An Analysis of Performance in Mathematics for Technology Undergraduates and an Investigation of Teaching Interventions for these Students

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For the award of Doctor of Philosophy

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Abstract
Declining standards in students’ mathematical competency levels has become a major issue in mathematics education both nationally and internationally (Smith 2004; Kajander and Lovric 2005; Gill et al 2010). This decline in standards, which has commonly become known as the ‘Mathematics Problem’, refers to issues such as poor numeracy skills in beginning undergraduates, difficulties with basic arithmetic and algebraic manipulations and an inability to cope with mathematics which is presented in unfamiliar formats (Hourigan and O’Donoghue 2007). Economists and educationalists agree that competent citizens in the area of mathematics and science are necessary for a successful economy (OECD 2006; Breen et al 2009; IBEC 2010). The need to try and overcome, or at least alleviate somewhat, the ‘Mathematics Problem’ has therefore been a priority of many third level institutions worldwide (Croft 2000; Tonkes et al 2005; Symonds et al 2008). Third level institutions have introduced a variety of different mathematical support structures in an attempt to support their mathematically less prepared students. One popular example of this is the introduction of diagnostic testing which aims to establish where students’ difficulties may lie and to identify the students within a particular cohort who are most ‘at risk’ of failing university mathematics courses. The University of Limerick (UL) introduced diagnostic testing in 1997. The same diagnostic test is still distributed today and so a large dataset has been created which currently consists of data on almost 8,000 students between 1997 and 2010. Diagnostic test data has been found to provide valuable research opportunities such as the profiling of mathematics students over time (Kannemeyer 2005; Wilson and MacGillivary 2007; Faulkner et al 2010). Another popular use of diagnostic testing, which is prevalent in international education literature, is the prediction of students’ mathematical achievement (Simonite 2004; Barry and Chapman 2007; McDonald 2008).

The wealth of data contained in the UL dataset and the examination of literature in the area of the ‘Mathematics Problem’ led the author to investigate the profile of third level mathematics students over time. An investigation into the profile of ‘at risk’ mathematics students over time enabled the author to create a predictive model of performance in mathematics. Finally the author used these research findings to inform a mathematics intervention which was implemented in UL. The intention of these investigations is to further quantify the ‘Mathematics Problem’ so as to inform and improve the teaching and learning of mathematics both nationally and internationally.
Declaration

This thesis is presented in fulfilment of the requirements for the degree of Doctorate of Philosophy. It is entirely my own work and has not been submitted to any other university or higher education institution, or any other academic award in this University. Where use has been made of the work of other people it has been fully acknowledged and fully referenced. Where use has been made of author’s work it has been fully acknowledged and fully referenced.

Signature: _______________________
Fiona Faulkner                                 January, 2012
Dedication

This thesis is dedicated to my parents, Bartle and Angela. Thank you for your unwavering support, love and guidance throughout the years. I could not have even dreamt of achieving what I have done if it wasn’t for both of you.
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Glossary of Terms Used

UL: University of Limerick.

Service Mathematics: Service mathematics refers to degree courses where mathematics is present, but is not the main focus of the degree (Gill 2006). In UL there are five different service mathematics strands: Science, Technology, Business, Engineering and Electronics. The two service mathematics strands which are referred to throughout this research project are Technology and Science mathematics.

Successful and Unsuccessful students: A Successful student refers to anyone who passed service mathematics and an Unsuccessful student refers to anyone who failed service mathematics in UL.

Standard student: A standard student refers to a student who entered UL service mathematics directly from second level education in Ireland i.e. through the CAO system.

Non-standard student: A non-standard student refers to a student who entered UL service mathematics through any means other than the CAO system i.e. they are either a mature student, a non-national student or they have completed a previous certificate, degree or diploma which has allowed them to gain access to UL.

Leaving Certificate (Grade, Points and Level): The Leaving Certificate is the final state examination taken by second level students in Ireland. It is the main entry route to third level education. The author refers to Leaving Certificate mathematics grades, points and level throughout this research project. Each grade in Leaving Certificate mathematics has a points equivalent. Each subject at Leaving Certificate can be taken at three levels; Higher Level, Ordinary Level and Foundation Level. A Higher Level A1 grade in Leaving Certificate mathematics equates to 100 points. Greater detail on the Leaving Certificate and how it dictates entry into third level education in Ireland can be found in chapter 2, section 2.3.2.
CAO overall points: This refers to the accumulation of points that students’ received from their best performance in 6 Leaving Certificate subjects. It is calculated through converting students’ highest 6 Leaving Certificate grades into points and calculating the total points. Greater detail on this can also be found in chapter 2, section 2.3.2 (e).

Grade Inflation: Grade Inflation refers to an increase in the number and percentage of students receiving high mathematics grades per year without a matched increase in mathematical competency levels.

Student Profiling: This refers to the process through which students characteristics and previous mathematical performances are summarised per year.

Discriminant Analysis: Discriminant analysis is a statistical method which aims to determine which variables best discriminate between two or more naturally occurring groups. Its main purpose is to predict group membership based on a linear combination of interval variables.

Probability Failure group/Student Classification: In chapter 5 and 6 the term probability of failure group and student classification are used to describe the group that a service mathematics student has been assigned to based on their predicted probability of failure as determined by the discriminant analysis.

Teaching Intervention: This is a structure which is designed to improve educators’ and students’ academic performances when problems have been found to be present. They can take many different forms several of which are discussed in chapter 2 of this research project.

Mixed Ability: In the case of this research mixed ability refers to a group of students who have varying mathematical backgrounds and abilities. Students’ mathematical backgrounds are determined based on their Leaving Certificate mathematics performance and/or their performance in the UL diagnostic test.

Intervention students and Non-intervention students: An intervention student refers to any student who took part in the intervention within this research project and consequently non-intervention students refer to those who did not take part in the intervention.
**Theme:** Themes are referred to throughout the qualitative analysis of the intervention and refer to any major reoccurring ideas/opinions that emerged from the data.

**Node and Sub-node:** Nodes are a sub-category of themes and sub-nodes are a sub-category of nodes. These terms are also used throughout the detailing of the qualitative analysis of the intervention.
1. Introduction

The ‘Mathematics Problem’ has become a commonly referred to, and universally acknowledged, issue in mathematics education circles worldwide (Tariq 2002; Kajander and Lovric 2005; Gill et al 2010). It refers to the declining standards in mathematical competency levels of third level students. The international documentation of the ‘Mathematics Problem’ details common characterisations of it such as:

- Numeracy skills which are inadequate to cope with everyday mathematical skills.
- Poor competency levels in algebraic manipulation and simplification.
- Difficulty when applying mathematics, particularly when met in unfamiliar formats.

(Hourigan and O’Donoghue 2007).

Students’ understanding of basic mathematical skills on entry to third level, which in the past could have been considered second nature, can therefore no longer be assumed as common knowledge (Mervis 2007; Commonwealth of Australia 2007).
The international literature which has documented the ‘Mathematics Problem’ reflects that which has been documented in Ireland (Chief Examiner’s Report 2000; Gill 2006; Liston 2008; Ni Fhlionn 2009; Mac an Bhaird and O’Shea 2010). This nationwide decline in students’ competency levels is also prevalent in the University of Limerick (UL) where much research has been focused on the documentation of the problem and the extent to which it has progressed over time (O’Donoghue 1999; Gill 2006; Gill et al 2010).

The continued reports of the ‘Mathematics Problem’ are extremely worrying due to the fact that:

*A knowledge economy depends on a strong supply of scientists and engineers, and for students to pursue these disciplines they must have a strong foundation in mathematics....
Maths is critical for the success of the high-value knowledge-intensive business sectors that will drive our economic recovery.*

(‘IBEC 2010).

A country’s competitiveness relies heavily on its ability to produce competent citizens in the area of mathematics and science (IBEC 2010). Mathematics is present in many careers and so an understanding in mathematics is important for effective functioning in everyday life (OECD 2006). The importance of an understanding in mathematics from an educational point of view is therefore undeniable and is something which has been acknowledged in many countries worldwide (Smith 2004; Senate State Committee on Employment 2007; National Mathematics Advisory Panel 2008).

To date a variety of different efforts have been made in an attempt to alleviate the ‘Mathematics Problem’. Third level institutions worldwide have introduced mathematics support structures such as bridging courses, support tutorials, computer assisted learning, peer assisted learning, mathematics drop-in centers and diagnostic testing in an effort to support their mathematically underprepared students (Hunt and Lawson 1996; Croft 2000; Booth and Cameron 2002; Tonkes et al 2005; Cleary 2008; O’Shea and Mac and Bhaird 2009).

Diagnostic testing has become a popular response to the ‘Mathematics Problem’ internationally (Lawson 1997; O’Donoghue 1999; Malcolm and McCoy 2007). It allows institutions to examine exactly where students’ mathematical weaknesses/misconceptions lie as well as allowing for an examination of the proportion of students in any one cohort who may require additional
mathematical support. Diagnostic testing has been implemented in UL since 1997. Students’ performances in the diagnostic test, as well as other demographic and mathematical background information relating to them, are documented in a UL dataset. This dataset currently consists of data on almost 8,000 students between the years 1997 and 2010. Datasets such as this can be used to profile mathematics students over time and predict their performance. To date few studies have been carried out in Ireland which attempted to predict the mathematical performance of third level students (Byrne and Lyons 2001; Bergin and Reilly 2005). Little has been carried out however on a large scale. The vast UL dataset provides an opportunity for a large scale study, which aims to predict failure in \(^{2}\) service mathematics, to be carried out. Such an examination could provide mathematics educationalists worldwide with an insight into the changes in mathematical competency levels of third level service mathematics students over time. It also has the potential to offer insights into the possible causes of such changes in competency levels over time, therefore helping all mathematical educationalists to quantify the ‘Mathematics Problem’ further. On a national and local level such findings could be hugely beneficial in providing insights for those involved in third level policy on appropriate admission requirements and to third level mathematics educators on the mathematical competency levels of current cohorts of students. An investigation of this nature has not yet been carried out in Ireland to date.

In this thesis the author uses the UL dataset to profile service mathematics students over a ten year period and to build a predictive model of failure in service mathematics. The predictive model of failure is in turn used to inform a mathematics teaching intervention.

\(^{1}\)IBEC is the Irish Business and Employers’ Confederation. IBEC provides training which aims to give people the information, skills and the motivation they need to do their jobs well and achieve their organisation's business goals.

\(^{2}\)Service mathematics refers to degree programmes where mathematics is present but is not the main focus of the degree (Gill 2006)
1.1 Origin of the Research

The origin of this research dates back to the 1990’s when anecdotal evidence of a decline in the mathematical competency levels of service mathematics students in UL was put forward by lecturers. Lecturers’ concerns surrounded students’ unease when faced with problems which were not presented in formats which they were used to and difficulties with basic arithmetic and algebraic manipulations (O’Donoghue 1999). Several responses to these concerns have since taken place. In 1997 O’Donoghue, a Professor of Mathematics Education in UL, designed a diagnostic test to determine exactly where students’ mathematical weaknesses lay. This initiative was supported by the introduction of a Mathematics Learning Centre in 2001 in which students, who performed poorly in the diagnostic test, were encouraged to go to seek support. Further insight was gained into the ‘Mathematics Problem’ and supporting ‘at risk’ service mathematics students when Gill (2006) investigated the ‘Mathematics Problem’ in Ireland with a particular focus on UL. This investigation quantified the ‘Mathematics Problem’ in service mathematics in Irish universities and detailed specific areas where gaps in students’ mathematical knowledge were present over time. This research project therefore aims to use all of this background information to further investigate the ‘Mathematics Problem’.
1.2 Rationale and Intent

The decline in students’ mathematics competency levels is something which is constantly referred to amongst those in mathematics education and the client departments it serves (i.e. Engineering, Business, Technology, Science and Electronics) (Gill 2006). Insights into some of the possible contributors to the decline in mathematical standards in Ireland have been put forth in recent times. For example, Ni Riordain and Hannigan (2009) found that almost 50% of Irish mathematics teachers surveyed were not qualified to teach mathematics. O’Grady (2009) put forward an argument that grade inflation is occurring in Leaving Certificate mathematics and is contributing to the poor performance of students in third level mathematics. A re-address of the issues and possible approaches to further informing and alleviating the problem are needed however. The author therefore intends to:

1. Determine if the ‘Mathematics Problem’ still exists in Ireland and if so to what extent.
2. Establish the factors contributing to the ‘Mathematics Problem’ and
3. Use statistical methodology to predict failure in service mathematics and to inform third level mathematical teaching practices.

Several institutions in Ireland have implemented diagnostic testing and, in turn, support services to aid the ever increasing group of ‘at risk’ students (Armstrong and Croft 1999; Simonite 2004; McDonald 2008). No institution in Ireland however has a unique vast dataset such as that which exists in UL. Rationale for the implementation of this research therefore is to exploit this wealth of data which has the potential to provide a positive influence on the provision of mathematics education in the third level education sector. Such contributions include:

- To give students specific feedback relating to their mathematical ability and the extent to which they may need support.
- To provide lecturers and tutors involved in teaching service mathematics with an estimate of the range of mathematical ability levels which are present in their classes. This could guide them in their approach to the delivery of lecturers/tutorials, the pace at which they teach particular topics and the depth and breadth of material they choose to expose their students to.
- To help to reduce the failure rate in service mathematics.
1.3 Aims and Objectives of the Research

The main aim of this research is to quantify the ‘Mathematics Problem’ in Ireland using the UL dataset which contains information on students from 1997 to 2010. The objectives of the research are therefore as follows:

- To investigate the changing profile of Science and Technological students between 1998 and 2008.
- To identify trends over time in mathematical competencies (1998-2008).
- To profile current ‘at risk’ students in service mathematics courses using examination data from 2006-2008.
- To build a predictive model of failure in service mathematics.
- To design an intervention strategy for students in service mathematics.
- To identify the probability of failure of all students in the academic year 2010/11 in order to implement an intervention strategy using the probability of failure to inform practice and to monitor the success of the intervention.

The research questions addressed in each phase of the research are detailed in section 1.3.1 and can also be found in chapter 3 (section 3.3).
### 1.3.1 Research Questions

This research project is broken down into 6 phases within which different research questions must be addressed. The research questions to be addressed in each phase are detailed in table 1.1 below. All of the questions to be answered have emerged from the objectives of the thesis as outlined in section 1.3.

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<tr>
<th>Research Phase</th>
<th>Research Questions</th>
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| **Phase 1**    | 1. How prevalent is the ‘Mathematics Problem’ internationally?  
                  2. How prevalent is the ‘Mathematics Problem’ in Ireland?  
                  3. What are the dominant documented causes of the ‘Mathematics Problem’?  
                  4. What support services and interventions have been put in place in an attempt to alleviate the ‘Mathematics Problem’?  
                  5. What does national and international educational research say about profiling ‘at risk’ students? |
| **Phase 2**    | 1. What is the profile of service mathematics students in UL between 1998 and 2008?  
                  2. What are the trends in the mathematical competency levels of UL students by Leaving Certificate mathematics grade?  
                  3. Is there evidence of grade inflation occurring in Leaving Certificate mathematics? |
| **Phase 3**    | 1. What is the profile of students who are ‘at risk’ of failing Technology mathematics?  
                  2. What is the profile of students who are ‘at risk’ of failing Science mathematics?  
                  3. What is the profile of a student who is ‘at risk’ of failing service mathematics?  
                  4. What is the most effective statistical method of prediction of failure within service mathematics in UL?  
                  5. Is discriminant analysis a more effective method of classifying failure in service mathematics than the measure which is currently in place i.e. the diagnostic test results? |
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<th>Phase</th>
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<td>6.</td>
<td>What are the challenges associated with predicting failure in third level service mathematics?</td>
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| Phase 4 | 1. What are the probabilities of failure of service mathematics students entering UL in September 2010/11 based on the discriminant function?  
2. What is an appropriate intervention design, implementation and evaluation method for service mathematics students which has been informed by the discriminant function? |
| Phase 5 | 1. Did students involved in the teaching intervention perform better than those who did not take part in the intervention?  
2. Did students involved in the intervention respond positively to the teaching strategies employed in the intervention? |
| Phase 6 | 1. What do the research findings in this thesis have to offer the university sector, the wider Irish education system and the international education system? |

**Table 1.1** Research questions per phase.
1.4 Scope and Significance of the Research

The proportion of second level students entering third level education in Ireland has been increasing steadily over the last 20 years, from 20% in 1980 to 54% in 2003 to almost 66% in 2010 (ERSI 2005; Higher Education Authority Report 2010). 30 years ago the vast majority of students entering third level education in Ireland came directly from Leaving Certificate, however today third level education can be accessed from several different routes such as post leaving certificate courses, FÁS courses (an Irish Training and Employment Authority), access courses, completion of certificate, diplomas and degrees. The current situation in Irish education makes the investigations carried out in this thesis extremely significant as it is no longer exclusively the elite secondary school students who progress to third level education. It is therefore vital that we re-assess the situation in relation to the mathematical preparedness of the recent cohorts of students entering third level education in Ireland. Third level students are less prepared mathematically than they were over a decade ago (Gill et al 2010) and further insight into why this might be needs to be investigated.

Another change to the Irish education system today which makes this thesis significant is the introduction of a new mathematics curriculum at second level entitled ‘Project Maths’. In Ireland the role of mathematics in developing students’ problem solving skills and reasoning skills has been acknowledged more so today than ever before. The introduction of this new mathematics curriculum aims to move away from the rote learning, examination focused mathematics teaching which has existed in second level education (NCCA 2006). It also intends to introduce students to real world mathematics in which they must problem solve and apply the theories and procedures they are studying in a way that is relevant to everyday life (see section 2.3.2 (e) for more detail on ‘Project Maths’). One of the catalysts for the introduction of ‘Project Maths’ was the reported difficulties which second level students face with mathematics when making the transition to third level education (Chief Examiner’s Report 2000; Liston 2008). It is pertinent that the changes in service mathematics students over time are understood at a time when new initiatives are being implemented in an attempt to alleviate the ‘Mathematics Problem’.

1.5 Overview of Thesis Chapters

The investigations which take place in each chapter of this thesis are as follows:

Chapter 2 This chapter presents a review of literature surrounding all related issues within the research. A documentation of the ‘Mathematics Problem’ forms the basis for the chapter. The chapter also details literature surrounding the probable causes of the ‘Mathematics Problem’ and some mathematical support services which have been implemented in an attempt to alleviate the problem. Finally literature detailing the prediction of students’ mathematical achievement is outlined along with the success of the predictions in each case.

Chapter 3 provides a description of the research methodologies carried out during each phase of the research. It details the research design, problem, objectives and questions. The research paradigms and the theoretical framework which informs the research are discussed. The author also provides a chronological phase by phase description of the objectives to be achieved during this research project. The theoretical framework for the intervention is discussed along with the quantitative and qualitative methods of data analysis which are used. Finally issues within the research such as validity, reliability and ethics are considered in light of the research being carried out.

Chapter 4 presents the first findings of the research into profiling service mathematics students between 1998 and 2008. This chapter explores the changes in competency levels of students entering UL between those years and examines if there is evidence of grade inflation being a possible contributor to the problem. A detailed profiling of Technology and Science mathematics students between 2006 and 2008 is then outlined to determine which student characteristics are statistically significantly related to performance in service mathematics. This chapter lays the foundations for the findings which are presented in chapter 5.

Chapter 5 explores a variety of methods for predicting failure in service mathematics. It provides a description of how discriminant analysis works and the results from a discriminant analysis of the Technology 2006-2008 dataset are presented. The success of the Technology discriminant analysis function in terms of predicting failure in service mathematics is outlined. The success of
the function is also determined based on its ability to predict the performance of students entering service mathematics in 2010. A comparison of the ability of the discriminant function and the diagnostic test to predict performance in service mathematics is carried out. The most effective method of predicting performance in service mathematics in UL is used to inform the intervention carried out in this research. Details of the challenges associated with predicting failure in third level service mathematics are outlined also.

**Chapter 6** provides details of the intervention design, implementation and evaluation. The rationale for the implementation of an intervention is outlined first along with a description of how the intervention type was decided upon and how the discriminant function was used to inform it. Some of the techniques used during the implementation of the intervention are detailed. Finally a description of how the intervention was evaluated is provided followed by a description of the qualitative and quantitative results which emerged.

**Chapter 7** provides a summary and the conclusions relating to the main findings of the thesis. It concludes by making recommendations based on the main findings and offers some suggestions for future research in the area.
Chapter 2: The ‘Mathematics Problem’ and Beyond: A Review of Literature

2. Introduction

The review of literature provides a background to this research project. This chapter explores existing research which examines the extent of the ‘Mathematics Problem’ internationally, nationally and in the University of Limerick. Literature examining related issues such as the causes of the ‘Mathematics Problem’, approaches through which it has been tackled in the past and more sophisticated newly established approaches are also discussed. The ‘Mathematics Problem’ in Ireland is discussed in light of the introduction of a new mathematics curriculum in Irish second level education; ‘Project Maths’. A detailed review of literature surrounding predicting mathematical achievement is also outlined. The examination of such literature provides the author with an overview of where UL students fit into the ‘Mathematics Problem’, therefore allowing for informed research practices to follow. The examinations of such issues are detailed in section 2.1 and throughout the entire chapter.
2.1 The ‘Mathematics Problem’: A Review of Literature

It has often been said that the economic success of any country is dependent upon the knowledge and skills of the individuals within it (Breen et al 2009). A ‘knowledge economy’ can be built through the development and distribution of well educated individuals, with particular emphasis on scientists, engineers and technologists, into the working world (IBEC 2010). These disciplines, on which a strong economy is so heavily dependent, are strongly reliant on mathematics and so the title given to the decline in standards in third level mathematics is quite fitting; ‘The Mathematics Problem’. The main concerns of this problem, which were first reported in the UK, relate to a steady decline of fluency in basic mathematical skills and the under-preparedness of students entering third level education to cope with the mathematical demands made on them (Engineering Council 2000).

Internationally (e.g. Australia, USA, UK, Ireland) ‘under-prepared’ students demonstrate some or all of the following characteristics:

- Large gaps in mathematical knowledge.
- Numeracy skills which are inadequate to cope with everyday mathematical skills.
- Poor competency levels in algebraic manipulation and simplification.
- Difficulty when applying mathematics, particularly when met in unfamiliar formats.
- Lack of ability to reason mathematically.

(Hourigan and O’Donoghue 2007).

Reports of such characteristics were found in many international studies, some of which are outlined in section 2.1.1.

2.1.1 International Studies

Concerns regarding the ‘Mathematics Problem’ have been reported internationally (London Mathematical Society 1995; TIMSS 1995; Kajander and Lovric 2005; Belward et al 2007). In Australia reports have been made over the last decade detailing concerns about the fact that declining mathematics standards are negatively impacting on the industry sector (Commonwealth of Australia 2007). The members of the committee responsible for this enquiry consisted of thirteen senators from all over Australia. The committee received more evidence
relating to the decline in mathematics teaching than any other aspect of the curriculum at third level. They came up with two major contributory factors to the decline in mathematical standards:

- The quality of aspiring teachers is on the decline. The inadequate treatment of mathematics content during teacher training is giving new teachers neither confidence nor enthusiasm to teach mathematics
- Too many children are unprepared at the end of primary school to learn algebra and without this algebraic knowledge studying mathematics at a higher level is made extremely difficult.

In 2008 a report entitled ‘Maths? Why Not?’ was carried out by the Australian Department of Education which encapsulates their concerns regarding mathematics education;

> *Australia will be unable to produce the next generation of students with an understanding of fundamental mathematical concepts, problem-solving abilities and training in modern developments to meet projected needs and remain globally competitive.*

>(Australian Academy of Science 2006, p.9)

Mathematics Professor Gavin Brown, former vice-chancellor of the University of Sydney, reviewed mathematics and science performance in Australian universities in 2009 and highlighted the under-preparedness of university entrants and the need for bridging courses. Professor Brown suggested the development of university bridging programs as a short-term fix to bring students up to standard (Trounson 2009). Belward et al (2007) discussed the “never ending downward spiral” of mathematics education in Queensland (p. 840). This study was motivated by the notable decline of students’ end of term mathematics examination performance over a ten year period. It took place in the School of Mathematics and Physics at James Cook University. Staff noted that first year undergraduates struggled with material which in the past was second nature on entry to university. Students’ performances on a Maths B examination, a standard pre-requisite preparatory course for students intending on enrolling in disciplines that require numeracy skills at university, were recorded. A total of 140 students involved in the study took an unannounced diagnostic test on entrance to university. Students’ performances in the diagnostic test, which tested basic school algebraic skills, were compared to their
performance in Maths B. Results showed that students’ performances in second level education was not a good indication of mathematical ability and that lecturers can no longer expect certain algebraic knowledge to be second nature to beginning undergraduates. Staff at James Cook University maintain that students leaving tertiary education struggle to achieve the expected level of mathematics for two reasons; (i) the intake of students are not prepared for the mathematics that they will face in third level and (ii) the indicators used to measure the ability of second level school-leavers can often be ineffective (Belward et al 2007). In Australia the decline in mathematical standards is matched with a shift in student interest towards less demanding mathematics subjects in HSC examinations, a problem which Louis and Page (1998) suggest could be remedied through “better teaching and more relevant mathematics courses” in second level education (p.1). This is something which is echoed in many other countries such as Ireland and the UK (Smith 2004; Hourigan and O’Donoghue 2007).

The ‘Mathematics Problem’ has also been highlighted in the United States with the announcement of results from the Third International Mathematics and Science Study (TIMSS) in 1995 which showed a continuing decline among American high school senior students in mathematics and science (Mervis 2007). This test examined the mathematical skills of graduating high school students. The US ranked 15th in mathematics out of the 16 countries who took part in the advanced tests. Other indicators of declining mathematics skills include data from SAT and ACT, two leading US college entrance examinations, highlighting that the majority of students are not as prepared as students in previous years and so are less likely to succeed in third level education (Von Neumann 2006). The mathematics score on the SAT for 2006 showed the largest decline in 31 years. The declining standards in performance is not a new phenomenon as William (1954) details how remedial algebra and geometry classes had to be set up in the University of Carolina for students upon entry to the university. Despite the fact that these students thought they were prepared for freshman mathematics, they did not have the necessary numeracy skills. Hilton (1983) highlights that the under-preparedness of American third level students can manifest itself in many forms; poor computational skills, an inability to manipulate algebraic expressions, an inability to think geometrically and poor intuition. His paper on the future of college mathematics also highlighted the difficulties that some students
have with mathematics due to poor verbal skills and he attributes the lack of effective second level teachers in America to:

*the unattractiveness of a career in secondary school teaching to bright graduates who could achieve double the starting salary, higher prestige and more job security in other industries.*

*(Hilton 1983, p.452)*

This is one suggestion as to why America performed poorly in TIMMS in 1995. The National Science Foundation (NSF) called for a funding boost in 2000 in order to reverse the decline in mathematics standards and make mathematics a more attractive profession (Smaglik 2000). The documentation of the poor mathematical performance of students in the US and the response to it however proved to have a positive effect on performance as outlined by the fourth TIMMS report which was published in 2007. Compared with 1995, the average mathematics scores for both US fourth and eighth grade students were higher in 2007. At fourth grade, the US average score in 2007 was 529, 11 points higher than its 1995 average. At eighth grade, the US average mathematics score in 2007 was 508, 16 points higher than its 1995 average score (National Centre for Education Statistics 2007).

Pitts and White (1996) carried out research entitled ‘The Impact of Underprepared Students on Regular College Faculty’. Fourteen faculties from core subjects, such as English, History and Mathematics, within two universities in southern America were involved. Staff members completed open-ended interviews on the effect of under-prepared students on them. The basic problems associated with academic under-preparedness such as: a lack of basic knowledge and skills among incoming students, lack of student motivation and a surface approach to learning were highlighted by the majority of staff. The approaches that the staff members tended to take when faced with this issue were: remedial instruction, flexibility, creative interaction with students, less breath and more depth in course material and lowering of standards. These coping strategies may in fact be adapted by those in mathematics education worldwide.

In a Canadian study Kajander and Lovric (2005) referred to the knowledge and skills of incoming university students as being “far from satisfactory” (p.149) when they discussed transitional issues from secondary to tertiary mathematics in McMaster University. Transitional issues will be discussed further in section 2.2.
In the UK in 1995 a report entitled “Tackling the Mathematics Problem” was published by the London Mathematical Society, the Institute of Mathematics and its Applications, and the Royal Statistical Society as a response to each party’s concerns regarding the mathematical preparedness of new undergraduates. This report, which aimed to “reflect the concerns of as wide a community as possible” (LMS p.4), highlighted the decline in students’ mathematical competency levels. Some suggested causes for the declining standards in mathematics highlighted in this report included:

- The introduction of time-consuming activities (investigations, problem solving) which are poorly focused. This is something which the developers of the new Irish second level mathematics curriculum ‘Project Maths’ need to be mindful of if it is to be a success (see section 2.3.2(e)).
- National curriculum suggesting a reduced emphasis on technical fluency which is necessary for future mathematical success.
- English students’ inability to complete questions involving fractions and decimals leading to future problems in mathematics.

Dearing (1995) also acknowledged this issue: “….the United Kingdom faces extremely serious problems to the supply and the mathematical preparation of entrants to university courses” (LMS 1995, p.5). These issues are re-addressed in another report entitled “Measuring the Mathematics Problem” by the Engineering Council in 2000 who again acknowledged the serious decline in students’ fundamental mathematics skills and consequently their preparedness for third level mathematics. This report outlined possible reasons for the ‘Mathematics Problem’ in the UK including changes to the school system leading to a variety of mathematical backgrounds upon entering third level as being a contributor. Another probable contributor suggested is the widening of access to university consequently leading to a more diverse range of mathematical backgrounds in third level mathematics courses. The consequences associated with these issues make it difficult to gauge an appropriate starting point for lecture material as well as appropriate pacing of lectures (Hunt and Lawson 1996; Gill 2006). A lack of acknowledgement by lecturers in relation to the changes in standards occurring in third level institutions in the UK and indeed world-wide is likely to lead to “a deterioration in the effectiveness of the learning” (Hunt and
Lawson 1996, p. 171). Acknowledgement also needs to be made that the problem is not just with the poorer A-level students. Beginning undergraduates with A grades in A-level mathematics displayed characteristics traditionally associated with weaker pupils such as unease when faced with unfamiliar mathematics (Micallef 1997).

In a concerted effort to reverse the decline in the standard of mathematics education in the UK a report was published by Professor Adrian Smith in 2004 entitled ‘Making Mathematics Count’. This enquiry into Post-14 Mathematics Education aimed to address the three main issues in the UK:

- A curriculum which is failing all of the stakeholders in mathematics education (higher education, employers and young people not proceeding to Post-16 maths),
- A lack of specialist mathematics teachers in schools
- A lack of support systems to enable continuing professional development.

In his report, Smith (2004) believed that reliable provision for teachers of mathematics in schools and colleges, such as resources and sustained access to professional support and development, was the first step necessary to reverse the problem of declining standards. The ‘Mathematics Problem’ in an Irish context is outlined in section 2.1.2 which follows.

2.1.2 The ‘Mathematics Problem’ in Ireland

Research surrounding the ‘Mathematics Problem’, and related issues, in Ireland have become more widespread in recent times (Hourigan and O’Donoghue 2007; Liston 2008; Hourigan 2009; Ni Fhloinn 2009; Ni Riordain and Hannigan 2009; Mac an Bhaird and O’Shea 2010; Carroll 2011; O’Keeffe 2011; O’Meara 2011; Prendergast 2011). These new studies and older studies however are in agreement that the problem is prevalent (e.g. Cork Institute of Technology, University College Cork, UL) (Hurley and Styles 1986; Department of Mathematics and Computing 1997; O’Donoghue 1999; O’Donoghue 2004; National Council for Curriculum and Assessment (NCCA) 2005; Gill 2006; Breen et al 2009). Similar to much of the international research in this area, Irish students who enter third level education and have a poor grasp of basic mathematical concepts are categorised as ‘under-prepared’ or ‘at risk’ (O’Donoghue 1999). Hourigan and O’Donoghue (2007) commented on how “under achievement within the Irish
context seems to defy all logic, as first year courses at tertiary level generally overlap and extend the school Leaving Certificate syllabus” (p.462). Despite this Irish students continue to demonstrate mathematical knowledge which is disjointed and insecure.

O’Donoghue (2004) gave an Irish perspective of the ‘Mathematics Problem’. He outlined some of the possible characteristics, which were very much in-line with the international characteristics mentioned previously, associated with under preparedness in third level education in Ireland:

- Students entering third level education lack numerical skills needed for everyday life “not to mention studying university mathematics” (Hurley and Stynes 1986; O’Donoghue 1999).
- Deficiencies in elementary algebra are pivotal to overall mathematical deficiencies in students (Lawson 1997).
- Difficulties in solving problems (Galbraith and Haines 2000).
- A lack of conceptual understanding and knowledge (Tall and Razili 1993).
- Syllabi without a common thread throughout resulting in fragmented learning (Galbraith and Haines 2000).

Gill (2006) suggested additional possible contributors of the mathematical issues in third level as being:

- Government policy of mass higher education.
- The points system used in the Irish Leaving Certificate Examination (see section 2.3.2(e) on the Irish educational system).
- Changes to the Irish second level syllabus (e.g. less emphasis on mental arithmetic and increased use of the calculator).
- Increasing class sizes in universities.

Much research which stems from the ‘Mathematics Problem’ has taken place in UL. Such studies offer further insight into the complexity of the problem and the vast number of areas which need to be considered if it is to be tackled holistically. Prendergast (2011) carried out research relating to promoting student interest in mathematics at second level, with a particular
emphasis on effective algebra teaching. One of the main aims of this work was to improve students’ conceptual understanding and interest in mathematics, an aim which is very much in line with trying to alleviate some of the issues related the ‘Mathematics Problem’. Carroll (2011) also carried out research which looked at methods of improving understanding in mathematics at second level. In his work he developed a framework for effectively teaching applications of mathematics to Leaving Certificate students. O’Meara (2011) examined issues related to the ‘Mathematics Problem’ in Ireland in terms of improving in-service mathematics teachers’ subject knowledge. Through the development and introduction of a Continuous Professional Development (CPD) initiative to in-service teachers, O’Meara (2011) maintained that this could potentially be a vehicle for “improving knowledge among teachers and in turn improving the teaching and learning of mathematics in Ireland” (p. i). O’Keeffe (2011) examined how Irish second level mathematics text books can impact on students’ conceptual understanding. O’Keeffe (2011) analysed Irish text books using a framework which was primarily based on the work of TIMMS which broke the analysis down into: Content, Structure and Expectation. Upon analysis of the most commonly used text books in Irish secondary schools she found that they were all weak in many areas such as motivation and comprehension. Problem solving exercises were very rarely featured in the text books and the presence of real life graphics was low throughout each also. The results of this research therefore found that the text books failed to “motivate students, or to provide for the comprehension and processing of the information provided and highlighted the role improved text books could play in student learning” (p.310). Ní Riordain and Hannigan (2009) highlighted the contribution that the large number of out-of-field mathematics teachers, present in post-primary education, were having on the ‘Mathematics Problem’, an out-of-field teacher being any teacher who was teaching mathematics yet not qualified to do so. In this study 48% of the teachers teaching mathematics did not have a mathematics teaching qualification. This finding was said to be likely to have a “negative effect on students’ mathematical learning” (p. 24). Ni Fhliomn (2009) detailed the steady decline in diagnostic test results of basic mathematical skills of first year students in Dublin City University. This study which examined students’ performances over a five year period (2004-2008) suggested that the decline in standards may be due to students’ increased inability to engage with mathematics which is “not of a routine and well-rehearsed type” (State Examinations Commission 2005, p.73). All of the recent Irish studies highlighted the complex
nature of the ‘Mathematics Problem’, the vast number of contributing factors to it and the huge amount of research and analysis needed in order to inform best practice in an attempt to alleviate it in the future.

In response to the reported decline in mathematics standards in third level education in Ireland many institutions put measures in place in an attempt to reverse the decline. For example the National University of Ireland Maynooth (NUIM) reported incidents of mathematical deficiencies of entering students and therefore put in place a diagnostic test and support services in an attempt to remedy this. Cork Institute of Technology also used diagnostic testing in an attempt to quantify the apparently increasing problem in the 1980’s. University College Cork (UCC) introduced diagnostic testing in 1985 also with the hope that the testing would identify where the mathematical weaknesses were most prevalent so that measures could be put in place to overcome these deficiencies (Hurley and Stynes 1986). It has also been used in other countries such as Australia, America and the UK (Tall and Razali 1993; Edwards 1996; Hunt and Lawson 1996; Malcolm and McCoy 2007). The data collected when diagnostic testing was carried out has also been used to profile student cohorts and predict student mathematical achievement (Barry and Chapman 2007; McDonald 2008; Faulkner et al 2010). UL also turned to diagnostic testing in 1997 upon O’Donoghue’s undertaking of a scheme to help “at risk” or mathematically insufficient students reach the standard of mathematics that was required of them in their service mathematics courses. Diagnostic testing as a method of alleviating the ‘Mathematics Problem’ is discussed in section 2.3.1(b).

One contributor to the mathematics problem which has been widely documented is the transitional issues which often exist between second and third level mathematics. This aspect of the ‘Mathematics Problem’ is discussed in section 2.2.
2.2 Contributing Factors to the ‘Mathematics Problem’

2.2.1 Transitional Issues

Difficulties experienced by third level students when making the transition from second to third level education have been found to contribute to the ‘Mathematics Problem’ (Pargetter et al 1999; Kantanis 2000; Liston 2008). Students’ difficulties have been shown to be both academic and social in nature and can arise due to the approaches used when learning mathematics and the mismatch between previous knowledge expectations and actualities upon entry to third level education (Jones and Frydenberg 1998; Liston 2008). These two elements within transitional issues will be discussed in terms of existing literature in sections 2.2.1(a) and 2.2.1(b).

2.2.1 (a) Approaches to Learning Mathematics

Students’ approaches to learning are influential on how successfully they make their transition from second to third level mathematics education (Anthony 2000). A student’s approach to learning is directly linked to how they experience and define their learning situation as well as “the strategies they use to learn and the motivation underlying their conduct” (Cano 2005, p.206). Liston (2008) determines students’ approaches to learning under two headings: deep-level and surface-level. These approaches which she adapted from the work of Biggs et al (2001) examined whether a student is inclined to learn for conceptual understanding (relational understanding) in which case they would be considered deep-level learners or are they learning for reproduction (instrumental understanding) making them surface-level learners. Those who adapt a deep-level learning approach have been found to be better able to cope with the academic demands of third level education (Ramsden 1992). Liston (2008) found that pre-service student teachers in UL tended to adapt a deep-level learning approach more than a surface-level approach. Scales which examined students’ learning approaches showed a mean of 29.8 (SD = 6.77) out of 50 for the deep-level scale and 24.3 (SD = 6.32) out of 50 for the surface-level scale. A high score on the deep-level scale was favourable in the case of this study as it suggested that the students favoured comprehension over the reproduction of knowledge when it came to studying mathematics. Although this is an encouraging finding several other Irish studies which consider approaches to learning suggest that in general Irish students adapt surface learning
approaches to mathematics (Chief Examiner’s Report 2005; NCCA Review 2005). Biggs (1993) maintained that the approach a student adopts is strongly influenced by their interest in a subject. Glaister and Glaister (2003) in the University of Reading, Oxford, introduced formative assessment in the form of a ‘buddy’ system in an attempt to help students to become more independent learners than they had been in school. This trial was conducted in 2002 and involved students taking first year Analysis and Calculus courses. Students paired up and worked on tutorial problems together before the tutorials took place each week. The results of all students involved in the ‘buddy’ system were compared against those who had taken the same modules in 2001. The students in 2001 were not involved in the ‘buddy’ system. The results showed slight improvements in the final average mark and the pass rate for Analysis in 2002 compared to 2001. A much larger improvement for Calculus occurred in 2002 making the new system a success. Another study which examined the methods through which undergraduates approach mathematics highlighted the reliance of undergraduates on an instrumental approach to studying mathematics as opposed to using relational understanding causing “difficulties in undergraduate problem-solving” (Anderson 1996, p.813). This finding emerged from an examination of 103 undergraduates in the University of Nottingham who were specialising in mathematics. Students attempted to solve problems which required them to make the transition from concrete to abstract mathematics. Despite the strong mathematical background of students they still tended to adapt instrumental approaches to the problems.

Hourigan and O’Donoghue (2007) investigated the effects of the pre-tertiary mathematical experiences on students’ ability to successfully make the transition to third level mathematics courses in Ireland. In this study the authors discuss Brousseau’s concept of the ‘didactical contract’ which suggests that an unspoken contract exists between the individuals in every classroom which determines the roles, behaviours and attitudes of all actors (students and teachers) within the classroom situation. Students’ ability to learn mathematics is determined by their level of intelligence as well as by the relationships present in the classroom (Balachel et al 1997). The study concluded that the narrow didactical contract that exists between teacher and pupil is hugely responsible for students’ success or failure in second level mathematics and consequently third level performance. A narrow didactical contract in this case was one in which:
exam driven practice results in regimental thinking, limited problem solving ability and a lack of self-confidence and perseverance in the face of unseen/different challenging problems.

(Hourigan and O’Donoghue 2007, p.473).

The findings in this study emerged from ten weeks of classroom observation in an Irish secondary school in which a checklist was used to analyse the existence of the unspoken contract. A reflective journal by the author and semi-formal interviews with a random selection of teachers and pupils supported the findings. The reflective journal and semi-formal interviews offered a means of confirmation for the observed behaviours. The authors mention that “learned helplessness” is a feature of Irish second level mathematics classrooms, a process which is in direct contrast to the ethos that most third level institutions try to adopt i.e. independent and discovery learning.

More recent studies on transitional issues in Ireland, such as one carried out by Liston and O’Donoghue (2009), have looked into the role that affective variables such as attitudes, beliefs, self-concept, conceptions of mathematics and approaches to learning of students have in the transition to university service mathematics in third level education. The research instrument in this study was a questionnaire consisting of 78 statements which students had to agree or disagree with on a scale of 1-5, 1 being strongly agree and 5 being strongly disagree. The questionnaire was completed by 607 service mathematics students in UL. Results showed a weak positive correlation between performance in service mathematics and enjoyment of mathematics ($r = 0.24$) and mathematics self-concept ($r = 0.22$). However neither conceptions of mathematics nor approaches to learning mathematics showed statistically significant relationships with performance in service mathematics.

Transitional issues are something which the Irish second level curriculum developers considered in 2008 when designing and implementing a new mathematics curriculum which aimed to improve relational understanding and discourage surface learning approaches. Further details on the new curriculum are provided in section 2.3.2(e).
2.2.1(b) Mismatch Between Previous Knowledge Expectations and Actualities

The discrepancies between the previous knowledge that students have leaving second level education and what lecturers expect that they have upon entry to third level education has commonly become known as the ‘gap’ between second level and university mathematics (Hoyles et al 2001; Kajander and Lovric 2005; Liston 2008). Although many agree that the problem exists with disciplines other than mathematics also, the mathematics ‘gap’ seems to be the most serious and problematic (Kajander and Lovric 2005).

In Australia, reports of transitional issues have been well documented. Hubbard (1986) described the ‘Remedial Mathematics Facility’ which was set up in Queensland Institute of Technology “to remedy the mathematical deficiencies of new undergraduates” through the use of specialised tutors and the provision of additional support (p. 247). Queensland Institute of Technology takes in approximately 1,200 undergraduates each year who take a mathematics subject. The Remedial Mathematics facility is available to all of these students. A report by Taylor et al (1998) on the ‘Perceived Mathematical Proficiencies Required for First Year Study at The University of Southern Queensland’ emphasised the mismatch between the expectations of mathematical skills required upon entry to university and the skills the students actually possessed. This mismatch was established through examination of a questionnaire which was developed using the Queensland second level education mathematics syllabi as a reference. A total of 77 questionnaires were completed by academics across six faculties in the university (Commerce, Business, Science, Engineering, Education and Arts). The questionnaire was broken down into two categories:

- Academics’ perceptions of mathematical topics and skills needed by commencing students.
- Academics’ perceptions of mathematical abilities in these topics and skills.

Recommendations were made as to how this mismatch may be remedied: firstly, more information should be given to academics regarding the pre-requisite knowledge of incoming students, and secondly more collaborative work between the Office of Preparatory and Continuing Studies (OPACS) and faculty staff in order to improve the standard of learning in a systematic way.
Hoyles et al (2001) discussed the general shift in the UK across all subjects including mathematics to a more utilitarian higher education. The authors explain their theory that mathematics has “two faces”; one in which it is an object in its own right and the other in which it is a tool needed to serve the sciences (p. 841). The ‘two faces’ referred to are described by Nardi (1996) as being either empirical and informal in nature or abstract and formal in nature. Nardi argues that upon entry to university most undergraduates have little idea of what mathematics is and assume it is an extension of school mathematics. Most students in the UK therefore arrive at university and subsequently meet a face of mathematics with which they have never come in contact with. This issue stems from the fact that the secondary school mathematics curriculum does not communicate with tertiary mathematics, which results in the under-preparedness of students (Hoyles et al 2001). This concern was also raised in Ireland in a report which stated that

\[
\text{the problem may lie with the perception that students at second level have an individual subject which is a self-contained area of study unconnected to other subjects or curriculum areas}
\]

(NCCA 2005, p.7.)

According to Holton (1998) these concerns can be dealt with if second level teachers begin to focus on “the process of mathematics rather than the skills” (p.49).

Many other studies in the UK examined the issue of transition such as that of Cox (2001) who highlighted the “mismatch between university teachers’ expectations of the knowledge and skills that the entrants bring with them and the students’ actual capabilities” (p.847). The probable preparedness of beginning undergraduates with specific A-level grades in Aston University was measured using diagnostic test results which examined 90 key mathematics skills. The expected mathematical knowledge by teachers was measured using a list of topics in which teachers assigned them as being ‘required’ (i.e. expected that students are 100% competent in this area), ‘preferred’ (i.e. most of the class should have command of this topic) and ‘not specified’ (i.e. do not expect students to be familiar with this topic). The expectations were set out by lecturers by A-level grade and 3 departments were involved: Mathematics, Electrical and Electronic Engineering and Mechanical Engineering. The expectations of staff were twice that which the students actually achieved.
In Ireland many studies have been carried out which focus on the ‘gap’ between secondary and tertiary mathematics education and the problems that come with this (Chief Examiner’s Report 2005; Gill 2006). In 2005 the NCCA carried out a ‘Review of Mathematics in Post-Primary Education’, which was similar to the report carried out by Smith (2004) in the UK, which explored the main concerns in post primary mathematics education and listed them as:

- The emphasis on procedural skills rather than on understanding.
- Poor applications of mathematics in a real-world context.
- Low uptake of Higher Level mathematics, especially in the Leaving Certificate.
- Low grades achieved at Ordinary Level, especially in the Leaving Certificate.
- Gender differences in uptake and achievement.
- Difficulties in mathematics experienced by some students in third level courses.

(NCCA 2005, p. 3).

The causes of the ‘Mathematics Problem’ and the extent of its effect may vary from country to country, however its existence is constant throughout many third level institutions worldwide. In Ireland, grade inflation has often been used as the scapegoat for the declining standards in third level education. Details of this will therefore be discussed in section 2.2.2.
2.2.2 Grade Inflation in Ireland

Grade inflation has been reported as a possible contributor to the decreases in competency levels in mathematics education in Ireland (Network for Irish Educational Standards 2007 & 2011). This claim has been made both in terms of the Leaving Certificate examination and the Irish university sector. Grade inflation is defined by O’Grady and Guilfoyle (2007) as a process which arises “when there is an improved trend in examination grades over time without an accompanying improvement in learning or academic achievement” (O’Grady and Guilfoyle, p.1).

The examination into the prospect of grade inflation occurring in the Leaving Certificate examination was influenced by numerous media reports over the last number of years (e.g. Downes (2006) in Irish Times; O’Kelly (2008) in Sunday Independent). Concerns surrounding the dramatically increasing numbers of students gaining honours in state examinations suggested the reason for this being that the examination has been made much less challenging than a decade ago. Figures from the State Examination Commission confirm this large increase in honours as it reported an increase of 16% in the numbers gaining honours in higher-level papers over the past decade (Flynn 2011). Walshe (2006) alluded to the steady upward trend in Leaving Certificate grades and also maintained that it may be due to grade inflation or ‘dumbing down’ as he stated that “….three times as many students now are getting at least 450 points compared with the early 1990s” (Irish Independent 2006). Following these reports a call for reform in the Leaving Certificate was set out which stated that “its emphasis on rote learning” needs changing. Particular attention was given to issues with Leaving Certificate mathematics stating that upon examining students’ examination scripts there was a clear “lack of understanding of basic maths among a huge number of students, many of whom had achieved very good grades at higher level in the Leaving Cert exam.” (Downes 2006). Walshe and Donnelly (2006) further emphasised the incidence of grade inflation around this time when they reported that Leaving Certificate grades had improved for fourteen successive years during which time the percentage of students getting 450 points or more had increased from 6.2% in 1992 to 17.7% in 2006 (Irish Independent 2006). O’Grady (2009) admitted that “the case for grade inflation is……the weakest in maths with no obvious pattern of grade increase since the early nineties” (p.24).

The Leaving Certificate is the final state examination which second level students take in Ireland. A detailed description of the Leaving Certificate can be found in section 2.3.2 (e).
However he also reported findings of the large increases in the percentage of A and B grades awarded in Leaving Certificate mathematics between 1992 and 2006. The percentage of A-grades received increased from 0.6% to 11.5% and B-grades increased from 9.6% to 27.0%. These increases were matched with a decline in the percentage of C-grades awarded which fell from 41.0% to 27.2% (p.13). In more recent times the former Minister for Education, Mr Batt O’Keeffe, expressed his concerns regarding the inflation of grades in the Leaving Certificate as well as the increase in the number of first-class honours qualifications being awarded by third level institutions. Mr O’Keeffe made reference to studies which had shown that the number of straight A Leaving Certificate results was up by 500% while the percentage of first-class honours degrees awarded by Irish universities has almost trebled in the past 15 years (RTE news 2010). The issue of grade inflation has also been examined in the UK by Lawson (2003) who set out to assess the situation regarding pre-tertiary level mathematical competencies. A-level students were assessed on entry to university using 50 multiple choice questions covering 7 topics: arithmetic, basic algebra, lines and curves, triangles, further algebra, trigonometry and basic calculus. The test did not change throughout the period of the study (1991-2001) and students received it without warning in their induction week. In this ten year study it was found that students in 1991 with particular A Level grades were better prepared mathematically on entrance to third level education than students entering with an identical A-level grade in 2001. Subsequently Lawson (2003) compared the two student profiles entering Coventry University in the UK and concluded that grade inflation had occurred over the ten year period. Contrasting conclusions were found when a similar investigation was carried out in Ireland. Faulkner et al (2010) analysed the profile of entrants to the University of Limerick in 1998 and 2008 in relation to their mathematical qualifications upon entering. There were 5,949 students involved in this study. A paper-based diagnostic test was used which tested students’ ability in 7 areas; arithmetic, algebra, co-ordinate geometry, trigonometry, complex numbers, calculus and modelling. Information on each student’s Leaving Certificate mathematics grade prior to entering UL was also documented. Although a larger number of students were found to have entered UL with lower Leaving Certificate mathematical qualifications, the performance of students in the diagnostic test by Leaving Certificate grade (The UK A-Level equivalent) remained the same over the ten year period being examined. All of the concerns relating to grade
inflation raised in the reports and articles mentioned here are matched with similar concerns in relation to grade inflation in third level education in Ireland.

Downes (2006) reported, in the Irish Times newspaper, concerns in relation to grade inflation in Irish universities. He highlighted that figures for 2004 revealed that the percentage of first-class and upper second-class honours degrees awarded to undergraduate students increased in each university involved in the study since 1998. For example Dublin City University (DCU) saw an increase of 10 per cent of students being awarded a first-class honours degree during this time period. A previous investigation into grade inflation in Irish universities within the period 1994-2004 also concluded that there is no evidence to suggest any improvement in ability or motivation among university entrants. Therefore grade inflation was found as the only explanation for the large increase in first class honours degrees being awarded (O’Grady and Guilfoyle 2007). Although this study did not focus on mathematics or any other subject or degree course specifically, a large proportion of the degree courses within the study had mathematical components. A report was recently prepared by the Insight Statistical Consulting for the Higher Education and Training Awards Council (HETAC) which provided supporting data for grade inflation in HETAC awards in Ireland. The report dealt with the period between 1998 and 2002 although it suggested that it was very probable that grade inflation long predated this period. Between 1998 and 2002 grades were found to have inflated significantly at certificate, diploma and degree level. The inflation was particularly significant at the upper end of the spectrum in all qualifications. At certificate level, the proportion of distinctions (average mark of 70% or higher) increased from around 12.5% to around 20% over the period. At Diploma level, the proportion of distinctions increased from roughly 12% to 18%. In HETAC degrees, the proportion of first class and 2:1 grades had increased while the proportion of 2:2 grades and passes had commensurately declined (Walsh, 2004). Walsh (2004) stated that these increases in first class awards between 1998 and 2002 were highly statistically significant. A study on grade inflation in eight colleges of nursing in Ireland by O’Grady (2008) highlighted that the minimum points on which students were allowed to enter third level courses in Ireland proved to act as a reliable indicator of average student ability on those courses and an even more accurate index of average student ability among students accepted to a course could be obtained from the median of their CAO points (O’Grady and Guilfoyle, 2007). Two of the colleges in this study were found to have a
large excess of first class and 2.1 awards; Waterford IT and Tralee IT, over all the other colleges of nursing. Because of this the prediction was that they would both have shown strikingly high median points. This was not found to be the case however as neither of them ranked highest out of the eight colleges. In general nursing, UCC students had higher median points than the Waterford Institute of Technology with UCC and UL both ranking higher than Tralee IT.

Of the most worrying findings in relation to grade inflation in Irish second and third level education was that the former Minister for education, Mr O’Keeffe, was forced to order an investigation into grade inflation in Irish universities after concerns regarding the quality of graduates was raised by the chiefs of some multi-national companies operating in Ireland (Breaking News 2010).

Reported declines in mathematical competencies in third level education in Ireland are met with increased numbers of students graduating with first class honours in the university sector. Research findings regarding the issue of grade inflation in Leaving Certificate mathematics grades on entry to UL are detailed in Chapter 4.

Upon acknowledgement that the ‘Mathematics Problem’ exists and some probable contributing factors to its existence most institutions attempt to alleviate the issue in some way. Some such attempts are examined next in section 2.3.
2.3 Mathematics Support Services in Third Level Education: Attempting to Alleviate the ‘Mathematics Problem’

Due to the realisation of the decline in mathematical competency levels of beginning undergraduates, many institutions put measures in place in an attempt to alleviate it. Whilst not wishing to turn away any prospective students many institutions recognised the need to be proactive in supporting its weaker ones. The decision surrounding the most appropriate means of addressing the issue of deficiencies in mathematics competencies necessary for third level education can only be effectively taken providing consideration for all existing types of interventions has taken place. Mathematical interventions of this nature have often been described in terms of being either a ‘stop gap’ solution or a ‘long term’ solution (O’Donoghue 2004). A variety of interventions, both ‘stop gap’ and ‘long term’, are detailed next along with the theoretical concepts underpinning them.

2.3.1 Stop Gap Solutions

O’Donoghue (2004) described any intervention which provided a short term solution to a long term problem as a ‘stop gap’ solution to the ‘Mathematics Problem’. He acknowledged the need and function of such interventions as aiding students to successfully complete service mathematics in third level education. Such interventions include drop-in centres, diagnostic testing, bridging courses and computer assisted learning (CAL). Details of these intervention types and where they have been used are found in section 2.3.1 (a) – 2.3.1 (d).

2.3.1(a) Drop-in Centres

Mathematics Drop-in Centres provide an extra resource for students in third level institutions. They generally consist of a comfortable room with a variety of resources including several trained tutors who are available to assist students on a one to one basis. Drop-in Centres are….

"....symptomatic of problems in the education system as a whole which result, for many reasons, in students embarking upon undergraduate degree programmes for which they are inadequately prepared. As such, provision of centres of this sort should be seen as one of a number of stop-gap measures intended to alleviate the situation in the short term while more radical longer-term solutions are put in place" (Croft 2000; p.431).
Drop-in Centres are becoming more and more common in third level institutions. A Learning and Teaching Support Network (LSTN)-funded study was carried out in 2001 and involved 95 higher education institutions. This study, which aimed to investigate the extent of mathematics support provision in the higher education community in the UK revealed that 46 out of the 95 institutions had some form of mathematics support services, the most common being drop-in centres (Perkin and Croft 2004).

Some of these centres, often called Mathematics Learning Centres, are places where students can call and are provided with free one to one help, mathematical materials are made available to students (computers, books etc) and independent learning is encouraged. Many of these centres are aimed at non-specialist mathematics students although no student is excluded (Perkin and Croft 2004). These centres have proven to be a very popular form of mathematics support (Symonds et al 2008; Gill and O’Donoghue 2009). They have been shown to not only improve mathematical competency levels but also to reduce anxiety levels and increase student confidence (Croft et al 2009). Drop-in centre data has often been used to carry out research on mathematical standards and performance in third level. Many institutions have carried out such research; for example UL, Dublin City University, National University of Ireland Maynooth, Loughborough University and De Montfort University. These institutions keep records of student attendance in the drop-in centre and the degree programs they are pursuing. This information is monitored to determine the problems which are presented most commonly in the centre, what cohort of students are most often in the centre allowing learning centres to liaise with students’ departments to address any underlying problems which may be present (Croft et al 2009).

Drop-in centres or Mathematics Learning Centres are often responsible for the implementation of diagnostic testing to beginning undergraduates; this will be discussed further in section 2.3.1(b).

**Theoretical Concepts Underpinning Drop-in Centres**

Much of the learning which takes place in drop-in centres is based on the learning theory of Constructivism. Constructivism has been described as the learning which takes place when one constructs their own knowledge from past experiences, i.e. it is a very personal type of learning (Kroll and LaBoskey 1996). It is based on the exploration of different ideas and involves a facilitator to aid the learning. All of this is provided for in a Mathematics Learning Centre environment. Students can access books, notes and various other resources such as internet
access to aid them in their learning. Various aspects of constructivism are prevalent in a drop-in centre setting such as self-directed learning and peer-assisted learning.

2.3.1 (b) Diagnostic Testing
Diagnostic testing has become an increasingly popular tool in third level education, both in Ireland and abroad, to help identify weaknesses in basic mathematical skills in students in mathematics intensive courses (Tall and Razali 1993; Edwards 1995; Edwards 1996; Hunt and Lawson 1996; Edwards 1997; Lawson 1997; Engineering council 2000; Todd 2001; Gill 2006; Malcolm and McCoy 2007; Gill 2009). Diagnostic tests can take many forms however generally they are paper based tests which consist of a variety of questions testing fundamental mathematics skills (Faulkner et al 2010). The results of diagnostic tests are utilised in order to improve mathematics provision within third level institutions (UL, Loughborough University, and Coventry University). One such use is to encourage students who have performed poorly in the test to visit the mathematics drop-in centre on a regular basis. Even students entering third level in Ireland with good Leaving Certificate grades or indeed UK universities “with good A-level grades may belie a weakness in some fundamental area of mathematics” (Booth et al 2002, p.101). For this reason universities do not solely rely on pre-university mathematics results to determine the mathematical ability of entering students, hence, the need for diagnostic testing.

Diagnostic testing has also been used to profile students. Diagnostic tests can provide more than just an insight into the mathematical knowledge and skills of individual students. They can also be used to document information such as gender, previous mathematical background, third level course, age etc helping those in mathematics education research to exploit the information by building up profiles of particular cohorts of students. For example Barry and Chapman (2007) compared Australian students’ end of school grades with their performance in a mathematics diagnostic test against their end of semester one grade in order to calculate if there was a better indicator of mathematical ability on entrance to university than end of school examination results. They found that students who took higher level mathematics courses in university, regardless of whether they had the same end of school examination result, got roughly 9 more marks than their colleagues doing lower level mathematics courses. Therefore the level of the mathematics course that students take can be used to help build a picture of students’ likely performance in end of semester examinations. Similar studies were carried out by McDonald
The prediction of student mathematical achievement through the use of diagnostic testing is considered an extremely valuable process which enables third level institutions to determine what profile of students they are dealing with and whether or not support services need to be put in place. In many cases in Ireland, the UK and other countries worldwide, based on the aforementioned research findings regarding the ‘Mathematics Problem’, support services are necessary (Lawson 1997; Gill et al 2010). The prediction of student achievement is an objective of this research project and is detailed in chapter 5.

Diagnostic testing has been used in UL since 1997 to help identify students who may be ‘at risk’ of failing service mathematics examinations and since then there has been a significant decline in mathematical competency (Gill 2006). The background of this test is detailed in chapter 3 followed by how the diagnostic test data is used to inform the research in chapter 4.

**Theoretical Concepts Underpinning Diagnostic Testing**

The learning theory most closely aligned with diagnostic testing as an intervention method is Cognitivism which is a psychological learning theory developed by Bode in 1929 (Hilgard 2005). The two major assumptions underlying this theory are:

- The memory system is an active organised processor of information and
- That prior knowledge plays an important role in learning.

This theory, which is also concerned with individuals’ ability to sort and code information, centres on the learner more than their environment. The need for students to rely on their memory of mathematics as well as draw on prior knowledge is central to their performance in a diagnostic test. Details of diagnostic testing in UL as well as students’ performance in it over time, can be found in chapter 4.

**2.3.1 (c) Bridging Courses**

Bridging courses are not unlike front-end tutorials, which are detailed in section 2.3.2 (a), as the main aim of them is to bring students up to speed with the mathematics necessary for third level education. They differ however in that they usually run for a longer period of time. For example
in UL a course entitled ‘Head Start Maths’ is offered to mature students. It is quite an intensive course and runs in the weeks before the start of semester one revising fundamental topics such as rational numbers, factorisation, graphing lines etc. It aims to refresh students who have been out of formal education for some time on mathematics fundamentals needed for successful completion of service mathematics courses within degree programs in UL. Research surrounding the effectiveness of this course in UL showed statistically significant improvements in students’ mathematics self-concept (p < 0.05) upon completing the course (Gill 2008).

The rationale for implementing a bridging course in third level institutions was described by Booth et al (2002) as a means “to provide a local equivalent to national entry qualifications for entering the higher education environment” (Booth et al 2002, p.106).

The University of New South Wales in Australia offered a course to bring students to the mathematical standard which was needed to complete the bridging course they offer. The course which was entitled the ‘Mathematics Skills Program’ (M.S.P) ran for 13 weeks, one semester, and cost $600 (UNSW 2009). The MSP brought students to the standard of HSC Mathematics. Upon successful completion of this students were eligible to enter the Mathematics Bridging Course which provided 40 hours of tuition at a fee of $300. Completion of the bridging course allowed students to gain access to degree programs with mathematical elements.

A bridging course was also offered in the University of New South Australia in mathematics, physics, chemistry and communication. This course provided alternative routes for prospective students to gain access to a Science or Engineering degree program. The mathematical component of the bridging program aims to fill some of the gaps in the students’ background as well as improving students’ confidence with mathematics (Boland 2002).

**Theoretical Concepts Underpinning Bridging Courses**

The nature of a bridging course involves quite a fast paced delivery of a high quantity of content. Time constraints generally result in the transmission of knowledge from teacher to learner and so it runs on the basis that knowledge can be transferred rather than constructed. This type of intervention shows links with the learning theory of Behaviourism (Skinner 1953) which centres around the belief that learning is the acquisition of new behaviour through conditioning.
2.3.1 (d) Computer Assisted Learning (CAL)

“Computer Assisted Learning seeks to provide an individualized learning environment for each student” (Yeo 1972; p.167). Generally an institutional program is stored on a computer and the student can interact with the program. Alternatively students can access web based resources provided by an institution and use them in their own time, on or off campus. These programs could include a wide variety of content and entail partaking in several learning experience such as drill and practice routines or a more complex tutorial type learning experience (Yeo 1972).

The University of Queensland used MATLAB, a software tool used for numerical analysis as well as visualization. This Computer Assisted Learning (CAL) support was implemented in an attempt to improve the provision of mathematics education for first year undergraduate students. This initiative yielded “higher confidence, improved understanding and better attendance levels” (Tonkes et al 2005, p.756). The workbook which accompanied the software also allowed for more independent and flexible learning for the students and allowed more time for tutors to prepare higher order teaching.

Forms of CAL support have also been adopted by other universities such as UL and Loughborough University through the use of packages such as MATHWISE and CALMAT. These packages aimed to re-enforce what was taught in mathematics lectures and tutorials (Gill 2006). Loughborough University also made use of CAL by using it in conjunction with diagnostic testing. Upon receiving results of mathematics diagnostic tests given to students on entry to Loughborough University, students could reattempt the test on-line and were given “feedback in the form of correct answers and helpful hints” (Armstrong and Croft 1999, p.65).

Another new initiative within CAL was used in Loughborough University in which every student in a lecture hall was given a ‘clicker’. The ‘clicker’ is a small electronic hand piece which allows students to anonymously answer questions in class and the teacher can then give immediate feedback. The aim of this introduction to the lecture hall was to explore the use of engagement through technology to overcome the ‘Mathematics Problem’. It was hoped that immediate feedback and direct participation in lectures would lead to improved learning (King 2009). The University of Loughborough also offered on-line support in the form of handouts, video tutorials and exercises on a wide range of topics which were known to cause difficulty (arithmetic, algebra and trigonometry). This service, which has also been offered by UL in the form of handouts and fact sheets specifically tailored to each service mathematics module offered in UL,
aimed to aid students whose preferred mode of learning may not be satisfied by other forms of mathematics support offered at university level (Croft et al 2009).
Dublin City University (DCU) created an on-line examination of core mathematics skills. Students can complete the test in their own time as many times as it takes to pass, the aim being that passing it would support engineering students’ progression in mathematics modules throughout their time in university (Carr and Ní Fhloinn 2009). Several other universities used on-line examinations which were taken as many times as was necessary and gave specific feedback to students if they had answered incorrectly. For example Kingston University used a program called ‘Finding Electronic Teaching, Learning and Assessment Resources’ (FETLAR), Brunel University used a program called ‘Computer Added Assessment’ (CAA) in which students were asked multiple choice questions consisting of deliberately set answers which students commonly made mistakes with. The Institute of Technology Sligo piloted an on-line form of mathematical assessment (Doyle 2009; Fletcher et al 2009; Greenhow 2009).
Ahmed and Love (2009) made use of technology through text messages to contact students to arrange meetings with both the mathematics learning centre and the mathematics advisor in the University of Glasgow. They found that this was a more effective means of contacting students than e-mailing. Mathematics examination dates, help resources and mathematics pod-casts were also put online for students in the University of Glasgow to help improve mathematical provision there. O’Shea and Mac an Bhaird (2009) of the National University of Ireland (NUI) Maynooth used touch screen technology in their support tutorials in which students were able to follow the workings of the tutor and ask questions where necessary. This material was then placed on-line and therefore students did not have to take down notes. Many students reported that they could actually listen to the teacher and had time to think about the mathematics. On-line video tutorials which focused on the applications of mathematics were also made available to students in NUI Maynooth.

**Theoretical Concepts Underpinning Computer Assisted Learning**

Computer Assisted Learning draws on many of the learning concepts within constructivism. This form of intervention often involves student engagement with self-directed learning, discovery learning, active learning and experiential learning depending on how the student chooses to engage with the CAL material in question. Students are required to construct their own thoughts
and understanding of particular subject matter through the use of prior knowledge and/or by researching new information themselves in an attempt to inform their learning.

This examination of ‘stop gap’ interventions is now followed by an analysis of ‘long term’ interventions which could serve as an alternative solution to the ‘Mathematics Problem’ depending on the specific needs and resources available to a particular institution (see section 2.3.2).
2.3.2 Long Term Solutions

Long term intervention solutions to the ‘Mathematics Problem’ as the name suggests, intend to serve as an ongoing solution to alleviate issues relating to mathematical competency levels in third level institutions. They intend to tackle the problem as a whole as opposed to artificially covering the cracks for the short term in order to aid students to pass an upcoming examination (Perkin and Croft 2004; Mac an Bhaird and O’Shea 2009; Gill et al 2010). Some long term interventions are detailed in this section.

2.3.2 (a) Tutorials

Tutorials are a very popular and practical method through which third level institutions have attempted to improve the mathematical knowledge of their students. These tutorials can take different forms:

• Front End Tutorials

Front end tutorials as the name suggests occur within a couple of weeks preceding the beginning of the first semester of the academic year. The purpose of these tutorials is to help students, with particular emphasis on adult learners, reach “a satisfactory level of competency in fundamental mathematical skills” to avoid them feeling overwhelmed when the semester begins (Gill and O’Donoghue 2007, p.15). The University of Limerick offers this type of support in the two weeks at the start of semester one and focuses on skills such as order of operations, fractions, indices, logarithms, manipulating formulae and solving simultaneous equations (Gill 2006).

• Support Tutorials

These tutorials run during the academic year and are offered to students who wish to gain extra support and practice in addition to their lectures and regular tutorials. These tutorials give students a chance to tackle specific difficulties they are experiencing. Because of this, support tutorials are often student-led in nature and have proven to be a very popular form of support in UL (Gill and O’Donoghue 2007). When this form of support was first implemented in UL, a study carried out by O’Donoghue (1999) showed a drop in the failure rate of students who attended the support tutorials over those students who were also advised to attend and chose not to.

Booth et al (2002) maintained that the rationale for this type of tutorial was to develop areas where skills were missing or needed strengthening. In NUI Maynooth, O’Shea and Mac an
Bhaird (2009) offered a Maths Proficiency course for students who failed a diagnostic test given at the start of the academic year. Similar to the student led format of support tutorials in UL, students in NUI Maynooth could e-mail tutors before the workshops and request particular topics they wished to be covered in class. They also offered a Maths Foundation Course which also ran as a support tutorial parallel to lectures. This covered very basic mathematics skills and was primarily attended by mature students.

Symonds et al (2007) described a non-traditional form of parallel tutorial in which Physics students in the University of Loughborough, who had been deemed less well-prepared mathematically than their counterparts, were taught in a separate tutorial group. This separate tutorial group engaged in the use of different teaching materials and teaching methods to that of their counterparts. These students were then given the same final assessment as everyone else, the result being “an increase in pass rate from 48% to 67%” compared to the previous year cohort of Physics students” (p. 140).

**Theoretical Concepts Underpinning Tutorials**

An intervention in a tutorial format could be executed using many different learning theories to inform the structure of the lessons. Learning theories such as Behaviourism, Cognitivism and Constructivism could all be implemented depending on the preference of the teacher/facilitator and the goals which they wanted to achieve in their lessons.
2.3.2 (b) Peer Assisted Learning

A technique which has been used by some primary schools in America, known as Peer Assisted Learning strategies (PALS), aims to assist students with varying learning histories to learn mathematics (Fuchs et al. 2002). This same strategy has been adopted by some third level institutions in the UK (University of Leicester, University College London). In 2003 the LSTN Maths TEAM reported on this practice in the UK in which second and third year students assisted first year students in mathematics modules which they have previously studied. This help came in the form of one-to-one or small group sessions. A similar practice was also set up in an Irish University, the Institute of Technology Tralee (IT Tralee), in which the aim was “to improve achievement in first year Mathematics, through provision of a student centred mentoring programme” (Cleary 2008). This initiative which was also entitled PALS was targeted at students who did not achieve what IT Tralee considered a satisfactory grade (> 50%) in their initial continuous assessment assignment. Those students who wished to be mentors were given training sessions of which there were seven altogether. At the end of each session the mentors were required to fill out evaluation sheets so that the PAL co-ordinator could assess how successful the session had been. Overall the co-ordinator found that the feedback from mentors and the students being mentored was positive towards the whole experience (Cleary 2008).

Theoretical Concepts Underpinning Peer Assisted Learning

Peer assisted learning (PAL) is a generic term for a group of strategies that involve the active and interactive mediation of learning through other learners who are not professional teachers (Topping and Ehly 2001; p.113).

It is implemented on the basis that students learn from each other regardless of the range of abilities in any one classroom (Fuchs and Fuchs 2001). Peer Assisted Learning can facilitate discovery learning, experimental learning, self directed learning and situational cognition if implemented effectively.
2.3.2 (c) Loughborough: University-Wide Strategy

In a recent study Croft et al (2009) described the development and implementation of a university-wide strategy to tackle the ‘Mathematics Problem’. This intervention strategy which was developed at Loughborough University involved an “integrated and holistic” vision of mathematics support in which the Mathematics Support Centre staff:

- Offer mathematics support to all university students.
- Teach mathematics modules to all users of mathematics within the university.
- Communicate on a regular basis with outside departments whose students are involved in mathematics to discuss possible issues that students may be having etc. and to make sure the mathematics they are providing is meeting the demands of students’ other subjects.
- Make contributions to national teaching and learning projects.
- Undertake mathematics pedagogical research.

(Croft et al 2009, p.112)

Loughborough University also has a centre for mathematically anxious students. This centre which is known as the Eureka Centre for Mathematical Confidence aims to build confidence in those students who lack confidence in their mathematics or statistics in order to help them in their study of engineering (Harrison 2008).

This proactive form of mathematics support could be used as an example to other institutions as Croft et al (2009) maintained that “the need for mathematics support is likely to continue into the future and universities need to be prepared for this” (Croft et al 2009, p.124). This holistic approach to mathematics support is likely to tackle many of the issues faced by widening access at third level as well as positively contributing to retention. As Thomas (2003) recommended, measures to improve retention need to be an institution-wide activity, something the practitioners at Loughborough University have implemented. This strategy differs slightly from that of many Mathematics Learning Centres as it deals with students’ emotional issues surrounding mathematics, as in the Eureka Centre for Mathematical Confidence, in addition to building on mathematical knowledge and approaches to studying mathematics.

Issues which the third level education sector have faced due to widening access such as those surrounding the increase in non-traditional students and declining competency levels in mathematics are discussed in detail in chapter 7.
Much of the thinking behind this university-wide strategy to tackle the ‘Mathematics Problem’ is reflected in the setting up of Loughborough’s National Centres for Excellence. Both approaches are long term solutions to the problem. Details of the establishment of several National Centres for Excellence are found in section 2.3.2 (d).

2.3.2 (d) The Establishment of National Centres for Excellence

A new support system which has emerged in Ireland and the UK is the introduction of National Centres for Excellence. The main aim of National Centres for Excellence in Mathematics is for long term improvements to the provision of mathematics teaching and learning by supporting those involved in mathematics education such as teachers, students, government bodies etc. This support could take many forms such as providing recent research findings on good practice in teaching and learning mathematics, running courses for secondary school teachers, and generally communicating with all stakeholders of mathematics education to develop a collaborative approach to improving mathematics provision in second and third level education. One such centre was established in California State University called ‘The Centre for Excellence in Science and Mathematics Education’ (C.E.S.M.E). This centre was established in 1992 with the aim of improving elementary and secondary teacher education. The centre worked in conjunction with the College of Natural Sciences and Mathematics and the College of Human Development and Community Service with the hope that collaborative activities among these parties would bring together those in education, science and mathematics faculties as well as outside bodies such as those involved in industry, schools and community organizations.

In response to issues raised in the Post-14 mathematics inquiry, Burghes and Hindle (2004) suggested the establishment of a National Centre for Excellence in Mathematics Education in the UK. They maintained that an establishment such as this could increase the support of all those involved in first and second level mathematics education in the UK.

In 2005 Loughborough and Coventry Universities established a collaborative Centre for Excellence in Teaching and Learning (CETL). The CETL programme which is sponsored by the Higher Education Funding Council for England is aimed solely at third level education. Although not specifically focused on mathematics, the CETL, which is directed by Professor Tony Croft (Loughborough University) and Professor Duncan Lawson (Coventry University), aims to strengthen further the mathematics support systems in these universities by providing
ongoing mathematics and statistics support to students and also to introduce new initiatives within departments at Loughborough and Coventry universities (http://www.sigma-cetl.ac.uk/).

June 2009 saw the official opening of the National Centre for Excellence in Mathematics and Science Teaching and Learning in the University of Limerick (NCE-MSTL). This centre works collaboratively with Mary Immaculate College, Limerick Institute of Technology and the Institute of Technology Tallaght. The NCE-MSTL, similar to the previously mentioned centres, aims to improve the provision of mathematics and science teaching and learning by supporting its stakeholders. It aims to encourage these stakeholders (teachers, academics, those in industry, government bodies) to get involved in whatever initiatives may be taking place within the centre. For example in August 2009, 2010 and 2011 the National Centre worked collaboratively with the National Council for Curriculum and Assessment (NCCA) to run a statistics course, trigonometry and geometry course and a number, algebra and functions course for second level mathematics teachers in Ireland. These courses were run as part of the training of second level teachers for the new mathematics curriculum in Ireland ‘Project Maths’. ‘Project Maths’ is a further example of a long term intervention aimed at addressing the ‘Mathematics Problem’, details of which are outlined in section 2.3.2 (e).

**Theoretical Concepts Underpinning National Centres for Excellence in Teaching and Learning**

Centres for Excellence in Teaching and Learning aim to promote best practice in teaching and learning in mathematics through the presentation and distribution of up to date evidence based research. These centres openly offer research findings to teachers, at all levels, who would benefit from the information. Although practitioners are informed about the services offered by the centre, through websites and newsletters, the onus is on the teachers to research what is available and determine how best to put particular findings into practice in their own classrooms. Because of this Centres for Excellence support the Constructivist learning theory in which the responsibility is on the teachers to direct their own learning.
2.3.2 (e) The Irish Education System and the Introduction of Project Mathematics

One of the most significant responses to the ‘Mathematics Problem’ in Ireland in recent times was the introduction of a new mathematics curriculum at second level; ‘Project Maths’. This curriculum aims to tackle some of the issues which the implementation of the previous curriculum had been accused of causing (e.g. rote learning leading to a lack of real understanding) and also ease the transition of students from second to third level education. In order to get a clear understanding of this new curriculum it is important that the Irish education system in its entirety is understood first.

2.3.2 (e) (i) Entry Into Third Level Education in Ireland

In Ireland second level education involves two programs: the Junior Certificate program, which takes three years to complete, and the Leaving Certificate program, which takes two years to complete. Students begin second level education at age 12-14 and finish at age 17-19 approximately. On completion of the Junior Certificate program students are equipped with the pre-requisite knowledge necessary for successful completion of the Leaving Certificate program. Both programs are completed through a summative state examination at the end of the three and two year periods respectively. Mathematics at Junior Certificate and Leaving Certificate can be taken at three levels; Higher Level, Ordinary Level and Foundation Level. Generally only students who have completed the Higher Level mathematics Junior Certificate examination take Higher Level mathematics for the Leaving Certificate. Currently 45% of students take Higher Level Junior Certificate mathematics and only 16% take Higher Level Leaving Certificate mathematics (Prendergast 2011). Students are however free to change level throughout either program as they wish. The Higher Level curriculum, as the name suggests, is the most advanced of the three levels and includes some topics which the Ordinary Level syllabus does not cover such as integration. Foundation mathematics is the least advanced of the three levels.

Direct entry to third level education in Ireland can be gained through students receiving sufficient Leaving Certificate grades at second level. Students who take Foundation Level Leaving Certificate mathematics are not entitled to direct entry to third level education. The minimum mathematics entry requirement for direct entry to universities is a grade C in Ordinary
Level Leaving Certificate mathematics. The minimum entry requirement for Institutes of Technology in Ireland is a grade D in Ordinary Level Leaving Certificate mathematics. The third level course which a student is eligible for is determined by a student’s performance in all six subjects taken in the Leaving Certificate examination. Overall performance in this examination, which is the final state examination taken by second level students, is based on a points system. Typically students take seven subjects at Leaving Certificate, three of which must be mathematics, Irish and English, providing students do not have an exemption from them. Approximately 82% of Irish students study mathematics from the age of 9 to at least 17 and 96% of Leaving Certificate students study mathematics (Breen et al 2008). Each grade received in the Leaving Certificate examination is awarded a specific number of points (see table 2.1). The total of a student’s highest six grades is calculated to give their CAO (Central Applications Office) points. The maximum points which can be awarded is therefore 600 points*. The CAO processes all standard applications to first year undergraduate courses in higher level institutions in Ireland. Entry to these courses is based solely on a student’s CAO points. Different degree programs have different CAO point requirements which are partly determined by the number of places available on a particular course and the level of demand for those places. The CAO points for different degree programs therefore vary from year to year.

The vast majority of subjects taken at Leaving Certificate are assessed through means of a terminal written examination worth 100% of the grade. Some of the terminal examinations are broken down into two examination papers, one such subject is mathematics. Different topics on the mathematics syllabus are examined on the two papers. The average performance of the two papers is calculated to determine the students’ Leaving Certificate mathematics grade. This method of assessment has been criticised saying it encourages examination focused teaching. This type of teaching has been shown to have detrimental effects on a student’s ability to learn essential mathematical knowledge and skills necessary to students’ future success in education and life (NCCA 2006). This was one of the reasons the new second level mathematics curriculum was introduced (see section 2.3.2 (e) (ii) which follows).
Table 2.1 Irish Leaving Certificate Examination Points Calculation Grid.

<table>
<thead>
<tr>
<th>Leaving Cert Grade</th>
<th>Higher Level</th>
<th>Ordinary Level</th>
<th>Bonus *</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>100</td>
<td>60</td>
<td>40</td>
</tr>
<tr>
<td>A2</td>
<td>90</td>
<td>50</td>
<td>35</td>
</tr>
<tr>
<td>B1</td>
<td>85</td>
<td>45</td>
<td>30</td>
</tr>
<tr>
<td>B2</td>
<td>80</td>
<td>40</td>
<td>25</td>
</tr>
<tr>
<td>B3</td>
<td>75</td>
<td>35</td>
<td>20</td>
</tr>
<tr>
<td>C1</td>
<td>70</td>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>C2</td>
<td>65</td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>C3</td>
<td>60</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>D1</td>
<td>55</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>D2</td>
<td>50</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>D3</td>
<td>45</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

*Bonus points are awarded by certain colleges in Ireland, such as UL, for the achievement of an honours grade (A1-C3) in Higher Level Leaving Certificate mathematics.

2.3.2 (e) (ii) Project Maths

A major review of the Junior and Leaving Certificate mathematics syllabus in Ireland began in 2008. This review, which was the first since the 1960s, aimed to turn the focus of second level mathematics teaching towards developing students’ understanding of the mathematical concepts by engaging them in problem solving with increased use of contexts and applications encouraging students to relate mathematics to everyday life. Changes were not solely focused on what students learn in mathematics however but also how they learn it and how they will be assessed. The changes to the curriculum were met with appropriate changes to the assessment procedure so that the level of understanding of the students is assessed as well as their subject matter knowledge.

The new curriculum entitled ‘Project Maths’ has been led by the NCCA and piloted in 24 schools around Ireland since September 2008. The introduction of the curriculum to all schools around the country took place in September 2010.

The mathematics syllabus was introduced to each of the pilot schools by strand as follows:

1. Statistics and Probability
2. Geometry and Trigonometry
3. Number
4. Algebra
5. Functions

By September of 2012 all five strands will be introduced to all secondary schools in Ireland and the current Leaving and Junior Certificate mathematics curriculum will no longer be delivered.
Introducing Mathematics Teachers to ‘Project Maths’

A ‘Project Maths’ Development Team (PMDT) was established by the Department of Education and Skills (DES) to help with the implementation of the new mathematics curriculum in all post-primary schools from September 2010. The project was directed by the National Council for Curriculum and Assessment (NCCA). The PMDT consisted of 1 National Coordinator, 1 Assistant National Coordinator and 16 Regional Development Officers (RDO). The PMDT team worked in conjunction with the NCCA, the National Centre for Excellence in Mathematics and Science Teaching and Learning Centre, University of Limerick (NCE-MSTL), the Irish Maths Teachers Association and the National Centre for Technology in Education.

The RDO’s received in-service on all strands of the ‘Project Maths’ curriculum from the National Centre for Excellence in Teaching and Learning in UL. The mathematics teachers in the 24 pilot schools also received this in-service as they had to deliver the new curriculum before any other schools in Ireland. A programme of teacher professional development was developed and the PMDT delivered workshops nationwide since September 2009.

The new curriculum, which focuses on understanding the mathematics and applying the mathematics, intends to tackle many of the contributors to the ‘Mathematics Problem’ such as: transitional issues, mismatch between lecturers expectations and students actual mathematical ability. It will take time before the success of this long term intervention can be assessed. Several reviews of the UK second level mathematics curriculum have taken place also in the past 50 years, some of which have had a negative impact on the mathematical performance of second and third level students, information on this is detailed next.

Changes to the Second Level Mathematics Curriculum in the UK

Some of the reviews of the second level mathematics curriculum which have taken place in the UK over the last 50 years have been found to be quite detrimental to the provision and uptake of mathematics (Engineering Council 2000). A-level mathematics was said to have been in its ‘Golden Years’ in the 1960’s when the curriculum was primarily directed by universities, and students entering third level education were said to have had an ideal foundation of pure mathematics and mechanics (Gill 2006). In 1988 however changes to O-Levels (the state examination which precedes A-Levels) took place, such as a reduction in mathematical content, which led to “a decline in students’ concept of proof and in their technical fluency and...
understanding of algebra” (The Engineering Council 2002, p. 2). This had a knock on effect as it caused the number of students taking mathematics at A-levels to fall dramatically (Gill 2006). In 1991 and again in 1994 the national mathematics curriculum was adjusted resulting in more essential mathematics skills being cut out of the curriculum. Consequently examination boards in the UK had to adjust assessment papers cutting out important elements within trigonometry, calculus, complex numbers and vectors all of which were essential skills necessary for third level mathematics (Institute of Mathematics and its Applications 1995).

Poorly thought out changes, such as those discussed here in the case of the UK, can have a negative effect on mathematical performance in third level education. It is hoped that the review of the Irish second level mathematics curriculum will have avoided these pitfalls and that the changes will improve the mathematical performance of second and third level Irish students.

### 2.3.3 Conclusion

Many different mathematical interventions have been tried and tested by different institutions worldwide. The appropriateness of an intervention whether it be a 'stop gap' or 'long term' solution, depends on the specific needs and resources available to a particular institution. Another practice which is often used by institutions to better inform itself in relation to ‘at risk’ students is profiling mathematics students. Studies relating to the profiling of ‘at risk’ students are outlined in section 2.4.
2.4 Profiling Mathematics Students: A Review of the Literature

The practice of student profiling has been used in tertiary mathematics education in countries such as New Zealand, Australia, South Africa, Canada, the UK and Ireland (Chansarkar and Michaeloudis 2001; Wilson and MacGillivray 2007; James et al 2008; Holton et al 2009; Faulkner et al 2010). As previously discussed diagnostic testing has been a major instrument in the diagnosing of students’ mathematical weaknesses to date however more sophisticated methods of determining whether or not students are ‘at risk’ of failing services mathematics have also been used. Such methods include profiling mathematics students in terms of the characteristics they possess and the relationship of those characteristics with success in mathematics examinations. The motives behind which institutions profile their students vary, however common rationale include:

- To build a picture of the suitability of certain pre-tertiary level education qualifications as preparation for tertiary mathematics (James et al 2008).
- To determine the general mathematical knowledge and complexities of the mathematical understanding of a particular cohort which can inform lecture content and delivery accordingly (Kannemeyer 2005; Wilson and MacGillivray 2007; Faulkner et al 2010).
- To determine whether external factors, i.e. non mathematical factors, such as motivation, nationality, distance of residence to place of study, attitudes, and beliefs are influential to success in tertiary mathematics (Chansarkar and Michaeloudis 2001; Carmichael and Taylor 2005; Liston and O’Donoghue 2009).
- To assess if grade inflation is present in pre-tertiary level examinations (Kitchen 1999; Lawson 2003; Faulkner et al 2010).
- To allow for the early address of possible mathematical weaknesses in a student’s university career (Gill et al 2010).
- As a method through which mathematical models can be created to predict student mathematical achievement in tertiary education (Wilson 1971; Murphy 1981).
- As a reference point from which entrance recommendations to tertiary courses involving mathematics can be revised and updated (James et al 2008).
The profiling of tertiary level mathematics students has produced some interesting findings which are outlined in detail in sections 2.4.1-2.4.3.

2.4.1 Student Profiling: A Means of Assessing the Suitability of Pre-Tertiary Mathematics Education as Preparation for Third Level Mathematics

In New Zealand a study was carried out which investigated the suitability of the National Certificate for Educational Achievement (NCEA) mathematics as preparation for tertiary mathematics. The NCEA is the formal secondary school qualification in New Zealand and was introduced in 2002. The NCEA system provides students and tertiary providers with a variety of useful information regarding the abilities of prospective students. It does so by offering information on students’ mathematics performance in areas such as differentiation, integration, trigonometric equations, algebra and complex numbers. This study involved 1329 beginning undergraduate students who were taking a 1st year core calculus module. All students involved in the study intended on majoring in engineering, mathematics and physics at the University of Canterbury. The analysis of the relationship between NCEA qualifications and first year tertiary mathematics performance concluded by stating that the “NCEA is a solid preparation for tertiary study” (James et al 2008, p.1048). There was a statistically significant correlation found between high achievement in NCEA and high achievement in first-year calculus (F=295, p < 0.001). In addition to this the two most important mathematics areas for predicting success in first year mathematics were found to be differentiation and integration. Note: The extremely large sample size in this study may have resulted in statistically significant relationship between NCEA and first year calculus, the $R^2$ value is not provided in this study.

An investigation carried out in Brisbane, Australia also aimed to come to some conclusion regarding the suitability of senior high-school algebra and calculus as preparation for the tertiary study of mathematics. However this study built up a student profile in a different manner. The authors created a multiple choice questionnaire to establish the basic mathematical skills and weaknesses of students enrolled in a first year data analysis subject. Each questionnaire also required students to provide information regarding their mathematical background and demographics. Questionnaires were completed by 552 students giving detailed information relating to each individuals pre-tertiary mathematics education. The assumed level of knowledge
of all students involved in the study was senior algebra and calculus, the final mathematics examination before leaving secondary level education, known as Maths B. Students were classified as ‘maths’ students or ‘non-maths’ students. ‘Maths’ students included all those who were studying a mathematics degree as well as applied science students majoring in mathematics. The significant predictor variables which were found to influence a student’s score on the questionnaire were: Maths B result (p < 0.001), maths students (p < 0.001), gender (p = 0.002), higher level maths (p = 0.002) and self-efficacy (p = 0.003). The variables which were also involved in the analyses and not statistically significantly related to students’ scores were: whether the students repeated or not, 1st semester student or not, year of study and years since school. The model which was created based on the statistically significant variables accounted for 30.5% of the variation in the skill questionnaire scores. When students’ answers were analysed it was found that students had difficulties with inequalities unless higher level mathematics was taken at second level and difficulties with multi-step problems, abstractions and the introduction of letters. The study concluded that in order to have full and confident use of basic mathematics skills, tertiary educators need greater awareness of the extent of consolidation needed at tertiary level for students to be able to apply these skills in new or multi-step situations which characterise so many tertiary areas of study (Wilson and MacGillivray 2007).

Different institutions have come to different conclusions in relation to the suitability of pre-tertiary mathematics education for third level mathematics in their respective countries. Each finding however is equally valuable in informing mathematics departments of the current student profile whom they are teaching.

### 2.4.2 Student Profiling: Addressing Mathematical Weaknesses

Upon establishing whether or not a student’s previous mathematics education is sufficient in preparing them for tertiary mathematics, many institutions use their knowledge of a student profile to address mathematical weaknesses. Some institutions use questionnaires and others diagnostic testing in an attempt to establish the common mathematical misunderstandings and shortcomings of a cohort (Lawson 2003; James et al 2008; Gill et al 2010). Another use of student profiling is emerging which attempts to predict student achievement.
2.4.3 Student Profiling: The Prediction of Student Achievement

The challenge of determining the most effective method to predict student performance has been of great interest to researchers and educational researchers alike for many years (Mush and Broder 1999; Cherney and Cooney 2005). The reasoning behind determining the most effective method of prediction has often been with the overall intention of designing an educational intervention in higher education. In order to establish what factors are related to success and failure many researchers have used methods such as discriminant analysis, logistic regression, multiple regression, classification trees etc. Studies which have used these approaches are outlined in this section. Different prediction variables have also been used in an attempt to successfully predict mathematical achievement such as previous mathematical performance, approach to learning and student demographics.

2.4.3.1 The Prediction of Student Achievement: An Examination of Students Previous Mathematical Performance as a Predictor Variable

The prediction of student achievement is generally focused on trying to establish the characteristics which students’ possess which are likely to result in their success or failure in tertiary mathematics modules. Different studies have found different variables to be effective in predicting performance in third level mathematics.

An Irish study carried out by Fahey (2009) built up a profile of 1st year Science students in the Limerick Institute of Technology in order to predict completion and progression of this cohort of students to the 2nd year of their degree program. Fahey (2009) found that a mathematical model involving CAO points, the level at which Leaving Certificate mathematics was taken, Leaving Certificate mathematics grade, attendance at lectures and performance in semester one of 1st year was highly successful in predicting completion and progression of these 1st year Science students. Multiple Regression analysis revealed that the variables examined accounted for 45% of the variance in 1st year performance.

Simonite (2004) used a logistic regression model to test if degree classification in mathematical science had a significant relationship with entry qualifications. This extensive study involved 22,433 students from 536 different cohorts graduating from 98 UK universities between 1995 and 2000. Results showed that the upward trend in the classification of degree awards within the
time being examined (1995-2000) were explained by increasing high grades achieved before entering higher education. Hence Simonite (2004) found that there was a significant relationship between degree classification and grades achieved prior to entering third level ($p < 0.001$).

Mathematics education in the US has also exploited student profiling in the form of predicting student achievement. Belcheir (2002) undertook a method of predicting mathematical success in Boise State University, Idaho. In her study, which involved 734 students, Belcheir focused on performance on an Intermediate Algebra course and found that pre-enrolment variables such as academic preparation in mathematics, and course-related variables such as class organisation and management were the best predictors of performance on this course (p.13). Variables which were found to be not so influential were attitudes and disposition, other commitments, study skills and time and attitude towards mathematics. Academic preparation was found to be a statistically significant predictor of performance on the course using multiple regression ($R^2 = 0.2047$, $p<0.001$) and accounted for 20% of the variance in the success criterion. Another list of influential variables to mathematical success was developed in Illinois by Frerichs and Eldersveld (1981). Data from students from 8 community colleges were analysed to test what variables were statistically significantly related to someone being a successful mathematics student. The study involved 513 university mathematics students. Discriminant analysis was carried out and it was revealed that successful students were more likely to have: higher numerical skills, be in a traditionally taught course (a traditionally taught course being one in which the instructor sets the pace for learning as opposed to a non-traditionally taught course in which the students set the pace for learning), be older, have higher perceptions of his/ her mathematical ability and have a positive attitude towards mathematics ($p < 0.01$). The variables which were tested and considered to be less influential on mathematical performance included: gender and reasons for taking the mathematics course in question. Another investigation was carried out in America by Sheel et al (2002) in Coastal Carolina University. In this research alternatives to mathematics placement examinations as a basis for assigning students to entry-level mathematics courses in universities were examined. Sheel et al (2002) recognise the significant importance of implementing a technique that is effective as well as accurate for college students’ placement in the appropriate mathematics course. Similar to a primary school study by Brown and Brown (2006), Sheel et al (2002) questioned the appropriateness of a current measure of mathematical
performance and therefore its effectiveness in informing educational policies and more specifically in this case entry standards. Over 650 students took part in this study. High school grade point average (GPA), class rank, Scholastic Aptitude Test (SAT) scores, grades earned in high school algebra I and II, geometry, and advanced mathematics courses were the predictor variables of performance on the mathematics placement examination. This research concluded by revealing that the accumulative high school grade point average, mathematics SAT scores and final grade in Algebra 2 were the best predictors of success on a mathematics placement examination.

Other countries around the world who examined student profiles in this manner included Canada, the West Indies and Australia. Kwanthlen University College in Canada undertook a study in which 300 first year mathematics students’ study skills and learning strategies were tested using the Learning and Study Skills Inventory (LASSI). This study aimed to determine the particular study skills which are considered predictive of academic success. It was found that time management (p < 0.001), motivation (p < 0.001), anxiety (p < 0.01), concentration (p < 0.01) and self-testing (p < 0.05) were five of the LASSI sub-scales which were vital to statistically significantly predicting the final grades of first year mathematics students (MacNamara and Penner 2005).

McDonald (2008) examined the relationship between mathematics results from the Caribbean Examinations Council (CXC) general proficiency examination and the results from the General Certificate of Education (GCE) advanced level examinations in an attempt to predict mathematical aptitude for higher education. Within the CXC examinations in the West Indies mathematics and English are compulsory to take. If students pass CXC they are chosen for GCE for two years prior to University. GCE is the equivalent of the A-Levels in the UK and the Leaving Certificate in Ireland. There were 177 high school students aged between 17 and 19 analysed in the study. Each student’s academic records from high school were documented along with students’ responses to a semi-structured interview. The relationship between CXC results and GCE mathematics results were found to be statistically significant (p < 0.01). MacDonald (2008) therefore concluded this work by stating that CXC results are a good predictor of mathematics aptitude for GCE.

In Australia an investigation was carried out into whether or not a better predictor of mathematics university performance than Entrance Ranking Scores could be found (Entrance
Ranking scores or TER scores are obtained based on all courses taken at secondary school similar to CAO points in the Irish context. Barry and Chapman (2007) used diagnostic testing of basic mathematics skills in 2006 on 200 incoming first year science and engineering students. Students had 50 minutes to complete the multiple choice test based on basic pre-calculus mathematics with an emphasis on basic algebra. It was found that the relationship between semester one grades and diagnostic test results ($R^2 = 43.5\%$) was stronger than that of semester one grades and TER results ($R^2 = 26.8\%$). Students doing higher level mathematics at university were also found to perform to a higher standard than those doing lower level university mathematics. A student’s diagnostic test performance is therefore more influential than TER results on a student’s semester one mathematics grades.

Breen et al (2009) investigated how Irish third level students performed when faced with a mathematical literacy test and what factors most influenced their performance. This study focused on the effect of three factors: gender, self-confidence and prior mathematical experience. The test was comparable to a PISA test. It consisted of 13 mathematical items which had to be completed in 30 minutes. Each student on completion of the literacy test also completed a questionnaire relating to five different areas: personal, second-level experience and examination results, third-level experience, study habits and attitude to mathematics. The test was administered to 316 students in three third-level institutions in Ireland. A stepwise multiple regression model was used in which the four independent variables were LC points, LC level, confidence (low or high), and gender. The dependent variable was the score in the literacy test. It was found that LC points ($p < 0.001$), confidence ($p < 0.005$) and gender ($p < 0.001$) were all significant predictors of literacy scores but LC level was not. The adjusted R-squared value was 0.244, meaning that the model explained about 25% of the variation in the literacy test score.

Mac an Bhaird and O’Shea (2010) conducted another Irish study in NUI Maynooth which aimed to examine factors that impacted upon students’ success in first year Science mathematics. A total of 267 students from the academic year 2008/09 were the subjects for this study. Regression analysis was carried out and revealed that students’ diagnostic test result and Leaving Certificate points were highly correlated with final Science mathematics results.

The strong influence of factors, other than previous mathematical performance, such as psychological factors on mathematics performance should also be acknowledged. Liston and O’Donoghue (2010) highlighted the impact of affective variables, conceptions of mathematics
and approaches to learning on students’ ability to learn mathematics in third level education. An examination of the influence of such factors, as well as demographical factors, on the prediction of student achievement is outlined in section 2.4.3.2.

2.4.3.2 The Prediction of Student Achievement: An Examination of Demographics, Affective Variables and Approaches to Learning Mathematics as Predictor Variables

A study in the UK which aimed to establish a prediction method for student mathematical performance chose to use less traditional student characteristics as independent variables. Chansarkar and Michaeloudis (2001) profiled first year students studying Quantitative Methods for Business and analysed their profiles in terms of their performance in this mathematics module. A questionnaire was administered by tutors on the program inquiring mainly about student profile. Each student was required to provide details relating to their; course, age, nationality, gender, academic qualifications, residence and travel details to university. Students were then grouped into ‘good’ (those who secured a first or upper class honours degree) and ‘poor’ (those who failed) achievers. The characteristics of the two groups were descriptively compared in terms of mean, standard deviations and percentages. There was no significant difference in age, sex and distance travelled between the two groups of students. However when the percentages of students were compared in terms of highest qualification at entry (A-level or other), qualification in the qualitative subjects and place of residence noticeable differences were revealed. A total of 497 questionnaires were completed.

Another such study which aimed to predict mathematical performance through the examination of a variable other than mathematical skills was carried out in the University of Queensland. Carmichael and Taylor (2005) examined the effect of confidence on drop-out and successful completion of a mathematics course. The study involved 129 prospective university students enrolled in a tertiary preparatory mathematics level-A course in 2005. Questionnaires and interviews were completed by all students involved. Findings from initial studies showed that “only specific measures of student confidence predict their performance and that both gender and age mediate the strength of this prediction” (p.713). Only prior knowledge (r = 0.33, p < 0.01) and question confidence (r = 0.2, p < 0.05) correlated (weakly) with academic performance. It must be highlighted that the effect of confidence in mathematics on student performance in a
tertiary mathematics context is a complex area and the authors stress the need for further investigation to be carried out in this area.

In an Irish context Liston and O’ Donoghue (2009 & 2010) investigated factors influencing the transition to service mathematics in UL. In this study 15 semi-structured interviews (with 5 randomly selected students from 3 different services mathematics groups) were carried out. A total of 607 service mathematics students, included those involved in the interview process, completed questionnaires in relation to their attitudes, beliefs and self concept. Students’ CAO mathematics points and diagnostic test results were also documented and examined. A student’s enjoyment of mathematics \((r = 0.24, p < 0.01)\) and mathematical self-concept \((r = 0.22, p< 0.01)\) revealed a positive, although not very strong, statistically significant relationship with service mathematics examination performance. A positive statistically significant relationship was also found between cohesive conception of mathematics and deep approaches to learning \((r = 0.32, p < 0.01)\). The study also showed that previous mathematical competencies, as measured by students’ CAO mathematics points, impact significantly on students’ performance in first year service mathematics examinations. Cohesive conceptions of mathematics and CAO mathematics points combined were the strongest predictors of variability in service mathematics performance accounting for 23.3% \((R^2 = 0.233)\) of the variation in examination results. An examination of the correlation between diagnostic test scores and service mathematics examination performance also revealed a positive relationship between the two variables \((r = 0.31, p < 0.001)\). This study therefore highlighted the potential predictive ability of service mathematics performance through examination of students’ mathematical competency levels (CAO mathematics points and diagnostic test performance), their enjoyment of mathematics, mathematical self concept and cohesive conceptions of mathematics.

### 2.4.4 Conclusion

All of the aforementioned student profile examinations have been exploited in an attempt to improve mathematics education worldwide. Some mathematics educationalists maintain that the changing profile of mathematics entrants can be attributed to the “lowering of admissions requirements” into third level education (Kitchen 1999, p.57). Whatever the reason for the changing profile the examination of it is undoubtedly valuable. In addition to all of the previously mentioned uses of student profiling it has also been suggested that it is a valuable
source of information through which mathematics lecture content, teaching style, pace of delivery and overall policies on entrance standards to mathematics based degree programs can be informed (Kannemeyer 2005; Wilson and MacGillivray 2007; James et al 2008; Faulkner et al 2010). All of the findings in relation to the prediction of student achievement are used to inform the course of this research. The existing research allows the author to determine which method is more appropriate in her specific situation.

2.5 Conclusion

The ‘Mathematics Problem’ is an international problem which is still prevalent today in Ireland (Gill et al 2010). Many causes of the decline in mathematical competency levels have been documented such as transitional issues and grade inflation in second and third level education. Both short term and long term intervention measures have been put in place, however in the current challenging economic times the main concern should be an effective, sustainable solution to the ‘Mathematics Problem’ in Ireland. There are many effective statistical methods through which the prediction of students’ mathematical performance can be carried out, the majority of which have very seldom been taken advantage of in Ireland. Upon establishing the significant characteristics to predicting students’ performance in third level education, research informed intervention practices can be implemented to exploit these findings to the benefit of the students involved.

Following the literature review, the methodologies which are used in each phase of the research are outlined in chapter 3 which follows.
Chapter 3: Methodology

3. Introduction

This chapter provides a description of the methodologies employed by the researcher throughout the project. The research questions to be answered dictate the types of research methodologies which were most appropriate to underpin the author’s work. Established research methodologies were investigated and a justification for the methodological choices made in relation to this research are presented. Qualitative research methods were used throughout the research in an attempt to describe and understand current situations in service mathematics in UL. Quantitative research methods were also employed in the investigation into which variables were most indicative of failure in service mathematics. The combination of this mixed method approach to research complement each other as one set of methodological findings are supported by a different set of methodological findings in a process known as ‘triangulation’ (McFee 1992). A breakdown of the research carried out in each phase of the author’s work is detailed next.
3.1 Chapter Outline

This chapter outlines all of the methodologies employed throughout this research in the following order:

- Section 3.2 considers the research design, discusses the research paradigms which were adhered to throughout, outlines the mixed method approach implemented and finally highlights the theoretical framework which underpins the research.

- The next section, section 3.3, outlines the research problem, the objectives of the research and the research questions which were addressed in an attempt to solve this problem.

- Following the research questions, section 3.4 details a chronology of the research by phase along with the methodologies employed during each phase. This section comprehensively details the considerations which were carried out in relation to the intervention and the theoretical framework and evaluation model decided upon for the intervention.

- Section 3.5 outlines the qualitative and quantitative research instruments which were used to evaluate the success of the intervention.

- The final section, section 3.6, of the methodology chapter discusses possible issues within the research such as validity and reliability, triangulation, ethics and limitations of the study.
3.2 Research Design

3.2.1 Research Paradigm

A theoretical framework in research has often been described as a paradigm (Mertens 2005; Bogdan and Biklen 2006). Research paradigms influence the manner in which knowledge is studied and interpreted (Cohen et al 2007). It is therefore “the choice of paradigm that sets down the intent, motivation and expectation for research” (Mackenzie and Knipe 2006 2.). It has also been described in more simple terms as ‘the way one views the world’. Several different theoretical paradigms are discussed in the literature such as:

- Positivist (Mertens 2005)
- Constructivist (Creswell 2003)
- Interpretivist (Creswell 2003)
- Transformative (Mertens 2005)
- Pragmatic (Somekh and Lewin 2005)
- Emancipatory

(cited in Mackenzie and Knipe 2006).

Two of the six approaches mentioned, however, are most commonly used in educational research (Lincoln and Guba 2000). These methods are positivist and interpretivist. Combined, these approaches result in a mixed methods approach of qualitative and quantitative research methods. Both of these approaches are implemented in this research project.

The positivist paradigm is most commonly aligned with quantitative methods of data collection and analysis and so is the dominant paradigm present throughout this research project. Positivists generally set out to test a theory or describe an experience “through observation and measurement in order to predict and control forces that surround us” (O’Leary 2004, p.5). It is often described as a ‘scientific method’ which is in line with the rationalist views which originated with Aristotle and “reflects a deterministic philosophy in which causes probably determine effects or outcomes” (Creswell 2003, p.7). In the case of this research project the declining mathematical standards on entry to UL are investigated through quantitative analysis of the UL database. The positivist paradigm is also reflected in the investigation into what characteristics best determine if a student is likely to be ‘at risk’ of failing service mathematics,
the development of a predictive function of failure in service mathematics and the evaluation of the teaching intervention in service mathematics.

The second paradigm present in this research project, the interpretivist paradigm, aims to understand “the world of human experience” (Cohen and Manion 1994, p.36). This paradigm generally involves the development of a theory or pattern of meaning throughout the research process (Creswell 2003). The paradigm relies on qualitative data collection methods and analysis. The interpretivist paradigm therefore calls on the researcher to develop an understanding of a situation or phenomenon from the subjects’ viewpoint (Lincoln and Guba 2000). The presence of the interpretivist paradigm in this research project is found throughout the interpretation of students’ feedback to the intervention tutorials. This form of qualitative research analysis has been described as being subjective and calls for the researcher to recognise the impact on the research of their own background and experiences. From the perspective of this research the author’s own experiences as a student in Technology mathematics tutorials, her experience of mathematics throughout her education to date as well as her experience as a mathematics teacher in second and third level education in Ireland have the potential to act as a source of bias on the research analysis and the interpretation of results. The author however aims to counteract this somewhat by establishing researchers distance and ‘objectivity’ as a result of considering and acknowledging the sources of potential bias due to her personal circumstances. The researcher distance and objectivity is further considered through the use of a variety of methods of data collection and through triangulation within the research (see section 3.6.2).

The two paradigms employed in this research project are therefore quite different by nature. The positivist approach is predominantly quantitative, objective and therefore it is possible for the researcher to remain detached from the study and the subjects. Contrastingly the interpretivist approach is predominantly qualitative in nature, is subjective and therefore the researcher is very much involved with the subjects and a heavy reliance is placed on the researcher’s interpretation of events or feedback (Creswell 2003). The combination of these paradigms therefore results in what is called a mixed methods approach, or triangulation, to educational research. Triangulation, and its prevalence in this research project, is discussed in detail in section 3.6.2 however details of a mixed method approach to research and its benefits are outlined next in section 3.2.2.
3.2.2 Mixed Method Approach

A mixed method approach to research is one in which both numerical (e.g. test results) and text information (e.g. interviews) are gathered so that the final findings represent both quantitative and qualitative information (Creswell 2003). Some have described the increased use of the mixed method approach in educational research as a key element to the improvements in research in this area (Gorard 2004). Some of the reasons for this viewpoint are due to a belief that mixed method research approaches:

- Require a greater level of skill.
- Can lead to less waste of potentially useful information.
- Create researchers with an increased ability to make appropriate criticisms of all types of research.
- Have a greater impact due to figures having a very persuasive influence on policy makers etc.

(Gorard 2004, p.7).

Qualitative and quantitative methods of research combined are therefore viewed by many to be complementary (Krathwohl 1993; Creswell 2003; Thomas 2003). Some even suggest that if research is to be fully effective both approaches need to be applied (Mackenzie and Knipe 2006). Table 3.1 highlights the presence of the qualitative and quantitative approaches used in each phase of this research.
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<td>Literature Review</td>
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<td>Phase 2</td>
<td>Creation of database 1, 2 &amp; 3</td>
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<td>Phase 3</td>
<td>Profiling ‘at risk’ students</td>
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<td>Phase 4</td>
<td>Building a Predictive Model of Failure</td>
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<td>Identifying ‘at risk’ students in September 2010/11</td>
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<td>Develop Theoretical Framework for design, implementation and evaluation of Intervention</td>
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<td>Phase 5</td>
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<td>Evaluation of intervention</td>
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<td>Phase 6</td>
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**Table 3.1** Presence of qualitative and quantitative research methods during each phase of the research.
3.2.3 Theoretical Framework

Upon analysis of the variety of research paradigms available an informed decision was made regarding the most appropriate theoretical framework for this research. Figure 3.1 gives a graphical representation of how the two paradigms, positivist and interpretivist, interact and influence the research design (Mackenzie and Knipe 2006). This mixed method approach involves the combination of both qualitative and quantitative research analysis, for example in the case of this research the qualitative analysis of students’ feedback on the intervention tutorials combined with the quantitative analysis regarding their Technology mathematics results. These findings were then subject to testing for validity and reliability with consideration also given to ethics and possible limitations of the research. Finally a report of the main findings were documented (see figure 3.1).
Figure 3.1 Summary of theoretical framework for this study
3.3 Research Problem, Objectives and Questions

3.3.1 Research Problem

The initial motivation for this research project was the need to exploit the information in the large database collected in UL over the last 12 years (1998-2010). Currently in Ireland, several other third level institutions have carried out diagnostic testing in mathematics and therefore have similar data to UL. However no university has access to such a large database, which currently consists of information on almost 8,000 students, a diagnostic test which has remained unchanged, a lecturer and course material (Technology mathematics) which have not changed throughout the period in question.

The UL database has been used in the past to investigate and document the marked decline in mathematical competency levels over time (O’Donoghue 1999; Gill 2006). However a comprehensive investigation has yet to be carried out into the possible causes of this decline. Although the documentation of the decline is extremely useful, only when the possible causes of the problem are identified can useful recommendations be made to improve mathematics education in UL. Anecdotal evidence, relating to the characteristics which students who are likely to fail service mathematics possess, exists in most universities. Very little research however has been carried out into the possibility of grade inflation in mathematics at second level being a contributory factor to the declining standards of Irish university entrants.

Many universities that have implemented mathematics diagnostic testing decided on a cut-off point in the test which determined whether the students were ‘at risk’ or not ‘at risk’ of failing service mathematics. Using more advanced statistical techniques to investigate differences between students who fail and students who pass would be beneficial in order to provide students and lecturers with a comprehensive warning system i.e. a probability of failure in service mathematics based on retrospective analysis of a large database of similar students.

Upon acknowledgement of the existence of declining standards in a university, many institutions have put support services in place to aid the students who are considered to be ‘at risk’ of failing. A less common practice however is to look to the traditional tutorial support which already exists in a university and attempt to improve its provision for all students using research to inform practice.

This research project therefore aims to investigate whether there is evidence of grade inflation in mathematics in Ireland and to offer an insight into alternative tutorial provision in third level
mathematics which was informed by a quantitative analysis of a large student database. The exact research objectives and questions which were used to achieve the objectives are detailed next in section 3.3.2.

3.3.2 Research Objectives and Research Questions

3.3.2.1 Research Objectives

The main objectives of this research project are as follows:

- To investigate the changing profile of Science and Technological students between 1998 and 2008.
- To identify trends over time in mathematical competencies (1998-2008).
- To profile current ‘at risk’ students in service mathematics courses using examination data from 2006-2008.
- To build a predictive model of failure in service mathematics.
- To design an intervention strategy for students in service mathematics.
- To identify the probability of failure of all students in the academic year 2010/11 in order to implement an intervention strategy using the probability of failures to inform practice and to monitor the success of the intervention.
### 3.3.2.2 Research Questions

The research questions which guided this study are detailed in table 3.2. The research questions were established through an examination of the objectives within each phase of the research.

<table>
<thead>
<tr>
<th>Research Phase</th>
<th>Research Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Phase 1</strong></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>How prevalent is the ‘Mathematics Problem’ internationally?</td>
</tr>
<tr>
<td>2.</td>
<td>How prevalent is the ‘Mathematics Problem’ in Ireland?</td>
</tr>
<tr>
<td>3.</td>
<td>What are the dominant documented causes of the ‘Mathematics Problem’?</td>
</tr>
<tr>
<td>4.</td>
<td>What support services and interventions have been put in place in an attempt to alleviate the ‘Mathematics Problem’?</td>
</tr>
<tr>
<td>5.</td>
<td>What does national and international educational research say about profiling ‘at risk’ students?</td>
</tr>
<tr>
<td><strong>Phase 2</strong></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>What is the profile of service mathematics students in UL between 1998 and 2008?</td>
</tr>
<tr>
<td>2.</td>
<td>What are the trends in the mathematical competency levels of UL students by Leaving Certificate mathematics grade?</td>
</tr>
<tr>
<td>3.</td>
<td>Is there evidence of grade inflation occurring in Leaving Certificate mathematics?</td>
</tr>
<tr>
<td><strong>Phase 3</strong></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>What is the profile of students who are ‘at risk’ of failing Technology mathematics?</td>
</tr>
<tr>
<td>2.</td>
<td>What is the profile of students who are ‘at risk’ of failing Science mathematics?</td>
</tr>
<tr>
<td>3.</td>
<td>What is the profile of a student who is ‘at risk’ of failing service mathematics?</td>
</tr>
<tr>
<td>4.</td>
<td>What is the most effective method of prediction of</td>
</tr>
</tbody>
</table>
| Phase 4 | 1. What are the probabilities of failure of service mathematics students entering UL in September 2010/11 based on the discriminant function?  
2. What is an appropriate intervention design, implementation and evaluation method for service mathematics students which has been informed by the discriminant function? |
|---|---|
| Phase 5 | 1. Did students involved in the teaching intervention perform better than those who did not take part in the intervention?  
2. Did students involved in the intervention respond positively to the teaching strategies employed in the intervention? |
| Phase 6 | 1. What do the research findings in this thesis have to offer the university sector, the wider Irish education system and the international education system? |

Table 3.2 Research questions by phase
3.4 Chronological Description of the Study

Introduction

Once the specific research questions to be addressed were outlined, the chronological order of the steps that need to be taken in order to gain an insight into each of these questions could be detailed. An overview of the tasks which were carried out during each phase of the research can be seen in the flow chart which follows. The time frame during which each phase was completed is also outlined (see figure 3.2). The statistical methodologies employed throughout the research are detailed in section 3.4.1 followed by a description of the diagnostic test in section 3.4.2 and finally the exact procedures carried out during each phase of the research are outlined in section 3.4.3.
Phase 1
(0-6 months)

Phase 2
(7-10 months)

Phase 3
(10-18 months)

Phase 4
(18-24 months)

Phase 5
(24-32 months)

Phase 6
(32-36 months)

Literature Review

Creation of:
- Dataset 1 (1997-2008 Science and Technology)
- Dataset 2 (2006-2008 Science Students)
- Dataset 3 (2006-2008 Technology Students)

Profiling ‘at risk’ Science and Technology Students (2006-2008) using datasets 2 & 3

Build a Predictive Model which will predict performance of service mathematics students

Identify probabilities of failure of students in September 2010/11

Select a theoretical framework and design a teaching intervention

Implementation of intervention

PhD Thesis

Figure 3.2 Chronological Description of the Study.
3.4.1 Statistical Methodology

The statistical software package SPSS for Windows (Version 15) was used for all statistical investigations carried out within this research. All of the analysis that took place throughout the research involved the use of either dataset 1, which consisted of data on 5,949 students, dataset 2, which consisted of data on 838 students, dataset 3, which contained data on 1,080 students, dataset 4, which consisted of 692 students or dataset 5, which consisted of information on 730 students within Science and Technological service mathematics. Details of variables and exact cohorts within each of these datasets can be found in table 3.3.

The information relating to trends by year were detailed using summary measures such as means, percentages, standard deviations and co-efficient of variation. Independent samples t-tests were used to test for statistically significant differences between the means of two groups (for example the changing student profile between 1998 and 2008). The equality of variances assumption was checked for all independent samples t-tests. Chi-Squared tests were used to test for statistically significant associations between the qualitative variables. Analysis of variance (ANOVA) was used to establish if statistically significant differences were present when three or more means were being compared. A 5% level of significance was used for all tests and no adjustment was made for multiple testing. All assumptions for each of the tests used were met before any analysis was carried out. Due to the large number of students in the database small differences between the means could be found to be statistically significant. The standard deviation and coefficient of variation (standard deviation/mean x 100) were used as measures of the spread/dispersion of results. Levene’s test for equality of variances was used to investigate significant differences between the variances of two groups.

The method used to calculate a function which would successfully predict performance in service mathematics was ‘discriminant analysis’ (other forms of prediction were also tested and proved to be less successful in the case of this research, see appendix E). The assumptions of normality and equal covariance for each of the two populations (success/failure) of the predictor variables were met. Exploratory data analysis was also carried out on the predictor variables to determine how different the means for the two populations were and how strong a correlation existed between them. A strong correlation between variables was considered to be associated with values of r between 0.50 and 1.00, a moderate correlation being between 0.30 and 0.49 and a weak correlation being between 0.10 and 0.29 (Cohen 1988). The discriminant function, Z, was
given by: $Z = a_1(x_1) + a_2(x_2) + \ldots + a_p(x_p)$ where $x_1, x_2, \ldots, x_p$ represent the independent variables and $a_1, a_2, \ldots, a_p$ are estimated from the dataset. The relative effect of each variable in the function was determined through information displayed in the Discriminant Analysis SPSS output. Finally, validation datasets were used to determine the success of the discriminant functions on new data (Note: discrepancies in total numbers within a dataset may appear throughout the tables in this chapter due to some students not sitting the diagnostic test, others not having a Leaving Certificate mathematics grade and others not having either). Details regarding the datasets used for the analysis described here, and the variables contained within them, can be found in section 3.4.3. An overview of the diagnostic test, one of the main sources of data within the research, is outlined first however in section 3.4.2.
3.4.2 Diagnostic Testing in the University of Limerick

3.4.2.1 Why was the test introduced?
In 1997 the diagnostic test was first implemented in UL due to mathematics lecturers’ anxiety regarding students’ mathematical competency levels. Students’ unease when faced with mathematics problems in unfamiliar formats became apparent to mathematics lecturers. Students’ work was analysed by mathematics education lecturers in UL and it emerged that many students were having difficulties with basic algebra, arithmetic, trigonometry, calculus and complex numbers (Gill 2006). As a result of this anecdotal evidence of declining mathematical competency levels the diagnostic test was introduced. The aim of the diagnostic test was to diagnose where problems existed so that high failure rates and diminishing degree standards could be avoided. The test therefore is a tool to identify those who are struggling with the basic mathematical concepts needed for third level. This information can then be used by lecturers to understand the ability of their students and for students to identify where their weaknesses may lie and if they need extra help (Gill 2006). The database in which all of the diagnostic test results are contained is described in detail in section 3.4.3.

3.4.2.2 When was the test introduced?
The first cohort of students to take the diagnostic test was Technology mathematics students in 1997. Technology mathematics students are one cohort of several service mathematics students in UL. Technology mathematics students are enrolled in technology-based degree programmes and are required to take at least one mathematics module. In 1998 the test was administered to Science service mathematics students and the two cohorts have received the test every year since then. These students were targeted as these are the students about which lecturers concerns were raised. The mathematics which they are required to study acts as a foundation for their other disciplines (chemistry, physics, construction etc.) and so it is important that they are proficient in their mathematics in order for them to be successful in other areas of their chosen disciplines.
Diagnostic testing was previously implemented in Ireland in 1986 by Hurley and Stynes. They used it to assess the mathematical skills of 682 students in University College Cork. Although this form of testing has been previously carried out in Ireland the scale of the UL study is much larger than any other with almost 8,000 students involved to date.
In UL the diagnostic test is administered to students in the first lecture of first year service mathematics modules, Science or Technology mathematics. The use of log tables and calculators is not permitted during the test however they are not required in order to complete it successfully. The students do not receive prior warning about the test, with the hope being that this approach increases the number of students taking the test.

### 3.4.2.3 How was the test designed?

The paper-based diagnostic test was developed in 1997 by John O’Donoghue, a Professor of Mathematics Education in UL. Originally the test was designed with a view to assessing the level of mathematical knowledge held by those in technology/engineering programmes in UL. As a result the test has a clear technology/engineering focus. It was then considered to be appropriate for Science students and consequently since 1998 both Science and Technology students have been given the test.

When the test was being designed a number of controls were used to ensure that the test was fit for purpose. The three controls used were:

- The Ordinary Level Leaving Certificate mathematics syllabus,
- the SEFI core level zero syllabus for engineers (Barry and Steele, 1993) and
- an extensive literature review.

All of these controls enabled O’Donoghue to decide upon the content and structure of the test. The diagnostic test was first piloted in several secondary schools around Ireland, with both students and teachers, to determine if it was pitched at an appropriate mathematical competency level. This piloting phase also gave some indication as to what result in the diagnostic test would deem a student to be ‘at risk’. The prototype of the test was then critically examined by 6 mathematics lecturers and upon receiving feedback from this, the test was adapted slightly and finalised (see Appendix A). Each test is corrected by hand which allows for closer examination of any paper if required (O’Donoghue 1999).
3.4.2.4 What does the test consist of?
There are 40 questions on the diagnostic test. Within these 40 questions there are nine topics: arithmetic (13 questions), algebra (8 questions), geometry (4 questions), trigonometry (3 questions), co-ordinate geometry (4 questions), complex numbers (2 questions), differentiation (3 questions), integration (2 questions) and modelling (1 question). The Ordinary Level Leaving Certificate examination was one of the main controls for the selection of appropriate questions and so the majority of questions on the test are aimed at Ordinary Level Leaving Certificate mathematics standard or below. Six questions are covered on the Higher Level Leaving Certificate mathematics examination only. Two of these questions focus on integration, two questions are on logarithms in the arithmetic section and two questions on differentiation. There is a rough work column to help students answer the questions and also to offer lecturers some further insight into students’ competency levels and possible misconceptions in a particular area. There have been no changes made to the diagnostic test, in content or structure, since it was first implemented in 1997 so comparisons across cohorts can be made.

3.4.2.5 Students’ performance in the diagnostic test
If students receive 19 out of 40 or below in the diagnostic test they are considered to be ‘at risk’ of failing service mathematics in UL. An ‘at risk’ student is regarded as someone who is deficient in the mathematical skills deemed necessary for successful completion of their mathematics module. The students who are categorised as ‘at risk’ are advised to avail of the mathematics support services made available to them by the university (O’Donoghue 1999). Details of each phase of the research, and where the diagnostic test data was applied, are outlined in section 3.4.3 next.
3.4.3 A Phase by Phase Description of the Study

This section outlines in detail the procedures carried out in each phase of the research.

3.4.3.1 Phase 1 (0-6 months) Review of Literature and Developing Familiarities with SPSS

In the first phase of the research the author engaged in a review of literature on the ‘Mathematics Problem’ in relation to its extent in Ireland and abroad. An examination into the literature detailing the profiling of ‘at risk’ students as well as interventions for ‘at risk’ students was carried out in this phase. The review of literature was ongoing throughout all other phases of the research, however, so that the most up to date findings in all areas were familiar. The aim of reviewing the literature was to aid in the establishment of what the current issues in mathematics education were, what methods have been tried and tested in terms of profiling ‘at risk’ students and developing interventions for them both in Ireland and elsewhere. This therefore outlined the common issues amongst mathematics educationalists in third level education and helped the author to determine what intervention worked best in order to see what was the most logical step to take next to help solve the ‘Mathematics Problem’.

The second objective of phase 1 was to become familiar with the structure of the diagnostic test dataset and the use of SPSS for describing quantitative and qualitative data. This familiarisation period involved summarising diagnostic test information for certain cohorts of students and carrying out comparisons between different year groups within the same dataset.

3.4.3.2 Phase 2 (7-10 months) The Identification of Trends Over Time in mathematical competencies

A global database (Dataset 1) was created which consists of all of the data from Science and Technology students from 1998-2008. This dataset was the first created by the author and was subsequently used in the creation of the other datasets used in this research. The individual cohorts of students were gathered from a researcher who had previously used them for investigations (Gill 2006). These individual cohorts were presented in the form of Excel files which consisted of information on each student such as id number, gender, diagnostic test results, performance in semester 1 service mathematics examination, degree programme of study, whether students were standard or non standard, Leaving Certificate mathematics grade
and level at which it was taken. The individual excel files were merged for each group for each year into one SPSS file. The dataset was checked to ensure that columns and variables matched with the original excel files and that there were no inconsistencies evident. The result of this was a clean dataset ready for analysis. The information contained in dataset 1 was used to identify 1) Trends over time in mathematical competencies of students on entry to UL and 2) Trends over time in mathematical competencies by Leaving Certificate mathematics grade. The first analysis required the author to summarise total diagnostic test scores for each student between 1998 and 2008 and also to summarise diagnostic test scores in each of the areas within the diagnostic test e.g. algebra, complex numbers, arithmetic. The second analysis involved a very similar examination except that total diagnostic test performances and performance in different diagnostic areas were summarised by students corresponding Leaving Certificate mathematics grades.

Two more datasets were then created from the global dataset (Dataset 1). The information contained within each dataset is detailed in table 3.3. Each dataset served a particular function. Dataset 2 and dataset 3 contain information on students within Science and Technology mathematics respectively who sat their service mathematics examination in 2006, 2007 or 2008. The students’ results in service mathematics were labelled as pass or fail. These datasets were needed in order to profile ‘at risk’ students in Science and Technology mathematics. Further details of the analysis which was carried out using these datasets is given in Phase 3, i.e. section 3.4.3.3.

All of the methodologies employed in the creation of the datasets in phase 2 of the research involved great care and attention to be given to ensure no inconsistencies were present throughout any of the datasets. Several checks were carried out such as comparisons of frequencies in different datasets against the original excel file to ensure no data was lost/duplicated in the creation of the new SPSS files. The exact variables contained in each dataset and their size can be found in table 3.3.

Note: The range of Leaving Certificate mathematics points in the dataset is from 0-100. Bonus points were not included as outlined in section 2.3.2(e) so as to make the findings in this thesis applicable to all third level institutions in Ireland.
<table>
<thead>
<tr>
<th>Dataset</th>
<th>Description</th>
<th>Variables within Dataset</th>
<th>Categories/Range</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Technology and Science students 1998-2008</td>
<td>Year</td>
<td>1998-2008</td>
<td>5,949</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Student Identification Numbers</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Semester 1 Result</td>
<td>0-100</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Semester 2 Result</td>
<td>0-100</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Support</td>
<td>Number of contacts students had with support services during a semester</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gender</td>
<td>Male/Female</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Leaving Certificate Level</td>
<td>Higher Level (HL) or Ordinary Level (OL)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Leaving Certificate Grade</td>
<td>HLA1-HLD3 and OLA1-OLC3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Leaving Certificate Points</td>
<td>0-100</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Program of Study</td>
<td>See table 4.2 (chapter 4) for degree programs within Science and Technology mathematics</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Diagnostic Test Result</td>
<td>0-40</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Performance in each area of the Diagnostic Test</td>
<td>Arithmetic (13 questions) Algebra (8 questions) Geometry (4 questions) Trigonometry (3 questions) Co-ordinate Geometry (4 questions) Complex Numbers (2 questions) Differentiation (3 questions) Integration (2 questions)</td>
<td></td>
</tr>
<tr>
<td>Dataset 2 and 3&lt;sup&gt;5&lt;/sup&gt;</td>
<td>Science&lt;sup&gt;1&lt;/sup&gt; and Technology&lt;sup&gt;2&lt;/sup&gt; 2006, 2007 and 2008</td>
<td>Success or Failure in Service Mathematics</td>
<td>Success/Failure</td>
<td>838&lt;sup&gt;1&lt;/sup&gt; 1,080&lt;sup&gt;2&lt;/sup&gt;</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sat Diagnostic Test</td>
<td>Yes/No</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Technology 2006-2008 discriminant function value</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Predicted Group</td>
<td>Predicted Success or Predicted Failure</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Probability of Success</td>
<td>0-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Probability of Failure</td>
<td>0-1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Probability Failure Group</td>
<td>Low, Medium or High risk</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Discriminant function 2 value (Leaving Certificate mathematics points as the only variable)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Predicted Group</td>
<td>Predicted Success or Predicted Failure</td>
<td></td>
</tr>
<tr>
<td>Dataset 4&lt;sup&gt;5&lt;/sup&gt;</td>
<td>Technology and Science 2009</td>
<td>Success or Failure in Service Mathematics</td>
<td>Success/Failure</td>
<td>692</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sat Diagnostic Test</td>
<td>Yes/No</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Technology 2006-2008 discriminant function value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Predicted Group</td>
<td>Predicted Success/Predicted Failure</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>-------------------------------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of Success</td>
<td>0-1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of Failure</td>
<td>0-1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability Failure Group</td>
<td>Low, Medium or High risk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discriminant Function 2 value</td>
<td>(Leaving Certificate mathematics points as the only variable)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Dataset 5**

| Technology and Science 2010 | Intervention Student | Yes or No | 730 |

**Table 3.3** Information contained in each dataset.

5Datasets 2, 3 and 4 contain all of the variables which dataset 1 contains in addition to the variables listed under dataset 2, 3 and 4 in table 3.3

6Dataset 5 contains all of the variables in datasets 2, 3 and 4 in addition to the variable listed under dataset 5 in table 3.3

Note: The discriminant functions referred to in table 3.3 will be outlined in chapter 5.

### 3.4.3.3 Phase 3 (10-18 months) Profiling ‘at risk’ students in Science and Technology mathematics (2006-2008)

Phase 3 of the research made use of datasets 2 and 3 which were created in phase 2. The author identified factors which put a student ‘at risk’ of failing service mathematics examinations in UL. This investigation involved separately summarising variables collected on each student within Science and Technological mathematics between 2006 and 2008. Information on each student such as the percentage of students who failed by year (2006, 2007 & 2008), the percentage of males and females who failed over the three year period combined, the percentage of students who failed and had Higher and Ordinary Level Leaving Certificate mathematics as pre-requisite knowledge, the percentage of students who failed who had specific Leaving Certificate mathematics grades as pre-requisite knowledge, the percentage of students who failed
within each sub-category of student and a breakdown of the percentage of students who failed within different degree programs were all examined and detailed. Upon summarising this data the characteristics which were statistically significantly associated with performance in service mathematics were outlined. Failure rates within each service mathematics course were analysed in terms of trends by year, gender, CAO points, student sub-category, diagnostic test mean results, mean performance in the arithmetic and algebra sections of the diagnostic test, whether a student was labelled ‘at risk’ or not and finally trends in relation to whether or not a student sat the diagnostic test.

Once the profiling of ‘at risk’ students was carried out different forms of prediction analysis were examined and tested using datasets 2 and 3. The most successful form of analysis, discriminant analysis, in terms of predicting performance in service mathematics was then used to develop a predictive function of performance in service mathematics. Dataset 4 (see table 3.3) served as a validation dataset during this process. Phase 4 of the research involved the use of the predictive function to predict the performance of incoming service mathematics students.

3.4.3.4 Phase 4 (18-24 months) Designing an Intervention Strategy for ‘at risk’ Students

The fourth phase of the research involved designing an intervention strategy for students in service mathematics. This intervention took the form of a teaching intervention. The students chosen for the intervention were identified in September of the academic year 2010/11 in the first week of term. Prior to the implementation of the intervention an extensive review of literature relating to teaching interventions was carried out. Best practice in the design, implementation and evaluation of interventions was investigated bearing in mind the specific situation regarding ‘at risk’ service mathematics students in UL. The author came to a conclusion as to what was the most appropriate form of teaching intervention for the students targeted; these conclusions are outlined further in section 3.4.4 and in chapter 6.

The discriminant function was used on students entering UL service mathematics courses in September 2010. All of the students’ details relating to their diagnostic test results, their Leaving Certificate mathematics results and the level at which they took Leaving Certificate mathematics were entered into dataset 5 (see table 3.3). A new variable was then created in SPSS which detailed the students’ prediction of performance (success/failure) along with their predicted
probability of failure in service mathematics. This information was then used to inform the methodologies involved in the design, implementation and evaluation of the teaching intervention (see section 3.4.4).

3.4.3.5 Phase 5 (24-32 months) *Implementation and Evaluation of the Intervention Strategy*

This phase of the research was dominated by the implementation of the teaching intervention, as decided upon in phase 4 of the research. The intervention implementation was initially undertaken by 3 tutors who worked together to facilitate the chosen students in a tutorial class once a week. The structure of each of the tutorials was dictated by the theoretical framework set out in the previous research phase (see figure 3.5). Constant monitoring of how effective the intervention was took place through feedback from students and tutors and adaptations were made during the course of the semester where necessary. On completion of the implementation of the teaching intervention, which lasted one semester (10 weeks), the next task was to assess the success of the intervention in terms of students’ performance in the end of term service mathematics examination. Using dataset 5, comparisons were made between end of term performance of those students who took part in the teaching intervention and those who did not take part in the intervention. The evaluation of the intervention also had a qualitative dimension to it which was informed by Shapiro’s model (1987) for evaluating interventions. The author aimed to determine if qualitative and/or quantitative improvements occurred for the students who took part in the 10 week teaching intervention. Further details on the methodologies used within the intervention are outlined in section 3.4.4.

3.4.3.6 Phase 6 (32-36 months) *PhD Thesis*

Phase 6 was primarily focused on the write up of the PhD findings.
3.4.4 Intervention: Methodologies Employed

Introduction

The main aim of the intervention was to design, implement and evaluate an effective teaching intervention for 1st year service mathematics tutorials. The tutorials were designed to promote understanding in mathematics through active learning methodologies and mixed ability group work. The intended outcome of the intervention was a reduction in failure rates of Technology mathematics students. It was also intended that the findings from the intervention could serve as a useful reference for others involved in educational practices worldwide. The structure of the intervention is outlined next.

3.4.4.1 Structure of the Intervention

The intervention had three distinct phases which were as follows:

1. Design

   The design of any intervention is said to involve the ideas, plans and rules of what needs to be done in order to develop the actual instruction (Dijkstra 1997). Careful attention therefore went into what teaching methodologies and student considerations needed to be addressed in order to design an effective intervention which would achieve the desired goals set out.

2. Implementation

   The implementation phase of the intervention involved taking what had been outlined in the design phase and putting it into action. The implementation planning centered on the notion that “mathematics is not a spectator’s sport” (Myers 1996) and consequently was heavily influenced by active learning methodologies. Guidelines set out by Felder, Bullard and Brent (2009) relating to effective teaching in education were also extremely influential in the implementation phase.

3. Evaluation

   Evaluation has become a central component to any teaching intervention. In fact “more and more programs are being launched with the proviso that their effectiveness be assessed” (Shapiro 1973, p.523). Shapiro (1987) discussed the need for the identification of critical parameters through which intervention research can be evaluated. Shapiro
(1987) outlined 4 such parameters which the author used in her research evaluation. The four parameters were as follows:

- Treatment effectiveness
- Treatment integrity
- Social validity
- Treatment acceptability

An outline of the sub categories of Shapiro’s parameters, all of which the author considered during the evaluation phase of the intervention, can be seen in figure 3.6. A detailing of what each parameter entails is outlined in section 3.4.4.5. The theoretical considerations and theories used to construct the intervention theoretical framework are detailed first in section 3.4.4.2.
3.4.4.2 Theoretical Consideration for Intervention

Throughout the literature review on interventions (see chapter 2) a detailed analysis was given of mathematics interventions which have been used worldwide. The interventions discussed all had one thing in common; their sole purpose was to improve the provision of mathematics education and hence the mathematical competency levels of the students within the respective institutions. The necessity for these support services was not under question however the manner in which they were executed must be carefully thought out. It was decided that a teaching intervention with an active learning group work focus was most appropriate to achieve the goals set out by the intervention. The learning theories which are used to underpin the active learning group work tutorials are Constructivism, Realistic Mathematics Education and Maths in Context. The issue of capturing interest and improving attitudes towards mathematics were also investigated due to an awareness that “many people……..appear frightened of mathematics or maintain that they hate it” (Rooney 1998, p.12). The rationale for using these theories to guide the tutorials and the investigation into methods of developing student interest is outlined in section 3.4.4.2.1.

The intervention took place during a regular tutorial slot and so students did not receive any extra support other than the change of teaching style which took place within their regular tutorial slot. One of the reasons for this being that many educationalists have discussed the importance of focusing “not so much on what mathematics we convey but how we convey it” (Holton 1998, p.49). This was a particularly relevant point in the case of this research as the mathematics tutorials involved in the intervention have not changed in the last ten years either in terms of the content or the didactic teaching style in which they are delivered. A tutorial was also chosen for practical reasons as if successful this tutorial could be sustained over time as opposed to extra support which may not be supported financially over time and can be difficult to organise in terms of logistics. The full rationale behind the implementation of a tutorial intervention as opposed to any other intervention is outlined in chapter 6.

Considerations were given to logistical issues, cost and students’ mathematical backgrounds (see chapter 6). This examination of literature was hoped to result in a theoretically sound intervention framework specific to the needs of this research. Details of this framework are outlined in section 3.4.4.3. The active learning methodologies and learning theories which guided the intervention design and implementation are outlined next in section 3.4.4.2.1.
3.4.4.2.1 Active Learning and Learning Theories

Classroom and cognitive research has been plentiful in recent decades and has given great insight into how learning occurs in classrooms (McKeachie et al. 1990). Felder and Brent (2003) highlighted that the only way to develop a skill, whatever the skill may be, is to practice the skill yourself. Students need to try something, analyse whether it was successful, decide how it could be improved upon and try it again, all of which is the basis of active learning. Due to the fact that active learning has been shown to encourage the mastery of skills and development of conceptual understanding (Redish and Steinberg 1997; Wiggins and McTighe 1998) it will be the focus of the intervention phase of the research. Active learning methodologies are the result of an accumulation of educational theories all brought together to form one model of learning. Many of the learning theories in question are detailed in sections 3.4.4.2.1(a) - 3.4.4.2.1(d) which follow.

3.4.4.2.1 (a) Constructivism

Constructivism is an epistemology, a learning meaning-making theory, which offers an explanation of the nature of knowledge and how humans learn. This theory stands firm on the notion that individuals create their own understandings/knowledge through the interaction of what they already know through involvement with content rather than through repetition and imitation (Kroll and LaBoskey 1996). Learning activities are very much involved with active engagement, inquiry, problem solving and collaboration all of which are also characteristics of active learning. Constructivist approaches are found to produce greater internalization and deeper understanding than traditional teaching methods (Abdal-Haq 1998).

Contemporary educators discuss two broad interpretations of Constructivism: (i) Psychological Constructivism associated with Piaget and (ii) Social Constructivism associated with Vygotsky. A brief outline of each is provided next.

Psychological Constructivism

Psychological Constructivism, often known as Piagetian Constructivism, maintains that the purpose of education is to educate an individual in a manner which supports the individual’s interests and needs. Cognitive development of the individual is the emphasis. This is a learner-centered approach that seeks to identify, through specific study, the natural path of cognitive
development (Vadeboncoeur 1997). This learning theory assumes that students enter the classroom with ideas, beliefs and opinions which are to be altered or modified and this is the role of the teacher. The teacher must devise tasks and problems for students; knowledge construction is thought to occur as a result of tackling these tasks/problems (Richarson 1997). Instructional practices which adhere to this theory may include discovery learning, use of manipulatives, student tasks and questioning techniques. Internal development is definitely the focus of Piagetian Constructivist teachers. Common criticisms of this approach center on its lack of attention to classroom culture as well as its disregard for power issues (Martin 1994; Vadeboncoeur 1997). Such issues are also acknowledged in the Social Constructivism learning theory which is detailed next.

**Social Constructivism**

Social Constructivism, often called Vygotskian Constructivism, is a learning theory which maintains that “Individuals construct knowledge in transaction with the environment, and in the process both the individual and the environment are changed” (Abdal-Haqq 1998, p.2). Richardson (1997) described this theory as a scenario in which individuals’ development derives from social interactions within which cultural meanings are shared in a group setting and eventually internalized by the individual. Within this theory of Constructivism there is an assumption that theory and practice do not develop in a vacuum but rather are hugely influenced by dominant cultural assumptions (Martin 1994; O’Loughlin 1995). There are many variations of social Constructivism such as Situated Constructivism, Socio-cultural Constructivism and Socio-historical Constructivism. Each variation however involves the learner as a constructor of his/her own knowledge based on their previous experience interacting with the content/situations to which they are exposed with a heavy influence on the social culture of their environment. Active learning methodologies are deeply founded in this theory of learning.
3.4.4.2.1 (b) Realistic Mathematics Education

Realistic Mathematics Education (RME) is an approach to teaching and learning in mathematics education developed in 1971 by the Freudenthal Institute in the Netherlands. RME is strongly influenced by Hans Freudenthal's view of mathematics in which he sees mathematics as something that has to be and is connected to reality, i.e. “mathematics as a human activity” (NCCA 2005, p.112). RME is described as a learning process which has a large connection and relevance to social and cultural aspects of life. Freudenthal was highly critical of mainstream mathematics education from the 1950’s due to its ‘instructional design’ emphasis and related hierarchical assumptions based on Bloom’s Taxonomy as well as its measurement focus (NCCA 2005). This theory of learning denotes that education should give students the "guided" opportunity to "re-invent" mathematics by doing it. This means that in mathematics education, the focal point should not be on mathematics as a closed system but on the activity, on the process of mathematisation (Freudenthal 1968). Freudenthal defined horizontal and vertical mathematisation as follows “Horizontal mathematisation leads from the world of life to the world of symbols” and contrastingly vertical mathematisation involves symbols being “shaped, reshaped, and manipulated, mechanically, comprehendingly and reflectingly” (Freudenthal 1991;41-42).

Freudenthal (1991, pp.133-37) outlined four types of curricular emphasis: mechanistic, empiricist, structuralist and realistic, each of which he characterised by vertical and/or horizontal mathematising as can be seen in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Horizontal</th>
<th>Vertical</th>
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<tbody>
<tr>
<td>Mechanistic</td>
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<tr>
<td>Empiricist</td>
<td>+</td>
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<tr>
<td>Structuralist</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Realistic</td>
<td>+</td>
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</tr>
</tbody>
</table>

(Source: NCCA 2005, p.113).

The **Mechanistic** approach possesses neither horizontal nor vertical mathematising. This type of mathematics course would promote mathematics being learned and practiced through involvement with step by step procedures. This would mean that each problem would be solved using the same procedure as the one before as well as the one that is to come after it.
An **Empiricist** approach involves horizontal mathematisation however excludes the involvement of vertical mathematisation. In this approach the students learn mathematics through exploration of the world around them only.

The **Structuralist** approach does the opposite and focuses solely on vertical mathematisation meaning that students schematise their knowledge paying no attention to relating mathematics to the real world.

**Realistic Maths Education**, as outlined above, involves both horizontal and vertical mathematisation. Mathematical tasks begin with real life problems therefore horizontal mathematisation is the basis for this method of mathematics education. Vertical mathematisation is then needed to solve the problems outlined (Gill 2006). If implemented successfully, using elements of RME in third level education could help to ease the documented transitional issues which exist between second and third level education (Liston and O’Donoghue 2008). Implementation of this learning theory into the classroom would involve students taking part in problem solving activities in real life contexts, a feature commonly used in classrooms in which active learning is taking place. Similar theoretical ideas are echoed in the Maths in Context curriculum developed by the University of Wisconsin (Madison) and the University of Utrecht (The Netherlands).

### 3.4.4.2.1 (c) Maths in Context

Maths in Context is a mathematics curriculum designed through collaborative work from the University of Wisconsin and the University of Utrecht. It is aimed at school children in grades 5-8 which is the equivalent of children of ages approximately 12-14 years. The curriculum program consists of 40 units which are organised into four content strands: number, algebra, geometry and statistics. The main aim of each of the units within this program is to “connect mathematical content both across mathematical domains and to the real world” (Conway et al 2005, p.143). A key feature of this curriculum is connections: connections between topics within mathematics, connections to outside disciplines and connections between mathematics and the real world. Similar to the ideas behind RME, Maths in Context emphasizes the ability mathematics can have to enable individuals to make sense of the world around them (Myers and Ludwig 1999). The process of implementing Maths in Context into the classroom calls for the reversing of how mathematics is traditionally taught. Generally teaching begins with the
generalization of an idea, moves to specific examples and finally to applications in context. Maths in Context however introduces concepts with realistic contexts that support mathematical abstractions. The nature of this curriculum calls for lessons to involve mathematical tasks and questions which will inherently stimulate mathematical thinking and promote discussion amongst students. The intervention theoretical framework considerations not only examined appropriate learning theories to guide the tutorials but also investigated how interest and attitudes towards mathematics could be improved.

3.4.4.2.1(d) Consideration for: Interest in, and Attitudes towards, Mathematics

There is concern that students will resist participation in the intervention. This concern is justified as it has been reported in literature detailing the introduction of active learning to the classroom: “If you start to use active learning in your classes, you can expect to see some initial hesitation among students” (Felder and Brent 2003, p.282). Through the effective implementation of interesting active learning strategies this initial resistance and awkwardness is hoped to change to a “rapidly increasing comfort level” for the majority of students (Felder and Brent 2003, p.282). In order to try to encourage this transition to occur as early as possible the author is mindful of the effects that attitudes towards mathematics and interest in mathematics will have on students’ ability to become actively involved in the lessons. These considerations are summarized in a model set out by Rooney (1998) regarding the development of positive perceptions in mathematics. This framework shows teaching’s influence on life-long perceptions of mathematics (see figure 3.3).
Anxieties within mathematics as well as other emotions such as frustration and disappointment are likely to feature within this intervention process. Evan (2000) draws attention to our emotional state being very much linked to our beliefs and attitudes. The development of a positive attitude in which students feel secure is therefore a priority of the author within this process. A framework for such a development is outlined in the next section.
Framework for Developing Interest and Improving Attitudes

Interest has a large influence on academic achievement (Hidi and Harackiewicz 2000). Not only has interest been found to aid students to remember particular material but it has an effect on an individual’s ability to understand material and it encourages a positive attitude towards a subject area (Mason and Bascolo 2004). The inter-relationship between interest and attitudes and the powerful influence they have over performance in a subject area have resulted in the author’s attention being turned to a model which could inform this. The delivery of the mathematics must be carried out in such a way that the facilitators’ interest is clearly evident (Bergin 1999). Resources that trigger interest in students should be utilised in the process also (Hidi 2006).

The author intends to incorporate the Hidi and Renninger (2006) model (see figure 3.4) of developing interest into the framework for the intervention. Hidi (2006) described situational interest as interest which is triggered by one’s environment and causes an affective reaction to occur. Situational interest is presented as the first phase of the model and is something which can be contrived by a facilitator in the early stages of interest developed in an attempt to smooth the way for individual interest (Hidi and Harackiewicz 2000). Both situational and individual interests are described as a two-stage process by the model. Situational interest must first be triggered and following this it must be maintained. In the case of individual interest; once situational interest is maintained emerging individual interest can occur followed by maintained/well-developed individual interest which is the ultimate goal of most educators within their respective disciplines. The aim of this model is that one phase will emerge from the previous one (Hidi and Harackiewicz 2000). The length of time it takes individuals to come through this process, if they do at all, can vary greatly depending on an individual’s past experience, genetic tendency and temperament (Hidi and Harackiewicz 2006).

Stimuli to progression through the phases set out by the model include external support and/or an individual’s approach to taking on challenges within a particular subject area. External support is crucial if interest is to be maintained throughout the process (Renninger and Hidi 2002). The author will attempt to develop interest and improve attitudes within the intervention tutorials through the use of resources such as you-tube, GeoGebra and interesting websites. It is hoped that the use of these resources, will not only trigger the students’ interest but will also demonstrate the tutor’s interest. Real world discussion will also be focused around what the author perceives the students will be interested in e.g. the movement of a soccer ball during a
soccer free kick and graphics in a computer game/movie when discussing vectors. A description of how the model, which aims to develop interest, is to be incorporated into the framework for the intervention can be seen in figure 3.5.

Figure 3.4 Hidi and Renninger (2006) Interest development: A four phase model.
3.4.4.3 Intervention Theoretical Framework

Active learning methodologies were employed during the intervention phase of this research. Active learning promotes understanding, encourages teaching in terms of problem solving, is conducive to mixed ability group work and allows for the injection of interest into a teaching situation (Barrow and Tamblyn 1980; McKeachie, et al; 1987; Webb 1991; Laws et al 1999). The examination of the three learning theories, which all have the common theme of students being central to the learning process and as constructers of their own knowledge, enabled the author to identify active learning group work methodologies, as outlined by Felder and Brent (2003), as a medium through which elements of the identified learning theories could be implemented. The learning theories discussed form the basis for the conceptual underpinnings which will work alongside the delivery and evaluation strategies decided upon to form the theoretical framework for the teaching intervention. Active learning strategies were used and adapted in an attempt to teach the students involved in the intervention for understanding rather than the traditional lecture style tutorial. Considerations such as attitudes towards, and interest in, mathematics were also incorporated in the theoretical framework. Real world examples, which were thought to be of interest to the intervention students, of the mathematics which was being taught were used in an attempt to improve students’ attitudes towards and interest in mathematics. A graphical image of how all of these elements came together to form the intervention framework can be seen in figure 3.5 which follows.
Figure 3.5 Intervention Theoretical Framework.
3.4.4.4 Intervention Theoretical Framework and Technological Mathematics

Syllabus

It was important that the structure of the intervention framework complemented the content of the Technology mathematics curriculum in order for the intervention to be a success. The Technology mathematics syllabus consists of the following topics:

- Functions
- Trigonometry
- The Derivative and its Applications
- Experimental Laws
- Linear Equations
- Vectors
- Complex Numbers

The applicable nature of these topics to the real world, and indeed all topics within mathematics, suggested that the structure of the theoretical framework which aimed to support the intervention implementation was appropriate (Freudenthal 1991). The conceptual underpinnings set out in the theoretical framework were all applied to the teaching of the aforementioned topics in Technological mathematics. For example functions have endless real world applications such as an x value representing time in years and a y value representing growth in inches of a teenager. Students were made aware that every function tells a story, in the case of this example it is the relationship between time and growth of teenagers.

It was also important that the aims and objectives of the Technological mathematics module could be achieved through the implementation of the intervention set out in this research. The aims/objectives are as follows:

- To introduce students to the fundamental concepts of calculus and linear algebra
- To develop and integrate the basic mathematical skills relevant to technology

The author was mindful of these objectives when designing and implementing each tutorial session. In terms of presenting real world problems to the students, the emphasis was on technology in the real world. The manner in which the intervention was evaluated and the model which informed the evaluation are detailed in section 3.4.4.5.
3.4.4.5 Evaluation of the Intervention

As previously mentioned the evaluation phase of the intervention being used in this research relied heavily on the work of Shapiro (1987). Shapiro’s intervention evaluation model was originally developed from a school psychology point of view however has been used successfully in several teaching intervention contexts (Regan 2005; Hourigan 2009; Prendergast 2011). Much of Shapiro’s more recent work looks at providing practical strategies for working with students who are struggling with mathematics. These strategies involve guidelines for assessing students’ learning and their instructional environments while also monitoring student progress (Shapiro 2010). A more comprehensive explanation of the components Shapiro’s (1987) model are now detailed:

The effectiveness of an intervention

The effectiveness of an intervention is a hugely important component in determining whether an intervention was successful or not. The degree of effectiveness is a quantitative measure which looks at the amount of change or improvement shown. The measure of effectiveness is partially determined by the differences found between the students who took part in the intervention and those who were part of the control group (Regan 2005). The impact of the intervention may be measured by calculating the immediacy of the improvements if any occurred. Shapiro (1987) highlighted how immediate changes are often associated with an effective intervention through an explanation of the probable issues which may occur in its absence “One problem with delayed treatment effects is ruling out potential threats to internal validity when behavior change does not occur immediately after treatment is implemented” (p.291). The effectiveness of an intervention is said to be achieved if the impact of the intervention is positive over a long period of time, in a number of different situations and for a large variety of different people.

The integrity of an intervention

If an intervention is to have integrity it is vital that it has transferrable results. The intervention execution is also required to be closely aligned with that which was outlined in the intervention documentation if the integrity of it is to be upheld. For these reasons a comprehensive documentation of the intervention is important if the integrity of the intervention is to be assessed.
Social Validity of an Intervention

Social validity refers to how those taking part in the intervention, and those who may choose to implement it, evaluate it (Shapiro 1987, p.293). The main question that individuals are faced with in this evaluation is: how well did the intervention meet its overall aim? The individuals involved in the intervention play a dominant role in this evaluation through their answering of questions which call for peoples’ feelings on the intervention and its success for them. Social validity can further be measured through a comparison of the control groups’ experiences and the “consumers” of the intervention. In the case of this research the author’s observations of the tutorials formed a large part of the evaluation.

Intervention Acceptability

The acceptability of the intervention works alongside the social validity of it and is a measure of how those involved in the intervention feel about the manner in which it was executed and delivered. This measure is largely centered on how “appropriate, fair and reasonable” the procedures are for those involved (Regan 2005). Acceptability is a component of evaluation which is influenced by factors such as immediacy of change, effort involved in the intervention implementation etc. The advantage of executing an evaluation such as this is that it may help a researcher to understand why an effective, socially valid intervention with integrity still failed. Many teachers attribute factors such as time required to put the intervention in place, cost, side-effects, treatment effectiveness and integrity to the acceptability of the intervention (Reimers et al 1987). In the case of this research, students’ reactions to the intervention were largely influential on the assessment of the intervention acceptability. The author’s observation of the perceived success of the intervention and the difficulties encountered when implementing it, in terms of time and cost etc., are also pivotal to the evaluation of its acceptability. The evaluation model which is presented in figure 3.6 is based on the four components of evaluating a teaching intervention by Shapiro (1987).
Figure 3.6 The Intervention Evaluation Model (Shapiro 1987).

The research instruments used in the analysis of the data are discussed next in section 3.5.
3.5 Intervention Data Analysis

3.5.1 Qualitative Analysis of the Intervention

The method of qualitative data analysis used in this research is based on Grounded Theory. This form of analysis, which was developed by Barney Glaser and Anselm Strauss in the late 60’s, involves theory emerging from the data rather than the other way around (Glaser and Strauss 1967). Patterns and theories are implicit in the data and are waiting to be discovered. As the author wishes to allow the data to reveal the findings and not to have predetermined notions of what may emerge from the data this theory is appropriate to use in the case of this research. Glaser (1996) states that forcing methodologies is too ascendent and grounded theory, particularly in the case of positivist research, causes one not to force a pre-existing theory onto the data.

The two main methods which will be used to analyse the data are therefore as follows:

1. **Coding**: Coding involves disassembling data and rearranging the fragments “to promote a new understanding that explores similarities, differences across a number of different cases” (Ezzy 2002, p.94). The early part of coding can be confusing as data can appear to be unrelated. Once this confusion subsides however themes should appear within the data. In the case of this research the author used open coding which involves exploring the data and identifying units of analysis to code for meanings, feelings, actions, events etc (Cohen et al 2007). The author first examined the student questionnaires and the author’s observations and colour-coded the data, highlighting themes and sub-themes until the coding was complete. The author then invited two mathematics education colleagues to do the same while giving them no insight into the codes and themes which she had developed. The three sets of themes and sub-themes which emerged from all analysis were compared. The comparison confirmed that the author did not have pre-concieved ideas of what would emerge from the data as all three sets of analysis revealed very similar results. The findings from the student observations were then entered into the qualitative analysis program NVivo to confirm the exact themes and subthemes which the data had to offer.

2. **Constant Comparison**: Constant comparison can be used in conjunction with any type of coding. In this process the researcher compares the codes and themes which have been
created to assess whether they are a perfect fit for the data. If they are not a good fit then reconsideration must be given to data which may have been overlooked or inaccurately coded. The codes and themes must then be reassessed and modified appropriately. Constant comparison is therefore the process by which “the properties and categories across the data are compared continuously until saturation occurs” (Glaser 1996). This approach has therefore been demonstrated in this research as the qualitative findings have been coded by 3 separate parties by hand, they have been coded using NVivo and finally upon completion of all coding, checks took place to assess if the themes and codes reflected the data and that no findings had been neglected or misinterpreted.

The research instruments used to gather the qualitative findings and the considerations undertaken in the design of them are outlined in section 3.5.1 (a).

3.5.1 (a) Intervention Evaluation: Qualitative Research Instruments

There were two mediums through which the qualitative intervention data emerged. The data was collected through student questionnaires which consisted of questions relating to the intervention and through journal entries made by the author. Details of each of these are outlined in the sections 3.5.1 (b) – 3.5.1 (d) along with information relating to the literature which informed them.

3.5.1 (b) Questionnaire: Design and Considerations

Before administering questionnaires to students many considerations must be made:

- **Ethics:** Ethical issues relating to questionnaire completion was firstly considered. Respondents were informed that they were not obliged to complete the questionnaires if they did not wish to do so. Students were informed that their responses would remain anonymous and that they would not affect their grades in service mathematics.
- **Design:** The design of the questionnaire and its potential value in providing insight into the success of the intervention was considered next. Following the advice of Cohen et al (2007) the questionnaire was piloted with the author’s supervisors and the items on it were refined before it was finalised.
Type of Questionnaire: After careful consideration it was decided that the questionnaire would take the form of a semi-structured questionnaire. This type was chosen as it is appropriate when dealing with smaller sample sizes.

Question Type: The questions decided upon were open ended questions. The reason for this being again as they are suitable for a smaller sample size and also because they allow the researcher to capture rich and personal data through the analysis of word-based responses as opposed to closed questions (which often take the form of dichotomous questions, multiple choice questions, rating scales etc).

There are however some risks which a researcher must be willing to take if deciding to use open-ended word-based questionnaires. Some of the risks include:

- Open-ended questions can often make it difficult for a researcher to make comparisons between respondents, as there may be little in common to compare.
- Open-ended questions take much longer to complete then closed ones and so time constraints may affect the quality of the questionnaire.
- An assumption is being made when open-ended questionnaires are administered that all respondents are equally capable of articulating their thoughts in the same amount of time.

Despite these possible risks open-ended questionnaires have much to offer and so there is many useful reasons to recommend them. Some of the reasons which highlight the positive aspects to questionnaires are:

- They are a widely used and useful research instrument which can be administered even in the absence of the researcher.
- They are comparatively straight forward to analyse (Wilson and McLeon 1994).
- They can be used for collecting survey information and providing structured data collection.

In light of the advantages of using questionnaires as a form of data collection, the small sample size within the intervention group and the “window of opportunity for a respondent to shed light on an issue” a questionnaire was considered an appropriate research instrument to use in this project (Cohen et al 2007, p.331). The questions asked in the two questionnaires administered during the intervention are outlined in section 3.5.1 (c).
3.5.1 (c) Student Questionnaires

On two occasions during the tutorial interventions students were asked to fill out feedback forms on the intervention to date. These feedback forms were given half way through the semester and at the end of the semester. The questions which students were asked in feedback sheet 1 were as follows:

Q1: What are your thoughts on the tutorials so far?
Q2: Do you think all tutorials should be taught in this format/style? Why? Why not?
Q3: Any other comments?

At the end of the semester students received feedback sheet 2 which consisted of the following questions:

Q1: What do you feel works in this tutorial best and should be kept?
Q2: What do you feel needs to be changed/ dropped from the tutorial?
Q3: Any other comments?

The concepts which the questions aimed to gain insight into and the phrasing of them were informed by Shapiro’s (1987) step 3 and step 4 of intervention evaluations. Step 3 is concerned with evaluating the social validity of the intervention and step 4 aimed to evaluate the acceptability, both of which were determined through an analysis of the opinions of the students involved in the intervention. Students’ responses to the questions asked therefore enabled the evaluation of the intervention’s social validity and acceptability. Students’ responses were also analysed using elements of Kitwood’s (1977) method of analysing accounts. Kitwood (1977) developed a qualitative technique for analyzing accounts. This technique consisted of eight methods for dealing with accounts, four of which were used in the analysis of students’ questionnaires. The four methods were as follows:

- **Similarities and differences**: this method of account analysis involved investigations into similarities and differences within the total sample of accounts according to some characteristics, for example in the case of this research by probability of failure category.
- **Grouping items together**: This process involved the bringing together of data which covered a similar subject matter, for example remarks relating to the benefits of group work.
- **Tracing a theme**: this method identified all material relevant to a particular topic (e.g. references to real world problems) regardless of where they appeared in the data.
The study of omissions: Here the researcher examined if there were topics/opinions which she expected to see from the literature however they were not present. The work of Shapiro (1987) therefore helped to inform the questions in the feedback sheets and both the work of Shapiro (1987) and Kitwood (1977) were used to analyse the qualitative data obtained from the student feedback form. This analysis was carried out using a qualitative research analysis program entitled NVivo, details of which are outlined in Chapter 6. The author’s observations of the intervention were also used to evaluate the intervention in a qualitative manner, details of which are outlined in section 3.5.1 (d) and 3.5.1 (e).

3.5.1 (d) Observation: Design and Considerations

Before the second qualitative research instrument was decided on considerations were given to the benefits of observation in research, the most appropriate type of observation specific to this project and the potential risks associated with this type of research instrument.

Benefits of Observations

Much of the literature concerned with educational research methodologies highlight the benefits of observation in qualitative research. Some such benefits include:

- The opportunity it provides to gather ‘live data’ from naturally occurring social situations.
- Its use of immediate documentation of events has the potential to yield more valid or authentic data when compared to inferential research methods.
- What people do may differ from what they say they do, therefore observation methods do not lie (Robson 2002).
- Everyday behaviors, which may otherwise go unnoticed, can be detailed and potentially provide rich insight into particular issues or topics (Cooper and Schindler 2001).
- It allows for the documentation of non-verbal data such as body language or a lack of interaction between participants (Bailey 1994).

All of the benefits mentioned above provide a good case for the use of observations in educational research, however consideration must still be given to the type of observation which is most appropriate to the research in question.
**Design Considerations**

The author considered the use of structured, semi-structured and un-structured observations. As the author did not know exactly what she was looking for in the intervention and was not testing any particular hypothesis but rather wanted a hypothesis to emerge from the data a structured observation was not appropriate to use (Cohen et al 2007). The author did however want the observations to be somewhat informed by research so that she did not fail to consider any one aspect of the intervention observations. Semi-structured observations were therefore used in which research informed the structure and content of the observations (the details of which are outlined section 3.5.1 (e)). As the observer was also the teacher within the intervention tutorials she did not have the luxury of taking notes as the class unfolded. She therefore chose an approach which was outlined by Lincoln and Guba (1985) in which the observation takes the form of a journal entry (or log or diary) in which the notes are written some time after the observation took place. Prior to undertaking the observations the author wished to be informed about the possible risks associated with observation. Some of these risks are detailed next.

**Observation Risks**

Observations carry several risk factors such as those outlined by Wilkinson 2000; Moyles 2002; Robson 2002; Shaughnessy et al 2003:

- **Selective attention of the observer:** What the observer notes is hugely influenced by their personal interests, attitudes, mood and past experiences.
- **Reactivity:** If students feel they are being observed or monitored they may react in a way which they normally would not.
- **Attention Deficit:** The observer’s attention could dip at points during the observation time.
- **Selective Memory:** An observation which is written up after the event may be subject to neglect and select of particular information
- **Expectancy Effects:** The observer’s expectations of what may occur might influence their documentation of events.

Upon consideration of the benefits, appropriate design, risks and specific situations present in this study the author deemed an observation which takes the form of a semi-structured journal
entry an appropriate means of gathering data. The details of the structure and content of the observations are outlined in section 3.5.1 (e).

3.5.1 (e) Author’s Observation of the Intervention: Journal Entries

Immediately after each intervention tutorial was completed, a journal entry was made based on the author’s observations of the class. Each journal entry was informed by a combination of three different theoretical guidelines relating to intervention observations. The three guidelines were as follows:

1. Spradley’s (1980) checklist for intervention observation entries: This checklist provided a list of headings for each observation to ensure that everything had been considered. The checklist consisted of the following components:
   - Space: The physical setting
   - Actors: The people in the situation
   - Activities: Acts taking place
   - Objects: The artefacts and physical things that are there
   - Acts: The specific actions that participants are doing
   - Events: The sets of activities that are taking place
   - Time: The sequence of acts, activities and events
   - Goals: What are people trying to achieve
   - Feelings: What people feel and how they express this

2. Spradley’s (1979) and Kirk and Miller’s (1986) Set of Observational Data were also used to guide the structure of the observations. These guidelines suggested that any observation should consist of 4 components including:
   - Notes made in situ
   - Expanded notes that are made as soon as possible after the initial observation
   - Journal notes to record issues, ideas, difficulties etc that arise during field-work
   - A developing, tentative running record of on-going analysis and interpretation.

Each of these components were used and adapted appropriate to the teaching intervention in this research.
3. Shapiro’s (1987) intervention research evaluation steps 1 and 2 were the final influence on the author’s observations. As partially detailed in figure 3.6 these two evaluation steps contain the following components:

- **Step 1: Treatment effectiveness**
  (i) Strength of change
  (ii) Immediacy of change
  (iii) Amount of change

- **Step 2: Treatment Integrity**
  (i) Significance of goals
  (ii) Social appropriateness
  (iii) Importance of outcomes

The observations provided the insight the author needed to evaluate the tutorial lessons in terms of Shapiro’s (1987) treatment integrity and treatment acceptability. This element of the qualitative findings of the teaching intervention was also analysed using NVivo, the findings of which are detailed in chapter 6. As highlighted previously in this chapter the intervention was also assessed in a quantitative manner, details of this are found in section 3.5.2 which follows.
3.5.2 Quantitative Analysis of the Intervention

The analysis of the intervention from a quantitative perspective involved the use of SPSS. Service mathematics students were given a probability of failure based on their Leaving Certificate mathematics grade on entrance to UL and their performance in the diagnostic test (details of exactly how these probabilities were calculated can be found in chapter 5). Based on this probability they were then categorised into one of three groups; Low risk, Medium risk or High risk of failure in service mathematics. One of the main objectives of the quantitative analysis of the intervention was to determine whether the students involved in the teaching intervention performed to a higher standard, by probability group, than the students who were involved in the traditional tutorial format. The pass rate within each of the two groups (intervention and non-intervention group) were analysed and compared. Quantitative analysis also involved an examination of the attendance at tutorials of those who passed service mathematics compared to those who failed and attendance by probability of failure group (i.e. low risk, medium risk, high risk). Details were also given relating to performance of the intervention students compared to the non-intervention students by Leaving Certificate mathematics grade. The statistical tests used in this analysis were outlined earlier in this chapter in section 3.4.1. The findings from the quantitative analysis are detailed in chapter 6.

The final considerations in the methodology chapter comes under the heading of ‘Issues within the Research’. Details of these potential issues are outlined next in section 3.6.
3.6 Issues within the Research

Every researcher must adhere to certain rules and procedures when carrying out both quantitative and qualitative research. Even when this is done, certain issues are likely to occur such as ones surrounding validity, reliability and ethics. These issues are considered next.

3.6.1 Validity and Reliability

The manner in which validity and reliability should be addressed varies depending on whether qualitative or quantitative research practices are being considered (Cohen et al 2007). Both will therefore be considered next in general and in the context of this research project.

3.6.1.1 Validity

The meaning of validity in educational research has varied over the years. In general terms it is the extent to which a measurement instrument measures what it is supposed to measure (Leedy and Ormrod 2005).

Validity and Qualitative Research

Common means of addressing validity in qualitative research involves ensuring honesty, depth, richness and scope within the data generated, the participants approached, the extent of triangulation and the objectivity of the researcher (Winter 2000). In the case of this research project, the interpretations of students’ feedback on the teaching intervention were carried out with honesty and depth. The author was mindful of staying as objective as possible throughout the analysis of both student feedback and the interpretation of her own observations documented as journal entries of the intervention tutorials. Gronlund (1981) highlighted that within qualitative data the subjectivity of respondents, their personal opinions, attitudes and perspectives will naturally combine to contribute to a certain degree of bias.
Validity and Quantitative Research

There are a number of measures which are said to contribute to the improvements of validity where quantitative data is concerned such as:

- Careful sampling
- Appropriate instrumentation
- Appropriate statistical treatment of data

(Cohen et al 2007, p.133).

In the context of this research project the very large sample size involved, the consistency of the instruments being analysed (diagnostic test, Leaving Certificate mathematics examination, Technology mathematics examination and lecturer) aid the levels of validity. Appropriate treatment of statistical data was also ensured through research into statistical methodologies available (e.g. most effective prediction method for this project) and careful implementation and documentation of descriptive and inferential statistical analysis throughout also ensured appropriate levels of validity.

Internal Validity

Internal validity seeks to demonstrate that the detailing of a particular situation or set of data which a piece of research provides can be backed up by the data. “The findings must describe accurately the phenomena being researched” (Cohen et al 2007, p.135). Internal validity can be applied to both quantitative and qualitative data analysis. It was addressed in this research in a qualitative context through a method suggested by LeCompte and Preissle (1993) by using peer examination of data. The author had two colleagues, both involved in mathematics education in UL, to review students’ feedback sheets and her personal journal observations to assess what themes emerged from the data. The author could then compare her own analysis with that of her colleagues. This research project also addressed internal validity in a quantitative manner through the input of multiple researchers when analyzing data such as that which compared the most effective method of prediction of performance in service mathematics.
External Validity

External validity is highly significant in any research study as it is effectively a measure of how well a study’s results are generalisable to the wider research community (Cohen et al 2007). From a qualitative perspective the intervention tutorial structure which took place in this project is one which could be replicated in universities worldwide as it did not require any specialist equipment and took place within the university timetable. The students involved in the tutorial had a variety of mathematical backgrounds; a diverse sample such as this is likely to be present in the majority of third level cohorts also. From a quantitative perspective the examination of the changing student profile, the possibility of grade inflation and the development of a predictive model of failure in service mathematics were all components of this body of work which are adaptable and generalisable to all countries worldwide who have a terminal mathematics examination on exit from second level education. Threats to external validity such as selection effects, setting effects, history effects and construct effects are therefore not likely to have been an issue (Lincoln and Guba 1985).

3.6.1.2 Reliability

For research to be reliable it must be able to demonstrate that similar results would occur if it was carried out again and again on similar subjects in a similar context. It is another term for dependability, consistency and replicability (Cohen et al 2007). Like validity it can be considered from a qualitative and quantitative perspective.

Reliability and Qualitative Research

In terms of the qualitative research analysis carried out in this research project, i.e. the analysis of students’ questionnaires and the author’s journal observations, the author can be confident that if analysed by another researcher using the same evaluation model that very similar findings would emerge. The fact that each student was asked the exact same questions satisfies Oppenheim’s (1992) issue with reliability in qualitative research and questioning. He suggested that changing the wording and emphasis of questions, for example in an interview setting, affected reliability as each participant was effectively being asked a different question.
Reliability and Quantitative Research

Reliability in relation to quantitative research is considered to be “concerned with precision and accuracy” (Cohen et al 2007, p.146). If the same tests were carried out on similar groups of respondents the results yielded should be very similar. In the context of this research project the statistics documented throughout, both inferential and descriptive, are accurate and precise as they have been carefully computed and considered in light of their limitations and statistical assumptions. The large sample sizes used throughout the research also improves the reliability of the quantitative research.

In addition to validity and reliability considerations an incorporation of triangulation into the research aids its integrity.

3.6.2 Triangulation

“Triangular techniques in the social sciences attempt to map out, or explain more fully, the richness and complexity of human behavior by studying it from more than one standpoint….by making use of quantitative and qualitative data” (Cohen et al 2007, p.141).

Triangulation is present in research when two or more methods of data collection are present. This mixed method approach has been used to demonstrate validity in research (Campbell and Fiske 1959). Exclusive reliance on either quantitative or qualitative research analysis may distort the researcher’s picture of a certain event/scenario being analysed (Lin 1976).

There are several types of triangulation presented in the literature, some of which are present in this research project:

- **Time triangulation**: this type takes into consideration the factors of change and process by utilizing cross-sectional and longitudinal design (Kirk and Miller 1986). This type of triangulation was present in a quantitative form in the comparison of mathematical competency levels of students entering UL between the years 1998-2008. It was also present in a qualitative form within this study in the comparisons of the student profile (in terms of gender, numbers sitting the diagnostic test, level at which Leaving Certificate mathematics was taken etc) between the years 1998 and 2008. Finally time triangulation presented itself from a qualitative point of view as students were given two different questionnaires at different times during the semester.
• **Data triangulation:** this type involves gathering data through several sampling strategies resulting in data being collected at different times on a variety of people (Denzin and Lincoln 1994). This form of triangulation was present in the quantitative data collection process as the information held in the database used in this research, which consist of information relating to students Leaving Certificate mathematics grades, diagnostic test results and end of term results etc, were gathered over time. It was also present in the qualitative data collection as feedback on the intervention was recorded through the use of students’ questionnaires and an observation journal taken by the author.

• **Methodological triangulation:** this refers to the use of more than one method for gathering data and was also used in the context of this research project. Data was gathered through diagnostic test corrections, examination of the online student record system in UL (i.e. to determine Leaving Certificate mathematics grades, standard or non standard student), through qualitative student feedback sheets on two occasions during the intervention and through the author’s journal observations.

As evidenced in the discussion in this section, triangulation is present in several different forms throughout the research, in the form of qualitative and quantitative findings. The presence of triangulation is vitally important to the validity of research as it helps to eliminate bias particularly where qualitative research is concerned as this can often have more room for bias (Brown 1996). The use of triangulation is therefore likely to have contributed positively to the validity of the research. Ethical issues are now considered in section 3.6.3.
3.6.3 Ethics

Researchers have the difficult task of trying to strike a balance between the demands which are placed on them by their research goals and the well being and rights of their subjects. This challenge is known as the ‘cost-benefit ratio’ and refers to the ethical considerations which must be carried out in all action research (Frankfort-Nachmias and Nachmias 1992). Ethical issues can arise at any stage in research due to many factors, for example:

- The nature of the research (differences in intelligence)
- The context of the research (someone’s home)
- The procedures to be adapted (causing high levels of anxiety)
- The nature of the participants (emotionally disturbed adolescents)
- The type of data collection (highly personal information)

(Cohen et al 2007, p. 51)

Such ethical issues were considered and ethical guidelines were adhered to throughout the course of this research project. The attention to ethical issues in this research are detailed in terms of diagnostic testing, qualitative analysis and quantitative analysis.

Diagnostic Testing

Students were not obliged to take the diagnostic test and were informed that if they did choose to take part and later wished for their results to be withdrawn that this was possible. Students were given clear information regarding the purpose of the test and they were informed that it would have no bearing on their grades in university mathematics education. The results of the diagnostic test were reported within one week of the test along with recommendations to engage in mathematics support.

Qualitative Study

Permission to conduct the intervention stage of this research was granted as the study received formal ethical approval from UL’s ethics committee prior to its commencement (see Appendix M). Permission was also granted by the Technology mathematics lecturer prior to commencement of the study. Participants of the intervention were asked to read and sign a consent form explaining what the intervention tutorial was about (see Appendix N). Students were not obliged to attend that tutorial and could withdraw and attend an alternative tutorial is
they so wished. No student taking part in the intervention, or those involved in any other aspect of the research, were identified in the write up of the project.

**Quantitative Study**

The manner in which student data was obtained from the UL student record system throughout the course of the research followed university guidelines. Although students were not anonymous within the datasets, due to their student identification numbers being listed, their names were not included and all data was only accessible by the investigators of this research project in-line with university guidelines. All quantitative analysis was carried out using the statistical package SPSS and was administered with careful adherence to theoretically sound statistical methodologies. The research therefore avoided many of the ethical dilemmas which can often occur, as highlighted by Robson (1993) such as involving people without their knowledge or consent, withholding information about the true nature of the research or exposing participants to physical or mental stress. Limitations of the research were also considered and are discussed next in section 3.6.4.

**3.6.4 Limitations of the Study**

Within this research project certain limitations presented themselves, the majority of which were out of the control of the researcher.

When determining the probability of failure of students within Science and Technology mathematics using the discriminant function some students were left out of the process. Students who did not have a Leaving Certificate mathematics grade or those who did not sit the diagnostic test or both were not presented with a probability of failure in service mathematics. These students were most often non-standard students who did not have a Leaving Certificate mathematics grade and students who were not present for their first service mathematics lecture and therefore did not sit the diagnostic test. This is a cause for concern as students who could be ‘at risk’ of failing service mathematics are not being assessed.

Some limitations also presented themselves during the intervention phase of the research. It was not compulsory for students to attend the intervention tutorials although they were encouraged as much as was possible to do so. This resulted in groups of students on occasion having only two
members present in their groups both of whom could have had high probabilities of failure. This went against the intention of the tutorials which was to have a mixed ability group of students problem solving together. This was managed as best as was possible on a week by week basis by grouping students into mixed ability groups i.e. for example combining two poorly attended groups.

The final limitation which was present in this study was the short period of time in which the intervention ran for. The intervention consisted of a total of ten tutorial sessions. This was quite a short amount of time to have an influence on students in terms of their mathematical ability and learning style considering the majority of them had come into UL from a system which encouraged rote learning and placed very little emphasis on understanding (O’Keeffe 2010). It was hoped that in spite of this short time period that the effort and research that went into the design and implementation of the intervention would result in an improvement in students’ understanding anyway, both in terms of their attitudes and interest in mathematics as well as improvements in their service mathematics grades.

3.7 Conclusion

This chapter outlined the theoretical framework which aimed to guide the author’s research. The methodologies employed in each phase of the research along with the research objectives and questions which drove the research were also detailed. The multi-method approach used throughout and the benefits of it are detailed. An extensive account of the methods used to evaluate the intervention and the theoretical models which informed the evaluation were set out. The chapter outlined the considerations which were undertaken by the author relating to validity, reliability, ethics and research limitations. The structure and methodologies employed throughout this research have therefore been outlined giving a clear chronology of the work which follows. Chapter 4 details the first of the major findings within the research relating to the changes over time in competency levels of service mathematics students on entry to UL.

4. Introduction

Examining student competency levels over time is a popular and widely carried out practice in third level education (Lawson 1997; Engineering Council 2000; Todd 2001; Gill 2006; Malcolm and McCoy 2007; Gill et al 2010). The aim of this procedure is often to establish if a problem exists within a particular cohort, to what extent it may exist and to establish whether or not there are strategies that can be implemented to alleviate such a problem. This chapter details an analysis of the UL database of service mathematics students between 1998 and 2008. An examination into the profile of ‘at risk’ students in Technology and Science mathematics is also detailed. A breakdown of the methods used to investigate these issues and the chronology of the investigations is detailed in section 4.1.
4.1 Chapter Outline

In this chapter the findings from the database relating to profiling students between 1998 and 2008 are presented. A profile of ‘at risk’ students between 2006 and 2008 is also detailed. The chapter is laid out as follows:

- The changing student profile of service mathematics students in UL (1998-2008) (section 4.2)
  - Increase in non-standard students to UL service mathematics (section 4.2.3)
- An Investigation into performance in the diagnostic test over time (1998-2008) (section 4.3)
- A profile of ‘at risk’ service mathematics students (2006-2008) (section 4.4)
- Discussion and Conclusions (section 4.5)
- Closing Remarks (section 4.6)

The analysis carried out in this chapter aims to answer the following research questions:

- What is the profile of service mathematics students in UL between 1998 and 2008?
- What are the trends in mathematical competency levels of UL service mathematics students by Leaving Certificate mathematics grade?
- Is there evidence of grade inflation occurring in Leaving Certificate mathematics?
- What is the profile of students who are ‘at risk’ of failing Technological mathematics?
- What is the profile of students who are ‘at risk’ of failing Science mathematics?
- What is the profile of a student who is ‘at risk’ of failing service mathematics?
4.2 The Changing Student Profile of Service Mathematics Students in UL (1998-2008)

The profile of those entering Science and Technology service mathematics courses in UL is consistently changing. Many aspects of the profile have changed between 1998 and 2008 such as gender breakdown, degree programmes contained in the two service mathematics groups and the mathematical background of the students. The findings, in relation to this, have been determined through an examination of the UL database. The database, and the information contained in it, was outlined in detail in Chapter 3 of this thesis. The analysis is based on the total number of students who registered for the modules in 1998 and 2008 i.e. 305 Technology students in 1998 and 374 Technology students in 2008, 202 Science students in 1998 and 303 Science students in 2008. Not all of these students sat the diagnostic test or the end of semester examinations.

4.2.1 Gender Balance

Within Technological mathematics the male dominance has remained largely unchanged between 1998, when 85.2% were male, and 2008, when 87.7% were male. The Science mathematics cohort, however, has seen a large change in gender balance from 1998 when 36.6% were male, to 2008 when the males made up more than half of the cohort, 53.5% (see table 4.1). This increase in male students in Science mathematics may be explained by the addition of degree programs such as Health and Safety and Physical Science to this mathematics module (see table 4.2). These degree programs are predominantly male.

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>2008</th>
<th>1998</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Technology mathematics</td>
<td>Science mathematics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Males</td>
<td>260 (85.2%)</td>
<td>328 (87.7%)</td>
<td>74 (36.6%)</td>
<td>162 (53.5%)</td>
</tr>
<tr>
<td>Females</td>
<td>45 (14.8%)</td>
<td>46 (12.3%)</td>
<td>128 (63.4%)</td>
<td>141 (46.5%)</td>
</tr>
</tbody>
</table>

Table 4.1 Gender Breakdown in Technology and Science mathematics (1998 & 2008).
4.2.2 Degree Programmes

There has been an increase in the number of degree programmes within Technological and Science mathematics modules between 1998 and 2008. Within Technological mathematics there has been an increase from 8 to 14 degree programmes with the addition of programmes of study such as Music, Media and Performance Technology; Materials and Engineering Technology; Digital Media and Design; Computer Systems and Engineering Science. Science mathematics has seen an increase from 8 degree programmes in 1998 to 11 in 2008 with courses such as Health and Safety, Physical Science, Law Plus and Joint Honours now being required to take Science mathematics (see table 4.2). The increase in degree programmes within the two service mathematics modules is likely to have had an influence on the profile of mathematical backgrounds of the students making up these cohorts. Some such changes are discussed next.

<table>
<thead>
<tr>
<th>Degree Programs in 1998</th>
<th>Degree programs in 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technological mathematics</td>
<td>Technological mathematics</td>
</tr>
<tr>
<td>Electronic Manufacturing</td>
<td>Electronic Systems</td>
</tr>
<tr>
<td>Information Technology and Telecommunications</td>
<td>Applied Computing and Network Technology</td>
</tr>
<tr>
<td>Material and Construction Technology</td>
<td>Materials and Construction Technology</td>
</tr>
<tr>
<td>Materials and Engineering Technology</td>
<td>Construction Management and Engineering</td>
</tr>
<tr>
<td>Manufacturing Technology</td>
<td>Manufacturing Systems</td>
</tr>
<tr>
<td>Physical Education</td>
<td>Physical Education</td>
</tr>
<tr>
<td>Production Management</td>
<td>Production Management</td>
</tr>
<tr>
<td>Wood Science and Technology</td>
<td>Wood Science and Technology</td>
</tr>
<tr>
<td></td>
<td>Music, Media and Performance Technology</td>
</tr>
<tr>
<td></td>
<td>Materials and Engineering Technology</td>
</tr>
<tr>
<td></td>
<td>Digital, Media Design</td>
</tr>
<tr>
<td></td>
<td>Product Design and Technology</td>
</tr>
<tr>
<td></td>
<td>Computer Systems</td>
</tr>
<tr>
<td></td>
<td>Engineering Science</td>
</tr>
<tr>
<td>Science mathematics</td>
<td>Science mathematics</td>
</tr>
<tr>
<td>Applied Physics</td>
<td>Biological Science</td>
</tr>
<tr>
<td>Biological Science</td>
<td>Environmental Science</td>
</tr>
<tr>
<td>Environmental Science</td>
<td>Food Science and Health</td>
</tr>
<tr>
<td>Food Technology</td>
<td>Pharmaceutical and Industrial Chemistry</td>
</tr>
<tr>
<td>Industrial Chemistry</td>
<td>Industrial Biochemistry</td>
</tr>
<tr>
<td>Industrial Biochemistry</td>
<td>Material Science</td>
</tr>
<tr>
<td>Material Science</td>
<td>Sport and Exercise Science</td>
</tr>
<tr>
<td>Sport and Exercise Science</td>
<td>Physical Science</td>
</tr>
<tr>
<td></td>
<td>Health and Safety</td>
</tr>
<tr>
<td></td>
<td>Law Plus</td>
</tr>
<tr>
<td></td>
<td>Joint Honours</td>
</tr>
</tbody>
</table>

Table 4.2 Degree Programs within Science and Technology mathematics (1998 & 2008).
4.2.3 Increase in Non-Standard Students to UL Service Mathematics

Another change to the profile of students entering UL to service mathematics courses is the increase in non-standard students. A non-standard student is defined as a student who did not enter UL through the CAO system. Therefore they consist of mature students (i.e. anyone over the age of 23), non-national students who have not done an Irish Leaving Certificate and those who have completed previous degree/diplomas/certificates and have used these as an entry point to UL. There has been an increase of 9.1 percentage points and 5.4 percentage points of non-standard students in Technological and Science mathematics respectively between 1998 and 2008 (see table 4.3). A shift in student intake like this has been shown to affect overall mathematical performance in UL as well as in other institutions such as Coventry University (Lawson 2003; Faulkner et al 2010). Due to the effect on overall performance that this group of students have, further investigation into this group and their performance in UL examinations is required. Information relating to this is outlined in section 4.2.3.1 (a), (b) and (c).

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>2008</th>
<th>1998</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Technology mathematics</td>
<td>Science mathematics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard students</td>
<td>304 (99.7%)</td>
<td>339 (90.6%)</td>
<td>199 (98.5%)</td>
<td>282 (93.1%)</td>
</tr>
<tr>
<td>Non-standard students</td>
<td>1 (0.3%)</td>
<td>35 (9.4%)</td>
<td>3 (1.5%)</td>
<td>21 (6.9%)</td>
</tr>
</tbody>
</table>


4.2.3.1 Non-Standard Students

4.2.3.1(a) Breakdown of Non-Standard Students in 2008

As previously mentioned non-standard students are made up of (i) mature students i.e. students who are 23 years of age or older, (ii) non-national students i.e. students who have reached a certain standard of schooling in their own country and are deemed to be of a similar standard to Leaving Certificate students in Ireland and (iii) students with previous qualifications such as a diploma, degree or a certificate who are returning to college.

Within the 2008 non-standard group the following breakdown occurs:
40 (71.4%) of the students are mature students all of whom have no previous qualifications in UL, are over 23 and have not completed the Leaving Certificate in the last 5 years. Some mature students gain access to UL degree programmes through a 1 year intensive access course which consists of a variety of subjects depending on the students’ choice of degree programme. Upon passing access course examinations and/or on the basis of an interview, mature students can gain places in degree programmes.

6 (10.7%) are students who have gained entry to UL in 2008 based on their completion of a previous degree/diploma/certificate in a variety of third level institutions around Ireland. The previous qualifications of these non-standard students range from engineering, industrial automation, automation technology and environmental science to areas of study which are likely to consist of less mathematics content such as sport and exercise science and archaeology.

The remaining 10 (17.9%) students are made up of non-national students who have completed an alternative school leaving qualification to the Leaving Certificate in another country.

Grouping these students together may not be appropriate if they are found to be non-homogenous in terms of their diagnostic test scores and service mathematics performance. The students within the non-standard category may perform to different standards due to differences for example in years since study of formal mathematics, age or motivation levels. The investigation which is outlined in section 4.2.3.1 (b) offers some insight into whether the group are homogenous or not.

4.2.3.1(b)Performance of Non-Standard Students in the Diagnostic Test in 2008

The figures mentioned in section 4.2.3.1(a) highlight that mature students make up the majority of the non-standard cohort in UL, therefore the majority of non-standard students have not studied mathematics for a number of years. This is evident in their diagnostic test results which can be seen in figures 4.1 and 4.2. Non-standard students have mean diagnostic test scores (expressed as a percentage of correct answers out of 40 questions) below that of the standard student coming directly from Leaving Certificate. For both Science and Technological non-standard students the vast majority, with the exception of a few outliers, are classified as being ‘at risk’ of failing their end of semester examinations. 117 (42.9%) of the 273 standard Technology mathematics students who sat the diagnostic test were classified as being ‘at risk’ of
failing Technology mathematics while 23 (76.7%) of the 30 non-standard Technology mathematics students who sat the diagnostic test were considered to be ‘at risk’ of failing. Similarly 96 (43.2%) of the 222 standard Science mathematics students who sat the diagnostic test and 13 (86.7%) of the 15 non-standard Science mathematics students who sat the diagnostic test were considered to be ‘at risk’ of failing service mathematics (see table 4.4). The box plots in figures 4.1 and 4.2 highlight that non-standard students are mathematically less prepared entering UL than standard students.


**4.2.3.1 (c) Performance of Non-Standard Students in Service Mathematics Examinations in 2008**

As was previously outlined a higher proportion of non-standard students are considered to be ‘at risk’ of failing service mathematics when compared to their standard counterparts in both Technology and Science mathematics. The non-standard students however have higher median performances than the standard students in service mathematics for both the Technology and Science cohorts (see figure 4.3 and 4.4). In spite of this the proportion of non-standard Technology students who failed their service mathematics examination is higher than that of the standard Technology students however it is lower than would be expected based on the analysis of the diagnostic test data (see section 4.2.3.1(b)). 78 (24.7%) of the 316 standard Technology students who sat the end of semester examination failed it while 10 (30.3%) of the 33 non-standard Technology students who sat the end of semester examination failed. The opposite was the case however for the Science cohort as 74 (27.7%) of the 267 standard students who sat the end of semester examination failed it compared to 2 (11.8%) of the 17 non-standard Science students who sat the end of semester examination (see table 4.4). Just like the Technology non-standard students, the Science non-standard students performed to a higher standard than would be expected based on their diagnostic test performance. These findings must be considered in light of the larger spread of results of the non-standard Technology students and the small number of non-standard students in both the Technology and the Science mathematics cohorts in 2008.
Note: 33/35 non-standard Technology students and 316/339 standard Technology students sat the end of term examination. 17/21 non-standard Science students and 267/282 of the standard Science students sat the end of term examination.

The non-standard students, the majority of whom are mature students, are clearly less prepared mathematically than the standard students on entry to UL however improve on average over the course of one semester. The fact that the non-standard students improve so much compared to the standard students over time is unexpected as generally a students’ diagnostic test performance is predictive of their service mathematics performance (see section 5.6.1, chapter 5). These findings therefore warrant further investigation. Section 4.2.4, which follows, investigates these findings by examining the performances of the each sub-category of non-standard and standard student in terms of diagnostic test performance and service mathematics performance.

<table>
<thead>
<tr>
<th></th>
<th>Technology Mathematics</th>
<th></th>
<th>Science Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>‘at risk’ n (%)</td>
<td>Failed Service Mathematics n (%)</td>
<td>‘at risk’ n (%)</td>
</tr>
<tr>
<td>Standard Students</td>
<td>117 (42.9%)</td>
<td>78 (24.7%)</td>
<td>96 (43.2%)</td>
</tr>
<tr>
<td>Non-Standard Students</td>
<td>23 (76.7%)</td>
<td>10 (30.3%)</td>
<td>13 (86.7%)</td>
</tr>
</tbody>
</table>

Table 4.4 Number and Percentage of students who were considered to be ‘at risk’ of failing service mathematics and those who failed service mathematics in Science and Technology Mathematics in 2008.
Figure 4.1 Comparison of diagnostic test performance for Standard (n=273) and Non-standard (n=30) Technology mathematics students in 2008.

Figure 4.2 Comparison of diagnostic test performance for Standard (n=222) and Non-standard (n=15) Science mathematics students in 2008.
Figure 4.3 Comparison of Technology service mathematics performance for Standard (n=316) and Non-standard (n=33) students in 2008.

Figure 4.4 Comparison of Science service mathematics performance for Standard (n=267) and Non-standard (n=17) students in 2008.
4.2.4. Standard and Non-Standard Students’ Performance in the Diagnostic Test and Service Mathematics: Further Analysis

4.2.4.1 Breakdown of Non-Standard Students’ Performance in Terms of 3 Sub-Categories in 2008

In order to attempt to determine why non-standard students in 2008 have a lower mean performance in the diagnostic test yet a higher mean performance in service mathematics compared to standard students, an analysis of non-standard students’ performance by sub-category was undertaken. Therefore the students were broken down into 1) mature students, 2) those who engaged in previous study and 3) international students (see table 4.5). The aim of this investigation was to determine if one particular category of non-standard student caused the mean to be particularly low in the diagnostic test or, alternatively, particularly high in service mathematics performance.

Each category within the non-standard group improved in their mean mathematics performance over time particularly the mature students and the students who have engaged in previous study (see table 4.5, figure 4.5 and figure 4.6). Initially 29 (85.3%) of the mature students and 4 (100%) of the students who have engaged in previous study were considered to be ‘at risk’ of failing service mathematics due to their diagnostic test performance. In spite of this 25 (71.4%) and 6 (100%) of the mature students and students who engaged in previous study respectively were successful in their service mathematics examination (see figure 4.6). Why then do these sub-categories of students perform so poorly on entry to UL and improve so much over the course of the semester?

Mature students are sometimes said to be deficient in the basic skills needed for effective study in higher education (Richardson 1995). They have also, in general, studied significantly less mathematics than their younger counterparts (Relich et al 1994). They tend to perform poorly in mathematics diagnostic tests on entry to higher education when compared to the standard students due to numeracy problems (O’Donoghue 1995; O’Donoghue 1996; Kaye 2002; Maguire et al 2002). Several reasons which have been suggested for this poor performance include:

- Being out of practice with formal mathematics (O’Donoghue 1999).
- A lack of confidence in a formal education setting (Bowl 2001).
• Weak mathematical backgrounds on entry to higher education (Golding and O’Donoghue 2005).

The initial challenges which mature students face however are likely to have been counteracted by their motivation to succeed. Mature students have been found to be more motivated by intrinsic goals than younger students in higher education (Pierce 1995; Forgasz 1996; Murphy and Roopchand 2003). In addition to this mature students tend to exhibit more desirable approaches to academic learning. They are less likely to adopt a surface learning approach in higher education than younger students. Younger students tend to acquire a surface learning approach to education due to their reliance on it in their final years of secondary school. The prior life experiences of mature students however promote their tendency to adapt a deep learning approach over a surface learning approach (Richardson 1995; Jacobs and Newstead 2000; Smith 2002).

As well as their tendency to adapt more desirable approaches to academic learning, mature students have been found to frequently avail of mathematics support services (Gill and O’Donoghue 2007). During the academic year 2008, UL provided support tutorials specifically for mature students in addition to one-to-one consultations with members of staff in the mathematics learning centre. The MLC data in UL highlights the frequent use of the centre by mature students in all service mathematics courses. Data collected over the time period 2009-2011 found that typically over 50% of attendances to the MLC are made by mature students (O’Keeffe 2011). Engagement with support services such as these have been found to have a positive impact on service mathematics grades for those students (Mac an Bhaird et al 2009; Gill et al 2010). Their research gives an insight into mature students’ progress with mathematics over the course of the semester.

In relation to the students who engaged in previous study, one of the possible reasons for their poor performance in the diagnostic test may, like mature students, be due to being out of practice with formal mathematics. Students within this category engaged in previous study such as a Sport and Leisure Diploma (n = 2), Environmental Science Diploma (n = 1) and BA Archaeology (n = 1). It is probable that these programmes did not have a strong mathematical content, if any. The move from studying mathematics at diploma level to degree level, for example, has been seen to be similar to the move between studying elementary mathematics to advanced mathematics. Tall (1991) describes this move as a significant transition from
describing to defining, from convincing to proving which requires a cognitive reconstruction often found to cause great difficulty for students. The fact that students within this sub-category successfully completed previous academic programmes however suggests that they are motivated and willing to work hard despite the fact that their mathematical literacy is not strong at the beginning of the semester. 100% (n = 6) of the students who engaged in previous study were successful in service mathematics however the small numbers makes it difficult to draw a definite conclusion from these findings.

The international students perform to a much higher mean standard in the diagnostic test when compared to the other two sub-categories of non-standard students (see table 4.5). This is likely to be attributed to the fact that many of the students in this sub-category are entering UL after completing the equivalent of the Leaving Certificate in their respective countries. Their mathematics is therefore likely to have been practiced in recent times when compared to the students within the other two sub-categories of non-standard students. The majority of the international students come from China (n = 4) with students also from Saudi Arabia (n = 2), South Africa (n = 2) and Spain (n = 2). The students in this sub-category also perform well in service mathematics with just 2 (22.2%) of them failing. Again the recent engagement with formal education and mathematics is likely to be a contributing factor to their success in service mathematics.

As mature students make up the majority of students in the non-standard category, their poor performance in the diagnostic test has the largest influence on the category’s performance as a whole. The improvements of the mature students and those who have engaged in previous study along with the maintenance of the high performance from the international students resulted in the increase in the performance of the non-standard students as a whole over time. Although this establishes some explanation as to why the non-standard category improve greatly over the course of the semester, the unexpected lower mean performance of the standard students in service mathematics is still to be examined. Note: See Appendix B for a separate breakdown of the non-standard students’ mathematical performance in Science and Technology mathematics.
<table>
<thead>
<tr>
<th></th>
<th>Mature Students</th>
<th>Previous Degree, Diploma, Certificate</th>
<th>International Students</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean Diagnostic Test Result</strong></td>
<td>28.3 (16.4)</td>
<td>27.5 (8.9)</td>
<td>52.9 (30.4)</td>
</tr>
<tr>
<td><strong>Expressed as a Percentage</strong></td>
<td>n=34</td>
<td>n=4</td>
<td>n=7</td>
</tr>
<tr>
<td><strong>(SD)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Mean End of Semester Result</strong></td>
<td>54.9 (27.9)</td>
<td>61.7 (17.4)</td>
<td>61.9 (22.8)</td>
</tr>
<tr>
<td><strong>Expressed as a Percentage</strong></td>
<td>n=35</td>
<td>n=6</td>
<td>n=9</td>
</tr>
<tr>
<td><strong>(SD)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Percentage who Passed Service Mathematics</strong></td>
<td>71.4%</td>
<td>100.0%</td>
<td>77.8%</td>
</tr>
<tr>
<td><strong>Total Number of Students in Each Category</strong></td>
<td>40</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 4.5 Mean Performance and Standard Deviation of 3 sub-categories of non-standard students in the diagnostic test and service mathematics for Technology and Science students in 2008.

**Note:** 45/56 non-standard students sat the diagnostic test and 50/56 non-standard students sat the end of semester examination.

**Figure 4.5** Diagnostic test performance of 3 sub-categories of non-standard students in Technology and Science mathematics in 2008.
Figure 4.6 Service mathematics performance of 3 sub-categories of non-standard students in Technology and Science mathematics in 2008.

Summary and Conclusions

- Non-standard students as a whole improve in their mathematics performance over the course of the semester.
- The majority of mature students and those who have engaged in previous study are considered to be ‘at risk’ of failing service mathematics due to their diagnostic test performance.
- International students perform to a much higher mean standard in the diagnostic test when compared to the other two sub-categories of non-standard students.
- The majority of non-standard students in each sub-category of both cohorts of students are successful in their end of term examination. The possible reasons for the improvement in performance of non-standard students are:
  - Their motivation to succeed (Murphy and Roopchand 2003).
  - Their tendency to adapt desirable approaches to learning (Smith 2002).
  - Their frequent use of available mathematical support services (Gill and O’Donoghue 2007).
The findings in this section shed some light on why non-standard students perform to a low standard in the diagnostic test but improve over time. Section 4.2.4.2 which follows, investigates why standard students do not outperform non-standard students in terms of mean service mathematics results.

4.2.4.2 Technology and Science Standard and Non-Standard Students Explored under Five Different Sub-Categories in 2008

Similar to the non-standard students, to perform a more in-depth analysis of their performance, a sub-categorisation of the standard students (into Higher Level and Ordinary Level) is also carried out. An examination into whether it is reasonable to group standard and non-standard students as homogenous groups consequently follows. Table 4.6 and figures 4.7 and 4.8 detail the performance of the 5 sub-categories of students (i.e. mature students, those who have engaged in previous study, international students, Ordinary Level students and Higher Level students) in the diagnostic test and service mathematics for Science and Technology mathematics students combined in 2008. An analysis of variance test (ANOVA) revealed that there is a statistically significant difference between the mean performances of the 5 sub-categories of students in both the diagnostic test and the end of semester examination (p < 0.001).

Upon carrying out an analysis of the standard students, it emerged that 194 (62.9%) of the Ordinary Level students were considered to be ‘at risk’ of failing service mathematics (see figure 4.7). In general, the Ordinary Level students’ performed poorly over the course of the semester with 142 (40.2%) of them failing service mathematics (see figure 4.8). Of the five sub-categories of students the largest proportion of students who failed service mathematics was contained in the Ordinary Level students’ category. Their mean service mathematics result is also considerably lower than any other sub-category of student and was found to be statistically significantly different (p < 0.001) (see table 4.6). This finding partially explains why standard students did not outperform the non-standard students in 2008. The Ordinary Level students negatively impact upon the mean performance of the standard students while every other sub-category of student improved in their performance over time.

19 (10.2%) of the Higher Level students were considered to be ‘at risk’ of failing service mathematics (see figure 4.7). Their mean performance in the diagnostic test is 65.7% and they maintain this high performance as they have a mean performance of 70.0% in service
mathematics. The Higher Level students have a failure rate of 17 (7.4%) confirming that they are definitely not the reason for the standard students having a lower mean performance than the non-standard students in service mathematics (see table 4.6 and figure 4.8).

These findings illustrate the large differences which exist between the sub-categories of the non-standard and standard groups. The Ordinary Level students have a high percentage of students who are ‘at risk’ of failing service mathematics and unlike the mature students and those who engaged in previous study they do not improve over the course of the semester as they have the highest failure rate in service mathematics of all 5 sub-categories (see figure 4.8). The Higher Level students on the other hand perform very well in the diagnostic test and maintain this high performance in service mathematics.

<table>
<thead>
<tr>
<th>Student Type</th>
<th>Mean Diagnostic Test Result (as a percentage)</th>
<th>Mean End of Semester Result</th>
<th>Percentage who Passed Service Mathematics</th>
<th>Total Number of Students in Each Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-standard Students</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mature Students</td>
<td>28.3 (16.4) n=34</td>
<td>54.9 (27.9) n=35</td>
<td>71.4%</td>
<td>40</td>
</tr>
<tr>
<td>Previous Degree,</td>
<td>27.5 (8.9) n=4</td>
<td>61.7 (17.4) n=6</td>
<td>100.0%</td>
<td>6</td>
</tr>
<tr>
<td>Diploma, Certificate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>International Students</td>
<td>52.9 (30.4) n=7</td>
<td>61.9 (22.8) n=9</td>
<td>77.8%</td>
<td>10</td>
</tr>
<tr>
<td>Standard Students</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ordinary Level</td>
<td>44.7 (11.8) n=308</td>
<td>43.5 (18.3) n=353</td>
<td>59.8%</td>
<td>382</td>
</tr>
<tr>
<td>Higher Level</td>
<td>65.7 (13.5) n=187</td>
<td>70.0 (18.3) n=230</td>
<td>92.6%</td>
<td>239</td>
</tr>
</tbody>
</table>

Table 4.6 Performance of different sub-categories of standard and non-standard students in the diagnostic test and end of semester examination for Science and Technology students in 2008.
Figure 4.7 Science and Technology students’ diagnostic test performance in 2008 by sub-category.

Figure 4.8 Science and Technology students’ service mathematics performance in 2008 by sub-category.

Note: 75/382 Ordinary Level students and 52/239 Higher Level students did not sit the diagnostic test. 29/382 Ordinary Level students and 9/239 Higher Level students did not sit their service mathematics examination.
Conclusion
Ordinary Level students’ lack of improvement in their performance between the diagnostic test and the service mathematics examination, which is in contrast to all other sub-categories of students, begs the following questions: What is the difference between Ordinary Level students and the other sub-categories of students in terms of mathematical ability, learning styles and motivation? Is there a need to consider using 5 sub-categories instead of non-standard and standard students? These questions will be discussed in section 4.2.4.3 and 4.2.4.4 which follow.

4.2.4.3 Consideration into Rearranging the Groupings of Standard and Non-Standard Students
The 3 sub-categories of non-standard students have several things in common. The mature students and those who have engaged in previous study perform to a similar low mean standard in the diagnostic test with similar proportions of ‘at risk’ students within the two sub-categories. All three sub-categories of non-standard students perform to a relatively similar mean standard in service mathematics. However there are several differences amongst the groups. The international students perform to a much higher mean standard in the diagnostic test with a smaller proportion of students being considered to be ‘at risk’. In addition to this the mature students have a very large spread of results in service mathematics resulting in a high failure rate of 28.6% (n = 10) while all of the students who have engaged in previous study were successful in service mathematics. Upon considering all of these similarities and differences, it was considered more useful to group and analyse all 3 sub-categories of non-standard students separately.

Upon analysing the standard students’ sub-categories, it could be seen that the Higher Level students perform to a high standard mathematically across the two examinations. The Higher Level students have a low failure rate of 7.4%. In contrast to this, the Ordinary Level students have the highest failure rate of 40.2% in service mathematics. They do not seem to behave like any of the other sub-categories. The Ordinary Level students outperform the mature students in the diagnostic test yet they have the highest failure rate in service mathematics of all 5 sub-categories. Due to the differing mathematical performances of Higher Level and Ordinary Level students, it may be appropriate to consider analysing them separately for future statistical
analysis also. The possible reasons for some Ordinary Level students performing as they do are discussed next in section 4.2.4.4.

4.2.4.4 Why Do Some Ordinary Level Students Underperform?
Leaving Certificate points in mathematics have been found to have a clear relationship with performance and progression in higher education (Hyland 2011). As would be expected the Higher Level service mathematics students in UL in 2008 have higher mean points in Leaving Certificate mathematics on entry to UL when compared to Ordinary Level students suggesting that they are more likely to perform well and progress in higher education when compared to their Ordinary Level counterparts. Even when this relationship is taken into account however the Ordinary Level students underperform in service mathematics when the relationship between diagnostic test performance and service mathematics performance is considered in the case of all other sub-categories of students (see table 4.6) (When the author uses the term underperformance in relation to Ordinary Level students she refers to their unexpected underperformance when their Leaving Certificate mathematics points and diagnostic test results are considered). Why then do some Ordinary Level students perform as they do? There are several possible reasons for this underperformance which are outlined next:

1. **Poor Teaching at Second Level Education**
   - Research has shown that students learn more when they engage with skilled and experienced teachers who know what and how to teach (Darling-Hammond 2000). A study conducted by Ni Ríordáin and Hannigan (2011) however, which focussed on out-of-field mathematics teachers, found that 48% of Irish second level teachers teaching mathematics are unqualified. These unqualified teachers were also found to be predominately teaching Junior Certificate classes and Ordinary Level students.
   - Ordinary Level students therefore, who may have found themselves being taught by an ill-prepared teacher are likely to have suffered in terms of their mathematical achievement and consequently their preparedness for third level mathematics education.

2. **Learning Style**
   - National and international studies suggest that Irish mathematics classrooms are largely traditional in nature (O’Murchu and O’Sullivan 1982; O’Donoghue 1999). One of the effects of this is that second level students in Ireland tend to rely on surface learning
approaches to mathematics such as using rote memory as an alternative to understanding (Hourigan and O’Donoghue 2007; Hyland 2011).

- Hourigan and O’Donoghue (2007) found that it was considered acceptable for Ordinary Level students in Irish second level mathematics classrooms to fail to recall basic elements of previously taught material. When such incidents did occur they found that no probing from the teacher to assist the student in recalling the material took place but rather the teacher was happy to re-administer the material to the students without question. Such findings suggest that surface learning approaches to mathematics are common practice in Ordinary Level Leaving Certificate classrooms and that the understanding of material is not a focus.

- Additionally surface learning approaches are negatively associated with academic achievement (Busato et al 2000). If the Ordinary Level students adopt these learning styles as the literature suggests, this may have negatively impacted on their service mathematics performance.

3. Perceived Ability

- Students’ perceptions about their ability to perform a task are believed to affect their motivation to learn (Pintrich et al 1993). Greene et al (1999) found a positive relationship between perceived ability in mathematics and mathematics achievement. Low perceived ability in a subject, level of past mathematical achievement and negative school mathematical experiences have also been found to be associated with mathematics anxiety and poor motivation (Eccles and Wigfield 1995).

- If Ordinary Level students have a poor perception of their mathematics ability and they perceive the mathematical demands made upon them in service mathematics to be too much it is likely to have a negative effect, not only on their mathematical achievement, but also their future motivation towards and persistence with learning mathematics. It is important to acknowledge at this point that poor perceived ability could also negatively affect Higher Level students’ mathematical performance.

4. Adjustment to Third Level Education

- The academic self-efficacy and optimism of first year students have been found to be strongly related to academic performance and ability to adjust to higher education (Chemers et al 2001). First year students however who are under external pressure due to
socialising and part-time work have been found to be associated with lower average grades (Trockel et al 2000). More specifically, the factors found to have the strongest negative influence on the academic performance of first year students were social-life (43.1%), part-time job (16.1%) and alcohol (15.1%) (Kuol et al 2006). The potential influence of external pressures, such as part-time work and socialising, on students’ academic performances is of course not confined to Ordinary Level students only.

- The poor attendance of the Ordinary Level students compared to Higher Level students in the intervention stage of this research (see chapter 6) suggests that they are not as involved in their mathematics studies (Ordinary Level: 24.1% attend no tutorials, 27.6% attended 5 or less and 48.3% attended 6 or more/Higher Level: 18.2% attended no tutorials and 9.1% attended 5 or less with 72.7% attending 6 or more).

4.2.4.5 Conclusion: Standard and Non-Standard Students’ Performance in the Diagnostic Test and Service Mathematics: Further Analysis

The three sub-categories of non-standard students were found to be varying in terms of their mathematical performances over time and the possible influences/reasons for these patterns of performance. Any further investigations involving these students should therefore take place using their sub-categories.

The sub-categories of standard students were also found to differ from each other in terms of their mathematical performances. The Ordinary Level students stood out in particular as their pattern of performance between the diagnostic test and their service mathematics examination did not follow the same trend as any of the other sub-categories i.e. they did not demonstrate any improvement in their mathematical performance over time. Although the discussed possible contributing factors to Ordinary Level students’ underperformance are just suggestions from literature and not based on research specific to these UL students, they may give an insight into the possible factors at play as well as the complexity of such an investigation. It is clear that the underperformance of Ordinary Level students in service mathematics requires further research in third level mathematics education in Ireland. The large differences in the failure rates of Higher Level and Ordinary Level students suggest that they too should not be grouped together into one category i.e. standard students. Issues surrounding the grouping of Higher and Ordinary Level students are examined further in chapter 5.
4.2.5 Leaving Certificate Level on Entrance to UL
From table 4.7, it can be seen that both Science and Technology degrees in UL attracted more students in 2008 than in 1998. Although the number of students in these degrees with Higher Level mathematics has not changed over time, the proportion of students with Higher Level mathematics has changed. When the two cohorts combined are examined, the decline in the percentage of students with Higher Level mathematics is from 46.7% in 1998 to 35.3% in 2008. The extra students attracted to these degrees are students with Ordinary Level mathematics (from 266 to 382 students) and the sub-categories of non-standard students (from 4 to 56 students). Within Technology mathematics there has been a drop of 7.8 percentage points of the total cohort entering with Higher Level mathematics and a much larger drop of 17.4 percentage points between 1998 and 2008 of Science mathematics students entering with Higher Level mathematics (see table 4.7). The bigger decrease in the proportion of students in Science mathematics with Higher Level mathematics, largely explains why Science students have seen bigger declines over time in diagnostic test performance compared to Technology students (Faulkner et al 2010). A similar change in student intake also caused declining mathematics standards to occur in Coventry University (Lawson 2003). The findings of Barry & Chapman (2007) also state that performance in mathematics in third level institutions has been shown to be better when students have Higher Level mathematics as pre-requisite knowledge.

Hourigan and O’Donoghue (2007) described the changing profile of third level mathematics students in Ireland as “the single most detrimental factor leading to under-preparedness within the mathematics-intensive courses in tertiary level” (Hourigan and O’Donoghue 2007, p. 463). A more in-depth investigation into UL students’ performance in the diagnostic test by Leaving Certificate mathematics grade is discussed in section 4.3.2.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohort</td>
<td>Whole Cohort</td>
<td>Technology</td>
<td>Science</td>
<td>Whole Cohort</td>
<td>Technology</td>
<td>Science</td>
</tr>
<tr>
<td>% doing HL</td>
<td>237 (46.7%)</td>
<td>239 (35.3%)</td>
<td>125 (41.0%)</td>
<td>124 (33.2%)</td>
<td>112 (55.4%)</td>
<td>115 (38.0%)</td>
</tr>
<tr>
<td>% doing OL</td>
<td>266 (52.5%)</td>
<td>382 (56.4%)</td>
<td>179 (58.7%)</td>
<td>215 (57.5%)</td>
<td>87 (43.1%)</td>
<td>167 (55.1%)</td>
</tr>
<tr>
<td>Non-Standard Students</td>
<td>4 (0.8%)</td>
<td>56 (8.3%)</td>
<td>1 (0.3%)</td>
<td>35 (9.4%)</td>
<td>3 (1.5%)</td>
<td>21 (6.9%)</td>
</tr>
</tbody>
</table>

Table 4.7 Percentages and numbers of students entering UL with higher level and ordinary level Leaving Certificate mathematics and non-standard students.
4.2.6 Increase in ‘at risk’ Service Mathematics Students

The percentage of students attending their first lecture and taking the diagnostic test has declined for both Technological and Science cohorts since 1998 when 100% of students registered for each module sat the test. For both cohorts in 2008 only 80% (approximately) of the registered students were present on the first day of their service mathematics lecture in first year (see table 4.8). This finding has been investigated further and details are found in chapter 5.

There has been an increase in the percentage of ‘at risk’ students i.e. students who receive 19/40 or below in the diagnostic test. A decrease in mathematical competency levels such as this, in the English education system, has been suggested by Hunt and Lawson (1996) to be partially due to heavier reliance in schools on calculators or possibly a shift in emphasis of A-level mathematics towards topics not covered on the diagnostic test. Although the suggestions of Hunt and Lawson (1996) may hold in an Irish context also, it is thought that the increase in ‘at risk’ students may be partially explained by the decrease in the proportion of students taking Higher Level Leaving Certificate mathematics, as highlighted in the previous section. This will be discussed in greater detail in section 4.3 of this chapter.

<table>
<thead>
<tr>
<th>Year</th>
<th>Technology mathematics</th>
<th>Science mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of total taking the test</td>
<td>305/305 (100.0%)</td>
<td>303/374 (81.0%)</td>
</tr>
<tr>
<td>% ‘at risk’</td>
<td>100 (32.8%)</td>
<td>140 (46.2%)</td>
</tr>
</tbody>
</table>

Table 4.8 Breakdown of students taking the diagnostic test and ‘at risk’ of failing service mathematics.
4.2.7 Conclusion

The student profile in Science and Technology mathematics has changed greatly in terms of gender, mathematics Leaving Certificate grade on entrance to UL, the proportion of standard and non-standard students, the percentage of ‘at risk’ students and the percentage of students taking the diagnostic test between 1998 and 2008 (see table 4.9). It is thought that the change in student profile over time has been influential on the increase in the percentage of ‘at risk’ students over time. A detailed investigation into the decline in diagnostic test performance in Science and Technology mathematics and an examination of beginning undergraduates’ performance by Leaving Certificate mathematics grade is carried out in section 4.3. The investigations which follow aim to offer specific explanations for the decline in diagnostic test performance over time with a view to being able to better profile the ‘at risk’ students in the two service mathematics groups (details of which are outlined in section 4.4).

<table>
<thead>
<tr>
<th></th>
<th>Technology mathematics</th>
<th>Science mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Year</strong></td>
<td><strong>1998</strong></td>
<td><strong>2008</strong></td>
</tr>
<tr>
<td><strong>Males</strong></td>
<td>260 (85.2%)</td>
<td>328 (87.7%)</td>
</tr>
<tr>
<td><strong>Females</strong></td>
<td>45 (14.8%)</td>
<td>46 (12.3%)</td>
</tr>
<tr>
<td><strong>% of total taking the test</strong></td>
<td>305/305 (100%)</td>
<td>303/374 (81%)</td>
</tr>
<tr>
<td><strong>% ‘at risk’</strong></td>
<td>100 (32.8%)</td>
<td>140 (46.2%)</td>
</tr>
<tr>
<td><strong>% doing HL</strong></td>
<td>125 (41.0%)</td>
<td>124 (33.2%)</td>
</tr>
<tr>
<td><strong>% doing OL</strong></td>
<td>179 (58.7%)</td>
<td>215 (57.5%)</td>
</tr>
<tr>
<td><strong>Standard Students</strong></td>
<td>304 (99.7%)</td>
<td>339 (90.6%)</td>
</tr>
<tr>
<td><strong>Non-Standard Students</strong></td>
<td>1 (0.3%)</td>
<td>35 (9.4%)</td>
</tr>
</tbody>
</table>

Table 4.9 Profile of students in Technological and Science mathematics (1998 and 2008).
4.3 An Investigation into Performance in the Diagnostic Test Over Time

There has been no change in the minimum mathematics entry requirement of standard entrants to Science and Technological service mathematics courses from 1998 to 2008. So what has caused the decline in the diagnostic test scores over time? The aim of this section of the research is to investigate the changing profile of third level mathematics students and to see if performance of students in the diagnostic test with the same Leaving Certificate mathematics grade has remained the same over time or is there evidence of grade inflation in the UL database?

The performance in the diagnostic test of both Science and Technological students in the core mathematical areas of arithmetic and algebra will also be investigated over the time period 1998-2008. Questions on arithmetic and algebra represent over half of the questions on the diagnostic test.

The most commonly occurring grades in the dataset used for this investigation (Dataset 1) are OLA2 (13.5% of students), OLB1 (11.9%) and OLA1 (11.8%). The most commonly occurring Higher Level Leaving Certificate mathematics grade is HLC1 (6.3%) and the minimum mathematics entry requirement for most courses is OLB3 (5.5%). The investigation into whether the UL database presents evidence of grade inflation will therefore focus on these five grades.

4.3.1 Decline in Diagnostic Test Results over Time

In 1998 the mean diagnostic test score for 507 students was 59.3 (SD = 16.5). In 2008 this declined to 50.8 (SD = 17.7) for the 540 students who sat the test. The mean diagnostic score is expressed as a percentage of correct answers out of the 40 questions. This represents a decline of 14\% in the mean diagnostic test score from the 1998 baseline. The difference between these means is statistically significant (p < 0.001). There is also more variation in the diagnostic test scores in 2008 compared to 1998. The difference between the variances however is not statistically significant.

The decline is larger for Science students (see table 4.10). In 1998, 202 Science mathematics students had a mean diagnostic test score of 63.1 (SD = 16.3). This had declined to 50.5 (SD = 17.9) for the 237 Science students who sat the test in 2008. A statistically significant difference exists between these means (p < 0.001) and the difference represents a decline of 20\% in the
mean diagnostic test score from the 1998 baseline. There is more variation in the diagnostic test
scores for Science students in 2008, however the variances were not found to be statistically
significantly different. Technological students also showed a decline in mean diagnostic test
performance, however it was not as dramatic as was the case for Science students. The mean
diagnostic test score was 56.8 (SD = 16.2) in 1998 for 305 students and 51.2 (SD = 17.3) in 2008
for 303 students, the difference between the means is also statistically significant (P < 0.001).
Again no statistically significant differences were found between the variances.
Science students were traditionally of a higher mathematical competency level entering third
level compared to Technological students, however both groups perform to a very similar mean
standard in 2008. Science mathematics students have a minimum entry requirement of an
Ordinary Level B3 compared to Technological students who have a lower minimum entry
requirement of an Ordinary Level C3. The overall decline over time is matched with a
fluctuation in variability of performances (see table 4.10). Although the variability fluctuates for
both cohorts of students the overall trend for the Science cohorts is that variability increases over
time. To summarise, in 2008 Science students are performing further from the mean value, above
and below, in the diagnostic test than they were in 1998. This increase in variability may be due
in part to the addition of new degree programs required to take Science mathematics over the
three years being examined (see section 4.4.6 (f) and Table 4.29).
The challenge to lecturers and other staff, particularly those involved in Science mathematics
teaching, to determine students’ prior knowledge depending on their performance in the
diagnostic test is made more difficult by the increase in the spread of results. This challenge has
been previously recognised by mathematics educators in Ireland when they state that it has
become “impossible to identify topics and prerequisite knowledge that all students are
guaranteed to be familiar with” (Hourigan and O’Donoghue 2007, p. 463). Decisions regarding
issues such as the pace of lectures are also more complicated. The trend, particularly in the last 4
years (2005-2008), seems to suggest that variation of performances from the mean may continue
to increase in years to come.
Table 4.10 Mean diagnostic test score (standard deviation) and sample size by year and group (Science and Technology).

4.3.2 Investigation into Students’ Performance over Time by Leaving Certificate Grade

Analysis of the most commonly occurring grades revealed that there is little change in performance in the diagnostic test, on average, for the five grades between 1998 and 2008 (Faulkner et al. 2010). There is also no crossover between any of the Leaving Certificate mathematics grades in terms of mean diagnostic test performance. This suggests that the diagnostic test does well in differentiating between students with different Leaving Certificate mathematics grades.

Although some slight changes in performance over time for all grades can be seen in figure 4.9, the overall changes in performance are extremely small and are not statistically significant. One exception however occurs in the case of OA1 grade students whose means were found to be statistically significantly different between 1998 and 2008 (P = 0.03). The mean for OA1 students went from 56.3% in 1998 to 52.8% in 2008. This equates to just over one fewer correct questions in the diagnostic test in 2008 (see table 4.11).

More variability exists for each Leaving Certificate mathematics grades in 2008 than was the case in 1998. This slight increase in the spread of results per Leaving Certificate grade can be seen in the case of OB1 students for example when the co-efficient of variation, which allows comparisons of variability across groups with different means, for this grade increased from 17.9% in 1998 to 24.8% in 2008. The increase in the coefficient of variation amongst HC1 students is also worth noting as it went from 15.3% in 1998 to 21.9% in 2008. Despite the increase in variation over time no statistically significant difference in variances were found for all grades examined.
Figure 4.9 shows that there is no crossover between performance in the diagnostic test for the Leaving Certificate mathematics grades over time highlighting the ability of both the diagnostic test and the Leaving Certificate mathematics examination to measure and differentiate between the mathematical proficiency of the students taking them. Figure 4.9 does not provide evidence that significant grade inflation is occurring within the Leaving Certificate mathematics examination and so this is unlikely to be a contributor to the declining mathematical standards of university entrants over time as highlighted by the diagnostic test performance in UL. The Network for Irish Educational Standards came to a conflicting conclusion however when they reported that significant grade inflation in the Leaving Certificate examinations between 1992 and 2006 had occurred. Although O’Grady (2009) puts forth a stronger case for grade inflation across the nine other most commonly taken Leaving Certificate subjects and admits that “the case for grade inflation...is weakest in mathematics with no obvious pattern of grade increases since the early nineties”, he still maintains that there is a strong argument which may cause one to doubt the validity of the increase in A and B grades in Higher Level Leaving Certificate mathematics (p.3). Figure 4.9 shows very little change, on average, in the mathematical ability of Leaving Certificate students entering UL with the grades; HC1, OA1, OA2, OB1 and OB3 in terms of their diagnostic test performance. This suggests that the Leaving Certificate mathematics examination is awarding the same grade for a very similar level of mathematical ability (as measured by the diagnostic test) in 2008 as it was in 1998. Grade inflation was not investigated within the UL dataset in terms of Higher Level A and B Leaving Certificate mathematics grades due to the small number of students with these grades in the UL dataset in 2008 (Higher Level A grades n=6, Higher Level B grades n=73 for Technology and Science cohorts combined in 2008). The possible consequences of this are highlighted in section 4.5.

In summary students’ performance in the diagnostic test according to Leaving Certificate grades over time has remained largely unchanged; the difference between performances for each grade between 1998 and 2008 is not statistically significant with the exception of the mean change for OA1 students. In general, the higher the Leaving Certificate grade a student enters UL service mathematics with the better the performance in the diagnostic test. In 2008, the mean diagnostic test score was 65.7 (SD = 13.5) for those with all Higher Level mathematics grades combined, 44.7 (SD = 11.8) for those with Ordinary Level mathematics grades and 30.7 (SD = 21.0) for those in the non-standard category. The significant decline in diagnostic test performance over
time can therefore be seen to have been inevitable due to the increase in admittance of Ordinary Level Leaving Certificate students and non-standard students by the university.

**Figure 4.9** Mean diagnostic test score (expressed as a percentage of correct answers out of 40 questions) from 1998 to 2008 for all students with grades HC1, OA1, OA2, OB1 and OB3.

<table>
<thead>
<tr>
<th>Year</th>
<th>HC1</th>
<th>OA1</th>
<th>OA2</th>
<th>OB1</th>
<th>OB3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>68.1 (10.4)</td>
<td>56.3 (9.3)</td>
<td>49.5 (11.2)</td>
<td>45.3 (8.1)</td>
<td>38.8 (6.5)</td>
</tr>
<tr>
<td>n=35</td>
<td>n=92</td>
<td>n=66</td>
<td>n=48</td>
<td>n=15</td>
<td>n=10</td>
</tr>
<tr>
<td>1999</td>
<td>67.9 (9.7)</td>
<td>57.4 (7.9)</td>
<td>48.5 (8.3)</td>
<td>47.2 (10.3)</td>
<td>38.0 (7.0)</td>
</tr>
<tr>
<td>n=36</td>
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<td>n=58</td>
<td>n=35</td>
<td>n=48</td>
<td>n=29</td>
</tr>
<tr>
<td>2000</td>
<td>67.1 (9.8)</td>
<td>55.8 (9.4)</td>
<td>48.3 (9.7)</td>
<td>43.1 (7.7)</td>
<td>40.6 (7.8)</td>
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<td>2001</td>
<td>61.8 (11.8)</td>
<td>57.4 (10.0)</td>
<td>48.5 (9.7)</td>
<td>44.5 (9.5)</td>
<td>40.6 (8.4)</td>
</tr>
<tr>
<td>n=15</td>
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<td>n=49</td>
<td>n=41</td>
<td>n=41</td>
<td>n=16</td>
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<td>2002</td>
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<td>50.0 (8.7)</td>
<td>47.0 (10.8)</td>
<td>40.2 (10.9)</td>
</tr>
<tr>
<td>n=23</td>
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<td>n=52</td>
<td>n=44</td>
<td>n=44</td>
<td>n=20</td>
</tr>
<tr>
<td>2003</td>
<td>66.6 (11.9)</td>
<td>55.0 (12.0)</td>
<td>52.3 (10.3)</td>
<td>47.2 (9.6)</td>
<td>39.3 (9.4)</td>
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<td>n=23</td>
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<td>n=41</td>
<td>n=44</td>
<td>n=50</td>
<td>n=27</td>
</tr>
<tr>
<td>2004</td>
<td>70.4 (10.2)</td>
<td>55.0 (9.9)</td>
<td>53.8 (8.2)</td>
<td>46.5 (10.9)</td>
<td>38.2 (6.8)</td>
</tr>
<tr>
<td>n=23</td>
<td>n=44</td>
<td>n=52</td>
<td>n=52</td>
<td>n=52</td>
<td>n=20</td>
</tr>
<tr>
<td>2005</td>
<td>65.9 (8.8)</td>
<td>53.6 (11.7)</td>
<td>47.9 (8.8)</td>
<td>43.3 (11.2)</td>
<td>37.1 (11.8)</td>
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<td>n=60</td>
<td>n=70</td>
<td>n=62</td>
<td>n=76</td>
<td>n=28</td>
</tr>
<tr>
<td>2006</td>
<td>64.5 (11.1)</td>
<td>54.8 (9.8)</td>
<td>48.9 (10.7)</td>
<td>42.8 (10.3)</td>
<td>39.3 (10.7)</td>
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<td>n=65</td>
<td>n=79</td>
<td>n=94</td>
<td>n=76</td>
<td>n=30</td>
</tr>
<tr>
<td>2007</td>
<td>64.9 (13.2)</td>
<td>52.5 (11.0)</td>
<td>50.4 (11.0)</td>
<td>45.1 (10.5)</td>
<td>38.4 (10.6)</td>
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<td>n=76</td>
<td>n=76</td>
<td>n=70</td>
<td>n=39</td>
</tr>
<tr>
<td>2008</td>
<td>64.9 (14.2)</td>
<td>52.8 (9.6)</td>
<td>48.5 (11.2)</td>
<td>42.4 (10.5)</td>
<td>39.4 (7.7)</td>
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<td>n=59</td>
<td>n=84</td>
<td>n=68</td>
<td>n=84</td>
<td>n=36</td>
</tr>
</tbody>
</table>

**Table 4.11** Mean diagnostic test score (standard deviation) and sample size by year and Leaving Certificate grade.
4.3.3 Investigation into Performance in Arithmetic and Algebra Sections of the Diagnostic Test

There are nine different topics covered on the UL diagnostic test. Combined arithmetic and algebra make up over half (21/40) of the questions on the test so they have been chosen for separate analyses in this section. Another reason for the separate analysis of these topics is that they are generic mathematics skills which are crucial to each other and to all other areas of mathematics covered in third level mathematics. Generally successfully learning algebraic skills is dependent on having already learnt arithmetic skills. Problems have been shown to occur when making the transition from arithmetic to algebra by researchers of mathematics education (Kieran 1992). A successful transition however to an understanding of algebraic concepts is hugely important as it provides proven benefits to one’s future success with mathematics (Gamoran and Hannigan 2000).

4.3.3.1 Arithmetic

A small decline in arithmetic performance was found for both Science and Technology students between 1998 and 2008. Similar to the analysis of overall diagnostic test performance the decline is greater in the case of the Science mathematics students. In 1998 Science students answered on average 66.6% of the questions correctly compared to 54.3% in 2008 (see table 4.12). This represents a drop of 12.3 percentage points of mean correctly answered arithmetic questions. This change in mean values over time was found to be statistically significant (p < 0.001). The decline in arithmetic performance for Technology mathematics students was from 62.4% in 1998 to 55.9% in 2008, which represents a drop of 6.5 percentage points of mean correctly answered questions (see table 4.12). The difference in mean values in the case of Technology students was also found to be statistically significant (p = 0.018).

Along with both cohorts experiencing significant declines in mean percentage of questions answered correctly, they also experienced an increase in the variability of results in arithmetic. For both cohorts there is a statistically significant difference between the variances in 1998 and 2008 (p = 0.004 for Science mathematics, p = 0.031 for Technology mathematics).

An examination of arithmetic performance over time by Leaving Certificate mathematics grades showed that there was no crossover of results between grades. There is however one exception in 2006, when Ordinary Level B3 students outperformed Ordinary Level B1 students in this section of the diagnostic test. Science mathematics students with HC1 grades in 2008 have a mean value
of 61.0% which is significantly different (p = 0.033) to the mean performance in 1998 of 71.2%. Other than this one exception, there are no other statistically significant mean differences in mean arithmetic performance over time by Leaving Certificate grades for both cohorts (see figure 4.10).

![Graph showing mean arithmetic score (expressed as a percentage of correct answers out of 13 questions) from 1998 to 2008 for all students with grades HC1, OA1, OA2, OB1 and OB3 in Science and Technological mathematics.]

Figure 4.10 Mean arithmetic score (expressed as a percentage of correct answers out of 13 questions) from 1998 to 2008 for all students with grades HC1, OA1, OA2, OB1 and OB3 in Science and Technological mathematics.

<table>
<thead>
<tr>
<th>Year</th>
<th>Science mathematics</th>
<th>Technology mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HC1</td>
<td>OA1</td>
</tr>
<tr>
<td></td>
<td>OA2</td>
<td>OB1</td>
</tr>
<tr>
<td></td>
<td>OB3</td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>66.6 (15.0)</td>
<td>62.4 (14.9)</td>
</tr>
<tr>
<td></td>
<td>n=202</td>
<td>n=305</td>
</tr>
<tr>
<td>1999</td>
<td>63.1 (14.5)</td>
<td>62.5 (15.2)</td>
</tr>
<tr>
<td></td>
<td>n=189</td>
<td>n=267</td>
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<tr>
<td>2000</td>
<td>65.0 (15.3)</td>
<td>60.4 (13.7)</td>
</tr>
<tr>
<td></td>
<td>n=193</td>
<td>n=304</td>
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<tr>
<td>2001</td>
<td>61.0 (17.4)</td>
<td>59.6 (14.7)</td>
</tr>
<tr>
<td></td>
<td>n=134</td>
<td>n=211</td>
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<tr>
<td>2002</td>
<td>62.7 (16.0)</td>
<td>61.8 (14.8)</td>
</tr>
<tr>
<td></td>
<td>n=141</td>
<td>n=212</td>
</tr>
<tr>
<td>2003</td>
<td>62.2 (14.3)</td>
<td>57.5 (16.5)</td>
</tr>
<tr>
<td></td>
<td>n=150</td>
<td>n=187</td>
</tr>
<tr>
<td>2004</td>
<td>62.9 (15.5)</td>
<td>58.6 (15.0)</td>
</tr>
<tr>
<td></td>
<td>n=181</td>
<td>n=225</td>
</tr>
<tr>
<td>2005</td>
<td>59.6 (17.3)</td>
<td>57.4 (15.3)</td>
</tr>
<tr>
<td></td>
<td>n=227</td>
<td>n=270</td>
</tr>
<tr>
<td>2006</td>
<td>56.6 (17.3)</td>
<td>55.6 (17.6)</td>
</tr>
<tr>
<td></td>
<td>n=266</td>
<td>n=360</td>
</tr>
<tr>
<td>2007</td>
<td>55.7 (17.3)</td>
<td>59.4 (15.5)</td>
</tr>
<tr>
<td></td>
<td>n=263</td>
<td>n=317</td>
</tr>
<tr>
<td>2008</td>
<td>54.3 (18.6)</td>
<td>55.9 (16.8)</td>
</tr>
<tr>
<td></td>
<td>n=237</td>
<td>n=303</td>
</tr>
</tbody>
</table>

Table 4.12 Mean arithmetic score (standard deviation) and sample size by year and group (Science and Technology).
4.3.3.2 Algebra

Algebra performance in the diagnostic test is more varied over time for all grades than was the case for arithmetic (see figure 4.11). Again Science mathematics students’ performance decreased more over time compared to the Technological students. In 1998, Science mathematics students had a mean percentage of correctly answered questions of 74.6% which declined to 58.3% in 2008. This represents a decline over time of 16.3 percentage points. The difference between these mean values was found to be statistically significant (P < 0.001). Technology students saw a decline from 63.8% in 1998 to 59.4% in 2008. This represents a decline over time of 4.4 percentage points which is also a statistically significant difference (p = 0.026).

An increase in the variability, as measured by the coefficient of variation, of results over time occurred for both groups of students. For Science students an increase of 14.9 percentage points in the variability of test scores occurred between 1998 and 2008 and an increase of 4.9 percentage points occurred for Technological students (see table 4.13). A statistically significant difference was found for the variance of Science mathematics results between 1998 and 2008 (p = 0.006), however the change in variance for Technology students was not found to be statistically significant.

As previously outlined, there is more fluctuation over time and some crossover in performance between grades in terms of performance in the algebra section of the diagnostic test. For example in 1999, OB1 grade students outperformed OA2 grade students and in 2004 OA2 students outperformed OA1 grade students (see figure 4.11). Although performance varied over time in terms of algebra performance by grade there was no statistically significant difference between performances by grade in 1998 compared to 2008.
Figure 4.11 Mean algebra (expressed as a percentage of correct answers out of 8 questions) from 1998 to 2008 for all students with grades HC1, OA1, OA2, OB1 and OB3.

<table>
<thead>
<tr>
<th>Year</th>
<th>Science mathematics</th>
<th>Technology mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>74.6 (20.5)</td>
<td>63.8 (23.6)</td>
</tr>
<tr>
<td></td>
<td>n=202</td>
<td>n=305</td>
</tr>
<tr>
<td>1999</td>
<td>71.4 (22.2)</td>
<td>66.3 (23.1)</td>
</tr>
<tr>
<td></td>
<td>n=189</td>
<td>n=267</td>
</tr>
<tr>
<td>2000</td>
<td>68.6 (22.8)</td>
<td>61.2 (21.7)</td>
</tr>
<tr>
<td></td>
<td>n=193</td>
<td>n=304</td>
</tr>
<tr>
<td>2001</td>
<td>67.3 (24.1)</td>
<td>61.0 (24.3)</td>
</tr>
<tr>
<td></td>
<td>n=134</td>
<td>n=211</td>
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<tr>
<td>2002</td>
<td>65.0 (24.1)</td>
<td>60.7 (22.4)</td>
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<td>n=141</td>
<td>n=212</td>
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<td>2003</td>
<td>65.8 (22.0)</td>
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<td>2004</td>
<td>69.3 (21.3)</td>
<td>62.0 (24.1)</td>
</tr>
<tr>
<td></td>
<td>n=181</td>
<td>n=225</td>
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<tr>
<td>2005</td>
<td>61.0 (21.8)</td>
<td>56.8 (23.4)</td>
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<td>n=227</td>
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<td>2006</td>
<td>56.1 (26.1)</td>
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<td>2007</td>
<td>58.1 (24.5)</td>
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<td></td>
<td>n=263</td>
<td>n=317</td>
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<tr>
<td>2008</td>
<td>58.3 (24.7)</td>
<td>59.4 (24.8)</td>
</tr>
<tr>
<td></td>
<td>n=237</td>
<td>n=303</td>
</tr>
</tbody>
</table>

Table 4.13 Mean algebra score (standard deviation) and sample size by year and group (Science and Technology).

4.3.3.3 Conclusion

There were no significant differences in mean performance between 1998 and 2008 for diagnostic test performance according to the five grades being analysed. A significant decline over time did occur however for Science and Technological mathematics students in UL in the areas of arithmetic and algebra along with an increase in the variability of results. A decline in performance in these particular areas highlights that students in 2008 do not have as good a knowledge of algebra and arithmetic as they did in 1998. This is likely to have negative effects
on students’ future achievement in mathematics and therefore their ability to successful complete service mathematics courses in UL.

The examination of the change in student profile in UL between 1998 and 2008 allowed for a detailed documentation of the changes in competency levels over time, an examination into the possibility of evidence of grade inflation in the Leaving Certificate mathematics grades examined and provided insights into students’ competency levels in particular areas of the diagnostic test over time. The decline in students’ diagnostic test performance has been found to be partially attributed to the changing profile of service mathematics students between 1998 and 2008. The examination of the UL database does not provide evidence that significant grade inflation has occurred in the Leaving Certificate grades investigated over the period being examined. This information informed the next major findings in this chapter which were concerned with profiling ‘at risk’ service mathematics students in UL. The next sections of this chapter detail the profile of students in Technology and Science mathematics in 2006-2008 which gives further insight into the common characteristics held by those who passed and those who failed service mathematics during those years.

Further examination is now needed to determine what characteristics this growing group of ‘at risk’ students may have which may be negatively influencing their progression in third level education. This examination is detailed in section 4.4 of this chapter.
4.4 Profiling Students Who Fail Service Mathematics

4.4.1 Introduction
In this section the characteristics of Technology and Science students, between the years 2006 and 2008, are outlined in terms of performance in service mathematics examinations. The variables which are compared are as follows: the percentage of students who failed service mathematics by year (2006, 2007 and 2008), the percentage of males and females who failed over the three year period combined, the percentage of students who failed and had Higher and Ordinary Level mathematics as pre-requisite knowledge, the percentage of students who failed who had specific Leaving Certificate mathematics grades as pre-requisite knowledge, the percentage of students who failed within each non-standard sub-category and a breakdown of the percentage of students who failed within different degree programmes. Aspects of diagnostic test performance against performance in service mathematics such as mean diagnostic test result, performance in the arithmetic and algebra sections of the test, an examination into the performance of students who were labelled ‘at risk’ and finally the percentage of students who failed and sat the diagnostic test compared to the percentage who failed and did not sit the diagnostic test are also examined.

4.4.2 Rationale for Analyses of Students within the Years 2006, 2007 and 2008.
The years 2006, 2007 and 2008 were chosen for this analysis as within this time period the lecturer of Technology and Science mathematics remained the same. The student body has remained constant in terms of students’ mean CAO overall points and mean mathematics points during this time period (see section 4.4.4(a) and 4.4.6(a)). The minimum Leaving Certificate mathematics grade required for entry remained the same also; an Ordinary Level C3 for Technology mathematics and an Ordinary Level B3 for Science mathematics. These consistencies therefore allowed for an examination of three quite similar conditions enabling a more powerful analysis of the characteristics which students possess that are associated with success in service mathematics and those which are associated with failure.
4.4.3 Aim of Profiling Technology and Science Mathematics Students Who Fail 2006-2008
The aim of this section was to determine the characteristics that students who failed had in common and the characteristics students who passed service mathematics had in common. When the characteristics which appeared to be negatively affecting students’ progression were determined, they were then used to inform the researchers on an appropriate model for predicting performance in service mathematics, the details of which can be found in chapter 5.

4.4.4 Profiling Technological Mathematics Students Who Fail (2006-2008)
4.4.4 (a) Failure Rates in Technological Mathematics Examinations (2006-2008): Trends by Year
Table 4.14 shows the failure rates in Technology mathematics examinations over the three years being examined. Although the failure rates fluctuate there is no significant change over time. A chi-square test carried out to establish if an association existed between year and performance in Technology mathematics showed no statistically significant association between the two variables. The mean overall CAO points and the mean mathematics CAO points changed very little over time. An analysis of variance test (ANOVA) revealed no statistically significant difference between the mean CAO overall points (p = 0.395) and mean CAO mathematics points (p = 0.604) over the three years.

The percentage of non-standard students decreased slightly over time. The number of new programmes of study and mid-term examinations have not changed throughout the three year period (see table 4.14).

In practical terms there is no real change in the students’ academic background (overall CAO points and CAO mathematics points) or in the delivery of the Technology mathematics module and the failure rate has varied only slightly over time. The following sections of this chapter aim to examine the students who have failed to see what characteristics may have left them more ‘at risk’ of failing than their counterparts who were successful.
Table 4.14 Number (%) of unsuccessful students in Technological mathematics and background information on individual cohorts (2006-2008).

### 4.4.4 (b) Failure Rates in Technological Mathematics Examinations (2006-2008): Trends by Gender

26.5% of females failed their end of semester examination compared to 23.0% of males. A chi-square test showed no statistically significant association between gender and success/failure in Technology mathematics examinations (p = 0.382).

<table>
<thead>
<tr>
<th>Year</th>
<th>Failure rate in end of term examination</th>
<th>Mean CAO points</th>
<th>Mean Mathematics CAO points</th>
<th>Number (%) of Non-standard students</th>
<th>New Programs of study within Technology maths</th>
<th>Number of midterm exams</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>76 (21.3%)</td>
<td>425</td>
<td>55.1</td>
<td>42 (11.8%)</td>
<td>13</td>
<td>1</td>
</tr>
<tr>
<td>2007</td>
<td>96 (25.6%)</td>
<td>427</td>
<td>55.5</td>
<td>26 (7.0%)</td>
<td>13</td>
<td>1</td>
</tr>
<tr>
<td>2008</td>
<td>82 (23.5%)</td>
<td>432</td>
<td>54.2</td>
<td>33 (9.5%)</td>
<td>13</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.15 Number of successful/unsuccessful males and females in Technological mathematics (2006-2008).

Table 4.16 shows that students with Higher Level Leaving Certificate mathematics as pre-requisite knowledge to services mathematics have a low failure rate in service mathematics of 4.9% between the years 2006-2008. Students who have Ordinary Level Leaving Certificate mathematics as pre-requisite knowledge had a higher failure rate of 32.9% between the years 2006 and 2008. A strong statistically significant association was found between Leaving Certificate level and success/failure in Technology mathematics examinations (p < 0.001). The statistically significant difference in failure rates between these two groups of standard students (Higher and Ordinary Level) reinforces the findings in section 4.2.4 that standard students are not homogenous and therefore possibly should not be grouped together. This finding will be taken into account when carrying out the prediction of student performance analysis in chapter 5.

<table>
<thead>
<tr>
<th>Leaving Certificate Level</th>
<th>Success</th>
<th>Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higher Level</td>
<td>350</td>
<td>18</td>
<td>368</td>
</tr>
<tr>
<td>(95.1%)</td>
<td>(4.9%)</td>
<td></td>
<td>(100.0%)</td>
</tr>
<tr>
<td>Ordinary Level</td>
<td>410</td>
<td>201</td>
<td>611</td>
</tr>
<tr>
<td>(67.1%)</td>
<td>(32.9%)</td>
<td></td>
<td>(100.0%)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>760</td>
<td>219</td>
<td>979</td>
</tr>
<tr>
<td>(77.6%)</td>
<td>(22.4%)</td>
<td></td>
<td>(100.0%)</td>
</tr>
</tbody>
</table>

Table 4.16 Number of successful/unsuccessful Ordinary and Higher Level Leaving Certificate students in Technological mathematics (2006-2008).


The highest percentages of failure rates are amongst students with Ordinary Level B1-C3 grades (Table 4.17). No students with Higher Level A1, A2 and B1 grades failed and failure rates are relatively low for all other Higher Level Leaving Certificate mathematics grade students with the exception of HD3 students. Ordinary Level B1-C3 students failed most frequently over the time period in 2006-2008. A strong statistically significant association between Leaving Certificate grade and success/failure in Technology mathematics was found (p < 0.001). The mean CAO mathematics points for successful students is 58 points compared to the mean for unsuccessful students which is 43 points. There are three sets of Leaving Certificate grade pairings which are assigned the same number of CAO points, i.e. OLA1 and HLC3, OLA2 and HLD2, OLB1 and
HLD3. By assigning students with grades with the same number of points one would assume that they have a similar level of mathematical competency. The findings in table 4.17 however show that the failure rates amongst students with these grades are quite different. An examination into how equivalent these grades are is outlined in chapter 5 as similar findings emerge in the case of the Science mathematics students also (see section 4.4.6 (d)).

<table>
<thead>
<tr>
<th>Leaving Certificate Grade</th>
<th>Success</th>
<th>Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>HA1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>HA2</td>
<td>15</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>HB1</td>
<td>27</td>
<td>0</td>
<td>27</td>
</tr>
<tr>
<td>HB2</td>
<td>45</td>
<td>1</td>
<td>46</td>
</tr>
<tr>
<td>HB3</td>
<td>51</td>
<td>4</td>
<td>55</td>
</tr>
<tr>
<td>HC1</td>
<td>61</td>
<td>1</td>
<td>62</td>
</tr>
<tr>
<td>HC2</td>
<td>47</td>
<td>2</td>
<td>49</td>
</tr>
<tr>
<td>HC3</td>
<td>36</td>
<td>1</td>
<td>37</td>
</tr>
<tr>
<td>HD1</td>
<td>38</td>
<td>2</td>
<td>40</td>
</tr>
<tr>
<td>HD2</td>
<td>20</td>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>HD3</td>
<td>9</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>OA1</td>
<td>111</td>
<td>13</td>
<td>124</td>
</tr>
<tr>
<td>OA2</td>
<td>116</td>
<td>27</td>
<td>143</td>
</tr>
<tr>
<td>OB1</td>
<td>92</td>
<td>53</td>
<td>145</td>
</tr>
<tr>
<td>OB2</td>
<td>59</td>
<td>65</td>
<td>124</td>
</tr>
<tr>
<td>OB3</td>
<td>25</td>
<td>37</td>
<td>62</td>
</tr>
<tr>
<td>OC1</td>
<td>4</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>OC2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>OC3</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>760</td>
<td>219</td>
<td>979</td>
</tr>
</tbody>
</table>

Table 4.17 Number of successful/unsuccessful students with particular Leaving Certificate grades in Technological mathematics (2006-2008).

Note: 101 students missing from table 4.16 and 4.17 as they do not have a Leaving Certificate grade.
4.4.4 (e) Failure Rates in Technological Mathematics Examinations (2006-2008): Trends by Sub-Category of Non-Standard Student

The failure rate amongst the sub-categories of non-standard students varies greatly with mature students having the largest failure rate of 42.2% (n = 27). A chi-square test revealed that there is a statistically significant association between the sub-categories of non-standard students and success/failure in Technology examinations (p = 0.042). This finding confirms those which were outlined in section 4.2.4.1 i.e. the mature students had the largest contribution to the failure rate of non-standard students in Technology mathematics. These findings confirm also that if Technology non-standard students are to be investigated further in terms of mathematical performance that they should be examined in terms of their individual sub-categorisation as they are evidently non-homogenous in terms of their service mathematics performance.

<table>
<thead>
<tr>
<th>Sub-Category of Student</th>
<th>Success</th>
<th>Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mature Students</td>
<td>37</td>
<td>27</td>
<td>64</td>
</tr>
<tr>
<td>Students with a Previous Degree, Diploma or Certificate</td>
<td>17</td>
<td>7</td>
<td>24</td>
</tr>
<tr>
<td>International Students</td>
<td>12</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>66</strong></td>
<td><strong>35</strong></td>
<td><strong>101</strong></td>
</tr>
</tbody>
</table>

Table 4.18 Number of successful/unsuccessful sub-categories of Non-standard students in Technological mathematics (2006-2008).

4.4.4 (f) Failure Rates in Technological Mathematics Examinations (2006-2008): Trends by Comparison of Students in Different Degree Programmes

Students in Digital Media Design and Wood Science and Technology have the highest failure rate for Technological mathematics examinations between 2006 and 2008 (Table 4.19). Other degree programmes which also had high percentages of failure rates (above 35%) are Production Management and Music, Media and Performance Technology. A chi-squared test revealed a statistically significant association between the degree programmes in which a student is enrolled and their performance in Technology mathematics examinations (p < 0.001).

In general the highest failure rates occur within degree programmes which have the lower mean overall CAO points. There is one exception however with the Music, Media and Performance degree which has the fourth highest mean overall CAO points. The interest of students within
this degree programmes in mathematics may be an influencing factor. Students within this degree programme may also have negative attitudes towards mathematics due to a lack of understanding as to its relevance to them (Hackett and Betz 1989). The three degree programmes with the highest failure rates (Digital Media and Design, Wood Science and Production Management) have the lowest mean overall CAO points.

<table>
<thead>
<tr>
<th>Degrees Program</th>
<th>Success</th>
<th>Failure</th>
<th>Total</th>
<th>Mean Mathematics CAO points for each Degree Programme</th>
<th>Mean Overall CAO points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronic Systems</td>
<td>5 (71.4%)</td>
<td>2 (28.6%)</td>
<td>7 (100.0%)</td>
<td>50.0</td>
<td>339.2</td>
</tr>
<tr>
<td>Applied Computing and Network Technology</td>
<td>12 (70.6%)</td>
<td>5 (29.4%)</td>
<td>17 (100.0%)</td>
<td>46.8</td>
<td>365.6</td>
</tr>
<tr>
<td>Materials and Construction Technology</td>
<td>212 (85.1%)</td>
<td>37 (14.9%)</td>
<td>249 (100.0%)</td>
<td>55.9</td>
<td>451.6</td>
</tr>
<tr>
<td>Construction Management and Engineering</td>
<td>127 (80.4%)</td>
<td>31 (19.6%)</td>
<td>158 (100.0%)</td>
<td>52.9</td>
<td>417.3</td>
</tr>
<tr>
<td>Manufacturing Systems</td>
<td>10 (76.9%)</td>
<td>3 (23.1%)</td>
<td>13 (100.0%)</td>
<td>52.7</td>
<td>368.2</td>
</tr>
<tr>
<td>Physical Education</td>
<td>77 (96.2%)</td>
<td>3 (3.8%)</td>
<td>80 (100.0%)</td>
<td>78.6</td>
<td>514.0</td>
</tr>
<tr>
<td>Production Management</td>
<td>17 (63.0%)</td>
<td>10 (37.0%)</td>
<td>27 (100.0%)</td>
<td>43.3</td>
<td>360.0</td>
</tr>
<tr>
<td>Wood Science and Technology</td>
<td>29 (61.7%)</td>
<td>18 (38.3%)</td>
<td>47 (100.0%)</td>
<td>49.1</td>
<td>372.4</td>
</tr>
<tr>
<td>Music, Media and Performance Technology</td>
<td>113 (64.9%)</td>
<td>61 (35.1%)</td>
<td>174 (100.0%)</td>
<td>53.4</td>
<td>418.8</td>
</tr>
<tr>
<td>Materials and Engineering Technology</td>
<td>124 (72.5%)</td>
<td>47 (27.5%)</td>
<td>171 (100.0%)</td>
<td>51.1</td>
<td>418.1</td>
</tr>
<tr>
<td>Digital Media Design</td>
<td>38 (57.6%)</td>
<td>28 (42.4%)</td>
<td>66 (100%)</td>
<td>46.6</td>
<td>388.1</td>
</tr>
<tr>
<td>Product Design and Technology</td>
<td>51 (86.4%)</td>
<td>8 (13.6%)</td>
<td>59 (100%)</td>
<td>58.8</td>
<td>432.8</td>
</tr>
<tr>
<td>Engineering Science</td>
<td>11 (91.7%)</td>
<td>1 (8.3%)</td>
<td>12 (100%)</td>
<td>61.4</td>
<td>412.3</td>
</tr>
<tr>
<td>Total</td>
<td>826 (76.5%)</td>
<td>254 (23.5%)</td>
<td>1080 (100.0%)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.19 Number of successful/unsuccesful students within particular degree programs required to take Technological mathematics (2006-2008).
4.4.4 (g) Failure Rates in Technological Mathematics Examinations (2006-2008): Trends by Mean Diagnostic Test Result

Students who were successful in Technology mathematics have a higher mean performance in the diagnostic test of 55.8 compared to the students who were unsuccessful who have a mean diagnostic test score of 38.7. The spread of results for the diagnostic test, given by the coefficient of variation, is 29.7% for successful students and 39.1% for unsuccessful students. The spread of results therefore is slightly greater amongst the students who failed Technology mathematics. An independent samples t-test showed a statistically significant difference between the mean diagnostic test results of students who were unsuccessful in end of term examinations against those who are not successful (p < 0.001). This finding suggests that if a student performs well in the diagnostic test they are more likely to be successful in Technology mathematics. Hence the diagnostic test must be performing reasonably well in categorising students as ‘at risk’ or not of failing Technology mathematics.

<table>
<thead>
<tr>
<th></th>
<th>Success</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean diagnostic test result</td>
<td>55.8</td>
<td>38.7</td>
</tr>
<tr>
<td>(Standard Deviation)</td>
<td>(16.6)</td>
<td>(13.2)</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>29.7%</td>
<td>39.1%</td>
</tr>
<tr>
<td>Numbers in group</td>
<td>715</td>
<td>199</td>
</tr>
</tbody>
</table>

Table 4.20 Mean diagnostic test result (expressed as a percentage out of 40) of successful/unsuccessful students in Technological mathematics (2006-2008).

Note: 166 students missing in table 4.20 as they did not sit the diagnostic test.
4.4.4 (h) Failure Rates in Technological Mathematics Examinations (2006-2008): Trends by Mean Arithmetic and Algebra Performance

Students who were successful in the end of semester examination have a higher mean performance in the arithmetic and algebra areas of the diagnostic test. The co-efficient of variation for the arithmetic performance for successful and unsuccessful students is 26.6% and 33.1% respectively. The co-efficient of variation for the algebra performance for students who were successful is 36.0% and 56.6% for those who were unsuccessful. The spread of results overall is quite large particularly in the case of unsuccessful students for algebra. Independent samples t-tests were carried out, for both arithmetic and algebra, and the mean values for successful students and those who were unsuccessful were statistically significantly different (p < 0.001). These findings highlight that there is a strong relationship between performing well in the arithmetic and algebra sections of the diagnostic test and being successful in Technology mathematics. Table 4.21 and 4.22 below highlight the results and numbers within each group.

<table>
<thead>
<tr>
<th></th>
<th>Success</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean arithmetic performance</td>
<td>59.8%</td>
<td>47.4%</td>
</tr>
<tr>
<td>(Standard deviation)</td>
<td>(15.9)</td>
<td>(15.7)</td>
</tr>
<tr>
<td>Co-efficient of Variation</td>
<td>26.6%</td>
<td>33.1%</td>
</tr>
<tr>
<td>Numbers in group</td>
<td>715</td>
<td>199</td>
</tr>
</tbody>
</table>

Table 4.21 Mean arithmetic result (expressed as a percentage out of 13) of successful/unsuccessful students in Technological mathematics (2006-2008).

<table>
<thead>
<tr>
<th></th>
<th>Success</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean algebra performance</td>
<td>64.0%</td>
<td>43.8%</td>
</tr>
<tr>
<td>(Standard deviation)</td>
<td>(23.0)</td>
<td>(24.8)</td>
</tr>
<tr>
<td>Co-efficient of Variation</td>
<td>36.0%</td>
<td>56.6%</td>
</tr>
<tr>
<td>Number within the group</td>
<td>715</td>
<td>199</td>
</tr>
</tbody>
</table>

Table 4.22 Mean algebra result (expressed as a percentage out of 8) of successful/unsuccessful students in Technological mathematics (2006-2008).

Note: 166 students are missing in table 4.21 & 4.22 as they did not sit the diagnostic test.
4.4.4 (i) Failure Rates in Technological Mathematics Examinations (2006-2008): Trends of Performance of Students who were Labelled ‘at risk’ and ‘not at risk’

Of the 375 students who were labelled ‘at risk’ of failing Technological mathematics only 146 (38.9%) students failed the examination. It was also found that 53 (9.8%) of the students who were labelled ‘not at risk’ went on to fail their end of semester examination (see table 4.23). It was assumed, based on their diagnostic test performance that they did not need to receive any extra mathematical support in order to pass Technology mathematics. The results of a chi-square test revealed a statistically significant association between success/failure in Technology mathematics and the category a student was given based on their performance in the diagnostic test ($p < 0.001$). Again this result highlights that the diagnostic test cut-off point which was put in place in an attempt to determine if a student was ‘at risk’ of failing service mathematics must be doing a reasonably effective job.

<table>
<thead>
<tr>
<th></th>
<th>Success</th>
<th>Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labelled ‘at risk’</td>
<td>229</td>
<td>146</td>
<td>375</td>
</tr>
<tr>
<td>% within group</td>
<td>(61.1%)</td>
<td>(38.9%)</td>
<td>(100.0%)</td>
</tr>
<tr>
<td>Labelled ‘not at risk’</td>
<td>486</td>
<td>53</td>
<td>539</td>
</tr>
<tr>
<td>% within group</td>
<td>(90.2%)</td>
<td>(9.8%)</td>
<td>(100%)</td>
</tr>
<tr>
<td>Total</td>
<td>715</td>
<td>199</td>
<td>914</td>
</tr>
</tbody>
</table>

Table 4.23 Number (and percentage) of successful/ unsuccessful students who were labelled ‘at risk’ and ‘not at risk’ in Technological mathematics (2006-2008).

Note: 166 students are missing in table 4.23 as they did not sit the diagnostic test.
4.4.5 Summary and Conclusion

The above findings therefore reveal that those who failed Technological mathematics are likely to:

- Have Ordinary Level Leaving Certificate mathematics as pre-requisite knowledge.
- Have obtained an Ordinary level B1-C3 grade in Leaving Certificate mathematics.
- Be a mature student.
- Those who are enrolled in Digital Media Design and Wood Science Technology degree programmes are also at a higher risk than those in all other degree programmes required to take Technology mathematics.

Finally the analysis of different aspects of diagnostic test performance highlights the following traits of an unsuccessful Technology mathematics student:

- They have a lower mean diagnostic test result when compared to successful students.
- Algebra and arithmetic performance on average is lower amongst students who are unsuccessful in Technology mathematics.
- Those who are labelled ‘at risk’ are not destined to fail nor are those who are labelled ‘not at risk’ destined to pass however a statistically significant association was found between being labelled ‘at risk’ or not and performance in Technology mathematics.

4.4.6 (a) Failure Rates in Science Mathematics Examinations (2006-2008): Trends by Year

Table 4.24 shows the increase in the failure rate in terms of end of semester examination performance over three years. Students have similar CAO overall points and CAO mathematics points. Analysis of variance (ANOVA) tests were carried out and revealed that there is no statistically significant difference between the mean CAO mathematics points and the mean CAO overall points over the three years being examined (p = 0.32 and p = 0.46 respectively). The two additional programmes of study added to Science mathematics in 2007 and 2008, Joint honours and Law plus, have the highest CAO points requirement of all the courses in Science mathematics and so are unlikely to have contributed to the increase in failure rates.

In 2008 there was a change made to the assessment method when 4 midterm examinations were given to students during one semester as opposed to 1 (see table 4.24). In addition to this, the lecturer also raised the pass mark threshold from 30% to 37% in 2008 and this caused a larger percentage of students to fail. If the pass mark had been kept at 30% the failure rate in 2008 would have been 14.2%.

There are little or no practical differences between students’ backgrounds on entry to UL within this three year period and a slight increase in failure rates over time occurred. An analysis of the relationship between certain student characteristics and performance in Science mathematics will now follow.

<table>
<thead>
<tr>
<th>Year</th>
<th>Failure rate in end of term examination</th>
<th>Mean overall CAO points</th>
<th>Mean Mathematics CAO points</th>
<th>Number (%) of Non-standard students</th>
<th>New Programs of study within Science maths</th>
<th>Number of midterm exams</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>29 (10.7%)</td>
<td>432</td>
<td>55.3</td>
<td>28 (10.3%)</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>n=271</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td>38 (13.4%)</td>
<td>445</td>
<td>53.3</td>
<td>16 (5.6%)</td>
<td>10 (addition of Joint Honours)</td>
<td>1</td>
</tr>
<tr>
<td>n=283</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td>77 (27.1%*)</td>
<td>446</td>
<td>54.1</td>
<td>17 (5.9%)</td>
<td>11 programmes (addition of Law plus)</td>
<td>4</td>
</tr>
<tr>
<td>n=284</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Table 4.24 Number of students successful/unsuccessful in Science mathematics (2006-2008).*

*If the pass mark had remained the same the failure rate in 2008 would have been 14.2%.
4.4.6 (b) Failure Rates in Science Mathematics Examinations (2006-2008): Trends by Gender

Within the years 2006, 2007 and 2008 combined, a higher percentage of males failed the Science mathematics examination. 19.3% of males failed their semester 1 examination compared to 15.2% of females. No statistically significant association was found between gender and success/failure in Science mathematics examinations.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Success</th>
<th>Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>326</td>
<td>78</td>
<td>404</td>
</tr>
<tr>
<td></td>
<td>(80.7%)</td>
<td>(19.3%)</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>368</td>
<td>66</td>
<td>434</td>
</tr>
<tr>
<td></td>
<td>(84.8%)</td>
<td>(15.2%)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>694</td>
<td>144</td>
<td>838</td>
</tr>
<tr>
<td></td>
<td>(82.8%)</td>
<td>(17.2%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.25 Number of successful/unsuccessful males and females in Science mathematics (2006-2008).
4.4.6 (c) Failure Rates in Science Mathematics Examinations (2006-2008): Trends by Leaving Certificate Level

Table 4.26 shows that students with Higher Level Leaving Certificate mathematics as pre-requisite knowledge had a low failure rate of 4.5% in service mathematics between the years 2006 and 2008. Students who have Ordinary Level Leaving Certificate mathematics as pre-requisite knowledge had a higher failure rate between the years 2006 and 2008 of 25.1%. A chi-square test revealed a statistically significant association between Leaving Certificate level and success/failure in Science mathematics (p < 0.001). These findings again reinforce those outlined in section 4.2.4 in relation to the non-homogeneous nature of the standard group of students. These findings will be of use in the prediction of student achievement in chapter 5.

<table>
<thead>
<tr>
<th></th>
<th>Success</th>
<th>Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Higher Level</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>297</td>
<td>14</td>
<td>311</td>
</tr>
<tr>
<td></td>
<td>(95.5%)</td>
<td>(4.5%)</td>
<td>(36.8%)</td>
</tr>
<tr>
<td><strong>Ordinary Level</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>349</td>
<td>117</td>
<td>466</td>
</tr>
<tr>
<td></td>
<td>(74.9%)</td>
<td>(25.1%)</td>
<td>(55.8%)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>646</td>
<td>131</td>
<td>777</td>
</tr>
<tr>
<td></td>
<td>(83.1%)</td>
<td>(16.9%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.26 Number of successful/unsuccesful Ordinary and Higher Level Leaving Certificate students in Science Mathematics (2006-2008).

4.4.6 (d) Failure Rates in Science Mathematics Examinations (2006-2008): Trends by Leaving Certificate Grades

Table 4.27 shows that the highest failure rates in Science mathematics are amongst students with Ordinary Level A1-C1 grades. No students with Higher Level A1 and A2 grades failed and between 4.5% of all students with all other Higher Level Leaving Certificate grades failed. There is a strong statistically significant association between Leaving Certificate grade and success/failure in Science mathematics examinations (p < 0.001). The mean number of mathematics points amongst students who were successful in Science mathematics was 57 points compared to a mean of 43 points for students who were unsuccessful. Students with Ordinary Level B1-C1 grades failed most frequently over the time period 2006-2008. This is a relevant finding as the minimum entry requirement with any degree program required to do Science mathematics is an Ordinary Level B3 grade. Again the issue of equivalent grades must be called into question as students with Ordinary Level A2 grades for example have a failure rate of 16.2%.
which is much higher than for Higher Level D2 which has a failure rate of 8.3% even though
both grades are awarded the same CAO points. An in-depth analysis into equivalent grades is
detailed in chapter 5.

<table>
<thead>
<tr>
<th>Leaving Certificate Grade</th>
<th>Success</th>
<th>Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>HLA1</td>
<td>4</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>HLA2</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>HLB1</td>
<td>25</td>
<td>2</td>
<td>27</td>
</tr>
<tr>
<td>HLB2</td>
<td>24</td>
<td>0</td>
<td>24</td>
</tr>
<tr>
<td>HLB3</td>
<td>39</td>
<td>2</td>
<td>41</td>
</tr>
<tr>
<td>HLC1</td>
<td>51</td>
<td>1</td>
<td>52</td>
</tr>
<tr>
<td>HLC2</td>
<td>55</td>
<td>2</td>
<td>57</td>
</tr>
<tr>
<td>HLC3</td>
<td>41</td>
<td>3</td>
<td>44</td>
</tr>
<tr>
<td>HLD1</td>
<td>26</td>
<td>1</td>
<td>27</td>
</tr>
<tr>
<td>HLD2</td>
<td>11</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>HLD3</td>
<td>11</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>OLA1</td>
<td>59</td>
<td>7</td>
<td>66</td>
</tr>
<tr>
<td>OLA2</td>
<td>93</td>
<td>18</td>
<td>111</td>
</tr>
<tr>
<td>OLB1</td>
<td>84</td>
<td>32</td>
<td>116</td>
</tr>
<tr>
<td>OLB2</td>
<td>61</td>
<td>27</td>
<td>88</td>
</tr>
<tr>
<td>OLB3</td>
<td>43</td>
<td>20</td>
<td>63</td>
</tr>
<tr>
<td>OLC1</td>
<td>6</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>OLC2</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>OLC3</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>OLD1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>OLD2</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>646</td>
<td>131</td>
<td>777</td>
</tr>
</tbody>
</table>

Table 4.27 Number of successful/unsuccessful students with particular Level Leaving Certificate

Students in Science mathematics with grades below the minimum entry grade (OLB3) have either transferred to a
course involving Science mathematics from another course in UL or have gained entry through an Access course.
4.4.6 (e) Failure Rates in Science Mathematics Examinations (2006-2008): Trends by Sub-Category of Non-Standard Student

The failure rate amongst the sub-categories of non-standard students is quite low in comparison to that of the Technology non-standard students particularly when the mature students and those who engaged in previous study are considered. A chi-square test revealed that there is no statistically significant association between sub-category of non-standard student and success/failure in the Science mathematics examination (p = 0.075). It is possible that the mature students in Science mathematics and those who have engaged in previous study were better able to cope with the mathematical demands made upon them in service mathematics. Possibly these students entered UL anticipating that engagement with mathematics would be an integral part of their study as opposed to the mature students in the Technology cohort who may not have had the same expectations due, for example, to enrolling in a degree which was predominantly Music based (see discussion in section 4.2.4.1 (b)).

<table>
<thead>
<tr>
<th>Sub-Category of Student</th>
<th>Success</th>
<th>Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mature Students</td>
<td>34</td>
<td>7</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>(82.9%)</td>
<td>(17.1%)</td>
<td></td>
</tr>
<tr>
<td>Students with a Previous Degree, Diploma or Certificate</td>
<td>9</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>(81.8%)</td>
<td>(18.2%)</td>
<td></td>
</tr>
<tr>
<td>International Students</td>
<td>5</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>(55.6%)</td>
<td>(44.4%)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>48</td>
<td>13</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>(78.7%)</td>
<td>(21.3%)</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.28 Number of successful/unsuccessful sub-categories of Non-standard students in Science mathematics (2006-2008).

4.4.6 (f) Failure Rates in Science Mathematics Examinations (2006-2008): Trends by Comparison of Students in Different Degree Programmes

Students in Sport and Exercise Science and Biomedical and Advanced Materials failed the Science mathematics examinations most frequently between 2006 and 2008. Other degree programmes which also saw high failure rates (above 20%) were Environmental Science and Health and Safety. The four degree programmes mentioned are amongst the lowest 6 mean CAO mathematics performance of all of the degree programmes within Science mathematics and so it
is unsurprising that they have the four highest failure rates. A chi-square test indicated that there is a statistically significant association between the degree programme in which a student is enrolled and their success/failure in Science mathematics examinations (p = 0.002).

<table>
<thead>
<tr>
<th>Degrees Program</th>
<th>Success</th>
<th>Failure</th>
<th>Total</th>
<th>Mean Mathematics CAO points for each Degree Programme</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA (Joint Honours)</td>
<td>18</td>
<td>2</td>
<td>20</td>
<td>65</td>
</tr>
<tr>
<td>Biological Science</td>
<td>181</td>
<td>24</td>
<td>205</td>
<td>53.75</td>
</tr>
<tr>
<td>Biomedical and Advanced Materials</td>
<td>18</td>
<td>6</td>
<td>24</td>
<td>54.1</td>
</tr>
<tr>
<td>Environmental Science</td>
<td>87</td>
<td>26</td>
<td>113</td>
<td>50</td>
</tr>
<tr>
<td>Food Science and Health</td>
<td>60</td>
<td>18</td>
<td>78</td>
<td>47.6</td>
</tr>
<tr>
<td>Health and Safety</td>
<td>54</td>
<td>14</td>
<td>68</td>
<td>48.5</td>
</tr>
<tr>
<td>Industrial Biochemistry</td>
<td>56</td>
<td>3</td>
<td>59</td>
<td>56.1</td>
</tr>
<tr>
<td>Law Plus</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>73.75</td>
</tr>
<tr>
<td>Pharmaceutical and Industrial Chemistry</td>
<td>50</td>
<td>5</td>
<td>55</td>
<td>62.9</td>
</tr>
<tr>
<td>Physical Science</td>
<td>28</td>
<td>4</td>
<td>32</td>
<td>70</td>
</tr>
<tr>
<td>Sport and Exercise Science</td>
<td>138</td>
<td>42</td>
<td>180</td>
<td>53.8</td>
</tr>
<tr>
<td>Total</td>
<td>694</td>
<td>144</td>
<td>838</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.29 Number of Successful/Unsuccessful students within particular degree programmes required to take Science mathematics (2006-2008).
4.4.6 (g) Failure Rates in Science Mathematics Examinations (2006-2008): Trends by Mean Diagnostic Test Result

An examination into the mean diagnostic test performance of students who were successful in Science mathematics against those who were unsuccessful shows that the successful students have a higher mean performance in the diagnostic test of 52.2% compared to the unsuccessful students who have a mean diagnostic test of 41.3%. The co-efficient of variation gives an indication of the spread of the data which is 32.6% for successful students and 38.0% for unsuccessful students. The spread of data is quite similar for each group. An independent samples t-test revealed that there is a statistically significant difference between the mean diagnostic test results of students who were successful in Science mathematics examinations against those who were unsuccessful (p < 0.001).

<table>
<thead>
<tr>
<th></th>
<th>Success</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean diagnostic test result</td>
<td>52.2%</td>
<td>41.3%</td>
</tr>
<tr>
<td>(Standard Deviation)</td>
<td>(17.1)</td>
<td>(15.0)</td>
</tr>
<tr>
<td>Co-efficient of Variation</td>
<td>32.6%</td>
<td>38.0%</td>
</tr>
<tr>
<td>Numbers in group</td>
<td>634</td>
<td>109</td>
</tr>
</tbody>
</table>

Table 4.30 Mean diagnostic test result of successful/unsuccessful students in Science mathematics (2006-2008).

Note: 95 students are missing from table 4.30 as they did not sit the diagnostic test.
4.4.6 (h) Failure Rates in Science Mathematics Examinations (2006-2008): Trends by Mean Arithmetic and Algebra Performance

Students who were successful in the end of semester examination have a higher mean performance in the arithmetic and algebra areas of the diagnostic test. The spread of results for arithmetic, given by the co-efficient of variation, is 29.4% for successful students and 39.1% for unsuccessful students. An independent samples t-test for both arithmetic and algebra revealed that the mean values for successful students and those who were unsuccessful were statistically significantly different (p < 0.001). Table 4.31 and 4.32 below highlight the results and numbers within each group. These findings suggest that a student’s performance in the arithmetic and algebra sections of the diagnostic test is an indication of their future success in Science mathematics.

<table>
<thead>
<tr>
<th></th>
<th>Success</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean arithmetic performance</strong></td>
<td>57.2%</td>
<td>48.6%</td>
</tr>
<tr>
<td>(Standard deviation)</td>
<td>(16.8)</td>
<td>(19.0)</td>
</tr>
<tr>
<td><strong>Co-efficient of Variation</strong></td>
<td>29.4%</td>
<td>39.1%</td>
</tr>
<tr>
<td><strong>Number in group</strong></td>
<td>634</td>
<td>109</td>
</tr>
</tbody>
</table>

Table 4.31 Mean arithmetic result (expressed as a percentage out of 13) of successful/unsuccessful students in Science mathematics (2006-2008).

<table>
<thead>
<tr>
<th></th>
<th>Success</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean algebra performance</strong></td>
<td>59.62%</td>
<td>48.2%</td>
</tr>
<tr>
<td>(Standard deviation)</td>
<td>(24.2)</td>
<td>(25.4)</td>
</tr>
<tr>
<td><strong>Co-efficient of Variation</strong></td>
<td>40.6%</td>
<td>52.7%</td>
</tr>
<tr>
<td><strong>Number in group</strong></td>
<td>634</td>
<td>109</td>
</tr>
</tbody>
</table>

Table 4.32 Mean algebra result (expressed as a percentage out of 8) of Successful/Unsuccessful students in Science mathematics (2006-2008).

Note: 95 students missing in table 4.31 & 4.32 as they did not sit the diagnostic test.
4.4.6 (i) Failure Rates in Science Mathematics Examinations (2006-2008): Trends of Performance of Students who were Labelled ‘at risk’ and ‘not at risk’

Of the 350 students who were labelled ‘at risk’ of failing Science mathematics in semester 1 only 80 (22.8%) students failed the examination. 7.4% of the students who failed were labelled ‘not at risk’ therefore it was assumed, based on their diagnostic test performance, that they did not require any extra mathematical support in order to pass their end of semester examination. A chi-square test showed a statistically significant association between success/failure and the category (at risk/not at risk) a student was given based on their performance in the diagnostic test ($p < 0.001$) with students in the ‘at risk’ category more likely to fail than those in the ‘not at risk’ category. This finding again emphasises the effectiveness of the current measure which is in place to determine if a student is likely to be ‘at risk’ of failing service mathematics or not.

<table>
<thead>
<tr>
<th>Labelled ‘at risk’ % within row</th>
<th>Success</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>270 (77.2%)</td>
<td>80      (22.8%)</td>
<td></td>
</tr>
<tr>
<td>Labelled ‘not at risk’ % within row</td>
<td>364      (92.6%)</td>
<td>29       (7.4%)</td>
</tr>
</tbody>
</table>

Table 4.33 Number (and percentage) of Successful/Unsuccessful students who were labelled ‘at risk’ and ‘not at risk’ in Science mathematics (2006-2008).

Note: 95 students missing in table 4.33 as they did not sit the diagnostic test.
4.4.7 Summary and Conclusion

From the above findings those who failed Science mathematics are likely to:

- Have Ordinary Level Leaving Certificate mathematics as pre-requisite knowledge.
- Have obtained an Ordinary Level B1-C1 grade in Leaving Certificate mathematics.
- Students who are enrolled in Sport and Exercise Science and Biological and Advanced Materials degree programmes are also at a higher risk of failing.

Finally the analysis of different aspects of diagnostic test performance highlights that successful students:

- Will have a higher mean diagnostic test result.
- On average will perform better in the algebra and arithmetic sections of the diagnostic test.

The analysis also highlighted that those who are labelled ‘at risk’ are not destined to fail nor are those who are labelled ‘not at risk’ destined to pass although there is a statistically significant association between these variables.
4.5 Discussion and Conclusions

The research findings in this chapter highlight the value and usefulness of the UL database. The vast database allows for the examination of the student profile in UL and an investigation into whether there is evidence of grade inflation in the Leaving Certificate mathematics examination between 1998 and 2008. Anecdotal ‘evidence’ from those involved in mathematics education regarding declining standards requires quantitative evidence in order to see exactly where the problems lie and how effective change could be implemented.

The investigation into the profile of service mathematics students revealed that the percentages of males and females in Technology mathematics have remained largely unchanged within the ten year period being examined however the percentage of males in Science mathematics increased by 16.5 percentage points in this time period. There has been an increase in the number of degree programmes within Technology and Science mathematics over time. This is likely to have contributed to the change in the mathematical backgrounds of students within these service mathematics groups. There was an increase of 9.1 and 5.4 percentage points of non-standard students in Technology and Science mathematics respectively. The increase in non-standard students was matched with a decrease in the percentage of students entering service mathematics in UL with Higher Level Leaving Certificate mathematics. A decrease of 7.8 percentage points occurred within the Technology cohort and a decrease of 17.4 percentage points occurred within the Science cohort. The percentage of ‘at risk’ students has increased within both service mathematics cohorts from 32.8% in 1998 to 46.4% in 2008 for Technology mathematics students and from 21.3% to 46.0% for Science mathematics students. The changing profile of Science and Technological mathematics students between 1998 and 2008 was found to be a major contributing factor to the declining standards in mathematical competency as measured by the UL diagnostic test of students entering UL (Faulkner et al 2010).

The analysis of standard students’ performance against non-standard students’ performance revealed that non-standard students’ diagnostic test score was below that of the standard students however their mean service mathematics performance was higher with a greater spread of results (however a larger proportion of non-standard Technology students failed service mathematics when compared to standard Technology students). This unexpected finding was found to be partially due to mature students and those who engaged in previous study improving in their mean mathematics performance over the course of the semester with international students.
maintaining what was already a strong mathematical performance in the diagnostic test. In addition to the large improvement in non-standard students’ performance over the course of the semester, the students entering UL with Ordinary Level Leaving Certificate mathematics underperformed in service mathematics bringing down the mean performance of standard students. These findings revealed that the standard and non-standard groups were in fact non-homogenous. Each sub-category was found to have a statistically significantly different mean performance in the diagnostic test ($p < 0.001$) and service mathematics ($p < 0.001$). In addition to this Ordinary Level students were found to have a statistically significantly different mean service mathematics performance to the four other sub-categories of students ($p < 0.001$). Ordinary Level students’ underperformance may be attributed to factors outside of their mathematics points such as poor teaching in second level education, adopting undesirable learning styles, poor perceived ability and motivation towards mathematics. The large differences in the performances of standard and non-standard students in service mathematics is of particular interest to the prediction of student achievement which is detailed in chapter 5.

There has been a decline in diagnostic test performance over time. A decline in the mean percentage achieved in the diagnostic test of 14% from the 1998 baseline occurred. In the case of Science mathematics students, a 20% decline in mean percentage achieved from the 1998 baseline occurred. An increase in variation of scores over time for both cohorts also occurred however the differences between the variances were not statistically significant. An increase in variability may cause difficulties for those involved in mathematics education. For example being able to establish a common level of competency amongst students may be difficult possibly resulting in issues with choosing lecture material and judging appropriate pacing of lectures.

A further investigation into the decline in diagnostic test performance investigated if changes in performance of students in the diagnostic test occurred amongst students with particular Leaving Certificate mathematics grade. No significant changes over time were found (with the exception of OA1 grades students) in performance by Leaving Certificate mathematics grade and there has been no crossover between grades i.e. in no instance did a lower Leaving Certificate grade outperform a higher one in the diagnostic test on average. This finding suggests that the decline in competency levels of students over time can be partially attributed to the increase in the number of Ordinary Level Leaving Certificate mathematics students attending the university.
The finding also indicates that the Leaving Certificate is producing the same level of competency of mathematics for the same Leaving Certificate grades and therefore the data does not provide evidence that grade inflation is responsible for the declining standards within Science and Technology mathematics in UL (Faulkner et al 2010). Contrasting findings were reported in the UK when Lawson (2003) reported a decline over time in the mathematical competency levels of A-level students entering university with the same A level grades. Lawson (2003) states that “A-level mathematics does not produce the same degree of competency as it did formerly” (p.174). The fact that the investigation carried out here did not include an examination of Higher Level A and B Leaving Certificate mathematics grades may have had an influence on the findings presented. It also may partially explain the contrasting findings in the UK and Ireland. Further discussion regarding this topic is detailed in chapter 7.

Statistically significant declines in performance over time as well as increases in variability were found when Science and Technological students’ performance in the algebra and arithmetic sections of the diagnostic test were examined. Science mathematics students experienced the largest decline in both areas of the test. This finding is very relevant as the algebra and arithmetic sections of the test make up over half of the questions on the paper and are vital for progression in other areas of mathematics (Gamoran & Hannigan 2000). A decline in these areas are likely, therefore, to have a negative effect on success in third level mathematics. An investigation into students with grades OA1, OA2, OB1, OB3 and HC1 was also carried out in terms of performance in the arithmetic and algebra sections of the diagnostic test. Small declines in performance were found in algebra and arithmetic along with an increase in the spread of results over time. The majority of grades however did not have statistically significant differences over time. There were two instances in which significant variability increases occurred.

From the analysis of the Technology and Science (2006-2008) databases, it was found that many of the variables being examined have a statistically significant association with performance in Science mathematics and Technological mathematics. For example independent sample t-tests revealed statistically significant differences between mean CAO points (p < 0.001), mean diagnostic test score (p < 0.001) and mean performance in arithmetic and algebra (p < 0.001) for those who were successful and those who were unsuccessful in service mathematics. A series of chi-squared tests carried out on the categorical data revealed statistically significant associations between success/failure in service mathematics and Leaving Certificate Level at which
mathematics was taken (p < 0.001), Leaving Certificate grade (p < 0.001), degree programme of study (p < 0.001 for Technology mathematics, p = 0.002 for Science mathematics) and whether a student was labelled at risk or not (p < 0.001). These significant associations are similar to those reported in much of the mathematics education research (Wilson and MacGillivray 2007; James et al 2008). For Technology mathematics students statistically significant associations were found between performance and a student’s non-standard sub-category (p = 0.042). No such association was found amongst the Science students. Gender showed no statistically significant association with performance in end of term mathematics examinations for either service mathematics group. All of the previously mentioned findings must be considered in light of the large sample sizes of 1,080 and 838 Technology and Science mathematics students respectively. These large samples may have influenced the prevalence of statistically significant associations/mean differences in some cases in which the practical significance of these associations may not be very noteworthy. There was no adjustment made for multiple testing.

4.6 Closing Remarks
It is important to analyse these findings in terms of practical significance to the teaching and learning of mathematics in third level education in Ireland. A different profile of students enters our lecture halls today compared to ten years ago. Students’ mathematical backgrounds are not as extensive as they were ten years ago and one can only conclude that some changes regarding the starting point of lecture material needs to be made or “in the absence of any change of starting point, a deterioration in the effectiveness of learning” (Hunt and Lawson 1996, p. 171) is likely to occur.

There is a changing student profile in service mathematics courses between the years 1998 and 2008. This changing profile is contributing to the declining standards in diagnostic test performance in UL (Faulkner et al 2010). Students’ Leaving Certificate mathematics performance and their diagnostic test performance is related to performance in service mathematics in UL. Some Ordinary Level mathematics students may therefore be considered to be inappropriate candidates for service mathematics however UL is unlikely to raise the entry qualification as this would eliminate a large number of students who presently enter third level with Ordinary Level Leaving Certificate mathematics as their prerequisite knowledge. So how can we use the information we have to improve mathematics education practice in UL? Many
qualitative and quantitative student characteristics are statistically significantly related to their performance in service mathematics. This information will now be used to inform the investigation which takes place in chapter 5. Chapter 5 details how the findings in chapter 4 were used to inform the creation of a predictive model of failure in service mathematics.
Chapter 5: Exploring Prediction Methods for Performance in Service Mathematics

5. Introduction

The current method for predicting performance in service mathematics in UL is the diagnostic test. Several statistical methods for predicting performance have been explored within this thesis. Of all of the statistical methods explored, discriminant analysis was considered the most appropriate to use in the case of the UL database. Rationale for this is detailed later in the chapter. Discriminant analysis is used to determine which variables discriminate between two or more naturally occurring groups. The main purpose of discriminant analysis is to predict group membership based on a linear combination of interval variables (Klecka 1980). The procedure begins with a set of observations where both group membership and the values of the interval
variables are known. The end result of this process is a function that allows the prediction of group membership when only the interval variables are known.

Discriminant analysis also allows researchers to develop a deeper understanding of a particular data set. An in-depth examination of the function can give insight into the independent variables used to predict group membership and the relationship they have with the dependent variable (Stockburger 2010). This form of analysis is often used in medicine. For example, it may be used to predict whether or not patients recovered from a particular illness based on combinations of demographic and treatment variables. Predictor variables can include age, gender, general health and time between the incident and arrival at the hospital. The creation of a classification model in this case would allow a medical professional to assess the chance of recovery based on observed variables.

To summarise, discriminant analysis is used in an attempt to achieve the following:

- Identify the variables that discriminate ‘best’ between two populations,
- Use the identified variables to develop a function for computing a new variable or index that will represent the differences between the two populations,
- Use the identified variables or the computed index to develop a rule to classify future observations into one of two populations.

With discriminant analysis, the population an individual belongs to is known for the sample initially being analysed. This sample is often called the training sample. In the case of this research the training sample is Technological mathematics students between the years 2006 and 2008. The rationale for choosing these students is outlined in section 5.4.1. Classification of future students into success and failure categories was the main aim. The continuous predictor variables which were used to achieve this are described in section 5.4.
5.1 Chapter Outline

In this chapter several methods of predicting student performance in service mathematics are explored. The chapter is divided into the following sections:

- Determining the most appropriate statistical prediction method (section 5.2).
- Description of how discriminant analysis is carried out (section 5.3).
- Discriminant analysis findings (section 5.4).
- An analysis of probability of failure by groups (section 5.5).
- Comparison of the diagnostic test and the discriminant functions’ ability to correctly classify service mathematics students (section 5.6).
- Challenges faced when predicting service mathematics performance in UL (section 5.7).
- Separate discriminant analysis for Ordinary Level and Higher Level students (section 5.8).
- Predicting performance: a comparison of approaches (section 5.9).
- Valuable findings which emerged due to prediction challenges (section 5.10).
- Summary and conclusion (section 5.11).
- Closing remarks (section 5.12).

The analysis carried out in this chapter aims to answer the following research questions:

- What is the most effective method of prediction of failure within service mathematics in UL?
- Is discriminant analysis a more effective method of classifying failure in service mathematics than the measure which is currently in place i.e. a cut-off in the diagnostic test?
- What are the challenges associated with predicting failure in third level service mathematics?
5.2 Determining the Most Appropriate Statistical Prediction Method

Various statistical prediction methods were examined within this thesis and discriminant analysis was chosen as the most appropriate prediction method in the case of the UL database for the following reasons:

- Logistic regression and classification tree analysis were carried out on the UL database. When compared to discriminant analysis however they were inferior methods of prediction in the case of predicting service mathematics performance for the UL database. Discriminant analysis correctly classified the highest percentage of students in terms of performance in service mathematics when compared to the other prediction methods tested (see Appendix E for descriptions of logistic regression and classification tree findings). Discriminant analysis also provides students with a probability of failure. Although prediction methods such as multiple regression provide a prediction of performance, discriminant analysis was considered more effective as it informed students as to whether they met the threshold (pass/fail) and also how close they were to it (probability of success/failure).

- Discriminant analysis uses numeric variables to predict group membership. The numeric variables, such as Leaving Certificate mathematics points and diagnostic test results, examined in the initial stages of profiling proved to have strong correlations with performance in service mathematics (see section 4.4).

- Discriminant analysis is a statistical technique designed to investigate the differences between two or more groups with respect to several underlying variables. This technique is therefore more appropriate than commonly used educational measures such as correlations and regression weights because the variable being predicted is categorical (success/failure).

- Due to the fact that discriminant analysis results in a predicted category membership and probability of category membership for each unit, it is also more useful in evaluating instructional interventions.

- The nature of discriminant analysis, i.e. its ability to determine what variables have a relationship with performance and categorise students accordingly, is of great benefit to the design, implementation and evaluation of any educational programme/policy (Thomas et al 1996).
Discriminant analysis can act as a tool for classifying future students. For the aforementioned reasons this form of prediction analysis was considered more appropriate than any other for this research project. The methodology for how discriminant analysis is carried out is detailed in section 5.3.

5.3 Description of How Discriminant Analysis is Carried Out

5.3.1 Introduction
When carrying out discriminant analysis it is vital that appropriate data is used for the analysis, that the assumptions underlying the methodology are considered and satisfied and that the discriminant analysis is examined and interpreted in the correct manner. An outline of the basic concepts which underpin it is given in this section followed by the discriminant analysis results (section 5.4).

5.3.2 Basic Concepts of Discriminant Analysis
With discriminant analysis any one individual belongs to just one of usually two possible populations. Discriminant analysis allows one to classify every individual into one of the populations based on one or more characteristics held by the individual.

5.3.2 (a) Classification of Individuals Based on a Single Characteristic
Discriminant analysis can be carried out using one or more independent variables. We will first consider an example in which just one independent variable is used. Take for example the classification of Technology students as being either successful or unsuccessful in their mathematics examination. The characteristic being used to classify the students is Leaving Certificate mathematics points (see figure 5.1). The distribution of Leaving Certificate mathematics points must be estimated from a representative group for each population, i.e. the spread of Leaving Certificate mathematics points of students who failed Technology mathematics from previous cohorts and the spread of Leaving Certificate mathematics points of students who were successful in Technology mathematics from previous cohorts. In the case of this example, low points in Leaving Certificate mathematics would result in a student being classified as unsuccessful (i.e. being from population I) and a high number of Leaving Certificate mathematics points would classify you as being successful (i.e. being from population II).
Figure 5.1 Hypothetical frequency distributions of two populations showing percentage of cases incorrectly classified.

A dividing point C must be selected so that what is considered a high and low value of Leaving Certificate mathematics points (x) can be decided upon. The dividing point allows for an exact classification prediction to be made for example, if a person had an X value in which $X \leq C$ that person is classified in population I and if they have a value of X (i.e. Leaving Certificate mathematics points) in which $X > C$, they must be classified in population II. A certain percentage of error, i.e. incorrect classification, is present for any given value of C. The incorrect classification sections can be seen in figure 5.1. Providing equal variance can be assumed for each population, C is calculated as follows:

$$C = \frac{\bar{X}_I + \bar{X}_{II}}{2}$$

The probabilities of error are the same for both populations. When analysing real data however, the degree of overlap between the distributions can be large and it is unlikely that the variances for each population will be the same. One method for reducing the misclassification of error is to use more than one variable. The method through which this can be carried out is described in section 5.3.2 (b).
5.3.2 (b) Classification of Individuals Using More Than One Variable

As previously mentioned, using more than one variable when trying to classify subjects into one of two populations can often reduce the misclassification error. When more than one variable is used for classification purposes a variable \( Z \) is created. \( Z \) is a linear combination of the variables being examined e.g. \( X_1 \) and \( X_2 \). Fisher (1936) developed the underlying concepts of classification behind the \( Z \) function and so it is often referred to as the Fisher Discriminant Function. For \( p \) predictor variables \( Z \) is therefore given by:

\[
Z = a_1x_1 + a_2x_2 + \ldots + a_p x_p
\]

where \( x_1, x_2, \ldots, x_p \) represent independent variables and \( a_1 \) and \( a_2 \) are estimated from the dataset. In order to measure how far apart two groups are in terms of their \( Z \) values we calculate \( D^2 \) which is given by:

\[
D^2 = \frac{(Z_I + Z_M)^2}{S_Z^2}
\]

where \( S_Z^2 \) represents the pooled sample variance of \( Z \). The co-efficients of \( a_1, a_2, \ldots, a_p \) are selected so that \( D^2 \) is maximised. A larger value of \( D^2 \) results in it being easier to discriminate between two populations.

For each individual, for each population (success/failure), the value of \( Z \) is calculated. This allows for the calculation of the frequency distribution for both populations of \( Z \) to be plotted (see figure 5.2). By creating the \( Z \) function, the multivariate classification problem is reduced to a uni-variate problem.
Figure 5.2 Frequency distributions of $Z$ for Populations I and II.

The concepts of discriminant analysis described in this section are implemented on the databases contained in this research in order to attempt to correctly classify as many service mathematics students as possible, the findings of which are detailed in section 5.4.

### 5.4 Discriminant Analysis Findings

Discriminant analysis was carried out on the Technology 2006-2008 dataset to create a discriminant function. If the function is found to be reasonably successful in terms of its classification of failure it will be used as a tool for predicting failure for future cohorts of students.

The predictor variables tested when logistic regression and classification tree analysis were carried out were as follows: gender, Leaving Certificate points, Leaving Certificate Level and grade, programme of study, non-standard student sub-category, whether a student sat the diagnostic test or not and performance in the algebra and arithmetic sections of the diagnostic test. Both prediction methods found Leaving Certificate mathematics points to be a statistically significant predictor of performance in service mathematics with classification trees also finding
diagnostic test result to be a statistically significant predictor (see Appendix E). These analyses therefore determined that all other variables considered added no predictive ability. Discriminant analysis was carried out using the predictor variables diagnostic test results, Leaving Certificate mathematics points and arithmetic and algebra performance. The only variables found to be statistically significant were diagnostic test results \( (p < 0.05) \) and Leaving Certificate mathematics points \( (p < 0.001) \). It was hoped that a more multi-dimensional model with more than two statistically significant predictor variables would have emerged when examining each of the prediction methods however that was not the case for this database.

A total of 8 steps were carried out during the discriminant analysis, the results of which are detailed in the section which follows.

5.4.1 Discriminant Analysis Findings: The Technology 2006-2008 database

5.4.1.1 Step 1: Selection of Appropriate Data

In order to create the discriminant function, the Technological mathematics 2006-2008 dataset was used. These students acted as the training sample. This data set consists of 1,080 students. Before the discriminant analysis took place checks were carried out on each of the predictor variables (Leaving Certificate mathematics points and diagnostic test results) to determine whether there was any missing data. If information was found to be missing for these variables, a search of the student records system in UL was carried out and any missing information was included. These two variables were chosen as they were found to have statistically significant relationships with success and failure in service mathematics. There were however some concerns in relation to using Leaving Certificate mathematics points and diagnostic test results as predictor variables which are outlined in section 5.4.1.5 (a).

The large dataset was useful for discriminant analysis as the violation of assumptions is not as problematic with large sample sizes (Pallent 2007). The reason for this is that in spite of a violation of the assumption of normality or equal variances, the assumptions can often be assumed to be met as small departures from normality give rise to statistically significant results when large sample sizes are being analysed. The Technology 2006-2008 dataset was checked and considered appropriate for discriminant analysis. As previously mentioned, the Technology 2006-2008 dataset consists of 1,080 students in total. Some students however do not have a Leaving Certificate mathematics grade, a diagnostic test result or an end of semester examination
result. The majority of the discriminant analysis in this chapter is based on data from students who have a Leaving Certificate mathematics grade with the exception of the analysis discussed in section 5.10.1 which also includes mature students, students who have engaged in previous study and international students who did not sit the Irish Leaving Certificate mathematics examination but who have a diagnostic test result.

5.4.1.2 Step 2: Satisfying Assumptions

Certain assumptions underlying discriminant analysis were satisfied before the analysis was carried out:

- **Assumption 1**: For each of the two populations (success/failure) the predictor variables had to have a multivariate normal distribution.
- **Assumption 2**: It was also assumed that the co-variance matrix is the same in the two populations.
- **Assumption 3**: That there was a representative sample from each of the two populations.

Checks on these assumptions were carried out to examine if they were satisfied for the Technology 2006-2008 dataset. The following results emerged:

**Assumption 1**: The two populations (success/failure) for each predictor variable were required to have multivariate normal distribution. This was assessed using normality tests in SPSS and through an analysis of histograms. Normality was satisfied when the p-value > 0.05.

When the Technology 2006-2008 cohort was examined in terms of the distribution of successful students for Leaving Certificate mathematics points the test for normality was not satisfied (p < 0.001). The same results emerged for unsuccessful students’ Leaving Certificate mathematics distribution and successful students’ diagnostic test distribution (p < 0.001). In spite of this, the shape of the histogram for these distributions showed relatively normally distributed data (see figures 5.3, 5.4 and 5.5). Normality will therefore be assumed. Small departures from normality give rise to statistically significant results in the normality tests when the sample sizes are very large. The distribution of unsuccessful students’ diagnostic test results was found to be normally distributed (p = 0.200) (see figure 5.6).
Figure 5.3 Distribution of Leaving Certificate mathematics points of successful Technology mathematics students 2006-2008.

Figure 5.4 Distribution of Leaving Certificate mathematics points of unsuccessful Technology mathematics students 2006-2008.
Figure 5.5 Distribution of diagnostic test results of successful Technology mathematics students 2006-2008.

Figure 5.6 Distribution of diagnostic test results of unsuccessful Technology mathematics students 2006-2008.

Note: Q-Q plots and normality tables relating to assumption 1 for the Technology 2006-2008 dataset can be found in Appendix G.
**Assumption 2:** A test for equal co-variance in the two populations (success/failure) for each predictor variable was carried out in order to test if assumption 2 was met. The results of these tests, which were carried out using SPSS, were found in the discriminant analysis output. The null hypothesis assumed equal variances. The large differences in the co-variances for the Technology dataset can be seen in table 5.1. Box’s M test examined whether the diagonal elements of the variance/co-variance matrix are equal in which case group variances can be considered the same (Field 2009). In the case of Technology students, the diagonal elements for both success and failure are not close in value. Box’s M test also states that if the off-diagonal elements are approximately zero the independent variables are not correlated (Field 2009). Table 5.1 shows off diagonal values of 59.537 for the successful students and 14.368 for the unsuccessful students neither of which are close to zero, hence suggesting that the independent variables are correlated. This is further examined in section 5.4.1.4.

<table>
<thead>
<tr>
<th>Performance in Technology mathematics</th>
<th>Points</th>
<th>Diagnostic test</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Success</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leaving Certificate Mathematics Points</td>
<td>192.816</td>
<td>59.537</td>
</tr>
<tr>
<td>Diagnostic test</td>
<td>59.537</td>
<td>37.970</td>
</tr>
<tr>
<td><strong>Failure</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Leaving Certificate Mathematics Points</td>
<td>39.772</td>
<td>14.368</td>
</tr>
<tr>
<td>Diagnostic test</td>
<td>14.368</td>
<td>20.342</td>
</tr>
</tbody>
</table>

**Table 5.1** Co-variance of successful and unsuccessful students according to Leaving Certificate mathematics points and diagnostic test results for Technology mathematics students 2006-2008.

The results found in Box’s M Test of Equality of Co-variance Matrices (see table 5.2) show that data from the Technology 2006-2008 cohort violated the assumptions of homogeneity of variance/co-variance matrices (p < 0.001). Tabachnick and Fidell (2007) warn however that Box’s M test is often too strict when large data sample sizes are being used and so this assumption will be assumed to be satisfied.
Table 5.2 Formal test for equal co-variance for Technology 2006-2008 successful and unsuccessful students.

<table>
<thead>
<tr>
<th>Box's M</th>
<th>114.664</th>
</tr>
</thead>
<tbody>
<tr>
<td>F</td>
<td>38.033</td>
</tr>
<tr>
<td>Approx.</td>
<td></td>
</tr>
<tr>
<td>df1</td>
<td>3.000</td>
</tr>
<tr>
<td>df2</td>
<td>1059696.636</td>
</tr>
<tr>
<td>Sig.</td>
<td>.000</td>
</tr>
</tbody>
</table>

If assumptions 1 and 2 are not satisfied the result may be a discriminant classification procedure which produces sub-optimal results. This could have a large influence on how effectively the results of the discriminant function can infer that what is happening for the training sample will happen also for the population. Recognition of the implications of the large sample size (Tabachnick and Fidell 2007) and a demonstration of caution throughout the discriminant analysis process however has led to a confidence that this situation (theoretically unsatisfied assumptions) will not negatively affect the overall results of the analysis being carried out.

**Assumption 3:** There is a representative sample of each of the two populations used in the analysis of the Technology 2006-2008 dataset. Each dataset being used has a large quantity of both successful and unsuccessful students in end of term service mathematics examinations over a three year period and so this assumption is met.
5.4.1.3 Step 3: Exploratory Data Analysis

Once all assumptions were examined and met, an exploratory data analysis was carried out. This analysis described each of the continuous variables (Leaving Certificate mathematics points and diagnostic test results) by success/failure. It established whether there are differences between the means of each independent variable for success and failure.

The Technological 2006-2008 dataset had a mean Leaving Certificate mathematics points for successful students of 58.3 (SD = 13.9) and for those who failed their end of term service mathematics examination it was 42.5 (SD = 6.9). In the case of mean diagnostic test results (expressed as a mark out of 40) for this group, successful students had a mean value of 22.3 (SD = 6.6) and unsuccessful students had a mean value of 16.2 (SD = 6.3).

The Technology dataset had relatively different mean values for each independent variable for success and failure. This finding suggested that the predictor variables have the ability to distinguish between success and failure within a discriminant function.

5.4.1.4 Step 4: An Examination of Predictor Variables

The correlations between the predictor variables were examined next. The following results were found:

The independent variables had a correlation value of \( r = 0.730 \). This indicated that the independent variables (Leaving Certificate mathematics points and diagnostic test results) were highly correlated and so the individual variables did not necessarily offer a huge amount of unique information to the discriminant function.

As the two predictor variables were highly correlated \( (r = 0.73) \) within the Technology 2006-2008 database they were likely to have contributed some shared information to the analysis. Uncorrelated variables are preferable in any discriminant analysis. This finding is something which will be considered when analysing the success of the functions ability to correctly classify students.
5.4.1.5 Step 5: Creating the Discriminant Function

Discriminant analysis can be carried out using SPSS once the checks and assumptions have been met. Once the discriminant analysis output had been created some computations must be carried out in order to obtain the co-efficients of the discriminant function. As highlighted in section 5.3.2 (b) variable Z is created, which is a linear combination of independent variables.

The dividing point C is also calculated using information in the discriminant analysis output (see Appendix H). In the case of this discriminant analysis students with values \( Z \geq C \) are classified in the successful group and those with \( Z < C \) are classified in the unsuccessful group.

When discriminant analysis was carried out in SPSS using the Technology 2006-2008 database the following Fisher’s Discriminant Function was computed:

\[
Z = 0.067 \text{(Leaving Certificate Mathematics Points)} + 0.087 \text{(Diagnostic Test Result)}, \text{ where } C = 5.078
\]

Therefore if an individual in this dataset had a Z value of \( Z \geq 5.078 \), the individual was classified as being successful and if \( Z < 5.078 \), the individual was classified as being unsuccessful.

Essentially if a student enters UL with more than 75 Leaving Certificate mathematics points they are guaranteed to be predicted to be successful in Technology mathematics by this discriminant function. Alternatively if a student enters with less than or equal to 75 points they can still be predicted to be successful in Technology mathematics if they complete some questions on the diagnostic test correctly. A student’s probability of failure is calculated using the following formula:

\[
1 - \frac{1}{1 + \exp(-Z + C)}
\]

For example, a student who enters UL with 90 Leaving Certificate mathematics points and 30 in the diagnostic test has a probability of failure of 0.03, while a student who enters with 40 Leaving Certificate mathematics points and 18 in the diagnostic test has a probability of failure of 0.70.

This discussion highlights that Leaving Certificate mathematics points are more influential on the prediction of failure in the case of this discriminant function (see section 5.4.1.7 for more details on this). The performance of this discriminant function in classifying the training sample, the relative effect of each independent variable and the ability of the discriminant function to
classify a validation dataset are outlined in section 5.4.1.6, 5.4.1.7 and 5.4.1.8. Prior to this however some discussion relating to the potential limitations which the predictor variables used may have on the accuracy of the discriminant function’s predictions are outlined.

5.4.1.5 (a) Potential Limitations of the Predictor Variables on the Discriminant Analysis

There were several concerns regarding the limitations which the chosen independent variables may have when attempting to predict performance in service mathematics. These concerns are outlined in this section.

1. Anticipated Limitations Arising from Leaving Certificate Mathematics Grades with a 50% Pass Rate

From chapter 4, it was evident that some of the Leaving Certificate grades, such as OLB2-OLC3, had approximately a 50% success and 50% failure rate in Technology mathematics (see table 4.17). Likewise similar results were presented in the case of the Science mathematics students (see table 4.27). This presented a potential limitation as the binary classification of results makes it difficult to decipher whether students should be predicted to pass or fail service mathematics with these grades. This could potentially affect the overall accuracy of the predictions. The presence of the probability of failure which is provided by discriminant analysis however partially eliminates this issue as students can be examined in terms of not only their binary classification but also their probability of failure. The probability of failure is therefore a major advantage of a discriminant function as a classification tool over the binary ‘at risk’/ not ‘at risk’ which is offered by the current system. The next section highlights some concrete examples of the potential benefits of having a student’s probability of failure when it comes to students with grades which tend to have a 50% pass rate in service mathematics (Note: the probabilities of performance detailed in table 5.3 were determined using the Technology 2006-2008 discriminant function).
Example Profile of Students with Leaving Certificate Grades which have a 50% Pass Rate in Service Mathematics

As previously mentioned, OLB2-OLC1 are examples of Leaving Certificate grades which have approximately a 50% pass rate in service mathematics in the Technology 2006-2008 database. The example profiles of students with these grades given in table 5.3 highlight the effectiveness of the probability of failure when dealing with Leaving Certificate grades which tend to have a 50% pass rate in service mathematics.

<table>
<thead>
<tr>
<th>Example Student 1</th>
<th>Leaving Certificate Grade</th>
<th>Diagnostic Test Result</th>
<th>At Risk/Not at Risk according to the Diagnostic Test</th>
<th>Probability of Failure</th>
<th>Performance in Technology Maths Exam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example Student 2</td>
<td>OB2</td>
<td>18</td>
<td>At risk</td>
<td>0.70</td>
<td>Failed 31%</td>
</tr>
<tr>
<td>Example Student 3</td>
<td>OB3</td>
<td>16</td>
<td>At risk</td>
<td>0.80</td>
<td>Failure 32%</td>
</tr>
<tr>
<td>Example Student 4</td>
<td>OB3</td>
<td>23</td>
<td>Not at risk</td>
<td>0.68</td>
<td>Failure 23%</td>
</tr>
<tr>
<td>Example Student 5</td>
<td>OC3</td>
<td>13</td>
<td>At risk</td>
<td>0.93</td>
<td>Failure 19%</td>
</tr>
</tbody>
</table>

Table 5.3 Example profile of Technology mathematics students with Leaving Certificate grades which have roughly 50% pass rate.

If one relies on Leaving Certificate mathematics grades alone to determine the probable performance of the students in table 5.3 for example, one essentially must guess whether a student is likely to pass or fail due to the roughly 50% pass rate of the grades in question. The probability of failure however which uses both Leaving Certificate grade and diagnostic test result gives an accurate indication of how the student is likely to perform (see table 5.3). Example student number 2 was not considered to be ‘at risk’ of failing service mathematics based on their diagnostic test result but they also had a grade which tended to have a 50% pass rate in service mathematics. The probability of failure given by the discriminant function however highlighted that the student was more likely to be unsuccessful in service mathematics which they evidently were. Although the borderline grades will present the discriminant function
with difficulties in terms of the accuracy of its binary classification of performance, the use of the diagnostic test and the calculation of the probability of failure which it also provides will eliminate this issue somewhat. The anticipated limitation which the CAO points system may have on the discriminant analysis is detailed next.

2. Anticipated Limitations of the CAO Points System

The CAO point system does not work on an equal interval scale. There is a ten points difference between an OLA1 grade and an OLA2 grade while all of the other increments between consecutive grades are just 5 points. This unusual scaling system may lead to issues in the discriminant function’s ability to accurately predict performance. These issues are explored and addressed further in section 5.8 of this chapter. The final concern which needs to be highlighted before the results of the discriminant analysis are outlined is the potential issues that using diagnostic test results as a predictor variable may raise.

3. Anticipated Limitations of Diagnostic Test Results as a Predictor Variable

Students’ diagnostic test scores have been divided into those who received between 0 and 10 (Group 1), 11-20 (Group 2), 21-30 (Group 3) and 31-40 (Group 4). Table 5.4 shows the relationship between performance in Technology mathematics and students’ diagnostic test score group. Similar to the case with Leaving Certificate mathematics grades as students’ performance in the diagnostic test increases so too does the prevalence of success in service mathematics examinations. Approximately half of the students in group 1 passed and half failed with a high proportion of group 2 students (37.7%) failing service mathematics also. This may make it difficult to predict performance using diagnostic test results as an independent variable for the students who are a part of groups 1 and 2 as it is not decisive either way as to whether these students have a greater tendency to pass or fail service mathematics. As was the case with Leaving Certificate grades which tended to have a pass rate of 50% however, the two dimensional discriminant analysis will provide a probability of failure which will give more insight into students’ probable performances than just the binary ‘at risk’/not ‘at risk’ method. Appendix F further emphasises the incidences in which students’ with Leaving Certificate
mathematics grades that have approximately a 50% pass rate and with diagnostic test scores in group 2 are difficult to predict for. For example, students with OLB1-OLB3 grades and who are in group 2 in Technology mathematics appear a similar amount of times in the success and the failure sections of this cross-tabulation (see Appendix F).

<table>
<thead>
<tr>
<th>Diagnostic Test Group</th>
<th>Technology Mathematics Performance</th>
<th>Success</th>
<th>Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Group 1</td>
<td>30</td>
<td>35</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>(46.2%)</td>
<td>(53.8%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Group 2</td>
<td>224</td>
<td>133</td>
<td>357</td>
</tr>
<tr>
<td></td>
<td>(65.7%)</td>
<td>(37.3%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Group 3</td>
<td>379</td>
<td>31</td>
<td>410</td>
</tr>
<tr>
<td></td>
<td>(92.4%)</td>
<td>(7.6%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Group 4</td>
<td>82</td>
<td>0</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>(100%)</td>
<td>(0%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>715</td>
<td>199</td>
<td>914</td>
</tr>
</tbody>
</table>

Table 5.4 Performance in Technology mathematics against Diagnostic Test group (2006-2008).

The potential limitations which using the predictor variables of Leaving Certificate mathematics points and diagnostic test results may have on the overall accuracy of the discriminant function’s predictions have been acknowledged in this section. These potential limitations and the potential means of addressing them will be considered in light of the discriminant analysis findings for Technology mathematics students which are outlined next in section 5.4.1.6.
5.4.1.6 Step 6: Determining How Well the Discriminant Function Performed

Upon creating the discriminant function it was important to determine how well it did in terms of correctly classifying people as being successful in service mathematics examinations or not. This was determined by interpreting the discriminant analysis output using the ‘Classification Results’ table. This table provided a breakdown of the percentage of correctly classified individuals within each category (success or failure) as well as the total of correctly classified individuals within the training sample. The Technology discriminant function correctly classified 68.2% of successful cases, 84.4% of unsuccessful cases and overall it correctly classified 71.3% of cases (see table 5.5). It is important that the function has a high percentage of correct classification of unsuccessful students as these are the students who need to be most aware of their probability of failure. The discriminant function was therefore moderately successful in its predictions of service mathematics performance. The predictor variables, Leaving Certificate mathematics points and diagnostic test results, had the ability to explain some variability in service mathematics performance but not all. The anticipated limitations of the discriminant function as outlined in section 5.4.1.5 (a) such as certain grades and diagnostic test results having a 50% pass rate as well as issues with the CAO points system are likely to have had an influence on the overall accuracy of the predictions made by the function. Information relating to the variable which had the largest impact on correct classification is detailed in section 5.4.1.7.

<table>
<thead>
<tr>
<th>Predicted Group Membership</th>
<th>Predicted Success</th>
<th>Predicted Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance in Technology mathematics</td>
<td>Success</td>
<td>448 (68.2%)</td>
<td>209 (31.8%)</td>
</tr>
<tr>
<td></td>
<td>Failure</td>
<td>24 (15.6%)</td>
<td>130 (84.4%)</td>
</tr>
</tbody>
</table>

Table 5.5 Technology discriminant function’s ability to correctly classify students within the training sample.
5.4.1.7 Step 7: Determining the Relative Effect of each Variable

Further insight was achieved from the discriminant analysis output when an examination of the ‘Pooled within Groups Matrices’ table was carried out. The relative effect of each variable in the function was calculated from the standardised discriminant co-efficients which were given in this table within the output. This value was calculated by determining the square root of the pooled standard deviation for each variable multiplied by its corresponding co-efficient which was calculated in step 5 (section 5.4.1.5). The absolute value of the resulting calculations determined which variable had the greatest effect on the discriminant function. The larger absolute value had the greatest effect on the function.

The Technological 2006-2008 discriminant function variables had standardised discriminant co-efficients of 0.857 for Leaving Certificate mathematics points and 0.512 for diagnostic test score. This highlighted that Leaving Certificate mathematics points had a larger relative effect on the discriminant function when compared to diagnostic test results. This finding was alluded to in the discussion in section 5.4.1.5 also as a cut-off point for the prediction of performance could be determined using Leaving Certificate mathematics points alone if desired (i.e. if students have more than 75 Leaving Certificate mathematics points they will be predicted to be successful by the discriminant function) with the diagnostic test element of the function helping to refine the prediction only.

5.4.1.8 Step 8: Testing the Discriminant Function on a Validation Dataset

Finally the discriminant function that was created was tested on another dataset, whose classifications were known, to see how well it performed in correctly classifying students in a validation dataset. The validation dataset used contained information on Technology 2009 students. The Technology 2009 dataset contained 282 students with both a Leaving Certificate mathematics grade and a diagnostic test result.

When the Technology 2006-2008 function was tested on the Technological 2009 dataset it misclassified 14.8% (9) of students who failed their end of term examination as being successful (see table 5.6). The discriminant function however correctly classified 85.2% of the unsuccessful Technology 2009 students. The function was therefore quite successful in its classification of unsuccessful students in a validation dataset.
### Table 5.6 Technology 2006-2008 discriminant function’s ability to classify Technological 2009 cohort.

<table>
<thead>
<tr>
<th></th>
<th>Predicted Success</th>
<th>Predicted Failure</th>
<th>Total</th>
<th>% Correctly Classified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success in Technology</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mathematics</td>
<td>130 (58.8%)</td>
<td>91 (41.2%)</td>
<td>221</td>
<td>182 (64.6%)</td>
</tr>
<tr>
<td>Failure in Technology</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mathematics</td>
<td>9 (14.8%)</td>
<td>52 (85.2%)</td>
<td>61</td>
<td></td>
</tr>
</tbody>
</table>

5.4.2 Discriminant Analysis Findings: A Summary

When the performance of the Technology 2006-2008 discriminant function was analysed, it was found to be moderately successful in its correct classification of the training sample and a validation dataset while being slightly more successful where the students who failed service mathematics were concerned. Although Leaving Certificate mathematics points were found to have a larger relative effect on the discriminant function with diagnostic test results helping to refine the prediction of performance, both predictor variables were only able to explain some of the variability in service mathematics performance. Concerns regarding the issues that the CAO points system and Leaving Certificate mathematics grades which have approximately 50% pass mark may be causing the discriminant function have been noted and an attempt to overcome these potential issues is outlined later in the chapter (see section 5.8). First however the Technology 2006-2008 discriminant function is used for a more in-depth analysis into ‘at risk’ service mathematics students (see section 5.5).
5.5 An Analysis of Probabilities of Failure by Groups

Although the Technology 2006-2008 discriminant function was only moderately successful in terms of its correct binary classification of service mathematics performance, an investigation into the probabilities of group membership was undertaken in an attempt to gain a deeper insight into students’ probable performances in service mathematics. A breakdown of the number and percentage of students who had a particular probability of failure can be seen in table 5.7. The probabilities of success and failure, which were determined by the Technology 2006-2008 discriminant function, were broken down into 3 groups:

- Group 1: 0 - 0.33 probability of failure, i.e. low risk
- Group 2: 0.34 - 0.66 probability of failure, i.e. medium risk
- Group 3: 0.67 - 1.00 probability of failure, i.e. high risk

Within the Technology 2006-2008 cohort, 42.3% of students could have been quite confident that they would be successful in their end of term examinations as their probability of failure was so low. 21.6% of students were unlikely to be successful in their service mathematics examinations as the discriminant function predicted that they had between 0.67-1.00 chance of failing (see table 5.7).

<table>
<thead>
<tr>
<th>Probability of Failure Group</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>343</td>
<td>42.3</td>
</tr>
<tr>
<td>Group 2</td>
<td>293</td>
<td>36.1</td>
</tr>
<tr>
<td>Group 3</td>
<td>175</td>
<td>21.6</td>
</tr>
<tr>
<td>Total</td>
<td>811</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 5.7 Breakdown of probability failure groups of Technological mathematics students between 2006 and 2008 as determined by the Technology discriminant function.

A breakdown of students into probability groupings such as this allows one to gain a more insightful perspective into the proportion of students who are ‘at risk’ of failing service mathematics and also to what extent they are ‘at risk’ (low, medium, high risk). This method of identifying the probability of failure has an advantage over the current ‘at risk’/ ‘not at risk’ system as students and lecturers can be clear on exactly how many students are falling into each probability group and therefore are aware of the extent of support which is required. For example, a student who is 90% likely to fail service mathematics is no longer put into the same...
category as a student who is 50% likely to fail. This allows for more specific teaching interventions to be implemented. The discriminant function also has the added benefit of being an evidence based cut off point for predicting failure as opposed to a subjective expert opinion which is the case with the current system (i.e. the diagnostic test). This information will therefore be considered when planning for and implementing the teaching intervention in this research project (see chapter 6). An examination into how accurate the discriminant function was in predicting performance by probability group is outlined in section 5.5.1.

5.5.1 The Success of the Technology 2006-2008 Discriminant Model by Probability Grouping

Closer analysis of the discriminant model may provide insight into which students the model performs well in classifying and those with which it is not as successful in classifying. This section therefore outlines the examination of the performance of the function by probability group. An analysis of its performance for the Technology 2006-2008 database was carried out. The Technology 2006-2008 discriminant function was highly successful in correctly classifying students with probabilities of failure between 0 and 0.33 (those in group 1). Of the 343 students in group 1, 336 (98.0%) were successful in passing Technology mathematics and so, the model was correct in assigning the majority of these students to the group with the lowest probability of failing service mathematics. The function however was not quite as successful in correctly classifying those in the highest probability of failure group (Group 3) with only 88 (50.3%) of the 175 students in this group failing Technology mathematics. 87 (49.7%) of the students in group 3 were successful in the examination in spite of the discriminant function assigning them a probability of failure between 0.67 and 1.00. This group however does have the highest proportion of failure students (57.2%). The majority of students in group 2 (i.e. those who were assigned a probability of failure between 0.34 and 0.66) were successful in Technology mathematics. A total of 234 (79.9%) of group 2 students were successful (see table 5.8). Group 2 students are those who have the probabilities of failure which lie around the peripheries of being successful or not. The advantage of knowing their probabilities of failure is that one is not relying on a binary classification of their performance.
Table 5.8 Technology 2006-2008 discriminant model prediction of Technology 2006-2008 cohort by probability group.

<table>
<thead>
<tr>
<th>Performance in Technology Mathematics</th>
<th>Probability Failure Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Success</td>
<td>336</td>
</tr>
<tr>
<td>%within Success</td>
<td>(51.2%)</td>
</tr>
<tr>
<td>% within Probability Failure Group</td>
<td>(98.0%)</td>
</tr>
<tr>
<td>Failure</td>
<td>7</td>
</tr>
<tr>
<td>%within Failure</td>
<td>(4.5%)</td>
</tr>
<tr>
<td>% within Probability Failure Group</td>
<td>(2.0%)</td>
</tr>
</tbody>
</table>

5.5.1.1 Summary

The Technology 2006-2008 discriminant function performed quite well in establishing that the students in group 1 were highly likely to pass service mathematics as the majority of these students were successful in Technology mathematics. As group 2 students had a probability of between 0.34 and 0.66, it was understandable that classifying these students as being successful or unsuccessful was not as straightforward as the case might be when classifying students from the other two groups. The actual performance of group 2 students in this database showed that the majority of them were successful in service mathematics. This is a finding in itself that the majority of students who lie around the 50% probability of failure mark are more likely to be successful than not in service mathematics. The function was cautious in its prediction of group 3 students however it correctly classified 50.3% of all students who failed within this group as being highly ‘at risk’ of failing service mathematics which they evidently were.

A comparison of the ability of the diagnostic test and the Technology 2006-2008 discriminant function to correctly classify students according to their service mathematics performance and a discussion on the pros and cons of each classification method is outlined in section 5.6.
5.6 Comparison of the Diagnostic Test and the Technology 2006-2008  
Discriminant Function’s Ability to Correctly Classify ‘at risk’ Students

One of the main objectives of this research was to find a more sophisticated method of predicting failure in service mathematics than the diagnostic test’s criteria of ≤ 19 out of 40. It was therefore necessary, after carrying out discriminant analysis, to compare the performance of both the discriminant function and the diagnostic test in predicting performance and to discuss the pros and cons of each classification method, the results of which are detailed in this section.

5.6.1 The Ability of the Diagnostic Test to Successfully Predict Performance

The diagnostic test’s ability to correctly classify Technology mathematics students between 2006-2008 was examined first. This examination revealed that 485 (67.8%) of the students who passed Technology mathematics were correctly classified as being ‘not at risk’ of failing by the diagnostic test cut-off (see figure 5.7). Of the 199 students who failed service mathematics, the diagnostic test results identified 145 (72.9%) of these students as being ‘at risk’. The diagnostic test therefore failed to forewarn 54 (27.1%) of the students who failed service mathematics that they were ‘at risk’ of failing. Overall 630 (68.9%) of the students in the Technology 2006-2008 dataset were correctly classified by the diagnostic test (see table 5.9).

![Figure 5.7](image-url)

**Figure 5.7:** Examination of the ability of the diagnostic test to correctly classifying students (success/failure) in the Technology 2006-2008 cohort.
### Table 5.9
Diagnostic tests ability to correctly classify Technology 2006-2008 students in terms of service mathematics performance.

<table>
<thead>
<tr>
<th>Performance in Technology Mathematics</th>
<th>Not at risk</th>
<th>At risk</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success</td>
<td>485</td>
<td>230</td>
<td>715</td>
</tr>
<tr>
<td></td>
<td>67.8%</td>
<td>32.2%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Failure</td>
<td>54</td>
<td>145</td>
<td>199</td>
</tr>
<tr>
<td></td>
<td>27.1%</td>
<td>72.9%</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

5.6.1.1 **Summary**

The diagnostic test was therefore reasonably successful in its prediction of failure in service mathematics and it was overcautious in its classification of ‘at risk’ students. Within the Technology 2006-2008 database, it correctly classified 630 (68.9%) students in total. The diagnostic test could not make a performance prediction for 166 (15.4%) of the Technology cohort due to these students not taking the diagnostic test. It is important to reiterate at this point that the diagnostic test classification method was developed by the expert subjective judgement of a Professor of Mathematics Education in UL. He determined what he considered to be a student who was ‘at risk’/not ‘at risk’ of failing service mathematics. The results reported in this section therefore suggest that the expert judgement which was used was reasonably accurate.
5.6.2 The Ability of the Technology 2006-2008 Discriminant Function to Successfully Predict Performance

The success of the Technology 2006-2008 discriminant function in its prediction of successful Technology students was quite similar to that of the diagnostic test. The discriminant function successfully classified 448 (68.2%) of the students who were successful in Technology mathematics (see Figure 5.8). Like the diagnostic test classification, it was overcautious in its prediction of failure as it classified 209 (31.8%) of students in the failure group who passed the examination. It was more successful in its prediction of failure, correctly classifying 130 (84.4%) of the students who failed Technology mathematics (see Table 5.10).

Figure 5.8 Examination of the ability of the Technology discriminant function to correctly classifying students (success/failure) in the Technology 2006-2008 dataset.

<table>
<thead>
<tr>
<th>Performance in Technology Mathematics</th>
<th>Technology 2006-2008 Discriminant Function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Success</td>
</tr>
<tr>
<td>Success</td>
<td>448</td>
</tr>
<tr>
<td>68.2%</td>
<td>31.8%</td>
</tr>
<tr>
<td>Failure</td>
<td>24</td>
</tr>
<tr>
<td>15.6%</td>
<td>84.4%</td>
</tr>
</tbody>
</table>

Table 5.10 Technology 2006-2008 discriminant function’s ability to correctly classify Technology 2006-2008 students in terms of service mathematics performance.
5.6.2.1 Summary
The Technology 2006-2008 discriminant function was slightly more successful in the percentage of correctly classified failure students than the diagnostic test was. It was also slightly more successful in terms of its overall correct classification of the Technology cohort as it correctly classified 578 (71.3%) of the students. The Technology 2006-2008 discriminant function however could not classify 269 (24.9%) of the students in this database, which is higher than that of the diagnostic test, due to students not sitting the diagnostic test and/or not having a Leaving Certificate mathematics grade.

5.6.3 Conclusion
Both methods of prediction were moderately successful in their prediction of students who were unsuccessful. The Technology 2006-2008 discriminant function correctly classified a slightly higher percentage of unsuccessful students within the Technology 2006-2008 database and it also correctly classified a slightly higher percentage of students overall. The discriminant analysis also has some advantages over the diagnostic test as a method of prediction. Discriminant analysis provides each student with a probability of failure as opposed to just a statement which tells them that they are ‘at risk’ or that they are not ‘at risk’. It provides a student with some insight into the extent to which they may need extra mathematical support, if at all, as opposed to telling them that they do or do not need it. In addition to this, the discriminant function is a classification method which is evidence based as opposed to the diagnostic test classification which is based on a subjective expert opinion only. Naturally the diagnostic test can only assess students who sit it. Likewise a prediction of performance can only be given to students who sat the diagnostic test and also have a Leaving Certificate mathematics grade. The diagnostic test has an advantage over the discriminant function in this sense as fewer students were left out of the prediction (i.e. 15.4% of the Technology 2006-2008 cohort were not classified by the diagnostic test compared to 24.9% unclassified students by the discriminant function). Both methods found the binary classification of students’ service mathematics performance difficult however. Some of the possible contributing factors to the prediction methods not performing better than they did are discussed in section 5.7.
5.7 Challenges Faced when Predicting Service Mathematics Performance in UL

From the discussion of performance of the various prediction methods it can be seen that the prediction of students’ service mathematics performance in UL is difficult. Although the diagnostic test and the Technology 2006-2008 discriminant function had a moderate level of success in terms of correct classification of students by service mathematics performance, there are several potential factors which may have negatively affected their ability to correctly classify students. Some such issues are discussed in this section.

5.7.1 Increasing Poor Attendance at the Diagnostic Test

Between the years 2006-2008, 166 students registered for Technological mathematics did not attend their first lecture and therefore did not sit the UL diagnostic test. The increased incidence of students not turning up to take the diagnostic test naturally affects the ability of the diagnostic test to forewarn certain students that they may need mathematical support. In addition to this, 33.1% of people who did not sit the diagnostic test failed the Technology mathematics examination compared to 21.8% of students who did sit the diagnostic test and also went on to fail Technology mathematics (see table 5.11). This suggests that if students do not turn up for their first lecture to take the diagnostic test they are at a higher risk of failing Technological mathematics than those who do sit the diagnostic test. A chi-square test showed that there is a statistically significant association between success/failure and whether or not a student sits the diagnostic test (p < 0.001). It would be interesting to establish if students who do not turn up to their first lecture have poor attendance throughout the semester. Unfortunately this data was not available however it may be an area of future research worth investigating. The profile of the students who did not sit the diagnostic test however has been examined. Students who did not sit the diagnostic test were examined in terms of their gender, their Leaving Certificate mathematics performance, the degree programme in which they are enrolled and their performance in all subjects in first year (i.e. an examination of QCA values, see Appendix D). The aim of this investigation was to attempt to decipher if students who did not sit the diagnostic test have common characteristics.
<table>
<thead>
<tr>
<th>Did not sit Test</th>
<th>Success</th>
<th>Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>111</td>
<td>55</td>
<td>166</td>
</tr>
<tr>
<td></td>
<td>(66.9%)</td>
<td>(33.1%)</td>
<td>(100.0%)</td>
</tr>
<tr>
<td>Sat Test</td>
<td>715</td>
<td>199</td>
<td>914</td>
</tr>
<tr>
<td></td>
<td>(78.2%)</td>
<td>(21.8%)</td>
<td>(100.0%)</td>
</tr>
<tr>
<td>Total</td>
<td>826</td>
<td>254</td>
<td>1080</td>
</tr>
<tr>
<td></td>
<td>(76.5%)</td>
<td>(23.5%)</td>
<td>(100.0%)</td>
</tr>
</tbody>
</table>

Table 5.11 Number (and percentage) of successful/unsuccessful students who sat/did not sit the diagnostic test in Technological mathematics (2006-2008).

Overall, the profile of the students who did not sit the diagnostic test and the corresponding Technology mathematics students who did sit the test were found to be very similar. However, the majority of students who did not sit the diagnostic test and failed their end of semester mathematics examination were also found to be performing poorly in other modules, i.e. they have low QCA values and many failed year 1 of their degree programme, indicating that the issue with these students was not solely with mathematics. These students however were found to be significantly more likely to fail service mathematics than those who did sit the diagnostic test and so the fact that they are not being advised on whether or not they should seek extra mathematical support is a cause for concern. Students not sitting the diagnostic test also have an implication on discriminant analysis as it too cannot predict performance for these students. The increased incidence of poor attendance at the diagnostic test is therefore a cause for concern for both prediction methods.

### 5.7.2 Challenges Faced By Statistical Prediction Methods

Findings in chapter 4 highlighted the large differences between Higher Level and Ordinary Level students in terms of both their diagnostic test performance and service mathematics performance. Due to the fact that the discriminant analysis did not perform as well as would have been hoped in terms of correct classification of students according to service mathematics performance, an examination into the potential influence of the using Leaving Certificate mathematics points with equivalent grades on the success of predictions was carried out. Before the examination of the
performance of students with equivalent Leaving Certificate grades is outlined an overview of how the Leaving Certificate points system was devised is detailed.

**How was the Leaving Certificate Points System Established?**

Prior to the points system being used as a point of entry to third level education in Ireland all institutions had their own entry system. This meant that even within institutions, different departments could have different entry requirements for beginning undergraduates. In 1990 the institutions offering third level education courses in Ireland and the Department of Education came together and set up the points system. The decisions on what points were to be awarded to each grade were decided upon by representative heads of admissions, registrars and academics from all institutions around Ireland in conjunction with the Department of Education. The Central Applications Office then applied this points system as a means of entry into third level courses (Gleeson 2011).

The breakdown of the allocation of points to different grade bands in Higher Level examinations were determined based on an equivalent percentage grading scheme i.e. 100% and 100 points for a completely correct examination script. The Ordinary Level grades were then awarded points based on how the standard of work compared to a Higher Level paper therefore those who implemented the system felt that a student with a Higher Level C3 grade had a similar ability level in a particular subject area to a student with an Ordinary Level A1 grade. Likewise the Higher Level D2 was deemed to be a similar standard to an Ordinary Level A2 grade and a Higher Level D3 similar to an Ordinary Level B1 grade. These decisions were aided by the examination of past Leaving Certificate scripts from different subject areas.

There are 35 subjects offered in the Leaving Certificate, the majority of which have the same syllabus for Higher Level and Ordinary Level. The Ordinary Level students do not cover all of the material on the syllabus and less detail and depth is expected. The mathematics syllabus, however, does not work in this manner. There is one syllabus for the Higher Level course and a different syllabus for the Ordinary Level Leaving Certificate mathematics course. There is naturally some overlap in the material which is covered however there are several instances in which material is covered on one syllabus and not the other. For example the Higher Level course covers integration and trigonometric differentiation and the Ordinary Level syllabus does not. The Ordinary Level syllabus covers Simpson’s Rule and the Higher Level syllabus does not.
The fact that the points system was established for all subject areas and different subjects are set out in different manners (i.e. 1 syllabus covered by both Higher and Ordinary Level or 2 different syllabi) means that the system may be more effective in appropriately grading some subjects over others. The overall goal however by the third level institutions in Ireland and the Department of Education was to standardise the entry system to all third level education courses which the points system does. The fact that the awarding of points to different grades is somewhat subjective and may facilitate some subject areas better than others caused the author to question how accurate it was in determining and differentiating between the mathematical literacy of students in the UL database. Discussion surrounding its effectiveness is detailed in the following section in terms of the failure rates in service mathematics of Higher Level and Ordinary Level mathematics students.

**Failure Rates of Higher Level and Ordinary Level Students in Service Mathematics: A Comparison of Equivalent Grades**

Findings in table 4.16 and 4.26 (see chapter 4) highlighted the considerable difference between Higher Level and Ordinary Level students in terms of their failure rates in service mathematics. The failure rate of 32.9% for Ordinary Level Technology students compared to 4.9% for Higher Level Technology students suggests that both groups are not equally prepared for third level mathematics. This finding is again emphasised by the data in table 4.17 and 4.27 (see chapter 4) which outlines failure rates specific to each Higher Level and Ordinary Level Leaving Certificate grade. For the most part the failure rates amongst the Ordinary Level grades are a lot higher than that of the Higher Level grades. Recall from table 2.1 that there are 3 pairings of Higher Level and Ordinary Level grades that are assigned the same number of CAO points (OA1 and HC3= 60 points; OA2 and HD2=50 points; OB1 and HD3= 45 points). With the exception of one grade pairing, these grades have very different failure rates in Technology mathematics. Higher Level C3 students have a failure rate of 2.7% while Ordinary Level A1 students have a failure rate of 10.5% in Technology mathematics between 2006 and 2008. Similarly, Higher Level D2 students have a failure rate of 4.8% while Ordinary Level A2 students have a failure rate of 18.9%. The final pair of equivalent grades do however have similar failure rates of 40.0% for Higher Level D3 students and 36.6% for Ordinary Level B1 students (see table 4.17, chapter 4). The performance of students with equivalent grades is investigated further in terms of their mean diagnostic test performance and their mean service mathematics performance.
An examination of equivalent grades for both cohorts combined took place. In each pairing of equivalent grades the students with the Higher Level grade have a higher mean performance in the diagnostic test (see table 5.12) for Science and Technology mathematics 2006-2008. The differences between the mean performances were found to be statistically significant ($p < 0.001$ for OA1-HC3 and OB1-HD3, $p < 0.05$ for OA1-HC3). The spread of results for each pairing, as measured by the standard deviation, were not found to be statistically significantly different.

One of the possible reasons for this statistically significant difference in mean diagnostic test performance could be the difference in the Ordinary Level and Higher Level Leaving Certificate syllabi. As highlighted in section 3.4.2 (chapter 3), the diagnostic test contains 6 questions which are covered on the Higher Level Leaving Certificate syllabus only. These questions are contained in the Arithmetic, Differentiation and Integration sections of the diagnostic test. For this reason a comparison of each equivalent grades performance by topic on the diagnostic test was carried out to determine if the Ordinary Level students’ lower mean performance could be attributed to their weaker performance in the topics on the test which they had not engaged with. This examination revealed that the Ordinary Level students performed to a lower mean standard in each topic on the diagnostic test with no exceptions. The differences between the means in each topic were found to be statistically significant ($p < 0.05$). These findings therefore suggest that Ordinary Level students are less mathematically prepared on entry to third level education when compared to their Higher Level equivalent grades.

A similar analysis was carried out on the Technology and Science 2006-2008 database to determine if there was a statistically significant difference between Ordinary Level and Higher Level students’ mean service mathematics performance. Table 5.12 highlights that all Higher Level grades perform to a higher mean standard when compared to their Ordinary Level grade equivalent. The analysis revealed that a statistically significant difference in mean service mathematics performance for all 3 grade pairings exists ($p < 0.05$).

Ordinary Level students were therefore found to be less mathematically prepared (as measured by the diagnostic test) than Higher Level students with equivalent points on entry to university and also less able to cope with third level mathematics examinations as highlighted by their mean service mathematics performance and higher failure rates. This begs the question is it correct to consider students with the same CAO points but having undertaken different levels of
mathematics at Leaving Certificate to have equivalent ability in terms of mathematical competency level in this database? Should there be consideration for reallocating points when it comes to Ordinary Level Leaving Certificate mathematics? While considering that the grade bands currently used in the Leaving Certificate Ordinary Level mathematics examinations may not be fully appropriate there are also other factors which must be considered such as those which were discussed in section 4.2.4.4 of chapter 4. These statistically significant differences in Ordinary Level and Higher Level students’ mean mathematics performance may be due, in part, to differences in mathematics teaching experiences at second level, learning styles, perceived ability and motivation.

<table>
<thead>
<tr>
<th></th>
<th>60 points</th>
<th></th>
<th>50 points</th>
<th></th>
<th>45 points</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OA1</td>
<td>HC3</td>
<td>OA2</td>
<td>HD2</td>
<td>OB1</td>
<td>HD3</td>
</tr>
<tr>
<td>Mean Diagnostic Test Results</td>
<td>21.39 (4.05)</td>
<td>24.86 (4.28)</td>
<td>19.71 (4.39)</td>
<td>21.84 (4.78)</td>
<td>17.38 (4.18)</td>
<td>22.71 (4.49)</td>
</tr>
<tr>
<td>n=181</td>
<td>n=71</td>
<td>n=239</td>
<td>n=31</td>
<td></td>
<td>n=238</td>
<td>n=24</td>
</tr>
<tr>
<td>End of Semester Performance</td>
<td>55.86 (18.20)</td>
<td>62.81 (17.76)</td>
<td>47.71 (16.15)</td>
<td>54.26 (17.27)</td>
<td>41.02 (15.83)</td>
<td>48.94 (21.33)</td>
</tr>
<tr>
<td>n=190</td>
<td>n=81</td>
<td>n=254</td>
<td>n=33</td>
<td></td>
<td>n=261</td>
<td>n=28</td>
</tr>
<tr>
<td>Failure Rates</td>
<td>10.5% (n=20)</td>
<td>4.9% (n=4)</td>
<td>17.7% (n=45)</td>
<td>6.1% (n=2)</td>
<td>32.6% (n=85)</td>
<td>28.6% (n=8)</td>
</tr>
</tbody>
</table>

Table 5.12 Mean Performance of equivalent grades of Science and Technology students between 2006 and 2008.

The fact that the equivalent grades equate to the same number of points but do not demonstrate the same level of mathematical competency is likely to have negatively impacted on the overall accuracy of the discriminant analysis’ predictions of failure.
5.7.3 Challenges Faced by All Prediction Methods in Irish Mathematics Education

Ordinary Level students were found to have a statistically significantly lower mean performance in service mathematics when compared to Higher Level, mature and international students as well as students who engaged in previous study prior to entering UL service mathematics (p < 0.001). Although a large proportion of Ordinary Level students are successful in service mathematics, their service mathematics performance is not always reflective of their diagnostic test performance or their Leaving Certificate mathematics points. As Leaving Certificate points and diagnostic test results were found to be the only statistically significant predictors of service mathematics performance within the UL database, and there is no consistent relationship between Ordinary Level students’ performance and these variables, it is an extremely difficult task to predict Ordinary Level students’ performance. Predicting third level service mathematics performance is therefore likely to be difficult for any Irish third level institution which admits large cohorts of Ordinary Level Leaving Certificate mathematics students.

Conclusion

There are several challenges for the prediction methods discussed in this chapter. Issues related to students’ poor attendance at the diagnostic test are something which neither the diagnostic test or any statistical prediction method has any control over however it is useful to know that there is a statistically significant association between students who do not sit the diagnostic test and service mathematics performance. The challenges faced by the discriminant analysis due to equivalent Leaving Certificate mathematics grades demonstrating statistically significantly different mathematical competency levels could be somewhat addressed however through a separate discriminant analysis for Higher Level and Ordinary Level students. A separate analysis may also provide further insights into the pattern of performance of Ordinary Level students in service mathematics in UL. A separate discriminant analysis has therefore been carried out the results of which are detailed in section 5.8.
5.8 Determining if a Separate Discriminant Function for Higher Level and Ordinary Level Students is More Effective in Predicting Performance than the Technology 2006-2008 Function.

Findings in chapter 4 highlighted the varying performances of Higher Level and Ordinary Level Leaving Certificate mathematics students in both the diagnostic test and service mathematics examinations. Discussion surrounding some possible reasons why Ordinary Level students do not follow the same trend of performance as the other sub-categories of students was also outlined in chapter 4, in that their performance in the diagnostic test is not reflective of their performance in service mathematics. A strong statistically significant association was found between Leaving Certificate mathematics level and success/failure in service mathematics (p < 0.001) which re-emphasised that these groups of students are non-homogenous. For these reasons it was appropriate to assess if individual predictor functions for Higher Level and Ordinary Level Leaving Certificate mathematics students would be more effective in predicting performance compared to when they are grouped together, as in the Technology 2006-2008 function.

Upon carrying out discriminant analysis on the Higher Level students in Technology mathematics between 2006 and 2008, the following function was created:

\[ Z = 0.112 \times \text{Diagnostic Test Result} + 0.156 \times \text{Leaving Certificate mathematics points}, \]

where \( C = 11.7 \).

The discriminant function created from Ordinary Level students in the Technology 2006-2008 database was as follows:

\[ Z = 0.079 \times \text{Diagnostic Test} + 0.084 \times \text{Leaving Certificate mathematics points}, \]

where \( C = 5.3 \).

From analysis of the performance of the Higher Level discriminant function in predicting performance (see table 5.13), it can be seen that it is more successful in its correct classification of student performance when compared to the Technology 2006-2008 discriminant function (see table 5.15). According to the Higher Level discriminant function if a student has 75 Leaving Certificate mathematics points or more, they are predicted to be successful in service mathematics. Alternatively they could have a slightly lesser amount of Leaving Certificate mathematics points and a reasonably high diagnostic test score, for example, 60 Leaving
Certificate mathematics points and a score of 21 in the diagnostic test. Evidently most students with Higher Level Leaving Certificate mathematics in the Technology 2006-2008 database do meet this threshold and those who fall below (i.e. those who are predicted to be unsuccessful) tend to pass the examination anyway. The fact that the Higher Level function outperforms the Technology 2006-2008 function in terms of correct classification may be due in part to the 60 points for Higher Level students being a stronger performance in Leaving Certificate mathematics than 60 points for Ordinary Level students (see section 5.7.2). The stronger performance by Higher Level students with all overlapping grades, combined with their more predictable nature (i.e. their diagnostic test results being reflective of their end of semester results) is likely to have made it easier for the Higher Level discriminant function to distinguish between successful and unsuccessful service mathematics students.

On the other hand the Ordinary Level discriminant function has a slightly lower correct classification of performance (see table 5.14) when compared to the Technology 2006-2008 discriminant function (see table 5.15). The Ordinary Level function predicts that if students have 65 Leaving Certificate mathematics points or more that they will be successful in service mathematics. One possible reason for the poorer correct classification by the Ordinary Level function may be due to Ordinary Level students’ unexpectedly poor service mathematics performance given their diagnostic test scores. The relationship between Ordinary Level students’ diagnostic test results and their service mathematics results is not as straightforward to determine as it is for the Higher Level students.

<table>
<thead>
<tr>
<th>Performance in Technology mathematics</th>
<th>Predicted Success</th>
<th>Predicted Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success</td>
<td>248 (82.4%)</td>
<td>53 (17.6%)</td>
<td>301</td>
</tr>
<tr>
<td>Failure</td>
<td>0 (0%)</td>
<td>7 (100%)</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 5.13 Higher Level discriminant function’s ability to correctly classify Higher Level students within the training sample.
### Table 5.14 Ordinary Level discriminant function’s ability to correctly classify Ordinary Level students within the training sample.

<table>
<thead>
<tr>
<th>Performance in Technology mathematics</th>
<th>Predicted Group Membership</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted Success</td>
</tr>
</tbody>
</table>
| Success                               | 240  
(67.4%)         | 116  
(32.6%)         | 356              |
| Failure                               | 40   
(27.2%)          | 107  
(72.8%)          | 147              |

Table 5.15 Technology 2006-2008 discriminant function’s ability to correctly classify students within the training sample.

<table>
<thead>
<tr>
<th>Performance in Technology mathematics</th>
<th>Predicted Group Membership</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted Success</td>
</tr>
</tbody>
</table>
| Success                               | 448  
(68.2%)         | 209  
(31.8%)         | 657              |
| Failure                               | 24   
(15.6%)          | 130  
(84.4%)          | 154              |

### 5.8.1 Diagnostic Test’s Ability to Correctly Classify Higher Level and Ordinary Level Students

Like the Technology 2006-2008 discriminant function, the diagnostic test treats Higher Level and Ordinary Level students as a homogenous group. The investigation into whether a separate discriminant analysis for Higher Level and Ordinary Level students would yield a higher correct classification therefore caused the author to consider how the diagnostic test would perform in a similar investigation. The results of this investigation are detailed in tables 5.16 and 5.17.

The diagnostic test has a very high correct classification of successful Higher Level students (94.3%) however it is not as strong as the discriminant analysis in determining which Higher Level students will fail service mathematics (see table 5.16). Possibly the discriminant function could decipher that students with lower Higher Level grades (for example Higher Level D grades) were likely to fail however these students may have gotten 20 in the diagnostic test and were therefore not considered to be ‘at risk’ of failing service mathematics. The diagnostic test correctly classified similar proportions of the Ordinary Level cohort to the Ordinary Level
discriminant function albeit with a slightly higher correct classification of unsuccessful students (74.5%).

These findings highlight that both the separate discriminant analysis and the diagnostic test are strong when predicting the performance of Higher Level students however they find it more difficult to predict for Ordinary Level students. These findings therefore re-emphasise the unpredictable nature of Ordinary Level students when it comes to service mathematics performance and its relationship with Leaving Certificate mathematics grades and diagnostic test performance. There is no clear pattern of performance for Ordinary Level students making them distinct from any other category of student analysed within this body of work. It would be valuable for future research to try to establish some reasoning behind this unpredictability as Ordinary Level students currently make up the largest proportion of students within Technology and Science mathematics in UL (see chapter 4, table 4.9) as well as the largest proportion of students who sit Leaving Certificate mathematics in Ireland (Prendergast 2011).

<table>
<thead>
<tr>
<th>Performance in Technology mathematics</th>
<th>Predicted Group Membership</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not ‘at Risk’</td>
<td>‘At Risk’</td>
<td>Total</td>
<td></td>
</tr>
<tr>
<td>Success</td>
<td>283</td>
<td>18</td>
<td>301</td>
<td></td>
</tr>
<tr>
<td>(94.3%)</td>
<td>(5.7%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Failure</td>
<td>12</td>
<td>3</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>(80.0%)</td>
<td>(20.0%)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.16 Diagnostic Test’s ability to correctly classify Higher Level students within the training sample.

<table>
<thead>
<tr>
<th>Performance in Technology mathematics</th>
<th>Predicted Group Membership</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Not ‘at Risk’</td>
<td>‘At Risk’</td>
<td>Total</td>
<td></td>
</tr>
<tr>
<td>Success</td>
<td>183</td>
<td>172</td>
<td>355</td>
<td></td>
</tr>
<tr>
<td>(51.5%)</td>
<td>48.5%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Failure</td>
<td>41</td>
<td>120</td>
<td>161</td>
<td></td>
</tr>
<tr>
<td>(25.5%)</td>
<td>74.5%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.17 Diagnostic Test’s ability to correctly classify Ordinary Level students within the training sample.

Note: More students are present in tables 5.16/5.17 when compared to tables 5.13/5.14 as tables 5.16/5.17 consists of standard as well as non-standard students, the other tables consists of predictions for standard students only.
Conclusion
The separate analysis of students highlights the impact that grouping Higher Level and Ordinary Level students together had on the results of the original discriminant analysis. The underperformance of Ordinary Level students when their diagnostic test results are considered was largely responsible for the misclassification of students in the original discriminant analysis and indeed in the Ordinary Level discriminant function. The high percentage of correct classification for the Higher Level function suggests that the Technology 2006-2008 discriminant function had difficulties due to the overlapping of Leaving Certificate mathematics grades/points. When the Higher Level students were analysed separately the function could clearly decipher that students with 60 Leaving Certificate mathematics points for example (and a reasonably high diagnostic test score) were highly likely to be successful in service mathematics. The Technology 2006-2008 function however had a higher demand on it as it had to treat students with an Ordinary Level 60 points and a Higher Level 60 points as one in spite of the fact that they do not have the same level of mathematical competency as demonstrated by the analysis in section 5.7.2. The separate analysis of Higher Level and Ordinary Level mathematics students according to correct classification by the diagnostic test also revealed the differences between each sub-category in terms of service mathematics performance. The analysis highlighted that possibly the two levels should not be treated as one group (i.e. determining whether a student is ‘at risk’/not ‘at risk’ regardless of Leaving Certificate mathematics level) and re-emphasised the difficulties in predicting performance for Ordinary Level students.

The results of the investigation into which discriminant analysis is more effective are not conclusive. The separate discriminant analysis of Higher Level and Ordinary Level Leaving Certificate mathematics students showed the strength of the discriminant analysis in predicting performance for Higher Level students and its difficulty when predicting for Ordinary Level students. The analysis does however prove very useful in attempting to understand the impact of grouping students according to Leaving Certificate mathematics level and that of overlapping grades on the accuracy of predictions.

A direct comparison will now be made between the different approaches’ ability to correctly classify service mathematics performance to determine which approach is the most effective and appropriate to use in this research.
5.9 Predicting Performance: A Comparison of Approaches

In order to decipher which method of classification of students in terms of service mathematics performance was the most effective, the correct classifications of the Technology 2006-2008 database were compared. The three methods being compared were the Technology 2006-2008 discriminant function, the diagnostic test and the separate discriminant analysis for Higher Level and Ordinary Level students. The performance of each method was also considered in light of the primary objective of this research project, the primary objective being to distinguish between those who passed and those who failed service mathematics. The performance of each method was also considered in terms of whether the classification was evidence based or not and if it offered more than just a binary classification of performance.

5.9.1 Correct Classification of the Technology 2006-2008 Database

All approaches to predicting performance in service mathematics performed reasonably well in their prediction of both successful and unsuccessful students (see table 5.18). Each method has strengths and weaknesses which can be summarised as follows:

- The Technology 2006-2008 discriminant function had the second highest percentage of correctly classified successful students and the second highest percentage of correctly classified students overall with the Higher Level discriminant function performing better than it in both areas. Similar to the separate discriminant analysis, the Technology 2006-2008 function was unable to classify the highest number and percentage of students because of missing data.

- The diagnostic test had the second lowest percentage of correctly classified unsuccessful students of all approaches.

- The separate discriminant analysis produced a Higher Level discriminant function which correctly classified the highest percentage of students in all categories when compared to all other prediction methods being analysed. The Ordinary Level discriminant function produced in this separate analysis of students however correctly classified the smallest percentage of unsuccessful students and the smallest percentage of successful students of all approaches.
When each method was compared in terms of whether the classification was evidence based or not and if it offered more than just a binary classification of performance, the following conclusions were drawn:

- All of the discriminant functions (i.e. Technology 2006-2008 discriminant function and the separate analysis functions) provided an evidence based cut off point to determine if a student is predicted to be successful or not in service mathematics in addition to providing a probability of failure for each student.
- The diagnostic test provided a binary classification only which is subjective and therefore not evidence based.

<table>
<thead>
<tr>
<th>Technology Mathematics</th>
<th>Successful Students</th>
<th>Unsuccessful Students</th>
<th>Overall % correctly classified</th>
<th>Students not classified</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagnostic Test</td>
<td>485 (67.8%)</td>
<td>145 (72.9%)</td>
<td>630 (68.9%)</td>
<td>166 (15.4%)</td>
</tr>
<tr>
<td>Technology 2006-2008 Discriminant Function</td>
<td>448 (68.2%)</td>
<td>130 (84.4%)</td>
<td>578 (71.3%)</td>
<td>269 (24.9%)</td>
</tr>
<tr>
<td>Separate Discriminant Analysis</td>
<td>Higher Level</td>
<td>248 (82.4%)</td>
<td>7 (100.0%)</td>
<td>255 (82.8%)</td>
</tr>
<tr>
<td></td>
<td>Ordinary Level</td>
<td>240 (67.4%)</td>
<td>107 (72.8%)</td>
<td>347 (68.9%)</td>
</tr>
</tbody>
</table>

Table 5.18 Correct classification of all approaches when classifying the Technology 2006-2008 mathematics students (n = 1,080).

### 5.9.2 Conclusion

The primary objective for classification within this research project was to distinguish between those who passed and those who failed. The Higher Level discriminant function was the most successful in achieving this of all approaches analysed. Despite this however, the Higher Level discriminant function’s accompanying function, i.e. the Ordinary Level discriminant function, was amongst the poorest performing approach in terms of correct classification of all approaches tested (see table 5.18). Much of the analysis within this thesis has revealed that the vast majority of Higher Level students are successful in service mathematics (see tables 5.13 and 5.16) and examinations have also highlighted the difficulties which arise when attempting to accurately predict the performance of Ordinary Level students. An ability to accurately predict the performances of both Higher Level and Ordinary Level students is therefore vital. As a result of
this, attention was drawn to the next most successful approach in terms of overall percentage of correctly classified students which was found to be the Technology 2006-2008 discriminant function. The Technology 2006-2008 discriminant function was therefore considered the most effective prediction method in this respect.

In addition to this the two dimensional Technology 2006-2008 discriminant function and the separate discriminant analyses were the only methods which provided evidence based cut off points for predicting performance in service mathematics and were able to advance on the binary classification of the diagnostic test by also providing each student with a probability of failure. The Technology 2006-2008 discriminant function was therefore the most effective in distinguishing between those who were successful and those who were unsuccessful in service mathematics while also providing an evidence based cut off point and probability for performance. It was therefore considered to be the most effective prediction method although it was recognised that none of the prediction methods tested were hugely successful.

### 5.9.3 Informing the Intervention

The author wishes to implement the discriminant function on students entering Technological service mathematics in the academic year 2010/11. The discriminant function will be applied to these students and based on their results in the diagnostic test and their Leaving Certificate mathematics points, they will be identified as potentially successful or unsuccessful service mathematics students. Students’ predictions of performance and probabilities of failure will be used to inform the grouping of students within the teaching intervention. The teaching intervention can then commence with the intended outcome that students predicted to be unsuccessful will have a greater chance of passing their end of term examination due in part to the intervention.

Due to the fact that the Technology 2006-2008 discriminant function emerged as the most effective prediction method of all tested, it will be used to group students according to probable performance within the teaching intervention which is to be carried out within this research. There was very little difference found, however, in how each function classified the students with the functions disagreeing on how they would classify 6 (16.7%) of the intervention students only. All functions assigned students in a very similar way in terms of ‘at risk’ groupings. The
author therefore made the decision to use the Technology 2006-2008 discriminant function to inform the teaching intervention as either choice would yield similar results.

5.10 Valuable Findings Which Emerged Due to Challenges in Prediction

Despite the challenges faced by the prediction methods analysed in this chapter several valuable insights have emerged from the examinations. Some such insights are discussed in this section.

5.10.1 Validating the Cut off Point for the Diagnostic Test

There are several statistical analysis methods which can be used to determine cut off points however when discriminant analysis was carried out, with diagnostic test results as its only predictor variable, it revealed some very useful information. Essentially the discriminant function which was created using diagnostic test score as the only predictor variable is a one dimensional model (see Appendix I). It found a cut-off point of 18 in diagnostic test performance and determined that a student who falls below this cut off point is likely to fail and one who falls above it is more likely to pass their end of semester examination. The cut off point which the discriminant function determined to be most effective at discriminating between students performance in service mathematics was very similar to that of the existing cut-off point of 19 in the diagnostic test. The establishment of the discriminant function with diagnostic test as the only predictor variable was therefore of value as it proved that the subjective expert judgement which was used to determine if students were likely to be ‘at risk’ or not ‘at risk’ of failing service mathematics was accurate. This finding therefore provides scientific evidence to those involved in diagnostic testing in the university that their cut-off point is effective. It also helps to refine the cut-off point slightly (to 18 as opposed to 19) and has the added benefit that students are provided not only with the binary classification of ‘at risk’ and not ‘at risk’ but can also establish how at risk they are through examining their probabilities of failure.
5.10.2 Overlapping Grades: Room for Improvement in the CAO Points system?
Ordinary Level students are less mathematically literate on entry to UL, as measured by the diagnostic test, as well as at the end of semester 1 when compared to their Higher Level grade equivalents. Some of the possible contributing factors to the Ordinary Level students’ weaker performance may include poor mathematics teaching at second level, adopting undesirable learning styles, poor perceived mathematical ability and poor adjustment to higher education. These possible contributing factors, which were discussed in detail in section 4.2.4.4, provide some rationale for Ordinary Level students not performing to the same standard as their Higher Level counterparts.

However additionally, one must consider the possibility that the points awarded for the Ordinary Level grades in question do not accurately reflect the mathematical competency levels of the students who enter third level education with them. Based on the evidence here, they do not demonstrate an equal level of mathematical competency to the Higher Level grade equivalents. These findings highlight that the points system may have a weakness when it comes to accurately assigning students’ mathematics points as it may misinform institutions of the mathematical preparedness of these students. Further discussion on this is detailed in chapter 7.

5.10.3 Ordinary Level Students: A Challenge for the Prediction of Service Mathematics Performance in Ireland
One of the results which emerged was the unusual pattern of performance by Ordinary Level Leaving Certificate mathematics students. Although some Ordinary Level students passed service mathematics, the majority of them did not perform as well as would be expected in service mathematics when their mean diagnostic test performance was taken into account. The difficulty with predicting for these students proved to be a challenge not only for the discriminant function but also for the current measure which is place in UL for predicting failure in service mathematics i.e. the diagnostic test. There may be something external to students’ mathematical competency levels affecting students’ service mathematics performance. The proportion of Ordinary Level Leaving Certificate mathematics students entering service mathematics in UL, and indeed other universities, is growing and so further investigation into the reasoning behind this unpredictable pattern of performance would be hugely beneficial if we are to cater for the specific needs of these students.
5.11 Summary and Conclusions

- The Technology 2006-2008 database was used to carry out discriminant analysis. The discriminant function created had a reasonable level of success when classifying unsuccessful service mathematics students within its training sample. It also proved to be moderately successful in classifying unsuccessful students when tested on a validation dataset; the Technological 2009 dataset. When the standardised discriminant co-efficient of each independent variable was calculated, Leaving Certificate mathematics points proved to have the largest relative influence on the discriminant function. The predictor variables, Leaving Certificate mathematics points and diagnostic test results, were only able to explain some variability in service mathematics performance however. Some concerns regarding the potential limitations of using Leaving Certificate mathematics points as an independent variable were raised. The possible effect of Leaving Certificate mathematics grades having a roughly 50% proportion of students passing on the overall accuracy of the binary predictions was amongst the concerns. The students’ diagnostic test results and probability of failure, however, gave further information giving a more accurate insight into the probable performance of each student.

- Students within the Technology 2006-2008 database were divided into three groups depending on their predicted probability of failure. This process revealed that the majority of students were in group 2 with a probability of failure of between 0.34 and 0.66 in service mathematics. In terms of predicted probability therefore the majority of this cohort lay around the peripheries of passing or failing. It was a finding of interest in itself that that majority (almost 80%) of students in this group passed service mathematics. The Technology 2006-2008 discriminant function was analysed in terms of its success in classifying students by probability group. The general findings revealed that it was highly successful in classifying group 1 students (low risk) however, group 3 students (high risk) proved to be more difficult to predict. Approximately half of the students in group 3 within Technology mathematics passed service mathematics and half failed it. This group was found to have the highest proportion of students who failed of the three groups and so it does in fact contain the most ‘at risk’ students.

- A comparison between the performance of the UL diagnostic test and the discriminant function’s ability to predict failure was carried out. The discriminant function was found
to be slightly more successful in all areas of prediction (success/failure) particularly in predicting unsuccessful students. It also was found to be more useful as it provides students with an insight into exactly how ‘at risk’ they are of failing service mathematics in that it provides students with a probability of failure. In addition to this, it provides an objective evidence based cut-off point for performance compared to the subjective cut-off point which exists in the case of the diagnostic test. It was acknowledged however that both methods of prediction were not overly successful in predicting service mathematics performance.

- Challenges which were faced when attempting to predict service mathematics performance in UL were outlined. The increasing proportion of students not sitting the diagnostic test has led to an increased proportion of students who do not have a prediction of performance. This is a challenge faced by both the diagnostic test and the discriminant analysis. The realisation that equivalent Leaving Certificate mathematics grades do not demonstrate the same level of mathematical competency levels in both the diagnostic test and service mathematics highlighted one potential reason for the discriminant analysis not being more successful in its binary predictions than it was. The lack of consistency between diagnostic test results, Leaving Certificate mathematics points and service mathematics performance is likely to have affected the discriminant analysis’ accuracy of predictions. The final challenge faced by all prediction methods discussed is that which may be caused due to Ordinary Level Leaving Certificate mathematics students’ unpredictable service mathematics performance. Prediction analysis relies on repeated patterns in data and for Ordinary Level students it is difficult to determine the likely pattern of performance.

- A final discriminant analysis was carried out due to the differences between the Higher Level and Ordinary Level students’ performance in both the diagnostic test and in service mathematics as outlined in chapter 4. The analysis was also carried out based on the concerns in relation to overlapping mathematics grades in the Leaving Certificate having statistically significant different mean competency levels (see section 5.7.2). Two separate discriminant functions were therefore created to predict for Higher Level students and Ordinary Level students. The Higher Level function was found to perform better in terms of its prediction of performance when compared to the Technology 2006-
2008 function. A possible reason for this was thought to be due to the stronger performance of students with Higher Level overlapping grades and Higher Level students’ more predictable performance (as measured by their diagnostic test performance) making them easier to predict for as an individual group. The Ordinary Level function however performed to a slightly lower standard in terms of correct predictions of performance when compared to the Technology 2006-2008 function. One probable contributing factor to this is the underperformance of Ordinary Level students in service mathematics. The Ordinary Level discriminant function therefore would have had difficulty in trying to decipher which diagnostic test performance and Leaving Certificate mathematics points combination would be likely to result in success in service mathematics. The difficulty in predicting performance for Ordinary Level students was further emphasised by the separate analysis of the diagnostic test’s ability to predict the performance of Ordinary Level and Higher Level students separately.

- A comparison of the approaches to predicting performances concluded by stating that the Technology 2006-2008 discriminant function was the most successful in terms of fulfilling the main classification objective of this research project i.e. to distinguish between those who passed and those who failed service mathematics and provide them with a probability of failure. In addition to this, it provided an objective evidence based cut-off point for performance. All prediction methods struggled however. The Technology 2006-2008 and the Higher Level and Ordinary Level discriminant functions were compared in terms of their classification of students into low, medium and high risk groups. The functions were found to group students very similarly according to probabilities of performance and so the author chose to use the Technology 2006-2008 function to categorise students for the teaching intervention which is detailed in full in chapter 6.

- Some valuable insights were gained as a result of the difficulties which presented themselves when attempting to predict service mathematics performance. A discriminant function which used diagnostic test result as the only predictor variable provided useful findings. The function, which reduces to a very simple rule, provided scientific evidence for the effectiveness of the subjective cut-off point for diagnostic test performance which
was being used. The function also refined the cut-off point slightly and provided a probability of failure in addition to the binary cut-off point of performance.

- The separate analysis of students, although it did not necessarily improve the prediction of performance, provided valuable insights into the difficulties which overlapping grades caused the discriminant analysis. The need for further investigations to be carried out into the reasoning behind Ordinary Level students’ unpredictable performances has also been highlighted as a result of the analysis carried out in this chapter.

**5.12 Closing Remarks**

The findings in this chapter bring to light several issues in the Irish education system. Difficulties arose in predicting the performance of service mathematics students due in part to the points system which is in place for the Leaving Certificate. The system in place involves overlapping grades which in terms of mathematical competency levels may not be equal and therefore could potentially be misinforming third level institutions of the actual mathematical competency levels of the students they are admitting.

The findings also highlight the underperformance of Ordinary Level students in service mathematics. This issue which makes predicting performance in service mathematics all the more difficult is worthy of further research. The effectiveness of the current ‘at risk’/not ‘at risk’ classification of students has been confirmed through evidence based research of retrospective data. This is a valuable finding for the university going forward. The unpredictability of Ordinary Level students in terms of service mathematics performance however also has implications for the current ‘at risk’/not ‘at risk’ cut-off point of the diagnostic test. The findings provide a rationale to consider a separate analysis of Ordinary Level Leaving Certificate mathematics students.
Chapter 6: Intervention Design, Implementation and Evaluation

6. Introduction
This chapter explores the definition of the term “intervention” from a wide variety of perspectives. Its proposed meaning in an educational context is examined and the rationale behind the implementation of an intervention in an educational context both in general and in UL is outlined.

6.1 What is an Intervention?
According to the Oxford English dictionary (2011) an intervention is “an action or act of coming between or interfering especially so as to modify or affect a result”. A report by Connors (2011) provides a definition of an intervention specifically from an educational perspective as being a process which intends to:

increase, improve, and/or enhance the performance of educators and their students. They are designed to provide services to educators and/or students at the earliest point in time possible, when difficulties first arise

(Connors 2011, Introduction, para. 1).

Intervention literature has described an intervention as a concept which aims to result in a positive outcome as opposed to the negative outcome which would have otherwise occurred. It
involves putting some change in place (a strategy or approach) in an existing relationship with the intention of improving it (Sandoval 1993). From the viewpoint of this research the author adopts the definition of intervention as:

* A course of action (teaching intervention) to address a problematic situation (mathematical competency levels in service mathematics) designed to change beliefs, attitudes and behaviour relating to performance in service mathematics end of term examinations

(Regan 2005).

Rationale for the introduction of a teaching intervention in third level education both internationally and specifically in the University of Limerick are outlined in section 6.2.

6.2 Rationale for the Implementation of an Educational Intervention

Research surrounding the design, implementation and evaluation of various teaching interventions has been documented across primary, secondary and tertiary education worldwide (Gortmaker et al 1999; Ghali et al 2001; Johns et al 2005). The rationale for the interventions being put in place however tend to vary depending on the specific needs of the individual institutions involved. Common rationale however includes:

- The establishment of a set of principles which can be applied in an educational setting to prevent and intervene if mathematical difficulties are evident in students (Fuchs and Fuchs 2001).
- To improve current educational conditions in an institution and if effective to re-structure practices accordingly (Bero et al 1998).
- To address the research-practice gap which is often present in universities i.e. practitioners having the knowledge and evidence that a certain teaching practice is more effective than another while knowingly pursuing the less effective method (Stoiber and Kratchwill 2000).
- To contribute successful/unsuccessful intervention findings to education literature with the intention of helping to improve educational standards worldwide (Kaiser and Willander 2005).
• In recognition of the need to improve mathematics learning (Walkden and James 2003). Non-mathematical intervention studies also call on interventions with the desired outcome of improving comprehension in particular subject areas (Rosenshine et al 1996).
• To attempt to improve the supply and retention of candidates to undergraduate degree courses involving mathematics (James et al 2008).
• To make a comparison between the effectiveness of different teaching methods implemented in different countries (Graham et al 1998).
• To improve/change third level students’ attitudes towards a particular subject area, for example mathematics (Falsetti and Rodriguez 2005).
• To help students to gain a deeper insight and better appreciation of a particular subject (Melendez et al 2009).
• To support students’ emotional needs as well as in terms of subject matter knowledge. This type of intervention often takes the form of a learning centre (Perkins and Croft 2004; Symonds et al 2008).
• To cater for the diversity of preferred learning styles which students possess, for example Computer Assisted Learning (Croft et al 2009) or Peer Assisted Learning (Cleary 2008).
• To provide for students who may have been absent from formal education for a period of time and wish to return to education. Interventions in the form of bridging courses are often put in place to achieve this (Boland 2002).
• To improve the provision of mathematics teaching and learning by supporting all of its stakeholders (primary, secondary and tertiary level teachers, students, governing bodies). This modern support system often takes the form of a National Centre for Excellence in teaching and learning mathematics

In order to achieve some/all of the goals that are often set out by institutions, it is pivotal that appropriate forms of intervention are chosen in order to fulfil the specific needs of the institution in question. Such intervention types and the theoretical concepts underpinning them are detailed in chapter 2. These interventions were considered in light of the situation in UL before a decision was made regarding the most appropriate intervention to implement. The rationale for the implementation of an intervention in UL is outlined next in section 6.3.
6.3 Rationale for the Implementation of an Intervention: The Situation in UL

As introduced in chapter 1 the UL database was initiated in the academic year 1997/98. This database is updated on a yearly basis and has formed the basis for all information regarding the mathematical situation in UL. An initial analysis of the situation in relation to mathematical preparedness was carried out between the academic years 1997/98 and 2002/03 and highlighted that mathematical under-preparedness was in fact evident. As outlined in chapter 4 over 30% of students in the database between the years being investigated were found to be ‘at risk’ as measured by the UL diagnostic test (Gill 2006). Confirmation that this problem still existed, which had previously been documented by O’Donoghue in 1999, paved the way for the delivery of support services in UL. A variety of support services/intervention initiatives were set up in response to the decline in mathematical standards such as:

- Front End tutorials.
- Support tutorials.
- Drop-In Centre (opened in the academic year 2001/02).
- Online Support: which is available from the Mathematics Learning Centre’s website.
- Fact Sheets: which are available from the Mathematics Learning Centre’s website.
- Exam Revision.
- Head Start Maths: an initiative created for adult learners returning to education (started in 2007).

All of these initiatives are detailed fully in the literature review on interventions (see chapter 2). The UL database was since revisited by Gill et al (2010) and revealed that the mathematical competency levels of service mathematics students in UL continues to decline. The situation which currently exists is one in which almost 50% of the students are ‘at risk’ of failing service mathematics.

Research carried out by Fuchs and Fuchs (2001) outlined principles for the prevention and intervention of mathematics difficulties. These principles involved three levels. The levels addressed are (1) primary prevention which focuses on effective instructional methods specific to mathematics (2) secondary prevention, otherwise known as peripheral intervention, which
involves making adaptations in your teaching to affect better student responsiveness and finally (3) tertiary prevention which is seen to be synonymous with intervention. Tertiary prevention in this context can be seen to be in line with the intervention which is to be carried out in this research as it is described as a process in which:

*Intensive and individual attention, which requires special resources, is brought to bear on problems that are severe and have proved resistant to other levels of prevention*  
*(Fachs and Fachs 2001, p.91).*

Although the current support systems in UL have been effective in helping to improve the problem of declining mathematical standards (Faulkner et al 2009) many of the reasons for the declining standards have proven resistant to the current support systems in place. The issues which are being referred to are those relating to the changing student profile of service mathematics students as presented in chapters 4 and 5. The existence of these findings provide rationale for further interventions aimed at improving standards in mathematics to be implemented in UL. A summary of these findings are as follows:

- There is a decline in mathematical competency levels as measured by the UL diagnostic test (see Chapter 4),
- There is an increase in the number of students entering service mathematics with Ordinary Level Leaving Certificate mathematics as pre-requisite knowledge for service mathematics between the years 1998 and 2008 i.e. there is a large change in the student profile within service mathematics over time (see Chapter 4),
- Strong statistically significant associations were found between performance in service mathematics; Leaving Certificate mathematics performance and diagnostic test results (see Chapter 4) and
- A discriminant function has been produced from the Technological 2006-2008 dataset which has proven to be highly successful in predicting performance in service mathematics examinations (see Chapter 5).

These findings are fully exploited through the implementation of an effective teaching intervention with the overall aim of improving the provision of mathematics education in UL and consequently reducing the failure rates in service mathematics.
6.3.1 Rationale for Choosing Technology Students as Intervention Participants

The Technology mathematics students were chosen to take part in the intervention over the Science mathematics students for a number of reasons:

- The Technology students were considered more appropriate as both the author and the additional tutors successfully passed Technology mathematics within their undergraduate degree programs and taught it for a number of years during their postgraduate studies.
- The minimum Leaving Certificate mathematics entry requirement for students entering UL to degree programs involving Technology mathematics is lower than that of Science mathematics students. Because of this it was taught that in general Technology mathematics students may have been more in need of the teaching intervention.
- A further justification for choosing Technology students was the willingness and support that the Technology lecturer offered when informed of the possibility of such an intervention taking place.

For all of the aforementioned reasons Technology mathematics students were chosen over Science mathematics students to take part in the intervention.

6.4 Addressing the Needs

Gill (2006), Corcoran (2005b) and others highlight the gaps which are present in service mathematics students’ knowledge, many of whom are training to be mathematics teachers. These findings call for measures to be put in place in order to alleviate some of the contributing problems rather than just merely identifying them. This is one of the intentions of this body of work. The diagnostic test results and Leaving Certificate mathematics points of UL service mathematics students, which form the foundation of the predictive model of failure, have acted as a form of ‘needs analyses’ for the intervention study. The author can be confident of students’ mathematical backgrounds (from Leaving Certificate mathematics grades) as well as where deficiencies currently lie (from diagnostic test data).

Based on the awareness that the recognition of deficiencies in subject matter knowledge, necessary for successful completion of service mathematics, will not make deficiencies simply disappear (LTSN 2003a) again compounds the need for action. The necessity for the researcher
to take the role of interventionist is further justified by the evidence that students, even upon receiving feedback of their deficiencies, are unlikely to remediate them independently (Hourigan 2009). There is a strong likelihood that many of the service mathematics students applied to courses which had more mathematical content than they expected and so may have “insufficient grounding in mathematics to be able to cope with the fast pace of course work” (Maths at third level-Qualifax 2011).

The author’s awareness of the characteristics of the problem allows for the development of an informed teaching intervention, which aims to address the specific needs of the service mathematics students involved (Robinson 1993; Atkinson 2004). The needs which are to be addressed are centered on:

- Raising awareness in students of the mathematical problems which exist and the consequences of not addressing them.
- Developing a deeper understanding in students’ mathematical knowledge with the hope of improving attitudes and appreciation levels towards the subject.
- From a practical viewpoint the logistics, cost, facilitators needed, lecturers’ willingness to participate need to be addressed also.

After recognising and addressing the needs of the intervention the author can set out to detail the theoretical framework which will underpin it. The consequences of brushing over an issue regarding mathematical competency levels such as this could affect those going on to work in industry. Alternatively a lack of acknowledgment that such an issue exists could have negative effects for those who are trainee student teachers. It is vital for errors in trainee student teachers’ subject knowledge to receive attention “if not to transfer to children in schools” (Ryan and McCrae 2005, p.647). The type of intervention best suited to achieve the needs set out has been considered through an examination of literature surrounding interventions, an examination of the rationale for the intervention and an analysis of the specific needs of the service mathematics students involved. The intervention type decided upon is detailed in section 6.5.
6.5 Intervention Type Decided Upon

The extensive review of literature which details interventions that have been implemented worldwide, and a review of the learning theories which are evident in each (see chapter 2), has allowed the author to make an informed decision on which type of intervention would be most conducive of achieving the goals set out by the intervention. After careful consideration of all forms of interventions the author decided that a tutorial format is most appropriate to achieve the goals set out. The reasons for this are as follows:

- This format allows the author to dictate the learning theory she wishes to implement in the classroom as the format of a tutorial is open to adaptations depending on the needs of the learners.
- Direct contact with the students allows the author to better identify possible misconceptions which are present amongst the group as well as encourage and facilitate learning. This would not have been the case with other interventions such as Computer Assisted Learning (CAL) or diagnostic testing.
- Within a module of tutorial sessions the facilitator has large scope to adapt the format of the lessons in order to develop interest and hold the attention of the students e.g. internet input, mathematics computer programs, group work, peer assisted learning etc.
- The intervention type chosen can also be justified for practical reasons such as the increased likelihood of students attending their timetabled tutorial over an additional one.
- The format which the tutorial will take is one which is sustainable over time.

The theoretical framework employed in this intervention is detailed in section 6.5.3. The focus of the tutorials however is detailed first in section 6.5.1.
6.5.1 Tutorial Focus: Active Learning and Mixed Ability Group Work

Active learning and group work has attracted much attention in education literature in the past two decades. While sceptical members of the education world may regard active learning and group work as “another in a long line of educational fads” many have used these techniques in their teaching and reported proven benefits of using them instead of traditional didactic teaching methods (Prince 2004, p.223). Such studies and their findings are outlined in section 6.5.1.1.

6.5.1.1 Active Learning

As highlighted in chapter 3, active learning is any instructional method which engages students in the learning process. Basically it is anything which involves the students in meaningful learning activities in which they must think about what they are doing (Bonwell and Eison 1991). Active Learning can take many forms including a pause during lectures to write down notes on the material being studied with a partner. This procedure has been used by several educationalists and has been shown to improve both short and long term knowledge retention (Di Vesta and Smith 1979; Ruhl et al 1987). Another method used within active learning which has also been found to improve knowledge retention is discussion (McKeachie 1972). Discussion has been found to promote better student attitudes towards learning, and improve motivation and cognitive skills (Bonwell and Eison 1991). Felder et al (2000) include active learning in their recommendations of teaching methods that work. They state one of the reasons for this as being its proven ability to engage student interest. Wanket (2002) discussed several studies which highlight how students’ attention span diminishes as a lesson goes on. Hartley and Davies (1978) found that immediately after a lecture students’ remember 70% of the information presented in the first ten minutes and just 20% of the information presented in the final ten minutes. Active learning has therefore been used as a means to avoid the occurrence of student attention loss and to promote engagement. Engaging students in their learning through active learning has been found to be a significant predictor of college success (Astin 1993). Many studies have been carried out which support the effectiveness of active engagement in the classroom (Redish et al 1997; Hake 1998; Laws et al 1999). For the reasons mentioned here it was thought that active learning methodologies implemented in the service mathematics classroom would be an effective method in improving understanding and performance in third level mathematics. Mixed ability
group work was considered an appropriate teaching method to team with active learning methodologies. The reasons for this are outlined next.

6.5.1.2 Mixed Ability Group Work

It has been reported that:

students working in small groups, regardless of the subject matter, tend to learn more about what is taught and retain it longer than when the same content is presented in other instructional formats (Gross Davis 1993, p.12).

Students working in small groups have also been found to be more content in their classes (Beckman 1990; Chickering and Gamason 1991; Johnson et al 1991). There are several titles given to the type of learning that is associated with group work such as co-operative learning, collaborative learning, peer teaching, team teaching and study groups however they all stem from the same concept i.e. students working together with a common goal in mind. Aspects of group work such as collaborative learning and cooperative learning are used in this research. Several longitudinal studies have been carried out which analyse the possible benefits of collaborative learning as opposed to traditional teaching methods. Improvements related to academic achievement, quality of interpersonal skills and perceptions of greater social support were found in these studies (Johnson et al 1998a; Johnson et al 1998b; Springer et al 1999).

Cooperative learning works on the premise that cooperation is better than competition among students for producing positive learning outcomes (Prince 2004, p. 227).

Other studies in this area highlighted the improvements in team skills and interpersonal skills (Panitz 1999; Terenzini et al 2001). Group work is also a very appropriate method of incorporating mixed ability learning into the classroom, something which the author wishes to include in the intervention. Several studies have shown the benefits of mixed ability mathematics classes such as improvements in behavior and increases in the level of respect and responsibility displayed among students. Mixed ability mathematics classrooms have also been found to outperform those in streamed ability groups (Webb et al 1997; Boaler 2006). This style of teaching has also been shown to benefit both high and low attaining students (Saleh et al 2005).
The potential benefits of active learning and mixed ability group work are clear from the detailing of literature in this section. Due to a belief in the potential effectiveness of these teaching strategies in improving mathematical understanding the author has chosen to implement them in her teaching intervention. Some of the strategies which will be used in the intervention tutorials are outlined next in section 6.5.1.3.

6.5.1.3 Active Learning Strategies

The teaching intervention will involve active learning strategies as set out by Felder et al (2009) and ideas stemming from the Centre for Teaching and Learning in the University of Minnesota. The strategies are as follows:

- **In-Class Teams**
  Arrange the class in teams of 2-4 and choose team recorders. Give teams 15 seconds-3 minutes to
  - Recall prior material
  - Answer or generate a question
  - Start a problem solution
  - Work out the next step in a derivation
  - Think of an example or application
  - Explain a concept
  - Discuss and determine why a given result may be wrong
  - Brainstorm a question
  - Summarise a lecture
  Select different individuals from different groups and request an answer to be presented and afterwards take any voluntary responses.

- **Think-Pair-Share**
  Students think individually for a minute about what they may know about a particular topic. They then form pairs to produce a joint answer and pairs may then confer with other pairs to see what they came up with. Finally a group discussion may ensue.
• **Co-operative Note-Taking Pairs**
  Students form pairs to work together during a particular class. After a particular segment of the class is over, one partner summarizes and explains their notes to the other who then contributes any information the summarizing partner may have missed. This should result in a more comprehensive set of notes for all parties involved.

• **Questioning**
  1) **Opening Question:** The tutor presents an opening question to a lesson, possibly on a power-point slide, and gives the students a few moments to think about it and then looks for feedback.
  2) **General Questions:** Particular times in a lesson are set aside for students to ask questions. The idea behind this is that students get time to have a breather and reflect on the material. This could possibly be done by inserting a blank slide into a power-point, immediately students’ attention will be grabbed, and the question time seems worthy of time in the tutorial rather than something which the facilitator really does not have time for.
  3) **Guided Reciprocal Peer Questioning:** Students, in small groups, are provided with a generic set of question stems for example:
   a. How does…..relate to what I’ve learned before?
   b. Explain why….?
   c. What conclusion can I draw about….?
   d. What is a new example of….?
   e. What if….?
   f. How are…. and…….similar?
  Each student is given a few minutes to prepare a thought provoking question and then all questions are attempted by members of the group. Any controversial or particularly interesting questions can be presented to the whole class.

• **Writing Assignments**
  Writing assignments can act as a means of helping students to make sense of new material both during and after class time. Felder et al (2009) advises that students are informed why they are being asked to carry out writing assignments and how they can
benefit from them. In and out of class times students can be asked to summarize what they have learnt about a particular topic. Alternatively before a topic is presented in class students can take a few moments to document what they already know about this topic so that connections can be made between new and old material. Students may be asked to write down a couple of applications of what they have been covering in class. Finally as a means of evaluation of teaching as well as a reflection for students on their learning, a short document of how students feel about the active learning strategies being employed and whether they feel they are effective or not can be carried out. These assignments could take the form of a journal which the students can build up over the course of a particular program.

- **Minute Paper**

Stop the lecture with two minutes to go and ask students to write down

1) The main points of the lecture/tutorial
2) The muddiest points for them

This can be carried out on an anonymous basis and left for the tutor to analyse after the class have departed. Future lessons can be informed by this feedback.

All of the strategies listed here are used and adapted to meet the needs of the author’s intervention strategy. Based on a thorough investigation of appropriate learning theories, and teaching strategies which could be used to implement these learning theories, the author was able to compile a theoretical framework for the intervention. This framework is detailed in section 6.5.3. Section 6.5.2 firstly details how the pedagogical values of the researcher may have impacted upon the design, implementation and evaluation of the intervention.
6.5.2 Pedagogical Values of the Researcher

Aspects of the author’s personal background are likely to have an impact on how elements of the thesis, such as the intervention, are approached and how the findings are interpreted. The immediate motivation for the thesis was the author’s qualification as a secondary school mathematics teacher and her involvement in mathematics education research in the final year of her undergraduate degree. Other influences include:

- Second level mathematics teaching experience.
- Third level mathematics teaching experience.
- Teaching experience in the Mathematics Learning Centre in UL.
- Teaching experience on two week bridging courses in UL.
- Lecturing experience with mature students.
- Member of the National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL) research team.

The author was also motivated by her interest in helping undergraduate students who struggle with basic mathematical skills and her desire to improve and investigate best practice in mathematics teaching and learning. The potential influence which the author’s personal circumstances and experiences may have had on the teaching intervention contained within this research must therefore be acknowledged.

The methods through which the author designed, implemented and evaluated the intervention were all influenced by her pedagogical values. The author believes in the importance of holistic mathematics education in which students’ personal, social, psychological and academic development be considered and this is reflected throughout the intervention. For example the design of the intervention involves consideration for students’ attitudes, interest, and interaction with other students as well as the incorporation of teaching strategies believed to improve academic performance. The implementation of the intervention was influenced by the author’s enthusiasm towards the subject matter and her interest in improving mathematical provision in UL for the sake of the subject as well as for the sake of her research. Finally the evaluation of the intervention using Shapiro’s model again demonstrates the author’s pedagogical standpoint which was driven by a belief in holistic education as it contains four evaluation elements: social
validity, intervention acceptability, intervention integrity and intervention effectiveness. The author’s pedagogical background therefore provides her with valuable insights into best practice in mathematics education however the potential limitations which her background may have must also be acknowledged.

6.5.2.1 Potential Limitations of the Author’s Observation Data

When data collection is interpersonal, humans interacting with humans, it is inevitable that the researcher will have some influence on the data (Hitchcock and Hughes 1989). As was acknowledged in chapter 3 (section 3.5.1 (c)) there are several risks associated with using observations as a means of data collection in educational research (Robson 2002; Shaughnessy et al 2003). These potential limitations are worthy of reiterating again in the context of this research intervention particularly as the author takes the role of both the intervention tutor and observer. Such potential limitations include:

- **Selective attention of the observer:** What is noted may be influenced by what the author is interested in i.e. the author may be more inclined to notice positive aspects of group work which may be occurring where someone with less vested interest in the area may notice something else such as students’ unease with it for example.

- **Attention Deficit and Selective Memory:** The author cannot take “on the spot” observations due to the fact that she is also the tutor. Post event recollection may result in certain aspects of the observation being left out due to a combination of attention deficit and selective memory (Cohen et al 2007).

- **Subjectivity:** The subjective nature of the observations means that the author is detailing what she feels is a successful intervention based solely on her opinion which is influenced by her pedagogical values. The author determined the success of the intervention tutorials based on the follow three factors:
  1. Was the appropriate mathematics material covered?
  2. Were the students engaged with the material in an active learning group work manner in which understanding of the material was a focal point?
  3. Did the students appear to enjoy the tutorial?
Conclusion
The author acknowledges that her background, experience and personal vested interest in the tutorials influenced the design, implementation and evaluation of the intervention. This is not to say the author’s background will have a negative influence but merely that it should be taken into consideration. If an external person undertook this intervention, who did not have the same interest in it as perhaps the author did, the results may be extremely different. Measures were however put in place to counter the potential bias which may be present i.e. triangulation (see chapter 3). The potential limitations of the author acting as both the tutor and the observer have been detailed and will be considered in the examination of the observation data.

6.5.3 Intervention Design: The Theoretical Framework
As outlined in chapter 3, the theoretical framework for the intervention design involves 3 conceptual underpinnings: Constructivism, Realistic Mathematics Education and Mathematics in Context. The conceptual underpinnings which all recognise the importance of problem solving in the learning process are delivered and guided using Felder’s (2009) active learning strategies and Hidi and Renninger’s (2006) Model for Interest and Attitudes.

The effectiveness of the intervention is evaluated using Shapiro’s (1987) model of intervention evaluation (see chapter 3) and the performance of the intervention students in Technology mathematics is also included in the evaluation. The success of the discriminant model in predicting performance is also used to assess the effectiveness of the intervention. Recall from chapter 3 that the qualitative aspect of the intervention is evaluated through the author’s observation entries, which assess the treatment integrity and acceptability, and student questionnaires, which assesses the social validity and also contributes to the evaluation of the acceptability of the intervention (see section 3.4.4.5 for details regarding Shapiro’s (1987) Intervention Evaluation Model).

A more comprehensive detailing of the aims, structure, theoretical framework and evaluation framework for the intervention is detailed in chapter 3. The structure of the intervention theoretical framework and evaluation framework can be found in chapter 3, figures 3.5 and 3.6.
6.6 Putting the Intervention into Action: Considerations

Once the appropriate intervention type was decided on and the theoretical framework for it was set out, a detailed plan of the tutorials was devised. An outline of the lesson plans for each tutorial included the aim of the lesson, the intended learning outcomes, the active learning strategies employed, the presence of Realistic Mathematics Education (RME) and the evaluation methods. The lesson plans are detailed in Appendix J. Many considerations were made prior to the implementation of the tutorials. These considerations included which class tutorial group would be chosen to take part in the intervention, how the students would be arranged into groups for the active learning tutorials and what tutors would be involved in facilitating the tutorials. Details of these considerations are outlined next in sections 6.6.1-6.6.6.

6.6.1 Tutorial Group Chosen for the Intervention

In order to choose an appropriate intervention group for this research project an analysis of the performance of students in each tutorial group in Leaving Certificate mathematics and the diagnostic test was carried out. The reason that students’ performance in these two tests were chosen as a measure of probable ability in service mathematics was because they were found to have a statistically significant relationship with performance in service mathematics (see chapter 4). The author wanted to ensure that a tutorial group containing a mixture of ability levels, as measured by the discriminant model, was chosen. Mixed ability tutorial groups in which problem solving took place were desired so that learning could be enhanced for all and so that students could learn more effectively by doing mathematics (Felder and Brent 2003). The examination of students within each tutorial group revealed that a mixture of abilities was present in all tutorials (Note: Students’ abilities were examined in terms of which probability of failure group (1, 2 or 3) they were assigned to by the discriminant model (see chapter 5, section 5.4.1.5). Due to this finding all tutorial groups were appropriate as they all had a mix of students who were highly ‘at risk’, i.e. assigned to group 3, and not ‘at risk’ students, i.e. those assigned to group 1. A further requirement however was that a tutorial time slot in which students were likely to attend regularly had to be chosen. Anecdotal evidence from lecturers and tutors in UL suggested that students were less likely to attend early morning tutorials or late evening tutorials if attendance was not compulsory. The 5 tutorial time slots were therefore analysed taking this into account, and a tutorial slot on Wednesday at 1-2 in the afternoon was considered the most likely to encourage attendance in the non-compulsory tutorial. Two tutorials were scheduled to take place.
on Wednesday from 1-2 in two different rooms. One of the two rooms consisted of movable tables and the other had set tables which were immovable. The tutorial room with the movable tables was chosen as it was considered more conducive to the active learning group work practices which the author intended on implementing (Felder et al 2009). Consideration was given to how participation in the intervention tutorials could be promoted. This has been documented in section 6.6.2.

6.6.2 Promoting Intervention Participation

Although the tutorial time slot was expected to facilitate reasonable attendance, there were still no guarantees that students would participate in the non-compulsory intervention tutorials. In general academic support which has been provided to students worldwide requires them to participate on a voluntary basis (Lawson 2000). On this same topic Ellis (2003) highlighted that often the students who needed the academic support the most were the most reluctant to take part in any initiatives which were offered to them. Bearing these findings in mind, and the fact that attendance at the tutorials resulted in no extra credit for students, the author attempted to “sell” the tutorials’ benefits to the students as much as possible. During the introduction of the intervention lessons the author therefore informed the students of the possibilities of improving their understanding and attitude towards mathematics and consequently their overall performance in UL through participation in the group work tutorials which were being offered to them. The students were informed that they were privileged to have been chosen to take part in these tutorials, however they were also given the opportunity to opt out and allow another student to take their place if they so wished. The intervention tutorial time slot was also advertised through the students’ service mathematics lectures and through college e-mails on several occasions at the beginning of the semester. Croft (2003) maintained that an intervention would never be optimally effective unless it was advertised effectively. All students were therefore aware that the intervention tutorials took place on a Wednesday from 1-2, that there were proven benefits to this form of tutorial style over the one that was currently in place. There was no facility for students who were not originally timetabled for the intervention slot to opt in if they wished. This was due to the fact that students were strategically organised into mixed ability groups and so accommodating students who chose to opt in at different times during the semester was not a feasible option. Although the literature highlighted the proven benefits which
the intervention outlined in this research had the potential to have, the intervention has never been carried out before. There was therefore no guarantee that the students who took part in the intervention were in practice going to have an advantage over those who did, equally the author was confident that they would not be at a disadvantage. The mathematical background and degree program of students within the tutorial group chosen was quite diverse. Details of this are outlined in section 6.6.3 which follows.

6.6.3 Tutorial Group Chosen: Background Information of Students and Comparisons with Non-Intervention Students

The tutorial group chosen for the intervention consisted of 43 students with varying mathematical backgrounds from a variety of degree programmes. The group was quite homogeneous however in terms of its gender breakdown. It was made up of 95.4% (41) males and just 4.6% (2) females. It also consisted mostly of standard students with just 9.3% (4) non-standard students present. There was a higher proportion of male students in the intervention group than there was in the non-intervention group (88.8%).

The students’ mathematical backgrounds were determined based on their Leaving Certificate mathematics grades and their diagnostic test results. Just over a fifth of the intervention students, 20.9% (9), had Higher Level Leaving Certificate mathematics grades on entry to UL. A further 69.8% (30) of intervention students therefore entered UL with Ordinary Level Leaving Certificate mathematics grades. There was therefore a slightly higher proportion (26.1%) of students with Higher Level Leaving Certificate mathematics within the non-intervention group than the intervention group (20.9%). The non-intervention cohort also had a higher proportion of non-standard students as it consists of 14.4% non-standard students compared to 9.3% in the intervention group. Overall the breakdown of mathematical backgrounds amongst the two cohorts is not extremely different. The mean mathematics points for both cohorts is 51.5.

Of the 37 intervention students who sat the diagnostic test 64.9% (24) of them scored 19 out of 40 or less classifying them as being ‘at risk’ of failing service mathematics. 56.6% of the non-intervention students were found to be ‘at risk’ of failing Technology mathematics. The students involved in the intervention therefore performed poorer in the diagnostic test than the non-intervention students. The intervention students had a mean diagnostic test result of 56.6% compared to the non-intervention students who had a mean of 64.0%. The basic mathematical
skills of the intervention students are therefore of a slightly lower standard than that of the non-intervention students.

There was a large variety of students from different degree programs within the intervention group which included students from 9 different degree programmes. The degree programmes with the largest proportion of students however were Materials and Architectural Technology which made up 20.9% (9) of the intervention students, Materials and Engineering Technology which made up 18.6% (8) of the intervention students and Product, Design and Technology students which made up 16.3% (7) of the intervention group. The other degree programs involved can be seen in figure 6.1. The breakdown of students in both cohorts in terms of degree programs of study is quite varied and there is no dominance of a particular degree program in the intervention group or the non-intervention group.

Figure 6.1 Breakdown of degree programmes represented in the intervention group.
Due to the variation present within the intervention group, both in terms of mathematical background and degree programme, this tutorial group was considered appropriate for participation in the intervention. The methods used to group these students into mixed ability groups are outlined in section 6.6.4.

### 6.6.4 Mixed Ability Grouping: Methodologies Employed

There were 43 students assigned to the Wednesday 1pm - 2pm intervention tutorial. Due to suggestions regarding group numbers made by Bullard et al (2009) it was considered appropriate to arrange students into groups of 5 within the tutorial. In order to ensure that students were arranged in groups with mixed mathematical backgrounds each student’s probability of failure was considered. Students were further categorised into 5 different probability groups which were assigned as follows:

- **Group 1**: students with a probability of failure between 0-0.2.
- **Group 2**: students with a probability of failure between 0.21-0.4.
- **Group 3**: students with a probability of failure between 0.41-0.6.
- **Group 4**: students with a probability of failure between 0.61-0.8.
- **Group 5**: students with a probability of failure between 0.81-1.00.

Students were arranged into groups which had one student from each probability failure group where possible (see table 6.1 for breakdown of students within each probability group).

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valid No Group</td>
<td>7</td>
<td>16.3</td>
</tr>
<tr>
<td>Group 1</td>
<td>7</td>
<td>16.3</td>
</tr>
<tr>
<td>Group 2</td>
<td>4</td>
<td>9.3</td>
</tr>
<tr>
<td>Group 3</td>
<td>6</td>
<td>14.0</td>
</tr>
<tr>
<td>Group 4</td>
<td>14</td>
<td>32.6</td>
</tr>
<tr>
<td>Group 5</td>
<td>5</td>
<td>11.5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>43</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

**Table 6.1** Breakdown of intervention students in probability groups 1-5.

If students did not have a group, due to not sitting the diagnostic test, approximations were made about their probable mathematical competency level based on their Leaving Certificate
mathematics grade. Students were also grouped in order to allow a wide variety of degree programs to be grouped together. Mature students were spread across different groups also in order to not have all of them in the one group. The rationale behind having mixed ability groups was that students had the opportunity to learn with and from each other (Griffiths et al 1995; Slavin 1995), develop self-directed learning skills and improve intrinsic interest in the subject matter (Barrow and Tamblyn 1980).

Students were informed of their groups during the first tutorial and the tables in the tutorial room were arranged so that students were sitting facing each other. Students were instructed to sit in their groups in a specified space in the room for every tutorial. They were however unaware of the methods used to group students.

6.6.5 Tutors involved in the intervention

Two tutors were involved in the intervention tutorials in addition to the author. These tutors both took a service mathematics tutor training course delivered in UL in September 2010. This training involved helping tutors to develop an awareness of the mathematical deficiencies that many of the beginning undergraduates have and also appropriate methods to use when teaching for understanding. The course was delivered by two lecturers of mathematics education. In addition to this training the two tutors are qualified secondary school mathematics teachers and have completed doctorates in mathematics education. They also have three years experience teaching Technology mathematics.

The author was also involved in the service mathematics tutor training course mentioned previously, is a qualified secondary school mathematics teacher, has taught Technology mathematics for two years prior to the intervention and completed a two day workshop in active learning methodologies in April 2010. All three tutors were therefore considered sufficiently capable of teaching the intervention students.

6.6.6 Change of Tutor Set Up During the Intervention

The provisional plan was to have 3 tutors involved in the intervention for the first three weeks and then assess how it was going and estimate if it could be run as effectively with just one tutor. In practical terms having just one tutor also lent itself to a more realistic tutorial set up which if successful could be replicated in future years. A teaching intervention which was transferrable
was desired. During the initial stages of the intervention the support of the 2 tutors was hugely beneficial in organising the group tables, facilitating group learning and providing the author with the experience and knowledge necessary to continue this format afterwards alone. The weeks in which the three tutors were present were also thought to act as an introductory period for students as they were getting to know their group members and becoming comfortable with working with them. It was hoped that students would have the tutors’ support when unsure of any tutorial work and that this support would then shift to their group members in the absence of the tutors.

During the evaluation of the teaching intervention, an intervention which was found to have transferrable skills was likely to score higher in terms of intervention integrity (Shapiro 1987). This approach was implemented and after 3 weeks of tutorials the author facilitated the classes alone for the remaining lessons. Some of the implications of this approach will be detailed in the Intervention Evaluation in section 6.7. The lesson plans set out before each tutorial took place are detailed in Appendix J.

6.6.7 Implementation of the Intervention: Intervention Lesson Plans
The implementation of the intervention involved bringing together all of the theoretical concepts studied and decided upon in the design phase in the form of concrete plans. The result of this coming together of ideas was a series of lesson plans which directly dictated what content was delivered during the mixed ability group work tutorials and the methods through which the content was delivered. The ten lesson plans which were used in the intervention are detailed in appendix J.
6.7 **Intervention Evaluation**

The evaluation of the intervention is carried out by examining both quantitative and qualitative findings. Details of the procedures used to carry out these evaluations as well as the results which emerged are outlined in section 6.7.1 and 6.7.2 which follow.

6.7.1 **Quantitative Evaluation**

The quantitative evaluation involves an analysis of the success of the Technology 2006-2008 discriminant function in predicting performance of service mathematics students in 2010. It also involves a comparison of the performance of the intervention students against the non-intervention students in service mathematics and a detailing of background information relating to both cohorts. Throughout this section there are varying totals in the tables due to students not taking the diagnostic test or having a Leaving Certificate mathematics grade and therefore not having a prediction of performance in Technology mathematics. There are 43 interventions students in total with 36 of them having a probability of failure and there are 344 non-intervention students in total with 272 of them having a probability of failure.

Results relating to the success of the Technology 2006-2008 discriminant function in predicting performance of both students involved in the intervention and those who were not are outlined next in section 6.7.1.1.

6.7.1.1 **Success of the Technology 2006-2008 Discriminant Function in Predicting Performance of Technology Mathematics in 2010**

The Technology 2006-2008 discriminant function was successful in correctly classifying 91.7% of the students who failed Technology mathematics in 2010. It was however extremely over-cautious in its prediction of failure as it predicted that another 110 students would fail however these students actually passed Technology mathematics. This equates to a prediction that 35.7% more of the Technology 2010 cohort would fail than actually did. Consequently the function correctly classified 53.4% of the students who passed as being successful. The function correctly classified 62.4% of the cohort overall.
The success of the Technology 2006-2008 discriminant function in predicting performance for the intervention group is very similar to its success in predicting performance of the entire Technology cohort. Again the function over estimated the number of students who would fail Technology mathematics. It estimated that 25 students would fail however just 8 students actually failed. It did however correctly classify these 8 students. In terms of its prediction of students who would pass the examination the model correctly predicted that 11 students within the intervention tutorial would pass and failed to correctly classify the remaining 17 students who also passed. The function therefore correctly classified 52.8% of the intervention students overall (see table 6.3).

Table 6.2 Technology 2006-2008 discriminant function’s success in predicting performance in Technology mathematics in 2010.

<table>
<thead>
<tr>
<th>Predicted Performance</th>
<th>Performance in Technology Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pass</td>
</tr>
<tr>
<td>Predicted Success</td>
<td></td>
</tr>
<tr>
<td>Count</td>
<td>126</td>
</tr>
<tr>
<td>% within Pass/Fail Group</td>
<td>53.4%</td>
</tr>
<tr>
<td>Predicted Failure</td>
<td></td>
</tr>
<tr>
<td>Count</td>
<td>110</td>
</tr>
<tr>
<td>% within Pass/Fail Group</td>
<td>46.6%</td>
</tr>
<tr>
<td>Total</td>
<td>236</td>
</tr>
</tbody>
</table>

Table 6.3 Technology 2006-2008 discriminant functions success in predicting performance of intervention students in 2010.
Conclusion
The Technology 2006-2008 discriminant function was reasonably successful in predicting the performance of Technology mathematics students. Although it was overcautious in its prediction of failure, predicting that more students would fail than actually did, it correctly classified the vast majority of students who did fail which is a strength of the function. This finding therefore also highlights the accuracy of the mixed ability groupings as the discriminant function did give a good indication of probable performance. An analysis of the attendance levels of the intervention students is outlined next in section 6.7.1.2.

6.7.1.2 Intervention Students’ Participation Levels
Before an analysis of the performance of intervention students is outlined, it is important to ascertain their level of participation in the tutorials. Table 6.4 details students’ attendance out of ten tutorials. 23.3% (10) of students failed to attend any tutorial while 20.9% (9) attended 5 or less (see table 6.4). The attendance at the intervention tutorials was therefore quite poor.

<table>
<thead>
<tr>
<th>Number of Tutorials Attended</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>23.3</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>4.7</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2.3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>7.0</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4.7</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>2.3</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>11.6</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>7.0</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>9.3</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>18.6</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>9.3</td>
</tr>
<tr>
<td>Total</td>
<td>43</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 6.4 Intervention students’ attendance at tutorials.
6.7.1.3 A Comparison of the Performance of Intervention Students Against Non-Intervention Students

One of the main indicators of intervention effectiveness is the performance of the intervention students in Technology mathematics. In this section their performance is compared against that of the students who did not take part in the intervention to assess if one group outperformed the other. The performance of the intervention group is also analysed by attendance and risk of failure classification.

6.7.1.3 (a) A Comparison of Failure Rates in Technology Mathematics: Intervention Students V's Non-Intervention Students

There is a small difference between the proportion of students who failed Technology mathematics and took part in the intervention tutorials and those who took part in traditional tutorials (i.e. non-intervention students). Of the 43 intervention students who sat the examination 23.3% of them failed Technology mathematics compared to 25.3% of the students who took part in the traditional tutorials (see table 6.5). Although the failure rate did move in the right direction the practical difference in the failure rate is negligible.

<table>
<thead>
<tr>
<th>Student Type</th>
<th>Pass</th>
<th>Fail</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Intervention Student</td>
<td>257</td>
<td>87</td>
<td>344</td>
</tr>
<tr>
<td>% within Non-Intervention Student</td>
<td>74.7%</td>
<td>25.3%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Intervention Student</td>
<td>33</td>
<td>10</td>
<td>43</td>
</tr>
<tr>
<td>% within Intervention Student</td>
<td>76.7%</td>
<td>23.3%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Total</td>
<td>290</td>
<td>97</td>
<td>387</td>
</tr>
</tbody>
</table>

Table 6.5 Comparison of failure rates of intervention students against non intervention students.

Logistic Regression Analysis

Section 6.6.3 outlined that there were small differences between the intervention group and the non-intervention group in terms of their mathematical backgrounds. A logistic regression analysis was carried out to establish if being involved in the intervention group or not had a statistically significant effect on performance in Technology mathematics, after adjusting for Leaving Certificate mathematics points and diagnostic test result. Of the three predictor variables
Leaving Certificate mathematics points was the only one found to be a statistically significant predictor of performance ($p < 0.001$) (Diagnostic test results: $p = 0.283$ and group $= 0.903$). Whether a student was involved in the intervention or not therefore has no significant influence on their performance in Technology mathematics. An examination of each groups’ performance by classification is outlined next.

### 6.7.1.3 (b) Performance of Intervention Students and Non-Intervention Students in Technology Mathematics by Probability Classification Group

Table 6.6 shows that the proportion of students in each classification group is reasonably similar in both the intervention and non-intervention group.

<table>
<thead>
<tr>
<th>Student Type</th>
<th>Risk of Failure in Technology Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Risk</td>
</tr>
<tr>
<td>Non-Intervention Student</td>
<td>Count</td>
</tr>
<tr>
<td></td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>28.3%</td>
</tr>
<tr>
<td>Intervention Student</td>
<td>Count</td>
</tr>
<tr>
<td></td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>22.2%</td>
</tr>
<tr>
<td>Total</td>
<td>Count</td>
</tr>
<tr>
<td></td>
<td>85</td>
</tr>
</tbody>
</table>

**Table 6.6** Proportion of intervention and non-intervention students in each probability failure group.

No students in the low risk category who were involved in the intervention tutorials failed Technology mathematics (see table 6.7). Of the two students who failed from the medium risk category; one of them never attended the intervention tutorials. 37.5% of the high risk students who took part in the intervention failed Technology mathematics. When this 37.5% was investigated further it was found that 4 out of the 6 of them attended the intervention tutorials 6 or more times. The remaining two students failed to attend any tutorials. A discussion regarding why the high risk students who had good attendance rates may have failed Technology mathematics is outlined in section 6.7.1.3 (c).
Table 6.7 Intervention students’ performance by classification.

Note: The overall failure rate of the intervention group appears larger here as the students without probability of failure groups are not included as they were in table 6.7

The vast majority of the low and medium risk students who were offered the traditional tutorial format passed Technology mathematics (Table 6.8). More high risk non-intervention students failed Technology mathematics when compared to their high risk intervention counterparts.

Table 6.8 Non-intervention students’ performance by classification.

It is a positive finding that no low risk intervention students failed Technology mathematics and that less high risk intervention students failed when compared to high risk non-intervention students. An examination into whether student attendance at intervention tutorials had a bearing on performance is outlined next in section 6.7.1.3 (c).
6.7.1.3 (c) Attendance at Intervention Tutorials and Technology Mathematics Performance

There is a large spread when it comes to the number of tutorials attended by students who both passed and failed the Technology mathematics examination (see table 6.9). A logistic regression analysis was therefore carried out to establish if attendance (expressed as a number out of ten possible tutorials) at the intervention tutorials had a statistically significant effect on performance in Technology mathematics, after adjusting for Leaving Certificate mathematics points and diagnostic test result. Attendance was not found to make a statistically significant contribution to the ability of the model to predict performance in Technology mathematics (p = 0.403).

Table 6.9 highlights the fact that 7 out of 10 students who never attended the intervention tutorials passed Technology mathematics. Upon examination of these students’ mathematics backgrounds it was revealed that 5 of them had Higher Level Leaving Certificate mathematics grades and 2 of them had an Ordinary Level Leaving Certificate A mathematics grade upon entry to UL. Their diagnostic test performance was also quite high with just 1 of them being considered to be ‘at risk’ of failing service mathematics based on their diagnostic test result. Of the 3 students who never attended and failed Technology mathematics 2 of them had Ordinary Level B mathematics grades and one was a mature student. 2 of them were also considered to be ‘at risk’ of failing service mathematics based on their diagnostic test result. The students who never attended and failed Technology mathematics therefore had weaker mathematical background than those who passed Technology mathematics and never attended.

These findings are supported by those which were outlined in appendix P in relation to the discriminant function 2 (i.e. the discriminant function with Leaving Certificate mathematics points as the only predictor variable). This discriminant function 2 found that students who received more than 50 points, which 5 of the students who never attended and passed Technology mathematics did, were more likely to pass service mathematics. None of the students who never attended and failed Technology mathematics had more than 50 Leaving Certificate mathematics points. Some further suggestions as to why the students who never attended the tutorials and passed Technology mathematics may not have needed to attend are discussed later in this section.

The poorer mathematical backgrounds of the students who never attended the tutorials and failed Technology mathematics on entry to UL may have influenced their decision not to engage with
the mathematics at all. Disengagement and dis-interest in mathematics have been found to be associated with previous achievement in mathematics (Fielding-Wells and Makar 2006). Mathematics education research has also found that students who choose not to engage with mathematical support often do so due to feeling that they have too many problems to address and feelings of being overwhelmed by the volume of mathematical material in a module (Symonds et al 2008). Another possible reason why these students may have chosen not to engage with the intervention tutorials may be due to a lack of awareness that they may have been ‘at risk’ of failing service mathematics (Symonds et al 2008) which they evidently were.

Nothing could be deduced in relation to the performance of these students in terms of their degree programme as there was a large mix of degree programmes amongst both the group of students who never attended and failed and those who never attended and passed Technology mathematics.

<table>
<thead>
<tr>
<th></th>
<th>Never Attended</th>
<th>Attended 5 or less</th>
<th>Attended 6 or more</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pass</td>
<td>7</td>
<td>6</td>
<td>20</td>
<td>33</td>
</tr>
<tr>
<td>% within Attendance</td>
<td>70.0%</td>
<td>66.7%</td>
<td>83.3%</td>
<td>75.0%</td>
</tr>
<tr>
<td>Fail</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>% within Attendance</td>
<td>30.0%</td>
<td>33.3%</td>
<td>16.7%</td>
<td>25.0%</td>
</tr>
<tr>
<td>Total</td>
<td>10</td>
<td>9</td>
<td>24</td>
<td>43</td>
</tr>
</tbody>
</table>

Table 6.9 Intervention students’ performance according to attendance at tutorials.

Logistic regression analyses were carried out to determine if ‘Probability Failure Group’ combined with attendance at tutorials had a statistically significant effect on performance (see table 6.10) or if there was a significant interaction effect between the two variables i.e. if the effect of attendance on performance in Technology mathematics may differ depending on whether a student was deemed to be a low, medium or high risk of failure. No statistically significant predictor variables were present in these analyses though care should be taken when interpreting the results because of the small sample size.

As mentioned in section 6.7.1.3 (b), of the 6 high risk students who failed Technology mathematics, 1 of them never attended the intervention tutorials however 4 of them attended 6 or more tutorials (see table 6.10). In an effort to try to understand why these high attenders failed
one must consider the possibility that their mathematical backgrounds may have been so far behind that, regardless of the support they received, they did not have the fundamental understanding needed to build upon and pass the Technology mathematics examination. In turn the active learning group work approach being implemented in the tutorials may not have served them as well as a rote learning, drill and practice approach may have considering the nature of the examination (i.e. the examination being almost the same paper every year, see section 6.8 for more details on this). This potential scenario could be viewed in light of the situation with the Leaving Certificate examination which has been criticised due to the points system having a negative impact on the type of teaching implemented and consequently the quality of learning which takes place (Hyland 2011). The attempts made in the intervention to improve the quality of students’ learning may not have been rewarded due to the examination style.

From the literature reviewed, it would seem that students undertaking university courses skip classes on a not infrequent basis (Rodgers 2002; Hughes 2005; Cohen and Johnson 2006). Both the high and low risk student categories however had relatively high attendance rates. It is possible that the high risk students have good attendance rates due to wanting to take up any opportunity that may prevent them from failing service mathematics and out of feelings that the material would be too difficult to make up if missed (Toohey 1999; Quinn 2000). Low risk students’ high attendance rates may be attributed to their general interest in learning and wanting to do as well as possible. The low attendance rate of the medium risk students is therefore noteworthy (see table 6.10). In spite of the fact that the medium risk group had a relatively low failure rate of 16.7% they were the poorest attenders of all 3 categories with 41.7% of them attending no tutorials and a further 25.0% attending 5 or less tutorials. The medium risk students appear to be generally unmotivated to engage with mathematics. Disengaged students have been found to engage more in surface learning strategies such as memorisation as distinct from deeper learning strategies (Connell and Wellborn 1991). The fact that the majority of these students pass the examination suggests that the nature of the examination supports those who engage in “learning-off” past examination papers (see section 6.8). This may be the rationale behind medium risk students’ lack of engagement with mathematics support combined with their high pass rate in Technology mathematics.
Note: The students who never attended the tutorials did not know that the intervention tutorials were any different from a regular tutorial. It is therefore assumed that they chose to disengage with mathematics support in third level as opposed to choosing to disengage with the intervention carried out in this research specifically.

<table>
<thead>
<tr>
<th>Attendance</th>
<th>Never Attended</th>
<th>Attended 5 or less</th>
<th>Attended 6 or more</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Low Risk</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Count</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>% within Probability Failure Group</td>
<td>12.5%</td>
<td>12.5%</td>
<td>75.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td><strong>Medium Risk</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Count</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>% within Probability Failure Group</td>
<td>41.7%</td>
<td>25.0%</td>
<td>33.3%</td>
<td>100.0%</td>
</tr>
<tr>
<td><strong>High Risk</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Count</td>
<td>1</td>
<td>4</td>
<td>11</td>
<td>16</td>
</tr>
<tr>
<td>% within Probability Failure Group</td>
<td>6.2%</td>
<td>25.0%</td>
<td>68.8%</td>
<td>100.0%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>7</td>
<td>8</td>
<td>21</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 6.10 Attendance of intervention students by classification.

6.7.1.4 Quantitative Findings: Summary and Conclusions

The quantitative findings outlined in section 6.7.1 reveal the moderate success of the Technology 2006-2008 discriminant function in predicting performance in Technology mathematics while also highlighting its tendency to be over cautious when predicting failure. A statistically significant association was found between a student’s performance in Technology mathematics and their classification (p < 0.001). This suggests that the grouping system used for intervention students was successful in achieving actual mixed ability groups by using the discriminant function as a point of reference.

Intervention students had a lower failure rate in service mathematics than non-intervention students by 2 percentage points. Logistic regression revealed that the group (intervention/non-intervention) that students were in had no significant influence on their performance in Technology mathematics after adjusting for Leaving Certificate mathematics points and diagnostic test result.

The intervention group consisted of 8 percentage points more high risk students than their counterparts attending the regular tutorials. When intervention students’ performance was compared to non-intervention students’ performance by classification they differed under several headings. No low risk intervention students failed Technology mathematics while 3.9% of the
low risk non-intervention students failed, 16.7% of medium risk students failed which was higher than that of the same group who attended traditional tutorials in which 14.0% failed. Finally 7.5 percentage points more high risk non-intervention students failed when compared to intervention students.

Attendance at the intervention tutorials was quite poor. When attendance was examined however it was revealed that the highest proportion of students who were successful were the best attendees. There were however a number of students who passed Technology mathematics having never attended any tutorials. These students were thought to have passed due to their strong mathematical backgrounds on entry to UL combined with the possibility that they engaged in rote learning of past Technology mathematics examination papers. The medium risk group were found to be the poorest attendees of all groups which is possibly attributed to their lack of motivation when compared to the other two groups and their ability to pass the examination in spite of not engaging with the mathematics support being offered. Logistic regression analysis was carried out to determine if attendance and ‘Probability Failure Group’ were statistically significant predictors of performance in Technology mathematics. An examination into the possibility that the effect of attendance on performance may differ depending on a student’s ‘Probability Failure Group’ was also carried out. Neither of these analyses revealed variables which were statistically significant predictors of performance in Technology mathematics.

Although the discriminant function performed reasonably well in predicting performance in Technology mathematics the quantitative analysis of the intervention reveals that there was no practical difference between the performances of the intervention students against the non-intervention students. Some possible reasons why the intervention may not have been more successful from a quantitative point of view are outlined in section 6.8. First however, an examination into whether the intervention was more successful from a qualitative point of view is detailed.
6.7.2 Qualitative Evaluation

The qualitative evaluation of the intervention involved an analysis of both the author’s observations of the intervention tutorials and the intervention students’ questionnaires. The method through which each of these tools for evaluation were created is outlined in detail in chapter 3, however a brief recap of the procedures used to create them is outlined in section 6.7.2.1. The results of the qualitative analysis are in section 6.7.2.2.

6.7.2.1 Author’s Observations and Student Questionnaires

The first element of the qualitative feedback, the author’s observations, took the form of a weekly journal entry. Each journal entry was completed immediately after each tutorial took place and the structure and content of the journal entries were guided by:

- Spradley’s (1980) Checklist for Intervention Observation Entries
- Spradley’s (1979) and Kirk and Miller’s (1986) Set of Observational Data and
- Shapiro’s (1987) Intervention Research Evaluation steps 2 and 4 i.e. Treatment Integrity and Treatment Acceptability

Student questionnaires were completed twice during the course of the semester. Students answered questions anonymously after 6 weeks and at the end of the semester giving their opinions on the intervention tutorials. The questions posed to them were all informed by Shapiro’s (1987) Intervention Research Evaluation steps 3 and 4 i.e. Social Validity and Acceptability of the intervention. The questions asked in each questionnaire were as follows:

**Questionnaire 1:**

**Q 1:** What are your thoughts on the tutorials so far?

**Q 2:** Do you think all tutorials should be taught in this format/style? Why? Why not?

**Q3:** Any other comments?

**Questionnaire 2:**

**Q1:** What do you feel works in this tutorial best and should be kept?

**Q2:** What do you feel needs to be changed/ dropped from the tutorial?

**Q3:** Any other comments?
The findings relating to these two methods of qualitative evaluation, which were analysed using the qualitative analysis program NVivo, are detailed next in section 6.7.2.2. (Note: the full transcripts of the qualitative findings can be found in Appendix K).

6.7.2.2 Qualitative Findings
In this section the most prominent themes and nodes emerging from the author’s observations and the student questionnaires are detailed. The analysis of the data revealed many commonalities across the two sets of findings as well as insights which emerged exclusively in one or other of the findings. The means through which these findings enabled the evaluation of the intervention using Shapiro’s (1987) Intervention Evaluation Model are outlined in section 6.7.3.

6.7.2.2 (a) Author’s Observations
The author’s observations of the tutorials, which can be found in Appendix K, were firstly manually coded by the author and two mathematics educationalists in UL. They were then analysed using NVivo software. After analysis two major themes emerged with a number of nodes within each theme. The interpretation and resulting discussion surrounding the emerging themes must be viewed in light of the researcher’s pedagogical values as outlined in section 6.5.2. The two major themes and their corresponding nodes are outlined next.

Theme 1: Teaching Techniques
The analysis of the author’s observations revealed constant references to the successfulness/unsuccessfulness of the intervention tutorials in relation to a particular ‘Teaching Technique’ used or which should have been used. Results of the analysis showed that if specific teaching techniques were used appropriately and effectively in the tutorials that the lessons were more likely to be successful both in terms of students’ understanding of the mathematics and their ease of comfort during the lessons. More specifically the nodes which emerged within this theme were:

Node 1: Time Management
Node 2: Teaching Medium and Realistic Mathematics Education
Node 1: Time Management

‘Time Management’ and issues surrounding effective use of time were frequent throughout the author’s observation data. Two sub-nodes emerged within this node when it was examined in greater detail:

Sub-node 1: Explanation Time V’s Student On-Task Time
Sub-node 2: Wait Time

Sub-node 1: Explanation Time V’s Student On-Task Time

Data relating to the struggle to effectively balance explanation time with student on-task time was common throughout the tutorial observations. This was a pivotal issue within these tutorials as the nature of them being active learning mixed ability group work called for constant time being given to students to work in groups. However some explanation and guidance in relation to each topic was essential if the appropriate mathematical material was to be covered. Within this sub-node the author reported several instances in which she felt more explanation was needed for students. For example in week 1 she noted that: “many students struggled with the same problems on the tutorial sheet suggesting that more explanation may have been needed” (Tutorial 1, September 22nd).

However this improved in week 3 when it was reported that: “short explanations seemed to have led to students discovering things for themselves and some noticeably grew in confidence” (Tutorial 3, October 6th).

The battle with the balance of explanation time and student on-task time was still present in later data as in tutorial 6 the author noted that “some students were confused and upset by the pace of the lesson” and the author felt that she “did students an injustice by not giving them enough of an explanation of trigonometric functions” (Tutorial 5, October 20th).

This was a node in which improvements over time were evident as positive reports relating to explanation time v’s student on task time were reported in tutorial 6 (October 27th), tutorial 7 (November 3rd), tutorial 9 (November 17th) and tutorial 10 (November 24th). In the latter the author reported satisfaction as she felt she:

struck a good balance between varying my mode of teaching (handouts, blackboard, discussion), my explanation time and the length of time I gave to student to practice questions

(Tutorial 10, November 24th).
The second sub-node which emerged within the ‘Time Management’ node was ‘Wait Time’ and is discussed next.

**Sub-node 2: Wait Time**

The consciousness of the author throughout her observations of the importance of allowing students sufficient time to process and consider a question posed in class before offering the answer was evident in the data. To begin with students were “not very responsive” to the questions posed however the author took the responsibility as she felt “more wait time was needed” in order for meaningful thought to take place (Tutorial 1, September 22nd). The confidence of both the author in her teaching methods and the students in their abilities could be seen to improve over time from the data as in tutorial 7 the author reported that she managed to leave “a long wait time until someone finally answered” (Tutorial 7, November 3rd).
The next node which emerged during the analysis of the data was not related to time management but rather to the effects of the medium through which the teaching took place. This node is discussed next.

**Node 2: Teaching Medium and Realistic Mathematics Education (RME)**

The mediums used to teach each lesson, for example the use of GeoGebra, blackboard explanations, you-tube clips, discussion and the references to Realistic Mathematics Education were all mentioned in reference to the success of each tutorial. This node revealed several references throughout the data regarding the mediums through which particular topics were taught all of them being positive: “Students seemed happy with the variation of teaching resources (GeoGebra, you-tube, blackboard explanations, discussion)……Students found the you-tube clip funny” (Tutorial 2, 29th September).

In tutorial 8 the topic: ‘differentiation by rule’, called for an increase in blackboard explanations and the author felt this helped the students’ comfort and confidence with the topic as they could self correct their work. The author continued to use a variety of teaching mediums where appropriate and did not dismiss the necessity at times for the more traditional teaching methods (blackboard explanations) as she mentions on several occasions in her observations requests from students for blackboard explanations.
Just as students’ interest and comfort with particular teaching mediums emerged from the analysis of the author’s observations, discussion surrounding RME was also prevalent. The continuous references to the relevance of the mathematics being studied to the real world were evident in the analysis also. In the observation of tutorial 3 the author felt “that the time spent explaining to students the use of a particular mathematical concept in the real world is worthwhile” (Tutorial 3, October 6th).

The author reiterates her belief that this aspect of each lesson is important in tutorial 8 and 9 when she states that “The inclusion of real world mathematics was pivotal to the success of the class”. The author made reference to how RME improved student engagement in tutorial 9 which dealt with matrices as students engaged in a discussion surrounding the topic of encryption.

The nodes relating to the theme ‘Teaching Techniques’ all presented positive relationships, according to the analysis of the author’s observations, between effective use of different teaching mediums and the inclusion of RME with successful intervention tutorials. The next theme which emerged upon analysis of the author’s observation data was related to group work.

**Theme 2: Group Work**

Discussion surrounding ‘Group Work’ was plentiful in the author’s journal observations and emerged as the second major theme when analysis was carried out. References were made throughout regarding many students increased ease with group work over time and to some students’ apparent discomfort with it throughout. These two nodes will therefore be discussed next.

**Node 1: Comfort with Group Work**

The majority of data which was present within the theme of ‘Group Work’ related to students apparent comfort with it. Initially, in tutorials 1 and 2, the author noted to her surprise that students were quite comfortable in their group setting which was displayed by the fact that the “majority of students seemed to communicate within their groups straight away” (Tutorial 1, September 22nd) and in tutorial 2 “the initial organisation of the tables by students” (Tutorial 2, September 29th) suggested they were quite comfortable with the set up. The comfort of students in groups was reiterated by the author throughout her observations. For example the author stated that “some groups were even conferring with each other to see if they had the same solutions for
different problems” (Tutorial 2, October 6th). This comfort progressed in tutorial 5 when students “displayed an ease of answering questions in front of the whole class” (Tutorial 5, October 20th). The findings within this node also highlighted how students took responsibility without instruction from the tutor if their group mates were not present on a particular day:

\[ \text{at the very beginning of the class some students who were in groups by themselves paired up with others in the same situation} \]

(Tutorial 6, October 27th).

The final major finding which emerged from the data within this node was in relation to the effectiveness of the work which occurred between “stronger standard students and mature students. This pairing seemed to work well” (Tutorial 9, November 17th). This finding, relating to the effective learning which can take place when standard and mature students work together, is one which is in keeping with findings in the work of Laws et al (1999).

The comfort of the majority of the intervention students with group work is very apparent from the data present in this node. Not all students however fell into this category as the analysis revealed a minority group of students who were not so comfortable with this form of learning. These students will be discussed next.

**Node 2: Discomfort with Group Work**

The major finding which emerged upon carrying out the analysis of this node was that some of the stronger students in the class along with some of the weaker students in the class, according to the discriminant function, “kept to themselves in relation to the work set rather than engaging with their group” (Tutorial 1, September 22nd).

The author noted and highlighted this again in her observations and felt that the students who were not engaging with the group work “may need more specific instruction to engage with the group” (Tutorial 2, September 29th). The same issue of disengaged students was prevalent in the data again in tutorial 5 and tutorial 10. It is clear from the examination of material relating to the node ‘Discomfort with Group Work’ that group work is not a comfortable means of learning for everyone, particularly in the case of the intervention students on the lower and higher end of the ability scale. However, simply encouraging these students to take part resulted in improved
engagement in group work (Tutorial 10, November 24\textsuperscript{th}). It is important to note that this theme emerged from the author’s observations only and was not backed up by the students’ feedback of the intervention.

**Summary**

The themes emerging from the author’s observations gave extensive insights into the success of the intervention tutorials. The main themes of ‘Teaching Techniques’ and ‘Group Work’ combined to highlight what the author felt were the most noteworthy and pivotal features to the success of the tutorials. The nodes within the ‘Teaching Techniques’ theme, i.e. Time Management and Teaching Mediums and RME, outlined how effective allocation of time to particular tasks and effective selection of appropriate mediums of delivery can have a huge influence on the success/failure of active learning group work mathematics tutorials. The contribution these findings made to the evaluation of the intervention are discussed in section 6.7.3. An analysis of the students’ questionnaires also took place and the findings are detailed in section 6.7.2.2 (b).
6.7.2.2 (b) Student Questionnaires

Throughout the analysis of the student questionnaires, which was also carried out using NVivo, several themes emerged. 55.8% (24) of the intervention students completed questionnaire 1; 7 low risk students, 5 medium risk students, 10 high risk students, 1 mature student and 1 student whose classification was unknown. 48.8% (21) of the students completed questionnaire 2; 8 low risk students, 5 medium risk students, 7 high risk students and 1 mature student. Before the major themes which emerged are outlined the bias, which is potentially present due to the fact that only students who chose to attend the intervention tutorials completed the questionnaires, must be acknowledged. As before the discussion surrounding the analysis must be viewed in light of the researcher’s pedagogical values. The three main themes which presented themselves after careful manual coding of the student questionnaires 1 and 2 in NVivo were:

**Theme 1: Believed Benefits of Group Work**

**Theme 2: Getting to ‘do’ Mathematics**

**Theme 3: Thorough Coverage of Questions**

**Theme 1: Believed Benefits of Group Work**

The emergence of data relating to ‘Believed Benefits of Group Work’ was the most prevalent throughout the analysis of the student questionnaires. 58.3% (14/24) of students who completed questionnaire 1 referred to the benefits of group work in their responses. 52.4% (11/21) of students who completed questionnaire 2 referred to the benefits of group work.

Generally students stated that the group work made the mathematics “easier to understand” for a variety of different reasons. Several low risk students referred to the notion that “when you’re explaining you are also learning” (Student 5, Classification 1) and acknowledged that “everyone has different problems we all learn from them as well” (Student 7, Classification 1). A third student in response to the question ‘What do you think works best about the tutorials?’ maintained that “the group work helps people to stay at a high standard” (Student 1, Classification 1).

Medium risk students, i.e. classification 2 students, appeared to have felt similarly in relation to group work as many also maintained the mathematics was easier to understand suggesting that “4 or 5 students working on a problem makes it a lot easier to understand” (Student 3,
Classification 2). A student responding to questionnaire 2 felt that the best thing about the tutorials was the “interaction between everyone in the group” (Student 2, Classification 2).

The support for the use of group work was emphasised again by the comments made by the classification 3 students who were in favour of it because they felt that “the faster students help the slower students” (Student 1, Classification 3), “working in groups makes it easier” (Student 6, Classification 3) and “we are in groups and can ask each other questions if we are not too sure and we don’t feel afraid to ask” (Student 10, Classification 3).

A small minority of students (3 students) had some problems with the group work as they stated they didn’t feel they got the same benefits as students in other groups as their group mates were poor attendees. The presence of positive comments in relation to group work which emerged from the analysis of the data however was far more common that that of the negative comments.

The next major theme which emerged from analysis of the data was relating to students’ comments regarding actually getting time to ‘do’ mathematics.

**Theme 2: Time to ‘do’ Mathematics**

The analysis, carried out in NVivo, also revealed that students were extremely satisfied that during their mathematics tutorials they got a chance to ‘do’ mathematics. 20.8% (5/24) of students who were asked what they thought of the tutorials so far stated that they were happy that they actually got to ‘do’ mathematics themselves in the tutorials rather than just being passive students. For example two low risk students stated that “I find the tutorials a good way of practicing my maths skills” (Student 1, Classification 1) and “they let us actually do the questions out to ensure you know how to do the problems” (Student 2, Classification 1).

Several high risk students also referred to this as a positive element to the tutorials as one person stated “it’s good, you do maths and put what you’re shown in lectures into practice” (Student 4, Classification 3) another student reiterated this by stating “they are handy because they give you a chance to put the maths from lectures into practice” (Student 5, Classification 3). A mature student agreed with this opinion when expressing his appreciation that “time is given to the student”.

Students’ appreciation for receiving on-task time to ‘do’ mathematics was a theme which emerged quite clearly from the data. Another aspect of the intervention tutorials which students referred to a lot was related to thorough coverage of questions.
Theme 3: Thorough Coverage of Questions

Analysis revealed that 23.8% (5/21) of the students who completed questionnaire 2 highlighted the importance for them that all questions be covered completely during class time. Throughout the analysis of this theme it became apparent that some students were uncomfortable with the pace of the lessons at times as well as the autonomy they were often given to work on questions. When asked what they like most about the tutorials, students within all classifications, referred to their preference for “showing how to do each part of the questions” (Student 6, Classification 1). A high risk student expressed an appreciation for “going through questions thoroughly” (Student 6, Classification 3) and another stated the importance for them of “going through each question slowly” (Student 3, Classification 3).

Upon realising students’ unease with incomplete tutorial sheet questions throughout the intervention the author introduced summary sheets for students, which contained sample questions and solutions and emphasised key concepts covered in different topics. Students were given this on exit of the class and encouraged to use them as a point of reference and for revision. A node which emerged within the theme of ‘Thorough Coverage of Questions’ was therefore ‘Usefulness of Hand-outs’.

Node 1: Usefulness of Hand-outs

Analysis of the second questionnaire revealed that one third of the students referred to the hand-outs they were provided with as something which they found extremely useful and something which they thought should not be changed about the tutorials: “Hand-outs of the key rules and formulae are very handy to refer to” (Student 4, Classification 1) and “the example sheets helped when I was stuck during study” (Students 3, Classification 1).

The simple concept of a hand-out to aid study gave students a comfort during tutorials that if they didn’t get all of the tutorial sheet questions completed that they would have a point of reference outside of class time.

Node 2: Longer Class Time

When asked in questionnaire 2 if students had any other comments the analysis of responses revealed that several students, all of whom had mentioned how thorough coverage of questions was important to them, requested longer class time: “I feel it would be more effective if the class lasted longer” (Student 6, Classification 3). A high risk student agreed with this point when they
mentioned “Do them more often or for longer because they are good (Student 4, Classification 3). One student suggested that “there should be two tutorials in one week and no lectures” (Student 2, Classification 2).

**Summary**

The themes which emerged from the student questionnaire data highlighted the positive response students had in relation to the intervention tutorials. Many students outlined how they believed group work benefitted their learning and understanding of the mathematics, much of which was in line with the reported benefits within the literature, as outlined in section 6.5.1.

A theme which emphasised the didactic nature of the majority of mathematics tutorials is the one which highlights students’ appreciation for actually getting to ‘do’ mathematics during class time. The fact that students outlined being able to engage with the mathematics as a benefit exposes the fact that this is probably not regular practice in Irish mathematics education. The study of mathematics is “not a spectator’s sport” and so this theme outlines a move in the right direction in terms of good practice in mathematics education (Myers and Ludwig 1999).

The final theme which emerged from the data was concerned with students’ request for ‘Thorough Coverage of Questions’. It is interesting that students’ main issue with the intervention tutorials was that there was not enough time to spend engaging with the mathematics. Two nodes emerged from this theme: ‘Appreciation for Hand-outs’ and ‘Longer Class Time’. Students’ concern for the coverage of questions was reinforced by their appreciation of the summary hand-outs. The author believes that these hand-outs were viewed as a sense of security to students that they would have support outside of class time. Some students’ suggestion to compensate for the lack of coverage of questions was to introduce more class time. This node was encouraging as it highlighted students’ interest in their learning and their willingness to spend more class time engaging in mathematics if it was made available to them.

Although the analysis of the author’s observations, and the students’ questionnaires, must be viewed in light of the author’s pedagogical values and the fact that only the students who attended the intervention tutorial completed the questionnaires they were still found to reveal a lot of positive feedback. How does this analysis contribute to the evaluation of the intervention in terms of social validity and treatment acceptability? The answers to this are outlined in the discussion in section 6.7.3 which follows.
6.7.3 Shapiro’s (1987) Model for Intervention Evaluation: Results

6.7.3.1 Treatment Effectiveness: Intervention Students V’s Non-Intervention Students’ Results

The extent of the effectiveness of an intervention refers to the degree of change/improvements present among the students partaking in the intervention when compared to those who are not partaking. This can be measured by considering factors such as the strength, immediacy and amount of change amongst the students involved in the intervention. The conveniently chosen intervention group were found to have similar credentials/ background to the non-intervention group (see section 6.6.3).

The effectiveness of the intervention was measured by quantitative means by assessing the amount of change evident when treatment was initiated compared to non-treatment. From section 6.7.1, it could be seen that the intervention group had a slightly lower failure rate when compared to that of the non-intervention students, although there is no statistically significant association between the group a student was in (intervention or non-intervention) and Technology mathematics results. The practical difference in the failure rate is negligible and so the intervention does not rate very strongly in terms of treatment effectiveness when strength and immediacy of change are considered. Some possible reasons for this are outlined in section 6.8.

6.7.3.2 Treatment Integrity: The Extent to which the Intervention Carried out was as Described

An evaluation of the integrity of an intervention is vitally important in order to allow one to be confident that researchers who intend to replicate research methods can be assured that the treatment was implemented precisely as described (Shapiro 1987). Research integrity has been said to compose of a range of good practice and conduct such as:

- Intellectual honesty in proposing, performing and reporting research.
- Accuracy in representing contributions to research proposals and reports.
- Transparency in conflicts of interests or potential conflicts of interest.

(National Research Council of the National Academics 2002).
In summary it calls upon the researcher to engage in responsible conduct. Shapiro (1987) refers to the significance of goals, social appropriateness and importance of outcomes when he discusses the integrity of an intervention. The integrity of the intervention carried out in this research project was therefore considered and increased through implementation of the following practices:

- A documentation of the lesson plans for each tutorial.
- An honest observation by the author immediately after each tutorial detailing exactly what occurred, what was successful and unsuccessful and how the execution of the lesson compared to the actual lesson plan. An accepted and widely used framework for observations in educational research was used to aid objectivity (see chapter 3).
- Detailing exact methods used to select the intervention group.
- Detailing statistical methods used throughout the process and reporting exact results.
- Allowing students involved in the intervention to offer anonymous feedback so as to ensure an honest reflection of the tutorials from different perspectives be presented in the write up of the thesis findings (see Appendix K).

The integrity of any intervention is also greatly influenced by the validity of the research being carried out. Validity was therefore considered during the initial stages of this research (see chapter 3, section 3.6). Such considerations include:

- An awareness of the importance of objectivity when analysing students’ questionnaires relating to the intervention.
- Outlining the potential influence of the author’s pedagogical values on the intervention.
- Peer examination of data (students’ questionnaires and the author’s observations) took place in order to improve the internal validity of the data.
- External validity was displayed in the design and implementation of the intervention as its equipment and resources were very minimal making it a generalisable intervention to the wider research community.

The integrity of the intervention was therefore upheld by a number of different practices as outlined in this section. One of the largest components however was the honest documentation of
the author’s observation of each lesson. These documentations highlighted the compliance, for
the vast majority of lessons, with the intended practices to be carried out as outlined in each
lesson plan (see Appendix K for the author’s observations and Appendix J for the intervention
lesson plans).

8The author was not aware of which students filled out the questionnaires despite knowing the
classifications.

6.7.3.3 Social Validity of the Intervention
“Social validity refers to the evaluation of treatment by consumers” (Shapiro 1978, p.293). This
stage of the evaluation involves an analysis of the social significance of the goals, social
appropriateness of the procedures and the social importance of treatment outcomes. Witt and
Martens (1983) suggest consumer questionnaires as an appropriate method to assess the social
validity of an intervention and so that is what the author implemented.

As highlighted in the analysis of the author’s observations the majority of students were content
with the active learning group work tutorial format from the beginning. This comfort was
displayed through the willingness of students to set up the classroom in groups and their
engagement with their group members from the very beginning. There was however a minority
of students who were at the upper and lower ends of the ability scales, as measured by the
discriminant function, who appeared to the author to be uncomfortable with the group work and
found it difficult to engage with their peers. Despite this however, the students still chose to
attend. It is important to note also that the students themselves did not give any indication of
being uncomfortable in their questionnaires. Over time the author realised that the students who
appeared to have found it difficult to engage needed some more encouragement and direction to
guide them to work with their groups and upon receipt of this the students did get more involved.
The author observed also that upon finding an appropriate balance of explanation time and
student on-task time in the tutorials, students were content that they received sufficient
explanations as well as enough time to ‘do’ mathematics themselves. The observations of
negative feelings towards the tutorials each week were infrequent with incidents being reported
in relation to the strongest and the weakest students’ unease with the tutorial format in weeks 1,
2 and 5 only (see Appendix K). From the author’s perspective therefore the majority of the
consumers of the intervention evaluated it quite positively.
The social validity however was primarily analysed through examination of the feedback given by students regarding the tutorials. The main themes which emerged from the student questionnaires firstly highlighted students’ support and belief in the positive effects of group work and active learning on mathematical understanding (see section 6.7.2.2 (b)). The second major theme emphasised students’ support for time being given to them doing mathematics:

_They are good as they let us actually do the questions out to ensure we know how to do the problems_

(Student 2, Classification 1).

The final major theme which emerged from students’ feedback was their request for thorough coverage of questions. Many students, particularly the high risk students, felt that being able to get all questions covered in class was very important and something which wasn’t always achieved:

_You don’t go through all tutorial sheet so I miss some questions …..try get all questions on the sheet covered_

(Student 9, Classification 3).

The most prevalent themes coming from the student questionnaires therefore highlighted the strength of the social validity of the intervention tutorials. Consideration however must be given to the fact that the students who completed the questionnaires were students who chose to attend the tutorials. The potential existence of additional factors which tend towards positive perceptions of interventions must also be acknowledged such as the author’s enthusiasm towards it. These factors are discussed in more detail in section 6.8. Negative feedback however did exist in terms of coverage of material. This issue may have stemmed from not having enough class time which was something which was out of the control of the author.

The poor attendance by some students must also be considered in light of the social validity of the intervention. The students who attended no tutorials at all may not have been aware that the tutorial set up was any different from a regular tutorial or alternatively they could have found out about it through word of mouth and chosen not to engage with it. The students who attended 5 or less tutorials may also have made a conscious decision not to attend the tutorials due to feelings of discomfort with the group work/active learning. Alternatively they may have felt that they
would be able to pass the examination without the tutorial input. The medium risk students were the poorest attendees of all the groups. They possibly decided that they could adopt a similar approach to that which many students adopt in the Leaving Certificate examination i.e. engage in rote learning of past examination papers (Hyland 2011). If this was the case it proved to be successful as the vast majority of medium risk students passed the examination. No qualitative data exists on the students who had poor attendance to decipher why they did not attend, nor was information gathered relating to whether the students who never attended were aware that the intervention was taking place. The poor attendance rate by some students however is something which negatively affects the social validity of the intervention and therefore must be acknowledged.

From the analysis of both the author’s feedback and the student questionnaires it is clear that the intervention scored highly in terms of social validity with the incidents of negative feedback being infrequent. The feedback however was only gathered from those students who did attend and the negative impact of the poor attendance rate by some students must also be considered. A generally socially valid intervention however does not necessarily mean that it was accepted to any great extent. The intervention acceptability is outlined next in section 6.7.3.4.

### 6.7.3.4 Intervention Acceptability

Treatment acceptability is closely related to social validity however it measures the extent to which clients receiving or giving treatment accepts the treatment. The acceptability of an intervention can be a complex issue as it is influenced by many variables such as: the degree of behavioural change evident, immediacy of change, type of change, type of intervention, effort involved in implementation of treatment and theoretical orientation of intervention (Witt and Martens 1983).

From the perspective of the clients receiving the treatment (i.e. the intervention students) the student feedback sheets provide a good insight into the intervention acceptability. The positivity regarding the intervention tutorials which is evident throughout the student questionnaires suggest that the vast majority of students who chose to attend the tutorials liked and were accepting of the treatment they received. Comments such as the tutorials are:
great-no real pressure atmosphere. Lots of help is available and there are good explanations. The visual aspects on the board and with GeoGebra I found them very good

(Student 6, Classification 1).

I feel I learn a lot more this way as some of my fellow students in other tutorials are not getting the same benefits as myself

(Student 1, Mature student).

highlight the students’ enthusiasm towards and interest in learning mathematics in this manner. Students’ support for active learning group work (55.6% of them mentioning it in a positive manner) suggests they were very accepting of the theoretical orientation upon which the intervention was based.

From the perspective of the client giving the treatment (i.e. the author) the intervention was largely accepted. Although the design and implementation of the intervention required extensive research, planning and preparation, the execution of the tutorials required little resources. Therefore the effort of implementation was no more time consuming than any other module of mathematics in university education. For the most part students who attended were very accepting of the intervention procedure and willing to engage with the lessons. As highlighted in the discussion surrounding the author’s observations in section 6.7.2.2 (a) the teaching techniques used were highly influential on the success of each lesson and as a result the author learned to adapt the lessons where appropriate/necessary.

In summary both the author and students who attended the tutorials were accepting of the intervention and from analysis it is clear that all parties felt that it was a worthwhile intervention to be involved in.

Summary

The intervention was considered successful from several aspects. The intervention was most overtly successful in terms of social validity and treatment acceptability, as demonstrated by the author’s observations and student questionnaire analysis in NVivo. It must be acknowledged however that this interpretation of success is subjective and should be viewed in light of the researcher’s pedagogical values as outlined in section 6.5.2. The outcome of the social validity of
the intervention may have been somewhat different if more feedback had been obtained from students who didn’t attend the tutorials as well as those who did.

The intervention was considered to be largely successful in terms of treatment integrity as clear documentation of all procedures within the intervention was detailed along with honest descriptions of the extent to which these procedures were executed. The intervention was not as successful however in terms of treatment effectiveness. The intervention students had a slightly lower failure rate compared to the non-intervention students however the difference was not statistically significant. This suggests that although the intervention was deemed to be reasonably socially valid, accepted by those who engaged with it and considered to have integrity it did not necessarily have any real impact on the students’ performances in Technology mathematics examinations.

The evaluation of an intervention such as this from a number of perspectives, i.e. treatment effectiveness, integrity, social validity and acceptability, is vital in order to determine where the strengths and weaknesses occurred exactly. This multi-dimensional evaluation therefore allows future education researchers to adapt some approaches so as to improve on the weaknesses and mimic the areas of success. A discussion outlining some of the possible reasons that the intervention was not more successful from a treatment effectiveness (quantitative) point of view along with discussion relating to the potential influences on it being considered successful from a qualitative point of view are outlined next.
6.8 Intervention Evaluation: A Discussion of Outcomes

Some aspects of the intervention did not have the desired outcomes such as the failure rate of the intervention students not being significantly lower than that of the non-intervention students. Other aspects emerged as being more successful such as the social validity and the intervention acceptability. The discussion which follows aims to determine why these outcomes may have occurred.

6.8.1 Possible Reasons for the Intervention not having the Desired Quantitative Outcome

There are several possible reasons why the intervention may not have been more successful from a quantitative point of view for example:

a. Time frame

Yeaton and Sechrest (1981) suggest that treatment effectiveness may be linked to the amount of treatment. The short time over which the intervention took place, i.e. ten weeks, may have had an influence on the degree of treatment effectiveness as set out by Shapiro (1987). Prendergast (2011) found that when implementing a mathematical intervention in second level education in Ireland, a period of time longer than 13 one hour lessons may have resulted in a greater realisation of his main aim; that was to increase the uptake of higher level mathematics in second level education in Ireland. Regan (2005) also alluded to this point when discussing issues which may have resulted in her chemistry intervention in second level education being less effective than was hoped. She maintained that it may have been “too short to make a lasting impression” (p.421). The teaching intervention carried out in this research therefore may have benefitted from having more than ten contact hours.

b. Examination Remaining Largely Unchanged

The Technology mathematics examination which students sat at the end of the first semester was very similar to that which previous cohorts of students have sat for the last ten years. The layout of the paper has remained the same for the last ten years and minimal changes have been made to the questions from year to year. For example, different functions and digits are introduced however the procedures used in each question remain the same (see Appendix L for a detailed analysis of past examination papers). This is likely to have lead to several issues which may have affected the intervention’s effectiveness such as:
### i. Support for Rote Learning

The examination paper remaining largely unchanged over the last ten years supports rote learning. Students have access to past examination papers online which includes solutions and so, the temptation is to engage in drill and practice procedures surrounding using these papers and regurgitate the material on the day of the examination. As was discussed in chapter 4, second level students tend to adapt surface learning approaches to education such as memorisation of procedures due to their reliance on these approaches in their final year of secondary school (Richardson 1995). This type of learning style which does not concern itself with the understanding of concepts can yield very good Leaving Certificate results. This point is often discussed in relation to one of the flaws of the Leaving Certificate programme as it has been said to sacrifice true learning to a public examination in a society in which final marks are the ultimate goal of education (Madaus and MacNamara 1970). The majority of Technology mathematics students are therefore coming directly from a second level education system in which surface learning approaches are dominant and lead to ‘success’ and so the nature of the Technology mathematics examination meets their needs as they can pass it using similar approaches.

### ii. Mismatch Between the Preparation for the Intervention and the Traditional Examination Style Being Used

The fact that students can perform very well in their final examination through engaging with rote learning may have downgraded the potential benefits which could have been gained through engagement with the intervention tutorials. The intervention practices may in-fact have been seen to waste students’ time as they could have been engaging in drill and practice procedures instead of trying to problem solve and discuss what the mathematics is actually telling them and how it can be used in everyday life. The non-intervention students who engaged in the traditional tutorials and the students who simply engaged with the past examination papers and solutions were potentially using their time more effectively by constantly taking part in drill and practice procedures. This is essentially what the examination rewards. The problem for these students occurs when they are faced with a more advanced mathematics module and their lack of understanding of concepts and previously memorised procedures and “tricks” are weak foundations for the new material. This approach to learning, which discourages critical thinking and real understanding, has been found to lead to problems for students on entering third level
education (Collins 2010), hence it is likely to negatively affect students progression throughout third level education. In summary students may not have benefitted in terms of increasing their preparation levels for the Technology mathematics examination due to a mismatch between the preparation for the intervention and the style of the Technology mathematics examination. For the student there is no reward in the examination for showing their understanding or detailing where the mathematics may be helpful in everyday life.

**c. Students Prior Mathematical Experiences**

Prior to entering third level education, the majority of students involved in the intervention would have been exposed to 13 years of didactical mathematics teaching (Smith 2002; Irish National Teachers’ Organisation 2006; Hourigan and O’Donoghue 2007). This is evidenced by the NCCA (2009) who describe the second level education system as being characterised by rote learning and procedural knowledge. It is evidently very difficult to change a student’s approach to learning mathematics over ten weeks when they have been exposed to and grown comfortable with an approach which they have been engaged with for 13 years. Zan (2008) found that students’ habits are difficult to break once students’ have gained a level of comfort with them. Liston (2008) found that students who enter university education with fragmented conceptions of mathematics and surface approaches to learning mathematics will carry these approaches with them into their third level mathematics education. This point further highlights that students’ prior mathematics experiences are hugely influential and often detrimental to their third level mathematics performance.

**d. More Experienced Tutor Needed? Attendance Issues?**

According to Clarke (2005), an effective facilitator of any intervention must have the ability to recognise the strengths and abilities of the individual participants involved. Although the author is a fully qualified teacher, perhaps if a more experienced tutor implemented the intervention they would have been better able to recognise students’ strengths and abilities and use this knowledge to improve the teaching practices. Cox (2006) highlights the need for third level mathematics teachers to have the ability to “excite and inspire students at all levels” (p.42). This is something which could be considered as a possible means of improving an intervention such as that which was carried out as part of this research. On the other hand the disengagement of students who never attended the tutorials had nothing to do with the author’s level of experience or ability to evoke interest in her students. As highlighted in table 6.9 the highest proportion of
students who passed Technology mathematics were amongst the best attendees (i.e. those who attended 6 or more tutorials). This is a finding which reflects positively on the intervention. However there were 7 students, the majority of whom were medium risk students, who never attended the tutorials, yet passed Technology mathematics. The initial reason for these students choosing not to engage with their mathematics may be due to previous negative mathematical experiences (Fielding-Wells and Makar 2006). Examination of these students, however, revealed that 5 of them had Leaving Certificate mathematics points equating to above 50 points (the other 2 students had 45 points each) demonstrating the effectiveness of the discriminant function 2 in classifying students with 50 points or more as predicted successful students (see appendix P). One might imagine however that in spite of their relatively strong mathematics backgrounds on entry to UL they would still need to attend the tutorials in order to have the necessary understanding to pass the module. This may be the case with other mathematics modules however one in which past examination papers are so similar (see Appendix L) may just require students to have a reasonable level of mathematical competency and an ability to engage in surface learning approaches such as the memorisation of procedures and ‘tricks’. It is possible that the Technology mathematics examination favours students with relatively high levels of mathematical competency and who are comfortable with summative, predictable assessment procedures.

In contrast to this poor attendance and success in Technology mathematics, 4 of the high risk students who were high attendees failed Technology mathematics. High risk students have often been found to be high attendees due to their fear of failure or feeling that there would be too much material to catch up on if the tutorials were missed (Toohey 1999; Quinn 2000). The students within this category (i.e. high attendees and failing) possibly had mathematical backgrounds that were too weak to cope with the demands of the Technology mathematics examination.

6.8.1.1 Conclusion

The attendance levels of the intervention students combined with the nature of the examination and the depth of students’ previous mathematical knowledge on entry to UL are a combination of factors that should be considered in light of the potential effects the intervention could have had on students’ mathematical performance.
The findings in relation to the quantitative analysis of the intervention demonstrate the complex and challenging nature of implementing a mathematical intervention. Further discussion on this is outlined in chapter 7.

6.8.2 Factors which Tend Towards Positive Perceptions of an Intervention

The evaluation and interpretation of the success of the intervention from a qualitative point of view is extremely subjective. Although much of the qualitative analysis was carried out in NVivo, the interpretation of the emerging themes and nodes should always be viewed in light of the author’s pedagogical views and her vested interest in the work. There are however several other factors which tend towards positive perceptions of an intervention. Such factors include:

- **The Author’s Enthusiasm:** The author’s interest in mathematics education and in best practice in teaching and learning mathematics along with her desire to implement an effective teaching intervention is likely to have had a positive effect on the intervention implementation and the students’ response to the intervention. A teacher’s enthusiasm has been found to mediate the relationship between teacher and student enjoyment in the mathematics classroom (Frenzel et al 2009; Frenzel et al 2010). This is likely to have positively influenced the acceptance of the intervention by the students who took part in it.

- **Change:** Research has shown that simply changing the conditions in which a class is conducted can elicit positive reactions from students. For example research which evaluated the experiences of students who were taught using various approaches to mathematics instruction highlighted the advantages of the variety of teaching methodologies in terms of students’ enjoyment and achievement (Sigurdson and Olson 1992). The small changes to the intervention students’ classroom (from what students are used to in a university setting) such as the room laid out in groups and the inclusion of different computer programmes to teach different mathematical concepts may have instantly made students more open to and interested in the learning experience than they normally would be.

- **The Hawthorn effect:** This refers to subjects in an educational setting improving their performance due to the awareness that they are being observed (Adair 1984). This therefore effectively leads to reactivity in students which may not be completely natural.
if the awareness of being observed was not present. Students may have made a bigger effort to engage in the group work and classroom activities just to comply with what they felt the researcher hoped to observe. It is possible that the view which the author had in relation to the intervention acceptability may have been slightly distorted due to the potential presence of the hawthorn effect.

- **Reactivity in Questionnaires:** Similar to the situation with students knowing they are being observed and reacting accordingly, the students may have responded to the questionnaires in a manner in which they felt would yield the best result for the author. Although the intervention students were informed on several occasions that the content of the questionnaires would not affect their grade and that they were anonymous, they may still have wanted to stay on the ‘good side’ of the tutor.

It is important to highlight however that researcher distance was maintained throughout the analysis of the intervention through triangulation, as was outlined in chapter 3.

### 6.9 Summary and Conclusions

Many institutions implement educational interventions for various different reasons (Stoiber and Kratchwill 2000; Fuchs and Fuchs 2001). The general aim however is to improve the educational provision and consequently student performances within a particular institution. The selection of an appropriate intervention type must be carefully considered in relation to the specific needs of the institution in question. Upon examination of a range of different intervention types and the theoretical concepts which underpinned each type, a tutorial intervention was considered the most appropriate to attempt to improve the provision of mathematics education in UL. The need for an intervention within UL service mathematics was undeniable due to the reported decline in mathematical competency levels of service mathematics students over the years (O’Donoghue 1999; Gill 2006; Gill et al 2010; Faulkner et al 2010). In an attempt to remedy, or at least alleviate somewhat, the documented decline in competency levels, intervention tutorials focused on active learning group work were designed and implemented. It was intended that a focus on active learning and teaching for understanding would reduce many of the misconceptions which service mathematics students had on entry to UL. Each tutorial’s lesson plan was guided by the
intervention design, implementation and evaluation theoretical framework in order to ensure that all teaching practices and facilitating roles were informed directly by research.

The tutorial group chosen to take part in the intervention was based on a decision regarding which tutorial slot was most likely to be attended regularly. Students within the intervention group had a variety of mathematical ability levels and came from a variety of degree programs. As a result of this it was very easy to arrange students into mixed ability groups. An observation of each tutorial was detailed by the author after each lesson. These observations were informed by the work of Spradley (1980), Spradley (1979) and Kirk and Miller (1986) and Shapiro (1987).

Two questionnaires were filled out by the intervention students. The questions the students were asked were informed by the work of Shapiro (1987) and Kitwood (1977).

The success of the intervention was analysed in terms of intervention students’ service mathematics performances compared to the performance of those who did not take part in the intervention. The success was also determined through a qualitative analysis of the author’s observations and the students’ questionnaire data. The design, implementation and evaluation of the intervention were all influenced by the author’s pedagogical values.

The quantitative analysis of the intervention revealed the ability of the Technology 2006-2008 discriminant function in predicting performance in Technology mathematics in 2010. The classification to which a student was assigned was found to be statistically significantly associated with performance in Technology mathematics (p < 0.001). This therefore suggested that the grouping of students in terms of probability of failure was quite accurate and is possibly a method of mixed ability grouping which can be used in the future.

The intervention students had a slightly different background to the non-intervention students in terms of diagnostic test performance and Leaving Certificate mathematics points on entry to UL. Logistic regression revealed however that the group (intervention/non-intervention) which a student belonged to had no significant influence on students’ performance in Technology mathematics. There was no statistically significant difference in the failure rate of intervention students and non-intervention students. The intervention was therefore found to be quite ineffective in terms of Shapiro’s (1987) intervention evaluation model. This may have been influenced by several factors. For example the short time which the tutorials ran for, the Technology mathematics examination remaining relatively unchanged for the last ten years or
the thirteen years of didactic mathematics teaching which the majority of students had been exposed to prior to entry to service mathematics in UL.

The qualitative results revealed how the author and the intervention students who attended the tutorials rated the tutorials. Some overlap was present in terms of what the analysis revealed as important to the author and the intervention students. For example all parties saw the benefits of mixed ability group work and having time in class for students to engage in mathematics. Although the intervention tutorials were not positively received by all, the vast majority of students who attended them did react favourably to them. Factors which tend towards positive perceptions of the intervention such as the author’s enthusiasm and the hawthorn effect must be considered however. The poor attendance rates of some students potentially suggested an intervention which was not socially valid to all. The qualitative analysis however revealed an intervention which was reasonably socially valid, accepted and was one that had integrity.

The results therefore revealed that the intervention did not have much of an impact on students’ examination performance, which is very important, however it did provide insights into an effective method to arrange mixed ability groups in a reasonably socially valid and accepted manner. The intervention therefore had strengths and weaknesses and the documentation of these in this chapter will serve as a point of reference for researchers looking to adopt similar approaches.

All of the findings outlined in this research project and the contributions they may have to the wider research community will now be discussed in chapter 7 which follows.
7. Introduction
This chapter outlines the main findings of this research study. The limitations and delimitations of this research are outlined first. The main findings are then discussed in relation to the research questions which they have addressed. The contributions which the main findings are likely to have made on both national and international mathematics education literature are also outlined. Finally the author’s recommendations in light of the conclusions drawn are discussed followed by some suggestions of possible future research which could be undertaken.
7.1 Delimitations and Limitations of the research

Before the research findings are presented and discussed the author feels it is vital to outline the delimitations and the limitations which are likely to have impacted on the findings and the analysis of them throughout this thesis.

The delimitations of any study are factors which can be controlled by the researcher. Decisions regarding the research aims, objectives and questions were all carried out based on the author’s idea of what the most pertinent issues in the research were. An awareness is present however that there are other methods through which the issues raised in this research could have been approached. The choices made were based on what the author and her supervisors believed to be the most valuable investigations. The author chose to initially profile Technology and Science mathematics students however in terms of targeting students in the intervention she only dealt with Technology mathematics students. The rationale for this choice was provided in chapter 6. The influence of the author’s pedagogical values were acknowledged and countered as much as possible through the use of triangulation in the research.

The limitations of any study are the factors which are out of the control of the researcher. Throughout the period of this research the Science mathematics lecturer and the manner in which Science mathematics was assessed changed (see chapter 4), this is likely to have had an effect on the research findings. Technology students in 2010 were informed of their probability of failure in service mathematics however the manner in which they used this information was out of the control of the researcher. The author encouraged students to use the information to their advantage, i.e. to seek mathematical support if necessary and to attend lectures; however this does not mean that that is how the information was used. The success of the intervention was based on a projected regular attendance at the tutorials. Attendance at tutorials was not compulsory however and so this aspect of the research was also out of the control of the researcher.
7.2 Summary and Discussion of Main Findings: Answering the Research Questions

The main findings are discussed in this section and are related to how they have addressed the research questions presented in chapter 1.

7.2.1 Phase 1: Research Questions

1. **How prevalent is the ‘Mathematics Problem’ internationally?**
   The ‘Mathematics Problem’ is widespread internationally. Reports of its existence have been documented worldwide, for example in Australia, the UK, USA and Canada (Smith 2004; Kajander and Lovric 2005; Commonwealth of Australia 2007). It continues to receive attention in the literature in recent times (Gill et al 2010; Mac an Bhaird and O’Shea 2010) and detail of efforts to try and tackle it are also common in international research in recent times (Cleary 2008; Croft et al 2009).

2. **How prevalent is the ‘Mathematics Problem’ in Ireland?**
   The ‘Mathematics Problem’ has been documented in Ireland since the 1980s and continues to be documented and analysed today (Hurley and Stynes 1986; O’Donoghue 1999; Hourigan and O’Donoghue 2007; Liston 2008; Gill et al 2010). The ‘Mathematics Problem’ in Ireland has been characterised by issues which are also common in other countries such as:
   - Students entering third level education lacking the numerical skills needed for everyday life, “not to mention studying university mathematics” (Hurley and Stynes 1986; O’Donoghue 1999).
   - Deficiencies in elementary algebra (Lawson 1996).
   - Difficulties in solving problems (Galbraith and Haines 2000).
   - A lack of conceptual understanding and knowledge (Tall and Razili 1993).
   - The existence of syllabi without a common thread throughout resulting in fragmented learning (Galbraith and Haines 2000).

Much of the research in this area maintains that these issues have been largely caused by the examination driven practices in second level education and the didactical contract which is
dominant in Irish second level mathematics classrooms (Hourigan and O’Donoghue 2007). Efforts are being made however to improve the situation in second level education in Ireland with the introduction of a new mathematics curriculum, Project Maths. Care needs to be taken with this introduction however so that issues such as those which arose in the UK second level system do not arise e.g. the introduction of poorly focused time-consuming activities and a lack of emphasis on necessary technical mathematical fluency (LMS 1995).

3. What are the dominant documented causes of the ‘Mathematics Problem’?

The dominant documented causes of the ‘Mathematics Problem’ surround the transitional issues which second level students often experience when entering third level education (Kantanis 2000; Kajander and Lovric 2005; Liston 2008) and grade inflation (Lawson 2003; O’Grady and Guilfoyle 2007). Studies relating to transitional issues often refer to the approach to learning mathematics that students adopt in second level education not transferring successfully to third level mathematics education (NCCA 2005). Another contribution to transitional issues which is also referred to regularly is the mismatch between the expected previous mathematical knowledge of students and the actualities of their mathematical knowledge by those in third level mathematics education (Liston 2008). This leads to great difficulties for students as they do not have the previous mathematical knowledge necessary to cope with the demands of third level mathematics (Kajander and Lovric 2005).

4. What support services and interventions have been put in place in an attempt to alleviate the ‘Mathematics Problem’?

The author’s review of literature on interventions implemented to alleviate the ‘Mathematics Problem’ revealed that there are two major intervention types: stop-gap solutions and long term solutions (O’Donoghue 2004). Stop-gap solutions refer to interventions that provide a short term solution to a long term problem, their main priority being to assist students in passing service mathematics examinations. Lawson (2006) described short term interventions as practices which treat the symptoms and not the causes. Such interventions include drop-in centres, diagnostic testing, bridging courses and computer assisted learning. The long term interventions which O’Donoghue (2004) refers to relate to any practice which seeks to improve mathematical competency levels on an on-going basis rather than just focussing on the passing of
examinations, important as examinations may be. The literature also revealed several examples of this intervention type such as; tutorials, peer assisted learning, national centres for excellence in mathematics teaching and learning and the introduction of new mathematics curricula. Such interventions aim to improve an institution’s mathematics education practices as a whole if implemented effectively over time. Both short term and long term interventions have proven to have positive influences on students’ mathematical performances. For example drop-in centres in the UK have been shown to improve students’ mathematical competency levels and also to reduce anxiety (Croft et al 2009). Bridging courses have been found to significantly improve students’ mathematical self-concept upon completion of a one week course (Gill 2008). Computer assisted learning proved a popular form of support in the mathematics education literature also (Doyle 2009; Fletcher et al 2009; Greenhow 2009). One study found that the introduction of CAL to their institutions’ mathematics tutorials enabled students to listen and concentrate more in tutorials as opposed to taking notes only (O’Shea and Mac an Bhaird 2009). Many of the long term interventions also revealed positive results. For example a tutorial implemented in the University of Loughborough in which student led classes were introduced in place of teacher led classes resulted in an increase in the pass rate from 48% to 67% compared to the previous year (Symonds et al 2007). National centres for excellence in mathematics teaching and learning have been established in both the UK and Ireland. These centres recognise the importance of supporting all stakeholders of mathematics in order to improve its provision in a real and long term way (Burghes and Hindle 2004).

Both short term and long term interventions proved to have benefits in improving the mathematical provisions in an institution. The specific goals which an institution wishes to achieve best dictates the most appropriate intervention to implement. The conclusion reached by the author was that if mathematical interventions are to significantly improve mathematical competency levels through provision and practice, then long term sustainable intervention practices are of greater value.
Section 5. What does national and international educational research say about profiling ‘at risk’ students?

Student profiling has been implemented and documented within mathematics education literature worldwide (Chansarkar and Michaelaudis 2001; James et al 2008; Holton et al 2009). The literature surrounding profiling students outlines several different reasons and motives for carrying it out. The three major uses which emerged from the literature were:

- Assessing the suitability of pre-tertiary mathematics education as preparation for third level mathematics (James et al 2008).
- Addressing mathematical weaknesses (Gill et al 2010).
- The prediction of student achievement (Murphy 1981).

The method of student profiling used varied in the literature depending on the specific aims of the research. For example, the studies which aimed to assess the suitability of pre-tertiary mathematics as preparation for third level mathematics commonly profiled students on the basis of their pre-tertiary level mathematics performance (Wilson and MacGillivray 2007; James et al 2008). To assess mathematical weaknesses however the majority of institutions chose to profile students based on their performance on a diagnostic test on entry to third level education (Lawson 2003; Gill et al 2010). Finally international research which used student profiling in an attempt to predict student achievement most commonly analysed either students’ previous mathematical performance, students’ demographics or students’ approach to learning mathematics, to predict mathematical performance (Carmichael and Taylor 2005; Fahey 2009; Liston and O’Donoghue 2009).

The international research which detailed profiling mathematics students therefore highlighted to the author the variety of motives which institutions may have for carrying it out. The research detailed the variety of prediction variables which have been used along with the level of success which different studies achieved (see chapter 2, section 2.4).
7.2.2 Phase 2: Research Questions

1. What is the profile of service mathematics students in UL between 1998 and 2008?

The profile of service mathematics students entering UL has changed considerably between 1998 and 2008. There has been a slight increase in the percentage of male students in Technology and Science mathematics over the ten year period. There has been a decline from 100% to approximately 80% of the registered students sitting the diagnostic test. Students’ performances in the diagnostic test have also changed over time with an increase in the percentage of ‘at risk’ students from 32.8% to 46.2% in Technology mathematics and from 21.3% to 46.0% in Science mathematics between 1998 and 2008. Students entering UL with Higher Level Leaving Certificate mathematics as pre-requisite knowledge declined from 41.0% to 33.2% in Technology mathematics and from 55.4% to 38.0% in Science mathematics between 1998 and 2008. This decline was matched with a change in the percentage of students entering UL with Ordinary Level mathematics. It declined from 58.7% to 57.5% in Technology mathematics and increased from 43.1% to 55.1% in Science mathematics. 2008 also saw a large increase in the percentage of non-standard students in service mathematics from 0.3% to 9.4% in Technology mathematics and from 1.5% to 6.9% in Science mathematics. The profile of service mathematics students in UL in 2008 is therefore notably different to that of the students in 1998.

Separate analysis of the sub-categories within the standard (Higher Level and Ordinary Level) and non-standard (Mature students, students who engaged in previous study and international students) student groups revealed that the sub-categories varied in their mathematical performances over time and therefore consideration was given to separate analysis of these non-homogenous groups in future. Ordinary Level Leaving Certificate mathematics students stood out in particular as their performance in the diagnostic test was not reflective of their end of semester examination results. Ordinary Level students’ unexpected underperformance in service mathematics may have been due to a number of reasons. Possible suggestions include receiving poor mathematics teaching at second level (Ni Riordáin and Hannigan 2011); having poor perceived abilities in mathematics due to previous mathematics experiences (Pintrich et al 1993); relying on rote learning practices in mathematics prior to entry to third level education (Hourigan and O’Donoghue 2007) or having difficulty adjusting to third level education (Chemers et al 2001). These suggestions could be influential factors on Ordinary Level students’ underperformance in service mathematics however further research is required in this area.
2. What are the trends in the mathematical competency levels of UL students by Leaving Certificate mathematics grade?

The mean diagnostic test results of students entering service mathematics in UL has declined over time. In 1998 the mean diagnostic test score was 59.3% which declined to 50.8% in 2008. The difference between the means was statistically significant (p < 0.001). The decline in diagnostic test performance was larger for Science mathematics students than Technology mathematics students.

When students’ performance by Leaving Certificate mathematics grade was analysed it was found that students’ performance in the diagnostic test by Leaving Certificate mathematics grade over time remained largely unchanged. There were no statistically significant differences in performance for each grade analysed (OLA1, OLA2, OLB1, OLB3, and HLC1) between 1998 and 2008 with the exception of students with an Ordinary Level A1 grade. This finding suggests that Leaving Certificate mathematics and the diagnostic test have the ability to measure a very similar level of mathematical competency over time.

Despite this positive finding surrounding the continuing ability of the Leaving Certificate mathematics examination to measure a similar level of mathematical competency over time, second level education in Ireland and the Leaving Certificate in particular continues to receive negative media coverage. After the Leaving Certificate results are announced every year newspaper articles and online blogs are full of discussion which castigates the Leaving Certificate system for its “outdated pedagogy, promotion of intellectual conformity, discouragement of critical enquiry and undermining of excellence in science and numeracy” (Von Prondzynski 2011, p.3). Although some of these criticisms may be justified and have been largely supported by the literature (NCCA 2005; NCCA 2009; Higher Education Authority 2010), the finding reported in this research provides a counter argument. Some credit must be given to a system which over a ten year period manages to measure almost an identical level of mathematical competency level (as measured by the diagnostic test) by mathematics grade. The Leaving Certificate mathematics grades in question therefore provide third level institutions with a consistent picture of the ability level of the students whom they are choosing to admit to different degree programmes. Some issues however were highlighted within this research in relation to Leaving Certificate mathematics grades with equivalent CAO points and their ability to distinguish between competency levels. This will be discussed further in question 6.
3. Is there evidence of grade inflation occurring in Leaving Certificate mathematics?

As highlighted in the previous research question, the Leaving Certificate mathematics examination and the diagnostic test have been consistent in their ability to measure and differentiate between the mathematical proficiency levels of the students taking them. If each Leaving Certificate mathematics grade is demonstrating a similar level of proficiency, the data does not suggest that grade inflation is occurring in Leaving Certificate mathematics. Figure 4.9 in chapter 4 graphically represents this finding. If the data does not suggest that grade inflation is occurring in Leaving Certificate mathematics then it is unlikely to be a contributory factor to the declining standards of mathematical competency levels in the case of this research. O’Grady (2009) however maintained that there were reasons to suspect that grade inflation was occurring in Leaving Certificate mathematics. His study particularly focused on the possibility of grade inflation in Leaving Certificate Higher Level A and B grades. In the case of this research however, the investigation into Leaving Certificate mathematics grades only included the most commonly occurring grades in the UL database which did not include Higher Level A and B grades. The Higher Level A and B grades were analysed later however in terms of the performance of students with these grades in the diagnostic test over time. It was found that there were no statistically significant mean differences over time in diagnostic test performance by Leaving Certificate grade. There was however a notable decline in the numbers of students entering UL service mathematics with these grades. For example in 1998, 17 students entered Technology and Science mathematics with a Higher Level A1 grade as pre-requisite knowledge which declined to just 1 student in 2008.

In the UK grade inflation was found to be present in second level mathematics grades. Lawson (2003) noted that students entering university in 1991 with a particular A-level mathematics grade displayed a higher level of mathematical competency than students entering with the same grade ten years later in 2001. The reasons for these contrasting findings in Ireland and the UK may be due to the changes which the UK A-level course went through during the period which Lawson’s (2003) study examined. Changes such as the A-level curriculum having several elements of topics within trigonometry, complex numbers and vectors being cut in 1991 and 1994 is thought to have resulted in fragmented learning (IMS 1995). The Irish Leaving Certificate syllabus however remained the same throughout the period of the examination in this research (1998-2008).
The data does not suggest that grade inflation has occurred in UL service mathematics students’ Leaving Certificate grades, however the profile of students entering third level education in Ireland has changed considerably between 1998 and 2008 (Faulkner et al 2010). The number of people attending higher education institutions is constantly growing. Improving access to higher education among those in lower socio-economic groups has been a government priority in Ireland in recent times (O’Reilly 2008). The most recent strategy which attempted to achieve equality of access to higher education in Ireland was focused on the years 2005-2007. This action plan had several goals, one of them being increased routes of access and progression to higher education (Department of Education and Science 2005). It was reported to have been a successful initiative (O’Reilly 2008) as the numbers grew steadily. This is also evident from the results presented on the changing student profile in chapter 4 of this research. Despite all of the positive aspects of the increased access for all socio-economic groups in Ireland, this move must be and has been considered in light of the declining standards in third level mathematics education (Gill 2006). The increase in the numbers entering higher education, a large proportion of whom are non-standard students, has led to a marked decrease in entry requirements in third level courses in Ireland (Childs 2002). Research has shown that for many non-standard students, studying in third level education is often characterised by a ‘struggle’ (Reay 2002; Leathwood and O’Connell 2003). This struggle often manifests itself in terms of financial issues, attitudinal barriers, qualifications, flexibility and language and learning difficulties (Watt and Paterson 2000). The difficulty met in this research in terms of non-standard students is that the discriminant function cannot predict mathematical performance for them because they do not have a Leaving Certificate mathematics grade. It should also be noted that third level institutions have little quantitative insight into the mathematical competency levels of the non-standard students they are admitting if diagnostic testing is not administered. The appropriateness of non-standard students’ ability for third level mathematics is therefore not always known prior to their entry to third level education.

The widening of access to third level education can be seen as contributing to the decline in mathematical standards in third level institutions (O’Connor 2006). If it is to continue, we must accept that a less mathematically prepared student body than in previous times must be catered for when third level access for all has been implemented.
7.2.3 Phase 3: Research Questions

1. **What is the profile of students who are ‘at risk’ of failing Technology mathematics?**

   Students who are most ‘at risk’ of failing Technology mathematics are likely to have Ordinary Level Leaving Certificate mathematics as pre-requisite knowledge, have obtained an Ordinary Level B1-C3 grade in Leaving Certificate mathematics or be a mature student. Students who are ‘at risk’ are also likely to have a lower mean diagnostic test result and perform poorly in the Algebra and Arithmetic sections of the diagnostic test.

2. **What is the profile of students who are ‘at risk’ of failing Science mathematics?**

   Students who are most ‘at risk’ of failing Science mathematics were more likely to have Ordinary Level Leaving Certificate mathematics grades between a B1-C1 as pre-requisite knowledge. Students enrolled in Sport and Exercise Science and Biological and Advanced Materials degree programmes are also at a higher risk of failing. Similar to the case with Technology ‘at risk’ students, Science ‘at risk’ students are likely to have a lower mean diagnostic test result compared to those who are not considered to be ‘at risk’ and perform poorly in the Algebra and Arithmetic sections of the diagnostic test.

   The failure rate of Science mathematics students in 2008 must be considered in light of the change of lecturer in this module and more importantly, the change in the pass mark. If the pass mark had remained constant over the 3 years being analysed, little change in terms of the failure rate in Science mathematics would have been present over the three years being examined (see section 4.4.6 (a)).
3. What is the profile of a student who are ‘at risk’ of failing service mathematics?
Statistically significant associations were found between performance in service mathematics and

- Mean CAO mathematics points (p < 0.001)
- Mean diagnostic test score (p < 0.001)
- Mean performance in Algebra and Arithmetic sections of the diagnostic test (p < 0.001)
- Leaving Certificate mathematics Level and Grade (p < 0.001)
- Degree Programme (p < 0.001)
- ‘at risk’/not ‘at risk’ (p < 0.001)

These statistically significant associations with service mathematics performance have practical significance for the provision of mathematics education in third level institutions. They can also be used to inform prediction analysis, as was the case in this thesis.

4. What is the most effective statistical method of prediction of failure within service mathematics in UL?
Several methods of prediction were considered and tested on the UL database such as logistic regression, classification trees and discriminant analysis. Classification trees did not classify a high enough percentage of unsuccessful students for it to be used on future cohorts in UL. It correctly classified 35.4% of the unsuccessful Technology 2006-2008 students. The logistic regression analysis also performed poorly in terms of the percentage of correctly classified unsuccessful students. It correctly classified 39.0% of the unsuccessful Technology 2006-2008 students. The most successful discriminant analysis function however, which was created using the Technology 2006-2008 dataset, correctly classified 68.2% of the successful students and 84.4% of the unsuccessful students in its training sample. Discriminant analysis was therefore found to be the most successful method of prediction of UL service mathematics performance.
Although each statistical prediction method had its advantages (See Appendix E), and none of them were extremely successful in terms of correct classification of students by service mathematics performance, in the case of UL database discriminant analysis was the most successful. It also had the advantage over the classification trees method as it provided a probability of failure. There was however some concerns regarding the potential limitations which the predictor variables used in the discriminant analysis, i.e. Leaving Certificate
mathematics points and diagnostic test result, may have had on the overall success of the function’s ability to correctly classify students’ service mathematics performance. These potential limitations, and the subsequent additional analysis which was carried out in light of them, are discussed next in section 7.2.3.1.

7.2.3.1 Potential Limitations of Predictor Variables

There were several concerns regarding the use of Leaving Certificate mathematics points and diagnostic test results as predictor variables. Firstly the Leaving Certificate points system which does not work on an equal interval scale may have led to issues in the discriminant function’s ability to accurately predict performance.

The discriminant analysis faced a further challenge due to the existence of grades with equivalent CAO points displaying statistically significantly different mean performances in the diagnostic test and service mathematics examinations. An examination into the potential impact which the ‘equivalent grades’ may have had on the ability of the discriminant function to accurately classify students according to service mathematics performance was undertaken. This examination, which took the form of a separate discriminant analysis for Higher Level and Ordinary Level students, highlighted the impact that grouping Higher Level and Ordinary Level students together had on the results of the original discriminant analysis (i.e. the Technology 2006-2008 discriminant function). The separate analysis of the Higher Level students produced a higher percentage of correctly classified students when compared to the function in which the two levels were analysed together. This is likely to be attributed to the function only having to deal with similar mathematical competency levels per Leaving Certificate points’ allocation. On the other hand the Ordinary Level discriminant function performed to a slightly lower standard in terms of correct classification of students when compared to the Technology 2006-2008 function. The unexpected underperformance of some of the Ordinary Level students and the lack of a consistent pattern in their mathematical performances over time are likely to have led to difficulties in both functions accurately predicting the performances of Ordinary Level students. Although the separate discriminant analysis of Higher Level and Ordinary Level students did not necessarily result in an overall higher correct classification of students when compared to the Technology 2006-2008 function, it did confirm some important insights i.e. the challenges faced by the original discriminant analysis due to ‘equivalent grades’ not displaying equivalent
mathematical competency levels and the unpredictable underperformance of Ordinary Level students in service mathematics.

In addition to this some of the Leaving Certificate grades, such as OLB2-OLC3, had approximately a 50% success and 50% failure rate in Technology mathematics. Likewise similar results were presented in the case of the students in groups 1 and 2 of diagnostic test results (i.e. students who received between 0-10 and 11-20 in the diagnostic test; see table 5.4). This presented a potential limitation as the binary classification of results makes it difficult to decipher whether students with these grades and diagnostic test groups should be predicted to pass or fail service mathematics. This could potentially have affected the overall accuracy of the predictions. The presence of the probability of failure which was provided by discriminant analysis however partially eliminated this issue as students could be examined in terms of not only their binary classification but also their probability of failure. The probability of failure is therefore a major advantage of a discriminant function as a classification tool over the binary ‘at risk’/ not ‘at risk’ which is offered by the current system.

5. Is discriminant analysis a more effective method of classifying failure in service mathematics than the measure which is currently in place i.e. the diagnostic test?

As outlined in chapter 5, both the diagnostic test and the Technology 2006-2008 discriminant function were reasonably successful in correctly classifying service mathematics students’ examination performance. Upon comparison however it was found that the Technology 2006-2008 discriminant function was slightly more effective in distinguishing between those who were successful and those who were unsuccessful in service mathematics while also providing an objective evidence based cut-off point and probability of failure. It was therefore considered to be the most effective prediction method although each method faced challenges which are outlined under research question 6 which follows.

The diagnostic test cut-off point was determined using the subjective expert judgement of the Professor of Mathematics Education in UL as outlined in chapter 3 (section 3.4.2.3). When discriminant analysis was carried out using diagnostic test results as the only predictor variable an insightful finding emerged. The cut-off point (of 18) which the discriminant function determined to be most effective at discriminating between students performance in service mathematics was very similar to the existing cut-off point of 19 in the diagnostic test. The
establishment of the discriminant function with diagnostic test as the only predictor variable was very important as it proved that the subjective expert judgement, which was used to determine if students were likely to be ‘at risk’ or not ‘at risk’ of failing service mathematics, was extremely accurate. Although this discriminant analysis reduces to a very simple rule, just like the diagnostic test, the finding provides scientific evidence to those involved in diagnostic testing in the university that their cut-off point is effective while refining it slightly also. The separate discriminant analysis for Higher Level and Ordinary Level students, and consequently the separate analysis for Higher Level and Ordinary Level students by the diagnostic test, brought to light that both means of prediction have an issue in predicting for Ordinary Level students due to their unpredictable mathematics performances over time. This finding opens up an area for further research into Ordinary Level students in mathematics education in Ireland. The factors which may be contributing to their underperformance have yet to be identified in the literature. Qualitative research in this area may provide valuable insights into the possible reasons for this unpredictable underperformance.

In summary the Technology 2006-2008 discriminant function was slightly more successful in classifying students’ service mathematics performance than the diagnostic test and also had the added advantages of being an objective evidence based cut-off point which provides a probability of failure.

The discriminant function highlights that a large proportion of students are highly ‘at risk’ of failing service mathematics upon entry to UL. In order to increase the competency levels of our third level students, does it make sense to increase the entry standards as opposed to lowering them? A report by the Royal Irish Academy (2009) entitled ‘Making the Best of Third-Level Science’ discusses how the measures of the past ten years to enhance the supply of graduates in science, mathematics and engineering have encouraged universities to substantially increase the number of undergraduate places in Science. The increase was therefore matched with a system of widening access as discussed in section 7.2.2. This expansion led:

*to the enrolment of a cohort of school-leavers whose levels of preparation and attainment are less than those of students who entered universities and institutions of technology in the early 1990’s*

(Royal Irish Academy 2009, p.4).
We do not want a situation in which the lowering of standards leads us to a larger number of undergraduates than ever before who are poorly qualified as naturally this “will not assist in Ireland’s economic development” (p.8). Adequately supporting these larger cohorts of students however should result in the maintenance of exit standards (Gill et al 2010). Alternatively suggestions which may be effective in maintaining exit standards in mathematics education and in the third level system as a whole include:

- **University Entrance**: To restrict the number of students entering third level science level 8 courses
- **Promotion of level 6 and 7 courses**: To encourage more students to apply for level 6 and 7 courses offering a guarantee to level 8 courses if a certain standard has been reached.
- **Co-ordination of enrolment**: This could involve offering the top 20% of students, according to Leaving Certificate performance, level 8 courses at university and the next 20% would be offered level 6 courses with an option to progress to level 8.

(Royal Irish Academy 2009, p.9).

The implementation of some/all of these suggestions may result in students entering into third level education in Ireland at a level more appropriate to their previous knowledge. Alternatively stronger support systems for those who enter third level education and who are not mathematically prepared should result in a win-win situation for all; increased numbers in third level education combined with the maintenance of degree standards.

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8Level 6, 7, and 8 courses refer to the National Framework of Qualifications (NGQ) in Ireland which is a system of ten levels which allows the different standards and levels of qualifications to be compared. e.g. Leaving Certificate = Level 5, Bachelor degree in university = Level 7, PhD = Level 10.
6. What are the challenges associated with predicting failure in third level service mathematics?

Chapter 5 details several challenges which negatively impacted upon the effectiveness of the prediction of failure in service mathematics, some of which have been partially discussed in this section already.

**Poor Attendance: A Challenge when Predicting Service Mathematics Performance**

The proportion of service mathematics students not sitting the diagnostic test has increased over the period being examined (1998-2008). Naturally the ability of the diagnostic test and the discriminant function to forewarn students as to whether they are ‘at risk’ of failing service mathematics or not is lessened when a smaller proportion of students sit the diagnostic test. In addition to this, students who did not sit the diagnostic test were found to be more likely to fail service mathematics than those who did sit it (p < 0.001). They were also found to be highly likely to fail their first year of college overall. The majority of students who did not sit the diagnostic test within Technology and Science mathematics performed poorly in all of the other modules they were studying in 1st year. These findings suggested that students’ difficulties were not solely related to mathematics. The issue of disengaged students has been documented as an increasing problem (Sax et al 1995). The problem has been characterised by learning habits of idleness, of getting by with the least possible effort and of cynicism about the very possibility of achievement (Owen 1995; Trout 1997). Another reported trait of disengaged students is an exhibition (to varying degrees) of “resentment towards attendance” (Trout 1997). This may be the case with the students who do not sit the diagnostic test. Trout (1997) concluded his paper on disengaged students by stating that we, as educators and parents, have not successfully conveyed to students:

*the personal and social benefits of learning, the fulfilment in being skilled rather than unskilled, to know than not to know, to inquire than to be self-satisfied, to strive than to be apathetic, to create than to be fallow*

*(p.52).*

He states that we have failed to socialise many students into taking responsibility for their own intellectual development or even to care about it. This led the author to question whether all
members of our society have to be leaders, creators and intellectuals. Are we forcing everyone to be academic instead of encouraging everyone to progress to areas that will aid economic development, in which they are competent, motivated and interested? These questions along with the characteristics of disengaged students listed in the literature are very complex and require further research.

Not sitting the diagnostic test was found to be statistically significantly related to performance in service mathematics however Leaving Certificate mathematics points were found to be the most significant predictor of all variables tested. Other recent Irish studies in this area have found similar predictors of performance. For example, a report on progression in Irish higher education found that the higher the points attained in mathematics in the Leaving Certificate by new entrants, the more likely it is that students will be present in the second year of their course of study. Attainment in Leaving Certificate English was also found to be a strong indicator of progression albeit to a much lesser extent than mathematics (Higher Education Authority 2010). This study found that there appears to be:

\[ \textit{a mismatch between the skills required for successful engagement with scientific and technological courses and the competencies of students enrolling in such courses} \]

\[(\text{HEA 2010, p.40}).\]

Prior attainment in Leaving Certificate mathematics was found to play an equally important role in student retention in both the university and institute of technology sectors. This signifies the importance of student ‘ability’ in meeting the academic demands of higher education (McCoy and Byrne 2010). The study concluded by revealing results of a multivariate analysis which showed that the influence of students’ gender and socio-economic background on their progression through higher education is mediated mostly through prior educational attainment with a particular emphasis on students’ mathematical ability (HEA 2010).

The strong relationship between prior mathematical achievement and performance in third level mathematics is therefore evident from the author’s findings and other Irish studies in the area as well as international research. In spite of this, indications that Leaving Certificate points did not always accurately reflect the mathematical competency levels of the students who attained them were brought to light in this research. Further discussion on this is outlined next.
Equivalent Grades: A Challenge when Predicting Service Mathematics Performance

The higher failure rates for Ordinary Level Leaving Certificate mathematics students in service mathematics compared to Higher Level Leaving Certificate mathematics students demonstrated that both groups are not equally prepared for third level mathematics. In almost all cases the failure rates in service mathematics amongst the Ordinary Level Leaving Certificate mathematics grades are higher than that of the Higher Level Leaving Certificate mathematics grades (see chapter 4, section 4.4.4(d) and 4.4.6(d)). Of particular interest were the findings of the examination of equivalent grades (OA1-HC3, OB1-HD3, OA1-HC3) for both cohorts combined. In each pairing of equivalent grades, the students with the Higher Level grade were found to have a statistically significantly higher mean performance in the diagnostic test for Science and Technology mathematics 2006-2008. This examination revealed that the Ordinary Level students also performed to a lower mean standard in each topic on the diagnostic test with no exceptions. The differences between the means in each topic were also found to be statistically significant (p < 0.05).

Ordinary Level students were therefore found to be less mathematically prepared than Higher Level students with equivalent points on entry to university and also less able to cope with third level mathematics examinations as highlighted by their mean service mathematics performance and higher failure rates (see table 5.12). These findings highlight that the points system may have a weakness when it comes to accurately assigning students’ mathematics points as it may misinform institutions of the mathematical preparedness of these students. Should there be consideration for reallocating points when it comes to Ordinary Level Leaving Certificate mathematics?

Although the findings in this research highlight that Ordinary Level students perform to a significantly lower mean standard than their Higher Level counterparts in two examinations only (i.e. the diagnostic test and service mathematics examinations), it must cause one to consider that this may be the case in many more incidents if further examinations were implemented. If the findings did prove to be repeatable in other instances there would be grounds to suggest that OLA1, OLA2 and consequently all other Ordinary Level Leaving Certificate mathematics grades be assigned a lower number of points then they currently receive. For example would it be more appropriate to assign 50 points to an OLA1 mathematics grade? The findings outlined in this
research would suggest that this number of points would better reflect the competency levels of a student with this grade (see table 5.12). The author is aware however that the Leaving Certificate points system is one which is implemented in a uniform fashion across all subject areas and it may be unreasonable to restructure the allocation of points to meet the needs of one subject area. Alternatively mathematics lecturers, researchers and those involved in determining entry requirements could be made aware that students with Ordinary Level Leaving Certificate grades, despite equivalent points, do not display the same level of mathematical competency as their Higher Level counterparts. This consideration and realisation may lead to more realistic expectations in terms of likely performance in end of term examinations of students being admitted to university courses with these grades.

**Ordinary Level Students: A Challenge when Predicting Service Mathematics Performance**

As previously discussed Ordinary Level students have a statistically significantly lower mean performance in service mathematics when compared to Higher Level Leaving Certificate mathematics, mature and international students as well as students who engaged in previous study prior to entering UL (p < 0.001). Some Ordinary Level students’ unpredictable service mathematics performances, when diagnostic test results are taken into account, are likely to have affected the discriminant function’s and the diagnostic test’s ability to predict performance. Further research into establishing somewhat the reasoning for this would be of interest to mathematics educationalists in Ireland and elsewhere.
7.2.4 Phase 4: Research Questions

1. What are the probabilities of failure of service mathematics students entering UL in September 2010/11 based on the discriminant function?

According to the Technology 2006-2008 discriminant function there was a large spread of students within each probability of failure group. The highest proportion of students occurred in the high risk group (38.3%). Approximately one third (34.1%) of the entire Technology 2010 cohort were considered to be of medium risk of failure with the smallest proportion of students being considered to have a low risk of failure (27.6%). These findings showed a definite rationale for the implementation of a mathematical intervention to reduce failure rates in service mathematics.

The Technology 2006-2008 discriminant function performed best when predicting the probabilities of students in the low and high risk groups. It correctly classified 96.5% of the students who passed as being low risk students. Of the 73 students who failed Technology mathematics altogether it correctly classified 61.8% of the students who failed as being high risk students which they evidently were. It was however overcautious in its prediction of high risk students with 53.4% of the students who were considered to be high risk passing the examination. The majority of students (85.7%) who were considered to be medium risk according to the Technology 2006-2008 discriminant function passed Technology mathematics. This is an interesting finding in itself that the majority of students who are considered to be on the boundary of passing and failing service mathematics are more likely to be successful.

2. What is an appropriate intervention design, implementation and evaluation method for service mathematics students which has been informed by the discriminant function?

The need for a mathematical intervention in UL was undeniable (Faulkner et al 2010; Gill et al 2010). As sustainability was one of the main concerns, as well as introducing research informed practices, an active learning group work tutorial was set up informed by the Technology 2006-2008 discriminant function. An active learning focus was adapted due to its proven benefits in short and long term knowledge retention (Laws et al 1999), in student motivation and cognitive skills (Bonwell and Eison 1991) and its ability to engage student interest (Wankat 2002). Group work was chosen due also to its proven benefits as outlined in the literature such as
improvements in student contentment with learning (Johnson et al 1998a), its ability to improve students’ academic achievement and the quality of students’ interpersonal skills (Springer et al 1999). The intervention group work style also allowed the author to make effective use of the Technology 2006-2008 discriminant function by using it to group students into mixed ability groups based on their probability of failure. All of the decisions made in relation to the intervention were also influenced by the author’s pedagogical values.

7.2.5 Phase 5: Research Questions

1. Did students involved in the teaching intervention perform better than those who did not take part in the intervention?

Overall the intervention was more successful from a qualitative point of view than a quantitative one. However the mixed ability grouping method used within the intervention was very effective. The mixed ability grouping method was informed by the Technology 2006-2008 discriminant function and students were grouped according to their probability of failure. As the prediction of performances of students by probability group was quite successful amongst the Technology 2010 cohort, particularly where the low and high risk groups were concerned, there is a strong argument for the introduction of such a grouping method in other institutions in Ireland and also in disciplines other than mathematics.

Quantitative analysis of intervention data, however, revealed no statistically significant differences in the failure rate of intervention students and non-intervention students. Although the failure rate of the intervention students of 23.3% was slightly lower than that of the non-intervention students (25.3%), in practical terms the difference was negligible. Raine et al (2010) found a similar result when they implemented a problem-based learning approach to mathematical support in the University of Leicester. They found that the PBL did not significantly improve students’ mathematical skills over one semester consisting of 2 contact hours per week: one lecture and one tutorial.

There are several possible reasons why the intervention may not have been more successful from a quantitative point of view. It is possible that the time frame over which the intervention was run was not long enough to result in any real change. This possibility is supported by the work of Yeaton and Sechrest (1981) who found that treatment effectiveness is often linked to the amount of treatment which in the case of this research was only ten tutorials. A second possible issue
may have stemmed from the fact that the Technology mathematics examination has remained largely unchanged over the last ten years (see Appendix L). The examination remaining very similar from year to year supports rote learning, with which students coming directly from second level education in Ireland are very familiar (NCCA 2009). As well as being predictable, the Technology mathematics examination assesses procedural knowledge as opposed to conceptual knowledge so it is quite sufficient to ‘learn off’ the last 2 years papers. Students are therefore moving from a system in second level into a very similar system at third level. The potential for students to engage in rote learning and perform well in the Technology mathematics examination may therefore have undermined the potential benefits which the intervention tutorials may have had on students’ understanding. Students who engaged in the intervention tutorials may have experienced improvements in their understanding of the mathematical concepts being studied however there was a mismatch between the preparation for the intervention and what the traditional examination rewarded. Such findings suggest that students who engaged very little with the Technology mathematics material but spent time engaging in drill and practice procedures relating to past examination papers could have performed as well as students who engaged with the mathematics. Another factor which has been considered is students’ prior mathematical experiences. Second level students in Ireland have been exposed to 13 years of predominantly didactical teaching in which the teacher is the transmitter of information and they are passive students who rarely engage with the material on more than a surface level (O’Murchu and O’Sullivan 1982; O’Donoghue 1999). It is difficult for students to move away from this style of education over such a short period of time and school habits can be extremely difficult to break (Zan 2008). Students have been found to adopt the same approach to learning mathematics in third level education as they did in second level education (Liston 2008).

The findings in relation to the quantitative analysis of the intervention demonstrate the complex and challenging nature of implementing a mathematical intervention. The significant amount of research which took place prior to the intervention and which informed intervention planning and practice is evidently not always powerful enough to help people progress academically in an intervention. These findings demonstrate that mathematical interventions do not always show positive results however that does not mean that lessons cannot be learnt along the way, particularly in terms of possible areas of improvement for future research. A study which
detailed similar issues with the implementation of a mathematical intervention found that less well prepared mathematics students who received additional mathematics support performed considerably worse in a mathematics examination than the students who did not receive additional support (Lawson et al. 2006). The students involved in the intervention however outperformed the other students in the coursework element of a module. The findings in this study give an indication of the complexity of mathematical support and suggest that consideration needs to be given to the comfort levels which ‘at risk’ students may have with summative examination procedures.

The author feels that one of the major factors at play in the case of her intervention, which led to an intervention which was not as successful as hoped, was the short time frame over which it took place and the nature of the Technology mathematics examination. The predictable nature of the Technology mathematics examination is reflective of a second level system which places all of its focus on the results of an examination (Madaus and MacNamara 1970) and engages in teaching practices which offend the principles of pedagogy (Von Prondzynski 2011) or in the case of this research offends the principles behind the implementation of the teaching intervention.

2. Did students involved in the intervention respond positively to the teaching strategies employed in the intervention?

The intervention was very well received by the students who attended the tutorials. Analysis of the students’ questionnaire data and the author’s observation data in NVivo revealed a reasonably socially valid and accepted intervention which had integrity. The themes which emerged from the analysis of the student questionnaires demonstrated their positivity towards the teaching strategies employed throughout. The majority of students involved alluded to how they felt the group work benefited their understanding of the mathematics. Their preference for the teaching strategies used over more traditional teaching styles was also demonstrated through their appreciation for actually getting to ‘do’ mathematics in class. From the quantitative analysis, it was clear that students did not make huge improvements in terms of their mathematics results, however from the qualitative analysis it appears that their persistence and enjoyment of the mathematics may have improved over the course of the tutorials. Results here are reflective of the findings of a study carried out by Barker (2008) who found that a problem
based learning approach to third level mathematics displayed increased positive attitudes towards mathematics and students expressed a greater confidence in tackling problems.

It is important to note also that the positive perceptions which students displayed towards the intervention, recorded through the author’s observations and the student questionnaires, could have been influenced by a number of different factors. Such factors include the Hawthorn effect i.e. students’ feeling “special” because they have been chosen for the intervention. Other factors which must be considered include the fact that only students who chose to take part in the intervention were involved in its evaluation. The potential influence which the enthusiasm of the author may have had on students’ evaluation of the intervention must also be considered (Frenzel et al 2009).

A final factor which must be considered when discussing the success of the intervention from a qualitative viewpoint is students’ attendance rates. The social validity and intervention acceptability can be partially determined through the evaluation of feedback and observations from those who take part in it. Consideration must also be given to the potential insights which the non-attendees/poor attendees may have given. The medium risk students were amongst the poorest attendees of all 3 groups, as discussed in chapter 6, yet they had a high success rate in service mathematics. Did these students choose not to attend the tutorials as they felt they were of no benefit to them in terms of passing the examination? Possibly the medium risk students were familiar with the nature of the Technology mathematics examination allowing for rote learning and lack of real engagement with the mathematics potentially resulting in success in the examination. Alternatively students who did not attend the tutorials did so out of disengagement with college in general and were unaware of any intervention being implemented in which case their non-attendance would give no real further insight into the evaluation of the intervention. The fact that the reasons for students’ non-attendance/poor attendance are not known leaves this as an area of potential future research.
7.3 Contributions to Research

The contributions to research which have resulted from this thesis are now presented. In doing so, the author is also answering the final research question (see section 7.3.1).

7.3.1 Phase 6: Research Questions

1. What do the research findings in this thesis have to offer the university sector, the wider Irish education system and the international education system?

- This thesis firstly highlights that the ‘Mathematics Problem’ is still prevalent nationally and internationally. It outlines the potential effectiveness of profiling mathematics students in order to a) assess the suitability of pre-tertiary mathematics education as preparation for third level mathematics b) address mathematical weaknesses and c) predict student achievement.

- One of the main findings is that the service mathematics student profile in UL has changed and this has been found to be a contributory factor to the declining standards in mathematical competency levels over time. This finding may offer both the national and international education community an insight into something which may also be occurring in their institutions. It is hoped that this will evoke an interest in mathematics educationalists which will lead to investigations of a similar nature in their respective institutions.

- Findings in chapter 4 brought to light the varying performances of non-standard students in the diagnostic test and service mathematics examination. The investigation revealed the non-homogenous nature of these students and in particular the large improvements made by the mature students over time. This may inform mathematics educationalists of the probable patterns of performance of similar groups of students within their universities.

- This research supports what most would naturally assume, that Ordinary Level Leaving Certificate mathematics students do not have as high a mathematical competency level as Higher Level Leaving Certificate students. It highlights not only the differences between these categories of students in terms of competency levels but also the unpredictable underperformance of some Ordinary Level mathematics students. This detailing of
underperformance may be found in other institutions around Ireland and further research into the area is needed.

- Another major finding within this thesis which is hoped to contribute to national and international mathematics education literature is the fact that the Leaving Certificate mathematics grades examined were found to be a consistent measure of mathematical competency levels and a strong predictor of performance in service mathematics. This information can be used in Ireland to inform policy on entry standards to third level courses involving service mathematics. It could also be used as a point of reference for our international counterparts who may wish to carry out investigations into whether similar correlations are present between performance in mathematics at the end of second level education in their country and third level mathematics performance.

- This research study found that the UL data did not suggest that grade inflation was present in the Leaving Certificate mathematics grades of service mathematics students over time. It is hoped that this finding will serve those in third level education internationally, nationally and locally in UL. In the case of UL, the increase in the number of Ordinary Level mathematics students and mature students was anecdotally being mistaken for grade inflation when declining standards in mathematical competency levels were being discussed. This indicates that care must be taken when making claims regarding declining academic standards. It is hoped that this finding can lead to a better informed community of mathematics educationalists regarding their students’ mathematical abilities.

- Despite testing many variables, the strongest predictor variables of performance in service mathematics in UL were found to be Leaving Certificate mathematics points and diagnostic test result. This finding may encourage those nationally and internationally to begin carrying out diagnostic testing and to develop records of such data in an attempt to better understand and better cater for specific cohorts of students.

- Analysis in this research found that students with equivalent Leaving Certificate mathematics grades (OA1-HC3; OB1-HD3; OA1-HC3) displayed significantly different mean performances in the diagnostic test and service mathematics examinations. Ordinary Level students, despite having the same Leaving Certificate mathematics points, are less mathematically prepared than their Higher Level counterparts. This finding may
encourage those in charge of entry requirements to change their requirements or those involved in the establishment of the points system to reconsider the allocation of points or raise awareness surrounding the finding.

- The discriminant analysis which was carried out using diagnostic test results as the only predictor variable confirmed the effectiveness of the cut-off point for ‘at risk’/not ‘at risk’ which is currently in place for the diagnostic test. The objective evidence-based cut-off point can therefore provide those involved in the implementation and evaluation of the diagnostic test in UL with confidence in the effectiveness of the subjective cut-off point which is currently being used. Similar analysis could take place in national and international third level institutes who are administering diagnostic testing to determine the effectiveness of their cut-off points.

- Some of the challenges faced when predicting service mathematics performance in Ireland have been addressed. It is hoped that these issues will help other Irish mathematics educationalists when predicting performance in similar contexts.

- It was found that students who did not sit the diagnostic test had one of the highest failure rates of all students. Although this thesis did not give an extensive insight into disengaged mathematics students, it is hoped that the insight which was provided may offer mathematics educationalists nationally and internationally another possible contributor to high failure rates and high drop-out rates among third level students.

- An effective method of grouping students into mixed ability groups is offered in this thesis. This is transferrable to any discipline and any educational institution. Although the method which was used involved one of the oldest statistical methods of prediction, it proved to be quite effective in predicting students’ mathematics performance and providing them with a probability of failure. The positive response of the 1st year service mathematics students to the active learning group work is also something which educationalists could take from this thesis and possibly consider incorporating into their teaching practices.

- Finally, the author would like to draw attention to the realisation that mathematical interventions are extremely difficult and complex to implement. The reasons why students may chose not to engage with mathematical support has been previously examined. Reasons such as a lack of awareness of the need for support, too many
problems and a fear of embarrassment were found to be amongst the most common reasons for students not to engage (Lawson et al 2006). The reasons why improvements in students’ performances do not occur could be due to a range of factors such as time frames not being adequate, examination styles which do not support the intervention style etc. Such findings serve as a reference to the complex nature of implementing mathematical support and give an indication of the large number of factors which should be considered extensively prior to the implementation of any intervention.

The contributions listed here are worthy of attention from a national point of view. In many cases it is the first time such examinations have been carried out in Ireland i.e. the changing service mathematics student profile over a ten year period; predicting service mathematics performance and highlighting related issues and mathematical interventions informed by retrospective data. These examinations shed light on the ‘Mathematics Problem’ in Ireland which is significant, particularly in service mathematics within the university and institute of technology sectors (Lynch et al 2003; Hourigan and O’Donoghue 2007). The findings presented here are therefore of value as they provide new insights and tools which may help in the understanding and addressing of the ‘Mathematics Problem’ in an Irish context.

This single study carried out in UL also adds to the international stock of knowledge on the ‘Mathematics Problem’. The examination of issues related to the changing service mathematics student profile, predicting service mathematics performance and implementing interventions highlights that these are worthwhile practices which can result in findings worthy of reporting. The concept of the database, which formed the basis of much of the findings in this research, is portable and so one viable method of attempting to address issues surrounding the ‘Mathematics Problem’ has been presented in this thesis.
7.4 Recommendations

Based on the main research findings, the author has the following recommendations:

• Considerations for a revision of the starting point of mathematics material in third level service mathematics must be acknowledged if institutions intend to continue to offer places to students with lower pre-requisite mathematical knowledge than before. Adjustments must be made if deterioration in the effectiveness of learning is to be avoided (Hunt and Lawson 1996).

• The author recommends that those in charge of third level access policy and practice be mindful of the potential differences in mathematical competency levels of Higher Level and Ordinary Level students who may have the same Leaving Certificate mathematics points and plan programmes accordingly.

• The author suggests that third level institutions consider the development of datasets containing information relating to students mathematical competency levels on entry to third level and their performance in service mathematics. This would allow changing student profiles to be analysed as well as changes in competency levels. If this was carried out on a national level, by third level institutions not currently doing so, it would allow for a useful cross comparison amongst all third level institutions.

• The author recommends that the effectiveness of Leaving Certificate mathematics grades in predicting third level mathematics performance be used to guide third level institutions on the level of mathematical support that may be required for specific cohorts of students prior to their entry. The varying mathematical competency level of students with the same number of Leaving Certificate mathematics points should also be considered.

• The author recommends that lecturers consider the potential underperformance of Ordinary Level Leaving Certificate students and possible reasons behind it in an attempt to better meet their needs.

• The author recommends that the confirmation of the effectiveness of the subjective cut-off point for the diagnostic test be used as a reason to continue the testing in the same manner with confidence.

• Finally third level mathematics examinations should be structured so that they do not encourage rote learning but rather assess understanding (see Appendix L). This would be
in keeping with the intentions of the new mathematics curriculum in second level education in Ireland, ‘Project Maths’.

7.5 Future Research
This study has provided an in-depth insight into the ‘Mathematics Problem’, profiling mathematics students and predicting performance from both a national and international perspective. Some areas of possible future research which have stemmed from these insights are as follows:

- This research provides insight into the changing profile of third level service mathematics students in Ireland. It highlights the increased incidence of students entering third level mathematics education with Ordinary Level Leaving Certificate mathematics and of mature students. Further research into the specific issues that mature students have on entry to service mathematics and how they overcome these issues is needed as these students are becoming an ever growing proportion of third level student bodies today.

- Due to the increase in the proportion of Ordinary Level students entering service mathematics courses, further insights into their underperformance in service mathematics is needed if more effective predictions of their performance are to be carried out and if institutions are to be better equipped to deal with their needs.

- The area of disengaged students was mentioned in this research in relation to students who did not sit the diagnostic test. In 1998 100% of students registered for Technology and Science mathematics sat the diagnostic test which declined to 80% in 2008. Disengagement is also referred to in relation to the poor attendance, in particular amongst the medium risk students, at the intervention tutorials. Further research into disengaged students both nationally and internationally could help educationalists to consider another probable contributory factor to the ‘Mathematics Problem’. The issue of disengaged students in mathematics has been examined in several European countries such as the UK, Norway and the Netherlands (Morris and Pullen 2007), however little has been documented on it in Ireland to date.

- The author alluded to some of the consequences of widening access to third level education in Ireland when discussing the changing student profile in UL in question 3, section 7.2.2. Further research into the direct influence widening access is having on
students’ mathematical performance and on degree standards is necessary to assess if policy needs to be changed in this area.

- This research demonstrated an effective method of grouping service mathematics students into mixed ability groups. It also demonstrated how a socially valid and accepted intervention, according to the students who engaged with it, can be conducted in third level mathematics education. The author suggests however that an intervention such as this be replicated over a much longer period of time to assess if its effectiveness is greater. In addition to this the intervention could be carried out with a mathematics module which currently has a traditional tutorial format and in which the examination is not predictable and tests understanding. This would establish if the nature of the examination did have an impact on the intervention’s effectiveness.

- Further examination into equivalent grades in terms of competency levels in other Leaving Certificate subjects should be carried out to establish if the results found here apply to other subjects or just mathematics.

7.6 Conclusion

This body of work has provided further insight into the ‘Mathematics Problem’. It is important to learn from international literature relating to best practice in mathematics education and localise it to suit the Irish context (NCCA 2005b). All mathematics education is local however, and so while we can learn from others’ situations and gain insights, it’s important to analyse our own situation as we often all have similar problems but for different reasons (Gill and O’Donoghue 2009). The changing student profile of those entering service mathematics in Ireland has been found to be a contributory factor to the increasing proportion of underprepared students discussed in much of the literature. A method to predict failure has been developed which has highlighted the effectiveness of Leaving Certificate mathematics grades and the diagnostic test in distinguishing between students who are likely to be successful and those who are likely to be unsuccessful in service mathematics. It is hoped that this body of work will serve to inform all stakeholders of mathematics education of the calibre of student now deemed appropriate to qualify for access to service mathematics. This information can then be used to determine how best to support these students mathematically so that degree standards can be maintained. It is
intended that these findings will contribute to improving mathematics policy and teaching practices nationally and internationally.
Appendix A
The UL Diagnostic Test

UNIVERSITY OF LIMERICK
DEPARTMENT OF MATHEMATICS AND STATISTICS
DIAGNOSTIC TEST

PURPOSE
The purpose of this test is to ascertain whether there are gaps in your knowledge of basic mathematics. A programme of extra voluntary tutorials is planned for students who need assistance in mathematics during this term.

NOTE
This assessment does not contribute in any way to your grade for this course.

INSTRUCTIONS
Please fill in the details requested and proceed to answer questions. Attempt each question and if you do not know how to do a question, simply tick the don't know box. There is no penalty. Calculators are not allowed.

TIME: 40 mins (approx)

Name __________________________ L.D. No __________________________

UL Programme __________________________

Please tick (✓) the appropriate grade:

LC (Maths)  Higher  A1 □ A2 □ B1 □ B2 □ B3 □ C1 □ C2 □ C3 □ D1 □ D2 □ D3 □
Grade  Ordinary  A1 □ A2 □ B1 □ B2 □ B3 □ C1 □ C2 □ C3 □ Other □

Mature Students:  Please enter grade in appropriate box

LC (Higher) □

Ordinary □
ARITHMETIC Q1 - Q13

1. Work out \((-8) + (-3)\)
   Ans ____________________ □ Don't know

2. Write down the value of \(10 - 8 + 2 + 9\)
   Ans ____________________ □ Don't know

3. Work out: \(\frac{1}{2} - \frac{1}{3}\)
   Ans ____________________ □ Don't know

4. Find the mean of the numbers 12, 14, 10
   Ans ____________________ □ Don't know

5. Work out: \(\frac{2}{3} \times \frac{4}{3}\)
   Ans ____________________ □ Don't know

6. Find 25% of 500
   Ans ____________________ □ Don't know

7. Write down the value of \(3^4\).
   Ans ____________________ □ Don't know

8. Write down the value of \(8^{\frac{1}{3}}\).
   Ans ____________________ □ Don't know

9. Write down the value of \(\frac{1}{2^4}\).
   Ans ____________________ □ Don't know

10. If \(x = 10^2\) then write down the value of \(\log x\).
    Ans ____________________ □ Don't know

11. If \(\log x = 5\) then write down the value of \(\log(x^2)\).
    Ans ____________________ □ Don't know
12. Express 0.01234 in Scientific Notation.
   Ans ___________________ □ Don't know

13. Divide 30 in the ratio 3:2
   Ans ___________________ □ Don't know

**ALGEBRA Q14 - Q21**

14. Solve for $h : V = \pi r^2 h$
   Ans ___________________ □ Don't know

15. Evaluate $ab + 2bc - 3ac$ when $a = 3$, $b = -2$ and $c = 4$.
   Ans ___________________ □ Don't know

16. Solve the equation: $3(x + 2) - 24 = 0$
   Ans ___________________ □ Don't know

17. Solve for $x : x^2 + x - 6 = 0$
   Ans ___________________ □ Don't know

18. Solve the set of equations:
   \[ \begin{align*}
   2x + y &= 7 \\
   x + 2y &= 5
   \end{align*} \]
   Ans ___________________ □ Don't know

19. Write out $(x + 3y)(a - 2b)$ in an equivalent form without brackets.
   Ans ___________________ □ Don't know

20. Solve for $x: 3 - 6x < 21$
   Ans ___________________ □ Don't know

21. Simplify $\frac{1}{x-1} - \frac{2}{x+1}$
   Ans ___________________ □ Don't know
31. Sketch the line \( y = 3x + 2 \) on the diagram.

32. Sketch the curve \( y = x^2 + 2 \) on the diagram.

33. If \( z \) is the complex number \( 1+2i \) find the modulus \( |z| \).
   
   Ans ________________  □ Don't know

34. Simplify: \( \frac{1+2i}{2-3i} \)
   
   Ans ________________  □ Don't know

35. If \( y = 2x^2 + 3 \) find \( \frac{dy}{dx} \)
   
   Ans ________________  □ Don't know

36. If \( y = x \sin x \) find \( \frac{dy}{dx} \)
   
   Ans ________________  □ Don't know

37. If \( y = e^{-2x} \) find \( \frac{dy}{dx} \)
   
   Ans ________________  □ Don't know
38. Evaluate \( \int (x^2 + 2x + 3)dx \)

Ans _________________  □ Don't know

39. Evaluate \( \int_{0}^{3} x^2 dx \)

Ans _________________  □ Don't know

**MODELLING (SPECIAL QUESTION)**

40. The ESB charges domestic users £0.92 per unit of electricity used. In addition each customer is also charged a standing charge of £6.50. Devise a formula for calculating the amount \( A \) of a customer's monthly bill when she uses \( x \) units. (Ignore VAT).

Ans _________________  □ Don't know
Appendix B
Breakdown of Technology and Science Non-Standard Students’ Performance in Terms of Three Sub-categories in 2008

In order to establish if one of the cohorts, Technology or Science, contributed more to the non-standard students’ poor mean diagnostic test performance or high mean service mathematics performance a separate analysis of each cohort took place. The details of this analysis are outlined below.

A Separate Analysis of Technology Non-Standard Students in 2008

The mean diagnostic test performance of each sub-category of non-standard student within Technology mathematics was similar to that of the entire Technology and Science cohort combined. The possible reasons for the performances of each sub-category in the diagnostic test which were outlined in section 4.2.4.1 should therefore also apply when examining the Technology students on their own. However each sub-category of non-standard students’ performance in Technology mathematics differs considerably from that of the entire Technology and Science cohort combined particularly in the case of the mature students. When the 2 cohorts were combined the mature students’ mean service mathematics performance was 54.9% however the mature students within the Technology mathematics on its own had a mean performance of 41.0% (see table 1B). The spread of results in service mathematics for mature students in Technology mathematics (SD = 32.7) is larger than that of the combined cohorts of mature students (SD = 27.9) (see figure 1B). It appears therefore that it was the mature students within the Technology cohort (as opposed to the other two sub-categories) who caused the non-standard students’ results to have a larger spread when compared to the standard students’ service mathematics performance as was outlined in the original analysis of non-standard students’ service mathematics performance (see section 4.2.3.1(c) and figure 4.3).

9 (40.9%) of the mature Technology students failed their service mathematics examination. The large spread of performance may be attributed to some mature students being over 50, for example, and never having sat a Leaving Certificate mathematics examination and others being 23 and having sat their Leaving Certificate four years previous to when they entered third level education. A study which was conducted in UL in 2008 highlighted that almost half (n = 12, 44.4%) of the mature students who took part in a mathematics bridging course in UL had not
studied mathematics for between 6 and 10 years and a further 6 (22.2%) having not studied it in 11 plus years (Gill 2008). In addition to these students who had not engaged in formal mathematics education for a number of years 9 (33.3%) of the students had studied mathematics more recently (in the last 5 years). Although this study only included 27 mature students in 2008 the findings give an insight into the probable varying mathematical backgrounds and ages of mature students in UL. The Technology mature students who failed Technology mathematics were enrolled in Materials and Engineering Technology (n = 4) and Music, Media and Performance Technology (n = 5). The Music, Media and Performance Technology students may not have anticipated engaging with mathematics due to their choice of degree programme, this may have negatively impacted on their performance. Research has shown that a negative attitude towards, or resentment of having to engage with, mathematics is statistically significantly correlated with performance in mathematics (Hackett and Betz 1989). In the case of the mature students who failed their examination and were enrolled in Materials and Engineering Technology these students may have found the work load of the mathematics too much when combined with that of their other subject areas. This concern in relation to the large workload was one which was highlighted by mature students in 2008 (Gill 2008). Although the mature students were on a relatively even plane on entry to UL, as demonstrated by their poor performance and small spread of results in the diagnostic test, the variations in the extent of their previous mathematical studies and the time frame between that and their entry to higher education is likely to be a contributory factor to the large spread of results and the high failure rate amongst this sub-category of Technology mathematics students in service mathematics (see figures 4.1 and 4.3, chapter 4).

The Technology non-standard students who engaged in previous study and the international students had high success rates in Technology mathematics. 3 (100%) of the students who engaged in previous study and 7 (87.5%) of the international students were successful in Technology mathematics (see figure 2B). As highlighted in section 4.2.4.1 this is likely to be partially due to the international students more recent engagement with mathematics and their familiarity with formal education. In the case of the students who engaged in previous study their success is likely to be attributed to their ability to commit to and successfully complete academic programmes of study. The high success rate of students within these two sub-categories is
therefore influential on the overall increase in mean mathematics performance of the non-standard student category over time.

<table>
<thead>
<tr>
<th></th>
<th>Diagnostic test mean result</th>
<th>End of Semester Result</th>
<th>Total Number of Students in each category</th>
<th>Percentage who Passed Technology Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mature Students</strong></td>
<td>28.8 (17.4) n=22</td>
<td>41.0 (32.7) n=22</td>
<td>24 (68.7%)</td>
<td>59.1%</td>
</tr>
<tr>
<td><strong>Previous degree/diploma/certificate</strong></td>
<td>28.8 (12.4) n=2</td>
<td>64.3 (21.4) n=3</td>
<td>3 (8.6%)</td>
<td>100%</td>
</tr>
<tr>
<td><strong>International Students</strong></td>
<td>47.9 (30.2) n=6</td>
<td>65.9 (20.8) n=8</td>
<td>8 (22.7%)</td>
<td>87.5%</td>
</tr>
</tbody>
</table>

**Table 1B** Mean Performance and Standard Deviation of 3 sub-categories of non-standard student in diagnostic test and Technology mathematics in 2008.

**Note:** Technology students: 30/35 sat diagnostic test and 33/35 sat Technology mathematics examination.

**Figure 1B** Diagnostic test performance of 3 non-standard student categories in Technology mathematics 2008.
A Separate Analysis of Non-Standard Science Students in 2008

Upon carrying out an analysis of the non-standard Science category it could be seen that the 3 sub-categories performed to a similar standard to the Technology students in the diagnostic test. That is the mature students and students who have engaged in previous study performed poorly with 11 (91.7%) and 2 (100%) being considered to be ‘at risk’ respectively and the international student performing to a higher standard and not being considered to be ‘at risk’ (n = 1) (see table 2B and figure 3B).

There is a contrast however in the performance of mature Science students in service mathematics when compared to those in Technology mathematics. 12 (92.3%) of the Science mature students were successful in service mathematics (see figure 4B) compared to the 13 (59.1%) success rate for the Technology mathematics mature students. All of the students who engaged in previous study were successful in Science mathematics, as was the case with the
Technology students. The one international Science student failed Science mathematics. Due to the fact there was only one student in this category it was hard to deduct anything from this finding. The pass rate amongst the non-standard Science mathematics students was therefore extremely high with just 2 (11.8%) of students failing (see table 2B and figure 4B).

From separate analysis of the 3 non-standard sub-categories it is clear that the mature students in the Technology cohort make up the majority of non-standard students who failed service mathematics. The mature Technology students have a failure rate of 40.9% (n = 9). It is therefore this sub-category of non-standard student who are responsible for the large spread of results in service mathematics performance as highlighted in the original analysis of non-standard students in section 4.2.3.

<table>
<thead>
<tr>
<th></th>
<th>Diagnostic test mean result</th>
<th>End of Semester Result</th>
<th>Total Number of Students in each category</th>
<th>Percentage who Passed Science Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mature Students</td>
<td>27.5 (15.0) n=12</td>
<td>61.2 (16.6) n=13</td>
<td>16 (76.2%)</td>
<td>92.3%</td>
</tr>
<tr>
<td>Previous degree/diploma/certificate</td>
<td>26.3 (8.8) n=2</td>
<td>59.0 (16.7) n=3</td>
<td>3 (14.3%)</td>
<td>100.0%</td>
</tr>
<tr>
<td>International Students</td>
<td>() n=1</td>
<td>() n=1</td>
<td>2 (9.5%)</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 2B Mean Performance and Standard Deviation of 3 categories of non-standard student in diagnostic test and Science mathematics in 2008.

Note: Science students: 15/21 sat diagnostic test and 17/21 sat Science mathematics examination.
**Figure 3B** Diagnostic test performance of 3 sub-categories of non-standard students in Science mathematics 2008.

**Figure 4B** Science mathematics performance of 3 sub-categories non-standard students in Science mathematics 2008.
Number and Percentage of Technology and Science Mathematics Students who did not sit the Diagnostic Test by Degree Programme

<table>
<thead>
<tr>
<th>Degree Programme</th>
<th>Frequency</th>
<th>Percentage of students who did not sit the diagnostic test within each degree programme</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applied Computing and Network Technology</td>
<td>1</td>
<td>0.6</td>
</tr>
<tr>
<td>Construction Management and Engineering</td>
<td>32</td>
<td>19.3</td>
</tr>
<tr>
<td>Digital, Media and Design</td>
<td>13</td>
<td>7.8</td>
</tr>
<tr>
<td>Engineering Science</td>
<td>2</td>
<td>1.2</td>
</tr>
<tr>
<td>Materials and Engineering Technology</td>
<td>22</td>
<td>13.3</td>
</tr>
<tr>
<td>Manufacturing Systems</td>
<td>3</td>
<td>1.8</td>
</tr>
<tr>
<td>Material and Construction Technology</td>
<td>17</td>
<td>10.2</td>
</tr>
<tr>
<td>Music, Media and Performance</td>
<td>26</td>
<td>15.6</td>
</tr>
<tr>
<td>Physical Education</td>
<td>14</td>
<td>8.4</td>
</tr>
<tr>
<td>Production Management</td>
<td>10</td>
<td>6.1</td>
</tr>
<tr>
<td>Product Design and Technology</td>
<td>16</td>
<td>9.6</td>
</tr>
<tr>
<td>Wood Science and Technology</td>
<td>10</td>
<td>6.1</td>
</tr>
<tr>
<td>Total</td>
<td>166</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 1C Number and percentage of Technology mathematics students who did not sit the diagnostic test within each degree programme
<table>
<thead>
<tr>
<th>Degree Programme</th>
<th>Frequency</th>
<th>Percentage of students who did not sit the diagnostic test within each degree programme</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA(Joint Honours)</td>
<td>6</td>
<td>6.3</td>
</tr>
<tr>
<td>Biological Science</td>
<td>14</td>
<td>14.7</td>
</tr>
<tr>
<td>Biomedical and Advanced Materials</td>
<td>2</td>
<td>2.1</td>
</tr>
<tr>
<td>Environmental Science</td>
<td>10</td>
<td>10.5</td>
</tr>
<tr>
<td>Food Science and Health</td>
<td>13</td>
<td>13.7</td>
</tr>
<tr>
<td>Health and Safety</td>
<td>11</td>
<td>11.6</td>
</tr>
<tr>
<td>Industrial Biochemistry</td>
<td>6</td>
<td>6.3</td>
</tr>
<tr>
<td>Pharmaceutical and Industrial Chemistry</td>
<td>7</td>
<td>7.4</td>
</tr>
<tr>
<td>Sport and Exercise Science</td>
<td>26</td>
<td>27.4</td>
</tr>
<tr>
<td>Total</td>
<td>95</td>
<td>100.0</td>
</tr>
</tbody>
</table>

*Table 2C Number and percentage of Science mathematics students who did not sit the diagnostic test within each degree programme*
Appendix D
The Quality Credit System (QCA) in UL

In the University of Limerick the system which is used to measure a student’s overall performance, i.e. the accumulation of their performance in all modules taken in a semester, uses a measure known as Quality Credit Average (QCA). This is a numerical average of a student’s performance in the credited modules which they attempt. A student’s QCA value is calculated on a semester and on a cumulative basis for each programme (O’Brien 2004).

The following table shows the QCA value awarded to each grade, 4 being the highest possible award for an A1 and 0 being awarded for a fail (see table 1D).

<table>
<thead>
<tr>
<th>Grade</th>
<th>Quality Point Value (QPV)</th>
<th>Final Award on basis of accumulated QCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>4.00</td>
<td>First Honours</td>
</tr>
<tr>
<td>A2</td>
<td>3.60</td>
<td>First Honours</td>
</tr>
<tr>
<td>B1</td>
<td>3.20</td>
<td>Honours 2.1</td>
</tr>
<tr>
<td>B2</td>
<td>3.00</td>
<td>Honours 2.1</td>
</tr>
<tr>
<td>B3</td>
<td>2.80</td>
<td>Honours 2.2</td>
</tr>
<tr>
<td>C1</td>
<td>2.60</td>
<td>Honours 2.2</td>
</tr>
<tr>
<td>C2</td>
<td>2.40</td>
<td>Third Class Honours</td>
</tr>
<tr>
<td>C3</td>
<td>2.00</td>
<td>Third Class Honours</td>
</tr>
<tr>
<td>D1</td>
<td>1.60</td>
<td>Compensating Fail</td>
</tr>
<tr>
<td>D2</td>
<td>1.20</td>
<td>Compensating Fail</td>
</tr>
<tr>
<td>F</td>
<td>0.00</td>
<td>Fail</td>
</tr>
</tbody>
</table>

Table 1D Breakdown of QCA value awarded for each grade (O’Brien 2004 slide 5).

In order to determine whether students who did not sit the diagnostic test were performing poorly in all areas of their chosen discipline of study or just mathematics an analyses of students’ QCA values for semester 1 was carried out. Of the 36 students who failed year 1 of their program of study 22 (61.1%) of them also failed Technology mathematics. This suggests that the majority of these students were struggling with all areas of their degree program including mathematics.
Appendix E
Alternative Prediction Methods: Classification Trees and Logistic Regression Results

1. Classification Trees

Classification trees are used to predict membership of a categorical dependant variable from using one or more predictor variables. It is an alternative to other prediction methods such as discriminant analysis and logistic regression. Classification trees are often called upon by researchers when the more traditional methods are not suitable. One of the reasons for this is that they can be used with categorical, discrete numeric or continuous data. Classification trees also do not require any assumptions about the distribution of the variables to be met.

1.1 How Do Classification Trees work?

Classification trees are built through a process known as binary recursive partitioning. This is a process of splitting the data into partitions and then splitting it up further on each of the branches (Berk 2008). Similar to discriminant analysis it starts with a training set in which the population (e.g. success or failure) is known. The algorithm then systematically tries breaking up the records into two parts, examining one variable at a time and splitting the records on the basis of a dividing line in that variable (e.g. > 50 Leaving Certificate mathematics points and ≤ 50 Leaving Certificate mathematics points). The aim then is to obtain as homogenous set of labels (e.g. success or failure) as possible in each partition. This splitting/partitioning is then applied to each of the new partitions. The process continues until no more useful splits can be found. The most important part of the algorithm is the rule that determines the initial split (see figure 1E overleaf for example) (http://www.resample.com/xliner/help/Ctree/ClassificationTree_intro.htm).
The results of the classification tree analysis which was carried out on the Technology 2006-2008 dataset is outlined in section 1.2.

**Figure 1E** Hypothetical Classification Tree.
1.2 Classification Tree Results: Technology 2006-2008 Dataset

When the Technology 2006-2008 dataset was used for analysis Leaving Certificate mathematics points and diagnostic test results were found to be the independent variables which best classify performance in service mathematics. From the classification table it can be seen that it was highly successful in correctly classifying success (93.5%) however it did not have the same degree of success when predicting failure (39.4%). Overall it correctly classified 79.8% of the students in the Technology 2006-2008 dataset (see table 3E).

<table>
<thead>
<tr>
<th>Model Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specifications</td>
</tr>
<tr>
<td>Dependent Variable</td>
</tr>
<tr>
<td>Independent Variables</td>
</tr>
<tr>
<td>Validation</td>
</tr>
<tr>
<td>Maximum Tree Depth</td>
</tr>
<tr>
<td>Minimum Cases in Parent Node</td>
</tr>
<tr>
<td>Minimum Cases in Child Node</td>
</tr>
<tr>
<td>Results</td>
</tr>
<tr>
<td>Number of Nodes</td>
</tr>
<tr>
<td>Number of Terminal Nodes</td>
</tr>
<tr>
<td>Depth</td>
</tr>
</tbody>
</table>

Table 1E Variables included in the model and the variables specified but not included in the model.
Figure 2E Classification Tree created using Technology 2006-2008 dataset.

Risk

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>.202</td>
<td>.012</td>
</tr>
</tbody>
</table>

Growing Method: CHAID
Dependent Variable: Success/Failure?

Table 2E Risk results: A measure of the tree’s predictive accuracy.

Classification

<table>
<thead>
<tr>
<th>Observed</th>
<th>Predicted</th>
<th>Success</th>
<th>Failure</th>
<th>Percent Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success</td>
<td>772</td>
<td>54</td>
<td>93.5%</td>
<td></td>
</tr>
<tr>
<td>Failure</td>
<td>164</td>
<td>90</td>
<td>35.4%</td>
<td></td>
</tr>
<tr>
<td>Overall Percentage</td>
<td>86.7%</td>
<td>13.3%</td>
<td>79.8%</td>
<td></td>
</tr>
</tbody>
</table>

Growing Method: CHAID
Dependent Variable: Success/Failure?

Table 3E The number of cases classified correctly and incorrectly for each category of the dependent variable.
2. Logistic Regression Analysis
Logistic regression is used to predict the probability of occurrence of an event by fitting data to a logistic curve. Similar to many types of regression analysis it makes use of several predictor variables. The aim of this is to assess the impact of a number of variables on the likelihood that students will fail service mathematics in UL. Failure is the outcome variable (yes/no) and all other variables act as explanatory variables. The variables being tested consist of: gender, Leaving Certificate points, programme of study, Leaving Certificate Level and Grade, Non standard student sub-category, whether a student sat the diagnostic test or not, diagnostic test results and performance in the algebra and arithmetic sections of the diagnostic test.

The author is interested in the combination of variables which will increase the ability of the logistic regression model to correctly classify students who will fail service mathematics. Many variations of variables are therefore tested in an attempt to increase the percentage of correctly classified failure students. Separate logistic regression analysis was carried out for the Science and Technological 2006-2008 datasets.

2.1 Logistic Regression Results: Technology 2006-2008
The logistic regression model created using the Technology 2006-2008 dataset found Leaving Certificate mathematics points to be the only significant variable in predicting performance in Technological mathematics students. The model was also found to be statistically significant (p > 0.001) indicating that it was able to distinguish between students who were unsuccessful in Technology mathematics and those who were successful. The model explains between 23.0% (Cox and Snell R Square) and 35.8% (Nagelkerke R squared) of variance in examination performance amongst this group of Technological mathematics students. It correctly classified 39.0% of the unsuccessful students in the dataset. See table 4E which highlights Leaving Certificate points as making a statistically significant contribution to the model (p < 0.001) and programme of study to a lesser extent (p < 0.05).
### Table 4E Individual significance of each predictor variable to the logistic regression model for the Technology 2006-2008 dataset.

<table>
<thead>
<tr>
<th>Variables in the Equation</th>
<th>B</th>
<th>S.E.</th>
<th>Wald</th>
<th>df</th>
<th>Sig.</th>
<th>Exp(B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1&lt;sup&gt;st&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LC</td>
<td>-.071</td>
<td>.014</td>
<td>27.528</td>
<td>1</td>
<td>.000</td>
<td>.931</td>
</tr>
<tr>
<td>Mathematics Points</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gender(1)</td>
<td>-.284</td>
<td>.316</td>
<td>.808</td>
<td>1</td>
<td>.369</td>
<td>.753</td>
</tr>
<tr>
<td>Programme of study</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diagnostic Test</td>
<td>-.023</td>
<td>.049</td>
<td>.221</td>
<td>1</td>
<td>.639</td>
<td>.977</td>
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<tr>
<td>Arithmetic</td>
<td>-.008</td>
<td>.012</td>
<td>.419</td>
<td>1</td>
<td>.517</td>
<td>.992</td>
</tr>
<tr>
<td>Algebra</td>
<td>-.005</td>
<td>.008</td>
<td>.368</td>
<td>1</td>
<td>.544</td>
<td>.995</td>
</tr>
<tr>
<td>LC Level</td>
<td>-.713</td>
<td>.368</td>
<td>3.760</td>
<td>1</td>
<td>.053</td>
<td>.490</td>
</tr>
<tr>
<td>Constant</td>
<td>4.006</td>
<td>.772</td>
<td>26.917</td>
<td>1</td>
<td>.000</td>
<td>54.901</td>
</tr>
</tbody>
</table>
3. Conclusion

Both methods of prediction, classification trees and logistic regression, performed reasonable well in terms of their overall correct classification of the Technology dataset. The classification trees however did not correctly classify a high enough percentage of unsuccessful students to justify using this method of prediction on future cohorts of UL service mathematics students. Despite the fact that a statistically significant logistic regression models emerged from the Technological dataset, the percentage of correctly classified failure students is not high enough for the author to use these models on future groups of service mathematics students in UL. Logistic regression may not have been as successful as hoped in this case as many of the variables could not be entered into the model at the same time. For example Leaving Certificate points and whether or not a student was standard or non-standard could not be entered into the model as non-standard students generally do not have Leaving Certificate points in the dataset being examined. Another pairing that could not be used was a student’s diagnostic test result/algebra performance/arithmetic performance along with whether or not they sat the diagnostic test. Obviously if a student did not sit the diagnostic test there is no diagnostic test result for them and so the model rejects this combination also. The author feels based on the descriptive statistics outlined in chapter 4 that these variables, if combined, may have lead to a model which was much better able to correctly classify the unsuccessful students in service mathematics (note: this is possible in classification tree analysis). This was not possible however and so another approach was tested. This approach is called discriminant analysis (Barry and Chapman 2007) and is detailed in chapter 5. Logistic Regression and classification trees are often used in place of discriminant analysis when data is not normally distributed or group sizes are very unequal. Discriminant analysis however is preferred when the assumptions underlying it are met since discriminant analysis has more statistical power than logistic regression (less chance of type 2 errors i.e. accepting a false null hypothesis).
Appendix F
Leaving Certificate Mathematics Grades against Diagnostic Test Group in Terms of Service Mathematics Performance.

<table>
<thead>
<tr>
<th>Performance in Technology Mathematics</th>
<th>Leaving Certificate Grade</th>
<th>Diagnostic Test Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success</td>
<td>Group 1</td>
<td>Group 2</td>
</tr>
<tr>
<td>HLA1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>.0%</td>
<td>.0%</td>
</tr>
<tr>
<td>HLA2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>.0%</td>
<td>.0%</td>
</tr>
<tr>
<td>HLB1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>.0%</td>
<td>.0%</td>
</tr>
<tr>
<td>HLB2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>.0%</td>
<td>.0%</td>
</tr>
<tr>
<td>HLB3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>.0%</td>
<td>2.2%</td>
</tr>
<tr>
<td>HLC1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>.0%</td>
<td>6.6%</td>
</tr>
<tr>
<td>HLC2</td>
<td>0</td>
<td>4</td>
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<td></td>
<td>.0%</td>
<td>10.5%</td>
</tr>
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<td>HLC3</td>
<td>0</td>
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<td>.0%</td>
<td>6.5%</td>
</tr>
<tr>
<td>HLD1</td>
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<td>.0%</td>
</tr>
<tr>
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<td>33.0%</td>
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<tr>
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<tr>
<td>OLB2</td>
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353
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<tr>
<th></th>
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<th></th>
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<tbody>
<tr>
<td></td>
<td>3</td>
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<td>%</td>
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<td>0</td>
<td>0</td>
<td>1</td>
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<td>100.0%</td>
<td>.0%</td>
<td>.0%</td>
<td>.0%</td>
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</tr>
<tr>
<td>OLC3</td>
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<td>0</td>
<td>0</td>
<td>2</td>
<td></td>
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<tr>
<td>%</td>
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<td>50.0%</td>
<td>.0%</td>
<td>.0%</td>
<td>.0%</td>
<td>100.0%</td>
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<tr>
<td>Total</td>
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<td>73</td>
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<td>31.4%</td>
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<td></td>
</tr>
<tr>
<td>Failure</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>HLD1</td>
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<td>1</td>
<td></td>
<td></td>
<td>1</td>
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<tr>
<td>%</td>
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<td>.0%</td>
<td>100.0%</td>
<td>.0%</td>
<td></td>
<td>100.0%</td>
</tr>
<tr>
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<td>1</td>
</tr>
<tr>
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<td>.0%</td>
<td>100.0%</td>
<td>.0%</td>
<td>.0%</td>
<td>.0%</td>
<td>100.0%</td>
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<td>60.0%</td>
<td>.0%</td>
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<td>4</td>
<td></td>
<td></td>
<td>6</td>
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<td>.0%</td>
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<td>33</td>
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<td></td>
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</tr>
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<td>%</td>
<td>11.5%</td>
<td>84.6%</td>
<td>3.8%</td>
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<td>100.0%</td>
</tr>
<tr>
<td>OLC1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>%</td>
<td>100.0%</td>
<td>.0%</td>
<td>.0%</td>
<td>.0%</td>
<td></td>
<td>100.0%</td>
</tr>
<tr>
<td>OLC3</td>
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<td>0</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>%</td>
<td>.0%</td>
<td>100.0%</td>
<td>.0%</td>
<td>.0%</td>
<td>.0%</td>
<td>100.0%</td>
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<tr>
<td>Total</td>
<td>13</td>
<td>111</td>
<td>30</td>
<td></td>
<td></td>
<td>154</td>
</tr>
<tr>
<td>%</td>
<td>8.4%</td>
<td>72.1%</td>
<td>19.5%</td>
<td>.0%</td>
<td></td>
<td>100.0%</td>
</tr>
</tbody>
</table>

Table 1F Leaving Certificate mathematics grades Against Diagnostic Test Group in Terms of Service Mathematics Performance.
Appendix G
Tests for Normality: Technology 2006-2008 Dataset

1. Technology Cohort: Assessing Assumption of Normality

Figure 1G Q-Q plot of successful Technology mathematics students 2006-2008 Leaving Certificate mathematics points.

![Normal Q-Q Plot of points](image)

Tests of Normality

<table>
<thead>
<tr>
<th>Success/Failure?</th>
<th>Kolmogorov-Smirnov&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Shapiro-Wilk</th>
</tr>
</thead>
<tbody>
<tr>
<td>points Success</td>
<td>Statistic: .150, df: 760, Sig.: .000</td>
<td>Statistic: .965, df: 760, Sig.: .000</td>
</tr>
</tbody>
</table>

Table 1G Normality test for successful Technology mathematics students 2006-2008.

Figure 2G Q-Q plot of unsuccessful Technology mathematics students 2006-2008 Leaving Certificate mathematics points.

![Normal Q-Q Plot of points](image)
Tests of Normality

<table>
<thead>
<tr>
<th>Success/Failure?</th>
<th>Kolmogorov-Smirnov Statistic</th>
<th>df</th>
<th>Sig.</th>
<th>Shapiro-Wilk Statistic</th>
<th>df</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>failure</td>
<td>.174</td>
<td>200</td>
<td>.000</td>
<td>.915</td>
<td>200</td>
<td>.000</td>
</tr>
</tbody>
</table>

**Table 2G** Normality test for unsuccessful Technology mathematics students 2006-2008.

![Normal Q-Q Plot of Diagtest](image)

**Figure 3G** Q-Q plot of successful Technology mathematics students 2006-2008 diagnostic test results.

Tests of Normality

<table>
<thead>
<tr>
<th>Success/Failure?</th>
<th>Kolmogorov-Smirnov Statistic</th>
<th>df</th>
<th>Sig.</th>
<th>Shapiro-Wilk Statistic</th>
<th>df</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>success</td>
<td>.051</td>
<td>716</td>
<td>.000</td>
<td>.994</td>
<td>716</td>
<td>.004</td>
</tr>
</tbody>
</table>

**Table 3G** Normality test for successful diagnostic test results Technology mathematics students 2006-2008.
Figure 4G Q-Q plot of unsuccessful Technology mathematics students 2006-2008 diagnostic test results.

<table>
<thead>
<tr>
<th>Success/Failure</th>
<th>Kolmogorov-Smirnov(^a)</th>
<th>Shapiro-Wilk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagtest Failure</td>
<td>Statistic</td>
<td>df</td>
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<tr>
<td></td>
<td>.056</td>
<td>177</td>
</tr>
</tbody>
</table>

Table 4G Normality test for unsuccessful diagnostic test results Technology mathematics students 2006-2008.
Appendix H
### Classification Function Co-efficient Tables

<table>
<thead>
<tr>
<th>Classification Function Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Success/Failure?</strong></td>
</tr>
<tr>
<td><strong>Success</strong></td>
</tr>
<tr>
<td><strong>Failure</strong></td>
</tr>
<tr>
<td>Diagnostic Test</td>
</tr>
<tr>
<td>Leaving Certificate Maths Points</td>
</tr>
<tr>
<td>(Constant)</td>
</tr>
</tbody>
</table>

*Fisher's linear discriminant functions*

**Table 1H** Technology 2006-2008 discriminant function variable’s co-efficients.

<table>
<thead>
<tr>
<th>Classification Function Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Success/Failure?</strong></td>
</tr>
<tr>
<td><strong>Success</strong></td>
</tr>
<tr>
<td><strong>Failure</strong></td>
</tr>
<tr>
<td>points</td>
</tr>
<tr>
<td>(Constant)</td>
</tr>
</tbody>
</table>

*Fisher's linear discriminant functions*

**Table 2H** Classification Function Coefficients of discriminant function 2.
Appendix I
**Discriminant Analysis Results with Diagnostic Test as only Variable**

### Classification Function Coefficients

<table>
<thead>
<tr>
<th>Success/Failure?</th>
<th>Success</th>
<th>Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagtest</td>
<td>.556</td>
<td>.387</td>
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<tr>
<td>(Constant)</td>
<td>-6.902</td>
<td>-3.706</td>
</tr>
</tbody>
</table>

Fisher's linear discriminant functions

**Table 1H** Classification Function Coefficients of discriminant function with diagnostic test as the only predictor variable

<table>
<thead>
<tr>
<th>Success/Failure?</th>
<th>Diagnostic Test Function Prediction</th>
<th>Predicted Success</th>
<th>Predicted Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success</td>
<td>Count</td>
<td>520</td>
<td>195</td>
<td>715</td>
</tr>
<tr>
<td></td>
<td>% within Success/Failure?</td>
<td>72.7%</td>
<td>27.3%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Failure</td>
<td>Count</td>
<td>64</td>
<td>135</td>
<td>199</td>
</tr>
<tr>
<td></td>
<td>% within Success/Failure?</td>
<td>32.2%</td>
<td>67.8%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Total</td>
<td>Count</td>
<td>584</td>
<td>330</td>
<td>914</td>
</tr>
</tbody>
</table>

**Table 2H** Discriminant analysis success with diagnostic test as the only independent variable.

**Discriminant Function:**

\[ Z = 0.169 \text{ (Diagnostic Test Result), where } C = 3.196. \]
Note: Appendix J (Intervention Lesson Plans) and Appendix K (Student Questionnaire Transcripts and Author’s Observation Transcripts) are stored on the cd at the back of the thesis.
UNIVERSITY of LIMERICK

OLLSCOIL LUIMNIGH

COLLEGE OF INFORMATICS AND ELECTRONICS

MODULE CODE : MA4701

MODULE TITLE : Technological Mathematics 1

SEMESTER : Autumn 1998/99

LECTURER : Ms. Maura Glynn

DURATION : 2½ hours

INSTRUCTIONS TO CANDIDATES : Question ONE is COMPULSORY and carries 40 marks. Answer ANY other FOUR worth 15 marks each.
Q.1  
(a) Find the distance between the points \((-1,6)\) and \((3,-2)\) and find the equation of the line passing through these points.

(b) Solve the following equation : \(5\ln(x + 10) = 20\).

(c) Find the derivative of the given functions, with respect to the appropriate variables.

(i) \(f(x) = \frac{1}{3}x^6 + \frac{7}{\sqrt{x}} - 5\)

(ii) \(g(t) = t^2 \ln(t)\)

(iii) \(P(Q) = (Q^3 - 2Q)^{1/2}\)

(iv) \(y = \frac{3x^2}{e^{2x} - 4}\)

(d) Find the equation of the tangent to the curve \(y = -x^2 + 3\) at \((1,2)\).

(e) Find \(\text{Re}(z)\) and \(\text{Im}(z)\) if \(z = \frac{(4 + 3i)(2 - i)}{1 + 2i}\).

(f) Use De Moivre’s Theorem to find \((-2 + 2i)^9\).

(g) Given the vectors \(\vec{a} = 2i - 3j + 2k\) and \(\vec{b} = -i + j + 4k\), find the direction cosines of both and hence, or otherwise, find the angle between them.

(h) If \(A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}\) find \(\det(A)\).

(i) From the following augmented matrix, state the solution to the linear system:

\[
\begin{bmatrix}
1 & -3 & 0 & | & 1 \\
0 & 1 & 2 & | & 5 \\
0 & 0 & 1 & | & 2 \\
\end{bmatrix}
\]
Q.2 (a) (i) Write down the largest domain for which the following function is defined:

\[ f(x) = \frac{3x}{x^2 - 4} \]

(ii) Determine if the function \( f(t) = \frac{1}{t^3 + 4} \) has an inverse. Find an expression for the inverse, if it exists.

(iii) Find an expression for \( f \circ g \) when \( f(x) = 3 - 7x \) and \( g(x) = \sin(x^2) \).

(iv) Find the acute angle between the straight lines \( y = -3x - 9 \) and \( y = 2x - 4 \).

(v) Sketch the function \( y = -x^2 - 3x - \frac{9}{4} \) without plotting points.

(ie. use concavity, roots, intercepts, shifting)

(b) Identify the variables you would plot in order to produce a straight line graph from:

(i) \( P = aQ^2 + b \) (iii) \( s = A + \frac{B}{t} \)

(ii) \( y = ax^2 + bx \) (iv) \( y = Ae^{Bx} \)

Q.3 (a) \( P(t) = 1000e^{0.037t} \) denotes the size of a colony of bacteria after \( t \) hours.

(i) Find the initial size of the colony.

(ii) Find its size after 2 days.

(iii) Sketch the graph of \( P(t) \).

Contd........
(b) Determine the angular frequency $\omega$, amplitude $A$ and period $T$ of the function $y = 3.5 \sin 4t$. Sketch the curve.

(c) A triangular component must be made using the dimensions shown in the diagram. Calculate the angles and side length that are not given.

![Triangle Diagram]

Q.4 (a) Let $f(x) = 2x^3 - 3x^2 - 12x + 10$. Determine the stationary points of $f(x)$ and classify them. Sketch the curve.

(b) The velocity, $v$ of a body moving through a resisting medium is given by $v(t) = 20(1 - e^{-0.04t})$ where $t$ is the time in seconds. Find the acceleration, $a$ after 5 seconds.

Q.5 (a) Given the complex numbers $z_1 = 2 - 3i$ and $z_2 = 5 + i$ find

(i) $\bar{z}_1 z_2$

(ii) $|z_1 - \bar{z}_2|$

(iii) $z_2$ in polar form

(iv) $\frac{\bar{z}_1}{z_2}$

(b) Find the cube roots of $z$ where $z = 5 + i$. Plot these cube roots on an Argand diagram.

Q.6 (a) Given the following vectors

$\vec{a} = 3i - 2j - k$

$\vec{b} = 4i + 3j - 2k$


\[ \overrightarrow{c} = -i - 2j + 5k \]

(i) Find \( \overrightarrow{b} - 2\overrightarrow{c} + 3\overrightarrow{a} \).

(ii) Find \( \overrightarrow{c} \times \overrightarrow{b} \).

(iii) Find the scalar projection of \( \overrightarrow{b} \) onto \( \overrightarrow{a} \).

(b) An object is experiencing two forces \( \overrightarrow{F}_1 = 50N \) and \( \overrightarrow{F}_2 = 30N \) as shown in the diagram. Calculate the magnitude of the resultant force \( \overrightarrow{F} \) and determine the angle, \( \theta \) that \( \overrightarrow{F} \) makes with the vertical.

![Diagram of forces](image)

**Q.7**

(a) Solve the following system of linear equations using Gaussian Elimination, or otherwise.

\[
\begin{align*}
x + 2y + 3z &= 5 \\
3x - y + 2z &= 8 \\
2x - 3y - 2z &= -1
\end{align*}
\]

(b) If \( A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -1 & 2 \\ 2 & -3 & -2 \end{bmatrix} \) find \( A^{-1} \).
END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4701  SEMESTER: Autumn 2002/03

MODULE TITLE: Technological Mathematics 1  DURATION OF EXAM: 2.5 hours

LECTURER: Mr J. O’Shea  GRADING SCHEME:
Examination: 85%

EXTERNAL EXAMINER: Prof. J.D. Gibbon

INSTRUCTIONS TO CANDIDATES

Question One (Q1) is compulsory and carries 40 marks.

Answer any other four questions worth 15 marks each.
1 (a) Find the slope and hence the equation of the line passing through a(3,4) and b(-1,2)  

(b) Solve the equation 5ln(x + 2) = 6  

(c) Find the first derivative of the following functions  

(i) $f(x) = 3x^2 + \frac{2}{x} - 5$  

(ii) $y = (2x^2 - 3x + 1)^4$  

(iii) $g(x) = \frac{3x}{\sin 2x}$  

(iv) $f(t) = e^{4t}.\ln 2t$  

(d) Find the slope of the tangent to the curve $y = 3x^2 - 6x + 1$ at the point (2,1)  

(e) Write in the form $a + bi$  

(i) $\frac{1+3i}{3+i}$  

(ii) $e^{i+\pi}$  

(f) Use De Moivre’s theorem to find $(1-i)^4$  

(g) Find the scalar product $\vec{a} \cdot \vec{b}$ where $\vec{a} = 2i + 3j + 4k$, $\vec{b} = i - 2j + 3k$  

(h) \[
\begin{bmatrix}
2 & 1 \\
1 & 2
\end{bmatrix}
\quad
\begin{bmatrix}
4 & 1 \\
1 & -1
\end{bmatrix}
\]

Find (i) AB  

(ii) $A^{-1}$, the inverse of A  

(i) \[
\begin{bmatrix}
3 & 2 & 1 \\
4 & 2 & 2 \\
1 & 3 & 1
\end{bmatrix}
\]  

Find det. (A).
2 (a) (i) Solve the inequality |3x + 5| < 2

(ii) Write down the largest domain for which the following real function is defined f(x) = \sqrt{x - 1}

(iii) f(x) = \frac{x - 1}{2}, find f^{-1}(x), the inverse of f

(iv) Find an expression for (f \circ g)(x) when f(x) = 3x - 2 and g(x) = 7 - 2x

(v) Determine if the function f(x) = \frac{x}{x^2 + 4} is odd, even or neither

(b) Identify the variables you would plot in order to produce a straight line graph

(i) y^2 = a + bx^2

(ii) P = a + \frac{b}{Q}

(iii) y = e^{a+bx}

(iv) S = a + b \sqrt{t}

3 (a) Y = Ae^{-0.0038t} denotes the process known as radioactive decay where A is the initial level of radioactivity and t is time in years. How long will it take 5g. of radium to reduce to 1g?

(b) Determine the angular velocity \omega, Amplitude A, Period T and frequency f of the function y = 5 \sin 2t Sketch the curve.

(c) A triangular component has the following dimensions

Find |\angle cab|
4 (a) Let \( f(x) = x^3 - 6x^2 + 9x - 10 \)
Determine the stationary points of \( f(x) \) and classify them, find also the inflection point.
Sketch the curve

(b) A particle moves in a straight line so that its distance \( S \) metres, travelled after \( t \) seconds is given by the formula \( S = t^3 - 4t^2 + 4t \)
Find (i) the speed and acceleration of the body after 3 seconds
(ii) find the value(s) of \( t \) when the object is at rest

5 (a) Given the complex numbers \( z_1 = 2 - 3i \)
\( z_2 = -1 + 4i \)
Find (i) \( z_1^2 \)
(ii) \( \frac{z_1}{z_2} \)
(iii) \( z_2 \) in polar form
(iv) \( |z_1 - z_2| \)

(b) Find all the cube roots of \(-1\) and show the roots graphically in the complex plane

6 (a) \( \vec{a} = i + 2j - k \)
\( \vec{b} = 2i + 3j + k \)
\( \vec{c} = -i + 3j - 2k \)
Find (i) \( \vec{a} - 2\vec{b} + 3\vec{c} \)
(ii) \( \vec{a} \times \vec{b} \)
(iii) Unit vector in the direction of \( \vec{a} \)
An object is experiencing a force $\vec{F}$ of 40N as in diagram. Express $\vec{F}$ in terms of components in the i and j directions.

7 (a) Solve the following system of linear equations using Gaussian elimination, or otherwise

\[
\begin{align*}
x + y + z &= 7 \\
x - y + 2z &= 9 \\
2x + y - z &= 1
\end{align*}
\]

(b) Verify by drawing a suitable graph that the values given in the table below satisfy a law of the form $Q = a + bp^2$

Determine the best values for $a$ and $b$ from the data, using the following

\[
b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} \quad \text{a} = \frac{\Sigma y}{n} - b\frac{\Sigma x}{n}
\]

(where $a$ and $b$ are the intercept and slope of the least square line
$y = a + bx$ plotting data $(x_i, y_i)$ )

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INSTRUCTIONS TO CANDIDATES

Question One (Q1) is compulsory and carries 40 marks.

Answer any other four questions worth 15 marks each.
Find the slope and hence the equation of the line passing through the points
(0, -1) and (2,4).

Solve the equation \(4 \ln (2x - 3) = 1\); \((\ln=\log_e)\)

Find the first derivative of the following functions

\[ y = 2x^3 + \ln 4x - 5 \]

\[ f(x) = (x^2 - 2x + 1)^4 \]

\[ g(x) = (4x - 1) \sin 2x \]

\[ f(t) = \frac{3t + 1}{t - 1} \]

Find the slope of the tangent to the curve \(y = x^3 - 9x\) at the
point \((1, -8)\)

Use De Moivre’s theorem to evaluate \((2 - 2i)^6\)

(i) Write \(e^{i\pi/6}\) in the form \(a + bi\)

(ii) Evaluate \(z_1z_2\) where \(z_1 = 1 + i\) and \(z_2 = 3 - 4i\)

Find the vector product \(\vec{a} \times \vec{b}\) where \(\vec{a} = i - 2j + k\), and \(\vec{b} = 4i + j - k\).

From the following augmented Matrix, state the solution to the following
linear system

\[
\begin{pmatrix}
2 & 1 & -1 & | & 1 \\
0 & 2 & -3 & | & 4 \\
0 & 0 & 1 & | & 2
\end{pmatrix}
\]

(i) Identify the variables you would plot in order to produce a straight line graph from

\[ s^2 = a + b\sqrt{t} \]

\[ P = a + \frac{b}{Q} \]
(iii) \( Y = \ln(a + bx) \)

MA4701 Technological Mathematics 1

Marks

2 (a) (i) Solve \( |4x + 1| < 9 \)

(ii) Write down the largest domain for which the following function is defined:
\( f(x) = \frac{3}{2x + 6} \)

(vi) \( f(x) = \sqrt{2x - 1} \), find the inverse function \( f^{-1}(x) \)

(vii) \( f(x) = 1 - 3x \), \( g(x) = x^2 - 1 \). Find the composite function \( f \circ g(x) \)

(viii) Determine whether the function \( f(x) = 2x^3 + x \) is odd, even or neither.

10

(b) (i) \( B = \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix} \) and \( C = \begin{pmatrix} -1 & 0 \\ 2 & 3 \end{pmatrix} \) Find \( BC \)

(ii) Write the simultaneous equations
\[ \begin{align*}
3x - 2y &= 10 \\
x + 2y &= 6
\end{align*} \]

in the form \( A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 6 \end{pmatrix} \) where \( A \) is a 2 x 2 matrix,

then find \( A^{-1} \) and use it to solve the equations for \( x \) and \( y \).
MA4701 Technological Mathematics 1

3 (a) The function \( y = 250e^{0.03t} \) represents the growth of €250 invested for \( t \) years at 3\% compound interest

Find (i) the amount present after 4 years,
(ii) how long it will take the investment to reach €400.

(d) Determine the amplitude \( A \), angular velocity \( \omega \), period \( T \) and frequency \( f \), of the function \( y = 3 \sin \pi t \).

Sketch the function.

(c) (i) Prove the identity \((1 - \cos A)(1 + \cos A) = \sin^2 A\)

(ii) A triangular component must be made using the dimensions shown in the diagram

\[
\begin{array}{c}
\text{a} \\
\text{b} \\
\text{c}
\end{array}
\]

\( \angle bac \)

Show that there are two possible solutions.

4 (a) Let \( f(x) = x^3 - 12x + 1 \)

Determine the stationary points of \( f(x) \) and classify them. Find also the inflection point.
Sketch the curve

(b) A ball is hit vertically upwards. The height, \( h \) in metres of the ball above the ground at any time \( t \) seconds is given by \( h(t) = 1 + 30t - 5t^2 \).

Find
(i) the rate at which the ball is rising after 2 seconds,
(ii) the time when the ball reaches its maximum height
(iii) the maximum height reached.

MA4701 Technological Mathematics 1
Marks

5 (a) Given \( z_1 = 2 + i \) and \( z_2 = 1 - 3i \)

Find

(i) \( 5z_1 - iz_2 \)  
(ii) \( \frac{1}{z_1} \) in the form \( a + bi \)  
(iv) \( z_1 - z_2 \)  
(iv) \( z_1 \) in polar form

(b) Express -16 in general polar form.
Find all the fourth roots of -16 (i.e. find \( (-16)^{\frac{1}{4}} \)).
Plot the roots in the complex plane.

6 (a) Given
\[
\begin{align*}
\vec{a} &= i + j - k \\
\vec{b} &= 2i - j + k \\
\vec{c} &= 3i + 2j - 2k
\end{align*}
\]

Find

(i) \( \vec{a} + \vec{b} - 2\vec{c} \)  
(ii) Show that \( \vec{a} \) and \( \vec{b} \) are perpendicular to each other  
(iii) Express \( \vec{a}\vec{c} \) in terms of \( i \) and \( j \).

(b)

An object is experiencing a force \( \vec{F} \) of 80N as in diagram. Express \( \vec{F} \) in terms of the components in the \( i \) and \( j \) directions.
7 (a) Solve the following system of linear equations using Gaussian elimination, or otherwise

\[
\begin{align*}
    x + y + z &= 2 \\
    3x - y + 2z &= 4 \\
    2x + 3y + z &= 7
\end{align*}
\]

8

(b) Verify by drawing a suitable graph that the values of P and T given in the table below satisfy a law of the form \( P^2 = a + bT^2 \)

Determine the best values for a and b using the following

\[
b = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}, \quad a = \frac{\Sigma y}{n} - \frac{b\Sigma x}{n}
\]

7

where a and b are the intercept and slope of the least square line \( y = a + bx \).

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</table>
INSTRUCTIONS TO CANDIDATES

Question One (Q1) is compulsory and carries 40 marks.

Answer any other four questions worth 15 marks each.
1 (a) The equation of the line $L$ is $2x - y + 4 = 0$. $L$ intersects the $x$ axis at $p$ and the $y$ axis at $q$.

Find (i) the slope of $L$.

(ii) the coordinates of $p$ and $q$ and sketch the line.

(b) Solve the equation $2\log_e(4x - 1) = 6$. ($\log_e = \log_{10}$)

(c) Find the first derivative of the following functions

(ix) $y = 1 + \frac{4}{x^2} + 6x$

(x) $f(x) = (3x^2 + 4x - 1)^3$

(xi) $g(x) = (5x + 1)\ln 3x$

(xii) $f(t) = \frac{1}{5t + 2}$.

(d) Find the slope and hence the equation of the tangent to the curve $y = x + e^{2x}$ at the point $(0,1)$.

(e) Write in the form $a + bi$

(i) $\frac{2i}{3 - i}$

(ii) $e^{1 + i \pi/6}$.

(f) Use De Moivre’s theorem to evaluate $(-1 - i)^6$.

(g) Show that the vectors $\vec{a} = i + 2j + 3k$ and $\vec{b} = 2i - 4j + 2k$ are perpendicular to each other.

(h) Find (i) $AB$ where $A = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$.

(ii) the value of $k \epsilon R$ if $\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 11 \\ 4 \end{pmatrix}$. 

-3-
From the following augmented matrix, state the solution to the linear system:

\[
\begin{bmatrix}
1 & 3 & -1 & | & 6 \\
0 & 1 & 2 & | & 1 \\
0 & 0 & 1 & | & 2
\end{bmatrix}
\]

2 (a) (i) Solve \(|2x - 5| \leq 1\).

(ii) \(f(x) = 3x + 1\) find the composite function \(f \circ f(x)\).

(ix) write down the largest domain for which the following function is defined:

\[f(x) = \frac{1}{\sqrt{x - 4}}\,.
\]

(x) Find \(f^{-1}(x)\) the inverse of the function \(f(x) = x^3 + 4\).

(xi) Identify the variables you would plot in order to produce a straight line graph from

(i) \(s = a + b \sqrt{t}\)

(ii) \(y = ax^2 + bx\).

(b) \(A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\)

(i) Find \(\det(A)\).

(ii) Find \(A^{-1}\), the inverse of \(A\).

(iii) Use matrix methods to solve the simultaneous equations

\[
\begin{align*}
3x + 2y &= 2 \\
2x + 3y &= 8
\end{align*}
\]
MA4701 Technological Mathematics 1

Marks

(a) \( y = 600e^{0.08t} \) denotes the size of a colony of bacteria after \( t \) hours.

(i) Find the initial size of the colony.

(ii) How many hours does it take the population to reach 1,000?

(e) Determine the amplitude \( A \), angular velocity \( \omega \), period \( T \) and frequency \( f \) of the function \( y = 2\sin 6t \).

(ii) A triangular component must be made using the dimensions as shown in the diagram.

\[
\begin{align*}
8 & \quad 5 \\
100^\circ & \\
\end{align*}
\]

Find \( |bc| \).

4

(a) Let \( f(x) = x^3 + 3x^2 - 2 \)
Determine the stationary points of \( f(x) \) and classify them.
Find also the inflection point.

Sketch the curve.

(b) (i) \( A = xy \) and \( 2x + y = 12 \), find the maximum value of \( A \) and the values of \( x \) and \( y \) which give this maximum.

(ii) The distance \( s \) metres travelled by a train in \( t \) seconds is given by \( s(t) = 3t^3 - 2t^2 + 4t - 1 \). Find the velocity of the train after 2 seconds.
MA4701 Technological Mathematics 1

**Marks**

5 (a)  Given \( z_1 = 5 + 2i \) and \( z_2 = 1 - 2i \)

Find

(i) \( z_1 z_2 \)

(ii) \( iz_1 + 3z_2 \)

(v) \( z_1 \) in polar form

(iv) \( |z_1 - z| \)

8

(b)  Express \(-1 + i\sqrt{3}\) in general polar form.

Find the four roots of \((-1 + i\sqrt{3})^{1/4}\).

Plot the roots in the complex plane.

7

6 (a)  Given

\( \bar{a} = i - 2j + 2k \)

\( \bar{b} = 2i + j + k \)

\( \bar{c} = 4i + 3j + k \)

Find

(i) \( 3\bar{a} - 2\bar{b} + \bar{c} \).

(ii) the unit vector in the direction of \( \bar{a} \).

(iii) the vector product \( \bar{a} \times \bar{b} \).

11

(b)

An object is experiencing a force \( \bar{F} \) of 65 N as in the diagram.

Express \( \bar{F} \) in terms of the components in the \( i \) and \( j \) directions.

4
Solve the following system of linear equations using Gaussian elimination, or otherwise

\[
\begin{align*}
&x - y - z = 0 \\
&2x - y + 3z = 24 \\
&3x + y + z = 16
\end{align*}
\]

Verify by drawing a suitable graph that the values of \( P \) and \( T \) given in the table below satisfy a law of the form \( P = a + bT^3 \).

Determine the best values for \( a \) and \( b \) using the following

\[
\begin{align*}
b & = \frac{n \Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2} \\
a & = \frac{\Sigma y - b \Sigma x}{n}
\end{align*}
\]

where \( a \) and \( b \) are the intercept and slope of the least squares line \( y = a + bx \).

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UNIVERSITY of LIMERICK
OLLSCOIL LUIMNIGH

FACULTY OF SCIENCE AND ENGINEERING
DEPARTMENT OF MATHEMATICS & STATISTICS

END OF SEMESTER ASSESSMENT PAPER

MODULE CODE: MA4701  SEMESTER: Autumn 2008/09

MODULE TITLE: Technological Mathematics 1  DURATION OF EXAM: 2.5 hours

LECTURER: Mr J. O’Shea  GRADING SCHEME: Examination: 85%

EXTERNAL EXAMINER:

INSTRUCTIONS TO CANDIDATES

Question One (Q1) is compulsory and carries 40 marks.
Answer any other four questions worth 15 marks each.
MA4701 Technological Mathematics 1

1 (a) Find the equation of the line passing through the point (1, -2) and which is perpendicular to the line $x - 2y + 4 = 0$.  

(b) Solve the equation $2 + 4\ln(x - 1) = 3$. ($\ln$=log$_e$)  

(c) Find the first derivative of the following functions

(i) $y = 3x^3 + 2\ln x - 1$
(ii) $f(x) = (x^3 - 2x + 1)^3$
(iii) $g(x) = (3x + 1)e^{4x^2}$
(iv) $f(t) = \frac{2t - 3}{4t + 1}$.  

(d) Find the equation of the tangent to the curve $y = \ln x + x - 2$ at the point (1,-1).  

(e) Use De Moivre’s theorem to evaluate $(\sqrt{3} + i)^{10}$.  

(f) Express in the form $a + bi$

(i) $\frac{4 - i}{1 + 2i}$
(ii) $e^{\pi i/3}$.  

(g) $\vec{a} = i + 2j - k$ find the unit vector in the direction of $\vec{a}$.  

(h) $A = \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$.

(i) Evaluate $AB$.

(ii) Find the value of $k \in \mathbb{R}$ if $\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} \frac{2}{2} \\ \frac{2}{2} \end{pmatrix} = k\begin{pmatrix} \frac{2}{2} \end{pmatrix}$.  

Marks

4

4

4

4

8

4

4

4
(i) From the following augmented matrix, state the solution to the linear system:

\[
\begin{pmatrix}
1 & 5 & 3 & | & 1 \\
0 & 1 & 2 & | & 4 \\
0 & 0 & 1 & | & 3
\end{pmatrix}
\]

2 (a) (i) Solve \(|3x + 1| \leq 10\).

(ii) \(f(x) = \sqrt{4x + 1}\), find \(f^{-1}(x)\) the inverse of the function.

(iii) \(f(x) = \cos x\) and \(g(x) = \sqrt{1 - x^2}\), find the composite function \(gof(x)\) and simplify answer.

(iv) Sketch the line \(5x + y = 0\) on a co-ordinate diagram.

(v) Identify the variables you would plot in order to produce a straight line graph from

\[
\begin{align*}
(iii) & \quad P^2 = a + b \quad Q^2 \\
(iv) & \quad s = a + \frac{b}{t}
\end{align*}
\]

(b) (i) \(A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\) and \(I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\)

Verify that \(A^2 + I = 2A\). \((A^2 = AA)\)

(ii) \(B = \begin{pmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \\ 3 & 4 & 2 \end{pmatrix}\)

find \(\det(B)\).
The function \( y = 200e^{-0.002t} \) denotes the process known as radioactive decay where 200 grams is the initial level of radium, and \( t \) is the time in years.

(i) What is the level of radium after 100 years?

(ii) How long will it take for the radium to reduce to 100 grams?

Find the amplitude \( A \), angular velocity \( \omega \), period \( T \) and frequency \( f \) of the function \( y = 3 \cos 4\pi t \).

Sketch the function.

A triangular component must be made using the dimensions as shown in the diagram.

Find \( \angle bac \).

MA4701 Technological Mathematics 1

4 (a) Let \( f(x) = x^3 + 3x^2 - 9x + 2 \)

Determine the stationary points of \( f(x) \) and classify them.

Find also the inflection point.

Sketch the curve.

(b) The distance \( s \) metres travelled by an object in \( t \) seconds is given by

\[ s = 2t^3 - 15t^2 + 40t. \]

Find (i) the velocity of the object after 1 second.

(ii) the times at which the velocity is 4 metres per second.

5 (a) Given \( z_1 = 4 + 3i \) and \( z_2 = 3 - 2i \)

Find (i) \( z_1 - iz_2 \).

(ii) \( \frac{z_1 + z_2}{iz} \)

(vi) The polar form of \( z_1 \).

(vii) \( k \in \mathbb{R} \) such that \( |k + 8i| = 10 \).

(b) Express -27i in general polar form.

Find the 3 roots of \((-27i)^{1/3}\).

Plot the roots in the complex plane.
(a) Given
\[ \vec{a} = 2i + j - k \]
\[ \vec{b} = 3i + 2j + 3k \]
Find
(i) \[ 2\vec{a} + \vec{b} \].
(ii) \( \vec{ab} \) in terms of \( i \) and \( j \).
(iii) the scalar product \( \vec{a} \cdot \vec{b} \).
(iv) the vector product \( \vec{a} \times \vec{b} \).

(b)

An object is experiencing two perpendicular forces \( \vec{F}_1 = 50 \text{N} \) and \( \vec{F}_2 = 20 \text{N} \) as in the diagram above. Calculate the magnitude of the resultant force \( \vec{F} \) and determine the angle \( \alpha \) that \( \vec{F} \) makes with the vertical.
7 (a) Solve the following system of linear equations using Gaussian elimination, or otherwise

\[
\begin{align*}
    x + 2y + z &= 1 \\
    2x + 3y + 3z &= 4 \\
    4x + y - 2z &= 5
\end{align*}
\]

(b) Verify by drawing a suitable graph that the values of \( P \) and \( Q \) given in the table below satisfy a law of the form \( P = a + bQ^2 \).

Determine the best values for \( a \) and \( b \) using the following

\[
b = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} \quad \text{and} \quad a = \frac{\Sigma y}{n} - \frac{b \Sigma x}{n}
\]

where \( a \) and \( b \) are the intercept and slope of the least squares line \( y = a + bx \).

<table>
<thead>
<tr>
<th>( Q )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>14</td>
</tr>
</tbody>
</table>
1. **The Layout of the Paper:**

The Technology mathematics examination consists of 7 questions in total. The topics covered in each question are detailed in this section and are extremely consistent between 1998 and 2008 (see Appendix L).

**Question 1:** consists of 9 parts (a-i) covering a variety of topics.

(a) **Co-ordinate Geometry:** This question requires you to either find the distance between 2 points or the equation of a line.
(b) **Algebra:** Solving an equation involving logs.
(c) **Differentiation:**
   - Basic Differentiation.
   - Chain Rule.
   - Product Rule.
   - Quotient Rule.
(d) **Differentiation:** Find the slope/equation of the tangent to a curve.
(e) and (f) **Complex Numbers:**
   - Applying De Moivre’s theorem.
   - Writing Complex Numbers in the form a +bi.
(g) **Vectors:** Finding the vector/scalar product of 2 vectors or finding the angle between two vectors.
(h) and (i) **Matrices:**
   - Finding the solution to the linear system of an augmented matrix if it exists.

**Question 2: Functions**

(a) The following concepts are tested within part (a)
   - Domain and range of a function.
   - Odd and even functions.
   - Inverse functions.
   - Composite functions.
   - Sketching/Graphing a function.
   - Solving Inequalities.
(b) Identify the variables you would use to plot a straight line graph (1998-2004).
Question 3: *Variety of Concepts*

(a) This question involves a function which denotes exponential growth/decay and related questions.
(b) Determine the angular velocity, amplitude, period and frequency of a trigonometric function and sketch the function.
(c) Calculate the missing sides/angles of a triangle using the cosine rule/sine rule.

**Question 4: Applications of Differentiation**

(a) Determine and classify stationary points of a function and sketch the curve.
(b) Finding velocity/ acceleration at a given time when given the distance.

**Question 5: Complex Numbers**

(a) This part of question 5 requires the candidate to carry out various operations with complex numbers such as the addition/multiplication of complex numbers and to express a complex number in polar form.
(b) Writing complex number in general polar form and plotting roots in the complex plane.

**Question 6: Vectors**

(a) This part of question 6 requires the candidate to carry out operations with vectors such as the addition/multiplication of vectors as well as finding scalar products and unit vectors.
(b) This question requires the candidate to calculate the magnitude of a resultant force.

**Question 7: System of Linear Equations and Least Square Lines**

(a) Solving systems of linear equations using Gaussian elimination.
(b) Sketching least squares lines.

As previously mentioned the topics covered in the 7 questions on the Technology mathematics examination are extremely predictable and vary very little from year to year. This is further evidenced by the sample questions detailed from 4 separate years of examination papers in table 1L which follows.

<table>
<thead>
<tr>
<th>Year</th>
<th>1998</th>
<th>2001</th>
<th>2004</th>
<th>2008</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Question 1(a)</strong></td>
<td>Find the distance between the points $(-1,6)$ and $(3,-2)$ and find the equation of the line passing through these points.</td>
<td>Find the equation of the line passing through the point $(2,-2)$ which is parallel to the line $4x + 3y = 5$.</td>
<td>Find the equation of the line passing through the point $(-1,4)$ and which is perpendicular to the line $3x - 2y + 6 = 0$.</td>
<td>Find the equation of the line passing through the point $(1,-2)$ and which is perpendicular to the line $x - 2y + 4 = 0$.</td>
</tr>
<tr>
<td><strong>Question 1 (d)</strong></td>
<td>Find the equation of the tangent to the curve $y = -x^2 + 3$ at $(1,2)$</td>
<td>Find the equation of the tangent to the curve $y = 2x^3 - 4$ at $(1,3)$.</td>
<td>Find the slope of the tangent to the curve $y = 5x^3 + 3\sin x + 4$ at the point $(0,4)$.</td>
<td>Find the equation of the tangent to the curve $y = \ln x + x - 2$ at the point $(1,-1)$.</td>
</tr>
</tbody>
</table>
### Question 3(a)

Given that \( P(t) = 1000e^{0.037t} \) denotes the size of a colony of bacteria after \( t \) hours.

(i) Find the initial size of the colony.

(ii) Find its size after 2 days.

(iii) Sketch the graph of \( P(t) \).

### Question 5 (a)

#### Given the complex numbers

\( z_1 = 2 - 3i \) and \( z_2 = 5 + i \)

Find

(i) \( \bar{z}_1z_2 \)

(ii) \( z_2 \) in polar form

(iii) \( |z_1 - z_2| \)

(iv) \( \frac{z_1}{z_2} \)

#### Given the complex numbers

\( z_1 = 3 + i \) and \( z_2 = -2 + 4i \)

Find

(i) \( |z_2 - z_1| \)

(ii) \( \frac{z_1}{|z_2 + z_1|} \)

(iii) \( \frac{z_2}{z_1} \)

(iv) \( \frac{z_1}{z_2} \)

#### Given \( z_1 = 2 + i \) and \( z_2 = 1 - 2i \)

Find

(i) \( 4z_1 + iz_2 \)

(ii) \( |z_1 + z_2| \)

### Question 3(b)

- Find the cube roots of \( z \) where \( z = 5 + i \). Plot these cube roots on an Argand diagram.
- Find the 3 roots of \((-27i)^{\frac{1}{3}}\). Plot the roots in the complex plane.

### Table II

Sample Examination Questions from 4 Technology Mathematics past papers.
2. Instructions to Candidates

The instructions to candidates have not changed between 1998 and 2008. Candidates are required to answer question 1, which is compulsory and carries 40 marks, and to answer any other 4 questions out of a possible 6 others which all carry 15 marks each. These instructions combined with the fact that all of the same topics appear in the same questions every year allows the candidates, if desired, to pick which questions/topics they will cover and which they will leave out.

3. Procedural Questions Lending Themselves to Rote Learning: An Examination

As the Technology mathematics papers between 1998 and 2008 are so similar (see Appendix x) any of the examinations papers within this period would be equally successful in demonstrating the procedural nature of the questions asked in the examination. The author examined the questions on the 2008 Technology mathematics examination in detail.

Question 1, as previously highlighted, consists of 9 parts (a-i) with the same topics being examined within this question each year. If a candidate wanted to successfully complete part (a) of question 1 in 2008 they could have memorised the relationship between the slopes of perpendicular/parallel lines along with the formula for the equation of a line. They could then substitute in the slope and the point to the equation of a line formula to successfully complete the question (Question 1(a) : Find the equation of the line passing through the point (1, -2) and which is perpendicular to the line x -2y + 4 = 0).

Memorisation of the relationship between logs and indices and practice in the procedures needed to solve an equation for an unknown variable would successfully see the candidate through part (b) of question 1 (Question 1(b): Solve the equation 2 + 4ln (x - 1) = 3). In order to successfully answer part (c) the candidate had to be able to carry out basic differentiation as well as be able to implement the chain, product and quotient rule. One could be quite confident as to what order they would even have to carry these differentiation procedures out in as this does not vary much from year to year (Question 1(c) : Find the first derivative of the following functions (i) $y = 3x^3 + 2\ln x -1$ (ii) $f(x) = (x^3 - 2x + 1)^3$).
(iii) \( g(x) = (3x + 1)e^{4x^2} \) \( f(t) = \frac{2t-3}{4t+1} \). Part (d) of question 1 required the candidate to know that to find the equation of the tangent to a curve at a particular point they must first differentiate the curve and substitute in the appropriate x value to get the slope and then use the equation of a line formula to find the equation of the tangent (Question 1(d): Find the equation of the tangent to the curve: \( y = \ln x + x^2 - 2 \) at the point \( (1,-1) \)). This pattern repeats itself throughout question 1 with each question, in the vast majority of cases, only changing the figures in the question from year to year (see table IL). No conceptual knowledge in which logical relationships between concepts needed to be demonstrated within this question. The candidate simply needed to familiarise themselves with the appropriate rules and procedures used to carry out routine mathematical tasks and they could be quite confident that the same routines and procedures would appear in the same questions each year. Although successfully completing question 1 may have demonstrated a certain level of mathematical fluency which is important it does not demonstrate understanding of mathematics in a real way (Ahlfors 1962). As question 1 carries 40 marks a strong ability to carry out the appropriate procedures correctly in this question could almost ensure the candidate of a pass mark in the examination.

Question 3 of the 2008 paper also gave a strong picture of how the Technology mathematics examination demonstrates its suitability for candidates who have engaged in rote learning. Part (a) has consistently given a function which denotes some form of exponential growth or decay and once the candidate can establish which information they have been given and which unknown variable they are trying to solve for they can obtain full marks in this part of the question without having any understanding of what they are really doing (Question 3(a): The function \( y = 200e^{-0.002t} \) denotes the process known as radioactive decay, where 200 grams is the initial level of radium, and \( t \) is the time in years. (i) What is the level of radium after 100 years? (ii) How long will it take for the radium to reduce to 100 grams?). Question 3 part (b) again lends itself to rote learning. In 2008 candidates were asked to sketch the function \( y = 3\cos \pi t \), in 2007 the function to be sketched was \( y = 2\sin 6t \) and in 1998 it was \( y = 3\sin 4t \). Once the student knows which ‘part’ of the function refers to the amplitude, the angular velocity, the period and the frequency it is just a matter of practicing sketching different cosine and sine functions (as no other trigonometric functions have ever been examined) to prepare you sufficiently for this question. Finally part (c) of this question requires students to apply the cosine
rule to solve for unknown elements of a given triangle. The candidate had to be familiar with the cosine rule and once they were clear on how to label the different elements of the triangle in question in terms of the cosine rule they then just needed to carry out procedural skills and manipulation to solve for the unknown angle in the case of the 2008 paper. In 2007 students were required to solve an unknown side of a triangle while in 2001 and 1998 candidates were required to solve for an unknown side and angle of a particular triangle. This question could be guaranteed full marks for students if they could successfully complete several years of past examination papers and memorise the procedures needed.

Although all questions lend themselves to rote learning, the final example question which the author will discuss is question 7 as this has not changed at all within the ten years being examined (with the exception of the 1998 examination in which the (b) part of question 7 involved matrices). Part (a) of this question gives the candidates 3 linear equations and requires them to solve them using Gaussian Elimination. This question is the exact same for the ten years of examination papers with different figures used in the linear equations. Once the candidate learns the procedures to complete this question they will have little trouble in completing it successfully and can be quite confident that it will appear as question 7 part (a) if the trend continues as it has done in the ten years being examined. Part (b) of question 7 does not require the candidate to understand anything about lines of best fit despite the fact that this concept is the basis of the question. Once the candidate remembers that they must plot the variable which the equation is equal to \( P = a + bQ \) \(^2\), in the case of the 2008 paper P, against whatever b is the coefficient of, i.e. \( Q^2 \) in this case, they will be able to successfully complete the first part of question 7 part (b). The second part of (b) requires the candidate to be familiar with the sigma (\( \Sigma \)) notation in order to create a table in which each of the elements of the given formulae for determining the best value for a and b are included. It is then up to the candidate to use basic arithmetic to solve for a and b.
Question 7, 2008 Technology mathematics examination:

(a) Solve the following system of linear equations using Gaussian elimination,
\[ \begin{align*}
    x + 2y + z &= 1 \\
    2x + 3y + 3z &= 4 \\
    4x + y - 2z &= 5
\end{align*} \]

(b) Verify by drawing a suitable graph that the values of \( P \) and \( Q \) given in the table below satisfy a law of the form \( P = a + bQ^2 \).

Determine the best values for \( a \) and \( b \) using the following:
\[
\begin{align*}
    b &= \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} \\
    a &= \frac{\Sigma y}{n} - \frac{b\Sigma x}{n}
\end{align*}
\]

where \( a \) and \( b \) are the intercept and slope of the least squares line \( y = a + bx \).

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The sample questions discussed from the 2008 Technology mathematics examination demonstrate the support which the Technology mathematics examinations papers may have had for rote learning. Despite the fact that the figures within the questions change from year to year, the sequence of questions, the instructions to candidates and concepts tested vary very little allowing the candidates to successfully pass the paper by relying on procedural learning alone if wished. Students are not required to demonstrate an understanding of the mathematical concepts but rather the focus is on carrying out routine mathematical tasks and the development of a familiarisation with the symbols used to represent the mathematics in question. The students can therefore learn procedural rules in the absence of a concept which is a discouraged practice in educational circles worldwide (Van De Walle 2007). Although there is place for procedural learning in mathematics, and it is a necessary element for real understanding in mathematics to take place, it should not take place in the absence of a student being able to apply mathematics and make connections to other areas/topics in mathematics (Hiebert 1992). This investigation offers some insight into the potential negative influence of the nature of the Technology mathematics examination on the success of the intervention carried out within this research.
Appendix M
UL Ethical Approval

16th July 2010

Dr. Olivia Gill
Manager
Mathematics Learning Centre
University of Limerick

RE: S&EO/47 The Implementation & Evaluation of Small Group, Problem Based Learning Approaches to Tutorial Support for a First Year Undergraduate Mathematics Module

Dear Olivia,

The Faculty of Science and Engineering Research Ethics Committee, at its meeting of 13th July 2010 approved the above application subject to the following:

• Some first year students may be under the age of 18 therefore a parental consent form will be required unless these students are omitted from the research.

Yours sincerely,

[Signature]

Dr. Séan Fair acting Chair Science & Engineering Research Ethics Committee

c.c. Fiona Faulkner, Ailish Hannigan
Appendix N
**Title of Project:** The Implementation & Evaluation Of Small Group, Problem Based Learning Approaches To Tutorial Support For A First Year Undergraduate Mathematics Module

**The Study:** this project involves the implementation of a small group, problem based tutorial strategy to enhance the mathematics education experience for you and your peers.

Students who participate are likely to benefit from problem based, active learning strategies which have proven to result in increased knowledge retention; the development of self directed learning skills and increased intrinsic interest in the subject matter. Students' experiences of mathematics and attitudes towards mathematics have been found to improve when they have been exposed to interesting content and good teaching.

**Participation Information:** You may participate in a tutorial group with approximately 40 or 50 of your peers should you wish to take part in the study.

Tutorials commence in week 3 of semester 1 and finish in week 12. Each tutorial session will last for no more than one hour.

You may agree or not to the following:

- Completion of a short journal at 3 stages throughout the semester.
There are no risks involved in this study. All information gathered will remain confidential and used only for the purpose of this study. No information re. subjects will be identified in the final report. It will be stored safely with access only available to the investigator.

You are under no obligation to participate in this study. Should you have any questions or do not understand something just ask the investigator to clarify the issue.

**Contact Details:**
Fiona Faulkner,
Doctoral Candidate,
Department of Mathematics & Statistics,
University of Limerick,
Limerick.
061-202512

**If you have concerns about this study and wish to contact someone independent, you may contact**

Dr. Olivia Gill,
Manager,
Mathematics Learning Centre,
University of Limerick
Tel: 061 202512.


Consent Form

Title of Project: The Implementation & Evaluation Of Small Group, Problem Based Learning Approaches To Tutorial Support For A First Year Undergraduate Mathematics Module.

You are under **no** obligation to participate in this study. If you agree to participate, but at a later stage feel the need to withdraw, you are free to do so. It will not affect you in any way.

**Please answer all of the following (tick the appropriate box):**

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have read and understood the subject information sheet.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I understand what the project is about, and what the results will be used for.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I am fully aware of all of the procedures involved and any risks and benefits associated with the study.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I know that my participation is voluntary and that I can withdraw from the project at any stage without giving any reason.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I am aware that my results will be kept confidential.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

I agree to participate in the above study

__________________________  __________
Signature of Participant     Date

__________________________  __________
Signature of Investigator    Date

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**Author’s Academic Publications and Presentations**

1. **Conferences Attended**

   - September 2009 MEI Conference in St Patricks College Dromcondra (Peer reviewed).
   - May 2010 MSSTL10 Conference in Carlow Institute of Technology (Presentation only).
   - August 2010 ECER Conference Helsinki, Finland. (Peer reviewed).
   - Feb 2011 CERME 7 Conference Poland (Peer reviewed).

2. **Journal papers**


3. **Published Conference Proceedings**


Appendix P
Discriminant Analysis with Leaving Certificate Mathematics Points as Only Independent Variable

A discriminant function was created from the Technology 2006-2008 dataset using Leaving Certificate mathematics points as the only predictor variable. The resulting discriminant function, which has been named discriminant function 2, is as follows:

\[ Z = 0.092 \text{ (Leaving Certificate mathematics points)} \text{ where } C = 4.667. \]

Just like the discriminant function containing diagnostic test results as the only predictor variable, the discriminant function 2 reduces to a simple rule. The discriminant value is found by multiplying a student’s Leaving Certificate mathematics points by 0.092 and determining whether the resultant value is above or below the cut off point of 4.667. If a student’s value falls above this dividing point, they are predicted to be successful and if it is below they are predicted to be unsuccessful in service mathematics. Essentially if a student receives more than 50 points in their Leaving Certificate mathematics examination, they are predicted to be successful in service mathematics. The discriminant function 2 however also provides each student with a probability of failure in addition to providing a binary prediction of success or failure which is all that can be achieved by using a cut off in Leaving Certificate mathematics points only. It also provides an evidence based cut off point for failure in service mathematics. This distinct advantage which the discriminant function has over the binary prediction method which could be carried out using Leaving Certificate grades only provides an argument for its usefulness in service mathematics in years to come.

The discriminant function 2 performed well in its classification of successful and unsuccessful students in the training sample. It correctly classified 57.0% of successful students, 89.0% of unsuccessful students resulting in an overall correct classification of 64.2% of students (see table 1P).
The discriminant function 2 outperformed the original Technology 2006-2008 discriminant function in its prediction of the training sample. The Technology 2006-2008 discriminant function correctly classified 84.4% of failure students while the discriminant function 2 correctly classified 89.0% of failure students. The new function also performed equally as well as the Technology 2006-2008 discriminant function in its correct classification of validation datasets. The discriminant function 2 outperformed the Technology 2006-2008 function in its correct classification of unsuccessful students within the Technology 2009 dataset (see table 2P).

Table 1P Discriminant function 2’s ability to correctly classify students within the training sample.

<table>
<thead>
<tr>
<th>Actual Performance</th>
<th>Predicted Success</th>
<th>Predicted Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success</td>
<td>433 (57.0%)</td>
<td>327 (43.0%)</td>
<td>760</td>
</tr>
<tr>
<td>Failure</td>
<td>24 (11.0%)</td>
<td>195 (89.0%)</td>
<td>219</td>
</tr>
</tbody>
</table>

Table 2P Discriminant function 2 ability to predict Tech 2009 dataset.

<table>
<thead>
<tr>
<th>Technology mathematics performance</th>
<th>Predicted Success</th>
<th>Predicted Failure</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success</td>
<td>115 52.0%</td>
<td>106 48.0%</td>
<td>221</td>
</tr>
<tr>
<td>Failure</td>
<td>8 13.1%</td>
<td>53 86.9%</td>
<td>61</td>
</tr>
</tbody>
</table>
Bibliography


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