion, what were the negative outcomes from completing the tasks?

4. What would you change about the approach used?

5. Do you think that the pupils learned more or less using these teaching methods than they would if the class was taught using traditional methods? Why?

6. What is your opinion of the teaching model employed? How would you improve on it?

7. Do you think integration of Mathematics and Science is beneficial? Why/Why not?

8. Do you think you will incorporate similar lessons into your Mathematics teaching in the future? Why/Why not?
Appendix G – Pupil Questionnaire: Aiken Value and Enjoyment Scales
### Pupil Questionnaire

**Gender:** ____________  **Age:** ______  **School Year:** ____

**Instructions:** Draw a circle around the answer which shows how closely you agree or disagree with each statement.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SD</strong> = Strongly disagree</td>
<td><strong>D</strong> = Disagree</td>
<td><strong>U</strong> = Not sure</td>
<td><strong>A</strong> = Agree</td>
<td><strong>SA</strong> = Strongly agree</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>I enjoy going beyond the assigned work and trying to solve new problems in mathematics.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>2.</td>
<td>Mathematics is enjoyable and stimulating to me.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>3.</td>
<td>Mathematics makes me feel uneasy and confused.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>4.</td>
<td>I am interested and willing to use mathematics outside school and on the job.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>5.</td>
<td>I have never liked mathematics, and it is my most dreaded subject.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>6.</td>
<td>I have always enjoyed studying mathematics in school.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>7.</td>
<td>I would like to develop my mathematical skills and study this subject more.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>Mathematics makes me feel uncomfortable and nervous.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>---</td>
<td>------------------------------------------------------</td>
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</tr>
<tr>
<td>8.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>I am interested and willing to acquire further knowledge of mathematics.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>10.</td>
<td>Mathematics is dull and boring because it leaves no room for personal opinion.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
<tr>
<td>11.</td>
<td>Mathematics is very interesting, and I have usually enjoyed courses in this subject.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
</tr>
</tbody>
</table>
**Instructions:** Draw a circle around the answer which shows how closely you agree or disagree with each statement.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<tr>
<td><strong>SD</strong> = Strongly disagree</td>
<td><strong>D</strong> = Disagree</td>
<td><strong>U</strong> = Not sure</td>
<td><strong>A</strong> = Agree</td>
<td><strong>SA</strong> =</td>
<td></td>
</tr>
</tbody>
</table>

1. Mathematics has contributed greatly to science and other fields of knowledge.  
   **SD** | **D** | **U** | **A** | **SA**

2. Mathematics is less important to people than art or literature.  
   **SD** | **D** | **U** | **A** | **SA**

3. Mathematics is not important for the advance of civilization and society.  
   **SD** | **D** | **U** | **A** | **SA**

4. Mathematics is a very worthwhile and necessary subject.  
   **SD** | **D** | **U** | **A** | **SA**

5. An understanding of mathematics is needed by artists and writers as well as scientists.  
   **SD** | **D** | **U** | **A** | **SA**

6. Mathematics helps develop a person's mind and teaches him to think.  
   **SD** | **D** | **U** | **A** | **SA**
<table>
<thead>
<tr>
<th></th>
<th>Mathematics is not important in everyday life.</th>
<th>SD</th>
<th>D</th>
<th>U</th>
<th>A</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.</td>
<td>Mathematics is needed in designing practically everything.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
</tr>
<tr>
<td>9.</td>
<td>Mathematics is needed in order to keep the world running.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
</tr>
<tr>
<td>10.</td>
<td>There is nothing creative about mathematics; it's just memorizing formulas and things.</td>
<td>SD</td>
<td>D</td>
<td>U</td>
<td>A</td>
<td>SA</td>
</tr>
</tbody>
</table>


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