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Modeling of semiconductor optical amplifier RIN and phase noise for optical PSK systems

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Abstract Phase modulation schemes are attracting much interest for use in ultra-fast optical communication systems because they are much less sensitive to fibre nonlinearities compared to conventional intensity modulation formats. Semiconductor optical amplifiers (SOAs) can be used to amplify and process phase modulated signals, but with a consequent addition of nonlinear phase noise (NLPN). Existing SOA NLPN models are simplistic. In this paper we show that a more accurate model can be used, which results in simple expressions for SOA nonlinear noise, in particular when used to amplify differential phase shift keyed modulated data. The model is used to calculate the optical signal to noise ratio introduced by a power booster SOA and the first inline amplifier of a 40 Gb/s NRZ-DQPSK single channel link.

Keywords Modelling · Semiconductor optical amplifier (SOA) · Optical noise · PSK

1 Introduction

Constant envelope modulation formats, in particular RZ- and NRZ-DPSK, are among the most promising candidates for SOA-based high bit rate systems because of their resilience to fiber non-linearities and pattern effects (Gnauck and Winzer 2005). Gain saturation in SOAs introduces NLPN that can be detrimental in PSK systems. The phase noise behaviour of saturated SOAs in DPSK systems has been analysed in several papers (Wei and Zhang 2005 and its references). The advantage of these NLPN models is that they are analytical and easy to apply, however they have limited accuracy, in that they do not consider the internal noise generated by the SOA or properly account for scattering losses. In this paper we show that these assumptions are not always correct. We also show that an existing computationally

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simple and more accurate noise model can be used that leads to simple NLPN expressions at the SOA for constant envelope modulation schemes (Shtaif et al. 1998). The model is used to calculate the optical signal to noise ratio of a power booster SOA and the first inline amplifier of a 40 Gb/s NRZ-DQPSK single channel link.

2 Theory

The propagation of the slowly varying envelope of the optical field $E(z,t)$ in an SOA, and the gain coefficient g rate equation are given by (Shtaif et al. 1998),

$$\frac{\partial E(z,t)}{\partial z} = \frac{1}{2}\left[g(1-j\alpha)-\gamma\right]E + f(z,t) \tag{1}$$

$$\frac{\partial g(z,t)}{\partial t} = \frac{g_0 - g}{\tau} - \frac{g\,|E|^2}{\tau} + F_g(z,t) \tag{2}$$

where α is the linewidth enhancement factor, γ the loss coefficient, τ the carrier lifetime and g_0 the unsaturated gain coefficient. g is taken to be a linear function of the carrier density (n), $g = a\,(n - n_t)$ where a is the differential gain coefficient and n_t the transparency carrier density. $f(z,t)$ and $F_g(z,t)$are Langevin (white) noise sources, which account for field fluctuations due to spontaneous emission, carrier noise and a term arising from their interaction. t is the time frame local to the propagating field and is equal to $t_r - z/v_g$ where t_r is the true time and z position. If it is assumed that the noise causes small variations in the optical field then

$$\begin{aligned} E = E_s\left(1+\frac{\delta E}{E_s}\right) &= (\rho_s + \delta\rho)\,e^{j(\phi_s+\delta\phi)} \\ &\approx E_s\left(1+\frac{\delta\rho}{\rho_s}+j\delta\phi\right) \end{aligned} \tag{3}$$

E_s, $\rho_s(z)$ and ϕ_s are the optical field, its amplitude (normalized to be equal to the square root of the optical power divided by SOA saturation energy) and phase in the absence of noise. $\delta\rho(z)$ and $\delta\phi$ are the amplitude and phase noise. Inserting (3) into (1–2) and taking Fourier transforms, the equations for the SOA relative amplitude noise $\delta\rho(z)/\rho_S(z)$ and phase noise as a function of z and frequency ω can be found (Shtaif et al. 1998).

$$\frac{\partial}{\partial z}\left(\frac{\delta\rho(z)}{\rho_S(z)}\right) = -\frac{g_s(z)\rho_S^2(z)}{1+\rho_S^2(z)+i\omega\tau}\left(\frac{\partial\rho(z)}{\rho_S(z)}\right) + N_\rho(z,\omega) \tag{4}$$

$$\frac{\partial\delta\phi}{\partial z} = \frac{\alpha g_s(z)\rho_S^2(z)}{1+\rho_S^2(z)+i\omega\tau}\left(\frac{\partial\rho(z)}{\rho_S(z)}\right) + N_\phi(z,\omega) \tag{5}$$

where

$$N_\rho(z,\omega) = \frac{1}{2}\left[\frac{\tau}{1+\rho_s^2(z)+j\omega\tau}F_g(\omega,z) + \frac{f(z,\omega)}{\rho_s(z)e^{j\phi_s(z)}} + \frac{f^*(z,-\omega)}{\rho_s(z)e^{-j\phi_s(z)}}\right] \tag{6}$$

$$N_\phi(z,\omega) = \frac{1}{2}\left\{\frac{-\alpha\tau}{1+\rho_s^2(z)+j\omega\tau}F_g(\omega,z) - j\left[\frac{f(z,\omega)}{\rho_s(z)e^{j\phi_s(z)}} - \frac{f^*(z,-\omega)}{\rho_s(z)e^{-j\phi_s(z)}}\right]\right\} \tag{7}$$

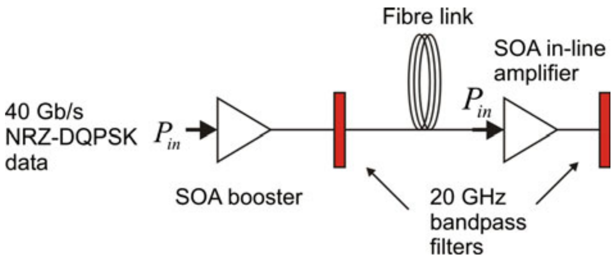

Fig. 1 40 Gb/s NRZ DQ-PSK link

$f(z,\omega)$ and $F_g(z,\omega)$ are Fourier transforms of $f(z,t)$ and $F_g(z,t)$, which have the correlation relationships

$$\begin{gathered}\left\langle f^*(z,t)f(z',t')\right\rangle = \frac{\hbar\omega_0}{P_{sat}} g_s n_{sp}\delta(z-z')\delta(t-t') \\ \left\langle f(z,t)f(z',t')\right\rangle = \left\langle f^*(z,t)f^*(z',t')\right\rangle = 0\end{gathered} \tag{8}$$

$$\begin{gathered}\left\langle F_g(z,t)f(z',t')\right\rangle = \frac{a}{\tau A}\left[\zeta g_0 + g_s + an_t(1+\zeta) + g_s\rho_s^2(2n_{sp}-1)\right]\delta(z-z')\delta(t-t') \\ \left\langle F_g(z,t)f(z',t')\right\rangle = -\frac{a\rho_s e^{j\phi_s} g_s n_{sp}}{A}\delta(z-z')\delta(t-t')\end{gathered} \tag{9}$$

(8) is due to additive spontaneous emission. (9) is due to current noise, non-radiative and radiative recombination processes. The inversion factor $n_{sp} = (g_s + an_t)/g_s$. The unperturbed distributions of $g_s(z)$ and $\rho_s(z)$ can be calculated by omitting the noise terms in (1–2), setting the time derivative in (2) to zero and using Runge–Kutta integration. These solutions can be inserted into (4–9), which can then be solved by numerical integration (Shtaif et al. 1998).

3 Numerical analysis

The geometrical and material parameters used in the model were determined for a 1 mm long tensile-strained SOA (Connelly 2007) with a 20 dB unsaturated gain, $\alpha = 2.5$, saturation power of 1.9 mW, $g_0 = 9,500\,\mathrm{m}^{-1}$ and $\gamma = 4,500\,\mathrm{m}^{-1}$. The model can be used to predict the relative intensity noise (RIN), which is equal to the square of the relative amplitude noise, and phase noise of a constant envelope phase modulated signal after amplification by a power booster SOA and subsequently by an identical an-line SOA at various levels of saturation. We consider the 40 Gb/s NRZ-DQPSK link shown in Fig. 1.

For the power booster SOA, it is assumed that the input signal has no noise. After first determining the unperturbed distributions then the noise equations can be solved to obtain the output signal RIN and phase noise, which are shown in Fig. 2 for an input signal with a normalized power=0.1. The dominant contribution to both the RIN and phase noise is the amplified spontaneous emission. Because the noise spectrum is not flat, its probability density function is not Gaussian as assumed in previous work (Ciaramella et al. 2008; Vocondio et al. 2010).

The phase noise variance σ_ϕ^2 can be determined by integrating the phase noise power spectrum over the signal bandwidth (for DQPSK the signal bandwidth is equal to half of the

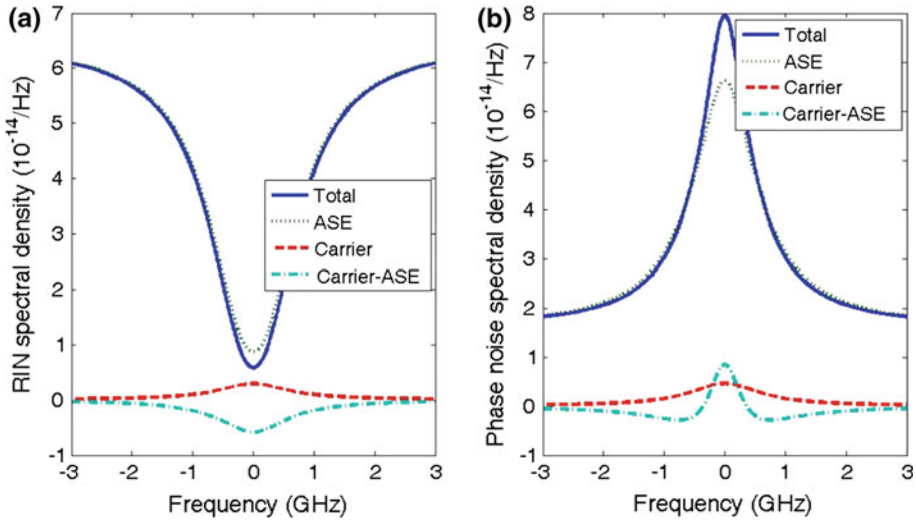

Fig. 2 SOA booster output, **a** total RIN and its components spectra, **b** total phase noise and its components spectra. The spectra are centered at the input optical frequency. The normalised input power is 0.1

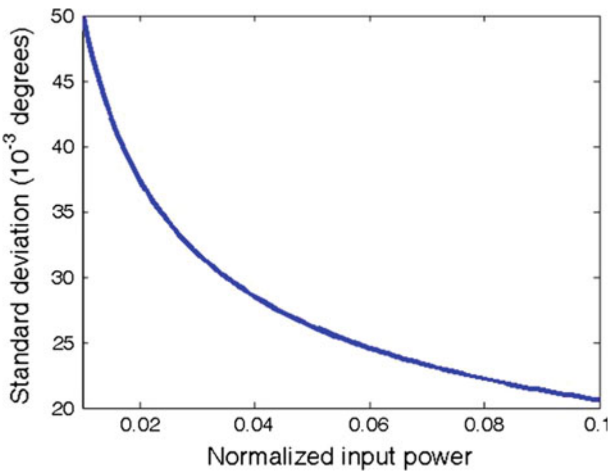

Fig. 3 SOA booster output phase noise SD versus normalised input power. The noise bandwidth is 20 GHz

bit rate). This is shown in Fig. 3 as a function of the normalised input power. Over this range of normalised power the gain decreases from 16 to 9 dB. The phase noise variance decreases with increase in the normalised power because the noise fluctuations are suppressed by the gain saturation. The probability density function of NRZ-DQPSK random data in the presence of NLPN has not yet been determined and so it is not presently possible to obtain an analytical expression for the bit-error-rate. To quantify the amplified signal noise, we use the differential phase Optical Signal-to-Noise Ratio $\mathrm{OSNR} = \pi^2/8\sigma_\phi^2$, assuming that phase noise is the dominant source of noise in the receiver (Eq. 32, Wei and Zhang 2005).

The output noise from the booster SOA can be used as the input signal noise to the in-line amplifier in the link. In the subsequent analysis we assume that the input power levels to

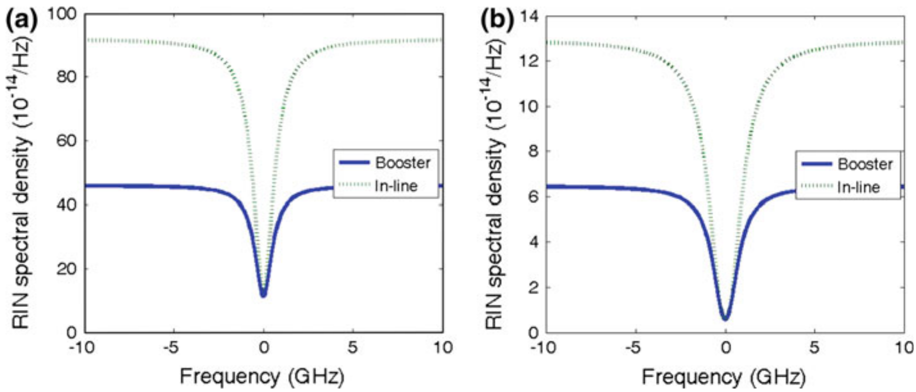

Fig. 4 Power booster and in-line SOA output RIN spectra. **a** Normalised power = 0.01. **b** Normalised power = 0.1

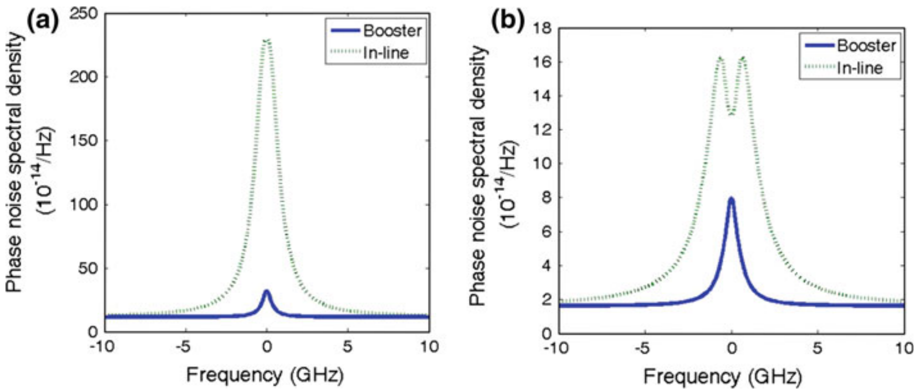

Fig. 5 Power booster and in-line SOA output phase noise spectra. **a** Normalised power = 0.01. **b** Normalised power = 0.1

both SOA are identical. Figures. 4 and 5 show a comparison between the noise spectra at the power booster and in-line SOA outputs. It is evident that the in-line amplifier noise is greatly enhanced due to the presence of the input noise, the most important component of which is the RIN. This is because the main contribution to the SOA additive RIN and phase noise are carrier density fluctuations due spontaneous emission and the input RIN. This noise source is not influenced by the input signal phase noise.

For high input powers to the in-line SOA the shape of its output phase noise is quite different to that of the power booster, in that it has a double-peak structure. The phase noise OSNR versus normalized input power is shown in Fig. 6, which shows that the OSNR increases with increasing saturation level. For normalized input powers greater than 0.02 the approximate OSNR power penalty of the in-line amplifier is 4 dB. The model can be used to determine the dependence of the OSNR on various SOA parameters. Figure 7 shows a typical dependency of the OSNR on the linewidth enhancement factor and the carrier lifetime. This shows that low linewidth enhancement factors are advantageous. This is to be expected since the linewidth enhancement factor is a measure of the coupling between the amplitude and phase noise. It is also advantageous to have short carrier lifetimes. Quantum-dot SOAs have been shown

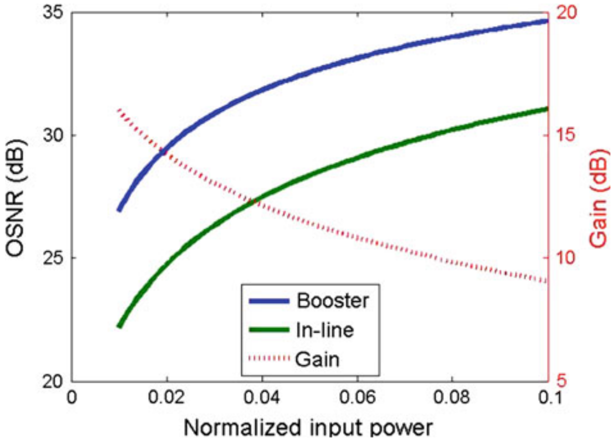

Fig. 6 OSNR versus normalized input power for the booster and in-line amplifiers

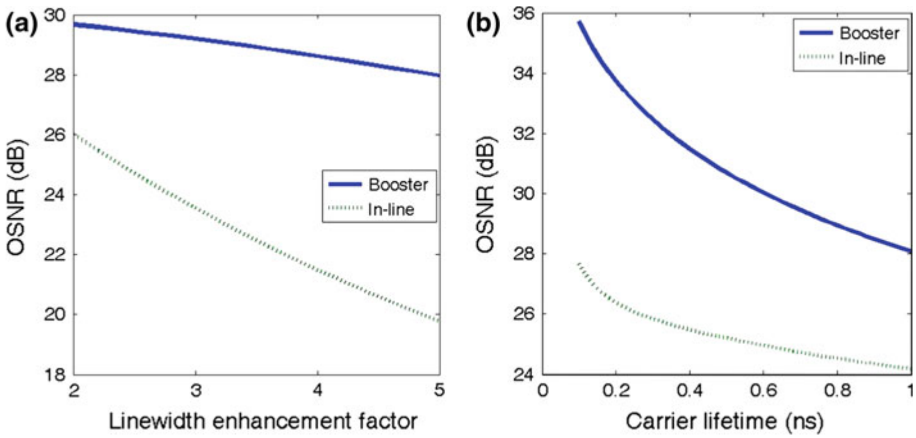

Fig. 7 OSNR versus **a** linewidth enhancement factor, and **b** carrier lifetime. The normalised input power is 0.02

to have applications in ultra-fast optical communication systems because of their resilient to pattern effects. In comparison to SOAs made from bulk or quantum-well material, QD-SOAs have very short carrier lifetimes, which as Fig. 7b shows is desirable for phase modulated systems.

4 Conclusions

Commonly used NLPN SOA models, for constant envelope DPSK systems, do not correctly describe its behavior. Internally generated noise must be taken into account and scattering losses have a strong influence on the OSNR degradation. We have demonstrated the use of an accurate model, which can be applied to constant envelope PSK systems and have shown how it can be used to calculate the OSNR degradation and its dependency on SOA parameters.

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