An Investigation into the Teaching and Learning of Probability at Senior Cycle

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For the award of Masters of Mathematics and Statistics

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Abstract

In Ireland at present, the National Council for Curriculum and Assessment is comprehensively overhauling the Second Level Mathematics curriculum. This reformed curriculum is known as Project Maths and is a response to concerns about how Irish students are taught and learn mathematics. These concerns are based around the achievement of Irish student’s in international studies (Close and Oldham 2005; Cosgrove, Shiel, Sofroniou, Zastrutzki and Shortt 2005; Perkins, Moran, Cosgrove and Shiel 2010; Oldham 2002, 2006) as well as domestic and international literature which, highlights the problems associated with the behaviourist methodology favoured by Irish teachers (Conway and Sloane 2006; English, O’Donoghue and Bajpai 1992; Lyons, Lynch, Close, Sheerin and Boland 2003; NCCA 2005). The aim of the study was to improve the teaching and learning of Probability through the development of a resource pack. Probability was chosen as the focus of the intervention due to the author’s experiences in the classroom, international literature highlighting its pedagogical difficulties (Shaughnessy 1992; Fischbein, Nello and Marino 1991; Ahlgren and Garfield 1988; Hawkins and Kapadia 1984) and its lack of popularity among Irish Leaving Certificate students (Chief Examiner 2000, 2005).

The study was designed to examine the benefits of the active learning methodologies and contextualised questions promoted by the Project Maths curriculum, specifically with regards to students’ attitudes and understanding through the implementation and evaluation of a resource pack designed by the author.

The evaluation process produced data, which was inconclusive in establishing a link between the promoted methodologies and students’ attitudes and understanding. The
only significant shift in students’ attitude was a negative one in response to the statement “I know I can do well in Maths”. A dip in students' confidence however is not unusual in studies involving changes in pedagogical style (Carpenter, Franke, Jacobs, Fennema and Epsom 1998; Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti and Perlwitz, 1991; Vershaffel and De Corte 1997; Fauzan et al. 2002; Van Reeuwijk 1992) and though not significant, student scores did improve in the three sub-categories of ‘Perception of Usefulness’, ‘Anxiety’ and ‘Effective Motivation’. There were also indications that these methodologies had a positive effect on understanding. The data also suggested that the resource pack, designed and developed by the author to support the teaching and learning of probability, will be of use to teachers who embrace Project Maths and what it is trying to achieve in Irish classrooms.
Declaration

This Thesis is presented in fulfilment of the requirements for the degree of Masters of Science. It is entirely my own work and has not been submitted to any other University or higher education institution, or for any other academic award in this University. Where use has been made of the work of other people it has been fully acknowledged and fully referenced.

Signature: __________________________

Conor Murphy

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Chapter 1 - Introduction

1.1 Introduction

“The needs and requirements of future citizens cannot be met by the traditional educational methodologies of the past”

(Dolgosa and Eliasa 1996, p.725)

This research study reviews the traditional educational methodologies employed in the classroom and looks at the methodologies, which are advocated by many to replace the status quo. With these methodologies in mind, the author designed, developed, implemented and evaluated a resource pack aimed at students of probability studying Higher Level mathematics at Senior Cycle in Ireland. The resource pack and the approaches adopted by it were heavily influenced by domestic and international mathematics education literature, which proposed the use of active learning methodologies, such as experiments and simulations in probability and the use of contextualised questions.

This chapter serves to describe the background against which mathematics education research is being undertaken in Ireland and provides a justification for this research study. The purpose of the research as well as its scope and significance are outlined. The research methodology is then discussed before an explanation of the Irish second level system and associated terms is given. Finally the author provides an overview of the thesis chapter by chapter.
1.2 Background

Mathematics education in Ireland is currently the focus of much attention not just among interested academics, but also in wider society. Media articles discussing mathematics education in Ireland are commonplace. The concern of the media has largely been fuelled by international comparative studies, which appear to show a relatively poor performance by Irish students as well as the low take-up of Higher Level mathematics and the high failure rate in Ordinary Level mathematics in state exams. The headlines in these newspaper and on-line articles paint a worrying picture, “Results spark new fears over standard of maths” (Irish Independent, 15/9/2010) “Alarm at Ireland’s falling maths performance in PISA study” (Silicon Republic, 7/12/2010) “Number of students taking Higher Maths hits record low” (Irish Times, 7/6/11).

The last quote was a headline from a front page article in the Irish Times, which revealed that only 16% of Leaving Certificate students were taking Higher Level mathematics in 2011 by far the lowest of any subject. It also stated that Irish teenagers’ performance in mathematics had fallen from 16th to 26th place in the international OCED rankings, the steepest decline of any of the participating countries and that Ireland was now ranked as below average in this comparative study. The rankings discussed are drawn up as of result Irish students’ performance in the PISA (Programme for International Student Assessment) tests of 2009. These test take place every three years and have been analysed and discussed in the literature (Close and Oldham 2005; Cosgrove, Shiel, Sofroniou, Zastrutzki and Shortt 2005; Perkins, Moran, Cosgrove and Shiel 2010; Oldham 2002, 2006). The statistics in the article
show that there is reason for concern in Irish mathematics education particularly at second level.

Inevitably, these concerns have led to questions about what is happening in Irish mathematics classrooms and how the situation can be improved. Academic studies have analysed the current situation in Irish schools and discussed possible new directions, which could help reverse the current trends in student performance in mathematics particularly at second level (Conway and Sloane 2005; Lyons, Lynch, Close, Sheerin and Boland 2003; NCCA 2005; Oldham 2006). These studies and the wider mathematics education literature are looked at in more detail in Chapter 2. Influenced by national as well as international research and perhaps media pressure the NCCA (National Council for Curriculum and Assessment) has developed and is currently implementing a new mathematics syllabus at second level, known as Project Maths.

“This change involves all aspects of mathematics education – its nature and content, the methods adopted in teaching and learning, how mathematics is taught, and the attitudes and perceptions associated with mathematics inside and outside the education system”

(NCCA 2007, p.13)
1.3 Justification of Research

As these issues regarding Irish mathematics education were being discussed in the media and by academics the author’s interest was sparked by personal experiences in the classroom specifically when teaching probability. The author received anecdotal feedback year after year that probability was not a popular topic among students. Some seemed to quickly achieve a reasonable grasp of the topic. However, most students, often those who excelled in other areas, struggled. On carrying out a little research it became clear that this was not a phenomenon strictly limited to the four walls of the author’s classroom. According to the last two Chief Examiners’ reports on Higher Level Leaving Certificate mathematics, the two probability and statistics questions were the least popular questions chosen on Paper 2. In 2000 (p.16) the chief examiner commented that “A drift away from discrete mathematics and statistics (questions 6 and 7)... has been noted in recent examinations” and though this had “stabilised” by 2005 (p.71) these questions were still the two least popular questions chosen on Paper 2. The extra probability and statistics question in the option section was chosen by less than 1 % of the examination candidates. Why did this topic, which has many every day applications, provoke such aversion in our Senior Cycle students?

The author was also struck by a line in the Chief Examiner’s report (2005, p.72) that noted that in general there was “a noticeable decline in the capacity of candidates to engage with problems that are not of a routine and well-rehearsed type”. The literature shows that due to its unique pedagogies, thinking and difficulties (Stohl 2005) “in no mathematical domain is blind faith in techniques more often denounced than in probability” (Freudenthal cited by Jones and Thornton 2005, p.84). As will be
outlined in Chapter 2 the current teaching approach in Ireland emphasises this “blind faith in techniques” approach. The author decided to examine if other pedagogical approaches, advocated within the mathematics education literature and by the Project Maths syllabus could lead to improvements in student understanding and attitude.

1.4 Purpose of Research

The author designed, developed and implemented a resource pack based on educational literature, which advocates the importance of attitudes and understanding and the use of active learning methodologies and contextualised questions in order to develop these. The aim of this research is to improve the teaching and learning of probability at Senior Cycle level in Ireland. The purpose of the author’s intervention is to assess whether

1. There is a link between the use of active learning methodologies and real life data and contexts and an improvement in students’ attitude towards mathematics.
2. The use of active learning methodologies and real life data and contexts develop students’ understanding of probability.

These are the study’s primary research questions and having outlined these the author would like to explain the significance of these questions within an Irish context.
1.5 Scope and Significance of the Research

“Mathematics Matters”

(NCCA 2005, p.2)

This short statement in the NCCA review of mathematics in Irish post-primary schools, while not illuminating, is certainly concrete and absolute in its opinion. That mathematics matters is beyond discussion. Mathematics is important to our education system in two key ways. It is important for all students as it is essential to the living of a “normal life” (Cockroft, 1982, p.1). The development of basic mathematical skills in terms of counting, measurement, and geometry provide people with an understanding that is a necessary to survive in our everyday lives. Ernest (2000, p.44) describes this as “functional numeracy” – the basic mathematical skills needed to function in society. Secondly, mathematics is important in terms of scientific advancement. The development of a “knowledge economy”, which has become a mantra for successive Irish governments, is predicated to a large degree on the mathematical abilities of its workforce as mathematics provides the “underpinning of the sciences, technology and engineering” (Smith 2004, p.14), which are vital to the economy. In 2009 an Expert Group on Future Skills Needs (EGFSN) reported that

“Raising our national mathematical achievement can improve Ireland’s competitiveness and each individual’s ability to contribute and participate in an increasingly globalised and technological society”

(EGFSN 2009, p.3)
This thesis examines the Irish mathematics classroom in its current incarnation as well as alternative approaches advocated in the mathematics literature. The author identifies two key areas in Irish students' performance, attitudes and understanding, in which improvement could be achieved. This study’s central research questions are whether the employment of active learning methodologies and the use of contextualised questions lead to an improvement in attitudes and understanding. The findings have significance as the research questions address the pedagogical tools advocated within the Project Maths syllabus and a good indication of their potential in the Irish mathematics classroom should be established. The study should also add to the research available on active learning, contextualised questions, attitudes and conceptual understanding as well as contributing to the research literature available on the Irish education system.

This investigation has potential benefits for practitioners. The new Project Maths syllabus in which Strand 1- “Probability and Statistics” has been introduced in the 2010/11 school year sees “Probability and Statistics” play a larger role within the syllabus and, due to changes in the exam structure (all questions will now have to be answered), probability can no longer be ignored or avoided by teachers and students. The resource pack developed will provide teachers with materials, which are in tune with the Project Maths syllabus and its demands that active learning and the use of contexts become part of the mathematics classroom (NCCA 2010).
1.6 Research Methodology

The research methodology will be discussed in Chapter 4 in more detail. Action research was chosen as the appropriate methodology and the research design included the following stages:

- An extensive literature review of the issues facing mathematics education and on the teaching of probability. The literature review served to clarify my research questions and inform the structure, content and approach of the resource pack.
- The development of the resource pack itself.
- The trialling of the resource pack using an expert panel and a Transition Year group before implementation with the target 5th year Higher Level mathematics class.
- Choosing appropriate data collection methods involving quantitative and qualitative instruments. This included a self-designed feedback form for the expert panel and critical friend as well as using an adapted Fennema-Sherman Mathematics Attitude Scale with students.
- Analysing the data collected, placing an emphasis on validity and reliability.
- Drawing conclusions from the findings of the research study regarding the impact the use of active learning methodologies and contextualised questions has on attitudes and understanding as well as commenting on any other interesting data that may have emerged.
1.7 Second Level Education in Ireland – System and Terms

This section provides a broad outline of the Irish education system explaining terms which are repeated throughout this document, which may be unique to Ireland. The traditional progression of Irish students is that they enter second level or post-primary level after finishing primary level education of eight years aged 12. The typical second level education is divided into two cycles. Junior Cycle, a three year cycle of study which ends in a terminal examination known as the Junior Certificate or Junior Cert. The examinations are independently created, supervised and corrected by a state body, the State Examinations Commission. The students then continue to Senior Cycle.

Senior Cycle can be a two or three year programme. It consists of 4th year, better known as Transition Year, followed by the academically focused 5th and 6th year. Transition Year was introduced in 1994 which provided students with a year which was focused on a vocational and holistic approach to education without the pressure of exams. It is available in the majority of schools in Ireland as an optional year for students. In some schools it is compulsory, while a minority of schools do not offer it at all. Senior cycle concludes with the Leaving Certificate, popularly known as the Leaving Cert. Currently 80% of Irish Students complete these examinations. Entry to third level is based on performance in these examinations. A points system based on the students’ best six subjects determines whether they meet the necessary requirements for entry into the third level courses they wish to study. Mathematics can be taken at three levels in both Junior Certificate and Leaving Certificate, Foundation Level, Ordinary Level and Higher Level.
1.8 Overview of Thesis

Chapter 2: This chapter reviews the general issues facing mathematics education today. It examines the current views on teaching and learning and analyses the position of mathematics education in Ireland within this context. The importance of attitudes and understanding are discussed as well as the difficulties encountered when educationalists attempt to implement reforms.

Chapter 3: This chapter looks specifically at teaching probability, its growing influence in international curricula and the rationale behind this. This chapter also looks at varying pedagogical approaches to probability as well as the difficulties, associated with teaching probability, which include issues such as teachers’ knowledge.

Chapter 4: This chapter outlines the methodology used in this research study. It explains action research methodology as well as outlining the research design and the instruments used to collect the data. The importance of validity and reliability are expounded upon and the ethical considerations involved in the study explained.

Chapter 5: In this chapter the format of the resource pack is explained and some of the materials are included. The resource pack is positioned within the mathematics education literature, while the benefits of active learning and contextualised questions are detailed.
**Chapter 6:** This chapter presents and analyses the main findings from the data. The data is presented from three perspectives, an outside perspective consisting of feedback from an expert panel and critical friend, a students’ perspective through an attitude questionnaire and class test as well as from the teacher’s perspective through a daily diary.

**Chapter 7:** In this chapter the main findings are discussed and conclusions are drawn. The author makes recommendation based on these conclusions and the research undertaken. The contribution made by the research is also outlined and the limitations of the study explained.
Chapter 2 – Issues in Mathematics Education Today

2.1 Introduction

“Project Maths aims to provide for an enhanced student learning experience and greater levels of achievement for all. Much greater emphasis will be placed on student understanding of mathematical contexts”

(Project Maths Development Team 2008)

In Ireland, we are at the present time experiencing an attempt to comprehensively overhaul how mathematics is taught and learned, at second level. This effort is not something that Ireland is attempting to do in isolation; in fact, we are following on the coat-tails of an international mathematics curricular reform movement. The 1990’s saw countries such as the U.K., U.S.A and Australia examining and dramatically altering their curricula at primary and second level education (Australian Education Council 1991, 1994; Department of Education and Science and the Welsh Office 1989, National Council of Teachers of Mathematics (NCTM) 1989, 2000).

Ireland has followed these international trends at primary level and is currently altering its approach to mathematics education at second level. The Irish education system is not however simply following what is internationally fashionable; it is attempting to adopt methods widely advocated in current research within the mathematics education community. The “Project Maths” reform, as it has been called,
is heavily influenced by those mathematics educators who persuasively argue that mathematics should be taught in a manner that promotes understanding. This literature also heavily influenced the author in the creation of the resource pack. Thus it is hoped that the resource pack, while valuable in its own right, will be consistent with the objectives of “Project Maths”. In order to fully understand why this new approach to teaching mathematics in Ireland is being advocated and why the author adopted this approach to the resource pack, it is important to delve backwards into mathematics education research literature.

2.2 Mathematics Education as a Research Field

Mathematics is a subject that has been taught and learned for millennia. In ancient Summeria, Egypt, Greece and Rome mathematics was seen as an important part of education. How to teach? What to teach? Whom to teach it to? When to teach it? All of these are questions, which have been discussed to varying degrees among mathematics educators since. Socrates, for example, felt his unique style, in which through a series of questions he would lead his students to make various mathematical observations and discoveries, drew the best out of his students (Fernandez 1994). Throughout the ages, people like Socrates, have held differing beliefs on mathematics education but it could not have been said to constitute an area of academic concern until relatively recently.

It has only been since the late 19th century as national school systems were set up that mathematics education and associated research in mathematics education began to
emerge as a professional field in its own right. At this time teachers began to be trained in higher education institutions and the training of these teachers became important. Alexander Bain’s book “Education as a Science” in 1879 marked a significant departure for this emerging field of study. In 1910, in Britain, W. McClelland authored a landmark first thesis in mathematics education for the purposes of obtaining a university degree from the University of Edinburgh. It was entitled “An Experimental Study of Different Methods of Subtraction” (Kilpatrick 1992). In the early stages, these professional educators may not have undertaken research and carried out studies as we would recognise them today (Howson 1988) but there were signs of the curiosity that would eventually bloom into modern educational research and already literature was beginning to affect practice.

In the first decades of the twentieth century there were various philosophical educational movements and trends which affected mathematics in schools. There was the measurement movement, the social utility movement and then a backlash to this reductionism. Questions of what should be taught and how it should be examined were being discussed. In the 1930’s and 1940’s different pedagogical questions on the teaching and learning of mathematics began to be researched. The traditional drill theory was attacked and more progressive teaching methods proposed. One extreme theory advanced was that of incidental learning, its advocates argued that arithmetic should not be systematically taught, and cited the research studies such as Harap and Mapes (1934, 1936). They conducted a study where a class of students learned operations, decimals and fractions by organising activities such as a candy sale or making furniture polish. Others such as Louis Benezet began investigating when children were ready to study topics. A school superintendent in Manchester, New
Hampshire Benezet (1935) described how “In the fall of 1929 I made up my mind to try the experiment of abandoning all formal instruction in arithmetic below the seventh grade” and after a year tested them against those who had undergone regular instruction and found no difference. Even in the 1930’s research literature read by teachers and those in charge of schools was beginning to affect the classroom (Kilpatrick 1992).

By the 1950’s the “New Maths” movement, which began in the U.S.A, had spread globally. This attempt at curriculum reform increased interest in mathematics education, and along with greater public funding for educational research in general this was a factor in leading to unprecedented levels of research into mathematics education around the world during the 1960’s and 1970’s (Kilpatrick 1992). Mathematics education had become a university discipline – a scientific discipline (Skovsmose 1985). The number of students studying in the area, and consequently the research in the area, ballooned. A scientific rigour was demanded in this new field, as Begle (1969, p.243) reminded fellow researchers, it is “only by becoming more scientific can we achieve the humanitarian goal of improving education for our children and everyone’s children”. While the need for strict methodologies was obvious to improve the standard of research it was becoming clear by the late 1970’s a more flexible approach may be needed. Many governments and public institutions felt that this field which had seemingly promised so much had delivered little in the form of tangible results. The need for education research to be conducted in the classroom not just in the library was becoming evident.
This feeling lead to increased co-operation between the practitioner and researcher, indeed the idea that a practitioner acting as a researcher was capable of producing valid research was accepted. The subsequent development in action research methodology, with the practitioner as researcher, is central to the methodology of this thesis. The author identified a problem with the teaching of probability in the post-primary mathematics classroom – the students did not like the topic because they simply did not understand it. On researching the issue the author concluded that probability was taught in a manner in Irish classrooms that did not promote understanding and the resource pack produced is an effort to change this.

What this short history on the emergence of research in mathematics education provides us with is an understanding that there never has been one single approach to mathematics education. Conflicting literature in the area has been both constant and to a degree influential. This fluid process of ever changing trends has led to the situation that prevails in mathematics education today, where three main views (behaviourist, cognitive and socio-cultural) of how learning best occurs in the classroom vie for predominance. The literature on these views of learning analyses current practice in Irish second level classrooms and informs a new and distinct type of practice, advocated by the Irish Department of Education and Skills (DES) through the National Council for Curriculum and Assessment (NCCA); and importantly for this thesis it informs the format and content of the resource pack developed.
2.3 Views on Learning

2.3.1 Behaviourist View

If mathematics educators and curriculum designers could understand how students learn then the best manner in which to teach them would follow. Unfortunately there is no consensus in the education literature on how students learn most effectively. Mathematics educators are in fact separated into many distinct groups or beliefs. These camps can be divided into three main approaches; behaviourist, cognitive and socio-cultural approaches.

In the behaviourist tradition, learning is the acquisition of new behaviour through conditioning. This approach has been championed by B.F. Skinner among others, and he is considered one of the most influential psychologists of the twentieth century. Of his many publications, the book “The Technology of Teaching” (1968) probably best outlined his views in the area. Behaviourism is focused on the students learning a skill-set. In order for students to learn, it is believed new tasks or information should be broken down into manageable ‘bite-size’ pieces. The learning then takes place in a step-by-step process where skills are practiced repetitively in routines. These routines progress from the easy to the more difficult, once the important basic skills have been grasped. Like Pavlov’s dog, good behaviour, i.e. demonstrating learning by moving up through the hierarchy of questions, should be rewarded with positive reinforcement. Bad behaviours should be rewarded with negative re-enforcement. Each lesson should have a set of objectives allowing achievement to be empirically assessed.
Many of the attributes of behaviourism feel inherently correct to teachers. The breaking down of skills into sub-skills allowing the teacher to support the students, building them up gradually from the simple to the complex, the rewarding of good behaviour etc. All these are characteristics that are associated with good teaching. Behaviourism is however, synonymous with the transmission approach (Conway and Sloane 2006). Rote learning of procedures and skills is demanded with little or no context of when, where or why these procedures could be used in the real world. Conceptual understanding takes a back seat to the ability of the student to repeat the appropriate procedure. Erlwanger (1975) demonstrated that students taught in this way were passing mastery tests but lacked any real understanding of what was being examined. As far back as 1933 Dewey (p.201) was railing against “the complete domination of instruction by rehearsing second-hand information, by memorising for the sake of producing correct replies at the proper time”. It has been variously described as a ‘Jug- and Mug’, or ‘Drill and kill’ approach. The teacher, who knows all, attempts to inscribe information onto the blank slates of students’ minds.

2.3.2 Cognitive View

The cognitive view of learning is significantly different from behaviourism. First and foremost learning is seen as occurring not by memorising new information but by the student assimilating or accommodating it with the information which they previously held. Learning does not take place by simply memorising all the knowledge, which is out there. True learning takes place when students interpret this new information in a manner which fits with their previous beliefs (Gardner 1985; Greeno Collins and Resnick 1996).
Secondly the cognitive perspective focuses more on authentic, problem type questions. It argues that recall, which is a signature of behaviourism, is only one step or type of learning. The influence of cognitive theory is evident in Bloom’s (1956) famous taxonomy. There are six levels of challenge - knowledge, comprehension, application, analysis, synthesis and evaluation. This taxonomy has undergone revision in recent times (Anderson and Krathwohl 2001) to become remembering, understanding, applying, analysing, evaluating and creating but still essentially promotes the view that reproduction is not sufficient evidence in the classroom or in examinations that true learning has taken place.

Figure 2.1: Revised Bloom's Taxonomy (Anderson et al. 2001)
The cognitive view of learning stems in a large part from the work of Jean Piaget (1971) and other psychologists who were proponents of the theory of constructivism, which essentially argues that knowledge of the world is made not found (Bruner 1997). Learning is an internal construction of meaning and in order to be meaningful, the child should be an active constructor of knowledge (De Corte, Greer, and Verschaffel 1996). Piaget (1971) believed that as students moved through progressive stages of development learning occurred. These theories have been developing since the 1950's and though most educationalists agree that they offer a progressive alternative to behaviourism this form of cognitive constructivism has been subject to criticisms of its own (Steffe and Tzur 1994). As Conway (2002 p.76) outlined “rather than viewing the learner as part of a family, community and social group embedded in a particular time and place, both the behavioural and cognitive perspectives portray learning as primarily a solo-undertaking.”

2.3.3 Socio-Cultural View

The socio-cultural view belongs within the umbrella of constructivism but emphasises the importance of both the cultural background in which the learning takes place and the importance of the group for effective learning. If Piaget is considered the most influential psychologist of the cognitive view of constructivism then Vygotsky (1978) and Bruner (1996) are the psychologists most associated with the socio-cultural view of constructivism. Bruner (1960) was clearly influenced by Piaget but his views evolved and became more in tune with those of Vygotsky. Vygotsky believed that we learn through our interactions and that "human activities take place in cultural settings and cannot be understood apart from these settings" (Woolfolk 2004 p.45).
According to Bruner (1996), learning and thinking are always situated within the given cultural setting. Therefore our culture influences how we learn. Our culture in this case is the language we use, as well as aids such as books and computers. Elements of culture that the students experience in everyday life should be brought into the classroom.

While all constructivists believe in active-discovery learning, Vygotsky believed in guided discovery. Two of the main principles of the socio-cultural view of learning are “scaffolding” and the “zone of proximal development”. Given a difficult problem students will discover the answer using appropriate and meaningful hints and clues offered by the teacher. The “scaffolding” the teacher provides through these hints brings students to a point, where new learning has to take place in order to solve the problem. The zone of proximal development is “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers” (Vygotsky 1978, p.86). In contrast to Piaget, learning comes before development rather than vice-versa. Finally the importance of collaboration, through peer and group learning cannot be overstated in the socio-cultural perspective (Bruner 1996, 1997). Students should be able to tease out problems together, both forwarding and critiquing possible answers in what is sometimes referred to as “communities of practice” (Brown 1994).

As we have seen there are three broad theories on student’s learning. Naturally, the proponents of each of these views on learning have developed and adopted teaching
approaches which are consistent with their opinions on how learning can best take place. The question at the centre of curricular reform movement is which teaching approach maximises learning for our students. The outline of the three underlying views of learning above is necessary, if the conflict between the differing approaches to teaching adopted in the classroom, specifically the conflict between current practices in Irish mathematics classrooms versus the approach adopted in the resource pack, is to be appreciated and understood.

2.4 Approaches to Teaching in Mathematics Education

There are a considerable number of pedagogical approaches to mathematics, which have as many overlapping aspects in common as distinctive features. The author will examine three pedagogical approaches, each one broadly in line with one of the three main views of learning. In each case, domestic or international examples of it in use will be discussed and its effect on the mathematics classroom outlined and in turn how this influenced the resource pack created.

2.4.1 New Maths

‘New Maths’ was born in the University of Illinnois in 1952. It was the master plan of The Committee on School Mathematics for Maths Education in post war U.S.A. and it was a genuine ‘attempt to radically improve mathematical attainment’ (Neyland 1996, p.34). The movement gained traction with the launching of Sputnik by the U.S.S.R. in 1957. The seemingly obvious fact that the U.S.A. was behind in terms of producing the type of mathematicians and scientists needed to compete with Soviet
technology provided the movement with the momentum and justification it needed. It was soon incorporated into the mathematics’ curricula in the U.S.A. and quickly became an international movement (Malatay 1988). ‘New Maths’ had a slightly different emphasis in terms of content; Euclidean Geometry was demoted and replaced by set theory as well as some probability and statistics. Indeed, ‘Euclid Must Go’ became the catch-phrase of the movement, a slogan coined by the French mathematician Dieudonne.

In terms of pedagogy ‘New Maths’ adopted what was amounted to a structural, behaviourist approach (English, O’Donoghue and Bajpai 1992). Mathematics in itself is important and should be taught in abstract conditions. In this type of pure mathematics, it is the rules, procedures and skills that are important, not where they come from or where they are used. Ernest (1985, p.606) argues that this method of learning “is without meaning, rules are learnt as purely formal devices to be followed without understanding or justification”. Essentially mathematics is taught and learned in a vacuum, without interference from the real world or the pupils’ daily lives. The learning process is organised in a teacher-orientated manner, with the emphasis on factual knowledge. Questions often require single word answers with little investigation of how this answer was reached if correct and none at all if the answer is wrong. Whole class activities based on writing are prescribed with little or no practical work carried out or collaboration among the students. There has been attempts to make ‘New Maths’ more relevant by the use of applications but most often “their applied problems are merely classroom stereotypes” (Treffers 1991, p.345), which pupils find “contrived and even confusing” (Neyland 1996, p.39)
The ‘New Maths’ movement reached Ireland in the 1960’s. The new Senior Cycle curriculum was introduced in 1964 with the first examination in 1966 and at an equivalent of Junior Cycle level the new curriculum was introduced in 1966 and first examined in 1969. The adoption of this approach in the Irish curriculum was important in that, although the content was slightly altered, it not only allowed teachers to continue teaching in the same manner they had always done but provided a justification for it. So in Ireland and many countries worldwide, teachers “simply taught modern mathematics in the same way they had taught the old mathematics” (Streffe and Wiegel 1992, p.445). In Ireland, this means that mathematics generally continues to be taught in a manner which emphasises rote learning, techniques and procedures rather than any great understanding of concepts. The ‘drill and practice’ routine of the mathematics classroom continued (Skemp, 1976). As Kapur (1997, p.266) described it

“While new mathematics has aroused great emotions, among both its protagonists and its opponents, most mathematics teachers have taken note of the movement and gone ahead with the task of teaching”

O’Donoghue (2002) explained that this approach has led to an inability among Irish third level mathematics students to use mathematics except in the practised, examination style manner (cited by the NCCA 2005, p.3). In 2003, in the mathematics test in the Programme for International Student Assessment (PISA) Irish students did not fare well. It has been suggested that this relatively poor performance could be attributed to the type of questions asked in PISA which are problem solving type questions placed within real-world contexts (Shiel, Perkins, Close, and Oldham
2007). Once Irish students were removed from their comfort zone of routine questions, they struggled. Essentially when Irish students were placed in a situation where rote learning of mathematics was not enough to answer the questions posed and actual understanding of the material involved was needed, they struggled.

A second resulting feature of ‘New Maths’ in Ireland is the negativity, which exists towards mathematics among students that does not attach itself in the same levels to other school subjects. There is a widespread public perception in Ireland of mathematics as a difficult subject (NCCA 2005; Conway and Sloane 2006). Smyth, McCoy and Darmody (2004) reported mathematics as the second least favourite subject in a study of Irish first year post-primary students. This is not just an Irish problem but rather seems to be a product of the ‘New Maths’ inspired behaviourist curricula internationally. The OECD (2004) found in their 2003 PISA study for example, that there was much lower levels of enjoyment reported in mathematics than in reading. It is these issues, of students’ poor attitude and a lack of understanding, which provoked the author into creating the resource pack to encourage mathematics educators in Ireland to examine other pedagogical approaches to teaching mathematics. One alternative to behaviourism and New Maths is the problem solving approach offered by those who take the cognitive view of learning.
2.4.2 Problem Solving and Investigation

“It is in fact nothing short of a miracle that the modern methods of instruction have not yet entirely strangled the holy curiosity of inquiry.

(Albert Einstein quoted in Time Magazine, March 21st, 1949)

It was this view of mathematics education that drove many of those interested in mathematics education to examine possible alternatives. As far back as 1933 Dewey was calling for mathematics education to be based around “reflective inquiry”. Though considered highly progressive, Dewey’s (1933) ideas were largely sidelined in terms of mainstream education until educators began to search around for alternatives in the backlash to “New Maths”. Several issues worried mathematics educators, chief among them was probably the gulf between students’ ability to acquire knowledge and to apply this knowledge.

Piaget and others who proposed what became known as the cognitive view offered a psychological basis for alternative pedagogical approaches. One of the alternative approaches proposed and examined which was consistent with this view, was “Problem Based learning”. This solution to the pedagogical difficulties facing mathematics education was widely lauded particularly in the 1980’s. In the U.S.A for example the NCTM designated the 1980’s as the decade of problem solving. The French mathematician Poincare said in 1895 that “it is by intuition that one creates” in mathematics (Kilpatrick 1992, p.5) and it is this creation of knowledge that the problem solving approach essentially tries to replicate in the classroom for students.
Hiebert, Carpenter, Fennema, Fuson, Human, Murray, Olivier, and Wearne (1996 p.14) synopsised Dewey’s reflective inquiry into three steps

- A problem is identified.
- It is studied using active engagement.
- Conclusions are reached, when the problem is solved (sometimes partially solved).

This three step approach is still essentially the problem-solving approach today. The lesson starts with the problem, the students create and explore, often in pairs or small groups, different solutions which are then discussed and analysed. The emphasis is not on the answer but on how the answer is arrived at. The process is considered crucial; there is no one correct way to achieve the desired answer. Piaget (1971 p.27) remarked “The essential functions of intelligence consist in understanding and inventing” and teachers now tried to promote both activities in their classroom.

The literature on the impact of problem solving in the classroom is for the most part positive. There is plenty of empirical evidence endorsing students exploring problems but an expectation that they can do this in a reasonably efficient manner without the active involvement of a teacher has been shown to be unrealistic (Carpenter, Franke, Jacobs, Fennema, and Epsom 1998; Resnick 1989). The increase in a teacher’s workload as well as the huge change in their role and the skill-set needed has also proved to be an obstacle in implementing problem solving approaches (Pehkonen 2007; Cai 2003). The difficulties, which teachers encounter in changing over to a problem based approach are very often psychological and their struggles are internal
(Smith 2000). This obstacle would however be encountered in varying degrees in any attempt at curricular reform.

Students have had similar problems in adapting their beliefs about what mathematics education should consist of but studies have found that the initially negative attitudes of students to problem-solving change with time (Carpenter et al. 1998; Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, and Perlwitz 1991; Vershaffel and De Corte 1997). In some cases these students now showed a more positive attitude towards the importance of understanding (Cobb et al. 1991). Finally studies conducted at primary level (Carpenter et al. 1998; Cobb et al. 1991; Fuson, Carroll and Drueck 2000) have demonstrated that students taught using a problem solving approach fare comparably to those taught using the traditional approach in basic arithmetic but have higher levels of understanding and problem-solving skills.

As mentioned earlier in the U.S.A in particular problem-solving was adopted as a potentially successful approach by many of those involved in mathematics education and was promoted as the way forward in mathematics education by the NCTM. Many other countries made similar attempts at change – Finland for example also adopted the problem-solving approach. The Canadian case is interesting. Five comparison tests carried out between their provinces from 1988 to 2000 showed that students in the province of Quebec were outperforming the students of British Columbia. When the curricula were analysed to see why this was the case, there were a few areas of difference, however the main difference was philosophical. Quebec’s curriculum had a “Cognitivist learning orientation” where problem solving was central to the
curriculum, whereas British Columbia’s curriculum reflected “a more behaviourist view of learning” (Raptis and Baxter 2006, p.2). In an attempt to replicate the success of Quebec, British Columbia made an unsuccessful attempt at curriculum reform, in which they attempted to adopt problem solving as their pedagogical approach. A change of governance at a political level subverted the reform process. In England, Wales and Northern Ireland, the related approach of investigation also became part of the curriculum.

By the year 2000 however a sense of realism about what could be achieved in the classroom using this method had prevailed (NCCA 2005). It was agreed that problem solving did have positive effects in the classroom and the NCTM Standards in 2000 stated that “Problem solving is an integral part of all mathematics learning” (p.52) but inadequacies and weaknesses in a problem solving or investigation approach alone had been exposed. Some educators felt for example some of the problems posed lacked any real context that the pupils could relate to (McMahon 1997; Brown, Collins and Duguid 1989). Others felt that unrealistic expectations had been placed on what students could achieve individually and the role of the teacher and classmates needed to be enhanced if the students were expected to learn. (Kirschner, Sweller and Clark 2006; Mayer 2004) These issues with a problem solving pedagogy and the cognitive view of learning in general meant that in recent years the socio-cultural view has become pre-eminent among mathematics educators, as the approach taken by those who oppose behaviourism and new mathematics (Conway and Sloane 2006). Problem solving and investigation have not been dismissed but rather co-opted as pedagogical tools into the socio-cultural view of mathematics education. This view, though similar, emphasises the need for mathematics to be placed in a context which
the students can relate to socially and culturally. There are a range of pedagogical approaches, which reflects the socio-cultural view such as situated cognition but perhaps the most influential approach of this kind is called Realistic Mathematics Education.

2.4.3 Realistic Mathematics Education

“The real world, context-focused mathematics education approach puts a premium on rich-socially-relevant contexts for learning, problem solving and the relevance of learners’ prior experiences in planning and engaging with learners in classrooms or in multimedia learning environments”

(Conway and Sloane 2006, p.6)

Realistic Mathematics Education (RME) is an approach conceived and pioneered by Dutch mathematician turned mathematics educator Hans Freudenthal. He became extremely disillusioned with how mathematics was being taught and learned in the 1950’s and the 1960’s and began developing his own ideas on how students should be taught. His idea that mathematics is a human activity situated in social and cultural contexts is consistent with the socio-cultural view of learning.

He first discussed his ideas in his 1973 book, *Mathematics as an Educational Task*. He built on his work with more publications following, most notably his *Revisiting Mathematics Education: China Lectures* (1991). Others have followed and built on his work, often supported by the Freudenthal Institute in the Netherlands, most notably Treffers (1991), Gravemeijer (1998) and DeLange (1996). Freudenthal argued that students, given problems in a real life context and with guidance from their
teachers, can “discover” mathematical solutions for themselves. Gravemeijer (1998 p.62) explains that authentic real-life situations can “evoke problem solving paradigms” which ignite students’ curiosity and interest.

Realistic mathematics education starts with a problem in context and moves from there. RME is not just limited to realistic situations as the title suggests but to problems which the students are capable of imagining (O’Donoghue, 2010 p.76). RME questions could therefore be based on fairytales. As Geoffree explains (1993 p.89) “reality does not only serve as an application area but also as the source of learning.” This approach has implications for the content taught. The content is taught in themes rather than strictly segregated topics. This thematic approach means that if a problem arises which involves other areas of mathematical content they too will be explored.

Mathematics lessons using the RME approach generally follow three steps.

- The students are given a problem that they can relate to, it is experientially real to them. This usually, but not exclusively means, that they are taken from real-life contexts. As Gravemeijer (1998) states it is important to note that “ once the students have mastered some mathematics, mathematics itself can become a realistic context” (p.58)

- Pupils then engage in a process of guided discovery or “re-invention”. Essentially the pupils try to solve the problem presented themselves rather than receiving ready-made answers. The process involves the students actively
interpreting and analysing the mathematics involved (Freudenthal 1973).

Much as Vygotsky (1978) advocated, the teacher acts as a guide in this process “striking a subtle balance between the freedom on inventing and the force of guiding” (Freudenthal 1991, p.48).

- The pupils then produce solutions based on their own informal and intuitive problem solving strategies. The process of transforming a problem from its context into mathematical terms is referred to as “horizontal mathematisation”. Gradually pupils will be expected to use more formal abstract standard strategies which involve manipulating questions in mathematical form, this is known as “vertical mathematisation” (Gravemeijer and Doorman 1999)

This approach has been successfully co-opted into the Dutch Education system. The Netherlands has ranked highly in the recent PISA (Mullis, Martin, Gonzalez, O’Connor, Chrostowski, Gregory, Garden, and Smith 2001) and TIMSS studies. This may be partly explained by the importance PISA placed on areas which the RME emphasises, mainly the importance of context in their tests. PMRI (Pendidikan Matematika Realistik Indonesia) an Indonesian version of RME has been gaining in popularity in Indonesian primary schools as the main pedagogical approach to mathematics education. This is due to the highly positive reactions of pupils in schools where it has been used since workshops for teachers were established in 2001 (Sembiring, Hadi and Dolk 2008).

Research carried out in Indonesia (Fauzan, Slettenhaar and Plomp 2002) in schools who adopted the RME approach found that
• Though pupils initially struggled with the new approach, the pupils themselves came to like the new approach and felt it had a positive impact on them.

• The pupils reasoning and creative thinking developed.

• The teachers’ felt that the pupils’ behaviour improved.

Martin Van Reeuwijk (1992) from the Freudental Institute, spent four weeks in Whitnall High School in Milwaukee, Wisconsin using RME as a pedagogical approach to teaching data visualisation. He found that once again despite initial struggles pupils soon became far more motivated and engaged in lessons. He noted that teachers struggled to a greater degree than the students to adapt to the new style. Their role had moved from being a lecturer to a facilitator. A curriculum project known as Mathematics in Context has been developed jointly by the Freudenthal Institute and a research team at the University of Wisconsin. This curriculum is a combination of the NCTM Curriculum and Evaluation Standards and a Realistic Mathematics Education approach and is being taught at schools in the U.S. It has been reported that it has been found to have a “significant positive impact on student learning” (Conway and Sloane 2006, p.161).

2.4.4 Conclusion

In the opening page of this chapter the current attempts at a radical overhaul of mathematics at second level in Ireland was outlined. This reform has been driven by the research into, the theories of learning and the approaches to teaching which, have been discussed in the intervening pages. It has been established that there are real and
credible problems with the current approach in Ireland. Teaching styles have been restricted to those that promote rote learning. (Conway and Sloane 2006; English et al. 1992, Lyons, Lynch, Close, Sheerin and Boland 2003; NCCA 2005) Essentially students do not understand the ‘how’ or ‘why’ of the mathematics they are supposed to be learning. This lack of understanding along with corresponding poor attitude levels towards mathematics has led to a poor uptake at the higher level of Senior Cycle and worries being expressed by industry figures that Ireland is not producing either the number or quality of mathematics graduates needed. In December 2010, Tony Donohoe, IBEC head of education policy, commented on the release of the PISA results for 2009, which showed that Ireland was below average in the PISA countries and was slipping down the table, that

“The mathematics results are particularly disappointing because Ireland’s aspiration to be a knowledge economy depends on a strong supply of engineers and technologists. Therefore, the marked decline in the number of high-achieving mathematics students is of particular concern to business”.

(Donohoe as quoted in Silicon Republic 7/6/11)

The objective of the resource pack developed by the author is to tackle the lack of understanding shown in the area of probability specifically and the poor attitudes exhibited by Irish students towards mathematics generally. The current reform effort of the Irish mathematics curriculum, Project Maths, is similarly driven. This desire for understanding and a positive attitude is sympathetic with a socio-cultural view of learning and it is clear Project Maths is an attempt to move away from the current behaviourist ‘New Maths’ approach towards an approach which has been heavily
influenced by teaching approaches such as RME and “Mathematics in Context”. The questions remain, why such an emphasis on improving understanding and attitudes? And why are these objectives considered sympathetic to the socio-cultural outlook?

2.5 Students’ Attitude to Mathematics

As far back as the 1950’s studies in the U.S. were finding that students had a negative attitude towards mathematics (Poffenberger and Norton 1959) compared with other school subjects. An attempt to improve this situation was and is one of the primary drivers in mathematics reform. The current reforms in mathematics education have placed an unprecedented emphasis on affective factors. The importance of student confidence and students appreciating the value of mathematics has been emphasised in reform documents in many countries (Australian Education Council 1991; Cockroft Report 1982; National Council of Teachers of Mathematics (NCTM) 1989, 2000). Indeed the NCTM (1989) and the National Research Council (1989) regarded it as one of the most immediate issues facing mathematics educators today.

The logic behind this emphasis is a fairly reasonable belief among educators, despite a lack of consensus in the literature, that pupils’ attitudes towards mathematics influences their success in it (Bell, Costello and Kuchemann 1983). What is referred to as attitudes, can be broken down into many separate areas; students’ confidence in their mathematical ability, students’ perception of its usefulness, students’ motivation or enjoyment and students’ emotional experiences. In PISA (2003) students were asked about four different factors in an effort to examine their influence on
performance. These were motivation, self-related beliefs, emotional response (anxiety) and learning strategies. The analysis of the data showed that there was a strong relationship between these factors and the students’ performance. Most importantly “analysis shows that students who are less anxious perform better regardless of other characteristics” and “that anxiety and interest and enjoyment of mathematics are closely interrelated” (OECD 2004, p.148). In terms of students’ perception of mathematics as useful “most students believe that success in mathematics will help them in their future work and study” (OECD 2004, p.121). This belief however diminishes as students move up through second level.

The relationship that exists between a positive attitude towards mathematics and performance in mathematics is emphasised repeatedly throughout the mathematics literature (Farooq and Shah 2008; McLeod 1992; OCED 2004). The direct relationship between attitude and achievement can be difficult to assess as there are so many other possible variables which can complicate any insight achieved in studies. As McLeod (1992, p.582) puts it the “research suggests that neither attitude nor achievement is dependent on each other; rather they interact with each other in complex and unpredictable ways”

Perhaps the best indicators of the link between attitude and achievement in mathematics are the meta-analysis studies that were undertaken in this area. Meta-analysis studies give the reader a general narrative on what the literature is saying. The meta-analysis undertaken by Ma and Kishor (1997) based on 113 studies contained some interesting results. They found that the literature demonstrated a
definite if slight link between attitudes towards mathematics and achievement in mathematics and that this relationship became more important as students progressed through school. The meta-analysis study undertaken by Hembree (1990) based on 151 studies, confirmed that mathematics anxiety is related to poor performance in mathematics.

It is interesting to note that despite the curriculum reform movement’s focus on aspects such as student motivation and enjoyment, students have proved at least initially resistant to new forms of pedagogy. Students tend to believe that only geniuses can be creative in mathematics. This belief subverts their ability to solve non-routine problems (Schoenfeld 1985). It takes time and experience for a change of mindsets in students. There is a generally held view that some people have a natural flair for mathematics and others do not, which limits peoples’ interest and motivation to learn mathematics and makes it acceptable to underachieve (NCCA 2005, National Research Council 1989). The Cockroft Report (1982) suggests that students seem to prefer and were certainly more comfortable with a presentation mode and routine questions. This brings us to Nickson’s (2000 p.194) “didactic contract”, where students feel comfortable with the current exam orientated pedagogy and informally push teachers in that direction. Studies have shown that positive attitudes to mathematics can flourish when students recover from their initial discomfort. Studies by Carpenter et al. (1998), Cobb et al. (1991), and Vershaffel and De Corte (1997) where a problem-solving curriculum was implemented mentioned student unhappiness in the beginning phases. Fauzan et al. (2002) and Van Reeuwijk (1992) encountered similar problems when assessing the implementation of RME type
curricula. In all these cases though, student attitude towards mathematics had improved by the end of the study.

In summary, the current behaviourist pedagogy in classrooms has led to the development of prejudices about and a poor attitude towards mathematics. Studies have shown that there is a link between this poor perception and attitude and poor performance (Farooq and Shah 2008; Hembree 1990; Ma and Kishor 1997; McLeod 1992; OCED 2004). The literature also shows that other pedagogical approaches have had a positive impact on the perception and attitudes held by students towards mathematics (Carpenter et al. 1998, Cobb et al. 1991, Fauzan et al. 2002; Van Reeuwijk 1992; Vershaffel and De Corte 1997). This improvement in attitudes can be harnessed to improve achievement within the subject. This reasoning is one of the primary drivers behind the resource pack and the methodologies promoted in it. Teaching mathematics in a manner which leads to improved attitude among students is one of its central objectives. This form of pedagogy, inspired by the socio-cultural view of learning places a premium not just on students’ attitude but also ‘Teaching for Understanding’ and the affect this can have on students’ attitudes.
2.6 Teaching for Understanding

2.6.1 Types of Understanding

What does ‘Teaching for Understanding’ actually mean? It could be argued that teachers have always taught mathematics in a manner which promotes understanding, if not students would not or could not have passed mathematics examinations. The reality is that teachers in the behaviourist ‘New maths’ tradition have in fact only catered for one type of understanding. Skemp (1976) divided understanding into two separate categories:

- Instrumental Understanding – students know how to perform calculations.
- Relational Understanding – students know why they perform calculations.

Hiebert (1986) had a similar view, distinguishing knowledge it into two different types

- Procedural knowledge - students know how to perform mathematical procedures
- Conceptual knowledge - students know why they perform mathematical procedures

Traditionally mathematics has been taught in a manner which promotes instrumental understanding or procedural knowledge. This type of teaching values rote learning, where the rules and procedures are learned off. This has been described as ‘rules without reasons’. Students know “how” to use these formulas and procedures to arrive at the correct solution but not “why” they are using them (Romberg and Kaput 1999). This has led to concerns outlined earlier where, problems are presented in a non-
routine manner or presented within a context. Students taught following the
behaviourist view struggle to solve these problems (Lyons et al. 2003; O’Donoghue
2002 cited by NCCA 2005). Both the cognitive and the socio-cultural view of
learning promote relational understanding or the development of conceptual
knowledge. When this is achieved students, understand why they are using these
procedures, what these procedures actually achieve and what the solution arrived at
actually means. Teaching for understanding means that both types of understanding
are promoted within the classroom (Carpenter and Hiebert 1992; Rittle-Johnson,
Siegler and Alibali 2001). Teaching in this manner promotes the formation of
connections between new ideas, representations and procedures and those that the
students already know. Carpenter and Hiebert (1992) explain that the degree of
students’ understanding is determined by the strength and number of connections
formed. The more extensive the network of relationships or connections created, the
greater the degree of student understanding. Teaching for understanding should
therefore involve a pedagogical approach, which promotes the construction of
connections rather than memorisation of procedures.
2.6.2 The Benefits of Teaching for Understanding

Carpenter and Hiebert (1992) list four benefits of teaching for understanding:

- Understanding is generative,
- Understanding promotes remembering,
- Understanding reduces the amount that must be remembered,
- Understanding enhances transfer.

Carpenter and Lehrer (1999) had a similar list outlining three distinct benefits of relational understanding; it is generative, it aids in remembering rules and procedures linked with instrumental understanding and it improves attitudes towards mathematics. Undoubtedly the most important benefit of conceptual understanding is that the knowledge acquired is generative. This means that students can use what they have learned and apply it in other situations to create more knowledge. It is in this area that Irish mathematics students have been identified as having difficulties (Lyons et al. 2003; O’Donoghue 2002 cited by NCCA 2005). Once they are placed in a situation which they are unfamiliar with, they cannot transfer previously learned skills into new contexts. When students truly understand what they have been taught, they are able to use the skills and knowledge learned and adapt it to solve new problems. This creativity and adaptability is highly valued in a workforce, where it is impossible given technological advances to predict today, the skills needed for future jobs (Donoghue 2010).

The second benefit of relational understanding is that it creates knowledge that is more easily recalled or as Carpenter and Lehrer (1999 p.21) explain it, this “structured...
knowledge is less susceptible to forgetting”. The “Structured knowledge” referred to by Carpenter and Lehrer (1999), describes a knowledge not just of isolated skills, routines or procedures but rather a knowledge created and integrated with contexts and applications. A knowledge that does not rely on rote memorisation to enable students to retrieve desired information and skills. Thirdly, teaching for understanding promotes an appreciation of the usefulness and importance of mathematics in the real world. This perception of mathematics has a positive influence on students’ attitudes towards the subject.

There are undoubted benefits in teaching for understanding (Carpenter and Hiebert 1992; Carpenter and Lehrer 1999). At its core, any attempt at teaching for understanding, has significant implications for classroom practice. The pack developed by the author is an attempt to create resource materials which would enable teachers, to bring both activity and real-life contexts into their probability classroom. These are the twin planks on which the resource pack promotes teaching for understanding and they will be discussed in more detail in Chapter 5.

The Project Maths curriculum also places a heavy emphasis on active learning and real-life contexts.

“Much greater emphasis will be placed on student understanding of mathematical concepts, with increased use of contexts and applications that will enable students to relate mathematics to everyday experience”

(Project Maths Development Team 2008)
However, this is not the first attempt at bringing these areas into second level mathematics in Ireland (Oldham and Close 2009; NCCA 2005). This current curricular reform drive is part of what Brown (1994 p.11), labels the great “conundrum” within mathematics education. It is generally agreed and has been for some time in the research literature that current school practice displays a heavy preference towards behaviourism and other out-dated learning theories (Lyons et al. NCCA 2005). There exist contemporary approaches, such as Problem Solving, Investigation and Realistic Mathematics Education which studies have shown are effective and mathematics educators feel have, enormous benefits for students’ mathematical learning. There has been, however, little real change in Irish school practice until now.

2.7 Curricular Reform in Mathematics Education

2.7.1 Introduction

“The failure of educational research to impact on teaching and learning has been lamented almost from the beginnings of educational research itself”

(William 2003, p.471)

There has been a wave of attempts at curriculum reform within mathematics education in recent times (Australian Education Council 1991; Department of Education and Science and the Welsh Office 1989; National Council of Teachers of mathematics (NCTM) 1989,2000). These have largely been attempts to change the curriculum in a move away from the behaviourist “New Maths” to one which has a cognitive or socio-cultural outlook. These reforms have been successful to varying degrees, the most recent however are still too new to analyse properly or to draw
conclusions with any degree of certainty. Unfortunately genuine educational reform has become notoriously difficult to achieve (Sarason 1993; William 2003). The title of Sarason’s 1993 text *The Predictable Failure of Educational Reform* sums up much of the frustration felt by educators in general. Why do these reforms fail to generate genuine change? There are varying reasons.

We do not have to look beyond Ireland for an example of failed reform in mathematics education. The Junior Cycle level syllabus was updated in 2000 but this was simply a change of words not the systemic review needed for a change of action (Oldham, 2002). Robataille and Travers (1992 p.693) explain that this was simply a change of the planned curriculum and there is in fact three levels at which the curriculum need to be worked on if real change is to occur. The other two levels are the implemented curriculum - what and how teachers actually teach, and the learned curriculum - what the students learn. In this Irish case the planned curriculum was changed but what was being taught and learned and how it was being taught and learned did not change. So when curriculum reforms are being implemented, it is not simply a case of altering the documented syllabus – it is much deeper than that.

Fullan (1991) differentiates between reform and change. He states that reform, may be accompanied by altered content but there is no change to practice and beliefs. Fullan (1991) explains how deep change will only occur if there is change on three levels; curriculum materials, pedagogy, and beliefs and values. He also explains that real change is generally accompanied by a “Bottom Up” approach where teachers play an active role in the change process. Superficial reforms on the other hand are
synonymous with a “Top Down” imposition of the new curriculum. Conway and Sloane (2006) claim, that in order for curriculum reform to work, the old curriculum needs to be attacked at three levels, the curriculum culture, the textbooks and the examinations. These are similar factors to those outlined by Fullan (1991) but they have combined “pedagogy” with “beliefs and values” to form “curricular culture” and added in “examinations” as a key factor in the change process. The author intends to examine these cornerstones of reform comparing and contrasting the success story of Realistic Mathematics Education in Holland with the failed attempt at reform of mathematics at Junior Cycle level in Ireland.

2.7.2 Curricular Culture

“Classroom approaches will be effective only if they give due weight to the essential role of teachers.”

(Steinbring 1991, p.155)

At the present time there are two competing and contrasting curricular cultures, which are heavily influencing mathematics education. They are the “New Maths” behaviourist culture and the socio-cultural context based culture. In recent times curriculum reform literature has by and large advocated a move away from ‘New Maths’ towards a context based mathematics education which also has constructivist elements. If this is to succeed however, it is not sufficient just to change a syllabus. In order to change the curriculum culture, the classroom culture also has to change. The single most important agent of any change in the classroom is the teacher.
Dossey states that “the conceptions of mathematics held by teachers may have a great deal to do with the way in which mathematics is characterised in classroom teaching” (1992 p.42) and he also discusses work by Cooney (1987) that suggests reforms can be difficult to achieve because of teachers beliefs. Many studies have shown that one of the biggest difficulties encountered in reform in the mathematics classroom is the ability of teachers to adapt. Van Reeuwijk (1992) discussed this when implementing an RME approach in schools in the U.S. and Pehkonen (2007) referred to similar difficulties with implementing a problem-solving approach in Finland. It has been shown that teachers’ beliefs do impact on their teaching (Thompson 1992). Teachers’ beliefs and conceptions of mathematics have to be consistent with the approach advocated in their curriculum – otherwise real curriculum reform will not occur as teachers will continue to teach in the manner consistent with previous approaches.

The reform literature reflects a view that mathematics is a dynamic, ever changing and growing field of study. A study by Lyons et al. (2003) of Irish classrooms showed that mathematics is often taught as a static abstract discipline with a set of skills and concepts to be learned. This was one of the major factors in the failure of the Junior Cycle reform, teachers’ beliefs and values of what is important in mathematics and mathematics education had not changed, consequently neither did their pedagogical approach. Teachers conception of mathematics education was handing “mathematics down to the pupil like a ready-made object in a simplified if piecemeal way” Steinbring (1991, p137). If the context laden, problem solving, active orientation of mathematics education is to become a reality “mathematics must be accepted as a human activity” (Dossey 1992, p.42) and “outdated assumptions” (Mathematical Sciences Education Board 1990, p.4) of mathematics must be abandoned by all
involved in the reform but perhaps most importantly by teachers. The introduction of 
the Realistic Mathematics Education (RME) curriculum in the Netherlands is an 
example of successful mathematics education reform. This would not have occurred 
without changes in teachers’ conceptions. These changes did not occur overnight. 
Conway and Sloane (2006) cite this as an example of how reforms can take time, up 
to a decade, to permeate an education system and filter into the classroom culture. 
This reform began with teacher educators and teachers and so is a case of practice 
influencing the theory rather than visa- versa. This bottom-up approach can be 
important in convincing sceptical teachers of the practicality and value of the new 
pedagogy and the conceptions of mathematics involved (Fullan 1991). The resource 
pack should be seen in the light of developing resources and momentum for change 
from the bottom up.

In the Netherlands, the government provided for the organisation of suitable support 
and guidance for teachers when the reform process was already effectively underway. 
These enabled teachers to discuss their experiences and disseminate the knowledge 
they had gained in the classroom. The decision to adopt new teaching methods needs 
professional development. This is not simply a case of preparing teaching material 
and handing it over to teachers just to deliver it. Teachers, as students do, learn 
experientially. Workshops where knowledge is simply transmitted are not enough; 
teachers need experiences of professional development that encourage the 
development of new skills and changes in beliefs if curriculum reform is to become a 
reality (Conway and Clark 2003; Sugrue 2002; Prendergast and O’Donoghue 2009).
2.7.3 Textbooks

Textbooks can be powerful agents of change or of status quo. The Royal Irish Academy (2006) felt that improved textbooks and resources would improve pupil performance in Ireland. Teachers rely heavily on their textbooks. O’Keeffe and O’Donoghue (2009), stated that in Ireland 75% of mathematics teachers use their textbook daily. The importance of textbooks is easily illustrated by our two case studies of reform. The first case was the attempted revision of the Irish Junior Certificate in 2000, which was first examined in 2003. In this case of curriculum reform, there was an effort to introduce a Problem-Solving approach into the curriculum. It was “hoped that the emphasis on meaningful and enjoyable learning will enrich the quality and enhance the effectiveness of our students’ mathematics education” (NCCA 2002, p.5).

Unfortunately this revision did not succeed in changing how mathematics was taught in the classroom. A contributing factor was the unchanged textbooks. Analysis of Irish Junior Certificate mathematics textbooks showed that the emphasis is on “proficiency and logic” and “understanding is not given the same importance as procedure and method throughout each of the textbooks” (O’Keeffe and O’Donoghue 2009, p.292). The textbooks still approached mathematics education from the behaviourist, ‘New Maths’ standpoint so classroom pedagogy did not change. Consequently, the reform was stillborn. O’Keeffe and O’Donoghue (2009) warned that any future curriculum reform of mathematics education in Ireland will need to see the production of heavily revised and re-orientated textbooks.
A contrasting example to the Irish failure is the case of mathematics in the Netherlands, which is an example of successful mathematics curriculum reform. The Dutch education system moved from a ‘New Maths’ approach to one that is now based largely around the Realistic Mathematics Education approach developed by Freudenthal. One of the drivers of this success was the development of appropriate textbooks (Moffett 2009). In 1996, De Lange estimated that 85% of primary school mathematics books bore characteristics of an RME approach and this process was now carrying on to second level. Van den Heuvel-Panhuizen (2000, p. 10) described the textbooks developed as the “most important tools” in guiding teachers in terms of new content but more importantly the new pedagogical approach.

It is also worth noting that the core activities and strands in these textbooks were first developed and trialled by teachers in collaboration with teacher educators and researchers in the classroom before they were included in the textbooks. Once again a sign of an organic desire for change rather than one forced upon teachers from above. Though not a textbook, the pack created would provide resource materials for teachers in the area of probability. These resource materials reviewed by practising teachers and trialled in the classroom have a degree of credibility which may convince teachers that this is a pedagogical approach worthy of consideration.
2.7.4 Examinations

“The combined effect of the behavioural focus on decontextualised skills and the ‘new’ mathematics focus on abstraction have lead to examinations and assessments that often pit the isolated learner against quite abstract tasks bearing little relation to real world challenges of a mathematical nature”

(Conway and Sloane 2006 p.94)

The link between assessment and what is being taught and how it is being taught is undeniable. Skemp (1976, p.5) refers to the difficulties created by the “backwash of examinations”. The major effect that examinations have on the curriculum provides a signal to those involved - students, teachers and parents - on what is valuable. The tradition of a terminal examination, which involves recall and regurgitation of facts, has a history which goes back to the formation of schools. If an examination signals to the stakeholders that recall rather than understanding, that using the correct procedure in an abstract question rather than analysing the context involved, is what is important, then the focus in the classroom will be on recall and procedure rather than on understanding and context. Beaton, Mullis, Martin, Gonzalez, Kelly and Smith (1996) acknowledge that didactic teaching can be rewarded in examinations of these type, so any genuine attempt at a move away from this type of pedagogy will have to see a change in examinations.
The influence of examinations can be counter-productive to the stated goals of the curriculum (Mehrens 1989). If reform is to be achieved it is important that assessment should reflect the desired content and approach. Teachers of mathematics, no more than any other subject, feel that their job is first and foremost to produce the best results possible not to produce enjoyable classes. Therefore, they teach to the examination, rather than necessarily aiming to fulfil some of the highly laudable goals in the syllabus (Lyons et al. 2003).

The failure to address this issue was one of the reasons that the attempted reform of mathematics education at Junior Cycle level in Ireland failed. Close (2006) analysed the terminal examination at the end of the Junior Cycle and found that only four of the ten objectives of the revised mathematics Junior Cycle syllabus were actually being examined. “Objectives relating to mathematics in unfamiliar contexts, creativity in mathematics, motor skills, communicating, appreciation, and history of mathematics” were not formally assessed (Close 2006, p.62). These objectives were promptly ignored by the majority of teachers as they were not seen as valuable. Once again, in contrast in the Netherlands, the influence of the RME approach is seen in the form questions take in the Dutch national exam at the end of second level (Appendix A). This provides a signal to the stakeholders that this approach to teaching is valuable in mathematics education and so is used in the classroom. Elwood and Carlisle (2003, p111) stated their belief that the current Irish examination structure promotes a view of mathematics “that does not sit comfortably with the aims and objectives outlined in the syllabuses on which the course of mathematics ..... are based”.

2.7.5 Conclusion – Superficial or Real Reform

In conclusion, new definitions of learning and teaching have come to be accepted among the wider educational community as well as by mathematics educators. One of the major issues facing mathematics education today is to ensure that the curriculum reforms that have ensued, succeed. To achieve more than this superficial reform, Conway and Sloane (2006) argue that the curriculum culture, textbooks and modes of assessment and examination all have to undergo changes. As contrasting examples the author showed how the Irish reform of Junior Cycle mathematics (2002) failed as it essentially ignored all three, but on the other hand Dutch reform of its mathematics curriculum succeeded as it dealt with all three points. In order to subvert the status quo it is important that the classroom culture is attacked on all three fronts. The resource pack created addresses the issue of resource materials being appropriate and consistent with the intended reform. By extension these resource materials can be used to influence teachers’ classroom behaviour and the culture of the classroom.

2.8 Conclusion

“To participate in a subject the following is required: - to engage in its form of enquiry, to apply the knowledge, to use the language of the subject and to think in the subject”

(Callan 1997, p.21)

As discussed, a behaviourist ‘New Maths’ approach is currently prevailing in Irish mathematics classrooms. The current curriculum reform, Project Maths, is clearly inspired by the “social turn” (Lerman 2001) that mathematics education research has
taken. The ideas of Hans Freudenthal first promoted in “Mathematics as an Educational Task” (1973) have heavily influenced thinking in curriculum development around the world and can be detected in national curriculum reviews such as “The Committee of Inquiry into the Teaching of Mathematics” (Cockcroft 1982) and “The Curriculum and Evaluation Standards for School Mathematics” (NCTM 1989). These reviews have not been just concerned with looking to the problems created by the current approach but looking forward at the needs of students in the future. In 2003 Kathleen Lynch was warning of the need to “examine what kind of mathematics education is best for a student’s future life as a citizen and a worker in this ‘global village’” (Lyons et al. 2003, p.1). Project Maths, is the NCCA’s answer to that warning.

The author has established, on examining the literature, that the behaviourist approach, so common place in Irish classrooms, has led to poor attitudes among students (English et al. 1992) and a lack of any real conceptual understanding (Romberg and Kaput 1999). There are important benefits to be gained from creating improved attitude towards mathematics (Carpenter et al. 1998, Cobb et al. 1991, Fauzan et al. 2002; Van Reeuwijk 1992; Verschaffel and De Corte 1997) and teaching students in a manner that promotes genuine conceptual understanding (Carpenter and Hiebert 1992; Carpenter and Lehrer 1999). The promotion of a socio-cultural outlook is important because of its emphasis on the creation of positive attitudes and an enhanced conceptual understanding. However, the author also established that reforming mathematics education in the classrooms is not easily achieved (Sarason 1993; William 2003). The failed reform at Junior Cycle level in 2000 was discussed and contrasted with the successful mathematics reform in Holland. The Irish attempt
was an example of what Fullan (1991) would describe as “superficial reform” as opposed to “real change” because it did not tackle the issues of classroom culture, textbooks and examinations.

This review of the literature had a significant impact on the author’s response to the problems experienced within his probability classroom. It was decided that any intervention could not follow the behaviourist approach but would instead adopt an approach influenced by the socio-cultural outlook, which involved students’ working together on activities and questions, which were of relevance to them. The resource pack would provide a set of materials which would allow the author to teach in a manner that promoted positive attitudes and real understanding. The need for resource materials created by teachers if Project Maths is to succeed also informed the authors’ approach. Project Maths is a radical change to mathematics education in Ireland but will struggle unless steps are taken from the “bottom-up” (Fullan 1991). The resource pack will help address the important factors of textbooks and classroom culture by providing teachers with resource materials which will ease them out of their pedagogical comfort zone and into an approach which emphasises student understanding and promotes positive attitudes towards mathematics. The resource pack specifically addresses the area of probability. After reviewing the literature on mathematics education in general, the author will now look at the literature on probability and examine how the research will inform the creation of the resource pack. Probability is an area of mathematics with its own distinct and colourful history but long associated with pedagogical difficulties, which will be discussed in the next chapter.
Chapter 3 – Teaching Probability

3.1 Introduction

“A major challenge to the field is harvesting what is known from this research to inform teaching”

(Greer and Mukhopadhyay 2005, p.315)

The resource pack provides teaching materials exclusively for the area of probability and in particular for the sub-topics of expected value, conditional probability and binomial distribution. The author has chosen this as the focus of the resource pack, not despite its relative newness as a branch of mathematics but perhaps because of it. It is a fresh and exciting topic but it also holds fears for teachers as they leave the comfort of the familiar. The tentative beginnings of probability as a field of study can be traced back a mere five hundred years, which pales in comparison with areas such as geometry and algebra, which have a history stretching millennia. In recent years probability has been pushing itself into mathematics curricula internationally despite a reputation among teachers as being difficult to teach. The intention of this chapter is to examine the research on probability teaching and learning and see how it is influenced by the wider issues discussed in Chapter 2, and how it influenced the construction of the resource pack.

Probability has unique pedagogical approaches, concepts and difficulties (Cobb and Moore 1997; Shaughnessy 1992; Fischbein, Nello and Marino 1991; Ahlgren and
Garfield 1988; Hawkins and Kapadia 1984) and it is important that any resource materials on the topic are cognisant of these issues and are formed in the light of the information provided by the research. Why, for example, is probability growing in significance within mathematics curricula world-wide yet a significant proportion of teachers nationally and internationally hold negative views of it? (Gattuso and Pannone’s 2002; Stohl 2005) The author first intends to look at the area of probability, its origins and uses as a mathematical discipline and the rationale behind the relatively recent inclusion of it in mathematics curricula internationally.

3.2 A History of Probability

3.2.1 Introduction

An understanding of the origins of probability as well as its development should afford us insights into its current uses and standing, in addition to its unique relationship with statistics. It is impossible to give a full or insightful historical background of probability as a mathematical topic within mathematics education research without giving some detail on statistics. This relationship is so strong that the term “stochastics” is commonly used by many in the mathematics world to refer to the study of statistics and probability (Shaughnessy 1992; Borovcnik and Peard 1996). They are viewed as one entity, developing as a mathematical discipline together, as well as being researched and taught underneath one umbrella (Shaughnessy 1992; Borovcnik and Peard 1996). The term “stochastics” is used interchangeably with the phrase “probability and statistics” throughout this document.
3.2.2 Probability

Probability theory originated with games of chance. Records exist, which show gambling on dice and card games for millennia. Roman soldiers famously gambled using an astragalus, a bone from the heel of sheep, while the Babylons used something very close to a modern dice as far back as 3,000 B.C (Chatterjee 2003). He also notes that probabilistic ideas are mentioned in Jewish Talmudic literature up to the 5th century AD and there is even a cryptic reference in the “Mahabharata”, an epic of Indian literature, which has its origins in the 5th century BC, which suggests some knowledge of probability and sampling. A 13th century French poem, called *De Vetula* which describes players betting on the sum of 3 rolled dice makes the oldest known connection between observed frequencies and probabilities. “Sixteen compound numbers are produced. They are not, however, of equal value, since the larger and smaller of them come rarely and the middle ones frequently” (Bellhouse 2000, p.134). The poem then proceeds to link probabilities to the 216 entries in, what we today call the sample space, as well as referring to the idea of expected value. This is an extraordinary display of probabilistic knowledge for medieval times.

The first evidence of probabilistic thinking emerges in Italy when the well known gambler Gerolamo Cardano wrote a book called “Liber de Ludo Aleae” (The Book on Games of Chance) in 1526 (David 1955) or 1564 (Chatterjee 2003). Whichever date, it was not published until 1663 long after his death. In it, Cardano makes various calculations and computations which clearly show an understanding of the principles of probability. Galileo also studied dice. Around 1620 the Grand Duke of Tuscany posed a problem to him based on a game similar to that mentioned in *De Vetula*,...
reaching similar conclusions to Cardano. It was not until in 1654, however that “probability was registered as a new born member in the family of mathematics” (Chatterjee 2003). A French nobleman, the Chevalier de Mere approached Blaise Pascal with some gambling puzzles. The resulting correspondence between Pascal and fellow French mathematician, Pierre de Fermat in which they solved many of these puzzles, put probability on a mathematical footing for the first time. It was left to a Dutch contemporary of the Frenchmen, Christian Huygens in his book “De Ratiociniis in Alae Ludo” (Computations in games of chance) in 1657 to set out their findings along with his own in a systematic manner. David (1955) argues that this along with his vision in seeing that probability had applications wider than just games of chance should earn Huygens the title “The Father of Probability Theory”.

3.2.3 Statistics

Though it is tempting to trace statistics back to ancient times it would be more apt to say that statistics, as we understand it today, has its origins in the seventeenth century. Data may have been collected in a large scale in the earliest civilisations such as ancient Egypt or in biblical times, when Augustus ordered a census of the empire, but to cite them or the Doomsday book of the middle ages as an early example of modern statistics is flawed (Hald 1990; Kendall 1960). Statistics however did evolve from such actions. The word statistics originates from the Italian word *statista*, which was someone who dealt with affairs of state and had knowledge of the characteristics of the state such as its population, geography etc. To claim these actions as statistics however, “is to fail to understand either the basis of the statistical approach or the nature of the statistical method” (Kendall, 1960, p.447). Moore (2000) asserts that
there are two key features to statistics as we know it today, data and inference. So while these biblical or medieval examples certainly involve collecting data and its tabulation they do not engage the other key concept of modern statistics – inference (Bessant and Macpherson 2002).

The origin of statistics in the modern sense can be traced to John Graunt’s (1662) `Natural and Political Observations on the London Bills of Mortality’ (Chatterjee 2003; Hald 1990). From 1604 onwards `Bills of Mortality` were published weekly by every parish in London detailing the number of births (christenings) and deaths (burials) held in the parish that week. Sex and cause of death were also listed. This gave Graunt (1662) a huge archive of source material from which he made conclusions on various demographic issues – such as, an estimation of the population of London, the composition of the population age-wise – a so called life table, variation in intensity of the plague, rural migration into the city and more (Chatterjee 2003; Hald 1990). This work lead to Graunt founding “The universal registration of births, marriages and sources of death in England. Shaughnessy, Garfield and Greer (1996, p.206) describe this as “the first such enterprise for purposes of state”. Thus the emphasis in statistics changed from simply amassing data for administration purposes to using this data to make inferences and conclusions.
3.2.4 Dual Development

Hacking (1975) explains how it was Huygens (1657), who first saw upon looking at Graunt’s (1662) work that probabilistic theories could be applied to statistical information. He demonstrated this using Graunt’s (1662) life table in 1669. So as Hacking (1975, p.109) stated, it was now for the first time that “problems about dicing and about mortality rate had been subsumed under one problem area”. Developments in the areas of statistics and probability were now linked together. James Bernouilli spent considerable time working on “theorem aureum” (golden theorem) or proving what we would now call the law of large numbers, which shows that relative frequencies converge to an underlying probability. This was published posthumously in 1713 in his book The Art of Conjecture. Thus he expanded the use of probability further, beyond its initial use in games, into working with data from the natural world. He showed that there are hidden probabilities in natural events, which manifest themselves in the data sets they produce, thus he underlined the ties between statistics and probability (Mlodinow 2008; Hacking 1975). His philosophy was that everything is ruled by a deterministic law, dice, the weather etc., and we only consider that an event occurs by chance because we do not understand the complicated process behind it (Borovcnik, Bentz, and Kapadia 1991).

In 1710, Arbuthnot used probabilities to, in what could be called a precursor to statistical inference, deduce as “God’s will in action” the fact that for the previous 80 years of birth statistics in London, more boys had been born than girls. This was backed up by his statement that the chances of more boys than girls being born in any one year should be a $\frac{1}{2}$. So the probability of this happening for 80 years in a row was
- a probability so small that to him the conclusion was obvious, this could only be
due of a higher power at play (Stigler 1986). This is a form of hypothesis testing that
is sometimes used but today only if comparing competing statements and it would not
be considered as proof.

Thomas Bayes (1774) an English preacher, whose work was also published
posthumously, developed the idea of conditional probability (Stigler, 1986). Bayes’
work was used as a basis for the inclusion of an individuals’ subjective view into the
calculation of probability, hence it is often referred to as Bayesian Probability.
LaPlace’s (1814) work over four decades began the process of inference, as we know
it today. Previously mathematicians worked out the theoretical probability in advance
but LaPlace was now working on the probability of getting specific results from
specific samples (Mlodinow 2008). He improved on the work of DeMoivre (1738)
and Gauss (1810), who had developed the idea of the continuous, normal and binomial
distributions (Rosenthal 2006). From this he developed the central limit theorem and
also gave us the classical definition that the probability of an event is the number of
favourable outcomes divided by the number of possible outcomes. Probability and
statistics were now becoming firmly entwined as essentially different sides of the one
mathematical coin.

Stochastics continued to develop through efforts of many more great mathematicians
such as Quetelet (1835), a Belgian, who produced evidence of the bell-curve in a
myriad of situations beginning with crime in France. From this he produced the idea
of “L’Homme Moyen” – the average man, which moved stochastics further away
from statecraft towards studying collective behaviour (Bellos 2010). Galton (1869), Darwin’s first cousin, studied hereditary traits. His studies produced the stochastical ideas of regression and correlation. His work also lead to the formation of a finger-print bank at Scotland Yard in 1901 (Mlodinow 2008) and unfortunately, to the development of the idea of Eugenics, which the Nazis adopted to give their idea of a super-race some quasi-scientific backing (Bellos 2010). Karl Pearson Galton’s protégé, continued his work on correlation and also developed the chi-square test which allowed mathematicians to examine whether data followed the distribution, they thought it did. He founded the world’s first university statistics department at the University College London in 1911 (Bellos 2010).

The continuing overlapping developments in both probability and statistics lead to the incorporation of statistical inference and probability into essentially one distinct discipline in the first half of the twentieth century (Pfannkuh 2005; Stigler 1986). In 1933, Kolmogorov succeeded in finally laying down a generally agreed upon set of axioms for probability, in the same way that they existed for geometry and algebra for hundreds of years (Bingham 2000). So while still a young topic when compared with topics such as geometry and algebra, stochastics had established itself in the wider mathematical world as a unique and interesting field its own right. The relative newness of statistics and probability means that new tools and applications are being continually developed and applied. Today probability is used in a multitude of scenarios. It is still used where it originated in games of chance but is now used in the worlds of insurance, banking, medicine, crime-detection, weather prediction, manufacturing, the travel industry, polling and financial decision making. This
explosion of applications in the real world has been accompanied by a gradual and sometimes bumpy introduction into the world of mathematics education.

3.3 Probability and Mathematics Education – A Rationale

3.3.1 Introduction

“There is no science more worthy of our contemplations nor a more useful one for admission to our system of public education”

(LePlace, 1825, cited by Batanero, Henry and Parzysz 2005, p.15)

The teaching of stochastics has been a major topic of discussion among mathematics educators around the world, particularly in western countries. This discussion has led to calls for statistics and probability to play a greater role in our secondary school mathematics curricula. Despite its relative newness to the mathematical world in 1948, John Wishart (p.215) was already posing the question “should an attempt be made to introduce an elementary course of instruction into the normal curriculum of the secondary school”. These isolated calls became part of a “vigorously growing movement” (Ahlgren and Garfield 1988, p. 44) gaining in significance from the 1960’s onwards with the inclusion of some statistics and probability in the Netherlands and the UK from 1963 and 1969 respectively. The publication of the NCTM yearbook in 1981 Teaching Statistics and Probability was another milestone, a signpost that probability and statistics were moving centre stream within mathematics education.
Since then probability has gained in importance as a topic. Internationally important syllabus reviews such as “The Committee of Inquiry into the Teaching of Mathematics” (Cockroft 1982), “The Curriculum and Evaluation Standards for School Mathematics” (NCTM 1989), its replacement “The Principles and Standards for School Mathematics” (NCTM, 2000), “Ready or Not: Creating a High School Diploma That Counts” (American Diploma Project, 2004) have placed probability and statistics, alongside numeracy, measurement, algebra and geometry in importance within school curricula. The new “Project Maths” curriculum in Ireland regards “statistics and probability” as one of five key strands of the curriculum. This major promotion within second level curricula would not have occurred without sound educational reasoning. The rationale for its promotion can be broken down into three broad headings; benefits to the mathematics curriculum, the wider benefits within education and thirdly, functionality or uses in everyday life (Greer and Mukhopadhyay (2005) and Pereira- Mendoza and Swift 1981).

3.3.2 Benefits to the Mathematics Curriculum

Probability allows for the use of two highly prized motivational tools in mathematics today – context and activity. Teaching stochastics can be relevant and motivating for students (Carr 2008). An argument exists that stochastics is different to other branches of mathematics because it is not just about numbers, but about numbers within a context (Scheaffer Watkins and Landwehr 1998). Whether this position is correct or not, statistics and probability are on the mathematics curriculum and the context inherent in stochastical questions is extremely valuable. Through using these naturally occurring contexts mathematics teachers’ can make mathematics relevant to students’
lives and excite their imaginations. As discussed in Chapter 2, inspired by Hans Freudenthal’s “Realistic Maths Education” (RME), mathematics curricula throughout the world are shifting towards the use of context to help motivate students in the classroom (Conway and Sloane 2006).

The Cockcroft report (Cockcroft 1982, p.234) states that stochastic “is essentially a practical subject” and like other practical subjects active learning should be intrinsic to the stochastic classroom. The opportunity to incorporate activity into the teaching of statistics and probability is huge. Whether through the collection, analysis and presentation of data or through simple experiments with dice and cards, statistics and probability lend themselves to investigation and exploration. Statistics and probability enables students to experience mathematics through action often in small groups in an enjoyable environment. This trait is an undoubted attraction of probability within mathematics classroom with a socio-cultural outlook. As discussed in Chapter 2, attitudes and understanding have suffered among students due to a traditional, behaviourist, “chalk and talk” rote learning focus (Gnanadesikan, Scheaffer, Watkins, and Witmer 1997). The idea of harnessing the motivational tools of context and active learning is central to the socio-culturally influenced resource pack developed as part of this study. Despite its huge potential, probability as it is currently taught in second level is a dry affair lacking any sense of usefulness. The resource pack aims to improve the situation in the probability classroom, hopefully helping to improve and develop students’ general attitudes towards mathematics and conceptual understanding of probability.
3.3.3 Wider Educational Benefits

As far back as 1959, Pieters and Kinsella were arguing for the need for a stochastical education. Bessant and MacPhersson (2002, p.24) cite them as describing how this area “is rapidly becoming more and more essential to both the general education of all citizens and to the vocational preparation of an increasing number of future specialists”. Today subjects such as geography, business and all the science subjects are replete with statistical and probabilistic ideas (Carr 2008; Vere-Jones 1995). A knowledge and understanding of these ideas is more than useful - it is simply necessary if one is to successfully study those areas. There are also huge possibilities for cross-curricular links with I.T. (Biehler 1991; Nooriafshar 2001; Noorifshar 2003; Engel 2002). At third level, modules in stochastics courses across many faculties are now commonplace. Statistics and probability therefore, need to be dealt with satisfactorily at a pre-college level before specialisation occurs at third level (Shaughnessy 1992). Once again, the emphasis placed on understanding rather than formula-filling in the resource pack should enable students to transfer the concepts learned in the probability classroom into these wider areas of study. This enhanced ability to transfer knowledge learned in one area to other areas is one of the benefits of teaching for understanding, as listed by Carpenter and Hiebert (1992) in Chapter 2. Understanding thus developed, allows students a greater insight into these subjects.
3.3.4 Functionality – Uses in Everyday Life

“Both professionals and lay adults from all walks of life have to interpret, react to, or cope with situations that involve probabilistic elements or different levels of unpredictability”

(Gal 2005, p.40)

We are living in an information age and this “makes it imperative for students to develop conceptual and practical tools to make sense of that information” (Shaughnessy et al. 1996, p.207). It is an age where this information is usually accompanied by statistics or probabilistic statements – “nine out of ten cats prefer Whiskas”. People are bombarded daily with such statements. What do these stochastic statements really mean? Can or should they be taken at face value? To process this information, to comprehend what has become part of the language of the everyday a good understanding and knowledge of statistics and probability is needed (Pereira-Mendoza and Swift 1981).

In an era of spin, stochastical statements are used to persuade us, confuse us and make us part with our money (Carr 2008). Whether from advertising companies or politicians, stochastical arguments are used in the selling of a product or a political point. Often these companies/politicians and their rivals both seem to have stochastical statements which back up their opposing pitches. They can’t both be right or can they? We as both citizens and consumers need to be able to decipher what we are being told and make our choices accordingly. As consumers of huge amounts of statistics and probabilistic statements, a stochastical literacy needs to be developed.
among the general population. Gal (2005, p.40) defined a literacy in probability as “the knowledge and dispositions that students may need to develop to be considered literate regarding real-world probabilistic matters.” If we are to develop informed citizens and responsible consumers, this kind of literacy is of the utmost importance (NCTM 1989; Shaughnessy 1992).

The resource pack acknowledges this need and seeks to address it by developing students’ understanding so it can be applied in situations other than those presented in the textbook and by placing questions within contexts that they will experience it, in real life. Despite this strong rationale for the inclusion of probability and statistics it has not been greeted into mathematics curricula with the unanimous acclamation within the mathematics education literature or by teachers.

3.3.5 Uneasy Relationships - Stochastics and Mathematics

The exact nature of the relationship between stochastics and mathematics has been discussed and debated liberally. Many have fought against the idea that stochastics belongs within the field of mathematics arguing instead that it is a science or field of study in its own right. Moore (1988) and Iverson (1992) among others have argued that the perception of stochastics as belonging to the field of mathematics is merely an illusion, a sham. Iverson (1992) and Scheaffer (2006) both refer to the relationship between mathematics and stochastics as being similar to a “marriage” of convenience. As you will read in more detail, both take differing views, Iverson (1992) focuses on the uneasiness, which exists between the two while Scheaffer (2006) though
acknowledging that problems exist, instead positively focuses on the mutual benefits of the inclusion of stochastics within primary and second level mathematics education.

Those leading the argument of differentiation, view stochastics as a separate discipline (Scheaffer 2006; Rossman, Chance and Medina 2006), which like other areas such as the science subjects uses mathematics heavily. It is a branch of study in itself, mathematical in nature, maybe, but not mathematics. They argue reasonably that, stochastics and mathematics differ in two main ways. First of all, the focus in stochastics is not just on using the data but interpreting it, and secondly the importance of the context within which the information is placed. The same data with differing contexts can lead to differing interpretations in stochastics, while in mathematics a change of context still gives rise to the same answer as long as the data remains unchanged (Cobb and Moore 1997; Rossman et al. 2006). This results in very different mindsets in mathematicians and statisticians. Mathematicians’ reasoning is deterministic – their thinking runs along the lines of, “there is a definite reason for this and here is the proof”. Whereas statisticians’ thinking is probabilistic – their thinking takes the form of “this result is probably caused by this factor or by a combination of these factors” (Scheaffer 2006).

These arguments are certainly valid and have a certain degree of logic to them; however, for the moment it is merely an interesting argument among mathematics educators. It has not been proposed seriously within the literature that statistics should be taught as a separate subject with the equivalent status of physics or chemistry at
primary or second level. Even the separatists acknowledge that at these levels “statistics cannot prosper without mathematics” (Rossman et al. 2006, p.323). It is generally agreed that the area of probability and statistics, though unique, is at home within the mathematics curriculum. Within the field of stochastics itself, however, the relationship between statistics and probability is also an uneasy one as each seeks to define the role it should play within the curriculum (Biehler 1994).

3.3.6 Uneasy relationships – Statistics and Probability

“I agree that only the probability essential for understanding statistics should be taught”

(Scheaffer, Watkins and Landwehr 1998, p.157)

Traditionally, when taught - stochastics has generally been broken down into three areas. Classically this has tended to be, descriptive statistics, probability theory and inferential statistics, in that order and often with little interplay between the three. At second and third level there has been a sluggishness in changing this status quo. This is partly a result of differing opinions on the value of aspects of stochastics. Some modern pedagogical approaches to the teaching of statistics and probability such as, Exploratory Data Analysis (EDA) or Data Handling – emphasise statistics and sideline probability only to those areas that are felt necessary to help understand statistics. Probability is being “turned into a tool that is used to approach problems that have arisen from statistical experiences” (Batanero et al. 2005, p.31). It is argued that probability should be minimised only to what is necessary to allow students grasp statistical concepts and that the focus should be on data analysis. They feel that many of the difficulties that students encounter in the area of probability have held statistics
back. Both Cobb and Moore (1997) and Ahlgren and Garfield (1988) suggest that an informal grasp of probability is all that is necessary for a conceptual understanding of statistical inference and that probability should be taught with the sole aim of enabling furthering statistical understanding.

Others feel that probability can play a role as the central core of stochastics. They argue aside from being crucial in the conceptual understanding of statistical inference (Batanero and Sanchez 2005) that probability develops its own distinctive approach to thinking and that students should be encouraged to think in this manner (Borovcnik and Peard 1996). While it is necessary, if students’ are to develop a conceptual understanding of statistical inference it offers more than that to students. Gal (2005) cites the importance of conditional probability in the real world as an example of why probability topics cannot be overlooked. The author would agree with the “complementarist” approach advocated by Biehler (1994) in his address to the International Conference on Teaching Statistics 4 (ICOTS 4) in which he argues that neither probability nor statistics should dominate the other but be taught in a manner in which each one enables a better understanding of the other.
3.3.7 Conclusion

There may be academic disagreement over the role probability plays within a stochastics education and over the placing of stochastics within mathematics curricula but it is agreed that probability has major educational benefits. The three most important being the benefits it brings to the mathematics curriculum (Carr 2008; Gnanadesikan et al. 1997), the benefits it brings to wider education (Bessant and MacPhersson 2002; Shaughnessy 1992) and thirdly, its increasing importance in everyday life (NCTM 1989; Shaughnessy 1992). This educational payoff has led to a dramatic increase in the profile of probability in international mathematics curricula.

The resource pack has attempted to harness these benefits by exploiting the experimental and practical side of probability to improve attitudes towards mathematics in general. These same active learning methods will enable students to develop a conceptual as well as a procedural knowledge of the topic. The use of questions in context also improves attitudes as it allows students to see its usefulness in everyday situations as well as in other fields of study. The questions in context also enable students’ to demonstrate that their understanding has gone beyond the procedural and can be used in unfamiliar situations. This in turn increases students’ confidence in their mathematical ability.

Internationally the process of introducing and enhancing probability within mathematics curricula has not been all plain sailing either at primary or second level. This is not due to academic arguments over whether stochastics is a part of mathematics or not, or whether it should be sidelined only to be drawn upon when needed by statistics, but rather practical difficulties at classroom level. Shaughnessy
(1992) stated that once stochastics had entered mainstream mathematical education there was a need to enhance teachers’ knowledge and confront students and teachers’ beliefs. He felt that if these obstacles were overcome it would lead to an improvement in stochastics teaching. The author has broken Shaughnessy’s “barriers for improvement of stochastics teaching” (1992, p.467) into three separate yet interlinked issues

- The best pedagogical approach for the teacher to adopt in the classroom,
- The unique conceptual difficulties facing students, and
- Teacher knowledge.

Each of these three practical considerations and how the resource pack overcomes and addresses them will be examined in the next three sections of this chapter.

3.4 Pedagogical approaches to Probability

3.4.1 Introduction

“Stochastics is really a mathematical theory about errors, deviations and interrelationships between an idealised forecast and real fluctuating behaviour”

(Steinbring 1991, p.147)

There are three differing philosophical views on how probabilities should be approached and calculated, the classical, the frequentist and the subjective approach (Albert 2003; Shaughnessy 1992; Chernoff 2008). These philosophical positions have a significant impact on the pedagogical approach adopted in the classroom. The
classical approach of theoretical probability, which is in tune with behaviourist ideas, has dominated curricula worldwide but in recent times a more than passing acknowledgement has been made both to the frequentist and subjective viewpoints in curricula internationally. If educators are to truly encourage an appreciation of stochastics, as Steinbring (1991, p.147) defines, it then the “idealised forecast” provided for by the classical approach will have to be mixed with the frequentist approach which offers the opportunities to experience the “real and fluctuating behaviour.” The resource pack adopts a mixture of philosophies, in a stance that is a pragmatic fusion rather than following a strict philosophical view.

3.4.2 The Classical View

In the classical view probability is viewed in terms of LaPlace’s (1774) definition; when presented with a sample space of equally likely outcomes, the probability of an outcome is defined as the number of favourable outcomes divided by the number of possible outcomes. This is known as a Priori approach – in that the probabilities are calculated theoretically, before or without any trials being carried out. It has the advantage of being easily understood and applicable to simple games of chance involving cards and dice for example. This approach can only be used when continuing into more difficult areas of probability where single outcomes in a sample space are not equally likely (Albert 2003). Bernoulli, one of the first major mathematicians to propose an alternative approach, noted such examples as epidemics and the weather.
This approach has become part of the traditional pedagogy when teaching probability in second level classrooms. It suits the behaviourist, “New Maths” axiomatic approach, students learn the formula:

\[
P = \frac{\text{No. of favourable outcomes}}{\text{No. of possible outcomes}}
\]

Students then substitute the given information into the formula and the question is answered. This type of pure behaviourist approach was discussed in chapter 2. The rules, routines and procedures are emphasised divorced from relevant contexts to the detriment of genuine understanding. Unfortunately this approach has led to significant difficulties. Probability in Ireland has become a topic to be avoided. According to the last two Chief Examiners’ reports on Higher Level Leaving Certificate mathematics, the probability and statistics questions were the least popular questions on Paper 2.

In 2000 (p.16) the Chief Examiner commented that “A drift away from discrete mathematics and statistics (questions 6 and 7).... has been noted in recent examinations” and though this had “stabilised” by 2005 (p.71) these questions were still the two least popular questions in Paper 2. There has been no study into the cause of this trend but the pedagogical approach favoured by teachers has at the very least done nothing to halt it. Probability is a topic where a lack of conceptual understanding is quickly exposed in examinations and so students found the examination questions difficult as they did not have any real conceptual understanding of the topic but rather had learnt off the rules or axioms. The appearance of numbers larger than 1, as probabilities in students’ answer scripts is one example of the lack of understanding in the basic concepts of probability. The resource pack will be useful to
the teacher in the classroom providing activities and contextualised questions, which build students’ relational knowledge.

3.4.3 The Frequentist View

This approach in contrast is a posterior one – in that the probabilities are calculated solely based on trials that have been carried out or already happened (Albert 2003). This is essentially experimental probability, where probabilities are calculated in terms of relative frequency. The probability of an event is obtained by repeating an experiment a large number of times and counting the number of times a particular result appears. The probability is the limit towards which the relative frequency tends. There are some criticisms of this viewpoint, it does not provide for the probability of an event where it is impossible to repeat the experiment a large number of times, though this can be overcome by simulation and modelling. Batanero et al. (2005, p.22) claim that “the most significant criticism of the frequentist definition of probability is the difficulty of confusing an abstract mathematical object with the empirical observed frequencies, which are experimentally obtained.” That is, students using this approach sometimes struggle to understand that a small number of experiments can produce misleading results and not necessarily, a correctly forecast probability.

Accepting that these are valid criticisms, it is necessary however that students’ understand the idea that probability is a limit of a stabilised frequency and an element of experimental probability is necessary in the classroom to demonstrate this.
effectively (Lane 2009). The frequentist approach by definition necessitates experimentation. This experimentation provides scope for active learning methodologies to be employed in the classroom. This is consistent with the constructivist based cognitive and socio-cultural theories. The socio–culturally influenced resource pack, brings in elements of the frequentist view incorporating experiments, investigations and simulations which involve students collecting, analysing and drawing inferences. Each of the three topics has two or more suggested questions which, involves this type of active learning. Active learning techniques allow students to develop their understanding beyond simply filling in the formula and improves students’ attitudes and motivation (Cockroft, 1982; Threlfall 2004).

3.4.4 The Subjectivist View

In this approach, probabilities are essentially personal calculations where people’s opinions become part of the mathematical calculations (Chernoff 2008). As well as taking both prior and empirical probabilities into account, it also includes peoples’ personal viewpoints. It is heavily linked with Bayesian probability. Its followers would argue that this form of probability is closer to peoples’ intuition, so more easily understood. Confusingly, however, since probability becomes personal what was random to one person may not be random to another or peoples’ opinions could simply differ, leading to differing probabilities being calculated. This view of probability is largely in line with the views of Ernest Von Glasersfeld (1991, 1995), the founding father of radical constructivism. Proponents of radical constructivism similarly to other constructivists discussed in Chapter 2, believe that students actively build their own knowledge. They however believe essentially that this knowledge,
which is formed from experiences, is valid even though it may or may not reflect reality.

‘radical constructivism ... is an unconventional approach to the problem of knowledge and knowing. It starts from the assumption that knowledge, no matter how it is defined, is in the heads of persons, and that the thinking subject has no alternative but to construct what he or she knows on the basis of his or her own experience. What we make of experience constitutes the only world we consciously live in. It can be sorted into many kinds, such as things, self, others, and so on. But all kinds of experience are essentially subjective, and though I may find reasons to believe that my experience may not be unlike yours, I have no way of knowing that it is the same.’

(Von Glasersfeld 1995, p.1)

Proponents of the subjectivist view claim that basing curricula solely around classical and frequentist notions is a mistake as these may “conflict with the children’s expectations and intuitions” (Hawkins and Kapadia 1984, p.372) and the subjectivist viewpoint naturally incorporates these intuitions. The fact that students’ think about probability subjectively, however, does not mean that students are subjectivists from a philosophical viewpoint (Ahlgren and Garfield 1991). It is important to understand our students’ conceptions and intuitions of mathematics and use this as a starting point, this does not mean we should base our pedagogical approach on the subjective viewpoint. This would lead to a “pedagogical nightmare” (Shaughnessy 1992, p.488) as the focus of much of teaching in probability is on “how theory is different from
subjectivist intuitions and why it is important to learn these theoretical ideas” (Ahlgren and Garfield 1991, p.121).

3.4.5 Conclusion

“The conceptual root of the pedagogical power that we gain from having students conduct simulations is the connection that they can make between the observed variation in data in repeated trials of an experiment, and the outcomes that they can expect based on knowledge of the underlying sample space or probability distribution”

(Shaughnessy and Ciancetta 2002, p.5)

The NCTM “Standards” (2000, p.48) said simply that students should be able to “understand and apply basic concepts of probability.” While it is clear that each approach has its drawbacks, the traditional classical approach has failed dramatically to achieve this. School textbooks and resource materials have been dominated by this approach. In Ireland, it has achieved little as students fled from the topic at examination time. (Chief Examiner 2000 and 2005) A pure experimental approach alone, however, is not sufficient either (Batanero et al. 2005). It fails to give students adequate appreciation of the fact that these experimental probabilities are only tending towards theoretical limits. It is obvious that the subjective view is unfeasible in second level mathematics (Shaughnessy 1992). The difficulties which exist with each approach, has pushed those developing curricula to form compromise solutions.
Most mathematics educators now feel that teachers should “avoid developing stochastics exclusively by means of relative frequency or via classical symmetry” (Steinbring 1991, p.144). They take the pragmatic view, that it is neither feasible nor advisable to abide strictly by one philosophical approach but rather that teachers should adopt the approach which suits the students’ needs in that lesson (Shaughnessy 1992; Ahlgren and Garfield 1991). Steinbring (1991), and Kavatinsky and Even (2002), argue vigorously in favour of mixing the classical and frequentist approaches. In real life, probabilities are not calculated strictly following one view or another but rather incorporate different approaches where needed. Gal (2005) gives the example of whether someone should or should not get life insurance, given personal factors such as their weight, they just had a child, your family’s medical history etc. One approach will not help you answer this question, a mixture is required.

The curricula of the current reform movement including “Project Maths” mix the frequentist with the classical approach, largely ignoring subjective probability at second level (NCCA 2010). This approach is recommended by Shaughnessy and Ciancetta (2002) in the opening quote of this conclusion. In creating the resource pack, the author adopted a similar approach linking the active learning methodologies and contexts, which are part of the frequentist approach and consistent with the socio-cultural views of education, with the constancy of the classical approach to enhance student’s conceptual understanding and improve their attitude towards the topic.
3.5 Difficulties in Teaching Probability

3.5.1 Introduction

“Probability is conceptually the hardest subject in elementary mathematics”

(Cobb and Moore 1997, p.821)

Probability has been viewed with some aversion by many in the teaching profession worldwide. Fischbein (1990, p.54), perhaps somewhat extremely but, pointedly stated that “we are afraid of probability” (cited by Greer and Mukhopadhyay, p.320). This aversion is exemplified in Ireland by their students’ choice of questions in the Leaving Certificate examination, which takes place at the end of second level. There are two papers in the exam, essentially consisting of eight questions from which the student chooses six. The probability and statistics questions appear in Paper 2. These two questions are the least two popular questions on the paper (Chief Examiner 2000, 2005). Anecdotally in Ireland, one of the main factors contributing to the poor reputation of probability among students and teachers is its unique conceptual and technical complications. This is in line with international research. The weakness of both teachers and students’ probabilistic intuitions is well documented (Shaughnessy 1992; Fischbein, Nello and Marino 1991; Ahlgren and Garfield 1988; Hawkins and Kapadia 1984). The author believes that many of these conceptual difficulties, which I will discuss in the forthcoming paragraphs, can be overcome by the employment of experiential learning in the classroom as well as the use of real life examples to force students to confront their probabilistic intuitions (Garfield 1995; Shaughnessy 1977).
Some mathematics educationalists advocate a “Data Analysis” approach which minimises probability to that which is necessary to teach statistics (Cobb and Moore 1997; Ahlgren and Garfeld 1988; Scheaffer et al. 1998). This is essentially a ‘head in the sand approach’ – if it cannot be seen, then there is no problem. Unfortunately this approach ignores the benefits outlined earlier in the thesis as well as diminishing students’ understanding of inferential statistics. Difficulties do exist but the advantages of a complementary approach when teaching probability and statistics outweigh the disadvantages of minimising one in favour of the other (Biehler 1994). The failure to confront, clarify and enhance these intuitions does not serve the students’ interests. These difficulties have to be addressed and the pack does this by, employing activities and questions in contextualised settings. The difficulties have been broken down under the sub-headings of language (Rossman et al. 2006; Gal 2005) specifically mentioning the “equiprobability bias”, (Lecoutre 1992; Pratt 2000 and Watson 2005), paradoxes and fallacies and the counter-intuitive problems of the “gamblers fallacy” and the “outcome approach” (Konold, Pollatsek, Well, Lohmeier, and Lipson 1993; Konold 1995; Fischbein and Schnarach 1997; Ahlgren and Garfield 1988). This is not an exhaustive list of potential areas of student difficulty but an examination of the most common difficulties and the tools the resource pack employs to overcome them. There exists an interplay between these difficulties, which means that they rarely exhibit themselves in isolation and all bar the language problem could be described as the conceptual teething troubles of adapting to a probabilistic mindset or way of thinking.
3.5.2 Language

Researcher: What is the chance of getting a “4” on a ten-sided die?

Student: Probable

Researcher: Could you give me the chances in a number form?

Student: 50-50. Even chance of getting a 6 or a 4 or a 3

Researcher: What is the chance of getting a number bigger than 6?

Student: Probable. 50-50

Researcher: And of getting an even number?

Student: Probable. 50-50

(Amir and Williams 1999, p.101)

In mathematics, terms used casually in our everyday language take on exact and specific meanings (Kotsopolus 2007). In probability in particular there are many stochastical terms such as bias, sample, significant, normal etc., which have seeped into everyday language and have taken on broader or looser meanings. These are technical terms and the loose way in which they are used in every day speech leads to students using them casually and imprecisely (Rossman et al. 2006; Gal 2005) often sowing confusion unless properly explained and understood. Phrases such as “probable”, “50-50”, “almost certain” or a “good chance” are widely used in day-to-day conversations. The fact that these and other similar phrases can be interpreted differently by different students means that it is not appropriate language to use in the
classroom (Gal 2005). It is important in these situations that decimals, fractions and percentages are used to quantify probabilities in a precise manner within the classroom if confusion is to be avoided. Research has shown, that as students progress through school, their ability to equate these quantities with colloquial probabilistic phrases improves, this may be linked with an increased understanding of decimals, fractions and percentages (Watson and Moritz 2003).

The opening piece of dialogue in this section is a classic example of the misuse of stochastic language. In this case the student exhibits the most common difficulty associated with language - the “equiprobability bias”. Though it is essentially a language problem, this is unique to probability due to the conceptual issues involved (Lecoutre 1992). This bias is based on a belief among some students that all events, which occur randomly have the same probability of occurring. The classic symptom is the incorrect use in probabilistic terms of the phrase “50:50”. To some the likelihood of two random events occurring is always “50:50”, no matter the probabilities involved. Pratt (2000) and Watson (2005) studied the “50:50” syndrome and identified the misuse of language as the key element in this issue. They found that for a lot of the students’ who erred, “50:50” was a way of saying “anything could happen” or “these two events are equally likely” rather than there is 0.5 probability of each of these events occurring. Again the imprecise use of a colloquial term in a probabilistic situation has implications which foster confusion in a student’s thinking.

Borovcnik and Bentz (1991, p.81) cite a study undertaken by Green (1982) in which pupils were presented with a simple hat lottery question. There are 13 boys’ and 16
girls’ names in a hat, one of which is drawn out. Is it more likely to be a boy’s name, a
girl’s name or equally likely to be either? 53% of 11 year olds and 25% of 16 year
olds answered equally likely since either a boy or girl will be drawn. They mistakenly
understood “50:50” as meaning that there were two options, so it was a “50:50”
probability as it had to be one or the other, a boy or girl. The resource pack addresses
the issue of language with the precise, correct use and explanation of probabilistic
terms in mathematical as well as everyday language. More importantly stochastical
terms, which have taken on a looser definition as they entered colloquial usage are
redefined and re-explained in their strict mathematical sense. This allows students to
concentrate on the conceptual issues at hand instead of being led astray by
misinterpretation of language.

The use of questions in context demonstrates to pupils that because mathematical and
specifically stochastical problems occur in everyday situations, the language we use in
asking and answering questions as well as examining claims is important (Kotsopolus,
2007; Garcia- Alonso and Garcia- Cruz 2007). The language of the questions is
specific and straightforward and the answers mirror this accordingly. In no place in
the pack, for example, is the answer to how probable an event is of occurring
answered with colloquial phrases but rather in specific fractions, decimals and
percentages first before any inference is drawn. The use of the questions in context
allows pupils to appreciate and experience the importance of using language correctly.
3.5.3 Paradoxes and Fallacies

Probability is rife with paradoxes and fallacies which, unless examined carefully, will only serve to confuse students’ thinking (Leviatan 2002). Paradoxes occur when there is an apparent contradiction in the statement but what is being said is true. The most famous probabilistic paradox of all is the “Monty Hall Problem”, which caused some of the most famous mathematicians of our time quite a bit of difficulty and indeed embarrassed many (Mlodinow 2008; Rosenthal 2006; Rosenhouse 2009). However, paradoxes such as this should not be avoided, they may cause initial confusion but if properly explained can lead to clearer thinking and a healthy wariness of their intuitions by the students. Fallacies are examples where a false outcome is arrived at by following what appear to be logical steps. Again these, can serve, if used properly to highlight how poorly developed people’s probabilistic intuition can be.

Many educators argue that using these fallacies and paradoxes can be a powerful pedagogical tool when teaching probability (Borovcnik et al. 1991). They certainly raise class interest and discussion, confronting peoples’ misconceptions and driving conceptual understanding in the area. The resource pack seeks to use both paradoxes and fallacies in confronting students’ intuitions. The previously mentioned paradox, the “Monty Hall” problem is included in the conditional probability section of the resource pack, together with an extensive explanation and examination as well as a history of the problem. An adaption of the less well known “Bertrand’s Box” fallacy is included in the expected value section, where the conduct of experiments by the students exposes that there is an error in the thinking process. Both of these serve the purpose of firing students’ interest while clarifying their thinking. They are also
important as they help students to develop a critical attitude that allows them to deconstruct fallacies and flawed probabilistic statements in the real world (Rasfeld 2001). These paradoxes and fallacies play on peoples’ intuitions. Peoples’ intuitions are notoriously weak and need to be confronted in the probability classroom if conceptual understanding is to occur (Konold et al. 1993; Konold 1995; Fischbein and Schnarach 1997; Ahlgren and Garfield 1988). The biggest intuitive blocks to relational understanding and conceptual knowledge are known respectively as the “gamblers fallacy” and “the outcome approach”.

3.5.4 Counter-Intuitive Problems

A) The Gambler’s Fallacy

The gambler’s fallacy is the classic example of a counter-intuitive problem at work and merits special mention due to its importance in probability (Shaughnessy 1992; Garfield 1995). The gamblers fallacy is so called because of the gamblers’ belief that their string of losses increases their chances of winning next time. The independence of each event is not recognised by the gambler. In simpler terms the odds of tails appearing is ½, every time a coin is tossed. This remains true no matter how many times heads may have appeared in the sequence because each new trial is independent of the previous trials. This fallacy demonstrates peoples’ desire for patterns and order where in fact none exist. To truly appreciate probability students have to understand what random really means.
Shaughnessy (1981, p.91) gave the example of a question he gave to his class of 70 college students in an introductory course in probability. The question was:

“The probability of a baby’s being born a boy is ½. Which of the following sequences is more likely to occur in having six children?

a) BGGBGB

b) BBBBGB

c) About the same chance for each of these sequences

Give a Reason for your answer.”

50 out of the 70 went with the incorrect answer “a” because they felt it would fit the correct ration of 50:50 boys and girls. 2 out of the 70 went for “b” and 18 out of the 70 chose “c” the correct answer that both are equally likely.

A study by Green (1986) of 11-16 year olds, demonstrated similar misconceptions.

The flat roof of a garden shed has 16 square sections. It begins to rain a little. After a while 16 raindrops have landed on the roof. Here are three pictures showing raindrops on the roof: Which picture best shows the pattern you expect to see?

Figure 3.1: Pattern of Raindrops (Green 1986)
Only 26% of 11 year olds and 18% of 16 year olds gave the correct answer c. Students did not appreciate the random nature of the event and chose options a and b because there was a pattern to the drops. Green (1986) also gave a question, which involved some black balls and some white balls in a bag. A boy picks out a ball four times in a row and replaces it. Each time it is black. Is it more likely the next ball will be black, white or either colour? Only 13% of 11 year olds and 25% of 16 year olds answered correctly. Borovcnik and Bentz (1991) and Biehler (1994) both discuss numerous studies of this phenomenon and come to the conclusion that intuitively people look to find patterns in everything often where there is none.

B) The “Outcome Approach”

This occurs when students’ beliefs and experiences of what the outcome of experiments or theoretical probabilities should be, influence their explanations, often despite the stochastic evidence to the contrary. Batanero, Estepa, Godino and Green (1996) found that students were often heavily influenced by their previous ideas of association and tended to ignore the empirical data that emerged. It was Konold in 1989 who labelled this problem the “outcome approach”. This name is appropriate as it essentially describes students’ belief that the “outcome” they experienced is the probable outcome no matter how unlikely that is. Konold (1989) used the example of the weatherman who says “there is a 70% chance of rain tomorrow”. This means, given these conditions 7 days out of 10 it will rain tomorrow and therefore 3 days out of 10 it will not rain. Those that fall into the “outcome approach” trap believe there to be something incorrect in the forecaster’s conclusion if it does not rain the following day. They may even understand to some degree that the outcome of one experiment is
not a good predictor but have a greater belief in what they experience casually than on sound stochastic explanations.

To overcome students’ intuitions and beliefs in relation to these two difficulties, they have to experience and “discover” the law of large numbers in action for themselves in the classroom. In classrooms where the classical approach is the sole mode of instruction, these widely held intuitions are ignored, not explained or examined though often surfacing, leading to conflict and confusion in the students’ stochastic reasoning. A frequentist approach, which shows that short term-runs and small scale experiments often give misleading results should be experienced first-hand by students of probability. The resource pack encourages the use of trials and experiments within the classroom. As groups of students’ work through the simulation used in the “Binomial distribution” section for example or the experiment in the “Expected Value” section the notion that any pattern exists apart from the fact that the probabilities involved are heading towards a limit will be fully thrashed out and visible. Confronted with this evidence, students’ intuitions are built upon and clarified allowing for the development of relational understanding.
3.5.5 Conclusion

“Instruction can lead students to actively experience the conflicts between their primary schemata and the particular types of reasoning specific to stochastic situations”

(Fischbein and Schnarch 1997, p.104)

In Chapter 2, the benefits of teaching for understanding were detailed (Carpenter and Hiebert 1992; Carpenter and Lehrer 1999). When teaching probability the conflict discussed by Fischbein and Schnarch (1997) above, needs to be experienced by pupils if true and deep learning is to occur (Garfield 1995; Shaughnessy 1977). The paradoxes, fallacies and counter-intuitive problems which are unique to stochastics should not be avoided and ignored for fear of creating confusion. The author was aware when creating the resource pack that these issues should be dealt with, analysed, examined and experienced by students in the probability classroom if conceptual understanding was to be achieved. If dealt with properly the paradoxes, fallacies and counter-intuitive problems can motivate and excite as well as clarify and illuminate students’ thinking. These unique difficulties can be overcome with the proper strategies. The resource pack uses signatures of the frequentist approach, such as activities and investigations to encourage not only instrumental learning of students’ knowing the formulas, rules and axioms emphasised by the classical approach but to develop a relational understanding that allows them to comprehend why these formulas are being used and how to apply them in other situations. In this manner these unique difficulties can be overcome and be turned into opportunities to excite and capture interest and improve attitudes.
3.6 The Teacher

“The biggest influence on secondary students’ mathematics experience and performance is the mathematics teacher.”

(O’Donoghue 2010, p.70)

The teacher in the mathematics classroom plays a significant role. A study by Midgley, Feldlaufer and Eccles (1989) showed a strong relationship between students’ attitudes and perception of mathematics, and their teacher. This especially holds true in the probability classroom, where the success of any curricula in this area will depend hugely upon the teachers’ knowledge of a topic with unique pedagogies, thinking and difficulties (Stohl 2005). The importance of teacher knowledge has been highlighted by many researchers, such as Bright and Friel (1998), Burgess (2002), Pereiara-Mendoza (2002), Speiser and Walter (2001), Watson (2001), Stohl (2005), Batanero, Godino, and Roa, (2004). Fennema and Franke (1992) argue that the “context specific knowledge” or the knowledge the teacher needs in the classroom is informed by four separate components the teacher’s beliefs, their knowledge of mathematics, their pedagogical knowledge and their knowledge of the learners cognitions in mathematics. By replacing the word mathematics with probability we have a model specifically adapted for the probability classroom.
In this section the author intends to use this model to examine how the issue of teacher knowledge is impacting on the teaching of probability specifically, why it is a concern for many educational researchers and how the resource pack helps to address these problems.

“Knowledge of probability” is perhaps better known in educational research as the teachers’ content knowledge. This is primarily, a knowledge of concepts and procedures and secondarily a knowledge of the role probability plays within the real world. A worry about a lack of content knowledge among teachers’ is common in the literature. Greer and Mukhopadhyay (2005) suggest that probability has been introduced into mainstream education without enough preparation for teachers. Jones,
Langrall and Mooney (2007), Lee (1999), Konold (1995), Howson (2002), Bantanero et al. (2004) all comment on the seriousness of this problem. This poor content knowledge can be ascribed to two factors, the first is that probability is a new topic on curricula and many teachers did not experience it in their mathematics education at primary, secondary and third level and therefore have not experienced probability in a systematic way as a student or in their teacher preparation. The second cause is that though teachers do have a knowledge of the procedures within probability, they often have poor conceptual knowledge and they themselves struggle to grasp major probabilistic ideas. They think deterministically and struggle with the mindset required to deal with uncertain situations (Stohl 2005).

A lack of teacher comfort and confidence with the content may account for the unpopularity of the probability questions in the Leaving Certificate examination. The fact that students in Ireland have avoided probability (Chief Examiner 2000; Chief Examiner 2005) certainly points in that direction but no Irish study has taken place to examine this though international research points at this being an issue (Stohl 2005). The resource pack allows teachers and students to experience probability in a manner that confronts their own misconceptions. This is important for teachers if they are to help their students (Lee 1999). Teachers may not use all the contextualised questions in the classroom but a knowledge of how and where the topic is used in the real world will broaden and deepen their understanding and consequently their ability to teach.

The knowledge of learners’ cognitions is another area of importance in probability. The author has already spent some time discussing the unique conceptual difficulties
encountered by students studying probability. Konold et al. (1993, p.413) wrote that “teachers become more effective as they increase their power to interpret student utterances, many of which initially seem incomprehensible”. In probability this is only possible if teachers are aware of how students are thinking and where their intuitions are likely to mislead them. Watson’s (2001) study showed that teachers tended to recognise and focus on procedural and computational difficulties but struggled to identify conceptual difficulties especially when using the classical approach. The use of paradoxes, fallacies, activities and experiments which promote conceptual understanding enable teachers not only to identify errors in learners’ conceptual thinking but confront them. For teachers to engage in this is vital if students are to progress in their probabilistic thinking.

The impact of teachers’ beliefs on how a topic is taught and learned is unquestionable.

“The conceptions of mathematics held by teachers may have a great deal to do with the way in which mathematics is characterised in classroom teaching. The subtle messages communicated to children about mathematics and its nature may in turn, affect the way they grow to view mathematics and its role in the world”.

(Dossey 1992, p.42)

This holds true in the probability classroom. Studies produced by Greer and Ritson (1994) and Gattuso and Pannone (2002) at respective International Conferences on the Teaching of Statistics reflect poorly on teachers’ beliefs about probability and how it should be taught. Both studies revealed that the majority of teachers surveyed in Northern Ireland and in Italy considered probability a relatively unimportant topic in
mathematics. This is a situation that Jones, Langrall and Mooney (2007, p.934) refer to as “worrisome”. While there are no studies of teachers’ beliefs regarding probability in Ireland, it is safe to say that students’ choice of questions at Leaving Certificate examination time suggest a similar pattern of beliefs. The “Project Maths” curriculum differs from the previous curricula in its conception of what mathematics is and consequently how it should be taught and learned. Teachers’ beliefs about the importance and relevance of probability need to change. The “Questions in Context” section address teachers’ beliefs in relation to the importance and the relevance of probability.

The importance of the pedagogical approach adopted by teachers in the classroom and the impact it can have on students has been discussed in both this and the preceding chapters. While the preceding chapter examined differing pedagogical approaches and their impact on mathematics as a whole, this chapter examined the approaches specific to the probability classroom. A study by Greer and Ritson (1994) in Northern Ireland, where teachers followed the same classical approach as used in the Republic, showed that teachers rarely used experiments when teaching probability. It is clear that this traditional rote learning approach no longer serves the aims of our mathematics curriculum which now states the need for both instrumental understanding (knowing how) and relational understanding (knowing why) and the need to develop flexible thinkers (NCCA 2010). “Project Maths” encourages the use of contexts and active learning methodologies. In probability specifically this means mixing the classical approach with the frequentist, which allows students to experiment and experience probability. In Chapter 2, the necessity to attack the curriculum at three levels if curriculum change is to be successful (Conway and
Sloane 2006) was discussed. The resource pack aids this curriculum change on two of those levels, classroom culture and textbooks. It encourages teachers to make this change in pedagogical approach by providing them with the resource materials to carry out experiments, investigations and the applications of three probability topics.

If the probability section of the “Project Maths” curriculum is to be successfully implemented then Irish teachers are going to have to examine their own probabilistic knowledge. Project Maths is addressing the problem of teachers’ avoiding the topic by stripping out any choice in the terminal examination papers. Research has shown a lack of confidence among teachers when teaching probability (Stohl 2005), a situation that can only be addressed by in-services and resources being made available for teachers. Continuing professional development on all four aspects of teachers’ knowledge is needed, where traditionally only content knowledge is addressed (Shaughnessy et al. 1996). Gattuso and Pannone’s (2002) research showed that teachers’ insecurities in teaching probability and statistics were largely accounted for by their lack of pedagogical knowledge rather than their content knowledge. The in-service provide by the Project Maths Development Team does address teachers’ pedagogical knowledge and beliefs as well as student cognitions. This resource pack also addresses this aspect. The active learning methodologies and the questions in context in the pack will have a positive impact on teachers’ knowledge of probability, teachers’ pedagogical knowledge, teachers’ beliefs and teachers’ knowledge of student cognitions.
3.7 Conclusion

This chapter provided an overview of some of the educational research regarding probability. It is clear that probability is a new mathematical topic both in and out of the classroom. A knowledge of its origins is interesting as well as useful in the classroom. Its presence in curricula worldwide has in the last twenty years become pervasive and this has led to an increase in research in the area in recent years (Jones, Langrall and Mooney 2007). It is clear from the research that the emergence of probability in curricula internationally is justified due to its increasing importance within mathematics, within other disciplines and in everyday life (Carr 2008; Bessant and MacPherson 2002; Shaughnessy et al. 1996; Gal 2005; Pereira-Mendoza and Swift 1981; NCTM 1989). There is debate in the literature over whether stochastics should be considered a part of mathematics but it is acknowledged that in primary and second level at least, it is at home within the mathematics fold. There is also disagreement on the amount of probability, which should be taught. Some argue that it should be reduced, only to that necessary to enable statistics to be taught but this argument has been successfully countered by those who maintain that probability deserves its central place (Biehler 1994; Borovcnik and Peard 1996; Batanero and Sanchez 2005). It is clear that the main issues facing the teaching of probability at second level are the pedagogical approaches to be adopted, the unique conceptual difficulties associated with probability and the probability teacher. All these issues are clearly interlinked and none can be approached without addressing the others.

From the author’s review of the literature it was obvious that only an approach that encompasses both classical and frequentist philosophies can help students overcome
misconceptions and alter their intuitions (Stohl 2005). This resulted in a complementary approach being adopted in the resource pack. The unique conceptual difficulties encountered by those studying probability were also examined. The conclusion reached in the literature is that these difficulties will only be overcome by confronting them (Borovcnik et al 1991; Fischbein and Schnarch 1997). Peoples’ probabilistic intuitions are misleading and they need to experience randomness in action first hand if these intuitions are to be altered. The experiments, simulations and investigations included in the pack should allow students to gain this experience (Shaughnessy and Ciancetta 2002) and are in keeping with the socio-cultural outlook adopted in the resource pack.

The last issue examined was the teacher in the classroom. A change in how mathematics in general but probability specifically is taught needs to occur but these changes cannot come about overnight. Earlier we looked at The Netherlands as an example of successful curriculum change but it took more than ten years for that change to bear fruit (Conway and Sloane 2006). A huge emphasis needs to be placed on teacher education in pre-service and in-service. All aspects of teacher knowledge need to be addressed, as teachers unsure of students’ thinking, the importance of the topic, the content of the topic they are teaching or the methodologies they are using will stick to the traditional method of teaching which they are comfortable with. In the Netherlands it was acknowledged that resources produced by teachers themselves explained content, helped to change teachers’ beliefs and encouraged them to adopt new pedagogies (Van den Heuvel-Panhuizen 2000; Moffett 2009; Conway and Sloane 2006). The resource pack informed by the literature follows in this tradition with the aim of enhancing students’ conceptual understanding in probability and their attitudes
towards mathematics in general. Having reviewed both the general mathematics education literature and then more specifically the literature on probability education and explained how they influenced the production of the resource pack, the author now wishes to describe the methodology used when researching the effect of the resource pack in detail in the next chapter. A detailed description of the resource pack developed is provided in Chapter 5.
Chapter 4 - Methodology

4.1 Introduction

“The improvement of teaching is a process of development ..... by the gradual elimination of failings through the systematic study of one’s own teaching.”

(Stenhouse 1975, p.39)

Stenhouse (1975) argues that the improvement of teaching is a process, which can only occur by examining current practice. This study is an examination by the author of current practice and the development of an intervention to see if it could be improved upon. The purpose of this chapter is to describe the methodology and varying research methods used in this study from its initial stages to its conclusion. Why this study was undertaken; the model of research followed, why this particular model was chosen, the characteristics of this model, the benefits and weaknesses of this model, the ethical issues involved and finally how the research was evaluated are all discussed in this chapter.

4.2 Rationale for Undertaking the Study

As described in the introductory chapter, the interest in an intervention was aroused due to the personal experiences of the author when teaching probability. The author received anecdotal feedback year-after-year that probability was not a popular topic among students. Some seemed to quickly achieve a reasonable grasp of the topic. However, most students, often those who excelled in other areas, struggled. Why does
a topic with so many practical, easily demonstrated, every day applications elicit such
distaste in our Senior Cycle students?

On carrying out a little research it became clear that this was not a phenomenon
strictly limited to the four walls of the author’s classroom. According to the last two
Chief Examiners’ reports on Higher Level Leaving Certificate mathematics, the
probability and statistics questions were the least popular questions chosen on Paper 2
(Chief Examiner 2000, 2005). In 2000 (p.16) the Chief Examiner commented that “A
drift away from discrete mathematics and statistics (questions 6 and 7).... has been
noted in recent examinations” and though this had “stabilised” by 2005 (p.71) these
questions were still the two least popular questions chosen on Paper 2.

The author decided to examine the teaching methods employed and observe whether a
change from the traditional, learn the axioms, “follow these steps” method lead to a
change in students’ feelings towards probability. The investigation has scope beyond
the author’s own classroom due to the new Project Maths syllabus. This new syllabus
in which Strand 1- “Probability and Statistics” has been introduced in the 2010/11
school year sees “Probability and Statistics” play a larger role within the syllabus and,
due to changes in the exam structure (all questions will now have to be answered),
probability can no longer be dismissed by teachers and students.
4.3 Research Aims

As outlined in the previous section this research was undertaken to solve a specific problem within the author’s probability classroom. The intention of the author was to create a resource pack, which would solve this problem. The first step in the creation of the resource pack was an examination of the literature of both general mathematics education and of probability education specifically, which the author has reviewed in the previous two chapters. This examination provided the author with a greater insight into what was occurring in the classroom and influenced the content and the pedagogical approach of the resource pack.

Figure 4.1 Influences from literature Review on the Resource Pack
The influences shown in figure 4.1 have been discussed in detail in the previous chapters. The use of active learning methodologies and questions in context in the resource pack will be outlined in the next chapter, Chapter 5. The author having thus grounded the resource pack within specifics approaches to mathematics and probability education developed two specific research questions.

1. To investigate if there is a link between the use of active learning methodologies and real life data and contexts and an improvement in students’ attitude towards mathematics.

2. To investigate if the use of active learning methodologies and real life data and contexts develop students’ understanding of probability.

4.4 The Research Design

An important part of any research study is the research design. The research design has many forms of general classification but the most prevalently used system is by the type of data created and the evaluation procedures used. Different types of studies value differing collection and analysis methods. Research studies are said to fall into one of the three categories of design

1. Quantitative

2. Qualitative

3. Mixed method

(Creswell 2010; Wiersma and Jurs 2009)
These design types should not necessarily be viewed as discrete categories but instead as a continuum in which the mixed method operates as a half-way house approach to the qualitative and quantitative approaches which occupy either end of the spectrum (Newman and Benz 1998). Quantitative studies are based on empiricism and demand scientific rigour. They generally involve either survey research or experiments and the ability to mathematically measure outcomes. The procedures are standardised and the results can be generalised to given populations. Quantitative data has the advantage that “numbers register the departure from theory with an authority and finesse that no qualitative technique can duplicate” (Kuhn 1961, p. 180). Qualitative studies on the other hand focus on people’s values and opinions and are context specific. Observations by those involved provide the focus rather than statistics. These observations are analysed and general themes emerge, which the researcher interprets within this context (Creswell 2010; Wiersma and Jurs 2009). Qualitative data is limited as these interpretations can only lead to hypotheses on the general population rather than specific conclusions. On the other hand qualitative data provide “thick descriptions” (Miles and Huberman 1994, p.10), which enable authors to elicit a more rounded, holistic picture than from quantitative data. Another advantage of qualitative research is that it provides researchers with data which can support, supplement, explain or force re-interpretation of quantitative data, which introduces the notion of mixed method research (Miles and Huberman 1994).

The mixed method approach is a combination of both types of inquiry. This method has become more acceptable in educational studies in recent years and has gained popularity among researchers who feel that it strengthens their study as it has the advantages of both qualitative and quantitative types of research design. It enables a
more complete understanding of difficult research topics (Firestone 1987; Plano Clark, Creswell, O’Neill, Green and Shope 1997). It also augments the research by providing a form of internal corroboration, known as triangulation (Bryman 1988). As well as providing greater breadth and insight, mixed method studies tend to have a wider appeal, with the ability to convince different audiences (Firestone 1987; Wiersma and Jurs 2009). Researchers who adopt the mixed-method approach tend to have a pragmatic philosophy, for them the overwhelming concern of the study is a solution to the problem, rather than committing to a strictly quantitative or qualitative methodology (Patton 1990).

This pragmatic mixed-method approach is in-line with the goal of the author’s study, to find a solution to the problem experienced in the classroom. The author employed a number of differing data collection instruments during the study. A self-designed questionnaire was given to an expert panel and a critical friend, a modified Fennema-Sherman Attitude Scale (1976) and a class test designed to test for understanding were given to students and finally a daily diary was kept by the author. These data collection instruments produced a mixture of qualitative and quantitative data, providing the author with rounded and important insights as well as enhancing the validity and reliability of the study by providing triangulation (Bryman 1988). Once a standard research design was chosen the author then needed to select a recognised and valid research methodology. Research methodologies are standardised models which provide the researcher with an outline of how they should approach their study and take many differing forms (Mertens 1998). This study followed an action research model.
4.5 What is Action Research?

“An action research report necessarily describes a sequence of events developing through time; its form is, therefore, essentially that of a narrative”

(Winter 2002, p.143)

There are many definitions of action research. It has been defined by the reasons it is undertaken, the characteristics the research takes, and the effects or benefits of the action. These three interlinked aspects combine to form what many researchers (Winter 2002; Heikkinen, Huttunen, and Syrjala 2006) refer to as the narrative of action research. Action research was first advocated by social psychologist Kurt Lewin (1948) in the U.S.A as a method of solving social and industrial problems and gradually spread into education. In essence it was a simple idea - a problem was identified, a good theoretical understanding of what was occurring was gained, a solution was proposed and trialled. This action was then evaluated, adjusted and re-triailed again and the process continued in a cyclical or spiral fashion until a satisfactory conclusion was reached.

It was not until the 1970s that action research began to be seen as an effective way of creating change in the classroom on this side of the Atlantic. British educationalist Lawrence Stenhouse (1975) in his influential book An Introduction to Curriculum Research and Development proposed the idea of “The Teacher as Researcher” in a chapter of the same name. This combined with work from other educationalists such as John Elliott and Clem Adelman, working with the Ford Teaching Project, ensured
that the idea of ‘practitioner as researcher’ gained in popularity and action research is now seen as a valid form of research.

Action research begins with self-reflection. All teachers are reflective, however the form this reflection takes can be varied both in nature and quality. Teachers reflections largely come back to one question; “How can I improve learning in this classroom?” Hustler, Cassidy and Cuff (1986, P.74) state that this “self-critical refusal to be complacent is a common thread” in action research projects and provides the beginning of the narrative of action research – the identification by the teacher of an on-going issue they feel needs addressing. The action research narrative is the story of identifying this issue, organisational or pedagogical, and the steps that were taken towards addressing it.

4.6 Rationale for Choosing an Action Research Approach

“The fundamental aim of action research is to improve practice rather than to produce knowledge”

( Elliott 1991, p.49)

As Elliott (1991) indicates, the central purpose of action research is to affect change in current practice. It both asks and empowers teachers to look to themselves to solve issues within the classroom rather than employing external prescriptions. This empowerment of the teacher to investigate in their own classroom attracted the author to the action research methodology. Action research felt like a natural approach, a comfortable fit with what the author was trying to achieve.
According to Cohen and Manion (1994, p.188-189), there are five general reasons why teachers pursue active research projects. These reasons, listed below, are both broad and overlapping:

1. They have identified a specific problem in certain situations or circumstances and are attempting to find a solution for this specific problem
2. They use the project as an in-service, forcing themselves to update their skill set
3. They want to try different or innovative teaching and learning methods
4. They are attempting to add something to the current research as a practising teacher rather than as an academic researcher
5. It provides an alternative method for teachers when problem-solving in the classroom.

This study began with a simple question; why do students dislike probability so much? This question led to further questions; why do some students find it difficult to grasp the concepts being taught? Would a different pedagogical approach improve my students’ understanding of probability and their perception of mathematics in general? These questions, which eventually led to research and an intervention, are broadly in-line with reasons one and three mentioned above by Cohen and Manion (1994). It is situational, a specific problem has been identified in the probability classroom and was addressed there. The attempt to solve it has pedagogical implication as the intervention involves innovative teaching methods. Though these are the two chief reasons for this investigation, the author’s research also lead to professional up-skilling and a contribution to the research from the perspective of a practicing teacher,
Cohen and Manion’s (1994) reason two and four. Before the author realised that action research existed the narrative of the investigation had already taken that form.

The fact that action research was complementary to the author’s goals was the main rationale for choosing the action research methodology. However, the author was also convinced by the literature of its many benefits. McNiff, Lomax and White (1996, p.8) detail three specific benefits that well conducted action research can lead to

1. To your own personal development,
2. To better professional practice,
3. To improvements in the institution in which you work.

The personal benefits to the practitioner who decides to research their own practice are many. Ultimately it leads to more effective educational practice (McKernan, 1996). This is sometimes achieved by simply fixing an organisational difficulty that has arisen. More often, teachers experiment with pedagogical approaches and find one that suits, not necessarily themselves but more importantly, their students. It leads to increased learning by the students and by the teacher of the students, who through the research undertaken now has a greater awareness of what is occurring in his classroom. It provides teachers with an in depth in-service into their area of expertise, updating their knowledge not just of pedagogical approaches but also familiarises themselves with recent teaching aids. It develops within the teacher, what is probably the most important habit of all – that of “informed” reflection (Furlong and Salisbury 2005, p.61). For the author, the driving force behind the action research was personal.
The author wanted to improve personal practice. An area where students struggled was identified and the purpose of the investigation was to address that.

The second benefit that McNiff et al. (1996) lists is professional development. This does not mean just the development of the individual practitioner as a professional, but on a wider scale the development of the profession as a whole. An intention to influence practice among a wider community of teachers was for the author a secondary consideration. However, the dynamic of practitioner as researcher has incredible implications both for practice and research. The notion that the practitioner i.e. the teacher is assigned the role of not only identifying problems but solving them through work in their own classroom is unique. Action research asserts that research is no longer just the domain of the academic; “In action-research theories are not validated independently and then applied to practice. They are validated through practice” (Elliott 1991 p.69). This has significance for curriculum reform. Reforms that are seen to be imposed onto the teaching profession by academics and others are generally hampered by the doubts of practitioners. Teachers are more likely to take action research, validated through practice, on its merits rather than educational research published by academics. This “bottom-up” approach with its organic roots in the classroom, strengthens its credibility among those who operate in the classroom (Fullan 1991).

The third benefit listed by McNiff et al. (1996) is to the institution with which the practitioner is working. In the case of education this usually refers to a school, either at primary or secondary level. A teacher or teachers pursuing action research is a
positive experience within a school or at least within a subject department within the school. Hutchinson and Whitehouse argue that “Action research is subversive in that it brings into question that which is taken for granted.” (1986, p.93) This desire to improve, the habit of questioning, the willingness to experiment even if not overtly pushed cannot but infiltrate a subject department’s thinking and hence its actions. Action research encourages teachers to solve their own problems and developing a problem solving attitude within a staff is of benefit to the institution on a much wider organisational basis. These benefits and the natural fit of the methodology provided the author with a clear rationale for choosing action research model.

4.7 Limitations of Action Research

“Action research should be judged by the three principles of, theoretical and methodological robustness, value-for-use and the potential to enable beneficial change”

(McMahon and Jefford 2009, p.361)

There are many significant benefits to action research but it does have limitations. Action research as a methodology does lack in scientific rigour. The very characteristics which make it unique - it is situational and specific - ensure that there is little or no control over the sample and other independent variables. This makes findings hard to generalise, though as action research projects become extensive in a specific area this becomes less of an issue (Cohen and Manion 1994).
These difficulties have led to repeated discussions on how to assess the validity and quality of action research (Hodgkinson 1957; Foster 1999; Hammersly 2004). McMahon and Jefford (2009), believe that by following the three criteria, which they refer to as “Elliott’s context-related criteria” (Elliott 2007) listed above, action research can be judged on its quality and validity. Action research, though it struggles to produce quantitative data that can be generalised for the whole population, can indisputably produce perfectly valid qualitative data. The question of validity and reliability will be discussed in more detail later in this chapter. A final problem worth mentioning is one raised by Street (1986). She acknowledges the undoubted benefits of self-reflection for practitioners engaged in action research but warns against “becoming so self-critical that it becomes destructive” (1986, p.131)

4.8 The Cyclical Nature of Action Research

“The cyclical nature of action research links the theoretical and the pragmatic, contributing to, and enriching both”

(Hustler et al. 1986, p.77)

Action research projects tend to be small in scale and situational, but just as the reasons action research projects are undertaken tend to fall into general categories, so too does the general procedure followed. Action research projects are cyclical or spiral in nature (Lewin 1948; Elliot 1991; Riding, Fowell and Levy 1995). A problem is identified, research in the relevant literature is carried out and some form of intervention is decided upon. This intervention or action is then trialled and evaluated. From this evaluation, problems with the intervention are identified, further research is
carried out and the intervention is refined. The process continues in a cyclical or spiral pattern, slowly circling upwards until it reaches a point where the researcher is content that he has produced an action that has resolved the problem initially identified satisfactorily (Lewin 1948; Elliot 1991; Riding, et al. 1995).

Figure 4.2 An Action Research Cycle (Riding, Fowell and Levy 1995)

4.9 Description of the Action Research Cycle of this Project

“It is not enough that teachers’ work should be studied: they should study it themselves”

(Stenhouse 1975, p.143)

In this section the author will describe the steps taken in this piece of action research, using the action research spiral/cycle of Riding, Fowell and Levy (1995) as a template. This intervention went through the action research cycle three times before the author was satisfied with the resource pack. It is important to note that the literature review continued through the three cycles. Each time the author planned or revised the plan, the literature was examined and the review in Chapters 2 and 3 was expanded upon. The methods of data collection are described in more detail later in this chapter, while the data collected is presented in Chapter 6.
Figure 4.3 The Three Cycles of this Action Research Study

**Cycle 1**

**Plan**
A Problem was identified and the literature was reviewed.

**Act**
Resource Pack was developed and given to an expert panel.

**Observe**
The Expert panel provided qualitative feedback in the form of a questionnaire.

**Reflect**
The author evaluated and analysed this data.

**Cycle 2**

**Revised Plan**
Using their feedback, the resource pack was adjusted.

**Act**
The Pack was taught to a Transition Year group.

**Observe**
Data in the form of a teacher diary, attitude questionnaire and a test for understanding were gathered.

**Reflect**
The author evaluated and analysed this data.

**Cycle 3**

**Revised Plan**
Using their feedback, the resource pack was adjusted.

**Act**
The pack was taught to a 5th year Higher level class.

**Observe**
Data in the form of a teacher diary, attitude questionnaire and a test for understanding were gathered.

**Reflect**
The author evaluated and analysed this data. Using their feedback, the resource pack was adjusted finally.
4.10 The Study Sample

There are two main types of sampling; probability and non-probability. In a sample using probabilistic methods, everyone in the wider population would have an equal chance of being selected. In this case the wider population would be all Senior Cycle students studying higher level mathematics. In a non-probabilistic sample a group is deliberately included or excluded (Cohen, Manion and Morrison 2007). This researcher/practitioner used a non-probabilistic sample. The author teaches in a mixed, rural, community college of circa 500 students and in this sample all the students who participated in this study, attended this school and were taught by the author. Though this raises issues of validity or ability to generalise the findings onto the wider population, in action-research studies the researcher’s primary goal is to solve the problem within the context or situation it arose (Cohen and Manion 1994), which is the author’s classroom.

As illustrated earlier in the description of this action research study (Section 4.9), Transition Year students were the first group taught using the resource pack, in the second stage of the action research cycle. They were a mixed ability group of 19 students who had done Ordinary and Higher Level at Junior Cert level. The third and final stage involved the teaching of the 14 students who form the only Higher Level 5th year mathematics class in the school. The make-up of the expert group as outlined earlier is once again a non-probabilistic sample. This judgemental or purposive sample consisted of a panel chosen by the author to reflect the perspectives of teachers and those involved in teacher education.
4.11 Evaluating Action Research

“Review, diagnosis, implementation, monitoring effects...proves the necessary link between self-evaluation and professional development”

(Elliot 1991, p.ii)

Action alone cannot be described as the central characteristic of action research. More accurately the central characteristic is action and reflection on action. Evaluation of the action is intrinsic to the action research. An intervention may be decided upon and implemented but only by close evaluation of this action can it first be said to be beneficial and, secondly, can areas for improvement within the initial intervention be highlighted. Cohen and Manion (1994) backed this view when they described action research as “a small scale intervention in the functioning of the real world and a close examination of the effects of such intervention” (1994, p.186). In Section 4.6 it was stated that one of the benefits of action research is that it validates theory through practice. The theory cannot be validated unless the practice is evaluated.

Evaluation through critical trialling or the need for triangulation in evaluation procedures appears repeatedly in the literature when evaluation of action research is discussed (Stenhouse 1975; Elliott 1991; McKernan 1996). Essentially triangulation demands mixed-method procedures; evaluation from multiple perspectives. These multiple perspectives serve to reinforce the research’s validity and reliability (Jick 1979). In this study this was achieved through the use of a modified Fennema-Sherman attitude scale to examine the students’ perspective, producing quantitative
data. The resource pack was also evaluated by an expert panel of neutral observers using a questionnaire. Finally, observations were made by a critical friend as well as in a class diary providing qualitative data on the pack from differing perspectives.

4.11.1 The Outside Perspective

Taking it, in order of occurrence the resource pack was first evaluated by an expert panel. This panel consisted of five people, a teacher from a Project Maths pilot school, a practising Higher Level mathematics teacher who had no experience of the new syllabus beyond in-service, a mathematics teacher seconded by the Department of Education and Skills to the Project Maths in-service panel, a former mathematics teacher, with a doctorate in Mathematics Education working with the University of Limerick and a doctoral student in Mathematics Education who comes from a pure mathematics background. This group of five were given a self-designed questionnaire (Appendix B) to fill in after examining the resource pack. These five were specifically picked so that a range of perspectives were included. All may have the same ideal lesson but each group has slightly different priorities reflecting their different backgrounds. For some the priority will be that the lessons reflect the aims and methodology of Project Maths, for others, the priority will be that these lessons will work well in the classroom. The use of a critical friend, a colleague within the schools mathematics department, who observed lessons as well as reviewing the material also provided an outsiders perspective and useful qualitative data.
4.11.2 The Students’ Perspective

In an attempt to evaluate the students’ perception of the resource pack all the students participating in this action research were given a survey, which used a modified Fennema-Sherman attitude scale (1976) (Appendix C). The questionnaire was administered before and after the intervention and evaluated the students attitude to mathematics in four key areas

- Personal Confidence on the subject matter,
- Usefulness of the subject’s content,
- Effective motivation of the student,
- Anxiety student feels when studying mathematics.

The students were asked to fill out all forty-eight questions honestly and were not allowed to confer with classmates. The fact that there was no right or wrong answer was explained and that though they should not spend much time with any statement, every statement had to be answered. The forty-eight questions were broken down into four sets of twelve questions, each set addressing one of the key areas mentioned above. The questions were simple statements, to which the students’ circled one of five answers from strongly agree to strongly disagree.

The attitude questionnaire allowed the first research question, “Would the resource pack help improve students’ attitudes”? to be evaluated in a valid manner. The second research question, “Would the resource pack help develop understanding”? was not as simple an issue. The difficulty in assessing understanding in students is well documented in the literature. Carpenter and Hiebert (1992), Hiebert and Wearne
(1996), Rittle-Johnson et al. (2001), Romero and Mari (2006), Konold (1995) and Skemp (1976) all agree that, assessing understanding, is a serious issue for researchers and one that there is no simple solution for. They all arrive at the conclusion that no system performs the task of assessing understanding in an ideal manner because while defining understanding in a clear yet comprehensive manner is a significant hurdle in itself, attempts to create tasks which assess this understanding satisfactorily are incredibly difficult.

Understanding has already been discussed in chapter two of this thesis but in summary it was concluded that teaching for understanding involves both procedural and conceptual knowledge (Carpenter and Hiebert 1992; Rittle-Johnson et al. 2001). This means that understanding involves the ability to perform a well rehearsed action as well as recognising the connections between this knowledge and the information at hand. Students need not only to know how to perform mathematical operations but also why they are performing these mathematical operations (Carpenter and Hiebert 1992). For the purposes of this study, the students undertook a written test, post intervention (Appendix D). The test is divided into two types of question. The first evaluates the students’ procedural knowledge through the use of routine, familiar tasks, which involve the use of well-practised skills. The second type evaluates the students’ conceptual knowledge by presenting questions in a non-routine or novel context. The use of novel contexts is advocated by Rittle-Johnson et al. (2001), Konold (1995) and Hiebert and Wearne (1996) as a valuable method of assessing conceptual understanding. It is useful as it tests the key element of understanding; the ability to see the connections between the procedures and concepts, which have been learnt and the information which they are confronted with. The ability to recognise the
connections in these situations is the hallmark of real understanding (Hiebert and Carpenter 1992).

The questions used were produced by the National Council for Curriculum and Assessment (NCCA) and are drawn from the 2010 Leaving Certificate Higher level papers, the 2011 Leaving Cert Higher Level Sample paper, the 2010 Pre-Leaving Certificate Higher Level paper and resource materials the NCCA (2009) produced for schools. Two of the questions were not produced by the NCCA, one was taken from teaching materials available on the Project Maths website and produced by the teachers in Colaiste na Sceilige (2010). The other question was produced for the “Statistics 101” course in Duke University (2010), North Carolina and was found on their website. These questions were designed to examine both the students’ instrumental (procedural) and relational (conceptual) understanding. The achievement of both these types of understanding is the stated objective of the “Project Maths” syllabus (NCCA 2010, p.6) and of this resource pack. The assessment tasks use questions set in well-practised and novel contexts to distinguish between procedural and conceptual knowledge (Rittle-Johnson et al. 2001; Konold 1995; Hiebert and Wearne 1996). Both have to be achieved if real understanding has occurred (Carpenter and Hiebert 1992; Rittle-Johnson et al. 2001).

The test consists of 8 problems, four of which have two parts. This leads to a total of 12 questions. These 12 questions are divided equally between the three sections, so there are four questions addressing each topic. These four questions are subsequently categorised as testing either procedural or conceptual knowledge, with two questions
addressing each type of knowledge. Each question is worth five marks, giving a total of 60. The table below outlines, what topics and type of understanding, the marks awarded in the test examine.

**Table 5.1 Outline of Class Test designed to Assess Understanding**

<table>
<thead>
<tr>
<th></th>
<th>Conditional</th>
<th>Binomial</th>
<th>Expected Value</th>
<th>Total</th>
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</thead>
<tbody>
<tr>
<td>Procedural</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Conceptual</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>60</td>
</tr>
</tbody>
</table>

The whole test can be found in Appendix D.

4.11.3 The Teacher/Researcher’s Perspective

"Action research is self-evaluative"

(Furlong and Salisbury 2005, p.48)

The story of action research in the classroom begins and ends with the teacher. It is the teacher who has identified the problem and above all it is the teacher who must be happy that the action has addressed the problem satisfactorily. Cohen and Manion (1994) stated that action research “is concerned with diagnosing a problem in a specific context and attempting to solve it in that context” (1994, p.186). If the teacher is satisfied that the problem identified is solved in his/her classroom then, for all intents and purposes, the action research has been a success and should be evaluated in this light. The author kept field notes during this process, as an attempt to make the evaluation of the pack as objective as possible. The field notes consisted of a class diary, class attendance and tests given to the class to ascertain their understanding of the topic.
4.11.4 Validity and Reliability

The use of multiple perspectives, triangulation, enhances the study’s validity and reliability (Jick 1979). Cohen et al., (2007), have two categories of validity – internal and external. Their definition of internal validity is that the explanation offered is actually borne out by the data collected. This is achieved through the use of multiple methods when accumulating qualitative data and the appropriate treatment of the quantitative data. These issues were both addressed in this study. The qualitative and quantitative data was collected using questionnaires, open ended feedback sheets, tests and a diary. The quantitative data was analysed using the SPSS data analysis programme and no claims were made unless a significance of P < 0.05 was achieved. External validity is the “degree to which the results can be generalised to wider population, cases or situations” (Cohen et al. 2007, p.136). Given the limited sample of this study, external validation is difficult to claim except in the most general of statements. This is a limitation not only of this study but in all but the largest of action research studies. By their very nature, they tend to be classroom specific (Cohen and Manion 1994).

Reliability is concerned with accuracy. According to Cohen et al. (2007, p.146) research is reliable if “it were carried out on a similar group of respondents in a similar context, then similar results would be found”. In terms of the quantitative data collected, the use of the internationally respected Fennema-Sherman (1976) attitude scale, allows the author to claim reliability. This attitude scale has been described as the “most influential” (Mcleod 1994, p.639) and “popular” (Tapia and Marsh 2004, p.1) scales developed in measuring the affective domain and has been used in
hundreds of studies. The degree of anonymity enjoyed as well as the 100% response rate, ensures a high degree of reliability with the data collected from both groups. The use of multiple sources of data helps ensure reliability in terms of the qualitative data (Jick 1979). The use of identical questionnaires and feedback forms, though including some open-ended questions, aids qualitative reliability.

4.12 Ethical Issues

“Because our practice affects other people, we have a moral obligation that any change in our practice is good for the others”

(Feldman 2007, p.31)

Action research projects have their own set of ethical issues, which have to be dealt with. The ethical guidelines laid down, are best followed for the sake of all participants, including the author. Ethical practice was ensured by the following:

1. Ethical approval was granted by the University of Limerick
2. Permission was received from the school principal and the parents of the participating students.
3. Confidentiality of students was ensured. Each student was allocated a number, by which they are identified and this number was maintained for correlation of results.
4. Permission was also obtained from the expert panel to quote their opinions and questionnaire results in my findings.
5. Of most importance, was the fact that this research project should not in any way have a negative effect on the participating students, particularly in terms
of performance in state examinations. The project was laid out so that the 5th year group was taught using the resource pack only when it had first been taught to transition years as well as reviewed by an expert panel and adjusted where appropriate.

6. Finally the author has to acknowledge the limitations of the study before making any claims. The sample population of students in this study is limited to two groups, which the author himself teaches. These groups are small and have a specific contextual nature, which raises concerns over the external validity or the ability to claim that the results can be generalised onto a wider population.

4.13 Conclusion

“Action research is an approach to improving education through changing it and learning from the consequences”

(Wells 2004, p.4)

Action research was the natural methodology to use in this inquiry. The author had unwittingly taken the first steps on the path of action research, by identifying a problem and researching the issues involved, before it was decided that action research was the most appropriate methodology to use in this investigation. It is the most suitable methodology to use in the author’s circumstances; that of the teacher who wants to change something within his/her classroom. This does not mean that every teacher doing research automatically falls into the category of ‘teacher as researcher’, in the action research sense. It does however provide a model for those teachers who wish to engage in research where the aim, “as opposed to much
traditional or fundamental research, is to solve the immediate and day-to-day problems of practitioners” (McKernan 1996, p.3)

The action research methodology enabled the author to move from simply identifying a problem in the classroom, to researching the problems and intervening in an attempt to solve the problem using the research as a support. The action research narrative followed the traditional path of evaluation from multiple perspectives. The multi-method design of evaluation ensured both the validity and reliability of the study. The small sample size does limit any attempts to generalise the results beyond the specific classrooms involved. The findings reached following these methods of evaluation will be discussed in a later chapter but first the resource-pack itself, its format and content needs to be introduced and explained.
5.1 Purpose of the Chapter

The purpose of this chapter is to introduce the resource pack to the reader and to describe how it is linked to the research literature examined. The content and format of the pack and its relationship to the theoretical frameworks discussed in Chapter 2 and 3 will be outlined. The author is cognisant of Kilpatrick’s 1981 article, entitled “The Reasonable Ineffectiveness of Research in Mathematics Education”. This article is essentially a warning, a warning also echoed by Romberg (1992) that research studies should be positioned within a specific field of thought within the mathematics education community. It is not credible to justify a position with selected quotes from research, whose overall findings and views may in fact be contradictory to each other and/or this resource pack. So having discussed some of the general issues in mathematics education and then more specifically in the area of probability, I intend in this chapter to

- Outline and discuss the objectives of this resource pack, positioning it within a specific view of teaching and learning of mathematics and probability.
- Outline the format and content of the resource pack.
- Include student section of the resource pack itself (the teacher section can be found in Appendix E).
5.2 The Resource Pack and its Standing in the Literature

The research questions of the resource pack are:

1. To develop students’ conceptual and procedural understanding of probability.
2. To improve students' attitude towards mathematics.

The importance of teaching for understanding was discussed in Chapter 2. Hiebert (1986) distinguished between two differing types of knowledge/understanding.

- Procedural knowledge - students know how to perform mathematical procedures,
- Conceptual knowledge - students know why they perform mathematical procedures.

Teaching for understanding means that both types of knowledge are promoted and developed within the classroom (Carpenter and Hiebert 1992; Rittle-Johnson et al. 2001). Carpenter and Hiebert (1992) list four benefits of teaching for understanding:

- Understanding is generative,
- Understanding promotes remembering,
- Understanding reduces the amount that must be remembered,
- Understanding enhances transfer.

Carpenter and Lehrer (1999) had a similar list outlining three distinct benefits of relational understanding; it is generative, it aids in remembering rules and procedures linked with instrumental understanding and it improves attitudes towards mathematics.

The reasons behind the author’s desire to improve students’ attitude were also discussed in Chapter 2 (Section 2.5). The importance of student confidence and
students appreciating the value of mathematics has been emphasised in reform documents in many countries (Australian Education Council 1991; The Cockroft Report 1982; National Council of Teachers of Mathematics (NCTM) 1989, 2000). The relationship that exists between a positive attitude towards mathematics and performance in mathematics is emphasised repeatedly throughout the mathematics literature (Farooq and Shah 2008; McLeod 1992; OECD 2004), though the exact nature of this relationship is complex (McLeod 1992). Many studies (Carpenter et al. 1998, Cobb et al. 1991, Fauzan et al. 2002; Van Reeuwijk 1992; Vershaffel and De Corte 1997) have shown a link between moving away from behaviourist methodologies and improvement in attitude. This view of mathematics education, which puts an emphasis on student understanding and attitudes, is inspired by a socio-cultural approach. According to Vygotsky (1978) learning is both an active and social process. This means that learning should not be a passive undertaking, where the student sits and listens but instead they are guided in constructing new knowledge themselves. This also implies that it should not be a solitary undertaking. Bruner (1996) also emphasises that the cultural setting of the students, what they experience in their life, should provide a context for the learning.

The resource pack was also influenced by the discussion of pedagogical approaches in the probability education literature. The author adopted the widely held view (Ahlgren and Garfield 1991; Kavatsinsky and Even 2002; Shaughnessy 1992; Shaughnessy and Ciancetta 2002; Steinbring 1991) that the classical approach was deficient in itself and that a combination of it and the frequentist approach was the best option. The Project Maths syllabus adopts this view, mixing the frequentist with the classical approach (NCCA 2010). The classical approach is an axiomatic behaviourist-orientated
approach, while the frequentist approach emphasises experimental probability (Lane 2009). This frequentist approach gives scope for experiments, investigations and simulations which involve students collecting, analysing and drawing inferences from data and is in-line with the socio-cultural orientation of the resource pack.

The pedagogical grounding of the resource pack is illustrated above in Figure 5.1 and provides the resource pack with a legitimate position within mathematics education research. The author’s motivation for creating a resource pack aligned with the socio-cultural and frequentist approaches was that, on reviewing the literature, the author understood that this position maximised the chances of achieving the resource pack’s objectives, of developing understanding and improving attitudes. This standpoint both
enabled and encouraged the author to employ two specific pedagogical tools to try to achieve these twin objectives of the resource pack, the use of active learning methodologies and real-life contexts.

5.3 Active Learning

“It is expected that the conduct of experiments (including simulations), both individually and in groups, will form the primary vehicle through which the knowledge, understanding and skills in probability are developed.”

(NCCA 2010, p.16)

In recent curriculum reviews, there has been a call for the increasing use of active learning methodologies. (Australian Education Council 1991; The Cockroft Report 1982; NCCA 2005; National Council of Teachers of Mathematics (NCTM) 1989, 2000). According to Felder and Brent (2009, p.2) “active learning is anything course-related that all students in a class session are called upon to do other than simply, watching, listening and taking notes.” Prince (2004, p.223) states “active learning is generally defined as an instructional method that engages students in the learning process”. Active learning methodologies enable students to construct, discover and understand important concepts while providing opportunities for collaboration (Garfield 1995). The basis for the argument in favour of active learning methodologies is summed up in the learning pyramid overleaf, which has been taken from the Department of Education and Skills’ (2010) website. As the students are increasingly challenged in a manner which is active, practical and multi-sensory their understanding of the topic they are studying increases.
The consistent mantra of those opposed to these methods of teaching is that these activities take time which the teachers do not have. Felder and Brent (2009, p.2) explain that active learning does not necessarily involve elaborate and lengthy lessons but taking five minutes out to challenge the students with a problem or question, which they are given time to work on individually or collaboratively and then asked to share their response. Teachers who do this regularly will see an improvement in their students’ conceptual understanding (Prince 2004). It should be acknowledged that employing active learning methodologies can be time consuming in and out of the classroom.

The author employed two active learning strategies in particular, experiments/simulations and investigations. The benefits of experiments in the teaching of probability have been studied by many (Threlfall 2004; Pratt 2005; Lane
2009). The differences between experimental probability and theoretical probability and the balance that needs to be struck between them has been discussed already in Chapter 3. Threlfall’s (2004) report indicated that the lack of experimentation in the probability curriculum of England and Wales impacted negatively on students’ understanding and their test scores. Pratt (2005) feels that students should be given the time needed to experiment within the probability curriculum to really understand concepts. Lane (2009) argues that experiments in probability help students to understand the basic concepts of sample space, likelihood and the relationship between experimental probability and theoretical probability.

The other main active learning strategy employed was investigations. The Cockcroft Report (1982, p.243) calls for “investigational work”. The influence of the Data Handling pedagogical approach can be seen in the path which investigations follow. The investigations follow the Data handling cycle suggested in the teacher support documents produced by the Project Maths Development Team (2010) when studying statistics and probability
The activities included in the resource pack are not activities for the sake of it. These activities are designed to either introduce students to concepts through a form of guided discovery, which is a hallmark of the socio-cultural view, or to enable students to experience the concepts in action, which is a hallmark of the frequentist approach. Guided discovery leads to better retention and increases students’ ability to transfer the new knowledge to other areas (Mayer 2002), while Engel (2002) explains that experiments and simulations are necessary in probability for conceptual understanding. To conclude there is a considerable body of evidence to support the use of active learning methodologies in the literature. Shaughnessy (1977) found that a practical activity based approach tended to result in higher student achievement. Active learning techniques develop students’ conceptual understanding and improve students’ attitudes. (Cockroft 1982; Gnanadesikan et al. 1997; Threalfall 2004).
5.4 Use of Real Life Contexts

“References should be made to the appropriate contexts and applications of probability.”

(NCCA 2010, p.16)

As well as containing possible activities, the resource pack also provides a “Questions in Context” section. The Project Maths syllabus quoted above values context and the resource pack gives teachers who wish to actively engage in the new syllabus the opportunity to bring contexts and applications into their probability classroom. Traditionally probability was taught in terms of a “new mathematics” approach and examples were based on coins, dice and cards. The resource pack, heavily influenced by the socio-cultural view, looks to move beyond that for reasons of understanding and attitude but it should also be noted that if the need to understand probabilistic statements in real life can be claimed as a valid rationale then students will have to experience probability questions in social settings (Schwartz and Goldman 1996). Teachers cannot claim that students who have only completed computational and decontextualised questions in class will be able to “interpret, reflect upon and think critically about diverse probabilistic situations and messages that they may encounter in real life” (Gal 2005, p.59).

Students should be introduced to probability using real life contexts and data (Shaughnessy 2003, Confrey and Kazak 2006). Teachers favour questions involving dice and cards because they are easy to manipulate not because they provide students
with a better understanding (Albert 2003). Gal (2005) states that the use of contexts in probability motivates students and provides them with learning experiences in a socially meaningful context and that “knowledge pertaining to the context is necessary both from a functional and educational standpoint” (p.52). Contexts, which challenge students’ probabilistic misconceptions are particularly useful. Saenz (1998) compared two sets of 14-15 year old Spanish students using a probability misconception test. The experimental group were taught using a method which focused on producing conceptual change by challenging students’ previously held beliefs in probability. The other group were taught traditionally. The experimental group performed far better in a misconception test and in their responses to counter-intuitive exercises. To conclude using real contexts, especially from and about the students themselves, increases the desire to learn, reduces misconceptions, and presents a solid base for meaningful understanding (Albert 2003; Gal 2005; Saenz 1998). However, setting questions within contexts does have its own issues which teachers need to be aware of. Students need to experience a variety of contexts, to ensure that the knowledge gained is not undermined by students’ understanding being limited solely within that context (Schwartz and Goldman, 1996). Contexts should also be chosen carefully to ensure it is educationally appropriate, while taking consideration of time constraints.
5.5 The Content and Format of the Resource Pack

The resource pack deals with three separate sub-topics of probability, Conditional Probability, Expected Value and Binomial Distribution. These three topics were chosen by the author as they are new concepts in terms of the Irish second level system, introduced as part of the Project Maths syllabus. The resource pack is pitched to those students studying the Higher Level course for their Leaving Certificate. These topics will be first examined in June 2012. Each topic is dealt with as an individual unit. Each unit is divided into four sections, which have a student and teacher component.

1. **Introductory Activity**

The opening activity in each section is an experiment, simulation or investigation designed to introduce students to the new concepts. This activity allows students to discover, with appropriate guidance from their teacher, the formulas and functions associated with the particular concept. The inclusion of this section was influenced by literature promoting guided discover (Bruner 1996; Vygotsky 1978) and active learning (Lane 2009; Prince 2004)

2. **Definitions and Examples (Teacher resource only)**

The learning outcomes, everyday uses, definitions, formulas and simple examples are outlined in this section (Appendix E).

3. **Student Investigation**

This investigation is a further activity which involves collecting data, either from the class group or through the conduction of a simulation. This allows the students to see the concept at work. The inclusion of this section was influenced by literature promoting the need for experiments and simulations in the probability classroom as
part of a mixed classical/frequentist approach (Kavatinsky and Even 2002; Shaughnessy and Ciancetta 2002)

4. Questions in Context

This section contains a number of questions, which are set in contexts where probability is encountered in everyday life. The inclusion of this section was based on the literature which promotes the use of context in the mathematics classroom (Shaughnessy 2003; Gal 2005)

The teacher component of each unit is essentially material which provides teachers with prompts, explanations and answers to the activities and questions in the student section. The only extra material in the teacher component is the “Definitions and Examples” section. The author has developed the materials in the teacher components so they can be delivered by teachers in a PowerPoint format, if they wish to do so. The resource pack does not have to be taught linearly or in totality. Teachers can use it as their sole source when teaching the topics or can just dip in and out of it at their discretion using it to complement their textbook rather than totally replace it.

5.6 The Evolution of the Resource Pack

Neither the format nor the content of the resource pack remained constant throughout this project. As is the norm with action research projects, at the end of each cycle the materials were evaluated and adjusted. The first cycle involved the creation of the resource pack, which was sent to an expert group for comment as described in the previous chapter in section 4.9.1. These comments were then evaluated and analysed,
which led to the alteration of the resource pack. The resource pack was then trialled with a group of 19 Transition Year students (data was only gathered from 17 due to non-attendance of two students for the entire intervention). Data was again collected and analysed from this group, which led to further changes in the resource pack format and content. The author has not included the data collected from the Transition Year group with the findings in Chapter 6, as only 9 of the 17 Transition Year students had studied Higher Level Junior Certificate mathematics. The author did not feel it appropriate to include this data as the pack was designed specifically for Higher Level mathematics students at Leaving Certificate level. The data generated did however have a strong influence on the content and format of the pack.

The purpose of this section is to illustrate how the cycles influenced the formation of the materials presented at the end of this chapter and in Appendix E. The author followed many suggestions made by the Expert Panel, one important example was a warning by Expert 2 on “the need to be very sensitive to events, which may have occurred in students’ lives”. This advice, led to changes in the conditional probability unit of the resource pack. The author removed an example from the Introduction section as well as a problem from the Questions in Context section. There are many other instances of the experts’ influence on the content of the resource pack, for example, the author followed the advice of Expert 4 and included more examples where binomial distribution could not be applied because he felt it “would help understanding”. Expert 2 also suggested that “maybe there needs to be more of a build up to the formula”. The author was unsure how to implement this advice and so the formation of the pack, which at that time was divided into three sections, Introduction,
Activities and Questions in Context remained unchanged. However this theme re-appeared in cycle 2 with the transition year group.

In the second cycle the author taught the resource pack to a group of transition year students. Data was collected in the form of a class diary, a test for understanding, a Fennema-Sherman attitude questionnaire as well as a review by the author’s critical friend, of a lesson. In the Diary, the author had a series of comments related to the introduction and how to maximise effectiveness of activities, which were in-line with Expert 2’s concerns about how the relevant formulae were introduced in the resource pack.

1/2/11  "Introduction is too teacher-orientated, not enough student activity."

9/2/11  "Is activity necessarily best to open topic? Sometimes activity more effective when pupils have practised easier examples."

10/2/11 "These activities are different in nature but both would probably have been more effective at an earlier stage."

The critical friend also made similar comments after witnessing one lesson, he stated that the “X card game leads to greater level of interest by all members of group. Better conceptual understanding?” The combined effect of this data was to cause the author to re-examine both the nature of the activities included and the format of the pack. On reflection and examination of the literature, the author decided that the activities included were positive features. These activities were in line with the frequentist philosophy, enabling students to experience probability at work, which is necessary
for conceptual understanding in probability (Engel 2002). However the activities did
not serve the purpose of guiding students to discover the pattern, formula or rule
which governed the concept being introduced. Guided discovery is a signature of the
socio-cultural constructivist approach, improving retention rates as well as students’
capacity to transfer knowledge to other areas (Mayer 2002).

The author thus divided the activities into two sections. The author altered previous
activities or created new ones to create an Introductory Activity section, which used
guided discovery to enable students to discover the pattern/formula involved in the
new topic. This became the first section of the resource pack. The second section was
now a Definitions and Examples section, which was an altered version of the old
Introduction section. The third section became a second activity section, a Student
Investigation, which involved an experiment or simulation allowing the students to
experience the concept in action. These simulations are powerful pedagogical tools,
which allow students to experience the variation between the observed data and the
expected outcome based on sample space (Shaughnessy and Ciancetta 2002).

The need for another major change to the format occurred during the second cycle.
The intention had been to provide the students with activity pages only. The author
realised in only the second lesson that this created problems when using the Questions
in Context section.

1/2/11 "I need to provide the students with questions on board/hand-out so
stronger students can progress.”
By not providing materials, which allowed students to work ahead the author could not keep all the students challenged. By the next lesson, the issue had been solved.

4/2/11  “I changed the format, I produced a hand-out of the PowerPoint slides with just the questions on it. This allowed students to progress at their own pace, which had the desired effect of constant work/discussion”

This change meant that three or four pages of Questions in Context slides were now added to each topic or unit of the student component of the resource pack.

The Fennema-Sherman Attitude Scale produced no conclusive indications. The mean score rose from 167.00 to 168.47 but this rise is not significant (p = .679). Significance is indicated by a probability < 0.05. None of the attitude subscales of Confidence, Anxiety, Perception of Usefulness, or Motivation registered significant changes. Only 7 students out of the 16 who completed the class test, scored a grade C or higher, indicating a strong understanding. These results would not be satisfactory in a Higher Level Leaving Certificate group. The author was not troubled by the data, which emerged from the attitudes scale or the class test. The context of the Transition Year class group is completely different from that of the a 5th year, Higher Level mathematics class, which the materials are aimed at. Only 9 of the Transition Year group did Higher Level at Junior Certificate and national statistics would suggest that less than half of these will go on to complete Higher Level mathematics at Leaving Certificate. Attendance is another factor, only 6 of the 19 students attended every lesson in the intervention due largely to involvement in other projects and extra-curricular activities. Homework is also not part of the Transition Year culture. Therefore, the lack of improvement in attitude or understanding indicated by the data,
did not lead directly to a major re-evaluation of the resource pack as the context of the Transition Year group is completely different to that of a Leaving Certificate Higher class. The format of the Resource Pack was however changed dramatically, as outlined earlier, due to the author’s experiences when teaching it to the Transition Year group. These alterations following the previous changes, which occurred after the expert panel review, meant that the format and content of the resource pack had now evolved to an extent that the author was satisfied to undertake the intervention with his 5th year Higher Level mathematics class.

5.7 Resource Materials

The materials included in the following pages are the student components of the resource pack, the teacher components of each unit can be found in Appendix E. As discussed in previous chapters the behaviourist approach aligned with the classical philosophy has led to students encountering difficulties when studying probability. In the opening chapter, Freudenthal (1970) was quoted as saying.

“In no mathematical domain is blind faith in techniques more often denounced than in probability, in no domain is critical thought more often required”

(Freudenthal cited by Jones and Thornton 2005, p.84)

This resource pack is an attempt by this author to move away from an approach which emphasises purely techniques to one which encourages “critical thought”, understanding and improved attitudes.
Senior Cycle
Probability
Resource Pack
Introductory Activity – Conditional Probability

Scenario:

You are given the Player profiles of the Irish rugby team who played against England in Croke Park in February 2007. Two pieces of categorical data have been chosen to be examined for a possible association (link) are

1) Does the player weigh less than 103 K.G.’s?
2) Is the player a back or a forward?

Activity:

1) Pose a Question

Can the answer given to one question help you predict the answer to another question? In this case “Given the weight of a player, does it affect the probability I randomly chose a back/forward? “

2) Collect the Data

Set up table like below, placing a tally mark in the right box
e.g. Girvan Dempsey is a back and weighs < 103 K.G.’s

<table>
<thead>
<tr>
<th>Position</th>
<th>Forward</th>
<th>Back</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In K.G.’s</td>
<td>≥103KG’s</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>&lt;103KG’s</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3) Analyse the data

When all the players have been analysed, count the ticks in each box and complete the Venn Diagram

![Venn Diagram](image)

Leaving Certificate – Higher Level Probability
1. What is the Probability that I randomly pick a forward from the panel?

2. What is the Probability that I randomly pick a player who weighs ≥ 103 K.G.’s. from the panel?

3. What is the Probability of picking a forward from the panel, if you chose from those who weigh ≥ 103 K.G.’s?

4. What is the Probability that I randomly pick a player who weigh ≥ 103 K.G.’s. from the panel, given he is a forward?

4) Interpret the data
   Can the occurrence of one event affect the probability of another event?

   Why do you think the probability is so high when I randomly pick a forward from the over 100 K.G.’s group? What is the association (link)?
NCE-MSTL Resource Pack

1. What is the Probability that I randomly pick a forward from the panel?


2. What is the Probability that I randomly pick a player who weighs ≥ 103 K.G.’s. from the panel?


3. What is the Probability of picking a forward from the panel, if you chose from those who weigh ≥ 103 K.G.’s?


4. What is the Probability that I randomly pick a player who weigh ≥ 103 K.G.’s. from the panel, given he is a forward?


4) Interpret the data

Can the occurrence of one event affect the probability of another event?


Why do you think the probability is so high when I randomly pick a forward from the over 100 K.G.’s group? What is the association (link)?


Leaving Certificate – Higher Level Probability
The Irish rugby panel from the Six Nations match against England in Croke Park in February '07.

In Rugby
- Players 1 - 8 and 16 – 19 are called Forwards
- Players 9 – 15 and 20 - 22 are called Backs

Leaving Certificate – Higher Level Probability
Note:

When a clause or condition is added when calculating a probability, it is called **Conditional Probability** e.g. What is the probability of A **given** B? (Mathematically written, What is \( P(A|B) \)?)

Use the empty Venn Diagram to answer the three questions below and try to establish a general rule for calculating conditional probability. (You can use the previous questions and Venn Diagrams as guides).

I. What is \( P(A|B) \)?

II. What is \( P(B|A) \)?

III. Does \( P(A|B) = P(B|A) \)?
Student Investigation – Conditional Probability

- As a class you could formulate some quick questions whose answers provide binary categorical data (data that fits into two possible answers e.g. Yes or No), and examine the conditional probabilities.

- Examples
  - “Is your favourite subject maths”
  - How many Males or Females?
  - Did you get an A in your Junior Certificate?
  - Do you like the “Twilight” movies?
  - Did you like the “Harry Potter” movies?
  - Do you send more than 10 texts a day?

Fill in the Venn Diagram below and then answer the questions below.

1. What is the probability of $A|B$?

2. What is the Probability of $B|A$?

3. Does $A|B = B|A$?

4. Is event A independent of event B? Why?
Conditional Probability

Q.1 - The Premiership

On February 8th, 2010 Liverpool F.C. were 4th in the premiership. This was their record

<table>
<thead>
<tr>
<th>Home</th>
<th>Draw</th>
<th>Loss</th>
<th>Away</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>9</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

Taken from the Irish Times, Monday February 8th

Is there an association between playing at home and Liverpool earning points?

Q.1 - The Premiership

Note: A win is worth three points and a draw 1 point. There are no points for a loss.

4. Draw a Venn Diagram to represent these figures and answer the following questions.
   Total played (25)

Q.2 Tossing two coins

A man tosses two coins. What is the probability that he gets

A) Two heads?

B) Two tails?

C) One head and one tail?

D) Given one of the coins is tails, what is the probability the man gets two tails?

Q.3 - The Swine Flu

According to the Irish Times on November 6th, 2009

3% of the Irish Population have contracted the H1N1 virus better known as Swine Flu. 665 people of these had to be hospitalised, of these 40% had an underlying condition. 14 of these patients died, all of whom had an underlying condition. The rate of infection in the general population currently stands at 178.5 cases per 100,000

Resource pack

NCE-MSTL
Q.3 - The Swine Flu
1. Construct a Venn Diagram of those who were hospitalised representing the connection between underlying condition and death.
2. At that date, what was the probability a patient hospitalised due to swine flu, will die given they have an underlying condition?
3. What was the probability that a patient hospitalised due to swine flu will die given the patient does not have an underlying condition?
   What do these probabilities suggest?

Q.4 - Unemployment Figures
The following information was taken from The CSO website in June 2010.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 25</td>
<td>57,000</td>
<td>20,300</td>
<td>77,300</td>
</tr>
<tr>
<td>Over 25</td>
<td>36,100</td>
<td>52,000</td>
<td>88,100</td>
</tr>
</tbody>
</table>

Examine the conditional probabilities involved without using Venn diagrams.

Q.4 - Unemployment Figures
1. What is the % probability of randomly picking a male on the live register given that they are under 25?
2. What is the % probability of randomly picking a female on the live register given that they are under 25?
3. What is the % probability of randomly picking a male on the live register given that they are over 25?
4. Why at this point in time do you think these probabilities are so high for males as compared to females?

Q.5 - Tennis
Serena Williams beat Vera Zvonareva 6–3, 6–2 in the Wimbledon ladies final on July 3rd 2010.
66% of Serena’s first serves were in, of which she won the point 94% of the time. She won 41% of her second serves.
Draw a tree diagram to represent this information.

Q.5 - Tennis
1. What is the probability of Serena winning a point during her service games?
2. What is the percentage probability of Serena losing a point during her service games given she has missed her first serve?
3. What is the probability of Serena serving her first serve out and losing the point?
4. What is the probability Serena’s first serve was in given she won the point?
Q. 6 – Medical Tests

There are a number of tests available to detect the presence of the AIDS virus.

Tests that give extremely accurate diagnosis take time to confirm but quick tests were developed to allow doctors to make a reasonably accurate rapid diagnosis.

These rapid tests have a sensitivity of 99.7% and a specificity of 98.5%

Note:

- Sensitivity which is 99.7% is the PP(Positive test result | Person actually has AIDS)
- Specificity which is 98.5% is the PN(Negative test result | Person doesn’t have AIDS)

In 2007, the rate of prevalence of the Aid’s virus was .2% in the Irish adult population. The Irish adult population was estimated at 2,750,000 in 2007.

Q. 6 – Medical Tests

Assuming the whole adult population was tested using the rapid tests, 1) Complete a Venn diagram to display this information by answering the following questions.

Has AIDS

Tested Positive

Adult Population (2,750,000)

Q. 6 – Medical Tests

2) How many people with AIDS tested positive?
3) How many with AIDS tested negative?
4) How many people do not have AIDS?
5) Of these how many tested negatively?
6) How many people do not have AIDS but tested positively?

Q. 6 – Medical Tests

7) How many people tested positively?
8) Perform a quick check to make sure your calculations add up.
9) What percentage of people who tested positive do not actually have AIDS?
10) Doctors are well aware of the conditional probabilities involved in these rapid tests. What action do you think they might take if you tested positive?

Prosecutor’s fallacy

The improper use of conditional probability in the courtroom has become notorious.

The “prosecutor’s fallacy” is probably the most famous error. This involves the prosecution reversing the condition. An example is the case of the Birmingham six.

In 1974 two Birmingham pubs were bombed by the IRA killing 21 people. Six men travelling to a funeral in Belfast were arrested.
Prosecutor's fallacy

- A forensic scientist testified that he was 99 percent certain the defendants had handled explosives, given the results of his tests.
- \( P(\text{Handled Explosives} \mid \text{Positive Test}) = 99\% \)
- What he should have said was the test is positive 99% of the time if someone has handled explosives or \( \frac{P(\text{Positive Test} \mid \text{Handled Explosives})}{P(\text{Positive Test})} = 99\% \)
- The \( P(\text{Handed Explosives} \mid \text{Positive Test}) = \frac{P(\text{Positive Test} \mid \text{Handled Explosives})}{P(\text{Positive Test})} \).

Prosecutor's fallacy

- There subtle but vital difference means the probability of them handling explosives was a lot lower than 99 out of 100.
- It was later revealed that handling many other everyday substances can produce positive test results, including playing cards, soil, cigarettes, soap and petrol.
- The defendants had been playing a game of cards on the train shortly before their arrest. The convictions of the Birmingham Six were overturned on appeal in 1991, after 18 years in prison.

The Monty Hall Problem

- *Let's Make a Deal* was a game show hosted by Monty Hall that ran on U.S. TV from 1966–77.
- In each episode a contestant was presented with 3 doors. Two doors hid goats behind them, the third hid a car. If the contestant guessed correctly they got to keep the car.
- The contestant would pick a door. The host would then open another door showing a goat and give the contestant the choice to switch with their original door or to swap.

The Monty Hall Problem

- This became one of the most famous mathematics problem in the world in 1990.
- Craig F. Whitaker wrote a letter to Marilyn vos Savant a Q&A columnist for *Parade* magazine asking "Is it to your advantage to switch your choice of doors?"
- She replied that "Yes, you should switch" and if you do you will win two-thirds of the time.

The Monty Hall Problem

- This answer caused a commotion with many mathematicians rushing to condemn her saying that the chances were clearly 50:50.
- Do you think that it would make a difference if the strategy is to stick or to swap?
- You can play the game on the Project Maths CD. Look up the "3 Doors Problem" in Java scripts.

Conditional Probability - Answers

1. 85%, 58%
2. \( \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \)
3. \( \frac{1}{16}, 0 \)
4. 63%, 37%, 66%, 34%
5. 76%, 59%, 20%, 82%
6. Adult Population (2,750,000)

Resource pack

NCE-MSTL

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Introductory Activity - Expected Value

Scenario:

You attend a local carnival and decide to play a game at a stand. Three cards are placed in a hat, one card has an "X" on both sides, a second has an "X" on one side and is blank on the other side and the last one is blank on both sides. The game involves you pulling one of the cards out of the hat and placing it on the table so only one side has been seen by the stall owner (who claims to have some psychic abilities). The stall owner then guesses what is on the side facing down.

<table>
<thead>
<tr>
<th>Card 1</th>
<th>Card 2</th>
<th>Card 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>X</td>
<td>≤</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If they guess correctly they take €1 if they guess wrong you get €1. Would you play this game?

Activity:

1) Pose a Question

Is this game fair? What is fair?

2) Collect the Data

To play, students pair off. One plays the part of the stall owner, the other is the player.

The player is given the 3 cards, shuffles them behind their back and places one of them on the table in front of the stall owner. The stall owner guesses that the underneath of the card is the same as the top

As they play the game the students should mark down how many times they win using the tally count in the activity worksheet.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Stall Owner Correct</th>
<th>Stall Owner Wrong</th>
</tr>
</thead>
<tbody>
<tr>
<td>My Tally</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative Frequency</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
NCE-MSTL Resource Pack

3) Analyse the data

Fill in the table and find the average amount of money won/lost using the mean of a frequency table formula

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Stall Owner</th>
<th>Player</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-€1</td>
<td>+€1</td>
</tr>
<tr>
<td>Class Frequency</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conclusion From experiment: The observed value for the player in this game is =

4) Interpret the data

Is this game fair? Why?

Having worked out the theoretical probability of the stall owner winning, find the average outcome using both tables.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Stall Owner Correct</th>
<th>Stall Owner Wrong</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>-€1</td>
<td>+€1</td>
</tr>
<tr>
<td>Frequency</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Stall Owner Correct</th>
<th>Stall Owner Wrong</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>-€1</td>
<td>+€1</td>
</tr>
<tr>
<td>P(X)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This average which predicts your average gain or loss is known as the Expected Value.

What would the expected value be in a fair game?

Leaving Certificate-Higher Level Probability
A charity sets up a game involving dice at their benefit dance. The game is simple and it costs only €1 to play.

The player picks a number from 1 to 6. They roll 3 dice at once. They are paid out €2 for each time their number turns up on a dice.

Will the charity make money on this game?

<table>
<thead>
<tr>
<th>Chance of rolling</th>
<th>Calculations</th>
<th>Probability</th>
<th>Value for charity</th>
<th>Expected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No 6's</td>
<td>( \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} )</td>
<td></td>
<td>125</td>
<td>216</td>
</tr>
<tr>
<td>1 6</td>
<td>( 3 \left( \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \right) )</td>
<td></td>
<td>75</td>
<td>216</td>
</tr>
<tr>
<td>2 6's</td>
<td>( 3 \left( \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \right) )</td>
<td></td>
<td>15</td>
<td>216</td>
</tr>
<tr>
<td>3 6's</td>
<td>( \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} )</td>
<td></td>
<td>1</td>
<td>216</td>
</tr>
</tbody>
</table>

The expected value for the charity =

Below are two empty tables, students should work together in pairs to change the game making it attractive to play yet raising the expected value for the charity.

<table>
<thead>
<tr>
<th>Chance of rolling</th>
<th>Calculations</th>
<th>Probability</th>
<th>Value</th>
<th>Expected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No 6's</td>
<td>( \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} )</td>
<td></td>
<td>125</td>
<td>216</td>
</tr>
<tr>
<td>1 6</td>
<td>( 3 \left( \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \right) )</td>
<td></td>
<td>75</td>
<td>216</td>
</tr>
<tr>
<td>2 6's</td>
<td>( 3 \left( \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \right) )</td>
<td></td>
<td>15</td>
<td>216</td>
</tr>
<tr>
<td>3 6's</td>
<td>( \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} )</td>
<td></td>
<td>1</td>
<td>216</td>
</tr>
</tbody>
</table>
The expected value for the player per game =

<table>
<thead>
<tr>
<th>Chance of rolling</th>
<th>Calculations</th>
<th>Probability</th>
<th>Value for charity</th>
</tr>
</thead>
<tbody>
<tr>
<td>No 6's</td>
<td>$\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$</td>
<td></td>
<td>$\frac{125}{216}$</td>
</tr>
<tr>
<td>1 6</td>
<td>$3 \left(\frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}\right)$</td>
<td></td>
<td>$\frac{75}{216}$</td>
</tr>
<tr>
<td>2 6's</td>
<td>$3 \left(\frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}\right)$</td>
<td></td>
<td>$\frac{15}{216}$</td>
</tr>
<tr>
<td>3 6's</td>
<td>$\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6}$</td>
<td></td>
<td>$\frac{1}{216}$</td>
</tr>
</tbody>
</table>

The expected value for the charity =

- Fill in the table below comparing each group in the class. Which one do you think is the best option for the charity?

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost to play</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prize for 1 No.</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Prize for 2 No.'s</td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Prize for 3 No.'s</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Expected Value for the charity</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Leaving Certificate-Higher Level Probability
Q.1 – The “Psychic”

A psychic runs the following ad in a magazine:

Expecting a baby? Renowned psychic will tell you the sex of the unborn child from are photograph of the pregnant mother. Cost €10. Money back guarantee if wrong.

- Considering the probability of a boy is 0.51 and the probability of a girl is 0.49. How will the psychic maximise their expected value?
- Taken from “Statistics – Concepts and Controversies” by David S. Moore

Q.2 Dice Game

A student has a game. He rolls 3 dice.

- If the dice sum to the first four possible numbers 3, 4, 5 or 6 or the last four possible numbers 15, 16, 17, 18 he will give you €2.
- If they sum from 7 to 14, you pay him €1.
- You each have 8 numbers, and you have a better payout but should you play this game?

<table>
<thead>
<tr>
<th>Dice sum to</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.005</td>
</tr>
<tr>
<td>4</td>
<td>0.014</td>
</tr>
<tr>
<td>5</td>
<td>0.028</td>
</tr>
<tr>
<td>6</td>
<td>0.046</td>
</tr>
<tr>
<td>7</td>
<td>0.069</td>
</tr>
<tr>
<td>8</td>
<td>0.097</td>
</tr>
<tr>
<td>9</td>
<td>0.116</td>
</tr>
<tr>
<td>10</td>
<td>0.125</td>
</tr>
<tr>
<td>11</td>
<td>0.125</td>
</tr>
<tr>
<td>12</td>
<td>0.116</td>
</tr>
<tr>
<td>13</td>
<td>0.097</td>
</tr>
<tr>
<td>14</td>
<td>0.069</td>
</tr>
</tbody>
</table>

Total

Q.3 Roulette Table

The wheel has 37 numbers from 0 to 36.
- Odd numbers are red, even ones are black.
- The number 0 is green.

(a) Bet A – “A straight” up involves picking a single number and betting on it. The odds on offer are 35 to 1. What is the expected value for the player if he bets €1?

(b) Bet C – “A Street” involves picking a row of 3 numbers. The odds on offer are 11 to 1. What is the expected value for a €1 bet?

(c) Bet J – “Even Chances”, involves backing red or black. The odds on offer are 1–1.
Q3 – Roulette

- If green turns up - the player loses half his stake. What is the expected value for a €1 bet?
- If you were playing roulette what would you bet on?
- In the American version of roulette, there is a 38th number, a second green 00. What does this do to the ‘house edge’ in the case of ‘A Straight’?

Car Recall

- In January 2010, a well known car manufacturer began recalling cars to fix them with new accelerators after complaints of accidents, some of which were fatal.
- The car manufacturer said that this recall which cost over a billion dollars demonstrated their commitment to quality.

Car Recall

- However others argued that the car manufacturer knew about the problem since late 2008 and only ordered the recall when the cost of settlements, individual repairs and customer loyalty meant that the expected value favoured a recall.

Q.4. “Tolerable Risk”

- A toy manufacturer has developed a product that is expected to sell really well. It has already produced 100,000 units of the toy before it discovers a flaw.
- A small part breaks off in certain conditions, it is estimated by their actuary department that the chance of a child dying by swallowing the piece is 0.00002
- It cost €20 to produce each unit, with a profit of €10 per unit.

Q.4. “Tolerable Risk”

- The manufacturer is given legal advice that a court settlement will cost €100,000.
- The manufacturer has fixed the flaw but what will they do with 100,000 units already produced?
- The company has two options either sell the stock or dump it. Work out the expected value for both options?

Q.5 – Multiple Choice Test

- Peter is sitting a multiple-choice test. He has not studied for the test so he picks the answers to the questions at random. He has to answer either Section A or Section B. Both sections are marked out of 100.
- In Section A, there are 20 questions with two possible answers. A correct answer is worth 5 marks but a question answered incorrectly is worth -1.5
Q.5 – Multiple Choice Test
- In Section B, there are 10 questions with 4 possible answers. Each question is worth 10 marks but an incorrect answer is worth -1.
- What is the best strategy for Peter?

Q.6 – A Warranty
- Ellen buys a new TV worth €1,000.
- It is guaranteed for a year but she can buy an extended warranty for a further 3 years for €100.
- She does some research and finds out that 10% of these TV’s experience trouble in these 3 years and the average repair costs €400.
- Should she get the warranty?

Q.7 “Yarborough” Insurance
- Lord Yarborough, (1809 – 1862) gave his name to a hand of cards dealt in bridge that has no card higher than a nine. (Ace is higher)
- High value cards are important in bridge and Lord Yarborough offered an insurance policy of £1,000 to any of his friends who were dealt such a hand – on condition they paid him a £1 insurance before each hand was dealt. There are 13 cards in a bridge hand.
- If you were Lord Yarborough’s friend would you have taken out his insurance policy?

Note on Insurance
- Insurance companies use expected values in the same way as casinos. They bet on you not having to claim based on probabilities arrived at from their data.
- The premiums are structured in a manner that gives the insurance company a “house edge”. This means over the long term the average customer will pay out more on insurance than they claim.

Q.8 Insurance Premium
- In 2008, there were 24,684 burglaries reported. There were 1,469,321 households in the country. You work for an insurance company with whom 10,000 people have house insurance.

Note on Insurance
- People get insurance to cover short term risk, e.g. car insurance may cost €500 a year but it is more affordable than the €3,000 it would take to replace the car if it was stolen.
- Insurance companies have to keep a large reserve of cash in case a short term streak works against them.
- In 2010 the failure of Quinn Insurance to do this meant the company was taken over by receivers put in place by the financial regulator.
Q.8. – Insurance
1. What is the probability that a house was burgled in 2008?
2. The average payout to a house that has been burgled is €2,000. The company likes to keep its Gross profit (its profit before it pays its employees etc.) at €3,000,000 a year. What will the average insurance premium be?

Q.9 All or Nothing
» All or Nothing is the latest National Lottery game. For €2 play for a chance to win €500,000
» Match 12 or None – Win €500,000
Match 11 Numbers – Win €5,000
Match 10 Numbers – Win €25
Match 9 Numbers – Win €10
Match 8 Numbers – Win €4

What is the National Lottery’s expected value?

Q.9 – All or Nothing

<table>
<thead>
<tr>
<th>Numbers Correct</th>
<th>Probability</th>
<th>Value X</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/12</td>
<td>1</td>
<td>4,704,156</td>
<td>507,087</td>
</tr>
<tr>
<td>11/12</td>
<td>1/12</td>
<td>4,704,156</td>
<td>481,993</td>
</tr>
<tr>
<td>10/12</td>
<td>1/120</td>
<td>4,704,156</td>
<td>39,163</td>
</tr>
<tr>
<td>9/12</td>
<td>1/1,200</td>
<td>4,704,156</td>
<td>39,163</td>
</tr>
<tr>
<td>8/12</td>
<td>1/7,200</td>
<td>4,704,156</td>
<td>21,217</td>
</tr>
<tr>
<td>7/12</td>
<td>1/47,520</td>
<td>4,704,156</td>
<td>12,445</td>
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<tr>
<td>6/12</td>
<td>1/252,000</td>
<td>4,704,156</td>
<td>5,232</td>
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<tr>
<td>5/12</td>
<td>1/1,546,000</td>
<td>4,704,156</td>
<td>3,652</td>
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<td>4/12</td>
<td>1/9,702,000</td>
<td>4,704,156</td>
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</tr>
<tr>
<td>3/12</td>
<td>1/60,850,000</td>
<td>4,704,156</td>
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<td>2/12</td>
<td>1/479,000,000</td>
<td>4,704,156</td>
<td>1,351</td>
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<tr>
<td>1/12</td>
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<td>4,704,156</td>
<td>1,126</td>
</tr>
<tr>
<td>0/12</td>
<td>1/2,431,025</td>
<td>4,704,156</td>
<td>1,035</td>
</tr>
</tbody>
</table>

Expected Value – Answers
1. Respond "boy" to all enquiries =€3.10 profit on average for every customer
2. = €-4.42
3. =-€0.027, -€0.027, -€0.0135, -€0.053
4. Using expected value as their only consideration, the best option for the business is to sell the product. There are other considerations

Expected Value – Answers
5. A = 35%, B = 17.5%
6. The warranty will cost customers €60 on average.
7. = -€0.45
8. 1.679% of houses, €333.58 = x – The premium charged
9. = €1.104
Introductory Activity – Binomial Distribution

Task:

Complete the second and third tables and try to establish the pattern.

### A Die is thrown twice

<table>
<thead>
<tr>
<th></th>
<th>6</th>
<th>Not a 6</th>
<th>No. of Possible Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (0 6's)</td>
<td>( \frac{1}{6} \times \frac{5}{6} )</td>
<td>( \frac{5}{6} \times \frac{6}{6} )</td>
<td>1</td>
</tr>
<tr>
<td>P (1 6's)</td>
<td>( \frac{1}{6} \times \frac{1}{6} )</td>
<td>( \frac{1}{6} \times \frac{6}{6} )</td>
<td>2</td>
</tr>
<tr>
<td>P (2 6's)</td>
<td>( \frac{1}{6} \times \frac{6}{6} )</td>
<td>( \frac{1}{6} \times \frac{6}{6} )</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
P(0 \text{ 6's}) = \frac{1}{6} \times \frac{5^2}{6} \]
\[
P(1 \text{ 6}) = \frac{1}{6} \times \frac{5^1}{6} \]
\[
P(2 \text{ 6's}) = \frac{1}{6} \times \frac{5^0}{6} \]

### A Die is thrown three times

<table>
<thead>
<tr>
<th></th>
<th>6</th>
<th>Not a 6</th>
<th>No. of Possible Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (0 6's)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P (1 6's)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P (2 6's)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P (3 6's)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
P(0 \text{ 6's}) = \]
\[
P(1 \text{ 6}) = \]
\[
P(2 \text{ 6's}) = \]
\[
P(3 \text{ 6's}) = \]

Leaving Certificate-Higher Level Probability
A Die is thrown four times

<table>
<thead>
<tr>
<th></th>
<th>6</th>
<th>Not a 6</th>
<th>No. of Possible Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (0 6's)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P (1 6's)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P (2 6’s)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P (3 6’s)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P (4 6’s)</td>
<td>x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ P(0 \text{ 6's}) = \]
\[ P(1 \text{ 6}) = \]
\[ P(2 \text{ 6's}) = \]
\[ P(3 \text{ 6's}) = \]
\[ P(4 \text{ 6's}) = \]

Can you write the pattern out in words?

__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________
__________________________________________________________________________

Leaving Certificate-Higher Level Probability
Student Investigation - Binomial Distribution

Scenario:

A company takes tourists on helicopter trips over Dublin. The helicopter only seats 6 people but the company discovered that 33% of all bookings do not show. In order to maximise profit they are thinking about taking 8 bookings for every trip.

Activity:

1) Pose a Question

What is the probability of their flights being overbooked? Is this good company policy?

2) Collect the Data

Using a die conduct a simulation with a 33% chance of success. Write 1, if 1 or 2 appears to represent the 33% chance of a no show. Write 3, if 3, 4, 5 and 6 appears to represent the 66% chance of the customer showing up.

The random number generator on your calculator could also be used.

<table>
<thead>
<tr>
<th>Trip 1</th>
<th>Booking 1</th>
<th>Booking 2</th>
<th>Booking 3</th>
<th>Booking 4</th>
<th>Booking 5</th>
<th>Booking 6</th>
<th>Booking 7</th>
<th>Booking 8</th>
<th>Extra Passengers</th>
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<tbody>
<tr>
<td>Trip 1</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Trip 2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

In trip 1 there was two 1’s, so two no-shows.

In trip 2 there was one 1, so only one booking did not show.

3) Analyse the data

<table>
<thead>
<tr>
<th>Booking</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>No. Of extra Passengers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Trip 2</td>
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<td></td>
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<tr>
<td>Trip 3</td>
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<td></td>
<td></td>
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<tr>
<td>Trip 4</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Trip 5</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Trip 6</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Trip 7</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Trip 8</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Trip 9</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Trip 10</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
### NCE-MSTL Resource Pack

<table>
<thead>
<tr>
<th>No. Of extra Passengers</th>
<th>My simulation</th>
<th>Class Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>/10</td>
<td>/</td>
</tr>
<tr>
<td>1</td>
<td>/10</td>
<td>/</td>
</tr>
<tr>
<td>2</td>
<td>/10</td>
<td>/</td>
</tr>
</tbody>
</table>

1. Using the class totals, what is the Probability
   (A) That exactly 7 show up?

   ____________________________

   (B) That 8 show up?

   ____________________________

   (C) Of being overbooked?

   ____________________________

2. Using the Bernoulli trials formula compare the probabilities to the simulation?
   (A) ____________________________

   (B) ____________________________

   (C) ____________________________

(4) Interpret the data
   (A) If you were advising the company, would you recommend this overbooking policy? Give 2 reasons.

   ____________________________

   ____________________________

   (B) Given the results from the data collected, are there other questions that you would like to examine?

   ____________________________

   ____________________________

   ____________________________

   ____________________________

   ____________________________

   ____________________________

Leaving Certificate-Higher Level Probability
**Q.1 Quality Control**

Records show that an MP3 player produced by a particular company has a defect rate of 1 in 200. The factory sells them to the shops in boxes of 20. The quality control technician tests 1 box selected at random every hour.

1. What is the probability that 1 or less of these 20 are defective?
2. If more than 1 was found to be defective what would this suggest? What might the quality control officer do?

**Q.2 Polygraph Test (Lie Detector)**

According to the American Polygraph Association website "Researchers conducted 41 studies involving the accuracy of 1,787 laboratory simulations of polygraph examinations, producing an average accuracy of 80%.

In 2003, the National Academy of Sciences issued a report entitled "The Polygraph and Lie Detection." The report concluded that this level of accuracy was overstated.

**Q.2 Polygraph Test**

If the police interviewed 10 innocent people in relation to a crime, what is the probability one or two of these will fail it?

Answer the question based on an 80% accuracy rate (Assuming this rate means that 2 out of 10 guilty people will appear innocent and 2 out of 10 innocent people will appear guilty).

**Q.3 Genetics**

Cystic Fibrosis is Ireland's most common life-threatening inherited disease. With over 1,100 CF Patients, Ireland has the highest proportion of CF people in the world.

Approximately 1 in 19 people are carriers of the CF gene and where two carriers parent a child together, there is a 1 in 4 chance of the baby developing Cystic Fibrosis.

1. What is the chance of two unaffected carriers parenting a child?
2. Two carriers have 3 children together, what is the probability that none of the children have CF?
3. What is the chance of at least one of their children having CF?
4. What is the chance that all three children have CF?
Q.4 Leaving Certificate Maths

In 2009 15% of those students who took Higher Level Mathematics got an A Grade

A school has one higher level leaving cert class of 20 students. The teacher tells her class of 20 students that she expects no more than 3 of them to get an A. What is the probability she will be correct? Round your answer to two decimal places.

Note: This assumes that class is representative of leaving certificate population

Q.5 Profit Optimisation

A certain airline knows that on average 10% of their customers do not turn up. It books its planes to a capacity of 105. Their standard plane seats 100 people.

What is the probability that they will be overbooked?

This is a basic example of what is called in the airline business "Profit Optimisation".

Q.6 Traffic Lights

On the way to work, John travels along a main street which has three traffic lights.

As it is the main street, the traffic lights are set so that they are green 70% of the time and red 30% of the time.

On Friday morning, John arrives into work a few minutes late again saying that every morning this week, the three traffic lights have been red when he got to them. Do you believe him?

Q.7 Multiple Choice Test

John says he has not studied for his exam so he is going to guess the answers at random.

It is a multiple choice test of 10 questions. There are four possible answers to each question. He needs 40% to pass.

Q.7 Multiple Choice Test

1. What is the chance of John passing this exam?

2. John got 70%. How probable is it that John got 70% or more without studying i.e. he is guessing at random?

3. If the test was changed to 20 questions, would the test be more or less difficult to pass?

Q.8. Psychic or Not?

Zener cards are a classic method of testing for extrasensory perception (ESP).

The Zener deck consists of 25 cards – five cards each of five symbols: a circle, a star, a square, a cross and wavy lines.

Resource pack

NCE-MSTL

168
Senior Cycle Higher Level Mathematics

Q.8. Psychic or Not?

- A tester pulls a card out of the deck and the subject tries to predict which of the 5 symbols is on the card.
- They have a 20% chance of being correct, the card is returned to the deck and the experiment repeated a set number of times.
- To prevent any bias the subject is not given their results until the end.

Q.8. Psychic or Not?

- What is the probability the subject gets 5 out of 25 correct guessing randomly?
- What is the probability the subject gets the 5th one correct on exactly the 25th go? Anyone who showed potential of ESP was tested further. This was defined as getting a score that had a less than 1% probability of happening by chance. (Though it has a chance of only .37%, which is less than 1%, scoring 0 does not count as it does not show potential for ESP)

Q.8. Psychic or Not?

- By trial and error, find how many correct answers out of 25 the subject has to have correct to be tested further?
- Joe got exactly 11 out of 25 correct, 3 times out of 5 what is the probability of this?

Q.9 – Oil Wells

- Suppose an oil exploration firm has identified 10 potential oil wells off the coast of Ireland. Each has a 10% probability of striking oil.
- On average it costs the firm €500,000 to drill each well but a successful well nets them a profit of €10,000,000.

Q.9 – Oil Wells

- What is the probability that the company will strike oil on all wells and lose money?
- What is the chance they will strike oil only on exactly the 10th drill?
- What is the expected value for the company on each well? What conclusions do you draw from this?

Binomial Distribution – Answers

1. 1.67%
2. 57% chance one or two will fail the lie detector test even though they are innocent.
3. 0.00277, 0.421875, 0.578125, 0.015625
4. 65%
5. 90.55%
6. 0.000800014
7. 22%, 0.34%, more difficult
8. 0.196, 0.0392, 11 or better, 0.000001
9. 35%, 4%, €550,000,
Chapter 6 – Research Findings

6.1 Introduction

In this chapter the author presents and analyses the main findings from the data collected during the study. The author adopted a mixed-method research design as described in Chapter 4, Section 4.4. Thus the data collected is a mix of both qualitative and quantitative. The author is concerned primarily with finding a solution to the particular problem of teaching probability in his own classroom and the mixed-method approach provided the author with the most pragmatic means of evaluating the intervention. The chapter includes a description of the evaluation procedures, an examination of the findings of each of the research questions posed, an examination of any other major themes which may have emerged in the data analysis and a summary of the findings. This chapter is limited to the presentation and analysis of the findings of the data collected during this study. Conclusions will be drawn in the next and final chapter, Chapter 7, where the findings discussed in this chapter will be assessed in light of the literature review conducted in Chapter 2 and Chapter 3.
6.2 Evaluation

The research questions, which the author examined in this action research project were:

1. Is there a link between the use of active learning methodologies and real life data and contexts and an improvement in students’ attitude towards mathematics?

2. Does the use of active learning methodologies and real life data and contexts develop students’ understanding of probability?

In Section 4.9 the need to examine the pack from more than one perspective, in order for the evaluation to be considered valid, was described (Stenhouse 1975; Elliott 1991; McKernan 1996). The author employed the strategy of analysing the resource pack from three separate perspectives.

1. The Outside Perspective

2. The Student Perspective

3. The Teacher’s perspective

This provides the author with the “holistic” or systematic overview (Miles and Huberman 1994), which is the hallmark of mixed method studies. It must also be noted that the resource pack was altered after each cycle in response to the evaluation from the three perspectives. Therefore, the resource pack reviewed by the expert panel has been changed by the time the fifth year group are responding to it.
6.2.1 The Outside Perspective

The outside perspective was provided by two distinct groups, an expert panel who provided feedback as part of the first action research cycle and a critical friend whose input was constant if largely informal throughout. The expert panel reviewed the resource pack in September 2010. Each expert brought slightly differing motivations to the panel and thus had different reasons for inclusion. The panel were given a self-designed questionnaire (Appendix B). This questionnaire contained 19 statements which the panel replied to using a Likert scale, though room for comment was allowed as well.

<table>
<thead>
<tr>
<th>Expert</th>
<th>Background</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expert 1</td>
<td>A practising Higher Level mathematics teacher who had no experience of the new syllabus beyond in-service</td>
</tr>
<tr>
<td>Expert 2</td>
<td>A mathematics teacher seconded by the Department of Education and Skills to the Project Maths in-service panel</td>
</tr>
<tr>
<td>Expert 3</td>
<td>A former mathematics teacher, with a doctorate in Mathematics Education working in the University of Limerick</td>
</tr>
<tr>
<td>Expert 4</td>
<td>A teacher of Higher Level mathematics in a Project Maths pilot school.</td>
</tr>
<tr>
<td>Expert 5</td>
<td>A doctoral student in Mathematics Education who comes from a pure mathematics background.</td>
</tr>
<tr>
<td>Critical Friend</td>
<td>A colleague within the mathematics department in the school</td>
</tr>
</tbody>
</table>
Consistent with an action research approach adjustments were made to the pack subsequent to expert panel review to improve certain areas, as described in Section 5.5. The critical friend review used the same 19 statements and was completed after seeing classes taught using the resource pack with the Transition Year group. Unfortunately it was not possible due to time-tabling constrictions for the critical friend to see the pack being taught with the 5th year group. Though he did re-examine the pack and provided the author with oral feed-back on the changes which were made in response to the Transition Year trialling. The author would also like to note that not all of the expert panel and critical friend comments have been included as some were made about specific content in the pack. These comments were extremely valuable when refining the pack as outlined in Section 5.5 but are not included below where they are not relevant to the specific research question being discussed. Where these comments could be interpreted as being relevant they are included for reasons of validity. The data from the outside perspective is essentially qualitative in nature, providing the author with important information. It makes no claim to be a representative sample, of teachers for example, but instead offers the researcher a rich insight into the views and perspectives of differing stakeholders in the Irish mathematics education system.

6.2.2 The Students’ Perspective

The student’s perspective was captured using two differing methods, see Section 4.9.2. They were given a pre and post intervention attitudes questionnaire (Appendix C). They were also given a written examination at the end of the intervention, which examined their understanding of the topics taught (Appendix D). The 5th year student data was collected in March/April 2011 and was analysed using SPSS. The 14
students are the only Higher Level 5th year mathematics class in a mixed, rural, community college of circa 500 students. Similar data was gathered from Transition Year students who were taught using the resource pack, as an earlier part of the action research cycle in February 2011. This data is not included here as this group as only 9 of these 17 students had studied Higher Level Junior Certificate mathematics. The author did not feel it appropriate to include data from these students as the pack was designed specifically for Higher Level mathematics students. The Transition Year group were invaluable in the formation of the resource pack as the second cycle in the action research approach centred around teaching the resource pack to them (see Section 5.5). The response rate in terms of completion of attitude questionnaires and a class test for the 5th year group was 100%. It should be noted one student had a high absenteeism rate, missing 3 out of 10 lessons. While no qualifications were made in terms of the attitude questionnaire, it was noted when discussing the results of the class test.

The Fennema-Sherman attitude questionnaire (1976, Appendix C) produced valid quantitative data allowing tests of significance to be carried out. The reliability of the scales was verified using a Cronbach Alpha score. The test for understanding on the other hand, is essentially a qualitative tool. A test for understanding following respected criteria was developed (Rittle-Johnson et al. 2001; Konold 1995; Hiebert and Wearne 1996) but since the data produced cannot be compared to a pre-test or control group no quantitative claims can be made in this respect. An examination of a correlation with Junior Cert results was however possible.
6.2.3 The Teacher’s Perspective

The perspective of the researcher/teacher was captured through the use of a class diary, which enabled the author to analyse and examine whether the data collected supported the findings of the other perspectives on the research questions. The author obviously completed the diary with the research questions in mind but it was very often the day-to-day practicalities which seemed to occupy a good share of the researcher’s thoughts. The data produced from the diary is qualitative in nature and while offering insights, it needs the corroboration of other perspectives to claim validity. In the case of each of the research questions the author will analyse the data produced from these three perspectives in turn before discussing any relevant findings.

6.3 Research Question 1:

Is there is a link between the use of active learning methodologies and real life data and contexts and an improvement in students’ attitude towards mathematics?

6.3.1 The Outside Perspective

The following are the responses of the expert panel and the critical friend to the relevant statements in the questionnaire (Appendix B). An opportunity to comment at the end of each section was given and where these comments are relevant to the question – they are included.
Statement: The resource pack encourages active learning

Two of the expert panel circled A, strongly agreeing with this statement and the other three circled B agreeing with this statement. On reviewing the lesson the critical friend circled A, strongly agreeing with the statement.

Figure 6.1 The Resource Pack encourages Active Learning

Additional comments:

Expert 2 noted “this section (Expected value) is excellent. Both the activities and example are very good”. Expert 4 wanted “another investigation/student activity for each of the three areas. It would encourage active learning”
Statement: The resource pack makes use of real life data and contexts

All 5 of the expert panel circled A, strongly agreeing with this statement. On reviewing the lesson the critical friend circled B, agreeing with the statement.

Figure 6.2 The Resource Pack makes use of real life Data and Contexts

Additional comments:

Expert 3 said of the Binomial Distribution section “Very good questions in context – applicable and of interest.” Expert 4 said “The examples/questions use real life everyday problems. It definitely helps to answer the question students ask – why are we doing this?” Expert 5 stated that “the use of real-life examples is a significant positive feature.”
**Statement:** The resource pack will engage and motivate students

Three of the expert panel circled A, strongly agreeing with this statement and the other two circled B agreeing with this statement. On reviewing the lesson the critical friend circled B, agreeing with the statement.

**Figure 6.3** The Resource Pack Will Engage And Motivate Students

Additional comments:
Expert 1 added that “I feel students will really enjoy the everyday examples which they will be able to relate to, not like the questions in the old textbooks e.g. replacing discs.” Expert 2 noted that the “students I tried the examples with were fully engaged and found them very interesting.” Expert 5 said that “the activities and games throughout the pack will help motivate and reinforce the learning that is occurring”. The Critical Friend commenting specifically on a lesson he witnessed that “the X card game leads to greater level of interest by all members of the group”
6.3.2 The Student’s Perspective

The Attitude Scales

The author used a modified version of the Fennema-Sherman attitude scale (1976, Appendix C) to test for any changes in students’ attitude towards mathematics. This attitude test was broken down into four subscales confidence, anxiety, perception of usefulness and effective motivation. There were 12 statements in each scale, 6 negative statements and 6 positive statements. Each respondent was asked to indicate their level of agreement or disagreement with each item. For the positive items, 1 = strongly disagree, 2 = disagree, 3 = not sure, 4 = agree, 5 = strongly agree. For negative worded statements the order was reversed (1 = strongly agree, 2 = agree, 3 = not sure, 4 = disagree, 5 = strongly disagree). Thus a high score indicated a favourable attitude. The highest possible score is a score of 60 (12 × 5). In the confidence subscales a score of 60 would indicate the highest possible confidence but in the anxiety subscales a score of 60 would indicate the lowest possible levels of anxiety. Each of the 12 statements in each of the four scales are listed in separate tables along with the mean score and standard deviation. The number the statement had in the questionnaire the students completed is provided in brackets. The data for the mean is given to three significant figures. The data for the standard deviation is given to two significant figures.
Reliability of Scales

A test of reliability or internal consistency of the data obtained using each of the scales, both pre and post-intervention, was undertaken. The value of Crobach’s alpha was calculated in each case and each is outlined in Table 6.2 below. The Cronbach alpha value indicated very good reliability (> 0.8) for Confidence (pre and post), Effective Motivation (pre), Anxiety (pre and post) and Usefulness (pre and post). Anxiety (post) had an acceptable reliability (> 0.7). Therefore all the data produced by the scales can be considered to be of a reliable quality.

Table 6.2 The Subscales’ Cronbach Alpha Reliability Value

<table>
<thead>
<tr>
<th></th>
<th>Pre-Intervention</th>
<th>Post-Intervention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confidence</td>
<td>.864</td>
<td>.907</td>
</tr>
<tr>
<td>Effective Motivation</td>
<td>.817</td>
<td>.774</td>
</tr>
<tr>
<td>Anxiety</td>
<td>.847</td>
<td>.840</td>
</tr>
<tr>
<td>Usefulness</td>
<td>.897</td>
<td>.939</td>
</tr>
</tbody>
</table>

Attitude Subscale 1: Confidence

The mean score and standard deviation for each statement are given overleaf in Table 6.3. As can be seen in both Table 6.3 and the Summary Statistics in Table 6.4 the mean score for confidence actually fell slightly from 45.9 to 45.6.
Table 6.3  Attitude Subscale – Personal Confidence in Subject Matter

<table>
<thead>
<tr>
<th>No.</th>
<th>Statement</th>
<th>N</th>
<th>Pre-Test Mean(SD)</th>
<th>Post-Test Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(1)</td>
<td>I am sure that I can learn maths</td>
<td>14</td>
<td>4.36 (.63)</td>
<td>4.43 (.65)</td>
</tr>
<tr>
<td>2(5)</td>
<td>I don’t think I could do advanced maths</td>
<td>14</td>
<td>3.79 (.89)</td>
<td>3.71 (.83)</td>
</tr>
<tr>
<td>3(9)</td>
<td>Maths is hard for me.</td>
<td>14</td>
<td>3.29 (.99)</td>
<td>3.29 (.83)</td>
</tr>
<tr>
<td>4(11)</td>
<td>I am sure of myself when I do maths</td>
<td>14</td>
<td>3.43 (.85)</td>
<td>3.43 (.76)</td>
</tr>
<tr>
<td>5(17)</td>
<td>I’m not the type to do well in maths</td>
<td>14</td>
<td>4.21 (.58)</td>
<td>4.00 (.68)</td>
</tr>
<tr>
<td>6(24)</td>
<td>Maths has been my worst subject</td>
<td>14</td>
<td>4.36 (.64)</td>
<td>4.36 (.63)</td>
</tr>
<tr>
<td>7(25)</td>
<td>I think I could handle more difficult maths</td>
<td>14</td>
<td>2.71 (.91)</td>
<td>3.00 (.96)</td>
</tr>
<tr>
<td>8(31)</td>
<td>Most subjects I can handle OK, but I just can’t do a good job with maths</td>
<td>14</td>
<td>4.07 (.83)</td>
<td>4.00 (.78)</td>
</tr>
<tr>
<td>9(32)</td>
<td>I can get good grades in maths</td>
<td>14</td>
<td>4.00 (.68)</td>
<td>4.00 (.68)</td>
</tr>
<tr>
<td>10(37)</td>
<td>I know I can do well in maths</td>
<td>14</td>
<td>4.29 (.47)</td>
<td>3.93 (.62)</td>
</tr>
<tr>
<td>11(41)</td>
<td>I am sure I could do advanced work in maths</td>
<td>14</td>
<td>3.21 (.67)</td>
<td>3.36 (1.0)</td>
</tr>
<tr>
<td>12(44)</td>
<td>I’m no good in maths</td>
<td>14</td>
<td>4.21 (.70)</td>
<td>4.14 (.66)</td>
</tr>
<tr>
<td>Total</td>
<td>Personal Confidence in Subject Matter</td>
<td>14</td>
<td>45.9 (5.9)</td>
<td>45.6 (6.5)</td>
</tr>
</tbody>
</table>

This drop in mean is accompanied by a small drop in the minimum value (38 to 36) but the maximum value remained the same (56) as can be seen in the Summary Statistics Table 6.4.

Table 6.4  Summary Statistics of Attitude Subscale – Confidence in Subject Matter

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Sum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest confidence score</td>
<td>14</td>
<td>38.00</td>
<td>56.00</td>
<td>643.00</td>
<td>45.9286</td>
<td>5.86337</td>
</tr>
<tr>
<td>Post-test confidence score</td>
<td>14</td>
<td>36.00</td>
<td>56.00</td>
<td>639.00</td>
<td>45.6429</td>
<td>6.46419</td>
</tr>
<tr>
<td>Valid N (listwise)</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A histogram of the differences in scores pre and post-test is shown overleaf in Figure 6.4. The histogram is negatively skewed dragging the mean difference down slightly
below 0. This decrease in confidence is not significant at a value of $p = .763$ (significance occurs where $p < .05$). The trend, however slight, is not desirable.

Of the 48 statements presented to students, only one showed a difference that could be counted as significant. This statement, belonging to the confidence subscale (see Table 6.3), was statement 37: *I know I can do well in maths*. This statement had a pre-test mean score of 4.29 but a post-test mean score of 3.93. This is a significant drop ($p = .019$) and demonstrates that the use of the resource pack and the accompanying adoption of new teaching methods shook the confidence of the students. The author will discuss this later from the perspective of the teacher but it was noted in the diary that the type of questions and questioning was troubling the students. The author felt that it was this move towards the Project Maths type questions, which attempt to assess conceptual knowledge rather than the usual routine, procedural type questions,
which lead to a slight drop in students’ overall confidence but a significant drop in their confidence to do well in these particular situations in mathematics.

**Attitude Subscale 2: Motivation of the students**

The mean score and standard deviation for each of the statements in the motivation subscale are given below. As can be seen from Table 6.5 and the Summary Statistics Table 6.6 the mean score rose slightly from 42.1 to 42.7.

<table>
<thead>
<tr>
<th>No.</th>
<th>Statement</th>
<th>N</th>
<th>Pre-Test Mean(SD)</th>
<th>Post-Test Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mathematics is enjoyable and stimulating to me</td>
<td>14</td>
<td>3.50 (1.1)</td>
<td>3.71 (.83)</td>
</tr>
<tr>
<td>2</td>
<td>I don’t understand how some people can spend so much time on maths and seem to enjoy it.</td>
<td>14</td>
<td>3.36 (.93)</td>
<td>3.64 (.63)</td>
</tr>
<tr>
<td>3</td>
<td>I am challenged by maths problems I can’t understand immediately.</td>
<td>14</td>
<td>3.64 (1.0)</td>
<td>3.79 (.97)</td>
</tr>
<tr>
<td>4</td>
<td>The challenge of maths problems does not appeal to me.</td>
<td>14</td>
<td>3.57 (1.0)</td>
<td>3.21 (1.1)</td>
</tr>
<tr>
<td>5</td>
<td>I would rather have someone give me the solution for a difficult maths problem than have to work it out myself.</td>
<td>14</td>
<td>3.43 (.85)</td>
<td>3.43 (.94)</td>
</tr>
<tr>
<td>6</td>
<td>I like maths puzzles.</td>
<td>14</td>
<td>3.64 (1.0)</td>
<td>3.64 (1.0)</td>
</tr>
<tr>
<td>7</td>
<td>I do as little work in maths as possible.</td>
<td>14</td>
<td>4.07 (.62)</td>
<td>4.00 (.78)</td>
</tr>
<tr>
<td>8</td>
<td>When a maths problem arises that I can’t immediately solve, I stick to it until I have the solution.</td>
<td>14</td>
<td>3.50 (.76)</td>
<td>3.71 (.61)</td>
</tr>
<tr>
<td>9</td>
<td>When a question is left unanswered in maths class, I continue to think about it afterwards.</td>
<td>14</td>
<td>3.21 (1.1)</td>
<td>2.86 (.95)</td>
</tr>
<tr>
<td>10</td>
<td>Once I start working on a maths puzzle I find it hard to stop</td>
<td>14</td>
<td>3.21 (1.1)</td>
<td>3.07 (1.1)</td>
</tr>
<tr>
<td>11</td>
<td>Figuring out mathematical problems does not appeal to me.</td>
<td>14</td>
<td>3.21 (1.1)</td>
<td>3.64 (.84)</td>
</tr>
<tr>
<td>12</td>
<td>Maths puzzles are boring</td>
<td>14</td>
<td>3.79 (.80)</td>
<td>4.00 (.87)</td>
</tr>
<tr>
<td>Total</td>
<td>Effective Motivation of the students</td>
<td>14</td>
<td>42.1 (.80)</td>
<td>42.7 (.88)</td>
</tr>
</tbody>
</table>
This rise in the mean was accompanied by an improvement in the minimum score (27 to 32) and maximum scores (49 to 54), which points to an overall trend of an increase in motivation as seen in Table 6.6.

Table 6.6  Summary Statistics of Attitude Subscale – Effective Motivation in Subject Matter

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Sum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest effective motivation score</td>
<td>14</td>
<td>27.00</td>
<td>49.00</td>
<td>590.00</td>
<td>42.1429</td>
<td>6.54989</td>
</tr>
<tr>
<td>Post-test effective motivation score</td>
<td>14</td>
<td>32.00</td>
<td>54.00</td>
<td>598.00</td>
<td>42.7143</td>
<td>5.78364</td>
</tr>
<tr>
<td>Valid N (listwise)</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A histogram of the differences in scores pre and post-test is shown in figure 6.5. The histogram shows a slight increase of .571 in the mean score but this is not significant (p= .547). The data has a large standard deviation of 35. This large standard deviation is of a result of large amounts of data being situated at the extremities rather than being centred on the mean.
While most of the individual statements reflect this slightly positive trend, no increase was of significance.
**Attitude Subscale 3: Anxiety of Students**

The mean score and standard deviation for each statement are given in Table 6.7. As can be seen in both the table and the Summary Statistics in Table 6.8, the mean score for anxiety actually rose slightly with a small improvement from 41.6 to 42.6.

**Table 6.7  Attitude Subscale – Anxiety felt towards Mathematics**

<table>
<thead>
<tr>
<th>No.</th>
<th>Statement</th>
<th>N</th>
<th>Pre-Test Mean(SD)</th>
<th>Post-Test Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (3)</td>
<td>Maths doesn’t scare me at all.</td>
<td>14</td>
<td>3.86 (.63)</td>
<td>3.57 (.65)</td>
</tr>
<tr>
<td>2 (7)</td>
<td>Mathematics usually makes me feel uncomfortable and nervous.</td>
<td>14</td>
<td>3.71 (1.1)</td>
<td>3.86 (.66)</td>
</tr>
<tr>
<td>3 (13)</td>
<td>It wouldn’t bother me at all to take more maths courses.</td>
<td>14</td>
<td>3.29 (1.1)</td>
<td>3.14 (1.1)</td>
</tr>
<tr>
<td>4 (15)</td>
<td>Mathematics me feel uncomfortable, restless, irritable and impatient.</td>
<td>14</td>
<td>3.64 (1.0)</td>
<td>4.07 (.83)</td>
</tr>
<tr>
<td>5 (21)</td>
<td>I haven’t usually worried about being able to solve maths problems.</td>
<td>14</td>
<td>3.36 (1.1)</td>
<td>3.64 (.93)</td>
</tr>
<tr>
<td>6 (23)</td>
<td>I get a sinking feeling when I think of doing maths problems.</td>
<td>14</td>
<td>4.07 (.73)</td>
<td>3.93 (.73)</td>
</tr>
<tr>
<td>7 (26)</td>
<td>My mind goes blank and I am unable to think clearly when working mathematics.</td>
<td>14</td>
<td>4.00 (.68)</td>
<td>3.93 (.27)</td>
</tr>
<tr>
<td>8 (30)</td>
<td>A maths test would scare me.</td>
<td>14</td>
<td>3.07 (1.1)</td>
<td>3.21 (1.1)</td>
</tr>
<tr>
<td>9 (34)</td>
<td>I almost never have got nervous during a maths exam.</td>
<td>14</td>
<td>2.36 (1.1)</td>
<td>2.50 (1.2)</td>
</tr>
<tr>
<td>10 (39)</td>
<td>Mathematics makes me feel uneasy and confused.</td>
<td>14</td>
<td>3.93 (.83)</td>
<td>3.93 (.73)</td>
</tr>
<tr>
<td>11 (47)</td>
<td>I usually have been at ease during maths tests.</td>
<td>14</td>
<td>2.71 (1.1)</td>
<td>2.86 (.86)</td>
</tr>
<tr>
<td>12 (48)</td>
<td>I usually have been at ease in maths classes.</td>
<td>14</td>
<td>3.64 (.84)</td>
<td>3.93 (.83)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>Anxiety felt towards mathematics</td>
<td>14</td>
<td>41.6 (7.1)</td>
<td>42.6 (6.4)</td>
</tr>
</tbody>
</table>

This rise in the mean was accompanied by an improvement in the minimum score (27 to 29), while the maximum score remained unchanged (53). These changes in data
scores could not be described as a trend but at least there seems to be an improvement in those with poor scores.

**Table 6.8** Summary Statistics of Attitude Subscale – Anxiety felt towards Subject Matter

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Sum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest anxiety</td>
<td>14</td>
<td>27.00</td>
<td>53.00</td>
<td>583.00</td>
<td>41.6429</td>
<td>7.08853</td>
</tr>
<tr>
<td>score</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post-test anxiety</td>
<td>14</td>
<td>29.00</td>
<td>53.00</td>
<td>596.00</td>
<td>42.5714</td>
<td>6.44162</td>
</tr>
<tr>
<td>score</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Valid N (listwise)</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A histogram of the differences in scores pre and post-test is shown below in Figure 6.6. The histogram shows a slight increase of .929 in the mean score but it is not significant (p=.530). The data has a large standard deviation of almost 5.4. This large standard deviation exists despite modal values existing around the mean due to two scores on the extremities.

**Figure 6.6** Histogram of difference in Mean Anxiety Score from Pre to Post Test

While some of the individual statements reflect this slightly positive trend, no increase was of significance.
**Attitude Subscale 4: Students’ Perception of the Usefulness of Mathematics**

The mean score and standard deviation for each statement are given below in Table 6.9. As can be seen in both Table 6.9 and in the Summary Statistics in Table 6.10, the mean score for perception of usefulness actually rose slightly with an improvement from 43.9 to 45.9.

**Table 6.9 Attitude subscale: Students’ Perception of Subject’s Usefulness**

<table>
<thead>
<tr>
<th>No.</th>
<th>Statement</th>
<th>N</th>
<th>Pre-Test Mean(SD)</th>
<th>Post-Test Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (4)</td>
<td>Knowing mathematics will help me earn a living.</td>
<td>14</td>
<td>3.86 (1.1)</td>
<td>3.93 (1.3)</td>
</tr>
<tr>
<td>2 (6)</td>
<td>Maths will not be important to me in my life's work</td>
<td>14</td>
<td>3.57 (1.2)</td>
<td>3.57 (1.2)</td>
</tr>
<tr>
<td>3 (10)</td>
<td>I'll need mathematics for my future work</td>
<td>14</td>
<td>3.21 (1.3)</td>
<td>3.57 (1.3)</td>
</tr>
<tr>
<td>4 (12)</td>
<td>I don't expect to use much maths when I get out of school</td>
<td>14</td>
<td>3.21 (1.3)</td>
<td>3.57 (1.1)</td>
</tr>
<tr>
<td>5 (16)</td>
<td>Maths is a worthwhile, necessary subject</td>
<td>14</td>
<td>4.21 (.58)</td>
<td>4.36 (.74)</td>
</tr>
<tr>
<td>6 (22)</td>
<td>Taking maths is a waste of time</td>
<td>14</td>
<td>4.64 (.50)</td>
<td>4.64 (.50)</td>
</tr>
<tr>
<td>7 (27)</td>
<td>I will use mathematics in many ways as an adult.</td>
<td>14</td>
<td>3.64 (1.1)</td>
<td>3.64 (1.2)</td>
</tr>
<tr>
<td>8 (29)</td>
<td>I see mathematics as something I won't use very often when I get out of school</td>
<td>14</td>
<td>3.43 (1.2)</td>
<td>3.64 (1.2)</td>
</tr>
<tr>
<td>9 (33)</td>
<td>I'll need a good understanding of maths for my future work</td>
<td>14</td>
<td>3.36 (1.0)</td>
<td>3.50 (.94)</td>
</tr>
<tr>
<td>10 (38)</td>
<td>Doing well in maths is not important for my future</td>
<td>14</td>
<td>3.64 (1.2)</td>
<td>3.93 (.83)</td>
</tr>
<tr>
<td>11 (42)</td>
<td>Maths is not important for my life</td>
<td>14</td>
<td>3.50 (1.2)</td>
<td>3.79 (1.2)</td>
</tr>
<tr>
<td>12 (46)</td>
<td>I study maths because I know how useful it is</td>
<td>14</td>
<td>3.57 (.85)</td>
<td>3.93 (.62)</td>
</tr>
</tbody>
</table>

**Total** Perception of the usefulness of mathematics 14 43.9 (8.7) 45.9 (9.7)
This rise in the mean was accompanied by an improvement in the minimum score (22 to 27) and in the maximum score from (54 to 60), which is a good overall trend of improvement in the students’ perception of the usefulness of mathematics.

Table 6.10  Summary Statistics of Attitude Subscale – Perception of Subject’s Usefulness

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Sum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest perception of</td>
<td>14</td>
<td>22.00</td>
<td>54.00</td>
<td>614.00</td>
<td>43.8571</td>
<td>8.66343</td>
</tr>
<tr>
<td>usefulness score</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post-test perception of</td>
<td>14</td>
<td>27.00</td>
<td>60.00</td>
<td>643.00</td>
<td>45.9286</td>
<td>9.70697</td>
</tr>
<tr>
<td>usefulness score</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Valid N (listwise)</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A histogram of the differences in scores pre and post-test is shown below. The histogram shows an increase of 2.07 in the mean score but it is not significant (p = .237). The histogram follows a reasonably normal distribution. However it is obvious from this graph that one student with a fairly dramatic decrease in perception of usefulness is preventing the data from being positively skewed and dragging the mean score down.
The perception of usefulness subscale showed the largest shift of any of the subscales and while most of the individual statements reflect this positive trend, no increase was of significance.

**Combined Subscales: Students’ Attitude to Mathematics**

The mean score and standard deviation for each subscale are given in Table 6.12. As can be seen in both Table 6.12 and the Summary Statistics Table 6.13, the mean score for attitude rose slightly with an improvement from 174 to 177 (rounded to the nearest whole number). This increase in mean score of 3 points in a total scale of 240 cannot be claimed to represent an improvement in attitude.
Table 6.11  Attitude towards mathematics – Subscale Summary

<table>
<thead>
<tr>
<th>No.</th>
<th>Subscale</th>
<th>N</th>
<th>Pre-Test Mean(SD)</th>
<th>Post-Test Mean(SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Personal Confidence in Subject Matter</td>
<td>14</td>
<td>45.9 (5.9)</td>
<td>45.6 (6.5)</td>
</tr>
<tr>
<td>2</td>
<td>Effective Motivation of the students</td>
<td>14</td>
<td>42.1 (.8)</td>
<td>42.7 (.88)</td>
</tr>
<tr>
<td>3</td>
<td>Anxiety felt towards mathematics</td>
<td>14</td>
<td>41.6 (7.1)</td>
<td>42.6 (6.4)</td>
</tr>
<tr>
<td>4</td>
<td>Perception of the usefulness of mathematics</td>
<td>14</td>
<td>43.9 (8.7)</td>
<td>45.9 (9.7)</td>
</tr>
<tr>
<td></td>
<td><strong>Total Attitude Towards mathematics</strong></td>
<td>14</td>
<td>174 (21.2)</td>
<td>177 (22.6)</td>
</tr>
</tbody>
</table>

This rise in the mean was accompanied by a one point improvement in the minimum score (127 to 128) and a twenty-four point improvement in the maximum score from (196 to 220). This improvement points towards a pattern which showed up when the individual student scores were analysed in the Motivation and Perception of Usefulness scales. In general those who had lower scores remained low but those on the higher end improved. The one exception to this was the confidence subscale.

Table 6.12  Summary Statistics of Combined Attitude Subscales

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Sum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest attitude score</td>
<td>14</td>
<td>127.00</td>
<td>196.00</td>
<td>2430.00</td>
<td>173.57</td>
<td>21.21942</td>
</tr>
<tr>
<td>Post-test attitude score</td>
<td>14</td>
<td>128.00</td>
<td>220.00</td>
<td>2476.00</td>
<td>176.86</td>
<td>22.60312</td>
</tr>
<tr>
<td>Valid N (listwise)</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A histogram of the differences in scores pre and post-test is shown below. The histogram shows an increase of 3.29 in the mean score, which is a small increase in the combined subscale score of 240. It is not a significant improvement (p = .326). The histogram follows a reasonably normal distribution of mean scores.
6.3.3 The Teacher’s Perspective

The author’s perceptions of the students’ attitudes pre and post intervention would be very similar to that which emerged from the Fennema-Sherman attitude survey. The author repeatedly mentions and discusses student “engagement” in the Diary. Here is a selection of quotes from the diary

28/3/2011  The activity certainly engaged students,

29/3/2011  Students started to ask questions, which is unusual for this class. This is good. It is in response to the Questions in Context,

31/3/2011  The students definitely find the topic engaging and motivating,

31/3/2011  Everyone stayed for the first 10 minutes of break to talk about a problem, even though I told them they could leave. I think that’s a first for me,

4/4/2011  We had a good class discussion,
Some students seem to really enjoy the Questions in Context. They keep reading on, attacking more of them”

However, while students liked the activities and saw a new relevance in mathematics through the questions in context, the author could see that their confidence was shaken. This especially applied to a particular type of student. Not necessarily the traditionally weaker student, some of those who got an A at Higher Level Junior Certificate found it disconcerting. It was the students who relied on learning off examples, who struggled. The following are quotes from the diary

31/3/2011 The weaker mathematical problem solver is struggling

5/4/2011 Some find the Questions in Context daunting simply because the questions are posed in English

5/4/2011 I’d worry about students’ confidence, they like to feel like they’ve seen every possible example . . . . . . The “Questions in Context” type questions rattle them sometimes.

Thus it seemed to the author that while in general their attitude towards mathematics improved, there was a concern among the students about their ability to solve the mathematics problems as they were now being put to them.

6.3.4 Discussion of Findings

The researcher’s first question was, “Is there a link between the use of active learning methodologies and real life data and contexts and an improvement in students’ attitude towards mathematics?” The data collected cannot conclusively provide an answer to that question. On first look the combined attitude score in the Fennema-Sherman Scales would suggest there is no link at all, either a positive or negative one,
between theses methodologies and student attitude. However inspection of the subscales and individual statements combined with the data collected from the other perspectives provides a much more nuanced picture.

There is no significant change in any of the sub-scales. In fact of the 48 statements responses pre and post-test only one was of significance. This statement, belonging to the confidence subscale (see Table 6.3), was statement 37: *I know I can do well in maths*. This statement had a pre-test mean score of 4.29 but a post-test mean score of 3.93. This is a significant drop (p = .019) and suggests that the use of active learning methodologies and real life data and contexts in the classroom may in fact have a negative impact on students’ attitude to mathematics. There is also a slight drop in the overall mean confidence score, however this is so slight that that there is a 76.3% of this occurring by chance, not the 5% needed to claim significance. The closely related sub-scale which measures anxiety levels had in fact improved marginally, though again not significantly (p=.547). It would seem then that it was a very particular aspect of students’ confidence that was negatively impacted on. This is corroborated in the teacher’s diary, where entries, detailed earlier, speak of shaken confidence. These suggest and the author would conclude that while a discomfort with new pedagogical approaches may have played a part, and that this negative impact is largely due to students’ fear of the new type of contextual real life question which appeared in the class test. The students were no longer so sure they could “do well in mathematics”, given the new approach adopted.

The most dramatic shift in any of the scales was in the perception of usefulness. This positive change was not significant (p=.273) but is in line with both the feedback from
the outside perspective as well as the teacher diary. The diary, Expert 2 (who used some materials in the classroom) and the critical friend all gave feedback which spoke in positive terms about students’ level of engagement and interest when using the activities and “Questions in Context”. The students could see the relevance of the mathematics as one expert; Expert 4 answered “It definitely helps to answer the question students ask – why are we doing this?” However it cannot be claimed that the students’ perception of the usefulness of mathematics was significantly improved as a result of the teaching methodologies employed.

6.4 Research Question 2:

Does the use of active learning methodologies and real life data and contexts develop students’ understanding of probability?

6.4.1 The Outside Perspective

The following are the responses of the expert panel and the critical friend to the relevant statements in the questionnaire. An opportunity was given to comment at the end and where these comments are relevant to the question – they are included.
Statement: The resource pack promotes conceptual understanding

Four of the expert panel circled A, strongly agreeing with this statement, while the remaining expert circled B agreeing with this statement. On reviewing the lesson the critical friend circled B, agreeing with the statement.

Figure 6.9 The Resource Pack promotes Conceptual Understanding

Additional comments:

Expert 5 commented that “the resource pack should have a positive effect on student conceptual understanding”.
Statement: The resource pack will increase student understanding of key concepts in probability.

Two of the expert panel circled A, strongly agreeing with this statement, while the remaining three experts circled B agreeing with this statement. On reviewing the lesson the critical friend circled A, agreeing with the statement.

Figure 6.10 The Resource Pack Will Increase Student Understanding of Key Concepts in Probability

Additional comments:
Expert 1 stated that the “everyday example enhance understanding”. Expert 3 said that “if used effectively, students will certainly benefit from this pack”.

6.4.2 The Students’ Perspective
As described in the methodology chapter the students were given a class test at the end of the two week intervention (see section 4.9.2). The aim of this test was to analyse the students’ understanding of three probability topics which had been taught using the resource pack (Chapter 5 and Appendix E), conditional probability, expected value and binomial distribution. Understanding and assessing understanding
have already been discussed in both Chapters two and four of this thesis. In these Chapters understanding was defined as involving both procedural and conceptual knowledge. Students need not only to know how to perform mathematical operations but also know why they are performing these mathematical operations (Hiebert and Carpenter 1992). By this definition real understanding occurs when students are able to make connections between the problem posed and the procedures they have learned. However, understanding is difficult to assess and there is no ideal system in place to do this (Hiebert and Carpenter 1992; Hiebert and Wearne 1996; Rittle-Johnson et al. 2001; Romero and Mari 2006; Konold 1995; Skemp 1976).

As described in the methodology chapter, Section 4.9.2, the class test was divided into two types of question. The assessment tasks use questions set in both well-practised and novel contexts to distinguish between procedural and conceptual knowledge (Rittle-Johnson et al. 2001; Konold 1995; Hiebert and Wearne 1996). Both have to be achieved if real understanding has occurred (Carpenter and Hiebert 1992; Rittle-Johnson et al. 2001). The tests were marked out of a possible score of 60, consisting of 30 marks each for the questions involving procedural and conceptual understanding.
**Table 6.13**  Participating Students’ Junior Certificate and Class Test Results

<table>
<thead>
<tr>
<th>Student</th>
<th>Junior Certificate Result</th>
<th>Procedural Understanding</th>
<th>Conceptual Understanding</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. *</td>
<td>A</td>
<td>15/30</td>
<td>6/30</td>
<td>E</td>
</tr>
<tr>
<td>2.</td>
<td>A</td>
<td>30/30</td>
<td>30/30</td>
<td>A</td>
</tr>
<tr>
<td>3.</td>
<td>C</td>
<td>25/30</td>
<td>8/30</td>
<td>C</td>
</tr>
<tr>
<td>4. **</td>
<td>C</td>
<td>20/30</td>
<td>8/30</td>
<td>D</td>
</tr>
<tr>
<td>5.</td>
<td>C</td>
<td>20/30</td>
<td>8/30</td>
<td>D</td>
</tr>
<tr>
<td>6.</td>
<td>A</td>
<td>25/30</td>
<td>21/30</td>
<td>B</td>
</tr>
<tr>
<td>7.</td>
<td>A</td>
<td>30/30</td>
<td>24/30</td>
<td>A</td>
</tr>
<tr>
<td>8.</td>
<td>B</td>
<td>25/30</td>
<td>20/30</td>
<td>B</td>
</tr>
<tr>
<td>9.</td>
<td>B</td>
<td>25/30</td>
<td>19/30</td>
<td>B</td>
</tr>
<tr>
<td>10.</td>
<td>C</td>
<td>10/30</td>
<td>0/30</td>
<td>F</td>
</tr>
<tr>
<td>11.</td>
<td>B</td>
<td>30/30</td>
<td>16/30</td>
<td>B</td>
</tr>
<tr>
<td>12.</td>
<td>A</td>
<td>30/30</td>
<td>28/30</td>
<td>A</td>
</tr>
<tr>
<td>13. **</td>
<td>N/A</td>
<td>25/30</td>
<td>13/30</td>
<td>B</td>
</tr>
<tr>
<td>14. **</td>
<td>N/A</td>
<td>30/30</td>
<td>15/30</td>
<td>B</td>
</tr>
</tbody>
</table>

* This student was absent from school for 3 of the 10 lessons, which the pack was taught

** These students were not in the Irish Education system until Senior Cycle, so there is no Junior Certificate Result available for them

These results show 10 of the 14 students achieving an honours grade C or better, showing a reasonable level of understanding. It should be noted of the four others, one of these, Student 1, missed the last three lessons during which the resource pack was used to teach these topics and was unaware that they were to be tested so it is likely that this had a major negative effect on their grade. Another student, Student 10, has been scoring consistently low grades for the year and for reasons which have no
relevance to this study, has despite regular attendance, essentially opted out of school work. Another interesting point to note, is that Student 14, a non-national has been to date an exceptional mathematics student. This is the first test they achieved a score below the high 90%’s. This student does however have English language difficulties. These difficulties had rarely shown in mathematics examinations before as the problems presented had always been routine in nature and she had learned to recognise “trigger-words” for certain procedures. This meant she scored an excellent, 30 out of 30 in the procedural type questions but scored only 15 out of 30 in the conceptual questions, which contained unfamiliar situations, contexts and words. It is also worth mentioning that the results support literature (Lyons et al. 2003; O’Donoghue 2002 cited by NCCA 2005), which suggests that Irish students are adept at procedural type questions but have greater difficulty with problems which require conceptual understanding.

Using SPSS, the author examined the correlation between the students’ results at Junior Certificate level and in this class test. In Table 6.14, it can be seen that there is a correlation co-efficient of .594, which means there is a moderate positive correlation between grade achievement at Junior Certificate level and achievement of grades in the post-intervention class test.
Table 6.14  Correlation between Junior Certificate Results and the Class Test

<table>
<thead>
<tr>
<th>Result in Junior Certificate</th>
<th>Result in Class Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson Correlation</td>
<td>1</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.594*</td>
</tr>
<tr>
<td>N</td>
<td>12</td>
</tr>
<tr>
<td>Sig. (2-tailed)</td>
<td>.042</td>
</tr>
<tr>
<td>N</td>
<td>12</td>
</tr>
</tbody>
</table>

*. Correlation is significant at the 0.05 level (2-tailed).

It is interesting to note that if Student 1 is removed, the student who missed a portion of the classes, this correlation jumps up to .853 which is considered a very high positive correlation.

6.4.3 The Teachers’ Perspective

Student understanding is difficult to gauge and on examining the diary it becomes apparent that the author was reluctant to make claims either way after lessons. In general though, where comments are made they are positive.

30/3/2011  Students do seem to understand what they are being asked and seem to enjoy the real-life contexts despite their messiness.

31/3/2011  Conceptual understanding is definitely being emphasised.

1/4/2011  Intro activity definitely shows student where formula comes from, allowing greater understanding of the concept.

1/4/2011  Chatting with a couple of students after . . . . . one said “Doing it this way, I understand why I’m doing something and I like that” though I feel some students would rather just get the formula.
4/4/2011  I really felt student understanding improved during this lesson, especially the roulette question. Light bulbs were turning on.

6/4/2011  The students studying the pattern that establishes formula rule should increase student understanding of the topic.

In as much as can be claimed from a teacher’s thoughts and feelings, the pattern to emerge in the diary is very much that the author/teacher feels that students do understand what is being taught and that the activities and questions in context are essential in giving students a real deep conceptual understanding of the topics rather than just strictly a procedural one.

6.4.4 Discussion of Findings

The data collected by the researcher when examining the question; ‘Does the use of activities and questions set in real life contexts develop students’ understanding of probability?’ is essentially all qualitative in nature. It is difficult to reach any specific or definite conclusions, though the corroboration of the multiple perspectives does allow general claims to be made. The results of the class test follow a reasonably normal distribution, skewed a little negatively towards the higher grades.
However, though the distribution is not unusual, the composition of the test is. The fact the students achieved these results in a paper, which was designed to test for real conceptual understanding, is important. The poor performance of Irish students in international studies, which are also designed using questions set in novel questions to test for conceptual understanding, would suggest that these teaching methodologies do improve understanding. However, since no direct comparison on these specific topics is possible no claim can be made. The expert panel and critical friend responses as well the diary entries would suggest that the methodologies employed did promote a high level of understanding in probability.
6.5 Other Themes to Emerge

On examining the data from the three perspectives other recurring issues emerged, outside of the two central research questions. These issues were largely concerned with the day to day practicalities of using the resource pack. Much of this regarded content and format, which though important in its own right in terms of the evolution of the pack through the cyclical action research process, has little implication beyond that. I have synopsised the remaining recurring issues into one overarching theme “Will the resource pack be of benefit to the author and other teachers of Higher Level Leaving Certificate probability, teaching the new Project Maths Syllabus?”

6.5.1 The Outside Perspective

The following are the responses of the expert panel and the critical friend to the relevant statements in the questionnaire. An opportunity was given to comment at the end and where these comments are relevant to the question – they are included.
Statement: The resource pack complements the aims of Project Maths

All 5 of the expert panel circled A, strongly agreeing with this statement. On reviewing the lesson the critical friend circled A, also strongly agreeing with the statement.

Figure 6.12 The Resource Pack Complements the Aims of Project Maths

Additional comments:
Expert 1 said she felt that “this pack will be an invaluable tool in teaching Project Maths”. Expert 2 commented that the resource pack was “fully in line with what Project Maths is trying to do”. Expert 3 felt that the “resource pack definitely complements Project Maths”. Expert 5 commented that it was a “very good resource over all......It definitely is a valuable resource with regards to the aims of Project Maths”.

205
The resource pack is feasible as a teaching aid with respect to time

Two of the expert panel circled A, strongly agreeing with this statement, 2 experts circled B agreeing with this statement, while the remaining expert circled C indicating he was unsure. On reviewing the lesson the critical friend circled A, strongly agreeing with the statement.

The Resource Pack is feasible as a teaching aid with respect to time

![Bar Chart]

Strongly Agree
Agree
Not Sure
Dissagree
Strongly Dissagree

The resource pack is feasible as a teaching with respect to time

Additional comments:

Expert 3 said that “teachers can use the resource pack at their leisure, therefore time shouldn’t be an issue.” Expert 5, who had circled C indicating he was unsure and further clarified “I’m not sure how much time is allocated to covering the Statistics and Probability strand so I’m not sure whether this pack is feasible as regards time. It is a substantial pack but as you suggested it does not have to be taught in a sequential fashion” The Critical Friend who Circled A, strongly agreeing with the statement added the comment that “teachers need to give sufficient time/questions for students to make connections”
Statement: The resource pack is feasible as a teaching aid with respect to classroom management

Three of the expert panel circled A, strongly agreeing with this statement, while the remaining 2 experts circled B agreeing with this statement. On reviewing the lesson the critical friend circled A, strongly agreeing with the statement.

Figure 6.14 The Resource Pack is feasible as a teaching aid with respect to classroom management

There were no additional comments regarding classroom management.
Statement:  The Resource pack is easy to use

All 5 of the expert panel circled A, strongly agreeing with this statement. On reviewing the lesson the critical friend circled B, agreeing with the statement.

Figure 6.15  The Resource Pack is easy to use

Additional comments:

Expert 3 stated that the resource pack was “well organised”. Expert 5 said “the format is clear and no teacher should have any trouble following or using this resource”.
Statement: This resource pack could have a positive influence on the teaching of higher level probability at senior cycle

All 5 of the expert panel circled A, strongly agreeing with this statement. On reviewing the lesson the critical friend circled B, agreeing with the statement.

Figure 6.16 This Resource Pack could have a positive influence on the teaching of higher level probability at Senior Cycle

Additional comments:

Expert 2 commented that he could “really see these shows/activities come alive in a classroom with an effective teacher”. Expert 3 said that it “will challenge Higher level students to think”.
Statement: Teachers will use this resource pack

Three of the expert panel circled A, strongly agreeing with this statement, while the remaining two experts circled B agreeing with the statement. On reviewing the lesson the critical friend circled C, indicating he was unsure.

Figure 6.17 Teachers will use this Resource Pack

Additional comments:

Expert 1 said that she “would definitely use the pack.” Expert 3 commented that “If I was teaching at second level I would certainly use it”. Expert 4 stated that “if teachers embrace ‘Project Maths’ using active methodologies and more emphasis on understanding, this package would be very useful for them”. Expert 5 noted that “it could prove a valuable Transition year resource for teachers”

There was a strong consensus that the resource pack and the accompanying methodologies were in harmony with the aims of Project Maths, would have little impact on classroom management, would have a positive influence on the teaching of probability for the Leaving Certificate and would be easy to use. However, there was
a note of dissension from this positive consensus with regards to the feasibility of the resource pack with respect to time, expert 5 expressed doubts due to the “substantial” nature of the pack. Critically the other area of concern is whether teachers would use this pack. The critical friend declared himself to be unsure if they would.

6.5.2 The Student’s Perspective

In this case the author’s data is limited as it was not an original research question. While the students are not in the best position to comment on whether the author or other teachers would use this pack or on its usefulness for the Project Maths syllabus, their views of the pack would have provided extremely interesting and useful material. In similar research undertaken in the future, the author would recommend that a process to collect this data would be incorporated into the methodology.

6.5.3 The Teacher’s Perspective

Much of the comment in the diary, though not a stated objective of the pack, reflects an underlying concern of the researcher/teacher - Will the resource pack be of benefit to the author and other teachers of higher level leaving certificate probability, teaching the new Project Maths Syllabus? In retrospect, these concerns are natural, action research was the author’s chosen methodology largely due to its practical outlook. Paraphrasing Kurt Lewin (1948) research that produces nothing but research will not solve the problem. In this case, the production of a resource pack that resulted in a significant improvement in attitudes and understanding would be pointless beyond its research implications if neither the author nor any other teacher will ever use it. This concern with usefulness is reflected in the diary entries. These entries, though on balance positive, do contain a mix of feelings.
28/3/2011 The Intro activity is very active and student lead.

28/3/2011 Time is a slight concern. Yes the students do seem to have a positive response but it does take longer to get through the material.

29/3/2011 It is achieving aim of Project Maths of getting the students to think more for themselves in class and experience the questions in real-life contexts.

29/3/2011 Time could become an issue. It is easy to use however, I’m very satisfied in that regard

30/3/2011 Students did well in homework, which meant that very little class time was spent on it. Maybe the extra-time in the introduction was worth it.

30/3/2011 Classroom management and usability are not an issue.

1/4/2011 Teachers may be a bit reluctant to try activity but once used I would be confident that teachers would use it again.

4/4/2011 It would be great to spend this kind of time on every topic. The extra class exploring possibilities. I am doing everything in the pack so a teacher dipping in and out would not have this issue.

6/4/2011 My “Critical Friend” said he would definitely use this (binomial introductory activity) with his class. This is what I need to hear.

7/4/2011 Time may become an issue but as for relevance, usability and effect on classroom environment are concerned it is positive.
8/4/2011  *Real-life contexts provide the students with the opportunity to experience binomial probability in the kind of setting that the Project Maths curriculum encourages*

In the diary, the perspective of the teacher is clearly that activities and questions in context work in a manner consistent with the Project Maths syllabus and is practically useful. Though concerns over time are raised, these are balanced with a perceived improvement in understanding.

### 6.5.4 Discussion of Findings

The suitability and usability of the resource pack was considered an important theme by the author in light of the curriculum reform movement currently being undertaken in Irish mathematics education. In Chapter 2, a previous failed attempt at reform was compared to how the successful adoption by the Dutch of their RME influenced their system. One reason for the failure was the textbooks and materials available to Irish teachers. O’Keeffe and O’Donoghue (2009) warned that any future curriculum reform of mathematics education in Ireland will need to see the production of heavily revised and re-orientated textbooks. Van den Heuvel-Panhuizen (2000, p. 10) described the textbooks developed in Holland as the “most important tools” in guiding teachers in terms of new content but more importantly the new pedagogical approach. The core activities and strands in the Dutch textbooks were first developed and trialled by teachers in the classroom before they were included in the textbooks. Similarly, the pack created will provide resource materials for teachers, trialled in the classroom and which involve both new content and methodologies. The author’s knowledge of the importance of such materials informed the concern which clearly emerged, as a theme
in the data; “Will the resource pack be of benefit to the author and other teachers of higher level leaving certificate probability, teaching the new Project Maths syllabus?”

The general consensus that emerges from the data is that yes it will be of use to teachers of Senior Cycle Project Maths. There is in fact a strong consensus but with two notable concerns. The first is time; while only one expert, Expert 5, expressed doubts over time, it is interesting to note the repetition of the word “time” in the teacher’s diary also. However, the pack is designed as a resource that teachers can dip in and out of if they wish rather than teach as a whole, negating the expert’s concerns over time. The second concern expressed and the more worrying one, was would teachers use the pack. The strong support the pack received in terms of suitability and usability was tempered with qualifications over whether teachers would actually use the pack. The critical friend declared himself unsure of this but when questioned orally he clarified that he himself would use the pack but he was unsure whether mathematics teachers in general would embrace the methodologies employed. Expert 3 notes that teachers will have “a strong bearing” on how beneficial the resource pack is. Expert 2, talks of the need for an “effective teacher” if the pack is to work, and Expert 4 warns that only teachers who “embrace” the methodologies advocated in Project Maths will use this resource pack.

To summarise, the consensus to emerge from the data is that teachers should use the resource pack but would they? This question mark forces the author to restrict any claims to its future use to himself. The author/researcher/teacher will use the resource pack in the future and it is worth noting that Cohen and Mannion (1994) argue, that in
action research projects it is the practitioner/researchers view, which is of greatest importance. They have identified the problem in their classroom, if they conclude the problem is solved then for all intents and purposes, the intervention can be claimed to have succeeded.

6.6 Summary of Findings

At the outset of this chapter the research questions were outlined. They were

1. To investigate if there is a link between the use of active learning methodologies and real life data and contexts and an improvement in students’ attitude towards mathematics.

2. To investigate if the use of active learning methodologies and real life data and contexts develop students’ understanding of probability.

A third research question or theme emerged on further examination of the data

3. Will the resource pack be of benefit to the author and other teachers of higher level leaving certificate probability, teaching the new Project Maths syllabus?

After examining the data the key findings to emerge are:

1. The data is inconclusive and contradictory as to whether there is a link between the use of active learning methodologies and real life data and contexts and an improvement in students’ general attitude towards mathematics.
2. The only significant (p < 0.5) shift in students’ responses to the 48 statements presented to them pre and post intervention was a negative one in response to the statement “I know I can do well in Maths”.

3. No claim of significance can be forwarded as to whether the use of active learning methodologies and real life contexts develop students’ understanding of probability but there were indications that these methodologies had a positive effect on understanding.

4. The general theme that emerges from the data is that the resource pack will be of use to teachers who embrace the Project Maths syllabus and what it is trying to achieve in Irish classrooms.

Each of these findings were elaborated on earlier in this chapter in the “Discussion of Findings” section, further conclusions and insights can be drawn from these findings and are discussed in the next chapter.
Chapter 7 - Conclusions, Contributions and Recommendations

7.1 Introduction

The aim of this research study was to improve the teaching and learning of probability in the author’s classroom. The research was focused around the development of a resource pack for the teaching of probability to Higher Level Leaving Certificate students, which promoted the use of active learning methodologies and real life contexts. It was hoped that the implementation of this resource pack in the classroom would lead to an improvement in students’ attitude towards mathematics and would develop conceptual as well as procedural understanding among the students. The author employed an action research methodology, the first step of which is the identification of a problem. The author recognised that an issue existed with probability within the mathematics classroom at Senior Cycle. The author’s next step was to undertake a reconnaissance of the issues involved. The issues facing mathematics education generally in Ireland and internationally were researched and discussed in Chapter 2. The focus of the author then moved specifically to the topic of probability and the literature on this topic is reviewed and presented in Chapter 3.

The third step in the action research cycle was the production and implementation of an intervention, in light of the research undertaken. The resource pack developed is outlined in Chapter 5. Upon implementation, it was necessary to evaluate the intervention, the fourth and final step in an action research cycle. The design methods
used in this evaluation and a justification for following an action research methodology are outlined in Chapter 4. In Chapter 6 the main findings from the analysis of the data collected were presented and analysed. In this chapter the author will discuss the links between the research questions posed in the introduction chapter, the theoretical frameworks discussed in Chapter 2, 3 and 5, the context of this study, in terms of the Irish Education system (Chapter 2) and the specific issues with probability as a topic (Chapter 3), and the key findings that emerged from the analysis of the qualitative and quantitative data in Chapter 6.

7.2 Conclusions

1. Is there a link between the use of active learning methodologies and real life data and contexts and an improvement in students’ attitude towards mathematics?

The first of the author’s research questions was “To investigate if there is a link between the use of active learning methodologies and real life contexts and an improvement in students’ attitude towards mathematics”. As to the existence of a link the analysis of the data collected proves inconclusive and contradictory. There was a drop in the Confidence subscale but an improvement in the Anxiety, Perception of Usefulness and Motivation subscales. None of these changes were significant and all tallied with the qualitative data collected from the expert panel, critical friend and teacher diary. The most dramatic shift in any of the scales was in the perception of usefulness. This positive change was not significant (p=.273) but is in line with both the feedback from the outside perspective as well as the teacher diary. However as stated already no claim of significance can be attached to this shift in attitude.
In fact the only finding of significance (p < 0.5) was found in one of the students’ responses to the 48 statements presented to them pre and post intervention. There was a negative change in response to the statement “I know I can do well in Maths”. This significant drop (p = .019) seems to suggest that the use of active learning methodologies and real life data and contexts in the classroom may in fact have a negative impact on students’ attitude. However given the trend of improvement in three of the four subscales it would seem then that it was students’ confidence specifically that was negatively impacted on. This is corroborated in the teacher’s diary, where entries, speak of shaken confidence. The author would suggest that this can be accounted for to some extent to the teaching methodologies employed but perhaps more importantly a fear on the students’ part of the new type of contextual real life question which appeared in the class test. The students were no longer so sure they could “do well in mathematics”, given the new approach.

This brings us back to Chapter 2, Section 2.5 and Nickson’s (2000, p.194) “didactic contract”. This term describes a phenomenon where students feel comfortable with the current exam orientated pedagogy and informally push teachers in that direction. Studies by Carpenter et al. (1998), Cobb et al. (1991), and Verschaffel and De Corte (1997) where a problem-solving curriculum was implemented mentioned student unhappiness in the beginning phases. Fauzan et al. (2002) and Van Reeuwijk (1992) encountered similar problems when assessing the implementation of RME type curricula. Given that these larger and longer studies, which had focused on a move away from a behaviourist orientated pedagogy and examination focused on procedural knowledge had encountered a dip in attitudes the author was aware that this may occur during his intervention. It is worth noting that in all the studies discussed
(Carpenter et al. 1998; Cobb et al. 1991; Vershaffel and De Corte 1997; Fauzan et al. 2002; Van Reeuwijk 1992) student attitude towards mathematics had improved by the end of the study once students recovered from their initial discomfort. One recent Irish study on active learning highlighted the need for an “extended period” of intervention (Hogan, Brosnan, DeRoiste, MacAilster, Malone, Quirke-Bolt and Smith 2007 p.43) and found that the use of these methodologies brought about a significant improvement in attitude. The author’s study was a short two-week intervention and a longer and wider intervention is needed to obtain an accurate measure of whether there is a link between the use of active learning methodologies and real life contexts and an improvement in students’ attitude towards mathematics. Given the Project Maths reforms currently underway in Ireland, the author feels that it is important that the teachers involved are aware that changes in syllabus are often accompanied by a transitory resistance and a temporary dip in attitude by students.

2. Does the use of active learning methodologies and real life data and contexts develop students’ understanding of probability?

The author, after analysing the data collected during this study, cannot make any claim of significance as to the existence of a link between the use of active learning methodologies and real life contexts and to the development of students’ understanding of probability. The author did however note in the findings that there were indications that these methodologies had a positive effect on understanding. The results of the test followed a reasonably normal distribution, slightly skewed toward the higher grades. These results, in a test designed to test for conceptual understanding, emphasising context and non-routine problems, contrasts with the poor
performance of Irish students in similarly designed international studies (Donohoe 2010; OCED 2004; Oldham 2002). This would suggest that these teaching methodologies do improve understanding. However, since no direct comparison on specific questions is possible no claim can be made. An improvement in understanding due to the implementation of pedagogical approaches which emphasise teaching for understanding is consistent with the studies the author discussed in chapter 2 (Carpenter et al. 1998; Cobb et al. 1991; Fauzan et al. 2002; Fuson et al. 2000). In the previous section the author noted that students’ confidence was shaken and that a fear of a new examination format was a contributing factor. It is important that teachers are aware of these fears and work to allay them. This study found that there was a significant positive correlation between the results of the class test and their Junior Certificate results. There was no marked difference between which students achieved which grade but there was a marked difference in the degree of understanding needed in the two tests due to the inclusion of non-routine contextualised questions, which accounted for half the marks available in the test.

3. Other Themes to Emerge

On analysis of the data the author realised the remaining data predominantly fell under one research question; “Will the resource pack be of benefit to the author and other teachers of Higher Level Leaving Certificate probability, teaching the new Project Maths Syllabus?” A strong consensus emerges from the qualitative data collected from the expert panel, critical friend and teacher diary that this resource pack will definitely be of use to teachers but only to those who embrace the Project Maths syllabus. Reservations are expressed by some of the expert panel regarding
teachers’ inclinations to use this resource pack as part of wider doubts regarding teachers adapting their practice to the Project Maths syllabus. These worries are not without foundation. Studies have shown that often it is the teachers who have greater difficulties than the students adapting to real syllabus change, involving new pedagogical approaches. Van Reeuwijk (1992) noted this in his study involving the implementation of R.M.E. based practise in Milwaukee in the U.S.A. Pehkonen (2007) and Cai (2003) also found that the increase in a teacher’s workload as well as the huge change in their role and the skill-set needed has also proved to be an obstacle in implementing problem solving approaches. Smith (2000) claimed that these difficulties, are very often psychological, due to beliefs and conceptions held by teachers about mathematics and mathematics teaching.

7.3 Thesis Contribution

The general objective of all research in mathematics education is to inform and improve both practice and learning (Hoyles, Ross and Kent 2004). In the case of this particular study an intervention was made in the everyday functioning of the classroom and an evaluation was made as to the effect of this intervention. The literature is replete with this form of action research, where the teacher involved takes on the dual role of researcher. These interventions and other research have advocated many changes in how classrooms function but the widespread failure of these reforms to take hold has been well detailed (Sarason 1993; William 2003). The Project Maths syllabus is an undertaking which if successfully implemented will bring major changes to the Irish mathematics classroom. Much greater emphasis will be placed on
the use of active learning methodologies and contexts to enhance student understanding and improve attitudes (NCCA 2010).

This study has two separate and significant contributions. The study has produced findings specifically regarding the research questions, which add to the literature available on the use of active learning methodologies and contexts and its effect on understanding and attitudes. The study has also produced resource materials, which are in themselves a contribution to mathematics education in Ireland. The initial research questions were

1. Is there a link between the use of active learning methodologies and real life data and contexts and an improvement in students’ attitude towards mathematics?

2. Does the use of active learning methodologies and real life data and contexts develop students’ understanding of probability?

The data is inconclusive on both of these questions but a finding of significance was made in one aspect. Students’ response to the statement “I know I can do well in Maths” was significantly lower. This adds to the body of evidence in the literature which highlights that students can experience discomfort when new pedagogical approaches along with corresponding assessment procedures are adopted.

The second contribution this study makes is directly to the Project Maths reforms. The importance of resource materials if these reforms are to be more than merely superficial was discussed in the previous section. Fullan (1991) differentiates between reform and change, explaining how deep change will only occur if there is change on
three levels; curriculum materials, pedagogy, and beliefs and values. He also explains that real change is generally accompanied by a “Bottom Up” approach where teachers play an active role in the change process. Superficial reforms on the other hand are synonymous with a “Top Down” imposition of the new curriculum and alterations to content only.

O’Keffe and O’Donoghue (2009) warned that any future curriculum reform of mathematics education in Ireland will need to see the production of heavily revised and re-orientated textbooks. The resource materials of this resource reviewed by practising teachers and trialled in the classroom have a degree of “Bottom Up” credibility which may convince teachers that this is a pedagogical approach worthy of consideration. The resource pack should aid the Project Maths reforms by providing appropriate curriculum materials, pushing teachers towards the desired pedagogical approach, and providing classroom experiences which impact on their practices and values. The resource pack thus forwards the Project Maths agenda on all three of Fullan’s (1991) criteria. It also provides teachers with resource materials outside the traditional textbook. Boaler (1997, 2002) found that students who worked predominantly through textbooks found it difficult to apply their mathematics in different situations.
7.4 Limitations of Study

The limitations of this study have been outlined earlier in this thesis, in Chapter Section 4.9 and earlier in this chapter, Section 7.2. There are two main limitations to this study.

- The first concern is that the student sample was small, limited to 14 students and all students attended one particular school with one particular teacher, meaning unique factors could be at play when analysing the data produced by the students. This raises concerns over the external validity or the ability to claim that the results can be generalised onto a wider population.

- The second limitation is the length of the intervention, a two week intervention does not provide enough time for students initial discomfort at changes in pedagogical and examination approach to be overcome, therefore the data gathered may not be a reliable guide as to long-term effect of these approaches on attitudes and understanding.

These limitations though noteworthy do not undermine the conclusions and recommendations made in this chapter. The limitations of this study were taken into account by the author when making both the conclusions and the recommendations.
7.5 Recommendations

The author wishes to make a number of recommendations based on the findings and conclusions reached during this research

1. The author recommends the adoption of active learning methodologies and the use of contexts in the classroom in-line with those envisaged by the Project Maths syllabus (NCCA 2010).

2. The author recommends the adoption of a pedagogy and content which mixes the classical and frequentist approaches in the probability classroom in-line with that envisaged by the Project Maths syllabus (NCCA 2010)

3. The author recommends that an intervention of a similar nature be undertaken in Ireland but on a wider scale, over a larger time-frame. A study involving multiple schools and multiple teachers covering more material would provide a more accurate picture of the impact that active learning methodologies and the use of contexts has on attitudes and understanding, overcoming the limitations of this study and any doubts over validity.

4. The author recommends that the unique elements of probability such as the paradoxes, fallacies and counter-intuitive problems, which were included in the activities and contextualised questions, should not be avoided and ignored by teachers for fear of creating confusion. They need to be worked through with students if true and deep learning is to occur (Garfield 1995; Shaughnessy 1977).
5. The author strongly recommends significant and continuous teacher in-service. This especially holds true in the case of probability, where successful teaching depends hugely upon the teachers’ knowledge of a topic with unique pedagogies, thinking and difficulties (Stohl 2005). Not alone has new content and materials alone to be disseminated to teachers but a change is needed in teacher mindset if new pedagogies are to be adopted (Garfield and Greer 1996; De Lange et al. 1993). In order for this to happen, hands on experience of learning and teaching in this new manner is needed, not just materials (Conway and Clark 2003; Sugrue 2002; Prendergast and O’Donoghue 2009).

6. The author recommends a continuation of the new Project Maths examinations which include contextualised non-routine type problems. Conway and Sloane (2006) have similar criteria to Fullan (1991) but have also included the need for a change in examinations. The author feels it is important to note that assessment should promote and support good instructional practice (Garfield and Greer 1996) and if the NCCA wish for a change to occur in practice, Project Maths examinations will have to be designed accordingly.

7. The author’s final recommendation is that mathematics teachers are actively encouraged to undertake action research studies such as this study. If real change is to occur there needs to be more “Bottom-up” input (Fullan 1991) and the trialling and development of materials in the classroom in a manner similar to what occurred in the Netherlands when they adopted an R.M.E. approach (Conway and Sloane 2006).
7.6 Final Comments

The conclusions the author drew from the findings of the research, the contribution of the research to mathematics education and the recommendations of the author have all been outlined in this chapter. The introduction of the Project Maths syllabus represents a significant change in how mathematics is taught in Ireland, much greater emphasis has been placed on the use of active learning methodologies and contexts to enhance student understanding and improve attitudes (NCCA 2010). This intervention while producing no conclusive data to support the methodologies advocated by Project Maths, does indicate that these methodologies are beneficial to students and should be adopted by teachers. This study was initiated after the identification of a specific problem within the author’s mathematics classroom, which the intervention succeeded in solving to the satisfaction of the practitioner/researcher. Cohen and Mannion (1994) argue that in action research projects it is the view of the practitioner/researcher, which is of greatest importance. Therefore, even though the research questions regarding attitude and understanding cannot be answered in a definitive manner, the author takes the view that this research project and intervention were at the very least a qualified success.
Appendix A

Sample Realistic Mathematics Education Questions
The following example is taken from the formal examination for students preparing for university in non-mathematical sciences. The first problem of the 1993 exam is about "Death of the Bass".

Figure 7

According to the introduction, a scientist named Courant counted the number of bass and the water temperature of the lake the bass were swimming in. This leads to (Figure 8):

1) Compute the percentage of the total cases in which the water was 22.5°C or above.

A colleague of Coutant postulated that the observed water temperatures were distributed in a normal way with an average of 19.5°C and a SD of 2.1°C, deviates quite a bit from the normal distribution that we might expect.

2) Compute how large this deviation is.

<table>
<thead>
<tr>
<th>Number of observations</th>
<th>water temperature</th>
<th>number observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>99</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>325</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>566</td>
<td></td>
</tr>
<tr>
<td>18.5</td>
<td>636</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>1201</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1555</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>990</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>918</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>495</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>241</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>total 7068</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Other experiments by Coutant showed that bass cannot survive a longer stay in water warmer than $25^\circ$ C. In a small lake, Lake Cherokee, the water became warmer and warmer during the summer. This resulted in the fact that the $25^\circ$ C level sank deeper and deeper into the lake (see Figure 9).

![Figure 9](image)

Coutant also discovered that bass will only survive in water containing at least 2 mg of oxygen per litre of water. During the summer, a layer of water with insufficient oxygen developed in Lake Cherokee. At the beginning of the summer the water of Lake Cherokee contained exactly 2 mg oxygen per litre (Figure 10a). Eighty days later 60 per cent of the water (the deeper part) consisted of water containing less than 2 mg of oxygen per litre of water (Figure 10b).

![Figure 10](image)

Assume that both processes develop in a linear fashion.

3) Make a graph representing the $25^\circ$ C level and the 2 mg/l level over time.

4) Compute after how many days, counting from the start of the summer, bass cannot live in Lake Cherokee.
Appendix B

Expert Panel/
Critical Friend
Questionnaire
Expert-Panel/Critical Friend Review

Format of the resource pack

Each of the three topics in the resource pack is examined in three sections.

Section 1 - Introduction

- Objectives
- Everyday uses
- Definitions
- Simple Examples

Section 2 – Active Learning

This includes questions based around activities such as experiments and investigations.

Section 3 – Questions in context

This includes questions which use real data or are set in contexts, which demonstrate the applications of the topic in everyday life.

The resource pack does not have to be taught linearly or in totality. Teachers can use it as their sole source when teaching the topics or they can just dip in and out of it at their discretion using it to complement their textbook rather than totally replacing them.

Aims and Objectives of the Resource Pack

The objectives of this pack are:

1. To enhance students' conceptual understanding of probability.
2. To improve students' attitude towards mathematics.

In order to achieve these objectives it is understood that,

- Conceptual understanding is more important than procedural skill
- Active learning occurs
- Real life data and contexts are used
- Technology is used as a teaching aid where appropriate (Franklin et al. 2005)
Aims and Objectives of Project Maths

“Project Maths aims to provide for an enhanced student learning experience and greater levels of achievement for all. Much greater emphasis will be placed on student understanding of mathematical concepts, with increased use of contexts and applications that will enable students to relate mathematics to everyday experience.”

http://www.projectmaths.ie/overview/

“The aim of the probability course is two-fold; it provides certain understandings intrinsic to problem solving and underpins the statistics course. It is expected that the conduct of experiments (including simulations), both individually and in groups, will form the primary vehicle through which the knowledge, understanding and skills in probability are developed. References should be made to the appropriate contexts and applications of probability.”

Please Circle the relevant response based on your reading of the resource pack.

Circle A – If you strongly agree with the statement

Circle B – If you agree with the statement

Circle C – If you are not sure

Circle D – If you disagree with the statement

Circle E – If you strongly disagree with the statement

**Section A- Aims and Objectives of the Resource Pack**

1) The resource pack complements the aims of the Project Maths Syllabus.
   
   A   B   C   D   E

2) The content of the resource pack complements the objectives of the resource pack?
   
   A   B   C   D   E

3) The resource pack encourages active learning.
   
   A   B   C   D   E

4) The resource pack promotes conceptual understanding.
   
   A   B   C   D   E

5) The resource pack encourages independent learning, using technology as an aid.
   
   A   B   C   D   E

6) The resource pack makes use of real life data and contexts.
   
   A   B   C   D   E

Comments/Suggestions:

_________________________________________________________________
_________________________________________________________________
Section B – The Teacher

7) The resource pack is feasible as a teaching aid with respect to time.
   A   B   C   D   E

8) The resource pack is feasible as a teaching aid with respect to classroom management.
   A   B   C   D   E

9) The resource pack is easy to use.
   A   B   C   D   E

10) This resource pack could have a positive influence on the teaching of higher level
    Probability at Senior Cycle.
    A   B   C   D   E

11) Teachers will use this resource pack.
    A   B   C   D   E

   Comments/Suggestions:

   __________________________________________________________
   __________________________________________________________
   __________________________________________________________
   __________________________________________________________

Section C – The Student

12) The students will find the questions in the resource pack relevant.
    A   B   C   D   E

13) This resource pack will engage and motivate students.
    A   B   C   D   E
14) This resource pack will increase student understanding of key concepts in probability?
   A     B     C     D     E

15) This resource pack could have a positive influence on the learning of higher level
    Probability at Senior Cycle?
   A     B     C     D     E

   Comments/Suggestions:

   ____________________________
   ____________________________
   ____________________________
   ____________________________

Section D – Format and Content

16) The three section format of: introduction, active learning and questions in context of
    each topic is suitable
   A     B     C     D     E

17) The content of the “Expected Value” section is educationally appropriate and useful.
   A     B     C     D     E

18) The content of the “Conditional Probability” section is educationally appropriate and
    useful.
   A     B     C     D     E

19) The content of the “Bernoulli Trials” section is educationally appropriate and useful.
   A     B     C     D     E
Comments/Suggestions:

Expected Value:


Conditional Probability:


Bernoulli Trials:


Thank you for your co-operation
Appendix C

Modified Fennema-Sherman Attitude Scale
A Modified Fennema-Sherman Mathematics Attitude Scale

No. □  

Class:  

Date:

Circle A – If you strongly agree with the statement
Circle B – If you agree with the statement
Circle C – If you are not sure
Circle D – If you disagree with the statement
Circle E – If you strongly disagree with the statement

1. I am sure that I can learn maths. A B C D E
2. Mathematics is enjoyable and stimulating to me. A B C D E
3. Maths doesn’t scare me at all. A B C D E
4. Knowing mathematics will help me earn a living. A B C D E
5. I don't think I could do advanced maths. A B C D E
6. Maths will not be important to me in my life's work. A B C D E
7. Mathematics usually makes me feel uncomfortable and nervous. A B C D E
8. I don’t understand how some people can spend so much time on maths and seem to enjoy it A B C D E
9. Maths is hard for me. A B C D E
10. I'll need mathematics for my future work. A B C D E
11. I am sure of myself when I do maths. A B C D E
12. I don’t expect to use much maths when I get out of school. A B C D E
13. It wouldn’t bother me at all to take more maths courses. A B C D E
14. I am challenged by maths problems I can’t understand immediately. A B C D E
15. Mathematics makes me feel uncomfortable, restless, irritable and impatient. A B C D E
16. Maths is a worthwhile, necessary subject. A B C D E
17. I’m not the type to do well in maths. A B C D E
18. The challenge of maths problems does not appeal to me. A B C D E
19. I would rather have someone give me the solution for a difficult maths problem than have to work it out myself. A B C D E
20. I like maths puzzles. A B C D E
21. I haven’t usually worried about being able to solve maths problems. A B C D E
22. Taking maths is a waste of time. A B C D E
23. I get a sinking feeling when I think of doing maths problems. A B C D E
24. Maths has been my worst subject. A B C D E
25. I think I could handle more difficult maths. A B C D E
Circle A – If you strongly agree with the statement
Circle B – If you agree with the statement
Circle C – If you are not sure
Circle D – If you disagree with the statement
Circle E – If you strongly disagree with the statement

26. My mind goes blank and I am unable to think clearly when working mathematics. A B C D E

27. I will use mathematics in many ways as an adult. A B C D E

28. I do as little work in maths as possible. A B C D E

29. I see mathematics as something I won't use very often when I get out of school. A B C D E

30. A maths test would scare me. A B C D E

31. Most subjects I can handle OK, but I just can't do a good job with maths. A B C D E

32. I can get good grades in maths. A B C D E

33. I'll need a good understanding of maths for my future work. A B C D E

34. I almost never have got nervous during a maths exam. A B C D E

35. When a maths problem arises that I can't immediately solve, I stick to it until I have the solution. A B C D E

36. When a question is left unanswered in maths class, I continue to think about it afterwards. A B C D E
<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>37.</td>
<td>I know I can do well in maths.</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>38.</td>
<td>Doing well in maths is not important for my future.</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>39.</td>
<td>Mathematics makes me feel uneasy and confused.</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>40.</td>
<td>Once I start working on a maths puzzle I find it hard to stop.</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>41.</td>
<td>I am sure I could do advanced work in maths.</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>42.</td>
<td>Maths is not important for my life.</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>43.</td>
<td>Figuring out mathematical problems does not appeal to me.</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>44.</td>
<td>I'm no good in maths.</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>45.</td>
<td>Maths puzzles are boring.</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>46.</td>
<td>I study maths because I know how useful it is.</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>47.</td>
<td>I usually have been at ease during maths tests.</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>48.</td>
<td>I usually have been at ease in maths classes.</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
</tbody>
</table>
**Key to Modified Fennema-Sherman Scale for Mathematics**

Confidence = Personal confidence about the subject matter.

Usefulness = Usefulness of the subject's content.

Anxiety = Anxiety felt when in mathematics situations involving mathematics.

Effectance motivation = What is motivating within the subject matter.

+ = Question reflects positive attitude

- = Question reflects negative attitude

<table>
<thead>
<tr>
<th>Question #</th>
<th>Category of Question</th>
<th>Attitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Confidence</td>
<td>+</td>
</tr>
<tr>
<td>2.</td>
<td>Effective Motivation</td>
<td>+</td>
</tr>
<tr>
<td>3.</td>
<td>Anxiety</td>
<td>+</td>
</tr>
<tr>
<td>4.</td>
<td>Usefulness</td>
<td>+</td>
</tr>
<tr>
<td>5.</td>
<td>Confidence</td>
<td>-</td>
</tr>
<tr>
<td>6.</td>
<td>Usefulness</td>
<td>-</td>
</tr>
<tr>
<td>7.</td>
<td>Anxiety</td>
<td>-</td>
</tr>
<tr>
<td>8.</td>
<td>Effective Motivation</td>
<td>-</td>
</tr>
<tr>
<td>9.</td>
<td>Confidence</td>
<td>-</td>
</tr>
<tr>
<td>10.</td>
<td>Usefulness</td>
<td>+</td>
</tr>
<tr>
<td>11.</td>
<td>Confidence</td>
<td>+</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>12.</td>
<td>Usefulness</td>
<td>-</td>
</tr>
<tr>
<td>13.</td>
<td>Anxiety</td>
<td>+</td>
</tr>
<tr>
<td>14.</td>
<td>Effective Motivation</td>
<td>+</td>
</tr>
<tr>
<td>15.</td>
<td>Anxiety</td>
<td>-</td>
</tr>
<tr>
<td>16.</td>
<td>Usefulness</td>
<td>+</td>
</tr>
<tr>
<td>17.</td>
<td>Confidence</td>
<td>-</td>
</tr>
<tr>
<td>18.</td>
<td>Effective Motivation</td>
<td>-</td>
</tr>
<tr>
<td>19.</td>
<td>Effective Motivation</td>
<td>-</td>
</tr>
<tr>
<td>20.</td>
<td>Effective Motivation</td>
<td>+</td>
</tr>
<tr>
<td>21.</td>
<td>Anxiety</td>
<td>+</td>
</tr>
<tr>
<td>22.</td>
<td>Usefulness</td>
<td>-</td>
</tr>
<tr>
<td>23.</td>
<td>Anxiety</td>
<td>-</td>
</tr>
<tr>
<td>24.</td>
<td>Confidence</td>
<td>-</td>
</tr>
<tr>
<td>25.</td>
<td>Confidence</td>
<td>+</td>
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<tr>
<td>26.</td>
<td>Anxiety</td>
<td>-</td>
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<tr>
<td>27.</td>
<td>Usefulness</td>
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<td>28.</td>
<td>Effective Motivation</td>
<td>-</td>
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<td>29.</td>
<td>Usefulness</td>
<td>-</td>
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<tr>
<td>30.</td>
<td>Anxiety</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Confidence</td>
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<tr>
<td>---</td>
<td>------------</td>
<td>---</td>
</tr>
<tr>
<td>31.</td>
<td>Confidence</td>
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<tr>
<td>32.</td>
<td>Confidence</td>
<td>+</td>
</tr>
<tr>
<td>33.</td>
<td>Usefulness</td>
<td>+</td>
</tr>
<tr>
<td>34.</td>
<td>Anxiety</td>
<td>+</td>
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<tr>
<td>35.</td>
<td>Effective Motivation</td>
<td>+</td>
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<tr>
<td>36.</td>
<td>Effective Motivation</td>
<td>+</td>
</tr>
<tr>
<td>37.</td>
<td>Confidence</td>
<td>+</td>
</tr>
<tr>
<td>38.</td>
<td>Usefulness</td>
<td></td>
</tr>
<tr>
<td>39.</td>
<td>Anxiety</td>
<td></td>
</tr>
<tr>
<td>40.</td>
<td>Effective Motivation</td>
<td>+</td>
</tr>
<tr>
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<td>Confidence</td>
<td>+</td>
</tr>
<tr>
<td>42.</td>
<td>Usefulness</td>
<td></td>
</tr>
<tr>
<td>43.</td>
<td>Effective Motivation</td>
<td></td>
</tr>
<tr>
<td>44.</td>
<td>Confidence</td>
<td></td>
</tr>
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<td>45.</td>
<td>Effective Motivation</td>
<td></td>
</tr>
<tr>
<td>46.</td>
<td>Usefulness</td>
<td>+</td>
</tr>
<tr>
<td>47.</td>
<td>Anxiety</td>
<td>+</td>
</tr>
<tr>
<td>48.</td>
<td>Anxiety</td>
<td>+</td>
</tr>
</tbody>
</table>
Probability Test

Problem 1 (10 marks)

Given the Venn Diagram below

(A) Find the conditional probability \( \Pr (A \mid B) \). (5 marks)

(B) State whether \( A \) and \( B \) are independent events and justify your answer. (5 marks)
Problem 2 (10 Marks)

In a certain type of archery competition, Laura hits the target with an average of two out of every three shots. The shots are independent of each other. During one such competition, she has ten shots at the target.

(A) Find the probability that Laura hits the target exactly nine times. (5 marks)

(B) Find the probability that Laura hits the target fewer than nine times. (5 marks)

Problem 3 (5 marks)

The table below gives motor insurance information for fully licensed drivers in Ireland in 2007. All drivers who had their own insurance policy are included.

<table>
<thead>
<tr>
<th></th>
<th>Number of Drivers</th>
<th>Number of claims</th>
<th>Average cost per claim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>9634</td>
<td>977</td>
<td>€6108</td>
</tr>
<tr>
<td>Female</td>
<td>6743</td>
<td>581</td>
<td>€6051</td>
</tr>
</tbody>
</table>

What is the expected value of the cost on male driver’s policy?
Problem 4  

The random variable $X$ has a discrete distribution. The probability that it takes a value other than 13, 14, 15, or 16 is negligible.

(A) Complete the probability distribution table below and hence calculate $E(X)$, the expected value of $X$.  

<table>
<thead>
<tr>
<th>$x$</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X=x)$</td>
<td>0.383</td>
<td>0.575</td>
<td>0.004</td>
<td></td>
</tr>
</tbody>
</table>

(B) If $X$ is the age in complete years, on 1 January 2010 of a student selected at random among all second year students in Irish schools, explain what $E(X)$ represents.  

(C) If ten students are selected at random from this population, find the probability that exactly six of them were 14 years old on 1 January 2010. Give your answer correct to three decimal places.
Problem 5

Suppose every child has an equal chance of being born a boy and a girl. You meet someone who tells you that they have two children and one of these is a girl. What is the probability the person has two girls?

Problem 6

A math teacher gave her class two tests. 25% of the class passed both tests and 42% of the class passed the first test. What percentage of those who passed the first test also passed the second test?
Problem 7  
(5 Marks)

On May 4th 1983, an Eastern Airlines Flight 855 took off from Miami to Nassau. 100 miles from Miami, Engine 1 failed so the pilot turned around. Engine 2 and 3 then failed shortly from Miami. The pilot managed to start engine 2 in time and land the aircraft safely.

If the probability of one of the independent jet engines on Flight 855 failing was .0001 or 1 in 10,000. What is the probability of exactly two engines failing?
Problem 8

Shaquille O’Neal is one of the best basketballers in the world. He plays for the LA Lakers, “Shaq” as he is nicknamed is 7’1” tall. Most of the shots he takes are close to the basket and because he is so tall the other players find it difficult to stop him making baskets. He makes 57% of these, close in shots, which are worth 2 points well above the average for a top player. In basketball if a player is fouled close to the basket while shooting, he gets two free shots each worth a point, from the free-throw line 15 yards out. The probability of “Shaq” making the two of these is 26%, one of them 51% of the time and none of them 23% of the time, which is poor for a top quality player. These probabilities have meant that some teams adopted a strategy called “Hack - a - Shaq”, which meant they fouled him when he was close to the basket. Using these probabilities only as a consideration, is this strategy worth it?
Appendix E

Slides of

Teacher Section

of

Resource Pack
Conditional Probability

Section 1

Introductory Activity

Student Activity

Scenario:
- You are given the player profiles from the programme of the Ireland v England rugby match held in Croke Park in February 2007.
- There were 22 players on the Irish panel that day.
- To view the current Irish Squad visit http://www.irishrugby.ie and click on player profiles.

1) Pose a Question
- Can the answer given to one question help you predict the answer to another question?
- Can examining different probabilities show an association (link) between two variables?

Q. Does the fact that given a player picked at random is a forward mean that they are more likely to weigh more than 103 K.G.’s?

Example - Class Survey

3) Analyse the data
- Place the data into a Venn diagram.
- Remembering to fill the intersection first.

Resource pack
Example - Class Survey

1. What is the probability that I randomly pick a forward from the panel?

\[ \frac{12}{22} \]

Panel (22)

Forward (12)

\[ \geq 103 \text{ K.G.'s} \]

(11)

2. What is the probability that I randomly pick a player \( \geq 103 \text{ K.G.'s} \) from the panel?

\[ \frac{11}{22} \]

Panel (22)

Forward (12)

\[ \geq 103 \text{ K.G.'s} \]

(11)

Example - Class Survey

3. What is the probability of picking a forward from the panel, if you chose from those who weigh \( \geq 103 \text{ K.G.'s} \)?

\[ \frac{10}{11} \]

Panel (22)

Forward (12)

\[ \geq 103 \text{ K.G.'s} \]

(11)

Example - Class Survey

4. What is the probability of picking a player who weighs \( \geq 103 \text{ K.G.'s} \) from the panel, if you chose from the forwards?

\[ \frac{10}{12} \]

Panel (22)

Forward (12)

\[ \geq 103 \text{ K.G.'s} \]

(11)

Student Activity

4) Interpret The Data

- Can the occurrence of event affect the probability of another event?

- Yes it can - clearly there is a link.

- This link is called an association. It suggests that if a player answers yes to the question they are over 103 K.G.'s, they are far more likely to be a forward.

Student Activity

- Why do you think the probability is so high when I randomly pick a forward given they weigh over 103 K.G.'s. What is the association (link)?

- Suggestion:

- Strength is highly valued if you are a forward whereas the backs have to be mobile and quick.
Conclusions from activity:
1. These events are said to be dependent – the occurrence of one affects the probability of the occurrence of the other.
2. When a clause or condition is added when calculating a probability, the affected probability is known as conditional probability.

Establishing Rules of Conditional Probability
1. What is probability of A given B?
   What is P(A|B)?
   = \#(A \cap B)
   \#B

2. What is probability of B given A?
   What is P(B|A)?
   = \#(A \cap B)
   \#A

3. Does P(A|B) = P(B|A)?
   No it does not as we learned from the rugby players.

Formula for conditional probability?
\( P(A|B) = \frac{\#(A \cap B)}{\#B} \)
N.B. P(A|B) ≠ P(B|A)
Learning Outcomes:

- Students will understand the concepts of independence and conditional probability.
- Students will be able to analyse categorical data for possible associations.
- Students will analyse conditional probabilities using Venn diagrams.

Recap – Definitions

Mutually Exclusive Events

- These are events which cannot happen at the same time, the occurrence of one precludes the occurrence of the other.

Example – Traffic Light

P(Green traffic light) = \( \frac{2}{3} \)

P (Red traffic light) = \( \frac{1}{3} \)

P (The Traffic light is red and green) = 0

Recap – Definitions

Note: In the Venn Diagram there is no intersection because the traffic lights cannot be green and red at the same time.

- These events are mutually exclusive

\[ P(G \cap R) = 0 \]

Recap – Definitions

Independent / Dependent Events

- Independent events are events, which do not affect the probability of the other occurring but which can occur at the same time (unlike mutually exclusive events).

Example – A deck of cards and a die

- Two cards are pulled from a deck without replacement and a die is rolled.

Recap – Definitions

- What is the probability that I pick a king from a full pack of cards?

\[ \frac{4}{52} = \frac{1}{13} \]

- What is the probability that I pick a king as my second card?

\[ \frac{3}{51} \]

- The first event has affected the probability of the second event. These events are dependent

Recap – Definitions

- What is the probability I roll a six?

\[ \frac{1}{6} \]

This event is independent of choosing the cards. So the probability of it occurring remains unchanged despite the other events having occurred.
Money Question ???

1. Thomas has placed a €10 note in one of his pockets, he can't remember which one. He has six pockets between his jacket and jeans. What is the probability that he picks the correct pocket first time?

   The possible sample space is 6 pockets
   Answer = \( \frac{1}{6} \)

Money Question ???

2. Given he has checked two pockets and they are empty, what is the probability that the €10 will be in the next pocket?

   The extra information, the "given" means we choose from a reduced sample space
   Answer = \( \frac{1}{4} \)

Money ???

- Why is the answer to part 2 different?
- It is different because these two events are not independent.
- In the first event, its in one of 6 pockets in the second extra information is "given" or has already occurred, which affects the probability of what we are looking for.
- This is called conditional probability.

Definition – Conditional Probability

- The probability assigned to an event, given we know another event has occurred is called Conditional Probability.
- Conditional probability involves figuring out "What is the probability of Event A occurring given Event B has occurred"
- Instead of "given" the phrases "if we only pick from" or "on condition that" could also be used

Everyday uses:

Conditional probability has many uses:
- Detectives use conditional probabilities based on crime statistics to help direct their investigations.
- Doctors also use conditional probability to help diagnose illnesses, given a set of symptoms.

Conditional Probability

- Conditional probability questions can be solved in two ways
- The probability of "Event A" given "Event B"

   Method 1
   The reduced sample space method.
   \[ P(A \mid B) = \frac{\#(A \cap B)}{\#(B)} \]
Conditional Probability

Method 2 - The formula

\[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} \]

Note: \( P(A|B) = P(B|A) \)

Example 1 - Class Survey

A teacher completes a quick survey of her Leaving Cert Maths class.

- She asks her students two quick questions
  - Do you drive to school?
  - Do you have a part-time job?

Of the class of twenty, six of them drive to school and eight have a part-time job.

Example 1 - Class Survey

Step 1 - Draw a Venn diagram.

Example 1 - Class Survey

Step 2 - Fill in the intersection (who does both)

The teacher checks, in this case, 5 do both.

Step 3 - Fill in the rest of the diagram.

Example 1 - Class Survey

1. What is the probability that a person picked at random from the class drives to school?

\[ \frac{6}{20} = \frac{3}{10} \]

Example 1 - Class Survey

2. What is the probability that a person picked at random drives to school given that they have a part-time job?

\[ \frac{15}{3} = \frac{5}{1} \]
Example 1 - Class Survey

1. Compare the probability that a person picked at random drives to school to the probability they drive to school given that they have a part-time job.

2. There is a higher probability of picking a student who drives to school given they have a part-time job than from the class as a whole.

3. Why are they different, in your opinion?

4. One interpretation is that those who drive to school need money to pay for petrol, insurance etc. and so they are more likely to need a part-time job.

Example 1 - Class Survey

4. What is the probability that a student picked at random has a part-time job?

5. Use the formula:

\[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} \]

Method 2 - Use the Formula

1. \[ P(A \mid B) = \frac{\frac{3}{5}}{\frac{8}{20}} \]

2. \[ P(A \mid B) = \frac{5}{8} \]

3. \[ P(A \mid B) = \frac{5}{8} \]

Example 1 - Class Survey

Resource pack

NCE-MSTL
Example 1 - Class Survey
6. What is the probability that a student picked at random has a part-time job given that they drive to school?

Ans: \( \frac{6}{20} \)

Example 1 - Class Survey
7. Does the probability of randomly picking a student who drives to school given they have a part-time job equal the probability of randomly choosing a student who has a part-time job given they drive to school?

Ans: No \( \frac{5}{11} \neq \frac{5}{15} \)

Remember: \( P(A|B) = P(B|A) \)

Example 2 - Cards
A) What is the probability I pick a red card from a full pack of cards?

\[ \frac{26}{52} = \frac{1}{2} \]

B) I replace the card, what is the probability that I pick a king from the full pack of cards?

\[ \frac{4}{52} = \frac{1}{13} \]

Example 2 - Cards
C) Draw a Venn diagram to represent this problem. Is the probability I pull a king mutually exclusive to the probability I pull a red card.

Example 2 - Cards
D) Are these events independent?

Yes they are, the probability of the first event does not affect the second

NB: To show events are independent use

1. \( P(A \mid B) = P(A) \)

or

2. \( P(A) \cdot P(B) = (A \cap B) \)

Resource pack

NCE-MSTL
Example 2 – Cards

1. The \( P(\text{Red Card} \mid \text{a king}) = P(\text{Red Card}) \)
   \[
   \frac{2}{52} \times \frac{1}{2} = \frac{1}{26}
   \]

2. \( P(\text{Red}) \times P(\text{King}) = (\text{Red} \cap \text{King}) \)
   \[
   \frac{2}{13} \times \frac{1}{2} = \frac{1}{26}
   \]

Possible Investigation 1
- As a class you could formulate some quick questions whose answers provide binary categorical data (data that fits into two possible answers e.g. Yes or No), and examine the conditional probabilities.
- Examples
  - "Is your favourite subject maths?"
  - "How many Males or Females?"
  - "Did you get an A in your Junior Certificate?"
  - "Do you like the "Twilight" movies?"
  - "Did you like the "Harry Potter" movies?"
  - "Do you send more than 10 texts a day?"

Possible Investigation 2
- Pick your favourite Premier League team and examine the relationship between losing and playing away from home. Is there conditional probabilities involved.
- I have given an example using Liverpool in Q1 of the "Questions in Context" section.
- This information can be found in Sunday’s and Monday’s newspapers.

Possible Investigation 3
- Is there an association between hand dominance and eye-dominance?
- Check your eye-dominance by making a circle with your thumb and forefinger and focusing with both eyes on an object on the wall. Close one eye at the time and see if the object moves. If the object does not move the open eye is the dominant one.

Section 3

Section 4

Resource pack

NCE-MSTL
Q.1 – The Premiership

On February 8th, 2010 Liverpool F.C. were 4th in the premiership. This was their record

<table>
<thead>
<tr>
<th>Played</th>
<th>Home</th>
<th>Away</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Won</td>
<td>Loss</td>
</tr>
<tr>
<td>25</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Taken from the Irish Times, Monday February 8th

Is there an association between playing at home and Liverpool earning points?

Q.1 – The Premiership

Note: A win is worth three points and a draw 1 point. There are no points for a loss.

Total played (25)

Home (13) 11

Points earned (18)

Q.1 – The Premiership

What is the probability in percentage terms of Liverpool picking up points given they are at home?

\[ \frac{11}{13} \approx 85\% \]

What is the probability in percentage terms of Liverpool picking up points given that they are away?

\[ \frac{7}{12} \approx 58\% \]

This is a trend true of all teams in all sports, why do you think this is so?

Suggestion:
Teams feel more comfortable at home, they train there so they know the grounds and tend to be more relaxed. They also have the majority of supporters cheering them on.

Q.2 Tossing two coins

A man tosses two coins. What is the probability that he gets

A) Two heads?
B) Two tails?
C) One head and one tail?
D) Given one of the coins is tails, what is the probability the man gets two tails?

A) Method 1 – Use Multiplication Rule

\[ \frac{1}{2} \times 1 = \frac{1}{4} \]

Method 2 – Use Sample Space

<table>
<thead>
<tr>
<th>1st Coin</th>
<th>2nd Coin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>Head</td>
</tr>
<tr>
<td>Head</td>
<td>Tails</td>
</tr>
<tr>
<td>Tails</td>
<td>Head, Tails</td>
</tr>
<tr>
<td>Tails</td>
<td>Tails, Tails</td>
</tr>
</tbody>
</table>

Resource pack

NCE-MSTL

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**Q.2 Tossing two coins**

- A) \( P(\text{Two Heads}) = \frac{1}{4} \)
- B) \( P(\text{Two Tails}) = \frac{1}{4} \)
- C) \( P(\text{One head and one tail}) = \frac{1}{2} \)

**Q.3 – The Swine Flu**

- According to the Irish Times on November 6th 2009
  - 3% of the Irish Population have contracted the H1N1 virus better known as Swine Flu.
  - 665 people of these had to be hospitalised, of these 40% had an underlying condition.
  - 14 of these patients died, all of whom had an underlying condition. The rate of infection in the general population currently stands at 178.5 cases per 100,000

- At that date, what was the probability a patient hospitalised due to swine flu, will die given they have an underlying condition?

\[
\frac{14}{266} = \frac{1}{19}
\]

- What was the probability that a patient hospitalised due to swine flu will die given the patient does not have an underlying condition?

\[
\frac{0}{399} = 0
\]
Q.3 - The Swine Flu

What do these probabilities suggest?

They tell us that although 14 people had died from swine flu unless you had an underlying condition the probability of you dying from swine flu was close to zero (it might not be actually 0, enough cases/experiments may not have occurred).

Q.4. - Unemployment Figures

The following information was taken from The CSO website in June 2010.

<table>
<thead>
<tr>
<th></th>
<th>Under 25</th>
<th>Over 25</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>15,000</td>
<td>275,000</td>
<td>300,000</td>
</tr>
<tr>
<td>Female</td>
<td>76,000</td>
<td>125,000</td>
<td>200,000</td>
</tr>
<tr>
<td>Total</td>
<td>91,000</td>
<td>400,000</td>
<td>490,000</td>
</tr>
</tbody>
</table>

Examine the conditional probabilities involved without using Venn diagrams.

Q.4. - Unemployment Figures

a) What is the % probability of randomly picking a male on the live register given that they are under 25?

b) What is the % probability of randomly picking a female on the live register given that they are under 25?

c) What is the % probability of randomly picking a male on the live register given that they are over 25?

d) What is the % probability of randomly picking a female on the live register given that they are over 25?

Why at this point in time do you think these probabilities are so high for males as compared to females?

Q.3. - Unemployment Figures

d) \( \frac{121,700}{361,200} = 34\% \)

e) One reason these probabilities might be so high is that there was a downturn in construction which employs a lot of men, there are other possibilities.
Q.5 – Tennis

1. What is the probability of Serena winning a point during her service games?
   \[ P(A) = (0.66 \times 0.94) + (0.34 \times 0.41) \]
   \[ = 0.62 + 0.14 \]
   \[ = 0.76 \text{ or } 76\% \]

2. What is the percentage probability of Serena losing a point during her service games given she has missed her first serve?
   \[ P(B | A) = \frac{P(A \cap B)}{P(B)} = \frac{0.62}{0.76} \]
   \[ = 0.82 \text{ or } 82\% \]

Q.6 – Medical Tests

There are a number of tests available to detect the presence of the AIDS virus.

Tests that give extremely accurate diagnosis take time to confirm but quick tests were developed to allow doctors to make a reasonably accurate rapid diagnosis.

These rapid tests have a sensitivity of 99.7% and a specificity of 98.5%.
Q.6 – Medical Tests

1) Assuming the whole adult population was tested using the rapid tests, complete a Venn diagram to display this information.
   Step 1) Has aids = 2,750,000 x 2% = 5,500
   Adult Population (2,750,000)
   
   Max Aids
   (5,500)
   
   Tested Positive

2) How many people with AIDS tested positive?
   5,500 x 99.7% = 5,484
   
   Adult Population (2,750,000)
   
   Max Aids
   (5,500)
   
   Tested Positive
   5,484

3) How many with AIDS tested negative?
   5,500 - 5,484 = 16
   
   Adult Population (2,750,000)
   
   Max Aids
   (5,500)
   
   Tested Positive
   16

4) How many people do not have AIDS?
   2,750,000 - 5,500 = 2,744,500
   
   Adult Population (2,750,000)
   
   Max Aids
   (5,500)
   
   Tested Positive
   16
   2,744,500

5) Of these how many tested negatively?
   2,744,500 x 98.5% = 2,703,332
   
   Adult Population (2,750,000)
   
   Max Aids
   (5,500)
   
   Tested Positive
   16
   2,703,332

6) How many people tested positively?
   5,484 + 41,186 = 46,652
   
   Adult Population (2,750,000)
   
   Max Aids
   (5,500)
   
   Tested Positive
   16
   5,484
   46,652
   2,703,332

7) Perform a quick check to make sure your calculations add up.
   16 + 5,484 + 41,186 + 2,703,332 = 2,750,000
   
   What percentage of people who tested positive do not actually have AIDS?
   41,168
   46,652
   = 88%
Q.6 – Medical Tests

9) Doctors are well aware of the conditional probabilities involved in these rapid tests. What action do you think they might take if you tested positive?

9) They will send them for further tests, which will give a conclusive result one way or the other.

Q.6 – Medical Tests

This is known as the false positive paradox.

It occurs because of the extreme rarity of the disease in the population and the fact that the whole population was tested.

This would never happen in real life. Only “at-risk” groups would be tested so the chances of false positives would be dramatically reduced.

Q.7 – A Cot Death

In November 1999, British solicitor Sally Clark was convicted of smothering her two infant sons, who died a year apart from sudden infant death syndrome (SIDS) or “Cot Death”.

At her murder trial, paediatrician Sir Roy Meadow testified that the probability of a single SIDS case in England in a professional non-smoking family is 1/8,543.

Q.7 – A Cot Death

Using this figure he testified that the probability of two cot deaths in one family is \( \frac{1}{8,543} \times \frac{1}{8,543} \) or 1 in 73,000,000.

Is there a problem with his reasoning?

Yes there is. He is assuming the events are independent.

Q.7 – A Cot Death

The cause of SIDS are extremely unclear but it is thought that it is a combination of unknown environmental and genetic factors.

So given a “Cot Death” has occurred once it means that it could be more likely to occur again.

In 2003 Sally Clark’s conviction was quashed, but she had developed problems with alcohol and died shortly afterwards as a result.

Prosecutor’s fallacy

The improper use of conditional probability in the courtroom has become notorious.

The “prosecutor’s fallacy” is probably the most famous error.

This involves, the prosecution reversing the condition. An example is the case of the Birmingham six.
Prosecutor's fallacy

- In 1974 two Birmingham pubs were bombed by the IRA killing 21 people. Six men travelling to a funeral in Belfast were arrested.
- A forensic scientist testified that he was 99 percent certain the defendants had handled explosives, given the results of his tests.
- \( P(\text{Handled Explosives} | \text{Positive Test}) = 99\% \)

Prosecutor's fallacy

- What he should have said was the test is positive 99% of the time if someone has handled explosives or \( P(\text{Positive Test} | \text{Handled Explosives}) = 99\% \).
- The \( P(\text{Handled Explosives} | \text{Positive Test}) = P(\text{Positive Test} | \text{Handled Explosives}) \).
- There is a subtle but vital difference. They did test positive but this does not mean that 99 out of 100 times they had been handling explosives, the probability is a lot lower.

Prosecutor's fallacy

- It was later revealed that handling many other everyday substances can produce positive test results, including playing cards, soil, cigarettes, soap and petrol.
- The defendants had been playing a game of cards on the train shortly before their arrest. The convictions of the Birmingham Six were overturned on appeal in 1991, after 18 years in prison.

The Monty Hall Problem

- *Let's Make A Deal* was a game show hosted by Monty Hall that ran on U.S. TV from 1966–77.
- In each episode a contestant was presented with 3 doors. Two doors hid goats behind them, the third hid a car. If the contestant guessed correctly they got to keep the car.
- The contestant would pick a door. The host would then open another door showing a goat and give the contestant the choice to stick with their original door or to swap.

The Monty Hall Problem

- This became one of the most famous mathematics problem in the world in 1990.
- Craig F. Whitaker wrote a letter to Marilyn vos Savant a Q&A columnist for *Parade* magazine asking "Is it to your advantage to switch your choice of doors?"
- She replied that "Yes, you should switch" and if you do you will win two-thirds of the time.

The Monty Hall Problem

- This answer caused a commotion with many mathematicians rushing to condemn her saying that the chances were clearly 50:50.
- Do you think that it would make a difference if the strategy is to stick or to swap?
- You can play the game on the Project Maths CD. Look up the "3 Doors Problem" in Java scripts.
The Monty Hall Problem

- There are many ways of solving this problem.

- In the table below examine the results of switching if the car is behind door A.

<table>
<thead>
<tr>
<th>Original Choice</th>
<th>Host opens</th>
<th>New Choice</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>C</td>
<td>Success</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>C</td>
<td>Success</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>C</td>
<td>Success</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>C</td>
<td>Success</td>
</tr>
</tbody>
</table>

- This holds true if similar tables were done if the car was behind Door B or Door C.

The Monty Hall Problem

- This is a video clip from the movie "21."

- In this clip, Monty Hall is explained.

The Monty Hall Problem

- It is clear that by using the strategy of switching you win 2 out of 3 times.

- The problem involves conditional probability. Something has occurred, which affects the probabilities. The choice to swap or stick is made given the host opens a door with a goat behind it.

- This is the condition that the mathematician’s ignored. Marilyn Vos Savant was correct – Switch
Section 1
Introductory Activity

Expected Value
The House Edge

Student Activity:

Scenario:
- You attend a local carnival and decide to play a game at a stand. It involves pulling one of three cards out of a hat.
- One card has an "X" on both sides.
- A second has an "X" on one side and blank on the other.
- The third one is blank on both sides.

Student Activity:

Card 1
Card 2
Card 3

- You place the card on the table so only one side has been seen by the stall owner (who claims to have some psychic abilities). The stall owner then guesses what is on the side facing down.

Student Activity:

- The stall owner says the probability of him guessing correctly is 50-50.
- For example, if a blank is turned up, the other side either has to be blank or have an X on it, since one card has an X on both sides.
- If the stall owner guesses correctly they take €1 off you, if they guess wrong you get €1.
- A "50-50 chance" sounds fair so you play.

Student Activity

1) Pose a Question
   Is this game fair? What is a fair game?

2) Collect the Data
   As they play the game the students should mark down how many times they win using the tally count in the activity worksheet

Resource Pack

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Student Activity

- To play, students pair off. One plays the part of the stall owner, the other is the player, who also records the data.
- The player is given the 3 cards, shuffles them behind their back and places one of them on the table in front of the stall owner.
- The stall owner guesses, either winning or losing €1. (But he has a strategy.)

Student Activity:

- The stall owner uses the strategy of guessing that the side hidden from view will be the same as what is showing.
- So if an “X” is facing up the stall owner guesses that there is an “X” underneath.
- Repeat this 15 times in your pairs, following the example on the next slide.

Student Activity

- In the example below, the stall owner won 5 times and the player won 10 times

<table>
<thead>
<tr>
<th>Stall Owner Correct</th>
<th>Stall Owner Wrong</th>
</tr>
</thead>
<tbody>
<tr>
<td>-€1</td>
<td>+€1</td>
</tr>
</tbody>
</table>

Tally: |||| | ||| |
Relative Frequency: 5/15 | 10/15

Student Activity

3) Analyse the Data
- Class Experiment: Fill in the frequency table and find the mean using the class frequency.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Stall Owner Correct</th>
<th>Stall Owner Wrong</th>
</tr>
</thead>
<tbody>
<tr>
<td>My Frequency</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class Frequency</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mean =

Student Activity

- This mean is known as the observed value for player.

4) Interpret the data
- Is this game fair? Why?

No it isn’t because clearly the observed value favours the stall owner. The probability of winning is in his favour.

Student Activity

- The stall owner claims that he has a “50-50 chance” of being correct. Our experiment suggests otherwise.
- One of the benefits of experimental probability is that it exposes errors in thinking.
- The strategy of guessing that the side hidden from view will be the same as the side showing, is the key for the stall owner.
Conclusions from activity:

- He is guessing that the card is the same on both sides, which is true in the case of 2 out of 3 of the cards.
- Using this strategy the stall owner will actually be right 2/3's of the time.
- Using conditional probability it could be explained as follows:

\[ P(\text{Both Sides have an } X \mid \text{ given an } X \text{ showing}) = \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{2} \]

Using this probability we can figure out the value for the player in advance.

Conclusions from activity:

- Fill in the frequency table and find the mean?

<table>
<thead>
<tr>
<th>Outcome</th>
<th>-€1</th>
<th>+€1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ \frac{2(-€1) + 1(+€1)}{3} = -€0.33 \]

Conclusions from activity:

- This means the player will lose an average of €0.33 every time they play the game.
- The mean could also be calculated by changing the frequency to the probability

<table>
<thead>
<tr>
<th>X</th>
<th>-€1</th>
<th>+€1</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>2/3</td>
<td>1/3</td>
</tr>
</tbody>
</table>

Conclusions from activity:

- This is how casinos work, the game seems fair but it is fixed so that the casino will win more than it loses provided enough customers come through the doors (i.e. enough experiments).
- These games are designed so that the customer wins enough to stay interested but over time the casino always wins. The expected value is in their favour.

Conclusions from activity:

- \((-€1 \times .33) + (-€1 \times .66)\)
- \(= €0.33 - €0.66\)
- \(= -€0.33\) per game

This average is known in probability as the Expected Value.

What would the expected value be in a fair game?

In a fair game the expected value would = 0
Section 2
Definitions and Examples

Learning Outcomes:
- Students will understand the concept of expected value as well as calculate the expected values involved in various scenarios.
- Students will use this knowledge to examine the fairness of given scenarios. This knowledge should inform their decision making in real life situations.

Question ???
- Two people toss a coin. If its heads, Mary wins €2, if tails John wins €1.
- Should John play this game?
- Using probability we can work out what John will expect to earn over a large number of trials. The average earned per trial is called the Expected Value.

Definition – Expected Value
- Expected value is a weighted average of all likely outcomes.
- Like all probabilities the observer/actual value and the expected value may differ over a short number of experiments but in the long term the real value will get closer and closer to expected value.

Definition – Expected Value
- To find the Expected Value:
  1. List all possible outcomes (distribution table).
  2. Multiply each outcome by the probability it will occur.
  3. Add to find the average outcome.

\[ \text{Expected Value} = \sum (\text{Outcomes} \times \text{Probability}) \]

Expected Value – Everyday uses:
- Expected value is commonplace in our lives.
- The two most obvious examples are gambling and insurance.
- Bookmakers and casinos work out their games so that the expected value favours them, this is known as "The House Edge".
- The "House-Edge" ensures that in the long run all gamblers will lose.
Expected Value – Everyday uses:

- Actuaries calculate insurance premiums using the same mathematical formulae.
- A branch of mathematics known as game theory is used in economics to make important business decisions. This has developed from expected value concepts.

Expected Value – Everyday uses:

- Expected value is also used when undertaking risk assessments. Risk assessments are carried out on a daily basis by government agencies and by private industry.
- Risk assessors investigate potential risks, calculate their probabilities, and either try to minimise the risk or establish the risk worth taking for the potential gain (expected value).

Recap - Definitions

- **Probability Distribution Table**
  - This is a table which lists the probability of events occurring. The sum of the probabilities listed should add to 1.
  - The distribution table for tossing a coin is:

<table>
<thead>
<tr>
<th>X</th>
<th>Heads</th>
<th>Tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>1/2</td>
<td>1/2</td>
</tr>
</tbody>
</table>

Note: If the probabilities do not sum to 1, either all the possible outcomes have not been listed or probabilities entered, are wrong.

Example 1 – Toss a Coin

Two people toss a coin. If its heads, Mary wins €2, if tails John wins €1. What is John’s Expected Value?

- Step 1 – Fill in a distribution table, to make sure all probabilities and outcomes are listed:

<table>
<thead>
<tr>
<th>X</th>
<th>Heads</th>
<th>Tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>1/2</td>
<td>1/2</td>
</tr>
</tbody>
</table>

Example 2 – A Die

- Find the expected value when throwing a die? Is it a value you can actually throw?

<table>
<thead>
<tr>
<th>X</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
<td>1/6</td>
</tr>
</tbody>
</table>
Example 2 – A Die

\[ E = \sum x \cdot P(x) \]

\[ = (1 \times \frac{1}{6}) + (2 \times \frac{1}{6}) + (3 \times \frac{1}{6}) + (4 \times \frac{1}{6}) + (5 \times \frac{1}{6}) + (6 \times \frac{1}{6}) \]

\[ = \frac{1 + 2 + 3 + 4 + 5 + 6}{6} \]

\[ = \frac{21}{6} \]

\[ = 3.5 \]

No, you cannot throw a 3.5

Charity Event – Investigation

A charity sets up a game involving dice at their benefit dance. The game is simple and it costs only €1 to play.

The player picks a number from 1 to 6. They roll 3 dice at once. They are paid out €2 for each time their number turns up on a dice.

Will the charity make money on this game?

<table>
<thead>
<tr>
<th>Number</th>
<th>Probability</th>
<th>Total Payout</th>
<th>Profit/loss</th>
<th>Expected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1/6</td>
<td>1</td>
<td>-€1</td>
<td>-€1</td>
</tr>
<tr>
<td>2</td>
<td>1/6</td>
<td>3</td>
<td>-€3</td>
<td>-€3</td>
</tr>
<tr>
<td>3</td>
<td>1/6</td>
<td>5</td>
<td>-€5</td>
<td>-€5</td>
</tr>
<tr>
<td>4</td>
<td>1/6</td>
<td>7</td>
<td>-€7</td>
<td>-€7</td>
</tr>
<tr>
<td>5</td>
<td>1/6</td>
<td>9</td>
<td>-€9</td>
<td>-€9</td>
</tr>
<tr>
<td>6</td>
<td>1/6</td>
<td>11</td>
<td>-€11</td>
<td>-€11</td>
</tr>
</tbody>
</table>

The expected value for the charity = -€13

Charity Event – Investigation

In pairs, you have 5 minutes to change this game so it raises money for the charity.

You can change
The entry fee and/or the prize money
Follow the format in the activity sheet

Hint: The key is to find a balance between a good expected value for the charity and making it attractive to play

Charity Event – Investigation

Sample Answer:

€2 to enter, 1 correct die entitles you to your money back, 2 correct dice doubles your money and 3 correct dice has a prize of €50.
Charity Event – Investigation

<table>
<thead>
<tr>
<th>Numbering</th>
<th>Total</th>
<th>Total 1</th>
<th>Total 2</th>
<th>Total 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>12.5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>6</td>
<td>6</td>
<td>21.6</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

The expected value of the charity per person: €45.98

Expected Value

The only incorrect answer is one that has a negative expected value for the charity.

Compare each group's answer using this table:

<table>
<thead>
<tr>
<th>Cost to play</th>
<th>Prize for 1 No.</th>
<th>Prize for 2 No.'s</th>
<th>Prize for 3 No.'s</th>
<th>Expected Value for charity</th>
</tr>
</thead>
<tbody>
<tr>
<td>€45.98</td>
<td>€10</td>
<td>€20</td>
<td>€30</td>
<td>€45.98</td>
</tr>
</tbody>
</table>

Discussion – Insurance Premiums

Noel Brett, chief executive of the Road Safety Authority, told the Daily Enterprise Committee:

"Young men were most at risk on our roads and those aged between 17 and 24 were between seven and eight times more likely to be killed or seriously injured than any other age group" (Irish Independent Thursday November 09 2006)

Given the probabilities above, if you were working for a car insurance company, would you charge every customer the same premium?

Section 4

Questions in Context

Q.1 – The “Psychic”

A psychic runs the following ad in a magazine:

Expecting a baby? Award-winning psychic will tell you the sex of the unborn child from any photograph of the pregnant mother. Cost €50. Money back guarantee if wrong.

Considering the probability of a boy is 0.51 and the probability of a girl is 0.49. How will the psychic maximise their expected value?

Taken from “Statistics – Concepts and Controversies” by David S. Moore

Q.1 – The “Psychic”

The psychic’s strategy will be simple, they will respond “boy” to all enquiries.

The probability distribution table for guessing “boys” looks like:

<table>
<thead>
<tr>
<th>Psychic right</th>
<th>Psychic wrong</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$-€10$</td>
</tr>
<tr>
<td>$P(x)$</td>
<td>0.51</td>
</tr>
<tr>
<td>$P(x)$</td>
<td>0.49</td>
</tr>
</tbody>
</table>
Q.1 - The "Psychic"

- $\Sigma x \cdot P(x)$
- $= (+€10 \times 0.51) + (€0 \times 0.49)$
- $= €5.10 - €0$
- $= €5.10$ profit on average for every customer

- The psychic will also count on the fact that some people will not bother to seek the money back to help boost their profit.

Q.2 Dice Game

- A student has a game. He rolls 3 dice.

- If the dice sum to the first four possible numbers 3, 4, 5 or 6 or the last four possible numbers 15, 16, 17, 18 he will give you €2.

- If they sum from 7 to 14, you pay him €1.

Q.2 Dice Game

- You each have 8 numbers, and you have a better payout but should you play this game?

- The probabilities are difficult to work out. The student's Project Maths CD will be of help to you. Look up "Dice" in the java scripts.

Q.2 Dice Game

<table>
<thead>
<tr>
<th>Dice sum to</th>
<th>Probability</th>
<th>Dice sum to</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>6</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>8</td>
<td>13</td>
</tr>
<tr>
<td>9</td>
<td>11</td>
<td>12</td>
<td>14</td>
</tr>
</tbody>
</table>

Total

Q.2 - Dice Game

- Expected value = $€2 \times 0.186 + (-€1 \times 0.814)$
  $= -€0.442$

- You will lose an average of 44 cent every time you play this game. This is not fair.

Q.3 Roulette Table

- The wheel has 37 numbers from 0 to 36.
- Odd numbers are red, even ones are black.
- The number 0 is green.
Q.3 Roulette Table

(A) Bet A - "A straight" up involves picking a single number and betting on it. The odds on offer are 35 to 1.
What is the expected value for the player if he bets €1?

<table>
<thead>
<tr>
<th></th>
<th>Number shows</th>
<th>Number doesn’t</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>-35</td>
<td>-1</td>
</tr>
<tr>
<td>P(Rou)</td>
<td>1/37</td>
<td>36/37</td>
</tr>
</tbody>
</table>

Q.3 Roulette Table

\[
\begin{align*}
&= (35 \times \frac{1}{37}) + (-1 \times \frac{36}{37}) \\
&= \frac{35}{37} - \frac{36}{37} \\
&= -\frac{1}{37} \\
&= -€0.027 \\
&= 2.7\% \text{ House Edge}
\end{align*}
\]

Q.3 Roulette Table

(B) Bet C - "A Street" involves picking a row of 3 numbers. The odds on offer are 11 to 1.
What is the expected value for a €1 bet?

<table>
<thead>
<tr>
<th></th>
<th>Number show</th>
<th>Number doesn’t</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>-11</td>
<td>-1</td>
</tr>
<tr>
<td>P(Rou)</td>
<td>3/37</td>
<td>34/37</td>
</tr>
</tbody>
</table>

Q.3 Roulette Table

\[
\begin{align*}
&= (11 \times \frac{3}{37}) + (-1 \times \frac{34}{37}) \\
&= \frac{33}{37} - \frac{34}{37} \\
&= -\frac{1}{37} \\
&= -€0.027 \\
&= 2.7\% \text{ House Edge}
\end{align*}
\]

Q3 – Roulette

(C) Bet J - "Even Chances", involves backing red or black. The odds on offer are 1-1. If green turns up - the player loses half his stake.
What is the expected value for a €1 bet?

<table>
<thead>
<tr>
<th></th>
<th>Order shown</th>
<th>Order doesn’t</th>
<th>Green</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>+1</td>
<td>-1</td>
<td>+0.05</td>
</tr>
<tr>
<td>P(Rou)</td>
<td>18/37</td>
<td>18/37</td>
<td>1/37</td>
</tr>
</tbody>
</table>

Q3 – Roulette

\[
\begin{align*}
&= (1 \times \frac{18}{37}) + (-1 \times \frac{18}{37}) + (-0.5 \times \frac{1}{37}) \\
&= \frac{18}{37} - \frac{18}{37} - \frac{5}{37} \\
&= -\frac{5}{37} \\
&= -€0.0135 \\
&= 1.35\% \text{ House Edge}
\end{align*}
\]
Q3 – Roulette

If you were playing roulette what would you bet on?

D. The bet with the Lowest "House Edge" is obviously 'Even Chance', people tend not to play it because of the small odds.

Q.3 Roulette Table

In the American version of roulette, there is a 38th number, a second green 00. What does this do to the "house edge" in the case of "A Straight"?

<table>
<thead>
<tr>
<th>Number show</th>
<th>Number doesn't</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>+35</td>
</tr>
<tr>
<td>0/0</td>
<td>-1</td>
</tr>
<tr>
<td>1/38</td>
<td>37</td>
</tr>
<tr>
<td>37/38</td>
<td>38</td>
</tr>
</tbody>
</table>

Q.3 Roulette Table

\[
\frac{1}{38} \times 35 + \frac{1}{38} \times -37 = \frac{35}{38} - \frac{37}{38} = -\frac{2}{38} = -0.053 = 5.3\% \text{ House Edge}
\]

"Tolerable Risk"

Expected value is also used in business. The main character in Fight Club is a recall co-ordinator. He describes his job in this clip.

Car Recall

In January 2010, a well known car manufacturer began recalling cars to fix them with new accelerators after complaints of accidents, some of which were fatal.

The car manufacturer said that this recall which cost over a billion dollars demonstrated their commitment to quality.

Car Recall

However others argued that the car manufacturer knew about the problem since late 2008 and only ordered the recall when the cost of settlements, individual repairs and customer loyalty meant that the expected value favoured a recall.
Q.4. “Tolerable Risk”

- A toy manufacturer has developed a product that is expected to sell really well. It has already produced 100,000 units of the toy before it discovers a flaw.
- A small part breaks off in certain conditions, it is estimated by their actuary department that the chance of a child dying by swallowing the piece is 0.00002
- It cost €20 to produce each unit, with a profit of €10 per unit.

Q.4. “Tolerable Risk”

- The manufacturer is given legal advice that a court settlement will cost €100,000.
- The manufacturer has fixed the flaw but what will they do with 100,000 units already produced?
- The company has two options either sell the stock or dump it. Work out the expected value for both options.

Q.4. “Tolerable Risk”

- Option A – Sell

<table>
<thead>
<tr>
<th>X</th>
<th>€10</th>
<th>-€99,990</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>.99998</td>
<td>.00002</td>
</tr>
</tbody>
</table>

Expected Value $E(X) = \sum x \cdot P(x)$

- $= +€10 \cdot (.99998) + (-€99,990) \cdot (.00002)$
- $= €10 - €2$
- $= €8$ profit per toy x 100,000
- $= €800,000$ profit

Q.4. “Tolerable Risk”

- Using expected value as their only consideration, the best option for the business is to sell the product.

Other Considerations
- Children would die (0.00002 x 100,000 = 2 children on average)

Q.4. “Tolerable Risk”

- The best option is obviously to dump the stock, but occasionally companies decide that the Expected Value of doing the wrong thing is too tempting.
- These decisions have been claimed be a “tolerable risk”. A risk the owner is prepared to tolerate or take.

Q.4. “Tolerable Risk”

- May affect future sales of the toy worth millions
- If the company’s knowledge of the problem was discovered,
  - Settlements would be larger
  - There would be massive fines for the company
  - The directors of the company may face jail

Q.4. “Tolerable Risk”

- Peoples lives are obviously not tolerable risks.
Q.5 – Multiple Choice Test

Peter is sitting a multiple-choice test. He has not studied for the test so he picks the answers to the questions at random. He has to answer either Section A or Section B. Both sections are marked out of 100.

In Section A, there are 20 questions with two possible answers. A correct answer is worth 5 marks but a question answered incorrectly is worth -1.5.

<table>
<thead>
<tr>
<th></th>
<th>Correct</th>
<th>Incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>5</td>
<td>-1.5</td>
</tr>
</tbody>
</table>

$E(X) = \Sigma x \cdot P(X)$

$= 20 \cdot 5 + 20 \cdot (-1.5)$

$= 2.5 \cdot -75$

$= 20 \cdot 1.75$

On average he will get 35 marks.

Q.5 – Multiple Choice Test

In Section B, there are 10 questions with 4 possible answers. Each question is worth 10 marks but an incorrect answer is worth -1.

What is the best strategy for Peter?

<table>
<thead>
<tr>
<th></th>
<th>Correct</th>
<th>Incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>10</td>
<td>-1</td>
</tr>
</tbody>
</table>

$E(X) = \Sigma x \cdot P(X)$

$= 10 \cdot 2.5 + 10 \cdot (-1) \cdot 0.75$

$= 2.5 \cdot -7.5$

$= 10 \cdot 1.75$

On average he will get 17.5 marks.

Q.5 – Multiple Choice Test

The best strategy for Peter is to attempt section A. His expected value is a 35 marks out of 100 or 35% which is better than the expected value of 17.5% if randomly guessing section B.

Q.6 – A Warranty

Ellen buys a new TV worth €1,000.

It is guaranteed for a year but she can buy an extended warranty for a further 3 years for €100.

She does some research and finds out that 10% of these TV’s experience trouble in these 3 years and the average repair costs €400.

Should she get the warranty?
Q. 6 – A Warranty
If she buys the warranty,

<table>
<thead>
<tr>
<th>X</th>
<th>+€100</th>
<th>&lt;€100</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X)</td>
<td>.10</td>
<td>.90</td>
</tr>
</tbody>
</table>

\[ \text{Expected Value} = \sum x \cdot P(x) \]
\[ = (-9 \times -€100) + (.1 \times €300) \]
\[ = -€90 + €30 \]
\[ = -€60 \]

The warranty will cost customers €60 on average. Therefore it will probably not be worth it.

Q. 7 “Yarborough” Insurance
- Lord Yarborough, (1809 – 1862) gave his name to a hand of cards dealt in bridge that has no card higher than a nine.
- High value cards (including aces) are important in bridge. Lord Yarborough offered an insurance policy of £1,000 to any of his friends who were dealt such a hand – on condition they paid him a £1 premium before the hand is dealt. A bridge hand has 13 cards
- If you were Lord Yarborough’s friend would you have taken out his insurance policy?

\[ \text{Probability of a Yarborough} = \frac{32}{52} = \frac{13}{52} = 0.25 \]
\[ = 635,013,599,600 \]
\[ = 5,470,595,19 \times 10^{-4} = 1,628 \]

The expected value for this “insurance” was
\[ = \sum x \cdot P(x) \]
\[ = (-3999 \times 1,628) + (€1 \times 1,628) \]

\[ = -€826 \]

This means for every pound paid to Lord Yarborough, he kept 45p. Put differently if he got 1,828 customers, he would make £828

This “insurance” was a real money maker

Note on Insurance
- Insurance companies use expected values in the same way as casinos. They bet on you not having to claim based on probabilities arrived at from their data.
- The premiums are structured in a manner that gives the insurance company a “house edge”. This means over the long term the average customer will pay out more on insurance then they claim.
Note on Insurance

- People get insurance to cover short term risk, e.g. car insurance may cost €500 a year but it is more affordable than the €5,000 it would take to replace the car if it was stolen.
- Insurance companies have to keep a large reserve of cash in case a short term streak works against them.
- In 2010 the failure of Quinn Insurance to do this meant the company was taken over by receivers put in place by the financial regulator.

Q.8 Insurance Premium

- In 2008, there were 24,684 burglaries reported. There were 1,469,521 households in the country. You work for an insurance company with whom 10,000 people have house insurance.
- What is the probability that a house was burgled in 2008?

\[
\text{Probability} = \frac{24,684}{1,469,521} = 0.01679 \text{ or } 1.679\% \text{ of houses}
\]

Q.8. - Insurance

2. The average payout to a house that has been burgled is €2,000. What is the expected value of the cost of claims per customer?
- The expected value per customer
  \[= \text{€2,000} \times 0.01679 = \text{€33.58}\]

Q.8. - Insurance

- The company likes to keep its Gross profit (its profit before it pays its employees etc.) at €3,000,000 a year. What will the average insurance premium be?
- \[\text{Premium} = \frac{3,000,000}{10,000} = \text{€300}\]
- The company needs to earn on average €300 from each customer.
- The premium will be €300 + €33.58 = €333.58

Q.9 All or Nothing

- All or Nothing is the latest National Lottery game. For €2 play for a chance to win €500,000
  - Match All 12 or None – Win €500,000
  - Match 11 Numbers – Win €5,000
  - Match 10 Numbers – Win €25
  - Match 9 Numbers – Win €10
  - Match 8 Numbers – Win €4

- What is the National Lottery’s expected value?

Q.9 - All or Nothing

<table>
<thead>
<tr>
<th>Numbers Matched</th>
<th>Probability</th>
<th>Value</th>
<th>Expected Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/12</td>
<td>1/221,001</td>
<td>€500,000</td>
<td>€221,000</td>
</tr>
<tr>
<td>11/12</td>
<td>1/121,421</td>
<td>€5,000</td>
<td>€605,000</td>
</tr>
<tr>
<td>10/12</td>
<td>1/10,000</td>
<td>€25</td>
<td>€250,000</td>
</tr>
<tr>
<td>9/12</td>
<td>1/112,990</td>
<td>€10</td>
<td>€112,990</td>
</tr>
<tr>
<td>8/12</td>
<td>1/490,000</td>
<td>€4</td>
<td>€1960,000</td>
</tr>
</tbody>
</table>
### Q.9 - All or Nothing

| Number/Variant | Calculation | Value | Mark/zhuis | Mark/zhuis
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22 x 12/7</td>
<td>457.04</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>24/12</td>
<td>2.704.156</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>22 x 12/8</td>
<td>245.92</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>24 x 12/10</td>
<td>2.704.156</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>22 x 12/9</td>
<td>45.400</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>24/12</td>
<td>2.704.156</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>22 x 12/10</td>
<td>4.356</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>24/12</td>
<td>2.704.156</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>22 x 12/11</td>
<td>181</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>24/12</td>
<td>2.704.156</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>22 x 12/12</td>
<td>1</td>
<td>400/106</td>
<td>42</td>
</tr>
<tr>
<td>12</td>
<td>24/12</td>
<td>2.704.156</td>
<td>42</td>
<td></td>
</tr>
</tbody>
</table>
Section 1

Student Activity

Task:
Complete the second and third tables and try to establish the pattern

<table>
<thead>
<tr>
<th>A Die is rolled twice</th>
<th>6</th>
<th>No 6</th>
<th>No. of Possible Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>MM 6's</td>
<td>x</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>M1 6's</td>
<td>1</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>M2 6's</td>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A Die is rolled three times</th>
<th>6</th>
<th>No 6</th>
<th>No. of Possible Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>MM 6's</td>
<td>x</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>M1 6's</td>
<td>1</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>M2 6's</td>
<td>2</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>M3 6's</td>
<td>3</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Student Activity

Task:
Complete the second and third tables and try to establish the pattern

- \( P(0 \text{ 6's}) = \binom{3}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^3 \)
- \( P(1 \text{ 6's}) = \binom{3}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2 \)
- \( P(2 \text{ 6's}) = \binom{3}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^1 \)
- \( P(3 \text{ 6's}) = \binom{3}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^0 \)
Student Activity

Task:
Complete the second and third tables and try to establish the pattern

<table>
<thead>
<tr>
<th>G</th>
<th>No of Possible Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1650</td>
</tr>
<tr>
<td>5</td>
<td>555</td>
</tr>
<tr>
<td>4</td>
<td>222</td>
</tr>
<tr>
<td>3</td>
<td>77</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Student Activity

Question:
Can you write the pattern in words?

\[
\begin{array}{ccc}
\text{No of Trials} & \text{Probability of Success} & \text{Probability of Failure} \\
\text{No of Successes} & \text{\# of All Patterns} & \text{\# of All Patterns} \\
\end{array}
\]

- The mathematical formula is
  \[
P(X) = \binom{n}{r} p^r q^{n-r}
\]
- This formula is known as the Binomial Distribution formula

Learning Outcomes:

- Students will recognise situations where the binomial distribution applies.
- Students will perform a simulation of a binomial experiment.
- Students will apply their knowledge and understanding of Bernoulli distributions to examples involving real life situations and data.

Section 2

Definitions and Examples

Question ???

Q. Three coins are tossed, what is the probability all 3 will be heads.

\[
\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}
\]

For this question we could use a second method called Binomial Distribution or sometimes Bernoulli Trials after Jacob Bernoulli.
Jacob Bernoulli

- Jacob Bernoulli (1654 – 1705) was a member of a famous family of Swiss Mathematicians.
- He was hugely interested in probability and produced work which showed how probability theory linked with games of chance as well as on the law of large numbers.
- He was also very interested in calculus and was the first to use the term integral.

Definition:

Bernoulli trials have the following characteristics:
1. The experiment has only two outcomes (normally success or failure).
2. It has a fixed number of trials.
3. The trials are independent.
4. The probabilities of success and failure remain constant throughout the trials.

Note: The sum of the probabilities of all the possible outcomes adds to 1.

Everyday uses:

Bernoulli trials or binomial distribution has a huge amount of applications in everyday life. Here are some of them:
- Manufacturing – quality control
- Sports statistics
- Weather forecasting
- Public opinion surveys
- Medical research
- Insurance
- Travel industry

Definition – Bernoulli Trials

- Formula: \[ P(X) = \binom{n}{r} p^r q^{n-r} \] (P.33 Binomial)
- \( P(x) \): The probability of getting \( x \) successes among \( n \) trials.
- \( n \): The fixed number of trials.
- \( r \): The specified number of successes.
- \( p \): The probability of success.
- \( q \): The probability of failure (\( q = 1 - p \))

Example 1 – Tossing Coins

Q. Three coins are tossed, what is the probability all 3 will be heads.

Method 1
\[ \frac{3}{6} \times \frac{3}{6} = \frac{9}{36} = \frac{1}{4} \]

Method 2
It is a Bernoulli trial because there is:
- A set number of trials = 3
- There are two outcomes – Heads or Tails
- The trials are independent of each other
- The Probability remains constant = \( \frac{1}{2} \)

Formula: \[ P(X) = \binom{n}{r} p^r q^{n-r} \]

\( n \): (The number of trials)
\( r \): (The number of successes, heads we want)
\( p \): (The probability of getting heads)
\( q \): (The probability of not getting heads)
Example 1 – Tossing Coins

Formula: \[ P(X) = \binom{n}{r} p^r q^{n-r} \]

- P(3 Heads) = \( \binom{3}{3} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \)
- P(3 Heads) = 1 \times \frac{1}{8} \times 1
- P(3 Heads) = \frac{1}{8}

Example 1 – Tossing Coins

- Be careful:
  - That the correct powers are given to p and q.
  - It is particularly important when the p = q

- E.G. if looking for a 6 when throwing a die
  \[ P = \frac{1}{6}, \quad Q = \frac{5}{6} \]
- If I toss a coin 3 times, what is the probability I get exactly 2 heads?

Example 1 – Tossing Coins

- Formula: \[ P(X) = \binom{n}{r} p^r q^{n-r} \]
- P(2 Heads) = \( \binom{3}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \)
- P(2 Heads) = \( 3 \times \frac{1}{4} \times \frac{1}{2} \)
- P(2 Heads) = \frac{3}{8}

Example 1 – Tossing Coins

- What is the probability of getting 1 head when I toss a coin 3 times?
  \[ \frac{3}{8} \]
- The Sum of a Binomial distribution sums to 1
  \[ \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1 \]
- Why does it not add to 1?

Example 1 – Tossing Coins

- I haven’t got the probability of 0 heads.
  \[ \frac{3}{8} \times \frac{1}{2} \times \frac{1}{2} \]
  \[ = \frac{1}{8} \]
- Note: The Sum of a Binomial distribution is 1
  P(0 heads) + P(1 head) + P(2 heads) + P(3 heads)
  \[ = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} \]
  \[ = 1 \]

Example 1 – Tossing Coins

- Click on the link below to check your answers to future questions and view them graphically

NCE-MSTL
**Example 2 – Deck of Cards**

- 10 Cards are drawn, one at a time from a pack of 52 cards. The cards are not replaced. What is the probability I get exactly 3 Kings?
- Can we use Bernoulli Trials to solve this problem?

- No, we cannot because the trials are not independent, the probability of drawing a king changes as each card is drawn and not replaced.

**Example 3 – The Premiership**

- After 10 games of the 2010/11 season Chelsea had
  - won 80% of their games
  - drawn 10% of their games and
  - lost 10% of their games.

- Using these statistics as probabilities for Chelsea’s performance for the rest of the year, can we use these to calculate binomial probabilities?

**Example 3 – The Premiership**

- No, not directly because there are 3 different outcomes so we cannot use the binomial probability formula.

- We can however adjust them to two outcomes e.g. Probability of losing = 10% e.g. Probability of not losing = 90%

- What is the probability of Chelsea losing 3 games in a row? What conclusion can you draw from this?

- \[ P(X) = \binom{n}{r} p^r q^{n-r} \]

- \[ = \binom{3}{3} \times 1.0 \times .9^3 \]

- \[ = 1 \times .001 \times 1 \]

- \[ = .001 \]

- \[ = .1% \]

- It is extremely unlikely that this will happen, its 1 in 1,000 chance.

**Student Activity – Simulation**

**Scenario:**

- A company takes tourists on helicopter trips over Dublin. The helicopter only seats 6 people but the company discovered that 33% or 1/3 of all bookings do not show. In order to maximise profit they are thinking about taking 8 bookings for every trip.

**Resource pack**

NCE-MSTL
Student Activity

1) Pose a Question
What is the probability of their flights being overbooked?

2) Collect the Data
In order to collect data on this we are going to run a simulation.
A simulation is an attempt to run an experiment which mirrors a real-life situation.

Student Activity

The key component of our simulation is to generate a probability of 33%.

The simplest method in which to generate a probability of 33% is to roll a die.

The chance of rolling a 1 or 2 is 33% which represents the 33% chance the customer does not show up.

Student Activity

The numbers 3, 4, 5, 6 represent the 66% chance the customer turns up.

If 1 or 2 appears, write down 1 representing a customer not turning up.

If 3, 4, 5, 6 appear, write down 3 representing a customer turning up.

Alternatively, you can use the random number generator in your calculator!

Student Activity

To use the random number generator on your calculator, follow the steps below:

1. Put in Line 1 mode
2. Press 3
3. 2nd
4. 6
5. 0
6. Press 2 (To get numbers 2, 3, 0)
7. Enter

Student Activity

Working in pairs, one student generates the number, while the other records it into their sheet.

Each group should repeat this 8 times per trip, each of the 8 trials represents a customer.

This is done for a total of 10 trips.

Student Activity

In the example below, a die was used.

In trip 1, 1 appears twice, so their was two no-shows and so no extra passengers.

In trip 2, 1 appears only once, so only one booking of the 8 did not show.

<table>
<thead>
<tr>
<th>Trip 1</th>
<th>Trip 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passenger</td>
<td>Passenger</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
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<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Resource pack
Student Activity

3) Analyse the data

<table>
<thead>
<tr>
<th>Number of people</th>
<th>Number of people</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Student Activity

Using the class totals, what is the probability
(A) That exactly 7 show up?
(B) That exactly 8 show up?
(C) Of being overbooked?

\[
\begin{align*}
\text{Binomial} & \quad V \\
(A) \quad \binom{7}{7} \times 0.66^7 \times 0.33^0 = 0.144 & \quad V \\
(B) \quad \binom{8}{7} \times 0.66^7 \times 0.33^1 = 0.036 & \quad V \\
(C) & = 0.18 = 18\% & \quad V
\end{align*}
\]

Student Activity

4) Interpret The Data
(A) If you were advising the company, would you recommend this overbooking policy?
   Give 2 reasons.
   
   Suggestion: No - Refunds eat into your profit
   - Poor customer satisfaction

Student Activity

(B) Given the results from the data collected, are there other questions that you would like to examine?

Suggestion: Examine probabilities involved in taking 7 bookings

Conclusions from activity:

1. A knowledge of probability is useful to assess situations in business.
2. Experimental probability can provide us with reasonably accurate data provided the number of trials is big enough.
3. The Binomial distribution formula is a much simpler and quicker method of establishing probabilities than actually performing the trials.

So why use simulations?

Conclusions from activity:

Why use simulations?

We could have found this answer theoretically using the binomial distribution formula.

The first instance where simulations are useful is to highlight errors of thinking. If the simulation and the theoretical probabilities are very different, it may highlight an error within the theoretical probability used.
Conclusions from activity:

- In many real-life situations there is no model for the theoretical probability or if there is, it is difficult to calculate. Running simulations might be the only way to go.
- Computerised models are created to help predict events, e.g.:
  - The weather
  - The spread of volcanic ash in the atmosphere
  - The melting of the polar ice-caps

Simulations

- In "The Day After Tomorrow" a scientist uses an advanced computer simulation to predict extreme climate change.
- In the 1st clip he explains how the probability of this event occurring has changed. In the 2nd clip he shows a model of his simulation.

Remember !!!

The binomial distribution formula is only appropriate if:

I. The experiment has only two outcomes.
II. It has a fixed number of trials.
III. The trials are independent.
IV. The probability of success or failure remain constant throughout the trials.

The sum of the probabilities of all the possible outcomes in Bernoulli trials is 1.

Possible Investigation

- It is thought that about 10% of the general population is left-handed.
- How many left-handed people in your class?
- Using the Binomial Distribution formula:
  what is the probability of this exact amount of people
  (A) being left-handed?
  (B) being right-handed?

Possible Investigation

- What is the probability of at least this amount of people being left-handed?
- What is the probability of at most this amount of people being left-handed?
Q.1 Quality Control

1. Records show that an MP3 player produced by a particular company has a defect rate of 1 in 200. The factory sells them to the shops in boxes of 20. The quality control technician tests 1 box selected at random every hour.

   1. What is the probability that 1 or less of these 20 are defective?

   2. If more than 1 was found to be defective, what would this suggest? What might the quality control officer do?

   \[ P(X \leq 1) = \binom{20}{0} \left(\frac{1}{200}\right)^0 \left(\frac{199}{200}\right)^{20} + \binom{20}{1} \left(\frac{1}{200}\right)^1 \left(\frac{199}{200}\right)^{19} \]

   \[ = 0.9046 + 0.0909 \]

   \[ = 0.9955 \]

   There is a 99.55% chance of 1 or less of the 20 products being defective.

Q.2. Polygraph Test (Lie Detector)

1. The probability of 2 or more of the MP3 players being defective is extremely unlikely at less than ½ a per cent.

2. The quality control technician would notify management and the whole batch might have to be checked to see if there is a problem on the production line.

According to the American Polygraph Association website, “Researchers conducted 41 studies involving the accuracy of 1,787 laboratory simulations of polygraph examinations, producing an average accuracy of 80%.”

In 2003, the National Academy of Sciences issued a report entitled “The Polygraph and Lie Detection.” The report concluded that this level of accuracy was overstated.

Q.2. Polygraph Test

1. If the police interviewed 10 innocent people in relation to a crime, what is the probability one or two of these will fail it?

2. Answer the question based on an 80% accuracy rate (Assuming this rate means that 2 out of 10 guilty people will appear innocent and 2 out of 10 innocent people will appear guilty).

\[ P(X = 1) + P(X = 2) = \binom{10}{1} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^9 + \binom{10}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^8 \]

\[ = 0.2684 + 0.3019 \]

\[ = 0.5703 \]

There is a 57% chance one or two will fail the lie detector test even though they are innocent.
Q. 3 Genetics

1. Cystic Fibrosis is Ireland's most common life-threatening inherited disease. With over 1,100 CF Patients, Ireland has the highest proportion of CF people in the world.

2. Approximately 1 in 19 people are carriers of the CF gene and where two carriers parent a child, there is a 1 in 4 chance of the baby developing Cystic Fibrosis.

3. What is the chance of two unaffected carriers parenting a child?

4. Two carriers have 3 children together, what is the probability that none of the children have CF?

5. What is the chance of at least one of their children having CF?

6. What is the chance that all three children have CF?

Q. 3 Genetics

1. \[ \frac{1}{19} \times \frac{1}{19} = \frac{1}{361} \]

   = 0.00277

   This is a less than 1% chance

2. \[ \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64} \]

   = 0.421875

   There is a 42% chance that none of their children will suffer from CF

Q. 4 Leaving Certificate Maths

1. In 2009 15% of those students who took Higher Level Mathematics got an A Grade

2. A school has one higher level leaving cert class of 20 students. The teacher tells her class of 20 students that she expects no more than 3 of them to get an A. What is the probability she will be correct? Round your answer to two decimal places.

   Note: This assumes that class is representative of Leaving certificate population

Q. 4 Leaving Cert Maths

"No more than 3" that means she will be correct if 0, 1, 2, or 3 students get an A

\[
P(X) = \binom{n}{x} p^x q^{n-x}
\]

\[
= \binom{20}{0} 0.15^{0} 0.85^{20} + \binom{20}{1} 0.15^{1} 0.85^{19} + \binom{20}{2} 0.15^{2} 0.85^{18} + \binom{20}{3} 0.15^{3} 0.85^{17}
\]

= 0.0388 + 0.1368 + 0.2293 + 0.2428

= 0.65 or a 65% chance of being correct
Q.5 Profit Optimisation

A certain airline knows that on average 10% of their customers do not turn up. It books its planes to a capacity of 105. Their standard plane seats 100 people.

What is the probability that they will be overbooked?

\[
\begin{align*}
&\frac{105\ \text{people}}{100\ \text{seats}} \times \frac{105\ \text{people}}{100\ \text{seats}} \times \frac{105\ \text{people}}{100\ \text{seats}}
\end{align*}
\]

This means for every 10,000 flights booked only 167 of them are overbooked.

Q.5 Profit Optimisation

This is a basic example of what is called in the airline business “Profit Optimisation”.

Each company runs a slightly different model, depending on, their average no-shows, the cost of compensation, can they bump people up to 1st class, importance of customer goodwill etc. in order to try maximise profit.

Some companies are more aggressive in overbooking than others.

Q.6 Traffic Lights

On the way to work, John travels along a main street which has three traffic lights.

As it is the main street, the traffic lights are set so that they are green 70% of the time and red 30% of the time.

On Friday morning, John arrives into work a few minutes late again saying that every morning this week, the three traffic lights have been red when he got to them. Do you believe him?

Q.6 Traffic Lights

The probability all three lights on a morning are red is

\[
\begin{align*}
&\left(\frac{3}{10}\right)^3
\end{align*}
\]

= 0.27

= 2.7%

The probability that all three were red 5 mornings in a row

= 0.27 \times 0.27 \times 0.27 \times 0.27 \times 0.27

= 0.000000014

That is a 14 in 1,000,000,000 or a 14 in a billion chance.

It is doubtful whether John is telling the truth, of course these calculations do ignore how heavy the traffic may be and other conditions so these calculations may be unrealistic.
Q.7 Multiple Choice Test

1. John says he has not studied for his exam so he is going to guess the answers at random.

2. It is a multiple choice test of 10 questions. There are four possible answers to each question. He needs 40% to pass.

3. What is the chance of John passing this exam?

4. John got 70%. How probable is it that John got 70% or more without studying i.e. he is guessing at random?

5. If the test was changed to 20 questions, would the test be harder or easier to pass?

Q.7 Multiple Choice Test

1. The probability of getting at least 40% = A(P(4) + P(5) + P(6) + P(7) + P(8) + P(9) + P(10)) or B1 - [P(0) + P(1) + P(2) + P(3)]

   = 1 - \( \binom{10}{0} \cdot 0.25^{10} \cdot 0.75^{0} + \binom{10}{1} \cdot 0.25^{9} \cdot 0.75^{1} + \binom{10}{2} \cdot 0.25^{8} \cdot 0.75^{2} + \binom{10}{3} \cdot 0.25^{7} \cdot 0.75^{3} \)

   = 1 - (0.0563 + 0.1877 + 0.2815 + 0.2502)

   = 1 - 0.7757

   = 0.2243

   = 22% Probability of passing the exam guessing randomly

Q.7 Multiple Choice Test

2. The probability John got 70% or more without study

   = \( \binom{10}{6} \cdot 0.25^{6} \cdot 0.75^{4} + \binom{10}{7} \cdot 0.25^{7} \cdot 0.75^{3} + \binom{10}{8} \cdot 0.25^{8} \cdot 0.75^{2} + \binom{10}{9} \cdot 0.25^{9} \cdot 0.75^{1} \)

   = 0.003 + 0.0004 + 0.00003 + 0.000001

   = 0.0034, which is less than a 1/2 of 1%

   For every 10,000 people who do it without studying only 34 will get 70% or better by guessing randomly.

Q.7 Multiple Choice Test

4. The probability of passing with 20 questions

   = 1 - [P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7)]

   = 1 - 0.8981

   = 0.1019

   There is close to 10% probability of passing the test with 20 questions compared with the 22% chance of passing it when there was only 10 questions.
Q.8. Psychic or Not?

Zener cards are a classic method of testing for extrasensory perception (ESP).

The Zener deck consists of 25 cards - five cards each of five symbols: a circle, a star, a square, a cross and wavy lines.

A tester pulls a card out of the deck and the subject tries to predict which of the 5 symbols is on the card.

They have a 20% chance of being correct, the card is returned to the deck and the experiment repeated a set number of times.

To prevent any bias the subject is not given their results until the end.

Q.8. Psychic or Not?

A) What is the probability the subject gets 5 out of 25 correct guessing randomly?

\[ P(X = 5) = \binom{25}{5} \cdot 0.2^5 \cdot 0.8^{20} \]

\[ = 0.196 \text{ or } 19\% \]

Q.8. Psychic or Not?

B) What is the probability the subject gets the 5th one correct on exactly the 25th go?

\[ P(X = 24) \cdot 0.2^{24} \cdot 0.8^1 \]

\[ = 0.0392 \text{ or a nearly 4\% chance} \]

Q.8. Psychic or Not?

Anyone who showed potential of ESP was tested further. This was defined as getting a score that had a less than 1% probability of happening by chance.

(Though it has a chance of only 0.37%, which is less than 1%, scoring 0 does not count as it does not show potential for ESP.)

Q.8. Psychic or Not?

C) By trial and error, find how many correct answers out of 25 the subject has to have correct to be tested further?

Try 10 correct answers

\[ P(X = 10) \cdot 0.2^{10} \cdot 0.8^{15} \]

\[ = 0.0118 \]

More than 1%, so not enough correct answers.
Q.8. Psychic or Not?

Try 11 correct answers next,
\[
\frac{25}{11} \times 2^{11} \times 0.11
\]
= 0.004

This less than 1% so you would have to get 11 or better before being tested further.

Q.9 – Oil Wells

Suppose an oil exploration firm has identified 10 potential oil wells off the coast of Ireland. Each has a 10% probability of striking oil.

On average it costs the firm €500,000 to drill each well but a successful well nets them a profit of €10,000,000.

Q.9 – Oil Wells

1. What is the probability that the company will strike out on all wells and lose money?

2. What is the chance they will strike oil only on exactly the 10th drill?

3. What is the expected value for the company on each well? What conclusions do you draw from this?

Q.9 – Oil Wells

1. The probability they will strike out on all 10 wells drills
\[
\frac{10}{10} \times 0.90 \times 1.0
\]
= 0.3486
= 35%

2. The probability they will strike oil on exactly the 10th drill is
\[
\frac{9}{10} \times 0.90 \times 1.0
\]
= 0.3067
= 4%
Q.9– Oil Wells

3. The expected value on each well is

\[
\frac{1}{10}(-\$10,000,000) + \frac{9}{10}(-\$500,000)
\]

= \$650,000

It will probably be worth investing in the oil wells.
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